Computer algebra independent integration tests

Summer 2022 edition

4-Trig-functions/4.1-Sine/70-4.1.1.2-g-cos-^p-a+b-sin-^m

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [653]. This is test number [70].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (653)	0.00 (0)
Mathematica	97.70 (638)	2.30 (15)
Maple	84.99 (555)	15.01 (98)
Fricas	79.79 (521)	20.21 (132)
Giac	47.63 (311)	52.37 (342)
Maxima	44.10 (288)	55.90 (365)
Mupad	39.51 (258)	60.49 (395)
Sympy	16.08 (105)	83.92 (548)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

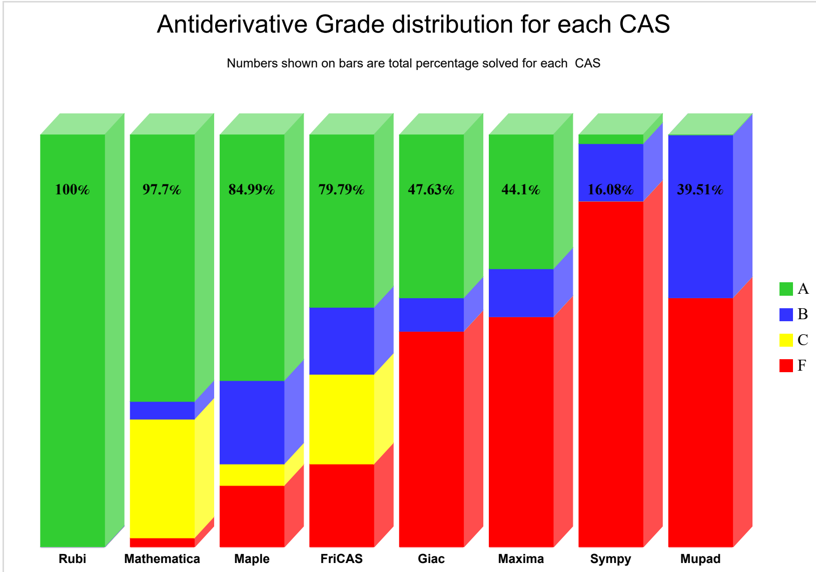
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

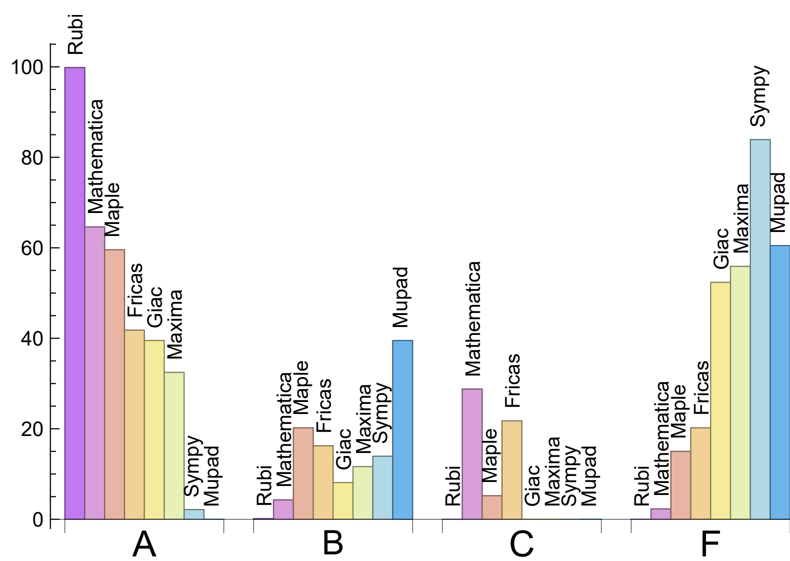
System	% A grade	% B grade	% C grade	% F grade
Rubi	99.85	0.15	0.00	0.00
Mathematica	64.62	4.29	28.79	2.30
Maple	59.57	20.21	5.21	15.01
Fricas	41.81	16.23	21.75	20.21
Giac	39.51	8.12	0.00	52.37
Maxima	32.47	11.64	0.00	55.90
Sympy	2.14	13.94	0.00	83.92
Mupad	N/A	39.51	0.00	60.49

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	15	100.00 %	0.00 %	0.00 %
Maple	98	100.00 %	0.00 %	0.00 %
Fricas	132	71.97 %	26.52 %	1.52 %
Giac	342	75.15 %	23.10 %	1.75 %
Maxima	365	84.93 %	4.11 %	10.96 %
Sympy	548	39.23 %	34.85 %	25.91 %
Mupad	395	98.73 %	1.27 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

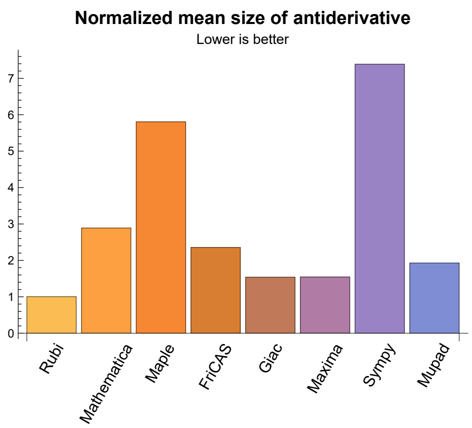
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

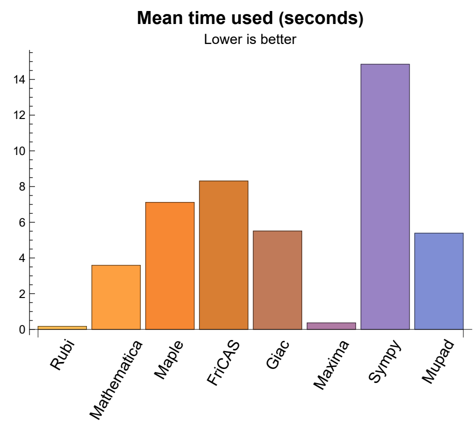
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.17	157.61	1.00	121.00	1.00
Mathematica	3.59	348.94	2.89	83.50	0.93
Maple	7.12	2569.97	5.81	146.00	1.33
Maxima	0.37	153.72	1.54	98.00	1.10
Fricas	8.31	393.92	2.36	136.00	1.33
Sympy	14.85	517.91	7.39	180.00	3.63
Giac	5.51	211.50	1.54	115.00	1.30
Mupad	5.39	250.26	1.93	105.00	1.26

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
0
1
2
3
4
5
6
7
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
0
2
4
6
8
10
12
14
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

{}

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi $\{615\}$

Mathematica $\{348, 349, 352, 353, 467, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 616, 619, 620, 621\}$

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

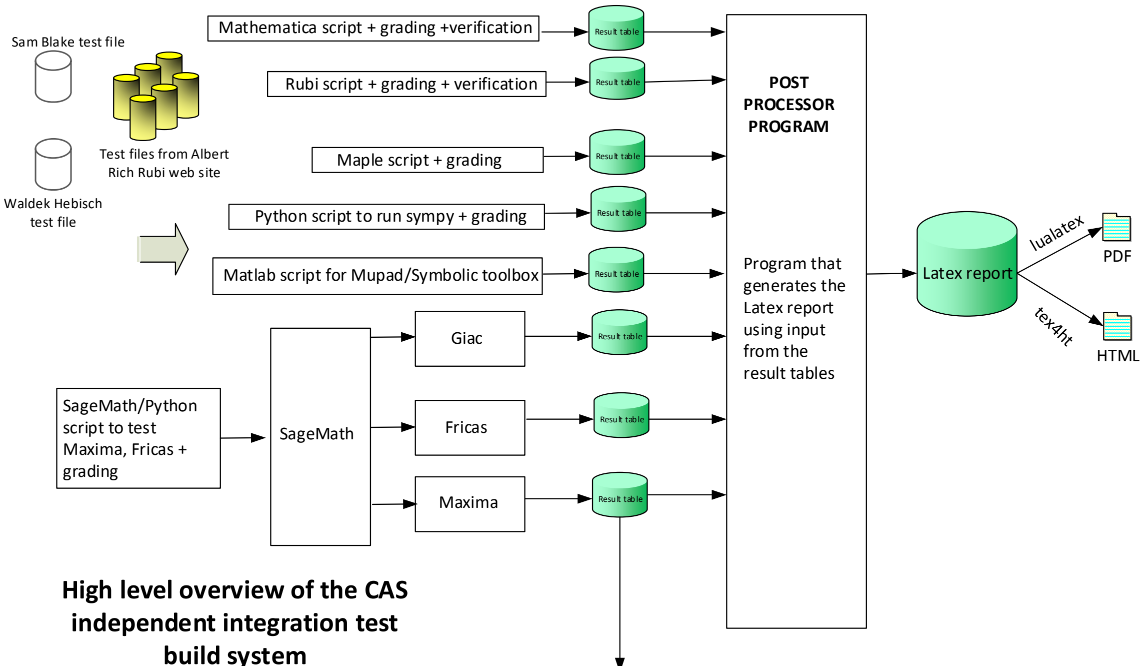
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified.

*The following fields are present only in **Rubi** Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form “{n,n,..}” which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

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August 26, 2022

Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653 }

B grade: { 615 }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 33, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 46, 48, 50, 51, 52, 54, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 69, 70, 71, 72, 73, 74, 75, 77, 78, 80, 81, 82, 83, 84, 85, 86, 87, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 114, 115, 116, 117, 118, 119, 120, 121, 123, 127, 128, 129, 130, 131, 132, 133, 134, 136, 139, 140, 141, 142, 143, 144, 145, 146, 147, 149, 150, 152, 156, 157, 158, 159, 160, 161, 162, 169, 170, 171, 172, 173, 174, 175, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 196, 198, 200, 202, 276, 277, 278, 279, 285, 286, 287, 288, 294, 295, 296, 297, 301, 302, 303, 304, 308, 309, 310, 311, 312, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 331, 332, 333, 334, 335, 336, 337, 339, 340, 341, 342, 344, 345, 346, 347, 350, 351, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 403, 404, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 447, 448, 449, 450, 451, 452, 453, 454, 455, 458, 459, 460, 461, 462, 463, 464, 465, 466, 468, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 522, 523, 524, 525, 526, 527, 528, 529, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 623, 624, 625, 626, 627, 628, 631, 632, 633, 634, 635, 636, 647, 648, 649, 650 }

B grade: { 32, 45, 53, 55, 68, 76, 88, 122, 135, 151, 389, 402, 405, 431, 432, 433, 444, 445, 446, 456, 457, 467, 469, 470, 619, 620, 621, 630 }

C grade: { 34, 47, 49, 79, 108, 109, 110, 111, 112, 113, 124, 125, 126, 137, 138, 148, 153, 154, 155, 163, 164, 165, 166, 167, 168, 176, 177, 178, 179, 180, 181, 192, 193, 194, 195, 197, 199, 201, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 280, 281, 282, 283, 284, 289, 290, 291, 292, 293, 298, 299, 300, 305, 306, 307, 313, 314, 315, 330, 338, 343, 348, 349, 352, 353, 519, 520, 521, 530, 531, 532, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618 }

F grade: { 622, 629, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 651, 652, 653 }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 22, 24, 25, 26, 27, 29, 31, 32, 33, 34, 38, 44, 45, 51, 52, 53, 54, 55, 56, 57, 58, 59, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 82, 83, 84, 85, 86, 87, 88, 89, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 204, 205, 206, 207, 208, 209, 210, 214, 215, 216, 217, 218, 219, 220, 224, 225, 226, 227, 228, 233, 234, 235, 236, 237, 238, 243, 244, 245, 246, 247, 253, 254, 255, 256, 257, 263, 264, 265, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 316, 317, 318, 319, 320, 346, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 415, 416, 417, 418, 419, 420, 421, 422, 423, 425, 426, 427, 428, 429, 430, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 473, 474, 475, 476, 477, 483, 484, 485, 494, 495, 496, 506, 507, 508, 509, 510, 516, 517, 518, 519, 520, 527, 528, 529, 530, 531, 539, 541, 542, 543, 544, 545, 551, 552, 559, 560, 568, 569, 633 }

B grade: { 21, 23, 28, 30, 35, 36, 37, 39, 40, 41, 42, 43, 46, 47, 48, 49, 50, 60, 81, 90, 203, 211, 212, 213, 221, 222, 223, 229, 230, 231, 232, 239, 240, 241, 242, 248, 249, 250, 251, 252, 258, 259, 260, 261, 262, 266, 267, 268, 269, 270, 271, 272, 292, 293, 314, 315, 413, 414, 424, 431, 444, 456, 467, 468, 469, 470, 471, 472, 478, 479, 480, 481, 482, 486, 487, 488, 489, 490, 491, 492, 493, 497, 498, 499, 500, 501, 502, 503, 504, 505, 511, 512, 513, 514, 515, 521, 522, 523, 524, 525, 526, 532, 533, 534, 535, 536, 537, 538, 540, 546, 547, 548, 549, 550, 553, 554, 555, 556, 557, 558, 561, 562, 563, 564, 565, 566, 567, 570, 571, 572, 573, 615 }

C grade: { 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 606, 608, 612, 613 }

F grade: { 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 604, 605, 607, 609, 610, 611, 614, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 37, 38, 39, 40, 42, 44, 45, 46, 47, 48, 49, 50, 52, 54, 56, 57, 59, 61, 63, 65, 67, 68, 69, 70, 72, 74, 78, 80, 82, 83, 85, 87, 91, 95, 96, 98, 100, 101, 103, 105, 107, 108, 110, 112, 114, 116, 118, 120, 121, 123, 125, 127, 129, 131, 132, 134, 136, 138, 139, 141, 143, 145, 146, 148, 150, 152, 154, 160, 162, 163, 165, 167, 169, 171, 173, 175, 176, 178, 180, 183, 185, 187, 189, 191, 192, 194, 346, 363, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 437, 438, 439, 440, 441, 442, 443, 449, 450, 451, 452, 453, 461, 462, 464, 473, 474, 475, 483, 484, 485, 494, 495, 496, 506, 507, 508, 516, 517, 518, 527, 528, 529, 631, 632, 633 }

B grade: { 36, 41, 43, 51, 53, 55, 58, 60, 62, 64, 66, 71, 73, 75, 76, 77, 79, 81, 84, 86, 88, 89, 90, 92, 93, 94, 97, 99, 122, 133, 135, 147, 149, 151, 156, 158, 276, 277, 278, 279, 285, 286, 287, 288, 294, 295, 296, 297, 301, 302, 303, 304, 308, 309, 310, 311, 312, 316, 317, 318, 319, 320, 343, 344, 345, 367, 368, 369, 413, 414, 454, 455, 463, 465, 466, 630 }

C grade: { }

F grade: { 102, 104, 106, 109, 111, 113, 115, 117, 119, 124, 126, 128, 130, 137, 140, 142, 144, 153, 155, 157, 159, 161, 164, 166, 168, 170, 172, 174, 177, 179, 181, 182, 184, 186, 188, 190, 193, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 280, 281, 282, 283, 284, 289, 290, 291, 292, 293, 298, 299, 300, 305, 306, 307, 313, 314, 315, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 364, 365, 366, 370, 371, 372, 373, 374, 375, 431, 432, 433, 434, 435, 436, 444, 445, 446, 447, 448, 456, 457, 458, 459, 460, 467, 468, 469, 470, 471, 472, 476, 477, 478, 479, 480, 481, 482, 486, 487, 488, 489, 490, 491, 492, 493, 497, 498, 499, 500, 501, 502, 503, 504, 505, 509, 510, 511, 512, 513, 514, 515, 519, 520, 521, 522, 523, 524, 525, 526, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 19, 20, 21, 22, 24, 25, 26, 27, 28, 29, 30, 31, 33, 34, 35, 37, 38, 40, 42, 44, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 77, 78, 79, 80, 82, 84, 85, 86, 87, 91, 94, 97, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 125, 126, 127, 128, 130, 132, 133, 134, 135, 136, 138, 139, 140, 142, 144, 146, 147, 148, 149, 150, 152, 154, 155, 156, 157, 158, 160, 162, 163, 165, 166, 167, 168, 169, 171, 173, 175, 176, 178, 179, 180, 181, 183, 185, 187, 189, 192, 193, 194, 195, 276, 277, 278, 279, 285, 286, 287, 288, 295, 296, 297, 301, 302, 303, 304, 309, 310, 311, 312, 318, 319, 320, 343, 344, 345, 346, 360, 361, 362, 363, 368, 369, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 444, 445, 446, 447, 448, 449, 450, 451, 456, 458, 461, 473, 474, 475, 483, 484, 494, 506, 507, 508, 516, 517, 518, 527, 528, 632, 633 }

B grade: { 18, 23, 32, 36, 39, 41, 43, 45, 76, 81, 83, 88, 89, 90, 92, 93, 95, 96, 98, 109, 124, 129, 131, 137, 141, 143, 145, 151, 153, 159, 161, 164, 170, 172, 174, 177, 182, 184, 186, 188, 190, 191, 273, 274, 275, 280, 281, 282, 283, 284, 289, 290, 291, 292, 293, 294, 298, 299, 300, 305, 306, 307, 308, 313, 314, 315, 316, 317, 367, 389, 402, 413, 414, 415, 442, 443, 452, 453, 454, 455, 457, 459, 460, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 476, 477, 485, 487, 495, 496, 498, 499, 529, 530, 630, 631 }

C grade: { 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 479, 480, 481, 482, 489, 490, 491, 492, 493, 500, 501, 502, 503, 504, 505, 512, 513, 514, 515, 522, 523, 524, 525, 526, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573 }

F grade: { 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 364, 365, 366, 370, 371, 372, 373, 374, 375, 478, 486, 488, 497, 509, 510, 511, 519, 520, 521, 531, 532, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653 }

2.1.6 Sympy

A grade: { 1, 3, 5, 6, 56, 69, 162, 376, 377, 383, 387, 388, 400, 407 }

B grade: { 2, 4, 7, 13, 14, 15, 16, 17, 18, 27, 28, 29, 30, 31, 32, 41, 42, 43, 44, 45, 51, 52, 53, 54, 55, 62, 63, 64, 65, 66, 67, 68, 75, 76, 77, 78, 79, 80, 81, 82, 89, 91, 92, 93, 94, 95, 107, 118, 120, 129, 131, 175, 189, 191, 344, 345, 346, 378, 382, 389, 393, 394, 395, 401, 402, 406, 413, 414, 415, 419, 427, 433, 439, 440, 451, 452, 461, 462, 463, 464, 475, 484, 485, 495, 496, 508, 518, 528, 529, 632, 633 }

C grade: { }

F grade: { 8, 9, 10, 11, 12, 19, 20, 21, 22, 23, 24, 25, 26, 33, 34, 35, 36, 37, 38, 39, 40, 46, 47, 48, 49, 50, 57, 58, 59, 60, 61, 70, 71, 72, 73, 74, 83, 84, 85, 86, 87, 88, 90, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 119, 121, 122, 123, 124, 125, 126, 127, 128, 130, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 190, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 379, 380, 381, 384, 385, 386, 390, 391, 392, 396, 397, 398, 399, 403, 404, 405, 408, 409, 410, 411, 412, 416, 417, 418, 420, 421, 422, 423, 424, 425, 426, 428, 429, 430, 431, 432, 434, 435, 436, 437, 438, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 453, 454, 455, 456, 457, 458, 459, 460, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 476, 477, 478, 479, 480, 481, 482, 483, 486, 487, 488, 489, 490, 491, 492, 493, 494, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 509, 510, 511, 512, 513, 514, 515, 516, 517, 519, 520, 521, 522, 523, 524, 525, 526, 527, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 20, 21, 22, 23, 24, 25, 27, 29, 30, 31, 32, 34, 36, 38, 39, 40, 42, 44, 45, 46, 47, 49, 51, 52, 53, 54, 55, 56, 57, 58, 59, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 77, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 191, 192, 193, 194, 195, 346, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 415, 416, 417, 418, 419, 421, 422, 423, 425, 426, 427, 428, 429, 430, 432, 433, 434, 437, 438, 439, 440, 441, 442, 443, 445, 446, 449, 450, 451, 452, 453, 454, 455, 458, 461, 462, 463, 464, 473, 474, 475, 476, 477, 478, 506, 507, 508, 509, 510, 633 }

B grade: { 8, 19, 26, 28, 33, 35, 37, 41, 43, 48, 50, 60, 76, 78, 90, 91, 163, 165, 176, 190, 343, 344, 345, 386, 399, 411, 412, 413, 414, 420, 424, 431, 435, 436, 444, 447, 448, 456, 457, 459, 460, 465, 466, 467, 468, 469, 470, 471, 472, 511, 630, 631, 632 }

C grade: { }

F grade: { 164, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 107, 120, 122, 131, 133, 135, 145, 149, 151, 162, 175, 191, 200, 276, 277, 278, 279, 285, 286, 287, 288, 294, 295, 296, 297, 301, 302, 303, 304, 308, 309, 310, 311, 312, 316, 317, 318, 319, 320, 343, 344, 345, 346, 360, 361, 362, 363, 367, 368, 369, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 475, 485, 496, 508, 518, 528, 529, 543, 630, 631, 632, 633 }

C grade: { }

F grade: { 101, 102, 103, 104, 105, 106, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 121, 123, 124, 125, 126, 127, 128, 129, 130, 132, 134, 136, 137, 138, 139, 140, 141, 142, 143, 144, 146, 147, 148, 150, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 192, 193, 194, 195, 196, 197, 198, 199, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 280, 281, 282, 283, 284, 289, 290, 291, 292, 293, 298, 299, 300, 305, 306, 307, 313, 314, 315, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 364, 365, 366, 370, 371, 372, 373, 374, 375, 472, 473, 474, 476, 477, 478, 479, 480, 481, 482, 483, 484, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 509, 510, 511, 512, 513, 514, 515, 516, 517, 519, 520, 521, 522, 523, 524, 525, 526, 527, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	79	56	92	62	105	118	90
N.S.	1	1.00	0.91	0.64	1.06	0.71	1.21	1.36	1.03
time (sec)	N/A	0.039	0.032	0.208	0.305	0.369	0.881	6.134	0.087

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	57	62	63	62	172	107	226
N.S.	1	1.00	0.66	0.71	0.72	0.71	1.98	1.23	2.60
time (sec)	N/A	0.044	0.112	0.165	0.298	0.359	0.712	7.094	8.231

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	63	46	70	51	83	88	68
N.S.	1	1.00	0.98	0.72	1.09	0.80	1.30	1.38	1.06
time (sec)	N/A	0.034	0.020	0.152	0.290	0.349	0.438	5.869	0.051

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	62	52	48	51	124	77	165
N.S.	1	1.00	0.95	0.80	0.74	0.78	1.91	1.18	2.54
time (sec)	N/A	0.034	0.058	0.116	0.306	0.376	0.260	5.328	7.991

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	44	36	48	39	60	48	46
N.S.	1	1.00	0.98	0.80	1.07	0.87	1.33	1.07	1.02
time (sec)	N/A	0.024	0.013	0.103	0.296	0.383	0.203	4.183	0.058

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	46	41	37	37	71	47	103
N.S.	1	1.00	1.07	0.95	0.86	0.86	1.65	1.09	2.40
time (sec)	N/A	0.024	0.038	0.083	0.293	0.350	0.121	4.299	6.761

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	28	39	23	20	25	34	25	20
N.S.	1	1.27	1.77	1.05	0.91	1.14	1.55	1.14	0.91
time (sec)	N/A	0.011	0.012	0.046	0.301	0.396	0.068	3.634	0.040

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	26	16	15	17	0	37	15
N.S.	1	1.00	1.53	0.94	0.88	1.00	0.00	2.18	0.88
time (sec)	N/A	0.013	0.013	0.068	0.303	0.360	0.000	4.060	0.047

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	24	23	40	0	19	19
N.S.	1	1.00	1.00	1.04	1.00	1.74	0.00	0.83	0.83
time (sec)	N/A	0.022	0.013	0.072	0.308	0.340	0.000	3.703	4.688

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	52	50	42	67	0	54	30
N.S.	1	1.00	1.33	1.28	1.08	1.72	0.00	1.38	0.77
time (sec)	N/A	0.031	0.016	0.105	0.298	0.357	0.000	5.115	0.061

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	41	38	35	52	0	66	63
N.S.	1	1.00	0.93	0.86	0.80	1.18	0.00	1.50	1.43
time (sec)	N/A	0.025	0.050	0.121	0.295	0.328	0.000	8.464	4.604

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	68	63	86	136	0	92	71
N.S.	1	1.00	0.81	0.75	1.02	1.62	0.00	1.10	0.85
time (sec)	N/A	0.045	0.060	0.151	0.304	0.374	0.000	6.166	4.501

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	171	129	115	85	398	123	461
N.S.	1	1.00	1.36	1.02	0.91	0.67	3.16	0.98	3.66
time (sec)	N/A	0.072	0.976	0.271	0.310	0.365	0.993	5.473	6.916

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	58	99	95	71	158	117	92
N.S.	1	1.00	0.87	1.48	1.42	1.06	2.36	1.75	1.37
time (sec)	N/A	0.042	0.050	0.210	0.334	0.369	0.658	4.284	4.536

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	151	109	89	72	287	106	349
N.S.	1	1.00	1.48	1.07	0.87	0.71	2.81	1.04	3.42
time (sec)	N/A	0.060	0.353	0.194	0.297	0.373	0.433	4.535	6.792

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	46	79	56	58	107	56	53
N.S.	1	1.00	1.02	1.76	1.24	1.29	2.38	1.24	1.18
time (sec)	N/A	0.031	0.060	0.176	0.293	0.352	0.262	7.269	0.064

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	131	87	65	59	180	72	237
N.S.	1	1.00	1.68	1.12	0.83	0.76	2.31	0.92	3.04
time (sec)	N/A	0.051	0.196	0.134	0.306	0.353	0.192	5.859	6.708

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	37	21	20	44	53	20	32
N.S.	1	1.00	1.68	0.95	0.91	2.00	2.41	0.91	1.45
time (sec)	N/A	0.017	0.123	0.082	0.290	0.359	0.125	3.746	4.545

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	29	63	30	32	0	91	26
N.S.	1	1.00	0.85	1.85	0.88	0.94	0.00	2.68	0.76
time (sec)	N/A	0.026	0.015	0.121	0.331	0.387	0.000	3.611	0.054

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	75	47	47	74	0	33	28
N.S.	1	1.00	1.97	1.24	1.24	1.95	0.00	0.87	0.74
time (sec)	N/A	0.053	0.041	0.112	0.524	0.355	0.000	4.844	4.561

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	32	100	18	19	0	30	18
N.S.	1	1.00	1.60	5.00	0.90	0.95	0.00	1.50	0.90
time (sec)	N/A	0.026	0.088	0.152	0.375	0.339	0.000	6.195	0.042

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	58	63	52	97	0	54	81
N.S.	1	1.00	0.92	1.00	0.83	1.54	0.00	0.86	1.29
time (sec)	N/A	0.048	0.007	0.168	0.304	0.351	0.000	5.874	4.558

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	56	132	71	125	0	77	58
N.S.	1	1.00	0.88	2.06	1.11	1.95	0.00	1.20	0.91
time (sec)	N/A	0.045	0.045	0.185	0.294	0.373	0.000	5.064	4.466

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	82	93	77	85	0	106	156
N.S.	1	1.00	1.28	1.45	1.20	1.33	0.00	1.66	2.44
time (sec)	N/A	0.039	0.008	0.216	0.307	0.343	0.000	3.976	4.634

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	85	160	108	203	0	119	94
N.S.	1	1.00	0.78	1.47	0.99	1.86	0.00	1.09	0.86
time (sec)	N/A	0.063	0.100	0.269	0.312	0.367	0.000	5.427	4.338

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	110	121	98	115	0	171	276
N.S.	1	1.00	1.34	1.48	1.20	1.40	0.00	2.09	3.37
time (sec)	N/A	0.040	0.017	0.214	0.313	0.338	0.000	5.552	5.066

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	181	163	141	98	439	157	501
N.S.	1	1.00	1.18	1.06	0.92	0.64	2.85	1.02	3.25
time (sec)	N/A	0.107	1.251	0.364	0.309	0.378	1.427	6.831	6.806

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	58	133	108	85	196	134	106
N.S.	1	1.00	0.87	1.99	1.61	1.27	2.93	2.00	1.58
time (sec)	N/A	0.043	0.063	0.323	0.289	0.357	0.957	6.184	4.539

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	161	143	115	85	335	123	389
N.S.	1	1.00	1.24	1.10	0.88	0.65	2.58	0.95	2.99
time (sec)	N/A	0.090	0.500	0.233	0.303	0.371	0.683	3.975	6.711

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	43	113	82	72	146	82	80
N.S.	1	1.00	0.96	2.51	1.82	1.60	3.24	1.82	1.78
time (sec)	N/A	0.029	0.097	0.209	0.318	0.357	0.426	3.719	4.472

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	141	121	91	72	226	89	277
N.S.	1	1.00	1.33	1.14	0.86	0.68	2.13	0.84	2.61
time (sec)	N/A	0.078	0.268	0.170	0.391	0.347	0.368	4.170	6.532

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	47	21	20	57	70	20	53
N.S.	1	1.00	2.14	0.95	0.91	2.59	3.18	0.91	2.41
time (sec)	N/A	0.016	0.273	0.114	0.287	0.355	0.170	5.672	0.056

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	41	88	43	45	0	128	36
N.S.	1	1.00	0.79	1.69	0.83	0.87	0.00	2.46	0.69
time (sec)	N/A	0.028	0.021	0.128	0.294	0.362	0.000	6.238	0.053

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	55	87	68	101	0	91	138
N.S.	1	1.00	1.10	1.74	1.36	2.02	0.00	1.82	2.76
time (sec)	N/A	0.087	0.025	0.135	0.537	0.369	0.000	5.608	4.776

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	59	124	33	51	0	92	35
N.S.	1	1.00	1.48	3.10	0.82	1.28	0.00	2.30	0.88
time (sec)	N/A	0.033	0.031	0.168	0.299	0.364	0.000	3.140	4.521

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	28	120	78	99	0	38	55
N.S.	1	1.00	0.90	3.87	2.52	3.19	0.00	1.23	1.77
time (sec)	N/A	0.055	0.019	0.201	0.338	0.358	0.000	4.744	4.592

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	35	154	28	30	0	63	18
N.S.	1	1.00	1.52	6.70	1.22	1.30	0.00	2.74	0.78
time (sec)	N/A	0.026	0.149	0.198	0.313	0.342	0.000	4.612	0.066

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	110	171	103	149	0	86	135
N.S.	1	1.00	1.20	1.86	1.12	1.62	0.00	0.93	1.47
time (sec)	N/A	0.062	0.010	0.200	0.416	0.360	0.000	7.260	4.694

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	67	202	96	185	0	90	81
N.S.	1	1.00	0.77	2.32	1.10	2.13	0.00	1.03	0.93
time (sec)	N/A	0.052	0.072	0.181	0.315	0.374	0.000	6.328	4.536

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	134	217	122	112	0	138	228
N.S.	1	1.00	1.35	2.19	1.23	1.13	0.00	1.39	2.30
time (sec)	N/A	0.060	0.009	0.186	0.308	0.384	0.000	4.502	4.865

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	58	513	173	149	558	219	134
N.S.	1	1.00	0.87	7.66	2.58	2.22	8.33	3.27	2.00
time (sec)	N/A	0.056	0.266	1.237	0.309	0.396	5.463	2.912	0.179

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	286	286	211	535	339	150	1280	208	684
N.S.	1	1.00	0.74	1.87	1.19	0.52	4.48	0.73	2.39
time (sec)	N/A	0.281	1.981	0.934	0.315	0.420	4.036	5.146	7.051

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	43	463	134	136	422	134	132
N.S.	1	1.00	0.96	10.29	2.98	3.02	9.38	2.98	2.93
time (sec)	N/A	0.032	0.658	0.688	0.316	0.391	2.837	6.633	0.117

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	191	480	319	137	1018	174	572
N.S.	1	1.00	0.73	1.83	1.22	0.52	3.89	0.66	2.18
time (sec)	N/A	0.253	0.908	0.493	0.318	0.404	2.124	5.697	7.258

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	97	21	20	122	148	20	118
N.S.	1	1.00	4.41	0.95	0.91	5.55	6.73	0.91	5.36
time (sec)	N/A	0.017	0.847	0.336	0.312	0.357	1.371	5.064	4.746

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	95	312	109	114	0	288	109
N.S.	1	1.00	0.59	1.93	0.67	0.70	0.00	1.78	0.67
time (sec)	N/A	0.053	0.118	0.220	0.313	0.375	0.000	4.955	4.648

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	55	389	331	231	0	231	513
N.S.	1	1.00	0.27	1.94	1.65	1.15	0.00	1.15	2.55
time (sec)	N/A	0.237	0.041	0.129	0.538	0.362	0.000	4.492	8.733

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	111	442	97	130	0	275	97
N.S.	1	1.00	0.92	3.65	0.80	1.07	0.00	2.27	0.80
time (sec)	N/A	0.062	0.176	0.206	0.293	0.381	0.000	5.321	4.623

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	59	478	311	247	0	200	437
N.S.	1	1.00	0.33	2.67	1.74	1.38	0.00	1.12	2.44
time (sec)	N/A	0.214	0.036	0.202	0.528	0.382	0.000	6.880	9.129

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	73	527	95	139	0	243	96
N.S.	1	1.00	0.66	4.79	0.86	1.26	0.00	2.21	0.87
time (sec)	N/A	0.063	0.295	0.205	0.313	0.391	0.000	6.204	4.602

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	141	114	258	50	1355	114	107
N.S.	1	1.00	1.93	1.56	3.53	0.68	18.56	1.56	1.47
time (sec)	N/A	0.047	0.505	0.178	0.551	0.368	15.820	6.435	8.155

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	46	45	47	37	530	47	54
N.S.	1	1.00	0.98	0.96	1.00	0.79	11.28	1.00	1.15
time (sec)	N/A	0.039	0.069	0.164	0.339	0.369	8.699	3.665	4.660

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	119	75	156	37	558	75	66
N.S.	1	1.00	2.43	1.53	3.18	0.76	11.39	1.53	1.35
time (sec)	N/A	0.037	0.214	0.134	0.700	0.394	4.859	4.838	6.874

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	24	28	25	25	158	25	22
N.S.	1	1.00	0.75	0.88	0.78	0.78	4.94	0.78	0.69
time (sec)	N/A	0.030	0.027	0.000	0.409	0.414	2.329	5.065	4.488

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	97	35	52	17	88	34	29
N.S.	1	1.00	5.11	1.84	2.74	0.89	4.63	1.79	1.53
time (sec)	N/A	0.027	0.083	0.091	0.556	0.351	1.250	4.703	4.532

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	19	18	16	24	19	16
N.S.	1	1.00	1.00	1.19	1.12	1.00	1.50	1.19	1.00
time (sec)	N/A	0.017	0.009	0.000	0.305	0.369	0.264	6.500	0.042

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	30	43	47	58	0	58	33
N.S.	1	1.00	0.81	1.16	1.27	1.57	0.00	1.57	0.89
time (sec)	N/A	0.037	0.028	0.000	0.309	0.355	0.000	5.427	0.066

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	45	70	129	49	0	67	71
N.S.	1	1.00	1.07	1.67	3.07	1.17	0.00	1.60	1.69
time (sec)	N/A	0.036	0.040	0.105	0.305	0.349	0.000	5.496	4.560

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	75	67	91	125	0	96	74
N.S.	1	1.00	0.97	0.87	1.18	1.62	0.00	1.25	0.96
time (sec)	N/A	0.054	0.070	0.152	0.303	0.369	0.000	3.858	4.656

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	66	130	294	75	0	119	125
N.S.	1	1.00	1.06	2.10	4.74	1.21	0.00	1.92	2.02
time (sec)	N/A	0.041	0.069	0.180	0.357	0.362	0.000	6.207	5.941

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	97	91	130	147	0	116	115
N.S.	1	1.00	0.81	0.76	1.08	1.22	0.00	0.97	0.96
time (sec)	N/A	0.075	0.105	0.227	0.511	0.361	0.000	6.792	0.142

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	151	181	393	60	2531	179	172
N.S.	1	1.00	1.45	1.74	3.78	0.58	24.34	1.72	1.65
time (sec)	N/A	0.079	0.737	0.156	0.853	0.358	78.335	6.888	8.223

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	46	45	47	47	1037	47	54
N.S.	1	1.00	0.98	0.96	1.00	1.00	22.06	1.00	1.15
time (sec)	N/A	0.035	0.109	0.276	0.384	0.355	45.756	4.871	4.662

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	131	129	267	50	1243	127	65
N.S.	1	1.00	1.64	1.61	3.34	0.62	15.54	1.59	0.81
time (sec)	N/A	0.066	0.327	0.232	0.525	0.368	29.084	5.009	4.746

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	39	19	35	37	394	35	32
N.S.	1	1.00	1.70	0.83	1.52	1.61	17.13	1.52	1.39
time (sec)	N/A	0.028	0.088	0.197	0.290	0.351	16.906	5.245	4.610

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	109	77	140	35	403	73	32
N.S.	1	1.00	1.95	1.38	2.50	0.62	7.20	1.30	0.57
time (sec)	N/A	0.055	0.116	0.201	0.503	0.349	9.436	6.030	4.653

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	26	28	30	27	150	54	27
N.S.	1	1.00	0.81	0.88	0.94	0.84	4.69	1.69	0.84
time (sec)	N/A	0.032	0.022	0.187	0.285	0.344	0.534	8.025	0.060

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	104	37	56	61	95	33	28
N.S.	1	1.00	3.06	1.09	1.65	1.79	2.79	0.97	0.82
time (sec)	N/A	0.026	0.113	0.198	0.530	0.370	2.671	7.748	4.641

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	31	21	20	21	32	20	18
N.S.	1	1.00	1.48	1.00	0.95	1.00	1.52	0.95	0.86
time (sec)	N/A	0.017	0.045	0.102	0.299	0.332	0.550	5.481	0.048

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	38	55	72	105	0	71	60
N.S.	1	1.00	0.63	0.92	1.20	1.75	0.00	1.18	1.00
time (sec)	N/A	0.039	0.065	0.214	0.304	0.366	0.000	6.296	4.519

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	53	98	204	79	0	93	156
N.S.	1	1.00	0.75	1.38	2.87	1.11	0.00	1.31	2.20
time (sec)	N/A	0.064	0.053	0.174	0.304	0.345	0.000	5.943	4.770

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	85	79	108	178	0	106	93
N.S.	1	1.00	0.82	0.76	1.04	1.71	0.00	1.02	0.89
time (sec)	N/A	0.058	0.078	0.236	0.299	0.353	0.000	3.665	0.103

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	78	158	396	103	0	145	276
N.S.	1	1.00	0.84	1.70	4.26	1.11	0.00	1.56	2.97
time (sec)	N/A	0.069	0.044	0.253	0.323	0.344	0.000	7.247	5.184

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	137	103	167	198	0	126	151
N.S.	1	1.00	0.94	0.71	1.14	1.36	0.00	0.86	1.03
time (sec)	N/A	0.081	0.213	0.340	0.305	0.422	0.000	6.554	0.186

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	141	142	310	60	1717	140	81
N.S.	1	1.00	1.37	1.38	3.01	0.58	16.67	1.36	0.79
time (sec)	N/A	0.079	0.671	0.149	0.537	0.403	120.347	6.943	4.721

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	48	19	45	45	654	45	53
N.S.	1	1.00	2.09	0.83	1.96	1.96	28.43	1.96	2.30
time (sec)	N/A	0.028	0.216	0.131	0.295	0.379	75.625	5.785	4.548

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	121	90	184	45	690	88	57
N.S.	1	1.00	1.57	1.17	2.39	0.58	8.96	1.14	0.74
time (sec)	N/A	0.067	0.311	0.280	0.510	0.348	48.296	5.829	4.638

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	38	38	41	36	564	115	36
N.S.	1	1.00	0.76	0.76	0.82	0.72	11.28	2.30	0.72
time (sec)	N/A	0.034	0.034	0.264	0.281	0.358	27.727	7.699	4.560

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	59	54	139	78	478	80	69
N.S.	1	1.00	1.20	1.10	2.84	1.59	9.76	1.63	1.41
time (sec)	N/A	0.055	0.027	0.252	0.506	0.333	16.447	6.446	4.852

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	58	32	37	41	299	35	36
N.S.	1	1.00	1.49	0.82	0.95	1.05	7.67	0.90	0.92
time (sec)	N/A	0.035	0.043	0.207	0.295	0.358	0.612	6.262	4.543

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	28	55	99	95	153	36	53
N.S.	1	1.00	1.04	2.04	3.67	3.52	5.67	1.33	1.96
time (sec)	N/A	0.028	0.014	0.211	0.277	0.352	5.091	5.901	4.580

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	33	21	20	36	51	20	18
N.S.	1	1.00	1.50	0.95	0.91	1.64	2.32	0.91	0.82
time (sec)	N/A	0.017	0.042	0.112	0.291	0.338	0.583	6.337	4.460

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	61	67	98	154	0	81	83
N.S.	1	1.00	0.74	0.82	1.20	1.88	0.00	0.99	1.01
time (sec)	N/A	0.043	0.057	0.224	0.277	0.344	0.000	5.515	4.595

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	63	130	310	106	0	119	228
N.S.	1	1.00	0.64	1.31	3.13	1.07	0.00	1.20	2.30
time (sec)	N/A	0.094	0.066	0.218	0.295	0.344	0.000	6.532	5.079

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	95	91	146	228	0	116	129
N.S.	1	1.00	0.75	0.72	1.16	1.81	0.00	0.92	1.02
time (sec)	N/A	0.070	0.105	0.322	0.285	0.383	0.000	8.380	0.149

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	85	190	482	130	0	171	167
N.S.	1	1.00	0.69	1.54	3.92	1.06	0.00	1.39	1.36
time (sec)	N/A	0.103	0.078	0.328	0.299	0.349	0.000	5.145	5.563

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	145	115	188	248	0	136	173
N.S.	1	1.00	0.85	0.67	1.10	1.45	0.00	0.80	1.01
time (sec)	N/A	0.092	0.301	0.406	0.301	0.392	0.000	4.532	4.765

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	275	110	295	244	0	99	91
N.S.	1	1.00	2.17	0.87	2.32	1.92	0.00	0.78	0.72
time (sec)	N/A	0.125	6.062	0.200	0.531	0.361	0.000	7.133	7.770

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	28	55	74	82	2006	68	64
N.S.	1	1.00	0.78	1.53	2.06	2.28	55.72	1.89	1.78
time (sec)	N/A	0.031	0.050	0.186	0.284	0.347	14.553	7.005	0.072

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	36	145	375	239	0	125	118
N.S.	1	1.00	0.62	2.50	6.47	4.12	0.00	2.16	2.03
time (sec)	N/A	0.058	0.057	0.215	0.316	0.337	0.000	7.323	6.628

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	58	43	93	100	1120	137	54
N.S.	1	1.00	0.89	0.66	1.43	1.54	17.23	2.11	0.83
time (sec)	N/A	0.043	0.080	0.217	0.317	0.358	14.305	6.565	4.712

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	58	175	461	291	2470	151	140
N.S.	1	1.00	0.49	1.48	3.91	2.47	20.93	1.28	1.19
time (sec)	N/A	0.120	0.054	0.250	0.328	0.376	201.182	6.195	7.148

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	43	33	96	105	493	28	28
N.S.	1	1.00	0.96	0.73	2.13	2.33	10.96	0.62	0.62
time (sec)	N/A	0.036	0.113	0.233	0.281	0.353	14.506	5.486	0.097

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	78	205	547	339	3405	177	162
N.S.	1	1.00	0.43	1.12	2.99	1.85	18.61	0.97	0.89
time (sec)	N/A	0.183	0.085	0.259	0.312	0.376	106.177	4.827	8.117

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	33	21	20	108	128	20	18
N.S.	1	1.00	1.50	0.95	0.91	4.91	5.82	0.91	0.82
time (sec)	N/A	0.018	0.151	0.286	0.284	0.345	14.202	6.679	4.668

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	122	127	213	374	0	131	198
N.S.	1	1.00	0.63	0.65	1.10	1.93	0.00	0.68	1.02
time (sec)	N/A	0.082	0.473	0.506	0.302	0.399	0.000	6.870	0.303

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	113	280	740	225	0	249	233
N.S.	1	1.00	0.46	1.14	3.02	0.92	0.00	1.02	0.95
time (sec)	N/A	0.285	0.218	0.249	0.351	0.359	0.000	7.840	8.035

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	238	175	151	248	446	0	166	231
N.S.	1	1.00	0.74	0.63	1.04	1.87	0.00	0.70	0.97
time (sec)	N/A	0.124	0.985	0.382	0.302	0.380	0.000	4.675	0.489

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	125	340	866	249	0	301	277
N.S.	1	1.00	0.45	1.22	3.10	0.89	0.00	1.08	0.99
time (sec)	N/A	0.302	0.270	0.339	0.406	0.377	0.000	4.769	9.231

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	284	195	175	305	466	0	186	290
N.S.	1	1.00	0.69	0.62	1.07	1.64	0.00	0.65	1.02
time (sec)	N/A	0.156	1.571	0.411	0.309	0.418	0.000	4.879	0.798

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	74	57	72	88	0	128	-1
N.S.	1	1.00	0.76	0.59	0.74	0.91	0.00	1.32	-0.01
time (sec)	N/A	0.055	2.594	0.392	0.302	0.362	0.000	7.787	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	99	75	0	172	0	201	-1
N.S.	1	1.00	0.78	0.59	0.00	1.35	0.00	1.58	-0.01
time (sec)	N/A	0.169	2.374	0.376	0.000	0.339	0.000	6.705	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	64	41	55	68	0	99	-1
N.S.	1	1.00	0.88	0.56	0.75	0.93	0.00	1.36	-0.01
time (sec)	N/A	0.049	0.608	0.266	0.309	0.353	0.000	7.522	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	89	65	0	132	0	147	-1
N.S.	1	1.00	0.94	0.68	0.00	1.39	0.00	1.55	-0.01
time (sec)	N/A	0.118	0.476	0.365	0.000	0.357	0.000	6.138	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	54	31	38	46	0	70	-1
N.S.	1	1.00	1.10	0.63	0.78	0.94	0.00	1.43	-0.02
time (sec)	N/A	0.041	0.145	0.270	0.305	0.357	0.000	5.467	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	79	55	0	92	0	93	-1
N.S.	1	1.00	1.25	0.87	0.00	1.46	0.00	1.48	-0.02
time (sec)	N/A	0.073	0.108	0.308	0.000	0.327	0.000	5.756	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	44	21	20	25	58	38	20
N.S.	1	1.00	1.83	0.88	0.83	1.04	2.42	1.58	0.83
time (sec)	N/A	0.020	0.057	0.037	0.291	0.351	0.124	5.796	4.566

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	95	32	58	92	0	59	-1
N.S.	1	1.00	2.38	0.80	1.45	2.30	0.00	1.48	-0.02
time (sec)	N/A	0.039	0.070	0.132	0.515	0.347	0.000	6.337	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	106	84	0	159	0	102	-1
N.S.	1	1.00	1.47	1.17	0.00	2.21	0.00	1.42	-0.01
time (sec)	N/A	0.054	0.159	0.462	0.000	0.353	0.000	4.821	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	271	90	117	99	0	112	-1
N.S.	1	1.00	2.85	0.95	1.23	1.04	0.00	1.18	-0.01
time (sec)	N/A	0.086	0.256	0.574	0.505	0.359	0.000	6.333	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	302	153	0	188	0	178	-1
N.S.	1	1.00	2.20	1.12	0.00	1.37	0.00	1.30	-0.01
time (sec)	N/A	0.111	0.283	0.461	0.000	0.365	0.000	8.191	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	179	118	168	121	0	146	-1
N.S.	1	1.00	1.20	0.79	1.13	0.81	0.00	0.98	-0.01
time (sec)	N/A	0.136	0.316	0.787	0.517	0.367	0.000	6.903	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	191	244	0	210	0	239	-1
N.S.	1	1.00	0.97	1.24	0.00	1.07	0.00	1.21	-0.01
time (sec)	N/A	0.206	0.405	0.521	0.000	0.373	0.000	7.260	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	61	57	72	110	0	132	-1
N.S.	1	1.00	0.63	0.59	0.74	1.13	0.00	1.36	-0.01
time (sec)	N/A	0.057	0.244	0.259	0.286	0.378	0.000	7.120	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	79	87	0	210	0	162	-1
N.S.	1	1.00	0.50	0.55	0.00	1.32	0.00	1.02	-0.01
time (sec)	N/A	0.209	0.388	0.490	0.000	0.347	0.000	5.585	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	51	41	55	88	0	102	-1
N.S.	1	1.00	0.70	0.56	0.75	1.21	0.00	1.40	-0.01
time (sec)	N/A	0.055	0.108	0.272	0.299	0.355	0.000	5.017	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	69	77	0	166	0	132	-1
N.S.	1	1.00	0.54	0.61	0.00	1.31	0.00	1.04	-0.01
time (sec)	N/A	0.158	0.102	0.375	0.000	0.360	0.000	5.425	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	41	31	38	66	252	72	-1
N.S.	1	1.00	0.84	0.63	0.78	1.35	5.14	1.47	-0.02
time (sec)	N/A	0.046	0.056	0.290	0.334	0.356	5.568	6.000	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	59	67	0	122	0	102	-1
N.S.	1	1.00	0.62	0.71	0.00	1.28	0.00	1.07	-0.01
time (sec)	N/A	0.112	0.095	0.366	0.000	0.367	0.000	6.078	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	21	20	40	90	38	20
N.S.	1	1.00	1.00	0.88	0.83	1.67	3.75	1.58	0.83
time (sec)	N/A	0.023	0.029	0.039	0.289	0.342	1.642	5.841	4.624

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	60	49	80	72	0	73	-1
N.S.	1	1.00	0.97	0.79	1.29	1.16	0.00	1.18	-0.02
time (sec)	N/A	0.045	0.069	0.204	0.562	0.359	0.000	6.770	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	67	37	98	26	0	38	37
N.S.	1	1.00	2.58	1.42	3.77	1.00	0.00	1.46	1.42
time (sec)	N/A	0.040	0.103	0.207	0.622	0.345	0.000	6.291	4.773

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	72	70	94	99	0	91	-1
N.S.	1	1.00	0.99	0.96	1.29	1.36	0.00	1.25	-0.01
time (sec)	N/A	0.079	0.164	0.399	0.519	0.361	0.000	6.851	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	B	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	130	107	0	215	0	95	-1
N.S.	1	1.00	1.21	1.00	0.00	2.01	0.00	0.89	-0.01
time (sec)	N/A	0.093	0.262	0.457	0.000	0.391	0.000	6.103	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	44	101	151	155	0	128	-1
N.S.	1	1.00	0.35	0.80	1.19	1.22	0.00	1.01	-0.01
time (sec)	N/A	0.128	0.052	0.821	0.557	0.368	0.000	4.445	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	288	172	0	248	0	143	-1
N.S.	1	1.00	1.70	1.02	0.00	1.47	0.00	0.85	-0.01
time (sec)	N/A	0.157	0.273	0.347	0.000	0.358	0.000	5.586	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	51	41	55	114	0	108	-1
N.S.	1	1.00	0.70	0.56	0.75	1.56	0.00	1.48	-0.01
time (sec)	N/A	0.050	0.123	0.288	0.301	0.363	0.000	7.287	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	79	87	0	219	0	172	-1
N.S.	1	1.00	0.50	0.55	0.00	1.38	0.00	1.08	-0.01
time (sec)	N/A	0.198	0.194	0.381	0.000	0.368	0.000	5.402	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	41	31	38	88	335	76	-1
N.S.	1	1.00	0.84	0.63	0.78	1.80	6.84	1.55	-0.02
time (sec)	N/A	0.044	0.080	0.336	0.300	0.361	48.105	4.745	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	69	77	0	167	0	140	-1
N.S.	1	1.00	0.54	0.61	0.00	1.31	0.00	1.10	-0.01
time (sec)	N/A	0.155	0.169	0.418	0.000	0.352	0.000	4.911	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	21	20	61	126	38	20
N.S.	1	1.00	1.00	0.88	0.83	2.54	5.25	1.58	0.83
time (sec)	N/A	0.026	0.039	0.040	0.282	0.336	16.919	4.895	4.782

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	73	66	97	89	0	91	-1
N.S.	1	1.00	0.85	0.77	1.13	1.03	0.00	1.06	-0.01
time (sec)	N/A	0.051	0.120	0.269	0.514	0.367	0.000	5.814	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	36	45	191	41	0	72	88
N.S.	1	1.00	0.65	0.82	3.47	0.75	0.00	1.31	1.60
time (sec)	N/A	0.080	3.125	0.336	0.580	0.355	0.000	6.916	5.460

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	75	66	94	102	0	91	-1
N.S.	1	1.00	1.09	0.96	1.36	1.48	0.00	1.32	-0.01
time (sec)	N/A	0.079	0.250	0.345	0.497	0.391	0.000	5.519	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	69	47	184	43	0	38	225
N.S.	1	1.00	2.30	1.57	6.13	1.43	0.00	1.27	7.50
time (sec)	N/A	0.038	5.103	0.262	0.698	0.334	0.000	5.323	7.589

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	110	107	134	147	0	112	-1
N.S.	1	1.00	1.07	1.04	1.30	1.43	0.00	1.09	-0.01
time (sec)	N/A	0.118	0.187	0.622	0.639	0.381	0.000	5.090	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	129	120	0	263	0	111	-1
N.S.	1	1.00	0.93	0.86	0.00	1.89	0.00	0.80	-0.01
time (sec)	N/A	0.140	5.205	0.461	0.000	0.376	0.000	5.976	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	44	113	185	208	0	144	-1
N.S.	1	1.00	0.28	0.71	1.16	1.31	0.00	0.91	-0.01
time (sec)	N/A	0.167	0.068	0.921	0.531	0.371	0.000	5.730	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	64	57	72	154	0	140	-1
N.S.	1	1.00	0.66	0.59	0.74	1.59	0.00	1.44	-0.01
time (sec)	N/A	0.056	0.376	0.313	0.319	0.376	0.000	5.617	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	102	107	0	296	0	236	-1
N.S.	1	1.00	0.46	0.48	0.00	1.33	0.00	1.06	-0.00
time (sec)	N/A	0.291	0.343	0.402	0.000	0.358	0.000	6.407	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	54	41	55	128	0	108	-1
N.S.	1	1.00	0.74	0.56	0.75	1.75	0.00	1.48	-0.01
time (sec)	N/A	0.054	0.174	0.274	0.286	0.376	0.000	7.043	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	92	97	0	244	0	204	-1
N.S.	1	1.00	0.48	0.51	0.00	1.28	0.00	1.07	-0.01
time (sec)	N/A	0.247	0.153	0.425	0.000	0.371	0.000	5.682	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	44	31	38	102	0	76	-1
N.S.	1	1.00	0.90	0.63	0.78	2.08	0.00	1.55	-0.02
time (sec)	N/A	0.047	0.087	0.319	0.279	0.352	0.000	7.038	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	82	87	0	192	0	172	-1
N.S.	1	1.00	0.52	0.55	0.00	1.21	0.00	1.08	-0.01
time (sec)	N/A	0.200	0.137	0.422	0.000	0.339	0.000	6.479	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	21	20	74	0	38	20
N.S.	1	1.00	1.00	0.88	0.83	3.08	0.00	1.58	0.83
time (sec)	N/A	0.024	0.057	0.047	0.291	0.351	0.000	2.256	4.844

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	85	83	115	102	0	107	-1
N.S.	1	1.00	0.77	0.75	1.05	0.93	0.00	0.97	-0.01
time (sec)	N/A	0.061	0.214	0.332	0.500	0.338	0.000	3.717	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	48	55	237	54	0	105	-1
N.S.	1	1.00	0.54	0.62	2.66	0.61	0.00	1.18	-0.01
time (sec)	N/A	0.119	5.303	0.340	0.567	0.349	0.000	4.301	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	42	83	112	116	0	107	-1
N.S.	1	1.00	0.46	0.91	1.23	1.27	0.00	1.18	-0.01
time (sec)	N/A	0.097	0.054	0.658	0.495	0.373	0.000	3.925	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	82	57	320	57	0	76	118
N.S.	1	1.00	1.34	0.93	5.25	0.93	0.00	1.25	1.93
time (sec)	N/A	0.083	5.190	0.411	0.585	0.338	0.000	5.540	8.531

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	108	75	132	145	0	149	-1
N.S.	1	1.00	1.02	0.71	1.25	1.37	0.00	1.41	-0.01
time (sec)	N/A	0.124	0.195	0.563	0.499	0.348	0.000	4.971	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	69	47	270	54	0	38	86
N.S.	1	1.00	2.30	1.57	9.00	1.80	0.00	1.27	2.87
time (sec)	N/A	0.038	5.175	0.312	0.564	0.365	0.000	3.849	8.427

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	120	144	168	193	0	128	-1
N.S.	1	1.00	0.89	1.07	1.24	1.43	0.00	0.95	-0.01
time (sec)	N/A	0.173	0.360	0.833	0.514	0.362	0.000	4.836	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	139	139	0	312	0	127	-1
N.S.	1	1.00	0.81	0.81	0.00	1.82	0.00	0.74	-0.01
time (sec)	N/A	0.185	5.324	0.473	0.000	0.394	0.000	3.159	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	44	129	219	254	0	160	-1
N.S.	1	1.00	0.23	0.68	1.15	1.33	0.00	0.84	-0.01
time (sec)	N/A	0.210	0.070	0.925	0.526	0.373	0.000	5.349	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	388	205	0	346	0	175	-1
N.S.	1	1.00	1.67	0.88	0.00	1.48	0.00	0.75	-0.00
time (sec)	N/A	0.245	5.429	0.557	0.000	0.378	0.000	6.646	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	61	57	281	82	0	112	-1
N.S.	1	1.00	0.63	0.59	2.90	0.85	0.00	1.15	-0.01
time (sec)	N/A	0.052	0.185	0.389	0.286	0.360	0.000	6.059	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	59	64	0	155	0	84	-1
N.S.	1	1.00	0.62	0.67	0.00	1.63	0.00	0.88	-0.01
time (sec)	N/A	0.120	0.160	0.392	0.000	0.338	0.000	6.487	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	51	41	160	62	0	90	-1
N.S.	1	1.00	0.70	0.56	2.19	0.85	0.00	1.23	-0.01
time (sec)	N/A	0.048	0.088	0.312	0.290	0.331	0.000	6.059	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	49	54	0	115	0	65	-1
N.S.	1	1.00	0.78	0.86	0.00	1.83	0.00	1.03	-0.02
time (sec)	N/A	0.076	0.044	0.460	0.000	0.344	0.000	5.927	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	34	31	75	40	0	68	-1
N.S.	1	1.00	0.69	0.63	1.53	0.82	0.00	1.39	-0.02
time (sec)	N/A	0.041	0.041	0.263	0.312	0.325	0.000	5.262	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	44	0	71	0	40	-1
N.S.	1	1.00	1.00	1.47	0.00	2.37	0.00	1.33	-0.03
time (sec)	N/A	0.034	0.045	0.342	0.000	0.334	0.000	6.326	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	20	20	32	38	20
N.S.	1	1.00	1.00	0.95	0.91	0.91	1.45	1.73	0.91
time (sec)	N/A	0.022	0.018	0.036	0.276	0.347	0.462	5.000	4.824

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	39	54	78	90	0	123	-1
N.S.	1	1.00	0.65	0.90	1.30	1.50	0.00	2.05	-0.02
time (sec)	N/A	0.042	0.033	0.262	0.500	0.380	0.000	4.736	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	118	130	0	200	0	0	-1
N.S.	1	1.00	1.16	1.27	0.00	1.96	0.00	0.00	-0.01
time (sec)	N/A	0.066	0.174	0.462	0.000	0.384	0.000	0.000	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	42	107	132	145	0	195	-1
N.S.	1	1.00	0.36	0.92	1.14	1.25	0.00	1.68	-0.01
time (sec)	N/A	0.096	0.044	0.668	0.500	0.362	0.000	6.637	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	117	231	0	230	0	144	-1
N.S.	1	1.00	0.72	1.43	0.00	1.42	0.00	0.89	-0.01
time (sec)	N/A	0.154	0.400	0.493	0.000	0.361	0.000	7.053	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	44	135	183	167	0	233	-1
N.S.	1	1.00	0.25	0.77	1.05	0.95	0.00	1.33	-0.01
time (sec)	N/A	0.185	0.054	0.824	0.496	0.372	0.000	6.370	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	140	308	0	250	0	262	-1
N.S.	1	1.00	0.63	1.39	0.00	1.13	0.00	1.19	-0.00
time (sec)	N/A	0.243	0.452	0.596	0.000	0.403	0.000	5.303	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	54	57	72	72	0	112	-1
N.S.	1	1.00	0.56	0.59	0.74	0.74	0.00	1.15	-0.01
time (sec)	N/A	0.058	0.146	0.316	0.287	0.352	0.000	3.313	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	49	57	0	142	0	68	-1
N.S.	1	1.00	0.78	0.90	0.00	2.25	0.00	1.08	-0.02
time (sec)	N/A	0.079	0.122	0.441	0.000	0.343	0.000	5.316	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	44	41	55	52	0	90	-1
N.S.	1	1.00	0.60	0.56	0.75	0.71	0.00	1.23	-0.01
time (sec)	N/A	0.050	0.063	0.302	0.289	0.328	0.000	4.052	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	42	47	0	98	0	40	-1
N.S.	1	1.00	1.40	1.57	0.00	3.27	0.00	1.33	-0.03
time (sec)	N/A	0.039	0.040	0.421	0.000	0.341	0.000	5.317	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	32	29	36	28	0	65	-1
N.S.	1	1.00	0.68	0.62	0.77	0.60	0.00	1.38	-0.02
time (sec)	N/A	0.046	0.035	0.257	0.297	0.351	0.000	4.715	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	84	95	0	196	0	114	-1
N.S.	1	1.00	1.11	1.25	0.00	2.58	0.00	1.50	-0.01
time (sec)	N/A	0.061	0.096	0.543	0.000	0.367	0.000	4.216	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	20	33	46	40	50
N.S.	1	1.00	1.00	0.95	0.91	1.50	2.09	1.82	2.27
time (sec)	N/A	0.025	0.022	0.043	0.304	0.341	0.850	3.716	4.876

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	41	71	91	132	0	142	-1
N.S.	1	1.00	0.46	0.80	1.02	1.48	0.00	1.60	-0.01
time (sec)	N/A	0.056	0.041	0.309	0.536	0.377	0.000	6.218	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	224	202	0	240	0	199	-1
N.S.	1	1.00	1.67	1.51	0.00	1.79	0.00	1.49	-0.01
time (sec)	N/A	0.113	0.185	0.560	0.000	0.372	0.000	8.809	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	42	124	146	187	0	211	-1
N.S.	1	1.00	0.28	0.83	0.97	1.25	0.00	1.41	-0.01
time (sec)	N/A	0.141	0.050	0.620	0.523	0.381	0.000	6.874	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	334	289	0	270	0	220	-1
N.S.	1	1.00	1.71	1.48	0.00	1.38	0.00	1.13	-0.01
time (sec)	N/A	0.200	0.232	0.615	0.000	0.397	0.000	6.817	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	44	152	197	207	0	249	-1
N.S.	1	1.00	0.21	0.72	0.93	0.98	0.00	1.18	-0.00
time (sec)	N/A	0.227	0.056	0.850	0.548	0.394	0.000	4.238	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	256	256	444	367	0	290	0	271	-1
N.S.	1	1.00	1.73	1.43	0.00	1.13	0.00	1.06	-0.00
time (sec)	N/A	0.301	0.907	0.694	0.000	0.411	0.000	4.026	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	59	67	0	201	0	90	-1
N.S.	1	1.00	0.62	0.71	0.00	2.12	0.00	0.95	-0.01
time (sec)	N/A	0.123	0.410	0.434	0.000	0.361	0.000	3.052	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	64	67	89	82	0	134	-1
N.S.	1	1.00	0.53	0.55	0.74	0.68	0.00	1.11	-0.01
time (sec)	N/A	0.063	0.177	0.352	0.300	0.367	0.000	3.483	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	49	57	0	161	0	68	-1
N.S.	1	1.00	0.78	0.90	0.00	2.56	0.00	1.08	-0.02
time (sec)	N/A	0.083	0.215	0.549	0.000	0.358	0.000	4.451	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	54	57	72	62	0	112	-1
N.S.	1	1.00	0.56	0.59	0.74	0.64	0.00	1.15	-0.01
time (sec)	N/A	0.058	0.115	0.336	0.289	0.344	0.000	7.417	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	42	47	0	117	0	40	-1
N.S.	1	1.00	1.40	1.57	0.00	3.90	0.00	1.33	-0.03
time (sec)	N/A	0.041	0.070	0.376	0.000	0.361	0.000	4.789	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	44	41	55	40	0	88	-1
N.S.	1	1.00	0.62	0.58	0.77	0.56	0.00	1.24	-0.01
time (sec)	N/A	0.051	0.048	0.298	0.295	0.343	0.000	4.828	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	96	112	0	215	0	140	-1
N.S.	1	1.00	0.89	1.04	0.00	1.99	0.00	1.30	-0.01
time (sec)	N/A	0.101	0.123	0.447	0.000	0.356	0.000	3.512	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	30	29	42	41	267	64	-1
N.S.	1	1.00	0.67	0.64	0.93	0.91	5.93	1.42	-0.02
time (sec)	N/A	0.046	0.037	0.251	0.273	0.350	2.835	6.801	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	100	123	0	252	0	139	-1
N.S.	1	1.00	1.33	1.64	0.00	3.36	0.00	1.85	-0.01
time (sec)	N/A	0.057	0.157	0.473	0.000	0.382	0.000	6.428	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	21	20	48	65	40	72
N.S.	1	1.00	1.00	0.88	0.83	2.00	2.71	1.67	3.00
time (sec)	N/A	0.025	0.026	0.035	0.294	0.371	2.863	6.133	7.521

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	41	88	114	169	0	158	-1
N.S.	1	1.00	0.36	0.78	1.01	1.50	0.00	1.40	-0.01
time (sec)	N/A	0.063	0.059	0.355	0.511	0.357	0.000	5.393	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	284	266	0	280	0	218	-1
N.S.	1	1.00	1.70	1.59	0.00	1.68	0.00	1.31	-0.01
time (sec)	N/A	0.162	0.272	0.645	0.000	0.373	0.000	5.531	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	42	141	167	225	0	227	-1
N.S.	1	1.00	0.23	0.76	0.90	1.22	0.00	1.23	-0.01
time (sec)	N/A	0.190	0.054	0.692	0.520	0.365	0.000	4.879	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	394	355	0	308	0	255	-1
N.S.	1	1.00	1.69	1.52	0.00	1.32	0.00	1.09	-0.00
time (sec)	N/A	0.256	0.341	0.602	0.000	0.412	0.000	5.927	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	98	249	0	109	0	0	-1
N.S.	1	1.00	0.79	2.01	0.00	0.88	0.00	0.00	-0.01
time (sec)	N/A	0.070	0.397	1.692	0.000	0.104	0.000	0.000	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	264	214	0	105	0	0	-1
N.S.	1	1.00	2.78	2.25	0.00	1.11	0.00	0.00	-0.01
time (sec)	N/A	0.049	1.791	1.774	0.000	0.102	0.000	0.000	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	75	179	0	93	0	0	-1
N.S.	1	1.00	0.79	1.88	0.00	0.98	0.00	0.00	-0.01
time (sec)	N/A	0.049	0.246	1.844	0.000	0.125	0.000	0.000	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	260	120	0	77	0	0	-1
N.S.	1	1.00	4.13	1.90	0.00	1.22	0.00	0.00	-0.02
time (sec)	N/A	0.032	0.634	1.886	0.000	0.095	0.000	0.000	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	48	103	0	66	0	0	45
N.S.	1	1.00	0.79	1.69	0.00	1.08	0.00	0.00	0.74
time (sec)	N/A	0.036	14.175	0.992	0.000	0.101	0.000	0.000	0.546

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	188	117	0	165	0	0	-1
N.S.	1	1.00	2.07	1.29	0.00	1.81	0.00	0.00	-0.01
time (sec)	N/A	0.050	0.636	1.825	0.000	0.118	0.000	0.000	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	86	189	0	108	0	0	-1
N.S.	1	1.00	0.89	1.95	0.00	1.11	0.00	0.00	-0.01
time (sec)	N/A	0.052	0.293	2.636	0.000	0.108	0.000	0.000	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	144	304	0	175	0	0	-1
N.S.	1	1.00	1.14	2.41	0.00	1.39	0.00	0.00	-0.01
time (sec)	N/A	0.065	0.820	4.076	0.000	0.100	0.000	0.000	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	66	295	0	134	0	0	-1
N.S.	1	1.00	0.39	1.76	0.00	0.80	0.00	0.00	-0.01
time (sec)	N/A	0.113	0.069	2.161	0.000	0.116	0.000	0.000	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	66	260	0	131	0	0	-1
N.S.	1	1.00	0.48	1.90	0.00	0.96	0.00	0.00	-0.01
time (sec)	N/A	0.088	0.074	2.093	0.000	0.111	0.000	0.000	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	66	203	0	118	0	0	-1
N.S.	1	1.00	0.48	1.48	0.00	0.86	0.00	0.00	-0.01
time (sec)	N/A	0.087	0.053	1.743	0.000	0.113	0.000	0.000	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	66	188	0	111	0	0	-1
N.S.	1	1.00	0.63	1.79	0.00	1.06	0.00	0.00	-0.01
time (sec)	N/A	0.068	0.032	1.804	0.000	0.107	0.000	0.000	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	64	152	0	86	0	0	-1
N.S.	1	1.00	0.61	1.45	0.00	0.82	0.00	0.00	-0.01
time (sec)	N/A	0.070	0.024	1.411	0.000	0.099	0.000	0.000	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	64	120	0	184	0	0	-1
N.S.	1	1.00	0.75	1.41	0.00	2.16	0.00	0.00	-0.01
time (sec)	N/A	0.098	0.040	2.439	0.000	0.108	0.000	0.000	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	66	193	0	120	0	0	-1
N.S.	1	1.00	0.74	2.17	0.00	1.35	0.00	0.00	-0.01
time (sec)	N/A	0.096	0.044	2.738	0.000	0.106	0.000	0.000	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	66	305	0	300	0	0	-1
N.S.	1	1.00	0.52	2.40	0.00	2.36	0.00	0.00	-0.01
time (sec)	N/A	0.131	0.054	4.414	0.000	0.116	0.000	0.000	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	66	375	0	176	0	0	-1
N.S.	1	1.00	0.58	3.29	0.00	1.54	0.00	0.00	-0.01
time (sec)	N/A	0.066	0.064	4.531	0.000	0.103	0.000	0.000	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	66	488	0	250	0	0	-1
N.S.	1	1.00	0.46	3.37	0.00	1.72	0.00	0.00	-0.01
time (sec)	N/A	0.082	0.098	6.983	0.000	0.107	0.000	0.000	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	66	321	0	149	0	0	-1
N.S.	1	1.00	0.33	1.58	0.00	0.73	0.00	0.00	-0.00
time (sec)	N/A	0.153	0.046	2.402	0.000	0.131	0.000	0.000	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	66	264	0	145	0	0	-1
N.S.	1	1.00	0.39	1.55	0.00	0.85	0.00	0.00	-0.01
time (sec)	N/A	0.136	0.064	2.442	0.000	0.142	0.000	0.000	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	66	251	0	134	0	0	-1
N.S.	1	1.00	0.38	1.46	0.00	0.78	0.00	0.00	-0.01
time (sec)	N/A	0.134	0.044	2.003	0.000	0.116	0.000	0.000	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	66	214	0	126	0	0	-1
N.S.	1	1.00	0.47	1.53	0.00	0.90	0.00	0.00	-0.01
time (sec)	N/A	0.106	0.026	1.882	0.000	0.114	0.000	0.000	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	64	178	0	99	0	0	-1
N.S.	1	1.00	0.47	1.31	0.00	0.73	0.00	0.00	-0.01
time (sec)	N/A	0.107	0.018	2.016	0.000	0.107	0.000	0.000	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	64	146	0	214	0	0	-1
N.S.	1	1.00	0.60	1.38	0.00	2.02	0.00	0.00	-0.01
time (sec)	N/A	0.138	0.029	2.938	0.000	0.110	0.000	0.000	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	66	219	0	134	0	0	-1
N.S.	1	1.00	0.60	1.99	0.00	1.22	0.00	0.00	-0.01
time (sec)	N/A	0.137	0.033	3.303	0.000	0.103	0.000	0.000	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	66	332	0	305	0	0	-1
N.S.	1	1.00	0.52	2.61	0.00	2.40	0.00	0.00	-0.01
time (sec)	N/A	0.142	0.044	4.990	0.000	0.098	0.000	0.000	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	66	401	0	176	0	0	-1
N.S.	1	1.00	0.52	3.16	0.00	1.39	0.00	0.00	-0.01
time (sec)	N/A	0.140	0.049	5.428	0.000	0.112	0.000	0.000	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	66	514	0	422	0	0	-1
N.S.	1	1.00	0.40	3.12	0.00	2.56	0.00	0.00	-0.01
time (sec)	N/A	0.173	0.107	7.637	0.000	0.122	0.000	0.000	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	66	295	0	149	0	0	-1
N.S.	1	1.00	0.31	1.40	0.00	0.71	0.00	0.00	-0.00
time (sec)	N/A	0.169	0.096	2.146	0.000	0.127	0.000	0.000	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	66	258	0	143	0	0	-1
N.S.	1	1.00	0.37	1.45	0.00	0.80	0.00	0.00	-0.01
time (sec)	N/A	0.138	0.057	2.105	0.000	0.106	0.000	0.000	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	64	222	0	115	0	0	-1
N.S.	1	1.00	0.36	1.25	0.00	0.65	0.00	0.00	-0.01
time (sec)	N/A	0.140	0.049	2.423	0.000	0.101	0.000	0.000	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	64	190	0	241	0	0	-1
N.S.	1	1.00	0.41	1.22	0.00	1.54	0.00	0.00	-0.01
time (sec)	N/A	0.157	0.070	3.057	0.000	0.130	0.000	0.000	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	66	263	0	146	0	0	-1
N.S.	1	1.00	0.43	1.73	0.00	0.96	0.00	0.00	-0.01
time (sec)	N/A	0.153	0.061	3.556	0.000	0.117	0.000	0.000	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	66	332	0	304	0	0	-1
N.S.	1	1.00	0.52	2.61	0.00	2.39	0.00	0.00	-0.01
time (sec)	N/A	0.141	0.083	5.535	0.000	0.120	0.000	0.000	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	66	401	0	179	0	0	-1
N.S.	1	1.00	0.52	3.16	0.00	1.41	0.00	0.00	-0.01
time (sec)	N/A	0.136	0.092	6.106	0.000	0.110	0.000	0.000	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	66	514	0	421	0	0	-1
N.S.	1	1.00	0.39	3.04	0.00	2.49	0.00	0.00	-0.01
time (sec)	N/A	0.177	0.128	7.484	0.000	0.124	0.000	0.000	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	66	583	0	241	0	0	-1
N.S.	1	1.00	0.39	3.45	0.00	1.43	0.00	0.00	-0.01
time (sec)	N/A	0.179	0.210	8.453	0.000	0.119	0.000	0.000	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	66	251	0	107	0	0	-1
N.S.	1	1.00	0.50	1.90	0.00	0.81	0.00	0.00	-0.01
time (sec)	N/A	0.078	0.145	2.351	0.000	0.106	0.000	0.000	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	66	216	0	104	0	0	-1
N.S.	1	1.00	0.65	2.14	0.00	1.03	0.00	0.00	-0.01
time (sec)	N/A	0.065	0.106	2.485	0.000	0.121	0.000	0.000	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	66	181	0	92	0	0	-1
N.S.	1	1.00	0.65	1.79	0.00	0.91	0.00	0.00	-0.01
time (sec)	N/A	0.066	0.079	2.115	0.000	0.123	0.000	0.000	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	66	122	0	77	0	0	-1
N.S.	1	1.00	0.97	1.79	0.00	1.13	0.00	0.00	-0.01
time (sec)	N/A	0.052	0.098	2.055	0.000	0.109	0.000	0.000	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	66	110	0	70	0	0	-1
N.S.	1	1.00	1.00	1.67	0.00	1.06	0.00	0.00	-0.02
time (sec)	N/A	0.053	0.074	1.528	0.000	0.106	0.000	0.000	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	66	115	0	170	0	0	-1
N.S.	1	1.00	0.89	1.55	0.00	2.30	0.00	0.00	-0.01
time (sec)	N/A	0.048	0.043	3.061	0.000	0.094	0.000	0.000	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	64	190	0	104	0	0	-1
N.S.	1	1.00	0.82	2.44	0.00	1.33	0.00	0.00	-0.01
time (sec)	N/A	0.056	0.039	3.864	0.000	0.094	0.000	0.000	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	63	304	0	168	0	0	-1
N.S.	1	1.00	0.56	2.71	0.00	1.50	0.00	0.00	-0.01
time (sec)	N/A	0.077	0.063	5.842	0.000	0.117	0.000	0.000	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	66	375	0	174	0	0	-1
N.S.	1	1.00	0.59	3.35	0.00	1.55	0.00	0.00	-0.01
time (sec)	N/A	0.071	0.072	6.177	0.000	0.120	0.000	0.000	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	66	488	0	202	0	0	-1
N.S.	1	1.00	0.46	3.41	0.00	1.41	0.00	0.00	-0.01
time (sec)	N/A	0.089	0.085	8.595	0.000	0.107	0.000	0.000	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	66	203	0	106	0	0	-1
N.S.	1	1.00	0.46	1.40	0.00	0.73	0.00	0.00	-0.01
time (sec)	N/A	0.079	0.137	2.429	0.000	0.100	0.000	0.000	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	66	190	0	102	0	0	-1
N.S.	1	1.00	0.58	1.67	0.00	0.89	0.00	0.00	-0.01
time (sec)	N/A	0.068	0.092	2.825	0.000	0.102	0.000	0.000	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	66	155	0	83	0	0	-1
N.S.	1	1.00	0.59	1.38	0.00	0.74	0.00	0.00	-0.01
time (sec)	N/A	0.067	0.068	1.939	0.000	0.092	0.000	0.000	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	66	120	0	179	0	0	-1
N.S.	1	1.00	0.84	1.52	0.00	2.27	0.00	0.00	-0.01
time (sec)	N/A	0.055	0.081	3.325	0.000	0.098	0.000	0.000	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	66	193	0	114	0	0	-1
N.S.	1	1.00	0.80	2.33	0.00	1.37	0.00	0.00	-0.01
time (sec)	N/A	0.055	0.076	4.037	0.000	0.107	0.000	0.000	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	66	303	0	290	0	0	-1
N.S.	1	1.00	0.57	2.61	0.00	2.50	0.00	0.00	-0.01
time (sec)	N/A	0.086	0.041	5.882	0.000	0.109	0.000	0.000	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	64	372	0	159	0	0	-1
N.S.	1	1.00	0.55	3.21	0.00	1.37	0.00	0.00	-0.01
time (sec)	N/A	0.094	0.042	6.602	0.000	0.122	0.000	0.000	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	66	488	0	226	0	0	-1
N.S.	1	1.00	0.44	3.25	0.00	1.51	0.00	0.00	-0.01
time (sec)	N/A	0.116	0.062	8.802	0.000	0.105	0.000	0.000	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	66	557	0	232	0	0	-1
N.S.	1	1.00	0.44	3.71	0.00	1.55	0.00	0.00	-0.01
time (sec)	N/A	0.111	0.078	10.106	0.000	0.132	0.000	0.000	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	66	670	0	258	0	0	-1
N.S.	1	1.00	0.36	3.70	0.00	1.43	0.00	0.00	-0.01
time (sec)	N/A	0.134	0.108	12.477	0.000	0.135	0.000	0.000	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	66	251	0	119	0	0	-1
N.S.	1	1.00	0.39	1.49	0.00	0.70	0.00	0.00	-0.01
time (sec)	N/A	0.123	0.374	2.785	0.000	0.111	0.000	0.000	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	66	216	0	114	0	0	-1
N.S.	1	1.00	0.48	1.57	0.00	0.83	0.00	0.00	-0.01
time (sec)	N/A	0.110	0.217	2.806	0.000	0.110	0.000	0.000	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	66	181	0	95	0	0	-1
N.S.	1	1.00	0.50	1.37	0.00	0.72	0.00	0.00	-0.01
time (sec)	N/A	0.108	0.141	2.616	0.000	0.103	0.000	0.000	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	66	146	0	204	0	0	-1
N.S.	1	1.00	0.64	1.42	0.00	1.98	0.00	0.00	-0.01
time (sec)	N/A	0.098	0.112	3.670	0.000	0.109	0.000	0.000	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	66	219	0	129	0	0	-1
N.S.	1	1.00	0.62	2.05	0.00	1.21	0.00	0.00	-0.01
time (sec)	N/A	0.094	0.078	4.316	0.000	0.113	0.000	0.000	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	66	330	0	295	0	0	-1
N.S.	1	1.00	0.56	2.80	0.00	2.50	0.00	0.00	-0.01
time (sec)	N/A	0.090	0.073	6.092	0.000	0.116	0.000	0.000	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	66	401	0	171	0	0	-1
N.S.	1	1.00	0.56	3.40	0.00	1.45	0.00	0.00	-0.01
time (sec)	N/A	0.094	0.070	6.816	0.000	0.098	0.000	0.000	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	66	512	0	408	0	0	-1
N.S.	1	1.00	0.43	3.35	0.00	2.67	0.00	0.00	-0.01
time (sec)	N/A	0.122	0.045	9.177	0.000	0.112	0.000	0.000	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	66	580	0	221	0	0	-1
N.S.	1	1.00	0.43	3.79	0.00	1.44	0.00	0.00	-0.01
time (sec)	N/A	0.126	0.052	10.258	0.000	0.115	0.000	0.000	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	66	696	0	283	0	0	-1
N.S.	1	1.00	0.35	3.72	0.00	1.51	0.00	0.00	-0.01
time (sec)	N/A	0.155	0.067	12.740	0.000	0.114	0.000	0.000	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	66	225	0	110	0	0	-1
N.S.	1	1.00	0.37	1.25	0.00	0.61	0.00	0.00	-0.01
time (sec)	N/A	0.128	0.301	2.555	0.000	0.109	0.000	0.000	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	66	190	0	231	0	0	-1
N.S.	1	1.00	0.44	1.28	0.00	1.55	0.00	0.00	-0.01
time (sec)	N/A	0.111	0.198	3.493	0.000	0.114	0.000	0.000	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	66	263	0	140	0	0	-1
N.S.	1	1.00	0.46	1.81	0.00	0.97	0.00	0.00	-0.01
time (sec)	N/A	0.114	0.133	4.632	0.000	0.110	0.000	0.000	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	66	332	0	294	0	0	-1
N.S.	1	1.00	0.55	2.77	0.00	2.45	0.00	0.00	-0.01
time (sec)	N/A	0.097	0.095	6.735	0.000	0.102	0.000	0.000	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	66	401	0	172	0	0	-1
N.S.	1	1.00	0.55	3.34	0.00	1.43	0.00	0.00	-0.01
time (sec)	N/A	0.097	0.076	7.545	0.000	0.115	0.000	0.000	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	66	514	0	407	0	0	-1
N.S.	1	1.00	0.43	3.34	0.00	2.64	0.00	0.00	-0.01
time (sec)	N/A	0.131	0.086	10.381	0.000	0.118	0.000	0.000	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	66	583	0	233	0	0	-1
N.S.	1	1.00	0.43	3.79	0.00	1.51	0.00	0.00	-0.01
time (sec)	N/A	0.131	0.076	11.227	0.000	0.100	0.000	0.000	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	66	694	0	520	0	0	-1
N.S.	1	1.00	0.35	3.63	0.00	2.72	0.00	0.00	-0.01
time (sec)	N/A	0.168	0.050	14.624	0.000	0.118	0.000	0.000	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	66	762	0	276	0	0	-1
N.S.	1	1.00	0.35	3.99	0.00	1.45	0.00	0.00	-0.01
time (sec)	N/A	0.170	0.062	15.400	0.000	0.121	0.000	0.000	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	66	878	0	338	0	0	-1
N.S.	1	1.00	0.29	3.90	0.00	1.50	0.00	0.00	-0.00
time (sec)	N/A	0.208	0.092	19.171	0.000	0.147	0.000	0.000	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	269	241	0	3208	0	0	-1
N.S.	1	1.00	1.14	1.02	0.00	13.59	0.00	0.00	-0.00
time (sec)	N/A	0.246	0.994	6.412	0.000	191.682	0.000	0.000	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	195	213	0	3259	0	0	-1
N.S.	1	1.00	1.01	1.10	0.00	16.80	0.00	0.00	-0.01
time (sec)	N/A	0.183	0.739	0.170	0.000	199.179	0.000	0.000	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	108	142	0	3105	0	0	-1
N.S.	1	1.00	0.67	0.88	0.00	19.29	0.00	0.00	-0.01
time (sec)	N/A	0.137	0.478	0.159	0.000	178.640	0.000	0.000	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	34	77	27	0	0	30
N.S.	1	1.00	1.00	1.00	2.26	0.79	0.00	0.00	0.88
time (sec)	N/A	0.044	0.120	0.178	0.553	0.346	0.000	0.000	5.311

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	46	44	187	37	0	0	61
N.S.	1	1.00	0.62	0.59	2.53	0.50	0.00	0.00	0.82
time (sec)	N/A	0.096	0.274	0.184	0.555	0.355	0.000	0.000	5.666

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	56	54	255	47	0	0	97
N.S.	1	1.00	0.49	0.47	2.22	0.41	0.00	0.00	0.84
time (sec)	N/A	0.153	0.362	0.181	0.550	0.344	0.000	0.000	6.060

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	74	70	321	59	0	0	129
N.S.	1	1.00	0.48	0.45	2.08	0.38	0.00	0.00	0.84
time (sec)	N/A	0.204	0.846	0.194	0.553	0.365	0.000	0.000	7.243

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	319	319	78	314	0	3472	0	0	-1
N.S.	1	1.00	0.24	0.98	0.00	10.88	0.00	0.00	-0.00
time (sec)	N/A	0.379	0.178	0.581	0.000	200.531	0.000	0.000	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	278	78	288	0	3365	0	0	-1
N.S.	1	1.00	0.28	1.04	0.00	12.10	0.00	0.00	-0.00
time (sec)	N/A	0.300	0.140	0.210	0.000	204.389	0.000	0.000	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	77	262	0	3443	0	0	-1
N.S.	1	1.00	0.32	1.08	0.00	14.17	0.00	0.00	-0.00
time (sec)	N/A	0.237	0.122	0.198	0.000	195.043	0.000	0.000	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	75	228	0	3274	0	0	-1
N.S.	1	1.00	0.38	1.15	0.00	16.54	0.00	0.00	-0.01
time (sec)	N/A	0.186	0.108	0.180	0.000	183.860	0.000	0.000	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	75	323	0	3313	0	0	-1
N.S.	1	1.00	0.36	1.54	0.00	15.78	0.00	0.00	-0.00
time (sec)	N/A	0.194	0.119	0.170	0.000	176.021	0.000	0.000	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	34	77	41	0	0	47
N.S.	1	1.00	1.00	0.94	2.14	1.14	0.00	0.00	1.31
time (sec)	N/A	0.047	0.103	0.143	0.535	0.364	0.000	0.000	5.622

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	72	44	189	65	0	0	71
N.S.	1	1.00	0.97	0.59	2.55	0.88	0.00	0.00	0.96
time (sec)	N/A	0.094	0.128	0.151	0.545	0.355	0.000	0.000	5.985

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	105	54	253	80	0	0	116
N.S.	1	1.00	0.93	0.48	2.24	0.71	0.00	0.00	1.03
time (sec)	N/A	0.152	0.199	0.155	0.576	0.351	0.000	0.000	6.810

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	74	70	321	94	0	0	261
N.S.	1	1.00	0.49	0.46	2.11	0.62	0.00	0.00	1.72
time (sec)	N/A	0.200	0.252	0.168	0.557	0.362	0.000	0.000	10.956

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	323	323	77	344	0	3429	0	0	-1
N.S.	1	1.00	0.24	1.07	0.00	10.62	0.00	0.00	-0.00
time (sec)	N/A	0.357	0.292	0.263	0.000	196.670	0.000	0.000	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	286	286	78	318	0	3502	0	0	-1
N.S.	1	1.00	0.27	1.11	0.00	12.24	0.00	0.00	-0.00
time (sec)	N/A	0.293	0.122	0.207	0.000	194.065	0.000	0.000	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	76	283	0	3330	0	0	-1
N.S.	1	1.00	0.31	1.15	0.00	13.48	0.00	0.00	-0.00
time (sec)	N/A	0.249	0.094	0.227	0.000	190.060	0.000	0.000	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	75	443	0	3371	0	0	-1
N.S.	1	1.00	0.31	1.85	0.00	14.10	0.00	0.00	-0.00
time (sec)	N/A	0.240	0.148	0.159	0.000	179.785	0.000	0.000	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	77	545	0	3497	0	0	-1
N.S.	1	1.00	0.38	2.67	0.00	17.14	0.00	0.00	-0.00
time (sec)	N/A	0.198	0.191	0.168	0.000	178.442	0.000	0.000	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-2)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	34	77	100	0	0	65
N.S.	1	1.00	1.00	0.94	2.14	2.78	0.00	0.00	1.81
time (sec)	N/A	0.048	0.131	0.163	0.544	0.352	0.000	0.000	6.058

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	54	44	189	70	0	0	96
N.S.	1	1.00	0.71	0.58	2.49	0.92	0.00	0.00	1.26
time (sec)	N/A	0.099	0.159	0.179	0.549	0.348	0.000	0.000	6.336

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	64	54	255	95	0	0	119
N.S.	1	1.00	0.57	0.48	2.26	0.84	0.00	0.00	1.05
time (sec)	N/A	0.154	0.190	0.158	0.543	0.351	0.000	0.000	6.694

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	74	70	321	115	0	0	232
N.S.	1	1.00	0.49	0.47	2.14	0.77	0.00	0.00	1.55
time (sec)	N/A	0.203	0.250	0.162	0.547	0.372	0.000	0.000	11.173

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	77	239	0	3304	0	0	-1
N.S.	1	1.00	0.32	0.98	0.00	13.54	0.00	0.00	-0.00
time (sec)	N/A	0.246	0.171	0.163	0.000	189.469	0.000	0.000	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	77	212	0	3207	0	0	-1
N.S.	1	1.00	0.38	1.06	0.00	16.04	0.00	0.00	-0.00
time (sec)	N/A	0.195	0.118	0.157	0.000	186.729	0.000	0.000	0.000

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	77	142	0	3171	0	0	-1
N.S.	1	1.00	0.46	0.84	0.00	18.76	0.00	0.00	-0.01
time (sec)	N/A	0.137	0.075	0.160	0.000	183.835	0.000	0.000	0.000

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	34	122	41	0	0	46
N.S.	1	1.00	1.00	1.00	3.59	1.21	0.00	0.00	1.35
time (sec)	N/A	0.044	0.064	0.337	0.560	0.353	0.000	0.000	5.662

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	46	44	189	63	0	0	77
N.S.	1	1.00	0.61	0.58	2.49	0.83	0.00	0.00	1.01
time (sec)	N/A	0.098	0.109	0.142	0.541	0.328	0.000	0.000	6.007

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	56	54	258	77	0	0	120
N.S.	1	1.00	0.49	0.47	2.24	0.67	0.00	0.00	1.04
time (sec)	N/A	0.144	0.107	0.154	0.566	0.325	0.000	0.000	6.680

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	66	70	325	89	0	0	261
N.S.	1	1.00	0.43	0.45	2.11	0.58	0.00	0.00	1.69
time (sec)	N/A	0.203	0.171	0.158	0.590	0.334	0.000	0.000	11.008

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	80	266	0	3361	0	0	-1
N.S.	1	1.00	0.32	1.08	0.00	13.61	0.00	0.00	-0.00
time (sec)	N/A	0.253	0.138	0.226	0.000	190.259	0.000	0.000	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	80	232	0	3431	0	0	-1
N.S.	1	1.00	0.37	1.08	0.00	15.96	0.00	0.00	-0.00
time (sec)	N/A	0.188	0.155	0.155	0.000	191.097	0.000	0.000	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	80	321	0	3519	0	0	-1
N.S.	1	1.00	0.34	1.36	0.00	14.91	0.00	0.00	-0.00
time (sec)	N/A	0.242	0.126	0.155	0.000	188.741	0.000	0.000	0.000

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	49	34	126	104	0	0	82
N.S.	1	1.00	1.36	0.94	3.50	2.89	0.00	0.00	2.28
time (sec)	N/A	0.046	0.072	0.145	0.539	0.341	0.000	0.000	5.992

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	59	44	197	72	0	0	95
N.S.	1	1.00	0.78	0.58	2.59	0.95	0.00	0.00	1.25
time (sec)	N/A	0.091	0.114	0.142	0.563	0.346	0.000	0.000	6.379

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	56	54	264	94	0	0	119
N.S.	1	1.00	0.49	0.47	2.30	0.82	0.00	0.00	1.03
time (sec)	N/A	0.149	0.105	0.145	0.550	0.351	0.000	0.000	6.823

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	66	70	335	110	0	0	230
N.S.	1	1.00	0.43	0.45	2.18	0.71	0.00	0.00	1.49
time (sec)	N/A	0.200	0.176	0.155	0.549	0.356	0.000	0.000	11.102

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	76	80	404	120	0	0	413
N.S.	1	1.00	0.39	0.41	2.09	0.62	0.00	0.00	2.14
time (sec)	N/A	0.248	0.283	0.188	0.547	0.353	0.000	0.000	11.651

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	80	282	0	3456	0	0	-1
N.S.	1	1.00	0.31	1.08	0.00	13.24	0.00	0.00	-0.00
time (sec)	N/A	0.303	0.203	0.244	0.000	194.478	0.000	0.000	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	80	443	0	3536	0	0	-1
N.S.	1	1.00	0.33	1.85	0.00	14.79	0.00	0.00	-0.00
time (sec)	N/A	0.247	0.122	0.144	0.000	195.179	0.000	0.000	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	80	545	0	3631	0	0	-1
N.S.	1	1.00	0.37	2.50	0.00	16.66	0.00	0.00	-0.00
time (sec)	N/A	0.197	0.136	0.152	0.000	187.758	0.000	0.000	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	49	34	126	70	0	0	102
N.S.	1	1.00	1.36	0.94	3.50	1.94	0.00	0.00	2.83
time (sec)	N/A	0.049	0.111	0.156	0.550	0.358	0.000	0.000	6.567

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	59	44	197	158	0	0	145
N.S.	1	1.00	0.78	0.58	2.59	2.08	0.00	0.00	1.91
time (sec)	N/A	0.092	0.102	0.183	0.542	0.353	0.000	0.000	7.047

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	69	54	266	100	0	0	137
N.S.	1	1.00	0.60	0.47	2.31	0.87	0.00	0.00	1.19
time (sec)	N/A	0.137	0.151	0.166	0.548	0.362	0.000	0.000	7.662

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	66	70	335	124	0	0	261
N.S.	1	1.00	0.43	0.45	2.18	0.81	0.00	0.00	1.69
time (sec)	N/A	0.200	0.145	0.153	0.557	0.361	0.000	0.000	11.167

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	76	80	404	138	0	0	379
N.S.	1	1.00	0.39	0.41	2.09	0.72	0.00	0.00	1.96
time (sec)	N/A	0.258	0.275	0.155	0.554	0.361	0.000	0.000	11.537

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	77	0	0	0	0	0	-1
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.070	0.158	0.099	0.000	0.000	0.000	0.000	0.000

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	77	0	0	0	0	0	-1
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.072	0.120	0.093	0.000	0.000	0.000	0.000	0.000

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	77	0	0	0	0	0	-1
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.070	0.086	0.089	0.000	0.000	0.000	0.000	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	77	0	0	0	0	0	-1
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.062	0.086	0.073	0.000	0.000	0.000	0.000	0.000

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	77	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.061	0.088	0.078	0.000	0.000	0.000	0.000	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	75	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.071	0.098	0.077	0.000	0.000	0.000	0.000	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	94	0	0	0	0	0	-1
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.058	0.202	0.459	0.000	0.000	0.000	0.000	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	94	0	0	0	0	0	-1
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.055	0.107	0.557	0.000	0.000	0.000	0.000	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	94	0	0	0	0	0	-1
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.055	0.108	0.536	0.000	0.000	0.000	0.000	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	266	0	0	0	0	0	-1
N.S.	1	1.00	2.86	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.041	1.538	0.230	0.000	0.000	0.000	0.000	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	94	0	0	0	0	0	-1
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.070	0.164	0.243	0.000	0.000	0.000	0.000	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	94	0	0	0	0	0	-1
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.065	0.175	0.604	0.000	0.000	0.000	0.000	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	94	0	0	0	0	0	-1
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.065	0.170	0.737	0.000	0.000	0.000	0.000	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	94	0	0	0	0	0	-1
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.064	0.195	2.879	0.000	0.000	0.000	0.000	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	102	0	0	0	0	0	-1
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.079	0.253	0.127	0.000	0.000	0.000	0.000	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	102	0	0	0	0	0	-1
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.077	0.199	0.128	0.000	0.000	0.000	0.000	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	101	0	0	0	0	0	-1
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.080	0.203	0.131	0.000	0.000	0.000	0.000	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	310	0	0	0	0	0	-1
N.S.	1	1.00	3.20	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.070	3.943	0.147	0.000	0.000	0.000	0.000	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	97	0	0	0	0	0	-1
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.073	0.109	0.105	0.000	0.000	0.000	0.000	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	101	0	0	0	0	0	-1
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.081	0.173	0.112	0.000	0.000	0.000	0.000	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	102	0	0	0	0	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.085	0.133	0.113	0.000	0.000	0.000	0.000	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	112	0	0	0	0	0	-1
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.081	0.190	0.128	0.000	0.000	0.000	0.000	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	B	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	796	0	520	153	0	517	555
N.S.	1	1.00	7.30	0.00	4.77	1.40	0.00	4.74	5.09
time (sec)	N/A	0.061	6.144	0.299	0.310	0.390	0.000	5.115	10.480

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	100	0	266	102	5534	294	195
N.S.	1	1.00	1.23	0.00	3.28	1.26	68.32	3.63	2.41
time (sec)	N/A	0.047	1.481	0.296	0.282	0.375	42.006	4.378	1.971

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	65	0	111	61	1114	137	85
N.S.	1	1.00	1.18	0.00	2.02	1.11	20.25	2.49	1.55
time (sec)	N/A	0.040	0.205	0.129	0.281	0.388	3.809	5.651	0.726

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	27	26	28	80	26	29
N.S.	1	1.00	1.00	1.04	1.00	1.08	3.08	1.00	1.12
time (sec)	N/A	0.021	0.045	3.103	0.276	0.357	0.458	5.616	0.215

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	63	0	0	0	0	0	-1
N.S.	1	1.00	1.58	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.034	0.099	0.165	0.000	0.000	0.000	0.000	0.000

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	10814	0	0	0	0	0	-1
N.S.	1	1.00	230.09	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.039	37.798	0.121	0.000	0.000	0.000	0.000	0.000

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	26029	0	0	0	0	0	-1
N.S.	1	1.00	510.37	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.040	47.338	0.136	0.000	0.000	0.000	0.000	0.000

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	78	0	0	0	0	0	-1
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.053	0.122	0.261	0.000	0.000	0.000	0.000	0.000

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	78	0	0	0	0	0	-1
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.052	0.104	0.192	0.000	0.000	0.000	0.000	0.000

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	3917	0	0	0	0	0	-1
N.S.	1	1.00	53.66	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.058	6.372	0.098	0.000	0.000	0.000	0.000	0.000

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	9400	0	0	0	0	0	-1
N.S.	1	1.00	113.25	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.058	6.655	0.116	0.000	0.000	0.000	0.000	0.000

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	85	0	0	0	0	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.064	0.198	0.161	0.000	0.000	0.000	0.000	0.000

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	85	0	0	0	0	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.063	0.140	0.141	0.000	0.000	0.000	0.000	0.000

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	85	0	0	0	0	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.061	0.093	0.156	0.000	0.000	0.000	0.000	0.000

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	83	0	0	0	0	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.061	0.086	0.125	0.000	0.000	0.000	0.000	0.000

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	82	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.064	0.112	0.128	0.000	0.000	0.000	0.000	0.000

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	85	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.066	0.125	0.131	0.000	0.000	0.000	0.000	0.000

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	101	0	0	105	0	0	137
N.S.	1	1.00	0.50	0.00	0.00	0.52	0.00	0.00	0.68
time (sec)	N/A	0.220	0.203	0.223	0.000	0.364	0.000	0.000	6.828

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	76	0	0	76	0	0	103
N.S.	1	1.00	0.54	0.00	0.00	0.54	0.00	0.00	0.73
time (sec)	N/A	0.147	0.138	0.217	0.000	0.390	0.000	0.000	6.126

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	53	0	0	62	0	0	71
N.S.	1	1.00	0.60	0.00	0.00	0.70	0.00	0.00	0.80
time (sec)	N/A	0.086	0.118	0.188	0.000	0.347	0.000	0.000	5.605

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	0	62	40	0	0	34
N.S.	1	1.00	1.00	0.00	1.82	1.18	0.00	0.00	1.00
time (sec)	N/A	0.034	0.058	0.204	0.493	0.369	0.000	0.000	0.288

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	108	0	0	0	0	0	-1
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.078	0.134	0.129	0.000	0.000	0.000	0.000	0.000

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	97	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.073	0.267	0.201	0.000	0.000	0.000	0.000	0.000

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	113	0	0	0	0	0	-1
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.081	0.252	0.201	0.000	0.000	0.000	0.000	0.000

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	105	0	689	315	0	0	601
N.S.	1	1.00	0.70	0.00	4.59	2.10	0.00	0.00	4.01
time (sec)	N/A	0.166	0.404	0.206	0.550	0.415	0.000	0.000	13.477

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	72	0	378	171	0	0	241
N.S.	1	1.00	0.77	0.00	4.02	1.82	0.00	0.00	2.56
time (sec)	N/A	0.096	0.220	0.213	0.512	0.361	0.000	0.000	8.768

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	43	0	155	81	0	0	58
N.S.	1	1.00	0.98	0.00	3.52	1.84	0.00	0.00	1.32
time (sec)	N/A	0.037	0.165	0.186	0.532	0.352	0.000	0.000	5.582

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	61	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.047	0.071	0.171	0.000	0.000	0.000	0.000	0.000

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	76	0	0	0	0	0	-1
N.S.	1	1.00	1.09	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.054	0.146	0.177	0.000	0.000	0.000	0.000	0.000

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	96	0	0	0	0	0	-1
N.S.	1	1.00	1.08	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.068	0.222	0.177	0.000	0.000	0.000	0.000	0.000

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	96	0	0	0	0	0	-1
N.S.	1	1.00	1.08	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.066	0.185	0.190	0.000	0.000	0.000	0.000	0.000

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	90	0	0	0	0	0	-1
N.S.	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.060	0.092	0.157	0.000	0.000	0.000	0.000	0.000

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	87	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.068	0.219	0.165	0.000	0.000	0.000	0.000	0.000

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	63	46	70	51	83	88	68
N.S.	1	1.00	1.05	0.77	1.17	0.85	1.38	1.47	1.13
time (sec)	N/A	0.030	0.024	0.279	0.295	0.341	0.401	5.951	0.065

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	36	48	39	60	48	46
N.S.	1	1.00	1.00	0.82	1.09	0.89	1.36	1.09	1.05
time (sec)	N/A	0.022	0.017	0.191	0.280	0.383	0.171	6.138	0.057

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	28	39	25	20	25	34	25	23
N.S.	1	1.27	1.77	1.14	0.91	1.14	1.55	1.14	1.05
time (sec)	N/A	0.011	0.017	0.100	0.290	0.339	0.070	6.456	0.042

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	26	32	35	37	0	37	54
N.S.	1	1.00	0.60	0.74	0.81	0.86	0.00	0.86	1.26
time (sec)	N/A	0.028	0.019	0.146	0.286	0.354	0.000	4.036	0.072

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	52	50	53	67	0	55	44
N.S.	1	1.00	1.27	1.22	1.29	1.63	0.00	1.34	1.07
time (sec)	N/A	0.027	0.029	0.220	0.364	0.363	0.000	3.614	0.075

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	68	63	78	82	0	70	64
N.S.	1	1.00	1.11	1.03	1.28	1.34	0.00	1.15	1.05
time (sec)	N/A	0.032	0.173	0.273	0.299	0.352	0.000	6.631	5.145

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	62	52	48	51	124	77	111
N.S.	1	1.00	0.95	0.80	0.74	0.78	1.91	1.18	1.71
time (sec)	N/A	0.033	0.118	0.219	0.289	0.343	0.259	6.651	8.694

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	46	41	37	37	71	47	68
N.S.	1	1.00	1.07	0.95	0.86	0.86	1.65	1.09	1.58
time (sec)	N/A	0.025	0.066	0.145	0.306	0.352	0.110	4.724	7.381

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	24	23	22	0	33	22
N.S.	1	1.00	1.00	1.04	1.00	0.96	0.00	1.43	0.96
time (sec)	N/A	0.023	0.017	0.148	0.318	0.327	0.000	4.049	5.132

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	41	38	35	35	0	76	42
N.S.	1	1.00	0.93	0.86	0.80	0.80	0.00	1.73	0.95
time (sec)	N/A	0.026	0.094	0.200	0.286	0.352	0.000	4.570	5.265

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	53	48	48	50	0	120	75
N.S.	1	1.00	0.88	0.80	0.80	0.83	0.00	2.00	1.25
time (sec)	N/A	0.030	0.186	0.260	0.306	0.341	0.000	6.154	5.299

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	104	98	106	87	158	136	104
N.S.	1	1.00	1.05	0.99	1.07	0.88	1.60	1.37	1.05
time (sec)	N/A	0.063	0.205	0.455	0.286	0.380	0.604	6.358	0.074

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	56	78	73	69	107	80	74
N.S.	1	1.00	0.73	1.01	0.95	0.90	1.39	1.04	0.96
time (sec)	N/A	0.049	0.132	0.348	0.279	0.348	0.266	4.482	0.050

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	46	21	20	48	53	20	39
N.S.	1	1.00	2.09	0.95	0.91	2.18	2.41	0.91	1.77
time (sec)	N/A	0.017	0.025	0.204	0.296	0.351	0.112	4.314	0.058

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	54	62	60	62	0	62	50
N.S.	1	1.00	0.89	1.02	0.98	1.02	0.00	1.02	0.82
time (sec)	N/A	0.056	0.065	0.282	0.281	0.407	0.000	4.143	5.156

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	113	99	78	90	0	86	62
N.S.	1	1.00	1.92	1.68	1.32	1.53	0.00	1.46	1.05
time (sec)	N/A	0.042	1.007	0.342	0.306	0.365	0.000	4.406	0.107

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	166	131	115	118	0	118	93
N.S.	1	1.00	1.68	1.32	1.16	1.19	0.00	1.19	0.94
time (sec)	N/A	0.061	0.785	0.422	0.288	0.361	0.000	7.979	5.098

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	141	128	114	108	398	162	178
N.S.	1	1.00	0.97	0.88	0.78	0.74	2.73	1.11	1.22
time (sec)	N/A	0.089	0.370	0.591	0.292	0.373	0.934	7.267	5.466

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	133	108	88	89	287	123	134
N.S.	1	1.00	1.15	0.93	0.76	0.77	2.47	1.06	1.16
time (sec)	N/A	0.076	0.208	0.424	0.295	0.361	0.440	4.977	5.307

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	85	86	64	70	180	76	71
N.S.	1	1.00	0.99	1.00	0.74	0.81	2.09	0.88	0.83
time (sec)	N/A	0.064	0.252	0.289	0.284	0.360	0.190	3.812	5.380

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	55	46	46	45	0	63	53
N.S.	1	1.00	1.12	0.94	0.94	0.92	0.00	1.29	1.08
time (sec)	N/A	0.033	0.063	0.207	0.507	0.331	0.000	5.140	5.207

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	105	62	51	52	0	102	71
N.S.	1	1.00	1.40	0.83	0.68	0.69	0.00	1.36	0.95
time (sec)	N/A	0.065	0.343	0.343	0.279	0.324	0.000	4.613	5.258

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	84	92	76	77	0	181	103
N.S.	1	1.00	0.82	0.89	0.74	0.75	0.00	1.76	1.00
time (sec)	N/A	0.071	0.456	0.406	0.276	0.352	0.000	5.833	5.364

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	110	120	97	99	0	260	135
N.S.	1	1.00	0.85	0.93	0.75	0.77	0.00	2.02	1.05
time (sec)	N/A	0.078	0.837	0.385	0.281	0.331	0.000	4.875	5.547

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	120	135	144	117	202	185	141
N.S.	1	1.00	0.83	0.94	1.00	0.81	1.40	1.28	0.98
time (sec)	N/A	0.091	0.535	0.661	0.272	0.345	0.898	3.659	0.092

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	56	115	100	95	151	112	98
N.S.	1	1.00	0.73	1.49	1.30	1.23	1.96	1.45	1.27
time (sec)	N/A	0.052	0.152	0.460	0.279	0.370	0.411	4.276	5.128

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	67	21	20	71	73	20	55
N.S.	1	1.00	3.05	0.95	0.91	3.23	3.32	0.91	2.50
time (sec)	N/A	0.017	0.029	0.234	0.271	0.323	0.175	6.925	0.060

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	67	90	91	93	0	93	65
N.S.	1	1.00	0.84	1.12	1.14	1.16	0.00	1.16	0.81
time (sec)	N/A	0.069	0.124	0.323	0.277	0.368	0.000	5.419	5.129

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	176	126	98	112	0	114	99
N.S.	1	1.00	1.59	1.14	0.88	1.01	0.00	1.03	0.89
time (sec)	N/A	0.089	1.377	0.399	0.281	0.369	0.000	5.187	5.211

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	318	156	136	138	0	139	114
N.S.	1	1.00	3.38	1.66	1.45	1.47	0.00	1.48	1.21
time (sec)	N/A	0.055	3.944	0.481	0.288	0.345	0.000	3.286	5.166

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	182	145	117	117	348	173	474
N.S.	1	1.00	1.15	0.92	0.74	0.74	2.20	1.09	3.00
time (sec)	N/A	0.152	0.396	0.517	0.278	0.393	0.647	4.610	6.926

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	107	123	93	98	236	113	356
N.S.	1	1.00	0.82	0.94	0.71	0.75	1.80	0.86	2.72
time (sec)	N/A	0.136	0.550	0.351	0.276	0.343	0.298	7.009	6.627

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	68	89	70	70	0	123	103
N.S.	1	1.00	0.86	1.13	0.89	0.89	0.00	1.56	1.30
time (sec)	N/A	0.049	0.330	0.233	0.525	0.326	0.000	7.311	5.814

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	136	122	80	77	0	128	81
N.S.	1	1.00	1.62	1.45	0.95	0.92	0.00	1.52	0.96
time (sec)	N/A	0.062	0.432	0.364	0.288	0.329	0.000	5.737	5.249

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	190	173	105	101	0	243	119
N.S.	1	1.00	1.41	1.28	0.78	0.75	0.00	1.80	0.88
time (sec)	N/A	0.132	0.612	0.471	0.280	0.337	0.000	3.636	5.407

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	245	219	124	124	0	358	152
N.S.	1	1.00	1.48	1.33	0.75	0.75	0.00	2.17	0.92
time (sec)	N/A	0.141	0.927	0.450	0.284	0.388	0.000	3.341	5.604

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	299	265	145	146	0	473	275
N.S.	1	1.00	1.56	1.38	0.76	0.76	0.00	2.46	1.43
time (sec)	N/A	0.153	1.536	0.598	0.341	0.353	0.000	5.207	6.132

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	120	530	311	356	614	464	306
N.S.	1	1.00	0.83	3.68	2.16	2.47	4.26	3.22	2.12
time (sec)	N/A	0.158	1.926	2.398	0.270	0.459	4.720	7.931	5.454

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	56	480	233	310	468	272	231
N.S.	1	1.00	0.73	6.23	3.03	4.03	6.08	3.53	3.00
time (sec)	N/A	0.106	0.833	1.414	0.275	0.380	2.569	7.394	5.374

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	20	257	168	20	135
N.S.	1	1.00	1.00	0.95	0.91	11.68	7.64	0.91	6.14
time (sec)	N/A	0.017	0.161	0.698	0.280	0.427	1.287	4.986	5.266

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	227	329	317	327	0	378	212
N.S.	1	1.00	0.93	1.34	1.29	1.33	0.00	1.54	0.87
time (sec)	N/A	0.122	0.235	0.542	0.287	0.396	0.000	4.559	5.362

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	284	366	459	323	368	0	408	257
N.S.	1	1.00	1.29	1.62	1.14	1.30	0.00	1.44	0.90
time (sec)	N/A	0.169	2.383	0.511	0.277	0.413	0.000	6.616	5.389

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	320	320	514	544	348	366	0	429	305
N.S.	1	1.00	1.61	1.70	1.09	1.14	0.00	1.34	0.95
time (sec)	N/A	0.214	4.092	0.597	0.276	0.396	0.000	3.623	5.484

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	423	423	457	497	336	315	1115	364	467
N.S.	1	1.00	1.08	1.17	0.79	0.74	2.64	0.86	1.10
time (sec)	N/A	0.800	1.019	1.060	0.281	0.399	1.963	6.158	7.344

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	349	349	313	406	348	266	0	799	767
N.S.	1	1.00	0.90	1.16	1.00	0.76	0.00	2.29	2.20
time (sec)	N/A	0.379	1.116	0.291	0.508	0.377	0.000	6.566	7.734

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	369	369	414	495	328	268	0	684	726
N.S.	1	1.00	1.12	1.34	0.89	0.73	0.00	1.85	1.97
time (sec)	N/A	0.434	1.154	0.478	0.486	0.378	0.000	4.872	7.823

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	381	381	472	544	315	281	0	663	665
N.S.	1	1.00	1.24	1.43	0.83	0.74	0.00	1.74	1.75
time (sec)	N/A	0.475	1.314	0.558	0.527	0.433	0.000	5.859	7.598

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	404	404	479	567	310	306	0	726	546
N.S.	1	1.00	1.19	1.40	0.77	0.76	0.00	1.80	1.35
time (sec)	N/A	0.541	1.426	0.668	0.489	0.366	0.000	7.937	8.853

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	313	662	315	336	0	892	659
N.S.	1	1.00	1.33	2.81	1.33	1.42	0.00	3.78	2.79
time (sec)	N/A	0.254	4.567	0.826	0.300	0.497	0.000	7.006	6.680

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	106	106	108	107	0	120	109
N.S.	1	1.00	0.90	0.90	0.92	0.91	0.00	1.02	0.92
time (sec)	N/A	0.071	0.271	0.346	0.287	0.355	0.000	5.792	5.072

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	55	54	55	53	0	56	55
N.S.	1	1.00	0.90	0.89	0.90	0.87	0.00	0.92	0.90
time (sec)	N/A	0.045	0.087	0.175	0.283	0.363	0.000	4.088	0.077

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	18	18	41	19	18
N.S.	1	1.00	1.00	1.06	1.00	1.00	2.28	1.06	1.00
time (sec)	N/A	0.018	0.009	0.089	0.265	0.345	0.287	2.096	5.085

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	64	71	64	62	0	71	69
N.S.	1	1.00	0.85	0.95	0.85	0.83	0.00	0.95	0.92
time (sec)	N/A	0.055	0.062	0.191	0.277	0.362	0.000	5.582	5.133

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	170	121	139	153	0	177	148
N.S.	1	1.00	1.38	0.98	1.13	1.24	0.00	1.44	1.20
time (sec)	N/A	0.115	0.599	0.430	0.268	0.388	0.000	6.238	5.388

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	266	190	278	253	0	332	322
N.S.	1	1.00	1.36	0.97	1.43	1.30	0.00	1.70	1.65
time (sec)	N/A	0.177	0.989	0.649	0.300	0.442	0.000	4.546	0.586

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	2827	382	0	483	0	496	2500
N.S.	1	1.00	15.04	2.03	0.00	2.57	0.00	2.64	13.30
time (sec)	N/A	0.293	6.279	0.480	0.000	0.395	0.000	3.205	7.655

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	428	203	0	332	0	226	364
N.S.	1	1.00	3.37	1.60	0.00	2.61	0.00	1.78	2.87
time (sec)	N/A	0.170	4.205	0.334	0.000	0.366	0.000	4.652	6.116

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	361	96	0	214	1923	95	318
N.S.	1	1.00	5.16	1.37	0.00	3.06	27.47	1.36	4.54
time (sec)	N/A	0.080	1.252	0.160	0.000	0.370	166.893	6.841	5.373

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	152	112	0	305	0	107	149
N.S.	1	1.00	1.81	1.33	0.00	3.63	0.00	1.27	1.77
time (sec)	N/A	0.066	0.302	0.211	0.000	0.358	0.000	6.968	5.255

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	202	215	0	466	0	273	387
N.S.	1	1.00	1.47	1.57	0.00	3.40	0.00	1.99	2.82
time (sec)	N/A	0.166	1.312	0.506	0.000	0.358	0.000	6.482	7.946

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	370	338	0	666	0	584	774
N.S.	1	1.00	1.88	1.72	0.00	3.38	0.00	2.96	3.93
time (sec)	N/A	0.315	2.534	0.787	0.000	0.395	0.000	5.172	8.061

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	165	205	190	243	0	251	259
N.S.	1	1.00	0.90	1.11	1.03	1.32	0.00	1.36	1.41
time (sec)	N/A	0.116	1.431	0.797	0.279	0.418	0.000	5.605	0.121

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	106	116	116	156	0	150	118
N.S.	1	1.00	0.88	0.97	0.97	1.30	0.00	1.25	0.98
time (sec)	N/A	0.068	0.815	0.606	0.296	0.360	0.000	3.844	0.083

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	52	60	61	78	221	91	69
N.S.	1	1.00	0.83	0.95	0.97	1.24	3.51	1.44	1.10
time (sec)	N/A	0.046	0.156	0.479	0.274	0.354	0.623	6.397	0.085

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	20	20	51	20	20
N.S.	1	1.00	1.00	1.05	1.00	1.00	2.55	1.00	1.00
time (sec)	N/A	0.018	0.032	0.239	0.263	0.328	0.504	5.820	5.065

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	102	93	118	188	0	147	98
N.S.	1	1.00	0.98	0.89	1.13	1.81	0.00	1.41	0.94
time (sec)	N/A	0.080	0.240	0.506	0.275	0.385	0.000	5.729	0.210

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	222	146	275	381	0	244	227
N.S.	1	1.00	1.25	0.82	1.55	2.15	0.00	1.38	1.28
time (sec)	N/A	0.148	1.871	0.818	0.285	0.416	0.000	5.548	5.474

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	406	213	505	527	0	460	449
N.S.	1	1.00	1.51	0.79	1.88	1.96	0.00	1.71	1.67
time (sec)	N/A	0.225	6.118	1.164	0.300	0.523	0.000	5.018	5.944

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	3679	395	0	599	0	469	2530
N.S.	1	1.00	19.67	2.11	0.00	3.20	0.00	2.51	13.53
time (sec)	N/A	0.249	6.504	0.819	0.000	0.415	0.000	4.611	7.583

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	448	220	0	411	0	235	601
N.S.	1	1.00	3.50	1.72	0.00	3.21	0.00	1.84	4.70
time (sec)	N/A	0.140	5.022	0.658	0.000	0.400	0.000	5.579	6.397

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	414	121	0	388	0	126	329
N.S.	1	1.00	4.93	1.44	0.00	4.62	0.00	1.50	3.92
time (sec)	N/A	0.076	2.539	0.471	0.000	0.375	0.000	6.073	5.559

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	162	158	0	538	0	271	303
N.S.	1	1.00	1.25	1.22	0.00	4.14	0.00	2.08	2.33
time (sec)	N/A	0.130	1.135	0.493	0.000	0.377	0.000	6.839	7.396

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	336	252	0	782	0	427	727
N.S.	1	1.00	1.74	1.31	0.00	4.05	0.00	2.21	3.77
time (sec)	N/A	0.237	1.860	0.934	0.000	0.394	0.000	3.919	8.511

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	165	196	200	304	0	245	234
N.S.	1	1.00	0.87	1.03	1.05	1.60	0.00	1.29	1.23
time (sec)	N/A	0.106	3.184	0.634	0.272	0.390	0.000	3.210	0.122

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	106	117	131	212	0	142	142
N.S.	1	1.00	0.83	0.92	1.03	1.67	0.00	1.12	1.12
time (sec)	N/A	0.072	1.136	0.905	0.284	0.373	0.000	2.845	5.136

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	56	66	76	110	398	62	80
N.S.	1	1.00	0.78	0.92	1.06	1.53	5.53	0.86	1.11
time (sec)	N/A	0.046	0.342	0.679	0.293	0.361	0.777	5.451	0.092

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	20	43	73	20	39
N.S.	1	1.00	1.00	0.95	0.91	1.95	3.32	0.91	1.77
time (sec)	N/A	0.017	0.035	0.353	0.263	0.346	0.728	5.435	0.058

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	135	130	223	462	0	242	169
N.S.	1	1.00	0.93	0.90	1.54	3.19	0.00	1.67	1.17
time (sec)	N/A	0.112	0.630	0.745	0.272	0.428	0.000	4.935	5.400

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	283	184	438	707	0	413	388
N.S.	1	1.00	1.25	0.81	1.94	3.13	0.00	1.83	1.72
time (sec)	N/A	0.192	4.081	1.177	0.290	0.520	0.000	5.413	5.776

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	328	328	388	252	725	895	0	575	688
N.S.	1	1.00	1.18	0.77	2.21	2.73	0.00	1.75	2.10
time (sec)	N/A	0.288	2.838	2.191	0.296	0.709	0.000	4.469	6.565

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	3889	378	0	752	0	457	1226
N.S.	1	1.00	19.74	1.92	0.00	3.82	0.00	2.32	6.22
time (sec)	N/A	0.241	6.582	1.169	0.000	0.416	0.000	5.788	8.583

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	2641	238	0	716	0	272	1360
N.S.	1	1.00	19.00	1.71	0.00	5.15	0.00	1.96	9.78
time (sec)	N/A	0.137	6.294	0.898	0.000	0.423	0.000	7.113	7.550

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	93	201	0	501	0	207	282
N.S.	1	1.00	0.81	1.75	0.00	4.36	0.00	1.80	2.45
time (sec)	N/A	0.087	0.274	0.639	0.000	0.360	0.000	4.842	7.369

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	193	254	0	894	0	385	650
N.S.	1	1.00	1.01	1.32	0.00	4.66	0.00	2.01	3.39
time (sec)	N/A	0.253	3.171	0.806	0.000	0.398	0.000	4.487	8.820

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	380	352	0	1200	0	622	1167
N.S.	1	1.00	1.44	1.33	0.00	4.55	0.00	2.36	4.42
time (sec)	N/A	0.420	2.862	1.533	0.000	0.430	0.000	5.149	9.369

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	179	208	279	382	2530	215	276
N.S.	1	1.00	0.86	1.00	1.35	1.85	12.22	1.04	1.33
time (sec)	N/A	0.117	0.453	3.348	0.278	0.396	15.571	6.992	0.242

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	107	127	206	309	1425	117	206
N.S.	1	1.00	0.76	0.90	1.46	2.19	10.11	0.83	1.46
time (sec)	N/A	0.076	0.334	3.214	0.289	0.385	14.993	4.946	0.136

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	54	67	151	254	636	52	152
N.S.	1	1.00	0.70	0.87	1.96	3.30	8.26	0.68	1.97
time (sec)	N/A	0.048	0.240	3.049	0.273	0.386	14.726	4.850	5.217

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	20	218	167	20	119
N.S.	1	1.00	1.00	0.95	0.91	9.91	7.59	0.91	5.41
time (sec)	N/A	0.018	0.087	3.228	0.276	0.379	14.713	6.279	5.203

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	385	385	365	356	1160	3165	0	1010	937
N.S.	1	1.00	0.95	0.92	3.01	8.22	0.00	2.62	2.43
time (sec)	N/A	0.371	2.681	5.879	0.350	1.572	0.000	6.256	7.635

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	527	527	770	433	1670	3678	0	1327	1443
N.S.	1	1.00	1.46	0.82	3.17	6.98	0.00	2.52	2.74
time (sec)	N/A	0.518	6.723	5.317	0.393	2.580	0.000	7.437	9.887

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	491	491	6570	1652	0	3721	0	2326	2500
N.S.	1	1.00	13.38	3.36	0.00	7.58	0.00	4.74	5.09
time (sec)	N/A	0.839	8.363	3.675	0.000	0.732	0.000	4.260	32.217

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	407	407	386	1389	0	2250	0	1650	1868
N.S.	1	1.00	0.95	3.41	0.00	5.53	0.00	4.05	4.59
time (sec)	N/A	0.512	5.789	2.414	0.000	0.494	0.000	3.436	12.465

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	411	411	1167	1660	0	2657	0	1932	2184
N.S.	1	1.00	2.84	4.04	0.00	6.46	0.00	4.70	5.31
time (sec)	N/A	0.521	6.086	2.095	0.000	0.523	0.000	5.286	10.854

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	422	422	1896	1872	0	2972	0	2207	2440
N.S.	1	1.00	4.49	4.44	0.00	7.04	0.00	5.23	5.78
time (sec)	N/A	0.501	6.201	2.062	0.000	0.568	0.000	8.749	10.339

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	529	529	494	1286	0	3882	0	2610	2500
N.S.	1	1.00	0.93	2.43	0.00	7.34	0.00	4.93	4.73
time (sec)	N/A	1.151	4.336	2.860	0.000	0.702	0.000	6.565	53.316

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	653	653	597	1380	0	4500	0	3047	-1
N.S.	1	1.00	0.91	2.11	0.00	6.89	0.00	4.67	-0.00
time (sec)	N/A	1.384	5.248	4.303	0.000	0.895	0.000	6.086	0.000

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	117	126	116	142	0	161	-1
N.S.	1	1.00	0.76	0.82	0.75	0.92	0.00	1.05	-0.01
time (sec)	N/A	0.076	0.331	1.826	0.275	0.364	0.000	5.082	0.000

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	58	55	61	78	0	72	-1
N.S.	1	1.00	0.70	0.66	0.73	0.94	0.00	0.87	-0.01
time (sec)	N/A	0.054	0.129	1.088	0.273	0.347	0.000	4.979	0.000

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	21	20	20	83	20	20
N.S.	1	1.00	1.00	0.88	0.83	0.83	3.46	0.83	0.83
time (sec)	N/A	0.022	0.019	0.050	0.270	0.339	0.145	6.578	5.204

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	74	61	0	1729	0	83	-1
N.S.	1	1.00	1.00	0.82	0.00	23.36	0.00	1.12	-0.01
time (sec)	N/A	0.073	0.057	1.125	0.000	0.602	0.000	5.068	0.000

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	143	185	0	2101	0	159	-1
N.S.	1	1.00	1.15	1.49	0.00	16.94	0.00	1.28	-0.01
time (sec)	N/A	0.111	0.746	1.797	0.000	0.619	0.000	5.948	0.000

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	224	471	0	0	0	358	-1
N.S.	1	1.00	1.08	2.28	0.00	0.00	0.00	1.73	-0.00
time (sec)	N/A	0.206	1.520	2.445	0.000	0.000	0.000	5.934	0.000

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	233	1189	0	534	0	0	-1
N.S.	1	1.00	0.78	3.99	0.00	1.79	0.00	0.00	-0.00
time (sec)	N/A	0.345	0.875	2.139	0.000	0.151	0.000	0.000	0.000

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	185	792	0	448	0	0	-1
N.S.	1	1.00	0.86	3.68	0.00	2.08	0.00	0.00	-0.00
time (sec)	N/A	0.166	0.822	1.813	0.000	0.121	0.000	0.000	0.000

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	127	614	0	418	0	0	-1
N.S.	1	1.00	0.85	4.12	0.00	2.81	0.00	0.00	-0.01
time (sec)	N/A	0.110	2.816	2.330	0.000	0.114	0.000	0.000	0.000

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	270	1261	0	526	0	0	-1
N.S.	1	1.00	1.09	5.08	0.00	2.12	0.00	0.00	-0.00
time (sec)	N/A	0.227	3.254	2.270	0.000	0.133	0.000	0.000	0.000

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	238	126	116	184	0	0	-1
N.S.	1	1.00	1.55	0.82	0.75	1.19	0.00	0.00	-0.01
time (sec)	N/A	0.075	0.992	1.577	0.265	0.386	0.000	0.000	0.000

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	58	55	61	111	314	0	-1
N.S.	1	1.00	0.70	0.66	0.73	1.34	3.78	0.00	-0.01
time (sec)	N/A	0.057	0.183	0.974	0.261	0.342	5.621	0.000	0.000

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	21	20	53	116	0	20
N.S.	1	1.00	1.00	0.88	0.83	2.21	4.83	0.00	0.83
time (sec)	N/A	0.024	0.023	0.052	0.269	0.336	1.653	0.000	5.410

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	89	201	0	0	0	0	-1
N.S.	1	1.00	0.95	2.14	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.112	0.097	1.269	0.000	0.000	0.000	0.000	0.000

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	121	279	0	1969	0	0	-1
N.S.	1	1.00	0.93	2.15	0.00	15.15	0.00	0.00	-0.01
time (sec)	N/A	0.174	0.714	1.936	0.000	0.619	0.000	0.000	0.000

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	297	409	0	0	0	0	-1
N.S.	1	1.00	1.58	2.18	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.201	2.486	2.052	0.000	0.000	0.000	0.000	0.000

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	329	329	278	1355	0	583	0	0	-1
N.S.	1	1.00	0.84	4.12	0.00	1.77	0.00	0.00	-0.00
time (sec)	N/A	0.444	1.094	2.586	0.000	0.149	0.000	0.000	0.000

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	222	943	0	491	0	0	-1
N.S.	1	1.00	0.90	3.82	0.00	1.99	0.00	0.00	-0.00
time (sec)	N/A	0.284	1.025	2.208	0.000	0.138	0.000	0.000	0.000

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	163	635	0	444	0	0	-1
N.S.	1	1.00	0.97	3.78	0.00	2.64	0.00	0.00	-0.01
time (sec)	N/A	0.128	0.663	2.292	0.000	0.108	0.000	0.000	0.000

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	211	938	0	481	0	0	-1
N.S.	1	1.00	0.97	4.30	0.00	2.21	0.00	0.00	-0.00
time (sec)	N/A	0.279	2.349	2.539	0.000	0.128	0.000	0.000	0.000

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	330	330	364	1519	0	612	0	0	-1
N.S.	1	1.00	1.10	4.60	0.00	1.85	0.00	0.00	-0.00
time (sec)	N/A	0.451	6.275	2.996	0.000	0.153	0.000	0.000	0.000

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	279	126	116	224	0	0	-1
N.S.	1	1.00	1.81	0.82	0.75	1.45	0.00	0.00	-0.01
time (sec)	N/A	0.073	1.391	1.928	0.288	0.409	0.000	0.000	0.000

Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	58	55	61	143	391	0	-1
N.S.	1	1.00	0.70	0.66	0.73	1.72	4.71	0.00	-0.01
time (sec)	N/A	0.059	0.116	1.195	0.280	0.358	48.744	0.000	0.000

Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	21	20	77	150	0	20
N.S.	1	1.00	1.00	0.88	0.83	3.21	6.25	0.00	0.83
time (sec)	N/A	0.024	0.033	0.071	0.270	0.351	17.318	0.000	5.569

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	105	286	0	0	0	0	-1
N.S.	1	1.00	0.90	2.44	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.148	0.164	1.583	0.000	0.000	0.000	0.000	0.000

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	147	356	0	2071	0	0	-1
N.S.	1	1.00	0.95	2.30	0.00	13.36	0.00	0.00	-0.01
time (sec)	N/A	0.175	0.886	2.109	0.000	0.652	0.000	0.000	0.000

Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	307	538	0	2229	0	0	-1
N.S.	1	1.00	1.54	2.70	0.00	11.20	0.00	0.00	-0.01
time (sec)	N/A	0.184	3.057	2.394	0.000	0.663	0.000	0.000	0.000

Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	398	398	321	1619	0	633	0	0	-1
N.S.	1	1.00	0.81	4.07	0.00	1.59	0.00	0.00	-0.00
time (sec)	N/A	0.588	1.268	2.596	0.000	0.166	0.000	0.000	0.000

Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	239	1190	0	536	0	0	-1
N.S.	1	1.00	0.80	3.98	0.00	1.79	0.00	0.00	-0.00
time (sec)	N/A	0.417	1.009	2.307	0.000	0.138	0.000	0.000	0.000

Problem 502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	203	1042	0	474	0	0	-1
N.S.	1	1.00	1.00	5.13	0.00	2.33	0.00	0.00	-0.00
time (sec)	N/A	0.176	0.870	2.402	0.000	0.130	0.000	0.000	0.000

Problem 503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	238	259	1249	0	522	0	0	-1
N.S.	1	1.00	1.09	5.25	0.00	2.19	0.00	0.00	-0.00
time (sec)	N/A	0.251	3.303	3.030	0.000	0.134	0.000	0.000	0.000

Problem 504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	322	351	1360	0	561	0	0	-1
N.S.	1	1.00	1.09	4.22	0.00	1.74	0.00	0.00	-0.00
time (sec)	N/A	0.416	6.264	2.749	0.000	0.142	0.000	0.000	0.000

Problem 505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	439	439	338	1887	0	697	0	0	-1
N.S.	1	1.00	0.77	4.30	0.00	1.59	0.00	0.00	-0.00
time (sec)	N/A	0.592	4.196	27.696	0.000	0.177	0.000	0.000	0.000

Problem 506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	161	126	160	111	0	161	-1
N.S.	1	1.00	1.06	0.83	1.05	0.73	0.00	1.06	-0.01
time (sec)	N/A	0.072	1.372	2.628	0.302	0.346	0.000	6.249	0.000

Problem 507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	108	55	75	54	0	72	-1
N.S.	1	1.00	1.33	0.68	0.93	0.67	0.00	0.89	-0.01
time (sec)	N/A	0.055	0.646	1.293	0.271	0.350	0.000	5.004	0.000

Problem 508	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	20	20	54	20	20
N.S.	1	1.00	1.00	0.95	0.91	0.91	2.45	0.91	0.91
time (sec)	N/A	0.022	0.017	0.076	0.302	0.343	0.547	6.396	6.224

Problem 509	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	74	60	0	0	0	75	-1
N.S.	1	1.00	1.00	0.81	0.00	0.00	0.00	1.01	-0.01
time (sec)	N/A	0.060	0.056	1.266	0.000	0.000	0.000	4.488	0.000

Problem 510	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F(-2)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	176	204	0	0	0	212	-1
N.S.	1	1.00	1.22	1.42	0.00	0.00	0.00	1.47	-0.01
time (sec)	N/A	0.202	0.539	2.313	0.000	0.000	0.000	2.207	0.000

Problem 511	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	244	580	0	0	0	419	-1
N.S.	1	1.00	1.06	2.52	0.00	0.00	0.00	1.82	-0.00
time (sec)	N/A	0.249	1.896	3.434	0.000	0.000	0.000	4.696	0.000

Problem 512	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	219	942	0	493	0	0	-1
N.S.	1	1.00	0.89	3.81	0.00	2.00	0.00	0.00	-0.00
time (sec)	N/A	0.240	1.070	2.114	0.000	0.136	0.000	0.000	0.000

Problem 513	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	145	462	0	408	0	0	-1
N.S.	1	1.00	0.83	2.64	0.00	2.33	0.00	0.00	-0.01
time (sec)	N/A	0.123	0.767	1.682	0.000	0.111	0.000	0.000	0.000

Problem 514	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	177	640	0	458	0	0	-1
N.S.	1	1.00	0.97	3.50	0.00	2.50	0.00	0.00	-0.01
time (sec)	N/A	0.148	0.653	2.263	0.000	0.113	0.000	0.000	0.000

Problem 515	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	306	1314	0	571	0	0	-1
N.S.	1	1.00	1.05	4.52	0.00	1.96	0.00	0.00	-0.00
time (sec)	N/A	0.289	4.036	8.312	0.000	0.133	0.000	0.000	0.000

Problem 516	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	165	116	124	125	0	0	-1
N.S.	1	1.00	1.10	0.77	0.83	0.83	0.00	0.00	-0.01
time (sec)	N/A	0.078	0.690	1.001	0.269	0.381	0.000	0.000	0.000

Problem 517	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	57	54	67	67	0	0	-1
N.S.	1	1.00	0.72	0.68	0.85	0.85	0.00	0.00	-0.01
time (sec)	N/A	0.062	0.066	0.825	0.268	0.381	0.000	0.000	0.000

Problem 518	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	20	32	56	0	51
N.S.	1	1.00	1.00	0.95	0.91	1.45	2.55	0.00	2.32
time (sec)	N/A	0.026	0.018	0.042	0.293	0.347	1.000	0.000	6.145

Problem 519	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	91	94	0	0	0	0	-1
N.S.	1	1.00	0.87	0.90	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.108	0.091	1.531	0.000	0.000	0.000	0.000	0.000

Problem 520	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	221	233	0	0	0	0	-1
N.S.	1	1.00	1.19	1.25	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.237	1.147	2.756	0.000	0.000	0.000	0.000	0.000

Problem 521	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	284	324	602	0	0	0	0	-1
N.S.	1	1.00	1.14	2.12	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.355	2.177	3.143	0.000	0.000	0.000	0.000	0.000

Problem 522	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	313	313	273	1195	0	723	0	0	-1
N.S.	1	1.00	0.87	3.82	0.00	2.31	0.00	0.00	-0.00
time (sec)	N/A	0.350	1.502	2.257	0.000	0.209	0.000	0.000	0.000

Problem 523	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	187	797	0	598	0	0	-1
N.S.	1	1.00	0.82	3.48	0.00	2.61	0.00	0.00	-0.00
time (sec)	N/A	0.220	1.069	2.026	0.000	0.150	0.000	0.000	0.000

Problem 524	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	125	434	0	470	0	0	-1
N.S.	1	1.00	0.78	2.71	0.00	2.94	0.00	0.00	-0.01
time (sec)	N/A	0.122	2.743	2.296	0.000	0.126	0.000	0.000	0.000

Problem 525	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	205	1064	0	685	0	0	-1
N.S.	1	1.00	0.82	4.24	0.00	2.73	0.00	0.00	-0.00
time (sec)	N/A	0.239	1.719	3.385	0.000	0.136	0.000	0.000	0.000

Problem 526	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	359	359	348	1646	0	878	0	0	-1
N.S.	1	1.00	0.97	4.58	0.00	2.45	0.00	0.00	-0.00
time (sec)	N/A	0.403	3.031	15.536	0.000	0.222	0.000	0.000	0.000

Problem 527	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	117	116	122	147	0	0	-1
N.S.	1	1.00	0.78	0.77	0.81	0.98	0.00	0.00	-0.01
time (sec)	N/A	0.079	0.298	1.091	0.267	0.378	0.000	0.000	0.000

Problem 528	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	56	55	64	91	304	0	1402
N.S.	1	1.00	0.71	0.70	0.81	1.15	3.85	0.00	17.75
time (sec)	N/A	0.058	0.061	0.832	0.297	0.354	3.088	0.000	11.891

Problem 529	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	21	20	55	87	0	157
N.S.	1	1.00	1.00	0.88	0.83	2.29	3.62	0.00	6.54
time (sec)	N/A	0.024	0.022	0.038	0.414	0.354	2.995	0.000	7.246

Problem 530	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	94	122	0	3225	0	0	-1
N.S.	1	1.00	0.68	0.88	0.00	23.20	0.00	0.00	-0.01
time (sec)	N/A	0.168	0.079	2.066	0.000	0.738	0.000	0.000	0.000

Problem 531	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	245	263	0	0	0	0	-1
N.S.	1	1.00	1.06	1.14	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.277	0.873	2.730	0.000	0.000	0.000	0.000	0.000

Problem 532	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	339	339	296	632	0	0	0	0	-1
N.S.	1	1.00	0.87	1.86	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.429	3.214	3.165	0.000	0.000	0.000	0.000	0.000

Problem 533	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	384	384	356	2253	0	1043	0	0	-1
N.S.	1	1.00	0.93	5.87	0.00	2.72	0.00	0.00	-0.00
time (sec)	N/A	0.493	1.740	3.032	0.000	0.413	0.000	0.000	0.000

Problem 534	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	293	244	1642	0	874	0	0	-1
N.S.	1	1.00	0.83	5.60	0.00	2.98	0.00	0.00	-0.00
time (sec)	N/A	0.336	1.216	2.233	0.000	0.247	0.000	0.000	0.000

Problem 535	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	174	1046	0	705	0	0	-1
N.S.	1	1.00	0.79	4.73	0.00	3.19	0.00	0.00	-0.00
time (sec)	N/A	0.207	1.035	2.235	0.000	0.171	0.000	0.000	0.000

Problem 536	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	167	864	0	711	0	0	-1
N.S.	1	1.00	0.76	3.95	0.00	3.25	0.00	0.00	-0.00
time (sec)	N/A	0.171	1.046	2.635	0.000	0.143	0.000	0.000	0.000

Problem 537	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	325	325	241	1653	0	993	0	0	-1
N.S.	1	1.00	0.74	5.09	0.00	3.06	0.00	0.00	-0.00
time (sec)	N/A	0.391	1.920	12.436	0.000	0.176	0.000	0.000	0.000

Problem 538	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	425	425	341	2585	0	1242	0	0	-1
N.S.	1	1.00	0.80	6.08	0.00	2.92	0.00	0.00	-0.00
time (sec)	N/A	0.564	2.500	18.655	0.000	0.270	0.000	0.000	0.000

Problem 539	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	104	259	0	109	0	0	-1
N.S.	1	1.00	0.84	2.09	0.00	0.88	0.00	0.00	-0.01
time (sec)	N/A	0.060	0.850	4.368	0.000	0.108	0.000	0.000	0.000

Problem 540	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	79	222	0	105	0	0	-1
N.S.	1	1.00	0.83	2.34	0.00	1.11	0.00	0.00	-0.01
time (sec)	N/A	0.048	0.496	4.201	0.000	0.111	0.000	0.000	0.000

Problem 541	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	79	185	0	93	0	0	-1
N.S.	1	1.00	0.83	1.95	0.00	0.98	0.00	0.00	-0.01
time (sec)	N/A	0.046	0.489	3.946	0.000	0.103	0.000	0.000	0.000

Problem 542	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	56	123	0	77	0	0	-1
N.S.	1	1.00	0.89	1.95	0.00	1.22	0.00	0.00	-0.02
time (sec)	N/A	0.033	0.109	3.092	0.000	0.101	0.000	0.000	0.000

Problem 543	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	50	106	0	66	0	0	47
N.S.	1	1.00	0.82	1.74	0.00	1.08	0.00	0.00	0.77
time (sec)	N/A	0.032	0.200	2.270	0.000	0.090	0.000	0.000	6.494

Problem 544	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	54	119	0	101	0	0	-1
N.S.	1	1.00	0.59	1.31	0.00	1.11	0.00	0.00	-0.01
time (sec)	N/A	0.052	0.129	5.523	0.000	0.099	0.000	0.000	0.000

Problem 545	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	55	193	0	100	0	0	-1
N.S.	1	1.00	0.57	1.99	0.00	1.03	0.00	0.00	-0.01
time (sec)	N/A	0.051	0.156	6.688	0.000	0.106	0.000	0.000	0.000

Problem 546	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	70	310	0	118	0	0	-1
N.S.	1	1.00	0.56	2.46	0.00	0.94	0.00	0.00	-0.01
time (sec)	N/A	0.061	0.345	10.352	0.000	0.103	0.000	0.000	0.000

Problem 547	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	160	473	0	165	0	0	-1
N.S.	1	1.00	0.85	2.52	0.00	0.88	0.00	0.00	-0.01
time (sec)	N/A	0.125	1.923	5.157	0.000	0.128	0.000	0.000	0.000

Problem 548	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	113	408	0	154	0	0	-1
N.S.	1	1.00	0.76	2.74	0.00	1.03	0.00	0.00	-0.01
time (sec)	N/A	0.106	0.922	5.080	0.000	0.111	0.000	0.000	0.000

Problem 549	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	115	343	0	142	0	0	-1
N.S.	1	1.00	0.77	2.30	0.00	0.95	0.00	0.00	-0.01
time (sec)	N/A	0.106	1.159	5.210	0.000	0.118	0.000	0.000	0.000

Problem 550	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	80	251	0	126	0	0	-1
N.S.	1	1.00	0.73	2.30	0.00	1.16	0.00	0.00	-0.01
time (sec)	N/A	0.084	0.336	4.523	0.000	0.115	0.000	0.000	0.000

Problem 551	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	75	210	0	99	0	0	-1
N.S.	1	1.00	0.69	1.93	0.00	0.91	0.00	0.00	-0.01
time (sec)	N/A	0.085	0.421	3.472	0.000	0.106	0.000	0.000	0.000

Problem 552	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	71	197	0	128	0	0	-1
N.S.	1	1.00	0.63	1.74	0.00	1.13	0.00	0.00	-0.01
time (sec)	N/A	0.091	0.245	5.944	0.000	0.117	0.000	0.000	0.000

Problem 553	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	72	333	0	127	0	0	-1
N.S.	1	1.00	0.61	2.80	0.00	1.07	0.00	0.00	-0.01
time (sec)	N/A	0.092	0.266	7.822	0.000	0.100	0.000	0.000	0.000

Problem 554	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	105	564	0	153	0	0	-1
N.S.	1	1.00	0.66	3.52	0.00	0.96	0.00	0.00	-0.01
time (sec)	N/A	0.118	0.566	14.655	0.000	0.103	0.000	0.000	0.000

Problem 555	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	205	618	0	193	0	0	-1
N.S.	1	1.00	0.86	2.61	0.00	0.81	0.00	0.00	-0.00
time (sec)	N/A	0.210	2.035	11.381	0.000	0.137	0.000	0.000	0.000

Problem 556	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	150	534	0	181	0	0	-1
N.S.	1	1.00	0.76	2.71	0.00	0.92	0.00	0.00	-0.01
time (sec)	N/A	0.187	1.417	9.688	0.000	0.138	0.000	0.000	0.000

Problem 557	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	153	450	0	169	0	0	-1
N.S.	1	1.00	0.78	2.28	0.00	0.86	0.00	0.00	-0.01
time (sec)	N/A	0.184	1.420	8.313	0.000	0.110	0.000	0.000	0.000

Problem 558	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	101	339	0	152	0	0	-1
N.S.	1	1.00	0.65	2.17	0.00	0.97	0.00	0.00	-0.01
time (sec)	N/A	0.156	0.646	6.868	0.000	0.111	0.000	0.000	0.000

Problem 559	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	94	279	0	124	0	0	-1
N.S.	1	1.00	0.62	1.84	0.00	0.82	0.00	0.00	-0.01
time (sec)	N/A	0.157	0.823	6.056	0.000	0.104	0.000	0.000	0.000

Problem 560	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	98	248	0	156	0	0	-1
N.S.	1	1.00	0.61	1.55	0.00	0.98	0.00	0.00	-0.01
time (sec)	N/A	0.159	0.438	8.197	0.000	0.111	0.000	0.000	0.000

Problem 561	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	103	384	0	150	0	0	-1
N.S.	1	1.00	0.63	2.34	0.00	0.91	0.00	0.00	-0.01
time (sec)	N/A	0.163	0.721	8.685	0.000	0.104	0.000	0.000	0.000

Problem 562	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	126	618	0	181	0	0	-1
N.S.	1	1.00	0.67	3.30	0.00	0.97	0.00	0.00	-0.01
time (sec)	N/A	0.171	0.758	17.228	0.000	0.106	0.000	0.000	0.000

Problem 563	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	140	750	0	176	0	0	-1
N.S.	1	1.00	0.74	3.99	0.00	0.94	0.00	0.00	-0.01
time (sec)	N/A	0.169	0.670	21.661	0.000	0.111	0.000	0.000	0.000

Problem 564	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	305	305	251	863	0	248	0	0	-1
N.S.	1	1.00	0.82	2.83	0.00	0.81	0.00	0.00	-0.00
time (sec)	N/A	0.359	4.558	12.358	0.000	0.158	0.000	0.000	0.000

Problem 565	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	258	209	776	0	228	0	0	-1
N.S.	1	1.00	0.81	3.01	0.00	0.88	0.00	0.00	-0.00
time (sec)	N/A	0.335	2.090	11.020	0.000	0.146	0.000	0.000	0.000

Problem 566	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	258	189	639	0	216	0	0	-1
N.S.	1	1.00	0.73	2.48	0.00	0.84	0.00	0.00	-0.00
time (sec)	N/A	0.330	2.777	9.994	0.000	0.143	0.000	0.000	0.000

Problem 567	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	137	525	0	196	0	0	-1
N.S.	1	1.00	0.65	2.50	0.00	0.93	0.00	0.00	-0.00
time (sec)	N/A	0.284	1.122	8.573	0.000	0.122	0.000	0.000	0.000

Problem 568	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	130	412	0	163	0	0	-1
N.S.	1	1.00	0.62	1.96	0.00	0.78	0.00	0.00	-0.00
time (sec)	N/A	0.299	1.179	6.463	0.000	0.105	0.000	0.000	0.000

Problem 569	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	135	378	0	194	0	0	-1
N.S.	1	1.00	0.62	1.73	0.00	0.89	0.00	0.00	-0.00
time (sec)	N/A	0.293	0.631	9.201	0.000	0.116	0.000	0.000	0.000

Problem 570	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	137	575	0	185	0	0	-1
N.S.	1	1.00	0.63	2.66	0.00	0.86	0.00	0.00	-0.00
time (sec)	N/A	0.293	1.198	9.879	0.000	0.116	0.000	0.000	0.000

Problem 571	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	152	874	0	210	0	0	-1
N.S.	1	1.00	0.64	3.69	0.00	0.89	0.00	0.00	-0.00
time (sec)	N/A	0.302	0.607	20.855	0.000	0.137	0.000	0.000	0.000

Problem 572	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	177	1067	0	206	0	0	-1
N.S.	1	1.00	0.73	4.43	0.00	0.85	0.00	0.00	-0.00
time (sec)	N/A	0.303	0.935	25.292	0.000	0.111	0.000	0.000	0.000

Problem 573	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	219	1416	0	243	0	0	-1
N.S.	1	1.00	0.83	5.36	0.00	0.92	0.00	0.00	-0.00
time (sec)	N/A	0.316	1.605	34.278	0.000	0.125	0.000	0.000	0.000

Problem 574	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	531	531	2035	1807	0	0	0	0	-1
N.S.	1	1.00	3.83	3.40	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.273	57.662	14.971	0.000	0.000	0.000	0.000	0.000

Problem 575	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	446	446	834	1583	0	0	0	0	-1
N.S.	1	1.00	1.87	3.55	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.833	46.910	10.346	0.000	0.000	0.000	0.000	0.000

Problem 576	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	461	461	1955	1246	0	0	0	0	-1
N.S.	1	1.00	4.24	2.70	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.868	58.417	11.989	0.000	0.000	0.000	0.000	0.000

Problem 577	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	384	384	709	1087	0	0	0	0	-1
N.S.	1	1.00	1.85	2.83	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.562	41.296	10.000	0.000	0.000	0.000	0.000	0.000

Problem 578	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	397	397	434	828	0	0	0	0	-1
N.S.	1	1.00	1.09	2.09	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.572	26.735	9.623	0.000	0.000	0.000	0.000	0.000

Problem 579	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	292	292	361	680	0	0	0	0	-1
N.S.	1	1.00	1.24	2.33	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.373	25.795	7.417	0.000	0.000	0.000	0.000	0.000

Problem 580	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	558	676	0	0	0	0	-1
N.S.	1	1.00	1.87	2.26	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.366	26.314	8.358	0.000	0.000	0.000	0.000	0.000

Problem 581	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	411	411	791	1056	0	0	0	0	-1
N.S.	1	1.00	1.92	2.57	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.621	42.633	13.110	0.000	0.000	0.000	0.000	0.000

Problem 582	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	434	434	1192	932	0	0	0	0	-1
N.S.	1	1.00	2.75	2.15	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.666	44.452	15.540	0.000	0.000	0.000	0.000	0.000

Problem 583	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	486	486	881	1510	0	0	0	0	-1
N.S.	1	1.00	1.81	3.11	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.864	26.511	22.537	0.000	0.000	0.000	0.000	0.000

Problem 584	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	543	543	2030	15532	0	0	0	0	-1
N.S.	1	1.00	3.74	28.60	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.034	57.494	45.068	0.000	0.000	0.000	0.000	0.000

Problem 585	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	459	459	835	19163	0	0	0	0	-1
N.S.	1	1.00	1.82	41.75	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.735	46.420	34.526	0.000	0.000	0.000	0.000	0.000

Problem 586	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	473	473	1956	11649	0	0	0	0	-1
N.S.	1	1.00	4.14	24.63	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.739	56.090	35.160	0.000	0.000	0.000	0.000	0.000

Problem 587	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	390	390	371	12860	0	0	0	0	-1
N.S.	1	1.00	0.95	32.97	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.540	86.897	23.792	0.000	0.000	0.000	0.000	0.000

Problem 588	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	404	404	614	7936	0	0	0	0	-1
N.S.	1	1.00	1.52	19.64	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.601	23.293	24.516	0.000	0.000	0.000	0.000	0.000

Problem 589	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	422	422	787	9239	0	0	0	0	-1
N.S.	1	1.00	1.86	21.89	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.568	36.134	27.313	0.000	0.000	0.000	0.000	0.000

Problem 590	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	429	429	1181	5088	0	0	0	0	-1
N.S.	1	1.00	2.75	11.86	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.579	43.496	27.169	0.000	0.000	0.000	0.000	0.000

Problem 591	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	492	492	777	7528	0	0	0	0	-1
N.S.	1	1.00	1.58	15.30	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.813	26.416	40.780	0.000	0.000	0.000	0.000	0.000

Problem 592	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	514	514	1258	4976	0	0	0	0	-1
N.S.	1	1.00	2.45	9.68	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.904	44.254	54.499	0.000	0.000	0.000	0.000	0.000

Problem 593	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	574	574	949	8202	0	0	0	0	-1
N.S.	1	1.00	1.65	14.29	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.087	26.628	82.797	0.000	0.000	0.000	0.000	0.000

Problem 594	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	575	575	932	109325	0	0	0	0	-1
N.S.	1	1.00	1.62	190.13	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.942	40.821	112.651	0.000	0.000	0.000	0.000	0.000

Problem 595	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	589	589	2024	78449	0	0	0	0	-1
N.S.	1	1.00	3.44	133.19	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.028	54.326	116.908	0.000	0.000	0.000	0.000	0.000

Problem 596	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	483	483	777	84670	0	0	0	0	-1
N.S.	1	1.00	1.61	175.30	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.679	28.170	91.504	0.000	0.000	0.000	0.000	0.000

Problem 597	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	497	497	1954	60431	0	0	0	0	-1
N.S.	1	1.00	3.93	121.59	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.737	51.091	90.852	0.000	0.000	0.000	0.000	0.000

Problem 598	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	505	505	831	60162	0	0	0	0	-1
N.S.	1	1.00	1.65	119.13	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.732	30.707	93.463	0.000	0.000	0.000	0.000	0.000

Problem 599	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	519	519	1211	42544	0	0	0	0	-1
N.S.	1	1.00	2.33	81.97	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.767	41.824	94.083	0.000	0.000	0.000	0.000	0.000

Problem 600	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	514	514	748	35480	0	0	0	0	-1
N.S.	1	1.00	1.46	69.03	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.776	30.139	93.590	0.000	0.000	0.000	0.000	0.000

Problem 601	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	520	520	1226	24488	0	0	0	0	-1
N.S.	1	1.00	2.36	47.09	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.807	42.438	91.417	0.000	0.000	0.000	0.000	0.000

Problem 602	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	F(-1)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	596	596	922	43749	0	0	0	0	-1
N.S.	1	1.00	1.55	73.40	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.050	16.367	170.834	0.000	0.000	0.000	0.000	0.000

Problem 603	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	614	614	1308	30052	0	0	0	0	-1
N.S.	1	1.00	2.13	48.94	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.181	41.868	228.924	0.000	0.000	0.000	0.000	0.000

Problem 604	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-1)	F(-1)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	685	685	1014	0	0	0	0	0	-1
N.S.	1	1.00	1.48	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.341	16.468	180.000	0.000	0.000	0.000	0.000	0.000

Problem 605	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-1)	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	671	671	2102	0	0	0	0	0	-1
N.S.	1	1.00	3.13	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.224	55.429	180.000	0.000	0.000	0.000	0.000	0.000

Problem 606	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	557	557	937	177735	0	0	0	0	-1
N.S.	1	1.00	1.68	319.09	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.866	33.783	284.540	0.000	0.000	0.000	0.000	0.000

Problem 607	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	571	571	2020	0	0	0	0	0	-1
N.S.	1	1.00	3.54	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.949	52.020	180.000	0.000	0.000	0.000	0.000	0.000

Problem 608	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	591	591	900	234955	0	0	0	0	-1
N.S.	1	1.00	1.52	397.55	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.976	33.527	277.040	0.000	0.000	0.000	0.000	0.000

Problem 609	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	597	597	1263	0	0	0	0	0	-1
N.S.	1	1.00	2.12	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.024	42.343	180.000	0.000	0.000	0.000	0.000	0.000

Problem 610	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	574	574	892	0	0	0	0	0	-1
N.S.	1	1.00	1.55	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.948	33.638	180.000	0.000	0.000	0.000	0.000	0.000

Problem 611	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	592	592	1263	0	0	0	0	0	-1
N.S.	1	1.00	2.13	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.995	41.936	180.000	0.000	0.000	0.000	0.000	0.000

Problem 612	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	579	579	900	109490	0	0	0	0	-1
N.S.	1	1.00	1.55	189.10	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.997	16.308	298.144	0.000	0.000	0.000	0.000	0.000

Problem 613	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	F(-1)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	593	593	1276	82867	0	0	0	0	-1
N.S.	1	1.00	2.15	139.74	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.065	42.321	293.566	0.000	0.000	0.000	0.000	0.000

Problem 614	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-1)	F(-1)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	674	674	996	0	0	0	0	0	-1
N.S.	1	1.00	1.48	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.304	16.425	180.000	0.000	0.000	0.000	0.000	0.000

Problem 615	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	C	B	F	F	F	F	F
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	374	117	751	0	0	0	0	-1
N.S.	1	2.04	0.64	4.10	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.312	0.221	11.441	0.000	0.000	0.000	0.000	0.000

Problem 616	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	17703	0	0	0	0	0	-1
N.S.	1	1.00	77.31	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.262	55.708	0.457	0.000	0.000	0.000	0.000	0.000

Problem 617	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	285	0	0	0	0	0	-1
N.S.	1	1.00	1.82	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.107	0.704	0.697	0.000	0.000	0.000	0.000	0.000

Problem 618	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	240	0	0	0	0	0	-1
N.S.	1	1.00	2.47	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.035	0.654	0.146	0.000	0.000	0.000	0.000	0.000

Problem 619	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	3815	0	0	0	0	0	-1
N.S.	1	1.00	24.15	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.058	19.157	0.164	0.000	0.000	0.000	0.000	0.000

Problem 620	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	4727	0	0	0	0	0	-1
N.S.	1	1.00	27.81	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.048	24.303	0.404	0.000	0.000	0.000	0.000	0.000

Problem 621	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	7781	0	0	0	0	0	-1
N.S.	1	1.00	45.77	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.050	25.702	0.448	0.000	0.000	0.000	0.000	0.000

Problem 622	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.046	46.111	0.361	0.000	0.000	0.000	0.000	0.000

Problem 623	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	187	0	0	0	0	0	-1
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.094	10.279	0.098	0.000	0.000	0.000	0.000	0.000

Problem 624	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	187	0	0	0	0	0	-1
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.082	2.065	0.093	0.000	0.000	0.000	0.000	0.000

Problem 625	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	187	0	0	0	0	0	-1
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.074	0.678	0.096	0.000	0.000	0.000	0.000	0.000

Problem 626	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	185	0	0	0	0	0	-1
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.081	2.051	0.073	0.000	0.000	0.000	0.000	0.000

Problem 627	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	185	0	0	0	0	0	-1
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.083	2.355	0.062	0.000	0.000	0.000	0.000	0.000

Problem 628	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	187	0	0	0	0	0	-1
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.080	2.641	0.062	0.000	0.000	0.000	0.000	0.000

Problem 629	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.072	1.574	0.105	0.000	0.000	0.000	0.000	0.000

Problem 630	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	854	0	558	814	0	3579	1196
N.S.	1	1.00	3.36	0.00	2.20	3.20	0.00	14.09	4.71
time (sec)	N/A	0.120	3.079	0.173	0.310	0.445	0.000	4.220	19.094

Problem 631	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	330	0	286	381	0	1410	641
N.S.	1	1.00	1.98	0.00	1.71	2.28	0.00	8.44	3.84
time (sec)	N/A	0.081	0.952	0.153	0.296	0.405	0.000	4.547	11.621

Problem 632	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	96	0	117	142	2837	373	197
N.S.	1	1.00	1.04	0.00	1.27	1.54	30.84	4.05	2.14
time (sec)	N/A	0.056	0.267	0.057	0.294	0.365	204.460	5.012	7.512

Problem 633	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	27	26	33	99	26	26
N.S.	1	1.00	1.00	1.04	1.00	1.27	3.81	1.00	1.00
time (sec)	N/A	0.019	0.030	0.115	0.278	0.355	0.533	3.941	6.322

Problem 634	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	99	0	0	0	0	0	-1
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.083	0.082	0.094	0.000	0.000	0.000	0.000	0.000

Problem 635	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	157	0	0	0	0	0	-1
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.166	0.426	0.081	0.000	0.000	0.000	0.000	0.000

Problem 636	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	305	305	260	0	0	0	0	0	-1
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.301	2.571	0.079	0.000	0.000	0.000	0.000	0.000

Problem 637	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.074	1.489	0.175	0.000	0.000	0.000	0.000	0.000

Problem 638	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.062	2.758	0.121	0.000	0.000	0.000	0.000	0.000

Problem 639	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.062	1.353	0.060	0.000	0.000	0.000	0.000	0.000

Problem 640	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.066	3.063	0.076	0.000	0.000	0.000	0.000	0.000

Problem 641	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-2)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.076	37.238	0.079	0.000	0.000	0.000	0.000	0.000

Problem 642	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.082	5.651	0.085	0.000	0.000	0.000	0.000	0.000

Problem 643	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.066	1.296	0.089	0.000	0.000	0.000	0.000	0.000

Problem 644	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.071	1.454	0.080	0.000	0.000	0.000	0.000	0.000

Problem 645	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.072	1.667	0.086	0.000	0.000	0.000	0.000	0.000

Problem 646	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.071	1.866	0.098	0.000	0.000	0.000	0.000	0.000

Problem 647	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	598	598	826	0	0	0	0	0	-1
N.S.	1	1.00	1.38	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.715	6.073	0.167	0.000	0.000	0.000	0.000	0.000

Problem 648	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	311	420	319	0	0	0	0	0	-1
N.S.	1	1.35	1.03	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.351	3.253	0.125	0.000	0.000	0.000	0.000	0.000

Problem 649	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	168	0	0	0	0	0	-1
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.203	0.618	0.144	0.000	0.000	0.000	0.000	0.000

Problem 650	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	132	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.046	0.272	0.131	0.000	0.000	0.000	0.000	0.000

Problem 651	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.081	1.117	0.095	0.000	0.000	0.000	0.000	0.000

Problem 652	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.067	3.604	0.116	0.000	0.000	0.000	0.000	0.000

Problem 653	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.073	2.852	0.125	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [273] had the largest ratio of [27]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	2	1.00	19	0.105
2	A	5	3	1.00	19	0.158
3	A	3	2	1.00	19	0.105
4	A	4	3	1.00	19	0.158
5	A	3	2	1.00	19	0.105
6	A	3	3	1.00	19	0.158
7	A	2	1	1.27	17	0.059
8	A	2	2	1.00	17	0.118
9	A	3	3	1.00	19	0.158
10	A	4	3	1.00	19	0.158
11	A	3	2	1.00	19	0.105
12	A	4	3	1.00	19	0.158
13	A	6	4	1.00	21	0.190
14	A	3	2	1.00	21	0.095
15	A	5	4	1.00	21	0.190
16	A	3	2	1.00	21	0.095
17	A	4	4	1.00	21	0.190
18	A	2	2	1.00	19	0.105
19	A	3	2	1.00	19	0.105
20	A	3	3	1.00	21	0.143
21	A	2	2	1.00	21	0.095
22	A	3	3	1.00	21	0.143
23	A	4	3	1.00	21	0.143
24	A	3	2	1.00	21	0.095
25	A	4	3	1.00	21	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	3	2	1.00	21	0.095
27	A	7	4	1.00	21	0.190
28	A	3	2	1.00	21	0.095
29	A	6	4	1.00	21	0.190
30	A	3	2	1.00	21	0.095
31	A	5	4	1.00	21	0.190
32	A	2	2	1.00	19	0.105
33	A	3	2	1.00	19	0.105
34	A	4	4	1.00	21	0.190
35	A	3	2	1.00	21	0.095
36	A	2	2	1.00	21	0.095
37	A	2	2	1.00	21	0.095
38	A	4	3	1.00	21	0.143
39	A	4	3	1.00	21	0.143
40	A	4	3	1.00	21	0.143
41	A	3	2	1.00	21	0.095
42	A	11	4	1.00	21	0.190
43	A	3	2	1.00	21	0.095
44	A	10	4	1.00	21	0.190
45	A	2	2	1.00	19	0.105
46	A	3	2	1.00	19	0.105
47	A	9	6	1.00	21	0.286
48	A	3	2	1.00	21	0.095
49	A	8	6	1.00	21	0.286
50	A	3	2	1.00	21	0.095
51	A	4	3	1.00	21	0.143
52	A	3	2	1.00	21	0.095
53	A	3	3	1.00	21	0.143
54	A	2	1	1.00	21	0.048
55	A	2	2	1.00	21	0.095
56	A	2	2	1.00	19	0.105
57	A	4	3	1.00	19	0.158
58	A	3	3	1.00	21	0.143
59	A	4	3	1.00	21	0.143
60	A	3	2	1.00	21	0.095

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	4	3	1.00	21	0.143
62	A	5	4	1.00	21	0.190
63	A	3	2	1.00	21	0.095
64	A	4	4	1.00	21	0.190
65	A	2	2	1.00	21	0.095
66	A	3	3	1.00	21	0.143
67	A	3	2	1.00	21	0.095
68	A	2	2	1.00	21	0.095
69	A	2	2	1.00	19	0.105
70	A	4	3	1.00	19	0.158
71	A	4	3	1.00	21	0.143
72	A	4	3	1.00	21	0.143
73	A	4	2	1.00	21	0.095
74	A	4	3	1.00	21	0.143
75	A	5	4	1.00	21	0.190
76	A	2	2	1.00	21	0.095
77	A	4	4	1.00	21	0.190
78	A	3	2	1.00	21	0.095
79	A	3	3	1.00	21	0.143
80	A	3	2	1.00	21	0.095
81	A	1	1	1.00	21	0.048
82	A	2	2	1.00	19	0.105
83	A	4	3	1.00	19	0.158
84	A	5	3	1.00	21	0.143
85	A	4	3	1.00	21	0.143
86	A	5	2	1.00	21	0.095
87	A	4	3	1.00	21	0.143
88	A	5	2	1.00	21	0.095
89	A	2	2	1.00	21	0.095
90	A	2	2	1.00	21	0.095
91	A	3	2	1.00	21	0.095
92	A	4	2	1.00	21	0.095
93	A	3	2	1.00	21	0.095
94	A	6	2	1.00	21	0.095
95	A	2	2	1.00	19	0.105

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	4	3	1.00	19	0.158
97	A	10	3	1.00	21	0.143
98	A	4	3	1.00	21	0.143
99	A	10	2	1.00	21	0.095
100	A	4	3	1.00	21	0.143
101	A	3	2	1.00	23	0.087
102	A	4	2	1.00	23	0.087
103	A	3	2	1.00	23	0.087
104	A	3	2	1.00	23	0.087
105	A	3	2	1.00	23	0.087
106	A	2	2	1.00	23	0.087
107	A	2	2	1.00	21	0.095
108	A	3	3	1.00	21	0.143
109	A	3	3	1.00	23	0.130
110	A	5	5	1.00	23	0.217
111	A	5	5	1.00	23	0.217
112	A	7	6	1.00	23	0.261
113	A	7	6	1.00	23	0.261
114	A	3	2	1.00	23	0.087
115	A	5	2	1.00	23	0.087
116	A	3	2	1.00	23	0.087
117	A	4	2	1.00	23	0.087
118	A	3	2	1.00	23	0.087
119	A	3	2	1.00	23	0.087
120	A	2	2	1.00	21	0.095
121	A	4	4	1.00	21	0.190
122	A	1	1	1.00	23	0.043
123	A	4	4	1.00	23	0.174
124	A	4	3	1.00	23	0.130
125	A	6	5	1.00	23	0.217
126	A	6	5	1.00	23	0.217
127	A	3	2	1.00	23	0.087
128	A	5	2	1.00	23	0.087
129	A	3	2	1.00	23	0.087
130	A	4	2	1.00	23	0.087

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
131	A	2	2	1.00	21	0.095
132	A	5	4	1.00	21	0.190
133	A	2	2	1.00	23	0.087
134	A	4	4	1.00	23	0.174
135	A	1	1	1.00	23	0.043
136	A	5	4	1.00	23	0.174
137	A	5	3	1.00	23	0.130
138	A	7	5	1.00	23	0.217
139	A	3	2	1.00	23	0.087
140	A	7	2	1.00	23	0.087
141	A	3	2	1.00	23	0.087
142	A	6	2	1.00	23	0.087
143	A	3	2	1.00	23	0.087
144	A	5	2	1.00	23	0.087
145	A	2	2	1.00	21	0.095
146	A	6	4	1.00	21	0.190
147	A	3	2	1.00	23	0.087
148	A	5	5	1.00	23	0.217
149	A	2	2	1.00	23	0.087
150	A	5	5	1.00	23	0.217
151	A	1	1	1.00	23	0.043
152	A	6	4	1.00	23	0.174
153	A	6	3	1.00	23	0.130
154	A	8	5	1.00	23	0.217
155	A	8	5	1.00	23	0.217
156	A	3	2	1.00	23	0.087
157	A	3	2	1.00	23	0.087
158	A	3	2	1.00	23	0.087
159	A	2	2	1.00	23	0.087
160	A	3	2	1.00	23	0.087
161	A	1	1	1.00	23	0.043
162	A	2	2	1.00	21	0.095
163	A	4	4	1.00	21	0.190
164	A	4	4	1.00	23	0.174
165	A	6	5	1.00	23	0.217

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
166	A	6	5	1.00	23	0.217
167	A	8	6	1.00	23	0.261
168	A	8	5	1.00	23	0.217
169	A	3	2	1.00	23	0.087
170	A	2	2	1.00	23	0.087
171	A	3	2	1.00	23	0.087
172	A	1	1	1.00	23	0.043
173	A	3	2	1.00	23	0.087
174	A	3	3	1.00	23	0.130
175	A	2	2	1.00	21	0.095
176	A	5	4	1.00	21	0.190
177	A	5	5	1.00	23	0.217
178	A	7	6	1.00	23	0.261
179	A	7	5	1.00	23	0.217
180	A	9	6	1.00	23	0.261
181	A	9	5	1.00	23	0.217
182	A	3	2	1.00	23	0.087
183	A	3	2	1.00	23	0.087
184	A	2	2	1.00	23	0.087
185	A	3	2	1.00	23	0.087
186	A	1	1	1.00	23	0.043
187	A	3	2	1.00	23	0.087
188	A	4	3	1.00	23	0.130
189	A	3	2	1.00	23	0.087
190	A	3	3	1.00	23	0.130
191	A	2	2	1.00	21	0.095
192	A	6	4	1.00	21	0.190
193	A	6	5	1.00	23	0.217
194	A	8	6	1.00	23	0.261
195	A	8	5	1.00	23	0.217
196	A	5	4	1.00	23	0.174
197	A	4	4	1.00	23	0.174
198	A	4	4	1.00	23	0.174
199	A	3	3	1.00	23	0.130
200	A	3	3	1.00	23	0.130

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
201	A	4	4	1.00	23	0.174
202	A	4	4	1.00	23	0.174
203	A	5	4	1.00	23	0.174
204	A	6	5	1.00	25	0.200
205	A	5	5	1.00	25	0.200
206	A	5	5	1.00	25	0.200
207	A	4	4	1.00	25	0.160
208	A	4	4	1.00	25	0.160
209	A	4	4	1.00	25	0.160
210	A	4	4	1.00	25	0.160
211	A	5	5	1.00	25	0.200
212	A	4	4	1.00	25	0.160
213	A	5	4	1.00	25	0.160
214	A	7	5	1.00	25	0.200
215	A	6	5	1.00	25	0.200
216	A	6	5	1.00	25	0.200
217	A	5	4	1.00	25	0.160
218	A	5	4	1.00	25	0.160
219	A	5	5	1.00	25	0.200
220	A	5	5	1.00	25	0.200
221	A	5	5	1.00	25	0.200
222	A	5	5	1.00	25	0.200
223	A	6	5	1.00	25	0.200
224	A	7	5	1.00	25	0.200
225	A	6	4	1.00	25	0.160
226	A	6	4	1.00	25	0.160
227	A	6	5	1.00	25	0.200
228	A	6	5	1.00	25	0.200
229	A	5	4	1.00	25	0.160
230	A	5	4	1.00	25	0.160
231	A	6	6	1.00	25	0.240
232	A	6	6	1.00	25	0.240
233	A	5	4	1.00	25	0.160
234	A	4	4	1.00	25	0.160
235	A	4	4	1.00	25	0.160

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
236	A	3	3	1.00	25	0.120
237	A	3	3	1.00	25	0.120
238	A	3	3	1.00	25	0.120
239	A	3	3	1.00	25	0.120
240	A	4	4	1.00	25	0.160
241	A	4	4	1.00	25	0.160
242	A	5	4	1.00	25	0.160
243	A	5	4	1.00	25	0.160
244	A	4	4	1.00	25	0.160
245	A	4	4	1.00	25	0.160
246	A	3	3	1.00	25	0.120
247	A	3	3	1.00	25	0.120
248	A	4	4	1.00	25	0.160
249	A	4	4	1.00	25	0.160
250	A	5	5	1.00	25	0.200
251	A	5	5	1.00	25	0.200
252	A	6	5	1.00	25	0.200
253	A	6	5	1.00	25	0.200
254	A	5	5	1.00	25	0.200
255	A	5	5	1.00	25	0.200
256	A	4	4	1.00	25	0.160
257	A	4	4	1.00	25	0.160
258	A	4	4	1.00	25	0.160
259	A	4	4	1.00	25	0.160
260	A	5	4	1.00	25	0.160
261	A	5	4	1.00	25	0.160
262	A	6	5	1.00	25	0.200
263	A	6	4	1.00	25	0.160
264	A	5	4	1.00	25	0.160
265	A	5	4	1.00	25	0.160
266	A	4	3	1.00	25	0.120
267	A	4	3	1.00	25	0.120
268	A	5	5	1.00	25	0.200
269	A	5	5	1.00	25	0.200
270	A	6	4	1.00	25	0.160

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
271	A	6	4	1.00	25	0.160
272	A	7	5	1.00	25	0.200
273	A	8	8	1.00	27	0.296
274	A	7	7	1.00	27	0.259
275	A	6	6	1.00	27	0.222
276	A	1	1	1.00	27	0.037
277	A	2	2	1.00	27	0.074
278	A	3	2	1.00	27	0.074
279	A	4	2	1.00	27	0.074
280	A	10	9	1.00	27	0.333
281	A	9	8	1.00	27	0.296
282	A	8	7	1.00	27	0.259
283	A	7	7	1.00	27	0.259
284	A	7	7	1.00	27	0.259
285	A	1	1	1.00	27	0.037
286	A	2	2	1.00	27	0.074
287	A	3	2	1.00	27	0.074
288	A	4	2	1.00	27	0.074
289	A	10	8	1.00	27	0.296
290	A	9	7	1.00	27	0.259
291	A	8	7	1.00	27	0.259
292	A	8	8	1.00	27	0.296
293	A	7	7	1.00	27	0.259
294	A	1	1	1.00	27	0.037
295	A	2	2	1.00	27	0.074
296	A	3	2	1.00	27	0.074
297	A	4	2	1.00	27	0.074
298	A	8	8	1.00	27	0.296
299	A	7	7	1.00	27	0.259
300	A	6	6	1.00	27	0.222
301	A	1	1	1.00	27	0.037
302	A	2	2	1.00	27	0.074
303	A	3	2	1.00	27	0.074
304	A	4	2	1.00	27	0.074
305	A	8	8	1.00	27	0.296

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
306	A	7	7	1.00	27	0.259
307	A	8	8	1.00	27	0.296
308	A	1	1	1.00	27	0.037
309	A	2	2	1.00	27	0.074
310	A	3	2	1.00	27	0.074
311	A	4	2	1.00	27	0.074
312	A	5	2	1.00	27	0.074
313	A	9	9	1.00	27	0.333
314	A	8	8	1.00	27	0.296
315	A	7	7	1.00	27	0.259
316	A	1	1	1.00	27	0.037
317	A	2	2	1.00	27	0.074
318	A	3	2	1.00	27	0.074
319	A	4	2	1.00	27	0.074
320	A	5	2	1.00	27	0.074
321	A	3	3	1.00	27	0.111
322	A	3	3	1.00	27	0.111
323	A	3	3	1.00	27	0.111
324	A	3	3	1.00	27	0.111
325	A	3	3	1.00	27	0.111
326	A	3	3	1.00	27	0.111
327	A	2	2	1.00	23	0.087
328	A	2	2	1.00	23	0.087
329	A	2	2	1.00	23	0.087
330	A	2	2	1.00	21	0.095
331	A	2	2	1.00	23	0.087
332	A	2	2	1.00	23	0.087
333	A	2	2	1.00	23	0.087
334	A	2	2	1.00	23	0.087
335	A	3	3	1.00	25	0.120
336	A	3	3	1.00	25	0.120
337	A	3	3	1.00	25	0.120
338	A	3	3	1.00	25	0.120
339	A	3	3	1.00	25	0.120
340	A	3	3	1.00	25	0.120

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
341	A	3	3	1.00	25	0.120
342	A	3	3	1.00	23	0.130
343	A	3	2	1.00	21	0.095
344	A	3	2	1.00	21	0.095
345	A	3	2	1.00	21	0.095
346	A	2	2	1.00	19	0.105
347	A	2	2	1.00	19	0.105
348	A	2	2	1.00	21	0.095
349	A	2	2	1.00	21	0.095
350	A	3	3	1.00	21	0.143
351	A	3	3	1.00	21	0.143
352	A	3	3	1.00	21	0.143
353	A	3	3	1.00	21	0.143
354	A	3	3	1.00	25	0.120
355	A	3	3	1.00	25	0.120
356	A	3	3	1.00	25	0.120
357	A	3	3	1.00	25	0.120
358	A	3	3	1.00	25	0.120
359	A	3	3	1.00	25	0.120
360	A	4	2	1.00	27	0.074
361	A	3	2	1.00	27	0.074
362	A	2	2	1.00	27	0.074
363	A	1	1	1.00	27	0.037
364	A	3	3	1.00	25	0.120
365	A	3	3	1.00	27	0.111
366	A	3	3	1.00	27	0.111
367	A	3	2	1.00	27	0.074
368	A	2	2	1.00	27	0.074
369	A	1	1	1.00	27	0.037
370	A	3	3	1.00	27	0.111
371	A	3	3	1.00	27	0.111
372	A	4	4	1.00	27	0.148
373	A	4	4	1.00	27	0.148
374	A	4	4	1.00	25	0.160
375	A	4	4	1.00	27	0.148

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
376	A	4	3	1.00	19	0.158
377	A	3	2	1.00	19	0.105
378	A	2	1	1.27	17	0.059
379	A	4	3	1.00	17	0.176
380	A	3	3	1.00	19	0.158
381	A	4	4	1.00	19	0.210
382	A	4	3	1.00	19	0.158
383	A	3	3	1.00	19	0.158
384	A	3	3	1.00	19	0.158
385	A	3	2	1.00	19	0.105
386	A	3	2	1.00	19	0.105
387	A	4	3	1.00	21	0.143
388	A	3	2	1.00	21	0.095
389	A	2	2	1.00	19	0.105
390	A	6	4	1.00	19	0.210
391	A	3	3	1.00	21	0.143
392	A	4	4	1.00	21	0.190
393	A	6	4	1.00	21	0.190
394	A	5	4	1.00	21	0.190
395	A	4	4	1.00	21	0.190
396	A	3	2	1.00	21	0.095
397	A	4	4	1.00	21	0.190
398	A	4	3	1.00	21	0.143
399	A	4	3	1.00	21	0.143
400	A	3	2	1.00	21	0.095
401	A	3	2	1.00	21	0.095
402	A	2	2	1.00	19	0.105
403	A	6	4	1.00	19	0.210
404	A	6	5	1.00	21	0.238
405	A	4	4	1.00	21	0.190
406	A	6	5	1.00	21	0.238
407	A	5	5	1.00	21	0.238
408	A	2	2	1.00	21	0.095
409	A	5	5	1.00	21	0.238
410	A	5	5	1.00	21	0.238

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
411	A	5	4	1.00	21	0.190
412	A	5	4	1.00	21	0.190
413	A	3	2	1.00	21	0.095
414	A	3	2	1.00	21	0.095
415	A	2	2	1.00	19	0.105
416	A	6	4	1.00	19	0.210
417	A	7	5	1.00	21	0.238
418	A	8	6	1.00	21	0.286
419	A	10	5	1.00	21	0.238
420	A	7	3	1.00	21	0.143
421	A	7	4	1.00	21	0.190
422	A	7	4	1.00	21	0.190
423	A	7	4	1.00	21	0.190
424	A	10	6	1.00	21	0.286
425	A	3	2	1.00	21	0.095
426	A	3	2	1.00	21	0.095
427	A	2	2	1.00	19	0.105
428	A	6	4	1.00	19	0.210
429	A	4	3	1.00	21	0.143
430	A	5	4	1.00	21	0.190
431	A	7	6	1.00	21	0.286
432	A	6	6	1.00	21	0.286
433	A	5	5	1.00	21	0.238
434	A	5	5	1.00	21	0.238
435	A	6	6	1.00	21	0.286
436	A	7	6	1.00	21	0.286
437	A	3	2	1.00	21	0.095
438	A	3	2	1.00	21	0.095
439	A	3	2	1.00	21	0.095
440	A	2	2	1.00	19	0.105
441	A	4	3	1.00	19	0.158
442	A	4	3	1.00	21	0.143
443	A	5	4	1.00	21	0.190
444	A	7	6	1.00	21	0.286
445	A	6	6	1.00	21	0.286

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
446	A	5	5	1.00	21	0.238
447	A	6	6	1.00	21	0.286
448	A	7	6	1.00	21	0.286
449	A	3	2	1.00	21	0.095
450	A	3	2	1.00	21	0.095
451	A	3	2	1.00	21	0.095
452	A	2	2	1.00	19	0.105
453	A	4	3	1.00	19	0.158
454	A	4	3	1.00	21	0.143
455	A	5	4	1.00	21	0.190
456	A	7	7	1.00	21	0.333
457	A	6	6	1.00	21	0.286
458	A	6	6	1.00	21	0.286
459	A	7	7	1.00	21	0.333
460	A	8	7	1.00	21	0.333
461	A	3	2	1.00	21	0.095
462	A	3	2	1.00	21	0.095
463	A	3	2	1.00	21	0.095
464	A	2	2	1.00	19	0.105
465	A	4	3	1.00	19	0.158
466	A	4	3	1.00	21	0.143
467	A	11	7	1.00	21	0.333
468	A	11	7	1.00	21	0.333
469	A	11	7	1.00	21	0.333
470	A	11	6	1.00	21	0.286
471	A	12	7	1.00	21	0.333
472	A	13	7	1.00	21	0.333
473	A	3	2	1.00	23	0.087
474	A	3	2	1.00	23	0.087
475	A	2	2	1.00	21	0.095
476	A	5	4	1.00	21	0.190
477	A	6	5	1.00	23	0.217
478	A	7	6	1.00	23	0.261
479	A	8	8	1.00	23	0.348
480	A	7	7	1.00	23	0.304

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
481	A	7	7	1.00	23	0.304
482	A	7	7	1.00	23	0.304
483	A	3	2	1.00	23	0.087
484	A	3	2	1.00	23	0.087
485	A	2	2	1.00	21	0.095
486	A	6	5	1.00	21	0.238
487	A	6	5	1.00	23	0.217
488	A	7	6	1.00	23	0.261
489	A	8	7	1.00	23	0.304
490	A	7	7	1.00	23	0.304
491	A	6	6	1.00	23	0.261
492	A	7	7	1.00	23	0.304
493	A	8	7	1.00	23	0.304
494	A	3	2	1.00	23	0.087
495	A	3	2	1.00	23	0.087
496	A	2	2	1.00	21	0.095
497	A	7	6	1.00	21	0.286
498	A	7	6	1.00	23	0.261
499	A	7	6	1.00	23	0.261
500	A	9	8	1.00	23	0.348
501	A	8	8	1.00	23	0.348
502	A	7	7	1.00	23	0.304
503	A	7	7	1.00	23	0.304
504	A	8	8	1.00	23	0.348
505	A	9	8	1.00	23	0.348
506	A	3	2	1.00	23	0.087
507	A	3	2	1.00	23	0.087
508	A	2	2	1.00	21	0.095
509	A	5	4	1.00	21	0.190
510	A	6	5	1.00	23	0.217
511	A	7	6	1.00	23	0.261
512	A	7	7	1.00	23	0.304
513	A	6	6	1.00	23	0.261
514	A	6	6	1.00	23	0.261
515	A	7	7	1.00	23	0.304

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
516	A	3	2	1.00	23	0.087
517	A	3	2	1.00	23	0.087
518	A	2	2	1.00	21	0.095
519	A	6	5	1.00	21	0.238
520	A	7	6	1.00	23	0.261
521	A	8	7	1.00	23	0.304
522	A	8	7	1.00	23	0.304
523	A	7	7	1.00	23	0.304
524	A	6	6	1.00	23	0.261
525	A	7	7	1.00	23	0.304
526	A	8	7	1.00	23	0.304
527	A	3	2	1.00	23	0.087
528	A	3	2	1.00	23	0.087
529	A	2	2	1.00	21	0.095
530	A	7	6	1.00	21	0.286
531	A	8	6	1.00	23	0.261
532	A	9	7	1.00	23	0.304
533	A	9	8	1.00	23	0.348
534	A	8	8	1.00	23	0.348
535	A	7	7	1.00	23	0.304
536	A	7	7	1.00	23	0.304
537	A	8	8	1.00	23	0.348
538	A	9	8	1.00	23	0.348
539	A	5	4	1.00	23	0.174
540	A	4	4	1.00	23	0.174
541	A	4	4	1.00	23	0.174
542	A	3	3	1.00	23	0.130
543	A	3	3	1.00	23	0.130
544	A	4	4	1.00	23	0.174
545	A	4	4	1.00	23	0.174
546	A	5	4	1.00	23	0.174
547	A	6	5	1.00	25	0.200
548	A	5	5	1.00	25	0.200
549	A	5	5	1.00	25	0.200
550	A	4	4	1.00	25	0.160

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
551	A	4	4	1.00	25	0.160
552	A	4	4	1.00	25	0.160
553	A	4	4	1.00	25	0.160
554	A	5	5	1.00	25	0.200
555	A	7	6	1.00	25	0.240
556	A	6	6	1.00	25	0.240
557	A	6	6	1.00	25	0.240
558	A	5	5	1.00	25	0.200
559	A	5	5	1.00	25	0.200
560	A	5	5	1.00	25	0.200
561	A	5	5	1.00	25	0.200
562	A	5	5	1.00	25	0.200
563	A	5	5	1.00	25	0.200
564	A	8	6	1.00	25	0.240
565	A	7	6	1.00	25	0.240
566	A	7	6	1.00	25	0.240
567	A	6	5	1.00	25	0.200
568	A	6	5	1.00	25	0.200
569	A	6	5	1.00	25	0.200
570	A	6	5	1.00	25	0.200
571	A	6	6	1.00	25	0.240
572	A	6	6	1.00	25	0.240
573	A	6	5	1.00	25	0.200
574	A	15	12	1.00	25	0.480
575	A	14	12	1.00	25	0.480
576	A	14	12	1.00	25	0.480
577	A	13	11	1.00	25	0.440
578	A	13	11	1.00	25	0.440
579	A	9	7	1.00	25	0.280
580	A	9	7	1.00	25	0.280
581	A	13	11	1.00	25	0.440
582	A	13	11	1.00	25	0.440
583	A	14	12	1.00	25	0.480
584	A	15	12	1.00	25	0.480
585	A	14	12	1.00	25	0.480

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
586	A	14	12	1.00	25	0.480
587	A	13	11	1.00	25	0.440
588	A	13	11	1.00	25	0.440
589	A	13	11	1.00	25	0.440
590	A	13	11	1.00	25	0.440
591	A	14	12	1.00	25	0.480
592	A	14	12	1.00	25	0.480
593	A	15	12	1.00	25	0.480
594	A	15	13	1.00	25	0.520
595	A	15	13	1.00	25	0.520
596	A	14	12	1.00	25	0.480
597	A	14	12	1.00	25	0.480
598	A	14	12	1.00	25	0.480
599	A	14	12	1.00	25	0.480
600	A	14	12	1.00	25	0.480
601	A	14	12	1.00	25	0.480
602	A	15	13	1.00	25	0.520
603	A	15	13	1.00	25	0.520
604	A	16	13	1.00	25	0.520
605	A	16	13	1.00	25	0.520
606	A	15	12	1.00	25	0.480
607	A	15	12	1.00	25	0.480
608	A	15	13	1.00	25	0.520
609	A	15	13	1.00	25	0.520
610	A	15	12	1.00	25	0.480
611	A	15	12	1.00	25	0.480
612	A	15	12	1.00	25	0.480
613	A	15	12	1.00	25	0.480
614	A	16	13	1.00	25	0.520
615	B	2	2	2.04	27	0.074
616	A	4	4	1.00	23	0.174
617	A	3	3	1.00	23	0.130
618	A	2	2	1.00	21	0.095
619	A	1	1	1.00	23	0.043
620	A	1	1	1.00	23	0.043

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
621	A	1	1	1.00	23	0.043
622	A	1	1	1.00	23	0.043
623	A	2	2	1.00	25	0.080
624	A	2	2	1.00	25	0.080
625	A	2	2	1.00	25	0.080
626	A	2	2	1.00	25	0.080
627	A	2	2	1.00	25	0.080
628	A	2	2	1.00	25	0.080
629	A	2	2	1.00	23	0.087
630	A	3	2	1.00	21	0.095
631	A	3	2	1.00	21	0.095
632	A	3	2	1.00	21	0.095
633	A	2	2	1.00	19	0.105
634	A	5	3	1.00	19	0.158
635	A	6	4	1.00	21	0.190
636	A	7	5	1.00	21	0.238
637	A	2	2	1.00	21	0.095
638	A	2	2	1.00	21	0.095
639	A	2	2	1.00	21	0.095
640	A	2	2	1.00	21	0.095
641	A	2	2	1.00	25	0.080
642	A	2	2	1.00	25	0.080
643	A	2	2	1.00	25	0.080
644	A	2	2	1.00	25	0.080
645	A	2	2	1.00	25	0.080
646	A	2	2	1.00	25	0.080
647	A	9	7	1.00	27	0.259
648	A	5	5	1.35	27	0.185
649	A	3	3	1.00	27	0.111
650	A	1	1	1.00	27	0.037
651	A	2	2	1.00	25	0.080
652	A	2	2	1.00	27	0.074
653	A	2	2	1.00	27	0.074

Chapter 3

Listing of integrals

Local contents

3.1 $\int \cos^7(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=87

$$\frac{8(a+a\sin(c+dx))^5}{5a^4d}-\frac{2(a+a\sin(c+dx))^6}{a^5d}+\frac{6(a+a\sin(c+dx))^7}{7a^6d}-\frac{(a+a\sin(c+dx))^8}{8a^7d}$$

```
[Out] 8/5*(a+a*sin(d*x+c))^5/a^4/d-2*(a+a*sin(d*x+c))^6/a^5/d+6/7*(a+a*sin(d*x+c)
)^7/a^6/d-1/8*(a+a*sin(d*x+c))^8/a^7/d
```

Rubi [A]

time = 0.04, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2746, 45}

$$-\frac{(a\sin(c+dx)+a)^8}{8a^7d}+\frac{6(a\sin(c+dx)+a)^7}{7a^6d}-\frac{2(a\sin(c+dx)+a)^6}{a^5d}+\frac{8(a\sin(c+dx)+a)^5}{5a^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*(a + a*Sin[c + d*x]),x]
```

```
[Out] (8*(a + a*Sin[c + d*x])^5)/(5*a^4*d) - (2*(a + a*Sin[c + d*x])^6)/(a^5*d) +
 (6*(a + a*Sin[c + d*x])^7)/(7*a^6*d) - (a + a*Sin[c + d*x])^8/(8*a^7*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \cos^7(c+dx)(a+a\sin(c+dx))\,dx &= \frac{\operatorname{Subst}\left(\int (a-x)^3(a+x)^4\,dx,x,a\sin(c+dx)\right)}{a^7d}\\ &= \frac{\operatorname{Subst}\left(\int \left(8a^3(a+x)^4-12a^2(a+x)^5+6a(a+x)^6-(a+x)^7\right)\,dx,x\right.}{a^7d}\\ &= \frac{8(a+a\sin(c+dx))^5}{5a^4d}-\frac{2(a+a\sin(c+dx))^6}{a^5d}+\frac{6(a+a\sin(c+dx))}{7a^6d}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 79, normalized size = 0.91

$$-\frac{a\cos^8(c+dx)}{8d}+\frac{35a\sin(c+dx)}{64d}+\frac{7a\sin(3(c+dx))}{64d}+\frac{7a\sin(5(c+dx))}{320d}+\frac{a\sin(7(c+dx))}{448d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/8*(a*Cos[c + d*x]^8)/d + (35*a*Sin[c + d*x])/(64*d) + (7*a*Sin[3*(c + d*
x)])/(64*d) + (7*a*Sin[5*(c + d*x)])/(320*d) + (a*Sin[7*(c + d*x)])/(448*d)
```

Maple [A]
time = 0.21, size = 56, normalized size = 0.64

method	result
derivativedivides	$\frac{-\frac{a\left(\cos^8(dx+c)\right)}{8}+\frac{a\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6\left(\cos^4(dx+c)\right)}{5}+\frac{8\left(\cos^2(dx+c)\right)}{5}\right)\sin(dx+c)}{7}}{d}$
default	$\frac{-\frac{a\left(\cos^8(dx+c)\right)}{8}+\frac{a\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6\left(\cos^4(dx+c)\right)}{5}+\frac{8\left(\cos^2(dx+c)\right)}{5}\right)\sin(dx+c)}{7}}{d}$
risch	$\frac{35a\sin(dx+c)}{64d}-\frac{a\cos(8dx+8c)}{1024d}+\frac{a\sin(7dx+7c)}{448d}-\frac{a\cos(6dx+6c)}{128d}+\frac{7a\sin(5dx+5c)}{320d}-\frac{7a\cos(4dx+4c)}{256d}+\frac{7a}{}$
norman	$\frac{\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{6a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{106a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5d}+\frac{1026a\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{35d}+\frac{1026a\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{35d}+\frac{106a\left(\tan^{11}\left(\frac{dx}{2}+\right.\right.}{5d}}{\left(1+\mathrm{ta}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/8*a*cos(d*x+c)^8+1/7*a*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(
d*x+c)^2)*sin(d*x+c))
```

Maxima [A]
time = 0.31, size = 92, normalized size = 1.06

$$-\frac{35\,a\sin(dx+c)^8+40\,a\sin(dx+c)^7-140\,a\sin(dx+c)^6-168\,a\sin(dx+c)^5+210\,a\sin(dx+c)^4+280\,a\sin(dx+c)^3-140\,a\sin(dx+c)^2-280\,a\sin(dx+c)}{280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/280*(35*a*sin(d*x + c)^8 + 40*a*sin(d*x + c)^7 - 140*a*sin(d*x + c)^6 -
168*a*sin(d*x + c)^5 + 210*a*sin(d*x + c)^4 + 280*a*sin(d*x + c)^3 - 140*a*
sin(d*x + c)^2 - 280*a*sin(d*x + c))/d
```

Fricas [A]
time = 0.37, size = 62, normalized size = 0.71

$$-\frac{35\,a\cos\left(dx+c\right)^{8}-8\left(5\,a\cos\left(dx+c\right)^{6}+6\,a\cos\left(dx+c\right)^{4}+8\,a\cos\left(dx+c\right)^{2}+16\,a\right)\sin\left(dx+c\right)}{280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/280*(35*a*cos(d*x + c)^8 - 8*(5*a*cos(d*x + c)^6 + 6*a*cos(d*x + c)^4 +
8*a*cos(d*x + c)^2 + 16*a)*sin(d*x + c))/d
```

Sympy [A]
time = 0.88, size = 105, normalized size = 1.21

$$\begin{cases}\frac{16a\sin^{7}(c+dx)}{35d}+\frac{8a\sin^{5}(c+dx)\cos^{2}(c+dx)}{5d}+\frac{2a\sin^{3}(c+dx)\cos^{4}(c+dx)}{d}+\frac{a\sin(c+dx)\cos^{6}(c+dx)}{d}-\frac{a\cos^{8}(c+dx)}{8d} & \text{for } d\neq 0\\ x(a\sin(c)+a)\cos^{7}(c) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((16*a*sin(c + d*x)**7/(35*d) + 8*a*sin(c + d*x)**5*cos(c + d*x)**
2/(5*d) + 2*a*sin(c + d*x)**3*cos(c + d*x)**4/d + a*sin(c + d*x)*cos(c + d*
x)**6/d - a*cos(c + d*x)**8/(8*d), Ne(d, 0)), (x*(a*sin(c) + a)*cos(c)**7,
True))
```

Giac [A]
time = 6.13, size = 118, normalized size = 1.36

$$-\frac{a\cos\left(8\,dx+8\,c\right)}{1024\,d}-\frac{a\cos\left(6\,dx+6\,c\right)}{128\,d}-\frac{7\,a\cos\left(4\,dx+4\,c\right)}{256\,d}-\frac{7\,a\cos\left(2\,dx+2\,c\right)}{128\,d}+\frac{a\sin\left(7\,dx+7\,c\right)}{448\,d}+\frac{7\,a\sin\left(5\,dx+5\,c\right)}{320\,d}+\frac{7\,a\sin\left(3\,dx+3\,c\right)}{64\,d}+\frac{35\,a\sin\left(dx+c\right)}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/1024*a*cos(8*d*x + 8*c)/d - 1/128*a*cos(6*d*x + 6*c)/d - 7/256*a*cos(4*d
*x + 4*c)/d - 7/128*a*cos(2*d*x + 2*c)/d + 1/448*a*sin(7*d*x + 7*c)/d + 7/3
20*a*sin(5*d*x + 5*c)/d + 7/64*a*sin(3*d*x + 3*c)/d + 35/64*a*sin(d*x + c)/
d
```

Mupad [B]
time = 0.09, size = 90, normalized size = 1.03

$$\frac{-\frac{a\sin(c+d\,x)^8}{8}-\frac{a\sin(c+d\,x)^7}{7}+\frac{a\sin(c+d\,x)^6}{2}+\frac{3\,a\sin(c+d\,x)^5}{5}-\frac{3\,a\sin(c+d\,x)^4}{4}-a\sin\left(c+d\,x\right)^3+\frac{a\sin(c+d\,x)^2}{2}+a\,\sin\left(c+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7*(a + a*sin(c + d*x)),x)
```

```
[Out] (a*sin(c + d*x) + (a*sin(c + d*x)^2)/2 - a*sin(c + d*x)^3 - (3*a*sin(c + d*
x)^4)/4 + (3*a*sin(c + d*x)^5)/5 + (a*sin(c + d*x)^6)/2 - (a*sin(c + d*x)^7
)/7 - (a*sin(c + d*x)^8)/8)/d
```

3.2 $\int \cos^6(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=87

$$\frac{5ax}{16}-\frac{a\cos^7(c+dx)}{7d}+\frac{5a\cos(c+dx)\sin(c+dx)}{16d}+\frac{5a\cos^3(c+dx)\sin(c+dx)}{24d}+\frac{a\cos^5(c+dx)\sin(c+dx)}{6d}$$

```
[Out] 5/16*a*x-1/7*a*cos(d*x+c)^7/d+5/16*a*cos(d*x+c)*sin(d*x+c)/d+5/24*a*cos(d*x
+c)^3*sin(d*x+c)/d+1/6*a*cos(d*x+c)^5*sin(d*x+c)/d
```

Rubi [A]
time = 0.04, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2748, 2715, 8}

$$-\frac{a\cos^7(c+dx)}{7d}+\frac{a\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{5a\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{5a\sin(c+dx)\cos(c+dx)}{16d}+\frac{5ax}{16}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*(a + a*Sin[c + d*x]),x]
```

```
[Out] (5*a*x)/16 - (a*Cos[c + d*x]^7)/(7*d) + (5*a*Cos[c + d*x]*Sin[c + d*x])/(16
*d) + (5*a*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) + (a*Cos[c + d*x]^5*Sin[c +
d*x])/(6*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\int \cos^6(c+dx)(a+a\sin(c+dx))\,dx = -\frac{a\cos^7(c+dx)}{7d} + a\int \cos^6(c+dx)\,dx$$
$$= -\frac{a\cos^7(c+dx)}{7d} + \frac{a\cos^5(c+dx)\sin(c+dx)}{6d} + \frac{1}{6}(5a)\int \cos^4(c+dx$$
$$= -\frac{a\cos^7(c+dx)}{7d} + \frac{5a\cos^3(c+dx)\sin(c+dx)}{24d} + \frac{a\cos^5(c+dx)\sin(c}{6d}$$
$$= -\frac{a\cos^7(c+dx)}{7d} + \frac{5a\cos(c+dx)\sin(c+dx)}{16d} + \frac{5a\cos^3(c+dx)\sin(c}{24d}$$
$$= \frac{5ax}{16} - \frac{a\cos^7(c+dx)}{7d} + \frac{5a\cos(c+dx)\sin(c+dx)}{16d} + \frac{5a\cos^3(c+dx}{24d}$$

Mathematica [A]
time = 0.11, size = 57, normalized size = 0.66

$$\frac{a(-192\cos^7(c+dx)+7(60c+60dx+45\sin(2(c+dx))+9\sin(4(c+dx))+\sin(6(c+dx))))}{1344d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(a + a*Sin[c + d*x]),x]
[Out] (a*(-192*Cos[c + d*x]^7 + 7*(60*c + 60*d*x + 45*Sin[2*(c + d*x)] + 9*Sin[4*
(c + d*x)] + Sin[6*(c + d*x)])))/(1344*d)
```

Maple [A]
time = 0.16, size = 62, normalized size = 0.71

method	result
derivativedivides	$\frac{-\frac{a\left(\cos^7(dx+c)\right)}{7}+a\left(\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)}{d}$
default	$\frac{-\frac{a\left(\cos^7(dx+c)\right)}{7}+a\left(\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)}{d}$
risch	$\frac{5ax}{16}-\frac{5a\cos(dx+c)}{64d}-\frac{a\cos(7dx+7c)}{448d}+\frac{a\sin(6dx+6c)}{192d}-\frac{a\cos(5dx+5c)}{64d}+\frac{3a\sin(4dx+4c)}{64d}-\frac{3a\cos(3dx+3c)}{64d}-$
norman	$\frac{\frac{5ax}{16}-\frac{2a}{7d}+\frac{11a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d}+\frac{7a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6d}+\frac{85a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d}-\frac{85a\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d}-\frac{7a\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6d}-\frac{11a\left(\tan^{13}\left(\frac{d}{}\right.\right.}{8d}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/7*a*cos(d*x+c)^7+a*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x
+c))*sin(d*x+c)+5/16*d*x+5/16*c))
```

Maxima [A]
time = 0.30, size = 63, normalized size = 0.72

$$-\frac{192\,a\cos\left(dx+c\right)^{7}+7\,\left(4\,\sin\left(2\,dx+2\,c\right)^{3}-60\,dx-60\,c-9\,\sin\left(4\,dx+4\,c\right)-48\,\sin\left(2\,dx+2\,c\right)\right)a}{1344\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/1344*(192*a*cos(d*x + c)^7 + 7*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9
*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*a)/d
```

Fricas [A]
time = 0.36, size = 62, normalized size = 0.71

$$-\frac{48\,a\cos\left(dx+c\right)^{7}-105\,adx-7\,\left(8\,a\cos\left(dx+c\right)^{5}+10\,a\cos\left(dx+c\right)^{3}+15\,a\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{336\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/336*(48*a*cos(d*x + c)^7 - 105*a*d*x - 7*(8*a*cos(d*x + c)^5 + 10*a*cos(
d*x + c)^3 + 15*a*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(82) = 164$.
time = 0.71, size = 172, normalized size = 1.98

$$\begin{cases} \frac{5ax\sin^{6}{\left(c+dx\right)}}{16} + \frac{15ax\sin^{4}{\left(c+dx\right)}\cos^{2}{\left(c+dx\right)}}{16} + \frac{15ax\sin^{2}{\left(c+dx\right)}\cos^{4}{\left(c+dx\right)}}{16} + \frac{5ax\cos^{6}{\left(c+dx\right)}}{16} + \frac{5a\sin^{5}{\left(c+dx\right)}\cos{\left(c+dx\right)}}{16d} + \frac{5a\sin^{3}{\left(c+dx\right)}\cos^{3}{\left(c+dx\right)}}{6d} + \frac{11a\sin{\left(c+dx\right)}\cos^{5}{\left(c+dx\right)}}{16d} - \frac{a\cos^{7}{\left(c+dx\right)}}{7d} & \text{for}\: d \neq 0 \\ x\left(a\sin{\left(c\right)}+a\right)\cos^{6}{\left(c\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((5*a*x*sin(c + d*x)**6/16 + 15*a*x*sin(c + d*x)**4*cos(c + d*x)**
2/16 + 15*a*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 5*a*x*cos(c + d*x)**6/16
 + 5*a*sin(c + d*x)**5*cos(c + d*x)/(16*d) + 5*a*sin(c + d*x)**3*cos(c + d*
x)**3/(6*d) + 11*a*sin(c + d*x)*cos(c + d*x)**5/(16*d) - a*cos(c + d*x)**7/
(7*d), Ne(d, 0)), (x*(a*sin(c) + a)*cos(c)**6, True))
```

Giac [A]
time = 7.09, size = 107, normalized size = 1.23

$$\frac{5}{16}\,ax-\frac{a\cos\left(7\,dx+7\,c\right)}{448\,d}-\frac{a\cos\left(5\,dx+5\,c\right)}{64\,d}-\frac{3\,a\cos\left(3\,dx+3\,c\right)}{64\,d}-\frac{5\,a\cos\left(dx+c\right)}{64\,d}+\frac{a\sin\left(6\,dx+6\,c\right)}{192\,d}+\frac{3\,a\sin\left(4\,dx+4\,c\right)}{64\,d}+\frac{15\,a\sin\left(2\,dx+2\,c\right)}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 5/16*a*x - 1/448*a*cos(7*d*x + 7*c)/d - 1/64*a*cos(5*d*x + 5*c)/d - 3/64*a*
cos(3*d*x + 3*c)/d - 5/64*a*cos(d*x + c)/d + 1/192*a*sin(6*d*x + 6*c)/d + 3
/64*a*sin(4*d*x + 4*c)/d + 15/64*a*sin(2*d*x + 2*c)/d
```

Mupad [B]
time = 8.23, size = 226, normalized size = 2.60

$$\frac{5\,a\,x}{16}+\frac{-\frac{11\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{8}+\left(\frac{a\,\left(735\,c+735\,d\,x-672\right)}{336}-\frac{35\,a\,\left(c+d\,x\right)}{16}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-\frac{7\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{6}-\frac{85\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{24}+\left(\frac{a\,\left(3675\,c+3675\,d\,x-3360\right)}{336}-\frac{175\,a\,\left(c+d\,x\right)}{16}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}+\frac{85\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{24}+\left(\frac{a\,\left(2205\,c+2205\,d\,x-2016\right)}{336}-\frac{105\,a\,\left(c+d\,x\right)}{16}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}+\frac{7\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{6}+\frac{11\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8}+\frac{a\,\left(105\,c+105\,d\,x-96\right)}{336}-\frac{5\,a\,\left(c+d\,x\right)}{16}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6*(a + a*sin(c + d*x)),x)
```

```
[Out] (5*a*x)/16 + ((a*(105*c + 105*d*x - 96))/336 + (11*a*tan(c/2 + (d*x)/2))/8
- (5*a*(c + d*x))/16 + tan(c/2 + (d*x)/2)^12*((a*(735*c + 735*d*x - 672))/3
36 - (35*a*(c + d*x))/16) + tan(c/2 + (d*x)/2)^4*((a*(2205*c + 2205*d*x - 2
016))/336 - (105*a*(c + d*x))/16) + tan(c/2 + (d*x)/2)^8*((a*(3675*c + 3675
*d*x - 3360))/336 - (175*a*(c + d*x))/16) + (7*a*tan(c/2 + (d*x)/2)^3)/6 +
(85*a*tan(c/2 + (d*x)/2)^5)/24 - (85*a*tan(c/2 + (d*x)/2)^9)/24 - (7*a*tan(
c/2 + (d*x)/2)^11)/6 - (11*a*tan(c/2 + (d*x)/2)^13)/8)/(d*(tan(c/2 + (d*x)/
2)^2 + 1)^7)
```

3.3 $\int \cos^5(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=64

$$\frac{(a+a\sin(c+dx))^4}{a^3d}-\frac{4(a+a\sin(c+dx))^5}{5a^4d}+\frac{(a+a\sin(c+dx))^6}{6a^5d}$$

```
[Out] (a+a*sin(d*x+c))^4/a^3/d-4/5*(a+a*sin(d*x+c))^5/a^4/d+1/6*(a+a*sin(d*x+c))^
6/a^5/d
```

Rubi [A]
time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2746, 45}

$$\frac{(a\sin(c+dx)+a)^6}{6a^5d}-\frac{4(a\sin(c+dx)+a)^5}{5a^4d}+\frac{(a\sin(c+dx)+a)^4}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a + a*Sin[c + d*x])^4/(a^3*d) - (4*(a + a*Sin[c + d*x])^5)/(5*a^4*d) + (a
+ a*Sin[c + d*x])^6/(6*a^5*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \cos^5(c+dx)(a+a\sin(c+dx))\,dx &= \frac{\text{Subst}\left(\int (a-x)^2(a+x)^3\,dx,x,a\sin(c+dx)\right)}{a^5d}\\ &= \frac{\text{Subst}\left(\int \left(4a^2(a+x)^3-4a(a+x)^4+(a+x)^5\right)\,dx,x,a\sin(c+dx)\right)}{a^5d}\\ &= \frac{(a+a\sin(c+dx))^4}{a^3d}-\frac{4(a+a\sin(c+dx))^5}{5a^4d}+\frac{(a+a\sin(c+dx))^6}{6a^5d}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 63, normalized size = 0.98

$$-\frac{a\cos^6(c+dx)}{6d}+\frac{5a\sin(c+dx)}{8d}+\frac{5a\sin(3(c+dx))}{48d}+\frac{a\sin(5(c+dx))}{80d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/6*(a*Cos[c + d*x]^6)/d + (5*a*Sin[c + d*x])/(8*d) + (5*a*Sin[3*(c + d*x)
])/(48*d) + (a*Sin[5*(c + d*x)])/(80*d)
```

Maple [A]
time = 0.15, size = 46, normalized size = 0.72

method	result
derivativedivides	$\frac{-\frac{a\left(\cos^6(dx+c)\right)}{6}+\frac{a\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}}{d}$
default	$\frac{-\frac{a\left(\cos^6(dx+c)\right)}{6}+\frac{a\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}}{d}$
risch	$\frac{5a\sin(dx+c)}{8d}-\frac{a\cos(6dx+6c)}{192d}+\frac{a\sin(5dx+5c)}{80d}-\frac{a\cos(4dx+4c)}{32d}+\frac{5a\sin(3dx+3c)}{48d}-\frac{5a\cos(2dx+2c)}{64d}$
norman	$\frac{\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{14a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{52a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5d}+\frac{52a\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5d}+\frac{14a\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{2a\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2a}{}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/6*a*cos(d*x+c)^6+1/5*a*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+
c))
```

Maxima [A]
time = 0.29, size = 70, normalized size = 1.09

$$\frac{5\,a\sin\left(dx+c\right)^6+6\,a\sin\left(dx+c\right)^5-15\,a\sin\left(dx+c\right)^4-20\,a\sin\left(dx+c\right)^3+15\,a\sin\left(dx+c\right)^2+30\,a\sin\left(dx+c\right)}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/30*(5*a*sin(d*x + c)^6 + 6*a*sin(d*x + c)^5 - 15*a*sin(d*x + c)^4 - 20*a*
sin(d*x + c)^3 + 15*a*sin(d*x + c)^2 + 30*a*sin(d*x + c))/d
```

Fricas [A]
time = 0.35, size = 51, normalized size = 0.80

$$-\frac{5\,a\cos\left(dx+c\right)^{6}-2\,\left(3\,a\cos\left(dx+c\right)^{4}+4\,a\cos\left(dx+c\right)^{2}+8\,a\right)\sin\left(dx+c\right)}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/30*(5*a*cos(d*x + c)^6 - 2*(3*a*cos(d*x + c)^4 + 4*a*cos(d*x + c)^2 + 8*
a)*sin(d*x + c))/d
```

Sympy [A]
time = 0.44, size = 83, normalized size = 1.30

$$\begin{cases} \frac{8a\sin^{5}\left(c+dx\right)}{15d}+\frac{4a\sin^{3}\left(c+dx\right)\cos^{2}\left(c+dx\right)}{3d}+\frac{a\sin\left(c+dx\right)\cos^{4}\left(c+dx\right)}{d}-\frac{a\cos^{6}\left(c+dx\right)}{6d} & \text{for } d\neq 0 \\ x(a\sin\left(c\right)+a)\cos^{5}\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((8*a*sin(c + d*x)**5/(15*d) + 4*a*sin(c + d*x)**3*cos(c + d*x)**2
/(3*d) + a*sin(c + d*x)*cos(c + d*x)**4/d - a*cos(c + d*x)**6/(6*d), Ne(d,
0)), (x*(a*sin(c) + a)*cos(c)**5, True))
```

Giac [A]
time = 5.87, size = 88, normalized size = 1.38

$$-\frac{a\cos\left(6\,dx+6\,c\right)}{192\,d}-\frac{a\cos\left(4\,dx+4\,c\right)}{32\,d}-\frac{5\,a\cos\left(2\,dx+2\,c\right)}{64\,d}+\frac{a\sin\left(5\,dx+5\,c\right)}{80\,d}+\frac{5\,a\sin\left(3\,dx+3\,c\right)}{48\,d}+\frac{5\,a\sin\left(dx+c\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/192*a*cos(6*d*x + 6*c)/d - 1/32*a*cos(4*d*x + 4*c)/d - 5/64*a*cos(2*d*x
+ 2*c)/d + 1/80*a*sin(5*d*x + 5*c)/d + 5/48*a*sin(3*d*x + 3*c)/d + 5/8*a*si
n(d*x + c)/d
```

Mupad [B]
time = 0.05, size = 68, normalized size = 1.06

$$\frac{\frac{a\,{\sin\left(c+d\,x\right)}^6}{6}+\frac{a\,{\sin\left(c+d\,x\right)}^5}{5}-\frac{a\,{\sin\left(c+d\,x\right)}^4}{2}-\frac{2\,a\,{\sin\left(c+d\,x\right)}^3}{3}+\frac{a\,{\sin\left(c+d\,x\right)}^2}{2}+a\,\sin\left(c+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(a + a*sin(c + d*x)),x)
```

```
[Out] (a*sin(c + d*x) + (a*sin(c + d*x)^2)/2 - (2*a*sin(c + d*x)^3)/3 - (a*sin(c
+ d*x)^4)/2 + (a*sin(c + d*x)^5)/5 + (a*sin(c + d*x)^6)/6)/d
```

3.4 $\int \cos^4(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=65

$$\frac{3ax}{8}-\frac{a\cos^5(c+dx)}{5d}+\frac{3a\cos(c+dx)\sin(c+dx)}{8d}+\frac{a\cos^3(c+dx)\sin(c+dx)}{4d}$$

```
[Out] 3/8*a*x-1/5*a*cos(d*x+c)^5/d+3/8*a*cos(d*x+c)*sin(d*x+c)/d+1/4*a*cos(d*x+c)
^3*sin(d*x+c)/d
```

Rubi [A]
time = 0.03, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.158$, Rules used = {2748, 2715, 8}

$$-\frac{a\cos^5(c+dx)}{5d}+\frac{a\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3a\sin(c+dx)\cos(c+dx)}{8d}+\frac{3ax}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] (3*a*x)/8 - (a*Cos[c + d*x]^5)/(5*d) + (3*a*Cos[c + d*x]*Sin[c + d*x])/(8*d
) + (a*Cos[c + d*x]^3*Sin[c + d*x])/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\int \cos^4(c+dx)(a+a\sin(c+dx))\,dx = -\frac{a\cos^5(c+dx)}{5d} + a\int \cos^4(c+dx)\,dx$$
$$= -\frac{a\cos^5(c+dx)}{5d} + \frac{a\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{1}{4}(3a)\int \cos^2(c+$$
$$= -\frac{a\cos^5(c+dx)}{5d} + \frac{3a\cos(c+dx)\sin(c+dx)}{8d} + \frac{a\cos^3(c+dx)\sin(}{4d}$$
$$= \frac{3ax}{8} - \frac{a\cos^5(c+dx)}{5d} + \frac{3a\cos(c+dx)\sin(c+dx)}{8d} + \frac{a\cos^3(c+dx}{4}$$

Mathematica [A]
time = 0.06, size = 62, normalized size = 0.95

$$\frac{3a(c+dx)}{8d} - \frac{a\cos^5(c+dx)}{5d} + \frac{a\sin(2(c+dx))}{4d} + \frac{a\sin(4(c+dx))}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] (3*a*(c + d*x))/(8*d) - (a*Cos[c + d*x]^5)/(5*d) + (a*Sin[2*(c + d*x)])/(4*
d) + (a*Sin[4*(c + d*x)])/(32*d)
```

Maple [A]
time = 0.12, size = 52, normalized size = 0.80

method	result
derivativedivides	$\frac{-\frac{a\left(\cos^5(dx+c)\right)}{5}+a\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}$
default	$\frac{-\frac{a\left(\cos^5(dx+c)\right)}{5}+a\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}$
risch	$\frac{3ax}{8}-\frac{a\cos(dx+c)}{8d}-\frac{a\cos(5dx+5c)}{80d}+\frac{a\sin(4dx+4c)}{32d}-\frac{a\cos(3dx+3c)}{16d}+\frac{a\sin(2dx+2c)}{4d}$
norman	$\frac{\frac{3ax}{8}-\frac{2a}{5d}+\frac{5a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}+\frac{a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-\frac{a\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-\frac{5a\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{15ax\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8}+\frac{15ax\left(\tan^4\left(\frac{dx}{2}\right.\right.}{4}}{\left(1+\tan^2\left(\frac{dx}{2}+\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/5*a*cos(d*x+c)^5+a*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/
8*d*x+3/8*c))
```

Maxima [A]
time = 0.31, size = 48, normalized size = 0.74

$$-\frac{32\,a\cos\left(dx+c\right)^{5}-5\,(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right))a}{160\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/160*(32*a*cos(d*x + c)^5 - 5*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2
*d*x + 2*c))*a)/d
```

Fricas [A]
time = 0.38, size = 51, normalized size = 0.78

$$-\frac{8\,a\cos\left(dx+c\right)^{5}-15\,adx-5\,\left(2\,a\cos\left(dx+c\right)^{3}+3\,a\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{40\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/40*(8*a*cos(d*x + c)^5 - 15*a*d*x - 5*(2*a*cos(d*x + c)^3 + 3*a*cos(d*x
+ c))*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 124 vs. 2(60) = 120.
time = 0.26, size = 124, normalized size = 1.91

$$\begin{cases} \frac{3ax\sin^{4}\left(c+dx\right)}{8}+\frac{3ax\sin^{2}\left(c+dx\right)\cos^{2}\left(c+dx\right)}{4}+\frac{3ax\cos^{4}\left(c+dx\right)}{8}+\frac{3a\sin^{3}\left(c+dx\right)\cos\left(c+dx\right)}{8d}+\frac{5a\sin\left(c+dx\right)\cos^{3}\left(c+dx\right)}{8d}-\frac{a\cos^{5}\left(c+dx\right)}{5d} & \text{for}\: d\neq 0 \\ x(a\sin\left(c\right)+a)\cos^{4}\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((3*a*x*sin(c + d*x)**4/8 + 3*a*x*sin(c + d*x)**2*cos(c + d*x)**2/
4 + 3*a*x*cos(c + d*x)**4/8 + 3*a*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 5*a*
sin(c + d*x)*cos(c + d*x)**3/(8*d) - a*cos(c + d*x)**5/(5*d), Ne(d, 0)), (x
*(a*sin(c) + a)*cos(c)**4, True))
```

Giac [A]
time = 5.33, size = 77, normalized size = 1.18

$$\frac{3}{8}\,ax-\frac{a\cos\left(5\,dx+5\,c\right)}{80\,d}-\frac{a\cos\left(3\,dx+3\,c\right)}{16\,d}-\frac{a\cos\left(dx+c\right)}{8\,d}+\frac{a\sin\left(4\,dx+4\,c\right)}{32\,d}+\frac{a\sin\left(2\,dx+2\,c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 3/8*a*x - 1/80*a*cos(5*d*x + 5*c)/d - 1/16*a*cos(3*d*x + 3*c)/d - 1/8*a*cos
(d*x + c)/d + 1/32*a*sin(4*d*x + 4*c)/d + 1/4*a*sin(2*d*x + 2*c)/d
```

Mupad [B]
time = 7.99, size = 165, normalized size = 2.54

$$\frac{3\,a\,x}{8}+\frac{-\frac{5\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{4}+\left(\frac{a\,(75\,c+75\,d\,x-80)}{40}-\frac{15\,a\,(c+d\,x)}{8}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-\frac{a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{2}+\left(\frac{a\,(150\,c+150\,d\,x-160)}{40}-\frac{15\,a\,(c+d\,x)}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\frac{a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{2}+\frac{5\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}+\frac{a\,(15\,c+15\,d\,x-16)}{40}-\frac{3\,a\,(c+d\,x)}{8}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(a + a*sin(c + d*x)),x)
```

```
[Out] (3*a*x)/8 + ((a*(15*c + 15*d*x - 16))/40 + (5*a*tan(c/2 + (d*x)/2))/4 - (3*
a*(c + d*x))/8 + tan(c/2 + (d*x)/2)^8*((a*(75*c + 75*d*x - 80))/40 - (15*a*
(c + d*x))/8) + tan(c/2 + (d*x)/2)^4*((a*(150*c + 150*d*x - 160))/40 - (15*
a*(c + d*x))/4) + (a*tan(c/2 + (d*x)/2)^3)/2 - (a*tan(c/2 + (d*x)/2)^7)/2 -
 (5*a*tan(c/2 + (d*x)/2)^9)/4)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^5)
```

3.5 $\int \cos^3(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=45

$$\frac{2(a+a\sin(c+dx))^3}{3a^2d}-\frac{(a+a\sin(c+dx))^4}{4a^3d}$$

```
[Out] 2/3*(a+a*sin(d*x+c))^3/a^2/d-1/4*(a+a*sin(d*x+c))^4/a^3/d
```

Rubi [A]
time = 0.02, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.105$, Rules used = {2746, 45}

$$\frac{2(a\sin(c+dx)+a)^3}{3a^2d}-\frac{(a\sin(c+dx)+a)^4}{4a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Sin[c + d*x]),x]
[Out] (2*(a + a*Sin[c + d*x])^3)/(3*a^2*d) - (a + a*Sin[c + d*x])^4/(4*a^3*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \cos^3(c+dx)(a+a\sin(c+dx))\,dx &= \frac{\text{Subst}\left(\int (a-x)(a+x)^2\,dx,x,a\sin(c+dx)\right)}{a^3d}\\ &= \frac{\text{Subst}\left(\int \left(2a(a+x)^2-(a+x)^3\right)\,dx,x,a\sin(c+dx)\right)}{a^3d}\\ &= \frac{2(a+a\sin(c+dx))^3}{3a^2d}-\frac{(a+a\sin(c+dx))^4}{4a^3d}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 44, normalized size = 0.98

$$-\frac{a\cos^4(c+dx)}{4d}+\frac{a\sin(c+dx)}{d}-\frac{a\sin^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/4*(a*Cos[c + d*x]^4)/d + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^3)/(3*d)
```

Maple [A]
time = 0.10, size = 36, normalized size = 0.80

method	result	s
derivativedivides	$\frac{-\frac{a\left(\cos^4(dx+c)\right)}{4}+\frac{a\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}}{d}$	3
default	$\frac{-\frac{a\left(\cos^4(dx+c)\right)}{4}+\frac{a\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}}{d}$	3
risch	$\frac{3a\sin(dx+c)}{4d}-\frac{a\cos(4dx+4c)}{32d}+\frac{a\sin(3dx+3c)}{12d}-\frac{a\cos(2dx+2c)}{8d}$	5
norman	$\frac{\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{10a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{10a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{2a\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2a\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}$	1

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/4*a*cos(d*x+c)^4+1/3*a*(2+cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A]
time = 0.30, size = 48, normalized size = 1.07

$$-\frac{3\,a\sin\left(dx+c\right)^4+4\,a\sin\left(dx+c\right)^3-6\,a\sin\left(dx+c\right)^2-12\,a\sin\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/12*(3*a*sin(d*x + c)^4 + 4*a*sin(d*x + c)^3 - 6*a*sin(d*x + c)^2 - 12*a*
sin(d*x + c))/d
```

Fricas [A]
time = 0.38, size = 39, normalized size = 0.87

$$-\frac{3\,a\cos\left(dx+c\right)^4-4\,\left(a\cos\left(dx+c\right)^2+2\,a\right)\sin\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/12*(3*a*cos(d*x + c)^4 - 4*(a*cos(d*x + c)^2 + 2*a)*sin(d*x + c))/d
```

Sympy [A]
time = 0.20, size = 60, normalized size = 1.33

$$\begin{cases} \frac{2a\sin^3{(c+dx)}}{3d} + \frac{a\sin{(c+dx)}\cos^2{(c+dx)}}{d} - \frac{a\cos^4{(c+dx)}}{4d} & \text{for } d \neq 0 \\ x(a\sin{(c)}+a)\cos^3{(c)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((2*a*sin(c + d*x)**3/(3*d) + a*sin(c + d*x)*cos(c + d*x)**2/d - a
*cos(c + d*x)**4/(4*d), Ne(d, 0)), (x*(a*sin(c) + a)*cos(c)**3, True))
```

Giac [A]
time = 4.18, size = 48, normalized size = 1.07

$$-\frac{3\,a\sin{(dx+c)}^4 + 4\,a\sin{(dx+c)}^3 - 6\,a\sin{(dx+c)}^2 - 12\,a\sin{(dx+c)}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/12*(3*a*sin(d*x + c)^4 + 4*a*sin(d*x + c)^3 - 6*a*sin(d*x + c)^2 - 12*a*
sin(d*x + c))/d
```

Mupad [B]
time = 0.06, size = 46, normalized size = 1.02

$$\frac{-\frac{a\sin(c+d\,x)^4}{4} - \frac{a\sin(c+d\,x)^3}{3} + \frac{a\sin(c+d\,x)^2}{2} + a\,\sin\left(c+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a + a*sin(c + d*x)),x)
```

```
[Out] (a*sin(c + d*x) + (a*sin(c + d*x)^2)/2 - (a*sin(c + d*x)^3)/3 - (a*sin(c +
d*x)^4)/4)/d
```

3.6 $\int \cos^2(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=43

$$\frac{ax}{2}-\frac{a\cos^3(c+dx)}{3d}+\frac{a\cos(c+dx)\sin(c+dx)}{2d}$$

```
[Out] 1/2*a*x-1/3*a*cos(d*x+c)^3/d+1/2*a*cos(d*x+c)*sin(d*x+c)/d
```

Rubi [A]
time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.158$, Rules used = {2748, 2715, 8}

$$-\frac{a\cos^3(c+dx)}{3d}+\frac{a\sin(c+dx)\cos(c+dx)}{2d}+\frac{ax}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Sin[c + d*x]),x]
[Out] (a*x)/2 - (a*Cos[c + d*x]^3)/(3*d) + (a*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\begin{aligned}\int \cos^2(c+dx)(a+a\sin(c+dx))\,dx &= -\frac{a\cos^3(c+dx)}{3d}+a\int \cos^2(c+dx)\,dx\\ &= -\frac{a\cos^3(c+dx)}{3d}+\frac{a\cos(c+dx)\sin(c+dx)}{2d}+\frac{1}{2}a\int 1\,dx\\ &= \frac{ax}{2}-\frac{a\cos^3(c+dx)}{3d}+\frac{a\cos(c+dx)\sin(c+dx)}{2d}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 46, normalized size = 1.07

$$\frac{a(c+dx)}{2d}-\frac{a\cos^3(c+dx)}{3d}+\frac{a\sin(2(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Sin[c + d*x]),x]
[Out] (a*(c + d*x))/(2*d) - (a*Cos[c + d*x]^3)/(3*d) + (a*Sin[2*(c + d*x)])/(4*d)
```

Maple [A]
time = 0.08, size = 41, normalized size = 0.95

method	result
derivativedivides	$\frac{-\frac{a\left(\cos^3(dx+c)\right)}{3}+a\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
default	$\frac{-\frac{a\left(\cos^3(dx+c)\right)}{3}+a\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
risch	$\frac{ax}{2}-\frac{a\cos(dx+c)}{4d}-\frac{a\cos(3dx+3c)}{12d}+\frac{a\sin(2dx+2c)}{4d}$
norman	$\frac{\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{ax}{2}-\frac{2a}{3d}-\frac{a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{3ax\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{3ax\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{ax\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-\frac{2a\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(-1/3*a*cos(d*x+c)^3+a*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))
```

Maxima [A]
time = 0.29, size = 37, normalized size = 0.86

$$-\frac{4\,a\cos\left(dx+c\right)^3-3\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)a}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="maxima")
[Out] -1/12*(4*a*cos(d*x + c)^3 - 3*(2*d*x + 2*c + sin(2*d*x + 2*c))*a)/d
```

Fricas [A]
time = 0.35, size = 37, normalized size = 0.86

$$-\frac{2\,a\cos\left(dx+c\right)^3-3\,adx-3\,a\cos\left(dx+c\right)\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/6*(2*a*cos(d*x + c)^3 - 3*a*d*x - 3*a*cos(d*x + c)*sin(d*x + c))/d
```

Sympy [A]
time = 0.12, size = 71, normalized size = 1.65

$$\begin{cases} \frac{ax\sin^2(c+dx)}{2} + \frac{ax\cos^2(c+dx)}{2} + \frac{a\sin(c+dx)\cos(c+dx)}{2d} - \frac{a\cos^3(c+dx)}{3d} & \text{for } d \neq 0 \\ x(a\sin(c)+a)\cos^2(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((a*x*sin(c + d*x)**2/2 + a*x*cos(c + d*x)**2/2 + a*sin(c + d*x)*c
os(c + d*x)/(2*d) - a*cos(c + d*x)**3/(3*d), Ne(d, 0)), (x*(a*sin(c) + a)*c
os(c)**2, True))
```

Giac [A]
time = 4.30, size = 47, normalized size = 1.09

$$\frac{1}{2}\,ax - \frac{a\cos(3\,dx + 3\,c)}{12\,d} - \frac{a\cos(dx + c)}{4\,d} + \frac{a\sin(2\,dx + 2\,c)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*a*x - 1/12*a*cos(3*d*x + 3*c)/d - 1/4*a*cos(d*x + c)/d + 1/4*a*sin(2*d*
x + 2*c)/d
```

Mupad [B]
time = 6.76, size = 103, normalized size = 2.40

$$\frac{a\,x}{2} + \frac{-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5 + \left(\frac{a\,(9\,c+9\,d\,x-12)}{6} - \frac{3\,a\,(c+d\,x)}{2}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 + a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right) + \frac{a\,(3\,c+3\,d\,x-4)}{6} - \frac{a\,(c+d\,x)}{2}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a + a*sin(c + d*x)),x)
```

```
[Out] (a*x)/2 + ((a*(3*c + 3*d*x - 4))/6 + a*tan(c/2 + (d*x)/2) - (a*(c + d*x))/2
 + tan(c/2 + (d*x)/2)^4*((a*(9*c + 9*d*x - 12))/6 - (3*a*(c + d*x))/2) - a*
tan(c/2 + (d*x)/2)^5)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^3)
```

3.7 $\int \cos(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=22

$$\frac{(a+a\sin(c+dx))^2}{2ad}$$

```
[Out] 1/2*(a+a*sin(d*x+c))^2/a/d
```

Rubi [A]

time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.27, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{2746\}$

$$\frac{a\sin^2(c+dx)}{2d}+\frac{a\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Sin[c + d*x])/d + (a*Sin[c + d*x]^2)/(2*d)
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a+a\sin(c+dx))\,dx &= \frac{\text{Subst}\left(\int (a+x)\,dx,x,a\sin(c+dx)\right)}{ad}\\ &= \frac{a\sin(c+dx)}{d}+\frac{a\sin^2(c+dx)}{2d}\end{aligned}$$

Mathematica [A]

time = 0.01, size = 39, normalized size = 1.77

$$-\frac{a\cos^2(c+dx)}{2d}+\frac{a\cos(dx)\sin(c)}{d}+\frac{a\cos(c)\sin(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Sin[c + d*x]),x]
```

[Out] -1/2*(a*Cos[c + d*x]^2)/d + (a*Cos[d*x]*Sin[c])/d + (a*Cos[c]*Sin[d*x])/d

Maple [A]
time = 0.05, size = 23, normalized size = 1.05

method	result	size
derivativedivides	$\frac{a\left(\frac{\left(\sin^2(dx+c)\right)}{2}+\sin(dx+c)\right)}{d}$	23
default	$\frac{a\left(\frac{\left(\sin^2(dx+c)\right)}{2}+\sin(dx+c)\right)}{d}$	23
risch	$\frac{a\sin(dx+c)}{d}-\frac{a\cos(2dx+2c)}{4d}$	28
norman	$\frac{\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}$	67

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)

[Out] 1/d*a*(1/2*sin(d*x+c)^2+sin(d*x+c))

Maxima [A]
time = 0.30, size = 20, normalized size = 0.91

$$\frac{(a\sin(dx+c)+a)^2}{2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="maxima")

[Out] 1/2*(a*sin(d*x + c) + a)^2/(a*d)

Fricas [A]
time = 0.40, size = 25, normalized size = 1.14

$$-\frac{a\cos(dx+c)^2-2\,a\sin(dx+c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="fricas")

[Out] -1/2*(a*cos(d*x + c)^2 - 2*a*sin(d*x + c))/d

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(15) = 30$.

time = 0.07, size = 34, normalized size = 1.55

$$\begin{cases} \frac{a\sin^2(c+dx)}{2d} + \frac{a\sin(c+dx)}{d} & \text{for } d \neq 0 \\ x(a\sin(c) + a)\cos(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((a*sin(c + d*x)**2/(2*d) + a*sin(c + d*x)/d, Ne(d, 0)), (x*(a*sin
(c) + a)*cos(c), True))
```

Giac [A]
time = 3.63, size = 25, normalized size = 1.14

$$\frac{a\sin(dx+c)^2 + 2\,a\sin(dx+c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(a*sin(d*x + c)^2 + 2*a*sin(d*x + c))/d
```

Mupad [B]
time = 0.04, size = 20, normalized size = 0.91

$$\frac{a\,\sin(c+d\,x)\,(\sin(c+d\,x)+2)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + a*sin(c + d*x)),x)
```

```
[Out] (a*sin(c + d*x)*(sin(c + d*x) + 2))/(2*d)
```

3.8 $\int \sec(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=17

$$-\frac{a\log(1-\sin(c+dx))}{d}$$

```
[Out] -a*ln(1-sin(d*x+c))/d
```

Rubi [A]
time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2746, 31}

$$-\frac{a\log(1-\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*Log[1 - Sin[c + d*x]])/d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \sec(c+dx)(a+a\sin(c+dx))\,dx &= \frac{a\text{Subst}\left(\int \frac{1}{a-x}\,dx, x, a\sin(c+dx)\right)}{d}\\ &= -\frac{a\log(1-\sin(c+dx))}{d}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 26, normalized size = 1.53

$$\frac{a\tanh^{-1}(\sin(c+dx))}{d}-\frac{a\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sin[c + d*x]),x]
[Out] (a*ArcTanh[Sin[c + d*x]])/d - (a*Log[Cos[c + d*x]])/d
```

Maple [A]
time = 0.07, size = 16, normalized size = 0.94

method	result	size
derivativedivides	$-\frac{a\ln(\sin(dx+c)-1)}{d}$	16
default	$-\frac{a\ln(\sin(dx+c)-1)}{d}$	16
risch	$iax+\frac{2iac}{d}-\frac{2a\ln(e^{i(dx+c)}-i)}{d}$	34
norman	$\frac{a\ln\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] -1/d*a*ln(sin(d*x+c)-1)
```

Maxima [A]
time = 0.30, size = 15, normalized size = 0.88

$$-\frac{a\log\left(\sin\left(dx+c\right)-1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="maxima")
[Out] -a*log(sin(d*x + c) - 1)/d
```

Fricas [A]
time = 0.36, size = 17, normalized size = 1.00

$$-\frac{a\log\left(-\sin\left(dx+c\right)+1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="fricas")
[Out] -a*log(-sin(d*x + c) + 1)/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\sin\left(c+dx\right)\sec\left(c+dx\right)dx+\int\sec\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c)),x)
```

```
[Out] a*(Integral(sin(c + d*x)*sec(c + d*x), x) + Integral(sec(c + d*x), x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(17) = 34$.
time = 4.06, size = 37, normalized size = 2.18

$$\frac{a\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)-2\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] (a*log(tan(1/2*d*x + 1/2*c)^2 + 1) - 2*a*log(abs(tan(1/2*d*x + 1/2*c) - 1))
)/d
```

Mupad [B]
time = 0.05, size = 15, normalized size = 0.88

$$-\frac{a\,\ln\left(\sin\left(c+d\,x\right)-1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))/cos(c + d*x),x)
```

```
[Out] -(a*log(sin(c + d*x) - 1))/d
```

3.9 $\int \sec^2(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=23

$$\frac{a\sec(c+dx)}{d}+\frac{a\tan(c+dx)}{d}$$

```
[Out] a*sec(d*x+c)/d+a*tan(d*x+c)/d
```

Rubi [A]

time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.158$, Rules used = {2748, 3852, 8}

$$\frac{a\tan(c+dx)}{d}+\frac{a\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Sec[c + d*x])/d + (a*Tan[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \sec^2(c+dx)(a+a\sin(c+dx))\,dx &= \frac{a\sec(c+dx)}{d}+a\int \sec^2(c+dx)\,dx\\
&= \frac{a\sec(c+dx)}{d}-\frac{a\text{Subst}\left(\int 1\,dx,x,-\tan(c+dx)\right)}{d}\\
&= \frac{a\sec(c+dx)}{d}+\frac{a\tan(c+dx)}{d}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 23, normalized size = 1.00

$$\frac{a\sec(c+dx)}{d}+\frac{a\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Sec[c + d*x])/d + (a*Tan[c + d*x])/d
```

Maple [A]
time = 0.07, size = 24, normalized size = 1.04

method	result	size
risch	$\frac{2a}{d\left(e^{i(dx+c)}-i\right)}$	21
derivativedivides	$\frac{\frac{a}{\cos(dx+c)}+a\tan(dx+c)}{d}$	24
default	$\frac{\frac{a}{\cos(dx+c)}+a\tan(dx+c)}{d}$	24
norman	$\frac{-\frac{2a}{d}-\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{2a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$	88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a/cos(d*x+c)+a*tan(d*x+c))
```

Maxima [A]
time = 0.31, size = 23, normalized size = 1.00

$$\frac{a\tan\left(dx+c\right)+\frac{a}{\cos(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] (a*tan(d*x + c) + a/cos(d*x + c))/d
```

Fricas [A]
time = 0.34, size = 40, normalized size = 1.74

$$\frac{a\cos\left(dx+c\right)+a\sin\left(dx+c\right)+a}{d\cos\left(dx+c\right)-d\sin\left(dx+c\right)+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] (a*cos(d*x + c) + a*sin(d*x + c) + a)/(d*cos(d*x + c) - d*sin(d*x + c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \sin\left(c+dx\right)\sec^2\left(c+dx\right)dx+\int \sec^2\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+a*sin(d*x+c)),x)
```

```
[Out] a*(Integral(sin(c + d*x)*sec(c + d*x)**2, x) + Integral(sec(c + d*x)**2, x)
)
```

Giac [A]
time = 3.70, size = 19, normalized size = 0.83

$$-\frac{2\,a}{d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -2*a/(d*(tan(1/2*d*x + 1/2*c) - 1))
```

Mupad [B]
time = 4.69, size = 19, normalized size = 0.83

$$-\frac{2\,a}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))/cos(c + d*x)^2,x)
```

```
[Out] -(2*a)/(d*(tan(c/2 + (d*x)/2) - 1))
```

3.10 $\int \sec^3(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=39

$$\frac{a\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^2}{2d(a-a\sin(c+dx))}$$

```
[Out] 1/2*a*arctanh(sin(d*x+c))/d+1/2*a^2/d/(a-a*sin(d*x+c))
```

Rubi [A]
time = 0.03, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.158$, Rules used = {2746, 46, 212}

$$\frac{a^2}{2d(a-a\sin(c+dx))}+\frac{a\tanh^{-1}(\sin(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/(2*d) + a^2/(2*d*(a - a*Sin[c + d*x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\int \sec^3(c+dx)(a+a\sin(c+dx))\,dx = \frac{a^3\text{Subst}\left(\int \frac{1}{(a-x)^2(a+x)}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^3\text{Subst}\left(\int \left(\frac{1}{2a(a-x)^2}+\frac{1}{2a(a^2-x^2)}\right)\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^2}{2d(a-a\sin(c+dx))}+\frac{a^2\text{Subst}\left(\int \frac{1}{a^2-x^2}\,dx, x, a\sin(c+dx)\right)}{2d}$$
$$= \frac{a\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^2}{2d(a-a\sin(c+dx))}$$

Mathematica [A]
time = 0.02, size = 52, normalized size = 1.33

$$\frac{a\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a\sec^2(c+dx)}{2d}+\frac{a\sec(c+dx)\tan(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/(2*d) + (a*Sec[c + d*x]^2)/(2*d) + (a*Sec[c + d*x
]*Tan[c + d*x])/(2*d)
```

Maple [A]
time = 0.10, size = 50, normalized size = 1.28

method	result
derivativedivides	$\frac{\frac{a}{2\cos(dx+c)^2}+a\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$
default	$\frac{\frac{a}{2\cos(dx+c)^2}+a\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$
risch	$-\frac{ia\,e^{i(dx+c)}}{\left(e^{i(dx+c)}-i\right)^2 d}-\frac{a\ln\left(e^{i(dx+c)}-i\right)}{2d}+\frac{a\ln\left(e^{i(dx+c)}+i\right)}{2d}$
norman	$\frac{\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2a\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}-\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2d}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2*a/cos(d*x+c)^2+a*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(
d*x+c))))
```

Maxima [A]
time = 0.30, size = 42, normalized size = 1.08

$$\frac{a\log\left(\sin\left(dx+c\right)+1\right)-a\log\left(\sin\left(dx+c\right)-1\right)-\frac{2\,a}{\sin\left(dx+c\right)-1}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*(a*log(sin(d*x + c) + 1) - a*log(sin(d*x + c) - 1) - 2*a/(sin(d*x + c)
- 1))/d
```

Fricas [A]
time = 0.36, size = 67, normalized size = 1.72

$$\frac{\left(a\sin\left(dx+c\right)-a\right)\log\left(\sin\left(dx+c\right)+1\right)-\left(a\sin\left(dx+c\right)-a\right)\log\left(-\sin\left(dx+c\right)+1\right)-2\,a}{4\,\left(d\sin\left(dx+c\right)-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*((a*sin(d*x + c) - a)*log(sin(d*x + c) + 1) - (a*sin(d*x + c) - a)*log(
-sin(d*x + c) + 1) - 2*a)/(d*sin(d*x + c) - d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\sin\left(c+dx\right)\sec^{3}\left(c+dx\right)\,dx+\int\sec^{3}\left(c+dx\right)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+a*sin(d*x+c)),x)
```

```
[Out] a*(Integral(sin(c + d*x)*sec(c + d*x)**3, x) + Integral(sec(c + d*x)**3, x)
)
```

Giac [A]
time = 5.12, size = 54, normalized size = 1.38

$$\frac{a\log\left(\left|\sin\left(dx+c\right)+1\right|\right)-a\log\left(\left|\sin\left(dx+c\right)-1\right|\right)+\frac{a\sin\left(dx+c\right)-3\,a}{\sin\left(dx+c\right)-1}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/4*(a*log(abs(sin(d*x + c) + 1)) - a*log(abs(sin(d*x + c) - 1)) + (a*sin(d
*x + c) - 3*a)/(sin(d*x + c) - 1))/d
```

Mupad [B]
time = 0.06, size = 30, normalized size = 0.77

$$\frac{a\,\mathrm{atanh}(\sin\left(c+d\,x\right))}{2\,d}-\frac{a}{2\,d\,\left(\sin\left(c+d\,x\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))/cos(c + d*x)^3,x)
[Out] (a*atanh(sin(c + d*x)))/(2*d) - a/(2*d*(sin(c + d*x) - 1))
```

3.11 $\int \sec^4(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=44

$$\frac{a\sec^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{a\tan^3(c+dx)}{3d}$$

```
[Out] 1/3*a*sec(d*x+c)^3/d+a*tan(d*x+c)/d+1/3*a*tan(d*x+c)^3/d
```

Rubi [A]
time = 0.03, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2748, 3852}

$$\frac{a\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{a\sec^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Sec[c + d*x]^3)/(3*d) + (a*Tan[c + d*x])/d + (a*Tan[c + d*x]^3)/(3*d)
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^4(c+dx)(a+a\sin(c+dx))\,dx &= \frac{a\sec^3(c+dx)}{3d}+a\int \sec^4(c+dx)\,dx\\ &= \frac{a\sec^3(c+dx)}{3d}-\frac{a\text{Subst}\left(\int (1+x^2)\,dx,x,-\tan(c+dx)\right)}{d}\\ &= \frac{a\sec^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{a\tan^3(c+dx)}{3d}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 41, normalized size = 0.93

$$\frac{a\sec^3(c+dx)}{3d}+\frac{a\left(\tan(c+dx)+\frac{1}{3}\tan^3(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + a*Sin[c + d*x]),x]
[Out] (a*Sec[c + d*x]^3)/(3*d) + (a*(Tan[c + d*x] + Tan[c + d*x]^3/3))/d
```

Maple [A]
time = 0.12, size = 38, normalized size = 0.86

method	result
derivativedivides	$\frac{\frac{a}{3\cos(dx+c)^3}-a\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)}{d}$
default	$\frac{\frac{a}{3\cos(dx+c)^3}-a\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)}{d}$
risch	$\frac{\frac{8a\,\mathrm{e}^{i(dx+c)}}{3}-\frac{4ia}{3}}{\left(\mathrm{e}^{i(dx+c)}-i\right)^3\left(\mathrm{e}^{i(dx+c)}+i\right)d}$
norman	$\frac{-\frac{2a}{3d}-\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{2a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{2a\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{2a\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(1/3*a/cos(d*x+c)^3-a*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c))
```

Maxima [A]
time = 0.30, size = 35, normalized size = 0.80

$$\frac{\left(\tan\left(dx+c\right)^3+3\,\tan\left(dx+c\right)\right)a+\frac{a}{\cos(dx+c)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="maxima")
[Out] 1/3*((tan(d*x + c)^3 + 3*tan(d*x + c))*a + a/cos(d*x + c)^3)/d
```

Fricas [A]
time = 0.33, size = 52, normalized size = 1.18

$$-\frac{2\,a\cos\left(dx+c\right)^2+2\,a\sin\left(dx+c\right)-a}{3\left(d\cos\left(dx+c\right)\sin\left(dx+c\right)-d\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/3*(2*a*cos(d*x + c)^2 + 2*a*sin(d*x + c) - a)/(d*cos(d*x + c)*sin(d*x +
c) - d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\sin\left(c+dx\right)\sec^4\left(c+dx\right)dx+\int\sec^4\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+a*sin(d*x+c)),x)
```

```
[Out] a*(Integral(sin(c + d*x)*sec(c + d*x)**4, x) + Integral(sec(c + d*x)**4, x)
)
```

Giac [A]
time = 8.46, size = 66, normalized size = 1.50

$$-\frac{\frac{3\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1}+\frac{9\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-12\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+7\,a}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(3*a/(tan(1/2*d*x + 1/2*c) + 1) + (9*a*tan(1/2*d*x + 1/2*c)^2 - 12*a*t
an(1/2*d*x + 1/2*c) + 7*a)/(tan(1/2*d*x + 1/2*c) - 1)^3)/d
```

Mupad [B]
time = 4.60, size = 63, normalized size = 1.43

$$\frac{2\,a\,\left(\cos\left(c+d\,x\right)+2\,\sin\left(c+d\,x\right)+\cos\left(2\,c+2\,d\,x\right)-\frac{\sin\left(2\,c+2\,d\,x\right)}{2}\right)}{3\,d\,\left(2\,\cos\left(c+d\,x\right)-\sin\left(2\,c+2\,d\,x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))/cos(c + d*x)^4,x)
```

```
[Out] (2*a*(cos(c + d*x) + 2*sin(c + d*x) + cos(2*c + 2*d*x) - sin(2*c + 2*d*x)/2
))/(3*d*(2*cos(c + d*x) - sin(2*c + 2*d*x)))
```

3.12 $\int \sec^5(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=84

$$\frac{3a\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^3}{8d(a-a\sin(c+dx))^2}+\frac{a^2}{4d(a-a\sin(c+dx))}-\frac{a^2}{8d(a+a\sin(c+dx))}$$

```
[Out] 3/8*a*arctanh(sin(d*x+c))/d+1/8*a^3/d/(a-a*sin(d*x+c))^2+1/4*a^2/d/(a-a*sin
(d*x+c))-1/8*a^2/d/(a+a*sin(d*x+c))
```

Rubi [A]

time = 0.04, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2746, 46, 212}

$$\frac{a^3}{8d(a-a\sin(c+dx))^2}+\frac{a^2}{4d(a-a\sin(c+dx))}-\frac{a^2}{8d(a\sin(c+dx)+a)}+\frac{3a\tanh^{-1}(\sin(c+dx))}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

```
[Out] (3*a*ArcTanh[Sin[c + d*x]])/(8*d) + a^3/(8*d*(a - a*Sin[c + d*x])^2) + a^2/
(4*d*(a - a*Sin[c + d*x])) - a^2/(8*d*(a + a*Sin[c + d*x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\int \sec^5(c+dx)(a+a\sin(c+dx))\,dx = \frac{a^5\text{Subst}\left(\int \frac{1}{(a-x)^3(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^5\text{Subst}\left(\int \left(\frac{1}{4a^2(a-x)^3} + \frac{1}{4a^3(a-x)^2} + \frac{1}{8a^3(a+x)^2} + \frac{3}{8a^3(a^2-x^2)}\right)\,dx, x, a\,\text{s}\right.}{d}$$

$$= \frac{a^3}{8d(a-a\sin(c+dx))^2} + \frac{a^2}{4d(a-a\sin(c+dx))} - \frac{a^2}{8d(a+a\sin(c+}$$

$$= \frac{3a\tanh^{-1}(\sin(c+dx))}{8d} + \frac{a^3}{8d(a-a\sin(c+dx))^2} + \frac{a^2}{4d(a-a\sin(c}$$

Mathematica [A]
time = 0.06, size = 68, normalized size = 0.81

$$\frac{a\sec^4(c+dx)}{4d} + \frac{a\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{3a\left(\tanh^{-1}(\sin(c+dx)) + \sec(c+dx)\tan(c+dx)\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Sec[c + d*x]^4)/(4*d) + (a*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (3*a*(Ar
cTanh[Sin[c + d*x]] + Sec[c + d*x]*Tan[c + d*x]))/(8*d)
```

Maple [A]
time = 0.15, size = 63, normalized size = 0.75

method	result
derivativedivides	$\frac{\frac{a}{4\cos(dx+c)^4} + a\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4} - \frac{3\sec(dx+c)}{8}\right)\tan(dx+c) + \frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)}{d}$
default	$\frac{\frac{a}{4\cos(dx+c)^4} + a\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4} - \frac{3\sec(dx+c)}{8}\right)\tan(dx+c) + \frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)}{d}$
risch	$-\frac{ia\left(-6i e^{4i(dx+c)} + 3\,e^{5i(dx+c)} + 6i e^{2i(dx+c)} + 2\,e^{3i(dx+c)} + 3\,e^{i(dx+c)}\right)}{4\left(e^{i(dx+c)}+i\right)^2\left(e^{i(dx+c)}-i\right)^4 d} + \frac{3a\ln\left(e^{i(dx+c)}+i\right)}{8d} - \frac{3a\ln\left(e^{i(dx+c)}-i\right)}{8d}$
norman	$\frac{\frac{2a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d} + \frac{2a\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d} + \frac{5a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d} + \frac{2a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d} + \frac{3a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d} + \frac{2a\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d} + \frac{5a\left(\text{ta}\right.}{}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/4*a/cos(d*x+c)^4+a*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3
/8*ln(sec(d*x+c)+tan(d*x+c))))
```

Maxima [A]
time = 0.30, size = 86, normalized size = 1.02

$$\frac{3\,a\log\left(\sin\left(dx+c\right)+1\right)-3\,a\log\left(\sin\left(dx+c\right)-1\right)-\frac{2\,\left(3\,a\sin\left(dx+c\right)^{2}-3\,a\sin\left(dx+c\right)-2\,a\right)}{\sin\left(dx+c\right)^{3}-\sin\left(dx+c\right)^{2}-\sin\left(dx+c\right)+1}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/16*(3*a*log(sin(d*x + c) + 1) - 3*a*log(sin(d*x + c) - 1) - 2*(3*a*sin(d*
x + c)^2 - 3*a*sin(d*x + c) - 2*a)/(sin(d*x + c)^3 - sin(d*x + c)^2 - sin(d
*x + c) + 1))/d
```

Fricas [A]
time = 0.37, size = 136, normalized size = 1.62

$$-\frac{6\,a\cos\left(dx+c\right)^{2}-3\,\left(a\cos\left(dx+c\right)^{2}\sin\left(dx+c\right)-a\cos\left(dx+c\right)^{2}\right)\log\left(\sin\left(dx+c\right)+1\right)+3\,\left(a\cos\left(dx+c\right)^{2}\sin\left(dx+c\right)-a\cos\left(dx+c\right)^{2}\right)\log\left(-\sin\left(dx+c\right)+1\right)+6\,a\sin\left(dx+c\right)-2\,a}{16\,\left(d\cos\left(dx+c\right)^{2}\sin\left(dx+c\right)-d\cos\left(dx+c\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/16*(6*a*cos(d*x + c)^2 - 3*(a*cos(d*x + c)^2*sin(d*x + c) - a*cos(d*x +
c)^2)*log(sin(d*x + c) + 1) + 3*(a*cos(d*x + c)^2*sin(d*x + c) - a*cos(d*x
+ c)^2)*log(-sin(d*x + c) + 1) + 6*a*sin(d*x + c) - 2*a)/(d*cos(d*x + c)^2*
sin(d*x + c) - d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\sin\left(c+dx\right)\sec^{5}\left(c+dx\right)\,dx+\int\sec^{5}\left(c+dx\right)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a+a*sin(d*x+c)),x)
```

```
[Out] a*(Integral(sin(c + d*x)*sec(c + d*x)**5, x) + Integral(sec(c + d*x)**5, x)
)
```

Giac [A]
time = 6.17, size = 92, normalized size = 1.10

$$\frac{6\,a\log\left(\left|\sin\left(dx+c\right)+1\right|\right)-6\,a\log\left(\left|\sin\left(dx+c\right)-1\right|\right)-\frac{2\,\left(3\,a\sin\left(dx+c\right)+5\,a\right)}{\sin\left(dx+c\right)+1}+\frac{9\,a\sin\left(dx+c\right)^{2}-26\,a\sin\left(dx+c\right)+21\,a}{\left(\sin\left(dx+c\right)-1\right)^{2}}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/32*(6*a*log(abs(sin(d*x + c) + 1)) - 6*a*log(abs(sin(d*x + c) - 1)) - 2*(
3*a*sin(d*x + c) + 5*a)/(sin(d*x + c) + 1) + (9*a*sin(d*x + c)^2 - 26*a*sin
(d*x + c) + 21*a)/(sin(d*x + c) - 1)^2)/d
```

Mupad [B]
time = 4.50, size = 71, normalized size = 0.85

$$\frac{3\,a\,\mathrm{atanh}(\sin\left(c+d\,x\right))}{8\,d}-\frac{-\frac{3\,a\,\sin\left(c+d\,x\right)^2}{8}+\frac{3\,a\,\sin\left(c+d\,x\right)}{8}+\frac{a}{4}}{d\,\left(-\sin\left(c+d\,x\right)^3+\sin\left(c+d\,x\right)^2+\sin\left(c+d\,x\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))/cos(c + d*x)^5,x)
```

```
[Out] (3*a*atanh(sin(c + d*x)))/(8*d) - (a/4 + (3*a*sin(c + d*x))/8 - (3*a*sin(c
+ d*x)^2)/8)/(d*(sin(c + d*x) + sin(c + d*x)^2 - sin(c + d*x)^3 - 1))
```

3.13 $\int \cos^6(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=126

$$\frac{45a^2x}{128}-\frac{9a^2\cos^7(c+dx)}{56d}+\frac{45a^2\cos(c+dx)\sin(c+dx)}{128d}+\frac{15a^2\cos^3(c+dx)\sin(c+dx)}{64d}+\frac{3a^2\cos^5(c+dx)\sin(}{16d}$$

```
[Out] 45/128*a^2*x-9/56*a^2*cos(d*x+c)^7/d+45/128*a^2*cos(d*x+c)*sin(d*x+c)/d+15/
64*a^2*cos(d*x+c)^3*sin(d*x+c)/d+3/16*a^2*cos(d*x+c)^5*sin(d*x+c)/d-1/8*cos
(d*x+c)^7*(a^2+a^2*sin(d*x+c))/d
```

Rubi [A]

time = 0.07, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2757, 2748, 2715, 8}

$$-\frac{9a^2\cos^7(c+dx)}{56d}-\frac{\cos^7(c+dx)\left(a^2\sin(c+dx)+a^2\right)}{8d}+\frac{3a^2\sin(c+dx)\cos^5(c+dx)}{16d}+\frac{15a^2\sin(c+dx)\cos^3(c+dx)}{64d}+\frac{45a^2\sin(c+dx)\cos(c+dx)}{128d}+\frac{45a^2x}{128}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (45*a^2*x)/128 - (9*a^2*Cos[c + d*x]^7)/(56*d) + (45*a^2*Cos[c + d*x]*Sin[c
 + d*x])/(128*d) + (15*a^2*Cos[c + d*x]^3*Sin[c + d*x])/(64*d) + (3*a^2*Cos
[c + d*x]^5*Sin[c + d*x])/(16*d) - (Cos[c + d*x]^7*(a^2 + a^2*Sin[c + d*x])
)/(8*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2757

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
```

```
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ
[2*m, 2*p]
```

Rubi steps

$$\begin{aligned}\int \cos^6(c+dx)(a+a\sin(c+dx))^2\,dx &= -\frac{\cos^7(c+dx)\left(a^2+a^2\sin(c+dx)\right)}{8d}+\frac{1}{8}(9a)\int \cos^6(c+dx)(a+a\\
&= -\frac{9a^2\cos^7(c+dx)}{56d}-\frac{\cos^7(c+dx)\left(a^2+a^2\sin(c+dx)\right)}{8d}+\frac{1}{8}\left(9a^2\right)\\
&= -\frac{9a^2\cos^7(c+dx)}{56d}+\frac{3a^2\cos^5(c+dx)\sin(c+dx)}{16d}-\frac{\cos^7(c+dx)\\
&= -\frac{9a^2\cos^7(c+dx)}{56d}+\frac{15a^2\cos^3(c+dx)\sin(c+dx)}{64d}+\frac{3a^2\cos^5(c+\\
&= -\frac{9a^2\cos^7(c+dx)}{56d}+\frac{45a^2\cos(c+dx)\sin(c+dx)}{128d}+\frac{15a^2\cos^3(c+\\
&= \frac{45a^2x}{128}-\frac{9a^2\cos^7(c+dx)}{56d}+\frac{45a^2\cos(c+dx)\sin(c+dx)}{128d}+\frac{15a^2\end{aligned}$$

Mathematica [A]
time = 0.98, size = 171, normalized size = 1.36

$$-\frac{a^2\cos^7(c+dx)\left(630\sin^{-1}\left(\frac{\sqrt{1-\sin(c+dx)}}{\sqrt{2}}\right)\sqrt{1-\sin(c+dx)}+\sqrt{1+\sin(c+dx)}\left(256-837\sin(c+dx)-187\sin^2(c+dx)+978\sin^3(c+dx)+558\sin^4(c+dx)-600\sin^5(c+dx)-424\sin^6(c+dx)+144\sin^7(c+dx)+112\sin^8(c+dx)\right)\right)}{896d(-1+\sin(c+dx))^4(1+\sin(c+dx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/896*(a^2*Cos[c + d*x]^7*(630*ArcSin[Sqrt[1 - Sin[c + d*x]]/Sqrt[2]]*Sqrt
[1 - Sin[c + d*x]] + Sqrt[1 + Sin[c + d*x]]*(256 - 837*Sin[c + d*x] - 187*S
in[c + d*x]^2 + 978*Sin[c + d*x]^3 + 558*Sin[c + d*x]^4 - 600*Sin[c + d*x]^
5 - 424*Sin[c + d*x]^6 + 144*Sin[c + d*x]^7 + 112*Sin[c + d*x]^8)))/(d*(-1
+ Sin[c + d*x])^4*(1 + Sin[c + d*x])^(7/2))
```

Maple [A]
time = 0.27, size = 129, normalized size = 1.02

method	result
risch	$\frac{45a^2x}{128}-\frac{5a^2\cos(dx+c)}{32d}-\frac{a^2\sin(8dx+8c)}{1024d}-\frac{a^2\cos(7dx+7c)}{224d}-\frac{a^2\cos(5dx+5c)}{32d}+\frac{5a^2\sin(4dx+4c)}{128d}-\frac{3a^2\cos}{3}$

derivativedivides	$\frac{a^2\left(-\frac{\sin(dx+c)\left(\cos^7(dx+c)\right)}{8}+\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{48}+\frac{5dx}{128}+\frac{5c}{128}\right)-\frac{2\left(\cos^7(dx+c)\right)a^2}{7}+a^2\left(\right.}{d}$
default	$\frac{a^2\left(-\frac{\sin(dx+c)\left(\cos^7(dx+c)\right)}{8}+\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{48}+\frac{5dx}{128}+\frac{5c}{128}\right)-\frac{2\left(\cos^7(dx+c)\right)a^2}{7}+a^2\left(\right.}{d}$
norman	$-\frac{83a^2\left(\tan^{15}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64d}-\frac{295a^2\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64d}+\frac{315a^2x\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16}+\frac{83a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{64d}+\frac{3a^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64d}+\frac{295a^2\left(\tan\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/8*sin(d*x+c)*cos(d*x+c)^7+1/48*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+
15/8*cos(d*x+c))*sin(d*x+c)+5/128*d*x+5/128*c)-2/7*cos(d*x+c)^7*a^2+a^2*(1/
6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/16*d*x+5/16*
c))
```

Maxima [A]
time = 0.31, size = 115, normalized size = 0.91

$$-\frac{6144\,a^2\cos\left(dx+c\right)^7-7\left(64\sin\left(2\,dx+2\,c\right)^3+120\,dx+120\,c-3\sin\left(8\,dx+8\,c\right)-24\sin\left(4\,dx+4\,c\right)\right)a^2+112\left(4\sin\left(2\,dx+2\,c\right)^3-60\,dx-60\,c-9\sin\left(4\,dx+4\,c\right)-48\sin\left(2\,dx+2\,c\right)\right)a^2}{21504\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/21504*(6144*a^2*cos(d*x + c)^7 - 7*(64*sin(2*d*x + 2*c)^3 + 120*d*x + 12
0*c - 3*sin(8*d*x + 8*c) - 24*sin(4*d*x + 4*c))*a^2 + 112*(4*sin(2*d*x + 2*
c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*a^2)/d
```

Fricas [A]
time = 0.37, size = 85, normalized size = 0.67

$$-\frac{256\,a^2\cos\left(dx+c\right)^7-315\,a^2dx+7\left(16\,a^2\cos\left(dx+c\right)^7-24\,a^2\cos\left(dx+c\right)^5-30\,a^2\cos\left(dx+c\right)^3-45\,a^2\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{896\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/896*(256*a^2*cos(d*x + c)^7 - 315*a^2*d*x + 7*(16*a^2*cos(d*x + c)^7 - 2
4*a^2*cos(d*x + c)^5 - 30*a^2*cos(d*x + c)^3 - 45*a^2*cos(d*x + c))*sin(d*x
 + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 398 vs. $2(119) = 238$.
time = 0.99, size = 398, normalized size = 3.16

$$\begin{cases} \frac{5a^2x\sin^8(c+dx)}{128} + \frac{5a^2x\sin^6(c+dx)\cos^2(c+dx)}{32} + \frac{5a^2x\sin^6(c+dx)}{16} + \frac{15a^2x\sin^4(c+dx)\cos^4(c+dx)}{64} + \frac{15a^2x\sin^4(c+dx)\cos^2(c+dx)}{16} + \frac{5a^2x\sin^2(c+dx)\cos^6(c+dx)}{32} + \frac{15a^2x\sin^2(c+dx)\cos^4(c+dx)}{16} + \frac{5a^2x\cos^8(c+dx)}{128} + \frac{5a^2x\cos^6(c+dx)}{16} + \frac{5a^2\sin^7(c+dx)\cos(c+dx)}{128d} + \frac{55a^2\sin^5(c+dx)\cos^3(c+dx)}{384d} + \frac{5a^2\sin^5(c+dx)\cos(c+dx)}{16d} + \frac{73a^2\sin^3(c+dx)\cos^5(c+dx)}{384d} + \frac{5a^2\sin^3(c+dx)\cos^3(c+dx)}{6d} - \frac{5a^2\sin(c+dx)\cos^7(c+dx)}{128d} + \frac{11a^2\sin(c+dx)\cos^5(c+dx)}{16d} - \frac{2a^2\cos^7(c+dx)}{7d} & \text{for } d \neq 0 \\ x(a\sin(c)+a)^2\cos^6(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((5*a**2*x*sin(c + d*x)**8/128 + 5*a**2*x*sin(c + d*x)**6*cos(c +
d*x)**2/32 + 5*a**2*x*sin(c + d*x)**6/16 + 15*a**2*x*sin(c + d*x)**4*cos(c
+ d*x)**4/64 + 15*a**2*x*sin(c + d*x)**4*cos(c + d*x)**2/16 + 5*a**2*x*sin(
c + d*x)**2*cos(c + d*x)**6/32 + 15*a**2*x*sin(c + d*x)**2*cos(c + d*x)**4/
16 + 5*a**2*x*cos(c + d*x)**8/128 + 5*a**2*x*cos(c + d*x)**6/16 + 5*a**2*si
n(c + d*x)**7*cos(c + d*x)/(128*d) + 55*a**2*sin(c + d*x)**5*cos(c + d*x)**
3/(384*d) + 5*a**2*sin(c + d*x)**5*cos(c + d*x)/(16*d) + 73*a**2*sin(c + d*
x)**3*cos(c + d*x)**5/(384*d) + 5*a**2*sin(c + d*x)**3*cos(c + d*x)**3/(6*d
) - 5*a**2*sin(c + d*x)*cos(c + d*x)**7/(128*d) + 11*a**2*sin(c + d*x)*cos(
c + d*x)**5/(16*d) - 2*a**2*cos(c + d*x)**7/(7*d), Ne(d, 0)), (x*(a*sin(c)
+ a)**2*cos(c)**6, True))
```

Giac [A]
time = 5.47, size = 123, normalized size = 0.98

$$\frac{45}{128}a^2x - \frac{a^2\cos(7dx+7c)}{224d} - \frac{a^2\cos(5dx+5c)}{32d} - \frac{3a^2\cos(3dx+3c)}{32d} - \frac{5a^2\cos(dx+c)}{32d} - \frac{a^2\sin(8dx+8c)}{1024d} + \frac{5a^2\sin(4dx+4c)}{128d} + \frac{a^2\sin(2dx+2c)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 45/128*a^2*x - 1/224*a^2*cos(7*d*x + 7*c)/d - 1/32*a^2*cos(5*d*x + 5*c)/d -
 3/32*a^2*cos(3*d*x + 3*c)/d - 5/32*a^2*cos(d*x + c)/d - 1/1024*a^2*sin(8*d
*x + 8*c)/d + 5/128*a^2*sin(4*d*x + 4*c)/d + 1/4*a^2*sin(2*d*x + 2*c)/d
```

Mupad [B]
time = 6.92, size = 461, normalized size = 3.66

$$\frac{45a^2x}{128} - \frac{\frac{815a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9}{64} - \frac{3a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{64} - \frac{815a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{64} - \frac{295a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{64} + \frac{3a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}}{64} + \frac{295a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}}{64} + \frac{83a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{15}}{64} + \frac{a^2(315c+315dx)}{896} - \frac{a^2(315c+315dx-512)}{896} + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(\frac{a^2(315c+315dx)}{112} - \frac{a^2(2520c+2520dx-512)}{896}\right) + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}\left(\frac{a^2(315c+315dx)}{112} - \frac{a^2(2520c+2520dx-3584)}{896}\right) + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}\left(\frac{a^2(315c+315dx)}{32} - \frac{a^2(8820c+8820dx-3584)}{896}\right) + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(\frac{a^2(315c+315dx)}{32} - \frac{a^2(8820c+8820dx-10752)}{896}\right) + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6\left(\frac{a^2(315c+315dx)}{16} - \frac{a^2(17640c+17640dx-10752)}{896}\right) + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}\left(\frac{a^2(315c+315dx)}{16} - \frac{a^2(17640c+17640dx-17920)}{896}\right) + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8\left(\frac{5a^2(315c+315dx)}{64} - \frac{a^2(22050c+22050dx-17920)}{896}\right) - \frac{83a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{64}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6*(a + a*sin(c + d*x))^2,x)
```

```
[Out] (45*a^2*x)/128 - ((815*a^2*tan(c/2 + (d*x)/2)^9)/64 - (3*a^2*tan(c/2 + (d*x
)/2)^5)/64 - (815*a^2*tan(c/2 + (d*x)/2)^7)/64 - (295*a^2*tan(c/2 + (d*x)/2
)^3)/64 + (3*a^2*tan(c/2 + (d*x)/2)^11)/64 + (295*a^2*tan(c/2 + (d*x)/2)^13
)/64 + (83*a^2*tan(c/2 + (d*x)/2)^15)/64 + (a^2*(315*c + 315*d*x))/896 - (a
```

```
^2*(315*c + 315*d*x - 512))/896 + tan(c/2 + (d*x)/2)^2*((a^2*(315*c + 315*d
*x))/112 - (a^2*(2520*c + 2520*d*x - 512))/896) + tan(c/2 + (d*x)/2)^14*((a
^2*(315*c + 315*d*x))/112 - (a^2*(2520*c + 2520*d*x - 3584))/896) + tan(c/2
 + (d*x)/2)^12*((a^2*(315*c + 315*d*x))/32 - (a^2*(8820*c + 8820*d*x - 3584
))/896) + tan(c/2 + (d*x)/2)^4*((a^2*(315*c + 315*d*x))/32 - (a^2*(8820*c +
 8820*d*x - 10752))/896) + tan(c/2 + (d*x)/2)^6*((a^2*(315*c + 315*d*x))/16
 - (a^2*(17640*c + 17640*d*x - 10752))/896) + tan(c/2 + (d*x)/2)^10*((a^2*(
315*c + 315*d*x))/16 - (a^2*(17640*c + 17640*d*x - 17920))/896) + tan(c/2 +
 (d*x)/2)^8*((5*a^2*(315*c + 315*d*x))/64 - (a^2*(22050*c + 22050*d*x - 179
20))/896) - (83*a^2*tan(c/2 + (d*x)/2))/64)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^8
)
```

3.14 $\int \cos^5(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=67

$$\frac{4(a+a\sin(c+dx))^5}{5a^3d}-\frac{2(a+a\sin(c+dx))^6}{3a^4d}+\frac{(a+a\sin(c+dx))^7}{7a^5d}$$

```
[Out] 4/5*(a+a*sin(d*x+c))^5/a^3/d-2/3*(a+a*sin(d*x+c))^6/a^4/d+1/7*(a+a*sin(d*x+
c))^7/a^5/d
```

Rubi [A]
time = 0.04, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.095$, Rules used = {2746, 45}

$$\frac{(a\sin(c+dx)+a)^7}{7a^5d}-\frac{2(a\sin(c+dx)+a)^6}{3a^4d}+\frac{4(a\sin(c+dx)+a)^5}{5a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (4*(a + a*Sin[c + d*x])^5)/(5*a^3*d) - (2*(a + a*Sin[c + d*x])^6)/(3*a^4*d)
 + (a + a*Sin[c + d*x])^7/(7*a^5*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \cos^5(c+dx)(a+a\sin(c+dx))^2\,dx &= \frac{\text{Subst}\left(\int (a-x)^2(a+x)^4\,dx,x,a\sin(c+dx)\right)}{a^5d}\\ &= \frac{\text{Subst}\left(\int \left(4a^2(a+x)^4-4a(a+x)^5+(a+x)^6\right)\,dx,x,a\sin(c+dx\right.}{a^5d}\\ &= \frac{4(a+a\sin(c+dx))^5}{5a^3d}-\frac{2(a+a\sin(c+dx))^6}{3a^4d}+\frac{(a+a\sin(c+dx\right.}{7a^5d}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 58, normalized size = 0.87

$$-\frac{a^2\cos^6(c+dx)(1+\sin(c+dx))^2\left(29-40\sin(c+dx)+15\sin^2(c+dx)\right)}{105d(-1+\sin(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/105*(a^2*Cos[c + d*x]^6*(1 + Sin[c + d*x])^2*(29 - 40*Sin[c + d*x] + 15*
Sin[c + d*x]^2))/(d*(-1 + Sin[c + d*x])^3)
```

Maple [A]
time = 0.21, size = 99, normalized size = 1.48

method	result
derivativedivides	$\frac{a^2\left(-\frac{\sin(dx+c)\left(\cos^6(dx+c)\right)}{7}+\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{35}\right)-\frac{\left(\cos^6(dx+c)\right)a^2}{3}+\frac{a^2\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(d\right.}{3}\right.}{5}}{d}$
default	$\frac{a^2\left(-\frac{\sin(dx+c)\left(\cos^6(dx+c)\right)}{7}+\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{35}\right)-\frac{\left(\cos^6(dx+c)\right)a^2}{3}+\frac{a^2\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(d\right.}{3}\right.}{5}}{d}$
risch	$\frac{45a^2\sin(dx+c)}{64d}-\frac{a^2\sin(7dx+7c)}{448d}-\frac{a^2\cos(6dx+6c)}{96d}+\frac{a^2\sin(5dx+5c)}{320d}-\frac{a^2\cos(4dx+4c)}{16d}+\frac{19a^2\sin(3dx+3c)}{192d}-$
norman	$\frac{4a^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{4a^2\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{4a^2\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{4a^2\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{28a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/7*sin(d*x+c)*cos(d*x+c)^6+1/35*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c
)^2)*sin(d*x+c))-1/3*cos(d*x+c)^6*a^2+1/5*a^2*(8/3+cos(d*x+c)^4+4/3*cos(d*x
+c)^2)*sin(d*x+c))
```

Maxima [A]
time = 0.33, size = 95, normalized size = 1.42

$$\frac{15\,a^2\sin(dx+c)^7+35\,a^2\sin(dx+c)^6-21\,a^2\sin(dx+c)^5-105\,a^2\sin(dx+c)^4-35\,a^2\sin(dx+c)^3+105\,a^2\sin(dx+c)^2+105\,a^2\sin(dx+c)}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/105*(15*a^2*sin(d*x + c)^7 + 35*a^2*sin(d*x + c)^6 - 21*a^2*sin(d*x + c)^
5 - 105*a^2*sin(d*x + c)^4 - 35*a^2*sin(d*x + c)^3 + 105*a^2*sin(d*x + c)^2
 + 105*a^2*sin(d*x + c))/d
```

Fricas [A]
time = 0.37, size = 71, normalized size = 1.06

$$-\frac{35\,a^2\cos\left(dx+c\right)^6+\left(15\,a^2\cos\left(dx+c\right)^6-24\,a^2\cos\left(dx+c\right)^4-32\,a^2\cos\left(dx+c\right)^2-64\,a^2\right)\sin\left(dx+c\right)}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/105*(35*a^2*cos(d*x + c)^6 + (15*a^2*cos(d*x + c)^6 - 24*a^2*cos(d*x + c
)^4 - 32*a^2*cos(d*x + c)^2 - 64*a^2)*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(58) = 116$.
time = 0.66, size = 158, normalized size = 2.36

$$\begin{cases} \frac{8a^2\sin^7(c+dx)}{105d}+\frac{4a^2\sin^5(c+dx)\cos^2(c+dx)}{15d}+\frac{8a^2\sin^5(c+dx)}{15d}+\frac{a^2\sin^3(c+dx)\cos^4(c+dx)}{3d}+\frac{4a^2\sin^3(c+dx)\cos^2(c+dx)}{3d}+\frac{a^2\sin(c+dx)\cos^4(c+dx)}{d}-\frac{a^2\cos^6(c+dx)}{3d} & \text{for } d\neq 0 \\ x(a\sin(c)+a)^2\cos^5(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((8*a**2*sin(c + d*x)**7/(105*d) + 4*a**2*sin(c + d*x)**5*cos(c +
d*x)**2/(15*d) + 8*a**2*sin(c + d*x)**5/(15*d) + a**2*sin(c + d*x)**3*cos(c
 + d*x)**4/(3*d) + 4*a**2*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + a**2*sin(
c + d*x)*cos(c + d*x)**4/d - a**2*cos(c + d*x)**6/(3*d), Ne(d, 0)), (x*(a*s
in(c) + a)**2*cos(c)**5, True))
```

Giac [A]
time = 4.28, size = 117, normalized size = 1.75

$$-\frac{a^2\cos\left(6\,dx+6\,c\right)}{96\,d}-\frac{a^2\cos\left(4\,dx+4\,c\right)}{16\,d}-\frac{5\,a^2\cos\left(2\,dx+2\,c\right)}{32\,d}-\frac{a^2\sin\left(7\,dx+7\,c\right)}{448\,d}+\frac{a^2\sin\left(5\,dx+5\,c\right)}{320\,d}+\frac{19\,a^2\sin\left(3\,dx+3\,c\right)}{192\,d}+\frac{45\,a^2\sin\left(dx+c\right)}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/96*a^2*cos(6*d*x + 6*c)/d - 1/16*a^2*cos(4*d*x + 4*c)/d - 5/32*a^2*cos(2
*d*x + 2*c)/d - 1/448*a^2*sin(7*d*x + 7*c)/d + 1/320*a^2*sin(5*d*x + 5*c)/d
 + 19/192*a^2*sin(3*d*x + 3*c)/d + 45/64*a^2*sin(d*x + c)/d
```

Mupad [B]
time = 4.54, size = 92, normalized size = 1.37

$$\frac{\frac{a^2\sin(c+d\,x)^7}{7}+\frac{a^2\sin(c+d\,x)^6}{3}-\frac{a^2\sin(c+d\,x)^5}{5}-a^2\sin\left(c+d\,x\right)^4-\frac{a^2\sin(c+d\,x)^3}{3}+a^2\sin\left(c+d\,x\right)^2+a^2\,\sin\left(c+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(a + a*sin(c + d*x))^2,x)
```

```
[Out] (a^2*sin(c + d*x) + a^2*sin(c + d*x)^2 - (a^2*sin(c + d*x)^3)/3 - a^2*sin(c
 + d*x)^4 - (a^2*sin(c + d*x)^5)/5 + (a^2*sin(c + d*x)^6)/3 + (a^2*sin(c +
d*x)^7)/7)/d
```

3.15 $\int \cos^4(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=102

$$\frac{7a^2x}{16}-\frac{7a^2\cos^5(c+dx)}{30d}+\frac{7a^2\cos(c+dx)\sin(c+dx)}{16d}+\frac{7a^2\cos^3(c+dx)\sin(c+dx)}{24d}-\frac{\cos^5(c+dx)\,(a^2+a^2\sin(c+dx))}{6d}$$

```
[Out] 7/16*a^2*x-7/30*a^2*cos(d*x+c)^5/d+7/16*a^2*cos(d*x+c)*sin(d*x+c)/d+7/24*a^
2*cos(d*x+c)^3*sin(d*x+c)/d-1/6*cos(d*x+c)^5*(a^2+a^2*sin(d*x+c))/d
```

Rubi [A]
time = 0.06, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2757, 2748, 2715, 8}

$$-\frac{7a^2\cos^5(c+dx)}{30d}-\frac{\cos^5(c+dx)\,(a^2\sin(c+dx)+a^2)}{6d}+\frac{7a^2\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{7a^2\sin(c+dx)\cos(c+dx)}{16d}+\frac{7a^2x}{16}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (7*a^2*x)/16 - (7*a^2*Cos[c + d*x]^5)/(30*d) + (7*a^2*Cos[c + d*x]*Sin[c +
d*x])/(16*d) + (7*a^2*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) - (Cos[c + d*x]^5
*(a^2 + a^2*Sin[c + d*x]))/(6*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2757

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
```

```
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ
[2*m, 2*p]
```

Rubi steps

$$\int \cos^4(c+dx)(a+a\sin(c+dx))^2\,dx = -\frac{\cos^5(c+dx)\left(a^2+a^2\sin(c+dx)\right)}{6d}+\frac{1}{6}(7a)\int \cos^4(c+dx)(a+a\,\mathrm{s}$$

$$= -\frac{7a^2\cos^5(c+dx)}{30d}-\frac{\cos^5(c+dx)\left(a^2+a^2\sin(c+dx)\right)}{6d}+\frac{1}{6}\left(7a^2\right)\int$$

$$= -\frac{7a^2\cos^5(c+dx)}{30d}+\frac{7a^2\cos^3(c+dx)\sin(c+dx)}{24d}-\frac{\cos^5(c+dx)\,(a$$

$$= -\frac{7a^2\cos^5(c+dx)}{30d}+\frac{7a^2\cos(c+dx)\sin(c+dx)}{16d}+\frac{7a^2\cos^3(c+dx)}{24d}$$

$$= \frac{7a^2x}{16}-\frac{7a^2\cos^5(c+dx)}{30d}+\frac{7a^2\cos(c+dx)\sin(c+dx)}{16d}+\frac{7a^2\cos^3(c}{}$$

Mathematica [A]

time = 0.35, size = 151, normalized size = 1.48

$$-\frac{a^2\cos^5(c+dx)\left(-210\sin^{-1}\left(\frac{\sqrt{1-\sin(c+dx)}}{\sqrt{2}}\right)\sqrt{1-\sin(c+dx)}+\sqrt{1+\sin(c+dx)}\left(-96+231\sin(c+dx)+57\sin^2(c+dx)-182\sin^3(c+dx)-106\sin^4(c+dx)+56\sin^5(c+dx)+40\sin^6(c+dx)\right)\right)}{240d(-1+\sin(c+dx))^3(1+\sin(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/240*(a^2*Cos[c + d*x]^5*(-210*ArcSin[Sqrt[1 - Sin[c + d*x]]/Sqrt[2]]*Sqr
t[1 - Sin[c + d*x]] + Sqrt[1 + Sin[c + d*x]]*(-96 + 231*Sin[c + d*x] + 57*S
in[c + d*x]^2 - 182*Sin[c + d*x]^3 - 106*Sin[c + d*x]^4 + 56*Sin[c + d*x]^5
 + 40*Sin[c + d*x]^6)))/(d*(-1 + Sin[c + d*x])^3*(1 + Sin[c + d*x])^(5/2))
```

Maple [A]

time = 0.19, size = 109, normalized size = 1.07

method	result
risch	$\frac{7a^2x}{16}-\frac{a^2\cos(dx+c)}{4d}-\frac{a^2\sin(6dx+6c)}{192d}-\frac{a^2\cos(5dx+5c)}{40d}+\frac{a^2\sin(4dx+4c)}{64d}-\frac{a^2\cos(3dx+3c)}{8d}+\frac{17a^2\sin(2dx}{64d}$
derivativedivides	$\frac{a^2\left(-\frac{\sin(dx+c)\left(\cos^5(dx+c)\right)}{6}+\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{24}+\frac{dx}{16}+\frac{c}{16}\right)-\frac{2\left(\cos^5(dx+c)\right)a^2}{5}+a^2\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(c}{2}\right.}{4}\right.}{d}$
default	$\frac{a^2\left(-\frac{\sin(dx+c)\left(\cos^5(dx+c)\right)}{6}+\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{24}+\frac{dx}{16}+\frac{c}{16}\right)-\frac{2\left(\cos^5(dx+c)\right)a^2}{5}+a^2\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(c}{2}\right.}{4}\right.}{d}$

norman	$\frac{7a^2x}{16}-\frac{4a^2}{5d}+\frac{9a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d}+\frac{89a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d}-\frac{11a^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{11a^2\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}-\frac{89a^2\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d}-\frac{9a}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/6*sin(d*x+c)*cos(d*x+c)^5+1/24*(cos(d*x+c)^3+3/2*cos(d*x+c))*s
in(d*x+c)+1/16*d*x+1/16*c)-2/5*cos(d*x+c)^5*a^2+a^2*(1/4*(cos(d*x+c)^3+3/2*
cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c))
```

Maxima [A]
time = 0.30, size = 89, normalized size = 0.87

$$-\frac{384\,a^2\cos\left(dx+c\right)^5-5\,\left(4\,\sin\left(2\,dx+2\,c\right)^3+12\,dx+12\,c-3\,\sin\left(4\,dx+4\,c\right)\right)a^2-30\,\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right)\right)a^2}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/960*(384*a^2*cos(d*x + c)^5 - 5*(4*sin(2*d*x + 2*c)^3 + 12*d*x + 12*c -
3*sin(4*d*x + 4*c))*a^2 - 30*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*
x + 2*c))*a^2)/d
```

Fricas [A]
time = 0.37, size = 72, normalized size = 0.71

$$-\frac{96\,a^2\cos\left(dx+c\right)^5-105\,a^2dx+5\,\left(8\,a^2\cos\left(dx+c\right)^5-14\,a^2\cos\left(dx+c\right)^3-21\,a^2\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/240*(96*a^2*cos(d*x + c)^5 - 105*a^2*d*x + 5*(8*a^2*cos(d*x + c)^5 - 14*
a^2*cos(d*x + c)^3 - 21*a^2*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 287 vs. $2(95) = 190$.
time = 0.43, size = 287, normalized size = 2.81

$$\begin{cases}\frac{a^2x\sin^6\left(c+dx\right)}{16}+\frac{3a^2x\sin^4\left(c+dx\right)\cos^2\left(c+dx\right)}{16}+\frac{3a^2x\sin^4\left(c+dx\right)}{8}+\frac{3a^2x\sin^2\left(c+dx\right)\cos^4\left(c+dx\right)}{16}+\frac{3a^2x\sin^2\left(c+dx\right)\cos^2\left(c+dx\right)}{4}+\frac{a^2x\cos^6\left(c+dx\right)}{16}+\frac{3a^2x\cos^4\left(c+dx\right)}{8}+\frac{a^2\sin^5\left(c+dx\right)\cos\left(c+dx\right)}{16d}+\frac{a^2\sin^3\left(c+dx\right)\cos^3\left(c+dx\right)}{6d}+\frac{3a^2\sin^3\left(c+dx\right)\cos\left(c+dx\right)}{8d}-\frac{a^2\sin\left(c+dx\right)\cos^5\left(c+dx\right)}{16d}+\frac{5a^2\sin\left(c+dx\right)\cos^3\left(c+dx\right)}{8d}-\frac{2a^2\cos^5\left(c+dx\right)}{5d} & \text{for}\; d\neq 0\\ x\left(a\sin\left(c\right)+a\right)^2\cos^4\left(c\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*x*sin(c + d*x)**6/16 + 3*a**2*x*sin(c + d*x)**4*cos(c + d*x
)**2/16 + 3*a**2*x*sin(c + d*x)**4/8 + 3*a**2*x*sin(c + d*x)**2*cos(c + d*x
```

```
)**4/16 + 3*a**2*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + a**2*x*cos(c + d*x)*
*6/16 + 3*a**2*x*cos(c + d*x)**4/8 + a**2*sin(c + d*x)**5*cos(c + d*x)/(16*
d) + a**2*sin(c + d*x)**3*cos(c + d*x)**3/(6*d) + 3*a**2*sin(c + d*x)**3*co
s(c + d*x)/(8*d) - a**2*sin(c + d*x)*cos(c + d*x)**5/(16*d) + 5*a**2*sin(c
+ d*x)*cos(c + d*x)**3/(8*d) - 2*a**2*cos(c + d*x)**5/(5*d), Ne(d, 0)), (x*
(a*sin(c) + a)**2*cos(c)**4, True))
```

Giac [A]

time = 4.54, size = 106, normalized size = 1.04

$$\frac{7}{16}\,a^2x-\frac{a^2\cos\left(5\,dx+5\,c\right)}{40\,d}-\frac{a^2\cos\left(3\,dx+3\,c\right)}{8\,d}-\frac{a^2\cos\left(dx+c\right)}{4\,d}-\frac{a^2\sin\left(6\,dx+6\,c\right)}{192\,d}+\frac{a^2\sin\left(4\,dx+4\,c\right)}{64\,d}+\frac{17\,a^2\sin\left(2\,dx+2\,c\right)}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 7/16*a^2*x - 1/40*a^2*cos(5*d*x + 5*c)/d - 1/8*a^2*cos(3*d*x + 3*c)/d - 1/4
*a^2*cos(d*x + c)/d - 1/192*a^2*sin(6*d*x + 6*c)/d + 1/64*a^2*sin(4*d*x + 4
*c)/d + 17/64*a^2*sin(2*d*x + 2*c)/d
```

Mupad [B]

time = 6.79, size = 349, normalized size = 3.42

$$\frac{7\,a^2\,x}{16}-\frac{\frac{11\,a^2\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5}{4}-\frac{89\,a^2\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3}{24}-\frac{11\,a^2\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7}{4}+\frac{89\,a^2\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^9}{24}+\frac{9\,a^2\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{11}}{8}+\frac{a^2\,\left(105\,c+105\,d\,x\right)}{240}-\frac{a^2\,\left(105\,c+105\,d\,x-192\right)}{240}+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(\frac{a^2\,\left(105\,c+105\,d\,x\right)}{40}-\frac{a^2\,\left(630\,c+630\,d\,x-192\right)}{240}\right)+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}\,\left(\frac{a^2\,\left(105\,c+105\,d\,x\right)}{40}-\frac{a^2\,\left(630\,c+630\,d\,x-960\right)}{240}\right)+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8\,\left(\frac{a^2\,\left(105\,c+105\,d\,x\right)}{16}-\frac{a^2\,\left(1575\,c+1575\,d\,x-960\right)}{240}\right)+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(\frac{a^2\,\left(105\,c+105\,d\,x\right)}{16}-\frac{a^2\,\left(1575\,c+1575\,d\,x-1920\right)}{240}\right)+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6\,\left(\frac{a^2\,\left(105\,c+105\,d\,x\right)}{12}-\frac{a^2\,\left(2100\,c+2100\,d\,x-1920\right)}{240}\right)-\frac{9\,a^2\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8}}{d\,{\left({\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(a + a*sin(c + d*x))^2,x)
```

```
[Out] (7*a^2*x)/16 - ((11*a^2*tan(c/2 + (d*x)/2)^5)/4 - (89*a^2*tan(c/2 + (d*x)/2
)^3)/24 - (11*a^2*tan(c/2 + (d*x)/2)^7)/4 + (89*a^2*tan(c/2 + (d*x)/2)^9)/2
4 + (9*a^2*tan(c/2 + (d*x)/2)^11)/8 + (a^2*(105*c + 105*d*x))/240 - (a^2*(1
05*c + 105*d*x - 192))/240 + tan(c/2 + (d*x)/2)^2*((a^2*(105*c + 105*d*x))/
40 - (a^2*(630*c + 630*d*x - 192))/240) + tan(c/2 + (d*x)/2)^10*((a^2*(105*
c + 105*d*x))/40 - (a^2*(630*c + 630*d*x - 960))/240) + tan(c/2 + (d*x)/2)^
8*((a^2*(105*c + 105*d*x))/16 - (a^2*(1575*c + 1575*d*x - 960))/240) + tan(
c/2 + (d*x)/2)^4*((a^2*(105*c + 105*d*x))/16 - (a^2*(1575*c + 1575*d*x - 19
20))/240) + tan(c/2 + (d*x)/2)^6*((a^2*(105*c + 105*d*x))/12 - (a^2*(2100*c
 + 2100*d*x - 1920))/240) - (9*a^2*tan(c/2 + (d*x)/2))/8)/(d*(tan(c/2 + (d*
x)/2)^2 + 1)^6)
```

3.16 $\int \cos^3(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=45

$$\frac{(a+a\sin(c+dx))^4}{2a^2d}-\frac{(a+a\sin(c+dx))^5}{5a^3d}$$

```
[Out] 1/2*(a+a*sin(d*x+c))^4/a^2/d-1/5*(a+a*sin(d*x+c))^5/a^3/d
```

Rubi [A]
time = 0.03, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.095$, Rules used = {2746, 45}

$$\frac{(a\sin(c+dx)+a)^4}{2a^2d}-\frac{(a\sin(c+dx)+a)^5}{5a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a + a*Sin[c + d*x])^4/(2*a^2*d) - (a + a*Sin[c + d*x])^5/(5*a^3*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \cos^3(c+dx)(a+a\sin(c+dx))^2\,dx &= \frac{\text{Subst}\left(\int (a-x)(a+x)^3\,dx,x,a\sin(c+dx)\right)}{a^3d}\\ &= \frac{\text{Subst}\left(\int \left(2a(a+x)^3-(a+x)^4\right)\,dx,x,a\sin(c+dx)\right)}{a^3d}\\ &= \frac{(a+a\sin(c+dx))^4}{2a^2d}-\frac{(a+a\sin(c+dx))^5}{5a^3d}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 46, normalized size = 1.02

$$-\frac{a^2 \sin(c+dx) \left(-10-10 \sin(c+dx)+5 \sin^3(c+dx)+2 \sin^4(c+dx)\right)}{10d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/10*(a^2*Sin[c + d*x]*(-10 - 10*Sin[c + d*x] + 5*Sin[c + d*x]^3 + 2*Sin[c
 + d*x]^4))/d
```

Maple [A]
time = 0.18, size = 79, normalized size = 1.76

method	result
derivativedivides	$\frac{a^2\left(-\frac{\sin(dx+c)\left(\cos^4(dx+c)\right)}{5}+\frac{\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{15}\right)-\frac{\left(\cos^4(dx+c)\right)a^2}{2}+\frac{a^2\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}}{d}$
default	$\frac{a^2\left(-\frac{\sin(dx+c)\left(\cos^4(dx+c)\right)}{5}+\frac{\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{15}\right)-\frac{\left(\cos^4(dx+c)\right)a^2}{2}+\frac{a^2\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}}{d}$
risch	$\frac{7a^2\sin(dx+c)}{8d}-\frac{a^2\sin(5dx+5c)}{80d}-\frac{a^2\cos(4dx+4c)}{16d}+\frac{a^2\sin(3dx+3c)}{16d}-\frac{a^2\cos(2dx+2c)}{4d}$
norman	$\frac{\frac{4a^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{4a^2\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{8a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{28a^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5d}+\frac{8a^2\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/5*sin(d*x+c)*cos(d*x+c)^4+1/15*(2+cos(d*x+c)^2)*sin(d*x+c))-1/
2*cos(d*x+c)^4*a^2+1/3*a^2*(2+cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A]
time = 0.29, size = 56, normalized size = 1.24

$$-\frac{2\,a^2\sin\left(dx+c\right)^5+5\,a^2\sin\left(dx+c\right)^4-10\,a^2\sin\left(dx+c\right)^2-10\,a^2\sin\left(dx+c\right)}{10\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/10*(2*a^2*sin(d*x + c)^5 + 5*a^2*sin(d*x + c)^4 - 10*a^2*sin(d*x + c)^2
- 10*a^2*sin(d*x + c))/d
```

Fricas [A]
time = 0.35, size = 58, normalized size = 1.29

$$-\frac{5 a^2 \cos (dx+c)^4 + 2 \left(a^2 \cos (dx+c)^4 - 2 a^2 \cos (dx+c)^2 - 4 a^2\right) \sin (dx+c)}{10 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/10*(5*a^2*cos(d*x + c)^4 + 2*(a^2*cos(d*x + c)^4 - 2*a^2*cos(d*x + c)^2
- 4*a^2)*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(36) = 72$.
time = 0.26, size = 107, normalized size = 2.38

$$\begin{cases} \frac{2a^2 \sin^5 (c+dx)}{15d} + \frac{a^2 \sin^3 (c+dx) \cos^2 (c+dx)}{3d} + \frac{2a^2 \sin^3 (c+dx)}{3d} + \frac{a^2 \sin (c+dx) \cos^2 (c+dx)}{d} - \frac{a^2 \cos^4 (c+dx)}{2d} & \text{for } d \neq 0 \\ x(a \sin (c) + a)^2 \cos^3 (c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((2*a**2*sin(c + d*x)**5/(15*d) + a**2*sin(c + d*x)**3*cos(c + d*x
)**2/(3*d) + 2*a**2*sin(c + d*x)**3/(3*d) + a**2*sin(c + d*x)*cos(c + d*x)*
*2/d - a**2*cos(c + d*x)**4/(2*d), Ne(d, 0)), (x*(a*sin(c) + a)**2*cos(c)**
3, True))
```

Giac [A]
time = 7.27, size = 56, normalized size = 1.24

$$-\frac{2 a^2 \sin (dx+c)^5 + 5 a^2 \sin (dx+c)^4 - 10 a^2 \sin (dx+c)^2 - 10 a^2 \sin (dx+c)}{10 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/10*(2*a^2*sin(d*x + c)^5 + 5*a^2*sin(d*x + c)^4 - 10*a^2*sin(d*x + c)^2
- 10*a^2*sin(d*x + c))/d
```

Mupad [B]
time = 0.06, size = 53, normalized size = 1.18

$$\frac{-\frac{a^2 \sin(c+d\,x)^5}{5} - \frac{a^2 \sin(c+d\,x)^4}{2} + a^2 \sin (c+d\,x)^2 + a^2\,\sin (c+d\,x)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a + a*sin(c + d*x))^2,x)
```

```
[Out] (a^2*sin(c + d*x) + a^2*sin(c + d*x)^2 - (a^2*sin(c + d*x)^4)/2 - (a^2*sin(
c + d*x)^5)/5)/d
```

3.17 $\int \cos^2(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=78

$$\frac{5a^2x}{8}-\frac{5a^2\cos^3(c+dx)}{12d}+\frac{5a^2\cos(c+dx)\sin(c+dx)}{8d}-\frac{\cos^3(c+dx)\,(a^2+a^2\sin(c+dx))}{4d}$$

```
[Out] 5/8*a^2*x-5/12*a^2*cos(d*x+c)^3/d+5/8*a^2*cos(d*x+c)*sin(d*x+c)/d-1/4*cos(d
*x+c)^3*(a^2+a^2*sin(d*x+c))/d
```

Rubi [A]
time = 0.05, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.190$, Rules used = {2757, 2748, 2715, 8}

$$-\frac{5a^2\cos^3(c+dx)}{12d}-\frac{\cos^3(c+dx)\,(a^2\sin(c+dx)+a^2)}{4d}+\frac{5a^2\sin(c+dx)\cos(c+dx)}{8d}+\frac{5a^2x}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (5*a^2*x)/8 - (5*a^2*Cos[c + d*x]^3)/(12*d) + (5*a^2*Cos[c + d*x]*Sin[c + d
*x])/(8*d) - (Cos[c + d*x]^3*(a^2 + a^2*Sin[c + d*x]))/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2757

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
```

```
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ
[2*m, 2*p]
```

Rubi steps

$$\begin{aligned}\int \cos^2(c+dx)(a+a\sin(c+dx))^2\,dx &= -\frac{\cos^3(c+dx)\left(a^2+a^2\sin(c+dx)\right)}{4d}+\frac{1}{4}(5a)\int \cos^2(c+dx)(a+a\\ &= -\frac{5a^2\cos^3(c+dx)}{12d}-\frac{\cos^3(c+dx)\left(a^2+a^2\sin(c+dx)\right)}{4d}+\frac{1}{4}\left(5a^2\right)\\ &= -\frac{5a^2\cos^3(c+dx)}{12d}+\frac{5a^2\cos(c+dx)\sin(c+dx)}{8d}-\frac{\cos^3(c+dx)\,(a\\ &= \frac{5a^2x}{8}-\frac{5a^2\cos^3(c+dx)}{12d}+\frac{5a^2\cos(c+dx)\sin(c+dx)}{8d}-\frac{\cos^3(c+\end{aligned}$$

Mathematica [A]
time = 0.20, size = 131, normalized size = 1.68

$$-\frac{a^2\cos^3(c+dx)\left(30\sin^{-1}\left(\frac{\sqrt{1-\sin(c+dx)}}{\sqrt{2}}\right)\sqrt{1-\sin(c+dx)}+\sqrt{1+\sin(c+dx)}\,(16-25\sin(c+dx)-7\sin^2(c+dx)+10\sin^3(c+dx)+6\sin^4(c+dx))\right)}{24d(-1+\sin(c+dx))^2(1+\sin(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
[Out] -1/24*(a^2*Cos[c + d*x]^3*(30*ArcSin[Sqrt[1 - Sin[c + d*x]]/Sqrt[2]]*Sqrt[1
 - Sin[c + d*x]] + Sqrt[1 + Sin[c + d*x]]*(16 - 25*Sin[c + d*x] - 7*Sin[c +
 d*x]^2 + 10*Sin[c + d*x]^3 + 6*Sin[c + d*x]^4)))/(d*(-1 + Sin[c + d*x])^2*
(1 + Sin[c + d*x])^(3/2))
```

Maple [A]
time = 0.13, size = 87, normalized size = 1.12

method	result
risch	$\frac{5a^2x}{8}-\frac{a^2\cos(dx+c)}{2d}-\frac{a^2\sin(4dx+4c)}{32d}-\frac{a^2\cos(3dx+3c)}{6d}+\frac{a^2\sin(2dx+2c)}{4d}$
derivativedivides	$\frac{a^2\left(-\frac{\sin(dx+c)\left(\cos^3(dx+c)\right)}{4}+\frac{\cos(dx+c)\sin(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)-\frac{2\left(\cos^3(dx+c)\right)a^2}{3}+a^2\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
default	$\frac{a^2\left(-\frac{\sin(dx+c)\left(\cos^3(dx+c)\right)}{4}+\frac{\cos(dx+c)\sin(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)-\frac{2\left(\cos^3(dx+c)\right)a^2}{3}+a^2\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$

norman	$\frac{\frac{5a^2x}{8}-\frac{4a^2}{3d}+\frac{3a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}+\frac{11a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}-\frac{11a^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}-\frac{3a^2\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{5a^2x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{15a^2}{}}{\left(1+\tan^2\left(\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/4*sin(d*x+c)*cos(d*x+c)^3+1/8*cos(d*x+c)*sin(d*x+c)+1/8*d*x+1/
8*c)-2/3*cos(d*x+c)^3*a^2+a^2*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))
```

Maxima [A]
time = 0.31, size = 65, normalized size = 0.83

$$-\frac{64\,a^2\cos\left(dx+c\right)^3-3\left(4\,dx+4\,c-\sin\left(4\,dx+4\,c\right)\right)a^2-24\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)a^2}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/96*(64*a^2*cos(d*x + c)^3 - 3*(4*d*x + 4*c - sin(4*d*x + 4*c))*a^2 - 24*
(2*d*x + 2*c + sin(2*d*x + 2*c))*a^2)/d
```

Fricas [A]
time = 0.35, size = 59, normalized size = 0.76

$$-\frac{16\,a^2\cos\left(dx+c\right)^3-15\,a^2dx+3\,\left(2\,a^2\cos\left(dx+c\right)^3-5\,a^2\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/24*(16*a^2*cos(d*x + c)^3 - 15*a^2*d*x + 3*(2*a^2*cos(d*x + c)^3 - 5*a^2
*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(71) = 142$.
time = 0.19, size = 180, normalized size = 2.31

$$\begin{cases}\frac{a^2x\sin^4\left(c+dx\right)}{8}+\frac{a^2x\sin^2\left(c+dx\right)\cos^2\left(c+dx\right)}{4}+\frac{a^2x\sin^2\left(c+dx\right)}{2}+\frac{a^2x\cos^4\left(c+dx\right)}{8}+\frac{a^2x\cos^2\left(c+dx\right)}{2}+\frac{a^2\sin^3\left(c+dx\right)\cos\left(c+dx\right)}{8d}-\frac{a^2\sin\left(c+dx\right)\cos^3\left(c+dx\right)}{8d}+\frac{a^2\sin\left(c+dx\right)\cos\left(c+dx\right)}{2d}-\frac{2a^2\cos^3\left(c+dx\right)}{3d} & \text{for}\: d\neq 0\\x\left(a\sin\left(c\right)+a\right)^2\cos^2\left(c\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*x*sin(c + d*x)**4/8 + a**2*x*sin(c + d*x)**2*cos(c + d*x)**
2/4 + a**2*x*sin(c + d*x)**2/2 + a**2*x*cos(c + d*x)**4/8 + a**2*x*cos(c +
d*x)**2/2 + a**2*sin(c + d*x)**3*cos(c + d*x)/(8*d) - a**2*sin(c + d*x)*cos
```

```
(c + d*x)**3/(8*d) + a**2*sin(c + d*x)*cos(c + d*x)/(2*d) - 2*a**2*cos(c +
d*x)**3/(3*d), Ne(d, 0)), (x*(a*sin(c) + a)**2*cos(c)**2, True))
```

Giac [A]
time = 5.86, size = 72, normalized size = 0.92

$$\frac{5}{8}\,a^2x - \frac{a^2\cos\left(3\,dx+3\,c\right)}{6\,d} - \frac{a^2\cos\left(dx+c\right)}{2\,d} - \frac{a^2\sin\left(4\,dx+4\,c\right)}{32\,d} + \frac{a^2\sin\left(2\,dx+2\,c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 5/8*a^2*x - 1/6*a^2*cos(3*d*x + 3*c)/d - 1/2*a^2*cos(d*x + c)/d - 1/32*a^2*
sin(4*d*x + 4*c)/d + 1/4*a^2*sin(2*d*x + 2*c)/d
```

Mupad [B]
time = 6.71, size = 237, normalized size = 3.04

$$\frac{5\,a^2\,x}{8} - \frac{\frac{11\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{4} - \frac{11\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{4} + \frac{3\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4} + \frac{a^2\,(15\,c+15\,d\,x)}{24} - \frac{a^2\,(15\,c+15\,d\,x-32)}{24} + \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\left(\frac{a^2\,(15\,c+15\,d\,x)}{6} - \frac{a^2\,(60\,c+60\,d\,x-32)}{24}\right) + \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\left(\frac{a^2\,(15\,c+15\,d\,x)}{6} - \frac{a^2\,(60\,c+60\,d\,x-96)}{24}\right) + \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\left(\frac{a^2\,(15\,c+15\,d\,x)}{4} - \frac{a^2\,(90\,c+90\,d\,x-96)}{24}\right) - \frac{3\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a + a*sin(c + d*x))^2,x)
```

```
[Out] (5*a^2*x)/8 - ((11*a^2*tan(c/2 + (d*x)/2)^5)/4 - (11*a^2*tan(c/2 + (d*x)/2)
^3)/4 + (3*a^2*tan(c/2 + (d*x)/2)^7)/4 + (a^2*(15*c + 15*d*x))/24 - (a^2*(1
5*c + 15*d*x - 32))/24 + tan(c/2 + (d*x)/2)^2*((a^2*(15*c + 15*d*x))/6 - (a
^2*(60*c + 60*d*x - 32))/24) + tan(c/2 + (d*x)/2)^6*((a^2*(15*c + 15*d*x))/
6 - (a^2*(60*c + 60*d*x - 96))/24) + tan(c/2 + (d*x)/2)^4*((a^2*(15*c + 15*
d*x))/4 - (a^2*(90*c + 90*d*x - 96))/24) - (3*a^2*tan(c/2 + (d*x)/2))/4)/(d
*(tan(c/2 + (d*x)/2)^2 + 1)^4)
```

3.18 $\int \cos(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=22

$$\frac{(a+a\sin(c+dx))^3}{3ad}$$

```
[Out] 1/3*(a+a*sin(d*x+c))^3/a/d
```

Rubi [A]
time = 0.02, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2746, 32}

$$\frac{(a\sin(c+dx)+a)^3}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a + a*Sin[c + d*x])^3/(3*a*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a+a\sin(c+dx))^2\,dx &= \frac{\text{Subst}\left(\int (a+x)^2\,dx, x, a\sin(c+dx)\right)}{ad}\\ &= \frac{(a+a\sin(c+dx))^3}{3ad}\end{aligned}$$

Mathematica [A]
time = 0.12, size = 37, normalized size = 1.68

$$-\frac{a^2(6\cos(2(c+dx))-15\sin(c+dx)+\sin(3(c+dx)))}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/12*(a^2*(6*Cos[2*(c + d*x)] - 15*Sin[c + d*x] + Sin[3*(c + d*x)]))/d
```

Maple [A]
time = 0.08, size = 21, normalized size = 0.95

method	result	size
derivativedivides	$\frac{(a+a\sin(dx+c))^3}{3da}$	21
default	$\frac{(a+a\sin(dx+c))^3}{3da}$	21
risch	$\frac{5a^2\sin(dx+c)}{4d} - \frac{a^2\sin(3dx+3c)}{12d} - \frac{a^2\cos(2dx+2c)}{2d}$	50
norman	$\frac{\frac{4a^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{4a^2\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{20a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{2a^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}$	111

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(a+a*sin(d*x+c))^3/d/a
```

Maxima [A]
time = 0.29, size = 20, normalized size = 0.91

$$\frac{(a\sin(dx+c)+a)^3}{3\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(a*sin(d*x + c) + a)^3/(a*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(20) = 40$.
time = 0.36, size = 44, normalized size = 2.00

$$-\frac{3\,a^2\cos(dx+c)^2+\left(a^2\cos(dx+c)^2-4\,a^2\right)\sin(dx+c)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(3*a^2*cos(d*x + c)^2 + (a^2*cos(d*x + c)^2 - 4*a^2)*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(15) = 30$.
time = 0.13, size = 53, normalized size = 2.41

$$\begin{cases} \frac{a^2 \sin^3(c+dx)}{3d} + \frac{a^2 \sin^2(c+dx)}{d} + \frac{a^2 \sin(c+dx)}{d} & \text{for } d \neq 0 \\ x(a\sin(c)+a)^2 \cos(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise(((a**2*sin(c + d*x)**3/(3*d) + a**2*sin(c + d*x)**2/d + a**2*sin(c
 + d*x)/d, Ne(d, 0)), (x*(a*sin(c) + a)**2*cos(c), True))
```

Giac [A]
time = 3.75, size = 20, normalized size = 0.91

$$\frac{(a\sin(dx+c)+a)^3}{3\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/3*(a*sin(d*x + c) + a)^3/(a*d)
```

Mupad [B]
time = 4.55, size = 32, normalized size = 1.45

$$\frac{a^2\,\sin(c+d\,x)\,\left(\sin(c+d\,x)^2+3\,\sin(c+d\,x)+3\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + a*sin(c + d*x))^2,x)
```

```
[Out] (a^2*sin(c + d*x)*(3*sin(c + d*x) + sin(c + d*x)^2 + 3))/(3*d)
```

3.19 $\int \sec(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=34

$$-\frac{2a^2\log(1-\sin(c+dx))}{d}-\frac{a^2\sin(c+dx)}{d}$$

```
[Out] -2*a^2*ln(1-sin(d*x+c))/d-a^2*sin(d*x+c)/d
```

Rubi [A]
time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2746, 45}

$$-\frac{a^2\sin(c+dx)}{d}-\frac{2a^2\log(1-\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-2*a^2*Log[1 - Sin[c + d*x]])/d - (a^2*Sin[c + d*x])/d
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}
\int \sec(c+dx)(a+a\sin(c+dx))^2\,dx &= \frac{a\text{Subst}\left(\int \frac{a+x}{a-x}\,dx,x,a\sin(c+dx)\right)}{d}\\
&= \frac{a\text{Subst}\left(\int \left(-1+\frac{2a}{a-x}\right)\,dx,x,a\sin(c+dx)\right)}{d}\\
&= -\frac{2a^2\log(1-\sin(c+dx))}{d}-\frac{a^2\sin(c+dx)}{d}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 29, normalized size = 0.85

$$\frac{a^2(-2\log(1-\sin(c+dx))-\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sin[c + d*x])^2,x]
[Out] (a^2*(-2*Log[1 - Sin[c + d*x]] - Sin[c + d*x]))/d
```

Maple [A]
time = 0.12, size = 63, normalized size = 1.85

method	result	size
derivativedivides	$\frac{a^2(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))-2a^2\ln(\cos(dx+c))+a^2\ln(\sec(dx+c)+\tan(dx+c))}{d}$	63
default	$\frac{a^2(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))-2a^2\ln(\cos(dx+c))+a^2\ln(\sec(dx+c)+\tan(dx+c))}{d}$	63
risch	$2ia^2x+\frac{ia^2e^{i(dx+c)}}{2d}-\frac{ia^2e^{-i(dx+c)}}{2d}+\frac{4ia^2c}{d}-\frac{4a^2\ln(e^{i(dx+c)}-i)}{d}$	76
norman	$\frac{-\frac{2a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{2a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{4a^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}+\frac{2a^2\ln\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}$	97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] 1/d*(a^2*(-sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))-2*a^2*ln(cos(d*x+c))+a^2*l
n(sec(d*x+c)+tan(d*x+c)))
```

Maxima [A]
time = 0.33, size = 30, normalized size = 0.88

$$-\frac{2a^2\log(\sin(dx+c)-1)+a^2\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
[Out] -(2*a^2*log(sin(d*x + c) - 1) + a^2*sin(d*x + c))/d
```

Fricas [A]
time = 0.39, size = 32, normalized size = 0.94

$$-\frac{2a^2\log(-\sin(dx+c)+1)+a^2\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -(2*a^2*log(-sin(d*x + c) + 1) + a^2*sin(d*x + c))/d
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$a^2 \left(\int 2\sin(c+dx)\sec(c+dx)\,dx + \int \sin^2(c+dx)\sec(c+dx)\,dx + \int \sec(c+dx)\,dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(2*sin(c + d*x)*sec(c + d*x), x) + Integral(sin(c + d*x)**2*s
ec(c + d*x), x) + Integral(sec(c + d*x), x))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 91 vs. 2(34) = 68.
time = 3.61, size = 91, normalized size = 2.68

$$\frac{2\left(a^2\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)-2\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 2*(a^2*log(tan(1/2*d*x + 1/2*c)^2 + 1) - 2*a^2*log(abs(tan(1/2*d*x + 1/2*c)
 - 1)) - (a^2*tan(1/2*d*x + 1/2*c)^2 + a^2*tan(1/2*d*x + 1/2*c) + a^2)/(tan
(1/2*d*x + 1/2*c)^2 + 1))/d
```

Mupad [**B**]
time = 0.05, size = 26, normalized size = 0.76

$$-\frac{a^2\,\left(2\,\ln\left(\sin\left(c+d\,x\right)-1\right)+\sin\left(c+d\,x\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^2/cos(c + d*x),x)
```

```
[Out] -(a^2*(2*log(sin(c + d*x) - 1) + sin(c + d*x)))/d
```

3.20 $\int \sec^2(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=38

$$-a^2x+\frac{2a^4\cos(c+dx)}{d\,(a^2-a^2\sin(c+dx))}$$

```
[Out] -a^2*x+2*a^4*cos(d*x+c)/d/(a^2-a^2*sin(d*x+c))
```

Rubi [A]
time = 0.05, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2749, 2759, 8}

$$\frac{2a^4\cos(c+dx)}{d\,(a^2-a^2\sin(c+dx))}-a^2x$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(a^2*x) + (2*a^4*Cos[c + d*x])/(d*(a^2 - a^2*Sin[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2749

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\int \sec^2(c+dx)(a+a\sin(c+dx))^2\,dx = a^4\int \frac{\cos^2(c+dx)}{(a-a\sin(c+dx))^2}\,dx$$
$$= \frac{2a^4\cos(c+dx)}{d\,(a^2-a^2\sin(c+dx))} - a^2\int 1\,dx$$
$$= -a^2x + \frac{2a^4\cos(c+dx)}{d\,(a^2-a^2\sin(c+dx))}$$

Mathematica [A]
time = 0.04, size = 75, normalized size = 1.97

$$\frac{2a^2\sec(c+dx)\sqrt{1+\sin(c+dx)}\left(\sin^{-1}\left(\frac{\sqrt{1-\sin(c+dx)}}{\sqrt{2}}\right)\sqrt{1-\sin(c+dx)}+\sqrt{1+\sin(c+dx)}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
[Out] (2*a^2*Sec[c + d*x]*Sqrt[1 + Sin[c + d*x]]*(ArcSin[Sqrt[1 - Sin[c + d*x]]/S
qrt[2]]*Sqrt[1 - Sin[c + d*x]] + Sqrt[1 + Sin[c + d*x]]))/d
```

Maple [A]
time = 0.11, size = 47, normalized size = 1.24

method	result
risch	$-a^2x+\frac{4a^2}{d\left(e^{i(dx+c)}-i\right)}$
derivativedivides	$\frac{a^2(\tan(dx+c)-dx-c)+\frac{2a^2}{\cos(dx+c)}+a^2\tan(dx+c)}{d}$
default	$\frac{a^2(\tan(dx+c)-dx-c)+\frac{2a^2}{\cos(dx+c)}+a^2\tan(dx+c)}{d}$
norman	$\frac{a^2x+a^2x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{4a^2\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{8a^2}{d}-\frac{4a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{8a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{4a^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-a^2x(t}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] 1/d*(a^2*(tan(d*x+c)-d*x-c)+2*a^2/cos(d*x+c)+a^2*tan(d*x+c))
```

Maxima [A]
time = 0.52, size = 47, normalized size = 1.24

$$-\frac{(dx+c-\tan(dx+c))a^2-a^2\tan(dx+c)-\frac{2a^2}{\cos(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -((d*x + c - tan(d*x + c))*a^2 - a^2*tan(d*x + c) - 2*a^2/cos(d*x + c))/d
```

Fricas [A]
time = 0.36, size = 74, normalized size = 1.95

$$-\frac{a^2dx - 2\,a^2 + (a^2dx - 2\,a^2)\cos\left(dx + c\right) - (a^2dx + 2\,a^2)\sin\left(dx + c\right)}{d\cos\left(dx + c\right) - d\sin\left(dx + c\right) + d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -(a^2*d*x - 2*a^2 + (a^2*d*x - 2*a^2)*cos(d*x + c) - (a^2*d*x + 2*a^2)*sin(
d*x + c))/(d*cos(d*x + c) - d*sin(d*x + c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int 2\sin\left(c + dx\right)\sec^2\left(c + dx\right)dx + \int \sin^2\left(c + dx\right)\sec^2\left(c + dx\right)dx + \int \sec^2\left(c + dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+a*sin(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(2*sin(c + d*x)*sec(c + d*x)**2, x) + Integral(sin(c + d*x)**
2*sec(c + d*x)**2, x) + Integral(sec(c + d*x)**2, x))
```

Giac [A]
time = 4.84, size = 33, normalized size = 0.87

$$-\frac{(dx + c)a^2 + \frac{4\,a^2}{\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -((d*x + c)*a^2 + 4*a^2/(tan(1/2*d*x + 1/2*c) - 1))/d
```

Mupad [B]
time = 4.56, size = 28, normalized size = 0.74

$$-a^2\,x - \frac{4\,a^2}{d\,\left(\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right) - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^2/cos(c + d*x)^2,x)
```

```
[Out] - a^2*x - (4*a^2)/(d*(tan(c/2 + (d*x)/2) - 1))
```

3.21 $\int \sec^3(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=20

$$\frac{a^3}{d(a-a\sin(c+dx))}$$

```
[Out] a^3/d/(a-a*sin(d*x+c))
```

Rubi [A]
time = 0.03, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2746, 32}

$$\frac{a^3}{d(a-a\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] a^3/(d*(a - a*Sin[c + d*x]))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \sec^3(c+dx)(a+a\sin(c+dx))^2\,dx &= \frac{a^3\text{Subst}\left(\int \frac{1}{(a-x)^2}\,dx, x, a\sin(c+dx)\right)}{d}\\ &= \frac{a^3}{d(a-a\sin(c+dx))}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 32, normalized size = 1.60

$$\frac{a^2}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] a^2/(d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(20) = 40$.
time = 0.15, size = 100, normalized size = 5.00

method	result
risch	$-\frac{2 i a^{2} e^{i (d x+c)}}{\left(e^{i (d x+c)}-i\right)^{2} d}$
derivativedivides	$\frac{a^{2} \left(\frac{\sin^{3}(d x+c)}{2 \cos (d x+c)^{2}}+\frac{\sin (d x+c)}{2}-\frac{\ln (\sec (d x+c)+\tan (d x+c))}{2}\right)+\frac{a^{2}}{\cos (d x+c)^{2}}+a^{2} \left(\frac{\sec (d x+c) \tan (d x+c)}{2}+\frac{\ln (\sec (d x+c)+\tan (d x+c))}{2}\right)}{d}$
default	$\frac{a^{2} \left(\frac{\sin^{3}(d x+c)}{2 \cos (d x+c)^{2}}+\frac{\sin (d x+c)}{2}-\frac{\ln (\sec (d x+c)+\tan (d x+c))}{2}\right)+\frac{a^{2}}{\cos (d x+c)^{2}}+a^{2} \left(\frac{\sec (d x+c) \tan (d x+c)}{2}+\frac{\ln (\sec (d x+c)+\tan (d x+c))}{2}\right)}{d}$
norman	$\frac{\frac{2 a^{2} \tan \left(\frac{d x}{2}+\frac{c}{2}\right)}{d}+\frac{4 a^{2} \left(\tan^{2}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)}{d}+\frac{6 a^{2} \left(\tan^{3}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)}{d}+\frac{6 a^{2} \left(\tan^{5}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)}{d}+\frac{4 a^{2} \left(\tan^{6}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2 a^{2} \left(\tan^{7}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)}{d}+}{\left(\tan^{2}\left(\frac{d x}{2}+\frac{c}{2}\right)-1\right)^{2} \left(1+\tan^{2}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(1/2*sin(d*x+c)^3/cos(d*x+c)^2+1/2*sin(d*x+c)-1/2*ln(sec(d*x+c)+ta
n(d*x+c)))+a^2/cos(d*x+c)^2+a^2*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c
)+tan(d*x+c))))
```

Maxima [A]
time = 0.38, size = 18, normalized size = 0.90

$$-\frac{a^2}{d(\sin(dx+c)-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -a^2/(d*(sin(d*x + c) - 1))
```

Fricas [A]
time = 0.34, size = 19, normalized size = 0.95

$$-\frac{a^2}{d\sin(dx+c)-d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -a^2/(d*sin(d*x + c) - d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int 2\sin\left(c+dx\right)\sec^3\left(c+dx\right)dx+\int \sin^2\left(c+dx\right)\sec^3\left(c+dx\right)dx+\int \sec^3\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+a*sin(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(2*sin(c + d*x)*sec(c + d*x)**3, x) + Integral(sin(c + d*x)**
2*sec(c + d*x)**3, x) + Integral(sec(c + d*x)**3, x))
```

Giac [**A**]
time = 6.19, size = 30, normalized size = 1.50

$$\frac{2\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 2*a^2*tan(1/2*d*x + 1/2*c)/(d*(tan(1/2*d*x + 1/2*c) - 1)^2)
```

Mupad [**B**]
time = 0.04, size = 18, normalized size = 0.90

$$-\frac{a^2}{d\,\left(\sin\left(c+d\,x\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^2/cos(c + d*x)^3,x)
```

```
[Out] -a^2/(d*(sin(c + d*x) - 1))
```

3.22 $\int \sec^4(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=63

$$\frac{a^4\cos(c+dx)}{3d(a-a\sin(c+dx))^2}+\frac{a^4\cos(c+dx)}{3d\left(a^2-a^2\sin(c+dx)\right)}$$

```
[Out] 1/3*a^4*cos(d*x+c)/d/(a-a*sin(d*x+c))^2+1/3*a^4*cos(d*x+c)/d/(a^2-a^2*sin(d
*x+c))
```

Rubi [A]
time = 0.05, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.143$, Rules used = {2749, 2729, 2727}

$$\frac{a^4\cos(c+dx)}{3d(a-a\sin(c+dx))^2}+\frac{a^4\cos(c+dx)}{3d\left(a^2-a^2\sin(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^4*Cos[c + d*x])/(3*d*(a - a*Sin[c + d*x])^2) + (a^4*Cos[c + d*x])/(3*d*(
a^2 - a^2*Sin[c + d*x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2749

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rubi steps

$$\int \sec^4(c+dx)(a+a\sin(c+dx))^2\,dx = a^4\int \frac{1}{(a-a\sin(c+dx))^2}\,dx$$
$$= \frac{a^4\cos(c+dx)}{3d(a-a\sin(c+dx))^2} + \frac{1}{3}a^3\int \frac{1}{a-a\sin(c+dx)}\,dx$$
$$= \frac{a^4\cos(c+dx)}{3d(a-a\sin(c+dx))^2} + \frac{a^3\cos(c+dx)}{3d(a-a\sin(c+dx))}$$

Mathematica [A]
time = 0.01, size = 58, normalized size = 0.92

$$\frac{2a^2\sec^3(c+dx)}{3d} + \frac{a^2\sec^2(c+dx)\tan(c+dx)}{d} - \frac{a^2\tan^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (2*a^2*Sec[c + d*x]^3)/(3*d) + (a^2*Sec[c + d*x]^2*Tan[c + d*x])/d - (a^2*T
an[c + d*x]^3)/(3*d)
```

Maple [A]
time = 0.17, size = 63, normalized size = 1.00

method	result
risch	$-\frac{2ia^2\left(-i+3\,\mathrm{e}^{i(dx+c)}\right)}{3d\left(\mathrm{e}^{i(dx+c)}-i\right)^3}$
derivativedivides	$\frac{\frac{a^2\left(\sin^3(dx+c)\right)}{3\cos(dx+c)^3}+\frac{2a^2}{3\cos(dx+c)^3}-a^2\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)}{d}$
default	$\frac{\frac{a^2\left(\sin^3(dx+c)\right)}{3\cos(dx+c)^3}+\frac{2a^2}{3\cos(dx+c)^3}-a^2\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)}{d}$
norman	$\frac{-\frac{4a^2}{3d}-\frac{2a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{16a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{20a^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{16a^2\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{2a^2\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{8a^2\left(\tan^6\right.}{}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/3*a^2*sin(d*x+c)^3/cos(d*x+c)^3+2/3*a^2/cos(d*x+c)^3-a^2*(-2/3-1/3*s
ec(d*x+c)^2)*tan(d*x+c))
```

Maxima [A]
time = 0.30, size = 52, normalized size = 0.83

$$\frac{a^2\tan\left(dx+c\right)^3 + \left(\tan\left(dx+c\right)^3 + 3\,\tan\left(dx+c\right)\right)a^2 + \frac{2\,a^2}{\cos(dx+c)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(a^2*tan(d*x + c)^3 + (tan(d*x + c)^3 + 3*tan(d*x + c))*a^2 + 2*a^2/cos
(d*x + c)^3)/d
```

Fricas [A]
time = 0.35, size = 97, normalized size = 1.54

$$-\frac{a^2\cos\left(dx+c\right)^2+2\,a^2\cos\left(dx+c\right)+a^2-\left(a^2\cos\left(dx+c\right)-a^2\right)\sin\left(dx+c\right)}{3\,\left(d\cos\left(dx+c\right)^2-d\cos\left(dx+c\right)+\left(d\cos\left(dx+c\right)+2\,d\right)\sin\left(dx+c\right)-2\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(a^2*cos(d*x + c)^2 + 2*a^2*cos(d*x + c) + a^2 - (a^2*cos(d*x + c) - a
^2)*sin(d*x + c))/(d*cos(d*x + c)^2 - d*cos(d*x + c) + (d*cos(d*x + c) + 2*
d)*sin(d*x + c) - 2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int 2\sin\left(c+dx\right)\sec^4\left(c+dx\right)dx+\int\sin^2\left(c+dx\right)\sec^4\left(c+dx\right)dx+\int\sec^4\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+a*sin(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(2*sin(c + d*x)*sec(c + d*x)**4, x) + Integral(sin(c + d*x)**
2*sec(c + d*x)**4, x) + Integral(sec(c + d*x)**4, x))
```

Giac [A]
time = 5.87, size = 54, normalized size = 0.86

$$-\frac{2\left(3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\,a^2\right)}{3\,d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -2/3*(3*a^2*tan(1/2*d*x + 1/2*c)^2 - 3*a^2*tan(1/2*d*x + 1/2*c) + 2*a^2)/(d
*(tan(1/2*d*x + 1/2*c) - 1)^3)
```

Mupad [B]
time = 4.56, size = 81, normalized size = 1.29

$$-\frac{2\,a^2\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{2\,a^2\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-3\right)}{3}}{d\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^2/cos(c + d*x)^4,x)
```

```
[Out] -(2*a^2*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2) + (2*a^2*cos(c/2 + (d*x)/2)
*(cos(c/2 + (d*x)/2)^2 - 3))/3)/(d*(cos(c/2 + (d*x)/2) - sin(c/2 + (d*x)/2)
)^3)
```

3.23 $\int \sec^5(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=64

$$\frac{a^2\tanh^{-1}(\sin(c+dx))}{4d}+\frac{a^4}{4d(a-a\sin(c+dx))^2}+\frac{a^3}{4d(a-a\sin(c+dx))}$$

```
[Out] 1/4*a^2*arctanh(sin(d*x+c))/d+1/4*a^4/d/(a-a*sin(d*x+c))^2+1/4*a^3/d/(a-a*s
in(d*x+c))
```

Rubi [A]
time = 0.05, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.143$, Rules used = {2746, 46, 212}

$$\frac{a^4}{4d(a-a\sin(c+dx))^2}+\frac{a^3}{4d(a-a\sin(c+dx))}+\frac{a^2\tanh^{-1}(\sin(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*ArcTanh[Sin[c + d*x]])/(4*d) + a^4/(4*d*(a - a*Sin[c + d*x])^2) + a^3/
(4*d*(a - a*Sin[c + d*x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\int \sec^5(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{a^5\text{Subst}\left(\int \frac{1}{(a-x)^3(a+x)}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^5\text{Subst}\left(\int \left(\frac{1}{2a(a-x)^3} + \frac{1}{4a^2(a-x)^2} + \frac{1}{4a^2(a^2-x^2)}\right)\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^4}{4d(a-a\sin(c+dx))^2} + \frac{a^3}{4d(a-a\sin(c+dx))} + \frac{a^3\text{Subst}\left(\int \frac{1}{a^2-x^2}\right.}{}$$

$$= \frac{a^2\tanh^{-1}(\sin(c+dx))}{4d} + \frac{a^4}{4d(a-a\sin(c+dx))^2} + \frac{a^3}{4d(a-a\sin(c}$$

Mathematica [A]
time = 0.05, size = 56, normalized size = 0.88

$$\frac{a^2\sec^4(c+dx)\left(2+\tanh^{-1}(\sin(c+dx))(-1+\sin(c+dx))^2-\sin(c+dx)\right)(1+\sin(c+dx))^2}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*Sec[c + d*x]^4*(2 + ArcTanh[Sin[c + d*x]]*(-1 + Sin[c + d*x])^2 - Sin[
c + d*x])*(1 + Sin[c + d*x])^2)/(4*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 131 vs. 2(58) = 116.
time = 0.18, size = 132, normalized size = 2.06

method	result
risch	$-\frac{i a^2\left(-4i\,\mathrm{e}^{2i(dx+c)}+\mathrm{e}^{3i(dx+c)}-\mathrm{e}^{i(dx+c)}\right)}{2d\left(\mathrm{e}^{i(dx+c)}-i\right)^4} - \frac{a^2\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{4d} + \frac{a^2\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{4d}$
derivativedivides	$\frac{a^2\left(\frac{\sin^3(dx+c)}{4\cos(dx+c)^4}+\frac{\sin^3(dx+c)}{8\cos(dx+c)^2}+\frac{\sin(dx+c)}{8}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+\frac{a^2}{2\cos(dx+c)^4}+a^2\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c}{8}\right.\right.}{d}$
default	$\frac{a^2\left(\frac{\sin^3(dx+c)}{4\cos(dx+c)^4}+\frac{\sin^3(dx+c)}{8\cos(dx+c)^2}+\frac{\sin(dx+c)}{8}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+\frac{a^2}{2\cos(dx+c)^4}+a^2\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c}{8}\right.\right.}{d}$
norman	$\frac{\frac{8a^2\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{8a^2\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{3a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d}+\frac{11a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}+\frac{9a^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{9a^2\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{d}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4\left(1+\mathrm{ta}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(1/4*sin(d*x+c)^3/cos(d*x+c)^4+1/8*sin(d*x+c)^3/cos(d*x+c)^2+1/8*s
in(d*x+c)-1/8*ln(sec(d*x+c)+tan(d*x+c)))+1/2*a^2/cos(d*x+c)^4+a^2*(-(-1/4*s
ec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c))))
```

Maxima [A]
time = 0.29, size = 71, normalized size = 1.11

$$\frac{a^2 \log\left(\sin\left(dx+c\right)+1\right) - a^2 \log\left(\sin\left(dx+c\right)-1\right) - \frac{2\,\left(a^2 \sin\left(dx+c\right)-2\,a^2\right)}{\sin\left(dx+c\right)^2-2\,\sin\left(dx+c\right)+1}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/8*(a^2*log(sin(d*x + c) + 1) - a^2*log(sin(d*x + c) - 1) - 2*(a^2*sin(d*x
 + c) - 2*a^2)/(sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(60) = 120$.
time = 0.37, size = 125, normalized size = 1.95

$$\frac{2\,a^2 \sin\left(dx+c\right) - 4\,a^2 + \left(a^2 \cos\left(dx+c\right)^2 + 2\,a^2 \sin\left(dx+c\right) - 2\,a^2\right) \log\left(\sin\left(dx+c\right)+1\right) - \left(a^2 \cos\left(dx+c\right)^2 + 2\,a^2 \sin\left(dx+c\right) - 2\,a^2\right) \log\left(-\sin\left(dx+c\right)+1\right)}{8\,\left(d \cos\left(dx+c\right)^2 + 2\,d \sin\left(dx+c\right) - 2\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/8*(2*a^2*sin(d*x + c) - 4*a^2 + (a^2*cos(d*x + c)^2 + 2*a^2*sin(d*x + c)
- 2*a^2)*log(sin(d*x + c) + 1) - (a^2*cos(d*x + c)^2 + 2*a^2*sin(d*x + c) -
 2*a^2)*log(-sin(d*x + c) + 1))/(d*cos(d*x + c)^2 + 2*d*sin(d*x + c) - 2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^2 \left(\int 2 \sin\left(c+dx\right) \sec^5\left(c+dx\right)\,dx + \int \sin^2\left(c+dx\right) \sec^5\left(c+dx\right)\,dx + \int \sec^5\left(c+dx\right)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a+a*sin(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(2*sin(c + d*x)*sec(c + d*x)**5, x) + Integral(sin(c + d*x)**
2*sec(c + d*x)**5, x) + Integral(sec(c + d*x)**5, x))
```

Giac [A]
time = 5.06, size = 77, normalized size = 1.20

$$\frac{2\,a^2 \log\left(\left|\sin\left(dx+c\right)+1\right|\right) - 2\,a^2 \log\left(\left|\sin\left(dx+c\right)-1\right|\right) + \frac{3\,a^2 \sin\left(dx+c\right)^2 - 10\,a^2 \sin\left(dx+c\right) + 11\,a^2}{\left(\sin\left(dx+c\right)-1\right)^2}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/16*(2*a^2*log(abs(sin(d*x + c) + 1)) - 2*a^2*log(abs(sin(d*x + c) - 1)) +
 (3*a^2*sin(d*x + c)^2 - 10*a^2*sin(d*x + c) + 11*a^2)/(sin(d*x + c) - 1)^2
)/d
```

Mupad [B]
time = 4.47, size = 58, normalized size = 0.91

$$\frac{a^2 \operatorname{atanh}(\sin(c+dx))}{4d} - \frac{\frac{a^2 \sin(c+dx)}{4} - \frac{a^2}{2}}{d\left(\sin(c+dx)^2 - 2\sin(c+dx) + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^2/cos(c + d*x)^5,x)
```

```
[Out] (a^2*atanh(sin(c + d*x)))/(4*d) - ((a^2*sin(c + d*x))/4 - a^2/2)/(d*(sin(c
+ d*x)^2 - 2*sin(c + d*x) + 1))
```

3.24 $\int \sec^6(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=64

$$\frac{2\sec^5(c+dx)\left(a^2+a^2\sin(c+dx)\right)}{5d}+\frac{3a^2\tan(c+dx)}{5d}+\frac{a^2\tan^3(c+dx)}{5d}$$

```
[Out] 2/5*sec(d*x+c)^5*(a^2+a^2*sin(d*x+c))/d+3/5*a^2*tan(d*x+c)/d+1/5*a^2*tan(d*
x+c)^3/d
```

Rubi [A]
time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2755, 3852}

$$\frac{a^2\tan^3(c+dx)}{5d}+\frac{3a^2\tan(c+dx)}{5d}+\frac{2\sec^5(c+dx)\left(a^2\sin(c+dx)+a^2\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (2*Sec[c + d*x]^5*(a^2 + a^2*Sin[c + d*x]))/(5*d) + (3*a^2*Tan[c + d*x])/(5
*d) + (a^2*Tan[c + d*x]^3)/(5*d)
```

Rule 2755

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[-2*b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(p + 1))), x] + Dist[b^2*((2*m + p - 1)/(g^2*(p + 1))),
Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[
{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && Inte
gersQ[2*m, 2*p]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \sec^6(c+dx)(a+a\sin(c+dx))^2\,dx &= \frac{2\sec^5(c+dx)\left(a^2+a^2\sin(c+dx)\right)}{5d}+\frac{1}{5}\left(3a^2\right)\int \sec^4(c+dx)\,dx\\
&= \frac{2\sec^5(c+dx)\left(a^2+a^2\sin(c+dx)\right)}{5d}-\frac{\left(3a^2\right)\mathrm{Subst}\left(\int \left(1+x^2\right)\,dx,x,\right.}{5d}\\
&= \frac{2\sec^5(c+dx)\left(a^2+a^2\sin(c+dx)\right)}{5d}+\frac{3a^2\tan(c+dx)}{5d}+\frac{a^2\tan^3(c-}{5d}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 82, normalized size = 1.28

$$\frac{2a^2\sec^5(c+dx)}{5d}+\frac{a^2\sec^4(c+dx)\tan(c+dx)}{d}-\frac{a^2\sec^2(c+dx)\tan^3(c+dx)}{d}+\frac{2a^2\tan^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (2*a^2*Sec[c + d*x]^5)/(5*d) + (a^2*Sec[c + d*x]^4*Tan[c + d*x])/d - (a^2*S
ec[c + d*x]^2*Tan[c + d*x]^3)/d + (2*a^2*Tan[c + d*x]^5)/(5*d)
```

Maple [A]
time = 0.22, size = 93, normalized size = 1.45

method	result
risch	$-\frac{4ia^2\left(-4i e^{i(dx+c)}+5\,e^{2i(dx+c)}-1\right)}{5\left(e^{i(dx+c)}-i\right)^5\left(e^{i(dx+c)}+i\right)d}$
derivativedivides	$\frac{a^2\left(\frac{\sin^3(dx+c)}{5\cos(dx+c)^5}+\frac{2\left(\sin^3(dx+c)\right)}{15\cos(dx+c)^3}\right)+\frac{2a^2}{5\cos(dx+c)^5}-a^2\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c)\right)}{5}-\frac{4\left(\sec^2(dx+c)\right)}{15}\right)\tan(dx+c)}{d}$
default	$\frac{a^2\left(\frac{\sin^3(dx+c)}{5\cos(dx+c)^5}+\frac{2\left(\sin^3(dx+c)\right)}{15\cos(dx+c)^3}\right)+\frac{2a^2}{5\cos(dx+c)^5}-a^2\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c)\right)}{5}-\frac{4\left(\sec^2(dx+c)\right)}{15}\right)\tan(dx+c)}{d}$
norman	$-\frac{4a^2}{5d}-\frac{2a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{4a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{54a^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5d}-\frac{88a^2\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5d}-\frac{54a^2\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5d}-4a^2(\tan$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(1/5*sin(d*x+c)^3/cos(d*x+c)^5+2/15*sin(d*x+c)^3/cos(d*x+c)^3)+2/5
*a^2/cos(d*x+c)^5-a^2*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c)
)
```

Maxima [A]
time = 0.31, size = 77, normalized size = 1.20

$$\frac{\left(3\,\tan\left(dx+c\right)^5+10\,\tan\left(dx+c\right)^3+15\,\tan\left(dx+c\right)\right)a^2+\left(3\,\tan\left(dx+c\right)^5+5\,\tan\left(dx+c\right)^3\right)a^2+\frac{6\,a^2}{\cos(dx+c)^5}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/15*((3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a^2 + (3*tan
(d*x + c)^5 + 5*tan(d*x + c)^3)*a^2 + 6*a^2/cos(d*x + c)^5)/d
```

Fricas [A]
time = 0.34, size = 85, normalized size = 1.33

$$-\frac{4\,a^{2}\cos\left(dx+c\right)^{2}-2\,a^{2}-\left(2\,a^{2}\cos\left(dx+c\right)^{2}-3\,a^{2}\right)\sin\left(dx+c\right)}{5\,\left(d\cos\left(dx+c\right)^{3}+2\,d\cos\left(dx+c\right)\sin\left(dx+c\right)-2\,d\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
[Out] -1/5*(4*a^2*cos(d*x + c)^2 - 2*a^2 - (2*a^2*cos(d*x + c)^2 - 3*a^2)*sin(d*x
 + c))/(d*cos(d*x + c)^3 + 2*d*cos(d*x + c)*sin(d*x + c) - 2*d*cos(d*x + c)
)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(a+a*sin(d*x+c))**2,x)
[Out] Timed out
```

Giac [A]
time = 3.98, size = 106, normalized size = 1.66

$$-\frac{\frac{5\,a^{2}}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1}+\frac{35\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-90\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+120\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-70\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+21\,a^{2}}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^{5}}}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+a*sin(d*x+c))^2,x, algorithm="giac")
[Out] -1/20*(5*a^2/(tan(1/2*d*x + 1/2*c) + 1) + (35*a^2*tan(1/2*d*x + 1/2*c)^4 -
90*a^2*tan(1/2*d*x + 1/2*c)^3 + 120*a^2*tan(1/2*d*x + 1/2*c)^2 - 70*a^2*tan
(1/2*d*x + 1/2*c) + 21*a^2)/(tan(1/2*d*x + 1/2*c) - 1)^5)/d
```

Mupad [B]
time = 4.63, size = 156, normalized size = 2.44

$$\frac{2\,a^{2}\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}-3\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+10\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}-10\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}+5\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}\right)}{5\,d\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)^{5}\,\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^2/cos(c + d*x)^6,x)
[Out] (2*a^2*cos(c/2 + (d*x)/2)*(2*cos(c/2 + (d*x)/2)^5 + 5*sin(c/2 + (d*x)/2)^5
- 10*cos(c/2 + (d*x)/2)*sin(c/2 + (d*x)/2)^4 - 3*cos(c/2 + (d*x)/2)^4*sin(c
/2 + (d*x)/2) + 10*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^3))/(5*d*(cos(c/
2 + (d*x)/2) - sin(c/2 + (d*x)/2))^5*(cos(c/2 + (d*x)/2) + sin(c/2 + (d*x)/
2)))
```

3.25 $\int \sec^7(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=109

$$\frac{a^2\tanh^{-1}(\sin(c+dx))}{4d}+\frac{a^5}{12d(a-a\sin(c+dx))^3}+\frac{a^4}{8d(a-a\sin(c+dx))^2}+\frac{3a^3}{16d(a-a\sin(c+dx))}-\frac{}{16d(a-}$$

```
[Out] 1/4*a^2*arctanh(sin(d*x+c))/d+1/12*a^5/d/(a-a*sin(d*x+c))^3+1/8*a^4/d/(a-a*
sin(d*x+c))^2+3/16*a^3/d/(a-a*sin(d*x+c))-1/16*a^3/d/(a+a*sin(d*x+c))
```

Rubi [A]
time = 0.06, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2746, 46, 212}

$$\frac{a^5}{12d(a-a\sin(c+dx))^3}+\frac{a^4}{8d(a-a\sin(c+dx))^2}+\frac{3a^3}{16d(a-a\sin(c+dx))}-\frac{a^3}{16d(a\sin(c+dx)+a)}+\frac{a^2\tanh^{-1}(\sin(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^7*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*ArcTanh[Sin[c + d*x]])/(4*d) + a^5/(12*d*(a - a*Sin[c + d*x])^3) + a^4
/(8*d*(a - a*Sin[c + d*x])^2) + (3*a^3)/(16*d*(a - a*Sin[c + d*x])) - a^3/(
16*d*(a + a*Sin[c + d*x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\int \sec^7(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{a^7\text{Subst}\left(\int \frac{1}{(a-x)^4(a+x)^2}\,dx,x,a\sin(c+dx)\right)}{d}$$
$$= \frac{a^7\text{Subst}\left(\int \left(\frac{1}{4a^2(a-x)^4}+\frac{1}{4a^3(a-x)^3}+\frac{3}{16a^4(a-x)^2}+\frac{1}{16a^4(a+x)^2}+\frac{1}{4a^4(a^2-x\right.}{d}$$
$$= \frac{a^5}{12d(a-a\sin(c+dx))^3}+\frac{a^4}{8d(a-a\sin(c+dx))^2}+\frac{3a^3}{16d(a-a\sin(c}$$
$$= \frac{a^2\tanh^{-1}(\sin(c+dx))}{4d}+\frac{a^5}{12d(a-a\sin(c+dx))^3}+\frac{a^4}{8d(a-a\sin(c}$$

Mathematica [A]
time = 0.10, size = 85, normalized size = 0.78

$$-\frac{a^2\sec^6(c+dx)(1+\sin(c+dx))^2\left(-4-\sin(c+dx)+6\sin^2(c+dx)-3\sin^3(c+dx)+3\tanh^{-1}(\sin(c+dx))(-1+\sin(c+dx))^3(1+\sin(c+dx))\right)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^7*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/12*(a^2*Sec[c + d*x]^6*(1 + Sin[c + d*x])^2*(-4 - Sin[c + d*x] + 6*Sin[c
 + d*x]^2 - 3*Sin[c + d*x]^3 + 3*ArcTanh[Sin[c + d*x]]*(-1 + Sin[c + d*x])^
3*(1 + Sin[c + d*x])))/d
```

Maple [A]
time = 0.27, size = 160, normalized size = 1.47

method	result
derivativedivides	$\frac{a^2\left(\frac{\sin^3(dx+c)}{6\cos(dx+c)^6}+\frac{\sin^3(dx+c)}{8\cos(dx+c)^4}+\frac{\sin^3(dx+c)}{16\cos(dx+c)^2}+\frac{\sin(dx+c)}{16}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{16}\right)+\frac{a^2}{3\cos(dx+c)^6}+a^2\left(-\left(-\frac{\left(\sec^5(dx+c\right.}{6}\right.\right.}{d}$
default	$\frac{a^2\left(\frac{\sin^3(dx+c)}{6\cos(dx+c)^6}+\frac{\sin^3(dx+c)}{8\cos(dx+c)^4}+\frac{\sin^3(dx+c)}{16\cos(dx+c)^2}+\frac{\sin(dx+c)}{16}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{16}\right)+\frac{a^2}{3\cos(dx+c)^6}+a^2\left(-\left(-\frac{\left(\sec^5(dx+c\right.}{6}\right.\right.}{d}$
risch	$-\frac{ia^2\left(-12ie^{6i(dx+c)}+3e^{7i(dx+c)}-8ie^{4i(dx+c)}-13e^{5i(dx+c)}-12ie^{2i(dx+c)}+13e^{3i(dx+c)}-3e^{i(dx+c)}\right)}{6\left(e^{i(dx+c)}+i\right)^2\left(e^{i(dx+c)}-i\right)^6d}-\frac{a^2\ln(e^{i(dx+c)}-}{4d}$
norman	$\frac{3a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d}+\frac{31a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6d}+\frac{77a^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6d}+\frac{139a^2\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6d}+\frac{139a^2\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6d}+\frac{77a^2\left(\tan^{11}\left(\frac{dx}{2}\right.\right.}{6d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(1/6*sin(d*x+c)^3/cos(d*x+c)^6+1/8*sin(d*x+c)^3/cos(d*x+c)^4+1/16*
sin(d*x+c)^3/cos(d*x+c)^2+1/16*sin(d*x+c)-1/16*ln(sec(d*x+c)+tan(d*x+c)))+1
```

```
/3*a^2/cos(d*x+c)^6+a^2*(-(-1/6*sec(d*x+c)^5-5/24*sec(d*x+c)^3-5/16*sec(d*x
+c))*tan(d*x+c)+5/16*ln(sec(d*x+c)+tan(d*x+c))))
```

Maxima [A]
time = 0.31, size = 108, normalized size = 0.99

$$\frac{3\,a^2\log\left(\sin\left(dx+c\right)+1\right)-3\,a^2\log\left(\sin\left(dx+c\right)-1\right)-\frac{2\,\left(3\,a^2\sin\left(dx+c\right)^3-6\,a^2\sin\left(dx+c\right)^2+a^2\sin\left(dx+c\right)+4\,a^2\right)}{\sin\left(dx+c\right)^4-2\,\sin\left(dx+c\right)^3+2\,\sin\left(dx+c\right)-1}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/24*(3*a^2*log(sin(d*x + c) + 1) - 3*a^2*log(sin(d*x + c) - 1) - 2*(3*a^2*
sin(d*x + c)^3 - 6*a^2*sin(d*x + c)^2 + a^2*sin(d*x + c) + 4*a^2)/(sin(d*x
+ c)^4 - 2*sin(d*x + c)^3 + 2*sin(d*x + c) - 1))/d
```

Fricas [A]
time = 0.37, size = 203, normalized size = 1.86

$$-\frac{12\,a^2\cos\left(dx+c\right)^2-4\,a^2-3\,\left(a^2\cos\left(dx+c\right)^4+2\,a^2\cos\left(dx+c\right)^2\sin\left(dx+c\right)-2\,a^2\cos\left(dx+c\right)^2\right)\log\left(\sin\left(dx+c\right)+1\right)+3\,\left(a^2\cos\left(dx+c\right)^4+2\,a^2\cos\left(dx+c\right)^2\sin\left(dx+c\right)-2\,a^2\cos\left(dx+c\right)^2\right)\log\left(-\sin\left(dx+c\right)+1\right)-2\,\left(3\,a^2\cos\left(dx+c\right)^2-4\,a^2\right)\sin\left(dx+c\right)}{24\,\left(d\cos\left(dx+c\right)^4+2\,d\cos\left(dx+c\right)^2\sin\left(dx+c\right)-2\,d\cos\left(dx+c\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/24*(12*a^2*cos(d*x + c)^2 - 4*a^2 - 3*(a^2*cos(d*x + c)^4 + 2*a^2*cos(d*
x + c)^2*sin(d*x + c) - 2*a^2*cos(d*x + c)^2)*log(sin(d*x + c) + 1) + 3*(a^
2*cos(d*x + c)^4 + 2*a^2*cos(d*x + c)^2*sin(d*x + c) - 2*a^2*cos(d*x + c)^2
)*log(-sin(d*x + c) + 1) - 2*(3*a^2*cos(d*x + c)^2 - 4*a^2)*sin(d*x + c))/(
d*cos(d*x + c)^4 + 2*d*cos(d*x + c)^2*sin(d*x + c) - 2*d*cos(d*x + c)^2)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [A]
time = 5.43, size = 119, normalized size = 1.09

$$\frac{6\,a^2\log\left(\left|\sin\left(dx+c\right)+1\right|\right)-6\,a^2\log\left(\left|\sin\left(dx+c\right)-1\right|\right)-\frac{3\,\left(2\,a^2\sin\left(dx+c\right)+3\,a^2\right)}{\sin\left(dx+c\right)+1}+\frac{11\,a^2\sin\left(dx+c\right)^3-42\,a^2\sin\left(dx+c\right)^2+57\,a^2\sin\left(dx+c\right)-30\,a^2}{\left(\sin\left(dx+c\right)-1\right)^3}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/48*(6*a^2*log(abs(sin(d*x + c) + 1)) - 6*a^2*log(abs(sin(d*x + c) - 1)) -
 3*(2*a^2*sin(d*x + c) + 3*a^2)/(sin(d*x + c) + 1) + (11*a^2*sin(d*x + c)^3
 - 42*a^2*sin(d*x + c)^2 + 57*a^2*sin(d*x + c) - 30*a^2)/(sin(d*x + c) - 1)
^3)/d
```

Mupad [B]
time = 4.34, size = 94, normalized size = 0.86

$$\frac{a^2\,\mathrm{atanh}(\sin\left(c+d\,x\right))}{4\,d}-\frac{\frac{a^2\,\sin(c+d\,x)^3}{4}-\frac{a^2\,\sin(c+d\,x)^2}{2}+\frac{a^2\,\sin(c+d\,x)}{12}+\frac{a^2}{3}}{d\,\left(\sin\left(c+d\,x\right)^4-2\sin\left(c+d\,x\right)^3+2\,\sin\left(c+d\,x\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^2/cos(c + d*x)^7,x)
```

```
[Out] (a^2*atanh(sin(c + d*x)))/(4*d) - ((a^2*sin(c + d*x))/12 + a^2/3 - (a^2*sin
(c + d*x)^2)/2 + (a^2*sin(c + d*x)^3)/4)/(d*(2*sin(c + d*x) - 2*sin(c + d*x
)^3 + sin(c + d*x)^4 - 1))
```

3.26 $\int \sec^8(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=82

$$\frac{2\sec^7(c+dx)\left(a^2+a^2\sin(c+dx)\right)}{7d}+\frac{5a^2\tan(c+dx)}{7d}+\frac{10a^2\tan^3(c+dx)}{21d}+\frac{a^2\tan^5(c+dx)}{7d}$$

```
[Out] 2/7*sec(d*x+c)^7*(a^2+a^2*sin(d*x+c))/d+5/7*a^2*tan(d*x+c)/d+10/21*a^2*tan(
d*x+c)^3/d+1/7*a^2*tan(d*x+c)^5/d
```

Rubi [A]

time = 0.04, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2755, 3852}

$$\frac{a^2\tan^5(c+dx)}{7d}+\frac{10a^2\tan^3(c+dx)}{21d}+\frac{5a^2\tan(c+dx)}{7d}+\frac{2\sec^7(c+dx)\left(a^2\sin(c+dx)+a^2\right)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^8*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (2*Sec[c + d*x]^7*(a^2 + a^2*Sin[c + d*x]))/(7*d) + (5*a^2*Tan[c + d*x])/(7
*d) + (10*a^2*Tan[c + d*x]^3)/(21*d) + (a^2*Tan[c + d*x]^5)/(7*d)
```

Rule 2755

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[-2*b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(p + 1))), x] + Dist[b^2*((2*m + p - 1)/(g^2*(p + 1))),
Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[
{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && Inte
gersQ[2*m, 2*p]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^8(c+dx)(a+a\sin(c+dx))^2\,dx &= \frac{2\sec^7(c+dx)\left(a^2+a^2\sin(c+dx)\right)}{7d}+\frac{1}{7}\left(5a^2\right)\int \sec^6(c+dx)\,dx\\ &= \frac{2\sec^7(c+dx)\left(a^2+a^2\sin(c+dx)\right)}{7d}-\frac{\left(5a^2\right)\operatorname{Subst}\left(\int \left(1+2x^2+x^4\right.\right.}{7d}\\ &= \frac{2\sec^7(c+dx)\left(a^2+a^2\sin(c+dx)\right)}{7d}+\frac{5a^2\tan(c+dx)}{7d}+\frac{10a^2\tan^3}{21}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 110, normalized size = 1.34

$$\frac{2a^2\sec^7(c+dx)}{7d}+\frac{a^2\sec^6(c+dx)\tan(c+dx)}{d}-\frac{5a^2\sec^4(c+dx)\tan^3(c+dx)}{3d}+\frac{4a^2\sec^2(c+dx)\tan^5(c+dx)}{3d}-\frac{8a^2\tan^7(c+dx)}{21d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^8*(a + a*Sin[c + d*x])^2,x]
[Out] (2*a^2*Sec[c + d*x]^7)/(7*d) + (a^2*Sec[c + d*x]^6*Tan[c + d*x])/d - (5*a^2
*Sec[c + d*x]^4*Tan[c + d*x]^3)/(3*d) + (4*a^2*Sec[c + d*x]^2*Tan[c + d*x]^
5)/(3*d) - (8*a^2*Tan[c + d*x]^7)/(21*d)
```

Maple [A]
time = 0.21, size = 121, normalized size = 1.48

method	result
risch	$-\frac{16ia^2\left(-8i\,e^{3i(dx+c)}+14\,e^{4i(dx+c)}-4i\,e^{i(dx+c)}+3\,e^{2i(dx+c)}-1\right)}{21\left(e^{i(dx+c)}-i\right)^7\left(e^{i(dx+c)}+i\right)^3 d}$
derivativedivides	$\frac{a^2\left(\frac{\sin^3(dx+c)}{7\cos(dx+c)^7}+\frac{4\left(\sin^3(dx+c)\right)}{35\cos(dx+c)^5}+\frac{8\left(\sin^3(dx+c)\right)}{105\cos(dx+c)^3}\right)+\frac{2a^2}{7\cos(dx+c)^7}-a^2\left(-\frac{16}{35}-\frac{\left(\sec^6(dx+c)\right)}{7}-\frac{6\left(\sec^4(dx+c)\right)}{35}-\frac{8\left(\sec^2(dx+c)\right)}{35}\right.}{d}$
default	$\frac{a^2\left(\frac{\sin^3(dx+c)}{7\cos(dx+c)^7}+\frac{4\left(\sin^3(dx+c)\right)}{35\cos(dx+c)^5}+\frac{8\left(\sin^3(dx+c)\right)}{105\cos(dx+c)^3}\right)+\frac{2a^2}{7\cos(dx+c)^7}-a^2\left(-\frac{16}{35}-\frac{\left(\sec^6(dx+c)\right)}{7}-\frac{6\left(\sec^4(dx+c)\right)}{35}-\frac{8\left(\sec^2(dx+c)\right)}{35}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^8*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] 1/d*(a^2*(1/7*sin(d*x+c)^3/cos(d*x+c)^7+4/35*sin(d*x+c)^3/cos(d*x+c)^5+8/10
5*sin(d*x+c)^3/cos(d*x+c)^3)+2/7*a^2/cos(d*x+c)^7-a^2*(-16/35-1/7*sec(d*x+c
)^6-6/35*sec(d*x+c)^4-8/35*sec(d*x+c)^2)*tan(d*x+c))
```

Maxima [A]
time = 0.31, size = 98, normalized size = 1.20

$$\frac{\left(15\tan\left(dx+c\right)^7+42\tan\left(dx+c\right)^5+35\tan\left(dx+c\right)^3\right)a^2+3\left(5\tan\left(dx+c\right)^7+21\tan\left(dx+c\right)^5+35\tan\left(dx+c\right)^3+35\tan\left(dx+c\right)\right)a^2+\frac{30a^2}{\cos(dx+c)^7}}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
[Out] 1/105*((15*tan(d*x + c)^7 + 42*tan(d*x + c)^5 + 35*tan(d*x + c)^3)*a^2 + 3*
(5*tan(d*x + c)^7 + 21*tan(d*x + c)^5 + 35*tan(d*x + c)^3 + 35*tan(d*x + c)
)*a^2 + 30*a^2/cos(d*x + c)^7)/d
```

Fricas [A]
time = 0.34, size = 115, normalized size = 1.40

$$-\frac{16\,a^2\cos\left(dx+c\right)^4-8\,a^2\cos\left(dx+c\right)^2-2\,a^2-\left(8\,a^2\cos\left(dx+c\right)^4-12\,a^2\cos\left(dx+c\right)^2-5\,a^2\right)\sin\left(dx+c\right)}{21\left(d\cos\left(dx+c\right)^5+2\,d\cos\left(dx+c\right)^3\sin\left(dx+c\right)-2\,d\cos\left(dx+c\right)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/21*(16*a^2*cos(d*x + c)^4 - 8*a^2*cos(d*x + c)^2 - 2*a^2 - (8*a^2*cos(d*
x + c)^4 - 12*a^2*cos(d*x + c)^2 - 5*a^2)*sin(d*x + c))/(d*cos(d*x + c)^5 +
 2*d*cos(d*x + c)^3*sin(d*x + c) - 2*d*cos(d*x + c)^3)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**8*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 171 vs. 2(74) = 148.
time = 5.55, size = 171, normalized size = 2.09

$$-\frac{\frac{7\left(9a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+15a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+8a^2\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)^3}+\frac{273a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6-1155a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+2450a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-2870a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+2037a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-791a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+152a^2}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)^7}}{168d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/168*(7*(9*a^2*tan(1/2*d*x + 1/2*c)^2 + 15*a^2*tan(1/2*d*x + 1/2*c) + 8*a
^2)/(tan(1/2*d*x + 1/2*c) + 1)^3 + (273*a^2*tan(1/2*d*x + 1/2*c)^6 - 1155*a
^2*tan(1/2*d*x + 1/2*c)^5 + 2450*a^2*tan(1/2*d*x + 1/2*c)^4 - 2870*a^2*tan(
1/2*d*x + 1/2*c)^3 + 2037*a^2*tan(1/2*d*x + 1/2*c)^2 - 791*a^2*tan(1/2*d*x
+ 1/2*c) + 152*a^2)/(tan(1/2*d*x + 1/2*c) - 1)^7)/d
```

Mupad [B]
time = 5.07, size = 276, normalized size = 3.37

$$\frac{2a^2\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\left(6\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^9-3\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^8\sin\left(\frac{c}{2}+\frac{dx}{2}\right)-24\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^7\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^2+76\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^6\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^3-28\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^5\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^4-42\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^4\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^5+56\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^3\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^6+28\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^7-42\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^8+21\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^9\right)}{21d\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^7\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^2/cos(c + d*x)^8,x)
```

```
[Out] (2*a^2*cos(c/2 + (d*x)/2)*(6*cos(c/2 + (d*x)/2)^9 + 21*sin(c/2 + (d*x)/2)^9
 - 42*cos(c/2 + (d*x)/2)*sin(c/2 + (d*x)/2)^8 - 3*cos(c/2 + (d*x)/2)^8*sin(
c/2 + (d*x)/2) + 28*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^7 + 56*cos(c/2
+ (d*x)/2)^3*sin(c/2 + (d*x)/2)^6 - 42*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)
/2)^5 - 28*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^4 + 76*cos(c/2 + (d*x)/2
)^6*sin(c/2 + (d*x)/2)^3 - 24*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2)^2))/(
21*d*(cos(c/2 + (d*x)/2) - sin(c/2 + (d*x)/2))^7*(cos(c/2 + (d*x)/2) + sin(
c/2 + (d*x)/2))^3)
```

3.27 $\int \cos^6(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=154

$$\frac{55a^3x}{128}-\frac{11a^3\cos^7(c+dx)}{56d}+\frac{55a^3\cos(c+dx)\sin(c+dx)}{128d}+\frac{55a^3\cos^3(c+dx)\sin(c+dx)}{192d}+\frac{11a^3\cos^5(c+dx)\,\mathrm{s}}{48d}$$

```
[Out] 55/128*a^3*x-11/56*a^3*cos(d*x+c)^7/d+55/128*a^3*cos(d*x+c)*sin(d*x+c)/d+55
/192*a^3*cos(d*x+c)^3*sin(d*x+c)/d+11/48*a^3*cos(d*x+c)^5*sin(d*x+c)/d-1/9*
a*cos(d*x+c)^7*(a+a*sin(d*x+c))^2/d-11/72*cos(d*x+c)^7*(a^3+a^3*sin(d*x+c))
/d
```

Rubi [A]
time = 0.11, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2757, 2748, 2715, 8}

$$-\frac{11a^3\cos^7(c+dx)}{56d}-\frac{11\cos^7(c+dx)\left(a^3\sin(c+dx)+a^3\right)}{72d}+\frac{11a^3\sin(c+dx)\cos^5(c+dx)}{48d}+\frac{55a^3\sin(c+dx)\cos^3(c+dx)}{192d}+\frac{55a^3\sin(c+dx)\cos(c+dx)}{128d}+\frac{55a^3x}{128}-\frac{a\cos^7(c+dx)(a\sin(c+dx)+a)^2}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (55*a^3*x)/128 - (11*a^3*Cos[c + d*x]^7)/(56*d) + (55*a^3*Cos[c + d*x]*Sin[
c + d*x])/(128*d) + (55*a^3*Cos[c + d*x]^3*Sin[c + d*x])/(192*d) + (11*a^3*
Cos[c + d*x]^5*Sin[c + d*x])/(48*d) - (a*Cos[c + d*x]^7*(a + a*Sin[c + d*x]
)^2)/(9*d) - (11*Cos[c + d*x]^7*(a^3 + a^3*Sin[c + d*x]))/(72*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2757

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ
[2*m, 2*p]
```

Rubi steps

$$\begin{aligned}
\int \cos^6(c+dx)(a+a\sin(c+dx))^3\,dx &= -\frac{a\cos^7(c+dx)(a+a\sin(c+dx))^2}{9d}+\frac{1}{9}(11a)\int \cos^6(c+dx)(a+ \\
&= -\frac{a\cos^7(c+dx)(a+a\sin(c+dx))^2}{9d}-\frac{11\cos^7(c+dx)\left(a^3+a^3\sin\right.}{72d} \\
&= -\frac{11a^3\cos^7(c+dx)}{56d}-\frac{a\cos^7(c+dx)(a+a\sin(c+dx))^2}{9d}-\frac{11\cos}{} \\
&= -\frac{11a^3\cos^7(c+dx)}{56d}+\frac{11a^3\cos^5(c+dx)\sin(c+dx)}{48d}-\frac{a\cos^7(c+}{} \\
&= -\frac{11a^3\cos^7(c+dx)}{56d}+\frac{55a^3\cos^3(c+dx)\sin(c+dx)}{192d}+\frac{11a^3\cos^5(c}{} \\
&= -\frac{11a^3\cos^7(c+dx)}{56d}+\frac{55a^3\cos(c+dx)\sin(c+dx)}{128d}+\frac{55a^3\cos^3(c}{} \\
&= \frac{55a^3x}{128}-\frac{11a^3\cos^7(c+dx)}{56d}+\frac{55a^3\cos(c+dx)\sin(c+dx)}{128d}+\frac{55a^3}{}
\end{aligned}$$

Mathematica [A]
time = 1.25, size = 181, normalized size = 1.18

$$-\frac{a^3\cos^7(c+dx)\left(6930\sin^{-1}\left(\frac{\sqrt{1-\sin(c+dx)}}{\sqrt{2}}\right)\sqrt{1-\sin(c+dx)}+\sqrt{1+\sin(c+dx)}\,(3712-8311\sin(c+dx)-5641\sin^2(c+dx)+7174\sin^3(c+dx)+11514\sin^4(c+dx)-1224\sin^5(c+dx)-8248\sin^6(c+dx)-2000\sin^7(c+dx)+2128\sin^8(c+dx)+896\sin^9(c+dx))\right)}{8064d(-1+\sin(c+dx))^4(1+\sin(c+dx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/8064*(a^3*Cos[c + d*x]^7*(6930*ArcSin[Sqrt[1 - Sin[c + d*x]]/Sqrt[2]]*Sq
rt[1 - Sin[c + d*x]] + Sqrt[1 + Sin[c + d*x]]*(3712 - 8311*Sin[c + d*x] - 5
641*Sin[c + d*x]^2 + 7174*Sin[c + d*x]^3 + 11514*Sin[c + d*x]^4 - 1224*Sin[
c + d*x]^5 - 8248*Sin[c + d*x]^6 - 2000*Sin[c + d*x]^7 + 2128*Sin[c + d*x]^
8 + 896*Sin[c + d*x]^9)))/(d*(-1 + Sin[c + d*x])^4*(1 + Sin[c + d*x])^(7/2)
)
```

Maple [A]
time = 0.36, size = 163, normalized size = 1.06

method	result
risch	$\frac{55a^3x}{128} - \frac{33a^3\cos(dx+c)}{128d} + \frac{a^3\cos(9dx+9c)}{2304d} - \frac{3a^3\sin(8dx+8c)}{1024d} - \frac{9a^3\cos(7dx+7c)}{1792d} - \frac{a^3\sin(6dx+6c)}{96d} - \frac{3a^3\cos}{6}$
derivativedivides	$\frac{a^3\left(-\frac{(\sin^2(dx+c))(\cos^7(dx+c))}{9} - \frac{2(\cos^7(dx+c))}{63}\right) + 3a^3\left(-\frac{\sin(dx+c)(\cos^7(dx+c))}{8} + \frac{\left(\cos^5(dx+c) + \frac{5(\cos^3(dx+c))}{4} + \frac{15\cos(dx}{8}\right.}{48}\right.}{d}$
default	$\frac{a^3\left(-\frac{(\sin^2(dx+c))(\cos^7(dx+c))}{9} - \frac{2(\cos^7(dx+c))}{63}\right) + 3a^3\left(-\frac{\sin(dx+c)(\cos^7(dx+c))}{8} + \frac{\left(\cos^5(dx+c) + \frac{5(\cos^3(dx+c))}{4} + \frac{15\cos(dx}{8}\right.}{48}\right.}{d}$
norman	$-\frac{6a^3\left(\tan^{16}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d} - \frac{36a^3\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d} - \frac{16a^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7d} + \frac{3465a^3x\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64} + \frac{495a^3x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{128} - \frac{949a^3}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-1/9*sin(d*x+c)^2*cos(d*x+c)^7-2/63*cos(d*x+c)^7)+3*a^3*(-1/8*sin
(d*x+c)*cos(d*x+c)^7+1/48*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*s
in(d*x+c)+5/128*d*x+5/128*c)-3/7*cos(d*x+c)^7*a^3+a^3*(1/6*(cos(d*x+c)^5+5/
4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/16*d*x+5/16*c))
```

Maxima [A]

time = 0.31, size = 141, normalized size = 0.92

$$-\frac{27648\,a^3\cos(dx+c)^7 - 1024\left(7\cos(dx+c)^9 - 9\cos(dx+c)^7\right)a^3 - 63\left(64\sin(2\,dx+2\,c)^3 + 120\,dx + 120\,c - 3\sin(8\,dx+8\,c) - 24\sin(4\,dx+4\,c)\right)a^3 + 336\left(4\sin(2\,dx+2\,c)^3 - 60\,dx - 60\,c - 9\sin(4\,dx+4\,c) - 48\sin(2\,dx+2\,c)\right)a^3}{64512\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/64512*(27648*a^3*cos(d*x + c)^7 - 1024*(7*cos(d*x + c)^9 - 9*cos(d*x + c
)^7)*a^3 - 63*(64*sin(2*d*x + 2*c)^3 + 120*d*x + 120*c - 3*sin(8*d*x + 8*c)
 - 24*sin(4*d*x + 4*c))*a^3 + 336*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9
*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*a^3)/d
```

Fricas [A]

time = 0.38, size = 98, normalized size = 0.64

$$\frac{896\,a^3\cos(dx+c)^9 - 4608\,a^3\cos(dx+c)^7 + 3465\,a^3dx - 21\left(144\,a^3\cos(dx+c)^7 - 88\,a^3\cos(dx+c)^5 - 110\,a^3\cos(dx+c)^3 - 165\,a^3\cos(dx+c)\right)\sin(dx+c)}{8064\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/8064*(896*a^3*cos(d*x + c)^9 - 4608*a^3*cos(d*x + c)^7 + 3465*a^3*d*x - 2
1*(144*a^3*cos(d*x + c)^7 - 88*a^3*cos(d*x + c)^5 - 110*a^3*cos(d*x + c)^3
- 165*a^3*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 439 vs. $2(146) = 292$.
time = 1.43, size = 439, normalized size = 2.85

$$\begin{cases} \frac{15a^3x\sin^8(c+dx)}{128} + \frac{15a^3x\sin^6(c+dx)\cos^2(c+dx)}{32} + \frac{5a^3x\sin^6(c+dx)}{16} + \frac{45a^3x\sin^4(c+dx)\cos^4(c+dx)}{64} + \frac{15a^3x\sin^4(c+dx)\cos^2(c+dx)}{16} + \frac{15a^3x\sin^2(c+dx)\cos^6(c+dx)}{32} + \frac{15a^3x\sin^2(c+dx)\cos^4(c+dx)}{16} + \frac{15a^3x\cos^8(c+dx)}{128} + \frac{5a^3x\cos^6(c+dx)}{16} + \frac{15a^3\sin^7(c+dx)\cos(c+dx)}{128d} + \frac{55a^3\sin^5(c+dx)\cos^3(c+dx)}{128d} + \frac{5a^3\sin^5(c+dx)\cos(c+dx)}{16d} + \frac{73a^3\sin^3(c+dx)\cos^5(c+dx)}{128d} + \frac{5a^3\sin^3(c+dx)\cos^3(c+dx)}{6d} - \frac{a^3\sin^2(c+dx)\cos^7(c+dx)}{7d} - \frac{15a^3\sin(c+dx)\cos^7(c+dx)}{128d} + \frac{11a^3\sin(c+dx)\cos^5(c+dx)}{16d} - \frac{2a^3\cos^9(c+dx)}{63d} - \frac{3a^3\cos^7(c+dx)}{7d} & \text{for } d \neq 0 \\ x(a\sin(c)+a)^3\cos^6(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((15*a**3*x*sin(c + d*x)**8/128 + 15*a**3*x*sin(c + d*x)**6*cos(c
+ d*x)**2/32 + 5*a**3*x*sin(c + d*x)**6/16 + 45*a**3*x*sin(c + d*x)**4*cos(
c + d*x)**4/64 + 15*a**3*x*sin(c + d*x)**4*cos(c + d*x)**2/16 + 15*a**3*x*s
in(c + d*x)**2*cos(c + d*x)**6/32 + 15*a**3*x*sin(c + d*x)**2*cos(c + d*x)*
*4/16 + 15*a**3*x*cos(c + d*x)**8/128 + 5*a**3*x*cos(c + d*x)**6/16 + 15*a*
*3*sin(c + d*x)**7*cos(c + d*x)/(128*d) + 55*a**3*sin(c + d*x)**5*cos(c + d
*x)**3/(128*d) + 5*a**3*sin(c + d*x)**5*cos(c + d*x)/(16*d) + 73*a**3*sin(c
 + d*x)**3*cos(c + d*x)**5/(128*d) + 5*a**3*sin(c + d*x)**3*cos(c + d*x)**3
/(6*d) - a**3*sin(c + d*x)**2*cos(c + d*x)**7/(7*d) - 15*a**3*sin(c + d*x)*
cos(c + d*x)**7/(128*d) + 11*a**3*sin(c + d*x)*cos(c + d*x)**5/(16*d) - 2*a
**3*cos(c + d*x)**9/(63*d) - 3*a**3*cos(c + d*x)**7/(7*d), Ne(d, 0)), (x*(a
*sin(c) + a)**3*cos(c)**6, True))
```

Giac [A]
time = 6.83, size = 157, normalized size = 1.02

$$\frac{55}{128}a^3x + \frac{a^3\cos(9dx+9c)}{2304d} - \frac{9a^3\cos(7dx+7c)}{1792d} - \frac{3a^3\cos(5dx+5c)}{64d} - \frac{29a^3\cos(3dx+3c)}{192d} - \frac{33a^3\cos(dx+c)}{128d} - \frac{3a^3\sin(8dx+8c)}{1024d} - \frac{a^3\sin(6dx+6c)}{96d} + \frac{3a^3\sin(4dx+4c)}{128d} + \frac{9a^3\sin(2dx+2c)}{32d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 55/128*a^3*x + 1/2304*a^3*cos(9*d*x + 9*c)/d - 9/1792*a^3*cos(7*d*x + 7*c)/
d - 3/64*a^3*cos(5*d*x + 5*c)/d - 29/192*a^3*cos(3*d*x + 3*c)/d - 33/128*a^
3*cos(d*x + c)/d - 3/1024*a^3*sin(8*d*x + 8*c)/d - 1/96*a^3*sin(6*d*x + 6*c
)/d + 3/128*a^3*sin(4*d*x + 4*c)/d + 9/32*a^3*sin(2*d*x + 2*c)/d
```

Mupad [B]
time = 6.81, size = 501, normalized size = 3.25

$$\frac{55a^3x}{128} - \frac{\frac{17a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{32} - \frac{949a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{96} - \frac{699a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{32} + \frac{699a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}}{32} - \frac{17a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}}{32} + \frac{949a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{15}}{96} + \frac{73a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{17}}{64} + \frac{a^3(3465c+3465dx)}{8064} - \frac{a^3(3465c+3465dx-7424)}{8064} + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(\frac{a^3(3465c+3465dx)}{896} - \frac{a^3(31185c+31185dx-18432)}{8064}\right) + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}\left(\frac{a^3(3465c+3465dx)}{896} - \frac{a^3(31185c+31185dx-48384)}{8064}\right) + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}\left(\frac{a^3(3465c+3465dx)}{224} - \frac{a^3(124740c+124740dx-129024)}{8064}\right) + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(\frac{a^3(3465c+3465dx)}{224} - \frac{a^3(124740c+124740dx-138240)}{8064}\right) + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}\left(\frac{a^3(3465c+3465dx)}{96} - \frac{a^3(291060c+291060dx-236544)}{8064}\right) + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6\left(\frac{a^3(3465c+3465dx)}{96} - \frac{a^3(291060c+291060dx-387072)}{8064}\right) + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8\left(\frac{a^3(3465c+3465dx)}{64} - \frac{a^3(436590c+436590dx-290304)}{8064}\right) + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}\left(\frac{a^3(3465c+3465dx)}{64} - \frac{a^3(436590c+436590dx-645120)}{8064}\right) - \frac{73a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{64}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6*(a + a*sin(c + d*x))^3,x)
```

```
[Out] (55*a^3*x)/128 - ((17*a^3*tan(c/2 + (d*x)/2)^5)/32 - (949*a^3*tan(c/2 + (d*
x)/2)^3)/96 - (699*a^3*tan(c/2 + (d*x)/2)^7)/32 + (699*a^3*tan(c/2 + (d*x)/
2)^11)/32 - (17*a^3*tan(c/2 + (d*x)/2)^13)/32 + (949*a^3*tan(c/2 + (d*x)/2)
^15)/96 + (73*a^3*tan(c/2 + (d*x)/2)^17)/64 + (a^3*(3465*c + 3465*d*x))/806
4 - (a^3*(3465*c + 3465*d*x - 7424))/8064 + tan(c/2 + (d*x)/2)^2*((a^3*(346
5*c + 3465*d*x))/896 - (a^3*(31185*c + 31185*d*x - 18432))/8064) + tan(c/2
+ (d*x)/2)^16*((a^3*(3465*c + 3465*d*x))/896 - (a^3*(31185*c + 31185*d*x -
48384))/8064) + tan(c/2 + (d*x)/2)^14*((a^3*(3465*c + 3465*d*x))/224 - (a^3
*(124740*c + 124740*d*x - 129024))/8064) + tan(c/2 + (d*x)/2)^4*((a^3*(3465
*c + 3465*d*x))/224 - (a^3*(124740*c + 124740*d*x - 138240))/8064) + tan(c/
2 + (d*x)/2)^12*((a^3*(3465*c + 3465*d*x))/96 - (a^3*(291060*c + 291060*d*x
 - 236544))/8064) + tan(c/2 + (d*x)/2)^6*((a^3*(3465*c + 3465*d*x))/96 - (a
^3*(291060*c + 291060*d*x - 387072))/8064) + tan(c/2 + (d*x)/2)^8*((a^3*(34
65*c + 3465*d*x))/64 - (a^3*(436590*c + 436590*d*x - 290304))/8064) + tan(c
/2 + (d*x)/2)^10*((a^3*(3465*c + 3465*d*x))/64 - (a^3*(436590*c + 436590*d*
x - 645120))/8064) - (73*a^3*tan(c/2 + (d*x)/2))/64)/(d*(tan(c/2 + (d*x)/2)
^2 + 1)^9)
```

3.28 $\int \cos^5(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=67

$$\frac{2(a+a\sin(c+dx))^6}{3a^3d}-\frac{4(a+a\sin(c+dx))^7}{7a^4d}+\frac{(a+a\sin(c+dx))^8}{8a^5d}$$

```
[Out] 2/3*(a+a*sin(d*x+c))^6/a^3/d-4/7*(a+a*sin(d*x+c))^7/a^4/d+1/8*(a+a*sin(d*x+
c))^8/a^5/d
```

Rubi [A]
time = 0.04, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2746, 45}

$$\frac{(a\sin(c+dx)+a)^8}{8a^5d}-\frac{4(a\sin(c+dx)+a)^7}{7a^4d}+\frac{2(a\sin(c+dx)+a)^6}{3a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (2*(a + a*Sin[c + d*x])^6)/(3*a^3*d) - (4*(a + a*Sin[c + d*x])^7)/(7*a^4*d)
 + (a + a*Sin[c + d*x])^8/(8*a^5*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \cos^5(c+dx)(a+a\sin(c+dx))^3\,dx &= \frac{\text{Subst}\left(\int (a-x)^2(a+x)^5\,dx, x, a\sin(c+dx)\right)}{a^5d}\\ &= \frac{\text{Subst}\left(\int \left(4a^2(a+x)^5-4a(a+x)^6+(a+x)^7\right)dx, x, a\sin(c+dx)\right)}{a^5d}\\ &= \frac{2(a+a\sin(c+dx))^6}{3a^3d}-\frac{4(a+a\sin(c+dx))^7}{7a^4d}+\frac{(a+a\sin(c+dx))^8}{8a^5d}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 58, normalized size = 0.87

$$-\frac{a^3\cos^6(c+dx)(1+\sin(c+dx))^3\left(37-54\sin(c+dx)+21\sin^2(c+dx)\right)}{168d(-1+\sin(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/168*(a^3*Cos[c + d*x]^6*(1 + Sin[c + d*x])^3*(37 - 54*Sin[c + d*x] + 21*
Sin[c + d*x]^2))/(d*(-1 + Sin[c + d*x])^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 132 vs. 2(61) = 122.
time = 0.32, size = 133, normalized size = 1.99

method	result
derivativedivides	$\frac{a^3\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^6(dx+c)\right)}{8}-\frac{\left(\cos^6(dx+c)\right)}{24}\right)+3a^3\left(-\frac{\sin(dx+c)\left(\cos^6(dx+c)\right)}{7}+\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx-}{35}\right.}{d}$
default	$\frac{a^3\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^6(dx+c)\right)}{8}-\frac{\left(\cos^6(dx+c)\right)}{24}\right)+3a^3\left(-\frac{\sin(dx+c)\left(\cos^6(dx+c)\right)}{7}+\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx-}{35}\right.}{d}$
risch	$\frac{55a^3\sin(dx+c)}{64d}+\frac{a^3\cos(8dx+8c)}{1024d}-\frac{3a^3\sin(7dx+7c)}{448d}-\frac{5a^3\cos(6dx+6c)}{384d}-\frac{a^3\sin(5dx+5c)}{64d}-\frac{25a^3\cos(4dx+4c)}{256d}$
norman	$\frac{2a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{50a^3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{70a^3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{1166a^3\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{21d}+\frac{1166a^3\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{21d}+\frac{70a^3\left(\tan^{11}\left(\right.\right.}{3d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-1/8*sin(d*x+c)^2*cos(d*x+c)^6-1/24*cos(d*x+c)^6)+3*a^3*(-1/7*sin
(d*x+c)*cos(d*x+c)^6+1/35*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))-1
/2*cos(d*x+c)^6*a^3+1/5*a^3*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A]
time = 0.29, size = 108, normalized size = 1.61

$$\frac{21\,a^3\sin(dx+c)^8+72\,a^3\sin(dx+c)^7+28\,a^3\sin(dx+c)^6-168\,a^3\sin(dx+c)^5-210\,a^3\sin(dx+c)^4+56\,a^3\sin(dx+c)^3+252\,a^3\sin(dx+c)^2+168\,a^3\sin(dx+c)}{168\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/168*(21*a^3*sin(d*x + c)^8 + 72*a^3*sin(d*x + c)^7 + 28*a^3*sin(d*x + c)^
6 - 168*a^3*sin(d*x + c)^5 - 210*a^3*sin(d*x + c)^4 + 56*a^3*sin(d*x + c)^3
 + 252*a^3*sin(d*x + c)^2 + 168*a^3*sin(d*x + c))/d
```

Fricas [A]
time = 0.36, size = 85, normalized size = 1.27

$$\frac{21\,a^{3}\cos\left(dx+c\right)^{8}-112\,a^{3}\cos\left(dx+c\right)^{6}-8\left(9\,a^{3}\cos\left(dx+c\right)^{6}-6\,a^{3}\cos\left(dx+c\right)^{4}-8\,a^{3}\cos\left(dx+c\right)^{2}-16\,a^{3}\right)\sin\left(dx+c\right)}{168\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/168*(21*a^3*cos(d*x + c)^8 - 112*a^3*cos(d*x + c)^6 - 8*(9*a^3*cos(d*x +
c)^6 - 6*a^3*cos(d*x + c)^4 - 8*a^3*cos(d*x + c)^2 - 16*a^3)*sin(d*x + c))/
d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 196 vs. 2(58) = 116.
time = 0.96, size = 196, normalized size = 2.93

$$\begin{cases} \frac{8a^{3}\sin^{7}\left(c+dx\right)}{35d}+\frac{4a^{3}\sin^{5}\left(c+dx\right)\cos^{2}\left(c+dx\right)}{5d}+\frac{8a^{3}\sin^{5}\left(c+dx\right)}{15d}+\frac{a^{3}\sin^{3}\left(c+dx\right)\cos^{4}\left(c+dx\right)}{d}+\frac{4a^{3}\sin^{3}\left(c+dx\right)\cos^{2}\left(c+dx\right)}{3d}-\frac{a^{3}\sin^{2}\left(c+dx\right)\cos^{6}\left(c+dx\right)}{6d}+\frac{a^{3}\sin\left(c+dx\right)\cos^{4}\left(c+dx\right)}{d}-\frac{a^{3}\cos^{8}\left(c+dx\right)}{24d}-\frac{a^{3}\cos^{6}\left(c+dx\right)}{2d} & \text{for}\: d\neq 0 \\ x\left(a\sin\left(c\right)+a\right)^{3}\cos^{5}\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((8*a**3*sin(c + d*x)**7/(35*d) + 4*a**3*sin(c + d*x)**5*cos(c + d
*x)**2/(5*d) + 8*a**3*sin(c + d*x)**5/(15*d) + a**3*sin(c + d*x)**3*cos(c +
 d*x)**4/d + 4*a**3*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) - a**3*sin(c + d*
x)**2*cos(c + d*x)**6/(6*d) + a**3*sin(c + d*x)*cos(c + d*x)**4/d - a**3*co
s(c + d*x)**8/(24*d) - a**3*cos(c + d*x)**6/(2*d), Ne(d, 0)), (x*(a*sin(c)
+ a)**3*cos(c)**5, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 134 vs. 2(61) = 122.
time = 6.18, size = 134, normalized size = 2.00

$$\frac{a^{3}\cos\left(8\,dx+8\,c\right)}{1024\,d}-\frac{5\,a^{3}\cos\left(6\,dx+6\,c\right)}{384\,d}-\frac{25\,a^{3}\cos\left(4\,dx+4\,c\right)}{256\,d}-\frac{33\,a^{3}\cos\left(2\,dx+2\,c\right)}{128\,d}-\frac{3\,a^{3}\sin\left(7\,dx+7\,c\right)}{448\,d}-\frac{a^{3}\sin\left(5\,dx+5\,c\right)}{64\,d}+\frac{17\,a^{3}\sin\left(3\,dx+3\,c\right)}{192\,d}+\frac{55\,a^{3}\sin\left(dx+c\right)}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/1024*a^3*cos(8*d*x + 8*c)/d - 5/384*a^3*cos(6*d*x + 6*c)/d - 25/256*a^3*c
os(4*d*x + 4*c)/d - 33/128*a^3*cos(2*d*x + 2*c)/d - 3/448*a^3*sin(7*d*x + 7
*c)/d - 1/64*a^3*sin(5*d*x + 5*c)/d + 17/192*a^3*sin(3*d*x + 3*c)/d + 55/64
*a^3*sin(d*x + c)/d
```

Mupad [B]

time = 4.54, size = 106, normalized size = 1.58

$$\frac{\frac{a^3\sin(c+d\,x)^8}{8}+\frac{3\,a^3\sin(c+d\,x)^7}{7}+\frac{a^3\sin(c+d\,x)^6}{6}-a^3\sin\left(c+d\,x\right)^5-\frac{5\,a^3\sin(c+d\,x)^4}{4}+\frac{a^3\sin(c+d\,x)^3}{3}+\frac{3\,a^3\sin(c+d\,x)^2}{2}+a^3\,\sin\left(c+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(a + a*sin(c + d*x))^3,x)
```

```
[Out] (a^3*sin(c + d*x) + (3*a^3*sin(c + d*x)^2)/2 + (a^3*sin(c + d*x)^3)/3 - (5*
a^3*sin(c + d*x)^4)/4 - a^3*sin(c + d*x)^5 + (a^3*sin(c + d*x)^6)/6 + (3*a^
3*sin(c + d*x)^7)/7 + (a^3*sin(c + d*x)^8)/8)/d
```

3.29 $\int \cos^4(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=130

$$\frac{9a^3x}{16}-\frac{3a^3\cos^5(c+dx)}{10d}+\frac{9a^3\cos(c+dx)\sin(c+dx)}{16d}+\frac{3a^3\cos^3(c+dx)\sin(c+dx)}{8d}-\frac{a\cos^5(c+dx)(a+a\sin}{7d}$$

```
[Out] 9/16*a^3*x-3/10*a^3*cos(d*x+c)^5/d+9/16*a^3*cos(d*x+c)*sin(d*x+c)/d+3/8*a^3
*cos(d*x+c)^3*sin(d*x+c)/d-1/7*a*cos(d*x+c)^5*(a+a*sin(d*x+c))^2/d-3/14*cos
(d*x+c)^5*(a^3+a^3*sin(d*x+c))/d
```

Rubi [A]
time = 0.09, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2757, 2748, 2715, 8}

$$-\frac{3a^3\cos^5(c+dx)}{10d}-\frac{3\cos^5(c+dx)\left(a^3\sin(c+dx)+a^3\right)}{14d}+\frac{3a^3\sin(c+dx)\cos^3(c+dx)}{8d}+\frac{9a^3\sin(c+dx)\cos(c+dx)}{16d}+\frac{9a^3x}{16}-\frac{a\cos^5(c+dx)(a\sin(c+dx)+a)^2}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (9*a^3*x)/16 - (3*a^3*Cos[c + d*x]^5)/(10*d) + (9*a^3*Cos[c + d*x]*Sin[c +
d*x])/(16*d) + (3*a^3*Cos[c + d*x]^3*Sin[c + d*x])/(8*d) - (a*Cos[c + d*x]^
5*(a + a*Sin[c + d*x])^2)/(7*d) - (3*Cos[c + d*x]^5*(a^3 + a^3*Sin[c + d*x]
))/(14*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2757

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
```

```
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ
[2*m, 2*p]
```

Rubi steps

$$\begin{aligned}\int \cos^4(c+dx)(a+a\sin(c+dx))^3\,dx &= -\frac{a\cos^5(c+dx)(a+a\sin(c+dx))^2}{7d}+\frac{1}{7}(9a)\int \cos^4(c+dx)(a+a\sin\\ &= -\frac{a\cos^5(c+dx)(a+a\sin(c+dx))^2}{7d}-\frac{3\cos^5(c+dx)\left(a^3+a^3\sin(c\right.}{14d}\\ &= -\frac{3a^3\cos^5(c+dx)}{10d}-\frac{a\cos^5(c+dx)(a+a\sin(c+dx))^2}{7d}-\frac{3\cos^5(c\right.}{}\\ &= -\frac{3a^3\cos^5(c+dx)}{10d}+\frac{3a^3\cos^3(c+dx)\sin(c+dx)}{8d}-\frac{a\cos^5(c+dx)(}{}\\ &= -\frac{3a^3\cos^5(c+dx)}{10d}+\frac{9a^3\cos(c+dx)\sin(c+dx)}{16d}+\frac{3a^3\cos^3(c+dx)}{8d}\\ &= \frac{9a^3x}{16}-\frac{3a^3\cos^5(c+dx)}{10d}+\frac{9a^3\cos(c+dx)\sin(c+dx)}{16d}+\frac{3a^3\cos^3(c}{}\end{aligned}$$

Mathematica [A]
time = 0.50, size = 161, normalized size = 1.24

$$-\frac{a^3\cos^5(c+dx)\left(-630\sin^{-1}\left(\frac{\sqrt{1-\sin(c+dx)}}{\sqrt{2}}\right)\sqrt{1-\sin(c+dx)}+\sqrt{1+\sin(c+dx)}\,(-368+613\sin(c+dx)+411\sin^2(c+dx)-306\sin^3(c+dx)-558\sin^4(c+dx)-72\sin^5(c+dx)+200\sin^6(c+dx)+80\sin^7(c+dx))\right)}{560d(-1+\sin(c+dx))^3(1+\sin(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/560*(a^3*Cos[c + d*x]^5*(-630*ArcSin[Sqrt[1 - Sin[c + d*x]]/Sqrt[2]]*Sqr
t[1 - Sin[c + d*x]] + Sqrt[1 + Sin[c + d*x]]*(-368 + 613*Sin[c + d*x] + 411
*Sin[c + d*x]^2 - 306*Sin[c + d*x]^3 - 558*Sin[c + d*x]^4 - 72*Sin[c + d*x]
^5 + 200*Sin[c + d*x]^6 + 80*Sin[c + d*x]^7)))/(d*(-1 + Sin[c + d*x])^3*(1
+ Sin[c + d*x])^(5/2))
```

Maple [A]
time = 0.23, size = 143, normalized size = 1.10

method	result
risch	$\frac{9a^3x}{16}-\frac{27a^3\cos(dx+c)}{64d}+\frac{a^3\cos(7dx+7c)}{448d}-\frac{a^3\sin(6dx+6c)}{64d}-\frac{11a^3\cos(5dx+5c)}{320d}-\frac{a^3\sin(4dx+4c)}{64d}-\frac{13a^3\cos}{6}$

derivativedivides	$\frac{a^3\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^5(dx+c)\right)}{7}-\frac{2\left(\cos^5(dx+c)\right)}{35}\right)+3a^3\left(-\frac{\sin(dx+c)\left(\cos^5(dx+c)\right)}{6}+\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{24}-\right.}{d}$
default	$\frac{a^3\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^5(dx+c)\right)}{7}-\frac{2\left(\cos^5(dx+c)\right)}{35}\right)+3a^3\left(-\frac{\sin(dx+c)\left(\cos^5(dx+c)\right)}{6}+\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{24}-\right.}{d}$
norman	$\frac{\frac{9a^3x}{16}-\frac{46a^3}{35d}+\frac{7a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d}+\frac{17a^3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-\frac{13a^3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}+\frac{13a^3\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}-\frac{17a^3\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-1/7*sin(d*x+c)^2*cos(d*x+c)^5-2/35*cos(d*x+c)^5)+3*a^3*(-1/6*sin
(d*x+c)*cos(d*x+c)^5+1/24*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+1/16*d*x
+1/16*c)-3/5*cos(d*x+c)^5*a^3+a^3*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*
x+c)+3/8*d*x+3/8*c))
```

Maxima [A]
time = 0.30, size = 115, normalized size = 0.88

$$-\frac{1344\,a^3\cos\left(dx+c\right)^5-64\left(5\cos\left(dx+c\right)^7-7\cos\left(dx+c\right)^5\right)a^3-35\left(4\sin\left(2\,dx+2\,c\right)^3+12\,dx+12\,c-3\sin\left(4\,dx+4\,c\right)\right)a^3-70\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\sin\left(2\,dx+2\,c\right)\right)a^3}{2240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2240*(1344*a^3*cos(d*x + c)^5 - 64*(5*cos(d*x + c)^7 - 7*cos(d*x + c)^5)
*a^3 - 35*(4*sin(2*d*x + 2*c)^3 + 12*d*x + 12*c - 3*sin(4*d*x + 4*c))*a^3 -
 70*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^3)/d
```

Fricas [A]
time = 0.37, size = 85, normalized size = 0.65

$$\frac{80\,a^3\cos\left(dx+c\right)^7-448\,a^3\cos\left(dx+c\right)^5+315\,a^3dx-35\left(8\,a^3\cos\left(dx+c\right)^5-6\,a^3\cos\left(dx+c\right)^3-9\,a^3\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{560\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/560*(80*a^3*cos(d*x + c)^7 - 448*a^3*cos(d*x + c)^5 + 315*a^3*d*x - 35*(8
*a^3*cos(d*x + c)^5 - 6*a^3*cos(d*x + c)^3 - 9*a^3*cos(d*x + c))*sin(d*x +
c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 335 vs. $2(122) = 244$.
time = 0.68, size = 335, normalized size = 2.58

$$\begin{cases}\frac{3a^3x\sin^6(c+dx)}{16}+\frac{9a^3x\sin^4(c+dx)\cos^2(c+dx)}{16}+\frac{3a^3x\sin^4(c+dx)}{8}+\frac{9a^3x\sin^2(c+dx)\cos^4(c+dx)}{16}+\frac{3a^3x\sin^2(c+dx)\cos^2(c+dx)}{4}+\frac{3a^3x\cos^6(c+dx)}{16}+\frac{3a^3x\cos^4(c+dx)}{8}+\frac{3a^3\sin^5(c+dx)\cos(c+dx)}{16d}+\frac{a^3\sin^3(c+dx)\cos^3(c+dx)}{2d}+\frac{3a^3\sin^3(c+dx)\cos(c+dx)}{8d}-\frac{a^3\sin^2(c+dx)\cos^5(c+dx)}{5d}-\frac{3a^3\sin(c+dx)\cos^5(c+dx)}{16d}+\frac{5a^3\sin(c+dx)\cos^3(c+dx)}{8d}-\frac{2a^3\cos^7(c+dx)}{35d}-\frac{3a^3\cos^5(c+dx)}{5d} & \text{for } d\neq 0\\ x(a\sin(c)+a)^3\cos^4(c) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((3*a**3*x*sin(c + d*x)**6/16 + 9*a**3*x*sin(c + d*x)**4*cos(c + d
*x)**2/16 + 3*a**3*x*sin(c + d*x)**4/8 + 9*a**3*x*sin(c + d*x)**2*cos(c + d
*x)**4/16 + 3*a**3*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 3*a**3*x*cos(c + d
*x)**6/16 + 3*a**3*x*cos(c + d*x)**4/8 + 3*a**3*sin(c + d*x)**5*cos(c + d*x
)/(16*d) + a**3*sin(c + d*x)**3*cos(c + d*x)**3/(2*d) + 3*a**3*sin(c + d*x)
**3*cos(c + d*x)/(8*d) - a**3*sin(c + d*x)**2*cos(c + d*x)**5/(5*d) - 3*a**
3*sin(c + d*x)*cos(c + d*x)**5/(16*d) + 5*a**3*sin(c + d*x)*cos(c + d*x)**3
/(8*d) - 2*a**3*cos(c + d*x)**7/(35*d) - 3*a**3*cos(c + d*x)**5/(5*d), Ne(d
, 0)), (x*(a*sin(c) + a)**3*cos(c)**4, True))
```

Giac [A]
time = 3.97, size = 123, normalized size = 0.95

$$\frac{9}{16}\,a^3x+\frac{a^3\cos\left(7\,dx+7\,c\right)}{448\,d}-\frac{11\,a^3\cos\left(5\,dx+5\,c\right)}{320\,d}-\frac{13\,a^3\cos\left(3\,dx+3\,c\right)}{64\,d}-\frac{27\,a^3\cos\left(dx+c\right)}{64\,d}-\frac{a^3\sin\left(6\,dx+6\,c\right)}{64\,d}-\frac{a^3\sin\left(4\,dx+4\,c\right)}{64\,d}+\frac{19\,a^3\sin\left(2\,dx+2\,c\right)}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 9/16*a^3*x + 1/448*a^3*cos(7*d*x + 7*c)/d - 11/320*a^3*cos(5*d*x + 5*c)/d -
 13/64*a^3*cos(3*d*x + 3*c)/d - 27/64*a^3*cos(d*x + c)/d - 1/64*a^3*sin(6*d
*x + 6*c)/d - 1/64*a^3*sin(4*d*x + 4*c)/d + 19/64*a^3*sin(2*d*x + 2*c)/d
```

Mupad [B]
time = 6.71, size = 389, normalized size = 2.99

$$\frac{9\,a^3\,x}{16}-\frac{\frac{13\,a^3\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5}{8}-\frac{17\,a^3\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3}{2}-\frac{13\,a^3\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^9}{8}+\frac{17\,a^3\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{11}}{2}+\frac{7\,a^3\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{13}}{8}+\frac{a^3\,\left(315\,c+315\,d\,x\right)}{560}-\frac{a^3\,\left(315\,c+315\,d\,x-736\right)}{560}+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(\frac{a^3\,\left(315\,c+315\,d\,x\right)}{80}-\frac{a^3\,\left(2205\,c+2205\,d\,x-1792\right)}{560}\right)+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{12}\,\left(\frac{a^3\,\left(315\,c+315\,d\,x\right)}{80}-\frac{a^3\,\left(2205\,c+2205\,d\,x-3360\right)}{560}\right)+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(\frac{3\,a^3\,\left(315\,c+315\,d\,x\right)}{80}-\frac{a^3\,\left(6615\,c+6615\,d\,x-6496\right)}{560}\right)+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}\,\left(\frac{3\,a^3\,\left(315\,c+315\,d\,x\right)}{80}-\frac{a^3\,\left(6615\,c+6615\,d\,x-8960\right)}{560}\right)+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8\,\left(\frac{a^3\,\left(315\,c+315\,d\,x\right)}{16}-\frac{a^3\,\left(11025\,c+11025\,d\,x-7840\right)}{560}\right)+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6\,\left(\frac{a^3\,\left(315\,c+315\,d\,x\right)}{16}-\frac{a^3\,\left(11025\,c+11025\,d\,x-17920\right)}{560}\right)-\frac{7\,a^3\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8}}{d\,{\left({\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(a + a*sin(c + d*x))^3,x)
```

```
[Out] (9*a^3*x)/16 - ((13*a^3*tan(c/2 + (d*x)/2)^5)/8 - (17*a^3*tan(c/2 + (d*x)/2
)^3)/2 - (13*a^3*tan(c/2 + (d*x)/2)^9)/8 + (17*a^3*tan(c/2 + (d*x)/2)^11)/2
 + (7*a^3*tan(c/2 + (d*x)/2)^13)/8 + (a^3*(315*c + 315*d*x))/560 - (a^3*(31
5*c + 315*d*x - 736))/560 + tan(c/2 + (d*x)/2)^2*((a^3*(315*c + 315*d*x))/8
0 - (a^3*(2205*c + 2205*d*x - 1792))/560) + tan(c/2 + (d*x)/2)^12*((a^3*(31
5*c + 315*d*x))/80 - (a^3*(2205*c + 2205*d*x - 3360))/560) + tan(c/2 + (d*x
)/2)^4*((3*a^3*(315*c + 315*d*x))/80 - (a^3*(6615*c + 6615*d*x - 6496))/560
) + tan(c/2 + (d*x)/2)^10*((3*a^3*(315*c + 315*d*x))/80 - (a^3*(6615*c + 66
15*d*x - 8960))/560) + tan(c/2 + (d*x)/2)^8*((a^3*(315*c + 315*d*x))/16 - (
a^3*(11025*c + 11025*d*x - 7840))/560) + tan(c/2 + (d*x)/2)^6*((a^3*(315*c
+ 315*d*x))/16 - (a^3*(11025*c + 11025*d*x - 17920))/560) - (7*a^3*tan(c/2
+ (d*x)/2))/8)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^7)
```

3.30 $\int \cos^3(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=45

$$\frac{2(a+a\sin(c+dx))^5}{5a^2d}-\frac{(a+a\sin(c+dx))^6}{6a^3d}$$

```
[Out] 2/5*(a+a*sin(d*x+c))^5/a^2/d-1/6*(a+a*sin(d*x+c))^6/a^3/d
```

Rubi [A]
time = 0.03, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.095$, Rules used = {2746, 45}

$$\frac{2(a\sin(c+dx)+a)^5}{5a^2d}-\frac{(a\sin(c+dx)+a)^6}{6a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (2*(a + a*Sin[c + d*x])^5)/(5*a^2*d) - (a + a*Sin[c + d*x])^6/(6*a^3*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \cos^3(c+dx)(a+a\sin(c+dx))^3\,dx&=\frac{\text{Subst}\left(\int (a-x)(a+x)^4\,dx,x,a\sin(c+dx)\right)}{a^3d}\\&=\frac{\text{Subst}\left(\int \left(2a(a+x)^4-(a+x)^5\right)\,dx,x,a\sin(c+dx)\right)}{a^3d}\\&=\frac{2(a+a\sin(c+dx))^5}{5a^2d}-\frac{(a+a\sin(c+dx))^6}{6a^3d}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 43, normalized size = 0.96

$$-\frac{a^3\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^{10}(-7+5\sin(c+dx))}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/30*(a^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^10*(-7 + 5*Sin[c + d*x]))/
d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(41) = 82$.
time = 0.21, size = 113, normalized size = 2.51

method	result
risch	$\frac{9a^3\sin(dx+c)}{8d}+\frac{a^3\cos(6dx+6c)}{192d}-\frac{3a^3\sin(5dx+5c)}{80d}-\frac{3a^3\cos(4dx+4c)}{32d}+\frac{a^3\sin(3dx+3c)}{48d}-\frac{27a^3\cos(2dx+2c)}{64d}$
derivativedivides	$\frac{a^3\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^4(dx+c)\right)}{6}-\frac{\left(\cos^4(dx+c)\right)}{12}\right)+3a^3\left(-\frac{\sin(dx+c)\left(\cos^4(dx+c)\right)}{5}+\frac{\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{15}\right)-\frac{3\left(\cos^4(dx+\right.}{4}}{d}$
default	$\frac{a^3\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^4(dx+c)\right)}{6}-\frac{\left(\cos^4(dx+c)\right)}{12}\right)+3a^3\left(-\frac{\sin(dx+c)\left(\cos^4(dx+c)\right)}{5}+\frac{\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{15}\right)-\frac{3\left(\cos^4(dx+\right.}{4}}{d}$
norman	$\frac{\frac{16a^3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{16a^3\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{46a^3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{84a^3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5d}+\frac{84a^3\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{5d}}{\left(1+\tan^2\left(\frac{dx}{2}+\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-1/6*sin(d*x+c)^2*cos(d*x+c)^4-1/12*cos(d*x+c)^4)+3*a^3*(-1/5*sin
(d*x+c)*cos(d*x+c)^4+1/15*(2+cos(d*x+c)^2)*sin(d*x+c))-3/4*cos(d*x+c)^4*a^3
+1/3*a^3*(2+cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A]
time = 0.32, size = 82, normalized size = 1.82

$$-\frac{5\,a^3\sin\left(dx+c\right)^6+18\,a^3\sin\left(dx+c\right)^5+15\,a^3\sin\left(dx+c\right)^4-20\,a^3\sin\left(dx+c\right)^3-45\,a^3\sin\left(dx+c\right)^2-30\,a^3\sin\left(dx+c\right)}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/30*(5*a^3*sin(d*x + c)^6 + 18*a^3*sin(d*x + c)^5 + 15*a^3*sin(d*x + c)^4
 - 20*a^3*sin(d*x + c)^3 - 45*a^3*sin(d*x + c)^2 - 30*a^3*sin(d*x + c))/d
```

Fricas [A]
time = 0.36, size = 72, normalized size = 1.60

$$\frac{5\,a^3\cos\left(dx+c\right)^6-30\,a^3\cos\left(dx+c\right)^4-2\left(9\,a^3\cos\left(dx+c\right)^4-8\,a^3\cos\left(dx+c\right)^2-16\,a^3\right)\sin\left(dx+c\right)}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/30*(5*a^3*cos(d*x + c)^6 - 30*a^3*cos(d*x + c)^4 - 2*(9*a^3*cos(d*x + c)^
4 - 8*a^3*cos(d*x + c)^2 - 16*a^3)*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(37) = 74$.
time = 0.43, size = 146, normalized size = 3.24

$$\begin{cases} \frac{2a^3\sin^5(c+dx)}{5d}+\frac{a^3\sin^3(c+dx)\cos^2(c+dx)}{d}+\frac{2a^3\sin^3(c+dx)}{3d}-\frac{a^3\sin^2(c+dx)\cos^4(c+dx)}{4d}+\frac{a^3\sin(c+dx)\cos^2(c+dx)}{d}-\frac{a^3\cos^6(c+dx)}{12d}-\frac{3a^3\cos^4(c+dx)}{4d} & \text{for } d\neq 0 \\ x(a\sin(c)+a)^3\cos^3(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((2*a**3*sin(c + d*x)**5/(5*d) + a**3*sin(c + d*x)**3*cos(c + d*x)
**2/d + 2*a**3*sin(c + d*x)**3/(3*d) - a**3*sin(c + d*x)**2*cos(c + d*x)**4
/(4*d) + a**3*sin(c + d*x)*cos(c + d*x)**2/d - a**3*cos(c + d*x)**6/(12*d)
- 3*a**3*cos(c + d*x)**4/(4*d), Ne(d, 0)), (x*(a*sin(c) + a)**3*cos(c)**3,
True))
```

Giac [A]
time = 3.72, size = 82, normalized size = 1.82

$$-\frac{5\,a^3\sin\left(dx+c\right)^6+18\,a^3\sin\left(dx+c\right)^5+15\,a^3\sin\left(dx+c\right)^4-20\,a^3\sin\left(dx+c\right)^3-45\,a^3\sin\left(dx+c\right)^2-30\,a^3\sin\left(dx+c\right)}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/30*(5*a^3*sin(d*x + c)^6 + 18*a^3*sin(d*x + c)^5 + 15*a^3*sin(d*x + c)^4
 - 20*a^3*sin(d*x + c)^3 - 45*a^3*sin(d*x + c)^2 - 30*a^3*sin(d*x + c))/d
```

Mupad [B]
time = 4.47, size = 80, normalized size = 1.78

$$\frac{-\frac{a^3\sin(c+d\,x)^6}{6}-\frac{3\,a^3\sin(c+d\,x)^5}{5}-\frac{a^3\sin(c+d\,x)^4}{2}+\frac{2\,a^3\sin(c+d\,x)^3}{3}+\frac{3\,a^3\sin(c+d\,x)^2}{2}+a^3\sin\left(c+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a + a*sin(c + d*x))^3,x)
```

```
[Out] (a^3*sin(c + d*x) + (3*a^3*sin(c + d*x)^2)/2 + (2*a^3*sin(c + d*x)^3)/3 - (
a^3*sin(c + d*x)^4)/2 - (3*a^3*sin(c + d*x)^5)/5 - (a^3*sin(c + d*x)^6)/6)/
d
```

3.31 $\int \cos^2(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=106

$$\frac{7a^3x}{8}-\frac{7a^3\cos^3(c+dx)}{12d}+\frac{7a^3\cos(c+dx)\sin(c+dx)}{8d}-\frac{a\cos^3(c+dx)(a+a\sin(c+dx))^2}{5d}-\frac{7\cos^3(c+dx)\,(a^3$$

```
[Out] 7/8*a^3*x-7/12*a^3*cos(d*x+c)^3/d+7/8*a^3*cos(d*x+c)*sin(d*x+c)/d-1/5*a*cos
(d*x+c)^3*(a+a*sin(d*x+c))^2/d-7/20*cos(d*x+c)^3*(a^3+a^3*sin(d*x+c))/d
```

Rubi [A]
time = 0.08, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2757, 2748, 2715, 8}

$$-\frac{7a^3\cos^3(c+dx)}{12d}-\frac{7\cos^3(c+dx)\left(a^3\sin(c+dx)+a^3\right)}{20d}+\frac{7a^3\sin(c+dx)\cos(c+dx)}{8d}+\frac{7a^3x}{8}-\frac{a\cos^3(c+dx)(a\sin(c+dx)+a)^2}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (7*a^3*x)/8 - (7*a^3*Cos[c + d*x]^3)/(12*d) + (7*a^3*Cos[c + d*x]*Sin[c + d
*x])/(8*d) - (a*Cos[c + d*x]^3*(a + a*Sin[c + d*x])^2)/(5*d) - (7*Cos[c + d
*x]^3*(a^3 + a^3*Sin[c + d*x]))/(20*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2757

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
```

```
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ
[2*m, 2*p]
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\sin(c+dx))^3\,dx = -\frac{a\cos^3(c+dx)(a+a\sin(c+dx))^2}{5d} + \frac{1}{5}(7a)\int \cos^2(c+dx)(a+$$

$$= -\frac{a\cos^3(c+dx)(a+a\sin(c+dx))^2}{5d} - \frac{7\cos^3(c+dx)\left(a^3+a^3\sin(\right.}{20d}$$

$$= -\frac{7a^3\cos^3(c+dx)}{12d} - \frac{a\cos^3(c+dx)(a+a\sin(c+dx))^2}{5d} - \frac{7\cos^3(}{}$$

$$= -\frac{7a^3\cos^3(c+dx)}{12d} + \frac{7a^3\cos(c+dx)\sin(c+dx)}{8d} - \frac{a\cos^3(c+dx)}{}$$

$$= \frac{7a^3x}{8} - \frac{7a^3\cos^3(c+dx)}{12d} + \frac{7a^3\cos(c+dx)\sin(c+dx)}{8d} - \frac{a\cos^3(c}{}$$

Mathematica [A]

time = 0.27, size = 141, normalized size = 1.33

$$-\frac{a^3\cos^3(c+dx)\left(210\sin^{-1}\left(\frac{\sqrt{1-\sin(c+dx)}}{\sqrt{2}}\right)\sqrt{1-\sin(c+dx)}+\sqrt{1+\sin(c+dx)}\left(136-151\sin(c+dx)-97\sin^2(c+dx)+22\sin^3(c+dx)+66\sin^4(c+dx)+24\sin^5(c+dx)\right)\right)}{120d(-1+\sin(c+dx))^2(1+\sin(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/120*(a^3*Cos[c + d*x]^3*(210*ArcSin[Sqrt[1 - Sin[c + d*x]]/Sqrt[2]]*Sqrt
[1 - Sin[c + d*x]] + Sqrt[1 + Sin[c + d*x]]*(136 - 151*Sin[c + d*x] - 97*Si
n[c + d*x]^2 + 22*Sin[c + d*x]^3 + 66*Sin[c + d*x]^4 + 24*Sin[c + d*x]^5)))
/(d*(-1 + Sin[c + d*x])^2*(1 + Sin[c + d*x])^(3/2))
```

Maple [A]

time = 0.17, size = 121, normalized size = 1.14

method	result
risch	$\frac{7a^3x}{8} - \frac{7a^3\cos(dx+c)}{8d} + \frac{a^3\cos(5dx+5c)}{80d} - \frac{3a^3\sin(4dx+4c)}{32d} - \frac{13a^3\cos(3dx+3c)}{48d} + \frac{a^3\sin(2dx+2c)}{4d}$
derivativedivides	$\frac{a^3\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^3(dx+c)\right)}{5}-\frac{2\left(\cos^3(dx+c)\right)}{15}\right)+3a^3\left(-\frac{\sin(dx+c)\left(\cos^3(dx+c)\right)}{4}+\frac{\cos(dx+c)\sin(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)-(\cos^3(}{d}$
default	$\frac{a^3\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^3(dx+c)\right)}{5}-\frac{2\left(\cos^3(dx+c)\right)}{15}\right)+3a^3\left(-\frac{\sin(dx+c)\left(\cos^3(dx+c)\right)}{4}+\frac{\cos(dx+c)\sin(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)-(\cos^3(}{d}$

norman	$\frac{\frac{7a^3x}{8}-\frac{34a^3}{15d}+\frac{a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}+\frac{13a^3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-\frac{13a^3\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-\frac{a^3\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{35a^3x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8}+\frac{35a^3}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-1/5*sin(d*x+c)^2*cos(d*x+c)^3-2/15*cos(d*x+c)^3)+3*a^3*(-1/4*sin
(d*x+c)*cos(d*x+c)^3+1/8*cos(d*x+c)*sin(d*x+c)+1/8*d*x+1/8*c)-cos(d*x+c)^3*
a^3+a^3*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))
```

Maxima [A]
time = 0.39, size = 91, normalized size = 0.86

$$-\frac{480\,a^3\cos\left(dx+c\right)^3-32\,\left(3\,\cos\left(dx+c\right)^5-5\,\cos\left(dx+c\right)^3\right)a^3-45\,\left(4\,dx+4\,c-\sin\left(4\,dx+4\,c\right)\right)a^3-120\,\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)a^3}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/480*(480*a^3*cos(d*x + c)^3 - 32*(3*cos(d*x + c)^5 - 5*cos(d*x + c)^3)*a
^3 - 45*(4*d*x + 4*c - sin(4*d*x + 4*c))*a^3 - 120*(2*d*x + 2*c + sin(2*d*x
 + 2*c))*a^3)/d
```

Fricas [A]
time = 0.35, size = 72, normalized size = 0.68

$$\frac{24\,a^3\cos\left(dx+c\right)^5-160\,a^3\cos\left(dx+c\right)^3+105\,a^3dx-15\,\left(6\,a^3\cos\left(dx+c\right)^3-7\,a^3\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/120*(24*a^3*cos(d*x + c)^5 - 160*a^3*cos(d*x + c)^3 + 105*a^3*d*x - 15*(6
*a^3*cos(d*x + c)^3 - 7*a^3*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(99) = 198$.
time = 0.37, size = 226, normalized size = 2.13

$$\begin{cases} \frac{3a^3x\sin^4\left(c+dx\right)}{8}+\frac{3a^3x\sin^2\left(c+dx\right)\cos^2\left(c+dx\right)}{4}+\frac{a^3x\sin^2\left(c+dx\right)}{2}+\frac{3a^3x\cos^4\left(c+dx\right)}{8}+\frac{a^3x\cos^2\left(c+dx\right)}{2}+\frac{3a^3\sin^3\left(c+dx\right)\cos\left(c+dx\right)}{8d}-\frac{a^3\sin^2\left(c+dx\right)\cos^3\left(c+dx\right)}{3d}-\frac{3a^3\sin\left(c+dx\right)\cos^3\left(c+dx\right)}{8d}+\frac{a^3\sin\left(c+dx\right)\cos\left(c+dx\right)}{2d}-\frac{2a^3\cos^5\left(c+dx\right)}{15d}-\frac{a^3\cos^3\left(c+dx\right)}{d} & \text{for}\: d \neq 0 \\ x\left(a\sin\left(c\right)+a\right)^3\cos^2\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((3*a**3*x*sin(c + d*x)**4/8 + 3*a**3*x*sin(c + d*x)**2*cos(c + d*
x)**2/4 + a**3*x*sin(c + d*x)**2/2 + 3*a**3*x*cos(c + d*x)**4/8 + a**3*x*co
```

```
s(c + d*x)**2/2 + 3*a**3*sin(c + d*x)**3*cos(c + d*x)/(8*d) - a**3*sin(c +
d*x)**2*cos(c + d*x)**3/(3*d) - 3*a**3*sin(c + d*x)*cos(c + d*x)**3/(8*d) +
 a**3*sin(c + d*x)*cos(c + d*x)/(2*d) - 2*a**3*cos(c + d*x)**5/(15*d) - a**
3*cos(c + d*x)**3/d, Ne(d, 0)), (x*(a*sin(c) + a)**3*cos(c)**2, True))
```

Giac [A]
time = 4.17, size = 89, normalized size = 0.84

$$\frac{7}{8}\,a^3x + \frac{a^3\cos\left(5\,dx+5\,c\right)}{80\,d} - \frac{13\,a^3\cos\left(3\,dx+3\,c\right)}{48\,d} - \frac{7\,a^3\cos\left(dx+c\right)}{8\,d} - \frac{3\,a^3\sin\left(4\,dx+4\,c\right)}{32\,d} + \frac{a^3\sin\left(2\,dx+2\,c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 7/8*a^3*x + 1/80*a^3*cos(5*d*x + 5*c)/d - 13/48*a^3*cos(3*d*x + 3*c)/d - 7/
8*a^3*cos(d*x + c)/d - 3/32*a^3*sin(4*d*x + 4*c)/d + 1/4*a^3*sin(2*d*x + 2*
c)/d
```

Mupad [B]
time = 6.53, size = 277, normalized size = 2.61

$$\frac{7\,a^3\,x}{8} - \frac{\frac{13\,a^3\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7}{2} - \frac{13\,a^3\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3}{2} + \frac{a^3\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^9}{4} + \frac{a^3\,\left(105\,c+105\,d\,x\right)}{120} - \frac{a^3\,\left(105\,c+105\,d\,x-272\right)}{120} + {\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(\frac{a^3\,\left(105\,c+105\,d\,x\right)}{24} - \frac{a^3\,\left(525\,c+525\,d\,x-640\right)}{120}\right) + {\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8\,\left(\frac{a^3\,\left(105\,c+105\,d\,x\right)}{24} - \frac{a^3\,\left(525\,c+525\,d\,x-720\right)}{120}\right) + {\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(\frac{a^3\,\left(105\,c+105\,d\,x\right)}{12} - \frac{a^3\,\left(1050\,c+1050\,d\,x-800\right)}{120}\right) + {\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6\,\left(\frac{a^3\,\left(105\,c+105\,d\,x\right)}{12} - \frac{a^3\,\left(1050\,c+1050\,d\,x-1920\right)}{120}\right) - \frac{a^3\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\,{\left({\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a + a*sin(c + d*x))^3,x)
```

```
[Out] (7*a^3*x)/8 - ((13*a^3*tan(c/2 + (d*x)/2)^7)/2 - (13*a^3*tan(c/2 + (d*x)/2)
^3)/2 + (a^3*tan(c/2 + (d*x)/2)^9)/4 + (a^3*(105*c + 105*d*x))/120 - (a^3*(
105*c + 105*d*x - 272))/120 + tan(c/2 + (d*x)/2)^2*((a^3*(105*c + 105*d*x))
/24 - (a^3*(525*c + 525*d*x - 640))/120) + tan(c/2 + (d*x)/2)^8*((a^3*(105*
c + 105*d*x))/24 - (a^3*(525*c + 525*d*x - 720))/120) + tan(c/2 + (d*x)/2)^
4*((a^3*(105*c + 105*d*x))/12 - (a^3*(1050*c + 1050*d*x - 800))/120) + tan(
c/2 + (d*x)/2)^6*((a^3*(105*c + 105*d*x))/12 - (a^3*(1050*c + 1050*d*x - 19
20))/120) - (a^3*tan(c/2 + (d*x)/2))/4)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^5)
```

3.32 $\int \cos(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=22

$$\frac{(a+a\sin(c+dx))^4}{4ad}$$

```
[Out] 1/4*(a+a*sin(d*x+c))^4/a/d
```

Rubi [A]
time = 0.02, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2746, 32}

$$\frac{(a\sin(c+dx)+a)^4}{4ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a + a*Sin[c + d*x])^4/(4*a*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a+a\sin(c+dx))^3\,dx &= \frac{\text{Subst}\left(\int (a+x)^3\,dx, x, a\sin(c+dx)\right)}{ad}\\ &= \frac{(a+a\sin(c+dx))^4}{4ad}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 47 vs. 2(22) = 44.
time = 0.27, size = 47, normalized size = 2.14

$$\frac{a^3(-28\cos(2(c+dx))+\cos(4(c+dx))+56\sin(c+dx)-8\sin(3(c+dx)))}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(-28*Cos[2*(c + d*x)] + Cos[4*(c + d*x)] + 56*Sin[c + d*x] - 8*Sin[3*(
c + d*x)]))/(32*d)
```

Maple [A]
time = 0.11, size = 21, normalized size = 0.95

method	result
derivativedivides	$\frac{(a+a\sin(dx+c))^4}{4da}$
default	$\frac{(a+a\sin(dx+c))^4}{4da}$
risch	$\frac{7a^3\sin(dx+c)}{4d}+\frac{a^3\cos(4dx+4c)}{32d}-\frac{a^3\sin(3dx+3c)}{4d}-\frac{7a^3\cos(2dx+2c)}{8d}$
norman	$\frac{\frac{2a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{14a^3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{14a^3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2a^3\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{6a^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{6a^3\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(a+a*sin(d*x+c))^4/d/a
```

Maxima [A]
time = 0.29, size = 20, normalized size = 0.91

$$\frac{(a\sin(dx+c)+a)^4}{4\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/4*(a*sin(d*x + c) + a)^4/(a*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(20) = 40$.
time = 0.36, size = 57, normalized size = 2.59

$$\frac{a^3\cos(dx+c)^4-8\,a^3\cos(dx+c)^2-4\left(a^3\cos(dx+c)^2-2\,a^3\right)\sin(dx+c)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(a^3*cos(d*x + c)^4 - 8*a^3*cos(d*x + c)^2 - 4*(a^3*cos(d*x + c)^2 - 2*
a^3)*sin(d*x + c))/d
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(15) = 30$.
time = 0.17, size = 70, normalized size = 3.18

$$\begin{cases} \frac{a^3\sin^4(c+dx)}{4d} + \frac{a^3\sin^3(c+dx)}{d} + \frac{3a^3\sin^2(c+dx)}{2d} + \frac{a^3\sin(c+dx)}{d} & \text{for } d \neq 0 \\ x(a\sin(c)+a)^3\cos(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((a**3*sin(c + d*x)**4/(4*d) + a**3*sin(c + d*x)**3/d + 3*a**3*sin
(c + d*x)**2/(2*d) + a**3*sin(c + d*x)/d, Ne(d, 0)), (x*(a*sin(c) + a)**3*c
os(c), True))
```

Giac [**A**]
time = 5.67, size = 20, normalized size = 0.91

$$\frac{(a\sin(dx+c)+a)^4}{4\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/4*(a*sin(d*x + c) + a)^4/(a*d)
```

Mupad [**B**]
time = 0.06, size = 53, normalized size = 2.41

$$\frac{\frac{a^3\sin(c+d\,x)^4}{4} + a^3\sin(c+d\,x)^3 + \frac{3\,a^3\sin(c+d\,x)^2}{2} + a^3\sin(c+d\,x)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + a*sin(c + d*x))^3,x)
```

```
[Out] (a^3*sin(c + d*x) + (3*a^3*sin(c + d*x)^2)/2 + a^3*sin(c + d*x)^3 + (a^3*si
n(c + d*x)^4)/4)/d
```

3.33 $\int \sec(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=52

$$-\frac{4a^3\log(1-\sin(c+dx))}{d}-\frac{3a^3\sin(c+dx)}{d}-\frac{a^3\sin^2(c+dx)}{2d}$$

```
[Out] -4*a^3*ln(1-sin(d*x+c))/d-3*a^3*sin(d*x+c)/d-1/2*a^3*sin(d*x+c)^2/d
```

Rubi [A]
time = 0.03, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2746, 45}

$$-\frac{a^3\sin^2(c+dx)}{2d}-\frac{3a^3\sin(c+dx)}{d}-\frac{4a^3\log(1-\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-4*a^3*Log[1 - Sin[c + d*x]])/d - (3*a^3*Sin[c + d*x])/d - (a^3*Sin[c + d*
x]^2)/(2*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \sec(c+dx)(a+a\sin(c+dx))^3\,dx &= \frac{a\text{Subst}\left(\int \frac{(a+x)^2}{a-x}\,dx, x, a\sin(c+dx)\right)}{d}\\ &= \frac{a\text{Subst}\left(\int \left(-3a+\frac{4a^2}{a-x}-x\right)\,dx, x, a\sin(c+dx)\right)}{d}\\ &= -\frac{4a^3\log(1-\sin(c+dx))}{d}-\frac{3a^3\sin(c+dx)}{d}-\frac{a^3\sin^2(c+dx)}{2d}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 41, normalized size = 0.79

$$\frac{a^3\left(-4\log(1-\sin(c+dx))-3\sin(c+dx)-\frac{1}{2}\sin^2(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sin[c + d*x])^3,x]
[Out] (a^3*(-4*Log[1 - Sin[c + d*x]] - 3*Sin[c + d*x] - Sin[c + d*x]^2/2))/d
```

Maple [A]
time = 0.13, size = 88, normalized size = 1.69

method	result
derivativedivides	$\frac{a^3\left(-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)+3a^3(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))-3a^3\ln(\cos(dx+c))+a^3\ln(\sec(dx+}{d}$
default	$\frac{a^3\left(-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)+3a^3(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))-3a^3\ln(\cos(dx+c))+a^3\ln(\sec(dx+}{d}$
risch	$4ia^3x+\frac{3ia^3e^{i(dx+c)}}{2d}-\frac{3ia^3e^{-i(dx+c)}}{2d}+\frac{8ia^3c}{d}-\frac{8a^3\ln\left(e^{i(dx+c)}-i\right)}{d}+\frac{a^3\cos(2dx+2c)}{4d}$
norman	$\frac{-\frac{6a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{12a^3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{6a^3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2a^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2a^3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}-\frac{8a^3\ln\left(\tan\left(\frac{dx}{2}\right.\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
[Out] 1/d*(a^3*(-1/2*sin(d*x+c)^2-ln(cos(d*x+c)))+3*a^3*(-sin(d*x+c)+ln(sec(d*x+c
)+tan(d*x+c)))-3*a^3*ln(cos(d*x+c))+a^3*ln(sec(d*x+c)+tan(d*x+c)))
```

Maxima [A]
time = 0.29, size = 43, normalized size = 0.83

$$-\frac{a^3\sin(dx+c)^2+8a^3\log(\sin(dx+c)-1)+6a^3\sin(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
[Out] -1/2*(a^3*sin(d*x + c)^2 + 8*a^3*log(sin(d*x + c) - 1) + 6*a^3*sin(d*x + c)
)/d
```

Fricas [A]
time = 0.36, size = 45, normalized size = 0.87

$$\frac{a^3\cos(dx+c)^2-8a^3\log(-\sin(dx+c)+1)-6a^3\sin(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(a^3*cos(d*x + c)^2 - 8*a^3*log(-sin(d*x + c) + 1) - 6*a^3*sin(d*x + c)
)/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^{3}\left(\int 3\sin\left(c+dx\right)\sec\left(c+dx\right)dx+\int 3\sin^{2}\left(c+dx\right)\sec\left(c+dx\right)dx+\int \sin^{3}\left(c+dx\right)\sec\left(c+dx\right)dx+\int \sec\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))**3,x)
```

```
[Out] a**3*(Integral(3*sin(c + d*x)*sec(c + d*x), x) + Integral(3*sin(c + d*x)**2
*sec(c + d*x), x) + Integral(sin(c + d*x)**3*sec(c + d*x), x) + Integral(se
c(c + d*x), x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 128 vs. 2(50) = 100.
time = 6.24, size = 128, normalized size = 2.46

$$\frac{2\left(2a^{3}\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+1\right)-4a^{3}\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)-\frac{3a^{3}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{4}+3a^{3}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}+7a^{3}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+3a^{3}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+3a^{3}}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+1\right)^{2}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 2*(2*a^3*log(tan(1/2*d*x + 1/2*c)^2 + 1) - 4*a^3*log(abs(tan(1/2*d*x + 1/2*
c) - 1)) - (3*a^3*tan(1/2*d*x + 1/2*c)^4 + 3*a^3*tan(1/2*d*x + 1/2*c)^3 + 7
*a^3*tan(1/2*d*x + 1/2*c)^2 + 3*a^3*tan(1/2*d*x + 1/2*c) + 3*a^3)/(tan(1/2*
d*x + 1/2*c)^2 + 1)^2)/d
```

Mupad [B]
time = 0.05, size = 36, normalized size = 0.69

$$-\frac{a^{3}\left(8\ln\left(\sin\left(c+d\,x\right)-1\right)+6\sin\left(c+d\,x\right)+{\sin\left(c+d\,x\right)}^{2}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^3/cos(c + d*x),x)
```

```
[Out] -(a^3*(8*log(sin(c + d*x) - 1) + 6*sin(c + d*x) + sin(c + d*x)^2))/(2*d)
```

3.34 $\int \sec^2(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=50

$$-3a^3x+\frac{3a^3\cos(c+dx)}{d}+\frac{2a^5\cos^3(c+dx)}{d(a-a\sin(c+dx))^2}$$

```
[Out] -3*a^3*x+3*a^3*cos(d*x+c)/d+2*a^5*cos(d*x+c)^3/d/(a-a*sin(d*x+c))^2
```

Rubi [A]
time = 0.09, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2749, 2759, 2761, 8}

$$\frac{2a^5\cos^3(c+dx)}{d(a-a\sin(c+dx))^2}+\frac{3a^3\cos(c+dx)}{d}-3a^3x$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -3*a^3*x + (3*a^3*Cos[c + d*x])/d + (2*a^5*Cos[c + d*x]^3)/(d*(a - a*Sin[c
+ d*x])^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2749

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
```

```
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int \sec^2(c+dx)(a+a\sin(c+dx))^3\,dx &= a^6\int \frac{\cos^4(c+dx)}{(a-a\sin(c+dx))^3}\,dx\\
&= \frac{2a^5\cos^3(c+dx)}{d(a-a\sin(c+dx))^2}-\left(3a^4\right)\int \frac{\cos^2(c+dx)}{a-a\sin(c+dx)}\,dx\\
&= \frac{3a^3\cos(c+dx)}{d}+\frac{2a^5\cos^3(c+dx)}{d(a-a\sin(c+dx))^2}-\left(3a^3\right)\int 1\,dx\\
&= -3a^3x+\frac{3a^3\cos(c+dx)}{d}+\frac{2a^5\cos^3(c+dx)}{d(a-a\sin(c+dx))^2}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.02, size = 55, normalized size = 1.10

$$\frac{4\sqrt{2}\,a^3\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{2};\frac{1}{2};\frac{1}{2}(1-\sin(c+dx))\right)\sec(c+dx)\sqrt{1+\sin(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
[Out] (4*Sqrt[2]*a^3*Hypergeometric2F1[-3/2, -1/2, 1/2, (1 - Sin[c + d*x])/2]*Sec
[c + d*x]*Sqrt[1 + Sin[c + d*x]])/d
```

Maple [A]
time = 0.14, size = 87, normalized size = 1.74

method	result
risch	$-3a^3x+\frac{a^3e^{i(dx+c)}}{2d}+\frac{a^3e^{-i(dx+c)}}{2d}+\frac{8a^3}{d\left(e^{i(dx+c)}-i\right)}$
derivativedivides	$\frac{a^3\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+\left(2+\sin^2(dx+c)\right)\cos(dx+c)\right)+3a^3(\tan(dx+c)-dx-c)+\frac{3a^3}{\cos(dx+c)}+a^3\tan(dx+c)}{d}$
default	$\frac{a^3\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+\left(2+\sin^2(dx+c)\right)\cos(dx+c)\right)+3a^3(\tan(dx+c)-dx-c)+\frac{3a^3}{\cos(dx+c)}+a^3\tan(dx+c)}{d}$
norman	$\frac{3a^3x-\frac{10a^3}{d}-\frac{8a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{24a^3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{22a^3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{24a^3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{8a^3\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+6a}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\left(1+ta\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d*x+c))+3*a^3*(tan(d
*x+c)-d*x-c)+3*a^3/cos(d*x+c)+a^3*tan(d*x+c))
```

Maxima [A]
time = 0.54, size = 68, normalized size = 1.36

$$-\frac{3\left(dx+c-\tan\left(dx+c\right)\right)a^3-a^3\left(\frac{1}{\cos(dx+c)}+\cos\left(dx+c\right)\right)-a^3\tan\left(dx+c\right)-\frac{3\,a^3}{\cos(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -(3*(d*x + c - tan(d*x + c))*a^3 - a^3*(1/cos(d*x + c) + cos(d*x + c)) - a^
3*tan(d*x + c) - 3*a^3/cos(d*x + c))/d
```

Fricas [A]
time = 0.37, size = 101, normalized size = 2.02

$$-\frac{3\,a^3dx-a^3\cos\left(dx+c\right)^2-4\,a^3+\left(3\,a^3dx-5\,a^3\right)\cos\left(dx+c\right)-\left(3\,a^3dx-a^3\cos\left(dx+c\right)+4\,a^3\right)\sin\left(dx+c\right)}{d\cos\left(dx+c\right)-d\sin\left(dx+c\right)+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -(3*a^3*d*x - a^3*cos(d*x + c)^2 - 4*a^3 + (3*a^3*d*x - 5*a^3)*cos(d*x + c)
 - (3*a^3*d*x - a^3*cos(d*x + c) + 4*a^3)*sin(d*x + c))/(d*cos(d*x + c) - d
*sin(d*x + c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int 3\sin\left(c+dx\right)\sec^2\left(c+dx\right)dx+\int 3\sin^2\left(c+dx\right)\sec^2\left(c+dx\right)dx+\int \sin^3\left(c+dx\right)\sec^2\left(c+dx\right)dx+\int \sec^2\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+a*sin(d*x+c))**3,x)
```

```
[Out] a**3*(Integral(3*sin(c + d*x)*sec(c + d*x)**2, x) + Integral(3*sin(c + d*x)
**2*sec(c + d*x)**2, x) + Integral(sin(c + d*x)**3*sec(c + d*x)**2, x) + In
tegral(sec(c + d*x)**2, x))
```

Giac [A]
time = 5.61, size = 91, normalized size = 1.82

$$-\frac{3\,(dx+c)a^3+\frac{2\left(4\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+5\,a^3\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -(3*(d*x + c)*a^3 + 2*(4*a^3*tan(1/2*d*x + 1/2*c)^2 - a^3*tan(1/2*d*x + 1/2
*c) + 5*a^3)/(tan(1/2*d*x + 1/2*c)^3 - tan(1/2*d*x + 1/2*c)^2 + tan(1/2*d*x
 + 1/2*c) - 1))/d
```

Mupad [B]
time = 4.78, size = 138, normalized size = 2.76

$$-3\,a^3\,x-\frac{3\,a^3\,(c+d\,x)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(3\,a^3\,(c+d\,x)-a^3\,(3\,c+3\,d\,x-2)\right)-a^3\,(3\,c+3\,d\,x-10)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(3\,a^3\,(c+d\,x)-a^3\,(3\,c+3\,d\,x-8)\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^3/cos(c + d*x)^2,x)
```

```
[Out] - 3*a^3*x - (3*a^3*(c + d*x) - tan(c/2 + (d*x)/2)*(3*a^3*(c + d*x) - a^3*(3
*c + 3*d*x - 2)) - a^3*(3*c + 3*d*x - 10) + tan(c/2 + (d*x)/2)^2*(3*a^3*(c
+ d*x) - a^3*(3*c + 3*d*x - 8)))/(d*(tan(c/2 + (d*x)/2) - 1)*(tan(c/2 + (d*
x)/2)^2 + 1))
```

3.35 $\int \sec^3(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=40

$$\frac{a^3\log(1-\sin(c+dx))}{d}+\frac{2a^4}{d(a-a\sin(c+dx))}$$

```
[Out] a^3*ln(1-sin(d*x+c))/d+2*a^4/d/(a-a*sin(d*x+c))
```

Rubi [A]

time = 0.03, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2746, 45}

$$\frac{2a^4}{d(a-a\sin(c+dx))}+\frac{a^3\log(1-\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*Log[1 - Sin[c + d*x]])/d + (2*a^4)/(d*(a - a*Sin[c + d*x]))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}
\int \sec^3(c+dx)(a+a\sin(c+dx))^3\,dx &= \frac{a^3\text{Subst}\left(\int \frac{a+x}{(a-x)^2}\,dx, x, a\sin(c+dx)\right)}{d}\\
&= \frac{a^3\text{Subst}\left(\int \left(\frac{2a}{(a-x)^2}+\frac{1}{-a+x}\right)\,dx, x, a\sin(c+dx)\right)}{d}\\
&= \frac{a^3\log(1-\sin(c+dx))}{d}+\frac{2a^4}{d(a-a\sin(c+dx))}
\end{aligned}$$

Mathematica [**A**]
time = 0.03, size = 59, normalized size = 1.48

$$\frac{a^3 \sec^2(c+dx)\left(\log(1-\sin(c+dx))+\frac{2}{1-\sin(c+dx)}\right)(1-\sin(c+dx))(1+\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*Sec[c + d*x]^2*(Log[1 - Sin[c + d*x]] + 2/(1 - Sin[c + d*x]))*(1 - Sin
[c + d*x])*(1 + Sin[c + d*x]))/d
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 123 vs. $2(40) = 80$.
time = 0.17, size = 124, normalized size = 3.10

method	result
risch	$-ia^3x-\frac{2ia^3c}{d}-\frac{4ia^3 e^{i(dx+c)}}{\left(e^{i(dx+c)}-i\right)^2 d}+\frac{2a^3\ln\left(e^{i(dx+c)}-i\right)}{d}$
derivativedivides	$\frac{a^3\left(\frac{\left(\tan^2(dx+c)\right)}{2}+\ln(\cos(dx+c))\right)+3a^3\left(\frac{\sin^3(dx+c)}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+\frac{3a^3}{2\cos(dx+c)^2}+a^3\left(\frac{\sec(dx}{}\right.}{d}$
default	$\frac{a^3\left(\frac{\left(\tan^2(dx+c)\right)}{2}+\ln(\cos(dx+c))\right)+3a^3\left(\frac{\sin^3(dx+c)}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+\frac{3a^3}{2\cos(dx+c)^2}+a^3\left(\frac{\sec(dx}{}\right.}{d}$
norman	$\frac{-\frac{8a^3}{d}+\frac{4a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{16a^3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{24a^3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{16a^3\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{4a^3\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{8a^3\left(\tan^1\right.}{}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(1/2*tan(d*x+c)^2+ln(cos(d*x+c)))+3*a^3*(1/2*sin(d*x+c)^3/cos(d*x+
c)^2+1/2*sin(d*x+c)-1/2*ln(sec(d*x+c)+tan(d*x+c)))+3/2*a^3/cos(d*x+c)^2+a^3
*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c))))
```

Maxima [**A**]
time = 0.30, size = 33, normalized size = 0.82

$$\frac{a^3\log\left(\sin\left(dx+c\right)-1\right)-\frac{2a^3}{\sin(dx+c)-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] (a^3*log(sin(d*x + c) - 1) - 2*a^3/(sin(d*x + c) - 1))/d
```

Fricas [A]
time = 0.36, size = 51, normalized size = 1.28

$$-\frac{2 a^3 - (a^3 \sin(dx+c) - a^3)\log(-\sin(dx+c)+1)}{d\sin(dx+c) - d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
[Out] -(2*a^3 - (a^3*sin(d*x + c) - a^3)*log(-sin(d*x + c) + 1))/(d*sin(d*x + c)
 - d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int 3\sin(c+dx)\sec^3(c+dx)\,dx + \int 3\sin^2(c+dx)\sec^3(c+dx)\,dx + \int \sin^3(c+dx)\sec^3(c+dx)\,dx + \int \sec^3(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+a*sin(d*x+c))**3,x)
[Out] a**3*(Integral(3*sin(c + d*x)*sec(c + d*x)**3, x) + Integral(3*sin(c + d*x)
**2*sec(c + d*x)**3, x) + Integral(sin(c + d*x)**3*sec(c + d*x)**3, x) + In
tegral(sec(c + d*x)**3, x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 92 vs. 2(41) = 82.
time = 3.14, size = 92, normalized size = 2.30

$$-\frac{a^3\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right) - 2a^3\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right) + \frac{3a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 - 10a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+3a^3}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="giac")
[Out] -(a^3*log(tan(1/2*d*x + 1/2*c)^2 + 1) - 2*a^3*log(abs(tan(1/2*d*x + 1/2*c)
 - 1)) + (3*a^3*tan(1/2*d*x + 1/2*c)^2 - 10*a^3*tan(1/2*d*x + 1/2*c) + 3*a^3
)/(tan(1/2*d*x + 1/2*c) - 1)^2)/d
```

Mupad [B]
time = 4.52, size = 35, normalized size = 0.88

$$\frac{a^3\ln(\sin(c+d\,x)-1)}{d} - \frac{2\,a^3}{d\,(\sin(c+d\,x)-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^3/cos(c + d*x)^3,x)
[Out] (a^3*log(sin(c + d*x) - 1))/d - (2*a^3)/(d*(sin(c + d*x) - 1))
```

3.36 $\int \sec^4(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=31

$$\frac{a^6\cos^3(c+dx)}{3d(a-a\sin(c+dx))^3}$$

```
[Out] 1/3*a^6*cos(d*x+c)^3/d/(a-a*sin(d*x+c))^3
```

Rubi [A]
time = 0.05, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2749, 2750}

$$\frac{a^6\cos^3(c+dx)}{3d(a-a\sin(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^6*Cos[c + d*x]^3)/(3*d*(a - a*Sin[c + d*x])^3)
```

Rule 2749

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2750

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^4(c+dx)(a+a\sin(c+dx))^3\,dx &= a^6\int \frac{\cos^2(c+dx)}{(a-a\sin(c+dx))^3}\,dx\\ &= \frac{a^6\cos^3(c+dx)}{3d(a-a\sin(c+dx))^3}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 28, normalized size = 0.90

$$\frac{a^3\sec^3(c+dx)(1+\sin(c+dx))^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*Sec[c + d*x]^3*(1 + Sin[c + d*x])^3)/(3*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(29) = 58$.
time = 0.20, size = 120, normalized size = 3.87

method	result
risch	$-\frac{2\left(3a^3 e^{2i(dx+c)}-a^3\right)}{3\left(e^{i(dx+c)}-i\right)^3 d}$
derivativedivides	$\frac{a^3\left(\frac{\sin^4(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^4(dx+c)}{3\cos(dx+c)}-\frac{\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}\right)+\frac{a^3\left(\sin^3(dx+c)\right)}{\cos(dx+c)^3}+\frac{a^3}{\cos(dx+c)^3}-a^3\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(da}{d}$
default	$\frac{a^3\left(\frac{\sin^4(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^4(dx+c)}{3\cos(dx+c)}-\frac{\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}\right)+\frac{a^3\left(\sin^3(dx+c)\right)}{\cos(dx+c)^3}+\frac{a^3}{\cos(dx+c)^3}-a^3\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(da}{d}$
norman	$\frac{-\frac{2a^3}{3d}-\frac{2a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{6a^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{38a^3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{28a^3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{92a^3\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{28a^3\left(\tan^7\right.}{d}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3\left(\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(1/3*sin(d*x+c)^4/cos(d*x+c)^3-1/3*sin(d*x+c)^4/cos(d*x+c)-1/3*(2+
sin(d*x+c)^2)*cos(d*x+c))+a^3*sin(d*x+c)^3/cos(d*x+c)^3+a^3/cos(d*x+c)^3-a^
3*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(30) = 60$.
time = 0.34, size = 78, normalized size = 2.52

$$\frac{3\,a^3\tan\left(dx+c\right)^3+\left(\tan\left(dx+c\right)^3+3\,\tan\left(dx+c\right)\right)a^3-\frac{\left(3\,\cos(dx+c)^2-1\right)a^3}{\cos(dx+c)^3}+\frac{3\,a^3}{\cos(dx+c)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/3*(3*a^3*tan(d*x + c)^3 + (tan(d*x + c)^3 + 3*tan(d*x + c))*a^3 - (3*cos(
d*x + c)^2 - 1)*a^3/cos(d*x + c)^3 + 3*a^3/cos(d*x + c)^3)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(30) = 60$.
time = 0.36, size = 99, normalized size = 3.19

$$\frac{a^3\cos\left(dx+c\right)^2-a^3\cos\left(dx+c\right)-2\,a^3-\left(a^3\cos\left(dx+c\right)+2\,a^3\right)\sin\left(dx+c\right)}{3\,\left(d\cos\left(dx+c\right)^2-d\cos\left(dx+c\right)+\left(d\cos\left(dx+c\right)+2\,d\right)\sin\left(dx+c\right)-2\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/3*(a^3*cos(d*x + c)^2 - a^3*cos(d*x + c) - 2*a^3 - (a^3*cos(d*x + c) + 2*
a^3)*sin(d*x + c))/(d*cos(d*x + c)^2 - d*cos(d*x + c) + (d*cos(d*x + c) + 2
*d)*sin(d*x + c) - 2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int 3\sin(c+dx)\sec^4(c+dx)\,dx + \int 3\sin^2(c+dx)\sec^4(c+dx)\,dx + \int \sin^3(c+dx)\sec^4(c+dx)\,dx + \int \sec^4(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+a*sin(d*x+c))**3,x)
```

```
[Out] a**3*(Integral(3*sin(c + d*x)*sec(c + d*x)**4, x) + Integral(3*sin(c + d*x)
**2*sec(c + d*x)**4, x) + Integral(sin(c + d*x)**3*sec(c + d*x)**4, x) + In
tegral(sec(c + d*x)**4, x))
```

Giac [A]
time = 4.74, size = 38, normalized size = 1.23

$$-\frac{2\left(3\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a^3\right)}{3\,d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -2/3*(3*a^3*tan(1/2*d*x + 1/2*c)^2 + a^3)/(d*(tan(1/2*d*x + 1/2*c) - 1)^3)
```

Mupad [B]
time = 4.59, size = 55, normalized size = 1.77

$$-\frac{2\,a^3\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-3\right)}{3\,d\,\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^3/cos(c + d*x)^4,x)
```

```
[Out] -(2*a^3*cos(c/2 + (d*x)/2)*(2*cos(c/2 + (d*x)/2)^2 - 3))/(3*d*(cos(c/2 + (d
*x)/2) - sin(c/2 + (d*x)/2))^3)
```

3.37 $\int \sec^5(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=23

$$\frac{a^5}{2d(a-a\sin(c+dx))^2}$$

```
[Out] 1/2*a^5/d/(a-a*sin(d*x+c))^2
```

Rubi [A]
time = 0.03, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2746, 32}

$$\frac{a^5}{2d(a-a\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] a^5/(2*d*(a - a*Sin[c + d*x])^2)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \sec^5(c+dx)(a+a\sin(c+dx))^3\,dx &= \frac{a^5\text{Subst}\left(\int \frac{1}{(a-x)^3}\,dx, x, a\sin(c+dx)\right)}{d}\\ &= \frac{a^5}{2d(a-a\sin(c+dx))^2}\end{aligned}$$

Mathematica [A]
time = 0.15, size = 35, normalized size = 1.52

$$\frac{a^3}{2d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] a^3/(2*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(21) = 42$.
time = 0.20, size = 154, normalized size = 6.70

method	result
risch	$-\frac{2a^3 e^{2i(dx+c)}}{\left(e^{i(dx+c)}-i\right)^4 d}$
derivativedivides	$\frac{\frac{a^3\left(\sin^4(dx+c)\right)}{4\cos(dx+c)^4}+3a^3\left(\frac{\sin^3(dx+c)}{4\cos(dx+c)^4}+\frac{\sin^3(dx+c)}{8\cos(dx+c)^2}+\frac{\sin(dx+c)}{8}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+\frac{3a^3}{4\cos(dx+c)^4}+a^3\left(-\left(-\frac{(\sec^3(\cdots}{}\right.\right.}{d}$
default	$\frac{\frac{a^3\left(\sin^4(dx+c)\right)}{4\cos(dx+c)^4}+3a^3\left(\frac{\sin^3(dx+c)}{4\cos(dx+c)^4}+\frac{\sin^3(dx+c)}{8\cos(dx+c)^2}+\frac{\sin(dx+c)}{8}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+\frac{3a^3}{4\cos(dx+c)^4}+a^3\left(-\left(-\frac{(\sec^3(\cdots}{}\right.\right.}{d}$
norman	$\frac{\frac{6a^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{6a^3\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{22a^3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{22a^3\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{12a^3\left(\tan^3\left(\frac{dx}{2}+\cdots\right.\right.}{d}}{(\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/4*a^3*sin(d*x+c)^4/cos(d*x+c)^4+3*a^3*(1/4*sin(d*x+c)^3/cos(d*x+c)^4
+1/8*sin(d*x+c)^3/cos(d*x+c)^2+1/8*sin(d*x+c)-1/8*ln(sec(d*x+c)+tan(d*x+c))
)+3/4*a^3/cos(d*x+c)^4+a^3*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+
3/8*ln(sec(d*x+c)+tan(d*x+c))))
```

Maxima [A]
time = 0.31, size = 28, normalized size = 1.22

$$\frac{a^3}{2\left(\sin\left(dx+c\right)^2-2\sin\left(dx+c\right)+1\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*a^3/((sin(d*x + c)^2 - 2*sin(d*x + c) + 1)*d)
```

Fricas [A]
time = 0.34, size = 30, normalized size = 1.30

$$-\frac{a^3}{2\left(d\cos\left(dx+c\right)^2+2\,d\sin\left(dx+c\right)-2\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/2*a^3/(d*cos(d*x + c)^2 + 2*d*sin(d*x + c) - 2*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(22) = 44$.
time = 4.61, size = 63, normalized size = 2.74

$$\frac{2\left(a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 2*(a^3*tan(1/2*d*x + 1/2*c)^3 - a^3*tan(1/2*d*x + 1/2*c)^2 + a^3*tan(1/2*d*
x + 1/2*c))/(d*(tan(1/2*d*x + 1/2*c) - 1)^4)
```

Mupad [B]
time = 0.07, size = 18, normalized size = 0.78

$$\frac{a^3}{2\,d\,\left(\sin\left(c+d\,x\right)-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^3/cos(c + d*x)^5,x)
```

```
[Out] a^3/(2*d*(sin(c + d*x) - 1)^2)
```

3.38 $\int \sec^6(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=92

$$\frac{a^6\cos(c+dx)}{5d(a-a\sin(c+dx))^3}+\frac{2a^5\cos(c+dx)}{15d(a-a\sin(c+dx))^2}+\frac{2a^6\cos(c+dx)}{15d\,(a^3-a^3\sin(c+dx))}$$

```
[Out] 1/5*a^6*cos(d*x+c)/d/(a-a*sin(d*x+c))^3+2/15*a^5*cos(d*x+c)/d/(a-a*sin(d*x+
c))^2+2/15*a^6*cos(d*x+c)/d/(a^3-a^3*sin(d*x+c))
```

Rubi [A]
time = 0.06, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2749, 2729, 2727}

$$\frac{a^6\cos(c+dx)}{5d(a-a\sin(c+dx))^3}+\frac{2a^5\cos(c+dx)}{15d(a-a\sin(c+dx))^2}+\frac{2a^6\cos(c+dx)}{15d\,(a^3-a^3\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^6*Cos[c + d*x])/(5*d*(a - a*Sin[c + d*x])^3) + (2*a^5*Cos[c + d*x])/(15*
d*(a - a*Sin[c + d*x])^2) + (2*a^6*Cos[c + d*x])/(15*d*(a^3 - a^3*Sin[c + d
*x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2749

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rubi steps

$$\int \sec^6(c+dx)(a+a\sin(c+dx))^3\,dx = a^6\int \frac{1}{(a-a\sin(c+dx))^3}\,dx$$
$$= \frac{a^6\cos(c+dx)}{5d(a-a\sin(c+dx))^3} + \frac{1}{5}\left(2a^5\right)\int \frac{1}{(a-a\sin(c+dx))^2}\,dx$$
$$= \frac{a^6\cos(c+dx)}{5d(a-a\sin(c+dx))^3} + \frac{2a^5\cos(c+dx)}{15d(a-a\sin(c+dx))^2} + \frac{1}{15}\left(2a^4\right)\int \frac{\quad}{a-}$$
$$= \frac{a^6\cos(c+dx)}{5d(a-a\sin(c+dx))^3} + \frac{2a^5\cos(c+dx)}{15d(a-a\sin(c+dx))^2} + \frac{2a^4\cos(c+}{15d(a-a\sin(c}$$

Mathematica [A]
time = 0.01, size = 110, normalized size = 1.20

$$\frac{7a^3\sec^5(c+dx)}{15d} + \frac{a^3\sec^4(c+dx)\tan(c+dx)}{d} + \frac{a^3\sec^3(c+dx)\tan^2(c+dx)}{3d} - \frac{a^3\sec^2(c+dx)\tan^3(c+dx)}{3d} + \frac{2a^3\tan^5(c+dx)}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (7*a^3*Sec[c + d*x]^5)/(15*d) + (a^3*Sec[c + d*x]^4*Tan[c + d*x])/d + (a^3*
Sec[c + d*x]^3*Tan[c + d*x]^2)/(3*d) - (a^3*Sec[c + d*x]^2*Tan[c + d*x]^3)/
(3*d) + (2*a^3*Tan[c + d*x]^5)/(15*d)
```

Maple [A]
time = 0.20, size = 171, normalized size = 1.86

method	result
risch	$-\frac{4\left(-a^3-5 i a^3 e^{i(dx+c)}+10 a^3 e^{2 i(dx+c)}\right)}{15\left(e^{i(dx+c)}-i\right)^5 d}$
derivativedivides	$\frac{a^3\left(\frac{\sin^4(dx+c)}{5\cos(dx+c)^5}+\frac{\sin^4(dx+c)}{15\cos(dx+c)^3}-\frac{\sin^4(dx+c)}{15\cos(dx+c)}-\frac{\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{15}\right)+3a^3\left(\frac{\sin^3(dx+c)}{5\cos(dx+c)^5}+\frac{2\left(\sin^3(dx+c)\right)}{15\cos(dx+c)^3}\right)+\frac{3a^3}{5\cos(dx}}{d}$
default	$\frac{a^3\left(\frac{\sin^4(dx+c)}{5\cos(dx+c)^5}+\frac{\sin^4(dx+c)}{15\cos(dx+c)^3}-\frac{\sin^4(dx+c)}{15\cos(dx+c)}-\frac{\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{15}\right)+3a^3\left(\frac{\sin^3(dx+c)}{5\cos(dx+c)^5}+\frac{2\left(\sin^3(dx+c)\right)}{15\cos(dx+c)^3}\right)+\frac{3a^3}{5\cos(dx}}{d}$
norman	$-\frac{14a^3}{15d}-\frac{2a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{34a^3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{494a^3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15d}-\frac{842a^3\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15d}-\frac{178a^3\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{842a^3}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(1/5*sin(d*x+c)^4/cos(d*x+c)^5+1/15*sin(d*x+c)^4/cos(d*x+c)^3-1/15
*sin(d*x+c)^4/cos(d*x+c)-1/15*(2+sin(d*x+c)^2)*cos(d*x+c))+3*a^3*(1/5*sin(d
```

```
*x+c)^3/cos(d*x+c)^5+2/15*sin(d*x+c)^3/cos(d*x+c)^3)+3/5*a^3/cos(d*x+c)^5-a
^3*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c))
```

Maxima [A]
time = 0.42, size = 103, normalized size = 1.12

$$\frac{\left(3\,\tan\left(dx+c\right)^{5}+10\,\tan\left(dx+c\right)^{3}+15\,\tan\left(dx+c\right)\right)a^{3}+3\left(3\,\tan\left(dx+c\right)^{5}+5\,\tan\left(dx+c\right)^{3}\right)a^{3}-\frac{\left(5\,\cos\left(dx+c\right)^{2}-3\right)a^{3}}{\cos\left(dx+c\right)^{5}}+\frac{9\,a^{3}}{\cos\left(dx+c\right)^{5}}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/15*((3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a^3 + 3*(3*t
an(d*x + c)^5 + 5*tan(d*x + c)^3)*a^3 - (5*cos(d*x + c)^2 - 3)*a^3/cos(d*x
+ c)^5 + 9*a^3/cos(d*x + c)^5)/d
```

Fricas [A]
time = 0.36, size = 149, normalized size = 1.62

$$\frac{2\,a^{3}\cos\left(dx+c\right)^{3}-4\,a^{3}\cos\left(dx+c\right)^{2}-9\,a^{3}\cos\left(dx+c\right)-3\,a^{3}+\left(2\,a^{3}\cos\left(dx+c\right)^{2}+6\,a^{3}\cos\left(dx+c\right)-3\,a^{3}\right)\sin\left(dx+c\right)}{15\left(d\cos\left(dx+c\right)^{3}+3\,d\cos\left(dx+c\right)^{2}-2\,d\cos\left(dx+c\right)-\left(d\cos\left(dx+c\right)^{2}-2\,d\cos\left(dx+c\right)-4\,d\right)\sin\left(dx+c\right)-4\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/15*(2*a^3*cos(d*x + c)^3 - 4*a^3*cos(d*x + c)^2 - 9*a^3*cos(d*x + c) - 3*
a^3 + (2*a^3*cos(d*x + c)^2 + 6*a^3*cos(d*x + c) - 3*a^3)*sin(d*x + c))/(d*
cos(d*x + c)^3 + 3*d*cos(d*x + c)^2 - 2*d*cos(d*x + c) - (d*cos(d*x + c)^2
- 2*d*cos(d*x + c) - 4*d)*sin(d*x + c) - 4*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [A]
time = 7.26, size = 86, normalized size = 0.93

$$-\frac{2\left(15\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-30\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+40\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-20\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+7\,a^{3}\right)}{15\,d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -2/15*(15*a^3*tan(1/2*d*x + 1/2*c)^4 - 30*a^3*tan(1/2*d*x + 1/2*c)^3 + 40*a
^3*tan(1/2*d*x + 1/2*c)^2 - 20*a^3*tan(1/2*d*x + 1/2*c) + 7*a^3)/(d*(tan(1/
2*d*x + 1/2*c) - 1)^5)
```

Mupad [B]
time = 4.69, size = 135, normalized size = 1.47

$$\frac{2\,a^3\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(7\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-20\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+40\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-30\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+15\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\right)}{15\,d\,\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^3/cos(c + d*x)^6,x)
```

```
[Out] (2*a^3*cos(c/2 + (d*x)/2)*(7*cos(c/2 + (d*x)/2)^4 + 15*sin(c/2 + (d*x)/2)^4
 - 30*cos(c/2 + (d*x)/2)*sin(c/2 + (d*x)/2)^3 - 20*cos(c/2 + (d*x)/2)^3*sin
(c/2 + (d*x)/2) + 40*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^2))/(15*d*(cos
(c/2 + (d*x)/2) - sin(c/2 + (d*x)/2))^5)
```

3.39 $\int \sec^7(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=87

$$\frac{a^3\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^6}{6d(a-a\sin(c+dx))^3}+\frac{a^5}{8d(a-a\sin(c+dx))^2}+\frac{a^4}{8d(a-a\sin(c+dx))}$$

```
[Out] 1/8*a^3*arctanh(sin(d*x+c))/d+1/6*a^6/d/(a-a*sin(d*x+c))^3+1/8*a^5/d/(a-a*s
in(d*x+c))^2+1/8*a^4/d/(a-a*sin(d*x+c))
```

Rubi [A]
time = 0.05, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2746, 46, 212}

$$\frac{a^6}{6d(a-a\sin(c+dx))^3}+\frac{a^5}{8d(a-a\sin(c+dx))^2}+\frac{a^4}{8d(a-a\sin(c+dx))}+\frac{a^3\tanh^{-1}(\sin(c+dx))}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^7*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*ArcTanh[Sin[c + d*x]])/(8*d) + a^6/(6*d*(a - a*Sin[c + d*x])^3) + a^5/
(8*d*(a - a*Sin[c + d*x])^2) + a^4/(8*d*(a - a*Sin[c + d*x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\int \sec^7(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{a^7\text{Subst}\left(\int \frac{1}{(a-x)^4(a+x)}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^7\text{Subst}\left(\int \left(\frac{1}{2a(a-x)^4} + \frac{1}{4a^2(a-x)^3} + \frac{1}{8a^3(a-x)^2} + \frac{1}{8a^3(a^2-x^2)}\right)\,dx, x, a\sin\right.}{d}$$
$$= \frac{a^6}{6d(a-a\sin(c+dx))^3} + \frac{a^5}{8d(a-a\sin(c+dx))^2} + \frac{a^4}{8d(a-a\sin(c+}$$
$$= \frac{a^3\tanh^{-1}(\sin(c+dx))}{8d} + \frac{a^6}{6d(a-a\sin(c+dx))^3} + \frac{a^5}{8d(a-a\sin(c+}$$

Mathematica [A]
time = 0.07, size = 67, normalized size = 0.77

$$-\frac{a^3\sec^6(c+dx)(1+\sin(c+dx))^3\left(-10+3\tanh^{-1}(\sin(c+dx))(-1+\sin(c+dx))^3+9\sin(c+dx)-3\sin^2(c+dx)\right)}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^7*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/24*(a^3*Sec[c + d*x]^6*(1 + Sin[c + d*x])^3*(-10 + 3*ArcTanh[Sin[c + d*x
]]*(-1 + Sin[c + d*x])^3 + 9*Sin[c + d*x] - 3*Sin[c + d*x]^2))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(79) = 158$.
time = 0.18, size = 202, normalized size = 2.32

method	result
risch	$-\frac{ia^3\left(-18i\,e^{4i(dx+c)}+3\,e^{5i(dx+c)}+18i\,e^{2i(dx+c)}-46\,e^{3i(dx+c)}+3\,e^{i(dx+c)}\right)}{12d\left(e^{i(dx+c)}-i\right)^6} - \frac{a^3\ln\left(e^{i(dx+c)}-i\right)}{8d} + \frac{a^3\ln\left(e^{i(dx+c)}+i\right)}{8d}$
derivativedivides	$\frac{a^3\left(\frac{\sin^4(dx+c)}{6\cos(dx+c)^6}+\frac{\sin^4(dx+c)}{12\cos(dx+c)^4}\right)+3a^3\left(\frac{\sin^3(dx+c)}{6\cos(dx+c)^6}+\frac{\sin^3(dx+c)}{8\cos(dx+c)^4}+\frac{\sin^3(dx+c)}{16\cos(dx+c)^2}+\frac{\sin(dx+c)}{16}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{16}\right)}{d}$
default	$\frac{a^3\left(\frac{\sin^4(dx+c)}{6\cos(dx+c)^6}+\frac{\sin^4(dx+c)}{12\cos(dx+c)^4}\right)+3a^3\left(\frac{\sin^3(dx+c)}{6\cos(dx+c)^6}+\frac{\sin^3(dx+c)}{8\cos(dx+c)^4}+\frac{\sin^3(dx+c)}{16\cos(dx+c)^2}+\frac{\sin(dx+c)}{16}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{16}\right)}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(1/6*sin(d*x+c)^4/cos(d*x+c)^6+1/12*sin(d*x+c)^4/cos(d*x+c)^4)+3*a
^3*(1/6*sin(d*x+c)^3/cos(d*x+c)^6+1/8*sin(d*x+c)^3/cos(d*x+c)^4+1/16*sin(d*
x+c)^3/cos(d*x+c)^2+1/16*sin(d*x+c)-1/16*ln(sec(d*x+c)+tan(d*x+c)))+1/2*a^3
```

```
/cos(d*x+c)^6+a^3*(-(-1/6*sec(d*x+c)^5-5/24*sec(d*x+c)^3-5/16*sec(d*x+c))*t
an(d*x+c)+5/16*ln(sec(d*x+c)+tan(d*x+c))))
```

Maxima [A]
time = 0.32, size = 96, normalized size = 1.10

$$\frac{3\,a^3\log\left(\sin\left(dx+c\right)+1\right)-3\,a^3\log\left(\sin\left(dx+c\right)-1\right)-\frac{2\left(3\,a^3\sin\left(dx+c\right)^2-9\,a^3\sin\left(dx+c\right)+10\,a^3\right)}{\sin\left(dx+c\right)^3-3\,\sin\left(dx+c\right)^2+3\,\sin\left(dx+c\right)-1}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/48*(3*a^3*log(sin(d*x + c) + 1) - 3*a^3*log(sin(d*x + c) - 1) - 2*(3*a^3*
sin(d*x + c)^2 - 9*a^3*sin(d*x + c) + 10*a^3)/(sin(d*x + c)^3 - 3*sin(d*x +
 c)^2 + 3*sin(d*x + c) - 1))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 185 vs. 2(82) = 164.
time = 0.37, size = 185, normalized size = 2.13

$$\frac{6\,a^3\cos\left(dx+c\right)^2+18\,a^3\sin\left(dx+c\right)-26\,a^3+3\left(3\,a^3\cos\left(dx+c\right)^2-4\,a^3-\left(a^3\cos\left(dx+c\right)^2-4\,a^3\right)\sin\left(dx+c\right)\right)\log\left(\sin\left(dx+c\right)+1\right)-3\left(3\,a^3\cos\left(dx+c\right)^2-4\,a^3-\left(a^3\cos\left(dx+c\right)^2-4\,a^3\right)\sin\left(dx+c\right)\right)\log\left(-\sin\left(dx+c\right)+1\right)}{48\left(3\,d\cos\left(dx+c\right)^2-\left(d\cos\left(dx+c\right)^2-4\,d\right)\sin\left(dx+c\right)-4\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/48*(6*a^3*cos(d*x + c)^2 + 18*a^3*sin(d*x + c) - 26*a^3 + 3*(3*a^3*cos(d*
x + c)^2 - 4*a^3 - (a^3*cos(d*x + c)^2 - 4*a^3)*sin(d*x + c))*log(sin(d*x +
 c) + 1) - 3*(3*a^3*cos(d*x + c)^2 - 4*a^3 - (a^3*cos(d*x + c)^2 - 4*a^3)*s
in(d*x + c))*log(-sin(d*x + c) + 1))/(3*d*cos(d*x + c)^2 - (d*cos(d*x + c)^
2 - 4*d)*sin(d*x + c) - 4*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep
```

Giac [A]
time = 6.33, size = 90, normalized size = 1.03

$$\frac{6\,a^3\log\left(\left|\sin\left(dx+c\right)+1\right|\right)-6\,a^3\log\left(\left|\sin\left(dx+c\right)-1\right|\right)+\frac{11\,a^3\sin\left(dx+c\right)^3-45\,a^3\sin\left(dx+c\right)^2+69\,a^3\sin\left(dx+c\right)-51\,a^3}{\left(\sin\left(dx+c\right)-1\right)^3}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/96*(6*a^3*log(abs(sin(d*x + c) + 1)) - 6*a^3*log(abs(sin(d*x + c) - 1)) +
 (11*a^3*sin(d*x + c)^3 - 45*a^3*sin(d*x + c)^2 + 69*a^3*sin(d*x + c) - 51*
a^3)/(sin(d*x + c) - 1)^3)/d
```

Mupad [B]
time = 4.54, size = 81, normalized size = 0.93

$$\frac{a^3\,\mathrm{atanh}(\sin\left(c+d\,x\right))}{8\,d}-\frac{\frac{a^3\,\sin(c+d\,x)^2}{8}-\frac{3\,a^3\,\sin(c+d\,x)}{8}+\frac{5\,a^3}{12}}{d\,\left(\sin\left(c+d\,x\right)^3-3\sin\left(c+d\,x\right)^2+3\,\sin\left(c+d\,x\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^3/cos(c + d*x)^7,x)
```

```
[Out] (a^3*atanh(sin(c + d*x)))/(8*d) - ((5*a^3)/12 - (3*a^3*sin(c + d*x))/8 + (a
^3*sin(c + d*x)^2)/8)/(d*(3*sin(c + d*x) - 3*sin(c + d*x)^2 + sin(c + d*x)^
3 - 1))
```

3.40 $\int \sec^8(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=99

$$\frac{3a^3\sec^5(c+dx)}{35d}+\frac{2a\sec^7(c+dx)(a+a\sin(c+dx))^2}{7d}+\frac{3a^3\tan(c+dx)}{7d}+\frac{2a^3\tan^3(c+dx)}{7d}+\frac{3a^3\tan^5(c+dx)}{35d}$$

[Out] 3/35*a^3*sec(d*x+c)^5/d+2/7*a*sec(d*x+c)^7*(a+a*sin(d*x+c))^2/d+3/7*a^3*tan(d*x+c)/d+2/7*a^3*tan(d*x+c)^3/d+3/35*a^3*tan(d*x+c)^5/d

Rubi [A]
time = 0.06, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2755, 2748, 3852}

$$\frac{3a^3\tan^5(c+dx)}{35d}+\frac{2a^3\tan^3(c+dx)}{7d}+\frac{3a^3\tan(c+dx)}{7d}+\frac{3a^3\sec^5(c+dx)}{35d}+\frac{2a\sec^7(c+dx)(a\sin(c+dx)+a)^2}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^8*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (3*a^3*Sec[c + d*x]^5)/(35*d) + (2*a*Sec[c + d*x]^7*(a + a*Sin[c + d*x])^2)
/(7*d) + (3*a^3*Tan[c + d*x])/(7*d) + (2*a^3*Tan[c + d*x]^3)/(7*d) + (3*a^3
*Tan[c + d*x]^5)/(35*d)
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2755

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[-2*b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(p + 1))), x] + Dist[b^2*((2*m + p - 1)/(g^2*(p + 1))),
Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[
{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && Inte
gersQ[2*m, 2*p]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \sec^8(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{2a\sec^7(c+dx)(a+a\sin(c+dx))^2}{7d} + \frac{1}{7}(3a^2)\int \sec^6(c+dx)(a+a$$
$$= \frac{3a^3\sec^5(c+dx)}{35d} + \frac{2a\sec^7(c+dx)(a+a\sin(c+dx))^2}{7d} + \frac{1}{7}(3a^3)\int$$
$$= \frac{3a^3\sec^5(c+dx)}{35d} + \frac{2a\sec^7(c+dx)(a+a\sin(c+dx))^2}{7d} - \frac{(3a^3)\,\text{Subs}}{}$$
$$= \frac{3a^3\sec^5(c+dx)}{35d} + \frac{2a\sec^7(c+dx)(a+a\sin(c+dx))^2}{7d} + \frac{3a^3\tan(c}{7d}$$

Mathematica [A]
time = 0.01, size = 134, normalized size = 1.35

$$\frac{13a^3\sec^7(c+dx)}{35d} + \frac{a^3\sec^6(c+dx)\tan(c+dx)}{d} + \frac{a^3\sec^5(c+dx)\tan^2(c+dx)}{5d} - \frac{a^3\sec^4(c+dx)\tan^3(c+dx)}{d} + \frac{4a^3\sec^2(c+dx)\tan^5(c+dx)}{5d} - \frac{8a^3\tan^7(c+dx)}{35d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^8*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (13*a^3*Sec[c + d*x]^7)/(35*d) + (a^3*Sec[c + d*x]^6*Tan[c + d*x])/d + (a^3
*Sec[c + d*x]^5*Tan[c + d*x]^2)/(5*d) - (a^3*Sec[c + d*x]^4*Tan[c + d*x]^3)
/d + (4*a^3*Sec[c + d*x]^2*Tan[c + d*x]^5)/(5*d) - (8*a^3*Tan[c + d*x]^7)/(
35*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(89) = 178$.
time = 0.19, size = 217, normalized size = 2.19

method	result
risch	$-\frac{16\left(-6a^3 e^{i(dx+c)}+ia^3+14a^3 e^{3i(dx+c)}-14ia^3 e^{2i(dx+c)}\right)}{35\left(e^{i(dx+c)}-i\right)^7\left(e^{i(dx+c)}+i\right)d}$
derivativedivides	$\frac{a^3\left(\frac{\sin^4(dx+c)}{7\cos(dx+c)^7}+\frac{3\left(\sin^4(dx+c)\right)}{35\cos(dx+c)^5}+\frac{\sin^4(dx+c)}{35\cos(dx+c)^3}-\frac{\sin^4(dx+c)}{35\cos(dx+c)}-\frac{\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{35}\right)+3a^3\left(\frac{\sin^3(dx+c)}{7\cos(dx+c)^7}+\frac{4\left(\sin^3(dx\right.}{35\cos(dx+}\right.}{d}$
default	$\frac{a^3\left(\frac{\sin^4(dx+c)}{7\cos(dx+c)^7}+\frac{3\left(\sin^4(dx+c)\right)}{35\cos(dx+c)^5}+\frac{\sin^4(dx+c)}{35\cos(dx+c)^3}-\frac{\sin^4(dx+c)}{35\cos(dx+c)}-\frac{\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{35}\right)+3a^3\left(\frac{\sin^3(dx+c)}{7\cos(dx+c)^7}+\frac{4\left(\sin^3(dx\right.}{35\cos(dx+}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^8*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(1/7*sin(d*x+c)^4/cos(d*x+c)^7+3/35*sin(d*x+c)^4/cos(d*x+c)^5+1/35
*sin(d*x+c)^4/cos(d*x+c)^3-1/35*sin(d*x+c)^4/cos(d*x+c)-1/35*(2+sin(d*x+c)^
```

```
2)*cos(d*x+c))+3*a^3*(1/7*sin(d*x+c)^3/cos(d*x+c)^7+4/35*sin(d*x+c)^3/cos(d
*x+c)^5+8/105*sin(d*x+c)^3/cos(d*x+c)^3)+3/7*a^3/cos(d*x+c)^7-a^3*(-16/35-1
/7*sec(d*x+c)^6-6/35*sec(d*x+c)^4-8/35*sec(d*x+c)^2)*tan(d*x+c))
```

Maxima [A]
time = 0.31, size = 122, normalized size = 1.23

$$\frac{\left(15\,\tan\left(dx+c\right)^{7}+42\,\tan\left(dx+c\right)^{5}+35\,\tan\left(dx+c\right)^{3}\right)a^{3}+\left(5\,\tan\left(dx+c\right)^{7}+21\,\tan\left(dx+c\right)^{5}+35\,\tan\left(dx+c\right)^{3}+35\,\tan\left(dx+c\right)\right)a^{3}-\frac{\left(7\,\cos(dx+c)^{2}-5\right)a^{3}}{\cos(dx+c)^{7}}+\frac{15\,a^{3}}{\cos(dx+c)^{7}}}{35\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/35*((15*tan(d*x + c)^7 + 42*tan(d*x + c)^5 + 35*tan(d*x + c)^3)*a^3 + (5*
tan(d*x + c)^7 + 21*tan(d*x + c)^5 + 35*tan(d*x + c)^3 + 35*tan(d*x + c))*a
^3 - (7*cos(d*x + c)^2 - 5)*a^3/cos(d*x + c)^7 + 15*a^3/cos(d*x + c)^7)/d
```

Fricas [A]
time = 0.38, size = 112, normalized size = 1.13

$$\frac{8\,a^{3}\cos\left(dx+c\right)^{4}-36\,a^{3}\cos\left(dx+c\right)^{2}+15\,a^{3}+4\left(6\,a^{3}\cos\left(dx+c\right)^{2}-5\,a^{3}\right)\sin\left(dx+c\right)}{35\left(3\,d\cos\left(dx+c\right)^{3}-4\,d\cos\left(dx+c\right)-\left(d\cos\left(dx+c\right)^{3}-4\,d\cos\left(dx+c\right)\right)\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/35*(8*a^3*cos(d*x + c)^4 - 36*a^3*cos(d*x + c)^2 + 15*a^3 + 4*(6*a^3*cos(
d*x + c)^2 - 5*a^3)*sin(d*x + c))/(3*d*cos(d*x + c)^3 - 4*d*cos(d*x + c) -
(d*cos(d*x + c)^3 - 4*d*cos(d*x + c))*sin(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**8*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

Giac [A]
time = 4.50, size = 138, normalized size = 1.39

$$-\frac{\frac{35\,a^{3}}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1}+\frac{525\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-1960\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+4025\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-4480\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+3143\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1176\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+243\,a^{3}}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^{7}}}{280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/280*(35*a^3/(tan(1/2*d*x + 1/2*c) + 1) + (525*a^3*tan(1/2*d*x + 1/2*c)^6
 - 1960*a^3*tan(1/2*d*x + 1/2*c)^5 + 4025*a^3*tan(1/2*d*x + 1/2*c)^4 - 4480
*a^3*tan(1/2*d*x + 1/2*c)^3 + 3143*a^3*tan(1/2*d*x + 1/2*c)^2 - 1176*a^3*ta
n(1/2*d*x + 1/2*c) + 243*a^3)/(tan(1/2*d*x + 1/2*c) - 1)^7)/d
```

Mupad [B]
time = 4.86, size = 228, normalized size = 2.30

$$\frac{2\,a^3\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(13\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7-43\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+77\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-7\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-105\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+175\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5-105\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+35\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\right)}{35\,d\,\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)^7\,\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^3/cos(c + d*x)^8,x)
```

```
[Out] (2*a^3*cos(c/2 + (d*x)/2)*(13*cos(c/2 + (d*x)/2)^7 + 35*sin(c/2 + (d*x)/2)^
7 - 105*cos(c/2 + (d*x)/2)*sin(c/2 + (d*x)/2)^6 - 43*cos(c/2 + (d*x)/2)^6*s
in(c/2 + (d*x)/2) + 175*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^5 - 105*cos
(c/2 + (d*x)/2)^3*sin(c/2 + (d*x)/2)^4 - 7*cos(c/2 + (d*x)/2)^4*sin(c/2 + (
d*x)/2)^3 + 77*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^2))/(35*d*(cos(c/2 +
 (d*x)/2) - sin(c/2 + (d*x)/2))^7*(cos(c/2 + (d*x)/2) + sin(c/2 + (d*x)/2))
)
```

3.41 $\int \cos^5(c+dx)(a+a\sin(c+dx))^8\,dx$

Optimal. Leaf size=67

$$\frac{4(a+a\sin(c+dx))^{11}}{11a^3d}-\frac{(a+a\sin(c+dx))^{12}}{3a^4d}+\frac{(a+a\sin(c+dx))^{13}}{13a^5d}$$

```
[Out] 4/11*(a+a*sin(d*x+c))^11/a^3/d-1/3*(a+a*sin(d*x+c))^12/a^4/d+1/13*(a+a*sin(
d*x+c))^13/a^5/d
```

Rubi [A]
time = 0.06, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2746, 45}

$$\frac{(a\sin(c+dx)+a)^{13}}{13a^5d}-\frac{(a\sin(c+dx)+a)^{12}}{3a^4d}+\frac{4(a\sin(c+dx)+a)^{11}}{11a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + a*Sin[c + d*x])^8,x]
```

```
[Out] (4*(a + a*Sin[c + d*x])^11)/(11*a^3*d) - (a + a*Sin[c + d*x])^12/(3*a^4*d)
+ (a + a*Sin[c + d*x])^13/(13*a^5*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \cos^5(c+dx)(a+a\sin(c+dx))^8\,dx &= \frac{\text{Subst}\left(\int (a-x)^2(a+x)^{10}\,dx,x,a\sin(c+dx)\right)}{a^5d}\\ &= \frac{\text{Subst}\left(\int \left(4a^2(a+x)^{10}-4a(a+x)^{11}+(a+x)^{12}\right)\,dx,x,a\sin(c+dx)\right)}{a^5d}\\ &= \frac{4(a+a\sin(c+dx))^{11}}{11a^3d}-\frac{(a+a\sin(c+dx))^{12}}{3a^4d}+\frac{(a+a\sin(c+dx))^{13}}{13a^5d}\end{aligned}$$

Mathematica [A]
time = 0.27, size = 58, normalized size = 0.87

$$-\frac{a^8 \cos^6(c+dx)(1+\sin(c+dx))^8 \left(46-77 \sin(c+dx)+33 \sin^2(c+dx)\right)}{429 d(-1+\sin(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + a*Sin[c + d*x])^8,x]
```

```
[Out] -1/429*(a^8*Cos[c + d*x]^6*(1 + Sin[c + d*x])^8*(46 - 77*Sin[c + d*x] + 33*
Sin[c + d*x]^2))/(d*(-1 + Sin[c + d*x])^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 512 vs. $2(61) = 122$.
time = 1.24, size = 513, normalized size = 7.66 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+a*sin(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^8*(-1/13*sin(d*x+c)^7*cos(d*x+c)^6-7/143*sin(d*x+c)^5*cos(d*x+c)^6-3
5/1287*sin(d*x+c)^3*cos(d*x+c)^6-5/429*sin(d*x+c)*cos(d*x+c)^6+1/429*(8/3+c
os(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))+8*a^8*(-1/12*sin(d*x+c)^6*cos(d*x
+c)^6-1/20*sin(d*x+c)^4*cos(d*x+c)^6-1/40*sin(d*x+c)^2*cos(d*x+c)^6-1/120*c
os(d*x+c)^6)+28*a^8*(-1/11*sin(d*x+c)^5*cos(d*x+c)^6-5/99*sin(d*x+c)^3*cos(
d*x+c)^6-5/231*sin(d*x+c)*cos(d*x+c)^6+1/231*(8/3+cos(d*x+c)^4+4/3*cos(d*x+
c)^2)*sin(d*x+c))+56*a^8*(-1/10*sin(d*x+c)^4*cos(d*x+c)^6-1/20*sin(d*x+c)^2
*cos(d*x+c)^6-1/60*cos(d*x+c)^6)+70*a^8*(-1/9*sin(d*x+c)^3*cos(d*x+c)^6-1/2
1*sin(d*x+c)*cos(d*x+c)^6+1/105*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x
+c))+56*a^8*(-1/8*sin(d*x+c)^2*cos(d*x+c)^6-1/24*cos(d*x+c)^6)+28*a^8*(-1/7
*sin(d*x+c)*cos(d*x+c)^6+1/35*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c
))-4/3*cos(d*x+c)^6*a^8+1/5*a^8*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x
+c))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(61) = 122$.
time = 0.31, size = 173, normalized size = 2.58

$$\frac{33 a^8 \sin(dx+c)^{13} + 286 a^8 \sin(dx+c)^{12} + 1014 a^8 \sin(dx+c)^{11} + 1716 a^8 \sin(dx+c)^{10} + 715 a^8 \sin(dx+c)^9 - 2574 a^8 \sin(dx+c)^8 - 5148 a^8 \sin(dx+c)^7 - 3432 a^8 \sin(dx+c)^6 + 1287 a^8 \sin(dx+c)^5 + 4290 a^8 \sin(dx+c)^4 + 3718 a^8 \sin(dx+c)^3 + 1716 a^8 \sin(dx+c)^2 + 429 a^8 \sin(dx+c)}{429 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 1/429*(33*a^8*sin(d*x + c)^13 + 286*a^8*sin(d*x + c)^12 + 1014*a^8*sin(d*x
+ c)^11 + 1716*a^8*sin(d*x + c)^10 + 715*a^8*sin(d*x + c)^9 - 2574*a^8*sin(
d*x + c)^8 - 5148*a^8*sin(d*x + c)^7 - 3432*a^8*sin(d*x + c)^6 + 1287*a^8*s
in(d*x + c)^5 + 4290*a^8*sin(d*x + c)^4 + 3718*a^8*sin(d*x + c)^3 + 1716*a^
8*sin(d*x + c)^2 + 429*a^8*sin(d*x + c))/d
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 149 vs. $2(61) = 122$.
time = 0.40, size = 149, normalized size = 2.22

$$\frac{286\,a^{8}\cos\left(dx+c\right)^{12}-3432\,a^{8}\cos\left(dx+c\right)^{10}+10296\,a^{8}\cos\left(dx+c\right)^{8}-9152\,a^{8}\cos\left(dx+c\right)^{6}+\left(33\,a^{8}\cos\left(dx+c\right)^{12}-1212\,a^{8}\cos\left(dx+c\right)^{10}+6280\,a^{8}\cos\left(dx+c\right)^{8}-8512\,a^{8}\cos\left(dx+c\right)^{6}+768\,a^{8}\cos\left(dx+c\right)^{4}+1024\,a^{8}\cos\left(dx+c\right)^{2}+2048\,a^{8}\right)\sin\left(dx+c\right)}{429\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/429*(286*a^8*cos(d*x + c)^12 - 3432*a^8*cos(d*x + c)^10 + 10296*a^8*cos(d
*x + c)^8 - 9152*a^8*cos(d*x + c)^6 + (33*a^8*cos(d*x + c)^12 - 1212*a^8*co
s(d*x + c)^10 + 6280*a^8*cos(d*x + c)^8 - 8512*a^8*cos(d*x + c)^6 + 768*a^8
*cos(d*x + c)^4 + 1024*a^8*cos(d*x + c)^2 + 2048*a^8)*sin(d*x + c))/d
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 558 vs. $2(56) = 112$.
time = 5.46, size = 558, normalized size = 8.33

$$\begin{cases} \frac{8a^{8}\sin^{13}{(c+dx)}}{1287d} + \frac{4a^{8}\sin^{11}{(c+dx)}\cos^{2}{(c+dx)}}{99d} + \frac{32a^{8}\sin^{11}{(c+dx)}}{99d} + \frac{a^{8}\sin^{9}{(c+dx)}\cos^{4}{(c+dx)}}{9d} + \frac{16a^{8}\sin^{9}{(c+dx)}\cos^{2}{(c+dx)}}{9d} + \frac{16a^{8}\sin^{9}{(c+dx)}}{9d} + \frac{4a^{8}\sin^{7}{(c+dx)}\cos^{4}{(c+dx)}}{d} + \frac{8a^{8}\sin^{7}{(c+dx)}\cos^{2}{(c+dx)}}{d} + \frac{32a^{8}\sin^{7}{(c+dx)}}{15d} - \frac{4a^{8}\sin^{6}{(c+dx)}\cos^{6}{(c+dx)}}{3d} + \frac{14a^{8}\sin^{5}{(c+dx)}\cos^{4}{(c+dx)}}{d} + \frac{112a^{8}\sin^{5}{(c+dx)}\cos^{2}{(c+dx)}}{15d} + \frac{8a^{8}\sin^{5}{(c+dx)}}{15d} - \frac{a^{8}\sin^{4}{(c+dx)}\cos^{8}{(c+dx)}}{d} - \frac{28a^{8}\sin^{4}{(c+dx)}\cos^{6}{(c+dx)}}{3d} + \frac{28a^{8}\sin^{3}{(c+dx)}\cos^{4}{(c+dx)}}{3d} + \frac{4a^{8}\sin^{3}{(c+dx)}\cos^{2}{(c+dx)}}{3d} - \frac{2a^{8}\sin^{2}{(c+dx)}\cos^{10}{(c+dx)}}{5d} - \frac{14a^{8}\sin^{2}{(c+dx)}\cos^{8}{(c+dx)}}{3d} - \frac{28a^{8}\sin^{2}{(c+dx)}\cos^{6}{(c+dx)}}{3d} + \frac{a^{8}\sin{(c+dx)}\cos^{4}{(c+dx)}}{d} - \frac{a^{8}\cos^{12}{(c+dx)}}{15d} - \frac{14a^{8}\cos^{10}{(c+dx)}}{15d} - \frac{7a^{8}\cos^{8}{(c+dx)}}{3d} - \frac{4a^{8}\cos^{6}{(c+dx)}}{3d} & \text{for}\: d \neq 0 \\ x\left(a\sin{(c)} + a\right)^{8}\cos^{5}{(c)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+a*sin(d*x+c))**8,x)
```

```
[Out] Piecewise((8*a**8*sin(c + d*x)**13/(1287*d) + 4*a**8*sin(c + d*x)**11*cos(c
 + d*x)**2/(99*d) + 32*a**8*sin(c + d*x)**11/(99*d) + a**8*sin(c + d*x)**9*
cos(c + d*x)**4/(9*d) + 16*a**8*sin(c + d*x)**9*cos(c + d*x)**2/(9*d) + 16*
a**8*sin(c + d*x)**9/(9*d) + 4*a**8*sin(c + d*x)**7*cos(c + d*x)**4/d + 8*a
**8*sin(c + d*x)**7*cos(c + d*x)**2/d + 32*a**8*sin(c + d*x)**7/(15*d) - 4*
a**8*sin(c + d*x)**6*cos(c + d*x)**6/(3*d) + 14*a**8*sin(c + d*x)**5*cos(c
+ d*x)**4/d + 112*a**8*sin(c + d*x)**5*cos(c + d*x)**2/(15*d) + 8*a**8*sin(
c + d*x)**5/(15*d) - a**8*sin(c + d*x)**4*cos(c + d*x)**8/d - 28*a**8*sin(c
 + d*x)**4*cos(c + d*x)**6/(3*d) + 28*a**8*sin(c + d*x)**3*cos(c + d*x)**4/
(3*d) + 4*a**8*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) - 2*a**8*sin(c + d*x)*
*2*cos(c + d*x)**10/(5*d) - 14*a**8*sin(c + d*x)**2*cos(c + d*x)**8/(3*d) -
 28*a**8*sin(c + d*x)**2*cos(c + d*x)**6/(3*d) + a**8*sin(c + d*x)*cos(c +
d*x)**4/d - a**8*cos(c + d*x)**12/(15*d) - 14*a**8*cos(c + d*x)**10/(15*d)
- 7*a**8*cos(c + d*x)**8/(3*d) - 4*a**8*cos(c + d*x)**6/(3*d), Ne(d, 0)), (
x*(a*sin(c) + a)**8*cos(c)**5, True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(61) = 122$.
time = 2.91, size = 219, normalized size = 3.27

$$\frac{a^{8}\cos\left(12\,dx+12\,c\right)}{3072\,d}-\frac{3\,a^{8}\cos\left(10\,dx+10\,c\right)}{256\,d}+\frac{27\,a^{8}\cos\left(8\,dx+8\,c\right)}{512\,d}+\frac{155\,a^{8}\cos\left(6\,dx+6\,c\right)}{768\,d}-\frac{475\,a^{8}\cos\left(4\,dx+4\,c\right)}{1024\,d}-\frac{323\,a^{8}\cos\left(2\,dx+2\,c\right)}{128\,d}+\frac{a^{8}\sin\left(13\,dx+13\,c\right)}{53248\,d}-\frac{115\,a^{8}\sin\left(11\,dx+11\,c\right)}{45056\,d}+\frac{205\,a^{8}\sin\left(9\,dx+9\,c\right)}{6144\,d}-\frac{7\,a^{8}\sin\left(7\,dx+7\,c\right)}{2048\,d}-\frac{2033\,a^{8}\sin\left(5\,dx+5\,c\right)}{4096\,d}-\frac{6137\,a^{8}\sin\left(3\,dx+3\,c\right)}{12288\,d}+\frac{4845\,a^{8}\sin\left(dx+c\right)}{1024\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/3072*a^8*cos(12*d*x + 12*c)/d - 3/256*a^8*cos(10*d*x + 10*c)/d + 27/512*a
^8*cos(8*d*x + 8*c)/d + 155/768*a^8*cos(6*d*x + 6*c)/d - 475/1024*a^8*cos(4
*d*x + 4*c)/d - 323/128*a^8*cos(2*d*x + 2*c)/d + 1/53248*a^8*sin(13*d*x + 1
3*c)/d - 115/45056*a^8*sin(11*d*x + 11*c)/d + 205/6144*a^8*sin(9*d*x + 9*c)
/d - 7/2048*a^8*sin(7*d*x + 7*c)/d - 2033/4096*a^8*sin(5*d*x + 5*c)/d - 613
7/12288*a^8*sin(3*d*x + 3*c)/d + 4845/1024*a^8*sin(d*x + c)/d
```

Mupad [B]
time = 0.18, size = 134, normalized size = 2.00

$$\frac{a^8\,\sin\left(c+d\,x\right)\,\left(33\sin\left(c+d\,x\right)^{12}+286\sin\left(c+d\,x\right)^{11}+1014\sin\left(c+d\,x\right)^{10}+1716\sin\left(c+d\,x\right)^9+715\sin\left(c+d\,x\right)^8-2574\sin\left(c+d\,x\right)^7-5148\sin\left(c+d\,x\right)^6-3432\sin\left(c+d\,x\right)^5+1287\sin\left(c+d\,x\right)^4+4290\sin\left(c+d\,x\right)^3+3718\sin\left(c+d\,x\right)^2+1716\,\sin\left(c+d\,x\right)+429\right)}{429\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(a + a*sin(c + d*x))^8,x)
```

```
[Out] (a^8*sin(c + d*x)*(1716*sin(c + d*x) + 3718*sin(c + d*x)^2 + 4290*sin(c + d
*x)^3 + 1287*sin(c + d*x)^4 - 3432*sin(c + d*x)^5 - 5148*sin(c + d*x)^6 - 2
574*sin(c + d*x)^7 + 715*sin(c + d*x)^8 + 1716*sin(c + d*x)^9 + 1014*sin(c
+ d*x)^10 + 286*sin(c + d*x)^11 + 33*sin(c + d*x)^12 + 429))/(429*d)
```

3.42 $\int \cos^4(c+dx)(a+a\sin(c+dx))^8\,dx$

Optimal. Leaf size=286

$$\frac{4199a^8x}{1024}-\frac{4199a^8\cos^5(c+dx)}{1920d}+\frac{4199a^8\cos(c+dx)\sin(c+dx)}{1024d}+\frac{4199a^8\cos^3(c+dx)\sin(c+dx)}{1536d}-\frac{323a^3\cos$$

```
[Out] 4199/1024*a^8*x-4199/1920*a^8*cos(d*x+c)^5/d+4199/1024*a^8*cos(d*x+c)*sin(d
*x+c)/d+4199/1536*a^8*cos(d*x+c)^3*sin(d*x+c)/d-323/1320*a^3*cos(d*x+c)^5*(
a+a*sin(d*x+c))^5/d-19/132*a^2*cos(d*x+c)^5*(a+a*sin(d*x+c))^6/d-1/12*a*cos
(d*x+c)^5*(a+a*sin(d*x+c))^7/d-4199/6336*a^2*cos(d*x+c)^5*(a^2+a^2*sin(d*x+
c))^3/d-323/792*cos(d*x+c)^5*(a^2+a^2*sin(d*x+c))^4/d-4199/4032*cos(d*x+c)^
5*(a^4+a^4*sin(d*x+c))^2/d-4199/2688*cos(d*x+c)^5*(a^8+a^8*sin(d*x+c))/d
```

Rubi [A]
time = 0.28, antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2757, 2748, 2715, 8}

$$-\frac{4199a^8\cos^5(c+dx)}{1920d}-\frac{4199\cos^5(c+dx)\left(a^8\sin(c+dx)+a^8\right)}{2688d}+\frac{4199a^8\sin(c+dx)\cos^3(c+dx)}{1536d}+\frac{4199a^8\sin(c+dx)\cos(c+dx)}{1024d}+\frac{4199a^8x}{1024}-\frac{4199\cos^5(c+dx)\left(a^4\sin(c+dx)+a^4\right)^2}{4032d}-\frac{323a^3\cos^5(c+dx)(a\sin(c+dx)+a)^5}{1320d}-\frac{19a^2\cos^5(c+dx)(a\sin(c+dx)+a)^6}{132d}-\frac{4199a^2\cos^5(c+dx)\left(a^2\sin(c+dx)+a^2\right)^3}{6336d}-\frac{323\cos^5(c+dx)\left(a^2\sin(c+dx)+a^2\right)^4}{792d}-\frac{a\cos^5(c+dx)(a\sin(c+dx)+a)^7}{12d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + a*Sin[c + d*x])^8,x]
```

```
[Out] (4199*a^8*x)/1024 - (4199*a^8*Cos[c + d*x]^5)/(1920*d) + (4199*a^8*Cos[c +
d*x]*Sin[c + d*x])/(1024*d) + (4199*a^8*Cos[c + d*x]^3*Sin[c + d*x])/(1536*
d) - (323*a^3*Cos[c + d*x]^5*(a + a*Sin[c + d*x])^5)/(1320*d) - (19*a^2*Cos
[c + d*x]^5*(a + a*Sin[c + d*x])^6)/(132*d) - (a*Cos[c + d*x]^5*(a + a*Sin[
c + d*x])^7)/(12*d) - (4199*a^2*Cos[c + d*x]^5*(a^2 + a^2*Sin[c + d*x])^3)/
(6336*d) - (323*Cos[c + d*x]^5*(a^2 + a^2*Sin[c + d*x])^4)/(792*d) - (4199*
Cos[c + d*x]^5*(a^4 + a^4*Sin[c + d*x])^2)/(4032*d) - (4199*Cos[c + d*x]^5*
(a^8 + a^8*Sin[c + d*x]))/(2688*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2757

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ
[2*m, 2*p]
```

Rubi steps

$$\int \cos^4(c+dx)(a+a\sin(c+dx))^8\,dx = -\frac{a\cos^5(c+dx)(a+a\sin(c+dx))^7}{12d} + \frac{1}{12}(19a)\int \cos^4(c+dx)(a+$$

$$= -\frac{19a^2\cos^5(c+dx)(a+a\sin(c+dx))^6}{132d} - \frac{a\cos^5(c+dx)(a+a\sin(}{12d}$$

$$= -\frac{323a^3\cos^5(c+dx)(a+a\sin(c+dx))^5}{1320d} - \frac{19a^2\cos^5(c+dx)(a+a}{132d}$$

$$= -\frac{323a^3\cos^5(c+dx)(a+a\sin(c+dx))^5}{1320d} - \frac{19a^2\cos^5(c+dx)(a+a}{132d}$$

$$= -\frac{4199a^5\cos^5(c+dx)(a+a\sin(c+dx))^3}{6336d} - \frac{323a^3\cos^5(c+dx)(a+}{1320d}$$

$$= -\frac{4199a^5\cos^5(c+dx)(a+a\sin(c+dx))^3}{6336d} - \frac{323a^3\cos^5(c+dx)(a+}{1320d}$$

$$= -\frac{4199a^5\cos^5(c+dx)(a+a\sin(c+dx))^3}{6336d} - \frac{323a^3\cos^5(c+dx)(a+}{1320d}$$

$$= -\frac{4199a^8\cos^5(c+dx)}{1920d} - \frac{4199a^5\cos^5(c+dx)(a+a\sin(c+dx))^3}{6336d} -$$

$$= -\frac{4199a^8\cos^5(c+dx)}{1920d} + \frac{4199a^8\cos^3(c+dx)\sin(c+dx)}{1536d} - \frac{4199a^5}{}$$

$$= -\frac{4199a^8\cos^5(c+dx)}{1920d} + \frac{4199a^8\cos(c+dx)\sin(c+dx)}{1024d} + \frac{4199a^8 c}{}$$

$$= \frac{4199a^8x}{1024} - \frac{4199a^8\cos^5(c+dx)}{1920d} + \frac{4199a^8\cos(c+dx)\sin(c+dx)}{1024d} +$$

Mathematica [A]

time = 1.98, size = 211, normalized size = 0.74

$$-\frac{a^8 \cos^5(c+dx) \left(-29099070 \sin^{-1}\left(\frac{\sqrt{1-\sin(c+dx)}}{\sqrt{2}}\right) \sqrt{1-\sin(c+dx)} + \sqrt{1+\sin(c+dx)} \left(-22470656 + 11469281 \sin(c+dx) + 13958687 \sin^2(c+dx) + 20459158 \sin^3(c+dx) + 14283114 \sin^4(c+dx) - 8321928 \sin^5(c+dx) - 26346616 \sin^6(c+dx) - 20428112 \sin^7(c+dx) - 1239728 \sin^8(c+dx) + 9086336 \sin^9(c+dx) + 6969984 \sin^{10}(c+dx) + 2284800 \sin^{11}(c+dx) + 295680 \sin^{12}(c+dx)\right)\right)}{3548160 d(-1+\sin(c+dx))^3(1+\sin(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + a*Sin[c + d*x])^8,x]
```

```
[Out] -1/3548160*(a^8*Cos[c + d*x]^5*(-29099070*ArcSin[Sqrt[1 - Sin[c + d*x]]/Sqr
t[2]]*Sqrt[1 - Sin[c + d*x]] + Sqrt[1 + Sin[c + d*x]]*(-22470656 + 11469281
*Sin[c + d*x] + 13958687*Sin[c + d*x]^2 + 20459158*Sin[c + d*x]^3 + 1428311
4*Sin[c + d*x]^4 - 8321928*Sin[c + d*x]^5 - 26346616*Sin[c + d*x]^6 - 20428
112*Sin[c + d*x]^7 - 1239728*Sin[c + d*x]^8 + 9086336*Sin[c + d*x]^9 + 6969
984*Sin[c + d*x]^10 + 2284800*Sin[c + d*x]^11 + 295680*Sin[c + d*x]^12)))/(
d*(-1 + Sin[c + d*x])^3*(1 + Sin[c + d*x])^(5/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 534 vs. $2(264) = 528$.
time = 0.93, size = 535, normalized size = 1.87 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+a*sin(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^8*(-1/12*sin(d*x+c)^7*cos(d*x+c)^5-7/120*sin(d*x+c)^5*cos(d*x+c)^5-7
/192*sin(d*x+c)^3*cos(d*x+c)^5-7/384*sin(d*x+c)*cos(d*x+c)^5+7/1536*(cos(d*
x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+7/1024*d*x+7/1024*c)+8*a^8*(-1/11*sin(d*x
+c)^6*cos(d*x+c)^5-2/33*sin(d*x+c)^4*cos(d*x+c)^5-8/231*sin(d*x+c)^2*cos(d*
x+c)^5-16/1155*cos(d*x+c)^5)+28*a^8*(-1/10*sin(d*x+c)^5*cos(d*x+c)^5-1/16*s
in(d*x+c)^3*cos(d*x+c)^5-1/32*sin(d*x+c)*cos(d*x+c)^5+1/128*(cos(d*x+c)^3+3
/2*cos(d*x+c))*sin(d*x+c)+3/256*d*x+3/256*c)+56*a^8*(-1/9*sin(d*x+c)^4*cos(
d*x+c)^5-4/63*sin(d*x+c)^2*cos(d*x+c)^5-8/315*cos(d*x+c)^5)+70*a^8*(-1/8*si
n(d*x+c)^3*cos(d*x+c)^5-1/16*sin(d*x+c)*cos(d*x+c)^5+1/64*(cos(d*x+c)^3+3/2
*cos(d*x+c))*sin(d*x+c)+3/128*d*x+3/128*c)+56*a^8*(-1/7*sin(d*x+c)^2*cos(d*
x+c)^5-2/35*cos(d*x+c)^5)+28*a^8*(-1/6*sin(d*x+c)*cos(d*x+c)^5+1/24*(cos(d*
x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+1/16*d*x+1/16*c)-8/5*cos(d*x+c)^5*a^8+a^8
*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c))
```

Maxima [**A**]
time = 0.32, size = 339, normalized size = 1.19

$$-\frac{45416448 a^8 \cos(dx+c)^5 - 196608 \left(105 \cos(dx+c)^{11} - 385 \cos(dx+c)^9 + 495 \cos(dx+c)^7 - 231 \cos(dx+c)^5\right) a^8 + 5046272 \left(35 \cos(dx+c)^9 - 90 \cos(dx+c)^7 + 63 \cos(dx+c)^5\right) a^8 - 45416448 \left(5 \cos(dx+c)^7 - 7 \cos(dx+c)^5\right) a^8 + 231 \left(384 \sin(2dx+2c)^5 + 20 \sin(4dx+4c)^3 - 840dx - 840c - 15 \sin(8dx+8c) + 240 \sin(4dx+4c)\right) a^8 + 77616 \left(32 \sin(2dx+2c)^5 - 120dx - 120c - 5 \sin(8dx+8c) + 40 \sin(4dx+4c)\right) a^8 - 4139520 \left(4 \sin(2dx+2c)^3 + 12dx + 12c - 3 \sin(4dx+4c)\right) a^8 - 1940400 \left(24dx + 24c + \sin(8dx+8c) - 8 \sin(4dx+4c)\right) a^8 - 887040 \left(12dx + 12c + \sin(4dx+4c) + 8 \sin(2dx+2c)\right) a^8}{28385280 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/28385280*(45416448*a^8*cos(d*x + c)^5 - 196608*(105*cos(d*x + c)^11 - 38
5*cos(d*x + c)^9 + 495*cos(d*x + c)^7 - 231*cos(d*x + c)^5)*a^8 + 5046272*(
```

```
35*cos(d*x + c)^9 - 90*cos(d*x + c)^7 + 63*cos(d*x + c)^5)*a^8 - 45416448*(
5*cos(d*x + c)^7 - 7*cos(d*x + c)^5)*a^8 + 231*(384*sin(2*d*x + 2*c)^5 + 20
*sin(4*d*x + 4*c)^3 - 840*d*x - 840*c - 15*sin(8*d*x + 8*c) + 240*sin(4*d*x
 + 4*c))*a^8 + 77616*(32*sin(2*d*x + 2*c)^5 - 120*d*x - 120*c - 5*sin(8*d*x
 + 8*c) + 40*sin(4*d*x + 4*c))*a^8 - 4139520*(4*sin(2*d*x + 2*c)^3 + 12*d*x
 + 12*c - 3*sin(4*d*x + 4*c))*a^8 - 1940400*(24*d*x + 24*c + sin(8*d*x + 8*
c) - 8*sin(4*d*x + 4*c))*a^8 - 887040*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8
*sin(2*d*x + 2*c))*a^8)/d
```

Fricas [A]
time = 0.42, size = 150, normalized size = 0.52

$$\frac{2580480\,a^8\cos\left(dx+c\right)^{11}-31539200\,a^8\cos\left(dx+c\right)^9+97320960\,a^8\cos\left(dx+c\right)^7-90832896\,a^8\cos\left(dx+c\right)^5+14549535\,a^8dx+231\,\left(1280\,a^8\cos\left(dx+c\right)^{11}-47744\,a^8\cos\left(dx+c\right)^9+253488\,a^8\cos\left(dx+c\right)^7-359624\,a^8\cos\left(dx+c\right)^5+41990\,a^8\cos\left(dx+c\right)^3+62985\,a^8\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{3548160\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/3548160*(2580480*a^8*cos(d*x + c)^11 - 31539200*a^8*cos(d*x + c)^9 + 9732
0960*a^8*cos(d*x + c)^7 - 90832896*a^8*cos(d*x + c)^5 + 14549535*a^8*d*x +
231*(1280*a^8*cos(d*x + c)^11 - 47744*a^8*cos(d*x + c)^9 + 253488*a^8*cos(d
*x + c)^7 - 359624*a^8*cos(d*x + c)^5 + 41990*a^8*cos(d*x + c)^3 + 62985*a^
8*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1280 vs. $2(270) = 540$.
time = 4.04, size = 1280, normalized size = 4.48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+a*sin(d*x+c))**8,x)
```

```
[Out] Piecewise((7*a**8*x*sin(c + d*x)**12/1024 + 21*a**8*x*sin(c + d*x)**10*cos(
c + d*x)**2/512 + 21*a**8*x*sin(c + d*x)**10/64 + 105*a**8*x*sin(c + d*x)**
8*cos(c + d*x)**4/1024 + 105*a**8*x*sin(c + d*x)**8*cos(c + d*x)**2/64 + 10
5*a**8*x*sin(c + d*x)**8/64 + 35*a**8*x*sin(c + d*x)**6*cos(c + d*x)**6/256
 + 105*a**8*x*sin(c + d*x)**6*cos(c + d*x)**4/32 + 105*a**8*x*sin(c + d*x)*
*6*cos(c + d*x)**2/16 + 7*a**8*x*sin(c + d*x)**6/4 + 105*a**8*x*sin(c + d*x
)**4*cos(c + d*x)**8/1024 + 105*a**8*x*sin(c + d*x)**4*cos(c + d*x)**6/32 +
 315*a**8*x*sin(c + d*x)**4*cos(c + d*x)**4/32 + 21*a**8*x*sin(c + d*x)**4*
cos(c + d*x)**2/4 + 3*a**8*x*sin(c + d*x)**4/8 + 21*a**8*x*sin(c + d*x)**2*
cos(c + d*x)**10/512 + 105*a**8*x*sin(c + d*x)**2*cos(c + d*x)**8/64 + 105*
a**8*x*sin(c + d*x)**2*cos(c + d*x)**6/16 + 21*a**8*x*sin(c + d*x)**2*cos(c
 + d*x)**4/4 + 3*a**8*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 7*a**8*x*cos(c
+ d*x)**12/1024 + 21*a**8*x*cos(c + d*x)**10/64 + 105*a**8*x*cos(c + d*x)**
8/64 + 7*a**8*x*cos(c + d*x)**6/4 + 3*a**8*x*cos(c + d*x)**4/8 + 7*a**8*sin
```

```
(c + d*x)**11*cos(c + d*x)/(1024*d) + 119*a**8*sin(c + d*x)**9*cos(c + d*x)
**3/(3072*d) + 21*a**8*sin(c + d*x)**9*cos(c + d*x)/(64*d) - 281*a**8*sin(c
 + d*x)**7*cos(c + d*x)**5/(2560*d) + 49*a**8*sin(c + d*x)**7*cos(c + d*x)*
*3/(32*d) + 105*a**8*sin(c + d*x)**7*cos(c + d*x)/(64*d) - 8*a**8*sin(c + d
*x)**6*cos(c + d*x)**5/(5*d) - 231*a**8*sin(c + d*x)**5*cos(c + d*x)**7/(25
60*d) - 14*a**8*sin(c + d*x)**5*cos(c + d*x)**5/(5*d) + 385*a**8*sin(c + d*
x)**5*cos(c + d*x)**3/(64*d) + 7*a**8*sin(c + d*x)**5*cos(c + d*x)/(4*d) -
48*a**8*sin(c + d*x)**4*cos(c + d*x)**7/(35*d) - 56*a**8*sin(c + d*x)**4*co
s(c + d*x)**5/(5*d) - 119*a**8*sin(c + d*x)**3*cos(c + d*x)**9/(3072*d) - 4
9*a**8*sin(c + d*x)**3*cos(c + d*x)**7/(32*d) - 385*a**8*sin(c + d*x)**3*co
s(c + d*x)**5/(64*d) + 14*a**8*sin(c + d*x)**3*cos(c + d*x)**3/(3*d) + 3*a*
*8*sin(c + d*x)**3*cos(c + d*x)/(8*d) - 64*a**8*sin(c + d*x)**2*cos(c + d*x
)**9/(105*d) - 32*a**8*sin(c + d*x)**2*cos(c + d*x)**7/(5*d) - 56*a**8*sin(
c + d*x)**2*cos(c + d*x)**5/(5*d) - 7*a**8*sin(c + d*x)*cos(c + d*x)**11/(1
024*d) - 21*a**8*sin(c + d*x)*cos(c + d*x)**9/(64*d) - 105*a**8*sin(c + d*x
)*cos(c + d*x)**7/(64*d) - 7*a**8*sin(c + d*x)*cos(c + d*x)**5/(4*d) + 5*a*
*8*sin(c + d*x)*cos(c + d*x)**3/(8*d) - 128*a**8*cos(c + d*x)**11/(1155*d)
- 64*a**8*cos(c + d*x)**9/(45*d) - 16*a**8*cos(c + d*x)**7/(5*d) - 8*a**8*c
os(c + d*x)**5/(5*d), Ne(d, 0)), (x*(a*sin(c) + a)**8*cos(c)**4, True))
```

Giac [A]
time = 5.15, size = 208, normalized size = 0.73

$$\frac{4199}{1024}a^8x+\frac{a^8\cos(11\,dx+11\,c)}{1408\,d}-\frac{31\,a^8\cos(9\,dx+9\,c)}{1152\,d}+\frac{139\,a^8\cos(7\,dx+7\,c)}{896\,d}+\frac{171\,a^8\cos(5\,dx+5\,c)}{640\,d}-\frac{323\,a^8\cos(3\,dx+3\,c)}{192\,d}-\frac{323\,a^8\cos(dx+c)}{64\,d}+\frac{a^8\sin(12\,dx+12\,c)}{24576\,d}-\frac{29\,a^8\sin(10\,dx+10\,c)}{5120\,d}+\frac{673\,a^8\sin(8\,dx+8\,c)}{8192\,d}-\frac{361\,a^8\sin(6\,dx+6\,c)}{3072\,d}-\frac{8721\,a^8\sin(4\,dx+4\,c)}{8192\,d}+\frac{323\,a^8\sin(2\,dx+2\,c)}{512\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 4199/1024*a^8*x + 1/1408*a^8*cos(11*d*x + 11*c)/d - 31/1152*a^8*cos(9*d*x +
 9*c)/d + 139/896*a^8*cos(7*d*x + 7*c)/d + 171/640*a^8*cos(5*d*x + 5*c)/d -
 323/192*a^8*cos(3*d*x + 3*c)/d - 323/64*a^8*cos(d*x + c)/d + 1/24576*a^8*s
in(12*d*x + 12*c)/d - 29/5120*a^8*sin(10*d*x + 10*c)/d + 673/8192*a^8*sin(8
*d*x + 8*c)/d - 361/3072*a^8*sin(6*d*x + 6*c)/d - 8721/8192*a^8*sin(4*d*x +
 4*c)/d + 323/512*a^8*sin(2*d*x + 2*c)/d
```

Mupad [B]
time = 7.05, size = 684, normalized size = 2.39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(a + a*sin(c + d*x))^8,x)
```

```
[Out] (4199*a^8*x)/1024 - ((1543*a^8*tan(c/2 + (d*x)/2)^3)/512 - (1068767*a^8*tan
(c/2 + (d*x)/2)^5)/2560 - (3297279*a^8*tan(c/2 + (d*x)/2)^7)/2560 - (168283
*a^8*tan(c/2 + (d*x)/2)^9)/3840 + (256139*a^8*tan(c/2 + (d*x)/2)^11)/256 -
```

```
(256139*a^8*tan(c/2 + (d*x)/2)^13)/256 + (168283*a^8*tan(c/2 + (d*x)/2)^15)
/3840 + (3297279*a^8*tan(c/2 + (d*x)/2)^17)/2560 + (1068767*a^8*tan(c/2 + (
d*x)/2)^19)/2560 - (1543*a^8*tan(c/2 + (d*x)/2)^21)/512 - (3175*a^8*tan(c/2
 + (d*x)/2)^23)/512 + a^8*((4199*c)/1024 + (4199*d*x)/1024) - a^8*((4199*c)
/1024 + (4199*d*x)/1024 - 43888/3465) + tan(c/2 + (d*x)/2)^22*(12*a^8*((419
9*c)/1024 + (4199*d*x)/1024) - a^8*((12597*c)/256 + (12597*d*x)/256 - 16))
+ tan(c/2 + (d*x)/2)^2*(12*a^8*((4199*c)/1024 + (4199*d*x)/1024) - a^8*((12
597*c)/256 + (12597*d*x)/256 - 157072/1155)) + tan(c/2 + (d*x)/2)^20*(66*a^
8*((4199*c)/1024 + (4199*d*x)/1024) - a^8*((138567*c)/512 + (138567*d*x)/51
2 - 336)) + tan(c/2 + (d*x)/2)^4*(66*a^8*((4199*c)/1024 + (4199*d*x)/1024)
- a^8*((138567*c)/512 + (138567*d*x)/512 - 52496/105)) + tan(c/2 + (d*x)/2)
^18*(220*a^8*((4199*c)/1024 + (4199*d*x)/1024) - a^8*((230945*c)/256 + (230
945*d*x)/256 - 5584/3)) + tan(c/2 + (d*x)/2)^6*(220*a^8*((4199*c)/1024 + (4
199*d*x)/1024) - a^8*((230945*c)/256 + (230945*d*x)/256 - 58288/63)) + tan(
c/2 + (d*x)/2)^14*(792*a^8*((4199*c)/1024 + (4199*d*x)/1024) - a^8*((415701
*c)/128 + (415701*d*x)/128 - 17696/5)) + tan(c/2 + (d*x)/2)^10*(792*a^8*((4
199*c)/1024 + (4199*d*x)/1024) - a^8*((415701*c)/128 + (415701*d*x)/128 - 2
27232/35)) + tan(c/2 + (d*x)/2)^12*(924*a^8*((4199*c)/1024 + (4199*d*x)/102
4) - a^8*((969969*c)/256 + (969969*d*x)/256 - 87776/15)) + tan(c/2 + (d*x)/
2)^16*(495*a^8*((4199*c)/1024 + (4199*d*x)/1024) - a^8*((2078505*c)/1024 +
(2078505*d*x)/1024 - 3504)) + tan(c/2 + (d*x)/2)^8*(495*a^8*((4199*c)/1024
+ (4199*d*x)/1024) - a^8*((2078505*c)/1024 + (2078505*d*x)/1024 - 19360/7))
 + (3175*a^8*tan(c/2 + (d*x)/2))/512)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^12)
```

3.43 $\int \cos^3(c+dx)(a+a\sin(c+dx))^8\,dx$

Optimal. Leaf size=45

$$\frac{(a+a\sin(c+dx))^{10}}{5a^2d}-\frac{(a+a\sin(c+dx))^{11}}{11a^3d}$$

```
[Out] 1/5*(a+a*sin(d*x+c))^10/a^2/d-1/11*(a+a*sin(d*x+c))^11/a^3/d
```

Rubi [A]
time = 0.03, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.095$, Rules used = {2746, 45}

$$\frac{(a\sin(c+dx)+a)^{10}}{5a^2d}-\frac{(a\sin(c+dx)+a)^{11}}{11a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Sin[c + d*x])^8,x]
```

```
[Out] (a + a*Sin[c + d*x])^10/(5*a^2*d) - (a + a*Sin[c + d*x])^11/(11*a^3*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}
\int \cos^3(c+dx)(a+a\sin(c+dx))^8\,dx &= \frac{\text{Subst}\left(\int (a-x)(a+x)^9\,dx,x,a\sin(c+dx)\right)}{a^3d}\\
&= \frac{\text{Subst}\left(\int \left(2a(a+x)^9-(a+x)^{10}\right)\,dx,x,a\sin(c+dx)\right)}{a^3d}\\
&= \frac{(a+a\sin(c+dx))^{10}}{5a^2d}-\frac{(a+a\sin(c+dx))^{11}}{11a^3d}
\end{aligned}$$

Mathematica [A]
time = 0.66, size = 43, normalized size = 0.96

$$-\frac{a^8\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^{20}(-6+5\sin(c+dx))}{55d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Sin[c + d*x])^8,x]
```

```
[Out] -1/55*(a^8*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^20*(-6 + 5*Sin[c + d*x]))/
d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 462 vs. $2(41) = 82$.
time = 0.69, size = 463, normalized size = 10.29 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*sin(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^8*(-1/11*sin(d*x+c)^7*cos(d*x+c)^4-7/99*sin(d*x+c)^5*cos(d*x+c)^4-5/
99*sin(d*x+c)^3*cos(d*x+c)^4-1/33*sin(d*x+c)*cos(d*x+c)^4+1/99*(2+cos(d*x+c
)^2)*sin(d*x+c))+8*a^8*(-1/10*sin(d*x+c)^6*cos(d*x+c)^4-3/40*sin(d*x+c)^4*c
os(d*x+c)^4-1/20*sin(d*x+c)^2*cos(d*x+c)^4-1/40*cos(d*x+c)^4)+28*a^8*(-1/9*
sin(d*x+c)^5*cos(d*x+c)^4-5/63*sin(d*x+c)^3*cos(d*x+c)^4-1/21*sin(d*x+c)*co
s(d*x+c)^4+1/63*(2+cos(d*x+c)^2)*sin(d*x+c))+56*a^8*(-1/8*sin(d*x+c)^4*cos(
d*x+c)^4-1/12*sin(d*x+c)^2*cos(d*x+c)^4-1/24*cos(d*x+c)^4)+70*a^8*(-1/7*sin
(d*x+c)^3*cos(d*x+c)^4-3/35*sin(d*x+c)*cos(d*x+c)^4+1/35*(2+cos(d*x+c)^2)*s
in(d*x+c))+56*a^8*(-1/6*sin(d*x+c)^2*cos(d*x+c)^4-1/12*cos(d*x+c)^4)+28*a^8
*(-1/5*sin(d*x+c)*cos(d*x+c)^4+1/15*(2+cos(d*x+c)^2)*sin(d*x+c))-2*cos(d*x+
c)^4*a^8+1/3*a^8*(2+cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(41) = 82$.
time = 0.32, size = 134, normalized size = 2.98

$$-\frac{5\,a^8\sin(dx+c)^{11}+44\,a^8\sin(dx+c)^{10}+165\,a^8\sin(dx+c)^9+330\,a^8\sin(dx+c)^8+330\,a^8\sin(dx+c)^7-462\,a^8\sin(dx+c)^5-660\,a^8\sin(dx+c)^4-495\,a^8\sin(dx+c)^3-220\,a^8\sin(dx+c)^2-55\,a^8\sin(dx+c)}{55\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/55*(5*a^8*sin(d*x + c)^11 + 44*a^8*sin(d*x + c)^10 + 165*a^8*sin(d*x + c
)^9 + 330*a^8*sin(d*x + c)^8 + 330*a^8*sin(d*x + c)^7 - 462*a^8*sin(d*x + c
)^5 - 660*a^8*sin(d*x + c)^4 - 495*a^8*sin(d*x + c)^3 - 220*a^8*sin(d*x + c
)^2 - 55*a^8*sin(d*x + c))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(41) = 82$.
time = 0.39, size = 136, normalized size = 3.02

$$\frac{44\,a^{8}\cos\left(dx+c\right)^{10}-550\,a^{8}\cos\left(dx+c\right)^{8}+1760\,a^{8}\cos\left(dx+c\right)^{6}-1760\,a^{8}\cos\left(dx+c\right)^{4}+\left(5\,a^{8}\cos\left(dx+c\right)^{10}-190\,a^{8}\cos\left(dx+c\right)^{8}+1040\,a^{8}\cos\left(dx+c\right)^{6}-1568\,a^{8}\cos\left(dx+c\right)^{4}+256\,a^{8}\cos\left(dx+c\right)^{2}+512\,a^{8}\right)\sin\left(dx+c\right)}{55\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/55*(44*a^8*cos(d*x + c)^10 - 550*a^8*cos(d*x + c)^8 + 1760*a^8*cos(d*x +
c)^6 - 1760*a^8*cos(d*x + c)^4 + (5*a^8*cos(d*x + c)^10 - 190*a^8*cos(d*x +
 c)^8 + 1040*a^8*cos(d*x + c)^6 - 1568*a^8*cos(d*x + c)^4 + 256*a^8*cos(d*x
 + c)^2 + 512*a^8)*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 422 vs. $2(36) = 72$.
time = 2.84, size = 422, normalized size = 9.38

$$\begin{cases} \frac{2a^{8}\sin^{11}(c+dx)}{99d}+\frac{a^{8}\sin^{9}(c+dx)\cos^{2}(c+dx)}{9d}+\frac{8a^{8}\sin^{9}(c+dx)}{9d}+\frac{4a^{8}\sin^{7}(c+dx)\cos^{2}(c+dx)}{d}+\frac{4a^{8}\sin^{7}(c+dx)}{d}-\frac{2a^{8}\sin^{6}(c+dx)\cos^{4}(c+dx)}{d}+\frac{14a^{8}\sin^{5}(c+dx)\cos^{2}(c+dx)}{d}+\frac{56a^{8}\sin^{5}(c+dx)}{15d}-\frac{2a^{8}\sin^{4}(c+dx)\cos^{6}(c+dx)}{d}-\frac{14a^{8}\sin^{4}(c+dx)\cos^{4}(c+dx)}{d}+\frac{28a^{8}\sin^{3}(c+dx)\cos^{2}(c+dx)}{3d}+\frac{2a^{8}\sin^{3}(c+dx)}{3d}-\frac{a^{8}\sin^{2}(c+dx)\cos^{8}(c+dx)}{d}-\frac{28a^{8}\sin^{2}(c+dx)\cos^{6}(c+dx)}{3d}-\frac{14a^{8}\sin^{2}(c+dx)\cos^{4}(c+dx)}{d}+\frac{a^{8}\sin(c+dx)\cos^{2}(c+dx)}{d}-\frac{a^{8}\cos^{10}(c+dx)}{5d}-\frac{7a^{8}\cos^{8}(c+dx)}{3d}-\frac{14a^{8}\cos^{6}(c+dx)}{3d}-\frac{2a^{8}\cos^{4}(c+dx)}{d} & \text{for } d \neq 0 \\ x(a\sin(c)+a)^{8}\cos^{3}(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*sin(d*x+c))**8,x)
```

```
[Out] Piecewise((2*a**8*sin(c + d*x)**11/(99*d) + a**8*sin(c + d*x)**9*cos(c + d*
x)**2/(9*d) + 8*a**8*sin(c + d*x)**9/(9*d) + 4*a**8*sin(c + d*x)**7*cos(c +
 d*x)**2/d + 4*a**8*sin(c + d*x)**7/d - 2*a**8*sin(c + d*x)**6*cos(c + d*x)
**4/d + 14*a**8*sin(c + d*x)**5*cos(c + d*x)**2/d + 56*a**8*sin(c + d*x)**5
/(15*d) - 2*a**8*sin(c + d*x)**4*cos(c + d*x)**6/d - 14*a**8*sin(c + d*x)**
4*cos(c + d*x)**4/d + 28*a**8*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + 2*a**
8*sin(c + d*x)**3/(3*d) - a**8*sin(c + d*x)**2*cos(c + d*x)**8/d - 28*a**8*
sin(c + d*x)**2*cos(c + d*x)**6/(3*d) - 14*a**8*sin(c + d*x)**2*cos(c + d*x
)**4/d + a**8*sin(c + d*x)*cos(c + d*x)**2/d - a**8*cos(c + d*x)**10/(5*d)
- 7*a**8*cos(c + d*x)**8/(3*d) - 14*a**8*cos(c + d*x)**6/(3*d) - 2*a**8*cos
(c + d*x)**4/d, Ne(d, 0)), (x*(a*sin(c) + a)**8*cos(c)**3, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(41) = 82$.
time = 6.63, size = 134, normalized size = 2.98

$$-\frac{5\,a^{8}\sin\left(dx+c\right)^{11}+44\,a^{8}\sin\left(dx+c\right)^{10}+165\,a^{8}\sin\left(dx+c\right)^{9}+330\,a^{8}\sin\left(dx+c\right)^{8}+330\,a^{8}\sin\left(dx+c\right)^{7}-462\,a^{8}\sin\left(dx+c\right)^{5}-660\,a^{8}\sin\left(dx+c\right)^{4}-495\,a^{8}\sin\left(dx+c\right)^{3}-220\,a^{8}\sin\left(dx+c\right)^{2}-55\,a^{8}\sin\left(dx+c\right)}{55\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -1/55*(5*a^8*sin(d*x + c)^11 + 44*a^8*sin(d*x + c)^10 + 165*a^8*sin(d*x + c
)^9 + 330*a^8*sin(d*x + c)^8 + 330*a^8*sin(d*x + c)^7 - 462*a^8*sin(d*x + c
```

```
)^5 - 660*a^8*sin(d*x + c)^4 - 495*a^8*sin(d*x + c)^3 - 220*a^8*sin(d*x + c
)^2 - 55*a^8*sin(d*x + c))/d
```

Mupad [B]
time = 0.12, size = 132, normalized size = 2.93

$$\frac{-\frac{a^8\sin(c+dx)^{11}}{11}-\frac{4a^8\sin(c+dx)^{10}}{5}-3a^8\sin(c+dx)^9-6a^8\sin(c+dx)^8-6a^8\sin(c+dx)^7+\frac{42a^8\sin(c+dx)^5}{5}+12a^8\sin(c+dx)^4+9a^8\sin(c+dx)^3+4a^8\sin(c+dx)^2+a^8\sin(c+dx)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a + a*sin(c + d*x))^8,x)
```

```
[Out] (a^8*sin(c + d*x) + 4*a^8*sin(c + d*x)^2 + 9*a^8*sin(c + d*x)^3 + 12*a^8*si
n(c + d*x)^4 + (42*a^8*sin(c + d*x)^5)/5 - 6*a^8*sin(c + d*x)^7 - 6*a^8*sin
(c + d*x)^8 - 3*a^8*sin(c + d*x)^9 - (4*a^8*sin(c + d*x)^10)/5 - (a^8*sin(c
 + d*x)^11)/11)/d
```

3.44 $\int \cos^2(c+dx)(a+a\sin(c+dx))^8\,dx$

<u>**Optimal**</u>. Leaf size=262

$$\frac{2431a^8x}{256}-\frac{2431a^8\cos^3(c+dx)}{384d}+\frac{2431a^8\cos(c+dx)\sin(c+dx)}{256d}-\frac{17a^3\cos^3(c+dx)(a+a\sin(c+dx))^5}{48d}-\frac{17}{}$$

```
[Out] 2431/256*a^8*x-2431/384*a^8*cos(d*x+c)^3/d+2431/256*a^8*cos(d*x+c)*sin(d*x+
c)/d-17/48*a^3*cos(d*x+c)^3*(a+a*sin(d*x+c))^5/d-17/90*a^2*cos(d*x+c)^3*(a+
a*sin(d*x+c))^6/d-1/10*a*cos(d*x+c)^3*(a+a*sin(d*x+c))^7/d-2431/2016*a^2*co
s(d*x+c)^3*(a^2+a^2*sin(d*x+c))^3/d-221/336*cos(d*x+c)^3*(a^2+a^2*sin(d*x+c
))^4/d-2431/1120*cos(d*x+c)^3*(a^4+a^4*sin(d*x+c))^2/d-2431/640*cos(d*x+c)^
3*(a^8+a^8*sin(d*x+c))/d
```

<u>**Rubi**</u> [A]
time = 0.25, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2757, 2748, 2715, 8}

$$-\frac{2431a^8\cos^3(c+dx)}{384d}-\frac{2431\cos^3(c+dx)\left(a^8\sin(c+dx)+a^8\right)}{640d}+\frac{2431a^8\sin(c+dx)\cos(c+dx)}{256d}+\frac{2431a^8x}{256}-\frac{2431\cos^3(c+dx)\left(a^4\sin(c+dx)+a^4\right)^2}{1120d}-\frac{17a^3\cos^3(c+dx)(a\sin(c+dx)+a)^5}{48d}-\frac{17a^2\cos^3(c+dx)(a\sin(c+dx)+a)^6}{90d}-\frac{2431a^2\cos^3(c+dx)\left(a^2\sin(c+dx)+a^2\right)^3}{2016d}-\frac{221\cos^3(c+dx)\left(a^2\sin(c+dx)+a^2\right)^4}{336d}-\frac{a\cos^3(c+dx)(a\sin(c+dx)+a)^7}{10d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Sin[c + d*x])^8,x]
```

```
[Out] (2431*a^8*x)/256 - (2431*a^8*Cos[c + d*x]^3)/(384*d) + (2431*a^8*Cos[c + d*
x]*Sin[c + d*x])/(256*d) - (17*a^3*Cos[c + d*x]^3*(a + a*Sin[c + d*x])^5)/(
48*d) - (17*a^2*Cos[c + d*x]^3*(a + a*Sin[c + d*x])^6)/(90*d) - (a*Cos[c +
d*x]^3*(a + a*Sin[c + d*x])^7)/(10*d) - (2431*a^2*Cos[c + d*x]^3*(a^2 + a^2
*Sin[c + d*x])^3)/(2016*d) - (221*Cos[c + d*x]^3*(a^2 + a^2*Sin[c + d*x])^4
)/(336*d) - (2431*Cos[c + d*x]^3*(a^4 + a^4*Sin[c + d*x])^2)/(1120*d) - (24
31*Cos[c + d*x]^3*(a^8 + a^8*Sin[c + d*x]))/(640*d)
```

<u>Rule 8</u>

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

<u>Rule 2715</u>

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

<u>Rule 2748</u>

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
```

```
Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2757

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ
[2*m, 2*p]
```

Rubi steps

$$\begin{aligned}\int \cos^2(c+dx)(a+a\sin(c+dx))^8\,dx &= -\frac{a\cos^3(c+dx)(a+a\sin(c+dx))^7}{10d}+\frac{1}{10}(17a)\int \cos^2(c+dx)(a+\\
&= -\frac{17a^2\cos^3(c+dx)(a+a\sin(c+dx))^6}{90d}-\frac{a\cos^3(c+dx)(a+a\sin(\\
&= -\frac{17a^3\cos^3(c+dx)(a+a\sin(c+dx))^5}{48d}-\frac{17a^2\cos^3(c+dx)(a+a\,\mathrm{s}\\
&= -\frac{17a^3\cos^3(c+dx)(a+a\sin(c+dx))^5}{48d}-\frac{17a^2\cos^3(c+dx)(a+a\,\mathrm{s}\\
&= -\frac{2431a^5\cos^3(c+dx)(a+a\sin(c+dx))^3}{2016d}-\frac{17a^3\cos^3(c+dx)(a+\\
&= -\frac{2431a^5\cos^3(c+dx)(a+a\sin(c+dx))^3}{2016d}-\frac{17a^3\cos^3(c+dx)(a+\\
&= -\frac{2431a^5\cos^3(c+dx)(a+a\sin(c+dx))^3}{2016d}-\frac{17a^3\cos^3(c+dx)(a+\\
&= -\frac{2431a^8\cos^3(c+dx)}{384d}-\frac{2431a^5\cos^3(c+dx)(a+a\sin(c+dx))^3}{2016d}-\\
&= -\frac{2431a^8\cos^3(c+dx)}{384d}+\frac{2431a^8\cos(c+dx)\sin(c+dx)}{256d}-\frac{2431a^5\,\mathrm{c}\\
&= \frac{2431a^8x}{256}-\frac{2431a^8\cos^3(c+dx)}{384d}+\frac{2431a^8\cos(c+dx)\sin(c+dx)}{256d}-\end{aligned}$$

Mathematica [A]
time = 0.91, size = 191, normalized size = 0.73

$$-\frac{a^8\cos^3(c+dx)\left(1531530\sin^{-1}\left(\frac{\sqrt{1-\sin(c+dx)}}{\sqrt{2}}\right)\sqrt{1-\sin(c+dx)}+\sqrt{1+\sin(c+dx)}\,(1193984-508859\sin(c+dx)-410693\sin^2(c+dx)-543442\sin^3(c+dx)-492846\sin^4(c+dx)-130728\sin^5(c+dx)+257704\sin^6(c+dx)+353648\sin^7(c+dx)+209552\sin^8(c+dx)+63616\sin^9(c+dx)+8064\sin^{10}(c+dx))\right)}{80640d(-1+\sin(c+dx))^2(1+\sin(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Sin[c + d*x])^8,x]
```

```
[Out] -1/80640*(a^8*Cos[c + d*x]^3*(1531530*ArcSin[Sqrt[1 - Sin[c + d*x]]/Sqrt[2]
]*Sqrt[1 - Sin[c + d*x]] + Sqrt[1 + Sin[c + d*x]]*(1193984 - 508859*Sin[c +
 d*x] - 410693*Sin[c + d*x]^2 - 543442*Sin[c + d*x]^3 - 492846*Sin[c + d*x]
^4 - 130728*Sin[c + d*x]^5 + 257704*Sin[c + d*x]^6 + 353648*Sin[c + d*x]^7
+ 209552*Sin[c + d*x]^8 + 63616*Sin[c + d*x]^9 + 8064*Sin[c + d*x]^10)))/(d
*(-1 + Sin[c + d*x])^2*(1 + Sin[c + d*x])^(3/2))
```

Maple [A]
time = 0.49, size = 480, normalized size = 1.83 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*sin(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^8*(-1/10*sin(d*x+c)^7*cos(d*x+c)^3-7/80*sin(d*x+c)^5*cos(d*x+c)^3-7/
96*sin(d*x+c)^3*cos(d*x+c)^3-7/128*sin(d*x+c)*cos(d*x+c)^3+7/256*cos(d*x+c)
*sin(d*x+c)+7/256*d*x+7/256*c)+8*a^8*(-1/9*sin(d*x+c)^6*cos(d*x+c)^3-2/21*s
in(d*x+c)^4*cos(d*x+c)^3-8/105*sin(d*x+c)^2*cos(d*x+c)^3-16/315*cos(d*x+c)^
3)+28*a^8*(-1/8*sin(d*x+c)^5*cos(d*x+c)^3-5/48*sin(d*x+c)^3*cos(d*x+c)^3-5/
64*sin(d*x+c)*cos(d*x+c)^3+5/128*cos(d*x+c)*sin(d*x+c)+5/128*d*x+5/128*c)+5
6*a^8*(-1/7*sin(d*x+c)^4*cos(d*x+c)^3-4/35*sin(d*x+c)^2*cos(d*x+c)^3-8/105*
cos(d*x+c)^3)+70*a^8*(-1/6*sin(d*x+c)^3*cos(d*x+c)^3-1/8*sin(d*x+c)*cos(d*x
+c)^3+1/16*cos(d*x+c)*sin(d*x+c)+1/16*d*x+1/16*c)+56*a^8*(-1/5*sin(d*x+c)^2
*cos(d*x+c)^3-2/15*cos(d*x+c)^3)+28*a^8*(-1/4*sin(d*x+c)*cos(d*x+c)^3+1/8*c
os(d*x+c)*sin(d*x+c)+1/8*d*x+1/8*c)-8/3*cos(d*x+c)^3*a^8+a^8*(1/2*cos(d*x+c
)*sin(d*x+c)+1/2*d*x+1/2*c))
```

Maxima [A]
time = 0.32, size = 319, normalized size = 1.22

$$-\frac{1720320\,a^{8}\cos\left(dx+c\right)^{3}-16384\,\left(35\,\cos\left(dx+c\right)^{9}-135\,\cos\left(dx+c\right)^{7}+189\,\cos\left(dx+c\right)^{5}-105\,\cos\left(dx+c\right)^{3}\right)a^{8}+344064\,\left(15\,\cos\left(dx+c\right)^{7}-42\,\cos\left(dx+c\right)^{5}+35\,\cos\left(dx+c\right)^{3}\right)a^{8}-2408448\,\left(3\,\cos\left(dx+c\right)^{5}-5\,\cos\left(dx+c\right)^{3}\right)a^{8}-21\,\left(96\,\sin\left(2\,dx+2\,c\right)^{5}-640\,\sin\left(2\,dx+2\,c\right)^{3}+840\,dx+840\,c-45\,\sin\left(8\,dx+8\,c\right)-120\,\sin\left(4\,dx+4\,c\right)\right)a^{8}+5880\,\left(64\,\sin\left(2\,dx+2\,c\right)^{3}-120\,dx-120\,c+3\,\sin\left(8\,dx+8\,c\right)+24\,\sin\left(4\,dx+4\,c\right)\right)a^{8}+235200\,\left(4\,\sin\left(2\,dx+2\,c\right)^{3}-12\,dx-12\,c+3\,\sin\left(4\,dx+4\,c\right)\right)a^{8}-564480\,\left(4\,dx+4\,c-\sin\left(4\,dx+4\,c\right)\right)a^{8}-161280\,\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)a^{8}}{645120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/645120*(1720320*a^8*cos(d*x + c)^3 - 16384*(35*cos(d*x + c)^9 - 135*cos(
d*x + c)^7 + 189*cos(d*x + c)^5 - 105*cos(d*x + c)^3)*a^8 + 344064*(15*cos(
d*x + c)^7 - 42*cos(d*x + c)^5 + 35*cos(d*x + c)^3)*a^8 - 2408448*(3*cos(d*
x + c)^5 - 5*cos(d*x + c)^3)*a^8 - 21*(96*sin(2*d*x + 2*c)^5 - 640*sin(2*d*
x + 2*c)^3 + 840*d*x + 840*c - 45*sin(8*d*x + 8*c) - 120*sin(4*d*x + 4*c))*
a^8 + 5880*(64*sin(2*d*x + 2*c)^3 - 120*d*x - 120*c + 3*sin(8*d*x + 8*c) +
24*sin(4*d*x + 4*c))*a^8 + 235200*(4*sin(2*d*x + 2*c)^3 - 12*d*x - 12*c + 3
*sin(4*d*x + 4*c))*a^8 - 564480*(4*d*x + 4*c - sin(4*d*x + 4*c))*a^8 - 1612
80*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^8)/d
```

Fricas [A]
time = 0.40, size = 137, normalized size = 0.52

$$\frac{71680\,a^{8}\cos\left(dx+c\right)^{9}-921600\,a^{8}\cos\left(dx+c\right)^{7}+3096576\,a^{8}\cos\left(dx+c\right)^{5}-3440640\,a^{8}\cos\left(dx+c\right)^{3}+765765\,a^{8}dx+63\left(128\,a^{8}\cos\left(dx+c\right)^{9}-4976\,a^{8}\cos\left(dx+c\right)^{7}+28328\,a^{8}\cos\left(dx+c\right)^{5}-46510\,a^{8}\cos\left(dx+c\right)^{3}+12155\,a^{8}\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{80640\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/80640*(71680*a^8*cos(d*x + c)^9 - 921600*a^8*cos(d*x + c)^7 + 3096576*a^8
*cos(d*x + c)^5 - 3440640*a^8*cos(d*x + c)^3 + 765765*a^8*d*x + 63*(128*a^8
*cos(d*x + c)^9 - 4976*a^8*cos(d*x + c)^7 + 28328*a^8*cos(d*x + c)^5 - 4651
0*a^8*cos(d*x + c)^3 + 12155*a^8*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1018 vs. $2(246) = 492$.
time = 2.12, size = 1018, normalized size = 3.89

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*sin(d*x+c))**8,x)
```

```
[Out] Piecewise((7*a**8*x*sin(c + d*x)**10/256 + 35*a**8*x*sin(c + d*x)**8*cos(c
+ d*x)**2/256 + 35*a**8*x*sin(c + d*x)**8/32 + 35*a**8*x*sin(c + d*x)**6*co
s(c + d*x)**4/128 + 35*a**8*x*sin(c + d*x)**6*cos(c + d*x)**2/8 + 35*a**8*x
*sin(c + d*x)**6/8 + 35*a**8*x*sin(c + d*x)**4*cos(c + d*x)**6/128 + 105*a*
*8*x*sin(c + d*x)**4*cos(c + d*x)**4/16 + 105*a**8*x*sin(c + d*x)**4*cos(c
+ d*x)**2/8 + 7*a**8*x*sin(c + d*x)**4/2 + 35*a**8*x*sin(c + d*x)**2*cos(c
+ d*x)**8/256 + 35*a**8*x*sin(c + d*x)**2*cos(c + d*x)**6/8 + 105*a**8*x*si
n(c + d*x)**2*cos(c + d*x)**4/8 + 7*a**8*x*sin(c + d*x)**2*cos(c + d*x)**2
+ a**8*x*sin(c + d*x)**2/2 + 7*a**8*x*cos(c + d*x)**10/256 + 35*a**8*x*cos(
c + d*x)**8/32 + 35*a**8*x*cos(c + d*x)**6/8 + 7*a**8*x*cos(c + d*x)**4/2 +
 a**8*x*cos(c + d*x)**2/2 + 7*a**8*sin(c + d*x)**9*cos(c + d*x)/(256*d) - 7
9*a**8*sin(c + d*x)**7*cos(c + d*x)**3/(384*d) + 35*a**8*sin(c + d*x)**7*co
s(c + d*x)/(32*d) - 8*a**8*sin(c + d*x)**6*cos(c + d*x)**3/(3*d) - 7*a**8*s
in(c + d*x)**5*cos(c + d*x)**5/(30*d) - 511*a**8*sin(c + d*x)**5*cos(c + d*
x)**3/(96*d) + 35*a**8*sin(c + d*x)**5*cos(c + d*x)/(8*d) - 16*a**8*sin(c +
 d*x)**4*cos(c + d*x)**5/(5*d) - 56*a**8*sin(c + d*x)**4*cos(c + d*x)**3/(3
*d) - 49*a**8*sin(c + d*x)**3*cos(c + d*x)**7/(384*d) - 385*a**8*sin(c + d*
x)**3*cos(c + d*x)**5/(96*d) - 35*a**8*sin(c + d*x)**3*cos(c + d*x)**3/(3*d
) + 7*a**8*sin(c + d*x)**3*cos(c + d*x)/(2*d) - 64*a**8*sin(c + d*x)**2*cos
(c + d*x)**7/(35*d) - 224*a**8*sin(c + d*x)**2*cos(c + d*x)**5/(15*d) - 56*
a**8*sin(c + d*x)**2*cos(c + d*x)**3/(3*d) - 7*a**8*sin(c + d*x)*cos(c + d*
x)**9/(256*d) - 35*a**8*sin(c + d*x)*cos(c + d*x)**7/(32*d) - 35*a**8*sin(c
 + d*x)*cos(c + d*x)**5/(8*d) - 7*a**8*sin(c + d*x)*cos(c + d*x)**3/(2*d) +
```

```
 a**8*sin(c + d*x)*cos(c + d*x)/(2*d) - 128*a**8*cos(c + d*x)**9/(315*d) -
64*a**8*cos(c + d*x)**7/(15*d) - 112*a**8*cos(c + d*x)**5/(15*d) - 8*a**8*c
os(c + d*x)**3/(3*d), Ne(d, 0)), (x*(a*sin(c) + a)**8*cos(c)**2, True))
```

Giac [A]
time = 5.70, size = 174, normalized size = 0.66

$$\frac{2431}{256}a^8x+\frac{a^8\cos(9dx+9c)}{288d}-\frac{33a^8\cos(7dx+7c)}{224d}+\frac{51a^8\cos(5dx+5c)}{40d}-\frac{17a^8\cos(3dx+3c)}{8d}-\frac{221a^8\cos(dx+c)}{16d}+\frac{a^8\sin(10dx+10c)}{5120d}-\frac{59a^8\sin(8dx+8c)}{2048d}+\frac{527a^8\sin(6dx+6c)}{1024d}-\frac{561a^8\sin(4dx+4c)}{256d}-\frac{663a^8\sin(2dx+2c)}{512d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 2431/256*a^8*x + 1/288*a^8*cos(9*d*x + 9*c)/d - 33/224*a^8*cos(7*d*x + 7*c)
/d + 51/40*a^8*cos(5*d*x + 5*c)/d - 17/8*a^8*cos(3*d*x + 3*c)/d - 221/16*a^
8*cos(d*x + c)/d + 1/5120*a^8*sin(10*d*x + 10*c)/d - 59/2048*a^8*sin(8*d*x
+ 8*c)/d + 527/1024*a^8*sin(6*d*x + 6*c)/d - 561/256*a^8*sin(4*d*x + 4*c)/d
 - 663/512*a^8*sin(2*d*x + 2*c)/d
```

Mupad [B]
time = 7.26, size = 572, normalized size = 2.18

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a + a*sin(c + d*x))^8,x)
```

```
[Out] (2431*a^8*x)/256 - ((11809*a^8*tan(c/2 + (d*x)/2)^3)/128 - (23647*a^8*tan(c
/2 + (d*x)/2)^5)/160 - (40749*a^8*tan(c/2 + (d*x)/2)^7)/32 - (70499*a^8*tan
(c/2 + (d*x)/2)^9)/64 + (70499*a^8*tan(c/2 + (d*x)/2)^11)/64 + (40749*a^8*t
an(c/2 + (d*x)/2)^13)/32 + (23647*a^8*tan(c/2 + (d*x)/2)^15)/160 - (11809*a
^8*tan(c/2 + (d*x)/2)^17)/128 - (2175*a^8*tan(c/2 + (d*x)/2)^19)/128 + a^8*
((2431*c)/256 + (2431*d*x)/256) - a^8*((2431*c)/256 + (2431*d*x)/256 - 9328
/315) + tan(c/2 + (d*x)/2)^18*(10*a^8*((2431*c)/256 + (2431*d*x)/256) - a^8
*((12155*c)/128 + (12155*d*x)/128 - 16)) + tan(c/2 + (d*x)/2)^2*(10*a^8*((2
431*c)/256 + (2431*d*x)/256) - a^8*((12155*c)/128 + (12155*d*x)/128 - 17648
/63)) + tan(c/2 + (d*x)/2)^14*(120*a^8*((2431*c)/256 + (2431*d*x)/256) - a^
8*((36465*c)/32 + (36465*d*x)/32 - 1984)) + tan(c/2 + (d*x)/2)^6*(120*a^8*(
(2431*c)/256 + (2431*d*x)/256) - a^8*((36465*c)/32 + (36465*d*x)/32 - 32960
/21)) + tan(c/2 + (d*x)/2)^16*(45*a^8*((2431*c)/256 + (2431*d*x)/256) - a^8
*((109395*c)/256 + (109395*d*x)/256 - 336)) + tan(c/2 + (d*x)/2)^4*(45*a^8*
((2431*c)/256 + (2431*d*x)/256) - a^8*((109395*c)/256 + (109395*d*x)/256 -
6976/7)) + tan(c/2 + (d*x)/2)^10*(252*a^8*((2431*c)/256 + (2431*d*x)/256) -
 a^8*((153153*c)/64 + (153153*d*x)/64 - 18656/5)) + tan(c/2 + (d*x)/2)^12*(
210*a^8*((2431*c)/256 + (2431*d*x)/256) - a^8*((255255*c)/128 + (255255*d*x
)/128 - 4288)) + tan(c/2 + (d*x)/2)^8*(210*a^8*((2431*c)/256 + (2431*d*x)/2
56) - a^8*((255255*c)/128 + (255255*d*x)/128 - 5792/3)) + (2175*a^8*tan(c/2
 + (d*x)/2))/128)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^10)
```

3.45 $\int \cos(c+dx)(a+a\sin(c+dx))^8\,dx$

Optimal. Leaf size=22

$$\frac{(a+a\sin(c+dx))^9}{9ad}$$

```
[Out] 1/9*(a+a*sin(d*x+c))^9/a/d
```

Rubi [A]
time = 0.02, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2746, 32}

$$\frac{(a\sin(c+dx)+a)^9}{9ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Sin[c + d*x])^8,x]
```

```
[Out] (a + a*Sin[c + d*x])^9/(9*a*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a+a\sin(c+dx))^8\,dx &= \frac{\text{Subst}\left(\int (a+x)^8\,dx, x, a\sin(c+dx)\right)}{ad}\\ &= \frac{(a+a\sin(c+dx))^9}{9ad}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 97 vs. 2(22) = 44.
time = 0.85, size = 97, normalized size = 4.41

$$\frac{a^8(-31824\cos(2(c+dx))+8568\cos(4(c+dx))-816\cos(6(c+dx))+18\cos(8(c+dx))+43758\sin(c+dx)-18564\sin(3(c+dx))+3060\sin(5(c+dx))-153\sin(7(c+dx))+\sin(9(c+dx)))}{2304d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Sin[c + d*x])^8,x]
```

```
[Out] (a^8*(-31824*Cos[2*(c + d*x)] + 8568*Cos[4*(c + d*x)] - 816*Cos[6*(c + d*x)
] + 18*Cos[8*(c + d*x)] + 43758*Sin[c + d*x] - 18564*Sin[3*(c + d*x)] + 306
0*Sin[5*(c + d*x)] - 153*Sin[7*(c + d*x)] + Sin[9*(c + d*x)]))/(2304*d)
```

Maple [A]
time = 0.34, size = 21, normalized size = 0.95

method	result
derivativedivides	$\frac{(a+a\sin(dx+c))^9}{9da}$
default	$\frac{(a+a\sin(dx+c))^9}{9da}$
risch	$\frac{2431a^8\sin(dx+c)}{128d}+\frac{a^8\sin(9dx+9c)}{2304d}+\frac{a^8\cos(8dx+8c)}{128d}-\frac{17a^8\sin(7dx+7c)}{256d}-\frac{17a^8\cos(6dx+6c)}{48d}+\frac{85a^8\sin(5d}{64d}$
norman	$\frac{16a^8\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{16a^8\left(\tan^{16}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2a^8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{272a^8\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{952a^8\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{3536a^8\left(\tan^7\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*sin(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*(a+a*sin(d*x+c))^9/d/a
```

Maxima [A]
time = 0.31, size = 20, normalized size = 0.91

$$\frac{(a\sin(dx+c)+a)^9}{9\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 1/9*(a*sin(d*x + c) + a)^9/(a*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 122 vs. $2(20) = 40$.
time = 0.36, size = 122, normalized size = 5.55

$$\frac{9\,a^8\cos(dx+c)^8-120\,a^8\cos(dx+c)^6+432\,a^8\cos(dx+c)^4-576\,a^8\cos(dx+c)^2+\left(a^8\cos(dx+c)^8-40\,a^8\cos(dx+c)^6+240\,a^8\cos(dx+c)^4-448\,a^8\cos(dx+c)^2+256\,a^8\right)\sin(dx+c)}{9\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/9*(9*a^8*cos(d*x + c)^8 - 120*a^8*cos(d*x + c)^6 + 432*a^8*cos(d*x + c)^4
 - 576*a^8*cos(d*x + c)^2 + (a^8*cos(d*x + c)^8 - 40*a^8*cos(d*x + c)^6 + 2
40*a^8*cos(d*x + c)^4 - 448*a^8*cos(d*x + c)^2 + 256*a^8)*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 148 vs. $2(15) = 30$.
time = 1.37, size = 148, normalized size = 6.73

$$\begin{cases} \frac{a^8 \sin^9(c+dx)}{9d} + \frac{a^8 \sin^8(c+dx)}{d} + \frac{4a^8 \sin^7(c+dx)}{d} + \frac{28a^8 \sin^6(c+dx)}{3d} + \frac{14a^8 \sin^5(c+dx)}{d} + \frac{14a^8 \sin^4(c+dx)}{d} + \frac{28a^8 \sin^3(c+dx)}{3d} + \frac{4a^8 \sin^2(c+dx)}{d} + \frac{a^8 \sin(c+dx)}{d} & \text{for } d \neq 0 \\ x(a\sin(c)+a)^8 \cos(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))**8,x)
```

```
[Out] Piecewise((a**8*sin(c + d*x)**9/(9*d) + a**8*sin(c + d*x)**8/d + 4*a**8*sin
(c + d*x)**7/d + 28*a**8*sin(c + d*x)**6/(3*d) + 14*a**8*sin(c + d*x)**5/d
+ 14*a**8*sin(c + d*x)**4/d + 28*a**8*sin(c + d*x)**3/(3*d) + 4*a**8*sin(c
+ d*x)**2/d + a**8*sin(c + d*x)/d, Ne(d, 0)), (x*(a*sin(c) + a)**8*cos(c),
True))
```

Giac [A]
time = 5.06, size = 20, normalized size = 0.91

$$\frac{(a\sin(dx+c)+a)^9}{9\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/9*(a*sin(d*x + c) + a)^9/(a*d)
```

Mupad [B]
time = 4.75, size = 118, normalized size = 5.36

$$\frac{\frac{a^8 \sin(c+d\,x)^9}{9} + a^8 \sin(c+d\,x)^8 + 4\,a^8 \sin(c+d\,x)^7 + \frac{28\,a^8 \sin(c+d\,x)^6}{3} + 14\,a^8 \sin(c+d\,x)^5 + 14\,a^8 \sin(c+d\,x)^4 + \frac{28\,a^8 \sin(c+d\,x)^3}{3} + 4\,a^8 \sin(c+d\,x)^2 + a^8 \sin(c+d\,x)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + a*sin(c + d*x))^8,x)
```

```
[Out] (a^8*sin(c + d*x) + 4*a^8*sin(c + d*x)^2 + (28*a^8*sin(c + d*x)^3)/3 + 14*a
^8*sin(c + d*x)^4 + 14*a^8*sin(c + d*x)^5 + (28*a^8*sin(c + d*x)^6)/3 + 4*a
^8*sin(c + d*x)^7 + a^8*sin(c + d*x)^8 + (a^8*sin(c + d*x)^9)/9)/d
```

3.46 $\int \sec(c+dx)(a+a\sin(c+dx))^8\,dx$

Optimal. Leaf size=162

$$-\frac{128a^8\log(1-\sin(c+dx))}{d}-\frac{64a^8\sin(c+dx)}{d}-\frac{16a^5(a+a\sin(c+dx))^3}{3d}-\frac{4a^3(a+a\sin(c+dx))^5}{5d}-\frac{a^2(a+}{}$$

```
[Out] -128*a^8*ln(1-sin(d*x+c))/d-64*a^8*sin(d*x+c)/d-16/3*a^5*(a+a*sin(d*x+c))^3
/d-4/5*a^3*(a+a*sin(d*x+c))^5/d-1/3*a^2*(a+a*sin(d*x+c))^6/d-1/7*a*(a+a*sin
(d*x+c))^7/d-2*(a^2+a^2*sin(d*x+c))^4/d-16*(a^4+a^4*sin(d*x+c))^2/d
```

Rubi [A]
time = 0.05, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = $\{2746, 45\}$

$$-\frac{64a^8\sin(c+dx)}{d}-\frac{128a^8\log(1-\sin(c+dx))}{d}-\frac{16a^5(a\sin(c+dx)+a)^3}{3d}-\frac{16(a^4\sin(c+dx)+a^4)^2}{d}-\frac{4a^3(a\sin(c+dx)+a)^5}{5d}-\frac{a^2(a\sin(c+dx)+a)^6}{3d}-\frac{2(a^2\sin(c+dx)+a^2)^4}{d}-\frac{a(a\sin(c+dx)+a)^7}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sin[c + d*x])^8,x]
```

```
[Out] (-128*a^8*Log[1 - Sin[c + d*x]])/d - (64*a^8*Sin[c + d*x])/d - (16*a^5*(a +
 a*Sin[c + d*x])^3)/(3*d) - (4*a^3*(a + a*Sin[c + d*x])^5)/(5*d) - (a^2*(a
+ a*Sin[c + d*x])^6)/(3*d) - (a*(a + a*Sin[c + d*x])^7)/(7*d) - (2*(a^2 + a
^2*Sin[c + d*x])^4)/d - (16*(a^4 + a^4*Sin[c + d*x])^2)/d
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\int \sec(c+dx)(a+a\sin(c+dx))^8\,dx = \frac{a\text{Subst}\left(\int \frac{(a+x)^7}{a-x}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a\text{Subst}\left(\int \left(-64a^6+\frac{128a^7}{a-x}-32a^5(a+x)-16a^4(a+x)^2-8a^3(a+x\right.\right.}{d}$$
$$= -\frac{128a^8\log(1-\sin(c+dx))}{d}-\frac{64a^8\sin(c+dx)}{d}-\frac{16a^5(a+a\sin(c-}{3d}$$

Mathematica [A]
time = 0.12, size = 95, normalized size = 0.59

$$\frac{a^8\left(-128\log(1-\sin(c+dx))-127\sin(c+dx)-60\sin^2(c+dx)-33\sin^3(c+dx)-16\sin^4(c+dx)-\frac{29}{5}\sin^5(c+dx)-\frac{4}{3}\sin^6(c+dx)-\frac{1}{7}\sin^7(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sin[c + d*x])^8,x]
```

```
[Out] (a^8*(-128*Log[1 - Sin[c + d*x]] - 127*Sin[c + d*x] - 60*Sin[c + d*x]^2 - 3
3*Sin[c + d*x]^3 - 16*Sin[c + d*x]^4 - (29*Sin[c + d*x]^5)/5 - (4*Sin[c + d
*x]^6)/3 - Sin[c + d*x]^7/7))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 311 vs. $2(154) = 308$.
time = 0.22, size = 312, normalized size = 1.93 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+a*sin(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^8*(-1/7*sin(d*x+c)^7-1/5*sin(d*x+c)^5-1/3*sin(d*x+c)^3-sin(d*x+c)+ln
(sec(d*x+c)+tan(d*x+c)))+8*a^8*(-1/6*sin(d*x+c)^6-1/4*sin(d*x+c)^4-1/2*sin(
d*x+c)^2-ln(cos(d*x+c)))+28*a^8*(-1/5*sin(d*x+c)^5-1/3*sin(d*x+c)^3-sin(d*x
+c)+ln(sec(d*x+c)+tan(d*x+c)))+56*a^8*(-1/4*sin(d*x+c)^4-1/2*sin(d*x+c)^2-l
n(cos(d*x+c)))+70*a^8*(-1/3*sin(d*x+c)^3-sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c
)))+56*a^8*(-1/2*sin(d*x+c)^2-ln(cos(d*x+c)))+28*a^8*(-sin(d*x+c)+ln(sec(d*
x+c)+tan(d*x+c)))-8*a^8*ln(cos(d*x+c))+a^8*ln(sec(d*x+c)+tan(d*x+c)))
```

Maxima [A]
time = 0.31, size = 109, normalized size = 0.67

$$-\frac{15\,a^8\sin(dx+c)^7+140\,a^8\sin(dx+c)^6+609\,a^8\sin(dx+c)^5+1680\,a^8\sin(dx+c)^4+3465\,a^8\sin(dx+c)^3+6300\,a^8\sin(dx+c)^2+13440\,a^8\log(\sin(dx+c)-1)+13335\,a^8\sin(dx+c)}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/105*(15*a^8*sin(d*x + c)^7 + 140*a^8*sin(d*x + c)^6 + 609*a^8*sin(d*x +
c)^5 + 1680*a^8*sin(d*x + c)^4 + 3465*a^8*sin(d*x + c)^3 + 6300*a^8*sin(d*x
 + c)^2 + 13440*a^8*log(sin(d*x + c) - 1) + 13335*a^8*sin(d*x + c))/d
```

Fricas [A]
time = 0.38, size = 114, normalized size = 0.70

$$\frac{140\,a^8\cos\left(dx+c\right)^6-2100\,a^8\cos\left(dx+c\right)^4+10080\,a^8\cos\left(dx+c\right)^2-13440\,a^8\log\left(-\sin\left(dx+c\right)+1\right)+3\left(5\,a^8\cos\left(dx+c\right)^6-218\,a^8\cos\left(dx+c\right)^4+1576\,a^8\cos\left(dx+c\right)^2-5808\,a^8\right)\sin\left(dx+c\right)}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/105*(140*a^8*cos(d*x + c)^6 - 2100*a^8*cos(d*x + c)^4 + 10080*a^8*cos(d*x
 + c)^2 - 13440*a^8*log(-sin(d*x + c) + 1) + 3*(5*a^8*cos(d*x + c)^6 - 218*
a^8*cos(d*x + c)^4 + 1576*a^8*cos(d*x + c)^2 - 5808*a^8)*sin(d*x + c))/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))**8,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [A]
time = 4.95, size = 288, normalized size = 1.78

$$\frac{2\left(6720\,a^8\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)-13440\,a^8\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{17424\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{14}+13335\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}+134568\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{12}+93870\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+442344\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}+265209\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+780640\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}+370308\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+780640\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+265209\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+442344\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+93870\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+134568\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+13335\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+17424\,a^8}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^7}\right)}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 2/105*(6720*a^8*log(tan(1/2*d*x + 1/2*c)^2 + 1) - 13440*a^8*log(abs(tan(1/2
*d*x + 1/2*c) - 1)) - (17424*a^8*tan(1/2*d*x + 1/2*c)^14 + 13335*a^8*tan(1/
2*d*x + 1/2*c)^13 + 134568*a^8*tan(1/2*d*x + 1/2*c)^12 + 93870*a^8*tan(1/2*
d*x + 1/2*c)^11 + 442344*a^8*tan(1/2*d*x + 1/2*c)^10 + 265209*a^8*tan(1/2*d
*x + 1/2*c)^9 + 780640*a^8*tan(1/2*d*x + 1/2*c)^8 + 370308*a^8*tan(1/2*d*x
+ 1/2*c)^7 + 780640*a^8*tan(1/2*d*x + 1/2*c)^6 + 265209*a^8*tan(1/2*d*x + 1
/2*c)^5 + 442344*a^8*tan(1/2*d*x + 1/2*c)^4 + 93870*a^8*tan(1/2*d*x + 1/2*c
)^3 + 134568*a^8*tan(1/2*d*x + 1/2*c)^2 + 13335*a^8*tan(1/2*d*x + 1/2*c) +
17424*a^8)/(tan(1/2*d*x + 1/2*c)^2 + 1)^7)/d
```

Mupad [B]
time = 4.65, size = 109, normalized size = 0.67

$$-\frac{128\,a^8\,\ln\left(\sin\left(c+d\,x\right)-1\right)+127\,a^8\,\sin\left(c+d\,x\right)+60\,a^8\sin\left(c+d\,x\right)^2+33\,a^8\sin\left(c+d\,x\right)^3+16\,a^8\sin\left(c+d\,x\right)^4+\frac{29\,a^8\sin\left(c+d\,x\right)^5}{5}+\frac{4\,a^8\sin\left(c+d\,x\right)^6}{3}+\frac{a^8\sin\left(c+d\,x\right)^7}{7}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^8/cos(c + d*x),x)
```

```
[Out] -(128*a^8*log(sin(c + d*x) - 1) + 127*a^8*sin(c + d*x) + 60*a^8*sin(c + d*x
)^2 + 33*a^8*sin(c + d*x)^3 + 16*a^8*sin(c + d*x)^4 + (29*a^8*sin(c + d*x)^
5)/5 + (4*a^8*sin(c + d*x)^6)/3 + (a^8*sin(c + d*x)^7)/7)/d
```

3.47 $\int \sec^2(c+dx)(a+a\sin(c+dx))^8\,dx$

Optimal. Leaf size=201

$$-\frac{3003a^8x}{16}+\frac{1001a^8\cos^5(c+dx)}{10d}-\frac{3003a^8\cos(c+dx)\sin(c+dx)}{16d}-\frac{1001a^8\cos^3(c+dx)\sin(c+dx)}{8d}+\frac{2a^{15}}{d(a-}$$

```
[Out] -3003/16*a^8*x+1001/10*a^8*cos(d*x+c)^5/d-3003/16*a^8*cos(d*x+c)*sin(d*x+c)
/d-1001/8*a^8*cos(d*x+c)^3*sin(d*x+c)/d+2*a^15*cos(d*x+c)^13/d/(a-a*sin(d*x
+c))^7+26*a^13*cos(d*x+c)^11/d/(a-a*sin(d*x+c))^5+286/3*a^14*cos(d*x+c)^9/d
/(a^2-a^2*sin(d*x+c))^3+143/2*a^16*cos(d*x+c)^7/d/(a^8-a^8*sin(d*x+c))
```

Rubi [A]
time = 0.24, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2749, 2759, 2758, 2761, 2715, 8}

$$\frac{2a^{15}\cos^{13}(c+dx)}{d(a-a\sin(c+dx))^7}+\frac{26a^{13}\cos^{11}(c+dx)}{d(a-a\sin(c+dx))^5}+\frac{1001a^8\cos^5(c+dx)}{10d}-\frac{1001a^8\sin(c+dx)\cos^3(c+dx)}{8d}-\frac{3003a^8\sin(c+dx)\cos(c+dx)}{16d}-\frac{3003a^8x}{16}+\frac{143a^{16}\cos^7(c+dx)}{2d\left(a^8-a^8\sin(c+dx)\right)}+\frac{286a^{14}\cos^9(c+dx)}{3d\left(a^2-a^2\sin(c+dx)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sin[c + d*x])^8,x]
```

```
[Out] (-3003*a^8*x)/16 + (1001*a^8*Cos[c + d*x]^5)/(10*d) - (3003*a^8*Cos[c + d*x
]*Sin[c + d*x])/(16*d) - (1001*a^8*Cos[c + d*x]^3*Sin[c + d*x])/(8*d) + (2*
a^15*Cos[c + d*x]^13)/(d*(a - a*Sin[c + d*x])^7) + (26*a^13*Cos[c + d*x]^11
)/(d*(a - a*Sin[c + d*x])^5) + (286*a^14*Cos[c + d*x]^9)/(3*d*(a^2 - a^2*Si
n[c + d*x])^3) + (143*a^16*Cos[c + d*x]^7)/(2*d*(a^8 - a^8*Sin[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2749

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2758

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(a*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rubi steps

$$\int \sec^2(c+dx)(a+a\sin(c+dx))^8\,dx = a^{16}\int \frac{\cos^{14}(c+dx)}{(a-a\sin(c+dx))^8}\,dx$$
$$= \frac{2a^{15}\cos^{13}(c+dx)}{d(a-a\sin(c+dx))^7} - (13a^{14})\int \frac{\cos^{12}(c+dx)}{(a-a\sin(c+dx))^6}\,dx$$
$$= \frac{2a^{15}\cos^{13}(c+dx)}{d(a-a\sin(c+dx))^7} + \frac{26a^{13}\cos^{11}(c+dx)}{d(a-a\sin(c+dx))^5} - (143a^{12})\int \frac{\mathrm{co}}{(a-}$$
$$= \frac{2a^{15}\cos^{13}(c+dx)}{d(a-a\sin(c+dx))^7} + \frac{26a^{13}\cos^{11}(c+dx)}{d(a-a\sin(c+dx))^5} + \frac{286a^{11}\cos^9(c+}{3d(a-a\sin(c+}$$
$$= \frac{2a^{15}\cos^{13}(c+dx)}{d(a-a\sin(c+dx))^7} + \frac{26a^{13}\cos^{11}(c+dx)}{d(a-a\sin(c+dx))^5} + \frac{286a^{11}\cos^9(c+}{3d(a-a\sin(c+}$$
$$= \frac{1001a^8\cos^5(c+dx)}{10d} + \frac{2a^{15}\cos^{13}(c+dx)}{d(a-a\sin(c+dx))^7} + \frac{26a^{13}\cos^{11}(c+dx}{d(a-a\sin(c+dx}$$
$$= \frac{1001a^8\cos^5(c+dx)}{10d} - \frac{1001a^8\cos^3(c+dx)\sin(c+dx)}{8d} + \frac{2a^{15}\mathrm{co}}{d(a-a}$$
$$= \frac{1001a^8\cos^5(c+dx)}{10d} - \frac{3003a^8\cos(c+dx)\sin(c+dx)}{16d} - \frac{1001a^8\mathrm{co}}{}$$
$$= -\frac{3003a^8x}{16} + \frac{1001a^8\cos^5(c+dx)}{10d} - \frac{3003a^8\cos(c+dx)\sin(c+dx}{16d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.04, size = 55, normalized size = 0.27

$$\frac{128\sqrt{2}\,a^8\,{}_2F_1\left(-\frac{13}{2},-\frac{1}{2};\frac{1}{2};\frac{1}{2}(1-\sin(c+dx))\right)\sec(c+dx)\sqrt{1+\sin(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sin[c + d*x])^8,x]
```

```
[Out] (128*Sqrt[2]*a^8*Hypergeometric2F1[-13/2, -1/2, 1/2, (1 - Sin[c + d*x])/2]*
Sec[c + d*x]*Sqrt[1 + Sin[c + d*x]])/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 388 vs. $2(189) = 378$.
time = 0.13, size = 389, normalized size = 1.94

method	result
risch	$-\frac{3003a^8x}{16} + \frac{173a^8e^{i(dx+c)}}{2d} + \frac{173a^8e^{-i(dx+c)}}{2d} + \frac{256a^8}{d(e^{i(dx+c)}-i)} + \frac{a^8\sin(6dx+6c)}{192d} + \frac{a^8\cos(5dx+5c)}{10d} - \frac{61a^8}{}$

derivativedivides	$\frac{a^8\left(\frac{\sin^9(dx+c)}{\cos(dx+c)}+\left(\sin^7(dx+c)+\frac{7\left(\sin^5(dx+c)\right)}{6}+\frac{35\left(\sin^3(dx+c)\right)}{24}+\frac{35\sin(dx+c)}{16}\right)\cos(dx+c)-\frac{35dx}{16}-\frac{35c}{16}\right)+8a^8\left(\frac{\sin^8(dx+c)}{\cos(dx+c)}\right.}{}$
default	$\frac{a^8\left(\frac{\sin^9(dx+c)}{\cos(dx+c)}+\left(\sin^7(dx+c)+\frac{7\left(\sin^5(dx+c)\right)}{6}+\frac{35\left(\sin^3(dx+c)\right)}{24}+\frac{35\sin(dx+c)}{16}\right)\cos(dx+c)-\frac{35dx}{16}-\frac{35c}{16}\right)+8a^8\left(\frac{\sin^8(dx+c)}{\cos(dx+c)}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+a*sin(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^8*(sin(d*x+c)^9/cos(d*x+c)+(sin(d*x+c)^7+7/6*sin(d*x+c)^5+35/24*sin(
d*x+c)^3+35/16*sin(d*x+c))*cos(d*x+c)-35/16*d*x-35/16*c)+8*a^8*(sin(d*x+c)^
8/cos(d*x+c)+(16/5+sin(d*x+c)^6+6/5*sin(d*x+c)^4+8/5*sin(d*x+c)^2)*cos(d*x+
c))+28*a^8*(sin(d*x+c)^7/cos(d*x+c)+(sin(d*x+c)^5+5/4*sin(d*x+c)^3+15/8*sin
(d*x+c))*cos(d*x+c)-15/8*d*x-15/8*c)+56*a^8*(sin(d*x+c)^6/cos(d*x+c)+(8/3+s
in(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c))+70*a^8*(sin(d*x+c)^5/cos(d*x+c)+(
sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)-3/2*d*x-3/2*c)+56*a^8*(sin(d*x+c)^4
/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d*x+c))+28*a^8*(tan(d*x+c)-d*x-c)+8*a^8/co
s(d*x+c)+a^8*tan(d*x+c))
```

Maxima [A]
time = 0.54, size = 331, normalized size = 1.65

$$\frac{384\left(\cos(dx+c)^5-5\cos(dx+c)^3+\frac{5}{\cos(dx+c)}+15\cos(dx+c)\right)a^8-4480\left(\cos(dx+c)^3-\frac{3}{\cos(dx+c)}-6\cos(dx+c)\right)a^8-5\left(105dx+105c-\frac{87\tan(dx+c)^5+136\tan(dx+c)^3+57\tan(dx+c)}{\tan(dx+c)^6+3\tan(dx+c)^4+3\tan(dx+c)^2+1}-48\tan(dx+c)\right)a^8-840\left(15dx+15c-\frac{9\tan(dx+c)^3+7\tan(dx+c)}{\tan(dx+c)^4+2\tan(dx+c)^2+1}-8\tan(dx+c)\right)a^8-8400\left(3dx+3c-\frac{\tan(dx+c)}{\tan(dx+c)^2+1}-2\tan(dx+c)\right)a^8-6720(dx+c-\tan(dx+c))a^8+13440a^8\left(\frac{1}{\cos(dx+c)}+\cos(dx+c)\right)+240a^8\tan(dx+c)+\frac{1920a^8}{\cos(dx+c)}}{240d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 1/240*(384*(cos(d*x + c)^5 - 5*cos(d*x + c)^3 + 5/cos(d*x + c) + 15*cos(d*x
 + c))*a^8 - 4480*(cos(d*x + c)^3 - 3/cos(d*x + c) - 6*cos(d*x + c))*a^8 -
5*(105*d*x + 105*c - (87*tan(d*x + c)^5 + 136*tan(d*x + c)^3 + 57*tan(d*x +
 c))/(tan(d*x + c)^6 + 3*tan(d*x + c)^4 + 3*tan(d*x + c)^2 + 1) - 48*tan(d*
x + c))*a^8 - 840*(15*d*x + 15*c - (9*tan(d*x + c)^3 + 7*tan(d*x + c))/(tan
(d*x + c)^4 + 2*tan(d*x + c)^2 + 1) - 8*tan(d*x + c))*a^8 - 8400*(3*d*x + 3
*c - tan(d*x + c)/(tan(d*x + c)^2 + 1) - 2*tan(d*x + c))*a^8 - 6720*(d*x +
c - tan(d*x + c))*a^8 + 13440*a^8*(1/cos(d*x + c) + cos(d*x + c)) + 240*a^8
*tan(d*x + c) + 1920*a^8/cos(d*x + c))/d
```

Fricas [A]
time = 0.36, size = 231, normalized size = 1.15

$$\frac{40a^8\cos(dx+c)^7+384a^8\cos(dx+c)^6-1526a^8\cos(dx+c)^5-6400a^8\cos(dx+c)^4+11865a^8\cos(dx+c)^3-45045a^8dx+46080a^8\cos(dx+c)^2+30720a^8-15\left(3003a^8dx-4027a^8\right)\cos(dx+c)+\left(40a^8\cos(dx+c)^6-344a^8\cos(dx+c)^5-1870a^8\cos(dx+c)^4+4530a^8\cos(dx+c)^3+45045a^8dx+16395a^8\cos(dx+c)^2-29685a^8\cos(dx+c)+30720a^8\right)\sin(dx+c)}{240\left(d\cos(dx+c)-d\sin(dx+c)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/240*(40*a^8*cos(d*x + c)^7 + 384*a^8*cos(d*x + c)^6 - 1526*a^8*cos(d*x +
c)^5 - 6400*a^8*cos(d*x + c)^4 + 11865*a^8*cos(d*x + c)^3 - 45045*a^8*d*x +
 46080*a^8*cos(d*x + c)^2 + 30720*a^8 - 15*(3003*a^8*d*x - 4027*a^8)*cos(d*
x + c) + (40*a^8*cos(d*x + c)^6 - 344*a^8*cos(d*x + c)^5 - 1870*a^8*cos(d*x
 + c)^4 + 4530*a^8*cos(d*x + c)^3 + 45045*a^8*d*x + 16395*a^8*cos(d*x + c)^
2 - 29685*a^8*cos(d*x + c) + 30720*a^8)*sin(d*x + c))/(d*cos(d*x + c) - d*s
in(d*x + c) + d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+a*sin(d*x+c))**8,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep
```

Giac [A]
time = 4.49, size = 231, normalized size = 1.15

$$-\frac{45045\,(dx+c)a^8+\frac{61440\,a^8}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1}+\frac{2\left(14565\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-28800\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}+50855\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-174720\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}+36930\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-400640\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-36930\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-426240\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-50855\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-211584\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-14565\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-40064\,a^8\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^6}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -1/240*(45045*(d*x + c)*a^8 + 61440*a^8/(tan(1/2*d*x + 1/2*c) - 1) + 2*(145
65*a^8*tan(1/2*d*x + 1/2*c)^11 - 28800*a^8*tan(1/2*d*x + 1/2*c)^10 + 50855*
a^8*tan(1/2*d*x + 1/2*c)^9 - 174720*a^8*tan(1/2*d*x + 1/2*c)^8 + 36930*a^8*
tan(1/2*d*x + 1/2*c)^7 - 400640*a^8*tan(1/2*d*x + 1/2*c)^6 - 36930*a^8*tan(
1/2*d*x + 1/2*c)^5 - 426240*a^8*tan(1/2*d*x + 1/2*c)^4 - 50855*a^8*tan(1/2*
d*x + 1/2*c)^3 - 211584*a^8*tan(1/2*d*x + 1/2*c)^2 - 14565*a^8*tan(1/2*d*x
+ 1/2*c) - 40064*a^8)/(tan(1/2*d*x + 1/2*c)^2 + 1)^6)/d
```

Mupad [B]
time = 8.73, size = 513, normalized size = 2.55

$$-\frac{3003\,a^8\,x}{16}-\frac{[illegible]}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^8/cos(c + d*x)^2,x)
```

```
[Out] - (3003*a^8*x)/16 - ((3003*a^8*(c + d*x))/16 - tan(c/2 + (d*x)/2)*((3003*a^
8*(c + d*x))/16 - (a^8*(45045*c + 45045*d*x - 50998))/240) - (a^8*(45045*c
+ 45045*d*x - 141568))/240 + tan(c/2 + (d*x)/2)^12*((3003*a^8*(c + d*x))/16
 - (a^8*(45045*c + 45045*d*x - 90570))/240) - tan(c/2 + (d*x)/2)^11*((9009*
```

```
a^8*(c + d*x))/8 - (a^8*(270270*c + 270270*d*x - 86730))/240) - tan(c/2 + (
d*x)/2)^3*((9009*a^8*(c + d*x))/8 - (a^8*(270270*c + 270270*d*x - 321458))/
240) + tan(c/2 + (d*x)/2)^10*((9009*a^8*(c + d*x))/8 - (a^8*(270270*c + 270
270*d*x - 527950))/240) + tan(c/2 + (d*x)/2)^2*((9009*a^8*(c + d*x))/8 - (a
^8*(270270*c + 270270*d*x - 762678))/240) - tan(c/2 + (d*x)/2)^9*((45045*a^
8*(c + d*x))/16 - (a^8*(675675*c + 675675*d*x - 451150))/240) - tan(c/2 + (
d*x)/2)^5*((45045*a^8*(c + d*x))/16 - (a^8*(675675*c + 675675*d*x - 778620)
)/240) - tan(c/2 + (d*x)/2)^7*((15015*a^8*(c + d*x))/4 - (a^8*(900900*c + 9
00900*d*x - 875140))/240) + tan(c/2 + (d*x)/2)^8*((45045*a^8*(c + d*x))/16
- (a^8*(675675*c + 675675*d*x - 1344900))/240) + tan(c/2 + (d*x)/2)^4*((450
45*a^8*(c + d*x))/16 - (a^8*(675675*c + 675675*d*x - 1672370))/240) + tan(c
/2 + (d*x)/2)^6*((15015*a^8*(c + d*x))/4 - (a^8*(900900*c + 900900*d*x - 19
56220))/240))/(d*(tan(c/2 + (d*x)/2) - 1)*(tan(c/2 + (d*x)/2)^2 + 1)^6)
```

3.48 $\int \sec^3(c+dx)(a+a\sin(c+dx))^8\,dx$

Optimal. Leaf size=121

$$\frac{192a^8\log(1-\sin(c+dx))}{d}+\frac{129a^8\sin(c+dx)}{d}+\frac{36a^8\sin^2(c+dx)}{d}+\frac{10a^8\sin^3(c+dx)}{d}+\frac{2a^8\sin^4(c+dx)}{d}+a^8 \ldots +$$

```
[Out] 192*a^8*ln(1-sin(d*x+c))/d+129*a^8*sin(d*x+c)/d+36*a^8*sin(d*x+c)^2/d+10*a^
8*sin(d*x+c)^3/d+2*a^8*sin(d*x+c)^4/d+1/5*a^8*sin(d*x+c)^5/d+64*a^9/d/(a-a*
sin(d*x+c))
```

Rubi [A]
time = 0.06, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2746, 45}

$$\frac{64a^9}{d(a-a\sin(c+dx))}+\frac{a^8\sin^5(c+dx)}{5d}+\frac{2a^8\sin^4(c+dx)}{d}+\frac{10a^8\sin^3(c+dx)}{d}+\frac{36a^8\sin^2(c+dx)}{d}+\frac{129a^8\sin(c+dx)}{d}+\frac{192a^8\log(1-\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sin[c + d*x])^8,x]
```

```
[Out] (192*a^8*Log[1 - Sin[c + d*x]])/d + (129*a^8*Sin[c + d*x])/d + (36*a^8*Sin[
c + d*x]^2)/d + (10*a^8*Sin[c + d*x]^3)/d + (2*a^8*Sin[c + d*x]^4)/d + (a^8
*Sin[c + d*x]^5)/(5*d) + (64*a^9)/(d*(a - a*Sin[c + d*x]))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\int \sec^3(c+dx)(a+a\sin(c+dx))^8\,dx = \frac{a^3\text{Subst}\left(\int \frac{(a+x)^6}{(a-x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^3\text{Subst}\left(\int \left(129a^4 + \frac{64a^6}{(a-x)^2} - \frac{192a^5}{a-x} + 72a^3x + 30a^2x^2 + 8ax^3 + x^4\right)\right.}{d}$$

$$= \frac{192a^8\log(1-\sin(c+dx))}{d} + \frac{129a^8\sin(c+dx)}{d} + \frac{36a^8\sin^2(c+dx)}{d}$$

Mathematica [A]
time = 0.18, size = 111, normalized size = 0.92

$$\frac{a^8\sec^2(c+dx)(1-\sin(c+dx))(1+\sin(c+dx))\left(192\log(1-\sin(c+dx)) + \frac{64}{1-\sin(c+dx)} + 129\sin(c+dx) + 36\sin^2(c+dx) + 10\sin^3(c+dx) + 2\sin^4(c+dx) + \frac{1}{5}\sin^5(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sin[c + d*x])^8,x]
```

```
[Out] (a^8*Sec[c + d*x]^2*(1 - Sin[c + d*x])*(1 + Sin[c + d*x])*(192*Log[1 - Sin[
c + d*x]] + 64/(1 - Sin[c + d*x]) + 129*Sin[c + d*x] + 36*Sin[c + d*x]^2 +
10*Sin[c + d*x]^3 + 2*Sin[c + d*x]^4 + Sin[c + d*x]^5/5))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 441 vs. $2(119) = 238$.
time = 0.21, size = 442, normalized size = 3.65

method	result
risch	$-192ia^8x - \frac{1093ia^8e^{i(dx+c)}}{16d} + \frac{1093ia^8e^{-i(dx+c)}}{16d} - \frac{384ia^8c}{d} - \frac{128ia^8e^{i(dx+c)}}{\left(e^{i(dx+c)}-i\right)^2d} + \frac{384a^8\ln\left(e^{i(dx+c)}-i\right)}{d} + $
derivativedivides	$\frac{a^8\left(\frac{\sin^9(dx+c)}{2\cos(dx+c)^2} + \frac{\left(\sin^7(dx+c)\right)}{2} + \frac{7\left(\sin^5(dx+c)\right)}{10} + \frac{7\left(\sin^3(dx+c)\right)}{6} + \frac{7\sin(dx+c)}{2} - \frac{7\ln(\sec(dx+c)+\tan(dx+c))}{2}\right) + 8a^8\left(\frac{\sin^8(dx+}{2\cos(dx+}\right.}{}$
default	$\frac{a^8\left(\frac{\sin^9(dx+c)}{2\cos(dx+c)^2} + \frac{\left(\sin^7(dx+c)\right)}{2} + \frac{7\left(\sin^5(dx+c)\right)}{10} + \frac{7\left(\sin^3(dx+c)\right)}{6} + \frac{7\sin(dx+c)}{2} - \frac{7\ln(\sec(dx+c)+\tan(dx+c))}{2}\right) + 8a^8\left(\frac{\sin^8(dx+}{2\cos(dx+}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+a*sin(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^8*(1/2*sin(d*x+c)^9/cos(d*x+c)^2+1/2*sin(d*x+c)^7+7/10*sin(d*x+c)^5+
7/6*sin(d*x+c)^3+7/2*sin(d*x+c)-7/2*ln(sec(d*x+c)+tan(d*x+c)))+8*a^8*(1/2*s
in(d*x+c)^8/cos(d*x+c)^2+1/2*sin(d*x+c)^6+3/4*sin(d*x+c)^4+3/2*sin(d*x+c)^2
+3*ln(cos(d*x+c)))+28*a^8*(1/2*sin(d*x+c)^7/cos(d*x+c)^2+1/2*sin(d*x+c)^5+5
/6*sin(d*x+c)^3+5/2*sin(d*x+c)-5/2*ln(sec(d*x+c)+tan(d*x+c)))+56*a^8*(1/2*s
```

```
in(d*x+c)^6/cos(d*x+c)^2+1/2*sin(d*x+c)^4+sin(d*x+c)^2+2*ln(cos(d*x+c)))+70
*a^8*(1/2*sin(d*x+c)^5/cos(d*x+c)^2+1/2*sin(d*x+c)^3+3/2*sin(d*x+c)-3/2*ln(
sec(d*x+c)+tan(d*x+c)))+56*a^8*(1/2*tan(d*x+c)^2+ln(cos(d*x+c)))+28*a^8*(1/
2*sin(d*x+c)^3/cos(d*x+c)^2+1/2*sin(d*x+c)-1/2*ln(sec(d*x+c)+tan(d*x+c)))+4
*a^8/cos(d*x+c)^2+a^8*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+
c))))
```

Maxima [A]
time = 0.29, size = 97, normalized size = 0.80

$$\frac{a^8 \sin(dx+c)^5 + 10\,a^8 \sin(dx+c)^4 + 50\,a^8 \sin(dx+c)^3 + 180\,a^8 \sin(dx+c)^2 + 960\,a^8 \log(\sin(dx+c)-1) + 645\,a^8 \sin(dx+c) - \frac{320\,a^8}{\sin(dx+c)-1}}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 1/5*(a^8*sin(d*x + c)^5 + 10*a^8*sin(d*x + c)^4 + 50*a^8*sin(d*x + c)^3 + 1
80*a^8*sin(d*x + c)^2 + 960*a^8*log(sin(d*x + c) - 1) + 645*a^8*sin(d*x + c
) - 320*a^8/(sin(d*x + c) - 1))/d
```

Fricas [A]
time = 0.38, size = 130, normalized size = 1.07

$$-\frac{4\,a^8 \cos(dx+c)^6 - 172\,a^8 \cos(dx+c)^4 + 2192\,a^8 \cos(dx+c)^2 - 1119\,a^8 - 3840\,(a^8 \sin(dx+c) - a^8) \log(-\sin(dx+c)+1) - (36\,a^8 \cos(dx+c)^4 - 592\,a^8 \cos(dx+c)^2 - 2399\,a^8) \sin(dx+c)}{20\,(d \sin(dx+c) - d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] -1/20*(4*a^8*cos(d*x + c)^6 - 172*a^8*cos(d*x + c)^4 + 2192*a^8*cos(d*x + c
)^2 - 1119*a^8 - 3840*(a^8*sin(d*x + c) - a^8)*log(-sin(d*x + c) + 1) - (36
*a^8*cos(d*x + c)^4 - 592*a^8*cos(d*x + c)^2 - 2399*a^8)*sin(d*x + c))/(d*s
in(d*x + c) - d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+a*sin(d*x+c))**8,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 275 vs. 2(120) = 240.
time = 5.32, size = 275, normalized size = 2.27

$$-\frac{2\left(480\,a^8 \log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right) - 960\,a^8 \log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) + \frac{160\left(9\,a^8 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 - 20\,a^8 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) + 9\,a^8\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^2} - \frac{1096\,a^8 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10} + 645\,a^8 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9 + 5840\,a^8 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8 + 2780\,a^8 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 + 12120\,a^8 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6 + 4286\,a^8 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 + 12120\,a^8 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4 + 2780\,a^8 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + 5840\,a^8 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 + 645\,a^8 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) + 1096\,a^8}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^5}\right)}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -2/5*(480*a^8*log(tan(1/2*d*x + 1/2*c)^2 + 1) - 960*a^8*log(abs(tan(1/2*d*x
 + 1/2*c) - 1)) + 160*(9*a^8*tan(1/2*d*x + 1/2*c)^2 - 20*a^8*tan(1/2*d*x +
1/2*c) + 9*a^8)/(tan(1/2*d*x + 1/2*c) - 1)^2 - (1096*a^8*tan(1/2*d*x + 1/2*
c)^10 + 645*a^8*tan(1/2*d*x + 1/2*c)^9 + 5840*a^8*tan(1/2*d*x + 1/2*c)^8 +
2780*a^8*tan(1/2*d*x + 1/2*c)^7 + 12120*a^8*tan(1/2*d*x + 1/2*c)^6 + 4286*a
^8*tan(1/2*d*x + 1/2*c)^5 + 12120*a^8*tan(1/2*d*x + 1/2*c)^4 + 2780*a^8*tan
(1/2*d*x + 1/2*c)^3 + 5840*a^8*tan(1/2*d*x + 1/2*c)^2 + 645*a^8*tan(1/2*d*x
 + 1/2*c) + 1096*a^8)/(tan(1/2*d*x + 1/2*c)^2 + 1)^5)/d
```

Mupad [B]
time = 4.62, size = 97, normalized size = 0.80

$$\frac{192\,a^8\,\ln\left(\sin\left(c+d\,x\right)-1\right)-\frac{64\,a^8}{\sin\left(c+d\,x\right)-1}+129\,a^8\,\sin\left(c+d\,x\right)+36\,a^8\,{\sin\left(c+d\,x\right)}^2+10\,a^8\,{\sin\left(c+d\,x\right)}^3+2\,a^8\,{\sin\left(c+d\,x\right)}^4+\frac{a^8\,{\sin\left(c+d\,x\right)}^5}{5}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^8/cos(c + d*x)^3,x)
```

```
[Out] (192*a^8*log(sin(c + d*x) - 1) - (64*a^8)/(sin(c + d*x) - 1) + 129*a^8*sin(
c + d*x) + 36*a^8*sin(c + d*x)^2 + 10*a^8*sin(c + d*x)^3 + 2*a^8*sin(c + d*
x)^4 + (a^8*sin(c + d*x)^5)/5)/d
```

3.49 $\int \sec^4(c+dx)(a+a\sin(c+dx))^8\,dx$

Optimal. Leaf size=179

$$\frac{1155a^8x}{8}-\frac{385a^8\cos^3(c+dx)}{4d}+\frac{1155a^8\cos(c+dx)\sin(c+dx)}{8d}+\frac{2a^{15}\cos^{11}(c+dx)}{3d(a-a\sin(c+dx))^7}-\frac{22a^{13}\cos^9(c+d}{3d(a-a\sin(c+d}$$

```
[Out] 1155/8*a^8*x-385/4*a^8*cos(d*x+c)^3/d+1155/8*a^8*cos(d*x+c)*sin(d*x+c)/d+2/
3*a^15*cos(d*x+c)^11/d/(a-a*sin(d*x+c))^7-22/3*a^13*cos(d*x+c)^9/d/(a-a*sin
(d*x+c))^5-66*a^14*cos(d*x+c)^7/d/(a^2-a^2*sin(d*x+c))^3-231/4*a^16*cos(d*x
+c)^5/d/(a^8-a^8*sin(d*x+c))
```

Rubi [A]
time = 0.21, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2749, 2759, 2758, 2761, 2715, 8}

$$\frac{2a^{15}\cos^{11}(c+dx)}{3d(a-a\sin(c+dx))^7}-\frac{22a^{13}\cos^9(c+dx)}{3d(a-a\sin(c+dx))^5}-\frac{385a^8\cos^3(c+dx)}{4d}+\frac{1155a^8\sin(c+dx)\cos(c+dx)}{8d}+\frac{1155a^8x}{8}-\frac{231a^{16}\cos^5(c+dx)}{4d\left(a^8-a^8\sin(c+dx)\right)}-\frac{66a^{14}\cos^7(c+dx)}{d\left(a^2-a^2\sin(c+dx)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + a*Sin[c + d*x])^8,x]
```

```
[Out] (1155*a^8*x)/8 - (385*a^8*Cos[c + d*x]^3)/(4*d) + (1155*a^8*Cos[c + d*x]*Si
n[c + d*x])/(8*d) + (2*a^15*Cos[c + d*x]^11)/(3*d*(a - a*Sin[c + d*x])^7) -
 (22*a^13*Cos[c + d*x]^9)/(3*d*(a - a*Sin[c + d*x])^5) - (66*a^14*Cos[c + d
*x]^7)/(d*(a^2 - a^2*Sin[c + d*x])^3) - (231*a^16*Cos[c + d*x]^5)/(4*d*(a^8
 - a^8*Sin[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2749

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2758

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(a*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rubi steps

$$\int \sec^4(c+dx)(a+a\sin(c+dx))^8\,dx = a^{16}\int \frac{\cos^{12}(c+dx)}{(a-a\sin(c+dx))^8}\,dx$$
$$= \frac{2a^{15}\cos^{11}(c+dx)}{3d(a-a\sin(c+dx))^7} - \frac{1}{3}(11a^{14})\int \frac{\cos^{10}(c+dx)}{(a-a\sin(c+dx))^6}\,dx$$
$$= \frac{2a^{15}\cos^{11}(c+dx)}{3d(a-a\sin(c+dx))^7} - \frac{22a^{13}\cos^9(c+dx)}{3d(a-a\sin(c+dx))^5} + (33a^{12})\int \frac{\cdots}{(a-\cdots}$$
$$= \frac{2a^{15}\cos^{11}(c+dx)}{3d(a-a\sin(c+dx))^7} - \frac{22a^{13}\cos^9(c+dx)}{3d(a-a\sin(c+dx))^5} - \frac{66a^{11}\cos^7(c+\cdots}{d(a-a\sin(c+\cdots}$$
$$= \frac{2a^{15}\cos^{11}(c+dx)}{3d(a-a\sin(c+dx))^7} - \frac{22a^{13}\cos^9(c+dx)}{3d(a-a\sin(c+dx))^5} - \frac{66a^{11}\cos^7(c+\cdots}{d(a-a\sin(c+\cdots}$$
$$= -\frac{385a^8\cos^3(c+dx)}{4d} + \frac{2a^{15}\cos^{11}(c+dx)}{3d(a-a\sin(c+dx))^7} - \frac{22a^{13}\cos^9(c+\cdots}{3d(a-a\sin(c+\cdots}$$
$$= -\frac{385a^8\cos^3(c+dx)}{4d} + \frac{1155a^8\cos(c+dx)\sin(c+dx)}{8d} + \frac{2a^{15}\cos\cdots}{3d(a-a\cdots}$$
$$= \frac{1155a^8x}{8} - \frac{385a^8\cos^3(c+dx)}{4d} + \frac{1155a^8\cos(c+dx)\sin(c+dx)}{8d} + \cdots$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.04, size = 59, normalized size = 0.33

$$\frac{64\sqrt{2}\,a^8\,{}_2F_1\left(-\frac{11}{2},-\frac{3}{2};-\frac{1}{2};\frac{1}{2}(1-\sin(c+dx))\right)\sec^3(c+dx)(1+\sin(c+dx))^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + a*Sin[c + d*x])^8,x]
```

```
[Out] (64*Sqrt[2]*a^8*Hypergeometric2F1[-11/2, -3/2, -1/2, (1 - Sin[c + d*x])/2]*
Sec[c + d*x]^3*(1 + Sin[c + d*x])^(3/2))/(3*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 477 vs. 2(167) = 334.
time = 0.20, size = 478, normalized size = 2.67

method	result
risch	$\frac{1155a^8x}{8}+\frac{31ia^8e^{2i(dx+c)}}{8d}-\frac{39a^8e^{i(dx+c)}}{d}-\frac{39a^8e^{-i(dx+c)}}{d}-\frac{31ia^8e^{-2i(dx+c)}}{8d}-\frac{128\left(-15ia^8e^{i(dx+c)}+9a^8e^{2i\cdots}\right.}{3\left(e^{i(dx+c)}-i\right)^3\cdots}$

derivativedivides	$\frac{a^8\left(\frac{\sin^9(dx+c)}{3\cos(dx+c)^3}-\frac{2\left(\sin^9(dx+c)\right)}{\cos(dx+c)}-2\left(\sin^7(dx+c)+\frac{7\left(\sin^5(dx+c)\right)}{6}+\frac{35\left(\sin^3(dx+c)\right)}{24}+\frac{35\sin(dx+c)}{16}\right)\cos(dx+c)+\frac{35dx}{8}+\frac{35c}{8}\right)}{}$
default	$\frac{a^8\left(\frac{\sin^9(dx+c)}{3\cos(dx+c)^3}-\frac{2\left(\sin^9(dx+c)\right)}{\cos(dx+c)}-2\left(\sin^7(dx+c)+\frac{7\left(\sin^5(dx+c)\right)}{6}+\frac{35\left(\sin^3(dx+c)\right)}{24}+\frac{35\sin(dx+c)}{16}\right)\cos(dx+c)+\frac{35dx}{8}+\frac{35c}{8}\right)}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+a*sin(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^8*(1/3*sin(d*x+c)^9/cos(d*x+c)^3-2*sin(d*x+c)^9/cos(d*x+c)-2*(sin(d*
x+c)^7+7/6*sin(d*x+c)^5+35/24*sin(d*x+c)^3+35/16*sin(d*x+c))*cos(d*x+c)+35/
8*d*x+35/8*c)+8*a^8*(1/3*sin(d*x+c)^8/cos(d*x+c)^3-5/3*sin(d*x+c)^8/cos(d*x
+c)-5/3*(16/5+sin(d*x+c)^6+6/5*sin(d*x+c)^4+8/5*sin(d*x+c)^2)*cos(d*x+c))+2
8*a^8*(1/3*sin(d*x+c)^7/cos(d*x+c)^3-4/3*sin(d*x+c)^7/cos(d*x+c)-4/3*(sin(d
*x+c)^5+5/4*sin(d*x+c)^3+15/8*sin(d*x+c))*cos(d*x+c)+5/2*d*x+5/2*c)+56*a^8*
(1/3*sin(d*x+c)^6/cos(d*x+c)^3-sin(d*x+c)^6/cos(d*x+c)-(8/3+sin(d*x+c)^4+4/
3*sin(d*x+c)^2)*cos(d*x+c))+70*a^8*(1/3*tan(d*x+c)^3-tan(d*x+c)+d*x+c)+56*a
^8*(1/3*sin(d*x+c)^4/cos(d*x+c)^3-1/3*sin(d*x+c)^4/cos(d*x+c)-1/3*(2+sin(d*
x+c)^2)*cos(d*x+c))+28/3*a^8/cos(d*x+c)^3*sin(d*x+c)^3+8/3*a^8/cos(d*x+c)^3
-a^8*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c))
```

Maxima [A]
time = 0.53, size = 311, normalized size = 1.74

$$\frac{224\,a^8\tan(dx+c)^3+64\left(\cos(dx+c)^3-\frac{9\cos(dx+c)^2-1}{\cos(dx+c)^3}-9\cos(dx+c)\right)a^8+\left(8\tan(dx+c)^3+105\,dx+105\,c-\frac{3\left(13\tan(dx+c)^3+11\tan(dx+c)\right)}{\tan(dx+c)^4+2\tan(dx+c)^2+1}-72\tan(dx+c)\right)a^8+112\left(2\tan(dx+c)^3+15\,dx+15\,c-\frac{3\tan(dx+c)}{\tan(dx+c)^2+1}-12\tan(dx+c)\right)a^8+560\left(\tan(dx+c)^3+3\,dx+3\,c-3\tan(dx+c)\right)a^8+8\left(\tan(dx+c)^3+3\tan(dx+c)\right)a^8-448\,a^8\left(\frac{6\cos(dx+c)^2-1}{\cos(dx+c)^3}+3\cos(dx+c)\right)-\frac{448\left(3\cos(dx+c)^2-1\right)a^8}{\cos(dx+c)^3}+\frac{64\,a^8}{\cos(dx+c)^3}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 1/24*(224*a^8*tan(d*x + c)^3 + 64*(cos(d*x + c)^3 - (9*cos(d*x + c)^2 - 1)/
cos(d*x + c)^3 - 9*cos(d*x + c))*a^8 + (8*tan(d*x + c)^3 + 105*d*x + 105*c
- 3*(13*tan(d*x + c)^3 + 11*tan(d*x + c))/(tan(d*x + c)^4 + 2*tan(d*x + c)^
2 + 1) - 72*tan(d*x + c))*a^8 + 112*(2*tan(d*x + c)^3 + 15*d*x + 15*c - 3*t
an(d*x + c)/(tan(d*x + c)^2 + 1) - 12*tan(d*x + c))*a^8 + 560*(tan(d*x + c)
^3 + 3*d*x + 3*c - 3*tan(d*x + c))*a^8 + 8*(tan(d*x + c)^3 + 3*tan(d*x + c)
)*a^8 - 448*a^8*((6*cos(d*x + c)^2 - 1)/cos(d*x + c)^3 + 3*cos(d*x + c)) -
448*(3*cos(d*x + c)^2 - 1)*a^8/cos(d*x + c)^3 + 64*a^8/cos(d*x + c)^3)/d
```

Fricas [A]
time = 0.38, size = 247, normalized size = 1.38

$$-\frac{6\,a^8\cos(dx+c)^6-52\,a^8\cos(dx+c)^5-317\,a^8\cos(dx+c)^4+1286\,a^8\cos(dx+c)^3+6930\,a^8dx+512\,a^8-\left(3465\,a^8dx+5641\,a^8\right)\cos(dx+c)^2+\left(3465\,a^8dx-6674\,a^8\right)\cos(dx+c)-\left(6\,a^8\cos(dx+c)^5+58\,a^8\cos(dx+c)^4-259\,a^8\cos(dx+c)^3+6930\,a^8dx-1545\,a^8\cos(dx+c)^2-512\,a^8+\left(3465\,a^8dx-7186\,a^8\right)\cos(dx+c)\right)\sin(dx+c)}{24\left(d\cos(dx+c)^2-d\cos(dx+c)+\left(d\cos(dx+c)+2\,d\right)\sin(dx+c)-2\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] -1/24*(6*a^8*cos(d*x + c)^6 - 52*a^8*cos(d*x + c)^5 - 317*a^8*cos(d*x + c)^
4 + 1286*a^8*cos(d*x + c)^3 + 6930*a^8*d*x + 512*a^8 - (3465*a^8*d*x + 5641
*a^8)*cos(d*x + c)^2 + (3465*a^8*d*x - 6674*a^8)*cos(d*x + c) - (6*a^8*cos(
d*x + c)^5 + 58*a^8*cos(d*x + c)^4 - 259*a^8*cos(d*x + c)^3 + 6930*a^8*d*x
- 1545*a^8*cos(d*x + c)^2 - 512*a^8 + (3465*a^8*d*x - 7186*a^8)*cos(d*x + c
))*sin(d*x + c))/(d*cos(d*x + c)^2 - d*cos(d*x + c) + (d*cos(d*x + c) + 2*d
)*sin(d*x + c) - 2*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+a*sin(d*x+c))**8,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8570 deep
```

Giac [A]
time = 6.88, size = 200, normalized size = 1.12

$$\frac{3465\,(dx+c)a^8+\frac{1024\left(6\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-15\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+7\,a^8\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^3}+\frac{2\left(369\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-1728\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+393\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-5568\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-393\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-5696\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-369\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1856\,a^8\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^4}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/24*(3465*(d*x + c)*a^8 + 1024*(6*a^8*tan(1/2*d*x + 1/2*c)^2 - 15*a^8*tan(
1/2*d*x + 1/2*c) + 7*a^8)/(tan(1/2*d*x + 1/2*c) - 1)^3 + 2*(369*a^8*tan(1/2
*d*x + 1/2*c)^7 - 1728*a^8*tan(1/2*d*x + 1/2*c)^6 + 393*a^8*tan(1/2*d*x + 1
/2*c)^5 - 5568*a^8*tan(1/2*d*x + 1/2*c)^4 - 393*a^8*tan(1/2*d*x + 1/2*c)^3
- 5696*a^8*tan(1/2*d*x + 1/2*c)^2 - 369*a^8*tan(1/2*d*x + 1/2*c) - 1856*a^8
)/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

Mupad [B]
time = 9.13, size = 437, normalized size = 2.44

$$\frac{1155\,a^8\,x}{8}+\frac{\frac{1155\,a^8\,(c+d\,x)}{8}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\frac{3465\,a^8\,(c+d\,x)}{8}-\frac{a^8\,(10395\,c+10395\,d\,x-25758)}{24}\right)-\frac{a^8\,(3465\,c+3465\,d\,x-10880)}{24}+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}\left(\frac{3465\,a^8\,(c+d\,x)}{8}-\frac{a^8\,(10395\,c+10395\,d\,x-6882)}{24}\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^9\left(\frac{8085\,a^8\,(c+d\,x)}{8}-\frac{a^8\,(24255\,c+24255\,d\,x-21030)}{24}\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\left(\frac{8085\,a^8\,(c+d\,x)}{8}-\frac{a^8\,(24255\,c+24255\,d\,x-55130)}{24}\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8\left(\frac{15015\,a^8\,(c+d\,x)}{8}-\frac{a^8\,(45045\,c+45045\,d\,x-45112)}{24}\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\left(\frac{15015\,a^8\,(c+d\,x)}{8}-\frac{a^8\,(45045\,c+45045\,d\,x-96328)}{24}\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7\left(\frac{10395\,a^8\,(c+d\,x)}{4}-\frac{a^8\,(62370\,c+62370\,d\,x-86940)}{24}\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\left(\frac{10395\,a^8\,(c+d\,x)}{4}-\frac{a^8\,(62370\,c+62370\,d\,x-109800)}{24}\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6\left(\frac{12705\,a^8\,(c+d\,x)}{4}-\frac{a^8\,(76230\,c+76230\,d\,x-103972)}{24}\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5\left(\frac{12705\,a^8\,(c+d\,x)}{4}-\frac{a^8\,(76230\,c+76230\,d\,x-135388)}{24}\right)}{d\,{\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)}^3\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^8/cos(c + d*x)^4,x)
```

```
[Out] (1155*a^8*x)/8 + ((1155*a^8*(c + d*x))/8 - tan(c/2 + (d*x)/2)*((3465*a^8*(c
 + d*x))/8 - (a^8*(10395*c + 10395*d*x - 25758))/24) - (a^8*(3465*c + 3465*
```

d*x - 10880))/24 + tan(c/2 + (d*x)/2)^10*((3465*a^8*(c + d*x))/8 - (a^8*(10395*c + 10395*d*x - 6882))/24) - tan(c/2 + (d*x)/2)^9*((8085*a^8*(c + d*x))/8 - (a^8*(24255*c + 24255*d*x - 21030))/24) + tan(c/2 + (d*x)/2)^2*((8085*a^8*(c + d*x))/8 - (a^8*(24255*c + 24255*d*x - 55130))/24) + tan(c/2 + (d*x)/2)^8*((15015*a^8*(c + d*x))/8 - (a^8*(45045*c + 45045*d*x - 45112))/24) - tan(c/2 + (d*x)/2)^3*((15015*a^8*(c + d*x))/8 - (a^8*(45045*c + 45045*d*x - 96328))/24) - tan(c/2 + (d*x)/2)^7*((10395*a^8*(c + d*x))/4 - (a^8*(62370*c + 62370*d*x - 86040))/24) + tan(c/2 + (d*x)/2)^4*((10395*a^8*(c + d*x))/4 - (a^8*(62370*c + 62370*d*x - 109800))/24) + tan(c/2 + (d*x)/2)^6*((12705*a^8*(c + d*x))/4 - (a^8*(76230*c + 76230*d*x - 103972))/24) - tan(c/2 + (d*x)/2)^5*((12705*a^8*(c + d*x))/4 - (a^8*(76230*c + 76230*d*x - 135388))/24))/(d*(tan(c/2 + (d*x)/2) - 1)^3*(tan(c/2 + (d*x)/2)^2 + 1)^4)

3.50 $\int \sec^5(c+dx)(a+a\sin(c+dx))^8\,dx$

Optimal. Leaf size=110

$$-\frac{80a^8\log(1-\sin(c+dx))}{d}-\frac{31a^8\sin(c+dx)}{d}-\frac{4a^8\sin^2(c+dx)}{d}-\frac{a^8\sin^3(c+dx)}{3d}+\frac{16a^{10}}{d(a-a\sin(c+dx))^2}-$$

[Out] -80*a^8*ln(1-sin(d*x+c))/d-31*a^8*sin(d*x+c)/d-4*a^8*sin(d*x+c)^2/d-1/3*a^8*sin(d*x+c)^3/d+16*a^10/d/(a-a*sin(d*x+c))^2-80*a^9/d/(a-a*sin(d*x+c))

Rubi [A]
time = 0.06, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2746, 45}

$$\frac{16a^{10}}{d(a-a\sin(c+dx))^2}-\frac{80a^9}{d(a-a\sin(c+dx))}-\frac{a^8\sin^3(c+dx)}{3d}-\frac{4a^8\sin^2(c+dx)}{d}-\frac{31a^8\sin(c+dx)}{d}-\frac{80a^8\log(1-\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d*x]^5*(a + a*Sin[c + d*x])^8,x]

[Out] (-80*a^8*Log[1 - Sin[c + d*x]])/d - (31*a^8*Sin[c + d*x])/d - (4*a^8*Sin[c + d*x]^2)/d - (a^8*Sin[c + d*x]^3)/(3*d) + (16*a^10)/(d*(a - a*Sin[c + d*x])^2) - (80*a^9)/(d*(a - a*Sin[c + d*x]))

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2])
```

Rubi steps

$$\int \sec^5(c+dx)(a+a\sin(c+dx))^8\,dx = \frac{a^5\text{Subst}\left(\int \frac{(a+x)^5}{(a-x)^3}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^5\text{Subst}\left(\int \left(-31a^2+\frac{32a^5}{(a-x)^3}-\frac{80a^4}{(a-x)^2}+\frac{80a^3}{a-x}-8ax-x^2\right)dx, x, a\sin\right.}{d}$$
$$= -\frac{80a^8\log(1-\sin(c+dx))}{d}-\frac{31a^8\sin(c+dx)}{d}-\frac{4a^8\sin^2(c+dx)}{d}$$

Mathematica [A]
time = 0.29, size = 73, normalized size = 0.66

$$\frac{a^8\left(-80\log(1-\sin(c+dx))-31\sin(c+dx)-4\sin^2(c+dx)-\frac{1}{3}\sin^3(c+dx)+\frac{16(-4+5\sin(c+dx))}{(-1+\sin(c+dx))^2}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*(a + a*Sin[c + d*x])^8,x]
```

```
[Out] (a^8*(-80*Log[1 - Sin[c + d*x]] - 31*Sin[c + d*x] - 4*Sin[c + d*x]^2 - Sin[
c + d*x]^3/3 + (16*(-4 + 5*Sin[c + d*x]))/(-1 + Sin[c + d*x])^2))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 526 vs. $2(108) = 216$.
time = 0.20, size = 527, normalized size = 4.79 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(a+a*sin(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^8*(1/4*sin(d*x+c)^9/cos(d*x+c)^4-5/8*sin(d*x+c)^9/cos(d*x+c)^2-5/8*s
in(d*x+c)^7-7/8*sin(d*x+c)^5-35/24*sin(d*x+c)^3-35/8*sin(d*x+c)+35/8*ln(sec
(d*x+c)+tan(d*x+c)))+8*a^8*(1/4*sin(d*x+c)^8/cos(d*x+c)^4-1/2*sin(d*x+c)^8/
cos(d*x+c)^2-1/2*sin(d*x+c)^6-3/4*sin(d*x+c)^4-3/2*sin(d*x+c)^2-3*ln(cos(d*
x+c)))+28*a^8*(1/4*sin(d*x+c)^7/cos(d*x+c)^4-3/8*sin(d*x+c)^7/cos(d*x+c)^2-
3/8*sin(d*x+c)^5-5/8*sin(d*x+c)^3-15/8*sin(d*x+c)+15/8*ln(sec(d*x+c)+tan(d*
x+c)))+56*a^8*(1/4*tan(d*x+c)^4-1/2*tan(d*x+c)^2-ln(cos(d*x+c)))+70*a^8*(1/
4*sin(d*x+c)^5/cos(d*x+c)^4-1/8*sin(d*x+c)^5/cos(d*x+c)^2-1/8*sin(d*x+c)^3-
3/8*sin(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c)))+14*a^8*sin(d*x+c)^4/cos(d*x+c
)^4+28*a^8*(1/4*sin(d*x+c)^3/cos(d*x+c)^4+1/8*sin(d*x+c)^3/cos(d*x+c)^2+1/8
*sin(d*x+c)-1/8*ln(sec(d*x+c)+tan(d*x+c)))+2*a^8/cos(d*x+c)^4+a^8*(-(-1/4*s
ec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c))))
```

Maxima [A]
time = 0.31, size = 95, normalized size = 0.86

$$-\frac{a^8\sin(dx+c)^3+12\,a^8\sin(dx+c)^2+240\,a^8\log(\sin(dx+c)-1)+93\,a^8\sin(dx+c)-\frac{48\,(5\,a^8\sin(dx+c)-4\,a^8)}{\sin(dx+c)^2-2\,\sin(dx+c)+1}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))^8,x, algorithm="maxima")

[Out] -1/3*(a^8*sin(d*x + c)^3 + 12*a^8*sin(d*x + c)^2 + 240*a^8*log(sin(d*x + c) - 1) + 93*a^8*sin(d*x + c) - 48*(5*a^8*sin(d*x + c) - 4*a^8)/(sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/d

Fricas [**A**]
time = 0.39, size = 139, normalized size = 1.26

$$\frac{10\,a^8\cos\left(dx+c\right)^4+160\,a^8\cos\left(dx+c\right)^2+16\,a^8-240\,\left(a^8\cos\left(dx+c\right)^2+2\,a^8\sin\left(dx+c\right)-2\,a^8\right)\log\left(-\sin\left(dx+c\right)+1\right)+\left(a^8\cos\left(dx+c\right)^4-72\,a^8\cos\left(dx+c\right)^2-64\,a^8\right)\sin\left(dx+c\right)}{3\,\left(d\cos\left(dx+c\right)^2+2\,d\sin\left(dx+c\right)-2\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))^8,x, algorithm="fricas")

[Out] 1/3*(10*a^8*cos(d*x + c)^4 + 160*a^8*cos(d*x + c)^2 + 16*a^8 - 240*(a^8*cos(d*x + c)^2 + 2*a^8*sin(d*x + c) - 2*a^8)*log(-sin(d*x + c) + 1) + (a^8*cos(d*x + c)^4 - 72*a^8*cos(d*x + c)^2 - 64*a^8)*sin(d*x + c))/(d*cos(d*x + c)^2 + 2*d*sin(d*x + c) - 2*d)

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**5*(a+a*sin(d*x+c))**8,x)

[Out] Timed out

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 243 vs. 2(110) = 220.
time = 6.20, size = 243, normalized size = 2.21

$$\frac{2\left(120\,a^8\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)-240\,a^8\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{220\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+93\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+684\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+190\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+684\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+93\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+220\,a^8}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3}+\frac{4\,\left(125\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-536\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+846\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-536\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+125\,a^8\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^4}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))^8,x, algorithm="giac")

[Out] 2/3*(120*a^8*log(tan(1/2*d*x + 1/2*c)^2 + 1) - 240*a^8*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - (220*a^8*tan(1/2*d*x + 1/2*c)^6 + 93*a^8*tan(1/2*d*x + 1/2*c)^5 + 684*a^8*tan(1/2*d*x + 1/2*c)^4 + 190*a^8*tan(1/2*d*x + 1/2*c)^3 + 684*a^8*tan(1/2*d*x + 1/2*c)^2 + 93*a^8*tan(1/2*d*x + 1/2*c) + 220*a^8)/(tan(1/2*d*x + 1/2*c)^2 + 1)^3 + 4*(125*a^8*tan(1/2*d*x + 1/2*c)^4 - 536*a^8*tan(1/2*d*x + 1/2*c)^3 + 846*a^8*tan(1/2*d*x + 1/2*c)^2 - 536*a^8*tan(1/2*d*x + 1/2*c) + 125*a^8)/(tan(1/2*d*x + 1/2*c) - 1)^4)/d

Mupad [B]
time = 4.60, size = 96, normalized size = 0.87

$$-\frac{80\,a^8\,\ln\left(\sin\left(c+d\,x\right)-1\right)+31\,a^8\,\sin\left(c+d\,x\right)-\frac{80\,a^8\,\sin\left(c+d\,x\right)-64\,a^8}{{\sin\left(c+d\,x\right)}^2-2\,\sin\left(c+d\,x\right)+1}+4\,a^8\,{\sin\left(c+d\,x\right)}^2+\frac{a^8\,{\sin\left(c+d\,x\right)}^3}{3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^8/cos(c + d*x)^5,x)
```

```
[Out] -(80*a^8*log(sin(c + d*x) - 1) + 31*a^8*sin(c + d*x) - (80*a^8*sin(c + d*x)
 - 64*a^8)/(sin(c + d*x)^2 - 2*sin(c + d*x) + 1) + 4*a^8*sin(c + d*x)^2 + (
a^8*sin(c + d*x)^3)/3)/d
```

3.51 $\int \frac{\cos^6(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=73

$$\frac{3x}{8a}+\frac{\cos^5(c+dx)}{5ad}+\frac{3\cos(c+dx)\sin(c+dx)}{8ad}+\frac{\cos^3(c+dx)\sin(c+dx)}{4ad}$$

```
[Out] 3/8*x/a+1/5*cos(d*x+c)^5/a/d+3/8*cos(d*x+c)*sin(d*x+c)/a/d+1/4*cos(d*x+c)^3
*sin(d*x+c)/a/d
```

Rubi [A]
time = 0.05, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2761, 2715, 8}

$$\frac{\cos^5(c+dx)}{5ad}+\frac{\sin(c+dx)\cos^3(c+dx)}{4ad}+\frac{3\sin(c+dx)\cos(c+dx)}{8ad}+\frac{3x}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6/(a + a*Sin[c + d*x]),x]
```

```
[Out] (3*x)/(8*a) + Cos[c + d*x]^5/(5*a*d) + (3*Cos[c + d*x]*Sin[c + d*x])/(8*a*d
) + (Cos[c + d*x]^3*Sin[c + d*x])/(4*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^6(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\cos^5(c+dx)}{5ad}+\frac{\int \cos^4(c+dx)\,dx}{a}\\
&= \frac{\cos^5(c+dx)}{5ad}+\frac{\cos^3(c+dx)\sin(c+dx)}{4ad}+\frac{3\int \cos^2(c+dx)\,dx}{4a}\\
&= \frac{\cos^5(c+dx)}{5ad}+\frac{3\cos(c+dx)\sin(c+dx)}{8ad}+\frac{\cos^3(c+dx)\sin(c+dx)}{4ad}+\frac{3\int 1\,dx}{8a}\\
&= \frac{3x}{8a}+\frac{\cos^5(c+dx)}{5ad}+\frac{3\cos(c+dx)\sin(c+dx)}{8ad}+\frac{\cos^3(c+dx)\sin(c+dx)}{4ad}
\end{aligned}
$$

Mathematica [A]
time = 0.51, size = 141, normalized size = 1.93

$$
-\frac{\cos^7(c+dx)\left(30\sin^{-1}\left(\frac{\sqrt{1-\sin(c+dx)}}{\sqrt{2}}\right)\sqrt{1-\sin(c+dx)}+\sqrt{1+\sin(c+dx)}\,(-8-17\sin(c+dx)+41\sin^2(c+dx)-6\sin^3(c+dx)-18\sin^4(c+dx)+8\sin^5(c+dx))\right)}{40ad(-1+\sin(c+dx))^4(1+\sin(c+dx))^{7/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/40*(Cos[c + d*x]^7*(30*ArcSin[Sqrt[1 - Sin[c + d*x]]/Sqrt[2]]*Sqrt[1 - S
in[c + d*x]] + Sqrt[1 + Sin[c + d*x]]*(-8 - 17*Sin[c + d*x] + 41*Sin[c + d*
x]^2 - 6*Sin[c + d*x]^3 - 18*Sin[c + d*x]^4 + 8*Sin[c + d*x]^5)))/(a*d*(-1
+ Sin[c + d*x])^4*(1 + Sin[c + d*x])^(7/2))
```

Maple [A]
time = 0.18, size = 114, normalized size = 1.56

method	result
risch	$\frac{3x}{8a}+\frac{\cos(dx+c)}{8ad}+\frac{\cos(5dx+5c)}{80ad}+\frac{\sin(4dx+4c)}{32da}+\frac{\cos(3dx+3c)}{16ad}+\frac{\sin(2dx+2c)}{4da}$
derivativedivides	$\frac{\frac{2\left(-\frac{5\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8}+\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}+2\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}+\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8}+\frac{1}{5}\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}+\frac{3\arctan(\tan(\cdots}{4}}{da}$
default	$\frac{\frac{2\left(-\frac{5\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8}+\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}+2\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}+\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8}+\frac{1}{5}\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}+\frac{3\arctan(\tan(\cdots}{4}}{da}$
norman	$\frac{\frac{19\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}+\frac{15x\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}+\frac{9x\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a}+\frac{193\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{20da}+\frac{9x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a}+\frac{93\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4da}+\frac{45\cdots}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

[Out] 2/d/a*((-5/8*tan(1/2*d*x+1/2*c)^9+tan(1/2*d*x+1/2*c)^8-1/4*tan(1/2*d*x+1/2*c)^7+2*tan(1/2*d*x+1/2*c)^4+1/4*tan(1/2*d*x+1/2*c)^3+5/8*tan(1/2*d*x+1/2*c)+1/5)/(1+tan(1/2*d*x+1/2*c)^2)^5+3/8*arctan(tan(1/2*d*x+1/2*c)))

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 258 vs. $2(65) = 130$.
time = 0.55, size = 258, normalized size = 3.53

$$\frac{\frac{\frac{25 \sin(dx+c)}{\cos(dx+c)+1}+\frac{10 \sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{80 \sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{10 \sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{40 \sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{25 \sin(dx+c)^9}{(\cos(dx+c)+1)^9}+8}{a+\frac{5 a \sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{10 a \sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{10 a \sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{5 a \sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{a \sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}}+\frac{15 \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}}{20 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/20*((25*sin(d*x + c)/(cos(d*x + c) + 1) + 10*sin(d*x + c)^3/(cos(d*x + c)
 + 1)^3 + 80*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 10*sin(d*x + c)^7/(cos(d
*x + c) + 1)^7 + 40*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 25*sin(d*x + c)^9
/(cos(d*x + c) + 1)^9 + 8)/(a + 5*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1
0*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 10*a*sin(d*x + c)^6/(cos(d*x + c)
 + 1)^6 + 5*a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + a*sin(d*x + c)^10/(cos(
d*x + c) + 1)^10) + 15*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a)/d
```

Fricas [A]
time = 0.37, size = 50, normalized size = 0.68

$$\frac{8 \cos(dx+c)^5 + 15 dx + 5 \left(2 \cos(dx+c)^3 + 3 \cos(dx+c)\right) \sin(dx+c)}{40 ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/40*(8*cos(d*x + c)^5 + 15*d*x + 5*(2*cos(d*x + c)^3 + 3*cos(d*x + c))*sin
(d*x + c))/(a*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1355 vs. $2(60) = 120$.
time = 15.82, size = 1355, normalized size = 18.56

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((15*d*x*tan(c/2 + d*x/2)**10/(40*a*d*tan(c/2 + d*x/2)**10 + 200*a
*d*tan(c/2 + d*x/2)**8 + 400*a*d*tan(c/2 + d*x/2)**6 + 400*a*d*tan(c/2 + d*
```

```
x/2)**4 + 200*a*d*tan(c/2 + d*x/2)**2 + 40*a*d) + 75*d*x*tan(c/2 + d*x/2)**
8/(40*a*d*tan(c/2 + d*x/2)**10 + 200*a*d*tan(c/2 + d*x/2)**8 + 400*a*d*tan(
c/2 + d*x/2)**6 + 400*a*d*tan(c/2 + d*x/2)**4 + 200*a*d*tan(c/2 + d*x/2)**2
 + 40*a*d) + 150*d*x*tan(c/2 + d*x/2)**6/(40*a*d*tan(c/2 + d*x/2)**10 + 200
*a*d*tan(c/2 + d*x/2)**8 + 400*a*d*tan(c/2 + d*x/2)**6 + 400*a*d*tan(c/2 +
d*x/2)**4 + 200*a*d*tan(c/2 + d*x/2)**2 + 40*a*d) + 150*d*x*tan(c/2 + d*x/2
)**4/(40*a*d*tan(c/2 + d*x/2)**10 + 200*a*d*tan(c/2 + d*x/2)**8 + 400*a*d*t
an(c/2 + d*x/2)**6 + 400*a*d*tan(c/2 + d*x/2)**4 + 200*a*d*tan(c/2 + d*x/2)
**2 + 40*a*d) + 75*d*x*tan(c/2 + d*x/2)**2/(40*a*d*tan(c/2 + d*x/2)**10 + 2
00*a*d*tan(c/2 + d*x/2)**8 + 400*a*d*tan(c/2 + d*x/2)**6 + 400*a*d*tan(c/2
+ d*x/2)**4 + 200*a*d*tan(c/2 + d*x/2)**2 + 40*a*d) + 15*d*x/(40*a*d*tan(c/
2 + d*x/2)**10 + 200*a*d*tan(c/2 + d*x/2)**8 + 400*a*d*tan(c/2 + d*x/2)**6
+ 400*a*d*tan(c/2 + d*x/2)**4 + 200*a*d*tan(c/2 + d*x/2)**2 + 40*a*d) - 50*
tan(c/2 + d*x/2)**9/(40*a*d*tan(c/2 + d*x/2)**10 + 200*a*d*tan(c/2 + d*x/2)
**8 + 400*a*d*tan(c/2 + d*x/2)**6 + 400*a*d*tan(c/2 + d*x/2)**4 + 200*a*d*t
an(c/2 + d*x/2)**2 + 40*a*d) + 80*tan(c/2 + d*x/2)**8/(40*a*d*tan(c/2 + d*x
/2)**10 + 200*a*d*tan(c/2 + d*x/2)**8 + 400*a*d*tan(c/2 + d*x/2)**6 + 400*a
*d*tan(c/2 + d*x/2)**4 + 200*a*d*tan(c/2 + d*x/2)**2 + 40*a*d) - 20*tan(c/2
 + d*x/2)**7/(40*a*d*tan(c/2 + d*x/2)**10 + 200*a*d*tan(c/2 + d*x/2)**8 + 4
00*a*d*tan(c/2 + d*x/2)**6 + 400*a*d*tan(c/2 + d*x/2)**4 + 200*a*d*tan(c/2
+ d*x/2)**2 + 40*a*d) + 160*tan(c/2 + d*x/2)**4/(40*a*d*tan(c/2 + d*x/2)**1
0 + 200*a*d*tan(c/2 + d*x/2)**8 + 400*a*d*tan(c/2 + d*x/2)**6 + 400*a*d*tan
(c/2 + d*x/2)**4 + 200*a*d*tan(c/2 + d*x/2)**2 + 40*a*d) + 20*tan(c/2 + d*x
/2)**3/(40*a*d*tan(c/2 + d*x/2)**10 + 200*a*d*tan(c/2 + d*x/2)**8 + 400*a*d
*tan(c/2 + d*x/2)**6 + 400*a*d*tan(c/2 + d*x/2)**4 + 200*a*d*tan(c/2 + d*x/
2)**2 + 40*a*d) + 50*tan(c/2 + d*x/2)/(40*a*d*tan(c/2 + d*x/2)**10 + 200*a*
d*tan(c/2 + d*x/2)**8 + 400*a*d*tan(c/2 + d*x/2)**6 + 400*a*d*tan(c/2 + d*x
/2)**4 + 200*a*d*tan(c/2 + d*x/2)**2 + 40*a*d) + 16/(40*a*d*tan(c/2 + d*x/2
)**10 + 200*a*d*tan(c/2 + d*x/2)**8 + 400*a*d*tan(c/2 + d*x/2)**6 + 400*a*d
*tan(c/2 + d*x/2)**4 + 200*a*d*tan(c/2 + d*x/2)**2 + 40*a*d), Ne(d, 0)), (x
*cos(c)**6/(a*sin(c) + a), True))
```

Giac [A]
time = 6.44, size = 114, normalized size = 1.56

$$\frac{\frac{15\,(dx+c)}{a} - \frac{2\left(25\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-40\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8+10\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-80\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-10\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-25\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-8\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^5 a}}{40\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/40*(15*(d*x + c)/a - 2*(25*tan(1/2*d*x + 1/2*c)^9 - 40*tan(1/2*d*x + 1/2*
c)^8 + 10*tan(1/2*d*x + 1/2*c)^7 - 80*tan(1/2*d*x + 1/2*c)^4 - 10*tan(1/2*d
*x + 1/2*c)^3 - 25*tan(1/2*d*x + 1/2*c) - 8)/((tan(1/2*d*x + 1/2*c)^2 + 1)^
5*a))/d
```

Mupad [B]
time = 8.15, size = 107, normalized size = 1.47

$$\frac{3\,x}{8\,a}+\frac{-\frac{5\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^9}{4}+2\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8-\frac{{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7}{2}+4\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+\frac{{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3}{2}+\frac{5\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}+\frac{2}{5}}{a\,d\,{\left({\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(a + a*sin(c + d*x)),x)
```

```
[Out] (3*x)/(8*a) + ((5*tan(c/2 + (d*x)/2))/4 + tan(c/2 + (d*x)/2)^3/2 + 4*tan(c/
2 + (d*x)/2)^4 - tan(c/2 + (d*x)/2)^7/2 + 2*tan(c/2 + (d*x)/2)^8 - (5*tan(c
/2 + (d*x)/2)^9)/4 + 2/5)/(a*d*(tan(c/2 + (d*x)/2)^2 + 1)^5)
```

3.52 $\int \frac{\cos^5(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=47

$$-\frac{2(a-a\sin(c+dx))^3}{3a^4d}+\frac{(a-a\sin(c+dx))^4}{4a^5d}$$

```
[Out] -2/3*(a-a*sin(d*x+c))^3/a^4/d+1/4*(a-a*sin(d*x+c))^4/a^5/d
```

Rubi [A]
time = 0.04, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2746, 45}

$$\frac{(a-a\sin(c+dx))^4}{4a^5d}-\frac{2(a-a\sin(c+dx))^3}{3a^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-2*(a - a*Sin[c + d*x])^3)/(3*a^4*d) + (a - a*Sin[c + d*x])^4/(4*a^5*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^5(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\text{Subst}\left(\int (a-x)^2(a+x)\,dx, x, a\sin(c+dx)\right)}{a^5d}\\
&= \frac{\text{Subst}\left(\int \left(2a(a-x)^2-(a-x)^3\right)\,dx, x, a\sin(c+dx)\right)}{a^5d}\\
&= -\frac{2(a-a\sin(c+dx))^3}{3a^4d}+\frac{(a-a\sin(c+dx))^4}{4a^5d}
\end{aligned}$$

Mathematica [A]
time = 0.07, size = 46, normalized size = 0.98

$$\frac{\sin(c+dx)\left(12-6\sin(c+dx)-4\sin^2(c+dx)+3\sin^3(c+dx)\right)}{12ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Sin[c + d*x]*(12 - 6*Sin[c + d*x] - 4*Sin[c + d*x]^2 + 3*Sin[c + d*x]^3))/
(12*a*d)
```

Maple [A]
time = 0.16, size = 45, normalized size = 0.96

method	result
derivativedivides	$\frac{\frac{\left(\sin^4(dx+c)\right)}{4}-\frac{\left(\sin^3(dx+c)\right)}{3}-\frac{\left(\sin^2(dx+c)\right)}{2}+\sin(dx+c)}{da}$
default	$\frac{\frac{\left(\sin^4(dx+c)\right)}{4}-\frac{\left(\sin^3(dx+c)\right)}{3}-\frac{\left(\sin^2(dx+c)\right)}{2}+\sin(dx+c)}{da}$
risch	$\frac{3\sin(dx+c)}{4ad}+\frac{\cos(4dx+4c)}{32ad}+\frac{\sin(3dx+3c)}{12da}+\frac{\cos(2dx+2c)}{8ad}$
norman	$\frac{\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{da}+\frac{2\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}+\frac{10\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3da}+\frac{10\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3da}+\frac{10\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3da}+\frac{10\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3da}+\frac{14\left(\tan^5 \right.}{}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(1/4*sin(d*x+c)^4-1/3*sin(d*x+c)^3-1/2*sin(d*x+c)^2+sin(d*x+c))
```

Maxima [A]
time = 0.34, size = 47, normalized size = 1.00

$$\frac{3\,\sin\left(dx+c\right)^4-4\,\sin\left(dx+c\right)^3-6\,\sin\left(dx+c\right)^2+12\,\sin\left(dx+c\right)}{12\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*(3*sin(d*x + c)^4 - 4*sin(d*x + c)^3 - 6*sin(d*x + c)^2 + 12*sin(d*x +
 c))/(a*d)
```

Fricas [A]
time = 0.37, size = 37, normalized size = 0.79

$$\frac{3\,\cos\left(dx+c\right)^4+4\,\left(\cos\left(dx+c\right)^2+2\right)\sin\left(dx+c\right)}{12\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(3*cos(d*x + c)^4 + 4*(cos(d*x + c)^2 + 2)*sin(d*x + c))/(a*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 530 vs. $2(37) = 74$.
time = 8.70, size = 530, normalized size = 11.28

$$\begin{cases} \frac{6\tan^{7}\left(\frac{c}{2}+\frac{dx}{2}\right)}{3ad\tan^{8}\left(\frac{c}{2}+\frac{dx}{2}\right)+12ad\tan^{6}\left(\frac{c}{2}+\frac{dx}{2}\right)+18ad\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)+12ad\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+3ad} - \frac{6\tan^{6}\left(\frac{c}{2}+\frac{dx}{2}\right)}{3ad\tan^{8}\left(\frac{c}{2}+\frac{dx}{2}\right)+12ad\tan^{6}\left(\frac{c}{2}+\frac{dx}{2}\right)+18ad\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)+12ad\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+3ad} + \frac{10\tan^{5}\left(\frac{c}{2}+\frac{dx}{2}\right)}{3ad\tan^{8}\left(\frac{c}{2}+\frac{dx}{2}\right)+12ad\tan^{6}\left(\frac{c}{2}+\frac{dx}{2}\right)+18ad\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)+12ad\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+3ad} + \frac{10\tan^{3}\left(\frac{c}{2}+\frac{dx}{2}\right)}{3ad\tan^{8}\left(\frac{c}{2}+\frac{dx}{2}\right)+12ad\tan^{6}\left(\frac{c}{2}+\frac{dx}{2}\right)+18ad\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)+12ad\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+3ad} - \frac{6\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)}{3ad\tan^{8}\left(\frac{c}{2}+\frac{dx}{2}\right)+12ad\tan^{6}\left(\frac{c}{2}+\frac{dx}{2}\right)+18ad\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)+12ad\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+3ad} + \frac{6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{3ad\tan^{8}\left(\frac{c}{2}+\frac{dx}{2}\right)+12ad\tan^{6}\left(\frac{c}{2}+\frac{dx}{2}\right)+18ad\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)+12ad\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+3ad} & \text{for}\ d \neq 0 \\ \frac{x\cos^{5}(c)}{a\sin(c)+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((6*tan(c/2 + d*x/2)**7/(3*a*d*tan(c/2 + d*x/2)**8 + 12*a*d*tan(c/
2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 12*a*d*tan(c/2 + d*x/2)**2 + 3
*a*d) - 6*tan(c/2 + d*x/2)**6/(3*a*d*tan(c/2 + d*x/2)**8 + 12*a*d*tan(c/2 +
 d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 12*a*d*tan(c/2 + d*x/2)**2 + 3*a*
d) + 10*tan(c/2 + d*x/2)**5/(3*a*d*tan(c/2 + d*x/2)**8 + 12*a*d*tan(c/2 + d
*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 12*a*d*tan(c/2 + d*x/2)**2 + 3*a*d)
 + 10*tan(c/2 + d*x/2)**3/(3*a*d*tan(c/2 + d*x/2)**8 + 12*a*d*tan(c/2 + d*x
/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 12*a*d*tan(c/2 + d*x/2)**2 + 3*a*d) -
 6*tan(c/2 + d*x/2)**2/(3*a*d*tan(c/2 + d*x/2)**8 + 12*a*d*tan(c/2 + d*x/2)
**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 12*a*d*tan(c/2 + d*x/2)**2 + 3*a*d) + 6*
tan(c/2 + d*x/2)/(3*a*d*tan(c/2 + d*x/2)**8 + 12*a*d*tan(c/2 + d*x/2)**6 +
18*a*d*tan(c/2 + d*x/2)**4 + 12*a*d*tan(c/2 + d*x/2)**2 + 3*a*d), Ne(d, 0))
, (x*cos(c)**5/(a*sin(c) + a), True))
```

Giac [A]
time = 3.67, size = 47, normalized size = 1.00

$$\frac{3\sin\left(dx+c\right)^{4}-4\sin\left(dx+c\right)^{3}-6\sin\left(dx+c\right)^{2}+12\sin\left(dx+c\right)}{12\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/12*(3*sin(d*x + c)^4 - 4*sin(d*x + c)^3 - 6*sin(d*x + c)^2 + 12*sin(d*x +
 c))/(a*d)
```

Mupad [B]
time = 4.66, size = 54, normalized size = 1.15

$$\frac{\frac{\sin(c+d\,x)}{a}-\frac{\sin(c+d\,x)^2}{2\,a}-\frac{\sin(c+d\,x)^3}{3\,a}+\frac{\sin(c+d\,x)^4}{4\,a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(a + a*sin(c + d*x)),x)
```

```
[Out] (sin(c + d*x)/a - sin(c + d*x)^2/(2*a) - sin(c + d*x)^3/(3*a) + sin(c + d*x
)^4/(4*a))/d
```

3.53 $\int \frac{\cos^4(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=49

$$\frac{x}{2a}+\frac{\cos^3(c+dx)}{3ad}+\frac{\cos(c+dx)\sin(c+dx)}{2ad}$$

```
[Out] 1/2*x/a+1/3*cos(d*x+c)^3/a/d+1/2*cos(d*x+c)*sin(d*x+c)/a/d
```

Rubi [A]
time = 0.04, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.143$, Rules used = {2761, 2715, 8}

$$\frac{\cos^3(c+dx)}{3ad}+\frac{\sin(c+dx)\cos(c+dx)}{2ad}+\frac{x}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + a*Sin[c + d*x]),x]
```

```
[Out] x/(2*a) + Cos[c + d*x]^3/(3*a*d) + (Cos[c + d*x]*Sin[c + d*x])/(2*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\cos^3(c+dx)}{3ad} + \frac{\int \cos^2(c+dx)\,dx}{a}$$
$$= \frac{\cos^3(c+dx)}{3ad} + \frac{\cos(c+dx)\sin(c+dx)}{2ad} + \frac{\int 1\,dx}{2a}$$
$$= \frac{x}{2a} + \frac{\cos^3(c+dx)}{3ad} + \frac{\cos(c+dx)\sin(c+dx)}{2ad}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 119 vs. $2(49) = 98$.

time = 0.21, size = 119, normalized size = 2.43

$$-\frac{\cos^5(c+dx)\left(-6\sin^{-1}\left(\frac{\sqrt{1-\sin(c+dx)}}{\sqrt{2}}\right)\sqrt{1-\sin(c+dx)}+\sqrt{1+\sin(c+dx)}\,(2+\sin(c+dx)-5\sin^2(c+dx)+2\sin^3(c+dx))\right)}{6ad(-1+\sin(c+dx))^3(1+\sin(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/6*(Cos[c + d*x]^5*(-6*ArcSin[Sqrt[1 - Sin[c + d*x]]/Sqrt[2]]*Sqrt[1 - Si
n[c + d*x]] + Sqrt[1 + Sin[c + d*x]]*(2 + Sin[c + d*x] - 5*Sin[c + d*x]^2 +
 2*Sin[c + d*x]^3)))/(a*d*(-1 + Sin[c + d*x])^3*(1 + Sin[c + d*x])^(5/2))
```

Maple [**A**]

time = 0.13, size = 75, normalized size = 1.53

method	result
risch	$\frac{x}{2a}+\frac{\cos(dx+c)}{4ad}+\frac{\cos(3dx+3c)}{12ad}+\frac{\sin(2dx+2c)}{4da}$
derivativedivides	$\frac{\frac{2\left(-\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}+\frac{1}{3}\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}$
default	$\frac{\frac{2\left(-\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}+\frac{1}{3}\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}$
norman	$-\frac{\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)}{da}+\frac{\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)}{da}+\frac{\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)}{da}+\frac{\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)}{da}+\frac{x}{2a}+\frac{x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}+\frac{2x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{2x\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/a*((-1/2*tan(1/2*d*x+1/2*c)^5+tan(1/2*d*x+1/2*c)^4+1/2*tan(1/2*d*x+1/2*
c)+1/3)/(1+tan(1/2*d*x+1/2*c)^2)^3+1/2*arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(43) = 86$.
time = 0.70, size = 156, normalized size = 3.18

$$\frac{\frac{\frac{3 \sin(dx+c)}{\cos(dx+c)+1} + \frac{6 \sin(dx+c)^4}{(\cos(dx+c)+1)^4} - \frac{3 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} + 2}{a + \frac{3 a \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{3 a \sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{a \sin(dx+c)^6}{(\cos(dx+c)+1)^6}} + \frac{3 \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/3*((3*sin(d*x + c)/(cos(d*x + c) + 1) + 6*sin(d*x + c)^4/(cos(d*x + c) +
1)^4 - 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 2)/(a + 3*a*sin(d*x + c)^2/(
cos(d*x + c) + 1)^2 + 3*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a*sin(d*x +
 c)^6/(cos(d*x + c) + 1)^6) + 3*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a)/
d
```

Fricas [A]
time = 0.39, size = 37, normalized size = 0.76

$$\frac{2 \cos(dx+c)^3 + 3\,dx + 3 \cos(dx+c) \sin(dx+c)}{6\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*cos(d*x + c)^3 + 3*d*x + 3*cos(d*x + c)*sin(d*x + c))/(a*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 558 vs. $2(36) = 72$.
time = 4.86, size = 558, normalized size = 11.39

$$\begin{cases} \frac{3dx\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)}{6ad\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)+18ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+18ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+6ad} + \frac{9dx\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{6ad\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)+18ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+18ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+6ad} + \frac{9dx\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{6ad\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)+18ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+18ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+6ad} + \frac{3dx}{6ad\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)+18ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+18ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+6ad} - \frac{6\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{6ad\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)+18ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+18ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+6ad} + \frac{12\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{6ad\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)+18ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+18ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+6ad} + \frac{6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{6ad\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)+18ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+18ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+6ad} + \frac{4}{6ad\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)+18ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+18ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+6ad} & \text{for}\: d \neq 0 \\ \frac{x\cos^4(c)}{a\sin(c)+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((3*d*x*tan(c/2 + d*x/2)**6/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*ta
n(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d) + 9*d*x*tan(c/2 + d
*x/2)**4/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*t
an(c/2 + d*x/2)**2 + 6*a*d) + 9*d*x*tan(c/2 + d*x/2)**2/(6*a*d*tan(c/2 + d*
x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d)
+ 3*d*x/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*ta
n(c/2 + d*x/2)**2 + 6*a*d) - 6*tan(c/2 + d*x/2)**5/(6*a*d*tan(c/2 + d*x/2)*
*6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d) + 12*
```

```
tan(c/2 + d*x/2)**4/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4
 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d) + 6*tan(c/2 + d*x/2)/(6*a*d*tan(c/2
+ d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a
*d) + 4/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*ta
n(c/2 + d*x/2)**2 + 6*a*d), Ne(d, 0)), (x*cos(c)**4/(a*sin(c) + a), True))
```

Giac [A]
time = 4.84, size = 75, normalized size = 1.53

$$\frac{\frac{3(dx+c)}{a}-\frac{2\left(3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{5}-6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{4}-3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+1\right)^{3}a}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(3*(d*x + c)/a - 2*(3*tan(1/2*d*x + 1/2*c)^5 - 6*tan(1/2*d*x + 1/2*c)^4
 - 3*tan(1/2*d*x + 1/2*c) - 2)/((tan(1/2*d*x + 1/2*c)^2 + 1)^3*a))/d
```

Mupad [B]
time = 6.87, size = 66, normalized size = 1.35

$$\frac{x}{2\,a}+\frac{-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}+2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{2}{3}}{a\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(a + a*sin(c + d*x)),x)
```

```
[Out] x/(2*a) + (tan(c/2 + (d*x)/2) + 2*tan(c/2 + (d*x)/2)^4 - tan(c/2 + (d*x)/2)
^5 + 2/3)/(a*d*(tan(c/2 + (d*x)/2)^2 + 1)^3)
```

3.54 $\int \frac{\cos^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=32

$$\frac{\sin(c+dx)}{ad}-\frac{\sin^2(c+dx)}{2ad}$$

```
[Out] sin(d*x+c)/a/d-1/2*sin(d*x+c)^2/a/d
```

Rubi [A]
time = 0.03, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.048$, Rules used = $\{2746\}$

$$\frac{\sin(c+dx)}{ad}-\frac{\sin^2(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + a*Sin[c + d*x]),x]
```

```
[Out] Sin[c + d*x]/(a*d) - Sin[c + d*x]^2/(2*a*d)
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^3(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\text{Subst}\left(\int (a-x)\,dx, x, a\sin(c+dx)\right)}{a^3d}\\ &= \frac{\sin(c+dx)}{ad}-\frac{\sin^2(c+dx)}{2ad}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 24, normalized size = 0.75

$$-\frac{(-2+\sin(c+dx))\sin(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + a*Sin[c + d*x]),x]
[Out] -1/2*((-2 + Sin[c + d*x])*Sin[c + d*x])/(a*d)
```

Maple [A]
time = 0.00, size = 28, normalized size = 0.88

method	result	size
derivativedivides	$-\frac{\frac{\left(\sin^2(dx+c)\right)}{2}-\sin(dx+c)}{ad}$	28
default	$-\frac{\frac{\left(\sin^2(dx+c)\right)}{2}-\sin(dx+c)}{ad}$	28
risch	$\frac{\sin(dx+c)}{ad}+\frac{\cos(2dx+2c)}{4ad}$	32
norman	$\frac{\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{da}+\frac{2\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}+\frac{2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}+\frac{2\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}$	105

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] -1/a/d*(1/2*sin(d*x+c)^2-sin(d*x+c))
```

Maxima [A]
time = 0.41, size = 25, normalized size = 0.78

$$-\frac{\sin\left(dx+c\right)^2-2\,\sin\left(dx+c\right)}{2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
[Out] -1/2*(sin(d*x + c)^2 - 2*sin(d*x + c))/(a*d)
```

Fricas [A]
time = 0.41, size = 25, normalized size = 0.78

$$\frac{\cos\left(dx+c\right)^2+2\,\sin\left(dx+c\right)}{2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
[Out] 1/2*(cos(d*x + c)^2 + 2*sin(d*x + c))/(a*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(22) = 44$.

time = 2.33, size = 158, normalized size = 4.94

$$\begin{cases} \frac{2\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad} - \frac{2\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad} + \frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad} & \text{for } d \neq 0 \\ \frac{x\cos^3(c)}{a\sin(c)+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((2*tan(c/2 + d*x/2)**3/(a*d*tan(c/2 + d*x/2)**4 + 2*a*d*tan(c/2 +
 d*x/2)**2 + a*d) - 2*tan(c/2 + d*x/2)**2/(a*d*tan(c/2 + d*x/2)**4 + 2*a*d*
tan(c/2 + d*x/2)**2 + a*d) + 2*tan(c/2 + d*x/2)/(a*d*tan(c/2 + d*x/2)**4 +
2*a*d*tan(c/2 + d*x/2)**2 + a*d), Ne(d, 0)), (x*cos(c)**3/(a*sin(c) + a), T
rue))
```

Giac [A]
time = 5.06, size = 25, normalized size = 0.78

$$-\frac{\sin(dx+c)^2 - 2\sin(dx+c)}{2ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(sin(d*x + c)^2 - 2*sin(d*x + c))/(a*d)
```

Mupad [B]
time = 4.49, size = 22, normalized size = 0.69

$$-\frac{\sin(c+d\,x)\,\left(\sin(c+d\,x)-2\right)}{2\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a + a*sin(c + d*x)),x)
```

```
[Out] -(sin(c + d*x)*(sin(c + d*x) - 2))/(2*a*d)
```

3.55 $\int \frac{\cos^2(c+dx)}{a+a\sin(c+dx)} dx$

Optimal. Leaf size=19

$$\frac{x}{a} + \frac{\cos(c+dx)}{ad}$$

```
[Out] x/a+cos(d*x+c)/a/d
```

Rubi [A]
time = 0.03, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2761, 8}

$$\frac{\cos(c+dx)}{ad} + \frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + a*Sin[c + d*x]),x]
```

```
[Out] x/a + Cos[c + d*x]/(a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned} \int \frac{\cos^2(c+dx)}{a+a\sin(c+dx)} dx &= \frac{\cos(c+dx)}{ad} + \frac{\int 1 dx}{a} \\ &= \frac{x}{a} + \frac{\cos(c+dx)}{ad} \end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 97 vs. 2(19) = 38.
time = 0.08, size = 97, normalized size = 5.11

$$-\frac{\cos^3(c+dx)\left(2\sin^{-1}\left(\frac{\sqrt{1-\sin(c+dx)}}{\sqrt{2}}\right)\sqrt{1-\sin(c+dx)} + (-1+\sin(c+dx))\sqrt{1+\sin(c+dx)}\right)}{ad(-1+\sin(c+dx))^2(1+\sin(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + a*Sin[c + d*x]),x]
```

```
[Out] -((Cos[c + d*x]^3*(2*ArcSin[Sqrt[1 - Sin[c + d*x]]/Sqrt[2]]*Sqrt[1 - Sin[c
+ d*x]] + (-1 + Sin[c + d*x])*Sqrt[1 + Sin[c + d*x]]))/(a*d*(-1 + Sin[c + d
*x])^2*(1 + Sin[c + d*x])^(3/2)))
```

Maple [A]
time = 0.09, size = 35, normalized size = 1.84

method	result
risch	$\frac{x}{a}+\frac{\cos(dx+c)}{ad}$
derivativedivides	$\frac{\frac{2}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}+2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}$
default	$\frac{\frac{2}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}+2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}$
norman	$\frac{\frac{x}{a}+\frac{x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a}+\frac{x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{x\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{2}{ad}+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{da}+\frac{2x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{2x\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{2\left(\tan^2\right.}{}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/a*(1/(1+tan(1/2*d*x+1/2*c)^2)+arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 52 vs. $2(19) = 38$.
time = 0.56, size = 52, normalized size = 2.74

$$\frac{2\left(\frac{\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{1}{a+\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 2*(arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a + 1/(a + a*sin(d*x + c)^2/(cos
(d*x + c) + 1)^2))/d
```

Fricas [A]
time = 0.35, size = 17, normalized size = 0.89

$$\frac{dx+\cos(dx+c)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] (d*x + cos(d*x + c))/(a*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(12) = 24$.
time = 1.25, size = 88, normalized size = 4.63

$$\begin{cases} \frac{dx\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad} + \frac{dx}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad} + \frac{2}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad} & \text{for } d \neq 0 \\ \frac{x\cos^2(c)}{a\sin(c)+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise(((d*x*tan(c/2 + d*x/2)**2/(a*d*tan(c/2 + d*x/2)**2 + a*d) + d*x/(a
*d*tan(c/2 + d*x/2)**2 + a*d) + 2/(a*d*tan(c/2 + d*x/2)**2 + a*d), Ne(d, 0)
), (x*cos(c)**2/(a*sin(c) + a), True))
```

Giac [A]
time = 4.70, size = 34, normalized size = 1.79

$$\frac{\frac{dx+c}{a} + \frac{2}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] ((d*x + c)/a + 2/((tan(1/2*d*x + 1/2*c)^2 + 1)*a))/d
```

Mupad [B]
time = 4.53, size = 29, normalized size = 1.53

$$\frac{x}{a} + \frac{2}{a\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a + a*sin(c + d*x)),x)
```

```
[Out] x/a + 2/(a*d*(tan(c/2 + (d*x)/2)^2 + 1))
```

3.56 $\int \frac{\cos(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=16

$$\frac{\log(1+\sin(c+dx))}{ad}$$

```
[Out] ln(1+sin(d*x+c))/a/d
```

Rubi [A]
time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2746, 31}

$$\frac{\log(\sin(c+dx)+1)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + a*Sin[c + d*x]),x]
```

```
[Out] Log[1 + Sin[c + d*x]]/(a*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, a\sin(c+dx)\right)}{ad}\\ &= \frac{\log(1+\sin(c+dx))}{ad}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 16, normalized size = 1.00

$$\frac{\log(1+\sin(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + a*Sin[c + d*x]),x]
[Out] Log[1 + Sin[c + d*x]]/(a*d)
```

Maple [A]
time = 0.00, size = 19, normalized size = 1.19

method	result	size
derivativedivides	$\frac{\ln(a+a\sin(dx+c))}{da}$	19
default	$\frac{\ln(a+a\sin(dx+c))}{da}$	19
risch	$-\frac{ix}{a}-\frac{2ic}{ad}+\frac{2\ln\left(e^{i(dx+c)}+i\right)}{ad}$	40
norman	$\frac{2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{ad}-\frac{\ln\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*ln(a+a*sin(d*x+c))/a
```

Maxima [A]
time = 0.30, size = 18, normalized size = 1.12

$$\frac{\log\left(a\sin\left(dx+c\right)+a\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
[Out] log(a*sin(d*x + c) + a)/(a*d)
```

Fricas [A]
time = 0.37, size = 16, normalized size = 1.00

$$\frac{\log\left(\sin\left(dx+c\right)+1\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
[Out] log(sin(d*x + c) + 1)/(a*d)
```

Sympy [A]
time = 0.26, size = 24, normalized size = 1.50

$$\begin{cases}\frac{\log\left(\sin\left(c+dx\right)+1\right)}{ad} & \text{for } d\neq 0\\ \frac{x\cos\left(c\right)}{a\sin\left(c\right)+a} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((log(sin(c + d*x) + 1)/(a*d), Ne(d, 0)), (x*cos(c)/(a*sin(c) + a)
, True))
```

Giac [A]
time = 6.50, size = 19, normalized size = 1.19

$$\frac{\log\left(|a\sin\left(dx+c\right)+a|\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] log(abs(a*sin(d*x + c) + a))/(a*d)
```

Mupad [B]
time = 0.04, size = 16, normalized size = 1.00

$$\frac{\ln\left(\sin\left(c+d\,x\right)+1\right)}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + a*sin(c + d*x)),x)
```

```
[Out] log(sin(c + d*x) + 1)/(a*d)
```

3.57 $\int \frac{\sec(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=37

$$\frac{\tanh^{-1}(\sin(c+dx))}{2ad}-\frac{1}{2d(a+a\sin(c+dx))}$$

```
[Out] 1/2*arctanh(sin(d*x+c))/a/d-1/2/d/(a+a*sin(d*x+c))
```

Rubi [A]
time = 0.04, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2746, 46, 212}

$$\frac{\tanh^{-1}(\sin(c+dx))}{2ad}-\frac{1}{2d(a\sin(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + a*Sin[c + d*x]),x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/(2*a*d) - 1/(2*d*(a + a*Sin[c + d*x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\int \frac{\sec(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{a\text{Subst}\left(\int \frac{1}{(a-x)(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a\text{Subst}\left(\int \left(\frac{1}{2a(a+x)^2}+\frac{1}{2a(a^2-x^2)}\right)\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{1}{2d(a+a\sin(c+dx))} + \frac{\text{Subst}\left(\int \frac{1}{a^2-x^2}\,dx, x, a\sin(c+dx)\right)}{2d}$$
$$= \frac{\tanh^{-1}(\sin(c+dx))}{2ad} - \frac{1}{2d(a+a\sin(c+dx))}$$

Mathematica [A]
time = 0.03, size = 30, normalized size = 0.81

$$\frac{\tanh^{-1}(\sin(c+dx)) - \frac{1}{1+\sin(c+dx)}}{2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + a*Sin[c + d*x]),x]
[Out] (ArcTanh[Sin[c + d*x]] - (1 + Sin[c + d*x])^(-1))/(2*a*d)
```

Maple [A]
time = 0.00, size = 43, normalized size = 1.16

method	result	size
derivativedivides	$\frac{-\frac{1}{2(1+\sin(dx+c))}+\frac{\ln(1+\sin(dx+c))}{4}-\frac{\ln(\sin(dx+c)-1)}{4}}{da}$	43
default	$\frac{-\frac{1}{2(1+\sin(dx+c))}+\frac{\ln(1+\sin(dx+c))}{4}-\frac{\ln(\sin(dx+c)-1)}{4}}{da}$	43
norman	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{da\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2ad}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2ad}$	71
risch	$-\frac{i e^{i(dx+c)}}{da\left(e^{i(dx+c)}+i\right)^2}-\frac{\ln\left(e^{i(dx+c)}-i\right)}{2ad}+\frac{\ln\left(e^{i(dx+c)}+i\right)}{2ad}$	76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d/a*(-1/2/(1+sin(d*x+c))+1/4*ln(1+sin(d*x+c))-1/4*ln(sin(d*x+c)-1))
```

Maxima [A]
time = 0.31, size = 47, normalized size = 1.27

$$\frac{\frac{\log(\sin(dx+c)+1)}{a} - \frac{\log(\sin(dx+c)-1)}{a} - \frac{2}{a\sin(dx+c)+a}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*(log(sin(d*x + c) + 1)/a - log(sin(d*x + c) - 1)/a - 2/(a*sin(d*x + c)
+ a))/d
```

Fricas [A]
time = 0.36, size = 58, normalized size = 1.57

$$\frac{(\sin(dx+c)+1)\log(\sin(dx+c)+1)-(\sin(dx+c)+1)\log(-\sin(dx+c)+1)-2}{4(ad\sin(dx+c)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*((sin(d*x + c) + 1)*log(sin(d*x + c) + 1) - (sin(d*x + c) + 1)*log(-sin
(d*x + c) + 1) - 2)/(a*d*sin(d*x + c) + a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)/(sin(c + d*x) + 1), x)/a
```

Giac [A]
time = 5.43, size = 58, normalized size = 1.57

$$\frac{\frac{\log(|\sin(dx+c)+1|)}{a}-\frac{\log(|\sin(dx+c)-1|)}{a}-\frac{\sin(dx+c)+3}{a(\sin(dx+c)+1)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/4*(log(abs(sin(d*x + c) + 1))/a - log(abs(sin(d*x + c) - 1))/a - (sin(d*x
 + c) + 3)/(a*(sin(d*x + c) + 1)))/d
```

Mupad [B]
time = 0.07, size = 33, normalized size = 0.89

$$\frac{\mathrm{atanh}(\sin(c+d\,x))}{2\,a\,d}-\frac{1}{2\,d\,(a+a\,\sin(c+d\,x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + a*sin(c + d*x))),x)
```

```
[Out] atanh(sin(c + d*x))/(2*a*d) - 1/(2*d*(a + a*sin(c + d*x)))
```

3.58 $\int \frac{\sec^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=42

$$-\frac{\sec(c+dx)}{3d(a+a\sin(c+dx))}+\frac{2\tan(c+dx)}{3ad}$$

```
[Out] -1/3*sec(d*x+c)/d/(a+a*sin(d*x+c))+2/3*tan(d*x+c)/a/d
```

Rubi [A]
time = 0.04, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2751, 3852, 8}

$$\frac{2\tan(c+dx)}{3ad}-\frac{\sec(c+dx)}{3d(a\sin(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/3*Sec[c + d*x]/(d*(a + a*Sin[c + d*x])) + (2*Tan[c + d*x])/(3*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\sec(c+dx)}{3d(a+a\sin(c+dx))} + \frac{2\int \sec^2(c+dx)\,dx}{3a}$$
$$= -\frac{\sec(c+dx)}{3d(a+a\sin(c+dx))} - \frac{2\text{Subst}(\int 1\,dx, x, -\tan(c+dx))}{3ad}$$
$$= -\frac{\sec(c+dx)}{3d(a+a\sin(c+dx))} + \frac{2\tan(c+dx)}{3ad}$$

Mathematica [A]
time = 0.04, size = 45, normalized size = 1.07

$$\frac{-\cos(2(c+dx))\sec(c+dx)+2\tan(c+dx)}{3ad(1+\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + a*Sin[c + d*x]),x]
[Out] (-(Cos[2*(c + d*x)]*Sec[c + d*x]) + 2*Tan[c + d*x])/(3*a*d*(1 + Sin[c + d*x
]))
```

Maple [A]
time = 0.10, size = 70, normalized size = 1.67

method	result	size
risch	$-\frac{4\left(2\,\mathrm{e}^{i(dx+c)}+i\right)}{3\left(\mathrm{e}^{i(dx+c)}+i\right)^{3}\left(\mathrm{e}^{i(dx+c)}-i\right)da}$	51
derivativedivides	$\frac{-\frac{2}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}+\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}-\frac{3}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}}{da}$	70
default	$\frac{-\frac{2}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}+\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}-\frac{3}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}}{da}$	70
norman	$\frac{-\frac{2\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}+\frac{2}{3ad}-\frac{2\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{3da}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}$	92

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 2/d/a*(-1/3/(tan(1/2*d*x+1/2*c)+1)^3+1/2/(tan(1/2*d*x+1/2*c)+1)^2-3/4/(tan(
1/2*d*x+1/2*c)+1)-1/4/(tan(1/2*d*x+1/2*c)-1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(38) = 76$.

time = 0.31, size = 129, normalized size = 3.07

$$\frac{2\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+\frac{3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-1\right)}{3\left(a+\frac{2a\sin(dx+c)}{\cos(dx+c)+1}-\frac{2a\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 2/3*(sin(d*x + c)/(cos(d*x + c) + 1) + 3*sin(d*x + c)^2/(cos(d*x + c) + 1)^
2 + 3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 1)/((a + 2*a*sin(d*x + c)/(cos(
d*x + c) + 1) - 2*a*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - a*sin(d*x + c)^4/
(cos(d*x + c) + 1)^4)*d)
```

Fricas [A]
time = 0.35, size = 49, normalized size = 1.17

$$-\frac{2\cos(dx+c)^2-2\sin(dx+c)-1}{3(ad\cos(dx+c)\sin(dx+c)+ad\cos(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/3*(2*cos(d*x + c)^2 - 2*sin(d*x + c) - 1)/(a*d*cos(d*x + c)*sin(d*x + c)
 + a*d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\sec^2(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**2/(sin(c + d*x) + 1), x)/a
```

Giac [A]
time = 5.50, size = 67, normalized size = 1.60

$$-\frac{\frac{3}{a\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)}+\frac{9\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+12\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+7}{a\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(3/(a*(tan(1/2*d*x + 1/2*c) - 1)) + (9*tan(1/2*d*x + 1/2*c)^2 + 12*tan
(1/2*d*x + 1/2*c) + 7)/(a*(tan(1/2*d*x + 1/2*c) + 1)^3))/d
```

Mupad [B]
time = 4.56, size = 71, normalized size = 1.69

$$-\frac{2\left(3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)}{3\,a\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)\,{\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + a*sin(c + d*x))),x)
```

```
[Out] -(2*(tan(c/2 + (d*x)/2) + 3*tan(c/2 + (d*x)/2)^2 + 3*tan(c/2 + (d*x)/2)^3 -
 1))/(3*a*d*(tan(c/2 + (d*x)/2) - 1)*(tan(c/2 + (d*x)/2) + 1)^3)
```

3.59 $\int \frac{\sec^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=77

$$\frac{3\tanh^{-1}(\sin(c+dx))}{8ad}+\frac{1}{8d(a-a\sin(c+dx))}-\frac{a}{8d(a+a\sin(c+dx))^2}-\frac{1}{4d(a+a\sin(c+dx))}$$

```
[Out] 3/8*arctanh(sin(d*x+c))/a/d+1/8/d/(a-a*sin(d*x+c))-1/8*a/d/(a+a*sin(d*x+c))
^2-1/4/d/(a+a*sin(d*x+c))
```

Rubi [A]
time = 0.05, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2746, 46, 212}

$$-\frac{a}{8d(a\sin(c+dx)+a)^2}+\frac{1}{8d(a-a\sin(c+dx))}-\frac{1}{4d(a\sin(c+dx)+a)}+\frac{3\tanh^{-1}(\sin(c+dx))}{8ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + a*Sin[c + d*x]),x]
```

```
[Out] (3*ArcTanh[Sin[c + d*x]])/(8*a*d) + 1/(8*d*(a - a*Sin[c + d*x])) - a/(8*d*(
a + a*Sin[c + d*x])^2) - 1/(4*d*(a + a*Sin[c + d*x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{a^3\text{Subst}\left(\int \frac{1}{(a-x)^2(a+x)^3}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^3\text{Subst}\left(\int \left(\frac{1}{8a^3(a-x)^2}+\frac{1}{4a^2(a+x)^3}+\frac{1}{4a^3(a+x)^2}+\frac{3}{8a^3(a^2-x^2)}\right)\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{1}{8d(a-a\sin(c+dx))}-\frac{a}{8d(a+a\sin(c+dx))^2}-\frac{1}{4d(a+a\sin(c+dx))}+\frac{3\text{Subs}}{}$$
$$= \frac{3\tanh^{-1}(\sin(c+dx))}{8ad}+\frac{1}{8d(a-a\sin(c+dx))}-\frac{a}{8d(a+a\sin(c+dx))^2}-\frac{}{4d(a}$$

Mathematica [A]
time = 0.07, size = 75, normalized size = 0.97

$$-\frac{\sec^2(c+dx)\left(2-3\sin(c+dx)-3\sin^2(c+dx)+3\tanh^{-1}(\sin(c+dx))(-1+\sin(c+dx))(1+\sin(c+dx))^2\right)}{8ad(1+\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/8*(Sec[c + d*x]^2*(2 - 3*Sin[c + d*x] - 3*Sin[c + d*x]^2 + 3*ArcTanh[Sin
[c + d*x]]*(-1 + Sin[c + d*x])*(1 + Sin[c + d*x])^2))/(a*d*(1 + Sin[c + d*x
]))
```

Maple [A]
time = 0.15, size = 67, normalized size = 0.87

method	result
derivativedivides	$\frac{-\frac{1}{8(\sin(dx+c)-1)}-\frac{3\ln(\sin(dx+c)-1)}{16}-\frac{1}{8(1+\sin(dx+c))^2}-\frac{1}{4(1+\sin(dx+c))}+\frac{3\ln(1+\sin(dx+c))}{16}}{da}$
default	$\frac{-\frac{1}{8(\sin(dx+c)-1)}-\frac{3\ln(\sin(dx+c)-1)}{16}-\frac{1}{8(1+\sin(dx+c))^2}-\frac{1}{4(1+\sin(dx+c))}+\frac{3\ln(1+\sin(dx+c))}{16}}{da}$
risch	$-\frac{i\left(6i e^{4i(dx+c)}+3\,e^{5i(dx+c)}-6i e^{2i(dx+c)}+2\,e^{3i(dx+c)}+3\,e^{i(dx+c)}\right)}{4\left(e^{i(dx+c)}+i\right)^4\left(e^{i(dx+c)}-i\right)^2 da}+\frac{3\ln\left(e^{i(dx+c)}+i\right)}{8ad}-\frac{3\ln\left(e^{i(dx+c)}-i\right)}{8ad}$
norman	$\frac{\frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{2da}+\frac{\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)}{2da}+\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{2da}+\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4da}+\frac{5\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4da}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}-\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{8ad}+\frac{3\ln\left(\tan\left(\frac{}{}\right.\right.}{8a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(-1/8/(sin(d*x+c)-1)-3/16*ln(sin(d*x+c)-1)-1/8/(1+sin(d*x+c))^2-1/4/(
1+sin(d*x+c))+3/16*ln(1+sin(d*x+c)))
```

Maxima [A]
time = 0.30, size = 91, normalized size = 1.18

$$-\frac{\frac{2\left(3\sin(dx+c)^2+3\sin(dx+c)-2\right)}{a\sin(dx+c)^3+a\sin(dx+c)^2-a\sin(dx+c)-a}-\frac{3\log(\sin(dx+c)+1)}{a}+\frac{3\log(\sin(dx+c)-1)}{a}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/16*(2*(3*sin(d*x + c)^2 + 3*sin(d*x + c) - 2)/(a*sin(d*x + c)^3 + a*sin(
d*x + c)^2 - a*sin(d*x + c) - a) - 3*log(sin(d*x + c) + 1)/a + 3*log(sin(d*
x + c) - 1)/a)/d
```

Fricas [A]
time = 0.37, size = 125, normalized size = 1.62

$$-\frac{6\cos\left(dx+c\right)^2-3\left(\cos\left(dx+c\right)^2\sin\left(dx+c\right)+\cos\left(dx+c\right)^2\right)\log\left(\sin\left(dx+c\right)+1\right)+3\left(\cos\left(dx+c\right)^2\sin\left(dx+c\right)+\cos\left(dx+c\right)^2\right)\log\left(-\sin\left(dx+c\right)+1\right)-6\sin\left(dx+c\right)-2}{16\left(ad\cos\left(dx+c\right)^2\sin\left(dx+c\right)+ad\cos\left(dx+c\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/16*(6*cos(d*x + c)^2 - 3*(cos(d*x + c)^2*sin(d*x + c) + cos(d*x + c)^2)*
log(sin(d*x + c) + 1) + 3*(cos(d*x + c)^2*sin(d*x + c) + cos(d*x + c)^2)*lo
g(-sin(d*x + c) + 1) - 6*sin(d*x + c) - 2)/(a*d*cos(d*x + c)^2*sin(d*x + c)
 + a*d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\sec^3\left(c+dx\right)}{\sin\left(c+dx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**3/(sin(c + d*x) + 1), x)/a
```

Giac [A]
time = 3.86, size = 96, normalized size = 1.25

$$\frac{\frac{6\log(|\sin(dx+c)+1|)}{a}-\frac{6\log(|\sin(dx+c)-1|)}{a}+\frac{2\,(3\sin(dx+c)-5)}{a(\sin(dx+c)-1)}-\frac{9\sin(dx+c)^2+26\sin(dx+c)+21}{a(\sin(dx+c)+1)^2}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/32*(6*log(abs(sin(d*x + c) + 1))/a - 6*log(abs(sin(d*x + c) - 1))/a + 2*(
3*sin(d*x + c) - 5)/(a*(sin(d*x + c) - 1)) - (9*sin(d*x + c)^2 + 26*sin(d*x
 + c) + 21)/(a*(sin(d*x + c) + 1)^2))/d
```

Mupad [B]
time = 4.66, size = 74, normalized size = 0.96

$$\frac{3\,\mathrm{atanh}(\sin\left(c+d\,x\right))}{8\,a\,d}+\frac{\frac{3\sin(c+d\,x)^2}{8}+\frac{3\,\sin(c+d\,x)}{8}-\frac{1}{4}}{d\,\left(-a\sin\left(c+d\,x\right)^3-a\sin\left(c+d\,x\right)^2+a\,\sin\left(c+d\,x\right)+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + a*sin(c + d*x))),x)
```

```
[Out] (3*atanh(sin(c + d*x)))/(8*a*d) + ((3*sin(c + d*x))/8 + (3*sin(c + d*x)^2)/
8 - 1/4)/(d*(a + a*sin(c + d*x) - a*sin(c + d*x)^2 - a*sin(c + d*x)^3))
```

3.60 $\int \frac{\sec^4(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=62

$$-\frac{\sec^3(c+dx)}{5d(a+a\sin(c+dx))}+\frac{4\tan(c+dx)}{5ad}+\frac{4\tan^3(c+dx)}{15ad}$$

```
[Out] -1/5*sec(d*x+c)^3/d/(a+a*sin(d*x+c))+4/5*tan(d*x+c)/a/d+4/15*tan(d*x+c)^3/a
/d
```

Rubi [A]
time = 0.04, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2751, 3852}

$$\frac{4\tan^3(c+dx)}{15ad}+\frac{4\tan(c+dx)}{5ad}-\frac{\sec^3(c+dx)}{5d(a\sin(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/5*Sec[c + d*x]^3/(d*(a + a*Sin[c + d*x])) + (4*Tan[c + d*x])/(5*a*d) + (
4*Tan[c + d*x]^3)/(15*a*d)
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec^4(c+dx)}{a+a\sin(c+dx)}\,dx &= -\frac{\sec^3(c+dx)}{5d(a+a\sin(c+dx))}+\frac{4\int \sec^4(c+dx)\,dx}{5a} \\
&= -\frac{\sec^3(c+dx)}{5d(a+a\sin(c+dx))}-\frac{4\text{Subst}\left(\int (1+x^2)\ dx, x, -\tan(c+dx)\right)}{5ad} \\
&= -\frac{\sec^3(c+dx)}{5d(a+a\sin(c+dx))}+\frac{4\tan(c+dx)}{5ad}+\frac{4\tan^3(c+dx)}{15ad}
\end{aligned}
$$

Mathematica [A]
time = 0.07, size = 66, normalized size = 1.06

$$
-\frac{\sec^3(c+dx)(2\cos(2(c+dx))+\cos(4(c+dx))-2(3\sin(c+dx)+\sin(3(c+dx))))}{15ad(1+\sin(c+dx))}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/15*(Sec[c + d*x]^3*(2*Cos[2*(c + d*x)] + Cos[4*(c + d*x)] - 2*(3*Sin[c +
 d*x] + Sin[3*(c + d*x)])))/(a*d*(1 + Sin[c + d*x]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(56) = 112$.
time = 0.18, size = 130, normalized size = 2.10

method	result
risch	$-\frac{16\left(6\,\mathrm{e}^{3i(dx+c)}+2i\mathrm{e}^{2i(dx+c)}+2\,\mathrm{e}^{i(dx+c)}+i\right)}{15\left(\mathrm{e}^{i(dx+c)}+i\right)^5\left(\mathrm{e}^{i(dx+c)}-i\right)^3 da}$
derivativedivides	$\frac{-\frac{2}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}+\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}-\frac{5}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}+\frac{3}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{11}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{1}{6\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\right.}}{da}$
default	$\frac{-\frac{2}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}+\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}-\frac{5}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}+\frac{3}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{11}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{1}{6\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\right.}}{da}$
norman	$\frac{\frac{10\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3da}+\frac{2}{5ad}-\frac{2\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}-\frac{2\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}-\frac{14\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5da}-\frac{6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{5da}+\frac{2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3da}-\frac{26(\mathrm{t}}{}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/a*(-1/5/(tan(1/2*d*x+1/2*c)+1)^5+1/2/(tan(1/2*d*x+1/2*c)+1)^4-5/6/(tan(
1/2*d*x+1/2*c)+1)^3+3/4/(tan(1/2*d*x+1/2*c)+1)^2-11/16/(tan(1/2*d*x+1/2*c)+
1)-1/12/(tan(1/2*d*x+1/2*c)-1)^3-1/8/(tan(1/2*d*x+1/2*c)-1)^2-5/16/(tan(1/2
*d*x+1/2*c)-1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 294 vs. $2(56) = 112$.

time = 0.36, size = 294, normalized size = 4.74

$$\frac{2\left(\frac{9\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{21\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{13\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{25\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{5\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{15\,\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{15\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-3\right)}{15\left(a+\frac{2\,a\sin(dx+c)}{\cos(dx+c)+1}-\frac{2\,a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{6\,a\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{6\,a\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{2\,a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{2\,a\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 2/15*(9*sin(d*x + c)/(cos(d*x + c) + 1) + 21*sin(d*x + c)^2/(cos(d*x + c) +
 1)^2 + 13*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 25*sin(d*x + c)^4/(cos(d*x
 + c) + 1)^4 - 5*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 15*sin(d*x + c)^6/(c
os(d*x + c) + 1)^6 + 15*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 3)/((a + 2*a*
sin(d*x + c)/(cos(d*x + c) + 1) - 2*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 -
 6*a*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 6*a*sin(d*x + c)^5/(cos(d*x + c)
 + 1)^5 + 2*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 2*a*sin(d*x + c)^7/(cos
(d*x + c) + 1)^7 - a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8)*d)
```

Fricas [A]

time = 0.36, size = 75, normalized size = 1.21

$$-\frac{8\,\cos\left(dx+c\right)^4-4\,\cos\left(dx+c\right)^2-4\left(2\,\cos\left(dx+c\right)^2+1\right)\sin\left(dx+c\right)-1}{15\left(ad\cos\left(dx+c\right)^3\sin\left(dx+c\right)+ad\cos\left(dx+c\right)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/15*(8*cos(d*x + c)^4 - 4*cos(d*x + c)^2 - 4*(2*cos(d*x + c)^2 + 1)*sin(d
*x + c) - 1)/(a*d*cos(d*x + c)^3*sin(d*x + c) + a*d*cos(d*x + c)^3)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\sec^4(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**4/(sin(c + d*x) + 1), x)/a
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(56) = 112$.

time = 6.21, size = 119, normalized size = 1.92

$$-\frac{\frac{5\left(15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-24\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+13\right)}{a\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^3}+\frac{165\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+480\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+650\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+400\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+113}{a\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^5}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/120*(5*(15*tan(1/2*d*x + 1/2*c)^2 - 24*tan(1/2*d*x + 1/2*c) + 13)/(a*(ta
n(1/2*d*x + 1/2*c) - 1)^3) + (165*tan(1/2*d*x + 1/2*c)^4 + 480*tan(1/2*d*x
+ 1/2*c)^3 + 650*tan(1/2*d*x + 1/2*c)^2 + 400*tan(1/2*d*x + 1/2*c) + 113)/(
a*(tan(1/2*d*x + 1/2*c) + 1)^5))/d
```

Mupad [B]
time = 5.94, size = 125, normalized size = 2.02

$$-\frac{2\left(15\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+15\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6-5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5-25\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+13\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+21\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+9\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-3\right)}{15\,a\,d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-1\right)^3\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a + a*sin(c + d*x))),x)
```

```
[Out] -(2*(9*tan(c/2 + (d*x)/2) + 21*tan(c/2 + (d*x)/2)^2 + 13*tan(c/2 + (d*x)/2)
^3 - 25*tan(c/2 + (d*x)/2)^4 - 5*tan(c/2 + (d*x)/2)^5 + 15*tan(c/2 + (d*x)/
2)^6 + 15*tan(c/2 + (d*x)/2)^7 - 3))/(15*a*d*(tan(c/2 + (d*x)/2) - 1)^3*(ta
n(c/2 + (d*x)/2) + 1)^5)
```

3.61 $\int \frac{\sec^5(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=120

$$\frac{5\tanh^{-1}(\sin(c+dx))}{16ad}+\frac{a}{32d(a-a\sin(c+dx))^2}+\frac{1}{8d(a-a\sin(c+dx))}-\frac{a^2}{24d(a+a\sin(c+dx))^3}-\frac{}{32d(a+a}$$

```
[Out] 5/16*arctanh(sin(d*x+c))/a/d+1/32*a/d/(a-a*sin(d*x+c))^2+1/8/d/(a-a*sin(d*x
+c))-1/24*a^2/d/(a+a*sin(d*x+c))^3-3/32*a/d/(a+a*sin(d*x+c))^2-3/16/d/(a+a*
sin(d*x+c))
```

Rubi [A]

time = 0.08, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2746, 46, 212}

$$-\frac{a^2}{24d(a\sin(c+dx)+a)^3}+\frac{a}{32d(a-a\sin(c+dx))^2}-\frac{3a}{32d(a\sin(c+dx)+a)^2}+\frac{1}{8d(a-a\sin(c+dx))}-\frac{3}{16d(a\sin(c+dx)+a)}+\frac{5\tanh^{-1}(\sin(c+dx))}{16ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + a*Sin[c + d*x]),x]
```

```
[Out] (5*ArcTanh[Sin[c + d*x]])/(16*a*d) + a/(32*d*(a - a*Sin[c + d*x])^2) + 1/(8
*d*(a - a*Sin[c + d*x])) - a^2/(24*d*(a + a*Sin[c + d*x])^3) - (3*a)/(32*d*
(a + a*Sin[c + d*x])^2) - 3/(16*d*(a + a*Sin[c + d*x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{a^5\text{Subst}\left(\int \frac{1}{(a-x)^3(a+x)^4}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^5\text{Subst}\left(\int \left(\frac{1}{16a^4(a-x)^3} + \frac{1}{8a^5(a-x)^2} + \frac{1}{8a^3(a+x)^4} + \frac{3}{16a^4(a+x)^3} + \frac{3}{16a^5(a+x)^2} + \frac{5}{16a^5(a^2-x^2)}\right.\right.}{d}$$
$$= \frac{a}{32d(a-a\sin(c+dx))^2} + \frac{1}{8d(a-a\sin(c+dx))} - \frac{a^2}{24d(a+a\sin(c+dx))^3} - \frac{32}{}$$
$$= \frac{5\tanh^{-1}(\sin(c+dx))}{16ad} + \frac{a}{32d(a-a\sin(c+dx))^2} + \frac{1}{8d(a-a\sin(c+dx))} - \frac{}{24d(}$$

Mathematica [A]
time = 0.10, size = 97, normalized size = 0.81

$$\frac{\sec^4(c+dx)\left(-8+25\sin(c+dx)+25\sin^2(c+dx)-15\sin^3(c+dx)-15\sin^4(c+dx)+15\tanh^{-1}(\sin(c+dx))(-1+\sin(c+dx))^2(1+\sin(c+dx))^3\right)}{48ad(1+\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a + a*Sin[c + d*x]),x]
[Out] (Sec[c + d*x]^4*(-8 + 25*Sin[c + d*x] + 25*Sin[c + d*x]^2 - 15*Sin[c + d*x]
^3 - 15*Sin[c + d*x]^4 + 15*ArcTanh[Sin[c + d*x]]*(-1 + Sin[c + d*x])^2*(1
+ Sin[c + d*x])^3))/(48*a*d*(1 + Sin[c + d*x]))
```

Maple [A]
time = 0.23, size = 91, normalized size = 0.76

method	result
derivativedivides	$\frac{\frac{1}{32(\sin(dx+c)-1)^2}-\frac{1}{8(\sin(dx+c)-1)}-\frac{5\ln(\sin(dx+c)-1)}{32}-\frac{1}{24(1+\sin(dx+c))^3}-\frac{3}{32(1+\sin(dx+c))^2}-\frac{3}{16(1+\sin(dx+c))}+\frac{5\ln(1+\sin(}{32}}{da}$
default	$\frac{\frac{1}{32(\sin(dx+c)-1)^2}-\frac{1}{8(\sin(dx+c)-1)}-\frac{5\ln(\sin(dx+c)-1)}{32}-\frac{1}{24(1+\sin(dx+c))^3}-\frac{3}{32(1+\sin(dx+c))^2}-\frac{3}{16(1+\sin(dx+c))}+\frac{5\ln(1+\sin(}{32}}{da}$
risch	$-\frac{i\left(30i e^{8i(dx+c)}+15\,e^{9i(dx+c)}+110i e^{6i(dx+c)}+40\,e^{7i(dx+c)}-110i e^{4i(dx+c)}+18\,e^{5i(dx+c)}-30i e^{2i(dx+c)}+40\,e^{3i(dx+c)}+\right.}{24\left(e^{i(dx+c)}+i\right)^6\left(e^{i(dx+c)}-i\right)^4 da}$
norman	$\frac{\frac{3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4da}+\frac{3\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4da}+\frac{11\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8da}+\frac{11\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8da}+\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{3da}-\frac{\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)}{12da}-\frac{\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)}{12da}+\frac{\mathrm{ta}}{}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^6\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(1/32/(sin(d*x+c)-1)^2-1/8/(sin(d*x+c)-1)-5/32*ln(sin(d*x+c)-1)-1/24/
(1+sin(d*x+c))^3-3/32/(1+sin(d*x+c))^2-3/16/(1+sin(d*x+c))+5/32*ln(1+sin(d*
x+c)))
```

Maxima [A]
time = 0.51, size = 130, normalized size = 1.08

$$-\frac{\frac{2\left(15\,\sin(dx+c)^4+15\,\sin(dx+c)^3-25\,\sin(dx+c)^2-25\,\sin(dx+c)+8\right)}{a\sin(dx+c)^5+a\sin(dx+c)^4-2\,a\sin(dx+c)^3-2\,a\sin(dx+c)^2+a\sin(dx+c)+a}-\frac{15\,\log(\sin(dx+c)+1)}{a}+\frac{15\,\log(\sin(dx+c)-1)}{a}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/96*(2*(15*sin(d*x + c)^4 + 15*sin(d*x + c)^3 - 25*sin(d*x + c)^2 - 25*si
n(d*x + c) + 8)/(a*sin(d*x + c)^5 + a*sin(d*x + c)^4 - 2*a*sin(d*x + c)^3 -
 2*a*sin(d*x + c)^2 + a*sin(d*x + c) + a) - 15*log(sin(d*x + c) + 1)/a + 15
*log(sin(d*x + c) - 1)/a)/d
```

Fricas [A]
time = 0.36, size = 147, normalized size = 1.22

$$-\frac{30\,\cos\left(dx+c\right)^4-10\,\cos\left(dx+c\right)^2-15\left(\cos\left(dx+c\right)^4\sin\left(dx+c\right)+\cos\left(dx+c\right)^4\right)\log\left(\sin\left(dx+c\right)+1\right)+15\left(\cos\left(dx+c\right)^4\sin\left(dx+c\right)+\cos\left(dx+c\right)^4\right)\log\left(-\sin\left(dx+c\right)+1\right)-10\left(3\,\cos\left(dx+c\right)^2+2\right)\sin\left(dx+c\right)-4}{96\left(ad\cos\left(dx+c\right)^4\sin\left(dx+c\right)+ad\cos\left(dx+c\right)^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/96*(30*cos(d*x + c)^4 - 10*cos(d*x + c)^2 - 15*(cos(d*x + c)^4*sin(d*x +
 c) + cos(d*x + c)^4)*log(sin(d*x + c) + 1) + 15*(cos(d*x + c)^4*sin(d*x +
c) + cos(d*x + c)^4)*log(-sin(d*x + c) + 1) - 10*(3*cos(d*x + c)^2 + 2)*sin
(d*x + c) - 4)/(a*d*cos(d*x + c)^4*sin(d*x + c) + a*d*cos(d*x + c)^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^5\left(c+dx\right)}{\sin\left(c+dx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**5/(sin(c + d*x) + 1), x)/a
```

Giac [A]
time = 6.79, size = 116, normalized size = 0.97

$$\frac{\frac{30\,\log(|\sin(dx+c)+1|)}{a}-\frac{30\,\log(|\sin(dx+c)-1|)}{a}+\frac{3\left(15\,\sin(dx+c)^2-38\,\sin(dx+c)+25\right)}{a(\sin(dx+c)-1)^2}-\frac{55\,\sin(dx+c)^3+201\,\sin(dx+c)^2+255\,\sin(dx+c)+117}{a(\sin(dx+c)+1)^3}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/192*(30*log(abs(sin(d*x + c) + 1))/a - 30*log(abs(sin(d*x + c) - 1))/a +
3*(15*sin(d*x + c)^2 - 38*sin(d*x + c) + 25)/(a*(sin(d*x + c) - 1)^2) - (55
*sin(d*x + c)^3 + 201*sin(d*x + c)^2 + 255*sin(d*x + c) + 117)/(a*(sin(d*x
+ c) + 1)^3))/d
```

Mupad [B]
time = 0.14, size = 115, normalized size = 0.96

$$\frac{5\,\mathrm{atanh}(\sin\left(c+d\,x\right))}{16\,a\,d}-\frac{\frac{5\sin(c+d\,x)^4}{16}+\frac{5\sin(c+d\,x)^3}{16}-\frac{25\sin(c+d\,x)^2}{48}-\frac{25\,\sin(c+d\,x)}{48}+\frac{1}{6}}{d\,\left(a\sin\left(c+d\,x\right)^5+a\sin\left(c+d\,x\right)^4-2\,a\sin\left(c+d\,x\right)^3-2\,a\sin\left(c+d\,x\right)^2+a\,\sin\left(c+d\,x\right)+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^5*(a + a*sin(c + d*x))),x)
```

```
[Out] (5*atanh(sin(c + d*x)))/(16*a*d) - ((5*sin(c + d*x)^3)/16 - (25*sin(c + d*x
)^2)/48 - (25*sin(c + d*x))/48 + (5*sin(c + d*x)^4)/16 + 1/6)/(d*(a + a*sin
(c + d*x) - 2*a*sin(c + d*x)^2 - 2*a*sin(c + d*x)^3 + a*sin(c + d*x)^4 + a*
sin(c + d*x)^5))
```

3.62 $\int \frac{\cos^8(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=104

$$\frac{7x}{16a^2}+\frac{7\cos^5(c+dx)}{30a^2d}+\frac{7\cos(c+dx)\sin(c+dx)}{16a^2d}+\frac{7\cos^3(c+dx)\sin(c+dx)}{24a^2d}+\frac{\cos^7(c+dx)}{6d\,(a^2+a^2\sin(c+dx))}$$

```
[Out] 7/16*x/a^2+7/30*cos(d*x+c)^5/a^2/d+7/16*cos(d*x+c)*sin(d*x+c)/a^2/d+7/24*co
s(d*x+c)^3*sin(d*x+c)/a^2/d+1/6*cos(d*x+c)^7/d/(a^2+a^2*sin(d*x+c))
```

Rubi [A]
time = 0.08, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2758, 2761, 2715, 8}

$$\frac{7\cos^5(c+dx)}{30a^2d}+\frac{\cos^7(c+dx)}{6d\,(a^2\sin(c+dx)+a^2)}+\frac{7\sin(c+dx)\cos^3(c+dx)}{24a^2d}+\frac{7\sin(c+dx)\cos(c+dx)}{16a^2d}+\frac{7x}{16a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^8/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (7*x)/(16*a^2) + (7*Cos[c + d*x]^5)/(30*a^2*d) + (7*Cos[c + d*x]*Sin[c + d*
x])/(16*a^2*d) + (7*Cos[c + d*x]^3*Sin[c + d*x])/(24*a^2*d) + Cos[c + d*x]^
7/(6*d*(a^2 + a^2*Sin[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2758

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(a*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{\cos^8(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\cos^7(c+dx)}{6d\,(a^2+a^2\sin(c+dx))} + \frac{7\int \frac{\cos^6(c+dx)}{a+a\sin(c+dx)}\,dx}{6a}$$

$$= \frac{7\cos^5(c+dx)}{30a^2d} + \frac{\cos^7(c+dx)}{6d\,(a^2+a^2\sin(c+dx))} + \frac{7\int \cos^4(c+dx)\,dx}{6a^2}$$

$$= \frac{7\cos^5(c+dx)}{30a^2d} + \frac{7\cos^3(c+dx)\sin(c+dx)}{24a^2d} + \frac{\cos^7(c+dx)}{6d\,(a^2+a^2\sin(c+dx))} + \frac{7\int c}{}$$

$$= \frac{7\cos^5(c+dx)}{30a^2d} + \frac{7\cos(c+dx)\sin(c+dx)}{16a^2d} + \frac{7\cos^3(c+dx)\sin(c+dx)}{24a^2d} + \frac{}{6d\,(}$$

$$= \frac{7x}{16a^2} + \frac{7\cos^5(c+dx)}{30a^2d} + \frac{7\cos(c+dx)\sin(c+dx)}{16a^2d} + \frac{7\cos^3(c+dx)\sin(c+dx}{24a^2d}$$

Mathematica [A]
time = 0.74, size = 151, normalized size = 1.45

$$-\frac{\cos^9(c+dx)\left(-210\sin^{-1}\left(\frac{\sqrt{1-\sin(c+dx)}}{\sqrt{2}}\right)\sqrt{1-\sin(c+dx)}+\sqrt{1+\sin(c+dx)}\,(96+39\sin(c+dx)-327\sin^2(c+dx)+202\sin^3(c+dx)+86\sin^4(c+dx)-136\sin^5(c+dx)+40\sin^6(c+dx))\right)}{240a^2d(-1+\sin(c+dx))^5(1+\sin(c+dx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^8/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/240*(Cos[c + d*x]^9*(-210*ArcSin[Sqrt[1 - Sin[c + d*x]]/Sqrt[2]]*Sqrt[1
- Sin[c + d*x]] + Sqrt[1 + Sin[c + d*x]]*(96 + 39*Sin[c + d*x] - 327*Sin[c
+ d*x]^2 + 202*Sin[c + d*x]^3 + 86*Sin[c + d*x]^4 - 136*Sin[c + d*x]^5 + 40
*Sin[c + d*x]^6)))/(a^2*d*(-1 + Sin[c + d*x])^5*(1 + Sin[c + d*x])^(9/2))
```

Maple [A]
time = 0.16, size = 181, normalized size = 1.74

method	result
risch	$\frac{7x}{16a^2}+\frac{\cos(dx+c)}{4a^2d}-\frac{\sin(6dx+6c)}{192a^2d}+\frac{\cos(5dx+5c)}{40a^2d}+\frac{\sin(4dx+4c)}{64a^2d}+\frac{\cos(3dx+3c)}{8a^2d}+\frac{17\sin(2dx+2c)}{64a^2d}$

derivativedivides	$\frac{2\left(-\frac{9\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16}+2\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{89\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{48}+2\left(\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{11\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8}+4\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{11(\tan}{}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}}{a^2 d}$
default	$\frac{2\left(-\frac{9\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16}+2\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{89\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{48}+2\left(\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{11\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8}+4\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{11(\tan}{}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}}{a^2 d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/a^2*((-9/16*tan(1/2*d*x+1/2*c)^11+2*tan(1/2*d*x+1/2*c)^10-89/48*tan(1/2
*d*x+1/2*c)^9+2*tan(1/2*d*x+1/2*c)^8+11/8*tan(1/2*d*x+1/2*c)^7+4*tan(1/2*d*
x+1/2*c)^6-11/8*tan(1/2*d*x+1/2*c)^5+4*tan(1/2*d*x+1/2*c)^4+89/48*tan(1/2*d
*x+1/2*c)^3+2/5*tan(1/2*d*x+1/2*c)^2+9/16*tan(1/2*d*x+1/2*c)+2/5)/(1+tan(1/
2*d*x+1/2*c)^2)^6+7/16*arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 393 vs. $2(94) = 188$.
time = 0.85, size = 393, normalized size = 3.78

$$\frac{\frac{\frac{135\sin(dx+c)}{\cos(dx+c)+1}+\frac{96\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{445\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{960\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{330\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{960\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{330\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{480\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{445\sin(dx+c)^9}{(\cos(dx+c)+1)^9}+\frac{480\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}-\frac{135\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}+96}{a^2+\frac{6a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{15a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{20a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{15a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{6a^2\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{a^2\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}}+\frac{105\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/120*((135*sin(d*x + c)/(cos(d*x + c) + 1) + 96*sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 + 445*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 960*sin(d*x + c)^4/(c
os(d*x + c) + 1)^4 - 330*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 960*sin(d*x
+ c)^6/(cos(d*x + c) + 1)^6 + 330*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 480
*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 445*sin(d*x + c)^9/(cos(d*x + c) + 1
)^9 + 480*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 - 135*sin(d*x + c)^11/(cos(
d*x + c) + 1)^11 + 96)/(a^2 + 6*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1
5*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 20*a^2*sin(d*x + c)^6/(cos(d*x
+ c) + 1)^6 + 15*a^2*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 6*a^2*sin(d*x +
c)^10/(cos(d*x + c) + 1)^10 + a^2*sin(d*x + c)^12/(cos(d*x + c) + 1)^12) +
105*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2)/d
```

Fricas [A]
time = 0.36, size = 60, normalized size = 0.58

$$\frac{96\cos(dx+c)^5+105\,dx-5\left(8\cos(dx+c)^5-14\cos(dx+c)^3-21\cos(dx+c)\right)\sin(dx+c)}{240\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/240*(96*cos(d*x + c)^5 + 105*d*x - 5*(8*cos(d*x + c)^5 - 14*cos(d*x + c)^
3 - 21*cos(d*x + c))*sin(d*x + c))/(a^2*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 2531 vs. $2(95) = 190$.
time = 78.33, size = 2531, normalized size = 24.34

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((105*d*x*tan(c/2 + d*x/2)**12/(240*a**2*d*tan(c/2 + d*x/2)**12 +
1440*a**2*d*tan(c/2 + d*x/2)**10 + 3600*a**2*d*tan(c/2 + d*x/2)**8 + 4800*a
**2*d*tan(c/2 + d*x/2)**6 + 3600*a**2*d*tan(c/2 + d*x/2)**4 + 1440*a**2*d*t
an(c/2 + d*x/2)**2 + 240*a**2*d) + 630*d*x*tan(c/2 + d*x/2)**10/(240*a**2*d
*tan(c/2 + d*x/2)**12 + 1440*a**2*d*tan(c/2 + d*x/2)**10 + 3600*a**2*d*tan(
c/2 + d*x/2)**8 + 4800*a**2*d*tan(c/2 + d*x/2)**6 + 3600*a**2*d*tan(c/2 + d
*x/2)**4 + 1440*a**2*d*tan(c/2 + d*x/2)**2 + 240*a**2*d) + 1575*d*x*tan(c/2
 + d*x/2)**8/(240*a**2*d*tan(c/2 + d*x/2)**12 + 1440*a**2*d*tan(c/2 + d*x/2
)**10 + 3600*a**2*d*tan(c/2 + d*x/2)**8 + 4800*a**2*d*tan(c/2 + d*x/2)**6 +
 3600*a**2*d*tan(c/2 + d*x/2)**4 + 1440*a**2*d*tan(c/2 + d*x/2)**2 + 240*a*
*2*d) + 2100*d*x*tan(c/2 + d*x/2)**6/(240*a**2*d*tan(c/2 + d*x/2)**12 + 144
0*a**2*d*tan(c/2 + d*x/2)**10 + 3600*a**2*d*tan(c/2 + d*x/2)**8 + 4800*a**2
*d*tan(c/2 + d*x/2)**6 + 3600*a**2*d*tan(c/2 + d*x/2)**4 + 1440*a**2*d*tan(
c/2 + d*x/2)**2 + 240*a**2*d) + 1575*d*x*tan(c/2 + d*x/2)**4/(240*a**2*d*ta
n(c/2 + d*x/2)**12 + 1440*a**2*d*tan(c/2 + d*x/2)**10 + 3600*a**2*d*tan(c/2
 + d*x/2)**8 + 4800*a**2*d*tan(c/2 + d*x/2)**6 + 3600*a**2*d*tan(c/2 + d*x/
2)**4 + 1440*a**2*d*tan(c/2 + d*x/2)**2 + 240*a**2*d) + 630*d*x*tan(c/2 + d
*x/2)**2/(240*a**2*d*tan(c/2 + d*x/2)**12 + 1440*a**2*d*tan(c/2 + d*x/2)**1
0 + 3600*a**2*d*tan(c/2 + d*x/2)**8 + 4800*a**2*d*tan(c/2 + d*x/2)**6 + 360
0*a**2*d*tan(c/2 + d*x/2)**4 + 1440*a**2*d*tan(c/2 + d*x/2)**2 + 240*a**2*d
) + 105*d*x/(240*a**2*d*tan(c/2 + d*x/2)**12 + 1440*a**2*d*tan(c/2 + d*x/2)
**10 + 3600*a**2*d*tan(c/2 + d*x/2)**8 + 4800*a**2*d*tan(c/2 + d*x/2)**6 +
3600*a**2*d*tan(c/2 + d*x/2)**4 + 1440*a**2*d*tan(c/2 + d*x/2)**2 + 240*a**
2*d) - 270*tan(c/2 + d*x/2)**11/(240*a**2*d*tan(c/2 + d*x/2)**12 + 1440*a**
2*d*tan(c/2 + d*x/2)**10 + 3600*a**2*d*tan(c/2 + d*x/2)**8 + 4800*a**2*d*ta
n(c/2 + d*x/2)**6 + 3600*a**2*d*tan(c/2 + d*x/2)**4 + 1440*a**2*d*tan(c/2 +
 d*x/2)**2 + 240*a**2*d) + 960*tan(c/2 + d*x/2)**10/(240*a**2*d*tan(c/2 + d
*x/2)**12 + 1440*a**2*d*tan(c/2 + d*x/2)**10 + 3600*a**2*d*tan(c/2 + d*x/2)
**8 + 4800*a**2*d*tan(c/2 + d*x/2)**6 + 3600*a**2*d*tan(c/2 + d*x/2)**4 + 1
440*a**2*d*tan(c/2 + d*x/2)**2 + 240*a**2*d) - 890*tan(c/2 + d*x/2)**9/(240
*a**2*d*tan(c/2 + d*x/2)**12 + 1440*a**2*d*tan(c/2 + d*x/2)**10 + 3600*a**2
*d*tan(c/2 + d*x/2)**8 + 4800*a**2*d*tan(c/2 + d*x/2)**6 + 3600*a**2*d*tan(
```

```
c/2 + d*x/2)**4 + 1440*a**2*d*tan(c/2 + d*x/2)**2 + 240*a**2*d) + 960*tan(c
/2 + d*x/2)**8/(240*a**2*d*tan(c/2 + d*x/2)**12 + 1440*a**2*d*tan(c/2 + d*x
/2)**10 + 3600*a**2*d*tan(c/2 + d*x/2)**8 + 4800*a**2*d*tan(c/2 + d*x/2)**6
 + 3600*a**2*d*tan(c/2 + d*x/2)**4 + 1440*a**2*d*tan(c/2 + d*x/2)**2 + 240*
a**2*d) + 660*tan(c/2 + d*x/2)**7/(240*a**2*d*tan(c/2 + d*x/2)**12 + 1440*a
**2*d*tan(c/2 + d*x/2)**10 + 3600*a**2*d*tan(c/2 + d*x/2)**8 + 4800*a**2*d*
tan(c/2 + d*x/2)**6 + 3600*a**2*d*tan(c/2 + d*x/2)**4 + 1440*a**2*d*tan(c/2
 + d*x/2)**2 + 240*a**2*d) + 1920*tan(c/2 + d*x/2)**6/(240*a**2*d*tan(c/2 +
 d*x/2)**12 + 1440*a**2*d*tan(c/2 + d*x/2)**10 + 3600*a**2*d*tan(c/2 + d*x/
2)**8 + 4800*a**2*d*tan(c/2 + d*x/2)**6 + 3600*a**2*d*tan(c/2 + d*x/2)**4 +
 1440*a**2*d*tan(c/2 + d*x/2)**2 + 240*a**2*d) - 660*tan(c/2 + d*x/2)**5/(2
40*a**2*d*tan(c/2 + d*x/2)**12 + 1440*a**2*d*tan(c/2 + d*x/2)**10 + 3600*a*
*2*d*tan(c/2 + d*x/2)**8 + 4800*a**2*d*tan(c/2 + d*x/2)**6 + 3600*a**2*d*ta
n(c/2 + d*x/2)**4 + 1440*a**2*d*tan(c/2 + d*x/2)**2 + 240*a**2*d) + 1920*ta
n(c/2 + d*x/2)**4/(240*a**2*d*tan(c/2 + d*x/2)**12 + 1440*a**2*d*tan(c/2 +
d*x/2)**10 + 3600*a**2*d*tan(c/2 + d*x/2)**8 + 4800*a**2*d*tan(c/2 + d*x/2)
**6 + 3600*a**2*d*tan(c/2 + d*x/2)**4 + 1440*a**2*d*tan(c/2 + d*x/2)**2 + 2
40*a**2*d) + 890*tan(c/2 + d*x/2)**3/(240*a**2*d*tan(c/2 + d*x/2)**12 + 144
0*a**2*d*tan(c/2 + d*x/2)**10 + 3600*a**2*d*tan(c/2 + d*x/2)**8 + 4800*a**2
*d*tan(c/2 + d*x/2)**6 + 3600*a**2*d*tan(c/2 + d*x/2)**4 + 1440*a**2*d*tan(
c/2 + d*x/2)**2 + 240*a**2*d) + 192*tan(c/2 + d*x/2)**2/(240*a**2*d*tan(c/2
 + d*x/2)**12 + 1440*a**2*d*tan(c/2 + d*x/2)**10 + 3600*a**2*d*tan(c/2 + d*
x/2)**8 + 4800*a**2*d*tan(c/2 + d*x/2)**6 + 3600*a**2*d*tan(c/2 + d*x/2)**4
 + 1440*a**2*d*tan(c/2 + d*x/2)**2 + 240*a**2*d) + 270*tan(c/2 + d*x/2)/(24
0*a**2*d*tan(c/2 + d*x/2)**12 + 1440*a**2*d*tan(c/2 + d*x/2)**10 + 3600*a**
2*d*tan(c/2 + d*x/2)**8 + 4800*a**2*d*tan(c/2 + d*x/2)**6 + 3600*a**2*d*tan
(c/2 + d*x/2)**4 + 1440*a**2*d*tan(c/2 + d*x/2)**2 + 240*a**2*d) + 192/(240
*a**2*d*tan(c/2 + d*x/2)**12 + 1440*a**2*d*tan(c/2 + d*x/2)**10 + 3600*a**2
*d*tan(c/2 + d*x/2)**8 + 4800*a**2*d*tan(c/2 + d*x/2)**6 + 3600*a**2*d*tan(
c/2 + d*x/2)**4 + 1440*a**2*d*tan(c/2 + d*x/2)**2 + 240*a**2*d), Ne(d, 0)),
 (x*cos(c)**8/(a*sin(c) + a)**2, True))
```

Giac [**A**]
time = 6.89, size = 179, normalized size = 1.72

$$\frac{\frac{105\,(dx+c)}{a^2} - \frac{2\left(135\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-480\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}+445\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-480\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}-330\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-960\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+330\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-960\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-445\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-96\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-135\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-96\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{6}a^2}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/240*(105*(d*x + c)/a^2 - 2*(135*tan(1/2*d*x + 1/2*c)^11 - 480*tan(1/2*d*x
 + 1/2*c)^10 + 445*tan(1/2*d*x + 1/2*c)^9 - 480*tan(1/2*d*x + 1/2*c)^8 - 33
0*tan(1/2*d*x + 1/2*c)^7 - 960*tan(1/2*d*x + 1/2*c)^6 + 330*tan(1/2*d*x + 1
/2*c)^5 - 960*tan(1/2*d*x + 1/2*c)^4 - 445*tan(1/2*d*x + 1/2*c)^3 - 96*tan(
1/2*d*x + 1/2*c)^2 - 135*tan(1/2*d*x + 1/2*c) - 96)/((tan(1/2*d*x + 1/2*c)^
2 + 1)^6*a^2))/d
```

Mupad [B]
time = 8.22, size = 172, normalized size = 1.65

$$\frac{7\,x}{16\,a^2}+\frac{-\frac{9\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{11}}{8}+4\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}-\frac{89\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^9}{24}+4\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8+\frac{11\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7}{4}+8\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6-\frac{11\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5}{4}+8\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+\frac{89\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3}{24}+\frac{4\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}{5}+\frac{9\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8}+\frac{4}{5}}{a^2\,d\,{\left({\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^8/(a + a*sin(c + d*x))^2,x)
```

```
[Out] (7*x)/(16*a^2) + ((9*tan(c/2 + (d*x)/2))/8 + (4*tan(c/2 + (d*x)/2)^2)/5 + (
89*tan(c/2 + (d*x)/2)^3)/24 + 8*tan(c/2 + (d*x)/2)^4 - (11*tan(c/2 + (d*x)/
2)^5)/4 + 8*tan(c/2 + (d*x)/2)^6 + (11*tan(c/2 + (d*x)/2)^7)/4 + 4*tan(c/2
+ (d*x)/2)^8 - (89*tan(c/2 + (d*x)/2)^9)/24 + 4*tan(c/2 + (d*x)/2)^10 - (9*
tan(c/2 + (d*x)/2)^11)/8 + 4/5)/(a^2*d*(tan(c/2 + (d*x)/2)^2 + 1)^6)
```

3.63 $\int \frac{\cos^7(c+dx)}{(a+a\sin(c+dx))^2} dx$

Optimal. Leaf size=47

$$-\frac{(a-a\sin(c+dx))^4}{2a^6d}+\frac{(a-a\sin(c+dx))^5}{5a^7d}$$

```
[Out] -1/2*(a-a*sin(d*x+c))^4/a^6/d+1/5*(a-a*sin(d*x+c))^5/a^7/d
```

Rubi [A]

time = 0.03, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2746, 45}

$$\frac{(a-a\sin(c+dx))^5}{5a^7d}-\frac{(a-a\sin(c+dx))^4}{2a^6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/2*(a - a*Sin[c + d*x])^4/(a^6*d) + (a - a*Sin[c + d*x])^5/(5*a^7*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^7(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{\text{Subst}\left(\int (a-x)^3(a+x)\,dx, x, a\sin(c+dx)\right)}{a^7d}\\
&= \frac{\text{Subst}\left(\int \left(2a(a-x)^3-(a-x)^4\right)\,dx, x, a\sin(c+dx)\right)}{a^7d}\\
&= -\frac{(a-a\sin(c+dx))^4}{2a^6d}+\frac{(a-a\sin(c+dx))^5}{5a^7d}
\end{aligned}$$

Mathematica [A]
time = 0.11, size = 46, normalized size = 0.98

$$-\frac{\sin(c+dx)\left(-10+10\sin(c+dx)-5\sin^3(c+dx)+2\sin^4(c+dx)\right)}{10a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/10*(Sin[c + d*x]*(-10 + 10*Sin[c + d*x] - 5*Sin[c + d*x]^3 + 2*Sin[c + d
*x]^4))/(a^2*d)
```

Maple [A]
time = 0.28, size = 45, normalized size = 0.96

method	result
derivativedivides	$\frac{-\frac{\left(\sin^5(dx+c)\right)}{5}+\frac{\left(\sin^4(dx+c)\right)}{2}-\left(\sin^2(dx+c)\right)+\sin(dx+c)}{d\,a^2}$
default	$\frac{-\frac{\left(\sin^5(dx+c)\right)}{5}+\frac{\left(\sin^4(dx+c)\right)}{2}-\left(\sin^2(dx+c)\right)+\sin(dx+c)}{d\,a^2}$
risch	$\frac{7\sin(dx+c)}{8a^2d}-\frac{\sin(5dx+5c)}{80a^2d}+\frac{\cos(4dx+4c)}{16a^2d}+\frac{\sin(3dx+3c)}{16a^2d}+\frac{\cos(2dx+2c)}{4a^2d}$
norman	$-\frac{2}{3ad}-\frac{2\left(\tan^{17}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3da}-\frac{14\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}-\frac{14\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}-\frac{14\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3da}-\frac{14\left(\tan^{15}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3da}-\frac{26\left(\tan^3\left(\frac{dx}{2}+\right.\right.}{3da}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^2*(-1/5*sin(d*x+c)^5+1/2*sin(d*x+c)^4-sin(d*x+c)^2+sin(d*x+c))
```

Maxima [A]
time = 0.38, size = 47, normalized size = 1.00

$$-\frac{2\sin\left(dx+c\right)^5-5\sin\left(dx+c\right)^4+10\sin\left(dx+c\right)^2-10\sin\left(dx+c\right)}{10\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/10*(2*sin(d*x + c)^5 - 5*sin(d*x + c)^4 + 10*sin(d*x + c)^2 - 10*sin(d*x
 + c))/(a^2*d)
```

Fricas [A]
time = 0.35, size = 47, normalized size = 1.00

$$\frac{5\cos\left(dx+c\right)^4-2\left(\cos\left(dx+c\right)^4-2\cos\left(dx+c\right)^2-4\right)\sin\left(dx+c\right)}{10\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/10*(5*cos(d*x + c)^4 - 2*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 - 4)*sin(d*x
+ c))/(a^2*d)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1037 vs. $2(36) = 72$.
time = 45.76, size = 1037, normalized size = 22.06

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((10*tan(c/2 + d*x/2)**9/(5*a**2*d*tan(c/2 + d*x/2)**10 + 25*a**2*
d*tan(c/2 + d*x/2)**8 + 50*a**2*d*tan(c/2 + d*x/2)**6 + 50*a**2*d*tan(c/2 +
 d*x/2)**4 + 25*a**2*d*tan(c/2 + d*x/2)**2 + 5*a**2*d) - 20*tan(c/2 + d*x/2
)**8/(5*a**2*d*tan(c/2 + d*x/2)**10 + 25*a**2*d*tan(c/2 + d*x/2)**8 + 50*a*
*2*d*tan(c/2 + d*x/2)**6 + 50*a**2*d*tan(c/2 + d*x/2)**4 + 25*a**2*d*tan(c/
2 + d*x/2)**2 + 5*a**2*d) + 40*tan(c/2 + d*x/2)**7/(5*a**2*d*tan(c/2 + d*x/
2)**10 + 25*a**2*d*tan(c/2 + d*x/2)**8 + 50*a**2*d*tan(c/2 + d*x/2)**6 + 50
*a**2*d*tan(c/2 + d*x/2)**4 + 25*a**2*d*tan(c/2 + d*x/2)**2 + 5*a**2*d) - 2
0*tan(c/2 + d*x/2)**6/(5*a**2*d*tan(c/2 + d*x/2)**10 + 25*a**2*d*tan(c/2 +
d*x/2)**8 + 50*a**2*d*tan(c/2 + d*x/2)**6 + 50*a**2*d*tan(c/2 + d*x/2)**4 +
 25*a**2*d*tan(c/2 + d*x/2)**2 + 5*a**2*d) + 28*tan(c/2 + d*x/2)**5/(5*a**2
*d*tan(c/2 + d*x/2)**10 + 25*a**2*d*tan(c/2 + d*x/2)**8 + 50*a**2*d*tan(c/2
 + d*x/2)**6 + 50*a**2*d*tan(c/2 + d*x/2)**4 + 25*a**2*d*tan(c/2 + d*x/2)**
2 + 5*a**2*d) - 20*tan(c/2 + d*x/2)**4/(5*a**2*d*tan(c/2 + d*x/2)**10 + 25*
a**2*d*tan(c/2 + d*x/2)**8 + 50*a**2*d*tan(c/2 + d*x/2)**6 + 50*a**2*d*tan(
c/2 + d*x/2)**4 + 25*a**2*d*tan(c/2 + d*x/2)**2 + 5*a**2*d) + 40*tan(c/2 +
d*x/2)**3/(5*a**2*d*tan(c/2 + d*x/2)**10 + 25*a**2*d*tan(c/2 + d*x/2)**8 +
50*a**2*d*tan(c/2 + d*x/2)**6 + 50*a**2*d*tan(c/2 + d*x/2)**4 + 25*a**2*d*t
an(c/2 + d*x/2)**2 + 5*a**2*d) - 20*tan(c/2 + d*x/2)**2/(5*a**2*d*tan(c/2 +
 d*x/2)**10 + 25*a**2*d*tan(c/2 + d*x/2)**8 + 50*a**2*d*tan(c/2 + d*x/2)**6
 + 50*a**2*d*tan(c/2 + d*x/2)**4 + 25*a**2*d*tan(c/2 + d*x/2)**2 + 5*a**2*d
) + 10*tan(c/2 + d*x/2)/(5*a**2*d*tan(c/2 + d*x/2)**10 + 25*a**2*d*tan(c/2
+ d*x/2)**8 + 50*a**2*d*tan(c/2 + d*x/2)**6 + 50*a**2*d*tan(c/2 + d*x/2)**4
 + 25*a**2*d*tan(c/2 + d*x/2)**2 + 5*a**2*d), Ne(d, 0)), (x*cos(c)**7/(a*si
n(c) + a)**2, True))
```

Giac [**A**]
time = 4.87, size = 47, normalized size = 1.00

$$-\frac{2\,\sin\left(dx+c\right)^{5}-5\,\sin\left(dx+c\right)^{4}+10\,\sin\left(dx+c\right)^{2}-10\,\sin\left(dx+c\right)}{10\,a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/10*(2*sin(d*x + c)^5 - 5*sin(d*x + c)^4 + 10*sin(d*x + c)^2 - 10*sin(d*x
 + c))/(a^2*d)
```

Mupad [B]
time = 4.66, size = 54, normalized size = 1.15

$$\frac{\frac{\sin(c+d\,x)}{a^2}-\frac{\sin(c+d\,x)^2}{a^2}+\frac{\sin(c+d\,x)^4}{2\,a^2}-\frac{\sin(c+d\,x)^5}{5\,a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7/(a + a*sin(c + d*x))^2,x)
```

```
[Out] (sin(c + d*x)/a^2 - sin(c + d*x)^2/a^2 + sin(c + d*x)^4/(2*a^2) - sin(c + d
*x)^5/(5*a^2))/d
```

3.64 $\int \frac{\cos^6(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=80

$$\frac{5x}{8a^2}+\frac{5\cos^3(c+dx)}{12a^2d}+\frac{5\cos(c+dx)\sin(c+dx)}{8a^2d}+\frac{\cos^5(c+dx)}{4d\left(a^2+a^2\sin(c+dx)\right)}$$

```
[Out] 5/8*x/a^2+5/12*cos(d*x+c)^3/a^2/d+5/8*cos(d*x+c)*sin(d*x+c)/a^2/d+1/4*cos(d
*x+c)^5/d/(a^2+a^2*sin(d*x+c))
```

Rubi [A]
time = 0.07, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.190$, Rules used = {2758, 2761, 2715, 8}

$$\frac{5\cos^3(c+dx)}{12a^2d}+\frac{\cos^5(c+dx)}{4d\left(a^2\sin(c+dx)+a^2\right)}+\frac{5\sin(c+dx)\cos(c+dx)}{8a^2d}+\frac{5x}{8a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (5*x)/(8*a^2) + (5*Cos[c + d*x]^3)/(12*a^2*d) + (5*Cos[c + d*x]*Sin[c + d*x
])/(8*a^2*d) + Cos[c + d*x]^5/(4*d*(a^2 + a^2*Sin[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2758

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(a*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^6(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{\cos^5(c+dx)}{4d\,(a^2+a^2\sin(c+dx))} + \frac{5\int \frac{\cos^4(c+dx)}{a+a\sin(c+dx)}\,dx}{4a} \\
&= \frac{5\cos^3(c+dx)}{12a^2d} + \frac{\cos^5(c+dx)}{4d\,(a^2+a^2\sin(c+dx))} + \frac{5\int \cos^2(c+dx)\,dx}{4a^2} \\
&= \frac{5\cos^3(c+dx)}{12a^2d} + \frac{5\cos(c+dx)\sin(c+dx)}{8a^2d} + \frac{\cos^5(c+dx)}{4d\,(a^2+a^2\sin(c+dx))} + \frac{5\int 1\,\ldots}{8a^2} \\
&= \frac{5x}{8a^2} + \frac{5\cos^3(c+dx)}{12a^2d} + \frac{5\cos(c+dx)\sin(c+dx)}{8a^2d} + \frac{\cos^5(c+dx)}{4d\,(a^2+a^2\sin(c+dx))}
\end{aligned}$$

Mathematica [A]
time = 0.33, size = 131, normalized size = 1.64

$$-\frac{\cos^7(c+dx)\left(30\sin^{-1}\left(\frac{\sqrt{1-\sin(c+dx)}}{\sqrt{2}}\right)\sqrt{1-\sin(c+dx)}+\sqrt{1+\sin(c+dx)}\,(-16+7\sin(c+dx)+25\sin^2(c+dx)-22\sin^3(c+dx)+6\sin^4(c+dx))\right)}{24a^2d(-1+\sin(c+dx))^4(1+\sin(c+dx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/24*(Cos[c + d*x]^7*(30*ArcSin[Sqrt[1 - Sin[c + d*x]]/Sqrt[2]]*Sqrt[1 - S
in[c + d*x]] + Sqrt[1 + Sin[c + d*x]]*(-16 + 7*Sin[c + d*x] + 25*Sin[c + d*
x]^2 - 22*Sin[c + d*x]^3 + 6*Sin[c + d*x]^4)))/(a^2*d*(-1 + Sin[c + d*x])^4
*(1 + Sin[c + d*x])^(7/2))
```

Maple [A]
time = 0.23, size = 129, normalized size = 1.61

method	result
risch	$\frac{5x}{8a^2}+\frac{\cos(dx+c)}{2a^2d}-\frac{\sin(4dx+4c)}{32a^2d}+\frac{\cos(3dx+3c)}{6a^2d}+\frac{\sin(2dx+2c)}{4a^2d}$
derivativedivides	$\frac{\frac{2\left(-\frac{3\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8}+2\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{11\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8}+2\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{11\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8}+\frac{2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\frac{3\tan(\ldots}{\ldots}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}}{a^2d}$

default	$\frac{\frac{2\left(-\frac{3\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8}+2\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{11\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8}+2\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{11\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8}+\frac{2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\frac{3\tan\left(\frac{d}{\cdot}\right.}{8}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}}{a^2d}$
norman	$\frac{415\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12da}+\frac{375x\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a}+\frac{95x\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a}+\frac{119\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6da}+\frac{45x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a}+\frac{177\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4da}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/a^2*((-3/8*tan(1/2*d*x+1/2*c)^7+2*tan(1/2*d*x+1/2*c)^6-11/8*tan(1/2*d*x
+1/2*c)^5+2*tan(1/2*d*x+1/2*c)^4+11/8*tan(1/2*d*x+1/2*c)^3+2/3*tan(1/2*d*x+
1/2*c)^2+3/8*tan(1/2*d*x+1/2*c)+2/3)/(1+tan(1/2*d*x+1/2*c)^2)^4+5/8*arctan(
tan(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(72) = 144$.
time = 0.52, size = 267, normalized size = 3.34

$$\frac{\frac{\frac{9\sin(dx+c)}{\cos(dx+c)+1}+\frac{16\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{33\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{48\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{33\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{48\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{9\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+16}{a^2+\frac{4a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{4a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}}+\frac{15\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/12*((9*sin(d*x + c)/(cos(d*x + c) + 1) + 16*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 + 33*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 48*sin(d*x + c)^4/(cos(d*
x + c) + 1)^4 - 33*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 48*sin(d*x + c)^6/
(cos(d*x + c) + 1)^6 - 9*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 16)/(a^2 + 4
*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*a^2*sin(d*x + c)^4/(cos(d*x +
c) + 1)^4 + 4*a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + a^2*sin(d*x + c)^8/
(cos(d*x + c) + 1)^8) + 15*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2)/d
```

Fricas [A]
time = 0.37, size = 50, normalized size = 0.62

$$\frac{16\cos(dx+c)^3+15\,dx-3\left(2\cos(dx+c)^3-5\cos(dx+c)\right)\sin(dx+c)}{24\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/24*(16*cos(d*x + c)^3 + 15*d*x - 3*(2*cos(d*x + c)^3 - 5*cos(d*x + c))*si
n(d*x + c))/(a^2*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1243 vs. $2(71) = 142$.
time = 29.08, size = 1243, normalized size = 15.54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((15*d*x*tan(c/2 + d*x/2)**8/(24*a**2*d*tan(c/2 + d*x/2)**8 + 96*a
**2*d*tan(c/2 + d*x/2)**6 + 144*a**2*d*tan(c/2 + d*x/2)**4 + 96*a**2*d*tan(
c/2 + d*x/2)**2 + 24*a**2*d) + 60*d*x*tan(c/2 + d*x/2)**6/(24*a**2*d*tan(c/
2 + d*x/2)**8 + 96*a**2*d*tan(c/2 + d*x/2)**6 + 144*a**2*d*tan(c/2 + d*x/2)
**4 + 96*a**2*d*tan(c/2 + d*x/2)**2 + 24*a**2*d) + 90*d*x*tan(c/2 + d*x/2)*
*4/(24*a**2*d*tan(c/2 + d*x/2)**8 + 96*a**2*d*tan(c/2 + d*x/2)**6 + 144*a**
2*d*tan(c/2 + d*x/2)**4 + 96*a**2*d*tan(c/2 + d*x/2)**2 + 24*a**2*d) + 60*d
*x*tan(c/2 + d*x/2)**2/(24*a**2*d*tan(c/2 + d*x/2)**8 + 96*a**2*d*tan(c/2 +
 d*x/2)**6 + 144*a**2*d*tan(c/2 + d*x/2)**4 + 96*a**2*d*tan(c/2 + d*x/2)**2
 + 24*a**2*d) + 15*d*x/(24*a**2*d*tan(c/2 + d*x/2)**8 + 96*a**2*d*tan(c/2 +
 d*x/2)**6 + 144*a**2*d*tan(c/2 + d*x/2)**4 + 96*a**2*d*tan(c/2 + d*x/2)**2
 + 24*a**2*d) - 18*tan(c/2 + d*x/2)**7/(24*a**2*d*tan(c/2 + d*x/2)**8 + 96*
a**2*d*tan(c/2 + d*x/2)**6 + 144*a**2*d*tan(c/2 + d*x/2)**4 + 96*a**2*d*tan
(c/2 + d*x/2)**2 + 24*a**2*d) + 96*tan(c/2 + d*x/2)**6/(24*a**2*d*tan(c/2 +
 d*x/2)**8 + 96*a**2*d*tan(c/2 + d*x/2)**6 + 144*a**2*d*tan(c/2 + d*x/2)**4
 + 96*a**2*d*tan(c/2 + d*x/2)**2 + 24*a**2*d) - 66*tan(c/2 + d*x/2)**5/(24*
a**2*d*tan(c/2 + d*x/2)**8 + 96*a**2*d*tan(c/2 + d*x/2)**6 + 144*a**2*d*tan
(c/2 + d*x/2)**4 + 96*a**2*d*tan(c/2 + d*x/2)**2 + 24*a**2*d) + 96*tan(c/2
+ d*x/2)**4/(24*a**2*d*tan(c/2 + d*x/2)**8 + 96*a**2*d*tan(c/2 + d*x/2)**6
+ 144*a**2*d*tan(c/2 + d*x/2)**4 + 96*a**2*d*tan(c/2 + d*x/2)**2 + 24*a**2*
d) + 66*tan(c/2 + d*x/2)**3/(24*a**2*d*tan(c/2 + d*x/2)**8 + 96*a**2*d*tan(
c/2 + d*x/2)**6 + 144*a**2*d*tan(c/2 + d*x/2)**4 + 96*a**2*d*tan(c/2 + d*x/
2)**2 + 24*a**2*d) + 32*tan(c/2 + d*x/2)**2/(24*a**2*d*tan(c/2 + d*x/2)**8
+ 96*a**2*d*tan(c/2 + d*x/2)**6 + 144*a**2*d*tan(c/2 + d*x/2)**4 + 96*a**2*
d*tan(c/2 + d*x/2)**2 + 24*a**2*d) + 18*tan(c/2 + d*x/2)/(24*a**2*d*tan(c/2
 + d*x/2)**8 + 96*a**2*d*tan(c/2 + d*x/2)**6 + 144*a**2*d*tan(c/2 + d*x/2)*
*4 + 96*a**2*d*tan(c/2 + d*x/2)**2 + 24*a**2*d) + 32/(24*a**2*d*tan(c/2 + d
*x/2)**8 + 96*a**2*d*tan(c/2 + d*x/2)**6 + 144*a**2*d*tan(c/2 + d*x/2)**4 +
 96*a**2*d*tan(c/2 + d*x/2)**2 + 24*a**2*d), Ne(d, 0)), (x*cos(c)**6/(a*sin
(c) + a)**2, True))
```

Giac [A]
time = 5.01, size = 127, normalized size = 1.59

$$\frac{\frac{15\,(dx+c)}{a^2}-\frac{2\left(9\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-48\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+33\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-48\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-33\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-16\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-9\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-16\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^4a^2}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(15*(d*x + c)/a^2 - 2*(9*tan(1/2*d*x + 1/2*c)^7 - 48*tan(1/2*d*x + 1/2
*c)^6 + 33*tan(1/2*d*x + 1/2*c)^5 - 48*tan(1/2*d*x + 1/2*c)^4 - 33*tan(1/2*
d*x + 1/2*c)^3 - 16*tan(1/2*d*x + 1/2*c)^2 - 9*tan(1/2*d*x + 1/2*c) - 16)/(
(tan(1/2*d*x + 1/2*c)^2 + 1)^4*a^2))/d
```

Mupad [B]
time = 4.75, size = 65, normalized size = 0.81

$$\frac{5\,x}{8\,a^2}+\frac{2\cos\left(c+d\,x\right)^3}{3\,a^2\,d}-\frac{\cos\left(c+d\,x\right)^3\,\sin\left(c+d\,x\right)}{4\,a^2\,d}+\frac{5\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{8\,a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(a + a*sin(c + d*x))^2,x)
```

```
[Out] (5*x)/(8*a^2) + (2*cos(c + d*x)^3)/(3*a^2*d) - (cos(c + d*x)^3*sin(c + d*x)
)/(4*a^2*d) + (5*cos(c + d*x)*sin(c + d*x))/(8*a^2*d)
```

3.65 $\int \frac{\cos^5(c+dx)}{(a+a\sin(c+dx))^2} dx$

Optimal. Leaf size=23

$$-\frac{(a-a\sin(c+dx))^3}{3a^5d}$$

```
[Out] -1/3*(a-a*sin(d*x+c))^3/a^5/d
```

Rubi [A]
time = 0.03, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2746, 32}

$$-\frac{(a-a\sin(c+dx))^3}{3a^5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/3*(a - a*Sin[c + d*x])^3/(a^5*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^5(c+dx)}{(a+a\sin(c+dx))^2} dx &= \frac{\text{Subst}\left(\int (a-x)^2 dx, x, a\sin(c+dx)\right)}{a^5d}\\ &= -\frac{(a-a\sin(c+dx))^3}{3a^5d}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 39, normalized size = 1.70

$$\frac{6\cos(2(c+dx))+15\sin(c+dx)-\sin(3(c+dx))}{12a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/(a + a*Sin[c + d*x])^2,x]
[Out] (6*Cos[2*(c + d*x)] + 15*Sin[c + d*x] - Sin[3*(c + d*x)])/(12*a^2*d)
```

Maple [A]
time = 0.20, size = 19, normalized size = 0.83

method	result
derivativedivides	$\frac{(\sin(dx+c)-1)^3}{3d\,a^2}$
default	$\frac{(\sin(dx+c)-1)^3}{3d\,a^2}$
risch	$\frac{5\sin(dx+c)}{4a^2d}-\frac{\sin(3dx+3c)}{12a^2d}+\frac{\cos(2dx+2c)}{2a^2d}$
norman	$\frac{\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{da}+\frac{2\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}+\frac{2\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}+\frac{2\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}+\frac{10\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}+\frac{10\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}+\frac{28\left(\tan^{5}\left(\frac{d}{}\right.\right.}{3dc}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5 a\left(\tan\left(\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] 1/3/d/a^2*(sin(d*x+c)-1)^3
```

Maxima [A]
time = 0.29, size = 35, normalized size = 1.52

$$\frac{\sin\left(dx+c\right)^3-3\,\sin\left(dx+c\right)^2+3\,\sin\left(dx+c\right)}{3\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
[Out] 1/3*(sin(d*x + c)^3 - 3*sin(d*x + c)^2 + 3*sin(d*x + c))/(a^2*d)
```

Fricas [A]
time = 0.35, size = 37, normalized size = 1.61

$$\frac{3\,\cos\left(dx+c\right)^2-\left(\cos\left(dx+c\right)^2-4\right)\sin\left(dx+c\right)}{3\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
[Out] 1/3*(3*cos(d*x + c)^2 - (cos(d*x + c)^2 - 4)*sin(d*x + c))/(a^2*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 394 vs. $2(19) = 38$.
time = 16.91, size = 394, normalized size = 17.13

$$\begin{cases} \frac{6\tan^{5}\left(\frac{c}{2}+\frac{dx}{2}\right)}{3a^{2}d\tan^{6}\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^{2}d\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^{2}d\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+3a^{2}d} - \frac{12\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)}{3a^{2}d\tan^{6}\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^{2}d\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^{2}d\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+3a^{2}d} + \frac{20\tan^{3}\left(\frac{c}{2}+\frac{dx}{2}\right)}{3a^{2}d\tan^{6}\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^{2}d\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^{2}d\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+3a^{2}d} - \frac{12\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)}{3a^{2}d\tan^{6}\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^{2}d\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^{2}d\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+3a^{2}d} + \frac{6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{3a^{2}d\tan^{6}\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^{2}d\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^{2}d\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+3a^{2}d} & \text{for}\: d \neq 0 \\ \frac{x\cos^{5}(c)}{(a\sin(c)+a)^{2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((6*tan(c/2 + d*x/2)**5/(3*a**2*d*tan(c/2 + d*x/2)**6 + 9*a**2*d*t
an(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2 + d*x/2)**2 + 3*a**2*d) - 12*tan(c/2
+ d*x/2)**4/(3*a**2*d*tan(c/2 + d*x/2)**6 + 9*a**2*d*tan(c/2 + d*x/2)**4 +
9*a**2*d*tan(c/2 + d*x/2)**2 + 3*a**2*d) + 20*tan(c/2 + d*x/2)**3/(3*a**2*d
*tan(c/2 + d*x/2)**6 + 9*a**2*d*tan(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2 + d*
x/2)**2 + 3*a**2*d) - 12*tan(c/2 + d*x/2)**2/(3*a**2*d*tan(c/2 + d*x/2)**6
+ 9*a**2*d*tan(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2 + d*x/2)**2 + 3*a**2*d) +
 6*tan(c/2 + d*x/2)/(3*a**2*d*tan(c/2 + d*x/2)**6 + 9*a**2*d*tan(c/2 + d*x/
2)**4 + 9*a**2*d*tan(c/2 + d*x/2)**2 + 3*a**2*d), Ne(d, 0)), (x*cos(c)**5/(
a*sin(c) + a)**2, True))
```

Giac [A]
time = 5.25, size = 35, normalized size = 1.52

$$\frac{\sin\left(dx+c\right)^{3}-3\,\sin\left(dx+c\right)^{2}+3\,\sin\left(dx+c\right)}{3\,a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/3*(sin(d*x + c)^3 - 3*sin(d*x + c)^2 + 3*sin(d*x + c))/(a^2*d)
```

Mupad [B]
time = 4.61, size = 32, normalized size = 1.39

$$\frac{\sin\left(c+d\,x\right)\,\left({\sin\left(c+d\,x\right)}^{2}-3\,\sin\left(c+d\,x\right)+3\right)}{3\,a^{2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(a + a*sin(c + d*x))^2,x)
```

```
[Out] (sin(c + d*x)*(sin(c + d*x)^2 - 3*sin(c + d*x) + 3))/(3*a^2*d)
```

3.66 $\int \frac{\cos^4(c+dx)}{(a+a\sin(c+dx))^2} dx$

Optimal. Leaf size=56

$$\frac{3x}{2a^2}+\frac{3\cos(c+dx)}{2a^2d}+\frac{\cos^3(c+dx)}{2d\left(a^2+a^2\sin(c+dx)\right)}$$

```
[Out] 3/2*x/a^2+3/2*cos(d*x+c)/a^2/d+1/2*cos(d*x+c)^3/d/(a^2+a^2*sin(d*x+c))
```

Rubi [A]
time = 0.06, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2758, 2761, 8}

$$\frac{3\cos(c+dx)}{2a^2d}+\frac{\cos^3(c+dx)}{2d\left(a^2\sin(c+dx)+a^2\right)}+\frac{3x}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (3*x)/(2*a^2) + (3*Cos[c + d*x])/(2*a^2*d) + Cos[c + d*x]^3/(2*d*(a^2 + a^2
*Sin[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2758

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(a*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\cos^3(c+dx)}{2d\,(a^2+a^2\sin(c+dx))} + \frac{3\int \frac{\cos^2(c+dx)}{a+a\sin(c+dx)}\,dx}{2a}$$
$$= \frac{3\cos(c+dx)}{2a^2d} + \frac{\cos^3(c+dx)}{2d\,(a^2+a^2\sin(c+dx))} + \frac{3\int 1\,dx}{2a^2}$$
$$= \frac{3x}{2a^2} + \frac{3\cos(c+dx)}{2a^2d} + \frac{\cos^3(c+dx)}{2d\,(a^2+a^2\sin(c+dx))}$$

Mathematica [A]
time = 0.12, size = 109, normalized size = 1.95

$$-\frac{\cos^5(c+dx)\left(-6\sin^{-1}\left(\frac{\sqrt{1-\sin(c+dx)}}{\sqrt{2}}\right)\sqrt{1-\sin(c+dx)}+\sqrt{1+\sin(c+dx)}\,(4-5\sin(c+dx)+\sin^2(c+dx))\right)}{2a^2d(-1+\sin(c+dx))^3(1+\sin(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/2*(Cos[c + d*x]^5*(-6*ArcSin[Sqrt[1 - Sin[c + d*x]]/Sqrt[2]]*Sqrt[1 - Si
n[c + d*x]] + Sqrt[1 + Sin[c + d*x]]*(4 - 5*Sin[c + d*x] + Sin[c + d*x]^2))
)/(a^2*d*(-1 + Sin[c + d*x])^3*(1 + Sin[c + d*x])^(5/2))
```

Maple [A]
time = 0.20, size = 77, normalized size = 1.38

method	result
risch	$\frac{3x}{2a^2}+\frac{2\cos(dx+c)}{a^2d}-\frac{\sin(2dx+2c)}{4a^2d}$
derivativedivides	$\frac{\frac{2\left(\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}+2\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+3\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2d}$
default	$\frac{\frac{2\left(\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}+2\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+3\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2d}$
norman	$\frac{4}{ad}+\frac{\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)}{da}+\frac{11\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{da}+\frac{16\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}+\frac{36\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}+\frac{3x}{2a}+\frac{9x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}+\frac{21x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/a^2*((1/2*tan(1/2*d*x+1/2*c)^3+2*tan(1/2*d*x+1/2*c)^2-1/2*tan(1/2*d*x+1
/2*c)+2)/(1+tan(1/2*d*x+1/2*c)^2)^2+3/2*arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 140 vs. $2(50) = 100$.

time = 0.50, size = 140, normalized size = 2.50

$$-\frac{\frac{\frac{\sin(dx+c)}{\cos(dx+c)+1}-\frac{4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-4}{a^2+\frac{2a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}-\frac{3\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -((sin(d*x + c)/(cos(d*x + c) + 1) - 4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2
- sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 4)/(a^2 + 2*a^2*sin(d*x + c)^2/(cos
(d*x + c) + 1)^2 + a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) - 3*arctan(sin(
d*x + c)/(cos(d*x + c) + 1))/a^2)/d
```

Fricas [A]

time = 0.35, size = 35, normalized size = 0.62

$$\frac{3\,dx-\cos\left(dx+c\right)\sin\left(dx+c\right)+4\,\cos\left(dx+c\right)}{2\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(3*d*x - cos(d*x + c)*sin(d*x + c) + 4*cos(d*x + c))/(a^2*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 403 vs. $2(48) = 96$.

time = 9.44, size = 403, normalized size = 7.20

$$\begin{cases}\frac{3dx\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{2a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+4a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+2a^2d}+\frac{6dx\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{2a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+4a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+2a^2d}+\frac{3dx}{2a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+4a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+2a^2d}+\frac{2\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{2a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+4a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+2a^2d}+\frac{8\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{2a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+4a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+2a^2d}-\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+4a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+2a^2d}+\frac{8}{2a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+4a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+2a^2d} & \text{for}\ d\neq 0\\ \frac{x\cos^4\left(c\right)}{\left(a\sin\left(c\right)+a\right)^2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((3*d*x*tan(c/2 + d*x/2)**4/(2*a**2*d*tan(c/2 + d*x/2)**4 + 4*a**2
*d*tan(c/2 + d*x/2)**2 + 2*a**2*d) + 6*d*x*tan(c/2 + d*x/2)**2/(2*a**2*d*ta
n(c/2 + d*x/2)**4 + 4*a**2*d*tan(c/2 + d*x/2)**2 + 2*a**2*d) + 3*d*x/(2*a**
2*d*tan(c/2 + d*x/2)**4 + 4*a**2*d*tan(c/2 + d*x/2)**2 + 2*a**2*d) + 2*tan(
c/2 + d*x/2)**3/(2*a**2*d*tan(c/2 + d*x/2)**4 + 4*a**2*d*tan(c/2 + d*x/2)**
2 + 2*a**2*d) + 8*tan(c/2 + d*x/2)**2/(2*a**2*d*tan(c/2 + d*x/2)**4 + 4*a**
2*d*tan(c/2 + d*x/2)**2 + 2*a**2*d) - 2*tan(c/2 + d*x/2)/(2*a**2*d*tan(c/2
+ d*x/2)**4 + 4*a**2*d*tan(c/2 + d*x/2)**2 + 2*a**2*d) + 8/(2*a**2*d*tan(c/
2 + d*x/2)**4 + 4*a**2*d*tan(c/2 + d*x/2)**2 + 2*a**2*d), Ne(d, 0)), (x*cos
(c)**4/(a*sin(c) + a)**2, True))
```

Giac [A]
time = 6.03, size = 73, normalized size = 1.30

$$\frac{\frac{3\,(dx+c)}{a^2}+\frac{2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+4\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+4\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2a^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(3*(d*x + c)/a^2 + 2*(tan(1/2*d*x + 1/2*c)^3 + 4*tan(1/2*d*x + 1/2*c)^2
 - tan(1/2*d*x + 1/2*c) + 4)/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^2))/d
```

Mupad [B]
time = 4.65, size = 32, normalized size = 0.57

$$\frac{4\,\cos\left(c+d\,x\right)-\frac{\sin\left(2\,c+2\,d\,x\right)}{2}+3\,d\,x}{2\,a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(a + a*sin(c + d*x))^2,x)
```

```
[Out] (4*cos(c + d*x) - sin(2*c + 2*d*x)/2 + 3*d*x)/(2*a^2*d)
```

3.67 $\int \frac{\cos^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=32

$$\frac{2\log(1+\sin(c+dx))}{a^2d}-\frac{\sin(c+dx)}{a^2d}$$

```
[Out] 2*ln(1+sin(d*x+c))/a^2/d-sin(d*x+c)/a^2/d
```

Rubi [A]
time = 0.03, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.095$, Rules used = {2746, 45}

$$\frac{2\log(\sin(c+dx)+1)}{a^2d}-\frac{\sin(c+dx)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (2*Log[1 + Sin[c + d*x]])/(a^2*d) - Sin[c + d*x]/(a^2*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{\text{Subst}\left(\int \frac{a-x}{a+x}\,dx,x,a\sin(c+dx)\right)}{a^3d}\\ &= \frac{\text{Subst}\left(\int \left(-1+\frac{2a}{a+x}\right)\,dx,x,a\sin(c+dx)\right)}{a^3d}\\ &= \frac{2\log(1+\sin(c+dx))}{a^2d}-\frac{\sin(c+dx)}{a^2d}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 26, normalized size = 0.81

$$-\frac{-2\log(1+\sin(c+dx))+\sin(c+dx)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -((-2*Log[1 + Sin[c + d*x]] + Sin[c + d*x])/(a^2*d))
```

Maple [A]
time = 0.19, size = 28, normalized size = 0.88

method	result
derivativedivides	$\frac{-\sin(dx+c)+2\ln(1+\sin(dx+c))}{d\,a^2}$
default	$\frac{-\sin(dx+c)+2\ln(1+\sin(dx+c))}{d\,a^2}$
risch	$-\frac{2ix}{a^2}+\frac{i\mathrm{e}^{i(dx+c)}}{2a^2d}-\frac{i\mathrm{e}^{-i(dx+c)}}{2a^2d}-\frac{4ic}{a^2d}+\frac{4\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{a^2d}$
norman	$\frac{-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{da}-\frac{2\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}-\frac{6\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}-\frac{10\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}-\frac{10\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}-\frac{6\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}-\frac{14\left(\tan^4\left(\cdots\right.\right.}{\cdots}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3 a\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^2*(-sin(d*x+c)+2*ln(1+sin(d*x+c)))
```

Maxima [A]
time = 0.28, size = 30, normalized size = 0.94

$$\frac{\frac{2\,\log(\sin(dx+c)+1)}{a^2}-\frac{\sin(dx+c)}{a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] (2*log(sin(d*x + c) + 1)/a^2 - sin(d*x + c)/a^2)/d
```

Fricas [A]
time = 0.34, size = 27, normalized size = 0.84

$$\frac{2\,\log\left(\sin\left(dx+c\right)+1\right)-\sin\left(dx+c\right)}{a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] (2*log(sin(d*x + c) + 1) - sin(d*x + c))/(a^2*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 150 vs. $2(27) = 54$.
time = 0.53, size = 150, normalized size = 4.69

$$\begin{cases} \frac{2\log\left(\sin\left(c+dx\right)+1\right)\sin\left(c+dx\right)}{a^2d\sin\left(c+dx\right)+a^2d} + \frac{2\log\left(\sin\left(c+dx\right)+1\right)}{a^2d\sin\left(c+dx\right)+a^2d} - \frac{2\sin^2\left(c+dx\right)}{a^2d\sin\left(c+dx\right)+a^2d} - \frac{\cos^2\left(c+dx\right)}{a^2d\sin\left(c+dx\right)+a^2d} + \frac{2}{a^2d\sin\left(c+dx\right)+a^2d} & \text{for } d \neq 0 \\ \frac{x\cos^3\left(c\right)}{\left(a\sin\left(c\right)+a\right)^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((2*log(sin(c + d*x) + 1)*sin(c + d*x)/(a**2*d*sin(c + d*x) + a**2
*d) + 2*log(sin(c + d*x) + 1)/(a**2*d*sin(c + d*x) + a**2*d) - 2*sin(c + d*
x)**2/(a**2*d*sin(c + d*x) + a**2*d) - cos(c + d*x)**2/(a**2*d*sin(c + d*x)
 + a**2*d) + 2/(a**2*d*sin(c + d*x) + a**2*d), Ne(d, 0)), (x*cos(c)**3/(a*s
in(c) + a)**2, True))
```

Giac [A]
time = 8.02, size = 54, normalized size = 1.69

$$-\frac{\frac{2\log\left(\frac{|a\sin(dx+c)+a|}{(a\sin(dx+c)+a)^2|a|}\right)}{a^2} + \frac{a\sin(dx+c)+a}{a^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -(2*log(abs(a*sin(d*x + c) + a)/((a*sin(d*x + c) + a)^2*abs(a)))/a^2 + (a*s
in(d*x + c) + a)/a^3)/d
```

Mupad [B]
time = 0.06, size = 27, normalized size = 0.84

$$\frac{2\,\ln\left(\sin\left(c+d\,x\right)+1\right)-\sin\left(c+d\,x\right)}{a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a + a*sin(c + d*x))^2,x)
```

```
[Out] (2*log(sin(c + d*x) + 1) - sin(c + d*x))/(a^2*d)
```

3.68 $\int \frac{\cos^2(c+dx)}{(a+a\sin(c+dx))^2} dx$

Optimal. Leaf size=34

$$-\frac{x}{a^2} - \frac{2\cos(c+dx)}{d\,(a^2+a^2\sin(c+dx))}$$

```
[Out] -x/a^2-2*cos(d*x+c)/d/(a^2+a^2*sin(d*x+c))
```

Rubi [A]
time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2759, 8}

$$-\frac{2\cos(c+dx)}{d\,(a^2\sin(c+dx)+a^2)} - \frac{x}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(x/a^2) - (2*Cos[c + d*x])/(d*(a^2 + a^2*Sin[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= -\frac{2\cos(c+dx)}{d\,(a^2+a^2\sin(c+dx))} - \frac{\int 1\,dx}{a^2}\\ &= -\frac{x}{a^2} - \frac{2\cos(c+dx)}{d\,(a^2+a^2\sin(c+dx))}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 104 vs. 2(34) = 68.

time = 0.11, size = 104, normalized size = 3.06

$$\frac{2\cos^3(c+dx)\left((-1+\sin(c+dx))\sqrt{1+\sin(c+dx)}+\sin^{-1}\left(\frac{\sqrt{1-\sin(c+dx)}}{\sqrt{2}}\right)\sqrt{1-\sin(c+dx)}\,(1+\sin(c+dx))\right)}{a^2d(-1+\sin(c+dx))^2(1+\sin(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (2*Cos[c + d*x]^3*((-1 + Sin[c + d*x])*Sqrt[1 + Sin[c + d*x]] + ArcSin[Sqrt
[1 - Sin[c + d*x]]/Sqrt[2]]*Sqrt[1 - Sin[c + d*x]]*(1 + Sin[c + d*x])))/(a^
2*d*(-1 + Sin[c + d*x])^2*(1 + Sin[c + d*x])^(5/2))
```

Maple [A]
time = 0.20, size = 37, normalized size = 1.09

method	result
risch	$-\frac{x}{a^2}-\frac{4}{d\,a^2\left(e^{i(dx+c)}+i\right)}$
derivativedivides	$\frac{-2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{4}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}}{a^2d}$
default	$\frac{-2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{4}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}}{a^2d}$
norman	$\frac{-\frac{4}{ad}-\frac{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{da}-\frac{4\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}-\frac{12\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}-\frac{8\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}-\frac{16\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}-\frac{12\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}-\frac{x}{a}-\ldots}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\ldots\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/a^2*(-arctan(tan(1/2*d*x+1/2*c))-2/(tan(1/2*d*x+1/2*c)+1))
```

Maxima [A]
time = 0.53, size = 56, normalized size = 1.65

$$-\frac{2\left(\frac{2}{a^2+\frac{a^2\sin(dx+c)}{\cos(dx+c)+1}}+\frac{\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -2*(2/(a^2 + a^2*sin(d*x + c)/(cos(d*x + c) + 1)) + arctan(sin(d*x + c)/(co
s(d*x + c) + 1))/a^2)/d
```

Fricas [A]
time = 0.37, size = 61, normalized size = 1.79

$$-\frac{dx + (dx + 2)\cos(dx + c) + (dx - 2)\sin(dx + c) + 2}{a^2 d\cos(dx + c) + a^2 d\sin(dx + c) + a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -(d*x + (d*x + 2)*cos(d*x + c) + (d*x - 2)*sin(d*x + c) + 2)/(a^2*d*cos(d*x
 + c) + a^2*d*sin(d*x + c) + a^2*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(29) = 58$.
time = 2.67, size = 95, normalized size = 2.79

$$\begin{cases} -\frac{dx\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{a^2d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a^2d} - \frac{dx}{a^2d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a^2d} - \frac{4}{a^2d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a^2d} & \text{for } d \neq 0 \\ \frac{x\cos^2(c)}{(a\sin(c)+a)^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((-d*x*tan(c/2 + d*x/2)/(a**2*d*tan(c/2 + d*x/2) + a**2*d) - d*x/(
a**2*d*tan(c/2 + d*x/2) + a**2*d) - 4/(a**2*d*tan(c/2 + d*x/2) + a**2*d), N
e(d, 0)), (x*cos(c)**2/(a*sin(c) + a)**2, True))
```

Giac [A]
time = 7.75, size = 33, normalized size = 0.97

$$-\frac{\frac{dx+c}{a^2} + \frac{4}{a^2(\tan(\frac{1}{2}dx+\frac{1}{2}c)+1)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -((d*x + c)/a^2 + 4/(a^2*(tan(1/2*d*x + 1/2*c) + 1)))/d
```

Mupad [B]
time = 4.64, size = 28, normalized size = 0.82

$$-\frac{x}{a^2} - \frac{4}{a^2\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a + a*sin(c + d*x))^2,x)
```

```
[Out] - x/a^2 - 4/(a^2*d*(tan(c/2 + (d*x)/2) + 1))
```

3.69 $\int \frac{\cos(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=21

$$-\frac{1}{d\left(a^2+a^2\sin(c+dx)\right)}$$

[Out] -1/d/(a^2+a^2*sin(d*x+c))

Rubi [A]
time = 0.02, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2746, 32}

$$-\frac{1}{d\left(a^2\sin(c+dx)+a^2\right)}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]/(a + a*Sin[c + d*x])^2,x]

[Out] -(1/(d*(a^2 + a^2*Sin[c + d*x])))

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{ad}\\ &= -\frac{1}{d\left(a^2+a^2\sin(c+dx)\right)}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 31, normalized size = 1.48

$$-\frac{1}{a^2d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + a*Sin[c + d*x])^2,x]
[Out] -(1/(a^2*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2))
```

Maple [A]
time = 0.10, size = 21, normalized size = 1.00

method	result	size
derivativedivides	$-\frac{1}{d(a+a\sin(dx+c))a}$	21
default	$-\frac{1}{d(a+a\sin(dx+c))a}$	21
risch	$-\frac{2i e^{i(dx+c)}}{d\,a^2\left(e^{i(dx+c)}+i\right)^2}$	33
norman	$\frac{\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{da}+\frac{2\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}+\frac{2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}+\frac{2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}}{a\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}$	108

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] -1/d/(a+a*sin(d*x+c))/a
```

Maxima [A]
time = 0.30, size = 20, normalized size = 0.95

$$-\frac{1}{(a\sin(dx+c)+a)ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
[Out] -1/((a*sin(d*x + c) + a)*a*d)
```

Fricas [A]
time = 0.33, size = 21, normalized size = 1.00

$$-\frac{1}{a^2d\sin(dx+c)+a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
[Out] -1/(a^2*d*sin(d*x + c) + a^2*d)
```

Sympy [A]
time = 0.55, size = 32, normalized size = 1.52

$$\begin{cases} -\frac{1}{a^2 d \sin(c+dx)+a^2 d} & \text{for } d \neq 0 \\ \frac{x \cos(c)}{(a \sin(c)+a)^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((-1/(a**2*d*sin(c + d*x) + a**2*d), Ne(d, 0)), (x*cos(c)/(a*sin(c
) + a)**2, True))
```

Giac [A]
time = 5.48, size = 20, normalized size = 0.95

$$-\frac{1}{(a \sin(dx+c)+a)ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/((a*sin(d*x + c) + a)*a*d)
```

Mupad [B]
time = 0.05, size = 18, normalized size = 0.86

$$-\frac{1}{a^2 d (\sin(c+dx)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + a*sin(c + d*x))^2,x)
```

```
[Out] -1/(a^2*d*(sin(c + d*x) + 1))
```

3.70 $\int \frac{\sec(c+dx)}{(a+a\sin(c+dx))^2} dx$

Optimal. Leaf size=60

$$\frac{\tanh^{-1}(\sin(c+dx))}{4a^2d} - \frac{1}{4d(a+a\sin(c+dx))^2} - \frac{1}{4d\,(a^2+a^2\sin(c+dx))}$$

```
[Out] 1/4*arctanh(sin(d*x+c))/a^2/d-1/4/d/(a+a*sin(d*x+c))^2-1/4/d/(a^2+a^2*sin(d
*x+c))
```

Rubi [A]

time = 0.04, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2746, 46, 212}

$$-\frac{1}{4d\,(a^2\sin(c+dx)+a^2)} + \frac{\tanh^{-1}(\sin(c+dx))}{4a^2d} - \frac{1}{4d(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/(4*a^2*d) - 1/(4*d*(a + a*Sin[c + d*x])^2) - 1/(4*d*(
a^2 + a^2*Sin[c + d*x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{a\text{Subst}\left(\int \frac{1}{(a-x)(a+x)^3}\,dx,x,a\sin(c+dx)\right)}{d}$$
$$= \frac{a\text{Subst}\left(\int \left(\frac{1}{2a(a+x)^3}+\frac{1}{4a^2(a+x)^2}+\frac{1}{4a^2(a^2-x^2)}\right)\,dx,x,a\sin(c+dx)\right)}{d}$$
$$= -\frac{1}{4d(a+a\sin(c+dx))^2}-\frac{1}{4d\,(a^2+a^2\sin(c+dx))}+\frac{\text{Subst}\left(\int \frac{1}{a^2-x^2}\,dx,x,a\sin\right.}{4ad}$$
$$= \frac{\tanh^{-1}(\sin(c+dx))}{4a^2d}-\frac{1}{4d(a+a\sin(c+dx))^2}-\frac{1}{4d\,(a^2+a^2\sin(c+dx))}$$

Mathematica [A]
time = 0.06, size = 38, normalized size = 0.63

$$\frac{\tanh^{-1}(\sin(c+dx))-\frac{2+\sin(c+dx)}{(1+\sin(c+dx))^2}}{4a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + a*Sin[c + d*x])^2,x]
[Out] (ArcTanh[Sin[c + d*x]] - (2 + Sin[c + d*x])/(1 + Sin[c + d*x])^2)/(4*a^2*d)
```

Maple [A]
time = 0.21, size = 55, normalized size = 0.92

method	result	size
derivativedivides	$\frac{-\frac{\ln(\sin(dx+c)-1)}{8}-\frac{1}{4(1+\sin(dx+c))^2}-\frac{1}{4(1+\sin(dx+c))}+\frac{\ln(1+\sin(dx+c))}{8}}{d\,a^2}$	55
default	$\frac{-\frac{\ln(\sin(dx+c)-1)}{8}-\frac{1}{4(1+\sin(dx+c))^2}-\frac{1}{4(1+\sin(dx+c))}+\frac{\ln(1+\sin(dx+c))}{8}}{d\,a^2}$	55
risch	$-\frac{i\left(4i\mathrm{e}^{2i(dx+c)}+\mathrm{e}^{3i(dx+c)}-\mathrm{e}^{i(dx+c)}\right)}{2d\,a^2\left(\mathrm{e}^{i(dx+c)}+i\right)^4}-\frac{\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{4a^2d}+\frac{\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{4a^2d}$	100
norman	$\frac{\frac{2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2da}+\frac{3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2da}}{a\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{4a^2d}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{4a^2d}$	115

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] 1/d/a^2*(-1/8*ln(sin(d*x+c)-1)-1/4/(1+sin(d*x+c))^2-1/4/(1+sin(d*x+c))+1/8*
ln(1+sin(d*x+c)))
```

Maxima [A]
time = 0.30, size = 72, normalized size = 1.20

$$-\frac{\frac{2(\sin(dx+c)+2)}{a^2\sin(dx+c)^2+2a^2\sin(dx+c)+a^2}-\frac{\log(\sin(dx+c)+1)}{a^2}+\frac{\log(\sin(dx+c)-1)}{a^2}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/8*(2*(sin(d*x + c) + 2)/(a^2*sin(d*x + c)^2 + 2*a^2*sin(d*x + c) + a^2)
- log(sin(d*x + c) + 1)/a^2 + log(sin(d*x + c) - 1)/a^2)/d
```

Fricas [A]
time = 0.37, size = 105, normalized size = 1.75

$$\frac{\left(\cos(dx+c)^2-2\sin(dx+c)-2\right)\log(\sin(dx+c)+1)-\left(\cos(dx+c)^2-2\sin(dx+c)-2\right)\log(-\sin(dx+c)+1)+2\sin(dx+c)+4}{8\left(a^2d\cos(dx+c)^2-2a^2d\sin(dx+c)-2a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/8*((cos(d*x + c)^2 - 2*sin(d*x + c) - 2)*log(sin(d*x + c) + 1) - (cos(d*x
 + c)^2 - 2*sin(d*x + c) - 2)*log(-sin(d*x + c) + 1) + 2*sin(d*x + c) + 4)/
(a^2*d*cos(d*x + c)^2 - 2*a^2*d*sin(d*x + c) - 2*a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec(c+dx)}{\sin^2(c+dx)+2\sin(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)/(sin(c + d*x)**2 + 2*sin(c + d*x) + 1), x)/a**2
```

Giac [A]
time = 6.30, size = 71, normalized size = 1.18

$$\frac{\frac{2\log(|\sin(dx+c)+1|)}{a^2}-\frac{2\log(|\sin(dx+c)-1|)}{a^2}-\frac{3\sin(dx+c)^2+10\sin(dx+c)+11}{a^2(\sin(dx+c)+1)^2}}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/16*(2*log(abs(sin(d*x + c) + 1))/a^2 - 2*log(abs(sin(d*x + c) - 1))/a^2 -
 (3*sin(d*x + c)^2 + 10*sin(d*x + c) + 11)/(a^2*(sin(d*x + c) + 1)^2))/d
```

Mupad [B]
time = 4.52, size = 60, normalized size = 1.00

$$\frac{\operatorname{atanh}(\sin(c+d\,x))}{4\,a^2\,d}-\frac{\frac{\sin(c+d\,x)}{4}+\frac{1}{2}}{d\,\left(a^2\sin(c+d\,x)^2+2\,a^2\,\sin(c+d\,x)+a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + a*sin(c + d*x))^2),x)
```

```
[Out] atanh(sin(c + d*x))/(4*a^2*d) - (sin(c + d*x)/4 + 1/2)/(d*(2*a^2*sin(c + d*
x) + a^2 + a^2*sin(c + d*x)^2))
```

3.71 $\int \frac{\sec^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=71

$$-\frac{\sec(c+dx)}{5d(a+a\sin(c+dx))^2}-\frac{\sec(c+dx)}{5d\,(a^2+a^2\sin(c+dx))}+\frac{2\tan(c+dx)}{5a^2d}$$

```
[Out] -1/5*sec(d*x+c)/d/(a+a*sin(d*x+c))^2-1/5*sec(d*x+c)/d/(a^2+a^2*sin(d*x+c))+
2/5*tan(d*x+c)/a^2/d
```

Rubi [A]
time = 0.06, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2751, 3852, 8}

$$\frac{2\tan(c+dx)}{5a^2d}-\frac{\sec(c+dx)}{5d\,(a^2\sin(c+dx)+a^2)}-\frac{\sec(c+dx)}{5d(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/5*Sec[c + d*x]/(d*(a + a*Sin[c + d*x])^2) - Sec[c + d*x]/(5*d*(a^2 + a^2
*Sin[c + d*x])) + (2*Tan[c + d*x])/(5*a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx = -\frac{\sec(c+dx)}{5d(a+a\sin(c+dx))^2}+\frac{3\int \frac{\sec^2(c+dx)}{a+a\sin(c+dx)}\,dx}{5a}$$
$$= -\frac{\sec(c+dx)}{5d(a+a\sin(c+dx))^2}-\frac{\sec(c+dx)}{5d\left(a^2+a^2\sin(c+dx)\right)}+\frac{2\int \sec^2(c+dx)\,dx}{5a^2}$$
$$= -\frac{\sec(c+dx)}{5d(a+a\sin(c+dx))^2}-\frac{\sec(c+dx)}{5d\left(a^2+a^2\sin(c+dx)\right)}-\frac{2\text{Subst}(\int 1\,dx,x,-\tan(c}{5a^2d}$$
$$= -\frac{\sec(c+dx)}{5d(a+a\sin(c+dx))^2}-\frac{\sec(c+dx)}{5d\left(a^2+a^2\sin(c+dx)\right)}+\frac{2\tan(c+dx)}{5a^2d}$$

Mathematica [A]
time = 0.05, size = 53, normalized size = 0.75

$$-\frac{\sec(c+dx)(4\cos(2(c+dx))-5\sin(c+dx)+\sin(3(c+dx)))}{10a^2d(1+\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/10*(Sec[c + d*x]*(4*Cos[2*(c + d*x)] - 5*Sin[c + d*x] + Sin[3*(c + d*x)]
))/(a^2*d*(1 + Sin[c + d*x])^2)
```

Maple [A]
time = 0.17, size = 98, normalized size = 1.38

method	result
risch	$-\frac{4i\left(4i\mathrm{e}^{i(dx+c)}+5\,\mathrm{e}^{2i(dx+c)}-1\right)}{5\left(\mathrm{e}^{i(dx+c)}+i\right)^5\left(\mathrm{e}^{i(dx+c)}-i\right)a^2d}$
derivativedivides	$\frac{-\frac{4}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}+\frac{2}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}-\frac{3}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}+\frac{5}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{7}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}}{a^2d}$
default	$\frac{-\frac{4}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}+\frac{2}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}-\frac{3}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}+\frac{5}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{7}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}}{a^2d}$
norman	$\frac{-\frac{4\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}-\frac{4\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}+\frac{4}{5ad}-\frac{2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}+\frac{6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{5da}}{a\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/a^2*(-2/5/(tan(1/2*d*x+1/2*c)+1)^5+1/(tan(1/2*d*x+1/2*c)+1)^4-3/2/(tan(
1/2*d*x+1/2*c)+1)^3+5/4/(tan(1/2*d*x+1/2*c)+1)^2-7/8/(tan(1/2*d*x+1/2*c)+1)
-1/8/(tan(1/2*d*x+1/2*c)-1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 204 vs. $2(65) = 130$.
time = 0.30, size = 204, normalized size = 2.87

$$-\frac{2\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}-\frac{10\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{10\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{5\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+2\right)}{5\left(a^2+\frac{4a^2\sin(dx+c)}{\cos(dx+c)+1}+\frac{5a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{5a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{4a^2\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -2/5*(3*sin(d*x + c)/(cos(d*x + c) + 1) - 10*sin(d*x + c)^3/(cos(d*x + c) +
 1)^3 - 10*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 5*sin(d*x + c)^5/(cos(d*x
+ c) + 1)^5 + 2)/((a^2 + 4*a^2*sin(d*x + c)/(cos(d*x + c) + 1) + 5*a^2*sin(
d*x + c)^2/(cos(d*x + c) + 1)^2 - 5*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4
 - 4*a^2*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - a^2*sin(d*x + c)^6/(cos(d*x
+ c) + 1)^6)*d)
```

Fricas [A]
time = 0.34, size = 79, normalized size = 1.11

$$\frac{4\cos(dx+c)^2+\left(2\cos(dx+c)^2-3\right)\sin(dx+c)-2}{5\left(a^2d\cos(dx+c)^3-2a^2d\cos(dx+c)\sin(dx+c)-2a^2d\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/5*(4*cos(d*x + c)^2 + (2*cos(d*x + c)^2 - 3)*sin(d*x + c) - 2)/(a^2*d*cos
(d*x + c)^3 - 2*a^2*d*cos(d*x + c)*sin(d*x + c) - 2*a^2*d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^2(c+dx)}{\sin^2(c+dx)+2\sin(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**2/(sin(c + d*x)**2 + 2*sin(c + d*x) + 1), x)/a**2
```

Giac [A]
time = 5.94, size = 93, normalized size = 1.31

$$-\frac{\frac{5}{a^2\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)}+\frac{35\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+90\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+120\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+70\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+21}{a^2\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)^5}}{20d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/20*(5/(a^2*(tan(1/2*d*x + 1/2*c) - 1)) + (35*tan(1/2*d*x + 1/2*c)^4 + 90
*tan(1/2*d*x + 1/2*c)^3 + 120*tan(1/2*d*x + 1/2*c)^2 + 70*tan(1/2*d*x + 1/2
*c) + 21)/(a^2*(tan(1/2*d*x + 1/2*c) + 1)^5))/d
```

Mupad [B]
time = 4.77, size = 156, normalized size = 2.20

$$\frac{2\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\left(-2\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^5-3\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^4\sin\left(\frac{c}{2}+\frac{dx}{2}\right)+10\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^3+10\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^4+5\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^5\right)}{5\,a^2\,d\,\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\,\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + a*sin(c + d*x))^2),x)
```

```
[Out] (2*cos(c/2 + (d*x)/2)*(5*sin(c/2 + (d*x)/2)^5 - 2*cos(c/2 + (d*x)/2)^5 + 10
*cos(c/2 + (d*x)/2)*sin(c/2 + (d*x)/2)^4 - 3*cos(c/2 + (d*x)/2)^4*sin(c/2 +
 (d*x)/2) + 10*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^3))/(5*a^2*d*(cos(c/
2 + (d*x)/2) - sin(c/2 + (d*x)/2))*(cos(c/2 + (d*x)/2) + sin(c/2 + (d*x)/2)
)^5)
```

3.72 $\int \frac{\sec^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=104

$$\frac{\tanh^{-1}(\sin(c+dx))}{4a^2d}-\frac{a}{12d(a+a\sin(c+dx))^3}-\frac{1}{8d(a+a\sin(c+dx))^2}+\frac{1}{16d\left(a^2-a^2\sin(c+dx)\right)}-\frac{}{16d\left(a^2\right.}$$

```
[Out] 1/4*arctanh(sin(d*x+c))/a^2/d-1/12*a/d/(a+a*sin(d*x+c))^3-1/8/d/(a+a*sin(d*
x+c))^2+1/16/d/(a^2-a^2*sin(d*x+c))-3/16/d/(a^2+a^2*sin(d*x+c))
```

Rubi [A]
time = 0.06, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2746, 46, 212}

$$\frac{1}{16d\left(a^2-a^2\sin(c+dx)\right)}-\frac{3}{16d\left(a^2\sin(c+dx)+a^2\right)}+\frac{\tanh^{-1}(\sin(c+dx))}{4a^2d}-\frac{a}{12d(a\sin(c+dx)+a)^3}-\frac{1}{8d(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/(4*a^2*d) - a/(12*d*(a + a*Sin[c + d*x])^3) - 1/(8*d*
(a + a*Sin[c + d*x])^2) + 1/(16*d*(a^2 - a^2*Sin[c + d*x])) - 3/(16*d*(a^2
+ a^2*Sin[c + d*x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{a^3\text{Subst}\left(\int \frac{1}{(a-x)^2(a+x)^4}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^3\text{Subst}\left(\int \left(\frac{1}{16a^4(a-x)^2}+\frac{1}{4a^2(a+x)^4}+\frac{1}{4a^3(a+x)^3}+\frac{3}{16a^4(a+x)^2}+\frac{1}{4a^4(a^2-x^2)}\right)\,dx, x, a\,s\right.}{d}$$
$$= -\frac{a}{12d(a+a\sin(c+dx))^3}-\frac{1}{8d(a+a\sin(c+dx))^2}+\frac{1}{16d\,(a^2-a^2\sin(c+dx))}$$
$$= \frac{\tanh^{-1}(\sin(c+dx))}{4a^2d}-\frac{a}{12d(a+a\sin(c+dx))^3}-\frac{1}{8d(a+a\sin(c+dx))^2}+\frac{}{16d}$$

Mathematica [A]
time = 0.08, size = 85, normalized size = 0.82

$$-\frac{\sec^2(c+dx)\left(4-\sin(c+dx)-6\sin^2(c+dx)-3\sin^3(c+dx)+3\tanh^{-1}(\sin(c+dx))(-1+\sin(c+dx))(1+\sin(c+dx))^3\right)}{12a^2d(1+\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/12*(Sec[c + d*x]^2*(4 - Sin[c + d*x] - 6*Sin[c + d*x]^2 - 3*Sin[c + d*x]
^3 + 3*ArcTanh[Sin[c + d*x]]*(-1 + Sin[c + d*x])*(1 + Sin[c + d*x])^3))/(a^
2*d*(1 + Sin[c + d*x])^2)
```

Maple [A]
time = 0.24, size = 79, normalized size = 0.76

method	result
derivativedivides	$\frac{-\frac{1}{16(\sin(dx+c)-1)}-\frac{\ln(\sin(dx+c)-1)}{8}-\frac{1}{12(1+\sin(dx+c))^3}-\frac{1}{8(1+\sin(dx+c))^2}-\frac{3}{16(1+\sin(dx+c))}+\frac{\ln(1+\sin(dx+c))}{8}}{d\,a^2}$
default	$\frac{-\frac{1}{16(\sin(dx+c)-1)}-\frac{\ln(\sin(dx+c)-1)}{8}-\frac{1}{12(1+\sin(dx+c))^3}-\frac{1}{8(1+\sin(dx+c))^2}-\frac{3}{16(1+\sin(dx+c))}+\frac{\ln(1+\sin(dx+c))}{8}}{d\,a^2}$
risch	$-\frac{i\left(12i e^{6i(dx+c)}+3\,e^{7i(dx+c)}+8i e^{4i(dx+c)}-13\,e^{5i(dx+c)}+12i e^{2i(dx+c)}+13\,e^{3i(dx+c)}-3\,e^{i(dx+c)}\right)}{6\left(e^{i(dx+c)}+i\right)^6\left(e^{i(dx+c)}-i\right)^2a^2d}-\frac{\ln\left(e^{i(dx+c)}-i\right)}{4a^2d}+$
norman	$\frac{\frac{2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}+\frac{2\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2da}+\frac{3\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2da}-\frac{4\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3da}+\frac{7\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6da}+\frac{7\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{6da}}{a\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^2*(-1/16/(sin(d*x+c)-1)-1/8*ln(sin(d*x+c)-1)-1/12/(1+sin(d*x+c))^3-1/
8/(1+sin(d*x+c))^2-3/16/(1+sin(d*x+c))+1/8*ln(1+sin(d*x+c)))
```

Maxima [A]
time = 0.30, size = 108, normalized size = 1.04

$$-\frac{\frac{2\left(3\sin\left(dx+c\right)^3+6\sin\left(dx+c\right)^2+\sin\left(dx+c\right)-4\right)}{a^2\sin\left(dx+c\right)^4+2a^2\sin\left(dx+c\right)^3-2a^2\sin\left(dx+c\right)-a^2}-\frac{3\log\left(\sin\left(dx+c\right)+1\right)}{a^2}+\frac{3\log\left(\sin\left(dx+c\right)-1\right)}{a^2}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/24*(2*(3*sin(d*x + c)^3 + 6*sin(d*x + c)^2 + sin(d*x + c) - 4)/(a^2*sin(
d*x + c)^4 + 2*a^2*sin(d*x + c)^3 - 2*a^2*sin(d*x + c) - a^2) - 3*log(sin(d
*x + c) + 1)/a^2 + 3*log(sin(d*x + c) - 1)/a^2)/d
```

Fricas [A]
time = 0.35, size = 178, normalized size = 1.71

$$\frac{12\cos\left(dx+c\right)^2+3\left(\cos\left(dx+c\right)^4-2\cos\left(dx+c\right)^2\sin\left(dx+c\right)-2\cos\left(dx+c\right)^2\right)\log\left(\sin\left(dx+c\right)+1\right)-3\left(\cos\left(dx+c\right)^4-2\cos\left(dx+c\right)^2\sin\left(dx+c\right)-2\cos\left(dx+c\right)^2\right)\log\left(-\sin\left(dx+c\right)+1\right)+2\left(3\cos\left(dx+c\right)^2-4\right)\sin\left(dx+c\right)-4}{24\left(a^2d\cos\left(dx+c\right)^4-2a^2d\cos\left(dx+c\right)^2\sin\left(dx+c\right)-2a^2d\cos\left(dx+c\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/24*(12*cos(d*x + c)^2 + 3*(cos(d*x + c)^4 - 2*cos(d*x + c)^2*sin(d*x + c)
 - 2*cos(d*x + c)^2)*log(sin(d*x + c) + 1) - 3*(cos(d*x + c)^4 - 2*cos(d*x
+ c)^2*sin(d*x + c) - 2*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) + 2*(3*cos(d
*x + c)^2 - 4)*sin(d*x + c) - 4)/(a^2*d*cos(d*x + c)^4 - 2*a^2*d*cos(d*x +
c)^2*sin(d*x + c) - 2*a^2*d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^3\left(c+dx\right)}{\sin^2\left(c+dx\right)+2\sin\left(c+dx\right)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**3/(sin(c + d*x)**2 + 2*sin(c + d*x) + 1), x)/a**2
```

Giac [A]
time = 3.66, size = 106, normalized size = 1.02

$$\frac{\frac{6\log\left(\left|\sin\left(dx+c\right)+1\right|\right)}{a^2}-\frac{6\log\left(\left|\sin\left(dx+c\right)-1\right|\right)}{a^2}+\frac{3\left(2\sin\left(dx+c\right)-3\right)}{a^2\left(\sin\left(dx+c\right)-1\right)}-\frac{11\sin\left(dx+c\right)^3+42\sin\left(dx+c\right)^2+57\sin\left(dx+c\right)+30}{a^2\left(\sin\left(dx+c\right)+1\right)^3}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/48*(6*log(abs(sin(d*x + c) + 1))/a^2 - 6*log(abs(sin(d*x + c) - 1))/a^2 +
 3*(2*sin(d*x + c) - 3)/(a^2*(sin(d*x + c) - 1)) - (11*sin(d*x + c)^3 + 42*
sin(d*x + c)^2 + 57*sin(d*x + c) + 30)/(a^2*(sin(d*x + c) + 1)^3))/d
```

Mupad [B]
time = 0.10, size = 93, normalized size = 0.89

$$\frac{\frac{\sin(c+d\,x)^3}{4}+\frac{\sin(c+d\,x)^2}{2}+\frac{\sin(c+d\,x)}{12}-\frac{1}{3}}{d\,\left(-a^2\sin\left(c+d\,x\right)^4-2\,a^2\sin\left(c+d\,x\right)^3+2\,a^2\,\sin\left(c+d\,x\right)+a^2\right)}+\frac{\mathrm{atanh}(\sin\left(c+d\,x\right))}{4\,a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + a*sin(c + d*x))^2),x)
```

```
[Out] (sin(c + d*x)/12 + sin(c + d*x)^2/2 + sin(c + d*x)^3/4 - 1/3)/(d*(2*a^2*sin
(c + d*x) + a^2 - 2*a^2*sin(c + d*x)^3 - a^2*sin(c + d*x)^4)) + atanh(sin(c
 + d*x))/(4*a^2*d)
```

3.73 $\int \frac{\sec^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=93

$$-\frac{\sec^3(c+dx)}{7d(a+a\sin(c+dx))^2}-\frac{\sec^3(c+dx)}{7d\left(a^2+a^2\sin(c+dx)\right)}+\frac{4\tan(c+dx)}{7a^2d}+\frac{4\tan^3(c+dx)}{21a^2d}$$

```
[Out] -1/7*sec(d*x+c)^3/d/(a+a*sin(d*x+c))^2-1/7*sec(d*x+c)^3/d/(a^2+a^2*sin(d*x+
c))+4/7*tan(d*x+c)/a^2/d+4/21*tan(d*x+c)^3/a^2/d
```

Rubi [A]
time = 0.07, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2751, 3852}

$$\frac{4\tan^3(c+dx)}{21a^2d}+\frac{4\tan(c+dx)}{7a^2d}-\frac{\sec^3(c+dx)}{7d\left(a^2\sin(c+dx)+a^2\right)}-\frac{\sec^3(c+dx)}{7d(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/7*Sec[c + d*x]^3/(d*(a + a*Sin[c + d*x])^2) - Sec[c + d*x]^3/(7*d*(a^2 +
 a^2*Sin[c + d*x])) + (4*Tan[c + d*x])/(7*a^2*d) + (4*Tan[c + d*x]^3)/(21*a
^2*d)
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx = -\frac{\sec^3(c+dx)}{7d(a+a\sin(c+dx))^2} + \frac{5\int \frac{\sec^4(c+dx)}{a+a\sin(c+dx)}\,dx}{7a}$$

$$= -\frac{\sec^3(c+dx)}{7d(a+a\sin(c+dx))^2} - \frac{\sec^3(c+dx)}{7d\left(a^2+a^2\sin(c+dx)\right)} + \frac{4\int \sec^4(c+dx)\,dx}{7a^2}$$

$$= -\frac{\sec^3(c+dx)}{7d(a+a\sin(c+dx))^2} - \frac{\sec^3(c+dx)}{7d\left(a^2+a^2\sin(c+dx)\right)} - \frac{4\text{Subst}\left(\int (1+x^2)\,dx, x,\right.}{7a^2d}$$

$$= -\frac{\sec^3(c+dx)}{7d(a+a\sin(c+dx))^2} - \frac{\sec^3(c+dx)}{7d\left(a^2+a^2\sin(c+dx)\right)} + \frac{4\tan(c+dx)}{7a^2d} + \frac{4\tan^3(c}{21a^2}$$

Mathematica [A]
time = 0.04, size = 78, normalized size = 0.84

$$-\frac{\sec^3(c+dx)\left(6-9\sin(c+dx)-24\sin^2(c+dx)-4\sin^3(c+dx)+16\sin^4(c+dx)+8\sin^5(c+dx)\right)}{21a^2d(1+\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/21*(Sec[c + d*x]^3*(6 - 9*Sin[c + d*x] - 24*Sin[c + d*x]^2 - 4*Sin[c + d
*x]^3 + 16*Sin[c + d*x]^4 + 8*Sin[c + d*x]^5))/(a^2*d*(1 + Sin[c + d*x])^2)
```

Maple [A]
time = 0.25, size = 158, normalized size = 1.70

method	result
risch	$-\frac{16i\left(8i e^{3i(dx+c)}+14\,e^{4i(dx+c)}+4i e^{i(dx+c)}+3\,e^{2i(dx+c)}-1\right)}{21\left(e^{i(dx+c)}+i\right)^7\left(e^{i(dx+c)}-i\right)^3 a^2 d}$
derivativedivides	$\frac{-\frac{4}{7\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^7}+\frac{2}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^6}-\frac{4}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}+\frac{5}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}-\frac{55}{12\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}+\frac{23}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right.}}{a^2 d}$
default	$\frac{-\frac{4}{7\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^7}+\frac{2}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^6}-\frac{4}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}+\frac{5}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}-\frac{55}{12\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}+\frac{23}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right.}}{a^2 d}$
norman	$\frac{\frac{4\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}+\frac{1}{14ad}+\frac{\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)}{2da}-\frac{5\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2da}-\frac{20\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3da}-\frac{12\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{7da}-\frac{68\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{21da}-\frac{5\left(\tan^6\right.}{}}{a\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^7\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/a^2*(-2/7/(tan(1/2*d*x+1/2*c)+1)^7+1/(tan(1/2*d*x+1/2*c)+1)^6-2/(tan(1/
2*d*x+1/2*c)+1)^5+5/2/(tan(1/2*d*x+1/2*c)+1)^4-55/24/(tan(1/2*d*x+1/2*c)+1)
^3+23/16/(tan(1/2*d*x+1/2*c)+1)^2-13/16/(tan(1/2*d*x+1/2*c)+1)-1/24/(tan(1/
```

```
2*d*x+1/2*c)-1)^3-1/16/(tan(1/2*d*x+1/2*c)-1)^2-3/16/(tan(1/2*d*x+1/2*c)-1)
)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 396 vs. $2(85) = 170$.
time = 0.32, size = 396, normalized size = 4.26

$$-\frac{2\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}-\frac{24\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{76\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{28\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{42\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{56\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{28\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{42\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{21\sin(dx+c)^9}{(\cos(dx+c)+1)^9}+6\right)}{21\left(a^2+\frac{4a^2\sin(dx+c)}{\cos(dx+c)+1}+\frac{3a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{8a^2\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{14a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{14a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{8a^2\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{3a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{4a^2\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{a^2\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -2/21*(3*sin(d*x + c)/(cos(d*x + c) + 1) - 24*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 - 76*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 28*sin(d*x + c)^4/(cos(d*
x + c) + 1)^4 + 42*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 56*sin(d*x + c)^6/
(cos(d*x + c) + 1)^6 - 28*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 42*sin(d*x
+ c)^8/(cos(d*x + c) + 1)^8 - 21*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 + 6)/(
(a^2 + 4*a^2*sin(d*x + c)/(cos(d*x + c) + 1) + 3*a^2*sin(d*x + c)^2/(cos(d*
x + c) + 1)^2 - 8*a^2*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 14*a^2*sin(d*x
+ c)^4/(cos(d*x + c) + 1)^4 + 14*a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 +
8*a^2*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 3*a^2*sin(d*x + c)^8/(cos(d*x +
 c) + 1)^8 - 4*a^2*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - a^2*sin(d*x + c)^1
0/(cos(d*x + c) + 1)^10)*d)
```

Fricas [A]
time = 0.34, size = 103, normalized size = 1.11

$$\frac{16\cos(dx+c)^4-8\cos(dx+c)^2+\left(8\cos(dx+c)^4-12\cos(dx+c)^2-5\right)\sin(dx+c)-2}{21\left(a^2d\cos(dx+c)^5-2a^2d\cos(dx+c)^3\sin(dx+c)-2a^2d\cos(dx+c)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/21*(16*cos(d*x + c)^4 - 8*cos(d*x + c)^2 + (8*cos(d*x + c)^4 - 12*cos(d*x
 + c)^2 - 5)*sin(d*x + c) - 2)/(a^2*d*cos(d*x + c)^5 - 2*a^2*d*cos(d*x + c)
^3*sin(d*x + c) - 2*a^2*d*cos(d*x + c)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^4(c+dx)}{\sin^2(c+dx)+2\sin(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**4/(sin(c + d*x)**2 + 2*sin(c + d*x) + 1), x)/a**2
```

Giac [A]
time = 7.25, size = 145, normalized size = 1.56

$$-\frac{\frac{7\left(9\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-15\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+8\right)}{a^{2}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)^{3}}+\frac{273\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{6}+1155\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{5}+2450\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{4}+2870\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}+2037\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+791\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+152}{a^{2}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)^{7}}}{168\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/168*(7*(9*tan(1/2*d*x + 1/2*c)^2 - 15*tan(1/2*d*x + 1/2*c) + 8)/(a^2*(ta
n(1/2*d*x + 1/2*c) - 1)^3) + (273*tan(1/2*d*x + 1/2*c)^6 + 1155*tan(1/2*d*x
 + 1/2*c)^5 + 2450*tan(1/2*d*x + 1/2*c)^4 + 2870*tan(1/2*d*x + 1/2*c)^3 + 2
037*tan(1/2*d*x + 1/2*c)^2 + 791*tan(1/2*d*x + 1/2*c) + 152)/(a^2*(tan(1/2*
d*x + 1/2*c) + 1)^7))/d
```

Mupad [B]
time = 5.18, size = 276, normalized size = 2.97

$$\frac{2\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\left(-6\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}-3\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)+24\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}+76\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}+28\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}-42\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}-56\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}+28\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}+42\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}+21\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}\right)}{21\,a^{2}\,d\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^{3}\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a + a*sin(c + d*x))^2),x)
```

```
[Out] (2*cos(c/2 + (d*x)/2)*(21*sin(c/2 + (d*x)/2)^9 - 6*cos(c/2 + (d*x)/2)^9 + 4
2*cos(c/2 + (d*x)/2)*sin(c/2 + (d*x)/2)^8 - 3*cos(c/2 + (d*x)/2)^8*sin(c/2
+ (d*x)/2) + 28*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^7 - 56*cos(c/2 + (d
*x)/2)^3*sin(c/2 + (d*x)/2)^6 - 42*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^
5 + 28*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^4 + 76*cos(c/2 + (d*x)/2)^6*
sin(c/2 + (d*x)/2)^3 + 24*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2)^2))/(21*a
^2*d*(cos(c/2 + (d*x)/2) - sin(c/2 + (d*x)/2))^3*(cos(c/2 + (d*x)/2) + sin(
c/2 + (d*x)/2))^7)
```

3.74 $\int \frac{\sec^5(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=146

$$\frac{15\tanh^{-1}(\sin(c+dx))}{64a^2d}+\frac{1}{64d(a-a\sin(c+dx))^2}-\frac{a^2}{32d(a+a\sin(c+dx))^4}-\frac{a}{16d(a+a\sin(c+dx))^3}-\frac{3}{32d(a+a\sin(c+dx))^2}+\frac{5}{64d(a^2-a^2\sin(c+dx))}-\frac{5}{32d(a^2+a^2\sin(c+dx))}$$

```
[Out] 15/64*arctanh(sin(d*x+c))/a^2/d+1/64/d/(a-a*sin(d*x+c))^2-1/32*a^2/d/(a+a*s
in(d*x+c))^4-1/16*a/d/(a+a*sin(d*x+c))^3-3/32/d/(a+a*sin(d*x+c))^2+5/64/d/(
a^2-a^2*sin(d*x+c))-5/32/d/(a^2+a^2*sin(d*x+c))
```

Rubi [A]
time = 0.08, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2746, 46, 212}

$$-\frac{a^2}{32d(a\sin(c+dx)+a)^4}+\frac{5}{64d\left(a^2-a^2\sin(c+dx)\right)}-\frac{5}{32d\left(a^2\sin(c+dx)+a^2\right)}+\frac{15\tanh^{-1}(\sin(c+dx))}{64a^2d}-\frac{a}{16d(a\sin(c+dx)+a)^3}+\frac{1}{64d(a-a\sin(c+dx))^2}-\frac{3}{32d(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (15*ArcTanh[Sin[c + d*x]])/(64*a^2*d) + 1/(64*d*(a - a*Sin[c + d*x])^2) - a
^2/(32*d*(a + a*Sin[c + d*x])^4) - a/(16*d*(a + a*Sin[c + d*x])^3) - 3/(32*
d*(a + a*Sin[c + d*x])^2) + 5/(64*d*(a^2 - a^2*Sin[c + d*x])) - 5/(32*d*(a^
2 + a^2*Sin[c + d*x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
```

])

Rubi steps

$$\int \frac{\sec^5(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{a^5\text{Subst}\left(\int \frac{1}{(a-x)^3(a+x)^5}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^5\text{Subst}\left(\int \left(\frac{1}{32a^5(a-x)^3} + \frac{5}{64a^6(a-x)^2} + \frac{1}{8a^3(a+x)^5} + \frac{3}{16a^4(a+x)^4} + \frac{3}{16a^5(a+x)^3} + \frac{5}{32a^6(a+\cdots}\right.\right.}{d}$$
$$= \frac{1}{64d(a-a\sin(c+dx))^2} - \frac{a^2}{32d(a+a\sin(c+dx))^4} - \frac{a}{16d(a+a\sin(c+dx))^3} - \cdots$$
$$= \frac{15\tanh^{-1}(\sin(c+dx))}{64a^2d} + \frac{1}{64d(a-a\sin(c+dx))^2} - \frac{a^2}{32d(a+a\sin(c+dx))^4} - \cdots$$

Mathematica [A]
time = 0.21, size = 137, normalized size = 0.94

$$\frac{\sec^4(c+dx)(1-\sin(c+dx))^2(1+\sin(c+dx))^2\left(\frac{15}{64}\tanh^{-1}(\sin(c+dx)) + \frac{1}{64(1-\sin(c+dx))^2} + \frac{5}{64(1-\sin(c+dx))} - \frac{1}{32(1+\sin(c+dx))^4} - \frac{1}{16(1+\sin(c+dx))^3} - \frac{3}{32(1+\sin(c+dx))^2} - \frac{5}{32(1+\sin(c+dx))}\right)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (Sec[c + d*x]^4*(1 - Sin[c + d*x])^2*(1 + Sin[c + d*x])^2*((15*ArcTanh[Sin[
c + d*x]])/64 + 1/(64*(1 - Sin[c + d*x])^2) + 5/(64*(1 - Sin[c + d*x])) - 1
/(32*(1 + Sin[c + d*x])^4) - 1/(16*(1 + Sin[c + d*x])^3) - 3/(32*(1 + Sin[c
 + d*x])^2) - 5/(32*(1 + Sin[c + d*x]))))/(a^2*d)
```

Maple [A]
time = 0.34, size = 103, normalized size = 0.71

method	result
derivativedivides	$\frac{\frac{1}{64(\sin(dx+c)-1)^2} - \frac{5}{64(\sin(dx+c)-1)} - \frac{15\ln(\sin(dx+c)-1)}{128} - \frac{1}{32(1+\sin(dx+c))^4} - \frac{1}{16(1+\sin(dx+c))^3} - \frac{3}{32(1+\sin(dx+c))^2} - \frac{5}{32(1+\sin(\cdots}}{d\,a^2}$
default	$\frac{\frac{1}{64(\sin(dx+c)-1)^2} - \frac{5}{64(\sin(dx+c)-1)} - \frac{15\ln(\sin(dx+c)-1)}{128} - \frac{1}{32(1+\sin(dx+c))^4} - \frac{1}{16(1+\sin(dx+c))^3} - \frac{3}{32(1+\sin(dx+c))^2} - \frac{5}{32(1+\sin(\cdots}}{d\,a^2}$
risch	$-\frac{i\left(60i\,\mathrm{e}^{10i(dx+c)} + 15\,\mathrm{e}^{11i(dx+c)} + 160i\,\mathrm{e}^{8i(dx+c)} - 35\,\mathrm{e}^{9i(dx+c)} + 72i\,\mathrm{e}^{6i(dx+c)} - 242\,\mathrm{e}^{7i(dx+c)} + 160i\,\mathrm{e}^{4i(dx+c)} + 242\,\mathrm{e}^{5i(dx+c)\cdots}\right.}{32\left(\mathrm{e}^{i(dx+c)}+i\right)^8\left(\mathrm{e}^{i(dx+c)}-i\right)^4a^2d}$
norman	$\frac{\frac{17\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8da} + \frac{17\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8da} - \frac{3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da} - \frac{3\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da} + \frac{23\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4da} + \frac{49\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{32da} + \frac{49\left(\tan^{11}\cdots\right.}{32\cdots}}{a\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\cdots\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^2*(1/64/(sin(d*x+c)-1)^2-5/64/(sin(d*x+c)-1)-15/128*ln(sin(d*x+c)-1)-
1/32/(1+sin(d*x+c))^4-1/16/(1+sin(d*x+c))^3-3/32/(1+sin(d*x+c))^2-5/32/(1+s
in(d*x+c))+15/128*ln(1+sin(d*x+c)))
```

Maxima [A]
time = 0.30, size = 167, normalized size = 1.14

$$-\frac{\frac{2\left(15\,\sin\left(dx+c\right)^{5}+30\,\sin\left(dx+c\right)^{4}-10\,\sin\left(dx+c\right)^{3}-50\,\sin\left(dx+c\right)^{2}-17\,\sin\left(dx+c\right)+16\right)}{a^{2}\sin\left(dx+c\right)^{6}+2\,a^{2}\sin\left(dx+c\right)^{5}-a^{2}\sin\left(dx+c\right)^{4}-4\,a^{2}\sin\left(dx+c\right)^{3}-a^{2}\sin\left(dx+c\right)^{2}+2\,a^{2}\sin\left(dx+c\right)+a^{2}}-\frac{15\,\log\left(\sin\left(dx+c\right)+1\right)}{a^{2}}+\frac{15\,\log\left(\sin\left(dx+c\right)-1\right)}{a^{2}}}{128\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/128*(2*(15*sin(d*x + c)^5 + 30*sin(d*x + c)^4 - 10*sin(d*x + c)^3 - 50*s
in(d*x + c)^2 - 17*sin(d*x + c) + 16)/(a^2*sin(d*x + c)^6 + 2*a^2*sin(d*x +
 c)^5 - a^2*sin(d*x + c)^4 - 4*a^2*sin(d*x + c)^3 - a^2*sin(d*x + c)^2 + 2*
a^2*sin(d*x + c) + a^2) - 15*log(sin(d*x + c) + 1)/a^2 + 15*log(sin(d*x + c
) - 1)/a^2)/d
```

Fricas [A]
time = 0.42, size = 198, normalized size = 1.36

$$\frac{60\,\cos\left(dx+c\right)^{4}-20\,\cos\left(dx+c\right)^{2}+15\left(\cos\left(dx+c\right)^{6}-2\,\cos\left(dx+c\right)^{4}\sin\left(dx+c\right)-2\,\cos\left(dx+c\right)^{4}\right)\log\left(\sin\left(dx+c\right)+1\right)-15\left(\cos\left(dx+c\right)^{6}-2\,\cos\left(dx+c\right)^{4}\sin\left(dx+c\right)-2\,\cos\left(dx+c\right)^{4}\right)\log\left(-\sin\left(dx+c\right)+1\right)+2\left(15\,\cos\left(dx+c\right)^{4}-20\,\cos\left(dx+c\right)^{2}-12\right)\sin\left(dx+c\right)-8}{128\left(a^{2}d\cos\left(dx+c\right)^{6}-2\,a^{2}d\cos\left(dx+c\right)^{4}\sin\left(dx+c\right)-2\,a^{2}d\cos\left(dx+c\right)^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/128*(60*cos(d*x + c)^4 - 20*cos(d*x + c)^2 + 15*(cos(d*x + c)^6 - 2*cos(d
*x + c)^4*sin(d*x + c) - 2*cos(d*x + c)^4)*log(sin(d*x + c) + 1) - 15*(cos(
d*x + c)^6 - 2*cos(d*x + c)^4*sin(d*x + c) - 2*cos(d*x + c)^4)*log(-sin(d*x
 + c) + 1) + 2*(15*cos(d*x + c)^4 - 20*cos(d*x + c)^2 - 12)*sin(d*x + c) -
8)/(a^2*d*cos(d*x + c)^6 - 2*a^2*d*cos(d*x + c)^4*sin(d*x + c) - 2*a^2*d*co
s(d*x + c)^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^{5}{\left(c+dx \right)}}{\sin^{2}{\left(c+dx \right)}+2\sin{\left(c+dx \right)}+1}\,dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**5/(sin(c + d*x)**2 + 2*sin(c + d*x) + 1), x)/a**2
```

Giac [A]
time = 6.55, size = 126, normalized size = 0.86

$$\frac{\frac{60 \log(|\sin(dx+c)+1|)}{a^2} - \frac{60 \log(|\sin(dx+c)-1|)}{a^2} + \frac{2\left(45 \sin(dx+c)^2 - 110 \sin(dx+c) + 69\right)}{a^2(\sin(dx+c)-1)^2} - \frac{125 \sin(dx+c)^4 + 580 \sin(dx+c)^3 + 1038 \sin(dx+c)^2 + 868 \sin(dx+c) + 301}{a^2(\sin(dx+c)+1)^4}}{512\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/512*(60*log(abs(sin(d*x + c) + 1))/a^2 - 60*log(abs(sin(d*x + c) - 1))/a^
2 + 2*(45*sin(d*x + c)^2 - 110*sin(d*x + c) + 69)/(a^2*(sin(d*x + c) - 1)^2
) - (125*sin(d*x + c)^4 + 580*sin(d*x + c)^3 + 1038*sin(d*x + c)^2 + 868*si
n(d*x + c) + 301)/(a^2*(sin(d*x + c) + 1)^4))/d
```

Mupad [B]
time = 0.19, size = 151, normalized size = 1.03

$$\frac{15\,\mathrm{atanh}(\sin(c+d\,x))}{64\,a^2\,d} + \frac{-\frac{15\sin(c+d\,x)^5}{64} - \frac{15\sin(c+d\,x)^4}{32} + \frac{5\sin(c+d\,x)^3}{32} + \frac{25\sin(c+d\,x)^2}{32} + \frac{17\sin(c+d\,x)}{64} - \frac{1}{4}}{d\,\left(a^2\sin(c+d\,x)^6 + 2\,a^2\sin(c+d\,x)^5 - a^2\sin(c+d\,x)^4 - 4\,a^2\sin(c+d\,x)^3 - a^2\sin(c+d\,x)^2 + 2\,a^2\,\sin(c+d\,x) + a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^5*(a + a*sin(c + d*x))^2),x)
```

```
[Out] (15*atanh(sin(c + d*x)))/(64*a^2*d) + ((17*sin(c + d*x))/64 + (25*sin(c + d
*x)^2)/32 + (5*sin(c + d*x)^3)/32 - (15*sin(c + d*x)^4)/32 - (15*sin(c + d*
x)^5)/64 - 1/4)/(d*(2*a^2*sin(c + d*x) + a^2 - a^2*sin(c + d*x)^2 - 4*a^2*s
in(c + d*x)^3 - a^2*sin(c + d*x)^4 + 2*a^2*sin(c + d*x)^5 + a^2*sin(c + d*x
)^6))
```

3.75 $\int \frac{\cos^8(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=103

$$\frac{7x}{8a^3}+\frac{7\cos^5(c+dx)}{15a^3d}+\frac{7\cos(c+dx)\sin(c+dx)}{8a^3d}+\frac{7\cos^3(c+dx)\sin(c+dx)}{12a^3d}+\frac{2\cos^7(c+dx)}{3ad(a+a\sin(c+dx))^2}$$

```
[Out] 7/8*x/a^3+7/15*cos(d*x+c)^5/a^3/d+7/8*cos(d*x+c)*sin(d*x+c)/a^3/d+7/12*cos(
d*x+c)^3*sin(d*x+c)/a^3/d+2/3*cos(d*x+c)^7/a/d/(a+a*sin(d*x+c))^2
```

Rubi [A]
time = 0.08, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2759, 2761, 2715, 8}

$$\frac{7\cos^5(c+dx)}{15a^3d}+\frac{7\sin(c+dx)\cos^3(c+dx)}{12a^3d}+\frac{7\sin(c+dx)\cos(c+dx)}{8a^3d}+\frac{7x}{8a^3}+\frac{2\cos^7(c+dx)}{3ad(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^8/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (7*x)/(8*a^3) + (7*Cos[c + d*x]^5)/(15*a^3*d) + (7*Cos[c + d*x]*Sin[c + d*x
])/(8*a^3*d) + (7*Cos[c + d*x]^3*Sin[c + d*x])/(12*a^3*d) + (2*Cos[c + d*x]
^7)/(3*a*d*(a + a*Sin[c + d*x])^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^8(c+dx)}{(a+a\sin(c+dx))^3}\,dx &= \frac{2\cos^7(c+dx)}{3ad(a+a\sin(c+dx))^2}+\frac{7\int \frac{\cos^6(c+dx)}{a+a\sin(c+dx)}\,dx}{3a^2}\\ &= \frac{7\cos^5(c+dx)}{15a^3d}+\frac{2\cos^7(c+dx)}{3ad(a+a\sin(c+dx))^2}+\frac{7\int \cos^4(c+dx)\,dx}{3a^3}\\ &= \frac{7\cos^5(c+dx)}{15a^3d}+\frac{7\cos^3(c+dx)\sin(c+dx)}{12a^3d}+\frac{2\cos^7(c+dx)}{3ad(a+a\sin(c+dx))^2}+\frac{7\int \cos\\ &= \frac{7\cos^5(c+dx)}{15a^3d}+\frac{7\cos(c+dx)\sin(c+dx)}{8a^3d}+\frac{7\cos^3(c+dx)\sin(c+dx)}{12a^3d}+\frac{}{3ad(}\\ &= \frac{7x}{8a^3}+\frac{7\cos^5(c+dx)}{15a^3d}+\frac{7\cos(c+dx)\sin(c+dx)}{8a^3d}+\frac{7\cos^3(c+dx)\sin(c+dx)}{12a^3d}\end{aligned}$$

Mathematica [A]

time = 0.67, size = 141, normalized size = 1.37

$$-\frac{\cos^9(c+dx)\left(-210\sin^{-1}\left(\frac{\sqrt{1-\sin(c+dx)}}{\sqrt{2}}\right)\sqrt{1-\sin(c+dx)}+\sqrt{1+\sin(c+dx)}\,(136-121\sin(c+dx)-127\sin^2(c+dx)+202\sin^3(c+dx)-114\sin^4(c+dx)+24\sin^5(c+dx))\right)}{120a^3d(-1+\sin(c+dx))^5(1+\sin(c+dx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^8/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/120*(Cos[c + d*x]^9*(-210*ArcSin[Sqrt[1 - Sin[c + d*x]]/Sqrt[2]]*Sqrt[1
- Sin[c + d*x]] + Sqrt[1 + Sin[c + d*x]]*(136 - 121*Sin[c + d*x] - 127*Sin[
c + d*x]^2 + 202*Sin[c + d*x]^3 - 114*Sin[c + d*x]^4 + 24*Sin[c + d*x]^5)))
/(a^3*d*(-1 + Sin[c + d*x])^5*(1 + Sin[c + d*x])^(9/2))
```

Maple [A]

time = 0.15, size = 142, normalized size = 1.38

method	result
risch	$\frac{7x}{8a^3}+\frac{7\cos(dx+c)}{8a^3d}-\frac{\cos(5dx+5c)}{80a^3d}-\frac{3\sin(4dx+4c)}{32a^3d}+\frac{13\cos(3dx+3c)}{48a^3d}+\frac{\sin(2dx+2c)}{4a^3d}$
derivativedivides	$\frac{\frac{2\left(-\frac{\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8}+3\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{13\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}+8\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{10\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\frac{13\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}+\frac{8\left(\tan^2\left(\right.\right.}{}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}}{a^3d}$

default	$\frac{\frac{2\left(-\frac{\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8}+3\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{13\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}+8\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{10\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\frac{13\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}+\frac{8\left(\tan\right.}{}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}}{a^3 d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/a^3*((-1/8*tan(1/2*d*x+1/2*c)^9+3*tan(1/2*d*x+1/2*c)^8-13/4*tan(1/2*d*x
+1/2*c)^7+8*tan(1/2*d*x+1/2*c)^6+10/3*tan(1/2*d*x+1/2*c)^4+13/4*tan(1/2*d*x
+1/2*c)^3+8/3*tan(1/2*d*x+1/2*c)^2+1/8*tan(1/2*d*x+1/2*c)+17/15)/(1+tan(1/2
*d*x+1/2*c)^2)^5+7/8*arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 310 vs. $2(93) = 186$.
time = 0.54, size = 310, normalized size = 3.01

$$\frac{\frac{\frac{15\sin(dx+c)}{\cos(dx+c)+1}+\frac{320\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{390\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{400\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{960\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{390\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{360\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{15\sin(dx+c)^9}{(\cos(dx+c)+1)^9}+136}{a^3+\frac{5a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{10a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{10a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{5a^3\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{a^3\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}}+\frac{105\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/60*((15*sin(d*x + c)/(cos(d*x + c) + 1) + 320*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 + 390*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 400*sin(d*x + c)^4/(co
s(d*x + c) + 1)^4 + 960*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 390*sin(d*x +
 c)^7/(cos(d*x + c) + 1)^7 + 360*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 15*s
in(d*x + c)^9/(cos(d*x + c) + 1)^9 + 136)/(a^3 + 5*a^3*sin(d*x + c)^2/(cos(
d*x + c) + 1)^2 + 10*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 10*a^3*sin(d
*x + c)^6/(cos(d*x + c) + 1)^6 + 5*a^3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8
+ a^3*sin(d*x + c)^10/(cos(d*x + c) + 1)^10) + 105*arctan(sin(d*x + c)/(cos
(d*x + c) + 1))/a^3)/d
```

Fricas [A]
time = 0.40, size = 60, normalized size = 0.58

$$-\frac{24\cos(dx+c)^5-160\cos(dx+c)^3-105\,dx+15\left(6\cos(dx+c)^3-7\cos(dx+c)\right)\sin(dx+c)}{120\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/120*(24*cos(d*x + c)^5 - 160*cos(d*x + c)^3 - 105*d*x + 15*(6*cos(d*x +
c)^3 - 7*cos(d*x + c))*sin(d*x + c))/(a^3*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1717 vs. $2(97) = 194$.

time = 120.35, size = 1717, normalized size = 16.67

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((105*d*x*tan(c/2 + d*x/2)**10/(120*a**3*d*tan(c/2 + d*x/2)**10 +
600*a**3*d*tan(c/2 + d*x/2)**8 + 1200*a**3*d*tan(c/2 + d*x/2)**6 + 1200*a**
3*d*tan(c/2 + d*x/2)**4 + 600*a**3*d*tan(c/2 + d*x/2)**2 + 120*a**3*d) + 52
5*d*x*tan(c/2 + d*x/2)**8/(120*a**3*d*tan(c/2 + d*x/2)**10 + 600*a**3*d*tan
(c/2 + d*x/2)**8 + 1200*a**3*d*tan(c/2 + d*x/2)**6 + 1200*a**3*d*tan(c/2 +
d*x/2)**4 + 600*a**3*d*tan(c/2 + d*x/2)**2 + 120*a**3*d) + 1050*d*x*tan(c/2
 + d*x/2)**6/(120*a**3*d*tan(c/2 + d*x/2)**10 + 600*a**3*d*tan(c/2 + d*x/2)
**8 + 1200*a**3*d*tan(c/2 + d*x/2)**6 + 1200*a**3*d*tan(c/2 + d*x/2)**4 + 6
00*a**3*d*tan(c/2 + d*x/2)**2 + 120*a**3*d) + 1050*d*x*tan(c/2 + d*x/2)**4/
(120*a**3*d*tan(c/2 + d*x/2)**10 + 600*a**3*d*tan(c/2 + d*x/2)**8 + 1200*a*
*3*d*tan(c/2 + d*x/2)**6 + 1200*a**3*d*tan(c/2 + d*x/2)**4 + 600*a**3*d*tan
(c/2 + d*x/2)**2 + 120*a**3*d) + 525*d*x*tan(c/2 + d*x/2)**2/(120*a**3*d*ta
n(c/2 + d*x/2)**10 + 600*a**3*d*tan(c/2 + d*x/2)**8 + 1200*a**3*d*tan(c/2 +
 d*x/2)**6 + 1200*a**3*d*tan(c/2 + d*x/2)**4 + 600*a**3*d*tan(c/2 + d*x/2)*
*2 + 120*a**3*d) + 105*d*x/(120*a**3*d*tan(c/2 + d*x/2)**10 + 600*a**3*d*ta
n(c/2 + d*x/2)**8 + 1200*a**3*d*tan(c/2 + d*x/2)**6 + 1200*a**3*d*tan(c/2 +
 d*x/2)**4 + 600*a**3*d*tan(c/2 + d*x/2)**2 + 120*a**3*d) - 30*tan(c/2 + d*
x/2)**9/(120*a**3*d*tan(c/2 + d*x/2)**10 + 600*a**3*d*tan(c/2 + d*x/2)**8 +
 1200*a**3*d*tan(c/2 + d*x/2)**6 + 1200*a**3*d*tan(c/2 + d*x/2)**4 + 600*a*
*3*d*tan(c/2 + d*x/2)**2 + 120*a**3*d) + 720*tan(c/2 + d*x/2)**8/(120*a**3*
d*tan(c/2 + d*x/2)**10 + 600*a**3*d*tan(c/2 + d*x/2)**8 + 1200*a**3*d*tan(c
/2 + d*x/2)**6 + 1200*a**3*d*tan(c/2 + d*x/2)**4 + 600*a**3*d*tan(c/2 + d*x
/2)**2 + 120*a**3*d) - 780*tan(c/2 + d*x/2)**7/(120*a**3*d*tan(c/2 + d*x/2)
**10 + 600*a**3*d*tan(c/2 + d*x/2)**8 + 1200*a**3*d*tan(c/2 + d*x/2)**6 + 1
200*a**3*d*tan(c/2 + d*x/2)**4 + 600*a**3*d*tan(c/2 + d*x/2)**2 + 120*a**3*
d) + 1920*tan(c/2 + d*x/2)**6/(120*a**3*d*tan(c/2 + d*x/2)**10 + 600*a**3*d
*tan(c/2 + d*x/2)**8 + 1200*a**3*d*tan(c/2 + d*x/2)**6 + 1200*a**3*d*tan(c/
2 + d*x/2)**4 + 600*a**3*d*tan(c/2 + d*x/2)**2 + 120*a**3*d) + 800*tan(c/2
+ d*x/2)**4/(120*a**3*d*tan(c/2 + d*x/2)**10 + 600*a**3*d*tan(c/2 + d*x/2)*
*8 + 1200*a**3*d*tan(c/2 + d*x/2)**6 + 1200*a**3*d*tan(c/2 + d*x/2)**4 + 60
0*a**3*d*tan(c/2 + d*x/2)**2 + 120*a**3*d) + 780*tan(c/2 + d*x/2)**3/(120*a
**3*d*tan(c/2 + d*x/2)**10 + 600*a**3*d*tan(c/2 + d*x/2)**8 + 1200*a**3*d*t
an(c/2 + d*x/2)**6 + 1200*a**3*d*tan(c/2 + d*x/2)**4 + 600*a**3*d*tan(c/2 +
 d*x/2)**2 + 120*a**3*d) + 640*tan(c/2 + d*x/2)**2/(120*a**3*d*tan(c/2 + d*
x/2)**10 + 600*a**3*d*tan(c/2 + d*x/2)**8 + 1200*a**3*d*tan(c/2 + d*x/2)**6
 + 1200*a**3*d*tan(c/2 + d*x/2)**4 + 600*a**3*d*tan(c/2 + d*x/2)**2 + 120*a
```

```
**3*d) + 30*tan(c/2 + d*x/2)/(120*a**3*d*tan(c/2 + d*x/2)**10 + 600*a**3*d*
tan(c/2 + d*x/2)**8 + 1200*a**3*d*tan(c/2 + d*x/2)**6 + 1200*a**3*d*tan(c/2
 + d*x/2)**4 + 600*a**3*d*tan(c/2 + d*x/2)**2 + 120*a**3*d) + 272/(120*a**3
*d*tan(c/2 + d*x/2)**10 + 600*a**3*d*tan(c/2 + d*x/2)**8 + 1200*a**3*d*tan(
c/2 + d*x/2)**6 + 1200*a**3*d*tan(c/2 + d*x/2)**4 + 600*a**3*d*tan(c/2 + d*
x/2)**2 + 120*a**3*d), Ne(d, 0)), (x*cos(c)**8/(a*sin(c) + a)**3, True))
```

Giac [A]
time = 6.94, size = 140, normalized size = 1.36

$$\frac{\frac{105\,(dx+c)}{a^3}-\frac{2\left(15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-360\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8+390\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-960\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-400\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-390\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-320\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-136\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^5a^3}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/120*(105*(d*x + c)/a^3 - 2*(15*tan(1/2*d*x + 1/2*c)^9 - 360*tan(1/2*d*x +
 1/2*c)^8 + 390*tan(1/2*d*x + 1/2*c)^7 - 960*tan(1/2*d*x + 1/2*c)^6 - 400*t
an(1/2*d*x + 1/2*c)^4 - 390*tan(1/2*d*x + 1/2*c)^3 - 320*tan(1/2*d*x + 1/2*
c)^2 - 15*tan(1/2*d*x + 1/2*c) - 136)/((tan(1/2*d*x + 1/2*c)^2 + 1)^5*a^3))
/d
```

Mupad [B]
time = 4.72, size = 81, normalized size = 0.79

$$\frac{7\,x}{8\,a^3}+\frac{4\cos\left(c+d\,x\right)^3}{3\,a^3\,d}-\frac{\cos\left(c+d\,x\right)^5}{5\,a^3\,d}-\frac{3\cos\left(c+d\,x\right)^3\,\sin\left(c+d\,x\right)}{4\,a^3\,d}+\frac{7\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{8\,a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^8/(a + a*sin(c + d*x))^3,x)
```

```
[Out] (7*x)/(8*a^3) + (4*cos(c + d*x)^3)/(3*a^3*d) - cos(c + d*x)^5/(5*a^3*d) - (
3*cos(c + d*x)^3*sin(c + d*x))/(4*a^3*d) + (7*cos(c + d*x)*sin(c + d*x))/(8
*a^3*d)
```

3.76 $\int \frac{\cos^7(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=23

$$-\frac{(a-a\sin(c+dx))^4}{4a^7d}$$

[Out] -1/4*(a-a*sin(d*x+c))^4/a^7/d

Rubi [A]
time = 0.03, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2746, 32}

$$-\frac{(a-a\sin(c+dx))^4}{4a^7d}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^7/(a + a*Sin[c + d*x])^3,x]

[Out] -1/4*(a - a*Sin[c + d*x])^4/(a^7*d)

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^7(c+dx)}{(a+a\sin(c+dx))^3}\,dx &= \frac{\text{Subst}\left(\int (a-x)^3\,dx, x, a\sin(c+dx)\right)}{a^7d}\\ &= -\frac{(a-a\sin(c+dx))^4}{4a^7d}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 48 vs. 2(23) = 46.

time = 0.22, size = 48, normalized size = 2.09

$$-\frac{-28\cos(2(c+dx))+\cos(4(c+dx))+8(-7\sin(c+dx)+\sin(3(c+dx)))}{32a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/32*(-28*Cos[2*(c + d*x)] + Cos[4*(c + d*x)] + 8*(-7*Sin[c + d*x] + Sin[3
*(c + d*x)]))/(a^3*d)
```

Maple [A]
time = 0.13, size = 19, normalized size = 0.83

method	result	size
derivativedivides	$-\frac{(\sin(dx+c)-1)^4}{4d\,a^3}$	19
default	$-\frac{(\sin(dx+c)-1)^4}{4d\,a^3}$	19
risch	$\frac{7\sin(dx+c)}{4a^3d}-\frac{\cos(4dx+4c)}{32a^3d}-\frac{\sin(3dx+3c)}{4a^3d}+\frac{7\cos(2dx+2c)}{8a^3d}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/d/a^3*(sin(d*x+c)-1)^4
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 45 vs. 2(22) = 44.
time = 0.29, size = 45, normalized size = 1.96

$$-\frac{\sin\left(dx+c\right)^4-4\,\sin\left(dx+c\right)^3+6\,\sin\left(dx+c\right)^2-4\,\sin\left(dx+c\right)}{4\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/4*(sin(d*x + c)^4 - 4*sin(d*x + c)^3 + 6*sin(d*x + c)^2 - 4*sin(d*x + c)
)/(a^3*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 45 vs. 2(22) = 44.
time = 0.38, size = 45, normalized size = 1.96

$$-\frac{\cos\left(dx+c\right)^4-8\,\cos\left(dx+c\right)^2+4\left(\cos\left(dx+c\right)^2-2\right)\sin\left(dx+c\right)}{4\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/4*(cos(d*x + c)^4 - 8*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 - 2)*sin(d*x +
c))/(a^3*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 654 vs. $2(19) = 38$.
time = 75.63, size = 654, normalized size = 28.43

$$\begin{cases} \frac{2\tan^7\left(\frac{c}{2}+\frac{dx}{2}\right)}{a^3d\tan^8\left(\frac{c}{2}+\frac{dx}{2}\right)+4a^3d\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^3d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+4a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+a^3d} - \frac{6\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)}{a^3d\tan^8\left(\frac{c}{2}+\frac{dx}{2}\right)+4a^3d\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^3d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+4a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+a^3d} + \frac{14\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{a^3d\tan^8\left(\frac{c}{2}+\frac{dx}{2}\right)+4a^3d\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^3d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+4a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+a^3d} - \frac{16\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{a^3d\tan^8\left(\frac{c}{2}+\frac{dx}{2}\right)+4a^3d\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^3d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+4a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+a^3d} + \frac{14\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{a^3d\tan^8\left(\frac{c}{2}+\frac{dx}{2}\right)+4a^3d\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^3d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+4a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+a^3d} - \frac{6\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{a^3d\tan^8\left(\frac{c}{2}+\frac{dx}{2}\right)+4a^3d\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^3d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+4a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+a^3d} + \frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{a^3d\tan^8\left(\frac{c}{2}+\frac{dx}{2}\right)+4a^3d\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^3d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+4a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+a^3d} & \text{for}\: d \neq 0 \\ \frac{x\cos^7(c)}{(a\sin(c)+a)^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((2*tan(c/2 + d*x/2)**7/(a**3*d*tan(c/2 + d*x/2)**8 + 4*a**3*d*tan
(c/2 + d*x/2)**6 + 6*a**3*d*tan(c/2 + d*x/2)**4 + 4*a**3*d*tan(c/2 + d*x/2)
**2 + a**3*d) - 6*tan(c/2 + d*x/2)**6/(a**3*d*tan(c/2 + d*x/2)**8 + 4*a**3*
d*tan(c/2 + d*x/2)**6 + 6*a**3*d*tan(c/2 + d*x/2)**4 + 4*a**3*d*tan(c/2 + d
*x/2)**2 + a**3*d) + 14*tan(c/2 + d*x/2)**5/(a**3*d*tan(c/2 + d*x/2)**8 + 4
*a**3*d*tan(c/2 + d*x/2)**6 + 6*a**3*d*tan(c/2 + d*x/2)**4 + 4*a**3*d*tan(c
/2 + d*x/2)**2 + a**3*d) - 16*tan(c/2 + d*x/2)**4/(a**3*d*tan(c/2 + d*x/2)*
*8 + 4*a**3*d*tan(c/2 + d*x/2)**6 + 6*a**3*d*tan(c/2 + d*x/2)**4 + 4*a**3*d
*tan(c/2 + d*x/2)**2 + a**3*d) + 14*tan(c/2 + d*x/2)**3/(a**3*d*tan(c/2 + d
*x/2)**8 + 4*a**3*d*tan(c/2 + d*x/2)**6 + 6*a**3*d*tan(c/2 + d*x/2)**4 + 4*
a**3*d*tan(c/2 + d*x/2)**2 + a**3*d) - 6*tan(c/2 + d*x/2)**2/(a**3*d*tan(c/
2 + d*x/2)**8 + 4*a**3*d*tan(c/2 + d*x/2)**6 + 6*a**3*d*tan(c/2 + d*x/2)**4
 + 4*a**3*d*tan(c/2 + d*x/2)**2 + a**3*d) + 2*tan(c/2 + d*x/2)/(a**3*d*tan(
c/2 + d*x/2)**8 + 4*a**3*d*tan(c/2 + d*x/2)**6 + 6*a**3*d*tan(c/2 + d*x/2)*
*4 + 4*a**3*d*tan(c/2 + d*x/2)**2 + a**3*d), Ne(d, 0)), (x*cos(c)**7/(a*sin
(c) + a)**3, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(22) = 44$.
time = 5.78, size = 45, normalized size = 1.96

$$-\frac{\sin\left(dx+c\right)^{4} - 4\,\sin\left(dx+c\right)^{3} + 6\,\sin\left(dx+c\right)^{2} - 4\,\sin\left(dx+c\right)}{4\,a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/4*(sin(d*x + c)^4 - 4*sin(d*x + c)^3 + 6*sin(d*x + c)^2 - 4*sin(d*x + c)
)/(a^3*d)
```

Mupad [B]
time = 4.55, size = 53, normalized size = 2.30

$$\frac{\frac{\sin(c+d\,x)}{a^3}-\frac{3\,\sin(c+d\,x)^2}{2\,a^3}+\frac{\sin(c+d\,x)^3}{a^3}-\frac{\sin(c+d\,x)^4}{4\,a^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7/(a + a*sin(c + d*x))^3,x)
```

```
[Out] (sin(c + d*x)/a^3 - (3*sin(c + d*x)^2)/(2*a^3) + sin(c + d*x)^3/a^3 - sin(c
 + d*x)^4/(4*a^3))/d
```

3.77 $\int \frac{\cos^6(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=77

$$\frac{5x}{2a^3}+\frac{5\cos^3(c+dx)}{3a^3d}+\frac{5\cos(c+dx)\sin(c+dx)}{2a^3d}+\frac{2\cos^5(c+dx)}{ad(a+a\sin(c+dx))^2}$$

```
[Out] 5/2*x/a^3+5/3*cos(d*x+c)^3/a^3/d+5/2*cos(d*x+c)*sin(d*x+c)/a^3/d+2*cos(d*x+
c)^5/a/d/(a+a*sin(d*x+c))^2
```

Rubi [A]
time = 0.07, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2759, 2761, 2715, 8}

$$\frac{5\cos^3(c+dx)}{3a^3d}+\frac{5\sin(c+dx)\cos(c+dx)}{2a^3d}+\frac{5x}{2a^3}+\frac{2\cos^5(c+dx)}{ad(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (5*x)/(2*a^3) + (5*Cos[c + d*x]^3)/(3*a^3*d) + (5*Cos[c + d*x]*Sin[c + d*x]
)/(2*a^3*d) + (2*Cos[c + d*x]^5)/(a*d*(a + a*Sin[c + d*x])^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^6(c+dx)}{(a+a\sin(c+dx))^3}\,dx &= \frac{2\cos^5(c+dx)}{ad(a+a\sin(c+dx))^2}+\frac{5\int \frac{\cos^4(c+dx)}{a+a\sin(c+dx)}\,dx}{a^2}\\ &= \frac{5\cos^3(c+dx)}{3a^3d}+\frac{2\cos^5(c+dx)}{ad(a+a\sin(c+dx))^2}+\frac{5\int \cos^2(c+dx)\,dx}{a^3}\\ &= \frac{5\cos^3(c+dx)}{3a^3d}+\frac{5\cos(c+dx)\sin(c+dx)}{2a^3d}+\frac{2\cos^5(c+dx)}{ad(a+a\sin(c+dx))^2}+\frac{5\int 1\,dx}{2a^3}\\ &= \frac{5x}{2a^3}+\frac{5\cos^3(c+dx)}{3a^3d}+\frac{5\cos(c+dx)\sin(c+dx)}{2a^3d}+\frac{2\cos^5(c+dx)}{ad(a+a\sin(c+dx))^2}\end{aligned}$$

Mathematica [A]
time = 0.31, size = 121, normalized size = 1.57

$$-\frac{\cos^7(c+dx)\left(30\sin^{-1}\left(\frac{\sqrt{1-\sin(c+dx)}}{\sqrt{2}}\right)\sqrt{1-\sin(c+dx)}+\sqrt{1+\sin(c+dx)}\,(-22+31\sin(c+dx)-11\sin^2(c+dx)+2\sin^3(c+dx))\right)}{6a^3d(-1+\sin(c+dx))^4(1+\sin(c+dx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/6*(Cos[c + d*x]^7*(30*ArcSin[Sqrt[1 - Sin[c + d*x]]/Sqrt[2]]*Sqrt[1 - Si
n[c + d*x]] + Sqrt[1 + Sin[c + d*x]]*(-22 + 31*Sin[c + d*x] - 11*Sin[c + d*
x]^2 + 2*Sin[c + d*x]^3)))/(a^3*d*(-1 + Sin[c + d*x])^4*(1 + Sin[c + d*x])^
(7/2))
```

Maple [A]
time = 0.28, size = 90, normalized size = 1.17

method	result
risch	$\frac{5x}{2a^3}+\frac{15\cos(dx+c)}{4a^3d}-\frac{\cos(3dx+3c)}{12a^3d}-\frac{3\sin(2dx+2c)}{4a^3d}$
derivativedivides	$\frac{\frac{2\left(\frac{3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+8\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}+\frac{11}{3}\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+5\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^3}$

default	$\frac{2\left(\frac{\frac{3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+8\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}+\frac{11}{3}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+5\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{d\,a^3}$
norman	$\frac{3055\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3da}+\frac{640x\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{100x\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{673\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3da}+\frac{40x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{1969\left(\tan^5\left(\frac{dx}{2}+\right.\right.}{3da}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^6/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)

[Out] 2/d/a^3*((3/2*tan(1/2*d*x+1/2*c)^5+3*tan(1/2*d*x+1/2*c)^4+8*tan(1/2*d*x+1/2*c)^2-3/2*tan(1/2*d*x+1/2*c)+11/3)/(1+tan(1/2*d*x+1/2*c)^2)^3+5/2*arctan(tan(1/2*d*x+1/2*c)))

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(71) = 142$.
time = 0.51, size = 184, normalized size = 2.39

$$-\frac{\frac{\frac{9\sin(dx+c)}{\cos(dx+c)+1}-\frac{48\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{18\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{9\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-22}{a^3+\frac{3a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}-\frac{15\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^6/(a+a*sin(d*x+c))^3,x, algorithm="maxima")

[Out] -1/3*((9*sin(d*x + c)/(cos(d*x + c) + 1) - 48*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 18*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 9*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 22)/(a^3 + 3*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) - 15*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3)/d

Fricas [A]
time = 0.35, size = 45, normalized size = 0.58

$$-\frac{2\cos(dx+c)^3-15\,dx+9\cos(dx+c)\sin(dx+c)-24\cos(dx+c)}{6\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^6/(a+a*sin(d*x+c))^3,x, algorithm="fricas")

[Out] -1/6*(2*cos(d*x + c)^3 - 15*d*x + 9*cos(d*x + c)*sin(d*x + c) - 24*cos(d*x + c))/(a^3*d)

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 690 vs. $2(71) = 142$.
time = 48.30, size = 690, normalized size = 8.96

$$\begin{cases} \frac{15dx\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)}{D}+\frac{45dx\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{D}+\frac{45dx\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{D}+\frac{15dx}{D}+\frac{18\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{D}+\frac{36\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{D}+\frac{96\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{D}-\frac{18\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{D}+\frac{44}{D} & \text{for}\ d\neq 0 \\ \frac{x\cos^6(c)}{(a\sin(c)+a)^3} & \text{otherwise} \end{cases}$$

where each denominator D is printed in full as $6a^3d\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)+18a^3d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+18a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^3d$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((15*d*x*tan(c/2 + d*x/2)**6/(6*a**3*d*tan(c/2 + d*x/2)**6 + 18*a*
*3*d*tan(c/2 + d*x/2)**4 + 18*a**3*d*tan(c/2 + d*x/2)**2 + 6*a**3*d) + 45*d
*x*tan(c/2 + d*x/2)**4/(6*a**3*d*tan(c/2 + d*x/2)**6 + 18*a**3*d*tan(c/2 +
d*x/2)**4 + 18*a**3*d*tan(c/2 + d*x/2)**2 + 6*a**3*d) + 45*d*x*tan(c/2 + d*
x/2)**2/(6*a**3*d*tan(c/2 + d*x/2)**6 + 18*a**3*d*tan(c/2 + d*x/2)**4 + 18*
a**3*d*tan(c/2 + d*x/2)**2 + 6*a**3*d) + 15*d*x/(6*a**3*d*tan(c/2 + d*x/2)*
*6 + 18*a**3*d*tan(c/2 + d*x/2)**4 + 18*a**3*d*tan(c/2 + d*x/2)**2 + 6*a**3
*d) + 18*tan(c/2 + d*x/2)**5/(6*a**3*d*tan(c/2 + d*x/2)**6 + 18*a**3*d*tan(
c/2 + d*x/2)**4 + 18*a**3*d*tan(c/2 + d*x/2)**2 + 6*a**3*d) + 36*tan(c/2 +
d*x/2)**4/(6*a**3*d*tan(c/2 + d*x/2)**6 + 18*a**3*d*tan(c/2 + d*x/2)**4 + 1
8*a**3*d*tan(c/2 + d*x/2)**2 + 6*a**3*d) + 96*tan(c/2 + d*x/2)**2/(6*a**3*d
*tan(c/2 + d*x/2)**6 + 18*a**3*d*tan(c/2 + d*x/2)**4 + 18*a**3*d*tan(c/2 +
d*x/2)**2 + 6*a**3*d) - 18*tan(c/2 + d*x/2)/(6*a**3*d*tan(c/2 + d*x/2)**6 +
 18*a**3*d*tan(c/2 + d*x/2)**4 + 18*a**3*d*tan(c/2 + d*x/2)**2 + 6*a**3*d)
+ 44/(6*a**3*d*tan(c/2 + d*x/2)**6 + 18*a**3*d*tan(c/2 + d*x/2)**4 + 18*a**
3*d*tan(c/2 + d*x/2)**2 + 6*a**3*d), Ne(d, 0)), (x*cos(c)**6/(a*sin(c) + a)
**3, True))
```

Giac [A]
time = 5.83, size = 88, normalized size = 1.14

$$\frac{\frac{15\,(dx+c)}{a^3}+\frac{2\left(9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+18\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+48\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+22\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3a^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/6*(15*(d*x + c)/a^3 + 2*(9*tan(1/2*d*x + 1/2*c)^5 + 18*tan(1/2*d*x + 1/2*
c)^4 + 48*tan(1/2*d*x + 1/2*c)^2 - 9*tan(1/2*d*x + 1/2*c) + 22)/((tan(1/2*d
*x + 1/2*c)^2 + 1)^3*a^3))/d
```

Mupad [B]
time = 4.64, size = 57, normalized size = 0.74

$$\frac{5\,x}{2\,a^3}+\frac{4\,\cos\left(c+d\,x\right)}{a^3\,d}-\frac{\cos\left(c+d\,x\right)^3}{3\,a^3\,d}-\frac{3\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{2\,a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(a + a*sin(c + d*x))^3,x)
```

```
[Out] (5*x)/(2*a^3) + (4*cos(c + d*x))/(a^3*d) - cos(c + d*x)^3/(3*a^3*d) - (3*co
s(c + d*x)*sin(c + d*x))/(2*a^3*d)
```

3.78 $\int \frac{\cos^5(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=50

$$\frac{4\log(1+\sin(c+dx))}{a^3d}-\frac{3\sin(c+dx)}{a^3d}+\frac{\sin^2(c+dx)}{2a^3d}$$

```
[Out] 4*ln(1+sin(d*x+c))/a^3/d-3*sin(d*x+c)/a^3/d+1/2*sin(d*x+c)^2/a^3/d
```

Rubi [A]
time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2746, 45}

$$\frac{\sin^2(c+dx)}{2a^3d}-\frac{3\sin(c+dx)}{a^3d}+\frac{4\log(\sin(c+dx)+1)}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (4*Log[1 + Sin[c + d*x]])/(a^3*d) - (3*Sin[c + d*x])/(a^3*d) + Sin[c + d*x]
^2/(2*a^3*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\int \frac{\cos^5(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^2}{a+x}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \left(-3a+x+\frac{4a^2}{a+x}\right)\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{4\log(1+\sin(c+dx))}{a^3 d} - \frac{3\sin(c+dx)}{a^3 d} + \frac{\sin^2(c+dx)}{2a^3 d}$$

Mathematica [A]
time = 0.03, size = 38, normalized size = 0.76

$$\frac{8\log(1+\sin(c+dx)) - 6\sin(c+dx) + \sin^2(c+dx)}{2a^3 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/(a + a*Sin[c + d*x])^3,x]
[Out] (8*Log[1 + Sin[c + d*x]] - 6*Sin[c + d*x] + Sin[c + d*x]^2)/(2*a^3*d)
```

Maple [A]
time = 0.26, size = 38, normalized size = 0.76

method	result
derivativedivides	$\frac{\frac{\left(\sin^2(dx+c)\right)}{2}-3\sin(dx+c)+4\ln(1+\sin(dx+c))}{d\,a^3}$
default	$\frac{\frac{\left(\sin^2(dx+c)\right)}{2}-3\sin(dx+c)+4\ln(1+\sin(dx+c))}{d\,a^3}$
risch	$-\frac{4ix}{a^3}+\frac{3i\mathrm{e}^{i(dx+c)}}{2a^3d}-\frac{3i\mathrm{e}^{-i(dx+c)}}{2a^3d}-\frac{8ic}{a^3d}+\frac{8\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{a^3d}-\frac{\cos(2dx+2c)}{4a^3d}$
norman	$-\frac{6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{da}-\frac{6\left(\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}-\frac{28\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}-\frac{28\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}-\frac{74\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}-\frac{74\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}-154$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
[Out] 1/d/a^3*(1/2*sin(d*x+c)^2-3*sin(d*x+c)+4*ln(1+sin(d*x+c)))
```

Maxima [A]
time = 0.28, size = 41, normalized size = 0.82

$$\frac{\frac{\sin(dx+c)^2-6\,\sin(dx+c)}{a^3}+\frac{8\,\log(\sin(dx+c)+1)}{a^3}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="maxima")

[Out] 1/2*((sin(d*x + c)^2 - 6*sin(d*x + c))/a^3 + 8*log(sin(d*x + c) + 1)/a^3)/d

Fricas [A]
time = 0.36, size = 36, normalized size = 0.72

$$-\frac{\cos\left(dx+c\right)^{2}-8\,\log\left(\sin\left(dx+c\right)+1\right)+6\,\sin\left(dx+c\right)}{2\,a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="fricas")

[Out] -1/2*(cos(d*x + c)^2 - 8*log(sin(d*x + c) + 1) + 6*sin(d*x + c))/(a^3*d)

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 564 vs. $2(44) = 88$.
time = 27.73, size = 564, normalized size = 11.28

$$\begin{cases} \frac{8\log\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)}{a^{3}d\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)+2a^{3}d\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+a^{3}d}+\frac{16\log\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)}{a^{3}d\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)+2a^{3}d\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+a^{3}d}+\frac{8\log\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)}{a^{3}d\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)+2a^{3}d\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+a^{3}d}-\frac{4\log\left(\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)}{a^{3}d\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)+2a^{3}d\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+a^{3}d}-\frac{8\log\left(\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)}{a^{3}d\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)+2a^{3}d\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+a^{3}d}-\frac{4\log\left(\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)}{a^{3}d\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)+2a^{3}d\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+a^{3}d}-\frac{6\tan^{3}\left(\frac{c}{2}+\frac{dx}{2}\right)}{a^{3}d\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)+2a^{3}d\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+a^{3}d}+\frac{2\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)}{a^{3}d\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)+2a^{3}d\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+a^{3}d}-\frac{6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{a^{3}d\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)+2a^{3}d\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+a^{3}d} & \text{for}\: d \neq 0 \\ \frac{x\cos^{5}\left(c\right)}{\left(a\sin\left(c\right)+a\right)^{3}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**5/(a+a*sin(d*x+c))**3,x)

[Out] Piecewise((8*log(tan(c/2 + d*x/2) + 1)*tan(c/2 + d*x/2)**4/(a**3*d*tan(c/2 + d*x/2)**4 + 2*a**3*d*tan(c/2 + d*x/2)**2 + a**3*d) + 16*log(tan(c/2 + d*x/2) + 1)*tan(c/2 + d*x/2)**2/(a**3*d*tan(c/2 + d*x/2)**4 + 2*a**3*d*tan(c/2 + d*x/2)**2 + a**3*d) + 8*log(tan(c/2 + d*x/2) + 1)/(a**3*d*tan(c/2 + d*x/2)**4 + 2*a**3*d*tan(c/2 + d*x/2)**2 + a**3*d) - 4*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2)**4/(a**3*d*tan(c/2 + d*x/2)**4 + 2*a**3*d*tan(c/2 + d*x/2)**2 + a**3*d) - 8*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2)**2/(a**3*d*tan(c/2 + d*x/2)**4 + 2*a**3*d*tan(c/2 + d*x/2)**2 + a**3*d) - 4*log(tan(c/2 + d*x/2)**2 + 1)/(a**3*d*tan(c/2 + d*x/2)**4 + 2*a**3*d*tan(c/2 + d*x/2)**2 + a**3*d) - 6*tan(c/2 + d*x/2)**3/(a**3*d*tan(c/2 + d*x/2)**4 + 2*a**3*d*tan(c/2 + d*x/2)**2 + a**3*d) + 2*tan(c/2 + d*x/2)**2/(a**3*d*tan(c/2 + d*x/2)**4 + 2*a**3*d*tan(c/2 + d*x/2)**2 + a**3*d) - 6*tan(c/2 + d*x/2)/(a**3*d*tan(c/2 + d*x/2)**4 + 2*a**3*d*tan(c/2 + d*x/2)**2 + a**3*d), Ne(d, 0)), (x*cos(c)**5/(a*sin(c) + a)**3, True))

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 115 vs. $2(48) = 96$.
time = 7.70, size = 115, normalized size = 2.30

$$-\frac{2\left(\frac{2\,\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)}{a^{3}}-\frac{4\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^{3}}-\frac{3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+7\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{2}a^{3}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -2*(2*log(tan(1/2*d*x + 1/2*c)^2 + 1)/a^3 - 4*log(abs(tan(1/2*d*x + 1/2*c)
+ 1))/a^3 - (3*tan(1/2*d*x + 1/2*c)^4 - 3*tan(1/2*d*x + 1/2*c)^3 + 7*tan(1/
2*d*x + 1/2*c)^2 - 3*tan(1/2*d*x + 1/2*c) + 3)/((tan(1/2*d*x + 1/2*c)^2 + 1
)^2*a^3))/d
```

Mupad [B]
time = 4.56, size = 36, normalized size = 0.72

$$\frac{8\ln\left(\sin\left(c+d\,x\right)+1\right)-6\,\sin\left(c+d\,x\right)+\sin\left(c+d\,x\right)^2}{2\,a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(a + a*sin(c + d*x))^3,x)
```

```
[Out] (8*log(sin(c + d*x) + 1) - 6*sin(c + d*x) + sin(c + d*x)^2)/(2*a^3*d)
```

3.79 $\int \frac{\cos^4(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=49

$$-\frac{3x}{a^3}-\frac{3\cos(c+dx)}{a^3d}-\frac{2\cos^3(c+dx)}{ad(a+a\sin(c+dx))^2}$$

```
[Out] -3*x/a^3-3*cos(d*x+c)/a^3/d-2*cos(d*x+c)^3/a/d/(a+a*sin(d*x+c))^2
```

Rubi [A]
time = 0.06, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2759, 2761, 8}

$$-\frac{3\cos(c+dx)}{a^3d}-\frac{3x}{a^3}-\frac{2\cos^3(c+dx)}{ad(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-3*x)/a^3 - (3*Cos[c + d*x])/(a^3*d) - (2*Cos[c + d*x]^3)/(a*d*(a + a*Sin[
c + d*x])^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{(a+a\sin(c+dx))^3}\,dx = -\frac{2\cos^3(c+dx)}{ad(a+a\sin(c+dx))^2} - \frac{3\int \frac{\cos^2(c+dx)}{a+a\sin(c+dx)}\,dx}{a^2}$$
$$= -\frac{3\cos(c+dx)}{a^3d} - \frac{2\cos^3(c+dx)}{ad(a+a\sin(c+dx))^2} - \frac{3\int 1\,dx}{a^3}$$
$$= -\frac{3x}{a^3} - \frac{3\cos(c+dx)}{a^3d} - \frac{2\cos^3(c+dx)}{ad(a+a\sin(c+dx))^2}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.03, size = 59, normalized size = 1.20

$$-\frac{\cos^5(c+dx)\,{}_2F_1\left(\frac{3}{2},\frac{5}{2};\frac{7}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{5\sqrt{2}\,a^3d(1+\sin(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/5*(Cos[c + d*x]^5*Hypergeometric2F1[3/2, 5/2, 7/2, (1 - Sin[c + d*x])/2]
)/(Sqrt[2]*a^3*d*(1 + Sin[c + d*x])^(5/2))
```

Maple [A]
time = 0.25, size = 54, normalized size = 1.10

method	result
derivativedivides	$\frac{-\frac{8}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-\frac{2}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}-6\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^3}$
default	$\frac{-\frac{8}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-\frac{2}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}-6\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^3}$
risch	$-\frac{3x}{a^3}-\frac{e^{i(dx+c)}}{2a^3d}-\frac{e^{-i(dx+c)}}{2a^3d}-\frac{8}{d\,a^3\left(e^{i(dx+c)}+i\right)}$
norman	$\frac{-\frac{186\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}-\frac{252x\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{90x\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{210\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}-\frac{42x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{412\left(\tan^5\left(\frac{dx}{2}+\right.\right.}{da}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/a^3*(-4/(tan(1/2*d*x+1/2*c)+1)-1/(1+tan(1/2*d*x+1/2*c)^2)-3*arctan(tan(
1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 139 vs. $2(49) = 98$.
time = 0.51, size = 139, normalized size = 2.84

$$-\frac{2\left(\frac{\frac{\sin(dx+c)}{\cos(dx+c)+1}+\frac{4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+5}{a^3+\frac{a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}+\frac{3\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -2*((sin(d*x + c)/(cos(d*x + c) + 1) + 4*sin(d*x + c)^2/(cos(d*x + c) + 1)^
2 + 5)/(a^3 + a^3*sin(d*x + c)/(cos(d*x + c) + 1) + a^3*sin(d*x + c)^2/(cos
(d*x + c) + 1)^2 + a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3) + 3*arctan(sin(
d*x + c)/(cos(d*x + c) + 1))/a^3)/d
```

Fricas [A]
time = 0.33, size = 78, normalized size = 1.59

$$-\frac{3\,dx+(3\,dx+5)\cos(dx+c)+\cos(dx+c)^2+(3\,dx+\cos(dx+c)-4)\sin(dx+c)+4}{a^3d\cos(dx+c)+a^3d\sin(dx+c)+a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -(3*d*x + (3*d*x + 5)*cos(d*x + c) + cos(d*x + c)^2 + (3*d*x + cos(d*x + c)
 - 4)*sin(d*x + c) + 4)/(a^3*d*cos(d*x + c) + a^3*d*sin(d*x + c) + a^3*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 478 vs. $2(46) = 92$.
time = 16.45, size = 478, normalized size = 9.76

$$\begin{cases} -\frac{3dx\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{a^3d\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)+a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+a^3d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a^3d}-\frac{3dx\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{a^3d\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)+a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+a^3d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a^3d}-\frac{3dx\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{a^3d\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)+a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+a^3d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a^3d}-\frac{3dx}{a^3d\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)+a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+a^3d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a^3d}-\frac{8\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{a^3d\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)+a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+a^3d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a^3d}-\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{a^3d\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)+a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+a^3d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a^3d}-\frac{10}{a^3d\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)+a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+a^3d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a^3d} & \text{for } d\neq 0 \\ \frac{x\cos^4(c)}{(a\sin(c)+a)^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((-3*d*x*tan(c/2 + d*x/2)**3/(a**3*d*tan(c/2 + d*x/2)**3 + a**3*d*
tan(c/2 + d*x/2)**2 + a**3*d*tan(c/2 + d*x/2) + a**3*d) - 3*d*x*tan(c/2 + d
*x/2)**2/(a**3*d*tan(c/2 + d*x/2)**3 + a**3*d*tan(c/2 + d*x/2)**2 + a**3*d*
tan(c/2 + d*x/2) + a**3*d) - 3*d*x*tan(c/2 + d*x/2)/(a**3*d*tan(c/2 + d*x/2
)**3 + a**3*d*tan(c/2 + d*x/2)**2 + a**3*d*tan(c/2 + d*x/2) + a**3*d) - 3*d
*x/(a**3*d*tan(c/2 + d*x/2)**3 + a**3*d*tan(c/2 + d*x/2)**2 + a**3*d*tan(c/
2 + d*x/2) + a**3*d) - 8*tan(c/2 + d*x/2)**2/(a**3*d*tan(c/2 + d*x/2)**3 +
a**3*d*tan(c/2 + d*x/2)**2 + a**3*d*tan(c/2 + d*x/2) + a**3*d) - 2*tan(c/2
```

```
+ d*x/2)/(a**3*d*tan(c/2 + d*x/2)**3 + a**3*d*tan(c/2 + d*x/2)**2 + a**3*d*
tan(c/2 + d*x/2) + a**3*d) - 10/(a**3*d*tan(c/2 + d*x/2)**3 + a**3*d*tan(c/
2 + d*x/2)**2 + a**3*d*tan(c/2 + d*x/2) + a**3*d), Ne(d, 0)), (x*cos(c)**4/
(a*sin(c) + a)**3, True))
```

Giac [A]
time = 6.45, size = 80, normalized size = 1.63

$$-\frac{\frac{3\,(dx+c)}{a^3}+\frac{2\left(4\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+5\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)a^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -(3*(d*x + c)/a^3 + 2*(4*tan(1/2*d*x + 1/2*c)^2 + tan(1/2*d*x + 1/2*c) + 5)
/((tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c)^2 + tan(1/2*d*x + 1/2*c) +
 1)*a^3))/d
```

Mupad [B]
time = 4.85, size = 69, normalized size = 1.41

$$-\frac{3\,x}{a^3}-\frac{8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+10}{a^3\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(a + a*sin(c + d*x))^3,x)
```

```
[Out] - (3*x)/a^3 - (2*tan(c/2 + (d*x)/2) + 8*tan(c/2 + (d*x)/2)^2 + 10)/(a^3*d*(
tan(c/2 + (d*x)/2) + 1)*(tan(c/2 + (d*x)/2)^2 + 1))
```

3.80 $\int \frac{\cos^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=39

$$-\frac{\log(1+\sin(c+dx))}{a^3d}-\frac{2}{d\,(a^3+a^3\sin(c+dx))}$$

```
[Out] -ln(1+sin(d*x+c))/a^3/d-2/d/(a^3+a^3*sin(d*x+c))
```

Rubi [A]
time = 0.03, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2746, 45}

$$-\frac{2}{d\,(a^3\sin(c+dx)+a^3)}-\frac{\log(\sin(c+dx)+1)}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + a*Sin[c + d*x])^3,x]
[Out] -(Log[1 + Sin[c + d*x]]/(a^3*d)) - 2/(d*(a^3 + a^3*Sin[c + d*x]))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{a-x}{(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{a^3d}\\
&= \frac{\operatorname{Subst}\left(\int \left(\frac{1}{-a-x}+\frac{2a}{(a+x)^2}\right)\,dx, x, a\sin(c+dx)\right)}{a^3d}\\
&= -\frac{\log(1+\sin(c+dx))}{a^3d}-\frac{2}{d\,(a^3+a^3\sin(c+dx))}
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 58, normalized size = 1.49

$$-\frac{2+\log(1+\sin(c+dx))+\log(1+\sin(c+dx))\sin(c+dx)}{a^3d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -((2 + Log[1 + Sin[c + d*x]] + Log[1 + Sin[c + d*x]]*Sin[c + d*x])/(a^3*d*(
Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2))
```

Maple [A]
time = 0.21, size = 32, normalized size = 0.82

method	result
derivativedivides	$\frac{-\frac{2}{1+\sin(dx+c)}-\ln(1+\sin(dx+c))}{d\,a^3}$
default	$\frac{-\frac{2}{1+\sin(dx+c)}-\ln(1+\sin(dx+c))}{d\,a^3}$
risch	$\frac{ix}{a^3}+\frac{2ic}{a^3d}-\frac{4i\mathrm{e}^{i(dx+c)}}{d\,a^3\left(\mathrm{e}^{i(dx+c)}+i\right)^2}-\frac{2\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{a^3d}$
norman	$\frac{\frac{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{da}+\frac{4\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}+\frac{12\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}+\frac{12\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}+\frac{24\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}+\frac{24\left(\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}+\frac{40(\tan}{}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3a^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^3*(-2/(1+sin(d*x+c))-ln(1+sin(d*x+c)))
```

Maxima [A]
time = 0.29, size = 37, normalized size = 0.95

$$-\frac{\frac{2}{a^3\sin(dx+c)+a^3}+\frac{\log(\sin(dx+c)+1)}{a^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -(2/(a^3*sin(d*x + c) + a^3) + log(sin(d*x + c) + 1)/a^3)/d
```

Fricas [A]
time = 0.36, size = 41, normalized size = 1.05

$$-\frac{(\sin(dx+c)+1)\log(\sin(dx+c)+1)+2}{a^3d\sin(dx+c)+a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -((sin(d*x + c) + 1)*log(sin(d*x + c) + 1) + 2)/(a^3*d*sin(d*x + c) + a^3*d
)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 299 vs. $2(32) = 64$.
time = 0.61, size = 299, normalized size = 7.67

$$\begin{cases} -\frac{2\log(\sin(c+dx)+1)\sin^2(c+dx)}{2a^3d\sin^2(c+dx)+4a^3d\sin(c+dx)+2a^3d} - \frac{4\log(\sin(c+dx)+1)\sin(c+dx)}{2a^3d\sin^2(c+dx)+4a^3d\sin(c+dx)+2a^3d} - \frac{2\log(\sin(c+dx)+1)}{2a^3d\sin^2(c+dx)+4a^3d\sin(c+dx)+2a^3d} - \frac{2\sin(c+dx)}{2a^3d\sin^2(c+dx)+4a^3d\sin(c+dx)+2a^3d} - \frac{\cos^2(c+dx)}{2a^3d\sin^2(c+dx)+4a^3d\sin(c+dx)+2a^3d} - \frac{2}{2a^3d\sin^2(c+dx)+4a^3d\sin(c+dx)+2a^3d} & \text{for } d \neq 0 \\ \frac{x\cos^3(c)}{(a\sin(c)+a)^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((-2*log(sin(c + d*x) + 1)*sin(c + d*x)**2/(2*a**3*d*sin(c + d*x)*
*2 + 4*a**3*d*sin(c + d*x) + 2*a**3*d) - 4*log(sin(c + d*x) + 1)*sin(c + d*
x)/(2*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x) + 2*a**3*d) - 2*log(si
n(c + d*x) + 1)/(2*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x) + 2*a**3*
d) - 2*sin(c + d*x)/(2*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x) + 2*a
**3*d) - cos(c + d*x)**2/(2*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x)
+ 2*a**3*d) - 2/(2*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x) + 2*a**3*
d), Ne(d, 0)), (x*cos(c)**3/(a*sin(c) + a)**3, True))
```

Giac [A]
time = 6.26, size = 35, normalized size = 0.90

$$-\frac{\frac{\log(|\sin(dx+c)+1|)}{a^3} + \frac{2}{a^3(\sin(dx+c)+1)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -(log(abs(sin(d*x + c) + 1))/a^3 + 2/(a^3*(sin(d*x + c) + 1)))/d
```

Mupad [B]
time = 4.54, size = 36, normalized size = 0.92

$$-\frac{2}{a^3\,d\,\left(\sin\left(c+d\,x\right)+1\right)}-\frac{\ln\left(\sin\left(c+d\,x\right)+1\right)}{a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a + a*sin(c + d*x))^3,x)
```

```
[Out] - 2/(a^3*d*(sin(c + d*x) + 1)) - log(sin(c + d*x) + 1)/(a^3*d)
```

3.81 $\int \frac{\cos^2(c+dx)}{(a+a\sin(c+dx))^3} dx$

Optimal. Leaf size=27

$$-\frac{\cos^3(c+dx)}{3d(a+a\sin(c+dx))^3}$$

```
[Out] -1/3*cos(d*x+c)^3/d/(a+a*sin(d*x+c))^3
```

Rubi [A]
time = 0.03, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {2750}

$$-\frac{\cos^3(c+dx)}{3d(a\sin(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/3*Cos[c + d*x]^3/(d*(a + a*Sin[c + d*x])^3)
```

Rule 2750

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a+a\sin(c+dx))^3} dx = -\frac{\cos^3(c+dx)}{3d(a+a\sin(c+dx))^3}$$

Mathematica [A]
time = 0.01, size = 28, normalized size = 1.04

$$-\frac{\cos^3(c+dx)}{3a^3d(1+\sin(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/3*Cos[c + d*x]^3/(a^3*d*(1 + Sin[c + d*x])^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(25) = 50$.
time = 0.21, size = 55, normalized size = 2.04

method	result
risch	$\frac{2\,e^{2i(dx+c)}-\frac{2}{3}}{d\,a^{3}\left(e^{i(dx+c)}+i\right)^{3}}$
derivativedivides	$\frac{-\frac{8}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}-\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+\frac{4}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}}{d\,a^{3}}$
default	$\frac{-\frac{8}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}-\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+\frac{4}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}}{d\,a^{3}}$
norman	$\frac{-\frac{2\left(\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}-\frac{2}{3ad}-\frac{4\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}-\frac{20\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3da}-\frac{20\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3da}-\frac{4\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}-\frac{8\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}-4}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}a^{2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{5}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/a^3*(-4/3/(tan(1/2*d*x+1/2*c)+1)^3-1/(tan(1/2*d*x+1/2*c)+1)+2/(tan(1/2*
d*x+1/2*c)+1)^2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(25) = 50$.
time = 0.28, size = 99, normalized size = 3.67

$$-\frac{2\left(\frac{3\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+1\right)}{3\left(a^{3}+\frac{3a^{3}\sin(dx+c)}{\cos(dx+c)+1}+\frac{3a^{3}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{a^{3}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -2/3*(3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)/((a^3 + 3*a^3*sin(d*x + c)
/(cos(d*x + c) + 1) + 3*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^3*sin(d
*x + c)^3/(cos(d*x + c) + 1)^3)*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(25) = 50$.
time = 0.35, size = 95, normalized size = 3.52

$$-\frac{\cos\left(dx+c\right)^{2}+\left(\cos\left(dx+c\right)+2\right)\sin\left(dx+c\right)-\cos\left(dx+c\right)-2}{3\left(a^{3}d\cos\left(dx+c\right)^{2}-a^{3}d\cos\left(dx+c\right)-2\,a^{3}d-\left(a^{3}d\cos\left(dx+c\right)+2\,a^{3}d\right)\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/3*(cos(d*x + c)^2 + (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2)/
(a^3*d*cos(d*x + c)^2 - a^3*d*cos(d*x + c) - 2*a^3*d - (a^3*d*cos(d*x + c)
+ 2*a^3*d)*sin(d*x + c))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(24) = 48$.
time = 5.09, size = 153, normalized size = 5.67

$$\begin{cases} -\frac{6\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{3a^3d\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^3d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+3a^3d} - \frac{2}{3a^3d\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^3d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+3a^3d} & \text{for } d \neq 0 \\ \frac{x\cos^2(c)}{(a\sin(c)+a)^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((-6*tan(c/2 + d*x/2)**2/(3*a**3*d*tan(c/2 + d*x/2)**3 + 9*a**3*d*
tan(c/2 + d*x/2)**2 + 9*a**3*d*tan(c/2 + d*x/2) + 3*a**3*d) - 2/(3*a**3*d*t
an(c/2 + d*x/2)**3 + 9*a**3*d*tan(c/2 + d*x/2)**2 + 9*a**3*d*tan(c/2 + d*x/
2) + 3*a**3*d), Ne(d, 0)), (x*cos(c)**2/(a*sin(c) + a)**3, True))
```

Giac [A]
time = 5.90, size = 36, normalized size = 1.33

$$-\frac{2\left(3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)}{3a^3d\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -2/3*(3*tan(1/2*d*x + 1/2*c)^2 + 1)/(a^3*d*(tan(1/2*d*x + 1/2*c) + 1)^3)
```

Mupad [B]
time = 4.58, size = 53, normalized size = 1.96

$$\frac{2\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\left(2\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2-3\right)}{3a^3d\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a + a*sin(c + d*x))^3,x)
```

```
[Out] (2*cos(c/2 + (d*x)/2)*(2*cos(c/2 + (d*x)/2)^2 - 3))/(3*a^3*d*(cos(c/2 + (d*
x)/2) + sin(c/2 + (d*x)/2))^3)
```

3.82 $\int \frac{\cos(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=22

$$-\frac{1}{2ad(a+a\sin(c+dx))^2}$$

[Out] -1/2/a/d/(a+a*sin(d*x+c))^2

Rubi [A]
time = 0.02, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2746, 32}

$$-\frac{1}{2ad(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]/(a + a*Sin[c + d*x])^3,x]

[Out] -1/2*1/(a*d*(a + a*Sin[c + d*x])^2)

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] || !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)}{(a+a\sin(c+dx))^3}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{(a+x)^3}\,dx, x, a\sin(c+dx)\right)}{ad}\\ &= -\frac{1}{2ad(a+a\sin(c+dx))^2}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 33, normalized size = 1.50

$$-\frac{1}{2a^3d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^4}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]/(a + a*Sin[c + d*x])^3,x]

[Out] -1/2*1/(a^3*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4)

Maple [A]
time = 0.11, size = 21, normalized size = 0.95

method	result	size
derivativedivides	$-\frac{1}{2ad(a+a\sin(dx+c))^{2}}$	21
default	$-\frac{1}{2ad(a+a\sin(dx+c))^{2}}$	21
risch	$\frac{2\,\mathrm{e}^{2i(dx+c)}}{d\,a^{3}\left(\mathrm{e}^{i(dx+c)}+i\right)^{4}}$	32
norman	$\frac{\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{da}+\frac{2\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}+\frac{4\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}+\frac{4\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}+\frac{6\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}+\frac{6\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^{2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{5}}$	146

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)

[Out] -1/2/a/d/(a+a*sin(d*x+c))^2

Maxima [A]
time = 0.29, size = 20, normalized size = 0.91

$$-\frac{1}{2\left(a\sin\left(dx+c\right)+a\right)^{2}ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="maxima")

[Out] -1/2/((a*sin(d*x + c) + a)^2*a*d)

Fricas [A]
time = 0.34, size = 36, normalized size = 1.64

$$\frac{1}{2\left(a^{3}d\cos\left(dx+c\right)^{2}-2\,a^{3}d\sin\left(dx+c\right)-2\,a^{3}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="fricas")

[Out] 1/2/(a^3*d*cos(d*x + c)^2 - 2*a^3*d*sin(d*x + c) - 2*a^3*d)

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(19) = 38$.
time = 0.58, size = 51, normalized size = 2.32

$$\begin{cases} -\frac{1}{2a^3d\sin^2(c+dx)+4a^3d\sin(c+dx)+2a^3d} & \text{for } d \neq 0 \\ \frac{x\cos(c)}{(a\sin(c)+a)^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((-1/(2*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x) + 2*a**3*d)
, Ne(d, 0)), (x*cos(c)/(a*sin(c) + a)**3, True))
```

Giac [A]
time = 6.34, size = 20, normalized size = 0.91

$$-\frac{1}{2(a\sin(dx+c)+a)^2ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2/((a*sin(d*x + c) + a)^2*a*d)
```

Mupad [B]
time = 4.46, size = 18, normalized size = 0.82

$$-\frac{1}{2a^3d(\sin(c+dx)+1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + a*sin(c + d*x))^3,x)
```

```
[Out] -1/(2*a^3*d*(sin(c + d*x) + 1)^2)
```

3.83 $\int \frac{\sec(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=82

$$\frac{\tanh^{-1}(\sin(c+dx))}{8a^3d}-\frac{1}{6d(a+a\sin(c+dx))^3}-\frac{1}{8ad(a+a\sin(c+dx))^2}-\frac{1}{8d\left(a^3+a^3\sin(c+dx)\right)}$$

```
[Out] 1/8*arctanh(sin(d*x+c))/a^3/d-1/6/d/(a+a*sin(d*x+c))^3-1/8/a/d/(a+a*sin(d*x
+c))^2-1/8/d/(a^3+a^3*sin(d*x+c))
```

Rubi [A]
time = 0.04, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2746, 46, 212}

$$-\frac{1}{8d\left(a^3\sin(c+dx)+a^3\right)}+\frac{\tanh^{-1}(\sin(c+dx))}{8a^3d}-\frac{1}{8ad(a\sin(c+dx)+a)^2}-\frac{1}{6d(a\sin(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/(8*a^3*d) - 1/(6*d*(a + a*Sin[c + d*x])^3) - 1/(8*a*d
*(a + a*Sin[c + d*x])^2) - 1/(8*d*(a^3 + a^3*Sin[c + d*x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{a\text{Subst}\left(\int \frac{1}{(a-x)(a+x)^4}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a\text{Subst}\left(\int \left(\frac{1}{2a(a+x)^4} + \frac{1}{4a^2(a+x)^3} + \frac{1}{8a^3(a+x)^2} + \frac{1}{8a^3(a^2-x^2)}\right)\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= -\frac{1}{6d(a+a\sin(c+dx))^3} - \frac{1}{8ad(a+a\sin(c+dx))^2} - \frac{1}{8d\,(a^3+a^3\sin(c+dx))} -$$

$$= \frac{\tanh^{-1}(\sin(c+dx))}{8a^3d} - \frac{1}{6d(a+a\sin(c+dx))^3} - \frac{1}{8ad(a+a\sin(c+dx))^2} - \frac{}{8d\,(}$$

Mathematica [A]
time = 0.06, size = 61, normalized size = 0.74

$$\frac{\frac{1}{8}\tanh^{-1}(\sin(c+dx)) - \frac{1}{6(1+\sin(c+dx))^3} - \frac{1}{8(1+\sin(c+dx))^2} - \frac{1}{8(1+\sin(c+dx))}}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (ArcTanh[Sin[c + d*x]]/8 - 1/(6*(1 + Sin[c + d*x])^3) - 1/(8*(1 + Sin[c + d
*x])^2) - 1/(8*(1 + Sin[c + d*x])))/(a^3*d)
```

Maple [A]
time = 0.22, size = 67, normalized size = 0.82

method	result
derivativedivides	$\frac{-\frac{\ln(\sin(dx+c)-1)}{16} - \frac{1}{6(1+\sin(dx+c))^3} - \frac{1}{8(1+\sin(dx+c))^2} - \frac{1}{8(1+\sin(dx+c))} + \frac{\ln(1+\sin(dx+c))}{16}}{d\,a^3}$
default	$\frac{-\frac{\ln(\sin(dx+c)-1)}{16} - \frac{1}{6(1+\sin(dx+c))^3} - \frac{1}{8(1+\sin(dx+c))^2} - \frac{1}{8(1+\sin(dx+c))} + \frac{\ln(1+\sin(dx+c))}{16}}{d\,a^3}$
risch	$-\frac{i\left(18i\,e^{4i(dx+c)} + 3\,e^{5i(dx+c)} - 18i\,e^{2i(dx+c)} - 46\,e^{3i(dx+c)} + 3\,e^{i(dx+c)}\right)}{12d\,a^3\left(e^{i(dx+c)}+i\right)^6} - \frac{\ln\left(e^{i(dx+c)}-i\right)}{8a^3d} + \frac{\ln\left(e^{i(dx+c)}+i\right)}{8a^3d}$
norman	$\frac{\frac{9\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2da} + \frac{9\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2da} + \frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4da} + \frac{7\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4da} + \frac{41\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6da}}{a^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^6} - \frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{8a^3d} + \frac{\ln\left(\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^3*(-1/16*ln(sin(d*x+c)-1)-1/6/(1+sin(d*x+c))^3-1/8/(1+sin(d*x+c))^2-1
/8/(1+sin(d*x+c))+1/16*ln(1+sin(d*x+c)))
```

Maxima [A]
time = 0.28, size = 98, normalized size = 1.20

$$-\frac{\frac{2\left(3\,\sin(dx+c)^2+9\,\sin(dx+c)+10\right)}{a^3\sin(dx+c)^3+3\,a^3\sin(dx+c)^2+3\,a^3\sin(dx+c)+a^3}-\frac{3\,\log(\sin(dx+c)+1)}{a^3}+\frac{3\,\log(\sin(dx+c)-1)}{a^3}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/48*(2*(3*sin(d*x + c)^2 + 9*sin(d*x + c) + 10)/(a^3*sin(d*x + c)^3 + 3*a
^3*sin(d*x + c)^2 + 3*a^3*sin(d*x + c) + a^3) - 3*log(sin(d*x + c) + 1)/a^3
 + 3*log(sin(d*x + c) - 1)/a^3)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 154 vs. 2(74) = 148.
time = 0.34, size = 154, normalized size = 1.88

$$-\frac{6\,\cos\left(dx+c\right)^2-3\left(3\,\cos\left(dx+c\right)^2+\left(\cos\left(dx+c\right)^2-4\right)\sin\left(dx+c\right)-4\right)\log\left(\sin\left(dx+c\right)+1\right)+3\left(3\,\cos\left(dx+c\right)^2+\left(\cos\left(dx+c\right)^2-4\right)\sin\left(dx+c\right)-4\right)\log\left(-\sin\left(dx+c\right)+1\right)-18\,\sin\left(dx+c\right)-26}{48\left(3\,a^3d\cos\left(dx+c\right)^2-4\,a^3d+\left(a^3d\cos\left(dx+c\right)^2-4\,a^3d\right)\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/48*(6*cos(d*x + c)^2 - 3*(3*cos(d*x + c)^2 + (cos(d*x + c)^2 - 4)*sin(d*
x + c) - 4)*log(sin(d*x + c) + 1) + 3*(3*cos(d*x + c)^2 + (cos(d*x + c)^2 -
 4)*sin(d*x + c) - 4)*log(-sin(d*x + c) + 1) - 18*sin(d*x + c) - 26)/(3*a^3
*d*cos(d*x + c)^2 - 4*a^3*d + (a^3*d*cos(d*x + c)^2 - 4*a^3*d)*sin(d*x + c)
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec\left(c+dx\right)}{\sin^3\left(c+dx\right)+3\sin^2\left(c+dx\right)+3\sin\left(c+dx\right)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)/(sin(c + d*x)**3 + 3*sin(c + d*x)**2 + 3*sin(c + d*x)
 + 1), x)/a**3
```

Giac [A]
time = 5.51, size = 81, normalized size = 0.99

$$\frac{\frac{6\,\log(|\sin(dx+c)+1|)}{a^3}-\frac{6\,\log(|\sin(dx+c)-1|)}{a^3}-\frac{11\,\sin(dx+c)^3+45\,\sin(dx+c)^2+69\,\sin(dx+c)+51}{a^3(\sin(dx+c)+1)^3}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/96*(6*log(abs(sin(d*x + c) + 1))/a^3 - 6*log(abs(sin(d*x + c) - 1))/a^3 -
 (11*sin(d*x + c)^3 + 45*sin(d*x + c)^2 + 69*sin(d*x + c) + 51)/(a^3*(sin(d
*x + c) + 1)^3))/d
```

Mupad [B]
time = 4.59, size = 83, normalized size = 1.01

$$\frac{\operatorname{atanh}(\sin(c+d\,x))}{8\,a^3\,d}-\frac{\frac{\sin(c+d\,x)^2}{8}+\frac{3\,\sin(c+d\,x)}{8}+\frac{5}{12}}{d\,\left(a^3\sin(c+d\,x)^3+3\,a^3\sin(c+d\,x)^2+3\,a^3\,\sin(c+d\,x)+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + a*sin(c + d*x))^3),x)
```

```
[Out] atanh(sin(c + d*x))/(8*a^3*d) - ((3*sin(c + d*x))/8 + sin(c + d*x)^2/8 + 5/
12)/(d*(3*a^3*sin(c + d*x) + a^3 + 3*a^3*sin(c + d*x)^2 + a^3*sin(c + d*x)^
3))
```

3.84 $\int \frac{\sec^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=99

$$-\frac{\sec(c+dx)}{7d(a+a\sin(c+dx))^3}-\frac{4\sec(c+dx)}{35ad(a+a\sin(c+dx))^2}-\frac{4\sec(c+dx)}{35d\,(a^3+a^3\sin(c+dx))}+\frac{8\tan(c+dx)}{35a^3d}$$

```
[Out] -1/7*sec(d*x+c)/d/(a+a*sin(d*x+c))^3-4/35*sec(d*x+c)/a/d/(a+a*sin(d*x+c))^2
-4/35*sec(d*x+c)/d/(a^3+a^3*sin(d*x+c))+8/35*tan(d*x+c)/a^3/d
```

Rubi [A]
time = 0.09, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2751, 3852, 8}

$$\frac{8\tan(c+dx)}{35a^3d}-\frac{4\sec(c+dx)}{35d\,(a^3\sin(c+dx)+a^3)}-\frac{4\sec(c+dx)}{35ad(a\sin(c+dx)+a)^2}-\frac{\sec(c+dx)}{7d(a\sin(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/7*Sec[c + d*x]/(d*(a + a*Sin[c + d*x])^3) - (4*Sec[c + d*x])/(35*a*d*(a
+ a*Sin[c + d*x])^2) - (4*Sec[c + d*x])/(35*d*(a^3 + a^3*Sin[c + d*x])) + (
8*Tan[c + d*x])/(35*a^3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx = -\frac{\sec(c+dx)}{7d(a+a\sin(c+dx))^3} + \frac{4\int \frac{\sec^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx}{7a}$$

$$= -\frac{\sec(c+dx)}{7d(a+a\sin(c+dx))^3} - \frac{4\sec(c+dx)}{35ad(a+a\sin(c+dx))^2} + \frac{12\int \frac{\sec^2(c+dx)}{a+a\sin(c+dx)}\,dx}{35a^2}$$

$$= -\frac{\sec(c+dx)}{7d(a+a\sin(c+dx))^3} - \frac{4\sec(c+dx)}{35ad(a+a\sin(c+dx))^2} - \frac{4\sec(c+dx)}{35d\left(a^3+a^3\sin(c+dx)\right)}$$

$$= -\frac{\sec(c+dx)}{7d(a+a\sin(c+dx))^3} - \frac{4\sec(c+dx)}{35ad(a+a\sin(c+dx))^2} - \frac{4\sec(c+dx)}{35d\left(a^3+a^3\sin(c+dx)\right)}$$

$$= -\frac{\sec(c+dx)}{7d(a+a\sin(c+dx))^3} - \frac{4\sec(c+dx)}{35ad(a+a\sin(c+dx))^2} - \frac{4\sec(c+dx)}{35d\left(a^3+a^3\sin(c+dx)\right)}$$

Mathematica [A]
time = 0.07, size = 63, normalized size = 0.64

$$\frac{\sec(c+dx)(-14\cos(2(c+dx))+\cos(4(c+dx))+14\sin(c+dx)-6\sin(3(c+dx)))}{35a^3d(1+\sin(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]*(-14*Cos[2*(c + d*x)] + Cos[4*(c + d*x)] + 14*Sin[c + d*x] -
6*Sin[3*(c + d*x)]))/(35*a^3*d*(1 + Sin[c + d*x])^3)
```

Maple [A]
time = 0.22, size = 130, normalized size = 1.31

method	result
risch	$\frac{-\frac{96\,e^{i(dx+c)}}{35}-\frac{16i}{35}+\frac{32\,e^{3i(dx+c)}}{5}+\frac{32i e^{2i(dx+c)}}{5}}{\left(e^{i(dx+c)}+i\right)^7\left(e^{i(dx+c)}-i\right)d\,a^3}$
derivativedivides	$\frac{-\frac{8}{7\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^7}+\frac{4}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^6}-\frac{38}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}+\frac{9}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}-\frac{15}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}+\frac{17}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}}{d\,a^3}$
default	$\frac{-\frac{8}{7\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^7}+\frac{4}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^6}-\frac{38}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}+\frac{9}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}-\frac{15}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}+\frac{17}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}}{d\,a^3}$
norman	$\frac{-\frac{6\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}+\frac{26}{35ad}-\frac{2\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}-\frac{10\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}-\frac{6\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}+\frac{22\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5da}+\frac{2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5da}+}{a^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^7}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/a^3*(-4/7/(tan(1/2*d*x+1/2*c)+1)^7+2/(tan(1/2*d*x+1/2*c)+1)^6-19/5/(tan
(1/2*d*x+1/2*c)+1)^5+9/2/(tan(1/2*d*x+1/2*c)+1)^4-15/4/(tan(1/2*d*x+1/2*c)+
1)^3+17/8/(tan(1/2*d*x+1/2*c)+1)^2-15/16/(tan(1/2*d*x+1/2*c)+1)-1/16/(tan(1
/2*d*x+1/2*c)-1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 310 vs. $2(91) = 182$.
time = 0.30, size = 310, normalized size = 3.13

$$-\frac{2\left(\frac{43\sin(dx+c)}{\cos(dx+c)+1}+\frac{77\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{7\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{105\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{175\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{105\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{35\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+13\right)}{35\left(a^3+\frac{6a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{14a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{14a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{14a^3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{14a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{6a^3\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{a^3\sin(dx+c)^8}{(\cos(dx+c)+1)^8}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -2/35*(43*sin(d*x + c)/(cos(d*x + c) + 1) + 77*sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 + 7*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 105*sin(d*x + c)^4/(cos(d
*x + c) + 1)^4 - 175*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 105*sin(d*x + c)
^6/(cos(d*x + c) + 1)^6 - 35*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 13)/((a^
3 + 6*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 14*a^3*sin(d*x + c)^2/(cos(d*x
+ c) + 1)^2 + 14*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 14*a^3*sin(d*x +
 c)^5/(cos(d*x + c) + 1)^5 - 14*a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 6
*a^3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - a^3*sin(d*x + c)^8/(cos(d*x + c)
 + 1)^8)*d)
```

Fricas [A]
time = 0.34, size = 106, normalized size = 1.07

$$-\frac{8\cos(dx+c)^4-36\cos(dx+c)^2-4\left(6\cos(dx+c)^2-5\right)\sin(dx+c)+15}{35\left(3a^3d\cos(dx+c)^3-4a^3d\cos(dx+c)+\left(a^3d\cos(dx+c)^3-4a^3d\cos(dx+c)\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/35*(8*cos(d*x + c)^4 - 36*cos(d*x + c)^2 - 4*(6*cos(d*x + c)^2 - 5)*sin(
d*x + c) + 15)/(3*a^3*d*cos(d*x + c)^3 - 4*a^3*d*cos(d*x + c) + (a^3*d*cos(
d*x + c)^3 - 4*a^3*d*cos(d*x + c))*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\sec^2(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**2/(sin(c + d*x)**3 + 3*sin(c + d*x)**2 + 3*sin(c + d
*x) + 1), x)/a**3
```

Giac [A]
time = 6.53, size = 119, normalized size = 1.20

$$-\frac{\frac{35}{a^3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}+\frac{525\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+1960\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+4025\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+4480\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3143\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1176\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+243}{a^3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^7}}{280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/280*(35/(a^3*(tan(1/2*d*x + 1/2*c) - 1)) + (525*tan(1/2*d*x + 1/2*c)^6 +
 1960*tan(1/2*d*x + 1/2*c)^5 + 4025*tan(1/2*d*x + 1/2*c)^4 + 4480*tan(1/2*d
*x + 1/2*c)^3 + 3143*tan(1/2*d*x + 1/2*c)^2 + 1176*tan(1/2*d*x + 1/2*c) + 2
43)/(a^3*(tan(1/2*d*x + 1/2*c) + 1)^7))/d
```

Mupad [B]
time = 5.08, size = 228, normalized size = 2.30

$$-\frac{2\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(13\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+43\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+77\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+7\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-105\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-175\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5-105\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-35\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\right)}{35\,a^3\,d\,\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + a*sin(c + d*x))^3),x)
```

```
[Out] -(2*cos(c/2 + (d*x)/2)*(13*cos(c/2 + (d*x)/2)^7 - 35*sin(c/2 + (d*x)/2)^7 -
 105*cos(c/2 + (d*x)/2)*sin(c/2 + (d*x)/2)^6 + 43*cos(c/2 + (d*x)/2)^6*sin(
c/2 + (d*x)/2) - 175*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^5 - 105*cos(c/
2 + (d*x)/2)^3*sin(c/2 + (d*x)/2)^4 + 7*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x
)/2)^3 + 77*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^2))/(35*a^3*d*(cos(c/2
+ (d*x)/2) - sin(c/2 + (d*x)/2))*(cos(c/2 + (d*x)/2) + sin(c/2 + (d*x)/2))^
7)
```

3.85 $\int \frac{\sec^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=126

$$\frac{5\tanh^{-1}(\sin(c+dx))}{32a^3d}-\frac{a}{16d(a+a\sin(c+dx))^4}-\frac{1}{12d(a+a\sin(c+dx))^3}-\frac{3}{32ad(a+a\sin(c+dx))^2}+\frac{}{32d\,(\ldots}$$

```
[Out] 5/32*arctanh(sin(d*x+c))/a^3/d-1/16*a/d/(a+a*sin(d*x+c))^4-1/12/d/(a+a*sin(
d*x+c))^3-3/32/a/d/(a+a*sin(d*x+c))^2+1/32/d/(a^3-a^3*sin(d*x+c))-1/8/d/(a^
3+a^3*sin(d*x+c))
```

Rubi [A]
time = 0.07, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2746, 46, 212}

$$\frac{1}{32d\,(a^3-a^3\sin(c+dx))}-\frac{1}{8d\,(a^3\sin(c+dx)+a^3)}+\frac{5\tanh^{-1}(\sin(c+dx))}{32a^3d}-\frac{a}{16d(a\sin(c+dx)+a)^4}-\frac{1}{12d(a\sin(c+dx)+a)^3}-\frac{3}{32ad(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (5*ArcTanh[Sin[c + d*x]])/(32*a^3*d) - a/(16*d*(a + a*Sin[c + d*x])^4) - 1/
(12*d*(a + a*Sin[c + d*x])^3) - 3/(32*a*d*(a + a*Sin[c + d*x])^2) + 1/(32*d
*(a^3 - a^3*Sin[c + d*x])) - 1/(8*d*(a^3 + a^3*Sin[c + d*x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{a^3\text{Subst}\left(\int \frac{1}{(a-x)^2(a+x)^5}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^3\text{Subst}\left(\int \left(\frac{1}{32a^5(a-x)^2} + \frac{1}{4a^2(a+x)^5} + \frac{1}{4a^3(a+x)^4} + \frac{3}{16a^4(a+x)^3} + \frac{1}{8a^5(a+x)^2} + \frac{5}{32a^5(a^2-x^2)}\right.\right.}{d}$$
$$= -\frac{a}{16d(a+a\sin(c+dx))^4} - \frac{1}{12d(a+a\sin(c+dx))^3} - \frac{3}{32ad(a+a\sin(c+dx))^2}$$
$$= \frac{5\tanh^{-1}(\sin(c+dx))}{32a^3d} - \frac{a}{16d(a+a\sin(c+dx))^4} - \frac{1}{12d(a+a\sin(c+dx))^3} - \frac{3}{}$$

Mathematica [A]
time = 0.11, size = 95, normalized size = 0.75

$$-\frac{\sec^2(c+dx)\left(32+15\sin(c+dx)-35\sin^2(c+dx)-45\sin^3(c+dx)-15\sin^4(c+dx)+15\tanh^{-1}(\sin(c+dx))(-1+\sin(c+dx))(1+\sin(c+dx))^4\right)}{96a^3d(1+\sin(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/96*(Sec[c + d*x]^2*(32 + 15*Sin[c + d*x] - 35*Sin[c + d*x]^2 - 45*Sin[c
+ d*x]^3 - 15*Sin[c + d*x]^4 + 15*ArcTanh[Sin[c + d*x]]*(-1 + Sin[c + d*x])
*(1 + Sin[c + d*x])^4))/(a^3*d*(1 + Sin[c + d*x])^3)
```

Maple [A]
time = 0.32, size = 91, normalized size = 0.72

method	result
derivativedivides	$\frac{-\frac{1}{32(\sin(dx+c)-1)}-\frac{5\ln(\sin(dx+c)-1)}{64}-\frac{1}{16(1+\sin(dx+c))^4}-\frac{1}{12(1+\sin(dx+c))^3}-\frac{3}{32(1+\sin(dx+c))^2}-\frac{1}{8(1+\sin(dx+c))}+\frac{5\ln(1+\sin(}{64}}{d\,a^3}$
default	$\frac{-\frac{1}{32(\sin(dx+c)-1)}-\frac{5\ln(\sin(dx+c)-1)}{64}-\frac{1}{16(1+\sin(dx+c))^4}-\frac{1}{12(1+\sin(dx+c))^3}-\frac{3}{32(1+\sin(dx+c))^2}-\frac{1}{8(1+\sin(dx+c))}+\frac{5\ln(1+\sin(}{64}}{d\,a^3}$
risch	$-\frac{i\left(90i\mathrm{e}^{8i(dx+c)}+15\,\mathrm{e}^{9i(dx+c)}-150i\mathrm{e}^{6i(dx+c)}-200\,\mathrm{e}^{7i(dx+c)}+150i\mathrm{e}^{4i(dx+c)}-142\,\mathrm{e}^{5i(dx+c)}-90i\mathrm{e}^{2i(dx+c)}-200\,\mathrm{e}^{3i(dx+c)}\right.}{48\left(\mathrm{e}^{i(dx+c)}+i\right)^8\left(\mathrm{e}^{i(dx+c)}-i\right)^2 d\,a^3}$
norman	$\frac{\frac{27\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16da}+\frac{27\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16da}+\frac{31\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6da}+\frac{31\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6da}+\frac{33\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8da}+\frac{33\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8da}-\frac{9\left(\tan^4\left(\right.\right.}{8}}{a^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^3*(-1/32/(sin(d*x+c)-1)-5/64*ln(sin(d*x+c)-1)-1/16/(1+sin(d*x+c))^4-1
/12/(1+sin(d*x+c))^3-3/32/(1+sin(d*x+c))^2-1/8/(1+sin(d*x+c))+5/64*ln(1+sin
(d*x+c)))
```

Maxima [A]
time = 0.29, size = 146, normalized size = 1.16

$$-\frac{\frac{2\left(15\sin\left(dx+c\right)^4+45\sin\left(dx+c\right)^3+35\sin\left(dx+c\right)^2-15\sin\left(dx+c\right)-32\right)}{a^3\sin\left(dx+c\right)^5+3a^3\sin\left(dx+c\right)^4+2a^3\sin\left(dx+c\right)^3-2a^3\sin\left(dx+c\right)^2-3a^3\sin\left(dx+c\right)-a^3}-\frac{15\log\left(\sin\left(dx+c\right)+1\right)}{a^3}+\frac{15\log\left(\sin\left(dx+c\right)-1\right)}{a^3}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/192*(2*(15*sin(d*x + c)^4 + 45*sin(d*x + c)^3 + 35*sin(d*x + c)^2 - 15*s
in(d*x + c) - 32)/(a^3*sin(d*x + c)^5 + 3*a^3*sin(d*x + c)^4 + 2*a^3*sin(d*
x + c)^3 - 2*a^3*sin(d*x + c)^2 - 3*a^3*sin(d*x + c) - a^3) - 15*log(sin(d*
x + c) + 1)/a^3 + 15*log(sin(d*x + c) - 1)/a^3)/d
```

Fricas [A]
time = 0.38, size = 228, normalized size = 1.81

$$-\frac{30\cos\left(dx+c\right)^4-130\cos\left(dx+c\right)^2-15\left(3\cos\left(dx+c\right)^4-4\cos\left(dx+c\right)^2+\left(\cos\left(dx+c\right)^4-4\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)\right)\log\left(\sin\left(dx+c\right)+1\right)+15\left(3\cos\left(dx+c\right)^4-4\cos\left(dx+c\right)^2+\left(\cos\left(dx+c\right)^4-4\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)\right)\log\left(-\sin\left(dx+c\right)+1\right)-30\left(3\cos\left(dx+c\right)^2-2\right)\sin\left(dx+c\right)+36}{192\left(3a^3d\cos\left(dx+c\right)^4-4a^3d\cos\left(dx+c\right)^2+\left(a^3d\cos\left(dx+c\right)^4-4a^3d\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/192*(30*cos(d*x + c)^4 - 130*cos(d*x + c)^2 - 15*(3*cos(d*x + c)^4 - 4*c
os(d*x + c)^2 + (cos(d*x + c)^4 - 4*cos(d*x + c)^2)*sin(d*x + c))*log(sin(d
*x + c) + 1) + 15*(3*cos(d*x + c)^4 - 4*cos(d*x + c)^2 + (cos(d*x + c)^4 -
4*cos(d*x + c)^2)*sin(d*x + c))*log(-sin(d*x + c) + 1) - 30*(3*cos(d*x + c)
^2 - 2)*sin(d*x + c) + 36)/(3*a^3*d*cos(d*x + c)^4 - 4*a^3*d*cos(d*x + c)^2
 + (a^3*d*cos(d*x + c)^4 - 4*a^3*d*cos(d*x + c)^2)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\sec^3\left(c+dx\right)}{\sin^3\left(c+dx\right)+3\sin^2\left(c+dx\right)+3\sin\left(c+dx\right)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**3/(sin(c + d*x)**3 + 3*sin(c + d*x)**2 + 3*sin(c + d
*x) + 1), x)/a**3
```

Giac [A]
time = 8.38, size = 116, normalized size = 0.92

$$\frac{\frac{60 \log(|\sin(dx+c)+1|)}{a^3} - \frac{60 \log(|\sin(dx+c)-1|)}{a^3} + \frac{12(5 \sin(dx+c)-7)}{a^3(\sin(dx+c)-1)} - \frac{125 \sin(dx+c)^4+596 \sin(dx+c)^3+1110 \sin(dx+c)^2+996 \sin(dx+c)+405}{a^3(\sin(dx+c)+1)^4}}{768\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/768*(60*log(abs(sin(d*x + c) + 1))/a^3 - 60*log(abs(sin(d*x + c) - 1))/a^
3 + 12*(5*sin(d*x + c) - 7)/(a^3*(sin(d*x + c) - 1)) - (125*sin(d*x + c)^4
+ 596*sin(d*x + c)^3 + 1110*sin(d*x + c)^2 + 996*sin(d*x + c) + 405)/(a^3*(
sin(d*x + c) + 1)^4))/d
```

Mupad [B]
time = 0.15, size = 129, normalized size = 1.02

$$\frac{5\,\mathrm{atanh}(\sin\left(c+d\,x\right))}{32\,a^3\,d}+\frac{\frac{5\sin(c+d\,x)^4}{32}+\frac{15\sin(c+d\,x)^3}{32}+\frac{35\sin(c+d\,x)^2}{96}-\frac{5\,\sin(c+d\,x)}{32}-\frac{1}{3}}{d\,\left(-a^3\sin\left(c+d\,x\right)^5-3\,a^3\sin\left(c+d\,x\right)^4-2\,a^3\sin\left(c+d\,x\right)^3+2\,a^3\sin\left(c+d\,x\right)^2+3\,a^3\,\sin\left(c+d\,x\right)+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + a*sin(c + d*x))^3),x)
```

```
[Out] (5*atanh(sin(c + d*x)))/(32*a^3*d) + ((35*sin(c + d*x)^2)/96 - (5*sin(c + d
*x))/32 + (15*sin(c + d*x)^3)/32 + (5*sin(c + d*x)^4)/32 - 1/3)/(d*(3*a^3*s
in(c + d*x) + a^3 + 2*a^3*sin(c + d*x)^2 - 2*a^3*sin(c + d*x)^3 - 3*a^3*sin
(c + d*x)^4 - a^3*sin(c + d*x)^5))
```

3.86 $\int \frac{\sec^4(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=123

$$-\frac{\sec^3(c+dx)}{9d(a+a\sin(c+dx))^3}-\frac{2\sec^3(c+dx)}{21ad(a+a\sin(c+dx))^2}-\frac{2\sec^3(c+dx)}{21d\,(a^3+a^3\sin(c+dx))}+\frac{8\tan(c+dx)}{21a^3d}+\frac{8\tan^3(c+dx)}{63a^3d}$$

```
[Out] -1/9*sec(d*x+c)^3/d/(a+a*sin(d*x+c))^3-2/21*sec(d*x+c)^3/a/d/(a+a*sin(d*x+c
))^2-2/21*sec(d*x+c)^3/d/(a^3+a^3*sin(d*x+c))+8/21*tan(d*x+c)/a^3/d+8/63*ta
n(d*x+c)^3/a^3/d
```

Rubi [A]
time = 0.10, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2751, 3852}

$$\frac{8\tan^3(c+dx)}{63a^3d}+\frac{8\tan(c+dx)}{21a^3d}-\frac{2\sec^3(c+dx)}{21d\,(a^3\sin(c+dx)+a^3)}-\frac{2\sec^3(c+dx)}{21ad(a\sin(c+dx)+a)^2}-\frac{\sec^3(c+dx)}{9d(a\sin(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/9*Sec[c + d*x]^3/(d*(a + a*Sin[c + d*x])^3) - (2*Sec[c + d*x]^3)/(21*a*d
*(a + a*Sin[c + d*x])^2) - (2*Sec[c + d*x]^3)/(21*d*(a^3 + a^3*Sin[c + d*x]
)) + (8*Tan[c + d*x])/(21*a^3*d) + (8*Tan[c + d*x]^3)/(63*a^3*d)
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{(a+a\sin(c+dx))^3}\,dx = -\frac{\sec^3(c+dx)}{9d(a+a\sin(c+dx))^3} + \frac{2\int \frac{\sec^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx}{3a}$$
$$= -\frac{\sec^3(c+dx)}{9d(a+a\sin(c+dx))^3} - \frac{2\sec^3(c+dx)}{21ad(a+a\sin(c+dx))^2} + \frac{10\int \frac{\sec^4(c+dx)}{a+a\sin(c+dx)}\,dx}{21a^2}$$
$$= -\frac{\sec^3(c+dx)}{9d(a+a\sin(c+dx))^3} - \frac{2\sec^3(c+dx)}{21ad(a+a\sin(c+dx))^2} - \frac{2\sec^3(c+dx)}{21d\,(a^3+a^3\sin(c+dx)}$$
$$= -\frac{\sec^3(c+dx)}{9d(a+a\sin(c+dx))^3} - \frac{2\sec^3(c+dx)}{21ad(a+a\sin(c+dx))^2} - \frac{2\sec^3(c+dx)}{21d\,(a^3+a^3\sin(c+dx)}$$
$$= -\frac{\sec^3(c+dx)}{9d(a+a\sin(c+dx))^3} - \frac{2\sec^3(c+dx)}{21ad(a+a\sin(c+dx))^2} - \frac{2\sec^3(c+dx)}{21d\,(a^3+a^3\sin(c+dx)}$$

Mathematica [A]
time = 0.08, size = 85, normalized size = 0.69

$$\frac{\sec^3(c+dx)(-27\cos(2(c+dx)) - 12\cos(4(c+dx)) + \cos(6(c+dx)) + 36\sin(c+dx) + 2\sin(3(c+dx)) - 6\sin(5(c+dx)))}{126a^3d(1+\sin(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]^3*(-27*Cos[2*(c + d*x)] - 12*Cos[4*(c + d*x)] + Cos[6*(c + d*
x)] + 36*Sin[c + d*x] + 2*Sin[3*(c + d*x)] - 6*Sin[5*(c + d*x)]))/(126*a^3*
d*(1 + Sin[c + d*x])^3)
```

Maple [A]
time = 0.33, size = 190, normalized size = 1.54

method	result
risch	$\frac{-\frac{64\,e^{i(dx+c)}}{21}+\frac{64\,e^{3i(dx+c)}}{63}+\frac{128\,e^{5i(dx+c)}}{7}-\frac{32i}{63}+\frac{128i e^{2i(dx+c)}}{21}+\frac{96i e^{4i(dx+c)}}{7}}{\left(e^{i(dx+c)}+i\right)^9\left(e^{i(dx+c)}-i\right)^3 d\,a^3}$
derivativedivides	$-\frac{8}{9\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^9}+\frac{4}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^8}-\frac{68}{7\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^7}+\frac{46}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^6}-\frac{35}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}+\frac{59}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\right.}$
default	$-\frac{8}{9\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^9}+\frac{4}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^8}-\frac{68}{7\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^7}+\frac{46}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^6}-\frac{35}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}+\frac{59}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\right.}$
norman	$\frac{\frac{28\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3da}+\frac{38}{63ad}-\frac{6\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}-\frac{26\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3da}+\frac{12\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}-\frac{2\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}+\frac{2\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}+}{a^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

[Out] 2/d/a^3*(-4/9/(tan(1/2*d*x+1/2*c)+1)^9+2/(tan(1/2*d*x+1/2*c)+1)^8-34/7/(tan(1/2*d*x+1/2*c)+1)^7+23/3/(tan(1/2*d*x+1/2*c)+1)^6-35/4/(tan(1/2*d*x+1/2*c)+1)^5+59/8/(tan(1/2*d*x+1/2*c)+1)^4-19/4/(tan(1/2*d*x+1/2*c)+1)^3+9/4/(tan(1/2*d*x+1/2*c)+1)^2-57/64/(tan(1/2*d*x+1/2*c)+1)-1/48/(tan(1/2*d*x+1/2*c)-1)^3-1/32/(tan(1/2*d*x+1/2*c)-1)^2-7/64/(tan(1/2*d*x+1/2*c)-1))

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 482 vs. $2(113) = 226$.

time = 0.30, size = 482, normalized size = 3.92

$$-\frac{2\left(\frac{51\sin(dx+c)}{\cos(dx+c)+1}+\frac{39\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{235\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{450\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{306\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{294\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{378\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{63\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{273\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{189\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}-\frac{63\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}+19\right)}{63\left(a^3+\frac{6a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{12a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{2a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{27a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{36a^3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{36a^3\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{27a^3\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{2a^3\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{12a^3\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}-\frac{6a^3\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}-\frac{a^3\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="maxima")

[Out] -2/63*(51*sin(d*x + c)/(cos(d*x + c) + 1) + 39*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 235*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 450*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 306*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 294*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 378*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 63*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 273*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 189*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 - 63*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 + 19)/((a^3 + 6*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 12*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 2*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 27*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 36*a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 36*a^3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 27*a^3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 2*a^3*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 12*a^3*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 - 6*a^3*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 - a^3*sin(d*x + c)^12/(cos(d*x + c) + 1)^12)*d)

Fricas [A]

time = 0.35, size = 130, normalized size = 1.06

$$-\frac{16\cos(dx+c)^6-72\cos(dx+c)^4+30\cos(dx+c)^2-2\left(24\cos(dx+c)^4-20\cos(dx+c)^2-7\right)\sin(dx+c)+7}{63\left(3a^3d\cos(dx+c)^5-4a^3d\cos(dx+c)^3+\left(a^3d\cos(dx+c)^5-4a^3d\cos(dx+c)^3\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="fricas")

[Out] -1/63*(16*cos(d*x + c)^6 - 72*cos(d*x + c)^4 + 30*cos(d*x + c)^2 - 2*(24*cos(d*x + c)^4 - 20*cos(d*x + c)^2 - 7)*sin(d*x + c) + 7)/(3*a^3*d*cos(d*x + c)^5 - 4*a^3*d*cos(d*x + c)^3 + (a^3*d*cos(d*x + c)^5 - 4*a^3*d*cos(d*x + c)^3)*sin(d*x + c))

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^4(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**4/(sin(c + d*x)**3 + 3*sin(c + d*x)**2 + 3*sin(c + d
*x) + 1), x)/a**3
```

Giac [A]
time = 5.15, size = 171, normalized size = 1.39

$$-\frac{\frac{21\left(21\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-36\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+19\right)}{a^{3}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)^{3}}+\frac{3591\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{8}+19656\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{7}+56196\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{6}+95760\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{5}+107730\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{4}+79464\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}+38484\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+10944\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1615}{a^{3}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)^{9}}}{2016\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2016*(21*(21*tan(1/2*d*x + 1/2*c)^2 - 36*tan(1/2*d*x + 1/2*c) + 19)/(a^3
*(tan(1/2*d*x + 1/2*c) - 1)^3) + (3591*tan(1/2*d*x + 1/2*c)^8 + 19656*tan(1
/2*d*x + 1/2*c)^7 + 56196*tan(1/2*d*x + 1/2*c)^6 + 95760*tan(1/2*d*x + 1/2*
c)^5 + 107730*tan(1/2*d*x + 1/2*c)^4 + 79464*tan(1/2*d*x + 1/2*c)^3 + 38484
*tan(1/2*d*x + 1/2*c)^2 + 10944*tan(1/2*d*x + 1/2*c) + 1615)/(a^3*(tan(1/2*
d*x + 1/2*c) + 1)^9))/d
```

Mupad [B]
time = 5.56, size = 167, normalized size = 1.36

$$\frac{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{63\cos\left(\frac{5c}{2}+\frac{5dx}{2}\right)}{8}-\frac{171\cos\left(\frac{3c}{2}+\frac{3dx}{2}\right)}{8}-\frac{145\cos\left(\frac{7c}{2}+\frac{7dx}{2}\right)}{16}+\frac{49\cos\left(\frac{9c}{2}+\frac{9dx}{2}\right)}{16}+\frac{\cos\left(\frac{11c}{2}+\frac{11dx}{2}\right)}{2}+\frac{617\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{16}-\frac{329\sin\left(\frac{3c}{2}+\frac{3dx}{2}\right)}{16}+\frac{145\sin\left(\frac{5c}{2}+\frac{5dx}{2}\right)}{32}-\frac{113\sin\left(\frac{7c}{2}+\frac{7dx}{2}\right)}{32}-\frac{115\sin\left(\frac{9c}{2}+\frac{9dx}{2}\right)}{32}+\frac{19\sin\left(\frac{11c}{2}+\frac{11dx}{2}\right)}{32}\right)}{2016\,a^{3}\,d\cos\left(\frac{c}{2}-\frac{\pi}{4}+\frac{dx}{2}\right)^{9}\cos\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a + a*sin(c + d*x))^3),x)
```

```
[Out] (cos(c/2 + (d*x)/2)*((63*cos((5*c)/2 + (5*d*x)/2))/8 - (171*cos((3*c)/2 + (
3*d*x)/2))/8 - (145*cos((7*c)/2 + (7*d*x)/2))/16 + (49*cos((9*c)/2 + (9*d*x
)/2))/16 + cos((11*c)/2 + (11*d*x)/2)/2 + (617*sin(c/2 + (d*x)/2))/16 - (32
9*sin((3*c)/2 + (3*d*x)/2))/16 + (145*sin((5*c)/2 + (5*d*x)/2))/32 - (113*s
in((7*c)/2 + (7*d*x)/2))/32 - (115*sin((9*c)/2 + (9*d*x)/2))/32 + (19*sin((
11*c)/2 + (11*d*x)/2))/32))/(2016*a^3*d*cos(c/2 - pi/4 + (d*x)/2)^9*cos(c/2
 + pi/4 + (d*x)/2)^3)
```

3.87 $\int \frac{\sec^5(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=171

$$\frac{21\tanh^{-1}(\sin(c+dx))}{128a^3d}+\frac{1}{128ad(a-a\sin(c+dx))^2}-\frac{a^2}{40d(a+a\sin(c+dx))^5}-\frac{3a}{64d(a+a\sin(c+dx))^4}-16$$

```
[Out] 21/128*arctanh(sin(d*x+c))/a^3/d+1/128/a/d/(a-a*sin(d*x+c))^2-1/40*a^2/d/(a
+a*sin(d*x+c))^5-3/64*a/d/(a+a*sin(d*x+c))^4-1/16/d/(a+a*sin(d*x+c))^3-5/64
/a/d/(a+a*sin(d*x+c))^2+3/64/d/(a^3-a^3*sin(d*x+c))-15/128/d/(a^3+a^3*sin(d
*x+c))
```

Rubi [A]
time = 0.09, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2746, 46, 212}

$$\frac{3}{64d\left(a^3-a^3\sin(c+dx)\right)}-\frac{15}{128d\left(a^3\sin(c+dx)+a^3\right)}+\frac{21\tanh^{-1}(\sin(c+dx))}{128a^3d}-\frac{a^2}{40d(a\sin(c+dx)+a)^5}-\frac{3a}{64d(a\sin(c+dx)+a)^4}-\frac{1}{16d(a\sin(c+dx)+a)^3}+\frac{1}{128ad(a-a\sin(c+dx))^2}-\frac{5}{64ad(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (21*ArcTanh[Sin[c + d*x]])/(128*a^3*d) + 1/(128*a*d*(a - a*Sin[c + d*x])^2)
 - a^2/(40*d*(a + a*Sin[c + d*x])^5) - (3*a)/(64*d*(a + a*Sin[c + d*x])^4)
- 1/(16*d*(a + a*Sin[c + d*x])^3) - 5/(64*a*d*(a + a*Sin[c + d*x])^2) + 3/(
64*d*(a^3 - a^3*Sin[c + d*x])) - 15/(128*d*(a^3 + a^3*Sin[c + d*x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
```

```
])
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{a^5\text{Subst}\left(\int \frac{1}{(a-x)^3(a+x)^6}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^5\text{Subst}\left(\int \left(\frac{1}{64a^6(a-x)^3} + \frac{3}{64a^7(a-x)^2} + \frac{1}{8a^3(a+x)^6} + \frac{3}{16a^4(a+x)^5} + \frac{3}{16a^5(a+x)^4} + \frac{5}{32a^6(a+\dots}\right.\right.}{d}$$

$$= \frac{1}{128ad(a-a\sin(c+dx))^2} - \frac{a^2}{40d(a+a\sin(c+dx))^5} - \frac{3a}{64d(a+a\sin(c+dx))^4}\dots$$

$$= \frac{21\tanh^{-1}(\sin(c+dx))}{128a^3d} + \frac{1}{128ad(a-a\sin(c+dx))^2} - \frac{a^2}{40d(a+a\sin(c+dx))^5}\dots$$

Mathematica [A]
time = 0.30, size = 145, normalized size = 0.85

$$\frac{\sec^4(c+dx)\left(-176+105\tanh^{-1}(\sin(c+dx))\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^4\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^{10}+7\sin(c+dx)+469\sin^2(c+dx)+420\sin^3(c+dx)-140\sin^4(c+dx)-315\sin^5(c+dx)-105\sin^6(c+dx)\right)}{640a^3d(1+\sin(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a + a*Sin[c + d*x])^3,x]
[Out] (Sec[c + d*x]^4*(-176 + 105*ArcTanh[Sin[c + d*x]]*(Cos[(c + d*x)/2] - Sin[(
c + d*x)/2])^4*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^10 + 7*Sin[c + d*x] +
469*Sin[c + d*x]^2 + 420*Sin[c + d*x]^3 - 140*Sin[c + d*x]^4 - 315*Sin[c +
d*x]^5 - 105*Sin[c + d*x]^6))/(640*a^3*d*(1 + Sin[c + d*x])^3)
```

Maple [A]
time = 0.41, size = 115, normalized size = 0.67

method	result
derivativedivides	$\frac{\frac{1}{128(\sin(dx+c)-1)^2}-\frac{3}{64(\sin(dx+c)-1)}-\frac{21\ln(\sin(dx+c)-1)}{256}-\frac{1}{40(1+\sin(dx+c))^5}-\frac{3}{64(1+\sin(dx+c))^4}-\frac{1}{16(1+\sin(dx+c))^3}-\frac{\dots}{64(1+\sin\dots}}{d\,a^3}$
default	$\frac{\frac{1}{128(\sin(dx+c)-1)^2}-\frac{3}{64(\sin(dx+c)-1)}-\frac{21\ln(\sin(dx+c)-1)}{256}-\frac{1}{40(1+\sin(dx+c))^5}-\frac{3}{64(1+\sin(dx+c))^4}-\frac{1}{16(1+\sin(dx+c))^3}-\frac{\dots}{64(1+\sin\dots}}{d\,a^3}$
risch	$-\frac{i(630i\,e^{12i(dx+c)}+105\,e^{13i(dx+c)}+210i\,e^{10i(dx+c)}-1190\,e^{11i(dx+c)}-4004i\,e^{8i(dx+c)}-3689\,e^{9i(dx+c)}+4004i\,e^{6i(dx+c)}-1\dots}{320\left(e^{i(dx+c)}+i\right)^{10}\left(e^{i(dx+c)}-i\right)\dots}$
norman	$\frac{\frac{137\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32da}+\frac{137\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32da}+\frac{129\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32da}+\frac{129\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32da}+\frac{107\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{64da}+\frac{107\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64da}\dots}{a^2\left(\tan\left(\dots\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^3*(1/128/(sin(d*x+c)-1)^2-3/64/(sin(d*x+c)-1)-21/256*ln(sin(d*x+c)-1)
-1/40/(1+sin(d*x+c))^5-3/64/(1+sin(d*x+c))^4-1/16/(1+sin(d*x+c))^3-5/64/(1+
sin(d*x+c))^2-15/128/(1+sin(d*x+c))+21/256*ln(1+sin(d*x+c)))
```

Maxima [A]
time = 0.30, size = 188, normalized size = 1.10

$$-\frac{\frac{2\left(105\,\sin\left(dx+c\right)^{6}+315\,\sin\left(dx+c\right)^{5}+140\,\sin\left(dx+c\right)^{4}-420\,\sin\left(dx+c\right)^{3}-469\,\sin\left(dx+c\right)^{2}-7\,\sin\left(dx+c\right)+176\right)}{a^{3}\sin\left(dx+c\right)^{7}+3\,a^{3}\sin\left(dx+c\right)^{6}+a^{3}\sin\left(dx+c\right)^{5}-5\,a^{3}\sin\left(dx+c\right)^{4}-5\,a^{3}\sin\left(dx+c\right)^{3}+a^{3}\sin\left(dx+c\right)^{2}+3\,a^{3}\sin\left(dx+c\right)+a^{3}}-\frac{105\,\log\left(\sin\left(dx+c\right)+1\right)}{a^{3}}+\frac{105\,\log\left(\sin\left(dx+c\right)-1\right)}{a^{3}}}{1280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/1280*(2*(105*sin(d*x + c)^6 + 315*sin(d*x + c)^5 + 140*sin(d*x + c)^4 -
420*sin(d*x + c)^3 - 469*sin(d*x + c)^2 - 7*sin(d*x + c) + 176)/(a^3*sin(d*
x + c)^7 + 3*a^3*sin(d*x + c)^6 + a^3*sin(d*x + c)^5 - 5*a^3*sin(d*x + c)^4
 - 5*a^3*sin(d*x + c)^3 + a^3*sin(d*x + c)^2 + 3*a^3*sin(d*x + c) + a^3) -
105*log(sin(d*x + c) + 1)/a^3 + 105*log(sin(d*x + c) - 1)/a^3)/d
```

Fricas [A]
time = 0.39, size = 248, normalized size = 1.45

$$-\frac{210\,\cos\left(dx+c\right)^{6}-910\,\cos\left(dx+c\right)^{4}+252\,\cos\left(dx+c\right)^{2}-105\,\left(3\,\cos\left(dx+c\right)^{6}-4\,\cos\left(dx+c\right)^{4}+\left(\cos\left(dx+c\right)^{6}-4\,\cos\left(dx+c\right)^{4}\right)\sin\left(dx+c\right)\right)\log\left(\sin\left(dx+c\right)+1\right)+105\,\left(3\,\cos\left(dx+c\right)^{6}-4\,\cos\left(dx+c\right)^{4}+\left(\cos\left(dx+c\right)^{6}-4\,\cos\left(dx+c\right)^{4}\right)\sin\left(dx+c\right)\right)\log\left(-\sin\left(dx+c\right)+1\right)-14\,\left(45\,\cos\left(dx+c\right)^{4}-30\,\cos\left(dx+c\right)^{2}-16\right)\sin\left(dx+c\right)+96}{1280\,\left(3\,a^{3}d\cos\left(dx+c\right)^{6}-4\,a^{3}d\cos\left(dx+c\right)^{4}+\left(a^{3}d\cos\left(dx+c\right)^{6}-4\,a^{3}d\cos\left(dx+c\right)^{4}\right)\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/1280*(210*cos(d*x + c)^6 - 910*cos(d*x + c)^4 + 252*cos(d*x + c)^2 - 105
*(3*cos(d*x + c)^6 - 4*cos(d*x + c)^4 + (cos(d*x + c)^6 - 4*cos(d*x + c)^4)
*sin(d*x + c))*log(sin(d*x + c) + 1) + 105*(3*cos(d*x + c)^6 - 4*cos(d*x +
c)^4 + (cos(d*x + c)^6 - 4*cos(d*x + c)^4)*sin(d*x + c))*log(-sin(d*x + c)
+ 1) - 14*(45*cos(d*x + c)^4 - 30*cos(d*x + c)^2 - 16)*sin(d*x + c) + 96)/(
3*a^3*d*cos(d*x + c)^6 - 4*a^3*d*cos(d*x + c)^4 + (a^3*d*cos(d*x + c)^6 - 4
*a^3*d*cos(d*x + c)^4)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^{5}\left(c+dx\right)}{\sin^{3}\left(c+dx\right)+3\sin^{2}\left(c+dx\right)+3\sin\left(c+dx\right)+1}\,dx}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**5/(sin(c + d*x)**3 + 3*sin(c + d*x)**2 + 3*sin(c + d
*x) + 1), x)/a**3
```

Giac [A]
time = 4.53, size = 136, normalized size = 0.80

$$\frac{\frac{420 \log\left(|\sin(dx+c)+1|\right)}{a^3} - \frac{420 \log\left(|\sin(dx+c)-1|\right)}{a^3} + \frac{10 \left(63 \sin(dx+c)^2 - 150 \sin(dx+c) + 91\right)}{a^3 (\sin(dx+c)-1)^2} - \frac{959 \sin(dx+c)^5 + 5395 \sin(dx+c)^4 + 12390 \sin(dx+c)^3 + 14710 \sin(dx+c)^2 + 9275 \sin(dx+c) + 2647}{a^3 (\sin(dx+c)+1)^5}}{5120\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/5120*(420*log(abs(sin(d*x + c) + 1))/a^3 - 420*log(abs(sin(d*x + c) - 1))
/a^3 + 10*(63*sin(d*x + c)^2 - 150*sin(d*x + c) + 91)/(a^3*(sin(d*x + c) -
1)^2) - (959*sin(d*x + c)^5 + 5395*sin(d*x + c)^4 + 12390*sin(d*x + c)^3 +
14710*sin(d*x + c)^2 + 9275*sin(d*x + c) + 2647)/(a^3*(sin(d*x + c) + 1)^5)
)/d
```

Mupad [B]
time = 4.77, size = 173, normalized size = 1.01

$$\frac{21\,\mathrm{atanh}(\sin(c+d\,x))}{128\,a^3\,d} - \frac{\frac{21\sin(c+d\,x)^6}{128} + \frac{63\sin(c+d\,x)^5}{128} + \frac{7\sin(c+d\,x)^4}{32} - \frac{21\sin(c+d\,x)^3}{32} - \frac{469\sin(c+d\,x)^2}{640} - \frac{7\sin(c+d\,x)}{640} + \frac{11}{40}}{d\,\left(a^3\sin(c+d\,x)^7 + 3\,a^3\sin(c+d\,x)^6 + a^3\sin(c+d\,x)^5 - 5\,a^3\sin(c+d\,x)^4 - 5\,a^3\sin(c+d\,x)^3 + a^3\sin(c+d\,x)^2 + 3\,a^3\,\sin(c+d\,x) + a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^5*(a + a*sin(c + d*x))^3),x)
```

```
[Out] (21*atanh(sin(c + d*x)))/(128*a^3*d) - ((7*sin(c + d*x)^4)/32 - (469*sin(c
+ d*x)^2)/640 - (21*sin(c + d*x)^3)/32 - (7*sin(c + d*x))/640 + (63*sin(c +
 d*x)^5)/128 + (21*sin(c + d*x)^6)/128 + 11/40)/(d*(3*a^3*sin(c + d*x) + a^
3 + a^3*sin(c + d*x)^2 - 5*a^3*sin(c + d*x)^3 - 5*a^3*sin(c + d*x)^4 + a^3*
sin(c + d*x)^5 + 3*a^3*sin(c + d*x)^6 + a^3*sin(c + d*x)^7))
```

3.88 $\int \frac{\cos^8(c+dx)}{(a+a\sin(c+dx))^8}\,dx$

Optimal. Leaf size=127

$$\frac{x}{a^8}-\frac{2\cos^7(c+dx)}{7ad(a+a\sin(c+dx))^7}+\frac{2\cos^5(c+dx)}{5a^3d(a+a\sin(c+dx))^5}-\frac{2\cos^3(c+dx)}{3a^2d\left(a^2+a^2\sin(c+dx)\right)^3}+\frac{2\cos(c+dx)}{d\left(a^8+a^8\sin(c+dx)\right)}$$

```
[Out] x/a^8-2/7*cos(d*x+c)^7/a/d/(a+a*sin(d*x+c))^7+2/5*cos(d*x+c)^5/a^3/d/(a+a*s
in(d*x+c))^5-2/3*cos(d*x+c)^3/a^2/d/(a^2+a^2*sin(d*x+c))^3+2*cos(d*x+c)/d/(
a^8+a^8*sin(d*x+c))
```

Rubi [A]
time = 0.13, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2759, 8}

$$\frac{2\cos(c+dx)}{d\left(a^8\sin(c+dx)+a^8\right)}+\frac{x}{a^8}+\frac{2\cos^5(c+dx)}{5a^3d(a\sin(c+dx)+a)^5}-\frac{2\cos^3(c+dx)}{3a^2d\left(a^2\sin(c+dx)+a^2\right)^3}-\frac{2\cos^7(c+dx)}{7ad(a\sin(c+dx)+a)^7}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^8/(a + a*Sin[c + d*x])^8,x]
```

```
[Out] x/a^8 - (2*Cos[c + d*x]^7)/(7*a*d*(a + a*Sin[c + d*x])^7) + (2*Cos[c + d*x]
^5)/(5*a^3*d*(a + a*Sin[c + d*x])^5) - (2*Cos[c + d*x]^3)/(3*a^2*d*(a^2 + a
^2*Sin[c + d*x])^3) + (2*Cos[c + d*x])/(d*(a^8 + a^8*Sin[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\int \frac{\cos^8(c+dx)}{(a+a\sin(c+dx))^8}\,dx = -\frac{2\cos^7(c+dx)}{7ad(a+a\sin(c+dx))^7} - \frac{\int \frac{\cos^6(c+dx)}{(a+a\sin(c+dx))^6}\,dx}{a^2}$$
$$= -\frac{2\cos^7(c+dx)}{7ad(a+a\sin(c+dx))^7} + \frac{2\cos^5(c+dx)}{5a^3d(a+a\sin(c+dx))^5} + \frac{\int \frac{\cos^4(c+dx)}{(a+a\sin(c+dx))^4}\,dx}{a^4}$$
$$= -\frac{2\cos^7(c+dx)}{7ad(a+a\sin(c+dx))^7} + \frac{2\cos^5(c+dx)}{5a^3d(a+a\sin(c+dx))^5} - \frac{2\cos^3(c+dx)}{3a^5d(a+a\sin(c+dx)}$$
$$= -\frac{2\cos^7(c+dx)}{7ad(a+a\sin(c+dx))^7} + \frac{2\cos^5(c+dx)}{5a^3d(a+a\sin(c+dx))^5} - \frac{2\cos^3(c+dx)}{3a^5d(a+a\sin(c+dx)}$$
$$= \frac{x}{a^8} - \frac{2\cos^7(c+dx)}{7ad(a+a\sin(c+dx))^7} + \frac{2\cos^5(c+dx)}{5a^3d(a+a\sin(c+dx))^5} - \frac{2\cos^3(c+dx)}{3a^5d(a+a\sin(c+}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 275 vs. 2(127) = 254.
time = 6.06, size = 275, normalized size = 2.17

$$-\frac{2\sqrt{2}\cos^9(c+dx)\left(-1+\frac{1}{2}(1-\sin(c+dx))\right)^4\left(\frac{\sin^{-1}\left(\frac{\sqrt{1-\sin(c+dx)}}{\sqrt{2}}\right)\sqrt{1-\sin(c+dx)}}{\sqrt{2}\sqrt{1+\frac{1}{2}(-1+\sin(c+dx))}}+\frac{1-\sin(c+dx)}{2\left(-1+\frac{1}{2}(1-\sin(c+dx))\right)}+\frac{(1-\sin(c+dx))^2}{12\left(-1+\frac{1}{2}(1-\sin(c+dx))\right)^2}+\frac{(1-\sin(c+dx))^3}{40\left(-1+\frac{1}{2}(1-\sin(c+dx))\right)^3}+\frac{(1-\sin(c+dx))^4}{112\left(-1+\frac{1}{2}(1-\sin(c+dx))\right)^4}\right)}{a^8d\left(1+\frac{1}{2}(-1+\sin(c+dx))\right)^{7/2}(1-\sin(c+dx))^5(1+\sin(c+dx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^8/(a + a*Sin[c + d*x])^8,x]
```

```
[Out] (-2*Sqrt[2]*Cos[c + d*x]^9*(-1 + (1 - Sin[c + d*x])/2)^4*((ArcSin[Sqrt[1 -
Sin[c + d*x]]/Sqrt[2]]*Sqrt[1 - Sin[c + d*x]])/(Sqrt[2]*Sqrt[1 + (-1 + Sin[
c + d*x])/2]) + (1 - Sin[c + d*x])/(2*(-1 + (1 - Sin[c + d*x])/2)) + (1 - S
in[c + d*x])^2/(12*(-1 + (1 - Sin[c + d*x])/2)^2) + (1 - Sin[c + d*x])^3/(4
0*(-1 + (1 - Sin[c + d*x])/2)^3) + (1 - Sin[c + d*x])^4/(112*(-1 + (1 - Sin
[c + d*x])/2)^4)))/(a^8*d*(1 + (-1 + Sin[c + d*x])/2)^(7/2)*(1 - Sin[c + d*
x])^5*(1 + Sin[c + d*x])^(9/2))
```

Maple [A]
time = 0.20, size = 110, normalized size = 0.87

method	result
risch	$\frac{x}{a^8}+\frac{48i\,e^{5i(dx+c)}+16\,e^{6i(dx+c)}-\frac{352i\,e^{3i(dx+c)}}{3}-\frac{352\,e^{4i(dx+c)}}{3}+\frac{464\,e^{2i(dx+c)}}{5}+\frac{464i\,e^{i(dx+c)}}{15}-\frac{704}{105}}{d\,a^8\left(e^{i(dx+c)}+i\right)^7}$
derivativedivides	$\frac{-\frac{256}{7\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^7}+\frac{128}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^6}-\frac{896}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}+\frac{128}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}-\frac{160}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}+\frac{16}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}}{d\,a^8}$

default	$\frac{-\frac{256}{7\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{7}}+\frac{128}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{6}}-\frac{896}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{5}}+\frac{128}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{4}}-\frac{160}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}+\frac{16}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\right.}}{d\,a^{8}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8/(a+a*sin(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/a^8*(-128/7/(tan(1/2*d*x+1/2*c)+1)^7+64/(tan(1/2*d*x+1/2*c)+1)^6-448/5/
(tan(1/2*d*x+1/2*c)+1)^5+64/(tan(1/2*d*x+1/2*c)+1)^4-80/3/(tan(1/2*d*x+1/2*
c)+1)^3+8/(tan(1/2*d*x+1/2*c)+1)^2+arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 295 vs. $2(121) = 242$.
time = 0.53, size = 295, normalized size = 2.32

$$\frac{2\left(\frac{8\left(\frac{133\sin(dx+c)}{\cos(dx+c)+1}+\frac{294\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{490\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{175\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{105\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+19\right)}{a^{8}+\frac{7a^{8}\sin(dx+c)}{\cos(dx+c)+1}+\frac{21a^{8}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{35a^{8}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{35a^{8}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{21a^{8}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+\frac{7a^{8}\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+\frac{a^{8}\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}}+\frac{105\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^{8}}\right)}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8/(a+a*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 2/105*(8*(133*sin(d*x + c)/(cos(d*x + c) + 1) + 294*sin(d*x + c)^2/(cos(d*x
 + c) + 1)^2 + 490*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 175*sin(d*x + c)^4
/(cos(d*x + c) + 1)^4 + 105*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 19)/(a^8
+ 7*a^8*sin(d*x + c)/(cos(d*x + c) + 1) + 21*a^8*sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 + 35*a^8*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 35*a^8*sin(d*x + c
)^4/(cos(d*x + c) + 1)^4 + 21*a^8*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 7*a
^8*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + a^8*sin(d*x + c)^7/(cos(d*x + c) +
 1)^7) + 105*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^8)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 244 vs. $2(121) = 242$.
time = 0.36, size = 244, normalized size = 1.92

$$\frac{(105\,dx-352)\cos(dx+c)^{4}-(315\,dx+568)\cos(dx+c)^{3}-24\,(35\,dx-31)\cos(dx+c)^{2}+840\,dx+60\,(7\,dx+12)\cos(dx+c)-\left((105\,dx+352)\cos(dx+c)^{3}+12\,(35\,dx-18)\cos(dx+c)^{2}-840\,dx-60\,(7\,dx+16)\cos(dx+c)-240\right)\sin(dx+c)-240}{105\left(a^{8}d\cos(dx+c)^{4}-3\,a^{8}d\cos(dx+c)^{3}-8\,a^{8}d\cos(dx+c)^{2}+4\,a^{8}d\cos(dx+c)+8\,a^{8}d-\left(a^{8}d\cos(dx+c)^{3}+4\,a^{8}d\cos(dx+c)^{2}-4\,a^{8}d\cos(dx+c)-8\,a^{8}d\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8/(a+a*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/105*((105*d*x - 352)*cos(d*x + c)^4 - (315*d*x + 568)*cos(d*x + c)^3 - 24
*(35*d*x - 31)*cos(d*x + c)^2 + 840*d*x + 60*(7*d*x + 12)*cos(d*x + c) - ((
105*d*x + 352)*cos(d*x + c)^3 + 12*(35*d*x - 18)*cos(d*x + c)^2 - 840*d*x -
 60*(7*d*x + 16)*cos(d*x + c) - 240)*sin(d*x + c) - 240)/(a^8*d*cos(d*x + c
)^4 - 3*a^8*d*cos(d*x + c)^3 - 8*a^8*d*cos(d*x + c)^2 + 4*a^8*d*cos(d*x + c
) + 8*a^8*d - (a^8*d*cos(d*x + c)^3 + 4*a^8*d*cos(d*x + c)^2 - 4*a^8*d*cos(
d*x + c) - 8*a^8*d)*sin(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8/(a+a*sin(d*x+c))**8,x)
```

```
[Out] Timed out
```

Giac [A]
time = 7.13, size = 99, normalized size = 0.78

$$\frac{\frac{105\,(dx+c)}{a^8}+\frac{16\left(105\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+175\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+490\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+294\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+133\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+19\right)}{a^8\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^7}}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8/(a+a*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/105*(105*(d*x + c)/a^8 + 16*(105*tan(1/2*d*x + 1/2*c)^5 + 175*tan(1/2*d*x
 + 1/2*c)^4 + 490*tan(1/2*d*x + 1/2*c)^3 + 294*tan(1/2*d*x + 1/2*c)^2 + 133
*tan(1/2*d*x + 1/2*c) + 19)/(a^8*(tan(1/2*d*x + 1/2*c) + 1)^7))/d
```

Mupad [B]
time = 7.77, size = 91, normalized size = 0.72

$$\frac{x}{a^8}+\frac{16\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\frac{80\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{3}+\frac{224\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}+\frac{224\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{5}+\frac{304\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{15}+\frac{304}{105}}{a^8\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^8/(a + a*sin(c + d*x))^8,x)
```

```
[Out] x/a^8 + ((304*tan(c/2 + (d*x)/2))/15 + (224*tan(c/2 + (d*x)/2)^2)/5 + (224*
tan(c/2 + (d*x)/2)^3)/3 + (80*tan(c/2 + (d*x)/2)^4)/3 + 16*tan(c/2 + (d*x)/
2)^5 + 304/105)/(a^8*d*(tan(c/2 + (d*x)/2) + 1)^7)
```

3.89 $\int \frac{\cos^7(c+dx)}{(a+a\sin(c+dx))^8}\,dx$

Optimal. Leaf size=36

$$-\frac{(a-a\sin(c+dx))^4}{8d\left(a^3+a^3\sin(c+dx)\right)^4}$$

```
[Out] -1/8*(a-a*sin(d*x+c))^4/d/(a^3+a^3*sin(d*x+c))^4
```

Rubi [A]

time = 0.03, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2746, 37}

$$-\frac{(a-a\sin(c+dx))^4}{8d\left(a^3\sin(c+dx)+a^3\right)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7/(a + a*Sin[c + d*x])^8,x]
```

```
[Out] -1/8*(a - a*Sin[c + d*x])^4/(d*(a^3 + a^3*Sin[c + d*x])^4)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^7(c+dx)}{(a+a\sin(c+dx))^8}\,dx &= \frac{\text{Subst}\left(\int \frac{(a-x)^3}{(a+x)^5}\,dx, x, a\sin(c+dx)\right)}{a^7d}\\ &= -\frac{(a-a\sin(c+dx))^4}{8d\left(a^3+a^3\sin(c+dx)\right)^4}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 28, normalized size = 0.78

$$-\frac{\cos^8(c+dx)}{8a^8d(1+\sin(c+dx))^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7/(a + a*Sin[c + d*x])^8,x]
[Out] -1/8*Cos[c + d*x]^8/(a^8*d*(1 + Sin[c + d*x])^8)
```

Maple [A]
time = 0.19, size = 55, normalized size = 1.53

method	result	size
derivativedivides	$\frac{-\frac{2}{(1+\sin(dx+c))^4}+\frac{1}{1+\sin(dx+c)}+\frac{4}{(1+\sin(dx+c))^3}-\frac{3}{(1+\sin(dx+c))^2}}{d\,a^8}$	55
default	$\frac{-\frac{2}{(1+\sin(dx+c))^4}+\frac{1}{1+\sin(dx+c)}+\frac{4}{(1+\sin(dx+c))^3}-\frac{3}{(1+\sin(dx+c))^2}}{d\,a^8}$	55
risch	$\frac{2i\left(e^{7i(dx+c)}-7\,e^{5i(dx+c)}+7\,e^{3i(dx+c)}-e^{i(dx+c)}\right)}{d\,a^8\left(e^{i(dx+c)}+i\right)^8}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7/(a+a*sin(d*x+c))^8,x,method=_RETURNVERBOSE)
[Out] 1/d/a^8*(-2/(1+sin(d*x+c))^4+1/(1+sin(d*x+c))+4/(1+sin(d*x+c))^3-3/(1+sin(d
*x+c))^2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(35) = 70$.
time = 0.28, size = 74, normalized size = 2.06

$$\frac{\sin(dx+c)^3+\sin(dx+c)}{\left(a^8\sin(dx+c)^4+4\,a^8\sin(dx+c)^3+6\,a^8\sin(dx+c)^2+4\,a^8\sin(dx+c)+a^8\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+a*sin(d*x+c))^8,x, algorithm="maxima")
[Out] (sin(d*x + c)^3 + sin(d*x + c))/((a^8*sin(d*x + c)^4 + 4*a^8*sin(d*x + c)^3
 + 6*a^8*sin(d*x + c)^2 + 4*a^8*sin(d*x + c) + a^8)*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(35) = 70$.
time = 0.35, size = 82, normalized size = 2.28

$$-\frac{\left(\cos(dx+c)^2-2\right)\sin(dx+c)}{a^8d\cos(dx+c)^4-8\,a^8d\cos(dx+c)^2+8\,a^8d-4\left(a^8d\cos(dx+c)^2-2\,a^8d\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+a*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] -(cos(d*x + c)^2 - 2)*sin(d*x + c)/(a^8*d*cos(d*x + c)^4 - 8*a^8*d*cos(d*x
+ c)^2 + 8*a^8*d - 4*(a^8*d*cos(d*x + c)^2 - 2*a^8*d)*sin(d*x + c))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 2006 vs. $2(31) = 62$.
time = 14.55, size = 2006, normalized size = 55.72

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7/(a+a*sin(d*x+c))**8,x)
```

```
[Out] Piecewise((16*sin(c + d*x)**6/(35*a**8*d*sin(c + d*x)**7 + 245*a**8*d*sin(c
 + d*x)**6 + 735*a**8*d*sin(c + d*x)**5 + 1225*a**8*d*sin(c + d*x)**4 + 122
5*a**8*d*sin(c + d*x)**3 + 735*a**8*d*sin(c + d*x)**2 + 245*a**8*d*sin(c +
d*x) + 35*a**8*d) + 77*sin(c + d*x)**5/(35*a**8*d*sin(c + d*x)**7 + 245*a**
8*d*sin(c + d*x)**6 + 735*a**8*d*sin(c + d*x)**5 + 1225*a**8*d*sin(c + d*x)
**4 + 1225*a**8*d*sin(c + d*x)**3 + 735*a**8*d*sin(c + d*x)**2 + 245*a**8*d
*sin(c + d*x) + 35*a**8*d) - 8*sin(c + d*x)**4*cos(c + d*x)**2/(35*a**8*d*s
in(c + d*x)**7 + 245*a**8*d*sin(c + d*x)**6 + 735*a**8*d*sin(c + d*x)**5 +
1225*a**8*d*sin(c + d*x)**4 + 1225*a**8*d*sin(c + d*x)**3 + 735*a**8*d*sin(
c + d*x)**2 + 245*a**8*d*sin(c + d*x) + 35*a**8*d) + 155*sin(c + d*x)**4/(3
5*a**8*d*sin(c + d*x)**7 + 245*a**8*d*sin(c + d*x)**6 + 735*a**8*d*sin(c +
d*x)**5 + 1225*a**8*d*sin(c + d*x)**4 + 1225*a**8*d*sin(c + d*x)**3 + 735*a
**8*d*sin(c + d*x)**2 + 245*a**8*d*sin(c + d*x) + 35*a**8*d) - 21*sin(c + d
*x)**3*cos(c + d*x)**2/(35*a**8*d*sin(c + d*x)**7 + 245*a**8*d*sin(c + d*x)
**6 + 735*a**8*d*sin(c + d*x)**5 + 1225*a**8*d*sin(c + d*x)**4 + 1225*a**8*
d*sin(c + d*x)**3 + 735*a**8*d*sin(c + d*x)**2 + 245*a**8*d*sin(c + d*x) +
35*a**8*d) + 168*sin(c + d*x)**3/(35*a**8*d*sin(c + d*x)**7 + 245*a**8*d*si
n(c + d*x)**6 + 735*a**8*d*sin(c + d*x)**5 + 1225*a**8*d*sin(c + d*x)**4 +
1225*a**8*d*sin(c + d*x)**3 + 735*a**8*d*sin(c + d*x)**2 + 245*a**8*d*sin(c
 + d*x) + 35*a**8*d) + 6*sin(c + d*x)**2*cos(c + d*x)**4/(35*a**8*d*sin(c +
 d*x)**7 + 245*a**8*d*sin(c + d*x)**6 + 735*a**8*d*sin(c + d*x)**5 + 1225*a
**8*d*sin(c + d*x)**4 + 1225*a**8*d*sin(c + d*x)**3 + 735*a**8*d*sin(c + d*
x)**2 + 245*a**8*d*sin(c + d*x) + 35*a**8*d) - 19*sin(c + d*x)**2*cos(c + d
*x)**2/(35*a**8*d*sin(c + d*x)**7 + 245*a**8*d*sin(c + d*x)**6 + 735*a**8*d
*sin(c + d*x)**5 + 1225*a**8*d*sin(c + d*x)**4 + 1225*a**8*d*sin(c + d*x)**
3 + 735*a**8*d*sin(c + d*x)**2 + 245*a**8*d*sin(c + d*x) + 35*a**8*d) + 104
*sin(c + d*x)**2/(35*a**8*d*sin(c + d*x)**7 + 245*a**8*d*sin(c + d*x)**6 +
735*a**8*d*sin(c + d*x)**5 + 1225*a**8*d*sin(c + d*x)**4 + 1225*a**8*d*sin(
c + d*x)**3 + 735*a**8*d*sin(c + d*x)**2 + 245*a**8*d*sin(c + d*x) + 35*a**
8*d) + 7*sin(c + d*x)*cos(c + d*x)**4/(35*a**8*d*sin(c + d*x)**7 + 245*a**8
```

```
*d*sin(c + d*x)**6 + 735*a**8*d*sin(c + d*x)**5 + 1225*a**8*d*sin(c + d*x)*
*4 + 1225*a**8*d*sin(c + d*x)**3 + 735*a**8*d*sin(c + d*x)**2 + 245*a**8*d*
sin(c + d*x) + 35*a**8*d) - 7*sin(c + d*x)*cos(c + d*x)**2/(35*a**8*d*sin(c
 + d*x)**7 + 245*a**8*d*sin(c + d*x)**6 + 735*a**8*d*sin(c + d*x)**5 + 1225
*a**8*d*sin(c + d*x)**4 + 1225*a**8*d*sin(c + d*x)**3 + 735*a**8*d*sin(c +
d*x)**2 + 245*a**8*d*sin(c + d*x) + 35*a**8*d) + 35*sin(c + d*x)/(35*a**8*d
*sin(c + d*x)**7 + 245*a**8*d*sin(c + d*x)**6 + 735*a**8*d*sin(c + d*x)**5
+ 1225*a**8*d*sin(c + d*x)**4 + 1225*a**8*d*sin(c + d*x)**3 + 735*a**8*d*si
n(c + d*x)**2 + 245*a**8*d*sin(c + d*x) + 35*a**8*d) - 5*cos(c + d*x)**6/(3
5*a**8*d*sin(c + d*x)**7 + 245*a**8*d*sin(c + d*x)**6 + 735*a**8*d*sin(c +
d*x)**5 + 1225*a**8*d*sin(c + d*x)**4 + 1225*a**8*d*sin(c + d*x)**3 + 735*a
**8*d*sin(c + d*x)**2 + 245*a**8*d*sin(c + d*x) + 35*a**8*d) + cos(c + d*x)
**4/(35*a**8*d*sin(c + d*x)**7 + 245*a**8*d*sin(c + d*x)**6 + 735*a**8*d*si
n(c + d*x)**5 + 1225*a**8*d*sin(c + d*x)**4 + 1225*a**8*d*sin(c + d*x)**3 +
 735*a**8*d*sin(c + d*x)**2 + 245*a**8*d*sin(c + d*x) + 35*a**8*d) - cos(c
+ d*x)**2/(35*a**8*d*sin(c + d*x)**7 + 245*a**8*d*sin(c + d*x)**6 + 735*a**
8*d*sin(c + d*x)**5 + 1225*a**8*d*sin(c + d*x)**4 + 1225*a**8*d*sin(c + d*x
)**3 + 735*a**8*d*sin(c + d*x)**2 + 245*a**8*d*sin(c + d*x) + 35*a**8*d) +
5/(35*a**8*d*sin(c + d*x)**7 + 245*a**8*d*sin(c + d*x)**6 + 735*a**8*d*sin(
c + d*x)**5 + 1225*a**8*d*sin(c + d*x)**4 + 1225*a**8*d*sin(c + d*x)**3 + 7
35*a**8*d*sin(c + d*x)**2 + 245*a**8*d*sin(c + d*x) + 35*a**8*d), Ne(d, 0))
, (x*cos(c)**7/(a*sin(c) + a)**8, True))
```

Giac [**A**]
time = 7.00, size = 68, normalized size = 1.89

$$\frac{2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+7\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+7\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{a^{8}d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+a*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 2*(tan(1/2*d*x + 1/2*c)^7 + 7*tan(1/2*d*x + 1/2*c)^5 + 7*tan(1/2*d*x + 1/2*
c)^3 + tan(1/2*d*x + 1/2*c))/(a^8*d*(tan(1/2*d*x + 1/2*c) + 1)^8)
```

Mupad [**B**]
time = 0.07, size = 64, normalized size = 1.78

$$\frac{\frac{1}{a^{8}\,\left(\sin(c+d\,x)+1\right)}-\frac{3}{a^{8}\,\left(\sin(c+d\,x)+1\right)^{2}}+\frac{4}{a^{8}\,\left(\sin(c+d\,x)+1\right)^{3}}-\frac{2}{a^{8}\,\left(\sin(c+d\,x)+1\right)^{4}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7/(a + a*sin(c + d*x))^8,x)
```

```
[Out] (1/(a^8*(sin(c + d*x) + 1)) - 3/(a^8*(sin(c + d*x) + 1)^2) + 4/(a^8*(sin(c
+ d*x) + 1)^3) - 2/(a^8*(sin(c + d*x) + 1)^4))/d
```

3.90 $\int \frac{\cos^6(c+dx)}{(a+a\sin(c+dx))^8}\,dx$

Optimal. Leaf size=58

$$-\frac{\cos^7(c+dx)}{9d(a+a\sin(c+dx))^8}-\frac{\cos^7(c+dx)}{63ad(a+a\sin(c+dx))^7}$$

```
[Out] -1/9*cos(d*x+c)^7/d/(a+a*sin(d*x+c))^8-1/63*cos(d*x+c)^7/a/d/(a+a*sin(d*x+c
))^7
```

Rubi [A]

time = 0.06, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2751, 2750}

$$-\frac{\cos^7(c+dx)}{63ad(a\sin(c+dx)+a)^7}-\frac{\cos^7(c+dx)}{9d(a\sin(c+dx)+a)^8}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6/(a + a*Sin[c + d*x])^8,x]
```

```
[Out] -1/9*Cos[c + d*x]^7/(d*(a + a*Sin[c + d*x])^8) - Cos[c + d*x]^7/(63*a*d*(a
+ a*Sin[c + d*x])^7)
```

Rule 2750

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{\cos^6(c+dx)}{(a+a\sin(c+dx))^8}\,dx = -\frac{\cos^7(c+dx)}{9d(a+a\sin(c+dx))^8} + \frac{\int \frac{\cos^6(c+dx)}{(a+a\sin(c+dx))^7}\,dx}{9a}$$
$$= -\frac{\cos^7(c+dx)}{9d(a+a\sin(c+dx))^8} - \frac{\cos^7(c+dx)}{63ad(a+a\sin(c+dx))^7}$$

Mathematica [A]
time = 0.06, size = 36, normalized size = 0.62

$$-\frac{\cos^7(c+dx)(8+\sin(c+dx))}{63a^8d(1+\sin(c+dx))^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6/(a + a*Sin[c + d*x])^8,x]
```

```
[Out] -1/63*(Cos[c + d*x]^7*(8 + Sin[c + d*x]))/(a^8*d*(1 + Sin[c + d*x])^8)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 144 vs. 2(54) = 108.
time = 0.22, size = 145, normalized size = 2.50

method	result
risch	$\frac{2i\left(-105i\,e^{6i(dx+c)}+63\,e^{7i(dx+c)}+189i\,e^{4i(dx+c)}-315\,e^{5i(dx+c)}-27i\,e^{2i(dx+c)}+189\,e^{3i(dx+c)}-i-9\,e^{i(dx+c)}\right)}{63d\,a^8\left(e^{i(dx+c)}+i\right)^9}$
derivativedivides	$\frac{-\frac{1856}{7\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^7}+\frac{152}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}+\frac{128}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^8}-\frac{172}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}+\frac{14}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{256}{9\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}}{d\,a^8}$
default	$\frac{-\frac{1856}{7\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^7}+\frac{152}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}+\frac{128}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^8}-\frac{172}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}+\frac{14}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{256}{9\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}}{d\,a^8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6/(a+a*sin(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/a^8*(-928/7/(tan(1/2*d*x+1/2*c)+1)^7+76/(tan(1/2*d*x+1/2*c)+1)^4+64/(ta
n(1/2*d*x+1/2*c)+1)^8-86/3/(tan(1/2*d*x+1/2*c)+1)^3+7/(tan(1/2*d*x+1/2*c)+1
)^2-128/9/(tan(1/2*d*x+1/2*c)+1)^9-136/(tan(1/2*d*x+1/2*c)+1)^5-1/(tan(1/2*
d*x+1/2*c)+1)+496/3/(tan(1/2*d*x+1/2*c)+1)^6)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 375 vs. 2(54) = 108.
time = 0.32, size = 375, normalized size = 6.47

$$-\frac{2\left(\frac{9\sin(dx+c)}{\cos(dx+c)+1}+\frac{225\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{189\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{693\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{315\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{483\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{63\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{63\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+8\right)}{63\left(a^8+\frac{9a^8\sin(dx+c)}{\cos(dx+c)+1}+\frac{36a^8\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{84a^8\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{126a^8\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{126a^8\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{84a^8\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{36a^8\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{9a^8\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{a^8\sin(dx+c)^9}{(\cos(dx+c)+1)^9}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+a*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -2/63*(9*sin(d*x + c)/(cos(d*x + c) + 1) + 225*sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 + 189*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 693*sin(d*x + c)^4/(cos
(d*x + c) + 1)^4 + 315*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 483*sin(d*x +
c)^6/(cos(d*x + c) + 1)^6 + 63*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 63*sin
(d*x + c)^8/(cos(d*x + c) + 1)^8 + 8)/((a^8 + 9*a^8*sin(d*x + c)/(cos(d*x +
 c) + 1) + 36*a^8*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 84*a^8*sin(d*x + c)
^3/(cos(d*x + c) + 1)^3 + 126*a^8*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 126
*a^8*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 84*a^8*sin(d*x + c)^6/(cos(d*x +
 c) + 1)^6 + 36*a^8*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 9*a^8*sin(d*x + c
)^8/(cos(d*x + c) + 1)^8 + a^8*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 239 vs. $2(54) = 108$.
time = 0.34, size = 239, normalized size = 4.12

$$\frac{\cos\left(dx+c\right)^{5}-4\,\cos\left(dx+c\right)^{4}+19\,\cos\left(dx+c\right)^{3}+52\,\cos\left(dx+c\right)^{2}-\left(\cos\left(dx+c\right)^{4}+5\,\cos\left(dx+c\right)^{3}+24\,\cos\left(dx+c\right)^{2}-28\,\cos\left(dx+c\right)-56\right)\sin\left(dx+c\right)-28\,\cos\left(dx+c\right)-56}{63\,\left(a^{8}d\cos\left(dx+c\right)^{5}+5\,a^{8}d\cos\left(dx+c\right)^{4}-8\,a^{8}d\cos\left(dx+c\right)^{3}-20\,a^{8}d\cos\left(dx+c\right)^{2}+8\,a^{8}d\cos\left(dx+c\right)+16\,a^{8}d+\left(a^{8}d\cos\left(dx+c\right)^{4}-4\,a^{8}d\cos\left(dx+c\right)^{3}-12\,a^{8}d\cos\left(dx+c\right)^{2}+8\,a^{8}d\cos\left(dx+c\right)+16\,a^{8}d\right)\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+a*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/63*(cos(d*x + c)^5 - 4*cos(d*x + c)^4 + 19*cos(d*x + c)^3 + 52*cos(d*x +
c)^2 - (cos(d*x + c)^4 + 5*cos(d*x + c)^3 + 24*cos(d*x + c)^2 - 28*cos(d*x
+ c) - 56)*sin(d*x + c) - 28*cos(d*x + c) - 56)/(a^8*d*cos(d*x + c)^5 + 5*a
^8*d*cos(d*x + c)^4 - 8*a^8*d*cos(d*x + c)^3 - 20*a^8*d*cos(d*x + c)^2 + 8*
a^8*d*cos(d*x + c) + 16*a^8*d + (a^8*d*cos(d*x + c)^4 - 4*a^8*d*cos(d*x + c
)^3 - 12*a^8*d*cos(d*x + c)^2 + 8*a^8*d*cos(d*x + c) + 16*a^8*d)*sin(d*x +
c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6/(a+a*sin(d*x+c))**8,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(54) = 108$.
time = 7.32, size = 125, normalized size = 2.16

$$-\frac{2\,\left(63\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}+63\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+483\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+315\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+693\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+189\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+225\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+8\right)}{63\,a^{8}d{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^6/(a+a*sin(d*x+c))^8,x, algorithm="giac")

[Out] -2/63*(63*tan(1/2*d*x + 1/2*c)^8 + 63*tan(1/2*d*x + 1/2*c)^7 + 483*tan(1/2*d*x + 1/2*c)^6 + 315*tan(1/2*d*x + 1/2*c)^5 + 693*tan(1/2*d*x + 1/2*c)^4 + 189*tan(1/2*d*x + 1/2*c)^3 + 225*tan(1/2*d*x + 1/2*c)^2 + 9*tan(1/2*d*x + 1/2*c) + 8)/(a^8*d*(tan(1/2*d*x + 1/2*c) + 1)^9)

Mupad [B]
time = 6.63, size = 118, normalized size = 2.03

$$-\frac{\sqrt{2}\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{63\,\sin\left(c+d\,x\right)}{2}-\frac{257\,\cos\left(c+d\,x\right)}{8}-\frac{113\,\cos\left(2\,c+2\,d\,x\right)}{4}+\frac{37\,\cos\left(3\,c+3\,d\,x\right)}{8}+\frac{7\,\cos\left(4\,c+4\,d\,x\right)}{16}-\frac{63\,\sin\left(2\,c+2\,d\,x\right)}{8}-\frac{9\,\sin\left(3\,c+3\,d\,x\right)}{2}+\frac{9\,\sin\left(4\,c+4\,d\,x\right)}{16}+\frac{1013}{16}\right)}{1008\,a^8\,d\,{\cos\left(\frac{c}{2}-\frac{\pi }{4}+\frac{d\,x}{2}\right)}^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^6/(a + a*sin(c + d*x))^8,x)

[Out] -(2^(1/2)*cos(c/2 + (d*x)/2)*((63*sin(c + d*x))/2 - (257*cos(c + d*x))/8 - (113*cos(2*c + 2*d*x))/4 + (37*cos(3*c + 3*d*x))/8 + (7*cos(4*c + 4*d*x))/16 - (63*sin(2*c + 2*d*x))/8 - (9*sin(3*c + 3*d*x))/2 + (9*sin(4*c + 4*d*x))/16 + 1013/16))/(1008*a^8*d*cos(c/2 - pi/4 + (d*x)/2)^9)

3.91 $\int \frac{\cos^5(c+dx)}{(a+a\sin(c+dx))^8}\,dx$

Optimal. Leaf size=65

$$-\frac{4}{5a^3d(a+a\sin(c+dx))^5}-\frac{1}{3a^5d(a+a\sin(c+dx))^3}+\frac{1}{d\left(a^2+a^2\sin(c+dx)\right)^4}$$

```
[Out] -4/5/a^3/d/(a+a*sin(d*x+c))^5-1/3/a^5/d/(a+a*sin(d*x+c))^3+1/d/(a^2+a^2*sin
(d*x+c))^4
```

Rubi [A]
time = 0.04, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2746, 45}

$$-\frac{1}{3a^5d(a\sin(c+dx)+a)^3}-\frac{4}{5a^3d(a\sin(c+dx)+a)^5}+\frac{1}{d\left(a^2\sin(c+dx)+a^2\right)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/(a + a*Sin[c + d*x])^8,x]
```

```
[Out] -4/(5*a^3*d*(a + a*Sin[c + d*x])^5) - 1/(3*a^5*d*(a + a*Sin[c + d*x])^3) +
1/(d*(a^2 + a^2*Sin[c + d*x])^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\int \frac{\cos^5(c+dx)}{(a+a\sin(c+dx))^8}\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^2}{(a+x)^6}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{4a^2}{(a+x)^6} - \frac{4a}{(a+x)^5} + \frac{1}{(a+x)^4}\right)dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= -\frac{4}{5a^3 d(a+a\sin(c+dx))^5} - \frac{1}{3a^5 d(a+a\sin(c+dx))^3} + \frac{1}{d\left(a^2+a^2\sin(c+dx)\right)}$$

Mathematica [A]
time = 0.08, size = 58, normalized size = 0.89

$$\frac{\cos^6(c+dx)\left(2-5\sin(c+dx)+5\sin^2(c+dx)\right)}{15a^8 d(-1+\sin(c+dx))^3(1+\sin(c+dx))^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/(a + a*Sin[c + d*x])^8,x]
```

```
[Out] (Cos[c + d*x]^6*(2 - 5*Sin[c + d*x] + 5*Sin[c + d*x]^2))/(15*a^8*d*(-1 + Si
n[c + d*x])^3*(1 + Sin[c + d*x])^8)
```

Maple [A]
time = 0.22, size = 43, normalized size = 0.66

method	result	size
derivativedivides	$\frac{-\frac{4}{5(1+\sin(dx+c))^5}+\frac{1}{(1+\sin(dx+c))^4}-\frac{1}{3(1+\sin(dx+c))^3}}{d\,a^8}$	43
default	$\frac{-\frac{4}{5(1+\sin(dx+c))^5}+\frac{1}{(1+\sin(dx+c))^4}-\frac{1}{3(1+\sin(dx+c))^3}}{d\,a^8}$	43
risch	$\frac{8i\left(-10i e^{6i(dx+c)}+5\,e^{7i(dx+c)}+10i e^{4i(dx+c)}-18\,e^{5i(dx+c)}+5\,e^{3i(dx+c)}\right)}{15d\,a^8\left(e^{i(dx+c)}+i\right)^{10}}$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a+a*sin(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^8*(-4/5/(1+sin(d*x+c))^5+1/(1+sin(d*x+c))^4-1/3/(1+sin(d*x+c))^3)
```

Maxima [A]
time = 0.32, size = 93, normalized size = 1.43

$$-\frac{5\,\sin\left(dx+c\right)^2-5\,\sin\left(dx+c\right)+2}{15\left(a^8\sin\left(dx+c\right)^5+5\,a^8\sin\left(dx+c\right)^4+10\,a^8\sin\left(dx+c\right)^3+10\,a^8\sin\left(dx+c\right)^2+5\,a^8\sin\left(dx+c\right)+a^8\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/15*(5*sin(d*x + c)^2 - 5*sin(d*x + c) + 2)/((a^8*sin(d*x + c)^5 + 5*a^8*
sin(d*x + c)^4 + 10*a^8*sin(d*x + c)^3 + 10*a^8*sin(d*x + c)^2 + 5*a^8*sin(
d*x + c) + a^8)*d)
```

Fricas [A]
time = 0.36, size = 100, normalized size = 1.54

$$\frac{5\cos(dx+c)^2+5\sin(dx+c)-7}{15\left(5a^8d\cos(dx+c)^4-20a^8d\cos(dx+c)^2+16a^8d+\left(a^8d\cos(dx+c)^4-12a^8d\cos(dx+c)^2+16a^8d\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/15*(5*cos(d*x + c)^2 + 5*sin(d*x + c) - 7)/(5*a^8*d*cos(d*x + c)^4 - 20*a
^8*d*cos(d*x + c)^2 + 16*a^8*d + (a^8*d*cos(d*x + c)^4 - 12*a^8*d*cos(d*x +
 c)^2 + 16*a^8*d)*sin(d*x + c))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1120 vs. $2(58) = 116$.
time = 14.30, size = 1120, normalized size = 17.23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a+a*sin(d*x+c))**8,x)
```

```
[Out] Piecewise((-8*sin(c + d*x)**4/(105*a**8*d*sin(c + d*x)**7 + 735*a**8*d*sin(
c + d*x)**6 + 2205*a**8*d*sin(c + d*x)**5 + 3675*a**8*d*sin(c + d*x)**4 + 3
675*a**8*d*sin(c + d*x)**3 + 2205*a**8*d*sin(c + d*x)**2 + 735*a**8*d*sin(c
 + d*x) + 105*a**8*d) - 21*sin(c + d*x)**3/(105*a**8*d*sin(c + d*x)**7 + 73
5*a**8*d*sin(c + d*x)**6 + 2205*a**8*d*sin(c + d*x)**5 + 3675*a**8*d*sin(c
+ d*x)**4 + 3675*a**8*d*sin(c + d*x)**3 + 2205*a**8*d*sin(c + d*x)**2 + 735
*a**8*d*sin(c + d*x) + 105*a**8*d) + 12*sin(c + d*x)**2*cos(c + d*x)**2/(10
5*a**8*d*sin(c + d*x)**7 + 735*a**8*d*sin(c + d*x)**6 + 2205*a**8*d*sin(c +
 d*x)**5 + 3675*a**8*d*sin(c + d*x)**4 + 3675*a**8*d*sin(c + d*x)**3 + 2205
*a**8*d*sin(c + d*x)**2 + 735*a**8*d*sin(c + d*x) + 105*a**8*d) - 19*sin(c
+ d*x)**2/(105*a**8*d*sin(c + d*x)**7 + 735*a**8*d*sin(c + d*x)**6 + 2205*a
**8*d*sin(c + d*x)**5 + 3675*a**8*d*sin(c + d*x)**4 + 3675*a**8*d*sin(c + d
*x)**3 + 2205*a**8*d*sin(c + d*x)**2 + 735*a**8*d*sin(c + d*x) + 105*a**8*d
) + 14*sin(c + d*x)*cos(c + d*x)**2/(105*a**8*d*sin(c + d*x)**7 + 735*a**8*
d*sin(c + d*x)**6 + 2205*a**8*d*sin(c + d*x)**5 + 3675*a**8*d*sin(c + d*x)*
*4 + 3675*a**8*d*sin(c + d*x)**3 + 2205*a**8*d*sin(c + d*x)**2 + 735*a**8*d
*sin(c + d*x) + 105*a**8*d) - 7*sin(c + d*x)/(105*a**8*d*sin(c + d*x)**7 +
735*a**8*d*sin(c + d*x)**6 + 2205*a**8*d*sin(c + d*x)**5 + 3675*a**8*d*sin(
c + d*x)**4 + 3675*a**8*d*sin(c + d*x)**3 + 2205*a**8*d*sin(c + d*x)**2 + 7
```

```
35*a**8*d*sin(c + d*x) + 105*a**8*d) - 15*cos(c + d*x)**4/(105*a**8*d*sin(c
 + d*x)**7 + 735*a**8*d*sin(c + d*x)**6 + 2205*a**8*d*sin(c + d*x)**5 + 367
5*a**8*d*sin(c + d*x)**4 + 3675*a**8*d*sin(c + d*x)**3 + 2205*a**8*d*sin(c
+ d*x)**2 + 735*a**8*d*sin(c + d*x) + 105*a**8*d) + 2*cos(c + d*x)**2/(105*
a**8*d*sin(c + d*x)**7 + 735*a**8*d*sin(c + d*x)**6 + 2205*a**8*d*sin(c + d
*x)**5 + 3675*a**8*d*sin(c + d*x)**4 + 3675*a**8*d*sin(c + d*x)**3 + 2205*a
**8*d*sin(c + d*x)**2 + 735*a**8*d*sin(c + d*x) + 105*a**8*d) - 1/(105*a**8
*d*sin(c + d*x)**7 + 735*a**8*d*sin(c + d*x)**6 + 2205*a**8*d*sin(c + d*x)*
*5 + 3675*a**8*d*sin(c + d*x)**4 + 3675*a**8*d*sin(c + d*x)**3 + 2205*a**8*
d*sin(c + d*x)**2 + 735*a**8*d*sin(c + d*x) + 105*a**8*d), Ne(d, 0)), (x*co
s(c)**5/(a*sin(c) + a)**8, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(61) = 122$.
time = 6.57, size = 137, normalized size = 2.11

$$\frac{2\left(15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+30\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}+140\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+170\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+282\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+170\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+140\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+30\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{15\,a^{8}d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 2/15*(15*tan(1/2*d*x + 1/2*c)^9 + 30*tan(1/2*d*x + 1/2*c)^8 + 140*tan(1/2*d
*x + 1/2*c)^7 + 170*tan(1/2*d*x + 1/2*c)^6 + 282*tan(1/2*d*x + 1/2*c)^5 + 1
70*tan(1/2*d*x + 1/2*c)^4 + 140*tan(1/2*d*x + 1/2*c)^3 + 30*tan(1/2*d*x + 1
/2*c)^2 + 15*tan(1/2*d*x + 1/2*c))/(a^8*d*(tan(1/2*d*x + 1/2*c) + 1)^10)
```

Mupad [B]
time = 4.71, size = 54, normalized size = 0.83

$$\frac{1}{a^{8}\,d\,{\left(\sin\left(c+d\,x\right)+1\right)}^{4}}-\frac{1}{3\,a^{8}\,d\,{\left(\sin\left(c+d\,x\right)+1\right)}^{3}}-\frac{4}{5\,a^{8}\,d\,{\left(\sin\left(c+d\,x\right)+1\right)}^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(a + a*sin(c + d*x))^8,x)
```

```
[Out] 1/(a^8*d*(sin(c + d*x) + 1)^4) - 1/(3*a^8*d*(sin(c + d*x) + 1)^3) - 4/(5*a^
8*d*(sin(c + d*x) + 1)^5)
```

3.92 $\int \frac{\cos^4(c+dx)}{(a+a\sin(c+dx))^8}\,dx$

Optimal. Leaf size=118

$$-\frac{\cos^5(c+dx)}{11d(a+a\sin(c+dx))^8}-\frac{\cos^5(c+dx)}{33ad(a+a\sin(c+dx))^7}-\frac{2\cos^5(c+dx)}{231a^2d(a+a\sin(c+dx))^6}-\frac{2\cos^5(c+dx)}{1155a^3d(a+a\sin(c+a$$

```
[Out] -1/11*cos(d*x+c)^5/d/(a+a*sin(d*x+c))^8-1/33*cos(d*x+c)^5/a/d/(a+a*sin(d*x+
c))^7-2/231*cos(d*x+c)^5/a^2/d/(a+a*sin(d*x+c))^6-2/1155*cos(d*x+c)^5/a^3/d
/(a+a*sin(d*x+c))^5
```

Rubi [A]
time = 0.12, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2751, 2750}

$$-\frac{2\cos^5(c+dx)}{1155a^3d(a\sin(c+dx)+a)^5}-\frac{2\cos^5(c+dx)}{231a^2d(a\sin(c+dx)+a)^6}-\frac{\cos^5(c+dx)}{33ad(a\sin(c+dx)+a)^7}-\frac{\cos^5(c+dx)}{11d(a\sin(c+dx)+a)^8}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + a*Sin[c + d*x])^8,x]
```

```
[Out] -1/11*Cos[c + d*x]^5/(d*(a + a*Sin[c + d*x])^8) - Cos[c + d*x]^5/(33*a*d*(a
 + a*Sin[c + d*x])^7) - (2*Cos[c + d*x]^5)/(231*a^2*d*(a + a*Sin[c + d*x])^
6) - (2*Cos[c + d*x]^5)/(1155*a^3*d*(a + a*Sin[c + d*x])^5)
```

Rule 2750

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{(a+a\sin(c+dx))^8}\,dx = -\frac{\cos^5(c+dx)}{11d(a+a\sin(c+dx))^8} + \frac{3\int \frac{\cos^4(c+dx)}{(a+a\sin(c+dx))^7}\,dx}{11a}$$
$$= -\frac{\cos^5(c+dx)}{11d(a+a\sin(c+dx))^8} - \frac{\cos^5(c+dx)}{33ad(a+a\sin(c+dx))^7} + \frac{2\int \frac{\cos^4(c+dx)}{(a+a\sin(c+dx))^6}\,dx}{33a^2}$$
$$= -\frac{\cos^5(c+dx)}{11d(a+a\sin(c+dx))^8} - \frac{\cos^5(c+dx)}{33ad(a+a\sin(c+dx))^7} - \frac{2\cos^5(c+dx)}{231a^2d(a+a\sin(c+d}$$
$$= -\frac{\cos^5(c+dx)}{11d(a+a\sin(c+dx))^8} - \frac{\cos^5(c+dx)}{33ad(a+a\sin(c+dx))^7} - \frac{2\cos^5(c+dx)}{231a^2d(a+a\sin(c+d}$$

Mathematica [A]
time = 0.05, size = 58, normalized size = 0.49

$$-\frac{\cos^5(c+dx)\left(152+61\sin(c+dx)+16\sin^2(c+dx)+2\sin^3(c+dx)\right)}{1155a^8d(1+\sin(c+dx))^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + a*Sin[c + d*x])^8,x]
```

```
[Out] -1/1155*(Cos[c + d*x]^5*(152 + 61*Sin[c + d*x] + 16*Sin[c + d*x]^2 + 2*Sin[
c + d*x]^3))/(a^8*d*(1 + Sin[c + d*x])^8)
```

Maple [A]
time = 0.25, size = 175, normalized size = 1.48

method	result
risch	$\frac{4i\left(-2079i e^{6i(dx+c)}+1155\,e^{7i(dx+c)}+825i e^{4i(dx+c)}-2541\,e^{5i(dx+c)}+55i e^{2i(dx+c)}+165\,e^{3i(dx+c)}-i-11\,e^{i(dx+c)}\right)}{1155d\,a^8\left(e^{i(dx+c)}+i\right)^{11}}$
derivativedivides	$\frac{-\frac{4752}{7\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^7}+\frac{14}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{256}{11\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{11}}+\frac{584}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^6}+\frac{128}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{10}}-\frac{60}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\right.}}{d\,a^8}$
default	$\frac{-\frac{4752}{7\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^7}+\frac{14}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{256}{11\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{11}}+\frac{584}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^6}+\frac{128}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{10}}-\frac{60}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\right.}}{d\,a^8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+a*sin(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/a^8*(-2376/7/(tan(1/2*d*x+1/2*c)+1)^7+7/(tan(1/2*d*x+1/2*c)+1)^2-128/11
/(tan(1/2*d*x+1/2*c)+1)^11+292/(tan(1/2*d*x+1/2*c)+1)^6+64/(tan(1/2*d*x+1/2
*c)+1)^10-30/(tan(1/2*d*x+1/2*c)+1)^3+288/(tan(1/2*d*x+1/2*c)+1)^8+88/(tan(
1/2*d*x+1/2*c)+1)^4-512/3/(tan(1/2*d*x+1/2*c)+1)^9-932/5/(tan(1/2*d*x+1/2*c
)+1)^5-1/(tan(1/2*d*x+1/2*c)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 461 vs. $2(110) = 220$.
time = 0.33, size = 461, normalized size = 3.91

$$-\frac{2\left(\frac{517\sin(dx+c)}{\cos(dx+c)+1}+\frac{4895\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{11220\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{27060\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{32802\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{37422\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{23100\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{13860\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{3465\sin(dx+c)^9}{(\cos(dx+c)+1)^9}+\frac{1155\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+152\right)}{1155\left(a^8+\frac{11a^8\sin(dx+c)}{\cos(dx+c)+1}+\frac{55a^8\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{165a^8\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{330a^8\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{462a^8\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{462a^8\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{330a^8\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{165a^8\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{55a^8\sin(dx+c)^9}{(\cos(dx+c)+1)^9}+\frac{11a^8\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{a^8\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -2/1155*(517*sin(d*x + c)/(cos(d*x + c) + 1) + 4895*sin(d*x + c)^2/(cos(d*x
 + c) + 1)^2 + 11220*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 27060*sin(d*x +
c)^4/(cos(d*x + c) + 1)^4 + 32802*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 374
22*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 23100*sin(d*x + c)^7/(cos(d*x + c)
 + 1)^7 + 13860*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 3465*sin(d*x + c)^9/(
cos(d*x + c) + 1)^9 + 1155*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + 152)/((a
^8 + 11*a^8*sin(d*x + c)/(cos(d*x + c) + 1) + 55*a^8*sin(d*x + c)^2/(cos(d*
x + c) + 1)^2 + 165*a^8*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 330*a^8*sin(d
*x + c)^4/(cos(d*x + c) + 1)^4 + 462*a^8*sin(d*x + c)^5/(cos(d*x + c) + 1)^
5 + 462*a^8*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 330*a^8*sin(d*x + c)^7/(c
os(d*x + c) + 1)^7 + 165*a^8*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 55*a^8*s
in(d*x + c)^9/(cos(d*x + c) + 1)^9 + 11*a^8*sin(d*x + c)^10/(cos(d*x + c) +
 1)^10 + a^8*sin(d*x + c)^11/(cos(d*x + c) + 1)^11)*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 291 vs. $2(110) = 220$.
time = 0.38, size = 291, normalized size = 2.47

$$\frac{2\cos(dx+c)^6+12\cos(dx+c)^5-25\cos(dx+c)^4-70\cos(dx+c)^3-245\cos(dx+c)^2+\left(2\cos(dx+c)^5-10\cos(dx+c)^4-35\cos(dx+c)^3+35\cos(dx+c)^2-210\cos(dx+c)-420\right)\sin(dx+c)+210\cos(dx+c)+420}{1155\left(a^8d\cos(dx+c)^6-5a^8d\cos(dx+c)^5-18a^8d\cos(dx+c)^4+20a^8d\cos(dx+c)^3+48a^8d\cos(dx+c)^2-16a^8d\cos(dx+c)-32a^8d-\left(a^8d\cos(dx+c)^5+6a^8d\cos(dx+c)^4-12a^8d\cos(dx+c)^3-32a^8d\cos(dx+c)^2+16a^8d\cos(dx+c)+32a^8d\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/1155*(2*cos(d*x + c)^6 + 12*cos(d*x + c)^5 - 25*cos(d*x + c)^4 - 70*cos(d
*x + c)^3 - 245*cos(d*x + c)^2 + (2*cos(d*x + c)^5 - 10*cos(d*x + c)^4 - 35
*cos(d*x + c)^3 + 35*cos(d*x + c)^2 - 210*cos(d*x + c) - 420)*sin(d*x + c)
+ 210*cos(d*x + c) + 420)/(a^8*d*cos(d*x + c)^6 - 5*a^8*d*cos(d*x + c)^5 -
18*a^8*d*cos(d*x + c)^4 + 20*a^8*d*cos(d*x + c)^3 + 48*a^8*d*cos(d*x + c)^2
 - 16*a^8*d*cos(d*x + c) - 32*a^8*d - (a^8*d*cos(d*x + c)^5 + 6*a^8*d*cos(d
*x + c)^4 - 12*a^8*d*cos(d*x + c)^3 - 32*a^8*d*cos(d*x + c)^2 + 16*a^8*d*co
s(d*x + c) + 32*a^8*d)*sin(d*x + c))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 2470 vs. $2(107) = 214$.
time = 201.18, size = 2470, normalized size = 20.93

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**4/(a+a*sin(d*x+c))**8,x)

[Out] Piecewise((-2310*tan(c/2 + d*x/2)**10/(1155*a**8*d*tan(c/2 + d*x/2)**11 + 1
2705*a**8*d*tan(c/2 + d*x/2)**10 + 63525*a**8*d*tan(c/2 + d*x/2)**9 + 19057
5*a**8*d*tan(c/2 + d*x/2)**8 + 381150*a**8*d*tan(c/2 + d*x/2)**7 + 533610*a
8*d*tan(c/2 + d*x/2)6 + 533610*a**8*d*tan(c/2 + d*x/2)**5 + 381150*a**8
*d*tan(c/2 + d*x/2)**4 + 190575*a**8*d*tan(c/2 + d*x/2)**3 + 63525*a**8*d*t
an(c/2 + d*x/2)**2 + 12705*a**8*d*tan(c/2 + d*x/2) + 1155*a**8*d) - 6930*ta
n(c/2 + d*x/2)**9/(1155*a**8*d*tan(c/2 + d*x/2)**11 + 12705*a**8*d*tan(c/2
+ d*x/2)**10 + 63525*a**8*d*tan(c/2 + d*x/2)**9 + 190575*a**8*d*tan(c/2 + d
*x/2)**8 + 381150*a**8*d*tan(c/2 + d*x/2)**7 + 533610*a**8*d*tan(c/2 + d*x/
2)**6 + 533610*a**8*d*tan(c/2 + d*x/2)**5 + 381150*a**8*d*tan(c/2 + d*x/2)*
*4 + 190575*a**8*d*tan(c/2 + d*x/2)**3 + 63525*a**8*d*tan(c/2 + d*x/2)**2 +
12705*a**8*d*tan(c/2 + d*x/2) + 1155*a**8*d) - 27720*tan(c/2 + d*x/2)**8/(
1155*a**8*d*tan(c/2 + d*x/2)**11 + 12705*a**8*d*tan(c/2 + d*x/2)**10 + 6352
5*a**8*d*tan(c/2 + d*x/2)**9 + 190575*a**8*d*tan(c/2 + d*x/2)**8 + 381150*a
8*d*tan(c/2 + d*x/2)7 + 533610*a**8*d*tan(c/2 + d*x/2)**6 + 533610*a**8
*d*tan(c/2 + d*x/2)**5 + 381150*a**8*d*tan(c/2 + d*x/2)**4 + 190575*a**8*d*
tan(c/2 + d*x/2)**3 + 63525*a**8*d*tan(c/2 + d*x/2)**2 + 12705*a**8*d*tan(c
/2 + d*x/2) + 1155*a**8*d) - 46200*tan(c/2 + d*x/2)**7/(1155*a**8*d*tan(c/2
+ d*x/2)**11 + 12705*a**8*d*tan(c/2 + d*x/2)**10 + 63525*a**8*d*tan(c/2 +
d*x/2)**9 + 190575*a**8*d*tan(c/2 + d*x/2)**8 + 381150*a**8*d*tan(c/2 + d*x
/2)**7 + 533610*a**8*d*tan(c/2 + d*x/2)**6 + 533610*a**8*d*tan(c/2 + d*x/2)
5 + 381150*a8*d*tan(c/2 + d*x/2)**4 + 190575*a**8*d*tan(c/2 + d*x/2)**3
+ 63525*a**8*d*tan(c/2 + d*x/2)**2 + 12705*a**8*d*tan(c/2 + d*x/2) + 1155*
a**8*d) - 74844*tan(c/2 + d*x/2)**6/(1155*a**8*d*tan(c/2 + d*x/2)**11 + 127
05*a**8*d*tan(c/2 + d*x/2)**10 + 63525*a**8*d*tan(c/2 + d*x/2)**9 + 190575*
a**8*d*tan(c/2 + d*x/2)**8 + 381150*a**8*d*tan(c/2 + d*x/2)**7 + 533610*a**
8*d*tan(c/2 + d*x/2)**6 + 533610*a**8*d*tan(c/2 + d*x/2)**5 + 381150*a**8*d
*tan(c/2 + d*x/2)**4 + 190575*a**8*d*tan(c/2 + d*x/2)**3 + 63525*a**8*d*tan
(c/2 + d*x/2)**2 + 12705*a**8*d*tan(c/2 + d*x/2) + 1155*a**8*d) - 65604*tan
(c/2 + d*x/2)**5/(1155*a**8*d*tan(c/2 + d*x/2)**11 + 12705*a**8*d*tan(c/2 +
d*x/2)**10 + 63525*a**8*d*tan(c/2 + d*x/2)**9 + 190575*a**8*d*tan(c/2 + d*
x/2)**8 + 381150*a**8*d*tan(c/2 + d*x/2)**7 + 533610*a**8*d*tan(c/2 + d*x/2
)**6 + 533610*a**8*d*tan(c/2 + d*x/2)**5 + 381150*a**8*d*tan(c/2 + d*x/2)**
4 + 190575*a**8*d*tan(c/2 + d*x/2)**3 + 63525*a**8*d*tan(c/2 + d*x/2)**2 +
12705*a**8*d*tan(c/2 + d*x/2) + 1155*a**8*d) - 54120*tan(c/2 + d*x/2)**4/(1
155*a**8*d*tan(c/2 + d*x/2)**11 + 12705*a**8*d*tan(c/2 + d*x/2)**10 + 63525
*a**8*d*tan(c/2 + d*x/2)**9 + 190575*a**8*d*tan(c/2 + d*x/2)**8 + 381150*a*
*8*d*tan(c/2 + d*x/2)**7 + 533610*a**8*d*tan(c/2 + d*x/2)**6 + 533610*a**8*
d*tan(c/2 + d*x/2)**5 + 381150*a**8*d*tan(c/2 + d*x/2)**4 + 190575*a**8*d*t
an(c/2 + d*x/2)**3 + 63525*a**8*d*tan(c/2 + d*x/2)**2 + 12705*a**8*d*tan(c/
2 + d*x/2) + 1155*a**8*d) - 22440*tan(c/2 + d*x/2)**3/(1155*a**8*d*tan(c/2

```
+ d*x/2)**11 + 12705*a**8*d*tan(c/2 + d*x/2)**10 + 63525*a**8*d*tan(c/2 + d
*x/2)**9 + 190575*a**8*d*tan(c/2 + d*x/2)**8 + 381150*a**8*d*tan(c/2 + d*x/
2)**7 + 533610*a**8*d*tan(c/2 + d*x/2)**6 + 533610*a**8*d*tan(c/2 + d*x/2)*
*5 + 381150*a**8*d*tan(c/2 + d*x/2)**4 + 190575*a**8*d*tan(c/2 + d*x/2)**3
+ 63525*a**8*d*tan(c/2 + d*x/2)**2 + 12705*a**8*d*tan(c/2 + d*x/2) + 1155*a
**8*d) - 9790*tan(c/2 + d*x/2)**2/(1155*a**8*d*tan(c/2 + d*x/2)**11 + 12705
*a**8*d*tan(c/2 + d*x/2)**10 + 63525*a**8*d*tan(c/2 + d*x/2)**9 + 190575*a*
*8*d*tan(c/2 + d*x/2)**8 + 381150*a**8*d*tan(c/2 + d*x/2)**7 + 533610*a**8*
d*tan(c/2 + d*x/2)**6 + 533610*a**8*d*tan(c/2 + d*x/2)**5 + 381150*a**8*d*t
an(c/2 + d*x/2)**4 + 190575*a**8*d*tan(c/2 + d*x/2)**3 + 63525*a**8*d*tan(c
/2 + d*x/2)**2 + 12705*a**8*d*tan(c/2 + d*x/2) + 1155*a**8*d) - 1034*tan(c/
2 + d*x/2)/(1155*a**8*d*tan(c/2 + d*x/2)**11 + 12705*a**8*d*tan(c/2 + d*x/2
)**10 + 63525*a**8*d*tan(c/2 + d*x/2)**9 + 190575*a**8*d*tan(c/2 + d*x/2)**
8 + 381150*a**8*d*tan(c/2 + d*x/2)**7 + 533610*a**8*d*tan(c/2 + d*x/2)**6 +
 533610*a**8*d*tan(c/2 + d*x/2)**5 + 381150*a**8*d*tan(c/2 + d*x/2)**4 + 19
0575*a**8*d*tan(c/2 + d*x/2)**3 + 63525*a**8*d*tan(c/2 + d*x/2)**2 + 12705*
a**8*d*tan(c/2 + d*x/2) + 1155*a**8*d) - 304/(1155*a**8*d*tan(c/2 + d*x/2)*
*11 + 12705*a**8*d*tan(c/2 + d*x/2)**10 + 63525*a**8*d*tan(c/2 + d*x/2)**9
+ 190575*a**8*d*tan(c/2 + d*x/2)**8 + 381150*a**8*d*tan(c/2 + d*x/2)**7 + 5
33610*a**8*d*tan(c/2 + d*x/2)**6 + 533610*a**8*d*tan(c/2 + d*x/2)**5 + 3811
50*a**8*d*tan(c/2 + d*x/2)**4 + 190575*a**8*d*tan(c/2 + d*x/2)**3 + 63525*a
**8*d*tan(c/2 + d*x/2)**2 + 12705*a**8*d*tan(c/2 + d*x/2) + 1155*a**8*d), N
e(d, 0)), (x*cos(c)**4/(a*sin(c) + a)**8, True))
```

Giac [A]
time = 6.20, size = 151, normalized size = 1.28

$$-\frac{2\left(1155\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{10}+3465\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{9}+13860\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{8}+23100\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{7}+37422\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{6}+32802\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{5}+27060\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{4}+11220\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}+4895\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+517\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+152\right)}{1155a^{8}d\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -2/1155*(1155*tan(1/2*d*x + 1/2*c)^10 + 3465*tan(1/2*d*x + 1/2*c)^9 + 13860
*tan(1/2*d*x + 1/2*c)^8 + 23100*tan(1/2*d*x + 1/2*c)^7 + 37422*tan(1/2*d*x
+ 1/2*c)^6 + 32802*tan(1/2*d*x + 1/2*c)^5 + 27060*tan(1/2*d*x + 1/2*c)^4 +
11220*tan(1/2*d*x + 1/2*c)^3 + 4895*tan(1/2*d*x + 1/2*c)^2 + 517*tan(1/2*d*
x + 1/2*c) + 152)/(a^8*d*(tan(1/2*d*x + 1/2*c) + 1)^11)
```

Mupad [B]
time = 7.15, size = 140, normalized size = 1.19

$$-\frac{\sqrt{2}\,\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{7623\sin(c+dx)}{4}-697\cos\left(c+dx\right)-\frac{3977\cos(2c+2dx)}{4}+\frac{3203\cos(3c+3dx)}{16}+\frac{461\cos(4c+4dx)}{8}-\frac{75\cos(5c+5dx)}{16}-462\sin\left(2c+2dx\right)-\frac{4983\sin(3c+3dx)}{16}+\frac{187\sin(4c+4dx)}{4}+\frac{77\sin(5c+5dx)}{16}+\frac{12721}{8}\right)}{36960\,a^{8}\,d\cos\left(\frac{c}{2}-\frac{\pi}{4}+\frac{dx}{2}\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(a + a*sin(c + d*x))^8,x)
```

```
[Out] -(2^(1/2)*cos(c/2 + (d*x)/2)*((7623*sin(c + d*x))/4 - 697*cos(c + d*x) - (3
977*cos(2*c + 2*d*x))/4 + (3203*cos(3*c + 3*d*x))/16 + (461*cos(4*c + 4*d*x
))/8 - (75*cos(5*c + 5*d*x))/16 - 462*sin(2*c + 2*d*x) - (4983*sin(3*c + 3*
d*x))/16 + (187*sin(4*c + 4*d*x))/4 + (77*sin(5*c + 5*d*x))/16 + 12721/8))/
(36960*a^8*d*cos(c/2 - pi/4 + (d*x)/2)^11)
```

3.93 $\int \frac{\cos^3(c+dx)}{(a+a\sin(c+dx))^8}\,dx$

Optimal. Leaf size=45

$$-\frac{1}{3a^2d(a+a\sin(c+dx))^6}+\frac{1}{5a^3d(a+a\sin(c+dx))^5}$$

```
[Out] -1/3/a^2/d/(a+a*sin(d*x+c))^6+1/5/a^3/d/(a+a*sin(d*x+c))^5
```

Rubi [A]
time = 0.04, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.095$, Rules used = {2746, 45}

$$\frac{1}{5a^3d(a\sin(c+dx)+a)^5}-\frac{1}{3a^2d(a\sin(c+dx)+a)^6}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + a*Sin[c + d*x])^8,x]
```

```
[Out] -1/3*1/(a^2*d*(a + a*Sin[c + d*x])^6) + 1/(5*a^3*d*(a + a*Sin[c + d*x])^5)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^3(c+dx)}{(a+a\sin(c+dx))^8}\,dx &= \frac{\text{Subst}\left(\int \frac{a-x}{(a+x)^7}\,dx,x,a\sin(c+dx)\right)}{a^3d}\\ &= \frac{\text{Subst}\left(\int \left(\frac{2a}{(a+x)^7}-\frac{1}{(a+x)^6}\right)\,dx,x,a\sin(c+dx)\right)}{a^3d}\\ &= -\frac{1}{3a^2d(a+a\sin(c+dx))^6}+\frac{1}{5a^3d(a+a\sin(c+dx))^5}\end{aligned}$$

Mathematica [A]
time = 0.11, size = 43, normalized size = 0.96

$$\frac{-2+3\sin(c+dx)}{15a^8d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^{12}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + a*Sin[c + d*x])^8,x]
```

```
[Out] (-2 + 3*Sin[c + d*x])/(15*a^8*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^12)
```

Maple [A]
time = 0.23, size = 33, normalized size = 0.73

method	result	size
derivativedivides	$\frac{-\frac{1}{3(1+\sin(dx+c))^6}+\frac{1}{5(1+\sin(dx+c))^5}}{d\,a^8}$	33
default	$\frac{-\frac{1}{3(1+\sin(dx+c))^6}+\frac{1}{5(1+\sin(dx+c))^5}}{d\,a^8}$	33
risch	$\frac{32i\left(-4i\,\mathrm{e}^{6i(dx+c)}+3\,\mathrm{e}^{7i(dx+c)}-3\,\mathrm{e}^{5i(dx+c)}\right)}{15d\,a^8\left(\mathrm{e}^{i(dx+c)}+i\right)^{12}}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+a*sin(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^8*(-1/3/(1+sin(d*x+c))^6+1/5/(1+sin(d*x+c))^5)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 96 vs. 2(41) = 82.
time = 0.28, size = 96, normalized size = 2.13

$$\frac{3\sin(dx+c)-2}{15\left(a^8\sin(dx+c)^6+6\,a^8\sin(dx+c)^5+15\,a^8\sin(dx+c)^4+20\,a^8\sin(dx+c)^3+15\,a^8\sin(dx+c)^2+6\,a^8\sin(dx+c)+a^8\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 1/15*(3*sin(d*x + c) - 2)/((a^8*sin(d*x + c)^6 + 6*a^8*sin(d*x + c)^5 + 15*
a^8*sin(d*x + c)^4 + 20*a^8*sin(d*x + c)^3 + 15*a^8*sin(d*x + c)^2 + 6*a^8*
sin(d*x + c) + a^8)*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 105 vs. 2(41) = 82.
time = 0.35, size = 105, normalized size = 2.33

$$-\frac{3\sin(dx+c)-2}{15\left(a^8d\cos(dx+c)^6-18\,a^8d\cos(dx+c)^4+48\,a^8d\cos(dx+c)^2-32\,a^8d-2\left(3\,a^8d\cos(dx+c)^4-16\,a^8d\cos(dx+c)^2+16\,a^8d\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] -1/15*(3*sin(d*x + c) - 2)/(a^8*d*cos(d*x + c)^6 - 18*a^8*d*cos(d*x + c)^4
+ 48*a^8*d*cos(d*x + c)^2 - 32*a^8*d - 2*(3*a^8*d*cos(d*x + c)^4 - 16*a^8*d
*cos(d*x + c)^2 + 16*a^8*d)*sin(d*x + c))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 493 vs. $2(39) = 78$.
time = 14.51, size = 493, normalized size = 10.96

$$\begin{cases} \frac{6\sin^2(c+dx)}{105a^8d\sin^7(c+dx)+735a^8d\sin^6(c+dx)+2205a^8d\sin^5(c+dx)+3675a^8d\sin^4(c+dx)+3675a^8d\sin^3(c+dx)+2205a^8d\sin^2(c+dx)+735a^8d\sin(c+dx)+105a^8d} + \frac{7\sin(c+dx)}{105a^8d\sin^7(c+dx)+735a^8d\sin^6(c+dx)+2205a^8d\sin^5(c+dx)+3675a^8d\sin^4(c+dx)+3675a^8d\sin^3(c+dx)+2205a^8d\sin^2(c+dx)+735a^8d\sin(c+dx)+105a^8d} - \frac{15\cos^2(c+dx)}{105a^8d\sin^7(c+dx)+735a^8d\sin^6(c+dx)+2205a^8d\sin^5(c+dx)+3675a^8d\sin^4(c+dx)+3675a^8d\sin^3(c+dx)+2205a^8d\sin^2(c+dx)+735a^8d\sin(c+dx)+105a^8d} + \frac{1}{105a^8d\sin^7(c+dx)+735a^8d\sin^6(c+dx)+2205a^8d\sin^5(c+dx)+3675a^8d\sin^4(c+dx)+3675a^8d\sin^3(c+dx)+2205a^8d\sin^2(c+dx)+735a^8d\sin(c+dx)+105a^8d} & \text{for}\ d \neq 0 \\ \frac{x\cos^3(c)}{(a\sin(c)+a)^8} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+a*sin(d*x+c))**8,x)
```

```
[Out] Piecewise((6*sin(c + d*x)**2/(105*a**8*d*sin(c + d*x)**7 + 735*a**8*d*sin(c
 + d*x)**6 + 2205*a**8*d*sin(c + d*x)**5 + 3675*a**8*d*sin(c + d*x)**4 + 36
75*a**8*d*sin(c + d*x)**3 + 2205*a**8*d*sin(c + d*x)**2 + 735*a**8*d*sin(c
+ d*x) + 105*a**8*d) + 7*sin(c + d*x)/(105*a**8*d*sin(c + d*x)**7 + 735*a**
8*d*sin(c + d*x)**6 + 2205*a**8*d*sin(c + d*x)**5 + 3675*a**8*d*sin(c + d*x
)**4 + 3675*a**8*d*sin(c + d*x)**3 + 2205*a**8*d*sin(c + d*x)**2 + 735*a**8
*d*sin(c + d*x) + 105*a**8*d) - 15*cos(c + d*x)**2/(105*a**8*d*sin(c + d*x)
**7 + 735*a**8*d*sin(c + d*x)**6 + 2205*a**8*d*sin(c + d*x)**5 + 3675*a**8*
d*sin(c + d*x)**4 + 3675*a**8*d*sin(c + d*x)**3 + 2205*a**8*d*sin(c + d*x)*
*2 + 735*a**8*d*sin(c + d*x) + 105*a**8*d) + 1/(105*a**8*d*sin(c + d*x)**7
+ 735*a**8*d*sin(c + d*x)**6 + 2205*a**8*d*sin(c + d*x)**5 + 3675*a**8*d*si
n(c + d*x)**4 + 3675*a**8*d*sin(c + d*x)**3 + 2205*a**8*d*sin(c + d*x)**2 +
 735*a**8*d*sin(c + d*x) + 105*a**8*d), Ne(d, 0)), (x*cos(c)**3/(a*sin(c) +
 a)**8, True))
```

Giac [A]
time = 5.49, size = 28, normalized size = 0.62

$$\frac{3\sin(dx+c)-2}{15\,a^8d(\sin(dx+c)+1)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/15*(3*sin(d*x + c) - 2)/(a^8*d*(sin(d*x + c) + 1)^6)
```

Mupad [B]
time = 0.10, size = 28, normalized size = 0.62

$$\frac{3\sin(c+d\,x)-2}{15\,a^8\,d\,(\sin(c+d\,x)+1)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a + a*sin(c + d*x))^8,x)
[Out] (3*sin(c + d*x) - 2)/(15*a^8*d*(sin(c + d*x) + 1)^6)
```

3.94 $\int \frac{\cos^2(c+dx)}{(a+a\sin(c+dx))^8}\,dx$

Optimal. Leaf size=183

$$-\frac{\cos^3(c+dx)}{13d(a+a\sin(c+dx))^8}-\frac{5\cos^3(c+dx)}{143ad(a+a\sin(c+dx))^7}-\frac{20\cos^3(c+dx)}{1287a^2d(a+a\sin(c+dx))^6}-\frac{20\cos^3(c+dx)}{3003a^3d(a+a\sin(c+dx))^5}$$

```
[Out] -1/13*cos(d*x+c)^3/d/(a+a*sin(d*x+c))^8-5/143*cos(d*x+c)^3/a/d/(a+a*sin(d*x
+c))^7-20/1287*cos(d*x+c)^3/a^2/d/(a+a*sin(d*x+c))^6-20/3003*cos(d*x+c)^3/a
^3/d/(a+a*sin(d*x+c))^5-8/3003*cos(d*x+c)^3/d/(a^2+a^2*sin(d*x+c))^4-8/9009
*cos(d*x+c)^3/a^2/d/(a^2+a^2*sin(d*x+c))^3
```

Rubi [A]

time = 0.18, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2751, 2750}

$$-\frac{20\cos^3(c+dx)}{3003a^3d(a\sin(c+dx)+a)^5}-\frac{8\cos^3(c+dx)}{9009a^2d\left(a^2\sin(c+dx)+a^2\right)^3}-\frac{8\cos^3(c+dx)}{3003d\left(a^2\sin(c+dx)+a^2\right)^4}-\frac{20\cos^3(c+dx)}{1287a^2d(a\sin(c+dx)+a)^6}-\frac{5\cos^3(c+dx)}{143ad(a\sin(c+dx)+a)^7}-\frac{\cos^3(c+dx)}{13d(a\sin(c+dx)+a)^8}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + a*Sin[c + d*x])^8,x]
```

```
[Out] -1/13*Cos[c + d*x]^3/(d*(a + a*Sin[c + d*x])^8) - (5*Cos[c + d*x]^3)/(143*a
*d*(a + a*Sin[c + d*x])^7) - (20*Cos[c + d*x]^3)/(1287*a^2*d*(a + a*Sin[c +
 d*x])^6) - (20*Cos[c + d*x]^3)/(3003*a^3*d*(a + a*Sin[c + d*x])^5) - (8*Co
s[c + d*x]^3)/(3003*d*(a^2 + a^2*Sin[c + d*x])^4) - (8*Cos[c + d*x]^3)/(900
9*a^2*d*(a^2 + a^2*Sin[c + d*x])^3)
```

Rule 2750

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a+a\sin(c+dx))^8}\,dx = -\frac{\cos^3(c+dx)}{13d(a+a\sin(c+dx))^8} + \frac{5\int \frac{\cos^2(c+dx)}{(a+a\sin(c+dx))^7}\,dx}{13a}$$

$$= -\frac{\cos^3(c+dx)}{13d(a+a\sin(c+dx))^8} - \frac{5\cos^3(c+dx)}{143ad(a+a\sin(c+dx))^7} + \frac{20\int \frac{\cos^2(c+dx)}{(a+a\sin(c+dx))^6}\,dx}{143a^2}$$

$$= -\frac{\cos^3(c+dx)}{13d(a+a\sin(c+dx))^8} - \frac{5\cos^3(c+dx)}{143ad(a+a\sin(c+dx))^7} - \frac{20\cos^3(c+dx)}{1287a^2d(a+a\sin(c+$$

$$= -\frac{\cos^3(c+dx)}{13d(a+a\sin(c+dx))^8} - \frac{5\cos^3(c+dx)}{143ad(a+a\sin(c+dx))^7} - \frac{20\cos^3(c+dx)}{1287a^2d(a+a\sin(c+$$

$$= -\frac{\cos^3(c+dx)}{13d(a+a\sin(c+dx))^8} - \frac{5\cos^3(c+dx)}{143ad(a+a\sin(c+dx))^7} - \frac{20\cos^3(c+dx)}{1287a^2d(a+a\sin(c+$$

$$= -\frac{\cos^3(c+dx)}{13d(a+a\sin(c+dx))^8} - \frac{5\cos^3(c+dx)}{143ad(a+a\sin(c+dx))^7} - \frac{20\cos^3(c+dx)}{1287a^2d(a+a\sin(c+$$

Mathematica [A]
time = 0.09, size = 78, normalized size = 0.43

$$-\frac{\cos^3(c+dx)\left(1240+911\sin(c+dx)+544\sin^2(c+dx)+236\sin^3(c+dx)+64\sin^4(c+dx)+8\sin^5(c+dx)\right)}{9009a^8d(1+\sin(c+dx))^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + a*Sin[c + d*x])^8,x]
```

```
[Out] -1/9009*(Cos[c + d*x]^3*(1240 + 911*Sin[c + d*x] + 544*Sin[c + d*x]^2 + 236
*Sin[c + d*x]^3 + 64*Sin[c + d*x]^4 + 8*Sin[c + d*x]^5))/(a^8*d*(1 + Sin[c
+ d*x])^8)
```

Maple [A]
time = 0.26, size = 205, normalized size = 1.12

method	result
risch	$\frac{16i\left(-4290i\,\mathrm{e}^{6i(dx+c)}+6006\,\mathrm{e}^{7i(dx+c)}-715i\,\mathrm{e}^{4i(dx+c)}-1287\,\mathrm{e}^{5i(dx+c)}+78i\,\mathrm{e}^{2i(dx+c)}+286\,\mathrm{e}^{3i(dx+c)}-i-13\,\mathrm{e}^{i(dx+c)}\right)}{9009d\,a^8\left(\mathrm{e}^{i(dx+c)}+i\right)^{13}}$
derivativedivides	$-\frac{9056}{7\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^7}+\frac{2672}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^6}+\frac{864}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{10}}-\frac{256}{13\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{13}}-\frac{4544}{11\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{11}}-\frac{188}{3\left(\tan\left(\frac{dx}{2}+\right.\right.}$
default	$-\frac{9056}{7\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^7}+\frac{2672}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^6}+\frac{864}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{10}}-\frac{256}{13\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{13}}-\frac{4544}{11\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{11}}-\frac{188}{3\left(\tan\left(\frac{dx}{2}+\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+a*sin(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/a^8*(-4528/7/(tan(1/2*d*x+1/2*c)+1)^7+1336/3/(tan(1/2*d*x+1/2*c)+1)^6+4
32/(tan(1/2*d*x+1/2*c)+1)^10-128/13/(tan(1/2*d*x+1/2*c)+1)^13-2272/11/(tan(
1/2*d*x+1/2*c)+1)^11-94/3/(tan(1/2*d*x+1/2*c)+1)^3-5840/9/(tan(1/2*d*x+1/2*
c)+1)^9+736/(tan(1/2*d*x+1/2*c)+1)^8-240/(tan(1/2*d*x+1/2*c)+1)^5+7/(tan(1/
2*d*x+1/2*c)+1)^2+100/(tan(1/2*d*x+1/2*c)+1)^4-1/(tan(1/2*d*x+1/2*c)+1)+64/
(tan(1/2*d*x+1/2*c)+1)^12)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 547 vs. $2(171) = 342$.
time = 0.31, size = 547, normalized size = 2.99

$$-\frac{2\left(\frac{7111\sin(dx+c)}{\cos(dx+c)+1}+\frac{51675\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{171457\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{451165\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{785070\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{1076790\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{1051050\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{810810\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{435435\sin(dx+c)^9}{(\cos(dx+c)+1)^9}+\frac{183183\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{45045\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}+\frac{9009\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}+1240\right)}{9009\left(a^8+\frac{13a^8\sin(dx+c)}{\cos(dx+c)+1}+\frac{78a^8\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{286a^8\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{715a^8\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{1287a^8\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{1716a^8\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{1716a^8\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{1287a^8\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{715a^8\sin(dx+c)^9}{(\cos(dx+c)+1)^9}+\frac{286a^8\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{78a^8\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}+\frac{13a^8\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}+\frac{a^8\sin(dx+c)^{13}}{(\cos(dx+c)+1)^{13}}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -2/9009*(7111*sin(d*x + c)/(cos(d*x + c) + 1) + 51675*sin(d*x + c)^2/(cos(d
*x + c) + 1)^2 + 171457*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 451165*sin(d*
x + c)^4/(cos(d*x + c) + 1)^4 + 785070*sin(d*x + c)^5/(cos(d*x + c) + 1)^5
+ 1076790*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 1051050*sin(d*x + c)^7/(cos
(d*x + c) + 1)^7 + 810810*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 435435*sin(
d*x + c)^9/(cos(d*x + c) + 1)^9 + 183183*sin(d*x + c)^10/(cos(d*x + c) + 1)
^10 + 45045*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 + 9009*sin(d*x + c)^12/(c
os(d*x + c) + 1)^12 + 1240)/((a^8 + 13*a^8*sin(d*x + c)/(cos(d*x + c) + 1)
+ 78*a^8*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 286*a^8*sin(d*x + c)^3/(cos(
d*x + c) + 1)^3 + 715*a^8*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 1287*a^8*si
n(d*x + c)^5/(cos(d*x + c) + 1)^5 + 1716*a^8*sin(d*x + c)^6/(cos(d*x + c) +
 1)^6 + 1716*a^8*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 1287*a^8*sin(d*x + c
)^8/(cos(d*x + c) + 1)^8 + 715*a^8*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 + 28
6*a^8*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + 78*a^8*sin(d*x + c)^11/(cos(d
*x + c) + 1)^11 + 13*a^8*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 + a^8*sin(d*
x + c)^13/(cos(d*x + c) + 1)^13)*d)
```

Fricas [A]
time = 0.38, size = 339, normalized size = 1.85

$$\frac{8\cos(dx+c)^7-48\cos(dx+c)^6-196\cos(dx+c)^5+280\cos(dx+c)^4+735\cos(dx+c)^3-378\cos(dx+c)^2-(8\cos(dx+c)^6+56\cos(dx+c)^5-140\cos(dx+c)^4-420\cos(dx+c)^3+315\cos(dx+c)^2+693\cos(dx+c)+1386)\sin(dx+c)+693\cos(dx+c)+1386}{9009\left(a^8d\cos(dx+c)^7+7a^8d\cos(dx+c)^6-18a^8d\cos(dx+c)^5-56a^8d\cos(dx+c)^4+48a^8d\cos(dx+c)^3+112a^8d\cos(dx+c)^2-32a^8d\cos(dx+c)-64a^8d+(a^8d\cos(dx+c)^6-6a^8d\cos(dx+c)^5-24a^8d\cos(dx+c)^4+32a^8d\cos(dx+c)^3+80a^8d\cos(dx+c)^2-32a^8d\cos(dx+c)-64a^8d)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/9009*(8*cos(d*x + c)^7 - 48*cos(d*x + c)^6 - 196*cos(d*x + c)^5 + 280*cos
(d*x + c)^4 + 735*cos(d*x + c)^3 - 378*cos(d*x + c)^2 - (8*cos(d*x + c)^6 +
 56*cos(d*x + c)^5 - 140*cos(d*x + c)^4 - 420*cos(d*x + c)^3 + 315*cos(d*x
```

```
+ c)^2 + 693*cos(d*x + c) + 1386)*sin(d*x + c) + 693*cos(d*x + c) + 1386)/(
a^8*d*cos(d*x + c)^7 + 7*a^8*d*cos(d*x + c)^6 - 18*a^8*d*cos(d*x + c)^5 - 5
6*a^8*d*cos(d*x + c)^4 + 48*a^8*d*cos(d*x + c)^3 + 112*a^8*d*cos(d*x + c)^2
 - 32*a^8*d*cos(d*x + c) - 64*a^8*d + (a^8*d*cos(d*x + c)^6 - 6*a^8*d*cos(d
*x + c)^5 - 24*a^8*d*cos(d*x + c)^4 + 32*a^8*d*cos(d*x + c)^3 + 80*a^8*d*co
s(d*x + c)^2 - 32*a^8*d*cos(d*x + c) - 64*a^8*d)*sin(d*x + c))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 3405 vs. $2(170) = 340$.
time = 106.18, size = 3405, normalized size = 18.61

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+a*sin(d*x+c))**8,x)
```

```
[Out] Piecewise((-18018*tan(c/2 + d*x/2)**12/(9009*a**8*d*tan(c/2 + d*x/2)**13 +
117117*a**8*d*tan(c/2 + d*x/2)**12 + 702702*a**8*d*tan(c/2 + d*x/2)**11 + 2
576574*a**8*d*tan(c/2 + d*x/2)**10 + 6441435*a**8*d*tan(c/2 + d*x/2)**9 + 1
1594583*a**8*d*tan(c/2 + d*x/2)**8 + 15459444*a**8*d*tan(c/2 + d*x/2)**7 +
15459444*a**8*d*tan(c/2 + d*x/2)**6 + 11594583*a**8*d*tan(c/2 + d*x/2)**5 +
 6441435*a**8*d*tan(c/2 + d*x/2)**4 + 2576574*a**8*d*tan(c/2 + d*x/2)**3 +
702702*a**8*d*tan(c/2 + d*x/2)**2 + 117117*a**8*d*tan(c/2 + d*x/2) + 9009*a
**8*d) - 90090*tan(c/2 + d*x/2)**11/(9009*a**8*d*tan(c/2 + d*x/2)**13 + 117
117*a**8*d*tan(c/2 + d*x/2)**12 + 702702*a**8*d*tan(c/2 + d*x/2)**11 + 2576
574*a**8*d*tan(c/2 + d*x/2)**10 + 6441435*a**8*d*tan(c/2 + d*x/2)**9 + 1159
4583*a**8*d*tan(c/2 + d*x/2)**8 + 15459444*a**8*d*tan(c/2 + d*x/2)**7 + 154
59444*a**8*d*tan(c/2 + d*x/2)**6 + 11594583*a**8*d*tan(c/2 + d*x/2)**5 + 64
41435*a**8*d*tan(c/2 + d*x/2)**4 + 2576574*a**8*d*tan(c/2 + d*x/2)**3 + 702
702*a**8*d*tan(c/2 + d*x/2)**2 + 117117*a**8*d*tan(c/2 + d*x/2) + 9009*a**8
*d) - 366366*tan(c/2 + d*x/2)**10/(9009*a**8*d*tan(c/2 + d*x/2)**13 + 11711
7*a**8*d*tan(c/2 + d*x/2)**12 + 702702*a**8*d*tan(c/2 + d*x/2)**11 + 257657
4*a**8*d*tan(c/2 + d*x/2)**10 + 6441435*a**8*d*tan(c/2 + d*x/2)**9 + 115945
83*a**8*d*tan(c/2 + d*x/2)**8 + 15459444*a**8*d*tan(c/2 + d*x/2)**7 + 15459
444*a**8*d*tan(c/2 + d*x/2)**6 + 11594583*a**8*d*tan(c/2 + d*x/2)**5 + 6441
435*a**8*d*tan(c/2 + d*x/2)**4 + 2576574*a**8*d*tan(c/2 + d*x/2)**3 + 70270
2*a**8*d*tan(c/2 + d*x/2)**2 + 117117*a**8*d*tan(c/2 + d*x/2) + 9009*a**8*d
) - 870870*tan(c/2 + d*x/2)**9/(9009*a**8*d*tan(c/2 + d*x/2)**13 + 117117*a
**8*d*tan(c/2 + d*x/2)**12 + 702702*a**8*d*tan(c/2 + d*x/2)**11 + 2576574*a
**8*d*tan(c/2 + d*x/2)**10 + 6441435*a**8*d*tan(c/2 + d*x/2)**9 + 11594583*
a**8*d*tan(c/2 + d*x/2)**8 + 15459444*a**8*d*tan(c/2 + d*x/2)**7 + 15459444
*a**8*d*tan(c/2 + d*x/2)**6 + 11594583*a**8*d*tan(c/2 + d*x/2)**5 + 6441435
*a**8*d*tan(c/2 + d*x/2)**4 + 2576574*a**8*d*tan(c/2 + d*x/2)**3 + 702702*a
**8*d*tan(c/2 + d*x/2)**2 + 117117*a**8*d*tan(c/2 + d*x/2) + 9009*a**8*d) -
 1621620*tan(c/2 + d*x/2)**8/(9009*a**8*d*tan(c/2 + d*x/2)**13 + 117117*a**
```

```
8*d*tan(c/2 + d*x/2)**12 + 702702*a**8*d*tan(c/2 + d*x/2)**11 + 2576574*a**
8*d*tan(c/2 + d*x/2)**10 + 6441435*a**8*d*tan(c/2 + d*x/2)**9 + 11594583*a*
*8*d*tan(c/2 + d*x/2)**8 + 15459444*a**8*d*tan(c/2 + d*x/2)**7 + 15459444*a
**8*d*tan(c/2 + d*x/2)**6 + 11594583*a**8*d*tan(c/2 + d*x/2)**5 + 6441435*a
**8*d*tan(c/2 + d*x/2)**4 + 2576574*a**8*d*tan(c/2 + d*x/2)**3 + 702702*a**
8*d*tan(c/2 + d*x/2)**2 + 117117*a**8*d*tan(c/2 + d*x/2) + 9009*a**8*d) - 2
102100*tan(c/2 + d*x/2)**7/(9009*a**8*d*tan(c/2 + d*x/2)**13 + 117117*a**8*
d*tan(c/2 + d*x/2)**12 + 702702*a**8*d*tan(c/2 + d*x/2)**11 + 2576574*a**8*
d*tan(c/2 + d*x/2)**10 + 6441435*a**8*d*tan(c/2 + d*x/2)**9 + 11594583*a**8
*d*tan(c/2 + d*x/2)**8 + 15459444*a**8*d*tan(c/2 + d*x/2)**7 + 15459444*a**
8*d*tan(c/2 + d*x/2)**6 + 11594583*a**8*d*tan(c/2 + d*x/2)**5 + 6441435*a**
8*d*tan(c/2 + d*x/2)**4 + 2576574*a**8*d*tan(c/2 + d*x/2)**3 + 702702*a**8*
d*tan(c/2 + d*x/2)**2 + 117117*a**8*d*tan(c/2 + d*x/2) + 9009*a**8*d) - 215
3580*tan(c/2 + d*x/2)**6/(9009*a**8*d*tan(c/2 + d*x/2)**13 + 117117*a**8*d*
tan(c/2 + d*x/2)**12 + 702702*a**8*d*tan(c/2 + d*x/2)**11 + 2576574*a**8*d*
tan(c/2 + d*x/2)**10 + 6441435*a**8*d*tan(c/2 + d*x/2)**9 + 11594583*a**8*d
*tan(c/2 + d*x/2)**8 + 15459444*a**8*d*tan(c/2 + d*x/2)**7 + 15459444*a**8*
d*tan(c/2 + d*x/2)**6 + 11594583*a**8*d*tan(c/2 + d*x/2)**5 + 6441435*a**8*
d*tan(c/2 + d*x/2)**4 + 2576574*a**8*d*tan(c/2 + d*x/2)**3 + 702702*a**8*d*
tan(c/2 + d*x/2)**2 + 117117*a**8*d*tan(c/2 + d*x/2) + 9009*a**8*d) - 15701
40*tan(c/2 + d*x/2)**5/(9009*a**8*d*tan(c/2 + d*x/2)**13 + 117117*a**8*d*ta
n(c/2 + d*x/2)**12 + 702702*a**8*d*tan(c/2 + d*x/2)**11 + 2576574*a**8*d*ta
n(c/2 + d*x/2)**10 + 6441435*a**8*d*tan(c/2 + d*x/2)**9 + 11594583*a**8*d*t
an(c/2 + d*x/2)**8 + 15459444*a**8*d*tan(c/2 + d*x/2)**7 + 15459444*a**8*d*
tan(c/2 + d*x/2)**6 + 11594583*a**8*d*tan(c/2 + d*x/2)**5 + 6441435*a**8*d*
tan(c/2 + d*x/2)**4 + 2576574*a**8*d*tan(c/2 + d*x/2)**3 + 702702*a**8*d*ta
n(c/2 + d*x/2)**2 + 117117*a**8*d*tan(c/2 + d*x/2) + 9009*a**8*d) - 902330*
tan(c/2 + d*x/2)**4/(9009*a**8*d*tan(c/2 + d*x/2)**13 + 117117*a**8*d*tan(c
/2 + d*x/2)**12 + 702702*a**8*d*tan(c/2 + d*x/2)**11 + 2576574*a**8*d*tan(c
/2 + d*x/2)**10 + 6441435*a**8*d*tan(c/2 + d*x/2)**9 + 11594583*a**8*d*tan(
c/2 + d*x/2)**8 + 15459444*a**8*d*tan(c/2 + d*x/2)**7 + 15459444*a**8*d*tan
(c/2 + d*x/2)**6 + 11594583*a**8*d*tan(c/2 + d*x/2)**5 + 6441435*a**8*d*tan
(c/2 + d*x/2)**4 + 2576574*a**8*d*tan(c/2 + d*x/2)**3 + 702702*a**8*d*tan(c
/2 + d*x/2)**2 + 117117*a**8*d*tan(c/2 + d*x/2) + 9009*a**8*d) - 342914*tan
(c/2 + d*x/2)**3/(9009*a**8*d*tan(c/2 + d*x/2)**13 + 117117*a**8*d*tan(c/2
+ d*x/2)**12 + 702702*a**8*d*tan(c/2 + d*x/2)**11 + 2576574*a**8*d*tan(c/2
+ d*x/2)**10 + 6441435*a**8*d*tan(c/2 + d*x/2)**9 + 11594583*a**8*d*tan(c/2
 + d*x/2)**8 + 15459444*a**8*d*tan(c/2 + d*x/2)...
```

Giac [A]
time = 4.83, size = 177, normalized size = 0.97

$$-\frac{2\left(9009\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{12}+45045\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{11}+183183\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{10}+435435\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{9}+810810\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{8}+1051050\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{7}+1076790\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{6}+785070\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{5}+451165\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{4}+171457\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}+51675\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+7111\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1240\right)}{9009\,a^{8}d\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2/(a+a*sin(d*x+c))^8,x, algorithm="giac")

[Out] -2/9009*(9009*tan(1/2*d*x + 1/2*c)^12 + 45045*tan(1/2*d*x + 1/2*c)^11 + 183183*tan(1/2*d*x + 1/2*c)^10 + 435435*tan(1/2*d*x + 1/2*c)^9 + 810810*tan(1/2*d*x + 1/2*c)^8 + 1051050*tan(1/2*d*x + 1/2*c)^7 + 1076790*tan(1/2*d*x + 1/2*c)^6 + 785070*tan(1/2*d*x + 1/2*c)^5 + 451165*tan(1/2*d*x + 1/2*c)^4 + 171457*tan(1/2*d*x + 1/2*c)^3 + 51675*tan(1/2*d*x + 1/2*c)^2 + 7111*tan(1/2*d*x + 1/2*c) + 1240)/(a^8*d*(tan(1/2*d*x + 1/2*c) + 1)^13)

Mupad [B]
time = 8.12, size = 162, normalized size = 0.89

$$\frac{\sqrt{2}\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{14983\,\cos\left(c+d\,x\right)}{2}-\frac{63921\,\sin\left(c+d\,x\right)}{2}+17605\,\cos\left(2\,c+2\,d\,x\right)-\frac{15365\,\cos\left(3\,c+3\,d\,x\right)}{4}-\frac{6943\,\cos\left(4\,c+4\,d\,x\right)}{4}+\frac{937\,\cos\left(5\,c+5\,d\,x\right)}{4}+\frac{77\,\cos\left(6\,c+6\,d\,x\right)}{4}+\frac{28743\,\sin\left(2\,c+2\,d\,x\right)}{4}+\frac{27027\,\sin\left(3\,c+3\,d\,x\right)}{4}-\frac{5005\,\sin\left(4\,c+4\,d\,x\right)}{4}-\frac{1079\,\sin\left(5\,c+5\,d\,x\right)}{4}+\frac{39\,\sin\left(6\,c+6\,d\,x\right)}{2}-21013\right)}{576576\,a^8\,d\,{\cos\left(\frac{c}{2}-\frac{\pi}{4}+\frac{d\,x}{2}\right)}^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^2/(a + a*sin(c + d*x))^8,x)

[Out] (2^(1/2)*cos(c/2 + (d*x)/2)*((14983*cos(c + d*x))/2 - (63921*sin(c + d*x))/2 + 17605*cos(2*c + 2*d*x) - (15365*cos(3*c + 3*d*x))/4 - (6943*cos(4*c + 4*d*x))/4 + (937*cos(5*c + 5*d*x))/4 + (77*cos(6*c + 6*d*x))/4 + (28743*sin(2*c + 2*d*x))/4 + (27027*sin(3*c + 3*d*x))/4 - (5005*sin(4*c + 4*d*x))/4 - (1079*sin(5*c + 5*d*x))/4 + (39*sin(6*c + 6*d*x))/2 - 21013))/(576576*a^8*d*cos(c/2 - pi/4 + (d*x)/2)^13)

3.95 $\int \frac{\cos(c+dx)}{(a+a\sin(c+dx))^8}\,dx$

Optimal. Leaf size=22

$$-\frac{1}{7ad(a+a\sin(c+dx))^7}$$

[Out] -1/7/a/d/(a+a*sin(d*x+c))^7

Rubi [A]
time = 0.02, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2746, 32}

$$-\frac{1}{7ad(a\sin(c+dx)+a)^7}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]/(a + a*Sin[c + d*x])^8,x]

[Out] -1/7*1/(a*d*(a + a*Sin[c + d*x])^7)

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)}{(a+a\sin(c+dx))^8}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{(a+x)^8}\,dx, x, a\sin(c+dx)\right)}{ad}\\ &= -\frac{1}{7ad(a+a\sin(c+dx))^7}\end{aligned}$$

Mathematica [A]
time = 0.15, size = 33, normalized size = 1.50

$$-\frac{1}{7a^8d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^{14}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + a*Sin[c + d*x])^8,x]
```

```
[Out] -1/7*1/(a^8*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^14)
```

Maple [A]
time = 0.29, size = 21, normalized size = 0.95

method	result
derivativedivides	$-\frac{1}{7ad(a+a\sin(dx+c))^7}$
default	$-\frac{1}{7ad(a+a\sin(dx+c))^7}$
risch	$\frac{128i\,e^{7i(dx+c)}}{7d\,a^8\left(e^{i(dx+c)}+i\right)^{14}}$
norman	$\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{da}+\frac{2\left(\tan^{16}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}+\frac{14\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}+\frac{14\left(\tan^{15}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}+\frac{66\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}+\frac{66\left(\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}+\frac{490(\tan$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+a*sin(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] -1/7/a/d/(a+a*sin(d*x+c))^7
```

Maxima [A]
time = 0.28, size = 20, normalized size = 0.91

$$-\frac{1}{7\left(a\sin\left(dx+c\right)+a\right)^{7}ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/7/((a*sin(d*x + c) + a)^7*a*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 108 vs. $2(20) = 40$.
time = 0.34, size = 108, normalized size = 4.91

$$\frac{1}{7\left(7\,a^8d\cos\left(dx+c\right)^6-56\,a^8d\cos\left(dx+c\right)^4+112\,a^8d\cos\left(dx+c\right)^2-64\,a^8d+\left(a^8d\cos\left(dx+c\right)^6-24\,a^8d\cos\left(dx+c\right)^4+80\,a^8d\cos\left(dx+c\right)^2-64\,a^8d\right)\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/7/(7*a^8*d*cos(d*x + c)^6 - 56*a^8*d*cos(d*x + c)^4 + 112*a^8*d*cos(d*x +
 c)^2 - 64*a^8*d + (a^8*d*cos(d*x + c)^6 - 24*a^8*d*cos(d*x + c)^4 + 80*a^8
*d*cos(d*x + c)^2 - 64*a^8*d)*sin(d*x + c))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(19) = 38$.
time = 14.20, size = 128, normalized size = 5.82

$$\begin{cases} -\frac{1}{7a^8d\sin^7(c+dx)+49a^8d\sin^6(c+dx)+147a^8d\sin^5(c+dx)+245a^8d\sin^4(c+dx)+245a^8d\sin^3(c+dx)+147a^8d\sin^2(c+dx)+49a^8d\sin(c+dx)+7a^8d} & \text{for } d \neq 0 \\ \frac{x\cos(c)}{(a\sin(c)+a)^8} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sin(d*x+c))**8,x)
```

```
[Out] Piecewise((-1/(7*a**8*d*sin(c + d*x)**7 + 49*a**8*d*sin(c + d*x)**6 + 147*a
**8*d*sin(c + d*x)**5 + 245*a**8*d*sin(c + d*x)**4 + 245*a**8*d*sin(c + d*x
)**3 + 147*a**8*d*sin(c + d*x)**2 + 49*a**8*d*sin(c + d*x) + 7*a**8*d), Ne(
d, 0)), (x*cos(c)/(a*sin(c) + a)**8, True))
```

Giac [A]
time = 6.68, size = 20, normalized size = 0.91

$$-\frac{1}{7\left(a\sin\left(dx+c\right)+a\right)^7ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -1/7/((a*sin(d*x + c) + a)^7*a*d)
```

Mupad [B]
time = 4.67, size = 18, normalized size = 0.82

$$-\frac{1}{7\,a^8\,d\,{\left(\sin\left(c+d\,x\right)+1\right)}^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + a*sin(c + d*x))^8,x)
```

```
[Out] -1/(7*a^8*d*(sin(c + d*x) + 1)^7)
```

3.96 $\int \frac{\sec(c+dx)}{(a+a\sin(c+dx))^8}\,dx$

Optimal. Leaf size=194

$$\frac{\tanh^{-1}(\sin(c+dx))}{256a^8d}-\frac{1}{16d(a+a\sin(c+dx))^8}-\frac{1}{28ad(a+a\sin(c+dx))^7}-\frac{1}{48a^2d(a+a\sin(c+dx))^6}-\frac{}{80a^3d}$$

```
[Out] 1/256*arctanh(sin(d*x+c))/a^8/d-1/16/d/(a+a*sin(d*x+c))^8-1/28/a/d/(a+a*sin
(d*x+c))^7-1/48/a^2/d/(a+a*sin(d*x+c))^6-1/80/a^3/d/(a+a*sin(d*x+c))^5-1/19
2/a^5/d/(a+a*sin(d*x+c))^3-1/128/d/(a^2+a^2*sin(d*x+c))^4-1/256/d/(a^4+a^4*
sin(d*x+c))^2-1/256/d/(a^8+a^8*sin(d*x+c))
```

Rubi [A]
time = 0.08, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2746, 46, 212}

$$-\frac{1}{256d\left(a^8\sin(c+dx)+a^8\right)}+\frac{\tanh^{-1}(\sin(c+dx))}{256a^8d}-\frac{1}{192a^5d(a\sin(c+dx)+a)^3}-\frac{1}{256d\left(a^4\sin(c+dx)+a^4\right)^2}-\frac{1}{80a^3d(a\sin(c+dx)+a)^5}-\frac{1}{128d\left(a^2\sin(c+dx)+a^2\right)^4}-\frac{1}{48a^2d(a\sin(c+dx)+a)^6}-\frac{1}{28ad(a\sin(c+dx)+a)^7}-\frac{1}{16d(a\sin(c+dx)+a)^8}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + a*Sin[c + d*x])^8,x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/(256*a^8*d) - 1/(16*d*(a + a*Sin[c + d*x])^8) - 1/(28
*a*d*(a + a*Sin[c + d*x])^7) - 1/(48*a^2*d*(a + a*Sin[c + d*x])^6) - 1/(80*
a^3*d*(a + a*Sin[c + d*x])^5) - 1/(192*a^5*d*(a + a*Sin[c + d*x])^3) - 1/(1
28*d*(a^2 + a^2*Sin[c + d*x])^4) - 1/(256*d*(a^4 + a^4*Sin[c + d*x])^2) - 1
/(256*d*(a^8 + a^8*Sin[c + d*x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
```

```
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a+a\sin(c+dx))^8}\,dx = \frac{a\text{Subst}\left(\int \frac{1}{(a-x)(a+x)^9}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a\text{Subst}\left(\int \left(\frac{1}{2a(a+x)^9} + \frac{1}{4a^2(a+x)^8} + \frac{1}{8a^3(a+x)^7} + \frac{1}{16a^4(a+x)^6} + \frac{1}{32a^5(a+x)^5} + \frac{1}{64a^6(a+x)^4}\right.\right.}{d}$$
$$= -\frac{1}{16d(a+a\sin(c+dx))^8} - \frac{1}{28ad(a+a\sin(c+dx))^7} - \frac{1}{48a^2d(a+a\sin(c+dx))}$$
$$= \frac{\tanh^{-1}(\sin(c+dx))}{256a^8d} - \frac{1}{16d(a+a\sin(c+dx))^8} - \frac{1}{28ad(a+a\sin(c+dx))^7} -$$

Mathematica [A]
time = 0.47, size = 122, normalized size = 0.63

$$-\frac{4096-105\tanh^{-1}(\sin(c+dx))\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^{16}+5993\sin(c+dx)+8008\sin^2(c+dx)+8351\sin^3(c+dx)+6160\sin^4(c+dx)+2975\sin^5(c+dx)+840\sin^6(c+dx)+105\sin^7(c+dx)}{26880a^8d(1+\sin(c+dx))^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + a*Sin[c + d*x])^8,x]
```

```
[Out] -1/26880*(4096 - 105*ArcTanh[Sin[c + d*x]]*(Cos[(c + d*x)/2] + Sin[(c + d*x
)/2])^16 + 5993*Sin[c + d*x] + 8008*Sin[c + d*x]^2 + 8351*Sin[c + d*x]^3 +
6160*Sin[c + d*x]^4 + 2975*Sin[c + d*x]^5 + 840*Sin[c + d*x]^6 + 105*Sin[c
+ d*x]^7)/(a^8*d*(1 + Sin[c + d*x])^8)
```

Maple [A]
time = 0.51, size = 127, normalized size = 0.65

method	result
derivativedivides	$\frac{-\frac{\ln(\sin(dx+c)-1)}{512}-\frac{1}{16(1+\sin(dx+c))^8}-\frac{1}{28(1+\sin(dx+c))^7}-\frac{1}{48(1+\sin(dx+c))^6}-\frac{1}{80(1+\sin(dx+c))^5}-\frac{1}{128(1+\sin(dx+c))^4}-\frac{}{192(1}}{d\,a^8}$
default	$\frac{-\frac{\ln(\sin(dx+c)-1)}{512}-\frac{1}{16(1+\sin(dx+c))^8}-\frac{1}{28(1+\sin(dx+c))^7}-\frac{1}{48(1+\sin(dx+c))^6}-\frac{1}{80(1+\sin(dx+c))^5}-\frac{1}{128(1+\sin(dx+c))^4}-\frac{}{192(1}}{d\,a^8}$
risch	$-\frac{i(1680i e^{14i(dx+c)}+105\,e^{15i(dx+c)}-59360i e^{12i(dx+c)}-12635\,e^{13i(dx+c)}+478576i e^{10i(dx+c)}+195321\,e^{11i(dx+c)}-1366}{}$
norman	$\frac{127\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8da}+\frac{127\left(\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8da}+\frac{255\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{128da}+\frac{255\left(\tan^{15}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{128da}+\frac{1049\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4da}+\frac{1049\left(\tan^{12}\left(\frac{dx}{2}+\right.\right.}{4da}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+a*sin(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^8*(-1/512*ln(sin(d*x+c)-1)-1/16/(1+sin(d*x+c))^8-1/28/(1+sin(d*x+c))^
7-1/48/(1+sin(d*x+c))^6-1/80/(1+sin(d*x+c))^5-1/128/(1+sin(d*x+c))^4-1/192/
(1+sin(d*x+c))^3-1/256/(1+sin(d*x+c))^2-1/256/(1+sin(d*x+c))+1/512*ln(1+sin
(d*x+c)))
```

Maxima [A]
time = 0.30, size = 213, normalized size = 1.10

$$-\frac{\frac{2\left(105\,\sin\left(dx+c\right)^{7}+840\,\sin\left(dx+c\right)^{6}+2975\,\sin\left(dx+c\right)^{5}+6160\,\sin\left(dx+c\right)^{4}+8351\,\sin\left(dx+c\right)^{3}+8008\,\sin\left(dx+c\right)^{2}+5993\,\sin\left(dx+c\right)+4096\right)}{a^{8}\sin\left(dx+c\right)^{8}+8\,a^{8}\sin\left(dx+c\right)^{7}+28\,a^{8}\sin\left(dx+c\right)^{6}+56\,a^{8}\sin\left(dx+c\right)^{5}+70\,a^{8}\sin\left(dx+c\right)^{4}+56\,a^{8}\sin\left(dx+c\right)^{3}+28\,a^{8}\sin\left(dx+c\right)^{2}+8\,a^{8}\sin\left(dx+c\right)+a^{8}}-\frac{105\,\log\left(\sin\left(dx+c\right)+1\right)}{a^{8}}+\frac{105\,\log\left(\sin\left(dx+c\right)-1\right)}{a^{8}}}{53760\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/53760*(2*(105*sin(d*x + c)^7 + 840*sin(d*x + c)^6 + 2975*sin(d*x + c)^5
+ 6160*sin(d*x + c)^4 + 8351*sin(d*x + c)^3 + 8008*sin(d*x + c)^2 + 5993*si
n(d*x + c) + 4096)/(a^8*sin(d*x + c)^8 + 8*a^8*sin(d*x + c)^7 + 28*a^8*sin(
d*x + c)^6 + 56*a^8*sin(d*x + c)^5 + 70*a^8*sin(d*x + c)^4 + 56*a^8*sin(d*x
 + c)^3 + 28*a^8*sin(d*x + c)^2 + 8*a^8*sin(d*x + c) + a^8) - 105*log(sin(d
*x + c) + 1)/a^8 + 105*log(sin(d*x + c) - 1)/a^8)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 374 vs. $2(176) = 352$.
time = 0.40, size = 374, normalized size = 1.93

$$\frac{1680\,\cos\left(dx+c\right)^{6}-17360\,\cos\left(dx+c\right)^{4}+45696\,\cos\left(dx+c\right)^{2}+105\left(\cos\left(dx+c\right)^{8}-32\,\cos\left(dx+c\right)^{6}+160\,\cos\left(dx+c\right)^{4}-256\,\cos\left(dx+c\right)^{2}-8\left(\cos\left(dx+c\right)^{6}-10\,\cos\left(dx+c\right)^{4}+24\,\cos\left(dx+c\right)^{2}-16\right)\sin\left(dx+c\right)+128\right)\log\left(\sin\left(dx+c\right)+1\right)-105\left(\cos\left(dx+c\right)^{8}-32\,\cos\left(dx+c\right)^{6}+160\,\cos\left(dx+c\right)^{4}-256\,\cos\left(dx+c\right)^{2}-8\left(\cos\left(dx+c\right)^{6}-10\,\cos\left(dx+c\right)^{4}+24\,\cos\left(dx+c\right)^{2}-16\right)\sin\left(dx+c\right)+128\right)\log\left(-\sin\left(dx+c\right)+1\right)+2\left(105\,\cos\left(dx+c\right)^{6}-3290\,\cos\left(dx+c\right)^{4}+14616\,\cos\left(dx+c\right)^{2}-17424\right)\sin\left(dx+c\right)-38208}{53760\left(a^{8}d\cos\left(dx+c\right)^{8}-32\,a^{8}d\cos\left(dx+c\right)^{6}+160\,a^{8}d\cos\left(dx+c\right)^{4}-256\,a^{8}d\cos\left(dx+c\right)^{2}+128\,a^{8}d-8\left(a^{8}d\cos\left(dx+c\right)^{6}-10\,a^{8}d\cos\left(dx+c\right)^{4}+24\,a^{8}d\cos\left(dx+c\right)^{2}-16\,a^{8}d\right)\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/53760*(1680*cos(d*x + c)^6 - 17360*cos(d*x + c)^4 + 45696*cos(d*x + c)^2
+ 105*(cos(d*x + c)^8 - 32*cos(d*x + c)^6 + 160*cos(d*x + c)^4 - 256*cos(d*
x + c)^2 - 8*(cos(d*x + c)^6 - 10*cos(d*x + c)^4 + 24*cos(d*x + c)^2 - 16)*
sin(d*x + c) + 128)*log(sin(d*x + c) + 1) - 105*(cos(d*x + c)^8 - 32*cos(d*
x + c)^6 + 160*cos(d*x + c)^4 - 256*cos(d*x + c)^2 - 8*(cos(d*x + c)^6 - 10
*cos(d*x + c)^4 + 24*cos(d*x + c)^2 - 16)*sin(d*x + c) + 128)*log(-sin(d*x
+ c) + 1) + 2*(105*cos(d*x + c)^6 - 3290*cos(d*x + c)^4 + 14616*cos(d*x + c
)^2 - 17424)*sin(d*x + c) - 38208)/(a^8*d*cos(d*x + c)^8 - 32*a^8*d*cos(d*x
 + c)^6 + 160*a^8*d*cos(d*x + c)^4 - 256*a^8*d*cos(d*x + c)^2 + 128*a^8*d -
 8*(a^8*d*cos(d*x + c)^6 - 10*a^8*d*cos(d*x + c)^4 + 24*a^8*d*cos(d*x + c)^
2 - 16*a^8*d)*sin(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sin(d*x+c))**8,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 6.87, size = 131, normalized size = 0.68

$$\frac{\frac{840 \log(|\sin(dx+c)+1|)}{a^8} - \frac{840 \log(|\sin(dx+c)-1|)}{a^8} - \frac{2283 \sin(dx+c)^8+19944 \sin(dx+c)^7+77364 \sin(dx+c)^6+175448 \sin(dx+c)^5+258370 \sin(dx+c)^4+261464 \sin(dx+c)^3+192052 \sin(dx+c)^2+114152 \sin(dx+c)+67819}{a^8(\sin(dx+c)+1)^8}}{430080\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/430080*(840*log(abs(sin(d*x + c) + 1))/a^8 - 840*log(abs(sin(d*x + c) - 1
))/a^8 - (2283*sin(d*x + c)^8 + 19944*sin(d*x + c)^7 + 77364*sin(d*x + c)^6
 + 175448*sin(d*x + c)^5 + 258370*sin(d*x + c)^4 + 261464*sin(d*x + c)^3 +
192052*sin(d*x + c)^2 + 114152*sin(d*x + c) + 67819)/(a^8*(sin(d*x + c) + 1
)^8))/d
```

Mupad [**B**]
time = 0.30, size = 198, normalized size = 1.02

$$\frac{\operatorname{atanh}(\sin(c+d\,x))}{256\,a^8\,d} - \frac{\frac{\sin(c+d\,x)^7}{256} + \frac{\sin(c+d\,x)^6}{32} + \frac{85\sin(c+d\,x)^5}{768} + \frac{11\sin(c+d\,x)^4}{48} + \frac{1193\sin(c+d\,x)^3}{3840} + \frac{143\sin(c+d\,x)^2}{480} + \frac{5993\sin(c+d\,x)}{26880} + \frac{16}{105}}{d\,\left(a^8\sin(c+d\,x)^8 + 8\,a^8\sin(c+d\,x)^7 + 28\,a^8\sin(c+d\,x)^6 + 56\,a^8\sin(c+d\,x)^5 + 70\,a^8\sin(c+d\,x)^4 + 56\,a^8\sin(c+d\,x)^3 + 28\,a^8\sin(c+d\,x)^2 + 8\,a^8\sin(c+d\,x) + a^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + a*sin(c + d*x))^8),x)
```

```
[Out] atanh(sin(c + d*x))/(256*a^8*d) - ((5993*sin(c + d*x))/26880 + (143*sin(c +
 d*x)^2)/480 + (1193*sin(c + d*x)^3)/3840 + (11*sin(c + d*x)^4)/48 + (85*si
n(c + d*x)^5)/768 + sin(c + d*x)^6/32 + sin(c + d*x)^7/256 + 16/105)/(d*(8*
a^8*sin(c + d*x) + a^8 + 28*a^8*sin(c + d*x)^2 + 56*a^8*sin(c + d*x)^3 + 70
*a^8*sin(c + d*x)^4 + 56*a^8*sin(c + d*x)^5 + 28*a^8*sin(c + d*x)^6 + 8*a^8
*sin(c + d*x)^7 + a^8*sin(c + d*x)^8))
```

3.97 $\int \frac{\sec^2(c+dx)}{(a+a\sin(c+dx))^8}\,dx$

Optimal. Leaf size=245

$$-\frac{\sec(c+dx)}{17d(a+a\sin(c+dx))^8}-\frac{3\sec(c+dx)}{85ad(a+a\sin(c+dx))^7}-\frac{24\sec(c+dx)}{1105a^2d(a+a\sin(c+dx))^6}-\frac{168\sec(c+dx)}{12155a^3d(a+a\sin(c+$$

```
[Out] -1/17*sec(d*x+c)/d/(a+a*sin(d*x+c))^8-3/85*sec(d*x+c)/a/d/(a+a*sin(d*x+c))^
7-24/1105*sec(d*x+c)/a^2/d/(a+a*sin(d*x+c))^6-168/12155*sec(d*x+c)/a^3/d/(a
+a*sin(d*x+c))^5-112/12155*sec(d*x+c)/d/(a^2+a^2*sin(d*x+c))^4-16/2431*sec(
d*x+c)/a^2/d/(a^2+a^2*sin(d*x+c))^3-64/12155*sec(d*x+c)/d/(a^4+a^4*sin(d*x+
c))^2-64/12155*sec(d*x+c)/d/(a^8+a^8*sin(d*x+c))+128/12155*tan(d*x+c)/a^8/d
```

Rubi [A]
time = 0.29, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2751, 3852, 8}

$$\frac{128\tan(c+dx)}{12155a^8d}-\frac{64\sec(c+dx)}{12155d\left(a^8\sin(c+dx)+a^8\right)}-\frac{64\sec(c+dx)}{12155d\left(a^4\sin(c+dx)+a^4\right)^2}-\frac{168\sec(c+dx)}{12155a^3d(a\sin(c+dx)+a)^5}-\frac{16\sec(c+dx)}{2431a^2d\left(a^2\sin(c+dx)+a^2\right)^3}-\frac{112\sec(c+dx)}{12155d\left(a^2\sin(c+dx)+a^2\right)^4}-\frac{24\sec(c+dx)}{1105a^2d(a\sin(c+dx)+a)^6}-\frac{3\sec(c+dx)}{85ad(a\sin(c+dx)+a)^7}-\frac{\sec(c+dx)}{17d(a\sin(c+dx)+a)^8}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + a*Sin[c + d*x])^8,x]
```

```
[Out] -1/17*Sec[c + d*x]/(d*(a + a*Sin[c + d*x])^8) - (3*Sec[c + d*x])/(85*a*d*(a
 + a*Sin[c + d*x])^7) - (24*Sec[c + d*x])/(1105*a^2*d*(a + a*Sin[c + d*x])^
6) - (168*Sec[c + d*x])/(12155*a^3*d*(a + a*Sin[c + d*x])^5) - (112*Sec[c +
 d*x])/(12155*d*(a^2 + a^2*Sin[c + d*x])^4) - (16*Sec[c + d*x])/(2431*a^2*d
*(a^2 + a^2*Sin[c + d*x])^3) - (64*Sec[c + d*x])/(12155*d*(a^4 + a^4*Sin[c
+ d*x])^2) - (64*Sec[c + d*x])/(12155*d*(a^8 + a^8*Sin[c + d*x])) + (128*Ta
n[c + d*x])/(12155*a^8*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(a+a\sin(c+dx))^8}\,dx = -\frac{\sec(c+dx)}{17d(a+a\sin(c+dx))^8} + \frac{9\int \frac{\sec^2(c+dx)}{(a+a\sin(c+dx))^7}\,dx}{17a}$$

$$= -\frac{\sec(c+dx)}{17d(a+a\sin(c+dx))^8} - \frac{3\sec(c+dx)}{85ad(a+a\sin(c+dx))^7} + \frac{24\int \frac{\sec^2(c+dx)}{(a+a\sin(c+dx))^6}\,dx}{85a^2}$$

$$= -\frac{\sec(c+dx)}{17d(a+a\sin(c+dx))^8} - \frac{3\sec(c+dx)}{85ad(a+a\sin(c+dx))^7} - \frac{24\sec(c+dx)}{1105a^2d(a+a\sin(c+$$

$$= -\frac{\sec(c+dx)}{17d(a+a\sin(c+dx))^8} - \frac{3\sec(c+dx)}{85ad(a+a\sin(c+dx))^7} - \frac{24\sec(c+dx)}{1105a^2d(a+a\sin(c+$$

$$= -\frac{\sec(c+dx)}{17d(a+a\sin(c+dx))^8} - \frac{3\sec(c+dx)}{85ad(a+a\sin(c+dx))^7} - \frac{24\sec(c+dx)}{1105a^2d(a+a\sin(c+$$

$$= -\frac{\sec(c+dx)}{17d(a+a\sin(c+dx))^8} - \frac{3\sec(c+dx)}{85ad(a+a\sin(c+dx))^7} - \frac{24\sec(c+dx)}{1105a^2d(a+a\sin(c+$$

$$= -\frac{\sec(c+dx)}{17d(a+a\sin(c+dx))^8} - \frac{3\sec(c+dx)}{85ad(a+a\sin(c+dx))^7} - \frac{24\sec(c+dx)}{1105a^2d(a+a\sin(c+$$

$$= -\frac{\sec(c+dx)}{17d(a+a\sin(c+dx))^8} - \frac{3\sec(c+dx)}{85ad(a+a\sin(c+dx))^7} - \frac{24\sec(c+dx)}{1105a^2d(a+a\sin(c+$$

$$= -\frac{\sec(c+dx)}{17d(a+a\sin(c+dx))^8} - \frac{3\sec(c+dx)}{85ad(a+a\sin(c+dx))^7} - \frac{24\sec(c+dx)}{1105a^2d(a+a\sin(c+$$

$$= -\frac{\sec(c+dx)}{17d(a+a\sin(c+dx))^8} - \frac{3\sec(c+dx)}{85ad(a+a\sin(c+dx))^7} - \frac{24\sec(c+dx)}{1105a^2d(a+a\sin(c+$$

Mathematica [A]

time = 0.22, size = 113, normalized size = 0.46

$$\frac{\sec(c+dx)(-7072\cos(2(c+dx))+3808\cos(4(c+dx))-544\cos(6(c+dx))+16\cos(8(c+dx))+4862\sin(c+dx)-6188\sin(3(c+dx))+1700\sin(5(c+dx))-119\sin(7(c+dx))+\sin(9(c+dx)))}{24310a^8d(1+\sin(c+dx))^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + a*Sin[c + d*x])^8,x]
```

```
[Out] (Sec[c + d*x]*(-7072*Cos[2*(c + d*x)] + 3808*Cos[4*(c + d*x)] - 544*Cos[6*(
c + d*x)] + 16*Cos[8*(c + d*x)] + 4862*Sin[c + d*x] - 6188*Sin[3*(c + d*x)]
```

```
 + 1700*Sin[5*(c + d*x)] - 119*Sin[7*(c + d*x)] + Sin[9*(c + d*x)]))/(24310
*a^8*d*(1 + Sin[c + d*x])^8)
```

Maple [A]
time = 0.25, size = 280, normalized size = 1.14

method	result
risch	$\frac{256i\left(7072ie^{7i(dx+c)}+4862\,e^{8i(dx+c)}-3808ie^{5i(dx+c)}-6188\,e^{6i(dx+c)}+544ie^{3i(dx+c)}+1700\,e^{4i(dx+c)}-16ie^{i(dx+c)}-119\,e\right.}{12155\left(e^{i(dx+c)}+i\right)^{17}\left(e^{i(dx+c)}-i\right)a^8d}$
derivativedivides	$-\frac{256}{17\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{17}}+\frac{128}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{16}}-\frac{2752}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{15}}+\frac{1568}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{14}}-\frac{42800}{13\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{13}}+\frac{53}{\left(\tan\left(\frac{dx}{2}+\right.\right.}$
default	$-\frac{256}{17\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{17}}+\frac{128}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{16}}-\frac{2752}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{15}}+\frac{1568}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{14}}-\frac{42800}{13\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{13}}+\frac{53}{\left(\tan\left(\frac{dx}{2}+\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+a*sin(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/a^8*(-128/17/(tan(1/2*d*x+1/2*c)+1)^17+64/(tan(1/2*d*x+1/2*c)+1)^16-137
6/5/(tan(1/2*d*x+1/2*c)+1)^15+784/(tan(1/2*d*x+1/2*c)+1)^14-21400/13/(tan(1
/2*d*x+1/2*c)+1)^13+2692/(tan(1/2*d*x+1/2*c)+1)^12-38954/11/(tan(1/2*d*x+1/
2*c)+1)^11+19109/5/(tan(1/2*d*x+1/2*c)+1)^10-6847/2/(tan(1/2*d*x+1/2*c)+1)^
9+10241/4/(tan(1/2*d*x+1/2*c)+1)^8-12799/8/(tan(1/2*d*x+1/2*c)+1)^7+13313/1
6/(tan(1/2*d*x+1/2*c)+1)^6-57083/160/(tan(1/2*d*x+1/2*c)+1)^5+7937/64/(tan(
1/2*d*x+1/2*c)+1)^4-4351/128/(tan(1/2*d*x+1/2*c)+1)^3+1793/256/(tan(1/2*d*x
+1/2*c)+1)^2-511/512/(tan(1/2*d*x+1/2*c)+1)-1/512/(tan(1/2*d*x+1/2*c)-1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 740 vs. $2(227) = 454$.
time = 0.35, size = 740, normalized size = 3.02

$$-\frac{2\left(\frac{18181\sin(dx+c)}{\cos(dx+c)+1}+\frac{128384\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{545224\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{1667360\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{3612364\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{5742464\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{6271096\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{3928496\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{850850\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{5289856\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}-\frac{7137416\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}-\frac{5989984\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}-\frac{3607604\sin(dx+c)^{13}}{(\cos(dx+c)+1)^{13}}-\frac{1555840\sin(dx+c)^{14}}{(\cos(dx+c)+1)^{14}}-\frac{486200\sin(dx+c)^{15}}{(\cos(dx+c)+1)^{15}}-\frac{97240\sin(dx+c)^{16}}{(\cos(dx+c)+1)^{16}}-\frac{12155\sin(dx+c)^{17}}{(\cos(dx+c)+1)^{17}}+1896\right)}{12155\left(a^8+\frac{16a^8\sin(dx+c)}{\cos(dx+c)+1}+\frac{119a^8\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{544a^8\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{1700a^8\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{3808a^8\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{6188a^8\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{7072a^8\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{4862a^8\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{4862a^8\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}-\frac{7072a^8\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}-\frac{6188a^8\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}-\frac{3808a^8\sin(dx+c)^{13}}{(\cos(dx+c)+1)^{13}}-\frac{1700a^8\sin(dx+c)^{14}}{(\cos(dx+c)+1)^{14}}-\frac{544a^8\sin(dx+c)^{15}}{(\cos(dx+c)+1)^{15}}-\frac{119a^8\sin(dx+c)^{16}}{(\cos(dx+c)+1)^{16}}-\frac{16a^8\sin(dx+c)^{17}}{(\cos(dx+c)+1)^{17}}-\frac{a^8\sin(dx+c)^{18}}{(\cos(dx+c)+1)^{18}}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -2/12155*(18181*sin(d*x + c)/(cos(d*x + c) + 1) + 128384*sin(d*x + c)^2/(co
s(d*x + c) + 1)^2 + 545224*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 1667360*si
n(d*x + c)^4/(cos(d*x + c) + 1)^4 + 3612364*sin(d*x + c)^5/(cos(d*x + c) +
1)^5 + 5742464*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 6271096*sin(d*x + c)^7
/(cos(d*x + c) + 1)^7 + 3928496*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 85085
0*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 5289856*sin(d*x + c)^10/(cos(d*x +
c) + 1)^10 - 7137416*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 - 5989984*sin(d*
x + c)^12/(cos(d*x + c) + 1)^12 - 3607604*sin(d*x + c)^13/(cos(d*x + c) + 1
)^13 - 1555840*sin(d*x + c)^14/(cos(d*x + c) + 1)^14 - 486200*sin(d*x + c)^
15/(cos(d*x + c) + 1)^15 - 97240*sin(d*x + c)^16/(cos(d*x + c) + 1)^16 - 12
```

```
155*sin(d*x + c)^17/(cos(d*x + c) + 1)^17 + 1896)/((a^8 + 16*a^8*sin(d*x +
c)/(cos(d*x + c) + 1) + 119*a^8*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 544*a
^8*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 1700*a^8*sin(d*x + c)^4/(cos(d*x +
 c) + 1)^4 + 3808*a^8*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 6188*a^8*sin(d*
x + c)^6/(cos(d*x + c) + 1)^6 + 7072*a^8*sin(d*x + c)^7/(cos(d*x + c) + 1)^
7 + 4862*a^8*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 4862*a^8*sin(d*x + c)^10
/(cos(d*x + c) + 1)^10 - 7072*a^8*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 - 6
188*a^8*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 - 3808*a^8*sin(d*x + c)^13/(c
os(d*x + c) + 1)^13 - 1700*a^8*sin(d*x + c)^14/(cos(d*x + c) + 1)^14 - 544*
a^8*sin(d*x + c)^15/(cos(d*x + c) + 1)^15 - 119*a^8*sin(d*x + c)^16/(cos(d*
x + c) + 1)^16 - 16*a^8*sin(d*x + c)^17/(cos(d*x + c) + 1)^17 - a^8*sin(d*x
 + c)^18/(cos(d*x + c) + 1)^18)*d)
```

Fricas [**A**]
time = 0.36, size = 225, normalized size = 0.92

$$\frac{1024\cos(dx+c)^8-10752\cos(dx+c)^6+29568\cos(dx+c)^4-27456\cos(dx+c)^2+\left(128\cos(dx+c)^8-4032\cos(dx+c)^6+18480\cos(dx+c)^4-24024\cos(dx+c)^2+6435\right)\sin(dx+c)+5720}{12155\left(a^8d\cos(dx+c)^9-32a^8d\cos(dx+c)^7+160a^8d\cos(dx+c)^5-256a^8d\cos(dx+c)^3+128a^8d\cos(dx+c)-8\left(a^8d\cos(dx+c)^7-10a^8d\cos(dx+c)^5+24a^8d\cos(dx+c)^3-16a^8d\cos(dx+c)\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/12155*(1024*cos(d*x + c)^8 - 10752*cos(d*x + c)^6 + 29568*cos(d*x + c)^4
- 27456*cos(d*x + c)^2 + (128*cos(d*x + c)^8 - 4032*cos(d*x + c)^6 + 18480*
cos(d*x + c)^4 - 24024*cos(d*x + c)^2 + 6435)*sin(d*x + c) + 5720)/(a^8*d*c
os(d*x + c)^9 - 32*a^8*d*cos(d*x + c)^7 + 160*a^8*d*cos(d*x + c)^5 - 256*a^
8*d*cos(d*x + c)^3 + 128*a^8*d*cos(d*x + c) - 8*(a^8*d*cos(d*x + c)^7 - 10*
a^8*d*cos(d*x + c)^5 + 24*a^8*d*cos(d*x + c)^3 - 16*a^8*d*cos(d*x + c))*sin
(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+a*sin(d*x+c))**8,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 7.84, size = 249, normalized size = 1.02

$$-\frac{\frac{12155}{a^8\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)}+\frac{6211205\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{16}+55791450\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{15}+303072770\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{14}+1091397450\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{13}+2909561798\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{12}+5901218466\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{11}+9405145178\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{10}+11877161010\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{9}+12017308160\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{8}+9710430158\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{7}+6263238566\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{6}+3172666718\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{5}+1247921210\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{4}+365303990\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}+77883902\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+10498214\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+982907}{a^8\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)^{17}}}{3111680\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -1/3111680*(12155/(a^8*(tan(1/2*d*x + 1/2*c) - 1)) + (6211205*tan(1/2*d*x +
 1/2*c)^16 + 55791450*tan(1/2*d*x + 1/2*c)^15 + 303072770*tan(1/2*d*x + 1/2
*c)^14 + 1091397450*tan(1/2*d*x + 1/2*c)^13 + 2909561798*tan(1/2*d*x + 1/2*
c)^12 + 5901218466*tan(1/2*d*x + 1/2*c)^11 + 9405145178*tan(1/2*d*x + 1/2*c
)^10 + 11877161010*tan(1/2*d*x + 1/2*c)^9 + 12017308160*tan(1/2*d*x + 1/2*c
)^8 + 9710430158*tan(1/2*d*x + 1/2*c)^7 + 6263238566*tan(1/2*d*x + 1/2*c)^6
 + 3172666718*tan(1/2*d*x + 1/2*c)^5 + 1247921210*tan(1/2*d*x + 1/2*c)^4 +
365303990*tan(1/2*d*x + 1/2*c)^3 + 77883902*tan(1/2*d*x + 1/2*c)^2 + 104982
14*tan(1/2*d*x + 1/2*c) + 982907)/(a^8*(tan(1/2*d*x + 1/2*c) + 1)^17))/d
```

Mupad [B]
time = 8.04, size = 233, normalized size = 0.95

$$\frac{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{519571\,\cos\left(\frac{5\,c}{2}+\frac{5\,d\,x}{2}\right)}{16}-\frac{576147\,\cos\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)}{16}+\frac{213707\,\cos\left(\frac{7\,c}{2}+\frac{7\,d\,x}{2}\right)}{16}-\frac{183243\,\cos\left(\frac{9\,c}{2}+\frac{9\,d\,x}{2}\right)}{16}-\frac{18207\,\cos\left(\frac{11\,c}{2}+\frac{11\,d\,x}{2}\right)}{16}+\frac{13855\,\cos\left(\frac{13\,c}{2}+\frac{13\,d\,x}{2}\right)}{16}+\frac{493\,\cos\left(\frac{15\,c}{2}+\frac{15\,d\,x}{2}\right)}{32}-\frac{237\,\cos\left(\frac{17\,c}{2}+\frac{17\,d\,x}{2}\right)}{32}+\frac{56425\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}-\frac{51563\,\sin\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)}{2}-\frac{53191\,\sin\left(\frac{5\,c}{2}+\frac{5\,d\,x}{2}\right)}{2}+\frac{47003\,\sin\left(\frac{7\,c}{2}+\frac{7\,d\,x}{2}\right)}{2}+\frac{9403\,\sin\left(\frac{9\,c}{2}+\frac{9\,d\,x}{2}\right)}{2}-\frac{7703\,\sin\left(\frac{11\,c}{2}+\frac{11\,d\,x}{2}\right)}{2}-\frac{355\,\sin\left(\frac{13\,c}{2}+\frac{13\,d\,x}{2}\right)}{2}+118\,\sin\left(\frac{15\,c}{2}+\frac{15\,d\,x}{2}\right)+\frac{\sin\left(\frac{17\,c}{2}+\frac{17\,d\,x}{2}\right)}{2}\right)}{3111680\,a^8\,d\,{\cos\left(\frac{c}{2}-\frac{\pi }{4}+\frac{d\,x}{2}\right)}^{17}\,\cos\left(\frac{c}{2}+\frac{\pi }{4}+\frac{d\,x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + a*sin(c + d*x))^8),x)
```

```
[Out] (cos(c/2 + (d*x)/2)*((519571*cos((5*c)/2 + (5*d*x)/2))/16 - (576147*cos((3*
c)/2 + (3*d*x)/2))/16 + (213707*cos((7*c)/2 + (7*d*x)/2))/16 - (183243*cos(
(9*c)/2 + (9*d*x)/2))/16 - (18207*cos((11*c)/2 + (11*d*x)/2))/16 + (13855*c
os((13*c)/2 + (13*d*x)/2))/16 + (493*cos((15*c)/2 + (15*d*x)/2))/32 - (237*
cos((17*c)/2 + (17*d*x)/2))/32 + (56425*sin(c/2 + (d*x)/2))/2 - (51563*sin(
(3*c)/2 + (3*d*x)/2))/2 - (53191*sin((5*c)/2 + (5*d*x)/2))/2 + (47003*sin((
7*c)/2 + (7*d*x)/2))/2 + (9403*sin((9*c)/2 + (9*d*x)/2))/2 - (7703*sin((11*
c)/2 + (11*d*x)/2))/2 - (355*sin((13*c)/2 + (13*d*x)/2))/2 + 118*sin((15*c)
/2 + (15*d*x)/2) + sin((17*c)/2 + (17*d*x)/2)/2))/(3111680*a^8*d*cos(c/2 -
pi/4 + (d*x)/2)^17*cos(c/2 + pi/4 + (d*x)/2))
```

3.98 $\int \frac{\sec^3(c+dx)}{(a+a\sin(c+dx))^8}\,dx$

Optimal. Leaf size=238

$$\frac{5\tanh^{-1}(\sin(c+dx))}{512a^8d}-\frac{a}{36d(a+a\sin(c+dx))^9}-\frac{1}{32d(a+a\sin(c+dx))^8}-\frac{3}{112ad(a+a\sin(c+dx))^7}-\frac{}{48a}$$

```
[Out] 5/512*arctanh(sin(d*x+c))/a^8/d-1/36*a/d/(a+a*sin(d*x+c))^9-1/32/d/(a+a*sin
(d*x+c))^8-3/112/a/d/(a+a*sin(d*x+c))^7-1/48/a^2/d/(a+a*sin(d*x+c))^6-1/64/
a^3/d/(a+a*sin(d*x+c))^5-7/768/a^5/d/(a+a*sin(d*x+c))^3-3/256/d/(a^2+a^2*si
n(d*x+c))^4-1/128/d/(a^4+a^4*sin(d*x+c))^2+1/1024/d/(a^8-a^8*sin(d*x+c))-9/
1024/d/(a^8+a^8*sin(d*x+c))
```

Rubi [A]
time = 0.12, antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2746, 46, 212}

$$\frac{1}{1024d(a^8-a^8\sin(c+dx))}-\frac{9}{1024d(a^8\sin(c+dx)+a^8)}+\frac{5\tanh^{-1}(\sin(c+dx))}{512a^8d}-\frac{7}{768a^5d(a\sin(c+dx)+a)^3}-\frac{1}{128d(a^4\sin(c+dx)+a^4)^2}-\frac{1}{64a^3d(a\sin(c+dx)+a)^5}-\frac{3}{256d(a^2\sin(c+dx)+a^2)^4}-\frac{1}{48a^2d(a\sin(c+dx)+a)^6}-\frac{a}{36d(a\sin(c+dx)+a)^9}-\frac{1}{32d(a\sin(c+dx)+a)^8}-\frac{3}{112ad(a\sin(c+dx)+a)^7}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + a*Sin[c + d*x])^8,x]
```

```
[Out] (5*ArcTanh[Sin[c + d*x]])/(512*a^8*d) - a/(36*d*(a + a*Sin[c + d*x])^9) - 1
/(32*d*(a + a*Sin[c + d*x])^8) - 3/(112*a*d*(a + a*Sin[c + d*x])^7) - 1/(48
*a^2*d*(a + a*Sin[c + d*x])^6) - 1/(64*a^3*d*(a + a*Sin[c + d*x])^5) - 7/(7
68*a^5*d*(a + a*Sin[c + d*x])^3) - 3/(256*d*(a^2 + a^2*Sin[c + d*x])^4) - 1
/(128*d*(a^4 + a^4*Sin[c + d*x])^2) + 1/(1024*d*(a^8 - a^8*Sin[c + d*x])) -
 9/(1024*d*(a^8 + a^8*Sin[c + d*x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
```

```
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a+a\sin(c+dx))^8}\,dx = \frac{a^3\text{Subst}\left(\int \frac{1}{(a-x)^2(a+x)^{10}}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^3\text{Subst}\left(\int \left(\frac{1}{1024a^{10}(a-x)^2} + \frac{1}{4a^2(a+x)^{10}} + \frac{1}{4a^3(a+x)^9} + \frac{3}{16a^4(a+x)^8} + \frac{1}{8a^5(a+x)^7} + \frac{5}{64a^6(a\cdots}\right.\right.}{\cdots}$$
$$= -\frac{a}{36d(a+a\sin(c+dx))^9} - \frac{1}{32d(a+a\sin(c+dx))^8} - \frac{3}{112ad(a+a\sin(c+dx)\cdots}$$
$$= \frac{5\tanh^{-1}(\sin(c+dx))}{512a^8d} - \frac{a}{36d(a+a\sin(c+dx))^9} - \frac{1}{32d(a+a\sin(c+dx))^8} - \cdots$$

Mathematica [A]
time = 0.99, size = 175, normalized size = 0.74

$$-\frac{\sec^2(c+dx)\left(5120-315\tanh^{-1}(\sin(c+dx))\left(\cos\left(\tfrac{1}{2}(c+dx)\right)-\sin\left(\tfrac{1}{2}(c+dx)\right)\right)^2\left(\cos\left(\tfrac{1}{2}(c+dx)\right)+\sin\left(\tfrac{1}{2}(c+dx)\right)\right)^{18}+9019\sin(c+dx)+11736\sin^2(c+dx)+7074\sin^3(c+dx)-5544\sin^4(c+dx)-16128\sin^5(c+dx)-15960\sin^6(c+dx)-8610\sin^7(c+dx)-2520\sin^8(c+dx)-315\sin^9(c+dx)\right)}{32256a^8d(1+\sin(c+dx))^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + a*Sin[c + d*x])^8,x]
```

```
[Out] -1/32256*(Sec[c + d*x]^2*(5120 - 315*ArcTanh[Sin[c + d*x]]*(Cos[(c + d*x)/2
] - Sin[(c + d*x)/2])^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^18 + 9019*Sin
[c + d*x] + 11736*Sin[c + d*x]^2 + 7074*Sin[c + d*x]^3 - 5544*Sin[c + d*x]^
4 - 16128*Sin[c + d*x]^5 - 15960*Sin[c + d*x]^6 - 8610*Sin[c + d*x]^7 - 252
0*Sin[c + d*x]^8 - 315*Sin[c + d*x]^9))/(a^8*d*(1 + Sin[c + d*x])^8)
```

Maple [A]
time = 0.38, size = 151, normalized size = 0.63

method	result
derivativedivides	$\frac{-\frac{1}{1024(\sin(dx+c)-1)}-\frac{5\ln(\sin(dx+c)-1)}{1024}-\frac{1}{36(1+\sin(dx+c))^9}-\frac{1}{32(1+\sin(dx+c))^8}-\frac{3}{112(1+\sin(dx+c))^7}-\frac{1}{48(1+\sin(dx+c))^6}-\frac{\cdots}{64(1+\cdots}}{d\,a^8}$
default	$\frac{-\frac{1}{1024(\sin(dx+c)-1)}-\frac{5\ln(\sin(dx+c)-1)}{1024}-\frac{1}{36(1+\sin(dx+c))^9}-\frac{1}{32(1+\sin(dx+c))^8}-\frac{3}{112(1+\sin(dx+c))^7}-\frac{1}{48(1+\sin(dx+c))^6}-\frac{\cdots}{64(1+\cdots}}{d\,a^8}$
risch	$-\frac{i\left(-168000i\,e^{4i(dx+c)}+315\,e^{19i(dx+c)}+5040i\,e^{2i(dx+c)}-37275\,e^{17i(dx+c)}-1404864i\,e^{12i(dx+c)}+510468\,e^{15i(dx+c)}-165\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+a*sin(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^8*(-1/1024/(sin(d*x+c)-1)-5/1024*ln(sin(d*x+c)-1)-1/36/(1+sin(d*x+c))
^9-1/32/(1+sin(d*x+c))^8-3/112/(1+sin(d*x+c))^7-1/48/(1+sin(d*x+c))^6-1/64/
(1+sin(d*x+c))^5-3/256/(1+sin(d*x+c))^4-7/768/(1+sin(d*x+c))^3-1/128/(1+sin
(d*x+c))^2-9/1024/(1+sin(d*x+c))+5/1024*ln(1+sin(d*x+c)))
```

Maxima [A]
time = 0.30, size = 248, normalized size = 1.04

$$-\frac{\frac{2\left(315 \sin(dx+c)^9+2520 \sin(dx+c)^8+8610 \sin(dx+c)^7+15960 \sin(dx+c)^6+16128 \sin(dx+c)^5+5544 \sin(dx+c)^4-7074 \sin(dx+c)^3-11736 \sin(dx+c)^2-9019 \sin(dx+c)-5120\right)}{a^8 \sin(dx+c)^{10}+8 a^8 \sin(dx+c)^9+27 a^8 \sin(dx+c)^8+48 a^8 \sin(dx+c)^7+42 a^8 \sin(dx+c)^6-42 a^8 \sin(dx+c)^4-48 a^8 \sin(dx+c)^3-27 a^8 \sin(dx+c)^2-8 a^8 \sin(dx+c)-a^8}-\frac{315 \log(\sin(dx+c)+1)}{a^8}+\frac{315 \log(\sin(dx+c)-1)}{a^8}}{64512\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/64512*(2*(315*sin(d*x + c)^9 + 2520*sin(d*x + c)^8 + 8610*sin(d*x + c)^7
 + 15960*sin(d*x + c)^6 + 16128*sin(d*x + c)^5 + 5544*sin(d*x + c)^4 - 7074
*sin(d*x + c)^3 - 11736*sin(d*x + c)^2 - 9019*sin(d*x + c) - 5120)/(a^8*sin
(d*x + c)^10 + 8*a^8*sin(d*x + c)^9 + 27*a^8*sin(d*x + c)^8 + 48*a^8*sin(d*
x + c)^7 + 42*a^8*sin(d*x + c)^6 - 42*a^8*sin(d*x + c)^4 - 48*a^8*sin(d*x +
 c)^3 - 27*a^8*sin(d*x + c)^2 - 8*a^8*sin(d*x + c) - a^8) - 315*log(sin(d*x
 + c) + 1)/a^8 + 315*log(sin(d*x + c) - 1)/a^8)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 446 vs. 2(217) = 434.
time = 0.38, size = 446, normalized size = 1.87

$$\frac{5040 \cos(dx+c)^8-52080 \cos(dx+c)^6+137088 \cos(dx+c)^4-114624 \cos(dx+c)^2+315\left(\cos(dx+c)^{10}-32 \cos(dx+c)^8+160 \cos(dx+c)^6-256 \cos(dx+c)^4+128 \cos(dx+c)^2-8\left(\cos(dx+c)^8-10 \cos(dx+c)^6+24 \cos(dx+c)^4-16 \cos(dx+c)^2\right)\sin(dx+c)\right)\log(\sin(dx+c)+1)-315\left(\cos(dx+c)^{10}-32 \cos(dx+c)^8+160 \cos(dx+c)^6-256 \cos(dx+c)^4+128 \cos(dx+c)^2-8\left(\cos(dx+c)^8-10 \cos(dx+c)^6+24 \cos(dx+c)^4-16 \cos(dx+c)^2\right)\sin(dx+c)\right)\log(-\sin(dx+c)+1)+2\left(315 \cos(dx+c)^8-9870 \cos(dx+c)^6+43848 \cos(dx+c)^4-52272 \cos(dx+c)^2+8960\right)\sin(dx+c)+14336}{64512\left(a^8 d \cos(dx+c)^{10}-32 a^8 d \cos(dx+c)^8+160 a^8 d \cos(dx+c)^6-256 a^8 d \cos(dx+c)^4+128 a^8 d \cos(dx+c)^2-8\left(a^8 d \cos(dx+c)^8-10 a^8 d \cos(dx+c)^6+24 a^8 d \cos(dx+c)^4-16 a^8 d \cos(dx+c)^2\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/64512*(5040*cos(d*x + c)^8 - 52080*cos(d*x + c)^6 + 137088*cos(d*x + c)^4
 - 114624*cos(d*x + c)^2 + 315*(cos(d*x + c)^10 - 32*cos(d*x + c)^8 + 160*c
os(d*x + c)^6 - 256*cos(d*x + c)^4 + 128*cos(d*x + c)^2 - 8*(cos(d*x + c)^8
 - 10*cos(d*x + c)^6 + 24*cos(d*x + c)^4 - 16*cos(d*x + c)^2)*sin(d*x + c))
*log(sin(d*x + c) + 1) - 315*(cos(d*x + c)^10 - 32*cos(d*x + c)^8 + 160*cos
(d*x + c)^6 - 256*cos(d*x + c)^4 + 128*cos(d*x + c)^2 - 8*(cos(d*x + c)^8 -
 10*cos(d*x + c)^6 + 24*cos(d*x + c)^4 - 16*cos(d*x + c)^2)*sin(d*x + c))*l
og(-sin(d*x + c) + 1) + 2*(315*cos(d*x + c)^8 - 9870*cos(d*x + c)^6 + 43848
*cos(d*x + c)^4 - 52272*cos(d*x + c)^2 + 8960)*sin(d*x + c) + 14336)/(a^8*d
*cos(d*x + c)^10 - 32*a^8*d*cos(d*x + c)^8 + 160*a^8*d*cos(d*x + c)^6 - 256
```

```
*a^8*d*cos(d*x + c)^4 + 128*a^8*d*cos(d*x + c)^2 - 8*(a^8*d*cos(d*x + c)^8
- 10*a^8*d*cos(d*x + c)^6 + 24*a^8*d*cos(d*x + c)^4 - 16*a^8*d*cos(d*x + c)
^2)*sin(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+a*sin(d*x+c))**8,x)
```

```
[Out] Timed out
```

Giac [A]
time = 4.67, size = 166, normalized size = 0.70

$$\frac{\frac{2520\log(|\sin(dx+c)+1|)}{a^8}-\frac{2520\log(|\sin(dx+c)-1|)}{a^8}+\frac{504\,(5\sin(dx+c)-6)}{a^8(\sin(dx+c)-1)}-\frac{7129\sin(dx+c)^9+68697\sin(dx+c)^8+296964\sin(dx+c)^7+758772\sin(dx+c)^6+1271214\sin(dx+c)^5+1465758\sin(dx+c)^4+1191540\sin(dx+c)^3+693828\sin(dx+c)^2+295425\sin(dx+c)+89553}{a^8(\sin(dx+c)+1)^9}}{516096\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/516096*(2520*log(abs(sin(d*x + c) + 1))/a^8 - 2520*log(abs(sin(d*x + c) -
 1))/a^8 + 504*(5*sin(d*x + c) - 6)/(a^8*(sin(d*x + c) - 1)) - (7129*sin(d*
x + c)^9 + 68697*sin(d*x + c)^8 + 296964*sin(d*x + c)^7 + 758772*sin(d*x +
c)^6 + 1271214*sin(d*x + c)^5 + 1465758*sin(d*x + c)^4 + 1191540*sin(d*x +
c)^3 + 693828*sin(d*x + c)^2 + 295425*sin(d*x + c) + 89553)/(a^8*(sin(d*x +
 c) + 1)^9))/d
```

Mupad [B]
time = 0.49, size = 231, normalized size = 0.97

$$\frac{\frac{5\sin(c+d\,x)^9}{512}+\frac{5\sin(c+d\,x)^8}{64}+\frac{205\sin(c+d\,x)^7}{768}+\frac{95\sin(c+d\,x)^6}{192}+\frac{\sin(c+d\,x)^5}{2}+\frac{11\sin(c+d\,x)^4}{64}-\frac{393\sin(c+d\,x)^3}{1792}-\frac{163\sin(c+d\,x)^2}{448}-\frac{9019\sin(c+d\,x)}{32256}-\frac{10}{63}}{d\,\left(-a^8\sin\left(c+d\,x\right)^{10}-8\,a^8\sin\left(c+d\,x\right)^9-27\,a^8\sin\left(c+d\,x\right)^8-48\,a^8\sin\left(c+d\,x\right)^7-42\,a^8\sin\left(c+d\,x\right)^6+42\,a^8\sin\left(c+d\,x\right)^4+48\,a^8\sin\left(c+d\,x\right)^3+27\,a^8\sin\left(c+d\,x\right)^2+8\,a^8\,\sin\left(c+d\,x\right)+a^8\right)}+\frac{5\,\mathrm{atanh}\left(\sin\left(c+d\,x\right)\right)}{512\,a^8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + a*sin(c + d*x))^8),x)
```

```
[Out] ((11*sin(c + d*x)^4)/64 - (163*sin(c + d*x)^2)/448 - (393*sin(c + d*x)^3)/1
792 - (9019*sin(c + d*x))/32256 + sin(c + d*x)^5/2 + (95*sin(c + d*x)^6)/19
2 + (205*sin(c + d*x)^7)/768 + (5*sin(c + d*x)^8)/64 + (5*sin(c + d*x)^9)/5
12 - 10/63)/(d*(8*a^8*sin(c + d*x) + a^8 + 27*a^8*sin(c + d*x)^2 + 48*a^8*s
in(c + d*x)^3 + 42*a^8*sin(c + d*x)^4 - 42*a^8*sin(c + d*x)^6 - 48*a^8*sin(
c + d*x)^7 - 27*a^8*sin(c + d*x)^8 - 8*a^8*sin(c + d*x)^9 - a^8*sin(c + d*x
)^10)) + (5*atanh(sin(c + d*x)))/(512*a^8*d)
```

3.99 $\int \frac{\sec^4(c+dx)}{(a+a\sin(c+dx))^8}\,dx$

Optimal. Leaf size=279

$$-\frac{\sec^3(c+dx)}{19d(a+a\sin(c+dx))^8}-\frac{11\sec^3(c+dx)}{323ad(a+a\sin(c+dx))^7}-\frac{22\sec^3(c+dx)}{969a^2d(a+a\sin(c+dx))^6}-\frac{66\sec^3(c+dx)}{4199a^3d(a+a\sin(c+$$

```
[Out] -1/19*sec(d*x+c)^3/d/(a+a*sin(d*x+c))^8-11/323*sec(d*x+c)^3/a/d/(a+a*sin(d*
x+c))^7-22/969*sec(d*x+c)^3/a^2/d/(a+a*sin(d*x+c))^6-66/4199*sec(d*x+c)^3/a
^3/d/(a+a*sin(d*x+c))^5-48/4199*sec(d*x+c)^3/d/(a^2+a^2*sin(d*x+c))^4-112/1
2597*sec(d*x+c)^3/a^2/d/(a^2+a^2*sin(d*x+c))^3-32/4199*sec(d*x+c)^3/d/(a^4+
a^4*sin(d*x+c))^2-32/4199*sec(d*x+c)^3/d/(a^8+a^8*sin(d*x+c))+128/4199*tan(
d*x+c)/a^8/d+128/12597*tan(d*x+c)^3/a^8/d
```

Rubi [A]
time = 0.30, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2751, 3852}

$$\frac{128\tan^3(c+dx)}{12597a^8d}+\frac{128\tan(c+dx)}{4199a^8d}-\frac{32\sec^3(c+dx)}{4199d\left(a^8\sin(c+dx)+a^8\right)}-\frac{32\sec^3(c+dx)}{4199d\left(a^4\sin(c+dx)+a^4\right)^2}-\frac{66\sec^3(c+dx)}{4199a^3d(a\sin(c+dx)+a)^5}-\frac{112\sec^3(c+dx)}{12597a^2d\left(a^2\sin(c+dx)+a^2\right)^3}-\frac{48\sec^3(c+dx)}{4199d\left(a^2\sin(c+dx)+a^2\right)^4}-\frac{22\sec^3(c+dx)}{969a^2d(a\sin(c+dx)+a)^6}-\frac{11\sec^3(c+dx)}{323ad(a\sin(c+dx)+a)^7}-\frac{\sec^3(c+dx)}{19d(a\sin(c+dx)+a)^8}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + a*Sin[c + d*x])^8,x]
```

```
[Out] -1/19*Sec[c + d*x]^3/(d*(a + a*Sin[c + d*x])^8) - (11*Sec[c + d*x]^3)/(323*
a*d*(a + a*Sin[c + d*x])^7) - (22*Sec[c + d*x]^3)/(969*a^2*d*(a + a*Sin[c +
 d*x])^6) - (66*Sec[c + d*x]^3)/(4199*a^3*d*(a + a*Sin[c + d*x])^5) - (48*S
ec[c + d*x]^3)/(4199*d*(a^2 + a^2*Sin[c + d*x])^4) - (112*Sec[c + d*x]^3)/(
12597*a^2*d*(a^2 + a^2*Sin[c + d*x])^3) - (32*Sec[c + d*x]^3)/(4199*d*(a^4
+ a^4*Sin[c + d*x])^2) - (32*Sec[c + d*x]^3)/(4199*d*(a^8 + a^8*Sin[c + d*x
])) + (128*Tan[c + d*x])/(4199*a^8*d) + (128*Tan[c + d*x]^3)/(12597*a^8*d)
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{(a+a\sin(c+dx))^8}\,dx = -\frac{\sec^3(c+dx)}{19d(a+a\sin(c+dx))^8} + \frac{11\int \frac{\sec^4(c+dx)}{(a+a\sin(c+dx))^7}\,dx}{19a}$$

$$= -\frac{\sec^3(c+dx)}{19d(a+a\sin(c+dx))^8} - \frac{11\sec^3(c+dx)}{323ad(a+a\sin(c+dx))^7} + \frac{110\int \frac{\sec^4(c+dx)}{(a+a\sin(c+dx))^6}\,dx}{323a^2}$$

$$= -\frac{\sec^3(c+dx)}{19d(a+a\sin(c+dx))^8} - \frac{11\sec^3(c+dx)}{323ad(a+a\sin(c+dx))^7} - \frac{22\sec^3(c+dx)}{969a^2d(a+a\sin(c+$$

$$= -\frac{\sec^3(c+dx)}{19d(a+a\sin(c+dx))^8} - \frac{11\sec^3(c+dx)}{323ad(a+a\sin(c+dx))^7} - \frac{22\sec^3(c+dx)}{969a^2d(a+a\sin(c+$$

$$= -\frac{\sec^3(c+dx)}{19d(a+a\sin(c+dx))^8} - \frac{11\sec^3(c+dx)}{323ad(a+a\sin(c+dx))^7} - \frac{22\sec^3(c+dx)}{969a^2d(a+a\sin(c+$$

$$= -\frac{\sec^3(c+dx)}{19d(a+a\sin(c+dx))^8} - \frac{11\sec^3(c+dx)}{323ad(a+a\sin(c+dx))^7} - \frac{22\sec^3(c+dx)}{969a^2d(a+a\sin(c+$$

$$= -\frac{\sec^3(c+dx)}{19d(a+a\sin(c+dx))^8} - \frac{11\sec^3(c+dx)}{323ad(a+a\sin(c+dx))^7} - \frac{22\sec^3(c+dx)}{969a^2d(a+a\sin(c+$$

$$= -\frac{\sec^3(c+dx)}{19d(a+a\sin(c+dx))^8} - \frac{11\sec^3(c+dx)}{323ad(a+a\sin(c+dx))^7} - \frac{22\sec^3(c+dx)}{969a^2d(a+a\sin(c+$$

$$= -\frac{\sec^3(c+dx)}{19d(a+a\sin(c+dx))^8} - \frac{11\sec^3(c+dx)}{323ad(a+a\sin(c+dx))^7} - \frac{22\sec^3(c+dx)}{969a^2d(a+a\sin(c+$$

$$= -\frac{\sec^3(c+dx)}{19d(a+a\sin(c+dx))^8} - \frac{11\sec^3(c+dx)}{323ad(a+a\sin(c+dx))^7} - \frac{22\sec^3(c+dx)}{969a^2d(a+a\sin(c+$$

Mathematica [A]
time = 0.27, size = 125, normalized size = 0.45

$$\frac{\sec^3(c+dx)(-10336\cos(2(c+dx))+2736\cos(6(c+dx))-512\cos(8(c+dx))+16\cos(10(c+dx))+8398\sin(c+dx)-5814\sin(3(c+dx))-2907\sin(5(c+dx))+1463\sin(7(c+dx))-117\sin(9(c+dx))+\sin(11(c+dx)))}{50388a^8d(1+\sin(c+dx))^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + a*Sin[c + d*x])^8,x]
[Out] (Sec[c + d*x]^3*(-10336*Cos[2*(c + d*x)] + 2736*Cos[6*(c + d*x)] - 512*Cos[
8*(c + d*x)] + 16*Cos[10*(c + d*x)] + 8398*Sin[c + d*x] - 5814*Sin[3*(c + d
*x)] - 2907*Sin[5*(c + d*x)] + 1463*Sin[7*(c + d*x)] - 117*Sin[9*(c + d*x)]
 + Sin[11*(c + d*x)]))/(50388*a^8*d*(1 + Sin[c + d*x])^8)
```

Maple [A]
time = 0.34, size = 340, normalized size = 1.22

method	result
risch	$\frac{512i\left(10336i\,e^{9i(dx+c)}+8398\,e^{10i(dx+c)}-5814\,e^{8i(dx+c)}-2736i\,e^{5i(dx+c)}-2907\,e^{6i(dx+c)}+512i\,e^{3i(dx+c)}+1463\,e^{4i(dx+c)}}{12597\left(e^{i(dx+c)}+i\right)^{19}\left(e^{i(dx+c)}-i\right)^{3}a^{8}d}$
derivativedivides	$\frac{-\frac{256}{19\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{19}}+\frac{128}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{18}}-\frac{10496}{17\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{17}}+\frac{1984}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{16}}-\frac{14192}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{15}}+\frac{}{\left(\tan\left(\right.\right.}}{}$
default	$\frac{-\frac{256}{19\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{19}}+\frac{128}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{18}}-\frac{10496}{17\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{17}}+\frac{1984}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{16}}-\frac{14192}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{15}}+\frac{}{\left(\tan\left(\right.\right.}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+a*sin(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/a^8*(-128/19/(tan(1/2*d*x+1/2*c)+1)^19+64/(tan(1/2*d*x+1/2*c)+1)^18-524
8/17/(tan(1/2*d*x+1/2*c)+1)^17+992/(tan(1/2*d*x+1/2*c)+1)^16-7096/3/(tan(1/
2*d*x+1/2*c)+1)^15+4428/(tan(1/2*d*x+1/2*c)+1)^14-87508/13/(tan(1/2*d*x+1/2
*c)+1)^13+25468/3/(tan(1/2*d*x+1/2*c)+1)^12-18011/2/(tan(1/2*d*x+1/2*c)+1)^
11+32417/4/(tan(1/2*d*x+1/2*c)+1)^10-6215/(tan(1/2*d*x+1/2*c)+1)^9+32525/8/
(tan(1/2*d*x+1/2*c)+1)^8-72425/32/(tan(1/2*d*x+1/2*c)+1)^7+204605/192/(tan(
1/2*d*x+1/2*c)+1)^6-26871/64/(tan(1/2*d*x+1/2*c)+1)^5+2177/16/(tan(1/2*d*x+
1/2*c)+1)^4-54229/1536/(tan(1/2*d*x+1/2*c)+1)^3+7181/1024/(tan(1/2*d*x+1/2*
c)+1)^2-509/512/(tan(1/2*d*x+1/2*c)+1)-1/1536/(tan(1/2*d*x+1/2*c)-1)^3-1/10
24/(tan(1/2*d*x+1/2*c)-1)^2-3/512/(tan(1/2*d*x+1/2*c)-1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 866 vs. $2(259) = 518$.
time = 0.41, size = 866, normalized size = 3.10

$$-\frac{2\left(\frac{19787\sin(dx+c)}{\cos(dx+c)+1}+\frac{136032\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{540806\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{1483064\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{2552175\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{2356608\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{1108536\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{6930288\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{10934842\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{7793344\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{1058148\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}+\frac{9204208\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}+\frac{9985222\sin(dx+c)^{13}}{(\cos(dx+c)+1)^{13}}+\frac{4837248\sin(dx+c)^{14}}{(\cos(dx+c)+1)^{14}}-\frac{1108536\sin(dx+c)^{15}}{(\cos(dx+c)+1)^{15}}-\frac{3527160\sin(dx+c)^{16}}{(\cos(dx+c)+1)^{16}}-\frac{2985489\sin(dx+c)^{17}}{(\cos(dx+c)+1)^{17}}-\frac{1478048\sin(dx+c)^{18}}{(\cos(dx+c)+1)^{18}}-\frac{495482\sin(dx+c)^{19}}{(\cos(dx+c)+1)^{19}}-\frac{100776\sin(dx+c)^{20}}{(\cos(dx+c)+1)^{20}}-\frac{12597\sin(dx+c)^{21}}{(\cos(dx+c)+1)^{21}}+2024\right)}{12597\left(a^8+\frac{16a^8\sin(dx+c)}{\cos(dx+c)+1}+\frac{117a^8\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{512a^8\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{1463a^8\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{2736a^8\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{2907a^8\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{5814a^8\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{10336a^8\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{8398a^8\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{8398a^8\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}+\frac{10336a^8\sin(dx+c)^{13}}{(\cos(dx+c)+1)^{13}}+\frac{5814a^8\sin(dx+c)^{14}}{(\cos(dx+c)+1)^{14}}-\frac{2907a^8\sin(dx+c)^{16}}{(\cos(dx+c)+1)^{16}}-\frac{2736a^8\sin(dx+c)^{17}}{(\cos(dx+c)+1)^{17}}-\frac{1463a^8\sin(dx+c)^{18}}{(\cos(dx+c)+1)^{18}}-\frac{512a^8\sin(dx+c)^{19}}{(\cos(dx+c)+1)^{19}}-\frac{117a^8\sin(dx+c)^{20}}{(\cos(dx+c)+1)^{20}}-\frac{16a^8\sin(dx+c)^{21}}{(\cos(dx+c)+1)^{21}}-\frac{a^8\sin(dx+c)^{22}}{(\cos(dx+c)+1)^{22}}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -2/12597*(19787*sin(d*x + c)/(cos(d*x + c) + 1) + 136032*sin(d*x + c)^2/(co
s(d*x + c) + 1)^2 + 540806*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 1483064*si
n(d*x + c)^4/(cos(d*x + c) + 1)^4 + 2552175*sin(d*x + c)^5/(cos(d*x + c) +
1)^5 + 2356608*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 1108536*sin(d*x + c)^7
/(cos(d*x + c) + 1)^7 - 6930288*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 10934
842*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 7793344*sin(d*x + c)^10/(cos(d*x
+ c) + 1)^10 + 1058148*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 + 9204208*sin(
d*x + c)^12/(cos(d*x + c) + 1)^12 + 9985222*sin(d*x + c)^13/(cos(d*x + c) +
 1)^13 + 4837248*sin(d*x + c)^14/(cos(d*x + c) + 1)^14 - 1108536*sin(d*x +
c)^15/(cos(d*x + c) + 1)^15 - 3527160*sin(d*x + c)^16/(cos(d*x + c) + 1)^16
```

```
 - 2985489*sin(d*x + c)^17/(cos(d*x + c) + 1)^17 - 1478048*sin(d*x + c)^18/
(cos(d*x + c) + 1)^18 - 495482*sin(d*x + c)^19/(cos(d*x + c) + 1)^19 - 1007
76*sin(d*x + c)^20/(cos(d*x + c) + 1)^20 - 12597*sin(d*x + c)^21/(cos(d*x +
 c) + 1)^21 + 2024)/((a^8 + 16*a^8*sin(d*x + c)/(cos(d*x + c) + 1) + 117*a^
8*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 512*a^8*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3 + 1463*a^8*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 2736*a^8*sin(d*x
+ c)^5/(cos(d*x + c) + 1)^5 + 2907*a^8*sin(d*x + c)^6/(cos(d*x + c) + 1)^6
- 5814*a^8*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 10336*a^8*sin(d*x + c)^9/(
cos(d*x + c) + 1)^9 - 8398*a^8*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + 8398
*a^8*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 + 10336*a^8*sin(d*x + c)^13/(cos
(d*x + c) + 1)^13 + 5814*a^8*sin(d*x + c)^14/(cos(d*x + c) + 1)^14 - 2907*a
^8*sin(d*x + c)^16/(cos(d*x + c) + 1)^16 - 2736*a^8*sin(d*x + c)^17/(cos(d*
x + c) + 1)^17 - 1463*a^8*sin(d*x + c)^18/(cos(d*x + c) + 1)^18 - 512*a^8*s
in(d*x + c)^19/(cos(d*x + c) + 1)^19 - 117*a^8*sin(d*x + c)^20/(cos(d*x + c
) + 1)^20 - 16*a^8*sin(d*x + c)^21/(cos(d*x + c) + 1)^21 - a^8*sin(d*x + c)
^22/(cos(d*x + c) + 1)^22)*d)
```

Fricas [A]
time = 0.38, size = 249, normalized size = 0.89

$$\frac{2048 \cos\left(dx+c\right)^{10} - 21504 \cos\left(dx+c\right)^{8} + 59136 \cos\left(dx+c\right)^{6} - 54912 \cos\left(dx+c\right)^{4} + 11440 \cos\left(dx+c\right)^{2} + \left(256 \cos\left(dx+c\right)^{10} - 8064 \cos\left(dx+c\right)^{8} + 36960 \cos\left(dx+c\right)^{6} - 48048 \cos\left(dx+c\right)^{4} + 12870 \cos\left(dx+c\right)^{2} + 2431\right) \sin\left(dx+c\right) + 1768}{12597 \left(a^{8} d \cos\left(dx+c\right)^{11} - 32 a^{8} d \cos\left(dx+c\right)^{9} + 160 a^{8} d \cos\left(dx+c\right)^{7} - 256 a^{8} d \cos\left(dx+c\right)^{5} + 128 a^{8} d \cos\left(dx+c\right)^{3} - 8 \left(a^{8} d \cos\left(dx+c\right)^{9} - 10 a^{8} d \cos\left(dx+c\right)^{7} + 24 a^{8} d \cos\left(dx+c\right)^{5} - 16 a^{8} d \cos\left(dx+c\right)^{3}\right) \sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/12597*(2048*cos(d*x + c)^10 - 21504*cos(d*x + c)^8 + 59136*cos(d*x + c)^6
 - 54912*cos(d*x + c)^4 + 11440*cos(d*x + c)^2 + (256*cos(d*x + c)^10 - 806
4*cos(d*x + c)^8 + 36960*cos(d*x + c)^6 - 48048*cos(d*x + c)^4 + 12870*cos(
d*x + c)^2 + 2431)*sin(d*x + c) + 1768)/(a^8*d*cos(d*x + c)^11 - 32*a^8*d*c
os(d*x + c)^9 + 160*a^8*d*cos(d*x + c)^7 - 256*a^8*d*cos(d*x + c)^5 + 128*a
^8*d*cos(d*x + c)^3 - 8*(a^8*d*cos(d*x + c)^9 - 10*a^8*d*cos(d*x + c)^7 + 2
4*a^8*d*cos(d*x + c)^5 - 16*a^8*d*cos(d*x + c)^3)*sin(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+a*sin(d*x+c))**8,x)
```

```
[Out] Timed out
```

Giac [A]
time = 4.77, size = 301, normalized size = 1.08

$$-\frac{\frac{4199 \left(18 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{2} - 33 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right) + 17\right)}{a^{8} \left(\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right) - 1\right)^{3}} + \frac{12823746 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{18} + 140368371 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{17} + 879644311 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{16} + 3693272440 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{15} + 11467502592 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{14} + 27403194676 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{13} + 51919375300 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{12} + 79183835016 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{11} + 98304418212 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{10} + 99750226290 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{9} + 82860874122 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{8} + 56110430792 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{7} + 30766700912 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{6} + 13462452660 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{5} + 4616712644 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{4} + 1197851960 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{3} + 226248618 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{2} + 27911475 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right) + 2143959}{a^{8} \left(\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right) + 1\right)^{19}}}{6449664 \, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -1/6449664*(4199*(18*tan(1/2*d*x + 1/2*c)^2 - 33*tan(1/2*d*x + 1/2*c) + 17)
/(a^8*(tan(1/2*d*x + 1/2*c) - 1)^3) + (12823746*tan(1/2*d*x + 1/2*c)^18 + 1
40368371*tan(1/2*d*x + 1/2*c)^17 + 879644311*tan(1/2*d*x + 1/2*c)^16 + 3693
272440*tan(1/2*d*x + 1/2*c)^15 + 11467502592*tan(1/2*d*x + 1/2*c)^14 + 2740
3194676*tan(1/2*d*x + 1/2*c)^13 + 51919375300*tan(1/2*d*x + 1/2*c)^12 + 791
83835016*tan(1/2*d*x + 1/2*c)^11 + 98304418212*tan(1/2*d*x + 1/2*c)^10 + 99
750226290*tan(1/2*d*x + 1/2*c)^9 + 82860874122*tan(1/2*d*x + 1/2*c)^8 + 561
10430792*tan(1/2*d*x + 1/2*c)^7 + 30766700912*tan(1/2*d*x + 1/2*c)^6 + 1346
2452660*tan(1/2*d*x + 1/2*c)^5 + 4616712644*tan(1/2*d*x + 1/2*c)^4 + 119785
1960*tan(1/2*d*x + 1/2*c)^3 + 226248618*tan(1/2*d*x + 1/2*c)^2 + 27911475*t
an(1/2*d*x + 1/2*c) + 2143959)/(a^8*(tan(1/2*d*x + 1/2*c) + 1)^19))/d
```

Mupad [B]
time = 9.23, size = 277, normalized size = 0.99

$$\frac{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{896971\,\cos\left(\frac{5\,c}{2}+\frac{5\,d\,x}{2}\right)}{64}-\frac{1062347\,\cos\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)}{64}-\frac{40375\,\cos\left(\frac{7\,c}{2}+\frac{7\,d\,x}{2}\right)}{16}+\frac{40375\,\cos\left(\frac{9\,c}{2}+\frac{9\,d\,x}{2}\right)}{16}+\frac{412471\,\cos\left(\frac{11\,c}{2}+\frac{11\,d\,x}{2}\right)}{128}-\frac{324919\,\cos\left(\frac{13\,c}{2}+\frac{13\,d\,x}{2}\right)}{128}-\frac{11305\,\cos\left(\frac{15\,c}{2}+\frac{15\,d\,x}{2}\right)}{32}+\frac{7209\,\cos\left(\frac{17\,c}{2}+\frac{17\,d\,x}{2}\right)}{32}+\frac{765\,\cos\left(\frac{19\,c}{2}+\frac{19\,d\,x}{2}\right)}{128}-\frac{253\,\cos\left(\frac{21\,c}{2}+\frac{21\,d\,x}{2}\right)}{128}+\frac{65033\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}-\frac{56635\,\sin\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)}{4}-6271\,\sin\left(\frac{5\,c}{2}+\frac{5\,d\,x}{2}\right)+\frac{9635\,\sin\left(\frac{7\,c}{2}+\frac{7\,d\,x}{2}\right)}{2}-\frac{9635\,\sin\left(\frac{9\,c}{2}+\frac{9\,d\,x}{2}\right)}{2}+\frac{16363\,\sin\left(\frac{11\,c}{2}+\frac{11\,d\,x}{2}\right)}{4}+\frac{10537\,\sin\left(\frac{13\,c}{2}+\frac{13\,d\,x}{2}\right)}{8}-\frac{7611\,\sin\left(\frac{15\,c}{2}+\frac{15\,d\,x}{2}\right)}{8}-\frac{485\,\sin\left(\frac{17\,c}{2}+\frac{17\,d\,x}{2}\right)}{8}+\frac{251\,\sin\left(\frac{19\,c}{2}+\frac{19\,d\,x}{2}\right)}{8}+\frac{\sin\left(\frac{21\,c}{2}+\frac{21\,d\,x}{2}\right)}{4}\right)}{12899328\,a^8\,d\,{\cos\left(\frac{c}{2}-\frac{\pi}{4}+\frac{d\,x}{2}\right)}^{19}\,{\cos\left(\frac{c}{2}+\frac{\pi}{4}+\frac{d\,x}{2}\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a + a*sin(c + d*x))^8),x)
```

```
[Out] (cos(c/2 + (d*x)/2)*((896971*cos((5*c)/2 + (5*d*x)/2))/64 - (1062347*cos((3
*c)/2 + (3*d*x)/2))/64 - (40375*cos((7*c)/2 + (7*d*x)/2))/16 + (40375*cos((
9*c)/2 + (9*d*x)/2))/16 + (412471*cos((11*c)/2 + (11*d*x)/2))/128 - (324919
*cos((13*c)/2 + (13*d*x)/2))/128 - (11305*cos((15*c)/2 + (15*d*x)/2))/32 +
(7209*cos((17*c)/2 + (17*d*x)/2))/32 + (765*cos((19*c)/2 + (19*d*x)/2))/128
 - (253*cos((21*c)/2 + (21*d*x)/2))/128 + (65033*sin(c/2 + (d*x)/2))/4 - (5
6635*sin((3*c)/2 + (3*d*x)/2))/4 - 6271*sin((5*c)/2 + (5*d*x)/2) + (9635*si
n((7*c)/2 + (7*d*x)/2))/2 - (9635*sin((9*c)/2 + (9*d*x)/2))/2 + (16363*sin(
(11*c)/2 + (11*d*x)/2))/4 + (10537*sin((13*c)/2 + (13*d*x)/2))/8 - (7611*si
n((15*c)/2 + (15*d*x)/2))/8 - (485*sin((17*c)/2 + (17*d*x)/2))/8 + (251*sin
((19*c)/2 + (19*d*x)/2))/8 + sin((21*c)/2 + (21*d*x)/2)/4))/(12899328*a^8*d
*cos(c/2 - pi/4 + (d*x)/2)^19*cos(c/2 + pi/4 + (d*x)/2)^3)
```

3.100 $\int \frac{\sec^5(c+dx)}{(a+a\sin(c+dx))^8}\,dx$

Optimal. Leaf size=284

$$\frac{33\tanh^{-1}(\sin(c+dx))}{2048a^8d}-\frac{a^2}{80d(a+a\sin(c+dx))^{10}}-\frac{a}{48d(a+a\sin(c+dx))^9}-\frac{3}{128d(a+a\sin(c+dx))^8}-224a$$

```
[Out] 33/2048*arctanh(sin(d*x+c))/a^8/d-1/80*a^2/d/(a+a*sin(d*x+c))^10-1/48*a/d/(
a+a*sin(d*x+c))^9-3/128/d/(a+a*sin(d*x+c))^8-5/224/a/d/(a+a*sin(d*x+c))^7-5
/256/a^2/d/(a+a*sin(d*x+c))^6-21/1280/a^3/d/(a+a*sin(d*x+c))^5-3/256/a^5/d/
(a+a*sin(d*x+c))^3-7/512/d/(a^2+a^2*sin(d*x+c))^4+1/4096/d/(a^4-a^4*sin(d*x
+c))^2-45/4096/d/(a^4+a^4*sin(d*x+c))^2+11/4096/d/(a^8-a^8*sin(d*x+c))-55/4
096/d/(a^8+a^8*sin(d*x+c))
```

Rubi [A]
time = 0.16, antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2746, 46, 212}

$$\frac{11}{4096d\left(a^8-a^8\sin(c+dx)\right)}-\frac{55}{4096d\left(a^8\sin(c+dx)+a^8\right)}+\frac{33\tanh^{-1}(\sin(c+dx))}{2048a^8d}-\frac{3}{256a^5d(a\sin(c+dx)+a)^3}+\frac{1}{4096d\left(a^4-a^4\sin(c+dx)\right)^2}-\frac{45}{4096d\left(a^4\sin(c+dx)+a^4\right)^2}-\frac{21}{1280a^3d(a\sin(c+dx)+a)^5}-\frac{a^2}{80d(a\sin(c+dx)+a)^{10}}-\frac{7}{512d\left(a^2\sin(c+dx)+a^2\right)^4}-\frac{5}{256a^2d(a\sin(c+dx)+a)^6}-\frac{a}{48d(a\sin(c+dx)+a)^9}-\frac{3}{128d(a\sin(c+dx)+a)^8}-\frac{5}{224ad(a\sin(c+dx)+a)^7}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + a*Sin[c + d*x])^8,x]
```

```
[Out] (33*ArcTanh[Sin[c + d*x]])/(2048*a^8*d) - a^2/(80*d*(a + a*Sin[c + d*x])^10
) - a/(48*d*(a + a*Sin[c + d*x])^9) - 3/(128*d*(a + a*Sin[c + d*x])^8) - 5/
(224*a*d*(a + a*Sin[c + d*x])^7) - 5/(256*a^2*d*(a + a*Sin[c + d*x])^6) - 2
1/(1280*a^3*d*(a + a*Sin[c + d*x])^5) - 3/(256*a^5*d*(a + a*Sin[c + d*x])^3
) - 7/(512*d*(a^2 + a^2*Sin[c + d*x])^4) + 1/(4096*d*(a^4 - a^4*Sin[c + d*x
])^2) - 45/(4096*d*(a^4 + a^4*Sin[c + d*x])^2) + 11/(4096*d*(a^8 - a^8*Sin[
c + d*x])) - 55/(4096*d*(a^8 + a^8*Sin[c + d*x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] || !IntegerQ[m + 1/2
])
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{(a+a\sin(c+dx))^8}\,dx = \frac{a^5\text{Subst}\left(\int \frac{1}{(a-x)^3(a+x)^{11}}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^5\text{Subst}\left(\int \left(\frac{1}{2048a^{11}(a-x)^3} + \frac{11}{4096a^{12}(a-x)^2} + \frac{1}{8a^3(a+x)^{11}} + \frac{3}{16a^4(a+x)^{10}} + \frac{3}{16a^5(a+x)^9} + \right.\right.}{}$$
$$= -\frac{a^2}{80d(a+a\sin(c+dx))^{10}} - \frac{a}{48d(a+a\sin(c+dx))^9} - \frac{3}{128d(a+a\sin(c+dx}$$
$$= \frac{33\tanh^{-1}(\sin(c+dx))}{2048a^8d} - \frac{a^2}{80d(a+a\sin(c+dx))^{10}} - \frac{a}{48d(a+a\sin(c+dx))^9}$$

Mathematica [A]
time = 1.57, size = 195, normalized size = 0.69

$$\frac{\sec^4(c+dx)\left(-34816+3465\tanh^{-1}(\sin(c+dx))\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^4\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^{20}-66953\sin(c+dx)-72776\sin^2(c+dx)+21395\sin^3(c+dx)+190080\sin^4(c+dx)+255222\sin^5(c+dx)+114576\sin^6(c+dx)-82698\sin^7(c+dx)-147840\sin^8(c+dx)-91245\sin^9(c+dx)-27720\sin^{10}(c+dx)-3465\sin^{11}(c+dx)\right)}{215040a^8d(1+\sin(c+dx))^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a + a*Sin[c + d*x])^8,x]
```

```
[Out] (Sec[c + d*x]^4*(-34816 + 3465*ArcTanh[Sin[c + d*x]]*(Cos[(c + d*x)/2] - Si
n[(c + d*x)/2])^4*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^20 - 66953*Sin[c +
d*x] - 72776*Sin[c + d*x]^2 + 21395*Sin[c + d*x]^3 + 190080*Sin[c + d*x]^4
+ 255222*Sin[c + d*x]^5 + 114576*Sin[c + d*x]^6 - 82698*Sin[c + d*x]^7 - 14
7840*Sin[c + d*x]^8 - 91245*Sin[c + d*x]^9 - 27720*Sin[c + d*x]^10 - 3465*S
in[c + d*x]^11))/(215040*a^8*d*(1 + Sin[c + d*x])^8)
```

Maple [A]
time = 0.41, size = 175, normalized size = 0.62

method	result
derivativedivides	$\frac{1}{4096(\sin(dx+c)-1)^2} - \frac{11}{4096(\sin(dx+c)-1)} - \frac{33\ln(\sin(dx+c)-1)}{4096} - \frac{1}{80(1+\sin(dx+c))^{10}} - \frac{1}{48(1+\sin(dx+c))^9} - \frac{3}{128(1+\sin(dx+c))^8} -$

default	$\frac{1}{4096(\sin(dx+c)-1)^2}-\frac{11}{4096(\sin(dx+c)-1)}-\frac{33\ln(\sin(dx+c)-1)}{4096}-\frac{1}{80(1+\sin(dx+c))^{10}}-\frac{1}{48(1+\sin(dx+c))^9}-\frac{3}{128(1+\sin(dx+c))^8}-\frac{}{224}$
risch	$-\frac{i\left(-1737120ie^{4i(dx+c)}+3465\,e^{23i(dx+c)}+55440ie^{2i(dx+c)}-403095\,e^{21i(dx+c)}-23276992ie^{12i(dx+c)}+4798563\,e^{19i(dx+c)}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+a*sin(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^8*(1/4096/(sin(d*x+c)-1)^2-11/4096/(sin(d*x+c)-1)-33/4096*ln(sin(d*x+
c)-1)-1/80/(1+sin(d*x+c))^10-1/48/(1+sin(d*x+c))^9-3/128/(1+sin(d*x+c))^8-5
/224/(1+sin(d*x+c))^7-5/256/(1+sin(d*x+c))^6-21/1280/(1+sin(d*x+c))^5-7/512
/(1+sin(d*x+c))^4-3/256/(1+sin(d*x+c))^3-45/4096/(1+sin(d*x+c))^2-55/4096/(
1+sin(d*x+c))+33/4096*ln(1+sin(d*x+c)))
```

Maxima [A]
time = 0.31, size = 305, normalized size = 1.07

$$-\frac{\frac{2\left(3465\sin(dx+c)^{11}+27720\sin(dx+c)^{10}+91245\sin(dx+c)^{9}+147840\sin(dx+c)^{8}+82698\sin(dx+c)^{7}-114576\sin(dx+c)^{6}-255222\sin(dx+c)^{5}-190080\sin(dx+c)^{4}-21395\sin(dx+c)^{3}+72776\sin(dx+c)^{2}+66953\sin(dx+c)+34816\right)}{a^8\sin(dx+c)^{12}+8a^8\sin(dx+c)^{11}+26a^8\sin(dx+c)^{10}+40a^8\sin(dx+c)^{9}+15a^8\sin(dx+c)^{8}-48a^8\sin(dx+c)^{7}-84a^8\sin(dx+c)^{6}-48a^8\sin(dx+c)^{5}+15a^8\sin(dx+c)^{4}+40a^8\sin(dx+c)^{3}+26a^8\sin(dx+c)^{2}+8a^8\sin(dx+c)+a^8}-\frac{3465\log(\sin(dx+c)+1)}{a^8}+\frac{3465\log(\sin(dx+c)-1)}{a^8}}{430080\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/430080*(2*(3465*sin(d*x + c)^11 + 27720*sin(d*x + c)^10 + 91245*sin(d*x
+ c)^9 + 147840*sin(d*x + c)^8 + 82698*sin(d*x + c)^7 - 114576*sin(d*x + c)
^6 - 255222*sin(d*x + c)^5 - 190080*sin(d*x + c)^4 - 21395*sin(d*x + c)^3 +
 72776*sin(d*x + c)^2 + 66953*sin(d*x + c) + 34816)/(a^8*sin(d*x + c)^12 +
8*a^8*sin(d*x + c)^11 + 26*a^8*sin(d*x + c)^10 + 40*a^8*sin(d*x + c)^9 + 15
*a^8*sin(d*x + c)^8 - 48*a^8*sin(d*x + c)^7 - 84*a^8*sin(d*x + c)^6 - 48*a^
8*sin(d*x + c)^5 + 15*a^8*sin(d*x + c)^4 + 40*a^8*sin(d*x + c)^3 + 26*a^8*s
in(d*x + c)^2 + 8*a^8*sin(d*x + c) + a^8) - 3465*log(sin(d*x + c) + 1)/a^8
+ 3465*log(sin(d*x + c) - 1)/a^8)/d
```

Fricas [A]
time = 0.42, size = 466, normalized size = 1.64

$$\frac{55440\cos(dx+c)^{10}-572880\cos(dx+c)^{8}+1507968\cos(dx+c)^{6}-1260864\cos(dx+c)^{4}+157696\cos(dx+c)^{2}+3465\left(\cos(dx+c)^{12}-32\cos(dx+c)^{10}+160\cos(dx+c)^{8}-256\cos(dx+c)^{6}+128\cos(dx+c)^{4}-8\left(\cos(dx+c)^{10}-10\cos(dx+c)^{8}+24\cos(dx+c)^{6}-16\cos(dx+c)^{4}\right)\sin(dx+c)\right)\log(\sin(dx+c)+1)-3465\left(\cos(dx+c)^{12}-32\cos(dx+c)^{10}+160\cos(dx+c)^{8}-256\cos(dx+c)^{6}+128\cos(dx+c)^{4}-8\left(\cos(dx+c)^{10}-10\cos(dx+c)^{8}+24\cos(dx+c)^{6}-16\cos(dx+c)^{4}\right)\sin(dx+c)\right)\log(-\sin(dx+c)+1)+2\left(3465\cos(dx+c)^{10}-108570\cos(dx+c)^{8}+482328\cos(dx+c)^{6}-574992\cos(dx+c)^{4}+98560\cos(dx+c)^{2}+32256\right)\sin(dx+c)+43008}{430080\left(a^8d\cos(dx+c)^{12}-32a^8d\cos(dx+c)^{10}+160a^8d\cos(dx+c)^{8}-256a^8d\cos(dx+c)^{6}+128a^8d\cos(dx+c)^{4}-8\left(a^8d\cos(dx+c)^{10}-10a^8d\cos(dx+c)^{8}+24a^8d\cos(dx+c)^{6}-16a^8d\cos(dx+c)^{4}\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/430080*(55440*cos(d*x + c)^10 - 572880*cos(d*x + c)^8 + 1507968*cos(d*x +
 c)^6 - 1260864*cos(d*x + c)^4 + 157696*cos(d*x + c)^2 + 3465*(cos(d*x + c)
^12 - 32*cos(d*x + c)^10 + 160*cos(d*x + c)^8 - 256*cos(d*x + c)^6 + 128*co
s(d*x + c)^4 - 8*(cos(d*x + c)^10 - 10*cos(d*x + c)^8 + 24*cos(d*x + c)^6 -
 16*cos(d*x + c)^4)*sin(d*x + c))*log(sin(d*x + c) + 1) - 3465*(cos(d*x + c
```

```
)^12 - 32*cos(d*x + c)^10 + 160*cos(d*x + c)^8 - 256*cos(d*x + c)^6 + 128*c
os(d*x + c)^4 - 8*(cos(d*x + c)^10 - 10*cos(d*x + c)^8 + 24*cos(d*x + c)^6
- 16*cos(d*x + c)^4)*sin(d*x + c))*log(-sin(d*x + c) + 1) + 2*(3465*cos(d*x
 + c)^10 - 108570*cos(d*x + c)^8 + 482328*cos(d*x + c)^6 - 574992*cos(d*x +
 c)^4 + 98560*cos(d*x + c)^2 + 32256)*sin(d*x + c) + 43008)/(a^8*d*cos(d*x
+ c)^12 - 32*a^8*d*cos(d*x + c)^10 + 160*a^8*d*cos(d*x + c)^8 - 256*a^8*d*c
os(d*x + c)^6 + 128*a^8*d*cos(d*x + c)^4 - 8*(a^8*d*cos(d*x + c)^10 - 10*a^
8*d*cos(d*x + c)^8 + 24*a^8*d*cos(d*x + c)^6 - 16*a^8*d*cos(d*x + c)^4)*sin
(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+a*sin(d*x+c))**8,x)
```

```
[Out] Timed out
```

Giac [A]
time = 4.88, size = 186, normalized size = 0.65

$$\frac{\frac{27720 \log(|\sin(dx+c)+1|)}{a^8} - \frac{27720 \log(|\sin(dx+c)-1|)}{a^8} + \frac{420 \left(99 \sin(dx+c)^2 - 220 \sin(dx+c) + 123\right)}{a^8 (\sin(dx+c)-1)^2} - \frac{81191 \sin(dx+c)^{10} + 858110 \sin(dx+c)^9 + 4107195 \sin(dx+c)^8 + 11748840 \sin(dx+c)^7 + 22318590 \sin(dx+c)^6 + 29583540 \sin(dx+c)^5 + 27983550 \sin(dx+c)^4 + 19002600 \sin(dx+c)^3 + 9206235 \sin(dx+c)^2 + 3108990 \sin(dx+c) + 648327}{a^8 (\sin(dx+c)+1)^{10}}}{3440640\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/3440640*(27720*log(abs(sin(d*x + c) + 1))/a^8 - 27720*log(abs(sin(d*x + c
) - 1))/a^8 + 420*(99*sin(d*x + c)^2 - 220*sin(d*x + c) + 123)/(a^8*(sin(d*
x + c) - 1)^2) - (81191*sin(d*x + c)^10 + 858110*sin(d*x + c)^9 + 4107195*s
in(d*x + c)^8 + 11748840*sin(d*x + c)^7 + 22318590*sin(d*x + c)^6 + 2958354
0*sin(d*x + c)^5 + 27983550*sin(d*x + c)^4 + 19002600*sin(d*x + c)^3 + 9206
235*sin(d*x + c)^2 + 3108990*sin(d*x + c) + 648327)/(a^8*(sin(d*x + c) + 1)
^10))/d
```

Mupad [B]
time = 0.80, size = 290, normalized size = 1.02

$$\frac{33\, \mathrm{atanh}(\sin(c+d\,x))}{2048\, a^8\, d} - \frac{\frac{33 \sin(c+d\,x)^{11}}{2048} + \frac{33 \sin(c+d\,x)^{10}}{256} + \frac{869 \sin(c+d\,x)^9}{2048} + \frac{11 \sin(c+d\,x)^8}{16} + \frac{1969 \sin(c+d\,x)^7}{5120} - \frac{341 \sin(c+d\,x)^6}{640} - \frac{42537 \sin(c+d\,x)^5}{35840} - \frac{99 \sin(c+d\,x)^4}{112} - \frac{4279 \sin(c+d\,x)^3}{43008} + \frac{9097 \sin(c+d\,x)^2}{26880} + \frac{66953 \sin(c+d\,x)}{215040} + \frac{17}{105}}{d \left(a^8 \sin(c+d\,x)^{12} + 8\, a^8 \sin(c+d\,x)^{11} + 26\, a^8 \sin(c+d\,x)^{10} + 40\, a^8 \sin(c+d\,x)^9 + 15\, a^8 \sin(c+d\,x)^8 - 48\, a^8 \sin(c+d\,x)^7 - 84\, a^8 \sin(c+d\,x)^6 - 48\, a^8 \sin(c+d\,x)^5 + 15\, a^8 \sin(c+d\,x)^4 + 40\, a^8 \sin(c+d\,x)^3 + 26\, a^8 \sin(c+d\,x)^2 + 8\, a^8 \sin(c+d\,x) + a^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^5*(a + a*sin(c + d*x))^8),x)
```

```
[Out] (33*atanh(sin(c + d*x)))/(2048*a^8*d) - ((66953*sin(c + d*x))/215040 + (909
7*sin(c + d*x)^2)/26880 - (4279*sin(c + d*x)^3)/43008 - (99*sin(c + d*x)^4)
```

```
/112 - (42537*sin(c + d*x)^5)/35840 - (341*sin(c + d*x)^6)/640 + (1969*sin(
c + d*x)^7)/5120 + (11*sin(c + d*x)^8)/16 + (869*sin(c + d*x)^9)/2048 + (33
*sin(c + d*x)^10)/256 + (33*sin(c + d*x)^11)/2048 + 17/105)/(d*(8*a^8*sin(c
 + d*x) + a^8 + 26*a^8*sin(c + d*x)^2 + 40*a^8*sin(c + d*x)^3 + 15*a^8*sin(
c + d*x)^4 - 48*a^8*sin(c + d*x)^5 - 84*a^8*sin(c + d*x)^6 - 48*a^8*sin(c +
 d*x)^7 + 15*a^8*sin(c + d*x)^8 + 40*a^8*sin(c + d*x)^9 + 26*a^8*sin(c + d*
x)^10 + 8*a^8*sin(c + d*x)^11 + a^8*sin(c + d*x)^12))
```

3.101 $\int \cos^7(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=97

$$\frac{16(a+a\sin(c+dx))^{9/2}}{9a^4d}-\frac{24(a+a\sin(c+dx))^{11/2}}{11a^5d}+\frac{12(a+a\sin(c+dx))^{13/2}}{13a^6d}-\frac{2(a+a\sin(c+dx))^{15/2}}{15a^7d}$$

```
[Out] 16/9*(a+a*sin(d*x+c))^(9/2)/a^4/d-24/11*(a+a*sin(d*x+c))^(11/2)/a^5/d+12/13
*(a+a*sin(d*x+c))^(13/2)/a^6/d-2/15*(a+a*sin(d*x+c))^(15/2)/a^7/d
```

Rubi [A]
time = 0.06, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2746, 45}

$$-\frac{2(a\sin(c+dx)+a)^{15/2}}{15a^7d}+\frac{12(a\sin(c+dx)+a)^{13/2}}{13a^6d}-\frac{24(a\sin(c+dx)+a)^{11/2}}{11a^5d}+\frac{16(a\sin(c+dx)+a)^{9/2}}{9a^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (16*(a + a*Sin[c + d*x])^(9/2))/(9*a^4*d) - (24*(a + a*Sin[c + d*x])^(11/2)
)/(11*a^5*d) + (12*(a + a*Sin[c + d*x])^(13/2))/(13*a^6*d) - (2*(a + a*Sin[
c + d*x])^(15/2))/(15*a^7*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\int \cos^7(c+dx)\sqrt{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int (a-x)^3(a+x)^{7/2}\,dx, x, a\sin(c+dx)\right)}{a^7d}$$
$$= \frac{\text{Subst}\left(\int \left(8a^3(a+x)^{7/2}-12a^2(a+x)^{9/2}+6a(a+x)^{11/2}-(a+x\right.}{a^7d}$$
$$= \frac{16(a+a\sin(c+dx))^{9/2}}{9a^4d} - \frac{24(a+a\sin(c+dx))^{11/2}}{11a^5d} + \frac{12(a+a\sin}{13}$$

Mathematica [A]
time = 2.59, size = 74, normalized size = 0.76

$$\frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^8\sqrt{a(1+\sin(c+dx))}\,(8330-3366\cos(2(c+dx))-10755\sin(c+dx)+429\sin(3(c+dx)))}{12870d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^8*Sqrt[a*(1 + Sin[c + d*x])]*(8330 -
 3366*Cos[2*(c + d*x)] - 10755*Sin[c + d*x] + 429*Sin[3*(c + d*x)]))/(12870
*d)
```

Maple [A]
time = 0.39, size = 57, normalized size = 0.59

method	result	size
default	$\frac{2(a+a\sin(dx+c))^{\frac{9}{2}}\left(429(\cos^2(dx+c))\sin(dx+c)-1683(\cos^2(dx+c))-2796\sin(dx+c)+2924\right)}{6435a^4d}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*(a+a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/6435/a^4*(a+a*sin(d*x+c))^(9/2)*(429*cos(d*x+c)^2*sin(d*x+c)-1683*cos(d*x
+c)^2-2796*sin(d*x+c)+2924)/d
```

Maxima [A]
time = 0.30, size = 72, normalized size = 0.74

$$-\frac{2\left(429\,(a\sin(dx+c)+a)^{\frac{15}{2}}-2970\,(a\sin(dx+c)+a)^{\frac{13}{2}}a+7020\,(a\sin(dx+c)+a)^{\frac{11}{2}}a^2-5720\,(a\sin(dx+c)+a)^{\frac{9}{2}}a^3\right)}{6435\,a^7d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -2/6435*(429*(a*sin(d*x + c) + a)^(15/2) - 2970*(a*sin(d*x + c) + a)^(13/2)
*a + 7020*(a*sin(d*x + c) + a)^(11/2)*a^2 - 5720*(a*sin(d*x + c) + a)^(9/2)
*a^3)/(a^7*d)
```

Fricas [A]
time = 0.36, size = 88, normalized size = 0.91

$$\frac{2\left(33\cos\left(dx+c\right)^{6}+56\cos\left(dx+c\right)^{4}+128\cos\left(dx+c\right)^{2}+\left(429\cos\left(dx+c\right)^{6}+504\cos\left(dx+c\right)^{4}+640\cos\left(dx+c\right)^{2}+1024\right)\sin\left(dx+c\right)+1024\right)\sqrt{a\sin\left(dx+c\right)+a}}{6435\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/6435*(33*cos(d*x + c)^6 + 56*cos(d*x + c)^4 + 128*cos(d*x + c)^2 + (429*c
os(d*x + c)^6 + 504*cos(d*x + c)^4 + 640*cos(d*x + c)^2 + 1024)*sin(d*x + c
) + 1024)*sqrt(a*sin(d*x + c) + a)/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 7.79, size = 128, normalized size = 1.32

$$-\frac{256\sqrt{2}\left(429\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{15}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)-1485\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{13}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)+1755\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{11}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)-715\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{9}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\right)\sqrt{a}}{6435\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -256/6435*sqrt(2)*(429*cos(-1/4*pi + 1/2*d*x + 1/2*c)^15*sgn(cos(-1/4*pi +
1/2*d*x + 1/2*c)) - 1485*cos(-1/4*pi + 1/2*d*x + 1/2*c)^13*sgn(cos(-1/4*pi
+ 1/2*d*x + 1/2*c)) + 1755*cos(-1/4*pi + 1/2*d*x + 1/2*c)^11*sgn(cos(-1/4*p
i + 1/2*d*x + 1/2*c)) - 715*cos(-1/4*pi + 1/2*d*x + 1/2*c)^9*sgn(cos(-1/4*p
i + 1/2*d*x + 1/2*c)))*sqrt(a)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^7\,\sqrt{a+a\,\sin\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7*(a + a*sin(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^7*(a + a*sin(c + d*x))^(1/2), x)
```

3.102 $\int \cos^6(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=127

$$-\frac{256a^4\cos^7(c+dx)}{3003d(a+a\sin(c+dx))^{7/2}}-\frac{64a^3\cos^7(c+dx)}{429d(a+a\sin(c+dx))^{5/2}}-\frac{24a^2\cos^7(c+dx)}{143d(a+a\sin(c+dx))^{3/2}}-\frac{2a\cos^7(c+dx)}{13d\sqrt{a+a\sin(c+dx)}}$$

```
[Out] -256/3003*a^4*cos(d*x+c)^7/d/(a+a*sin(d*x+c))^(7/2)-64/429*a^3*cos(d*x+c)^7
/d/(a+a*sin(d*x+c))^(5/2)-24/143*a^2*cos(d*x+c)^7/d/(a+a*sin(d*x+c))^(3/2)-
2/13*a*cos(d*x+c)^7/d/(a+a*sin(d*x+c))^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2753, 2752}

$$-\frac{256a^4\cos^7(c+dx)}{3003d(a\sin(c+dx)+a)^{7/2}}-\frac{64a^3\cos^7(c+dx)}{429d(a\sin(c+dx)+a)^{5/2}}-\frac{24a^2\cos^7(c+dx)}{143d(a\sin(c+dx)+a)^{3/2}}-\frac{2a\cos^7(c+dx)}{13d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-256*a^4*Cos[c + d*x]^7)/(3003*d*(a + a*Sin[c + d*x])^(7/2)) - (64*a^3*Cos
[c + d*x]^7)/(429*d*(a + a*Sin[c + d*x])^(5/2)) - (24*a^2*Cos[c + d*x]^7)/(
143*d*(a + a*Sin[c + d*x])^(3/2)) - (2*a*Cos[c + d*x]^7)/(13*d*Sqrt[a + a*S
in[c + d*x]])
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2753

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && N
eQ[m + p, 0]
```

Rubi steps

$$\int \cos^6(c+dx)\sqrt{a+a\sin(c+dx)}\,dx = -\frac{2a\cos^7(c+dx)}{13d\sqrt{a+a\sin(c+dx)}} + \frac{1}{13}(12a)\int \frac{\cos^6(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$$
$$= -\frac{24a^2\cos^7(c+dx)}{143d(a+a\sin(c+dx))^{3/2}} - \frac{2a\cos^7(c+dx)}{13d\sqrt{a+a\sin(c+dx)}} + \frac{1}{143}(9$$
$$= -\frac{64a^3\cos^7(c+dx)}{429d(a+a\sin(c+dx))^{5/2}} - \frac{24a^2\cos^7(c+dx)}{143d(a+a\sin(c+dx))^{3/2}} - \frac{}{13d}$$
$$= -\frac{256a^4\cos^7(c+dx)}{3003d(a+a\sin(c+dx))^{7/2}} - \frac{64a^3\cos^7(c+dx)}{429d(a+a\sin(c+dx))^{5/2}} - \frac{}{143}$$

Mathematica [A]
time = 2.37, size = 99, normalized size = 0.78

$$\frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^7\sqrt{a(1+\sin(c+dx))}\,(-5230+1890\cos(2(c+dx))-6377\sin(c+dx)+231\sin(3(c+dx)))}{6006d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] ((Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^7*Sqrt[a*(1 + Sin[c + d*x])]*(-5230
+ 1890*Cos[2*(c + d*x)] - 6377*Sin[c + d*x] + 231*Sin[3*(c + d*x)]))/(6006*
d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

Maple [A]
time = 0.38, size = 75, normalized size = 0.59

method	result	size
default	$-\frac{2(1+\sin(dx+c))a(\sin(dx+c)-1)^4\left(231\left(\sin^3(dx+c)\right)+945\left(\sin^2(dx+c)\right)+1421\sin(dx+c)+835\right)}{3003\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$	75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(a+a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3003*(1+sin(d*x+c))*a*(sin(d*x+c)-1)^4*(231*sin(d*x+c)^3+945*sin(d*x+c)^
2+1421*sin(d*x+c)+835)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*cos(d*x + c)^6, x)
```

Fricas [A]
time = 0.34, size = 172, normalized size = 1.35

$$-\frac{2\,\left(231\,\cos\left(dx+c\right)^{7}-21\,\cos\left(dx+c\right)^{6}+28\,\cos\left(dx+c\right)^{5}-40\,\cos\left(dx+c\right)^{4}+64\,\cos\left(dx+c\right)^{3}-128\,\cos\left(dx+c\right)^{2}-\left(231\,\cos\left(dx+c\right)^{6}+252\,\cos\left(dx+c\right)^{5}+280\,\cos\left(dx+c\right)^{4}+320\,\cos\left(dx+c\right)^{3}+384\,\cos\left(dx+c\right)^{2}+512\,\cos\left(dx+c\right)+1024\right)\sin\left(dx+c\right)+512\,\cos\left(dx+c\right)+1024\right)\sqrt{a\sin\left(dx+c\right)+a}}{3003\,\left(d\cos\left(dx+c\right)+d\sin\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3003*(231*cos(d*x + c)^7 - 21*cos(d*x + c)^6 + 28*cos(d*x + c)^5 - 40*co
s(d*x + c)^4 + 64*cos(d*x + c)^3 - 128*cos(d*x + c)^2 - (231*cos(d*x + c)^6
 + 252*cos(d*x + c)^5 + 280*cos(d*x + c)^4 + 320*cos(d*x + c)^3 + 384*cos(d
*x + c)^2 + 512*cos(d*x + c) + 1024)*sin(d*x + c) + 512*cos(d*x + c) + 1024
)*sqrt(a*sin(d*x + c) + a)/(d*cos(d*x + c) + d*sin(d*x + c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sin\left(c+dx\right)+1\right)}\,\cos^{6}\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(c + d*x) + 1))*cos(c + d*x)**6, x)
```

Giac [A]
time = 6.71, size = 201, normalized size = 1.58

$$\frac{\sqrt{2}\left(60060\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)-15015\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{3}{4}\pi+\frac{3}{2}dx+\frac{3}{2}c\right)-9009\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{5}{4}\pi+\frac{5}{2}dx+\frac{5}{2}c\right)+2574\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{7}{4}\pi+\frac{7}{2}dx+\frac{7}{2}c\right)+2002\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{9}{4}\pi+\frac{9}{2}dx+\frac{9}{2}c\right)-273\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{11}{4}\pi+\frac{11}{2}dx+\frac{11}{2}c\right)-231\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{13}{4}\pi+\frac{13}{2}dx+\frac{13}{2}c\right)\right)\sqrt{a}}{96096\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/96096*sqrt(2)*(60060*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/
2*d*x + 1/2*c) - 15015*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-3/4*pi + 3/
2*d*x + 3/2*c) - 9009*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-5/4*pi + 5/2
*d*x + 5/2*c) + 2574*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-7/4*pi + 7/2*
d*x + 7/2*c) + 2002*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-9/4*pi + 9/2*d
*x + 9/2*c) - 273*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-11/4*pi + 11/2*d
*x + 11/2*c) - 231*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-13/4*pi + 13/2*
d*x + 13/2*c))*sqrt(a)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^6\,\sqrt{a+a\,\sin\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6*(a + a*sin(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^6*(a + a*sin(c + d*x))^(1/2), x)
```

3.103 $\int \cos^5(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=73

$$\frac{8(a+a\sin(c+dx))^{7/2}}{7a^3d}-\frac{8(a+a\sin(c+dx))^{9/2}}{9a^4d}+\frac{2(a+a\sin(c+dx))^{11/2}}{11a^5d}$$

```
[Out] 8/7*(a+a*sin(d*x+c))^(7/2)/a^3/d-8/9*(a+a*sin(d*x+c))^(9/2)/a^4/d+2/11*(a+a
*sin(d*x+c))^(11/2)/a^5/d
```

Rubi [A]
time = 0.05, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2746, 45}

$$\frac{2(a\sin(c+dx)+a)^{11/2}}{11a^5d}-\frac{8(a\sin(c+dx)+a)^{9/2}}{9a^4d}+\frac{8(a\sin(c+dx)+a)^{7/2}}{7a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (8*(a + a*Sin[c + d*x])^(7/2))/(7*a^3*d) - (8*(a + a*Sin[c + d*x])^(9/2))/(
9*a^4*d) + (2*(a + a*Sin[c + d*x])^(11/2))/(11*a^5*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \cos^5(c+dx)\sqrt{a+a\sin(c+dx)}\,dx &= \frac{\text{Subst}\left(\int (a-x)^2(a+x)^{5/2}\,dx,x,a\sin(c+dx)\right)}{a^5d}\\ &= \frac{\text{Subst}\left(\int \left(4a^2(a+x)^{5/2}-4a(a+x)^{7/2}+(a+x)^{9/2}\right)\,dx,x,a\sin(c\right.}{a^5d}\\ &= \frac{8(a+a\sin(c+dx))^{7/2}}{7a^3d}-\frac{8(a+a\sin(c+dx))^{9/2}}{9a^4d}+\frac{2(a+a\sin(c+}{11a^5d}\end{aligned}$$

Mathematica [A]
time = 0.61, size = 64, normalized size = 0.88

$$-\frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^6\sqrt{a(1+\sin(c+dx))}\,(-365+63\cos(2(c+dx))+364\sin(c+dx))}{693d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -1/693*((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6*Sqrt[a*(1 + Sin[c + d*x])]*
(-365 + 63*Cos[2*(c + d*x)] + 364*Sin[c + d*x]))/d
```

Maple [A]
time = 0.27, size = 41, normalized size = 0.56

method	result	size
default	$-\frac{2(a+a\sin(dx+c))^{\frac{7}{2}}\left(63\left(\cos^2(dx+c)\right)+182\sin(dx+c)-214\right)}{693a^3d}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/693/a^3*(a+a*sin(d*x+c))^(7/2)*(63*cos(d*x+c)^2+182*sin(d*x+c)-214)/d
```

Maxima [A]
time = 0.31, size = 55, normalized size = 0.75

$$\frac{2\left(63\,(a\sin(dx+c)+a)^{\frac{11}{2}}-308\,(a\sin(dx+c)+a)^{\frac{9}{2}}a+396\,(a\sin(dx+c)+a)^{\frac{7}{2}}a^2\right)}{693\,a^5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/693*(63*(a*sin(d*x + c) + a)^(11/2) - 308*(a*sin(d*x + c) + a)^(9/2)*a +
396*(a*sin(d*x + c) + a)^(7/2)*a^2)/(a^5*d)
```

Fricas [A]
time = 0.35, size = 68, normalized size = 0.93

$$\frac{2\left(7\cos(dx+c)^4+16\cos(dx+c)^2+\left(63\cos(dx+c)^4+80\cos(dx+c)^2+128\right)\sin(dx+c)+128\right)\sqrt{a\sin(dx+c)+a}}{693\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/693*(7*cos(d*x + c)^4 + 16*cos(d*x + c)^2 + (63*cos(d*x + c)^4 + 80*cos(d
*x + c)^2 + 128)*sin(d*x + c) + 128)*sqrt(a*sin(d*x + c) + a)/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 7.52, size = 99, normalized size = 1.36

$$\frac{64\sqrt{2}\left(63\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{11}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)-154\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{9}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)+99\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{7}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\right)\sqrt{a}}{693\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 64/693*sqrt(2)*(63*cos(-1/4*pi + 1/2*d*x + 1/2*c)^11*sgn(cos(-1/4*pi + 1/2*
d*x + 1/2*c)) - 154*cos(-1/4*pi + 1/2*d*x + 1/2*c)^9*sgn(cos(-1/4*pi + 1/2*
d*x + 1/2*c)) + 99*cos(-1/4*pi + 1/2*d*x + 1/2*c)^7*sgn(cos(-1/4*pi + 1/2*d
*x + 1/2*c)))*sqrt(a)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^5\,\sqrt{a+a\,\sin\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(a + a*sin(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^5*(a + a*sin(c + d*x))^(1/2), x)
```

3.104 $\int \cos^4(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=95

$$-\frac{64a^3\cos^5(c+dx)}{315d(a+a\sin(c+dx))^{5/2}}-\frac{16a^2\cos^5(c+dx)}{63d(a+a\sin(c+dx))^{3/2}}-\frac{2a\cos^5(c+dx)}{9d\sqrt{a+a\sin(c+dx)}}$$

```
[Out] -64/315*a^3*cos(d*x+c)^5/d/(a+a*sin(d*x+c))^(5/2)-16/63*a^2*cos(d*x+c)^5/d/
(a+a*sin(d*x+c))^(3/2)-2/9*a*cos(d*x+c)^5/d/(a+a*sin(d*x+c))^(1/2)
```

Rubi [A]

time = 0.12, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2753, 2752}

$$-\frac{64a^3\cos^5(c+dx)}{315d(a\sin(c+dx)+a)^{5/2}}-\frac{16a^2\cos^5(c+dx)}{63d(a\sin(c+dx)+a)^{3/2}}-\frac{2a\cos^5(c+dx)}{9d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-64*a^3*Cos[c + d*x]^5)/(315*d*(a + a*Sin[c + d*x])^(5/2)) - (16*a^2*Cos[c
 + d*x]^5)/(63*d*(a + a*Sin[c + d*x])^(3/2)) - (2*a*Cos[c + d*x]^5)/(9*d*Sq
rt[a + a*Sin[c + d*x]])
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2753

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && N
eQ[m + p, 0]
```

Rubi steps

$$\int \cos^4(c+dx)\sqrt{a+a\sin(c+dx)}\,dx = -\frac{2a\cos^5(c+dx)}{9d\sqrt{a+a\sin(c+dx)}}+\frac{1}{9}(8a)\int\frac{\cos^4(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$$
$$= -\frac{16a^2\cos^5(c+dx)}{63d(a+a\sin(c+dx))^{3/2}}-\frac{2a\cos^5(c+dx)}{9d\sqrt{a+a\sin(c+dx)}}+\frac{1}{63}(32a^2)$$
$$= -\frac{64a^3\cos^5(c+dx)}{315d(a+a\sin(c+dx))^{5/2}}-\frac{16a^2\cos^5(c+dx)}{63d(a+a\sin(c+dx))^{3/2}}-\frac{2a}{9d\sqrt{a}}$$

Mathematica [**A**]
time = 0.48, size = 89, normalized size = 0.94

$$-\frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^5\sqrt{a(1+\sin(c+dx))}\,(249-35\cos(2(c+dx))+220\sin(c+dx))}{315d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -1/315*((Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^5*Sqrt[a*(1 + Sin[c + d*x])]*
(249 - 35*Cos[2*(c + d*x)] + 220*Sin[c + d*x]))/(d*(Cos[(c + d*x)/2] + Sin[
(c + d*x)/2]))
```

Maple [**A**]
time = 0.36, size = 65, normalized size = 0.68

method	result	size
default	$\frac{2(1+\sin(dx+c))a(\sin(dx+c)-1)^3(35(\sin^2(dx+c))+110\sin(dx+c)+107)}{315\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/315*(1+sin(d*x+c))*a*(sin(d*x+c)-1)^3*(35*sin(d*x+c)^2+110*sin(d*x+c)+107
)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*cos(d*x + c)^4, x)
```

Fricas [A]
time = 0.36, size = 132, normalized size = 1.39

$$-\frac{2\left(35\cos\left(dx+c\right)^{5}-5\cos\left(dx+c\right)^{4}+8\cos\left(dx+c\right)^{3}-16\cos\left(dx+c\right)^{2}-\left(35\cos\left(dx+c\right)^{4}+40\cos\left(dx+c\right)^{3}+48\cos\left(dx+c\right)^{2}+64\cos\left(dx+c\right)+128\right)\sin\left(dx+c\right)+64\cos\left(dx+c\right)+128\right)\sqrt{a\sin\left(dx+c\right)+a}}{315\left(d\cos\left(dx+c\right)+d\sin\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/315*(35*cos(d*x + c)^5 - 5*cos(d*x + c)^4 + 8*cos(d*x + c)^3 - 16*cos(d*
x + c)^2 - (35*cos(d*x + c)^4 + 40*cos(d*x + c)^3 + 48*cos(d*x + c)^2 + 64*
cos(d*x + c) + 128)*sin(d*x + c) + 64*cos(d*x + c) + 128)*sqrt(a*sin(d*x +
c) + a)/(d*cos(d*x + c) + d*sin(d*x + c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\left(\sin\left(c+dx\right)+1\right)}\,\cos^{4}\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(c + d*x) + 1))*cos(c + d*x)**4, x)
```

Giac [A]
time = 6.14, size = 147, normalized size = 1.55

$$\frac{\sqrt{2}\left(1890\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)-420\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{3}{4}\pi+\frac{3}{2}dx+\frac{3}{2}c\right)-252\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{5}{4}\pi+\frac{5}{2}dx+\frac{5}{2}c\right)+45\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{7}{4}\pi+\frac{7}{2}dx+\frac{7}{2}c\right)+35\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{9}{4}\pi+\frac{9}{2}dx+\frac{9}{2}c\right)\right)\sqrt{a}}{2520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2520*sqrt(2)*(1890*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*
d*x + 1/2*c) - 420*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-3/4*pi + 3/2*d*
x + 3/2*c) - 252*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-5/4*pi + 5/2*d*x
+ 5/2*c) + 45*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-7/4*pi + 7/2*d*x + 7
/2*c) + 35*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-9/4*pi + 9/2*d*x + 9/2*
c))*sqrt(a)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\cos\left(c+d\,x\right)^{4}\,\sqrt{a+a\,\sin\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(a + a*sin(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^4*(a + a*sin(c + d*x))^(1/2), x)
```

3.105 $\int \cos^3(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=49

$$\frac{4(a+a\sin(c+dx))^{5/2}}{5a^2d}-\frac{2(a+a\sin(c+dx))^{7/2}}{7a^3d}$$

```
[Out] 4/5*(a+a*sin(d*x+c))^(5/2)/a^2/d-2/7*(a+a*sin(d*x+c))^(7/2)/a^3/d
```

Rubi [A]
time = 0.04, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2746, 45}

$$\frac{4(a\sin(c+dx)+a)^{5/2}}{5a^2d}-\frac{2(a\sin(c+dx)+a)^{7/2}}{7a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Sqrt[a + a*Sin[c + d*x]],x]
[Out] (4*(a + a*Sin[c + d*x])^(5/2))/(5*a^2*d) - (2*(a + a*Sin[c + d*x])^(7/2))/(
7*a^3*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \cos^3(c+dx)\sqrt{a+a\sin(c+dx)}\,dx &= \frac{\operatorname{Subst}\left(\int (a-x)(a+x)^{3/2}\,dx,x,a\sin(c+dx)\right)}{a^3d}\\ &= \frac{\operatorname{Subst}\left(\int \left(2a(a+x)^{3/2}-(a+x)^{5/2}\right)\,dx,x,a\sin(c+dx)\right)}{a^3d}\\ &= \frac{4(a+a\sin(c+dx))^{5/2}}{5a^2d}-\frac{2(a+a\sin(c+dx))^{7/2}}{7a^3d}\end{aligned}$$

Mathematica [A]
time = 0.15, size = 54, normalized size = 1.10

$$-\frac{2\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^4\sqrt{a(1+\sin(c+dx))}\,(-9+5\sin(c+dx))}{35d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4*Sqrt[a*(1 + Sin[c + d*x])]*(-9
+ 5*Sin[c + d*x]))/(35*d)
```

Maple [A]
time = 0.27, size = 31, normalized size = 0.63

method	result	size
default	$-\frac{2(a+a\sin(dx+c))^{\frac{5}{2}}(5\sin(dx+c)-9)}{35a^2d}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/35/a^2*(a+a*sin(d*x+c))^(5/2)*(5*sin(d*x+c)-9)/d
```

Maxima [A]
time = 0.31, size = 38, normalized size = 0.78

$$-\frac{2\left(5\left(a\sin\left(dx+c\right)+a\right)^{\frac{7}{2}}-14\left(a\sin\left(dx+c\right)+a\right)^{\frac{5}{2}}a\right)}{35\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -2/35*(5*(a*sin(d*x + c) + a)^(7/2) - 14*(a*sin(d*x + c) + a)^(5/2)*a)/(a^3
*d)
```

Fricas [A]
time = 0.36, size = 46, normalized size = 0.94

$$\frac{2\left(\cos\left(dx+c\right)^2+\left(5\,\cos\left(dx+c\right)^2+8\right)\sin\left(dx+c\right)+8\right)\sqrt{a\sin\left(dx+c\right)+a}}{35\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/35*(cos(d*x + c)^2 + (5*cos(d*x + c)^2 + 8)*sin(d*x + c) + 8)*sqrt(a*sin(
d*x + c) + a)/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 5.47, size = 70, normalized size = 1.43

$$-\frac{16\sqrt{2}\left(5\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{7}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)-7\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{5}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\right)\sqrt{a}}{35d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -16/35*sqrt(2)*(5*cos(-1/4*pi + 1/2*d*x + 1/2*c)^7*sgn(cos(-1/4*pi + 1/2*d*
x + 1/2*c)) - 7*cos(-1/4*pi + 1/2*d*x + 1/2*c)^5*sgn(cos(-1/4*pi + 1/2*d*x
+ 1/2*c)))*sqrt(a)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \cos\left(c+d\,x\right)^3\,\sqrt{a+a\,\sin\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a + a*sin(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^3*(a + a*sin(c + d*x))^(1/2), x)
```

3.106 $\int \cos^2(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=63

$$-\frac{8a^2\cos^3(c+dx)}{15d(a+a\sin(c+dx))^{3/2}}-\frac{2a\cos^3(c+dx)}{5d\sqrt{a+a\sin(c+dx)}}$$

```
[Out] -8/15*a^2*cos(d*x+c)^3/d/(a+a*sin(d*x+c))^(3/2)-2/5*a*cos(d*x+c)^3/d/(a+a*s
in(d*x+c))^(1/2)
```

Rubi [A]

time = 0.07, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2753, 2752}

$$-\frac{8a^2\cos^3(c+dx)}{15d(a\sin(c+dx)+a)^{3/2}}-\frac{2a\cos^3(c+dx)}{5d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-8*a^2*Cos[c + d*x]^3)/(15*d*(a + a*Sin[c + d*x])^(3/2)) - (2*a*Cos[c + d*
x]^3)/(5*d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2753

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && N
eQ[m + p, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^2(c+dx)\sqrt{a+a\sin(c+dx)}\,dx &= -\frac{2a\cos^3(c+dx)}{5d\sqrt{a+a\sin(c+dx)}}+\frac{1}{5}(4a)\int \frac{\cos^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx\\ &= -\frac{8a^2\cos^3(c+dx)}{15d(a+a\sin(c+dx))^{3/2}}-\frac{2a\cos^3(c+dx)}{5d\sqrt{a+a\sin(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.11, size = 79, normalized size = 1.25

$$-\frac{2\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3\sqrt{a(1+\sin(c+dx))}\,(7+3\sin(c+dx))}{15d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sqrt[a + a*Sin[c + d*x]],x]
[Out] (-2*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3*Sqrt[a*(1 + Sin[c + d*x])]*(7 +
 3*Sin[c + d*x]))/(15*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

Maple [A]
time = 0.31, size = 55, normalized size = 0.87

method	result	size
default	$-\frac{2(1+\sin(dx+c))a(\sin(dx+c)-1)^2(3\sin(dx+c)+7)}{15\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/15*(1+sin(d*x+c))*a*(sin(d*x+c)-1)^2*(3*sin(d*x+c)+7)/cos(d*x+c)/(a+a*si
n(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a*sin(d*x + c) + a)*cos(d*x + c)^2, x)
```

Fricas [A]
time = 0.33, size = 92, normalized size = 1.46

$$-\frac{2\left(3\cos(dx+c)^3-\cos(dx+c)^2-\left(3\cos(dx+c)^2+4\cos(dx+c)+8\right)\sin(dx+c)+4\cos(dx+c)+8\right)\sqrt{a\sin(dx+c)+a}}{15\left(d\cos(dx+c)+d\sin(dx+c)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
[Out] -2/15*(3*cos(d*x + c)^3 - cos(d*x + c)^2 - (3*cos(d*x + c)^2 + 4*cos(d*x +
c) + 8)*sin(d*x + c) + 4*cos(d*x + c) + 8)*sqrt(a*sin(d*x + c) + a)/(d*cos(
d*x + c) + d*sin(d*x + c) + d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sin\left(c+dx\right)+1\right)}\,\cos^2\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(c + d*x) + 1))*cos(c + d*x)**2, x)
```

Giac [**A**]
time = 5.76, size = 93, normalized size = 1.48

$$\frac{\sqrt{2}\left(30\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-5\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{3}{4}\,\pi+\frac{3}{2}\,dx+\frac{3}{2}\,c\right)-3\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{5}{4}\,\pi+\frac{5}{2}\,dx+\frac{5}{2}\,c\right)\right)\sqrt{a}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/30*sqrt(2)*(30*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*d*x
+ 1/2*c) - 5*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-3/4*pi + 3/2*d*x + 3/
2*c) - 3*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-5/4*pi + 5/2*d*x + 5/2*c)
)*sqrt(a)/d
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int {\cos\left(c+d\,x\right)}^2\,\sqrt{a+a\,\sin\left(c+d\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a + a*sin(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^2*(a + a*sin(c + d*x))^(1/2), x)
```

3.107 $\int \cos(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=24

$$\frac{2(a+a\sin(c+dx))^{3/2}}{3ad}$$

```
[Out] 2/3*(a+a*sin(d*x+c))^(3/2)/a/d
```

Rubi [A]
time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2746, 32}

$$\frac{2(a\sin(c+dx)+a)^{3/2}}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (2*(a + a*Sin[c + d*x])^(3/2))/(3*a*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)\sqrt{a+a\sin(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \sqrt{a+x}\,dx, x, a\sin(c+dx)\right)}{ad}\\ &= \frac{2(a+a\sin(c+dx))^{3/2}}{3ad}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 44, normalized size = 1.83

$$\frac{2\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2\sqrt{a(1+\sin(c+dx))}}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sqrt[a + a*Sin[c + d*x]],x]
[Out] (2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2*Sqrt[a*(1 + Sin[c + d*x])])/(3*d
)
```

Maple [A]
time = 0.04, size = 21, normalized size = 0.88

method	result	size
derivativedivides	$\frac{2(a+a\sin(dx+c))^{\frac{3}{2}}}{3da}$	21
default	$\frac{2(a+a\sin(dx+c))^{\frac{3}{2}}}{3da}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/3*(a+a*sin(d*x+c))^(3/2)/d/a
```

Maxima [A]
time = 0.29, size = 20, normalized size = 0.83

$$\frac{2\,(a\sin\left(dx+c\right)+a)^{\frac{3}{2}}}{3\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
[Out] 2/3*(a*sin(d*x + c) + a)^(3/2)/(a*d)
```

Fricas [A]
time = 0.35, size = 25, normalized size = 1.04

$$\frac{2\,\sqrt{a\sin\left(dx+c\right)+a}\,(\sin\left(dx+c\right)+1)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
[Out] 2/3*sqrt(a*sin(d*x + c) + a)*(sin(d*x + c) + 1)/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(19) = 38$.
time = 0.12, size = 58, normalized size = 2.42

$$\begin{cases} \frac{2\sqrt{a\sin\left(c+dx\right)+a}\,\sin\left(c+dx\right)}{3d}+\frac{2\sqrt{a\sin\left(c+dx\right)+a}}{3d} & \text{for}\ d\neq 0 \\ x\sqrt{a\sin\left(c\right)+a}\,\cos\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Piecewise((2*sqrt(a*sin(c + d*x) + a)*sin(c + d*x)/(3*d) + 2*sqrt(a*sin(c +
 d*x) + a)/(3*d), Ne(d, 0)), (x*sqrt(a*sin(c) + a)*cos(c), True))
```

Giac [A]
time = 5.80, size = 38, normalized size = 1.58

$$\frac{4\sqrt{2}\sqrt{a}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{3}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 4/3*sqrt(2)*sqrt(a)*cos(-1/4*pi + 1/2*d*x + 1/2*c)^3*sgn(cos(-1/4*pi + 1/2*
d*x + 1/2*c))/d
```

Mupad [B]
time = 4.57, size = 20, normalized size = 0.83

$$\frac{2\left(a\left(\sin\left(c+d\,x\right)+1\right)\right)^{3/2}}{3\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + a*sin(c + d*x))^(1/2),x)
```

```
[Out] (2*(a*(sin(c + d*x) + 1))^(3/2))/(3*a*d)
```

3.108 $\int \sec(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=40

$$\frac{\sqrt{2}\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}$$

```
[Out] arctanh(1/2*(a+a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)*a^(1/2)/d
```

Rubi [A]
time = 0.04, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2746, 65, 212}

$$\frac{\sqrt{2}\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (Sqrt[2]*Sqrt[a]*ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])])/d
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\int \sec(c+dx)\sqrt{a+a\sin(c+dx)}\,dx = \frac{a\text{Subst}\left(\int \frac{1}{(a-x)\sqrt{a+x}}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{(2a)\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \sqrt{a+a\sin(c+dx)}\right)}{d}$$
$$= \frac{\sqrt{2}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.07, size = 95, normalized size = 2.38

$$-\frac{(2-2i)\sqrt[4]{-1}\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\sec\left(\frac{dx}{4}\right)\left(\cos\left(\frac{1}{4}(2c+dx)\right)+\sin\left(\frac{1}{4}(2c+dx)\right)\right)\right)\sqrt{a(1+\sin(c+dx))}}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*Sqrt[a + a*Sin[c + d*x]],x]
[Out] ((-2 + 2*I)*(-1)^(1/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*Sec[(d*x)/4]*(Cos[(2*
c + d*x)/4] + Sin[(2*c + d*x)/4])]*Sqrt[a*(1 + Sin[c + d*x])])/(d*(Cos[(c +
 d*x)/2] + Sin[(c + d*x)/2]))
```

Maple [A]
time = 0.13, size = 32, normalized size = 0.80

method	result	size
default	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+a\sin(dx+c)}\sqrt{2}}{2\sqrt{a}}\right)\sqrt{2}\sqrt{a}}{d}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] arctanh(1/2*(a+a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)*a^(1/2)/d
```

Maxima [A]
time = 0.51, size = 58, normalized size = 1.45

$$-\frac{\sqrt{2}\sqrt{a}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{a\sin(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{a\sin(dx+c)+a}}\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(2)*sqrt(a)*log(-(sqrt(2)*sqrt(a) - sqrt(a*sin(d*x + c) + a))/(sqr
t(2)*sqrt(a) + sqrt(a*sin(d*x + c) + a)))/d
```

Fricas [A]
time = 0.35, size = 92, normalized size = 2.30

$$\left[\frac{\sqrt{2}\,\sqrt{a}\,\log\left(-\frac{a\sin(dx+c)+2\,\sqrt{2}\,\sqrt{a\sin\left(dx+c\right)+a}\,\sqrt{a}+3\,a}{\sin(dx+c)-1}\right)}{2\,d},-\frac{\sqrt{2}\,\sqrt{-a}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{-a}}{\sqrt{a\sin\left(dx+c\right)+a}}\right)}{d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(2)*sqrt(a)*log(-(a*sin(d*x + c) + 2*sqrt(2)*sqrt(a*sin(d*x + c) +
 a)*sqrt(a) + 3*a)/(sin(d*x + c) - 1))/d, -sqrt(2)*sqrt(-a)*arctan(sqrt(2)*
sqrt(-a)/sqrt(a*sin(d*x + c) + a))/d]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\left(\sin\left(c+dx\right)+1\right)}\,\sec\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(c + d*x) + 1))*sec(c + d*x), x)
```

Giac [A]
time = 6.34, size = 59, normalized size = 1.48

$$\frac{\sqrt{2}\,\sqrt{a}\,\left(\log\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-\log\left(-\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*sqrt(a)*(log(cos(-1/4*pi + 1/2*d*x + 1/2*c) + 1) - log(-cos(-1/
4*pi + 1/2*d*x + 1/2*c) + 1))*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{\sqrt{a+a\,\sin\left(c+d\,x\right)}}{\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(1/2)/cos(c + d*x),x)
[Out] int((a + a*sin(c + d*x))^(1/2)/cos(c + d*x), x)
```

3.109 $\int \sec^2(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=72

$$-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{2}\,d}+\frac{\sec(c+dx)\sqrt{a+a\sin(c+dx)}}{d}$$

```
[Out] -1/2*arctanh(1/2*cos(d*x+c)*a^(1/2)*2^(1/2)/(a+a*sin(d*x+c))^(1/2))*a^(1/2)
/d*2^(1/2)+sec(d*x+c)*(a+a*sin(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.05, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {2754, 2728, 212}

$$\frac{\sec(c+dx)\sqrt{a\sin(c+dx)+a}}{d}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{\sqrt{2}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -((Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]]
)])/(Sqrt[2]*d)) + (Sec[c + d*x]*Sqrt[a + a*Sin[c + d*x]])/d
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^m/(a*f*g*(p + 1))), x] + Dist[a*((m + p + 1)/(g^2*(p + 1))), Int[(g*C
os[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e
, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ[
m + 1/2, 2*p]
```

Rubi steps

$$\int \sec^2(c+dx)\sqrt{a+a\sin(c+dx)}\,dx = \frac{\sec(c+dx)\sqrt{a+a\sin(c+dx)}}{d}+\frac{1}{2}a\int \frac{1}{\sqrt{a+a\sin(c+dx)}}\,dx$$
$$= \frac{\sec(c+dx)\sqrt{a+a\sin(c+dx)}}{d}-\frac{a\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx,x,\frac{a\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d}$$
$$= -\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{2}\,d}+\frac{\sec(c+dx)\sqrt{a+a\sin(c+dx)}}{d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.16, size = 106, normalized size = 1.47

$$\frac{\sec(c+dx)\left(1-(1+i)(-1)^{3/4}\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\sec\left(\frac{dx}{4}\right)\left(\cos\left(\frac{1}{4}(2c+dx)\right)-\sin\left(\frac{1}{4}(2c+dx)\right)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)\sqrt{a(1+\sin(c+dx))}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (Sec[c + d*x]*(1 - (1 + I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*Sec[(d
*x)/4]*(Cos[(2*c + d*x)/4] - Sin[(2*c + d*x)/4])]*(Cos[(c + d*x)/2] - Sin[(
c + d*x)/2]))*Sqrt[a*(1 + Sin[c + d*x])])/d
```

Maple [A]
time = 0.46, size = 84, normalized size = 1.17

method	result	size
default	$\frac{(1+\sin(dx+c))\left(-\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)a\sqrt{a-a\sin(dx+c)}+2a^{\frac{3}{2}}\right)}{2\sqrt{a}\,\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(1+sin(d*x+c))*(-2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(
1/2))*a*(a-a*sin(d*x+c))^(1/2)+2*a^(3/2))/a^(1/2)/cos(d*x+c)/(a+a*sin(d*x+c
))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*sec(d*x + c)^2, x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 159 vs. 2(61) = 122.
time = 0.35, size = 159, normalized size = 2.21

$$\frac{\sqrt{2}\,\sqrt{a}\,\cos\left(dx+c\right)\log\left(-\frac{a\cos(dx+c)^{2}-2\,\sqrt{2}\,\sqrt{a\sin\left(dx+c\right)+a}\,\sqrt{a}\,(\cos(dx+c)-\sin(dx+c)+1)+3\,a\cos(dx+c)-(a\cos(dx+c)-2\,a)\sin(dx+c)+2\,a}{\cos(dx+c)^{2}-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2}\right)+4\,\sqrt{a\sin\left(dx+c\right)+a}}{4\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*sqrt(a)*cos(d*x + c)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a
*sin(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - sin(d*x + c) + 1) + 3*a*cos(d*x
+ c) - (a*cos(d*x + c) - 2*a)*sin(d*x + c) + 2*a)/(cos(d*x + c)^2 - (cos(d*
x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2)) + 4*sqrt(a*sin(d*x + c) + a))
/(d*cos(d*x + c))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\left(\sin\left(c+dx\right)+1\right)}\,\sec^{2}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(c + d*x) + 1))*sec(c + d*x)**2, x)
```

Giac [**A**]
time = 4.82, size = 102, normalized size = 1.42

$$\frac{\sqrt{2}\left(\log\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)-\log\left(-\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)-\frac{2\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}\right)\sqrt{a}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*(log(sin(-1/4*pi + 1/2*d*x + 1/2*c) + 1)*sgn(cos(-1/4*pi + 1/2*
d*x + 1/2*c)) - log(-sin(-1/4*pi + 1/2*d*x + 1/2*c) + 1)*sgn(cos(-1/4*pi +
1/2*d*x + 1/2*c)) - 2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))/sin(-1/4*pi + 1/2
*d*x + 1/2*c))*sqrt(a)/d
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+a\,\sin\left(c+d\,x\right)}}{{\cos\left(c+d\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(1/2)/cos(c + d*x)^2,x)
[Out] int((a + a*sin(c + d*x))^(1/2)/cos(c + d*x)^2, x)
```

3.110 $\int \sec^3(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=95

$$\frac{3\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}\,d}-\frac{3a}{4d\sqrt{a+a\sin(c+dx)}}+\frac{\sec^2(c+dx)\sqrt{a+a\sin(c+dx)}}{2d}$$

```
[Out] 3/8*arctanh(1/2*(a+a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)*a^(1/2)/d-3
/4*a/d/(a+a*sin(d*x+c))^(1/2)+1/2*sec(d*x+c)^2*(a+a*sin(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.09, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2754, 2746, 53, 65, 212}

$$-\frac{3a}{4d\sqrt{a\sin(c+dx)+a}}+\frac{3\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}\,d}+\frac{\sec^2(c+dx)\sqrt{a\sin(c+dx)+a}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (3*Sqrt[a]*ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(4*Sqrt[2]*
d) - (3*a)/(4*d*Sqrt[a + a*Sin[c + d*x]]) + (Sec[c + d*x]^2*Sqrt[a + a*Sin[
c + d*x]])/(2*d)
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2754

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^m/(a*f*g*(p + 1))), x] + Dist[a*((m + p + 1)/(g^2*(p + 1))), Int[(g*C
os[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e
, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ[
m + 1/2, 2*p]
```

Rubi steps

$$\begin{aligned}
\int \sec^3(c+dx)\sqrt{a+a\sin(c+dx)}\,dx &= \frac{\sec^2(c+dx)\sqrt{a+a\sin(c+dx)}}{2d}+\frac{1}{4}(3a)\int \frac{\sec(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,d \\
&= \frac{\sec^2(c+dx)\sqrt{a+a\sin(c+dx)}}{2d}+\frac{(3a^2)\,\text{Subst}\left(\int \frac{1}{(a-x)(a+x)^{3/2}}\,dx,\right.}{4d} \\
&= -\frac{3a}{4d\sqrt{a+a\sin(c+dx)}}+\frac{\sec^2(c+dx)\sqrt{a+a\sin(c+dx)}}{2d}+\frac{(3a}{} \\
&= -\frac{3a}{4d\sqrt{a+a\sin(c+dx)}}+\frac{\sec^2(c+dx)\sqrt{a+a\sin(c+dx)}}{2d}+\frac{(3a}{} \\
&= \frac{3\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}\,d}-\frac{3a}{4d\sqrt{a+a\sin(c+dx)}}+
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.26, size = 271, normalized size = 2.85

$$\frac{\left(-2-(3-3i)\sqrt[4]{-1}\,\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\sec\left(\frac{dx}{4}\right)\left(\cos\left(\frac{1}{4}(2c+dx)\right)+\sin\left(\frac{1}{4}(2c+dx)\right)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+\frac{2\sin\left(\frac{dx}{2}\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}+\frac{\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}\right)\sqrt{a(1+\sin(c+dx))}}{4d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] ((-2 - (3 - 3*I)*(-1)^(1/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*Sec[(d*x)/4]*(Co
s[(2*c + d*x)/4] + Sin[(2*c + d*x)/4])]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2
]) + (2*Sin[(d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))/((Cos[c/2] - Si
n[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2) + ((Cos[c/2] + Sin[c/2])*(
Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))/((Cos[c/2] - Sin[c/2])*(Cos[(c + d*x)
/2] - Sin[(c + d*x)/2])))*Sqrt[a*(1 + Sin[c + d*x])])/(4*d*(Cos[(c + d*x)/2
] + Sin[(c + d*x)/2])^2)
```

Maple [A]
time = 0.57, size = 90, normalized size = 0.95

method	result
default	$\frac{2a^{3}\left(-\frac{\frac{\sqrt{a+a\sin\left(dx+c\right)}}{2a\sin\left(dx+c\right)-2a}-\frac{3\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+a\sin\left(dx+c\right)}\,\sqrt{2}}{2\sqrt{a}}\right)}{4\sqrt{a}}}{4a^{2}}-\frac{1}{4a^{2}\sqrt{a+a\sin\left(dx+c\right)}}\right)}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*a^3*(-1/4/a^2*(1/2*(a+a*sin(d*x+c))^(1/2)/(a*sin(d*x+c)-a)-3/4*2^(1/2)/a^
(1/2)*arctanh(1/2*(a+a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2)))-1/4/a^2/(a+a*sin
(d*x+c))^(1/2))/d
```

Maxima [A]
time = 0.50, size = 117, normalized size = 1.23

$$-\frac{3\sqrt{2}\,a^{\frac{3}{2}}\log\left(-\frac{\sqrt{2}\,\sqrt{a}-\sqrt{a\sin\left(dx+c\right)+a}}{\sqrt{2}\,\sqrt{a}+\sqrt{a\sin\left(dx+c\right)+a}}\right)+\frac{4\,\left(3\,\left(a\sin\left(dx+c\right)+a\right)a^{2}-4\,a^{3}\right)}{\left(a\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}-2\,\sqrt{a\sin\left(dx+c\right)+a}\,a}}{16\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/16*(3*sqrt(2)*a^(3/2)*log(-(sqrt(2)*sqrt(a) - sqrt(a*sin(d*x + c) + a))/
(sqrt(2)*sqrt(a) + sqrt(a*sin(d*x + c) + a))) + 4*(3*(a*sin(d*x + c) + a)*a
^2 - 4*a^3)/((a*sin(d*x + c) + a)^(3/2) - 2*sqrt(a*sin(d*x + c) + a)*a))/(a
*d)
```

Fricas [A]
time = 0.36, size = 99, normalized size = 1.04

$$\frac{3\,\sqrt{2}\,\sqrt{a}\,\cos\left(dx+c\right)^{2}\log\left(-\frac{a\sin(dx+c)+2\,\sqrt{2}\,\sqrt{a\sin\left(dx+c\right)+a}\,\sqrt{a}+3\,a}{\sin(dx+c)-1}\right)+4\,\sqrt{a\sin\left(dx+c\right)+a}\,\left(3\,\sin\left(dx+c\right)-1\right)}{16\,d\cos\left(dx+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/16*(3*sqrt(2)*sqrt(a)*cos(d*x + c)^2*log(-(a*sin(d*x + c) + 2*sqrt(2)*sqr
t(a*sin(d*x + c) + a)*sqrt(a) + 3*a)/(sin(d*x + c) - 1)) + 4*sqrt(a*sin(d*x
 + c) + a)*(3*sin(d*x + c) - 1))/(d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\left(\sin\left(c+dx\right)+1\right)}\,\sec^{3}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(c + d*x) + 1))*sec(c + d*x)**3, x)
```

Giac [A]
time = 6.33, size = 112, normalized size = 1.18

$$-\frac{\sqrt{2}\,\sqrt{a}\left(\frac{2\left(3\,\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-2\right)}{\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-3\,\log\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)+3\,\log\left(-\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/16*sqrt(2)*sqrt(a)*(2*(3*cos(-1/4*pi + 1/2*d*x + 1/2*c)^2 - 2)/(cos(-1/4
*pi + 1/2*d*x + 1/2*c)^3 - cos(-1/4*pi + 1/2*d*x + 1/2*c)) - 3*log(cos(-1/4
*pi + 1/2*d*x + 1/2*c) + 1) + 3*log(-cos(-1/4*pi + 1/2*d*x + 1/2*c) + 1))*s
gn(cos(-1/4*pi + 1/2*d*x + 1/2*c))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\sqrt{a+a\,\sin\left(c+d\,x\right)}}{{\cos\left(c+d\,x\right)}^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(1/2)/cos(c + d*x)^3,x)
```

```
[Out] int((a + a*sin(c + d*x))^(1/2)/cos(c + d*x)^3, x)
```

3.111 $\int \sec^4(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=137

$$-\frac{5\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(c+dx)}{\sqrt{2}\,\sqrt{a+a\sin(c+dx)}}\right)}{8\sqrt{2}\,d}-\frac{5a^2\cos(c+dx)}{8d(a+a\sin(c+dx))^{3/2}}+\frac{5a\sec(c+dx)}{6d\sqrt{a+a\sin(c+dx)}}+\frac{\sec^3(c+d$$

```
[Out] -5/8*a^2*cos(d*x+c)/d/(a+a*sin(d*x+c))^(3/2)-5/16*arctanh(1/2*cos(d*x+c)*a^
(1/2)*2^(1/2)/(a+a*sin(d*x+c))^(1/2))*a^(1/2)/d*2^(1/2)+5/6*a*sec(d*x+c)/d/
(a+a*sin(d*x+c))^(1/2)+1/3*sec(d*x+c)^3*(a+a*sin(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.11, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2754, 2766, 2729, 2728, 212}

$$-\frac{5a^2\cos(c+dx)}{8d(a\sin(c+dx)+a)^{3/2}}+\frac{\sec^3(c+dx)\sqrt{a\sin(c+dx)+a}}{3d}+\frac{5a\sec(c+dx)}{6d\sqrt{a\sin(c+dx)+a}}-\frac{5\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(c+dx)}{\sqrt{2}\,\sqrt{a\sin(c+dx)+a}}\right)}{8\sqrt{2}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-5*Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]
])])/(8*Sqrt[2]*d) - (5*a^2*Cos[c + d*x])/(8*d*(a + a*Sin[c + d*x])^(3/2))
+ (5*a*Sec[c + d*x])/(6*d*Sqrt[a + a*Sin[c + d*x]]) + (Sec[c + d*x]^3*Sqrt[
a + a*Sin[c + d*x]])/(3*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
```

```
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2754

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^m/(a*f*g*(p + 1))), x] + Dist[a*((m + p + 1)/(g^2*(p + 1))), Int[(g*C
os[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e
, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ[
m + 1/2, 2*p]
```

Rule 2766

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(a*f*g*(p + 1)*S
qrt[a + b*Sin[e + f*x]])), x] + Dist[a*((2*p + 1)/(2*g^2*(p + 1))), Int[(g*
Cos[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e,
 f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\int \sec^4(c+dx)\sqrt{a+a\sin(c+dx)}\,dx &= \frac{\sec^3(c+dx)\sqrt{a+a\sin(c+dx)}}{3d}+\frac{1}{6}(5a)\int\frac{\sec^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,d \\
&= \frac{5a\sec(c+dx)}{6d\sqrt{a+a\sin(c+dx)}}+\frac{\sec^3(c+dx)\sqrt{a+a\sin(c+dx)}}{3d}+\frac{1}{4}(5a \\
&= -\frac{5a^2\cos(c+dx)}{8d(a+a\sin(c+dx))^{3/2}}+\frac{5a\sec(c+dx)}{6d\sqrt{a+a\sin(c+dx)}}+\frac{\sec^3(c+dx}{} \\
&= -\frac{5a^2\cos(c+dx)}{8d(a+a\sin(c+dx))^{3/2}}+\frac{5a\sec(c+dx)}{6d\sqrt{a+a\sin(c+dx)}}+\frac{\sec^3(c+dx}{} \\
&= -\frac{5\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{8\sqrt{2}d}-\frac{5a^2\cos(c+dx)}{8d(a+a\sin(c+dx}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.28, size = 302, normalized size = 2.20

$$
\frac{\left(\frac{6\sin\left(\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)}-\frac{3\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)}-(15+15i)(-1)^{3/4}\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\sec\left(\frac{dx}{4}\right)\left(\cos\left(\frac{1}{4}(2c+dx)\right)-\sin\left(\frac{1}{4}(2c+dx)\right)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2+\frac{4\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3}+\frac{12\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)}\right)\sqrt{a(1+\sin(c+dx))}}{24d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (((6*Sin[(d*x)/2])/(Cos[c/2] + Sin[c/2]) - (3*(Cos[c/2] - Sin[c/2])*(Cos[(c
 + d*x)/2] + Sin[(c + d*x)/2]))/(Cos[c/2] + Sin[c/2]) - (15 + 15*I)*(-1)^(3
/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*Sec[(d*x)/4]*(Cos[(2*c + d*x)/4] - Sin[(
2*c + d*x)/4])]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + (4*(Cos[(c + d*x)
/2] + Sin[(c + d*x)/2])^2)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3 + (12*(C
os[(c + d*x)/2] + Sin[(c + d*x)/2])^2)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]
))*Sqrt[a*(1 + Sin[c + d*x])])/(24*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^
3)
```

Maple [A]
time = 0.46, size = 153, normalized size = 1.12

method	result
default	$\frac{\sin(dx+c)\left(15(a-a\sin(dx+c))^{\frac{3}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)a-20a^{\frac{5}{2}}\right)-30a^{\frac{5}{2}}\left(\cos^2(dx+c)\right)+15(a-a\sin(}{48a^{\frac{3}{2}}\left(\sin(dx+c)-1\right)\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/48/a^(3/2)*(sin(d*x+c)*(15*(a-a*sin(d*x+c))^(3/2)*2^(1/2)*arctanh(1/2*(a-
a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a-20*a^(5/2))-30*a^(5/2)*cos(d*x+c)^2+
15*(a-a*sin(d*x+c))^(3/2)*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2
)/a^(1/2))*a+4*a^(5/2))/(sin(d*x+c)-1)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*sec(d*x + c)^4, x)
```

Fricas [A]
time = 0.37, size = 188, normalized size = 1.37

$$\frac{15\sqrt{2}\sqrt{a}\cos(dx+c)^3\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{a\sin(dx+c)+a}\left(\sqrt{2}\cos(dx+c)-\sqrt{2}\sin(dx+c)+\sqrt{2}\right)\sqrt{a}+3a\cos(dx+c)-(a\cos(dx+c)-2a)\sin(dx+c)+2a}{\cos(dx+c)^2-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2}\right)+4\left(15\cos(dx+c)^2+10\sin(dx+c)-2\right)\sqrt{a\sin(dx+c)+a}}{96\,d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/96*(15*sqrt(2)*sqrt(a)*cos(d*x + c)^3*log(-(a*cos(d*x + c)^2 - 2*sqrt(a*s
in(d*x + c) + a)*(sqrt(2)*cos(d*x + c) - sqrt(2)*sin(d*x + c) + sqrt(2))*sq
rt(a) + 3*a*cos(d*x + c) - (a*cos(d*x + c) - 2*a)*sin(d*x + c) + 2*a)/(cos(
d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2)) + 4*(15*c
os(d*x + c)^2 + 10*sin(d*x + c) - 2)*sqrt(a*sin(d*x + c) + a))/(d*cos(d*x +
 c)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sin\left(c+dx\right)+1\right)}\,\sec^4\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(c + d*x) + 1))*sec(c + d*x)**4, x)
```

Giac [A]
time = 8.19, size = 178, normalized size = 1.30

$$\frac{\sqrt{2}\left(15\,\log\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)-15\,\log\left(-\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)-\frac{6\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}-\frac{4\left(6\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\right)}{\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}\right)\sqrt{a}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/96*sqrt(2)*(15*log(sin(-1/4*pi + 1/2*d*x + 1/2*c) + 1)*sgn(cos(-1/4*pi +
1/2*d*x + 1/2*c)) - 15*log(-sin(-1/4*pi + 1/2*d*x + 1/2*c) + 1)*sgn(cos(-1/
4*pi + 1/2*d*x + 1/2*c)) - 6*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*p
i + 1/2*d*x + 1/2*c)/(sin(-1/4*pi + 1/2*d*x + 1/2*c)^2 - 1) - 4*(6*sgn(cos(
-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*d*x + 1/2*c)^2 + sgn(cos(-1/4
*pi + 1/2*d*x + 1/2*c)))/sin(-1/4*pi + 1/2*d*x + 1/2*c)^3)*sqrt(a)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+a\,\sin\left(c+d\,x\right)}}{{\cos\left(c+d\,x\right)}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(1/2)/cos(c + d*x)^4,x)
```

```
[Out] int((a + a*sin(c + d*x))^(1/2)/cos(c + d*x)^4, x)
```

3.112 $\int \sec^5(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=149

$$\frac{35\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{64\sqrt{2}\,d}-\frac{35a^2}{96d(a+a\sin(c+dx))^{3/2}}-\frac{35a}{64d\sqrt{a+a\sin(c+dx)}}+\frac{7a\sec^2(}{16d\sqrt{a+a\,s}}$$

```
[Out] -35/96*a^2/d/(a+a*sin(d*x+c))^(3/2)+35/128*arctanh(1/2*(a+a*sin(d*x+c))^(1/
2)*2^(1/2)/a^(1/2))*2^(1/2)*a^(1/2)/d-35/64*a/d/(a+a*sin(d*x+c))^(1/2)+7/16
*a*sec(d*x+c)^2/d/(a+a*sin(d*x+c))^(1/2)+1/4*sec(d*x+c)^4*(a+a*sin(d*x+c))^
(1/2)/d
```

Rubi [A]
time = 0.14, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2754, 2766, 2746, 53, 65, 212}

$$-\frac{35a^2}{96d(a\sin(c+dx)+a)^{3/2}}-\frac{35a}{64d\sqrt{a\sin(c+dx)+a}}+\frac{35\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{64\sqrt{2}\,d}+\frac{\sec^4(c+dx)\sqrt{a\sin(c+dx)+a}}{4d}+\frac{7a\sec^2(c+dx)}{16d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (35*Sqrt[a]*ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(64*Sqrt[2
]*d) - (35*a^2)/(96*d*(a + a*Sin[c + d*x])^(3/2)) - (35*a)/(64*d*Sqrt[a + a
*Sin[c + d*x]]) + (7*a*Sec[c + d*x]^2)/(16*d*Sqrt[a + a*Sin[c + d*x]]) + (S
ec[c + d*x]^4*Sqrt[a + a*Sin[c + d*x]])/(4*d)
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2754

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^m/(a*f*g*(p + 1))), x] + Dist[a*((m + p + 1)/(g^2*(p + 1))), Int[(g*C
os[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e
, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ[
m + 1/2, 2*p]
```

Rule 2766

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(a*f*g*(p + 1)*S
qrt[a + b*Sin[e + f*x]])), x] + Dist[a*((2*p + 1)/(2*g^2*(p + 1))), Int[(g*
Cos[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e,
 f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \sec^5(c+dx)\sqrt{a+a\sin(c+dx)}\,dx &= \frac{\sec^4(c+dx)\sqrt{a+a\sin(c+dx)}}{4d}+\frac{1}{8}(7a)\int\frac{\sec^3(c+dx)}{\sqrt{a+a\sin(c+dx)}} \\ &= \frac{7a\sec^2(c+dx)}{16d\sqrt{a+a\sin(c+dx)}}+\frac{\sec^4(c+dx)\sqrt{a+a\sin(c+dx)}}{4d}+\frac{1}{32} \\ &= \frac{7a\sec^2(c+dx)}{16d\sqrt{a+a\sin(c+dx)}}+\frac{\sec^4(c+dx)\sqrt{a+a\sin(c+dx)}}{4d}+\frac{(3}{} \\ &= -\frac{35a^2}{96d(a+a\sin(c+dx))^{3/2}}+\frac{7a\sec^2(c+dx)}{16d\sqrt{a+a\sin(c+dx)}}+\frac{\sec^4(c+}{} \\ &= -\frac{35a^2}{96d(a+a\sin(c+dx))^{3/2}}-\frac{35a}{64d\sqrt{a+a\sin(c+dx)}}+\frac{7a\,s}{16d\sqrt{a}} \\ &= -\frac{35a^2}{96d(a+a\sin(c+dx))^{3/2}}-\frac{35a}{64d\sqrt{a+a\sin(c+dx)}}+\frac{7a\,s}{16d\sqrt{a}} \\ &= \frac{35\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{64\sqrt{2}\,d}-\frac{35a^2}{96d(a+a\sin(c+dx))^3}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.32, size = 179, normalized size = 1.20

$$\frac{\sqrt{a(1+\sin(c+dx))}\left((-420+420i)\sqrt[4]{-1}\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\sec\left(\frac{dx}{4}\right)\left(\cos\left(\frac{1}{4}(2c+dx)\right)+\sin\left(\frac{1}{4}(2c+dx)\right)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3+\frac{-102-70\cos(2(c+dx))+329\sin(c+dx)+105\sin(3(c+dx))}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^4}\right)}{768d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (Sqrt[a*(1 + Sin[c + d*x])]*((-420 + 420*I)*(-1)^(1/4)*ArcTanh[(1/2 + I/2)*
(-1)^(3/4)*Sec[(d*x)/4]*(Cos[(2*c + d*x)/4] + Sin[(2*c + d*x)/4])]*(Cos[(c
+ d*x)/2] + Sin[(c + d*x)/2])^3 + (-102 - 70*Cos[2*(c + d*x)] + 329*Sin[c +
 d*x] + 105*Sin[3*(c + d*x)])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^4))/(76
8*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4)
```

Maple [A]
time = 0.79, size = 118, normalized size = 0.79

method	result

default	$-\frac{2a^5\left(\frac{\frac{\sqrt{a+a\sin(dx+c)}\,a(11\sin(dx+c)-15)}{8(a\sin(dx+c)-a)^2}-\frac{35\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+a\sin(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{16\sqrt{a}}}{16a^4}+\frac{3}{16a^4\sqrt{a+a\sin}}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(a+a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*a^5*(1/16/a^4*(1/8*(a+a*sin(d*x+c))^(1/2)*a*(11*sin(d*x+c)-15)/(a*sin(d*
x+c)-a)^2-35/16*2^(1/2)/a^(1/2)*arctanh(1/2*(a+a*sin(d*x+c))^(1/2)*2^(1/2)/
a^(1/2)))+3/16/a^4/(a+a*sin(d*x+c))^(1/2)+1/24/a^3/(a+a*sin(d*x+c))^(3/2))/
d
```

Maxima [A]
time = 0.52, size = 168, normalized size = 1.13

$$-\frac{105\sqrt{2}\,a^{\frac{3}{2}}\log\left(-\frac{\sqrt{2}\,\sqrt{a}-\sqrt{a\sin(dx+c)+a}}{\sqrt{2}\,\sqrt{a}+\sqrt{a\sin(dx+c)+a}}\right)+\frac{4\left(105\,(a\sin(dx+c)+a)^3a^2-350\,(a\sin(dx+c)+a)^2a^3+224\,(a\sin(dx+c)+a)a^4+64\,a^5\right)}{(a\sin(dx+c)+a)^{\frac{7}{2}}-4\,(a\sin(dx+c)+a)^{\frac{5}{2}}a+4\,(a\sin(dx+c)+a)^{\frac{3}{2}}a^2}}{768\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/768*(105*sqrt(2)*a^(3/2)*log(-(sqrt(2)*sqrt(a) - sqrt(a*sin(d*x + c) + a
))/(sqrt(2)*sqrt(a) + sqrt(a*sin(d*x + c) + a))) + 4*(105*(a*sin(d*x + c) +
 a)^3*a^2 - 350*(a*sin(d*x + c) + a)^2*a^3 + 224*(a*sin(d*x + c) + a)*a^4 +
 64*a^5)/((a*sin(d*x + c) + a)^(7/2) - 4*(a*sin(d*x + c) + a)^(5/2)*a + 4*(
a*sin(d*x + c) + a)^(3/2)*a^2))/(a*d)
```

Fricas [A]
time = 0.37, size = 121, normalized size = 0.81

$$\frac{105\sqrt{2}\,\sqrt{a}\,\cos(dx+c)^4\log\left(-\frac{a\sin(dx+c)+2\sqrt{2}\,\sqrt{a\sin(dx+c)+a}\,\sqrt{a}+3a}{\sin(dx+c)-1}\right)-4\left(35\cos(dx+c)^2-7\left(15\cos(dx+c)^2+8\right)\sin(dx+c)+8\right)\sqrt{a\sin(dx+c)+a}}{768\,d\cos(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/768*(105*sqrt(2)*sqrt(a)*cos(d*x + c)^4*log(-(a*sin(d*x + c) + 2*sqrt(2)*
sqrt(a*sin(d*x + c) + a)*sqrt(a) + 3*a)/(sin(d*x + c) - 1)) - 4*(35*cos(d*x
 + c)^2 - 7*(15*cos(d*x + c)^2 + 8)*sin(d*x + c) + 8)*sqrt(a*sin(d*x + c) +
 a))/(d*cos(d*x + c)^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sin\left(c+dx\right)+1\right)}\,\sec^{5}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(c + d*x) + 1))*sec(c + d*x)**5, x)
```

Giac [A]
time = 6.90, size = 146, normalized size = 0.98

$$-\frac{\sqrt{2}\,\sqrt{a}\left(\frac{6\left(11\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{3}-13\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-1\right)^{2}}+\frac{16\left(9\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{3}}-105\log\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)+105\log\left(-\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)\right)\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{768\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/768*sqrt(2)*sqrt(a)*(6*(11*cos(-1/4*pi + 1/2*d*x + 1/2*c)^3 - 13*cos(-1/
4*pi + 1/2*d*x + 1/2*c))/(cos(-1/4*pi + 1/2*d*x + 1/2*c)^2 - 1)^2 + 16*(9*c
os(-1/4*pi + 1/2*d*x + 1/2*c)^2 + 1)/cos(-1/4*pi + 1/2*d*x + 1/2*c)^3 - 105
*log(cos(-1/4*pi + 1/2*d*x + 1/2*c) + 1) + 105*log(-cos(-1/4*pi + 1/2*d*x +
 1/2*c) + 1))*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+a\,\sin\left(c+d\,x\right)}}{{\cos\left(c+d\,x\right)}^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(1/2)/cos(c + d*x)^5,x)
```

```
[Out] int((a + a*sin(c + d*x))^(1/2)/cos(c + d*x)^5, x)
```

3.113 $\int \sec^6(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=197

$$-\frac{63\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{128\sqrt{2}\,d}-\frac{63a^2\cos(c+dx)}{128d(a+a\sin(c+dx))^{3/2}}-\frac{21a^2\sec(c+dx)}{80d(a+a\sin(c+dx))^{3/2}}+\frac{2}{32d\sqrt{}}$$

```
[Out] -63/128*a^2*cos(d*x+c)/d/(a+a*sin(d*x+c))^(3/2)-21/80*a^2*sec(d*x+c)/d/(a+a
*sin(d*x+c))^(3/2)-63/256*arctanh(1/2*cos(d*x+c)*a^(1/2)*2^(1/2)/(a+a*sin(d
*x+c))^(1/2))*a^(1/2)/d*2^(1/2)+21/32*a*sec(d*x+c)/d/(a+a*sin(d*x+c))^(1/2)
+3/10*a*sec(d*x+c)^3/d/(a+a*sin(d*x+c))^(1/2)+1/5*sec(d*x+c)^5*(a+a*sin(d*x
+c))^(1/2)/d
```

Rubi [A]
time = 0.21, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2754, 2766, 2760, 2729, 2728, 212}

$$-\frac{63a^2\cos(c+dx)}{128d(a\sin(c+dx)+a)^{3/2}}-\frac{21a^2\sec(c+dx)}{80d(a\sin(c+dx)+a)^{3/2}}+\frac{\sec^5(c+dx)\sqrt{a\sin(c+dx)+a}}{5d}+\frac{3a\sec^3(c+dx)}{10d\sqrt{a\sin(c+dx)+a}}+\frac{21a\sec(c+dx)}{32d\sqrt{a\sin(c+dx)+a}}-\frac{63\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{128\sqrt{2}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-63*Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x
]])])/(128*Sqrt[2]*d) - (63*a^2*Cos[c + d*x])/(128*d*(a + a*Sin[c + d*x])^(
3/2)) - (21*a^2*Sec[c + d*x])/(80*d*(a + a*Sin[c + d*x])^(3/2)) + (21*a*Sec
[c + d*x])/(32*d*Sqrt[a + a*Sin[c + d*x]]) + (3*a*Sec[c + d*x]^3)/(10*d*Sqr
t[a + a*Sin[c + d*x]]) + (Sec[c + d*x]^5*Sqrt[a + a*Sin[c + d*x]])/(5*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
```

```
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2754

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^m/(a*f*g*(p + 1))), x] + Dist[a*((m + p + 1)/(g^2*(p + 1))), Int[(g*C
os[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e
, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ[
m + 1/2, 2*p]
```

Rule 2760

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*(2*m + p + 1))), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2766

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(a*f*g*(p + 1)*S
qrt[a + b*Sin[e + f*x]])), x] + Dist[a*((2*p + 1)/(2*g^2*(p + 1))), Int[(g*
Cos[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e,
 f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\int \sec^6(c+dx)\sqrt{a+a\sin(c+dx)}\,dx = \frac{\sec^5(c+dx)\sqrt{a+a\sin(c+dx)}}{5d} + \frac{1}{10}(9a)\int \frac{\sec^4(c+dx)}{\sqrt{a+a\sin(c+dx)}}$$

$$= \frac{3a\sec^3(c+dx)}{10d\sqrt{a+a\sin(c+dx)}} + \frac{\sec^5(c+dx)\sqrt{a+a\sin(c+dx)}}{5d} + \frac{1}{20}($$

$$= -\frac{21a^2\sec(c+dx)}{80d(a+a\sin(c+dx))^{3/2}} + \frac{3a\sec^3(c+dx)}{10d\sqrt{a+a\sin(c+dx)}} + \frac{\sec^5(c+}{}$$

$$= -\frac{21a^2\sec(c+dx)}{80d(a+a\sin(c+dx))^{3/2}} + \frac{21a\sec(c+dx)}{32d\sqrt{a+a\sin(c+dx)}} + \frac{3a\,\mathrm{se}}{10d\sqrt{a+}}$$

$$= -\frac{63a^2\cos(c+dx)}{128d(a+a\sin(c+dx))^{3/2}} - \frac{21a^2\sec(c+dx)}{80d(a+a\sin(c+dx))^{3/2}} + \frac{21}{32d\sqrt{a}}$$

$$= -\frac{63a^2\cos(c+dx)}{128d(a+a\sin(c+dx))^{3/2}} - \frac{21a^2\sec(c+dx)}{80d(a+a\sin(c+dx))^{3/2}} + \frac{21}{32d\sqrt{a}}$$

$$= -\frac{63\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{128\sqrt{2}d} - \frac{63a^2\cos(c+}{128d(a+a\sin(c+}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.41, size = 191, normalized size = 0.97

$$\frac{\sqrt{a(1+\sin(c+dx))}\left((-2520-2520i)(-1)^{3/4}\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\sec\left(\frac{dx}{4}\right)\left(\cos\left(\frac{1}{4}(2c+dx)\right)-\sin\left(\frac{1}{4}(2c+dx)\right)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^4+\frac{649+1092\cos(2(c+dx))+315\cos(4(c+dx))+1572\sin(c+dx)+420\sin(3(c+dx))}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^5}\right)}{5120d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (Sqrt[a*(1 + Sin[c + d*x])]*((-2520 - 2520*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2
)*(-1)^(3/4)*Sec[(d*x)/4]*(Cos[(2*c + d*x)/4] - Sin[(2*c + d*x)/4])]*(Cos[(
c + d*x)/2] + Sin[(c + d*x)/2])^4 + (649 + 1092*Cos[2*(c + d*x)] + 315*Cos[
4*(c + d*x)] + 1572*Sin[c + d*x] + 420*Sin[3*(c + d*x)])/(Cos[(c + d*x)/2]
- Sin[(c + d*x)/2])^5))/(5120*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5)
```

Maple [A]
time = 0.52, size = 244, normalized size = 1.24

method	result

default	$-\frac{-420a^{\frac{9}{2}}\sin(dx+c)\left(\cos^2(dx+c)\right)+\left(630(a-a\sin(dx+c))^{\frac{5}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)a^2-288a^{\frac{9}{2}}\right)\sin(}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(a+a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/1280/a^(7/2)*(-420*a^(9/2)*sin(d*x+c)*cos(d*x+c)^2+(630*(a-a*sin(d*x+c))
^(5/2)*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^2-288*
a^(9/2))*sin(d*x+c)-630*a^(9/2)*cos(d*x+c)^4+(-315*(a-a*sin(d*x+c))^(5/2)*2
^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^2+84*a^(9/2))*
cos(d*x+c)^2+630*(a-a*sin(d*x+c))^(5/2)*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c)
)^(1/2)*2^(1/2)/a^(1/2))*a^2+32*a^(9/2))/(sin(d*x+c)-1)^2/(1+sin(d*x+c))/co
s(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*sec(d*x + c)^6, x)
```

Fricas [A]
time = 0.37, size = 210, normalized size = 1.07

$$\frac{315\sqrt{2}\sqrt{a}\cos(dx+c)^5\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{a\sin(dx+c)+a}\left(\sqrt{2}\cos(dx+c)-\sqrt{2}\sin(dx+c)+\sqrt{2}\right)\sqrt{a}+3a\cos(dx+c)-(a\cos(dx+c)-2a)\sin(dx+c)+2a}{\cos(dx+c)^2-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2}\right)+4\left(315\cos(dx+c)^4-42\cos(dx+c)^2+6\left(35\cos(dx+c)^2+24\right)\sin(dx+c)-16\right)\sqrt{a\sin(dx+c)+a}}{2560\,d\cos(dx+c)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2560*(315*sqrt(2)*sqrt(a)*cos(d*x + c)^5*log(-(a*cos(d*x + c)^2 - 2*sqrt(
a*sin(d*x + c) + a)*(sqrt(2)*cos(d*x + c) - sqrt(2)*sin(d*x + c) + sqrt(2))
*sqrt(a) + 3*a*cos(d*x + c) - (a*cos(d*x + c) - 2*a)*sin(d*x + c) + 2*a)/(c
os(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2)) + 4*(3
15*cos(d*x + c)^4 - 42*cos(d*x + c)^2 + 6*(35*cos(d*x + c)^2 + 24)*sin(d*x
+ c) - 16)*sqrt(a*sin(d*x + c) + a))/(d*cos(d*x + c)^5)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

Giac [A]
time = 7.26, size = 239, normalized size = 1.21

$$\frac{\sqrt{2}\left(315 \log\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right) + 1\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right)\right) - 315 \log\left(-\sin\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right) + 1\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right)\right) - \frac{10\left(15\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right)^3 - 17\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right)\right)}{\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right)^2 - 1\right)^2} - \frac{16\left(30\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right)^4 + 5\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right)^2 + \mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right)\right)\right)}{\sin\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right)^5}\right)\sqrt{a}}{2560\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2560*sqrt(2)*(315*log(sin(-1/4*pi + 1/2*d*x + 1/2*c) + 1)*sgn(cos(-1/4*pi
 + 1/2*d*x + 1/2*c)) - 315*log(-sin(-1/4*pi + 1/2*d*x + 1/2*c) + 1)*sgn(cos
(-1/4*pi + 1/2*d*x + 1/2*c)) - 10*(15*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*s
in(-1/4*pi + 1/2*d*x + 1/2*c)^3 - 17*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*si
n(-1/4*pi + 1/2*d*x + 1/2*c))/(sin(-1/4*pi + 1/2*d*x + 1/2*c)^2 - 1)^2 - 16
*(30*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*d*x + 1/2*c)^4 +
 5*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*d*x + 1/2*c)^2 + s
gn(cos(-1/4*pi + 1/2*d*x + 1/2*c)))/sin(-1/4*pi + 1/2*d*x + 1/2*c)^5)*sqrt(
a)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+a\,\sin\left(c+d\,x\right)}}{{\cos\left(c+d\,x\right)}^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(1/2)/cos(c + d*x)^6,x)
```

```
[Out] int((a + a*sin(c + d*x))^(1/2)/cos(c + d*x)^6, x)
```

3.114 $\int \cos^7(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=97

$$\frac{16(a+a\sin(c+dx))^{11/2}}{11a^4d}-\frac{24(a+a\sin(c+dx))^{13/2}}{13a^5d}+\frac{4(a+a\sin(c+dx))^{15/2}}{5a^6d}-\frac{2(a+a\sin(c+dx))^{17/2}}{17a^7d}$$

```
[Out] 16/11*(a+a*sin(d*x+c))^(11/2)/a^4/d-24/13*(a+a*sin(d*x+c))^(13/2)/a^5/d+4/5
*(a+a*sin(d*x+c))^(15/2)/a^6/d-2/17*(a+a*sin(d*x+c))^(17/2)/a^7/d
```

Rubi [A]
time = 0.06, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2746, 45}

$$-\frac{2(a\sin(c+dx)+a)^{17/2}}{17a^7d}+\frac{4(a\sin(c+dx)+a)^{15/2}}{5a^6d}-\frac{24(a\sin(c+dx)+a)^{13/2}}{13a^5d}+\frac{16(a\sin(c+dx)+a)^{11/2}}{11a^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (16*(a + a*Sin[c + d*x])^(11/2))/(11*a^4*d) - (24*(a + a*Sin[c + d*x])^(13/
2))/(13*a^5*d) + (4*(a + a*Sin[c + d*x])^(15/2))/(5*a^6*d) - (2*(a + a*Sin[
c + d*x])^(17/2))/(17*a^7*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\int \cos^7(c+dx)(a+a\sin(c+dx))^{3/2}\,dx = \frac{\mathrm{Subst}\left(\int (a-x)^3(a+x)^{9/2}\,dx, x, a\sin(c+dx)\right)}{a^7 d}$$
$$= \frac{\mathrm{Subst}\left(\int \left(8a^3(a+x)^{9/2} - 12a^2(a+x)^{11/2} + 6a(a+x)^{13/2} - (a+\right.\right.}{a^7 d}$$
$$= \frac{16(a+a\sin(c+dx))^{11/2}}{11a^4 d} - \frac{24(a+a\sin(c+dx))^{13/2}}{13a^5 d} + \frac{4(a+a\,\mathrm{si}}{}$$

Mathematica [A]
time = 0.24, size = 61, normalized size = 0.63

$$-\frac{2(1+\sin(c+dx))^4(a(1+\sin(c+dx)))^{3/2}\left(-1767+3641\sin(c+dx)-2717\sin^2(c+dx)+715\sin^3(c+dx)\right)}{12155d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*(1 + Sin[c + d*x])^4*(a*(1 + Sin[c + d*x]))^(3/2)*(-1767 + 3641*Sin[c +
 d*x] - 2717*Sin[c + d*x]^2 + 715*Sin[c + d*x]^3))/(12155*d)
```

Maple [A]
time = 0.26, size = 57, normalized size = 0.59

method	result	size
default	$\frac{2(a+a\sin(dx+c))^{\frac{11}{2}}\left(715\left(\cos^2(dx+c)\right)\sin(dx+c)-2717\left(\cos^2(dx+c)\right)-4356\sin(dx+c)+4484\right)}{12155a^4 d}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*(a+a*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/12155/a^4*(a+a*sin(d*x+c))^(11/2)*(715*cos(d*x+c)^2*sin(d*x+c)-2717*cos(d
*x+c)^2-4356*sin(d*x+c)+4484)/d
```

Maxima [A]
time = 0.29, size = 72, normalized size = 0.74

$$-\frac{2\left(715\,(a\sin(dx+c)+a)^{\frac{17}{2}} - 4862\,(a\sin(dx+c)+a)^{\frac{15}{2}}a + 11220\,(a\sin(dx+c)+a)^{\frac{13}{2}}a^2 - 8840\,(a\sin(dx+c)+a)^{\frac{11}{2}}a^3\right)}{12155\,a^7 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -2/12155*(715*(a*sin(d*x + c) + a)^(17/2) - 4862*(a*sin(d*x + c) + a)^(15/2
)*a + 11220*(a*sin(d*x + c) + a)^(13/2)*a^2 - 8840*(a*sin(d*x + c) + a)^(11
/2)*a^3)/(a^7*d)
```

Fricas [A]
time = 0.38, size = 110, normalized size = 1.13

$$-\frac{2\left(715\,a\cos\left(dx+c\right)^{8}-66\,a\cos\left(dx+c\right)^{6}-112\,a\cos\left(dx+c\right)^{4}-256\,a\cos\left(dx+c\right)^{2}-2\left(429\,a\cos\left(dx+c\right)^{6}+504\,a\cos\left(dx+c\right)^{4}+640\,a\cos\left(dx+c\right)^{2}+1024\,a\right)\sin\left(dx+c\right)-2048\,a\right)\sqrt{a\sin\left(dx+c\right)+a}}{12155\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/12155*(715*a*cos(d*x + c)^8 - 66*a*cos(d*x + c)^6 - 112*a*cos(d*x + c)^4
 - 256*a*cos(d*x + c)^2 - 2*(429*a*cos(d*x + c)^6 + 504*a*cos(d*x + c)^4 +
640*a*cos(d*x + c)^2 + 1024*a)*sin(d*x + c) - 2048*a)*sqrt(a*sin(d*x + c) +
 a)/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 7.12, size = 132, normalized size = 1.36

$$-\frac{512\,\sqrt{2}\left(715\,a\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{17}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)-2431\,a\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{15}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)+2805\,a\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)-1105\,a\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\right)\sqrt{a}}{12155\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -512/12155*sqrt(2)*(715*a*cos(-1/4*pi + 1/2*d*x + 1/2*c)^17*sgn(cos(-1/4*pi
 + 1/2*d*x + 1/2*c)) - 2431*a*cos(-1/4*pi + 1/2*d*x + 1/2*c)^15*sgn(cos(-1/
4*pi + 1/2*d*x + 1/2*c)) + 2805*a*cos(-1/4*pi + 1/2*d*x + 1/2*c)^13*sgn(cos
(-1/4*pi + 1/2*d*x + 1/2*c)) - 1105*a*cos(-1/4*pi + 1/2*d*x + 1/2*c)^11*sgn
(cos(-1/4*pi + 1/2*d*x + 1/2*c)))*sqrt(a)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^7\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7*(a + a*sin(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^7*(a + a*sin(c + d*x))^(3/2), x)
```

3.115 $\int \cos^6(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=159

$$-\frac{4096a^5\cos^7(c+dx)}{45045d(a+a\sin(c+dx))^{7/2}}-\frac{1024a^4\cos^7(c+dx)}{6435d(a+a\sin(c+dx))^{5/2}}-\frac{128a^3\cos^7(c+dx)}{715d(a+a\sin(c+dx))^{3/2}}-\frac{32a^2\cos^7(c+}{195d\sqrt{a+a\sin(}}$$

```
[Out] -4096/45045*a^5*cos(d*x+c)^7/d/(a+a*sin(d*x+c))^(7/2)-1024/6435*a^4*cos(d*x
+c)^7/d/(a+a*sin(d*x+c))^(5/2)-128/715*a^3*cos(d*x+c)^7/d/(a+a*sin(d*x+c))^
(3/2)-32/195*a^2*cos(d*x+c)^7/d/(a+a*sin(d*x+c))^(1/2)-2/15*a*cos(d*x+c)^7*
(a+a*sin(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.21, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2753, 2752}

$$-\frac{4096a^5\cos^7(c+dx)}{45045d(a\sin(c+dx)+a)^{7/2}}-\frac{1024a^4\cos^7(c+dx)}{6435d(a\sin(c+dx)+a)^{5/2}}-\frac{128a^3\cos^7(c+dx)}{715d(a\sin(c+dx)+a)^{3/2}}-\frac{32a^2\cos^7(c+dx)}{195d\sqrt{a\sin(c+dx)+a}}-\frac{2a\cos^7(c+dx)\sqrt{a\sin(c+dx)+a}}{15d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-4096*a^5*Cos[c + d*x]^7)/(45045*d*(a + a*Sin[c + d*x])^(7/2)) - (1024*a^4
*Cos[c + d*x]^7)/(6435*d*(a + a*Sin[c + d*x])^(5/2)) - (128*a^3*Cos[c + d*x
]^7)/(715*d*(a + a*Sin[c + d*x])^(3/2)) - (32*a^2*Cos[c + d*x]^7)/(195*d*Sq
rt[a + a*Sin[c + d*x]]) - (2*a*Cos[c + d*x]^7*Sqrt[a + a*Sin[c + d*x]])/(15
*d)
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2753

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && N
eQ[m + p, 0]
```

Rubi steps

$$\int \cos^6(c+dx)(a+a\sin(c+dx))^{3/2}\,dx = -\frac{2a\cos^7(c+dx)\sqrt{a+a\sin(c+dx)}}{15d} + \frac{1}{15}(16a)\int \cos^6(c+dx)$$
$$= -\frac{32a^2\cos^7(c+dx)}{195d\sqrt{a+a\sin(c+dx)}} - \frac{2a\cos^7(c+dx)\sqrt{a+a\sin(c+dx}}{15d}$$
$$= -\frac{128a^3\cos^7(c+dx)}{715d(a+a\sin(c+dx))^{3/2}} - \frac{32a^2\cos^7(c+dx)}{195d\sqrt{a+a\sin(c+dx)}} - \frac{2a\,\mathrm{c}}{}$$
$$= -\frac{1024a^4\cos^7(c+dx)}{6435d(a+a\sin(c+dx))^{5/2}} - \frac{128a^3\cos^7(c+dx)}{715d(a+a\sin(c+dx))^{3/2}} - \frac{}{19}$$
$$= -\frac{4096a^5\cos^7(c+dx)}{45045d(a+a\sin(c+dx))^{7/2}} - \frac{1024a^4\cos^7(c+dx)}{6435d(a+a\sin(c+dx))^{5/2}} -$$

Mathematica [A]
time = 0.39, size = 79, normalized size = 0.50

$$-\frac{2\cos^7(c+dx)(a(1+\sin(c+dx)))^{3/2}\left(16363+34748\sin(c+dx)+33138\sin^2(c+dx)+15708\sin^3(c+dx)+3003\sin^4(c+dx)\right)}{45045d(1+\sin(c+dx))^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*Cos[c + d*x]^7*(a*(1 + Sin[c + d*x]))^(3/2)*(16363 + 34748*Sin[c + d*x]
 + 33138*Sin[c + d*x]^2 + 15708*Sin[c + d*x]^3 + 3003*Sin[c + d*x]^4))/(450
45*d*(1 + Sin[c + d*x])^5)
```

Maple [A]
time = 0.49, size = 87, normalized size = 0.55

method	result	si
default	$-\frac{2(1+\sin(dx+c))a^2(\sin(dx+c)-1)^4\left(3003(\sin^4(dx+c))+15708(\sin^3(dx+c))+33138(\sin^2(dx+c))+34748\sin(dx+c)+16363\right)}{45045\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$	87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(a+a*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/45045*(1+sin(d*x+c))*a^2*(sin(d*x+c)-1)^4*(3003*sin(d*x+c)^4+15708*sin(d
*x+c)^3+33138*sin(d*x+c)^2+34748*sin(d*x+c)+16363)/cos(d*x+c)/(a+a*sin(d*x+
c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^6*(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")

[Out] integrate((a*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^6, x)

Fricas [A]
time = 0.35, size = 210, normalized size = 1.32

$$-\frac{2\,{\left(3003\,a\cos\left(dx+c\right)^{8}+6699\,a\cos\left(dx+c\right)^{7}-336\,a\cos\left(dx+c\right)^{6}+448\,a\cos\left(dx+c\right)^{5}-640\,a\cos\left(dx+c\right)^{4}+1024\,a\cos\left(dx+c\right)^{3}-2048\,a\cos\left(dx+c\right)^{2}+8192\,a\cos\left(dx+c\right)+{\left(3003\,a\cos\left(dx+c\right)^{7}-3696\,a\cos\left(dx+c\right)^{6}-4032\,a\cos\left(dx+c\right)^{5}-4480\,a\cos\left(dx+c\right)^{4}-5120\,a\cos\left(dx+c\right)^{3}-6144\,a\cos\left(dx+c\right)^{2}-8192\,a\cos\left(dx+c\right)-16384\,a\right)}\sin\left(dx+c\right)+16384\,a\right)}\sqrt{a\sin\left(dx+c\right)+a}}{45045\,{\left(d\cos\left(dx+c\right)+d\sin\left(dx+c\right)+d\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^6*(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")

[Out] -2/45045*(3003*a*cos(d*x + c)^8 + 6699*a*cos(d*x + c)^7 - 336*a*cos(d*x + c)^6 + 448*a*cos(d*x + c)^5 - 640*a*cos(d*x + c)^4 + 1024*a*cos(d*x + c)^3 - 2048*a*cos(d*x + c)^2 + 8192*a*cos(d*x + c) + (3003*a*cos(d*x + c)^7 - 3696*a*cos(d*x + c)^6 - 4032*a*cos(d*x + c)^5 - 4480*a*cos(d*x + c)^4 - 5120*a*cos(d*x + c)^3 - 6144*a*cos(d*x + c)^2 - 8192*a*cos(d*x + c) - 16384*a)*sin(d*x + c) + 16384*a)*sqrt(a*sin(d*x + c) + a)/(d*cos(d*x + c) + d*sin(d*x + c) + d)

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**6*(a+a*sin(d*x+c))**(3/2),x)

[Out] Timed out

Giac [A]
time = 5.59, size = 162, normalized size = 1.02

$$\frac{256\,\sqrt{2}\,{\left(3003\,a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{15}-13860\,a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}+24570\,a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-20020\,a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+6435\,a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}\right)}\sqrt{a}}{45045\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^6*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")

[Out] 256/45045*sqrt(2)*(3003*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*d*x + 1/2*c)^15 - 13860*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*d*x + 1/2*c)^13 + 24570*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*d*x + 1/2*c)^11 - 20020*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*d*x + 1/2*c)^9 + 6435*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*d*x + 1/2*c)^7)*sqrt(a)/d

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^6\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6*(a + a*sin(c + d*x))^(3/2),x)
[Out] int(cos(c + d*x)^6*(a + a*sin(c + d*x))^(3/2), x)
```

3.116 $\int \cos^5(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=73

$$\frac{8(a+a\sin(c+dx))^{9/2}}{9a^3d}-\frac{8(a+a\sin(c+dx))^{11/2}}{11a^4d}+\frac{2(a+a\sin(c+dx))^{13/2}}{13a^5d}$$

```
[Out] 8/9*(a+a*sin(d*x+c))^(9/2)/a^3/d-8/11*(a+a*sin(d*x+c))^(11/2)/a^4/d+2/13*(a
+a*sin(d*x+c))^(13/2)/a^5/d
```

Rubi [A]

time = 0.05, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2746, 45}

$$\frac{2(a\sin(c+dx)+a)^{13/2}}{13a^5d}-\frac{8(a\sin(c+dx)+a)^{11/2}}{11a^4d}+\frac{8(a\sin(c+dx)+a)^{9/2}}{9a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (8*(a + a*Sin[c + d*x])^(9/2))/(9*a^3*d) - (8*(a + a*Sin[c + d*x])^(11/2))/
(11*a^4*d) + (2*(a + a*Sin[c + d*x])^(13/2))/(13*a^5*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \cos^5(c+dx)(a+a\sin(c+dx))^{3/2}\,dx &= \frac{\text{Subst}\left(\int (a-x)^2(a+x)^{7/2}\,dx,x,a\sin(c+dx)\right)}{a^5d}\\ &= \frac{\text{Subst}\left(\int \left(4a^2(a+x)^{7/2}-4a(a+x)^{9/2}+(a+x)^{11/2}\right)\,dx,x,a\sin(\right.}{a^5d}\\ &= \frac{8(a+a\sin(c+dx))^{9/2}}{9a^3d}-\frac{8(a+a\sin(c+dx))^{11/2}}{11a^4d}+\frac{2(a+a\sin(c}{13a}\end{aligned}$$

Mathematica [A]
time = 0.11, size = 51, normalized size = 0.70

$$\frac{2(1+\sin(c+dx))^3(a(1+\sin(c+dx)))^{3/2}\left(203-270\sin(c+dx)+99\sin^2(c+dx)\right)}{1287d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + a*Sin[c + d*x])^(3/2),x]
[Out] (2*(1 + Sin[c + d*x])^3*(a*(1 + Sin[c + d*x]))^(3/2)*(203 - 270*Sin[c + d*x
] + 99*Sin[c + d*x]^2))/(1287*d)
```

Maple [A]
time = 0.27, size = 41, normalized size = 0.56

method	result	size
default	$-\frac{2(a+a\sin(dx+c))^{\frac{9}{2}}\left(99\left(\cos^2(dx+c)\right)+270\sin(dx+c)-302\right)}{1287a^3d}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+a*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
[Out] -2/1287/a^3*(a+a*sin(d*x+c))^(9/2)*(99*cos(d*x+c)^2+270*sin(d*x+c)-302)/d
```

Maxima [A]
time = 0.30, size = 55, normalized size = 0.75

$$\frac{2\left(99\left(a\sin\left(dx+c\right)+a\right)^{\frac{13}{2}}-468\left(a\sin\left(dx+c\right)+a\right)^{\frac{11}{2}}a+572\left(a\sin\left(dx+c\right)+a\right)^{\frac{9}{2}}a^2\right)}{1287\,a^5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
[Out] 2/1287*(99*(a*sin(d*x + c) + a)^(13/2) - 468*(a*sin(d*x + c) + a)^(11/2)*a
+ 572*(a*sin(d*x + c) + a)^(9/2)*a^2)/(a^5*d)
```

Fricas [A]
time = 0.35, size = 88, normalized size = 1.21

$$-\frac{2\left(99\,a\cos\left(dx+c\right)^6-14\,a\cos\left(dx+c\right)^4-32\,a\cos\left(dx+c\right)^2-2\left(63\,a\cos\left(dx+c\right)^4+80\,a\cos\left(dx+c\right)^2+128\,a\right)\sin\left(dx+c\right)-256\,a\right)\sqrt{a\sin\left(dx+c\right)+a}}{1287\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
[Out] -2/1287*(99*a*cos(d*x + c)^6 - 14*a*cos(d*x + c)^4 - 32*a*cos(d*x + c)^2 -
2*(63*a*cos(d*x + c)^4 + 80*a*cos(d*x + c)^2 + 128*a)*sin(d*x + c) - 256*a)
*sqrt(a*sin(d*x + c) + a)/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 5.02, size = 102, normalized size = 1.40

$$\frac{128\sqrt{2}\left(99\,a\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)-234\,a\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)+143\,a\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\right)\sqrt{a}}{1287\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 128/1287*sqrt(2)*(99*a*cos(-1/4*pi + 1/2*d*x + 1/2*c)^13*sgn(cos(-1/4*pi +
1/2*d*x + 1/2*c)) - 234*a*cos(-1/4*pi + 1/2*d*x + 1/2*c)^11*sgn(cos(-1/4*pi
 + 1/2*d*x + 1/2*c)) + 143*a*cos(-1/4*pi + 1/2*d*x + 1/2*c)^9*sgn(cos(-1/4*
pi + 1/2*d*x + 1/2*c)))*sqrt(a)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^5\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(a + a*sin(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^5*(a + a*sin(c + d*x))^(3/2), x)
```

3.117 $\int \cos^4(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=127

$$-\frac{256a^4\cos^5(c+dx)}{1155d(a+a\sin(c+dx))^{5/2}}-\frac{64a^3\cos^5(c+dx)}{231d(a+a\sin(c+dx))^{3/2}}-\frac{8a^2\cos^5(c+dx)}{33d\sqrt{a+a\sin(c+dx)}}-\frac{2a\cos^5(c+dx)\sqrt{a+a\sin(c+dx)}}{11d}$$

```
[Out] -256/1155*a^4*cos(d*x+c)^5/d/(a+a*sin(d*x+c))^(5/2)-64/231*a^3*cos(d*x+c)^5
/d/(a+a*sin(d*x+c))^(3/2)-8/33*a^2*cos(d*x+c)^5/d/(a+a*sin(d*x+c))^(1/2)-2/
11*a*cos(d*x+c)^5*(a+a*sin(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.16, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2753, 2752}

$$-\frac{256a^4\cos^5(c+dx)}{1155d(a\sin(c+dx)+a)^{5/2}}-\frac{64a^3\cos^5(c+dx)}{231d(a\sin(c+dx)+a)^{3/2}}-\frac{8a^2\cos^5(c+dx)}{33d\sqrt{a\sin(c+dx)+a}}-\frac{2a\cos^5(c+dx)\sqrt{a\sin(c+dx)+a}}{11d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-256*a^4*Cos[c + d*x]^5)/(1155*d*(a + a*Sin[c + d*x])^(5/2)) - (64*a^3*Cos
[c + d*x]^5)/(231*d*(a + a*Sin[c + d*x])^(3/2)) - (8*a^2*Cos[c + d*x]^5)/(3
3*d*Sqrt[a + a*Sin[c + d*x]]) - (2*a*Cos[c + d*x]^5*Sqrt[a + a*Sin[c + d*x]
])/(11*d)
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2753

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && N
eQ[m + p, 0]
```

Rubi steps

$$\int \cos^4(c+dx)(a+a\sin(c+dx))^{3/2}\,dx = -\frac{2a\cos^5(c+dx)\sqrt{a+a\sin(c+dx)}}{11d} + \frac{1}{11}(12a)\int \cos^4(c+dx)\sqrt{}$$
$$= -\frac{8a^2\cos^5(c+dx)}{33d\sqrt{a+a\sin(c+dx)}} - \frac{2a\cos^5(c+dx)\sqrt{a+a\sin(c+dx)}}{11d}$$
$$= -\frac{64a^3\cos^5(c+dx)}{231d(a+a\sin(c+dx))^{3/2}} - \frac{8a^2\cos^5(c+dx)}{33d\sqrt{a+a\sin(c+dx)}} - \frac{2a\cos^5}{}$$
$$= -\frac{256a^4\cos^5(c+dx)}{1155d(a+a\sin(c+dx))^{5/2}} - \frac{64a^3\cos^5(c+dx)}{231d(a+a\sin(c+dx))^{3/2}} - \frac{}{33d}$$

Mathematica [A]
time = 0.10, size = 69, normalized size = 0.54

$$-\frac{2\cos^5(c+dx)(a(1+\sin(c+dx)))^{3/2}\left(533+755\sin(c+dx)+455\sin^2(c+dx)+105\sin^3(c+dx)\right)}{1155d(1+\sin(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*Cos[c + d*x]^5*(a*(1 + Sin[c + d*x]))^(3/2)*(533 + 755*Sin[c + d*x] + 4
55*Sin[c + d*x]^2 + 105*Sin[c + d*x]^3))/(1155*d*(1 + Sin[c + d*x])^4)
```

Maple [A]
time = 0.38, size = 77, normalized size = 0.61

method	result	size
default	$\frac{2(1+\sin(dx+c))a^2(\sin(dx+c)-1)^3\left(105\left(\sin^3(dx+c)\right)+455\left(\sin^2(dx+c)\right)+755\sin(dx+c)+533\right)}{1155\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+a*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/1155*(1+sin(d*x+c))*a^2*(sin(d*x+c)-1)^3*(105*sin(d*x+c)^3+455*sin(d*x+c)
^2+755*sin(d*x+c)+533)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^4, x)
```

Fricas [A]
time = 0.36, size = 166, normalized size = 1.31

$$-\frac{2\,{\left(105\,a\cos\left(dx+c\right)^{6}+245\,a\cos\left(dx+c\right)^{5}-20\,a\cos\left(dx+c\right)^{4}+32\,a\cos\left(dx+c\right)^{3}-64\,a\cos\left(dx+c\right)^{2}+256\,a\cos\left(dx+c\right)+{\left(105\,a\cos\left(dx+c\right)^{5}-140\,a\cos\left(dx+c\right)^{4}-160\,a\cos\left(dx+c\right)^{3}-192\,a\cos\left(dx+c\right)^{2}-256\,a\cos\left(dx+c\right)-512\,a\right)}\sin\left(dx+c\right)+512\,a\right)}\sqrt{a\sin\left(dx+c\right)+a}}{1155\,{\left(d\cos\left(dx+c\right)+d\sin\left(dx+c\right)+d\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/1155*(105*a*cos(d*x + c)^6 + 245*a*cos(d*x + c)^5 - 20*a*cos(d*x + c)^4
+ 32*a*cos(d*x + c)^3 - 64*a*cos(d*x + c)^2 + 256*a*cos(d*x + c) + (105*a*c
os(d*x + c)^5 - 140*a*cos(d*x + c)^4 - 160*a*cos(d*x + c)^3 - 192*a*cos(d*x
 + c)^2 - 256*a*cos(d*x + c) - 512*a)*sin(d*x + c) + 512*a)*sqrt(a*sin(d*x
+ c) + a)/(d*cos(d*x + c) + d*sin(d*x + c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sin\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}\cos^{4}\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral((a*(sin(c + d*x) + 1))**(3/2)*cos(c + d*x)**4, x)
```

Giac [A]
time = 5.42, size = 132, normalized size = 1.04

$$-\frac{64\,\sqrt{2}\left(105\,a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-385\,a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+495\,a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-231\,a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}\right)\sqrt{a}}{1155\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -64/1155*sqrt(2)*(105*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1
/2*d*x + 1/2*c)^11 - 385*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi
+ 1/2*d*x + 1/2*c)^9 + 495*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*p
i + 1/2*d*x + 1/2*c)^7 - 231*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4
*pi + 1/2*d*x + 1/2*c)^5)*sqrt(a)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\cos\left(c+d\,x\right)}^4\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(a + a*sin(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^4*(a + a*sin(c + d*x))^(3/2), x)
```

3.118 $\int \cos^3(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=49

$$\frac{4(a+a\sin(c+dx))^{7/2}}{7a^2d}-\frac{2(a+a\sin(c+dx))^{9/2}}{9a^3d}$$

```
[Out] 4/7*(a+a*sin(d*x+c))^(7/2)/a^2/d-2/9*(a+a*sin(d*x+c))^(9/2)/a^3/d
```

Rubi [A]
time = 0.05, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.087$, Rules used = {2746, 45}

$$\frac{4(a\sin(c+dx)+a)^{7/2}}{7a^2d}-\frac{2(a\sin(c+dx)+a)^{9/2}}{9a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (4*(a + a*Sin[c + d*x])^(7/2))/(7*a^2*d) - (2*(a + a*Sin[c + d*x])^(9/2))/(
9*a^3*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \cos^3(c+dx)(a+a\sin(c+dx))^{3/2}\,dx&=\frac{\text{Subst}\left(\int (a-x)(a+x)^{5/2}\,dx,x,a\sin(c+dx)\right)}{a^3d}\\&=\frac{\text{Subst}\left(\int \left(2a(a+x)^{5/2}-(a+x)^{7/2}\right)\,dx,x,a\sin(c+dx)\right)}{a^3d}\\&=\frac{4(a+a\sin(c+dx))^{7/2}}{7a^2d}-\frac{2(a+a\sin(c+dx))^{9/2}}{9a^3d}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 41, normalized size = 0.84

$$-\frac{2(1+\sin(c+dx))^2(a(1+\sin(c+dx)))^{3/2}(-11+7\sin(c+dx))}{63d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Sin[c + d*x])^(3/2),x]
[Out] (-2*(1 + Sin[c + d*x])^2*(a*(1 + Sin[c + d*x]))^(3/2)*(-11 + 7*Sin[c + d*x]
))/(63*d)
```

Maple [A]
time = 0.29, size = 31, normalized size = 0.63

method	result	size
default	$-\frac{2(a+a\sin(dx+c))^{\frac{7}{2}}(7\sin(dx+c)-11)}{63a^2d}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
[Out] -2/63/a^2*(a+a*sin(d*x+c))^(7/2)*(7*sin(d*x+c)-11)/d
```

Maxima [A]
time = 0.33, size = 38, normalized size = 0.78

$$-\frac{2\left(7\left(a\sin\left(dx+c\right)+a\right)^{\frac{9}{2}}-18\left(a\sin\left(dx+c\right)+a\right)^{\frac{7}{2}}a\right)}{63\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
[Out] -2/63*(7*(a*sin(d*x + c) + a)^(9/2) - 18*(a*sin(d*x + c) + a)^(7/2)*a)/(a^3
*d)
```

Fricas [A]
time = 0.36, size = 66, normalized size = 1.35

$$-\frac{2\left(7\,a\cos\left(dx+c\right)^4-2\,a\cos\left(dx+c\right)^2-2\left(5\,a\cos\left(dx+c\right)^2+8\,a\right)\sin\left(dx+c\right)-16\,a\right)\sqrt{a\sin\left(dx+c\right)+a}}{63\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
[Out] -2/63*(7*a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^2 - 2*(5*a*cos(d*x + c)^2 + 8*
a)*sin(d*x + c) - 16*a)*sqrt(a*sin(d*x + c) + a)/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(42) = 84$.
time = 5.57, size = 252, normalized size = 5.14

$$\begin{cases} \frac{8a\sqrt{a\sin(c+dx)+a}\sin^4(c+dx)}{45d} + \frac{152a\sqrt{a\sin(c+dx)+a}\sin^3(c+dx)}{315d} + \frac{2a\sqrt{a\sin(c+dx)+a}\sin^2(c+dx)\cos^2(c+dx)}{5d} + \frac{8a\sqrt{a\sin(c+dx)+a}\sin^2(c+dx)}{21d} + \frac{4a\sqrt{a\sin(c+dx)+a}\sin(c+dx)\cos^2(c+dx)}{5d} + \frac{8a\sqrt{a\sin(c+dx)+a}\sin(c+dx)}{315d} + \frac{2a\sqrt{a\sin(c+dx)+a}\cos^2(c+dx)}{5d} - \frac{16a\sqrt{a\sin(c+dx)+a}}{315d} & \text{for } d \neq 0 \\ x(a\sin(c)+a)^{\frac{3}{2}}\cos^3(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Piecewise((8*a*sqrt(a*sin(c + d*x) + a)*sin(c + d*x)**4/(45*d) + 152*a*sqrt
(a*sin(c + d*x) + a)*sin(c + d*x)**3/(315*d) + 2*a*sqrt(a*sin(c + d*x) + a)
*sin(c + d*x)**2*cos(c + d*x)**2/(5*d) + 8*a*sqrt(a*sin(c + d*x) + a)*sin(c
 + d*x)**2/(21*d) + 4*a*sqrt(a*sin(c + d*x) + a)*sin(c + d*x)*cos(c + d*x)*
*2/(5*d) + 8*a*sqrt(a*sin(c + d*x) + a)*sin(c + d*x)/(315*d) + 2*a*sqrt(a*s
in(c + d*x) + a)*cos(c + d*x)**2/(5*d) - 16*a*sqrt(a*sin(c + d*x) + a)/(315
*d), Ne(d, 0)), (x*(a*sin(c) + a)**(3/2)*cos(c)**3, True))
```

Giac [A]
time = 6.00, size = 72, normalized size = 1.47

$$-\frac{32\sqrt{2}\left(7\,a\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)-9\,a\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\right)\sqrt{a}}{63\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -32/63*sqrt(2)*(7*a*cos(-1/4*pi + 1/2*d*x + 1/2*c)^9*sgn(cos(-1/4*pi + 1/2*
d*x + 1/2*c)) - 9*a*cos(-1/4*pi + 1/2*d*x + 1/2*c)^7*sgn(cos(-1/4*pi + 1/2*
d*x + 1/2*c)))*sqrt(a)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \cos\left(c+d\,x\right)^3\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a + a*sin(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^3*(a + a*sin(c + d*x))^(3/2), x)
```

3.119 $\int \cos^2(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=95

$$-\frac{64a^3\cos^3(c+dx)}{105d(a+a\sin(c+dx))^{3/2}}-\frac{16a^2\cos^3(c+dx)}{35d\sqrt{a+a\sin(c+dx)}}-\frac{2a\cos^3(c+dx)\sqrt{a+a\sin(c+dx)}}{7d}$$

```
[Out] -64/105*a^3*cos(d*x+c)^3/d/(a+a*sin(d*x+c))^(3/2)-16/35*a^2*cos(d*x+c)^3/d/
(a+a*sin(d*x+c))^(1/2)-2/7*a*cos(d*x+c)^3*(a+a*sin(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.11, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2753, 2752}

$$-\frac{64a^3\cos^3(c+dx)}{105d(a\sin(c+dx)+a)^{3/2}}-\frac{16a^2\cos^3(c+dx)}{35d\sqrt{a\sin(c+dx)+a}}-\frac{2a\cos^3(c+dx)\sqrt{a\sin(c+dx)+a}}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-64*a^3*Cos[c + d*x]^3)/(105*d*(a + a*Sin[c + d*x])^(3/2)) - (16*a^2*Cos[c
 + d*x]^3)/(35*d*Sqrt[a + a*Sin[c + d*x]]) - (2*a*Cos[c + d*x]^3*Sqrt[a + a
*Sin[c + d*x]])/(7*d)
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2753

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && N
eQ[m + p, 0]
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\sin(c+dx))^{3/2}\,dx = -\frac{2a\cos^3(c+dx)\sqrt{a+a\sin(c+dx)}}{7d} + \frac{1}{7}(8a)\int \cos^2(c+dx)\sqrt{a$$

$$= -\frac{16a^2\cos^3(c+dx)}{35d\sqrt{a+a\sin(c+dx)}} - \frac{2a\cos^3(c+dx)\sqrt{a+a\sin(c+dx)}}{7d}$$

$$= -\frac{64a^3\cos^3(c+dx)}{105d(a+a\sin(c+dx))^{3/2}} - \frac{16a^2\cos^3(c+dx)}{35d\sqrt{a+a\sin(c+dx)}} - \frac{2a\cos^3}{}$$

Mathematica [A]
time = 0.10, size = 59, normalized size = 0.62

$$-\frac{2\cos^3(c+dx)(a(1+\sin(c+dx)))^{3/2}\,(71+54\sin(c+dx)+15\sin^2(c+dx))}{105d(1+\sin(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*Cos[c + d*x]^3*(a*(1 + Sin[c + d*x]))^(3/2)*(71 + 54*Sin[c + d*x] + 15*
Sin[c + d*x]^2))/(105*d*(1 + Sin[c + d*x])^3)
```

Maple [A]
time = 0.37, size = 67, normalized size = 0.71

method	result	size
default	$-\frac{2(1+\sin(dx+c))a^2(\sin(dx+c)-1)^2(15(\sin^2(dx+c))+54\sin(dx+c)+71)}{105\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/105*(1+sin(d*x+c))*a^2*(sin(d*x+c)-1)^2*(15*sin(d*x+c)^2+54*sin(d*x+c)+7
1)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^2, x)
```

Fricas [A]
time = 0.37, size = 122, normalized size = 1.28

$$-\frac{2\,\left(15\,a\cos\left(dx+c\right)^{4}+39\,a\cos\left(dx+c\right)^{3}-8\,a\cos\left(dx+c\right)^{2}+32\,a\cos\left(dx+c\right)+\left(15\,a\cos\left(dx+c\right)^{3}-24\,a\cos\left(dx+c\right)^{2}-32\,a\cos\left(dx+c\right)-64\,a\right)\sin\left(dx+c\right)+64\,a\right)\sqrt{a\sin\left(dx+c\right)+a}}{105\,\left(d\cos\left(dx+c\right)+d\sin\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/105*(15*a*cos(d*x + c)^4 + 39*a*cos(d*x + c)^3 - 8*a*cos(d*x + c)^2 + 32
*a*cos(d*x + c) + (15*a*cos(d*x + c)^3 - 24*a*cos(d*x + c)^2 - 32*a*cos(d*x
 + c) - 64*a)*sin(d*x + c) + 64*a)*sqrt(a*sin(d*x + c) + a)/(d*cos(d*x + c)
 + d*sin(d*x + c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sin\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}\cos^{2}\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral((a*(sin(c + d*x) + 1))**(3/2)*cos(c + d*x)**2, x)
```

Giac [A]
time = 6.08, size = 102, normalized size = 1.07

$$\frac{16\,\sqrt{2}\,\left(15\,a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-42\,a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+35\,a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}\right)\sqrt{a}}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 16/105*sqrt(2)*(15*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*
d*x + 1/2*c)^7 - 42*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2
*d*x + 1/2*c)^5 + 35*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/
2*d*x + 1/2*c)^3)*sqrt(a)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^2\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a + a*sin(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^2*(a + a*sin(c + d*x))^(3/2), x)
```

3.120 $\int \cos(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=24

$$\frac{2(a+a\sin(c+dx))^{5/2}}{5ad}$$

```
[Out] 2/5*(a+a*sin(d*x+c))^(5/2)/a/d
```

Rubi [A]
time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2746, 32}

$$\frac{2(a\sin(c+dx)+a)^{5/2}}{5ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (2*(a + a*Sin[c + d*x])^(5/2))/(5*a*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a+a\sin(c+dx))^{3/2}\,dx &= \frac{\text{Subst}\left(\int (a+x)^{3/2}\,dx, x, a\sin(c+dx)\right)}{ad}\\ &= \frac{2(a+a\sin(c+dx))^{5/2}}{5ad}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 24, normalized size = 1.00

$$\frac{2(a+a\sin(c+dx))^{5/2}}{5ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (2*(a + a*Sin[c + d*x])^(5/2))/(5*a*d)
```

Maple [A]
time = 0.04, size = 21, normalized size = 0.88

method	result	size
derivativedivides	$\frac{2(a+a\sin(dx+c))^{\frac{5}{2}}}{5da}$	21
default	$\frac{2(a+a\sin(dx+c))^{\frac{5}{2}}}{5da}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*(a+a*sin(d*x+c))^(5/2)/d/a
```

Maxima [A]
time = 0.29, size = 20, normalized size = 0.83

$$\frac{2\,(a\sin\left(dx+c\right)+a)^{\frac{5}{2}}}{5\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/5*(a*sin(d*x + c) + a)^(5/2)/(a*d)
```

Fricas [A]
time = 0.34, size = 40, normalized size = 1.67

$$-\frac{2\,\left(a\cos\left(dx+c\right)^{2}-2\,a\sin\left(dx+c\right)-2\,a\right)\sqrt{a\sin\left(dx+c\right)+a}}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/5*(a*cos(d*x + c)^2 - 2*a*sin(d*x + c) - 2*a)*sqrt(a*sin(d*x + c) + a)/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(19) = 38$.
time = 1.64, size = 90, normalized size = 3.75

$$\begin{cases} \frac{2a\sqrt{a\sin\left(c+dx\right)+a}\,\sin^{2}\left(c+dx\right)}{5d}+\frac{4a\sqrt{a\sin\left(c+dx\right)+a}\,\sin\left(c+dx\right)}{5d}+\frac{2a\sqrt{a\sin\left(c+dx\right)+a}}{5d} & \text{for}\ d\neq 0 \\ x(a\sin\left(c\right)+a)^{\frac{3}{2}}\cos\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Piecewise((2*a*sqrt(a*sin(c + d*x) + a)*sin(c + d*x)**2/(5*d) + 4*a*sqrt(a*
sin(c + d*x) + a)*sin(c + d*x)/(5*d) + 2*a*sqrt(a*sin(c + d*x) + a)/(5*d),
Ne(d, 0)), (x*(a*sin(c) + a)**(3/2)*cos(c), True))
```

Giac [A]
time = 5.84, size = 38, normalized size = 1.58

$$\frac{8\sqrt{2}\,a^{\frac{3}{2}}\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 8/5*sqrt(2)*a^(3/2)*cos(-1/4*pi + 1/2*d*x + 1/2*c)^5*sgn(cos(-1/4*pi + 1/2*
d*x + 1/2*c))/d
```

Mupad [B]
time = 4.62, size = 20, normalized size = 0.83

$$\frac{2\,{\left(a\,\left(\sin\left(c+d\,x\right)+1\right)\right)}^{5/2}}{5\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + a*sin(c + d*x))^(3/2),x)
```

```
[Out] (2*(a*(sin(c + d*x) + 1))^(5/2))/(5*a*d)
```

3.121 $\int \sec(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=62

$$\frac{2\sqrt{2}\,a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{2}\,\sqrt{a}}\right)}{d}-\frac{2a\sqrt{a+a\sin(c+dx)}}{d}$$

```
[Out] 2*a^(3/2)*arctanh(1/2*(a+a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)/d-2*a
*(a+a*sin(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.04, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2746, 52, 65, 212}

$$\frac{2\sqrt{2}\,a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\,\sqrt{a}}\right)}{d}-\frac{2a\sqrt{a\sin(c+dx)+a}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (2*Sqrt[2]*a^(3/2)*ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])])/d -
 (2*a*Sqrt[a + a*Sin[c + d*x]])/d
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\int \sec(c+dx)(a+a\sin(c+dx))^{3/2}\,dx = \frac{a\text{Subst}\left(\int \frac{\sqrt{a+x}}{a-x}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{2a\sqrt{a+a\sin(c+dx)}}{d} + \frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{(a-x)\sqrt{a+x}}\,dx, x, a\sin\right.}{d}$$
$$= -\frac{2a\sqrt{a+a\sin(c+dx)}}{d} + \frac{(4a^2)\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \sqrt{a+a\sin(\right.}{d}$$
$$= \frac{2\sqrt{2}\,a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d} - \frac{2a\sqrt{a+a\sin(c+dx)}}{d}$$

Mathematica [A]
time = 0.07, size = 60, normalized size = 0.97

$$\frac{2a\left(\sqrt{2}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{2}\sqrt{a}}\right) - \sqrt{a+a\sin(c+dx)}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sin[c + d*x])^(3/2),x]
[Out] (2*a*(Sqrt[2]*Sqrt[a]*ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])] -
 Sqrt[a + a*Sin[c + d*x]]))/d
```

Maple [A]
time = 0.20, size = 49, normalized size = 0.79

method	result	size

default	$-\frac{2a\left(\sqrt{a+a\sin\left(dx+c\right)}-\sqrt{a}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+a\sin\left(dx+c\right)}\,\sqrt{2}}{2\sqrt{a}}\right)\right)}{d}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+a*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*a*((a+a*sin(d*x+c))^(1/2)-a^(1/2)*2^(1/2)*arctanh(1/2*(a+a*sin(d*x+c))^(
1/2)*2^(1/2)/a^(1/2)))/d
```

Maxima [A]
time = 0.56, size = 80, normalized size = 1.29

$$-\frac{\sqrt{2}\,a^{\frac{5}{2}}\log\left(-\frac{\sqrt{2}\,\sqrt{a}-\sqrt{a\sin\left(dx+c\right)+a}}{\sqrt{2}\,\sqrt{a}+\sqrt{a\sin\left(dx+c\right)+a}}\right)+2\,\sqrt{a\sin\left(dx+c\right)+a}\,a^2}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -(sqrt(2)*a^(5/2)*log(-(sqrt(2)*sqrt(a) - sqrt(a*sin(d*x + c) + a))/(sqrt(2
)*sqrt(a) + sqrt(a*sin(d*x + c) + a))) + 2*sqrt(a*sin(d*x + c) + a)*a^2)/(a
*d)
```

Fricas [A]
time = 0.36, size = 72, normalized size = 1.16

$$\frac{\sqrt{2}\,a^{\frac{3}{2}}\log\left(-\frac{a\sin(dx+c)+2\,\sqrt{2}\,\sqrt{a\sin\left(dx+c\right)+a}\,\sqrt{a}+3\,a}{\sin(dx+c)-1}\right)-2\,\sqrt{a\sin\left(dx+c\right)+a}\,a}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] (sqrt(2)*a^(3/2)*log(-(a*sin(d*x + c) + 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*
sqrt(a) + 3*a)/(sin(d*x + c) - 1)) - 2*sqrt(a*sin(d*x + c) + a)*a)/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\left(\sin\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}\sec\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))**(3/2),x)
```

[Out] Integral((a*(sin(c + d*x) + 1))**(3/2)*sec(c + d*x), x)

Giac [A]
time = 6.77, size = 73, normalized size = 1.18

$$-\frac{\sqrt{2}\,a^{\frac{3}{2}}\left(2\,\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\log\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)+\log\left(-\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")

[Out] -sqrt(2)*a^(3/2)*(2*cos(-1/4*pi + 1/2*d*x + 1/2*c) - log(cos(-1/4*pi + 1/2*d*x + 1/2*c) + 1) + log(-cos(-1/4*pi + 1/2*d*x + 1/2*c) + 1))*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))/d

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{3/2}}{\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a*sin(c + d*x))^(3/2)/cos(c + d*x),x)

[Out] int((a + a*sin(c + d*x))^(3/2)/cos(c + d*x), x)

3.122 $\int \sec^2(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=26

$$\frac{2a\sec(c+dx)\sqrt{a+a\sin(c+dx)}}{d}$$

```
[Out] 2*a*sec(d*x+c)*(a+a*sin(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.04, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = $\{2752\}$

$$\frac{2a\sec(c+dx)\sqrt{a\sin(c+dx)+a}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (2*a*Sec[c + d*x]*Sqrt[a + a*Sin[c + d*x]])/d
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\int \sec^2(c+dx)(a+a\sin(c+dx))^{3/2}\,dx = \frac{2a\sec(c+dx)\sqrt{a+a\sin(c+dx)}}{d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 67 vs. 2(26) = 52.
time = 0.10, size = 67, normalized size = 2.58

$$\frac{2(a(1+\sin(c+dx)))^{3/2}}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (2*(a*(1 + Sin[c + d*x]))^(3/2))/(d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(
Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3)
```

Maple [A]
time = 0.21, size = 37, normalized size = 1.42

method	result	size
default	$\frac{2a^2(1+\sin(dx+c))}{\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*a^2*(1+sin(d*x+c))/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 98 vs. 2(24) = 48.
time = 0.62, size = 98, normalized size = 3.77

$$-\frac{2\left(a^{\frac{3}{2}}+\frac{2\,a^{\frac{3}{2}}\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^{\frac{3}{2}}\sin(dx+c)^4}{(\cos(dx+c)+1)^4}\right)}{d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -2*(a^(3/2) + 2*a^(3/2)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^(3/2)*sin(d
*x + c)^4/(cos(d*x + c) + 1)^4)/(d*(sin(d*x + c)/(cos(d*x + c) + 1) - 1)*(s
in(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^(3/2))
```

Fricas [A]
time = 0.34, size = 26, normalized size = 1.00

$$\frac{2\sqrt{a\sin(dx+c)+a}\,a}{d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(a*sin(d*x + c) + a)*a/(d*cos(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

Giac [A]
time = 6.29, size = 38, normalized size = 1.46

$$-\frac{\sqrt{2}\, a^{\frac{3}{2}} \mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi + \frac{1}{2}\,dx + \frac{1}{2}\,c\right)\right)}{d\sin\left(-\frac{1}{4}\,\pi + \frac{1}{2}\,dx + \frac{1}{2}\,c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -sqrt(2)*a^(3/2)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))/(d*sin(-1/4*pi + 1/2*d
*x + 1/2*c))
```

Mupad [B]
time = 4.77, size = 37, normalized size = 1.42

$$\frac{4\,a\,\cos\left(c+d\,x\right)\,\sqrt{a\,\left(\sin\left(c+d\,x\right)+1\right)}}{d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(3/2)/cos(c + d*x)^2,x)
```

```
[Out] (4*a*cos(c + d*x)*(a*(sin(c + d*x) + 1))^(1/2))/(d*(cos(2*c + 2*d*x) + 1))
```

3.123 $\int \sec^3(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=73

$$\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}\,d}+\frac{\sec^2(c+dx)(a+a\sin(c+dx))^{3/2}}{2d}$$

```
[Out] 1/2*sec(d*x+c)^2*(a+a*sin(d*x+c))^(3/2)/d+1/4*a^(3/2)*arctanh(1/2*(a+a*sin(
d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)/d
```

Rubi [A]
time = 0.08, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2754, 2746, 65, 212}

$$\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}\,d}+\frac{\sec^2(c+dx)(a\sin(c+dx)+a)^{3/2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (a^(3/2)*ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(2*Sqrt[2]*d)
 + (Sec[c + d*x]^2*(a + a*Sin[c + d*x])^(3/2))/(2*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
```

```
])
```

Rule 2754

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^m/(a*f*g*(p + 1))), x] + Dist[a*((m + p + 1)/(g^2*(p + 1))), Int[(g*C
os[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e
, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ[
m + 1/2, 2*p]
```

Rubi steps

$$\int \sec^3(c+dx)(a+a\sin(c+dx))^{3/2}\,dx = \frac{\sec^2(c+dx)(a+a\sin(c+dx))^{3/2}}{2d}+\frac{1}{4}a\int \sec(c+dx)\sqrt{a+a\sin$$

$$=\frac{\sec^2(c+dx)(a+a\sin(c+dx))^{3/2}}{2d}+\frac{a^2\text{Subst}\left(\int \frac{1}{(a-x)\sqrt{a+x}}\,dx\right.}{4d}$$

$$=\frac{\sec^2(c+dx)(a+a\sin(c+dx))^{3/2}}{2d}+\frac{a^2\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx,x,\sqrt{a}\right.}{2d}$$

$$=\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}\,d}+\frac{\sec^2(c+dx)(a+a\sin(c}{2d}$$

Mathematica [A]
time = 0.16, size = 72, normalized size = 0.99

$$\frac{a\left(\sqrt{2}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a(1+\sin(c+dx))}}{\sqrt{2}\sqrt{a}}\right)-\frac{2\sqrt{a(1+\sin(c+dx))}}{-1+\sin(c+dx)}\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sin[c + d*x])^(3/2),x]
[Out] (a*(Sqrt[2]*Sqrt[a]*ArcTanh[Sqrt[a*(1 + Sin[c + d*x])]/(Sqrt[2]*Sqrt[a])] -
 (2*Sqrt[a*(1 + Sin[c + d*x])])/(-1 + Sin[c + d*x])))/(4*d)
```

Maple [A]
time = 0.40, size = 70, normalized size = 0.96

method	result	size

default	$\frac{2a^3\left(-\frac{\sqrt{a+a\sin(dx+c)}}{4a(a\sin(dx+c)-a)}+\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+a\sin(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{8a^{\frac{3}{2}}}\right)}{d}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+a*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*a^3*(-1/4*(a+a*sin(d*x+c))^(1/2)/a/(a*sin(d*x+c)-a)+1/8/a^(3/2)*2^(1/2)*a
rctanh(1/2*(a+a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2)))/d
```

Maxima [A]
time = 0.52, size = 94, normalized size = 1.29

$$-\frac{\sqrt{2}\,a^{\frac{5}{2}}\log\left(-\frac{\sqrt{2}\,\sqrt{a}-\sqrt{a\sin(dx+c)+a}}{\sqrt{2}\,\sqrt{a}+\sqrt{a\sin(dx+c)+a}}\right)+\frac{4\sqrt{a\sin(dx+c)+a}\,a^3}{a\sin(dx+c)-a}}{8\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/8*(sqrt(2)*a^(5/2)*log(-(sqrt(2)*sqrt(a) - sqrt(a*sin(d*x + c) + a))/(sq
rt(2)*sqrt(a) + sqrt(a*sin(d*x + c) + a))) + 4*sqrt(a*sin(d*x + c) + a)*a^3
/(a*sin(d*x + c) - a))/(a*d)
```

Fricas [A]
time = 0.36, size = 99, normalized size = 1.36

$$\frac{\left(\sqrt{2}\,a\sin(dx+c)-\sqrt{2}\,a\right)\sqrt{a}\,\log\left(-\frac{a\sin(dx+c)+2\sqrt{2}\,\sqrt{a\sin(dx+c)+a}\,\sqrt{a}+3a}{\sin(dx+c)-1}\right)-4\sqrt{a\sin(dx+c)+a}\,a}{8\,(d\sin(dx+c)-d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/8*((sqrt(2)*a*sin(d*x + c) - sqrt(2)*a)*sqrt(a)*log(-(a*sin(d*x + c) + 2*
sqrt(2)*sqrt(a*sin(d*x + c) + a)*sqrt(a) + 3*a)/(sin(d*x + c) - 1)) - 4*sqr
t(a*sin(d*x + c) + a)*a)/(d*sin(d*x + c) - d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5006 deep
```

Giac [A]
time = 6.85, size = 91, normalized size = 1.25

$$-\frac{\sqrt{2}\,a^{\frac{3}{2}}\left(\frac{2\,\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1}-\log\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)+\log\left(-\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(2)*a^(3/2)*(2*cos(-1/4*pi + 1/2*d*x + 1/2*c)/(cos(-1/4*pi + 1/2*d
*x + 1/2*c)^2 - 1) - log(cos(-1/4*pi + 1/2*d*x + 1/2*c) + 1) + log(-cos(-1/
4*pi + 1/2*d*x + 1/2*c) + 1))*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{3/2}}{{\cos\left(c+d\,x\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(3/2)/cos(c + d*x)^3,x)
```

```
[Out] int((a + a*sin(c + d*x))^(3/2)/cos(c + d*x)^3, x)
```

3.124 $\int \sec^4(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=107

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{2\sqrt{2}\,d}+\frac{a\sec(c+dx)\sqrt{a+a\sin(c+dx)}}{2d}+\frac{\sec^3(c+dx)(a+a\sin(c+dx)}{3d}$$

```
[Out] 1/3*sec(d*x+c)^3*(a+a*sin(d*x+c))^(3/2)/d-1/4*a^(3/2)*arctanh(1/2*cos(d*x+c
)*a^(1/2)*2^(1/2)/(a+a*sin(d*x+c))^(1/2))/d*2^(1/2)+1/2*a*sec(d*x+c)*(a+a*s
in(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.09, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {2754, 2728, 212}

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{2\sqrt{2}\,d}+\frac{\sec^3(c+dx)(a\sin(c+dx)+a)^{3/2}}{3d}+\frac{a\sec(c+dx)\sqrt{a\sin(c+dx)+a}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -1/2*(a^(3/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*
x]])])/(Sqrt[2]*d) + (a*Sec[c + d*x]*Sqrt[a + a*Sin[c + d*x]])/(2*d) + (Sec
[c + d*x]^3*(a + a*Sin[c + d*x])^(3/2))/(3*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^m/(a*f*g*(p + 1))), x] + Dist[a*((m + p + 1)/(g^2*(p + 1))), Int[(g*C
os[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e
, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ[
```

```
m + 1/2, 2*p]
```

Rubi steps

$$\int \sec^4(c+dx)(a+a\sin(c+dx))^{3/2}\,dx = \frac{\sec^3(c+dx)(a+a\sin(c+dx))^{3/2}}{3d} + \frac{1}{2}a\int \sec^2(c+dx)\sqrt{a+a\,\mathrm{s}}$$

$$= \frac{a\sec(c+dx)\sqrt{a+a\sin(c+dx)}}{2d} + \frac{\sec^3(c+dx)(a+a\sin(c+a}{3d}$$

$$= \frac{a\sec(c+dx)\sqrt{a+a\sin(c+dx)}}{2d} + \frac{\sec^3(c+dx)(a+a\sin(c+a}{3d}$$

$$= -\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{2\sqrt{2}\,d} + \frac{a\sec(c+dx)\sqrt{a+}}{2d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.26, size = 130, normalized size = 1.21

$$\frac{\left(\frac{1}{12}+\frac{i}{12}\right)a\sec^3(c+dx)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2\sqrt{a(1+\sin(c+dx))}\left(6(-1)^{3/4}\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(c+dx)\right)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3-(1-i)(-5+3\sin(c+dx))\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((1/12 + I/12)*a*Sec[c + d*x]^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2*Sqr
t[a*(1 + Sin[c + d*x])]*(6*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 +
Tan[(c + d*x)/4])]*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3 - (1 - I)*(-5 +
3*Sin[c + d*x])))/d
```

Maple [A]
time = 0.46, size = 107, normalized size = 1.00

method	result	size
default	$\frac{(1+\sin(dx+c))\left(6a^{\frac{7}{2}}\sin(dx+c)-10a^{\frac{7}{2}}+3\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)a^2(a-a\sin(dx+c))^{\frac{3}{2}}\right)}{12a^{\frac{3}{2}}(\sin(dx+c)-1)\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$	107

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+a*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12/a^(3/2)*(1+sin(d*x+c))/(sin(d*x+c)-1)*(6*a^(7/2)*sin(d*x+c)-10*a^(7/2)
+3*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^2*(a-a*sin
(d*x+c))^(3/2))/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 215 vs. 2(88) = 176.
time = 0.39, size = 215, normalized size = 2.01

$$\frac{3\left(\sqrt{2}\,a\cos\left(dx+c\right)\sin\left(dx+c\right)-\sqrt{2}\,a\cos\left(dx+c\right)\right)\sqrt{a}\,\log\left(-\frac{a\cos(dx+c)^2-2\,\sqrt{a\sin\left(dx+c\right)+a}\,\left(\sqrt{2}\,\cos(dx+c)-\sqrt{2}\,\sin(dx+c)+\sqrt{2}\right)\sqrt{a}+3\,a\cos(dx+c)-(a\cos(dx+c)-2\,a)\sin(dx+c)+2\,a}{\cos(dx+c)^2-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2}\right)+4\left(3\,a\sin\left(dx+c\right)-5\,a\right)\sqrt{a\sin\left(dx+c\right)+a}}{24\left(d\cos\left(dx+c\right)\sin\left(dx+c\right)-d\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/24*(3*(sqrt(2)*a*cos(d*x + c)*sin(d*x + c) - sqrt(2)*a*cos(d*x + c))*sqrt
(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(a*sin(d*x + c) + a)*(sqrt(2)*cos(d*x +
c) - sqrt(2)*sin(d*x + c) + sqrt(2))*sqrt(a) + 3*a*cos(d*x + c) - (a*cos(d*
x + c) - 2*a)*sin(d*x + c) + 2*a)/(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(
d*x + c) - cos(d*x + c) - 2)) + 4*(3*a*sin(d*x + c) - 5*a)*sqrt(a*sin(d*x +
 c) + a))/(d*cos(d*x + c)*sin(d*x + c) - d*cos(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8009 deep
```

Giac [A]
time = 6.10, size = 95, normalized size = 0.89

$$-\frac{\sqrt{2}\,a^{\frac{3}{2}}\left(\frac{2\left(3\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)}{\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}-3\,\log\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)+3\,\log\left(-\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -1/24*sqrt(2)*a^(3/2)*(2*(3*sin(-1/4*pi + 1/2*d*x + 1/2*c)^2 + 1)/sin(-1/4*
pi + 1/2*d*x + 1/2*c)^3 - 3*log(sin(-1/4*pi + 1/2*d*x + 1/2*c) + 1) + 3*log
(-sin(-1/4*pi + 1/2*d*x + 1/2*c) + 1))*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))/
d
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\,\sin(c+d\,x))^{3/2}}{\cos(c+d\,x)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(3/2)/cos(c + d*x)^4,x)
```

```
[Out] int((a + a*sin(c + d*x))^(3/2)/cos(c + d*x)^4, x)
```

3.125 $\int \sec^5(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=127

$$\frac{15a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{32\sqrt{2}\,d}-\frac{15a^2}{32d\sqrt{a+a\sin(c+dx)}}+\frac{5a\sec^2(c+dx)\sqrt{a+a\sin(c+dx)}}{16d}+\sec^4$$

```
[Out] 1/4*sec(d*x+c)^4*(a+a*sin(d*x+c))^(3/2)/d+15/64*a^(3/2)*arctanh(1/2*(a+a*si
n(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)/d-15/32*a^2/d/(a+a*sin(d*x+c))^(1/
2)+5/16*a*sec(d*x+c)^2*(a+a*sin(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.13, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2754, 2746, 53, 65, 212}

$$\frac{15a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{32\sqrt{2}\,d}-\frac{15a^2}{32d\sqrt{a\sin(c+dx)+a}}+\frac{\sec^4(c+dx)(a\sin(c+dx)+a)^{3/2}}{4d}+\frac{5a\sec^2(c+dx)\sqrt{a\sin(c+dx)+a}}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (15*a^(3/2)*ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(32*Sqrt[2
]*d) - (15*a^2)/(32*d*Sqrt[a + a*Sin[c + d*x]]) + (5*a*Sec[c + d*x]^2*Sqrt[
a + a*Sin[c + d*x]])/(16*d) + (Sec[c + d*x]^4*(a + a*Sin[c + d*x])^(3/2))/(
4*d)
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2754

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^m/(a*f*g*(p + 1))), x] + Dist[a*((m + p + 1)/(g^2*(p + 1))), Int[(g*C
os[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e
, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ[
m + 1/2, 2*p]
```

Rubi steps

$$\int \sec^5(c+dx)(a+a\sin(c+dx))^{3/2}\,dx = \frac{\sec^4(c+dx)(a+a\sin(c+dx))^{3/2}}{4d}+\frac{1}{8}(5a)\int \sec^3(c+dx)\sqrt{a+}$$

$$= \frac{5a\sec^2(c+dx)\sqrt{a+a\sin(c+dx)}}{16d}+\frac{\sec^4(c+dx)(a+a\sin(c+}{4d}$$

$$= \frac{5a\sec^2(c+dx)\sqrt{a+a\sin(c+dx)}}{16d}+\frac{\sec^4(c+dx)(a+a\sin(c+}{4d}$$

$$= -\frac{15a^2}{32d\sqrt{a+a\sin(c+dx)}}+\frac{5a\sec^2(c+dx)\sqrt{a+a\sin(c+dx)}}{16d}$$

$$= -\frac{15a^2}{32d\sqrt{a+a\sin(c+dx)}}+\frac{5a\sec^2(c+dx)\sqrt{a+a\sin(c+dx)}}{16d}$$

$$= \frac{15a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{32\sqrt{2}\,d}-\frac{15a^2}{32d\sqrt{a+a\sin(c+dx)}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.05, size = 44, normalized size = 0.35

$$-\frac{a^2\,{}_2F_1\left(-\frac{1}{2},3;\frac{1}{2};\frac{1}{2}(1+\sin(c+dx))\right)}{4d\sqrt{a+a\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -1/4*(a^2*Hypergeometric2F1[-1/2, 3, 1/2, (1 + Sin[c + d*x])/2])/(d*Sqrt[a
+ a*Sin[c + d*x]])
```

Maple [A]
time = 0.82, size = 101, normalized size = 0.80

method	result
default	$-\frac{2a^5\left(\frac{\frac{\sqrt{a+a\sin(dx+c)}\,a(7\sin(dx+c)-11)}{8(a\sin(dx+c)-a)^2}-\frac{15\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+a\sin(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{16\sqrt{a}}}{8a^3}+\frac{1}{8a^3\sqrt{a+a\sin(\cdots}}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(a+a*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*a^5*(1/8/a^3*(1/8*(a+a*sin(d*x+c))^(1/2)*a*(7*sin(d*x+c)-11)/(a*sin(d*x+
c)-a)^2-15/16*2^(1/2)/a^(1/2)*arctanh(1/2*(a+a*sin(d*x+c))^(1/2)*2^(1/2)/a^
(1/2)))+1/8/a^3/(a+a*sin(d*x+c))^(1/2))/d
```

Maxima [A]
time = 0.56, size = 151, normalized size = 1.19

$$-\frac{15\sqrt{2}\,a^{\frac{5}{2}}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{a\sin(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{a\sin(dx+c)+a}}\right)+\frac{4\left(15\,(a\sin(dx+c)+a)^2a^3-50\,(a\sin(dx+c)+a)a^4+32\,a^5\right)}{(a\sin(dx+c)+a)^{\frac{5}{2}}-4\,(a\sin(dx+c)+a)^{\frac{3}{2}}a+4\sqrt{a\sin(dx+c)+a}\,a^2}}{128\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/128*(15*sqrt(2)*a^(5/2)*log(-(sqrt(2)*sqrt(a) - sqrt(a*sin(d*x + c) + a)
)/(sqrt(2)*sqrt(a) + sqrt(a*sin(d*x + c) + a))) + 4*(15*(a*sin(d*x + c) + a
)^2*a^3 - 50*(a*sin(d*x + c) + a)*a^4 + 32*a^5)/((a*sin(d*x + c) + a)^(5/2)
 - 4*(a*sin(d*x + c) + a)^(3/2)*a + 4*sqrt(a*sin(d*x + c) + a)*a^2))/(a*d)
```

Fricas [A]
time = 0.37, size = 155, normalized size = 1.22

$$\frac{15\left(\sqrt{2}\,a\cos\left(dx+c\right)^{2}\sin\left(dx+c\right)-\sqrt{2}\,a\cos\left(dx+c\right)^{2}\right)\sqrt{a}\,\log\left(-\frac{a\sin(dx+c)+2\sqrt{2}\,\sqrt{a\sin\left(dx+c\right)+a}\,\sqrt{a}+3a}{\sin(dx+c)-1}\right)-4\left(15\,a\cos\left(dx+c\right)^{2}+20\,a\sin\left(dx+c\right)-12\,a\right)\sqrt{a\sin\left(dx+c\right)+a}}{128\left(d\cos\left(dx+c\right)^{2}\sin\left(dx+c\right)-d\cos\left(dx+c\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/128*(15*(sqrt(2)*a*cos(d*x + c)^2*sin(d*x + c) - sqrt(2)*a*cos(d*x + c)^2
)*sqrt(a)*log(-(a*sin(d*x + c) + 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*sqrt(a)
 + 3*a)/(sin(d*x + c) - 1)) - 4*(15*a*cos(d*x + c)^2 + 20*a*sin(d*x + c) -
12*a)*sqrt(a*sin(d*x + c) + a))/(d*cos(d*x + c)^2*sin(d*x + c) - d*cos(d*x
+ c)^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 4.44, size = 128, normalized size = 1.01

$$-\frac{\sqrt{2}\,a^{\frac{3}{2}}\left(\frac{2\left(7\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{3}-9\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-1\right)^{2}}+\frac{16}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)}-15\log\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)+15\log\left(-\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{128\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -1/128*sqrt(2)*a^(3/2)*(2*(7*cos(-1/4*pi + 1/2*d*x + 1/2*c)^3 - 9*cos(-1/4*
pi + 1/2*d*x + 1/2*c))/(cos(-1/4*pi + 1/2*d*x + 1/2*c)^2 - 1)^2 + 16/cos(-1
/4*pi + 1/2*d*x + 1/2*c) - 15*log(cos(-1/4*pi + 1/2*d*x + 1/2*c) + 1) + 15*
log(-cos(-1/4*pi + 1/2*d*x + 1/2*c) + 1))*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c
))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{3/2}}{{\cos\left(c+d\,x\right)}^5} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(3/2)/cos(c + d*x)^5,x)
```

```
[Out] int((a + a*sin(c + d*x))^(3/2)/cos(c + d*x)^5, x)
```

3.126 $\int \sec^6(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=169

$$-\frac{7a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(c+dx)}{\sqrt{2}\,\sqrt{a+a\sin(c+dx)}}\right)}{16\sqrt{2}\,d}-\frac{7a^3\cos(c+dx)}{16d(a+a\sin(c+dx))^{3/2}}+\frac{7a^2\sec(c+dx)}{12d\sqrt{a+a\sin(c+dx)}}+\frac{7a\sec^3(c$$

```
[Out] -7/16*a^3*cos(d*x+c)/d/(a+a*sin(d*x+c))^(3/2)+1/5*sec(d*x+c)^5*(a+a*sin(d*x
+c))^(3/2)/d-7/32*a^(3/2)*arctanh(1/2*cos(d*x+c)*a^(1/2)*2^(1/2)/(a+a*sin(d
*x+c))^(1/2))/d*2^(1/2)+7/12*a^2*sec(d*x+c)/d/(a+a*sin(d*x+c))^(1/2)+7/30*a
*sec(d*x+c)^3*(a+a*sin(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.16, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2754, 2766, 2729, 2728, 212}

$$-\frac{7a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(c+dx)}{\sqrt{2}\,\sqrt{a\sin(c+dx)+a}}\right)}{16\sqrt{2}\,d}-\frac{7a^3\cos(c+dx)}{16d(a\sin(c+dx)+a)^{3/2}}+\frac{7a^2\sec(c+dx)}{12d\sqrt{a\sin(c+dx)+a}}+\frac{\sec^5(c+dx)(a\sin(c+dx)+a)^{3/2}}{5d}+\frac{7a\sec^3(c+dx)\sqrt{a\sin(c+dx)+a}}{30d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-7*a^(3/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]
])])/(16*Sqrt[2]*d) - (7*a^3*Cos[c + d*x])/(16*d*(a + a*Sin[c + d*x])^(3/2)
) + (7*a^2*Sec[c + d*x])/(12*d*Sqrt[a + a*Sin[c + d*x]]) + (7*a*Sec[c + d*x
]^3*Sqrt[a + a*Sin[c + d*x]])/(30*d) + (Sec[c + d*x]^5*(a + a*Sin[c + d*x])
^(3/2))/(5*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
```

```
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2754

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^m/(a*f*g*(p + 1))), x] + Dist[a*((m + p + 1)/(g^2*(p + 1))), Int[(g*C
os[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e
, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ[
m + 1/2, 2*p]
```

Rule 2766

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(a*f*g*(p + 1)*S
qrt[a + b*Sin[e + f*x]])), x] + Dist[a*((2*p + 1)/(2*g^2*(p + 1))), Int[(g*
Cos[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e,
 f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\int \sec^6(c+dx)(a+a\sin(c+dx))^{3/2}\,dx = \frac{\sec^5(c+dx)(a+a\sin(c+dx))^{3/2}}{5d} + \frac{1}{10}(7a)\int \sec^4(c+dx)\sqrt{a\cdots}$$

$$= \frac{7a\sec^3(c+dx)\sqrt{a+a\sin(c+dx)}}{30d} + \frac{\sec^5(c+dx)(a+a\sin(c+\cdots}{5d}$$

$$= \frac{7a^2\sec(c+dx)}{12d\sqrt{a+a\sin(c+dx)}} + \frac{7a\sec^3(c+dx)\sqrt{a+a\sin(c+dx)}}{30d} \cdots$$

$$= -\frac{7a^3\cos(c+dx)}{16d(a+a\sin(c+dx))^{3/2}} + \frac{7a^2\sec(c+dx)}{12d\sqrt{a+a\sin(c+dx)}} + \frac{7a\sec^3\cdots}{}$$

$$= -\frac{7a^3\cos(c+dx)}{16d(a+a\sin(c+dx))^{3/2}} + \frac{7a^2\sec(c+dx)}{12d\sqrt{a+a\sin(c+dx)}} + \frac{7a\sec^3\cdots}{}$$

$$= -\frac{7a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{16\sqrt{2}\,d} - \frac{7a^3\cos(c+d\cdots}{16d(a+a\sin(c+\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.27, size = 288, normalized size = 1.70

$$\frac{\left(30\sin\left(\frac{1}{2}(c+dx)\right)-15\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+(105+105i)(-1)^{3/4}\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(c+dx)\right)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2+\frac{24\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^5}+\frac{40\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3}+\frac{90\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)}\right)(a(1+\sin(c+dx)))^{3/2}}{240d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((30*Sin[(c + d*x)/2] - 15*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) + (105 + 1
05*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/4])]*(C
os[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + (24*(Cos[(c + d*x)/2] + Sin[(c + d*
x)/2])^2)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^5 + (40*(Cos[(c + d*x)/2] +
 Sin[(c + d*x)/2])^2)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3 + (90*(Cos[(c
 + d*x)/2] + Sin[(c + d*x)/2])^2)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]))*(a
*(1 + Sin[c + d*x]))^(3/2))/(240*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5)
```

Maple [A]
time = 0.35, size = 172, normalized size = 1.02

method	result
default	$-\frac{210a^{\frac{7}{2}}\sin(dx+c)\left(\cos^2(dx+c)\right)+\left(105(a-a\sin(dx+c))^{\frac{5}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)a-168a^{\frac{7}{2}}\right)\sin(dx+c}{480a^{\frac{3}{2}}(\sin(dx+c)-1)^2\cos(dx+c)\sqrt{a+}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(a+a*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/480/a^(3/2)*(210*a^(7/2)*sin(d*x+c)*cos(d*x+c)^2+(105*(a-a*sin(d*x+c))^(
5/2)*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a-168*a^(7
/2))*sin(d*x+c)-350*a^(7/2)*cos(d*x+c)^2+105*(a-a*sin(d*x+c))^(5/2)*2^(1/2)
*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a+72*a^(7/2))/(sin(d*x
+c)-1)^2/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A]
time = 0.36, size = 248, normalized size = 1.47

$$\frac{105\left(\sqrt{2}\,a\cos(dx+c)^3\sin(dx+c)-\sqrt{2}\,a\cos(dx+c)^3\right)\sqrt{a}\,\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{a\sin(dx+c)+a}\left(\sqrt{2}\cos(dx+c)-\sqrt{2}\sin(dx+c)+\sqrt{2}\right)\sqrt{a}+3a\cos(dx+c)-(a\cos(dx+c)-2a)\sin(dx+c)+2a}{\cos(dx+c)^2-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2}\right)-4\left(175a\cos(dx+c)^2-21\left(5a\cos(dx+c)^2-4a\right)\sin(dx+c)-36a\right)\sqrt{a\sin(dx+c)+a}}{960\left(d\cos(dx+c)^3\sin(dx+c)-d\cos(dx+c)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/960*(105*(sqrt(2)*a*cos(d*x + c)^3*sin(d*x + c) - sqrt(2)*a*cos(d*x + c)^
3)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(a*sin(d*x + c) + a)*(sqrt(2)*cos
(d*x + c) - sqrt(2)*sin(d*x + c) + sqrt(2))*sqrt(a) + 3*a*cos(d*x + c) - (a
*cos(d*x + c) - 2*a)*sin(d*x + c) + 2*a)/(cos(d*x + c)^2 - (cos(d*x + c) +
2)*sin(d*x + c) - cos(d*x + c) - 2)) - 4*(175*a*cos(d*x + c)^2 - 21*(5*a*co
s(d*x + c)^2 - 4*a)*sin(d*x + c) - 36*a)*sqrt(a*sin(d*x + c) + a))/(d*cos(d
*x + c)^3*sin(d*x + c) - d*cos(d*x + c)^3)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 5.59, size = 143, normalized size = 0.85

$$-\frac{\sqrt{2}\, a^{\frac{3}{2}} \left(\frac{30 \sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1}+\frac{4\left(45 \sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+10 \sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+3\right)}{\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}}-105\,\log\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)+105\,\log\left(-\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -1/960*sqrt(2)*a^(3/2)*(30*sin(-1/4*pi + 1/2*d*x + 1/2*c)/(sin(-1/4*pi + 1/
2*d*x + 1/2*c)^2 - 1) + 4*(45*sin(-1/4*pi + 1/2*d*x + 1/2*c)^4 + 10*sin(-1/
4*pi + 1/2*d*x + 1/2*c)^2 + 3)/sin(-1/4*pi + 1/2*d*x + 1/2*c)^5 - 105*log(s
in(-1/4*pi + 1/2*d*x + 1/2*c) + 1) + 105*log(-sin(-1/4*pi + 1/2*d*x + 1/2*c
) + 1))*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))/d
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{3/2}}{{\cos\left(c+d\,x\right)}^6} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(3/2)/cos(c + d*x)^6,x)
```

```
[Out] int((a + a*sin(c + d*x))^(3/2)/cos(c + d*x)^6, x)
```

3.127 $\int \cos^5(c+dx)(a+a\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=73

$$\frac{8(a+a\sin(c+dx))^{11/2}}{11a^3d}-\frac{8(a+a\sin(c+dx))^{13/2}}{13a^4d}+\frac{2(a+a\sin(c+dx))^{15/2}}{15a^5d}$$

```
[Out] 8/11*(a+a*sin(d*x+c))^(11/2)/a^3/d-8/13*(a+a*sin(d*x+c))^(13/2)/a^4/d+2/15*
(a+a*sin(d*x+c))^(15/2)/a^5/d
```

Rubi [A]

time = 0.05, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2746, 45}

$$\frac{2(a\sin(c+dx)+a)^{15/2}}{15a^5d}-\frac{8(a\sin(c+dx)+a)^{13/2}}{13a^4d}+\frac{8(a\sin(c+dx)+a)^{11/2}}{11a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (8*(a + a*Sin[c + d*x])^(11/2))/(11*a^3*d) - (8*(a + a*Sin[c + d*x])^(13/2)
)/(13*a^4*d) + (2*(a + a*Sin[c + d*x])^(15/2))/(15*a^5*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\int \cos^5(c+dx)(a+a\sin(c+dx))^{5/2}\,dx = \frac{\text{Subst}\left(\int (a-x)^2(a+x)^{9/2}\,dx, x, a\sin(c+dx)\right)}{a^5d}$$

$$= \frac{\text{Subst}\left(\int \left(4a^2(a+x)^{9/2}-4a(a+x)^{11/2}+(a+x)^{13/2}\right)\,dx, x, a\sin\right.}{a^5d}$$

$$= \frac{8(a+a\sin(c+dx))^{11/2}}{11a^3d}-\frac{8(a+a\sin(c+dx))^{13/2}}{13a^4d}+\frac{2(a+a\sin(}{15a}$$

Mathematica [A]
time = 0.12, size = 51, normalized size = 0.70

$$\frac{2(1+\sin(c+dx))^3(a(1+\sin(c+dx)))^{5/2}\left(263-374\sin(c+dx)+143\sin^2(c+dx)\right)}{2145d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (2*(1 + Sin[c + d*x])^3*(a*(1 + Sin[c + d*x]))^(5/2)*(263 - 374*Sin[c + d*x
] + 143*Sin[c + d*x]^2))/(2145*d)
```

Maple [A]
time = 0.29, size = 41, normalized size = 0.56

method	result	size
default	$-\frac{2(a+a\sin(dx+c))^{\frac{11}{2}}\left(143\left(\cos^2(dx+c)\right)+374\sin(dx+c)-406\right)}{2145a^3d}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+a*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/2145/a^3*(a+a*sin(d*x+c))^(11/2)*(143*cos(d*x+c)^2+374*sin(d*x+c)-406)/d
```

Maxima [A]
time = 0.30, size = 55, normalized size = 0.75

$$\frac{2\left(143\left(a\sin\left(dx+c\right)+a\right)^{\frac{15}{2}}-660\left(a\sin\left(dx+c\right)+a\right)^{\frac{13}{2}}a+780\left(a\sin\left(dx+c\right)+a\right)^{\frac{11}{2}}a^2\right)}{2145\,a^5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 2/2145*(143*(a*sin(d*x + c) + a)^(15/2) - 660*(a*sin(d*x + c) + a)^(13/2)*a
 + 780*(a*sin(d*x + c) + a)^(11/2)*a^2)/(a^5*d)
```

Fricas [A]
time = 0.36, size = 114, normalized size = 1.56

$$-\frac{2\left(341\,a^2\cos\left(dx+c\right)^6-28\,a^2\cos\left(dx+c\right)^4-64\,a^2\cos\left(dx+c\right)^2-512\,a^2+\left(143\,a^2\cos\left(dx+c\right)^6-252\,a^2\cos\left(dx+c\right)^4-320\,a^2\cos\left(dx+c\right)^2-512\,a^2\right)\sin\left(dx+c\right)\right)\sqrt{a\sin\left(dx+c\right)+a}}{2145\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/2145*(341*a^2*cos(d*x + c)^6 - 28*a^2*cos(d*x + c)^4 - 64*a^2*cos(d*x +
c)^2 - 512*a^2 + (143*a^2*cos(d*x + c)^6 - 252*a^2*cos(d*x + c)^4 - 320*a^2
*cos(d*x + c)^2 - 512*a^2)*sin(d*x + c))*sqrt(a*sin(d*x + c) + a)/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3876 deep
```

Giac [A]
time = 7.29, size = 108, normalized size = 1.48

$$\frac{256\,\sqrt{2}\,\left(143\,a^{2}\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{15}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)-330\,a^{2}\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)+195\,a^{2}\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\right)\sqrt{a}}{2145\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 256/2145*sqrt(2)*(143*a^2*cos(-1/4*pi + 1/2*d*x + 1/2*c)^15*sgn(cos(-1/4*pi
 + 1/2*d*x + 1/2*c)) - 330*a^2*cos(-1/4*pi + 1/2*d*x + 1/2*c)^13*sgn(cos(-1
/4*pi + 1/2*d*x + 1/2*c)) + 195*a^2*cos(-1/4*pi + 1/2*d*x + 1/2*c)^11*sgn(c
os(-1/4*pi + 1/2*d*x + 1/2*c)))*sqrt(a)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^5\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(a + a*sin(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^5*(a + a*sin(c + d*x))^(5/2), x)
```

3.128 $\int \cos^4(c+dx)(a+a\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=159

$$-\frac{4096a^5\cos^5(c+dx)}{15015d(a+a\sin(c+dx))^{5/2}}-\frac{1024a^4\cos^5(c+dx)}{3003d(a+a\sin(c+dx))^{3/2}}-\frac{128a^3\cos^5(c+dx)}{429d\sqrt{a+a\sin(c+dx)}}-\frac{32a^2\cos^5(c+dx)}{}$$

```
[Out] -4096/15015*a^5*cos(d*x+c)^5/d/(a+a*sin(d*x+c))^(5/2)-1024/3003*a^4*cos(d*x
+c)^5/d/(a+a*sin(d*x+c))^(3/2)-2/13*a*cos(d*x+c)^5*(a+a*sin(d*x+c))^(3/2)/d
-128/429*a^3*cos(d*x+c)^5/d/(a+a*sin(d*x+c))^(1/2)-32/143*a^2*cos(d*x+c)^5*
(a+a*sin(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.20, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2753, 2752}

$$-\frac{4096a^5\cos^5(c+dx)}{15015d(a\sin(c+dx)+a)^{5/2}}-\frac{1024a^4\cos^5(c+dx)}{3003d(a\sin(c+dx)+a)^{3/2}}-\frac{128a^3\cos^5(c+dx)}{429d\sqrt{a\sin(c+dx)+a}}-\frac{32a^2\cos^5(c+dx)\sqrt{a\sin(c+dx)+a}}{143d}-\frac{2a\cos^5(c+dx)(a\sin(c+dx)+a)^{3/2}}{13d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-4096*a^5*Cos[c + d*x]^5)/(15015*d*(a + a*Sin[c + d*x])^(5/2)) - (1024*a^4
*Cos[c + d*x]^5)/(3003*d*(a + a*Sin[c + d*x])^(3/2)) - (128*a^3*Cos[c + d*x
]^5)/(429*d*Sqrt[a + a*Sin[c + d*x]]) - (32*a^2*Cos[c + d*x]^5*Sqrt[a + a*S
in[c + d*x]])/(143*d) - (2*a*Cos[c + d*x]^5*(a + a*Sin[c + d*x])^(3/2))/(13
*d)
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2753

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && N
eQ[m + p, 0]
```

Rubi steps

$$\int \cos^4(c+dx)(a+a\sin(c+dx))^{5/2}\,dx = -\frac{2a\cos^5(c+dx)(a+a\sin(c+dx))^{3/2}}{13d} + \frac{1}{13}(16a)\int \cos^4(c+dx)$$
$$= -\frac{32a^2\cos^5(c+dx)\sqrt{a+a\sin(c+dx)}}{143d} - \frac{2a\cos^5(c+dx)(a+a\,\mathrm{s}}{13d}$$
$$= -\frac{128a^3\cos^5(c+dx)}{429d\sqrt{a+a\sin(c+dx)}} - \frac{32a^2\cos^5(c+dx)\sqrt{a+a\sin(c+d}}{143d}$$
$$= -\frac{1024a^4\cos^5(c+dx)}{3003d(a+a\sin(c+dx))^{3/2}} - \frac{128a^3\cos^5(c+dx)}{429d\sqrt{a+a\sin(c+dx)}} - \frac{32a^2}{}$$
$$= -\frac{4096a^5\cos^5(c+dx)}{15015d(a+a\sin(c+dx))^{5/2}} - \frac{1024a^4\cos^5(c+dx)}{3003d(a+a\sin(c+dx))^{3/2}} - \frac{}{4}$$

Mathematica [A]
time = 0.19, size = 79, normalized size = 0.50

$$-\frac{2\cos^5(c+dx)(a(1+\sin(c+dx)))^{5/2}(9683+16700\sin(c+dx)+14210\sin^2(c+dx)+6300\sin^3(c+dx)+1155\sin^4(c+dx))}{15015d(1+\sin(c+dx))^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*Cos[c + d*x]^5*(a*(1 + Sin[c + d*x]))^(5/2)*(9683 + 16700*Sin[c + d*x]
+ 14210*Sin[c + d*x]^2 + 6300*Sin[c + d*x]^3 + 1155*Sin[c + d*x]^4))/(15015
*d*(1 + Sin[c + d*x])^5)
```

Maple [A]
time = 0.38, size = 87, normalized size = 0.55

method	result	size
default	$\frac{2(1+\sin(dx+c))a^3(\sin(dx+c)-1)^3(1155(\sin^4(dx+c))+6300(\sin^3(dx+c))+14210(\sin^2(dx+c))+16700\sin(dx+c)+9683)}{15015\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$	87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+a*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15015*(1+sin(d*x+c))*a^3*(sin(d*x+c)-1)^3*(1155*sin(d*x+c)^4+6300*sin(d*x
+c)^3+14210*sin(d*x+c)^2+16700*sin(d*x+c)+9683)/cos(d*x+c)/(a+a*sin(d*x+c))
^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(5/2)*cos(d*x + c)^4, x)
```

Fricas [A]
time = 0.37, size = 219, normalized size = 1.38

$$\frac{2\left(1155\,a^{2}\cos\left(dx+c\right)^{7}-2835\,a^{2}\cos\left(dx+c\right)^{6}-6230\,a^{2}\cos\left(dx+c\right)^{5}+320\,a^{2}\cos\left(dx+c\right)^{4}-512\,a^{2}\cos\left(dx+c\right)^{3}+1024\,a^{2}\cos\left(dx+c\right)^{2}-4096\,a^{2}\cos\left(dx+c\right)-8192\,a^{2}-\left(1155\,a^{2}\cos\left(dx+c\right)^{6}+3990\,a^{2}\cos\left(dx+c\right)^{5}-2240\,a^{2}\cos\left(dx+c\right)^{4}-2560\,a^{2}\cos\left(dx+c\right)^{3}-3072\,a^{2}\cos\left(dx+c\right)^{2}-4096\,a^{2}\cos\left(dx+c\right)-8192\,a^{2}\right)\sin\left(dx+c\right)\right)\sqrt{a\sin\left(dx+c\right)+a}}{15015\left(d\cos\left(dx+c\right)+d\sin\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/15015*(1155*a^2*cos(d*x + c)^7 - 2835*a^2*cos(d*x + c)^6 - 6230*a^2*cos(d
*x + c)^5 + 320*a^2*cos(d*x + c)^4 - 512*a^2*cos(d*x + c)^3 + 1024*a^2*cos(
d*x + c)^2 - 4096*a^2*cos(d*x + c) - 8192*a^2 - (1155*a^2*cos(d*x + c)^6 +
3990*a^2*cos(d*x + c)^5 - 2240*a^2*cos(d*x + c)^4 - 2560*a^2*cos(d*x + c)^3
 - 3072*a^2*cos(d*x + c)^2 - 4096*a^2*cos(d*x + c) - 8192*a^2)*sin(d*x + c)
)*sqrt(a*sin(d*x + c) + a)/(d*cos(d*x + c) + d*sin(d*x + c) + d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3060 deep
```

Giac [A]
time = 5.40, size = 172, normalized size = 1.08

$$\frac{128\,\sqrt{2}\left(1155\,a^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}-5460\,a^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+10010\,a^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-8580\,a^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+3003\,a^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}\right)\sqrt{a}}{15015\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 128/15015*sqrt(2)*(1155*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi
 + 1/2*d*x + 1/2*c)^13 - 5460*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-
1/4*pi + 1/2*d*x + 1/2*c)^11 + 10010*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c)
)*sin(-1/4*pi + 1/2*d*x + 1/2*c)^9 - 8580*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1
/2*c))*sin(-1/4*pi + 1/2*d*x + 1/2*c)^7 + 3003*a^2*sgn(cos(-1/4*pi + 1/2*d*
x + 1/2*c))*sin(-1/4*pi + 1/2*d*x + 1/2*c)^5)*sqrt(a)/d
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \cos (c+d x)^4 (a+a \sin (c+d x))^{5/2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(a + a*sin(c + d*x))^(5/2),x)
[Out] int(cos(c + d*x)^4*(a + a*sin(c + d*x))^(5/2), x)
```

3.129 $\int \cos^3(c+dx)(a+a\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=49

$$\frac{4(a+a\sin(c+dx))^{9/2}}{9a^2d}-\frac{2(a+a\sin(c+dx))^{11/2}}{11a^3d}$$

```
[Out] 4/9*(a+a*sin(d*x+c))^(9/2)/a^2/d-2/11*(a+a*sin(d*x+c))^(11/2)/a^3/d
```

Rubi [A]
time = 0.04, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.087$, Rules used = {2746, 45}

$$\frac{4(a\sin(c+dx)+a)^{9/2}}{9a^2d}-\frac{2(a\sin(c+dx)+a)^{11/2}}{11a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (4*(a + a*Sin[c + d*x])^(9/2))/(9*a^2*d) - (2*(a + a*Sin[c + d*x])^(11/2))/
(11*a^3*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \cos^3(c+dx)(a+a\sin(c+dx))^{5/2}\,dx &= \frac{\text{Subst}\left(\int (a-x)(a+x)^{7/2}\,dx,x,a\sin(c+dx)\right)}{a^3d}\\ &= \frac{\text{Subst}\left(\int \left(2a(a+x)^{7/2}-(a+x)^{9/2}\right)\,dx,x,a\sin(c+dx)\right)}{a^3d}\\ &= \frac{4(a+a\sin(c+dx))^{9/2}}{9a^2d}-\frac{2(a+a\sin(c+dx))^{11/2}}{11a^3d}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 41, normalized size = 0.84

$$-\frac{2(1+\sin(c+dx))^2(a(1+\sin(c+dx)))^{5/2}(-13+9\sin(c+dx))}{99d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Sin[c + d*x])^(5/2),x]
[Out] (-2*(1 + Sin[c + d*x])^2*(a*(1 + Sin[c + d*x]))^(5/2)*(-13 + 9*Sin[c + d*x]
))/(99*d)
```

Maple [A]
time = 0.34, size = 31, normalized size = 0.63

method	result	size
default	$-\frac{2(a+a\sin(dx+c))^{\frac{9}{2}}(9\sin(dx+c)-13)}{99a^2d}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
[Out] -2/99/a^2*(a+a*sin(d*x+c))^(9/2)*(9*sin(d*x+c)-13)/d
```

Maxima [A]
time = 0.30, size = 38, normalized size = 0.78

$$-\frac{2\left(9\left(a\sin\left(dx+c\right)+a\right)^{\frac{11}{2}}-22\left(a\sin\left(dx+c\right)+a\right)^{\frac{9}{2}}a\right)}{99\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
[Out] -2/99*(9*(a*sin(d*x + c) + a)^(11/2) - 22*(a*sin(d*x + c) + a)^(9/2)*a)/(a^
3*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 88 vs. 2(41) = 82.
time = 0.36, size = 88, normalized size = 1.80

$$-\frac{2\left(23\,a^2\cos\left(dx+c\right)^4-4\,a^2\cos\left(dx+c\right)^2-32\,a^2+\left(9\,a^2\cos\left(dx+c\right)^4-20\,a^2\cos\left(dx+c\right)^2-32\,a^2\right)\sin\left(dx+c\right)\right)\sqrt{a\sin\left(dx+c\right)+a}}{99\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/99*(23*a^2*cos(d*x + c)^4 - 4*a^2*cos(d*x + c)^2 - 32*a^2 + (9*a^2*cos(d
*x + c)^4 - 20*a^2*cos(d*x + c)^2 - 32*a^2)*sin(d*x + c))*sqrt(a*sin(d*x +
c) + a)/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 335 vs. $2(42) = 84$.
time = 48.11, size = 335, normalized size = 6.84

$$\begin{cases} \frac{8a^2\sqrt{a\sin(c+dx)+a}\sin^5(c+dx)}{77d} + \frac{272a^2\sqrt{a\sin(c+dx)+a}\sin^4(c+dx)}{693d} + \frac{2a^2\sqrt{a\sin(c+dx)+a}\sin^3(c+dx)\cos^2(c+dx)}{7d} + \frac{368a^2\sqrt{a\sin(c+dx)+a}\sin^3(c+dx)}{693d} + \frac{6a^2\sqrt{a\sin(c+dx)+a}\sin^2(c+dx)\cos^2(c+dx)}{7d} + \frac{64a^2\sqrt{a\sin(c+dx)+a}\sin^2(c+dx)}{231d} + \frac{6a^2\sqrt{a\sin(c+dx)+a}\sin(c+dx)\cos^2(c+dx)}{7d} + \frac{8a^2\sqrt{a\sin(c+dx)+a}\sin(c+dx)}{693d} + \frac{2a^2\sqrt{a\sin(c+dx)+a}\cos^2(c+dx)}{7d} - \frac{16a^2\sqrt{a\sin(c+dx)+a}}{693d} & \text{for } d \neq 0 \\ x(a\sin(c)+a)^{\frac{5}{2}}\cos^3(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Piecewise((8*a**2*sqrt(a*sin(c + d*x) + a)*sin(c + d*x)**5/(77*d) + 272*a**
2*sqrt(a*sin(c + d*x) + a)*sin(c + d*x)**4/(693*d) + 2*a**2*sqrt(a*sin(c +
d*x) + a)*sin(c + d*x)**3*cos(c + d*x)**2/(7*d) + 368*a**2*sqrt(a*sin(c + d
*x) + a)*sin(c + d*x)**3/(693*d) + 6*a**2*sqrt(a*sin(c + d*x) + a)*sin(c +
d*x)**2*cos(c + d*x)**2/(7*d) + 64*a**2*sqrt(a*sin(c + d*x) + a)*sin(c + d*
x)**2/(231*d) + 6*a**2*sqrt(a*sin(c + d*x) + a)*sin(c + d*x)*cos(c + d*x)**
2/(7*d) + 8*a**2*sqrt(a*sin(c + d*x) + a)*sin(c + d*x)/(693*d) + 2*a**2*sqr
t(a*sin(c + d*x) + a)*cos(c + d*x)**2/(7*d) - 16*a**2*sqrt(a*sin(c + d*x) +
 a)/(693*d), Ne(d, 0)), (x*(a*sin(c) + a)**(5/2)*cos(c)**3, True))
```

Giac [A]
time = 4.75, size = 76, normalized size = 1.55

$$-\frac{64\sqrt{2}\left(9a^2\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{11}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)-11a^2\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{9}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\right)\sqrt{a}}{99d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -64/99*sqrt(2)*(9*a^2*cos(-1/4*pi + 1/2*d*x + 1/2*c)^11*sgn(cos(-1/4*pi + 1
/2*d*x + 1/2*c)) - 11*a^2*cos(-1/4*pi + 1/2*d*x + 1/2*c)^9*sgn(cos(-1/4*pi
+ 1/2*d*x + 1/2*c)))*sqrt(a)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \cos(c+dx)^3\,(a+a\,\sin(c+dx))^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a + a*sin(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^3*(a + a*sin(c + d*x))^(5/2), x)
```

3.130 $\int \cos^2(c+dx)(a+a\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=127

$$-\frac{256a^4\cos^3(c+dx)}{315d(a+a\sin(c+dx))^{3/2}}-\frac{64a^3\cos^3(c+dx)}{105d\sqrt{a+a\sin(c+dx)}}-\frac{8a^2\cos^3(c+dx)\sqrt{a+a\sin(c+dx)}}{21d}-\frac{2a\cos^3(c+$$

```
[Out] -256/315*a^4*cos(d*x+c)^3/d/(a+a*sin(d*x+c))^(3/2)-2/9*a*cos(d*x+c)^3*(a+a*
sin(d*x+c))^(3/2)/d-64/105*a^3*cos(d*x+c)^3/d/(a+a*sin(d*x+c))^(1/2)-8/21*a
^2*cos(d*x+c)^3*(a+a*sin(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.15, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2753, 2752}

$$-\frac{256a^4\cos^3(c+dx)}{315d(a\sin(c+dx)+a)^{3/2}}-\frac{64a^3\cos^3(c+dx)}{105d\sqrt{a\sin(c+dx)+a}}-\frac{8a^2\cos^3(c+dx)\sqrt{a\sin(c+dx)+a}}{21d}-\frac{2a\cos^3(c+dx)(a\sin(c+dx)+a)^{3/2}}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-256*a^4*Cos[c + d*x]^3)/(315*d*(a + a*Sin[c + d*x])^(3/2)) - (64*a^3*Cos[
c + d*x]^3)/(105*d*Sqrt[a + a*Sin[c + d*x]]) - (8*a^2*Cos[c + d*x]^3*Sqrt[a
 + a*Sin[c + d*x]])/(21*d) - (2*a*Cos[c + d*x]^3*(a + a*Sin[c + d*x])^(3/2)
)/(9*d)
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2753

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && N
eQ[m + p, 0]
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\sin(c+dx))^{5/2}\,dx = -\frac{2a\cos^3(c+dx)(a+a\sin(c+dx))^{3/2}}{9d} + \frac{1}{3}(4a)\int \cos^2(c+dx)($$
$$= -\frac{8a^2\cos^3(c+dx)\sqrt{a+a\sin(c+dx)}}{21d} - \frac{2a\cos^3(c+dx)(a+a\,\mathrm{s}}{9d}$$
$$= -\frac{64a^3\cos^3(c+dx)}{105d\sqrt{a+a\sin(c+dx)}} - \frac{8a^2\cos^3(c+dx)\sqrt{a+a\sin(c+d}}{21d}$$
$$= -\frac{256a^4\cos^3(c+dx)}{315d(a+a\sin(c+dx))^{3/2}} - \frac{64a^3\cos^3(c+dx)}{105d\sqrt{a+a\sin(c+dx)}} - \frac{8a^2}{}$$

Mathematica [A]
time = 0.17, size = 69, normalized size = 0.54

$$-\frac{2\cos^3(c+dx)(a(1+\sin(c+dx)))^{5/2}\left(319+321\sin(c+dx)+165\sin^2(c+dx)+35\sin^3(c+dx)\right)}{315d(1+\sin(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*Cos[c + d*x]^3*(a*(1 + Sin[c + d*x]))^(5/2)*(319 + 321*Sin[c + d*x] + 1
65*Sin[c + d*x]^2 + 35*Sin[c + d*x]^3))/(315*d*(1 + Sin[c + d*x])^4)
```

Maple [A]
time = 0.42, size = 77, normalized size = 0.61

method	result	size
default	$-\frac{2(1+\sin(dx+c))a^3(\sin(dx+c)-1)^2\left(35\left(\sin^3(dx+c)\right)+165\left(\sin^2(dx+c)\right)+321\sin(dx+c)+319\right)}{315\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/315*(1+sin(d*x+c))*a^3*(sin(d*x+c)-1)^2*(35*sin(d*x+c)^3+165*sin(d*x+c)^
2+321*sin(d*x+c)+319)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(5/2)*cos(d*x + c)^2, x)
```

Fricas [A]
time = 0.35, size = 167, normalized size = 1.31

$$\frac{2\left(35\,a^{2}\cos\left(dx+c\right)^{5}-95\,a^{2}\cos\left(dx+c\right)^{4}-226\,a^{2}\cos\left(dx+c\right)^{3}+32\,a^{2}\cos\left(dx+c\right)^{2}-128\,a^{2}\cos\left(dx+c\right)-256\,a^{2}-\left(35\,a^{2}\cos\left(dx+c\right)^{4}+130\,a^{2}\cos\left(dx+c\right)^{3}-96\,a^{2}\cos\left(dx+c\right)^{2}-128\,a^{2}\cos\left(dx+c\right)-256\,a^{2}\right)\sin\left(dx+c\right)\right)\sqrt{a\sin\left(dx+c\right)+a}}{315\left(d\cos\left(dx+c\right)+d\sin\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*a^2*cos(d*x + c)^5 - 95*a^2*cos(d*x + c)^4 - 226*a^2*cos(d*x + c)
^3 + 32*a^2*cos(d*x + c)^2 - 128*a^2*cos(d*x + c) - 256*a^2 - (35*a^2*cos(d
*x + c)^4 + 130*a^2*cos(d*x + c)^3 - 96*a^2*cos(d*x + c)^2 - 128*a^2*cos(d*
x + c) - 256*a^2)*sin(d*x + c))*sqrt(a*sin(d*x + c) + a)/(d*cos(d*x + c) +
d*sin(d*x + c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \left(\sin{\left(c + dx \right)} + 1\right)\right)^{\frac{5}{2}} \cos^{2}{\left(c + dx \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Integral((a*(sin(c + d*x) + 1))**(5/2)*cos(c + d*x)**2, x)
```

Giac [A]
time = 4.91, size = 140, normalized size = 1.10

$$-\frac{32\,\sqrt{2}\left(35\,a^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-135\,a^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+189\,a^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-105\,a^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}\right)\sqrt{a}}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -32/315*sqrt(2)*(35*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1
/2*d*x + 1/2*c)^9 - 135*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi
 + 1/2*d*x + 1/2*c)^7 + 189*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/
4*pi + 1/2*d*x + 1/2*c)^5 - 105*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin
(-1/4*pi + 1/2*d*x + 1/2*c)^3)*sqrt(a)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\cos\left(c+d\,x\right)}^2\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a + a*sin(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^2*(a + a*sin(c + d*x))^(5/2), x)
```

3.131 $\int \cos(c+dx)(a+a\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=24

$$\frac{2(a+a\sin(c+dx))^{7/2}}{7ad}$$

```
[Out] 2/7*(a+a*sin(d*x+c))^(7/2)/a/d
```

Rubi [A]
time = 0.03, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2746, 32}

$$\frac{2(a\sin(c+dx)+a)^{7/2}}{7ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (2*(a + a*Sin[c + d*x])^(7/2))/(7*a*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a+a\sin(c+dx))^{5/2}\,dx &= \frac{\text{Subst}\left(\int (a+x)^{5/2}\,dx, x, a\sin(c+dx)\right)}{ad}\\ &= \frac{2(a+a\sin(c+dx))^{7/2}}{7ad}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 24, normalized size = 1.00

$$\frac{2(a+a\sin(c+dx))^{7/2}}{7ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (2*(a + a*Sin[c + d*x])^(7/2))/(7*a*d)
```

Maple [A]
time = 0.04, size = 21, normalized size = 0.88

method	result	size
derivativedivides	$\frac{2(a+a\sin(dx+c))^{\frac{7}{2}}}{7da}$	21
default	$\frac{2(a+a\sin(dx+c))^{\frac{7}{2}}}{7da}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/7*(a+a*sin(d*x+c))^(7/2)/d/a
```

Maxima [A]
time = 0.28, size = 20, normalized size = 0.83

$$\frac{2\,(a\sin\left(dx+c\right)+a)^{\frac{7}{2}}}{7\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 2/7*(a*sin(d*x + c) + a)^(7/2)/(a*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(20) = 40.
time = 0.34, size = 61, normalized size = 2.54

$$-\frac{2\,\left(3\,a^2\cos\left(dx+c\right)^2-4\,a^2+\left(a^2\cos\left(dx+c\right)^2-4\,a^2\right)\sin\left(dx+c\right)\right)\sqrt{a\sin\left(dx+c\right)+a}}{7\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/7*(3*a^2*cos(d*x + c)^2 - 4*a^2 + (a^2*cos(d*x + c)^2 - 4*a^2)*sin(d*x +
 c))*sqrt(a*sin(d*x + c) + a)/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 126 vs. 2(19) = 38.
time = 16.92, size = 126, normalized size = 5.25

$$\begin{cases} \frac{2a^2\sqrt{a\sin\left(c+dx\right)+a}\,\sin^3\left(c+dx\right)}{7d}+\frac{6a^2\sqrt{a\sin\left(c+dx\right)+a}\,\sin^2\left(c+dx\right)}{7d}+\frac{6a^2\sqrt{a\sin\left(c+dx\right)+a}\,\sin\left(c+dx\right)}{7d}+\frac{2a^2\sqrt{a\sin\left(c+dx\right)+a}}{7d} & \text{for}\ d\neq 0\\ x(a\sin\left(c\right)+a)^{\frac{5}{2}}\cos\left(c\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Piecewise((2*a**2*sqrt(a*sin(c + d*x) + a)*sin(c + d*x)**3/(7*d) + 6*a**2*s
qrt(a*sin(c + d*x) + a)*sin(c + d*x)**2/(7*d) + 6*a**2*sqrt(a*sin(c + d*x)
+ a)*sin(c + d*x)/(7*d) + 2*a**2*sqrt(a*sin(c + d*x) + a)/(7*d), Ne(d, 0)),
 (x*(a*sin(c) + a)**(5/2)*cos(c), True))
```

Giac [A]
time = 4.89, size = 38, normalized size = 1.58

$$\frac{16\sqrt{2}\,a^{\frac{5}{2}}\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{7\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 16/7*sqrt(2)*a^(5/2)*cos(-1/4*pi + 1/2*d*x + 1/2*c)^7*sgn(cos(-1/4*pi + 1/2
*d*x + 1/2*c))/d
```

Mupad [B]
time = 4.78, size = 20, normalized size = 0.83

$$\frac{2\,\left(a\,\left(\sin\left(c+d\,x\right)+1\right)\right)^{7/2}}{7\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + a*sin(c + d*x))^(5/2),x)
```

```
[Out] (2*(a*(sin(c + d*x) + 1))^(7/2))/(7*a*d)
```

3.132 $\int \sec(c+dx)(a+a\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=86

$$\frac{4\sqrt{2}\,a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{2}\,\sqrt{a}}\right)}{d}-\frac{4a^2\sqrt{a+a\sin(c+dx)}}{d}-\frac{2a(a+a\sin(c+dx))^{3/2}}{3d}$$

```
[Out] -2/3*a*(a+a*sin(d*x+c))^(3/2)/d+4*a^(5/2)*arctanh(1/2*(a+a*sin(d*x+c))^(1/2
)*2^(1/2)/a^(1/2))*2^(1/2)/d-4*a^2*(a+a*sin(d*x+c))^(1/2)/d
```

Rubi [A]

time = 0.05, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2746, 52, 65, 212}

$$\frac{4\sqrt{2}\,a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\,\sqrt{a}}\right)}{d}-\frac{4a^2\sqrt{a\sin(c+dx)+a}}{d}-\frac{2a(a\sin(c+dx)+a)^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (4*Sqrt[2]*a^(5/2)*ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])])/d -
 (4*a^2*Sqrt[a + a*Sin[c + d*x]])/d - (2*a*(a + a*Sin[c + d*x])^(3/2))/(3*d
)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\int \sec(c+dx)(a+a\sin(c+dx))^{5/2}\,dx = \frac{a\text{Subst}\left(\int \frac{(a+x)^{3/2}}{a-x}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= -\frac{2a(a+a\sin(c+dx))^{3/2}}{3d} + \frac{(2a^2)\,\text{Subst}\left(\int \frac{\sqrt{a+x}}{a-x}\,dx, x, a\sin(\right.}{d}$$

$$= -\frac{4a^2\sqrt{a+a\sin(c+dx)}}{d} - \frac{2a(a+a\sin(c+dx))^{3/2}}{3d} + \frac{(4a^3)\,\text{Sub}}{}$$

$$= -\frac{4a^2\sqrt{a+a\sin(c+dx)}}{d} - \frac{2a(a+a\sin(c+dx))^{3/2}}{3d} + \frac{(8a^3)\,\text{Sub}}{}$$

$$= \frac{4\sqrt{2}\,a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d} - \frac{4a^2\sqrt{a+a\sin(c+d}}{d}$$

Mathematica [A]
time = 0.12, size = 73, normalized size = 0.85

$$\frac{12\sqrt{2}\,a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a(1+\sin(c+dx))}}{\sqrt{2}\sqrt{a}}\right) - 2a^2\sqrt{a(1+\sin(c+dx))}\,(7+\sin(c+dx))}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (12*Sqrt[2]*a^(5/2)*ArcTanh[Sqrt[a*(1 + Sin[c + d*x])]/(Sqrt[2]*Sqrt[a])] -
 2*a^2*Sqrt[a*(1 + Sin[c + d*x])]*(7 + Sin[c + d*x]))/(3*d)
```

Maple [A]
time = 0.27, size = 66, normalized size = 0.77

method	result	size
default	$-\frac{2a\left(\frac{(a+a\sin(dx+c))^{\frac{3}{2}}}{3}+2a\sqrt{a+a\sin(dx+c)}-2a^{\frac{3}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+a\sin(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)\right)}{d}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+a*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*a*(1/3*(a+a*sin(d*x+c))^(3/2)+2*a*(a+a*sin(d*x+c))^(1/2)-2*a^(3/2)*2^(1/
2)*arctanh(1/2*(a+a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2)))/d
```

Maxima [A]
time = 0.51, size = 97, normalized size = 1.13

$$-\frac{2\left(3\sqrt{2}\,a^{\frac{7}{2}}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{a\sin(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{a\sin(dx+c)+a}}\right)+(a\sin(dx+c)+a)^{\frac{3}{2}}a^{2}+6\sqrt{a\sin(dx+c)+a}\,a^{3}\right)}{3\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3*(3*sqrt(2)*a^(7/2)*log(-(sqrt(2)*sqrt(a) - sqrt(a*sin(d*x + c) + a))/(
sqrt(2)*sqrt(a) + sqrt(a*sin(d*x + c) + a))) + (a*sin(d*x + c) + a)^(3/2)*a
^2 + 6*sqrt(a*sin(d*x + c) + a)*a^3)/(a*d)
```

Fricas [A]
time = 0.37, size = 89, normalized size = 1.03

$$\frac{2\left(3\sqrt{2}\,a^{\frac{5}{2}}\log\left(-\frac{a\sin(dx+c)+2\sqrt{2}\sqrt{a\sin(dx+c)+a}\,\sqrt{a}+3a}{\sin(dx+c)-1}\right)-(a^{2}\sin(dx+c)+7a^{2})\sqrt{a\sin(dx+c)+a}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(3*sqrt(2)*a^(5/2)*log(-(a*sin(d*x + c) + 2*sqrt(2)*sqrt(a*sin(d*x + c)
 + a)*sqrt(a) + 3*a)/(sin(d*x + c) - 1)) - (a^2*sin(d*x + c) + 7*a^2)*sqrt(
a*sin(d*x + c) + a))/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 5.81, size = 91, normalized size = 1.06

$$-\frac{2\sqrt{2}\left(2\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{3}+6\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)-3\log\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)+3\log\left(-\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)\right)a^{\frac{5}{2}}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -2/3*sqrt(2)*(2*cos(-1/4*pi + 1/2*d*x + 1/2*c)^3 + 6*cos(-1/4*pi + 1/2*d*x
+ 1/2*c) - 3*log(cos(-1/4*pi + 1/2*d*x + 1/2*c) + 1) + 3*log(-cos(-1/4*pi +
 1/2*d*x + 1/2*c) + 1))*a^(5/2)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+a\,\sin\left(c+d\,x\right)\right)^{5/2}}{\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(5/2)/cos(c + d*x),x)
```

```
[Out] int((a + a*sin(c + d*x))^(5/2)/cos(c + d*x), x)
```

3.133 $\int \sec^2(c+dx)(a+a\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=55

$$\frac{8a^2\sec(c+dx)\sqrt{a+a\sin(c+dx)}}{d}-\frac{2a\sec(c+dx)(a+a\sin(c+dx))^{3/2}}{d}$$

```
[Out] -2*a*sec(d*x+c)*(a+a*sin(d*x+c))^(3/2)/d+8*a^2*sec(d*x+c)*(a+a*sin(d*x+c))^
(1/2)/d
```

Rubi [A]
time = 0.08, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.087$, Rules used = {2753, 2752}

$$\frac{8a^2\sec(c+dx)\sqrt{a\sin(c+dx)+a}}{d}-\frac{2a\sec(c+dx)(a\sin(c+dx)+a)^{3/2}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (8*a^2*Sec[c + d*x]*Sqrt[a + a*Sin[c + d*x]])/d - (2*a*Sec[c + d*x]*(a + a*
Sin[c + d*x])^(3/2))/d
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2753

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && N
eQ[m + p, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^2(c+dx)(a+a\sin(c+dx))^{5/2}\,dx&=-\frac{2a\sec(c+dx)(a+a\sin(c+dx))^{3/2}}{d}+(4a)\int \sec^2(c+dx)(a+\\&=\frac{8a^2\sec(c+dx)\sqrt{a+a\sin(c+dx)}}{d}-\frac{2a\sec(c+dx)(a+a\sin(c-}{d}\end{aligned}$$

Mathematica [**A**]
time = 3.13, size = 36, normalized size = 0.65

$$-\frac{2a^2\sec(c+dx)(-3+\sin(c+dx))\sqrt{a(1+\sin(c+dx))}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sin[c + d*x])^(5/2),x]
[Out] (-2*a^2*Sec[c + d*x]*(-3 + Sin[c + d*x])*Sqrt[a*(1 + Sin[c + d*x])])/d
```

Maple [**A**]
time = 0.34, size = 45, normalized size = 0.82

method	result	size
default	$-\frac{2a^3(1+\sin(dx+c))(\sin(dx+c)-3)}{\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+a*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
[Out] -2*a^3*(1+sin(d*x+c))*(sin(d*x+c)-3)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 191 vs. $2(51) = 102$.
time = 0.58, size = 191, normalized size = 3.47

$$-\frac{2\left(3a^{\frac{5}{2}}-\frac{2a^{\frac{5}{2}}\sin(dx+c)}{\cos(dx+c)+1}+\frac{9a^{\frac{5}{2}}\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{4a^{\frac{5}{2}}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{9a^{\frac{5}{2}}\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{2a^{\frac{5}{2}}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{3a^{\frac{5}{2}}\sin(dx+c)^6}{(\cos(dx+c)+1)^6}\right)}{d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
[Out] -2*(3*a^(5/2) - 2*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) + 9*a^(5/2)*sin(d
*x + c)^2/(cos(d*x + c) + 1)^2 - 4*a^(5/2)*sin(d*x + c)^3/(cos(d*x + c) + 1
)^3 + 9*a^(5/2)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 2*a^(5/2)*sin(d*x + c
)^5/(cos(d*x + c) + 1)^5 + 3*a^(5/2)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6)/(
d*(sin(d*x + c)/(cos(d*x + c) + 1) - 1)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^
2 + 1)^(5/2))
```

Fricas [**A**]
time = 0.35, size = 41, normalized size = 0.75

$$-\frac{2\left(a^2\sin(dx+c)-3a^2\right)\sqrt{a\sin(dx+c)+a}}{d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -2*(a^2*sin(d*x + c) - 3*a^2)*sqrt(a*sin(d*x + c) + a)/(d*cos(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 6.92, size = 72, normalized size = 1.31

$$-\frac{2\sqrt{2}\left(a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+\frac{a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)}\right)\sqrt{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(2)*(a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*d*x +
 1/2*c) + a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))/sin(-1/4*pi + 1/2*d*x + 1
/2*c))*sqrt(a)/d
```

Mupad [**B**]
time = 5.46, size = 88, normalized size = 1.60

$$\frac{2\,a^2\,\sqrt{a\,\left(\sin\left(c+d\,x\right)+1\right)}\,\left(-22\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-2\sin\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)^2+4\,\sin\left(2\,c+2\,d\,x\right)+12\right)}{d\,\left(-4\sin\left(c+d\,x\right)^2+\sin\left(c+d\,x\right)+\sin\left(3\,c+3\,d\,x\right)+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(5/2)/cos(c + d*x)^2,x)
```

```
[Out] (2*a^2*(a*(sin(c + d*x) + 1))^(1/2)*(4*sin(2*c + 2*d*x) - 22*sin(c/2 + (d*x
)/2)^2 - 2*sin((3*c)/2 + (3*d*x)/2)^2 + 12))/(d*(sin(c + d*x) + sin(3*c + 3
*d*x) - 4*sin(c + d*x)^2 + 4))
```

3.134 $\int \sec^3(c+dx)(a+a\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=69

$$-\frac{a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}\,d}+\frac{a\sec^2(c+dx)(a+a\sin(c+dx))^{3/2}}{d}$$

```
[Out] a*sec(d*x+c)^2*(a+a*sin(d*x+c))^(3/2)/d-1/2*a^(5/2)*arctanh(1/2*(a+a*sin(d*
x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)/d
```

Rubi [A]
time = 0.08, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2755, 2746, 65, 212}

$$\frac{a\sec^2(c+dx)(a\sin(c+dx)+a)^{3/2}}{d}-\frac{a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] -((a^(5/2)*ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(Sqrt[2]*d)
) + (a*Sec[c + d*x]^2*(a + a*Sin[c + d*x])^(3/2))/d
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
```

```
])
```

Rule 2755

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[-2*b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(p + 1))), x] + Dist[b^2*((2*m + p - 1)/(g^2*(p + 1))),
Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[
{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && Inte
gersQ[2*m, 2*p]
```

Rubi steps

$$\int \sec^3(c+dx)(a+a\sin(c+dx))^{5/2}\,dx = \frac{a\sec^2(c+dx)(a+a\sin(c+dx))^{3/2}}{d} - \frac{1}{2}a^2\int \sec(c+dx)\sqrt{a+a\,\mathrm{s}}$$

$$= \frac{a\sec^2(c+dx)(a+a\sin(c+dx))^{3/2}}{d} - \frac{a^3\mathrm{Subst}\left(\int \frac{1}{(a-x)\sqrt{a+x}}\,d\right.}{2d}$$

$$= \frac{a\sec^2(c+dx)(a+a\sin(c+dx))^{3/2}}{d} - \frac{a^3\mathrm{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \sqrt{a}\right.}{d}$$

$$= -\frac{a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}\,d} + \frac{a\sec^2(c+dx)(a+a\sin(}{d}$$

Mathematica [A]
time = 0.25, size = 75, normalized size = 1.09

$$\frac{a^2\left(-\sqrt{2}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a(1+\sin(c+dx))}}{\sqrt{2}\sqrt{a}}\right) - \frac{2\sqrt{a(1+\sin(c+dx))}}{-1+\sin(c+dx)}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (a^2*(-(Sqrt[2]*Sqrt[a]*ArcTanh[Sqrt[a*(1 + Sin[c + d*x])]/(Sqrt[2]*Sqrt[a]
)]) - (2*Sqrt[a*(1 + Sin[c + d*x])])/(-1 + Sin[c + d*x])))/(2*d)
```

Maple [A]
time = 0.34, size = 66, normalized size = 0.96

method	result	size

default	$-\frac{a^{3}\left(\frac{\sqrt{a+a\sin\left(dx+c\right)}}{a\sin\left(dx+c\right)-a}+\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+a\sin\left(dx+c\right)}\,\sqrt{2}}{2\sqrt{a}}\right)}{2\sqrt{a}}\right)}{d}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+a*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -a^3*((a+a*sin(d*x+c))^(1/2)/(a*sin(d*x+c)-a)+1/2*2^(1/2)/a^(1/2)*arctanh(1
/2*(a+a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2)))/d
```

Maxima [A]
time = 0.50, size = 94, normalized size = 1.36

$$\frac{\sqrt{2}\,a^{\frac{7}{2}}\log\left(-\frac{\sqrt{2}\,\sqrt{a}-\sqrt{a\sin\left(dx+c\right)+a}}{\sqrt{2}\,\sqrt{a}+\sqrt{a\sin\left(dx+c\right)+a}}\right)-\frac{4\,\sqrt{a\sin\left(dx+c\right)+a}\,a^{4}}{a\sin\left(dx+c\right)-a}}{4\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/4*(sqrt(2)*a^(7/2)*log(-(sqrt(2)*sqrt(a) - sqrt(a*sin(d*x + c) + a))/(sqr
t(2)*sqrt(a) + sqrt(a*sin(d*x + c) + a))) - 4*sqrt(a*sin(d*x + c) + a)*a^4/
(a*sin(d*x + c) - a))/(a*d)
```

Fricas [A]
time = 0.39, size = 102, normalized size = 1.48

$$\frac{\sqrt{2}\left(a^{2}\sin\left(dx+c\right)-a^{2}\right)\sqrt{a}\,\log\left(-\frac{a\sin\left(dx+c\right)-2\,\sqrt{2}\,\sqrt{a\sin\left(dx+c\right)+a}\,\sqrt{a}+3\,a}{\sin\left(dx+c\right)-1}\right)-4\,\sqrt{a\sin\left(dx+c\right)+a}\,a^{2}}{4\left(d\sin\left(dx+c\right)-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*(a^2*sin(d*x + c) - a^2)*sqrt(a)*log(-(a*sin(d*x + c) - 2*sqrt
(2)*sqrt(a*sin(d*x + c) + a)*sqrt(a) + 3*a)/(sin(d*x + c) - 1)) - 4*sqrt(a*
sin(d*x + c) + a)*a^2)/(d*sin(d*x + c) - d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 5.52, size = 91, normalized size = 1.32

$$-\frac{\sqrt{2}\,a^{\frac{5}{2}}\left(\frac{2\,\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1}+\log\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-\log\left(-\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*a^(5/2)*(2*cos(-1/4*pi + 1/2*d*x + 1/2*c)/(cos(-1/4*pi + 1/2*d
*x + 1/2*c)^2 - 1) + log(cos(-1/4*pi + 1/2*d*x + 1/2*c) + 1) - log(-cos(-1/
4*pi + 1/2*d*x + 1/2*c) + 1))*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\,\sin\left(c+d\,x\right))^{5/2}}{\cos\left(c+d\,x\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(5/2)/cos(c + d*x)^3,x)
```

```
[Out] int((a + a*sin(c + d*x))^(5/2)/cos(c + d*x)^3, x)
```

3.135 $\int \sec^4(c+dx)(a+a\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=30

$$\frac{2a\sec^3(c+dx)(a+a\sin(c+dx))^{3/2}}{3d}$$

```
[Out] 2/3*a*sec(d*x+c)^3*(a+a*sin(d*x+c))^(3/2)/d
```

Rubi [A]
time = 0.04, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = $\{2752\}$

$$\frac{2a\sec^3(c+dx)(a\sin(c+dx)+a)^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (2*a*Sec[c + d*x]^3*(a + a*Sin[c + d*x])^(3/2))/(3*d)
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\int \sec^4(c+dx)(a+a\sin(c+dx))^{5/2}\,dx = \frac{2a\sec^3(c+dx)(a+a\sin(c+dx))^{3/2}}{3d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 69 vs. 2(30) = 60.
time = 5.10, size = 69, normalized size = 2.30

$$\frac{2(a(1+\sin(c+dx)))^{5/2}}{3d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (2*(a*(1 + Sin[c + d*x]))^(5/2))/(3*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])
^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5)
```

Maple [A]
time = 0.26, size = 47, normalized size = 1.57

method	result	size
default	$-\frac{2a^{3}(1+\sin(dx+c))}{3(\sin(dx+c)-1)\cos(dx+c)\sqrt{a+a\sin\left(dx+c\right)}\, d}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+a*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*a^3*(1+sin(d*x+c))/(sin(d*x+c)-1)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 184 vs. 2(26) = 52.
time = 0.70, size = 184, normalized size = 6.13

$$-\frac{2\left(a^{\frac{5}{2}}+\frac{4\,a^{\frac{5}{2}}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{6\,a^{\frac{5}{2}}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{4\,a^{\frac{5}{2}}\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+\frac{a^{\frac{5}{2}}\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}\right)}{3\,d\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}-\frac{3\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-1\right)\left(\frac{\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+1\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3*(a^(5/2) + 4*a^(5/2)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*a^(5/2)*s
in(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*a^(5/2)*sin(d*x + c)^6/(cos(d*x + c)
 + 1)^6 + a^(5/2)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8)/(d*(3*sin(d*x + c)/(
cos(d*x + c) + 1) - 3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + sin(d*x + c)^3/
(cos(d*x + c) + 1)^3 - 1)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^(5/2))
```

Fricas [A]
time = 0.33, size = 43, normalized size = 1.43

$$-\frac{2\sqrt{a\sin\left(dx+c\right)+a}\,a^{2}}{3\left(d\cos\left(dx+c\right)\sin\left(dx+c\right)-d\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(a*sin(d*x + c) + a)*a^2/(d*cos(d*x + c)*sin(d*x + c) - d*cos(d*x
+ c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 5.32, size = 38, normalized size = 1.27

$$-\frac{\sqrt{2}\, a^{\frac{5}{2}} \mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi + \frac{1}{2}\,dx + \frac{1}{2}\,c\right)\right)}{6\,d \sin\left(-\frac{1}{4}\,\pi + \frac{1}{2}\,dx + \frac{1}{2}\,c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(2)*a^(5/2)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))/(d*sin(-1/4*pi + 1
/2*d*x + 1/2*c)^3)
```

Mupad [B]
time = 7.59, size = 225, normalized size = 7.50

$$\frac{4\,a^2\,\sqrt{a\,\left(\sin\left(c+d\,x\right)+1\right)}\,\left(\sin\left(c+d\,x\right)^2\,4{}\mathrm{i}+\sin\left(c+d\,x\right)\,1{}\mathrm{i}-2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,\sin\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)^2-2\,\sin\left(2\,c+2\,d\,x\right)+\sin\left(3\,c+3\,d\,x\right)\,1{}\mathrm{i}-4{}\mathrm{i}\right)}{3\,d\,\left(8\,\sin\left(c+d\,x\right)^2+4\,\sin\left(c+d\,x\right)-2\,\sin\left(2\,c+2\,d\,x\right)^2+4\,\sin\left(3\,c+3\,d\,x\right)-8\right)}+\frac{4\,a^2\,\sqrt{a\,\left(\sin\left(c+d\,x\right)+1\right)}\,\left(\sin\left(2\,c+2\,d\,x\right)+4\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-\sin\left(c+d\,x\right)^2\,2{}\mathrm{i}-2+2{}\mathrm{i}\right)}{3\,d\,\left(4\,\sin\left(c+d\,x\right)^2+\sin\left(c+d\,x\right)+\sin\left(3\,c+3\,d\,x\right)-4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(5/2)/cos(c + d*x)^4,x)
```

```
[Out] (4*a^2*(a*(sin(c + d*x) + 1))^(1/2)*(sin(c + d*x)*1i - 2*sin(2*c + 2*d*x) +
 sin(3*c + 3*d*x)*1i - 2*sin(c/2 + (d*x)/2)^2 + 2*sin((3*c)/2 + (3*d*x)/2)^
2 + sin(c + d*x)^2*4i - 4i))/(3*d*(4*sin(c + d*x) + 4*sin(3*c + 3*d*x) - 2*
sin(2*c + 2*d*x)^2 + 8*sin(c + d*x)^2 - 8)) + (4*a^2*(a*(sin(c + d*x) + 1))
^(1/2)*(sin(2*c + 2*d*x) + 4*sin(c/2 + (d*x)/2)^2 - sin(c + d*x)^2*2i - (2
- 2i)))/(3*d*(sin(c + d*x) + sin(3*c + 3*d*x) + 4*sin(c + d*x)^2 - 4))
```

3.136 $\int \sec^5(c+dx)(a+a\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=103

$$\frac{3a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{16\sqrt{2}\,d}+\frac{3a\sec^2(c+dx)(a+a\sin(c+dx))^{3/2}}{16d}+\frac{\sec^4(c+dx)(a+a\sin(c+dx)}{4d}$$

```
[Out] 3/16*a*sec(d*x+c)^2*(a+a*sin(d*x+c))^(3/2)/d+1/4*sec(d*x+c)^4*(a+a*sin(d*x+
c))^(5/2)/d+3/32*a^(5/2)*arctanh(1/2*(a+a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2)
)*2^(1/2)/d
```

Rubi [A]

time = 0.12, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2754, 2746, 65, 212}

$$\frac{3a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{16\sqrt{2}\,d}+\frac{\sec^4(c+dx)(a\sin(c+dx)+a)^{5/2}}{4d}+\frac{3a\sec^2(c+dx)(a\sin(c+dx)+a)^{3/2}}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (3*a^(5/2)*ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(16*Sqrt[2]
*d) + (3*a*Sec[c + d*x]^2*(a + a*Sin[c + d*x])^(3/2))/(16*d) + (Sec[c + d*x
]^4*(a + a*Sin[c + d*x])^(5/2))/(4*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
```

```
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2754

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^m/(a*f*g*(p + 1))), x] + Dist[a*((m + p + 1)/(g^2*(p + 1))), Int[(g*C
os[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e
, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ[
m + 1/2, 2*p]
```

Rubi steps

$$\int \sec^5(c+dx)(a+a\sin(c+dx))^{5/2}\,dx = \frac{\sec^4(c+dx)(a+a\sin(c+dx))^{5/2}}{4d}+\frac{1}{8}(3a)\int \sec^3(c+dx)(a+a$$

$$= \frac{3a\sec^2(c+dx)(a+a\sin(c+dx))^{3/2}}{16d}+\frac{\sec^4(c+dx)(a+a\sin(c}{4d}$$

$$= \frac{3a\sec^2(c+dx)(a+a\sin(c+dx))^{3/2}}{16d}+\frac{\sec^4(c+dx)(a+a\sin(c}{4d}$$

$$= \frac{3a\sec^2(c+dx)(a+a\sin(c+dx))^{3/2}}{16d}+\frac{\sec^4(c+dx)(a+a\sin(c}{4d}$$

$$= \frac{3a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{16\sqrt{2}\,d}+\frac{3a\sec^2(c+dx)(a+a\sin}{16d}$$

Mathematica [A]
time = 0.19, size = 110, normalized size = 1.07

$$\frac{3\sqrt{2}\,a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a(1+\sin(c+dx))}}{\sqrt{2}\sqrt{a}}\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^4+2a^2(7-3\sin(c+dx))\sqrt{a(1+\sin(c+dx))}}{32d(-1+\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (3*Sqrt[2]*a^(5/2)*ArcTanh[Sqrt[a*(1 + Sin[c + d*x])]/(Sqrt[2]*Sqrt[a])]*(C
os[(c + d*x)/2] - Sin[(c + d*x)/2])^4 + 2*a^2*(7 - 3*Sin[c + d*x])*Sqrt[a*(
1 + Sin[c + d*x])])/(32*d*(-1 + Sin[c + d*x])^2)
```

Maple [A]
time = 0.62, size = 107, normalized size = 1.04

method	result
default	$-\frac{2a^5\left(-\frac{\sqrt{a+a\sin(dx+c)}}{8a(a\sin(dx+c)-a)^2}-\frac{3\left(-\frac{\sqrt{a+a\sin(dx+c)}}{4a(a\sin(dx+c)-a)}+\frac{\sqrt{2}\,\mathrm{arctanh}\left(\frac{\sqrt{a+a\sin(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{8a^{\frac{3}{2}}}\right)}{8a}\right)}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(a+a*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*a^5*(-1/8*(a+a*sin(d*x+c))^(1/2)/a/(a*sin(d*x+c)-a)^2-3/8/a*(-1/4*(a+a*s
in(d*x+c))^(1/2)/a/(a*sin(d*x+c)-a)+1/8/a^(3/2)*2^(1/2)*arctanh(1/2*(a+a*si
n(d*x+c))^(1/2)*2^(1/2)/a^(1/2))))/d
```

Maxima [A]
time = 0.64, size = 134, normalized size = 1.30

$$-\frac{3\sqrt{2}\,a^{\frac{7}{2}}\log\left(-\frac{\sqrt{2}\,\sqrt{a}-\sqrt{a\sin(dx+c)+a}}{\sqrt{2}\,\sqrt{a}+\sqrt{a\sin(dx+c)+a}}\right)+\frac{4\left(3\,(a\sin(dx+c)+a)^{\frac{3}{2}}a^4-10\,\sqrt{a\sin(dx+c)+a}\,a^5\right)}{(a\sin(dx+c)+a)^2-4\,(a\sin(dx+c)+a)a+4\,a^2}}{64\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -1/64*(3*sqrt(2)*a^(7/2)*log(-(sqrt(2)*sqrt(a) - sqrt(a*sin(d*x + c) + a))/
(sqrt(2)*sqrt(a) + sqrt(a*sin(d*x + c) + a))) + 4*(3*(a*sin(d*x + c) + a)^(
3/2)*a^4 - 10*sqrt(a*sin(d*x + c) + a)*a^5)/((a*sin(d*x + c) + a)^2 - 4*(a*
sin(d*x + c) + a)*a + 4*a^2))/(a*d)
```

Fricas [A]
time = 0.38, size = 147, normalized size = 1.43

$$\frac{3\left(\sqrt{2}\,a^2\cos(dx+c)^2+2\sqrt{2}\,a^2\sin(dx+c)-2\sqrt{2}\,a^2\right)\sqrt{a}\,\log\left(-\frac{a\sin(dx+c)+2\sqrt{2}\,\sqrt{a\sin(dx+c)+a}\,\sqrt{a}+3a}{\sin(dx+c)-1}\right)+4\left(3\,a^2\sin(dx+c)-7\,a^2\right)\sqrt{a\sin(dx+c)+a}}{64\left(d\cos(dx+c)^2+2\,d\sin(dx+c)-2\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/64*(3*(sqrt(2)*a^2*cos(d*x + c)^2 + 2*sqrt(2)*a^2*sin(d*x + c) - 2*sqrt(2
)*a^2)*sqrt(a)*log(-(a*sin(d*x + c) + 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*sq
```

```
rt(a) + 3*a)/(sin(d*x + c) - 1)) + 4*(3*a^2*sin(d*x + c) - 7*a^2)*sqrt(a*si
n(d*x + c) + a))/(d*cos(d*x + c)^2 + 2*d*sin(d*x + c) - 2*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a+a*sin(d*x+c))**(5/2),x)
[Out] Timed out
```

Giac [**A**]
time = 5.09, size = 112, normalized size = 1.09

$$-\frac{\sqrt{2}\,a^{\frac{5}{2}}\left(\frac{2\left(3\,\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-5\,\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{2}}-3\,\log\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)+3\,\log\left(-\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
[Out] -1/64*sqrt(2)*a^(5/2)*(2*(3*cos(-1/4*pi + 1/2*d*x + 1/2*c)^3 - 5*cos(-1/4*p
i + 1/2*d*x + 1/2*c))/(cos(-1/4*pi + 1/2*d*x + 1/2*c)^2 - 1)^2 - 3*log(cos(
-1/4*pi + 1/2*d*x + 1/2*c) + 1) + 3*log(-cos(-1/4*pi + 1/2*d*x + 1/2*c) + 1
))*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))/d
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+a\,\sin\left(c+d\,x\right)\right)^{5/2}}{\cos\left(c+d\,x\right)^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(5/2)/cos(c + d*x)^5,x)
[Out] int((a + a*sin(c + d*x))^(5/2)/cos(c + d*x)^5, x)
```

3.137 $\int \sec^6(c+dx)(a+a\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=139

$$-\frac{a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(c+dx)}{\sqrt{2}\,\sqrt{a+a\sin(c+dx)}}\right)}{4\sqrt{2}\,d}+\frac{a^2\sec(c+dx)\sqrt{a+a\sin(c+dx)}}{4d}+\frac{a\sec^3(c+dx)(a+a\sin(c+}{6d}$$

```
[Out] 1/6*a*sec(d*x+c)^3*(a+a*sin(d*x+c))^(3/2)/d+1/5*sec(d*x+c)^5*(a+a*sin(d*x+c
))^(5/2)/d-1/8*a^(5/2)*arctanh(1/2*cos(d*x+c)*a^(1/2)*2^(1/2)/(a+a*sin(d*x+
c))^(1/2))/d*2^(1/2)+1/4*a^2*sec(d*x+c)*(a+a*sin(d*x+c))^(1/2)/d
```

Rubi [A]

time = 0.14, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {2754, 2728, 212}

$$-\frac{a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(c+dx)}{\sqrt{2}\,\sqrt{a\sin(c+dx)+a}}\right)}{4\sqrt{2}\,d}+\frac{a^2\sec(c+dx)\sqrt{a\sin(c+dx)+a}}{4d}+\frac{\sec^5(c+dx)(a\sin(c+dx)+a)^{5/2}}{5d}+\frac{a\sec^3(c+dx)(a\sin(c+dx)+a)^{3/2}}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] -1/4*(a^(5/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*
x]])])/(Sqrt[2]*d) + (a^2*Sec[c + d*x]*Sqrt[a + a*Sin[c + d*x]])/(4*d) + (a
*Sec[c + d*x]^3*(a + a*Sin[c + d*x])^(3/2))/(6*d) + (Sec[c + d*x]^5*(a + a*
Sin[c + d*x])^(5/2))/(5*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^m/(a*f*g*(p + 1))), x] + Dist[a*((m + p + 1)/(g^2*(p + 1))), Int[(g*C
os[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e
```

```
, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ[
m + 1/2, 2*p]
```

Rubi steps

$$\int \sec^6(c+dx)(a+a\sin(c+dx))^{5/2}\,dx = \frac{\sec^5(c+dx)(a+a\sin(c+dx))^{5/2}}{5d} + \frac{1}{2}a\int \sec^4(c+dx)(a+a\sin$$

$$= \frac{a\sec^3(c+dx)(a+a\sin(c+dx))^{3/2}}{6d} + \frac{\sec^5(c+dx)(a+a\sin(c+}{5d}$$

$$= \frac{a^2\sec(c+dx)\sqrt{a+a\sin(c+dx)}}{4d} + \frac{a\sec^3(c+dx)(a+a\sin(c-}{6d}$$

$$= \frac{a^2\sec(c+dx)\sqrt{a+a\sin(c+dx)}}{4d} + \frac{a\sec^3(c+dx)(a+a\sin(c-}{6d}$$

$$= -\frac{a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{4\sqrt{2}\,d} + \frac{a^2\sec(c+dx)\sqrt{a-}{4d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 5.21, size = 129, normalized size = 0.93

$$\frac{\left((15+15i)(-1)^{3/4}\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(c+dx)\right)\right)\right)+\frac{89-15\cos(2(c+dx))-80\sin(c+dx)}{2\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^5}\right)(a(1+\sin(c+dx)))^{5/2}}{60d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6*(a + a*Sin[c + d*x])^(5/2),x]
[Out] (((15 + 15*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)
/4])] + (89 - 15*Cos[2*(c + d*x)] - 80*Sin[c + d*x])/(2*(Cos[(c + d*x)/2] -
 Sin[(c + d*x)/2])^5))*(a*(1 + Sin[c + d*x]))^(5/2))/(60*d*(Cos[(c + d*x)/2
] + Sin[(c + d*x)/2])^5)
```

Maple [A]
time = 0.46, size = 120, normalized size = 0.86

method	result
default	$-\frac{(1+\sin(dx+c))\left(30a^{\frac{11}{2}}(\cos^2(dx+c))+80a^{\frac{11}{2}}\sin(dx+c)-104a^{\frac{11}{2}}+15\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}\sqrt{2}}{2\sqrt{a}}\right)a^3(a-}{120a^{\frac{5}{2}}(\sin(dx+c)-1)^2\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(a+a*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/120*(1+sin(d*x+c))*(30*a^(11/2)*cos(d*x+c)^2+80*a^(11/2)*sin(d*x+c)-104*
a^(11/2)+15*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^3
*(a-a*sin(d*x+c))^(5/2))/a^(5/2)/(sin(d*x+c)-1)^2/cos(d*x+c)/(a+a*sin(d*x+c
))^(1/2)/d
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 263 vs. 2(116) = 232.
time = 0.38, size = 263, normalized size = 1.89

$$\frac{15\left(\sqrt{2}\,a^2\cos(dx+c)^3+2\sqrt{2}\,a^2\cos(dx+c)\sin(dx+c)-2\sqrt{2}\,a^2\cos(dx+c)\right)\sqrt{a}\,\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{a\sin(dx+c)+a}\left(\sqrt{2}\cos(dx+c)-\sqrt{2}\sin(dx+c)+\sqrt{2}\right)\sqrt{a}+3a\cos(dx+c)-(a\cos(dx+c)-2a)\sin(dx+c)+2a}{\cos(dx+c)^2-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2}\right)+4\left(15a^2\cos(dx+c)^2+40a^2\sin(dx+c)-52a^2\right)\sqrt{a\sin(dx+c)+a}}{240\left(d\cos(dx+c)^3+2d\cos(dx+c)\sin(dx+c)-2d\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/240*(15*(sqrt(2)*a^2*cos(d*x + c)^3 + 2*sqrt(2)*a^2*cos(d*x + c)*sin(d*x
+ c) - 2*sqrt(2)*a^2*cos(d*x + c))*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(
a*sin(d*x + c) + a)*(sqrt(2)*cos(d*x + c) - sqrt(2)*sin(d*x + c) + sqrt(2))
*sqrt(a) + 3*a*cos(d*x + c) - (a*cos(d*x + c) - 2*a)*sin(d*x + c) + 2*a)/(c
os(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2)) + 4*(1
5*a^2*cos(d*x + c)^2 + 40*a^2*sin(d*x + c) - 52*a^2)*sqrt(a*sin(d*x + c) +
a))/(d*cos(d*x + c)^3 + 2*d*cos(d*x + c)*sin(d*x + c) - 2*d*cos(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(a+a*sin(d*x+c))**(5/2),x)
```

[Out] Timed out

Giac [A]
time = 5.98, size = 111, normalized size = 0.80

$$-\frac{\sqrt{2}\,a^{\frac{5}{2}}\left(\frac{2\left(15\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+5\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+3\right)}{\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}}-15\,\log\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)+15\,\log\left(-\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -1/240*sqrt(2)*a^(5/2)*(2*(15*sin(-1/4*pi + 1/2*d*x + 1/2*c)^4 + 5*sin(-1/4
*pi + 1/2*d*x + 1/2*c)^2 + 3)/sin(-1/4*pi + 1/2*d*x + 1/2*c)^5 - 15*log(sin
(-1/4*pi + 1/2*d*x + 1/2*c) + 1) + 15*log(-sin(-1/4*pi + 1/2*d*x + 1/2*c) +
 1))*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{5/2}}{{\cos\left(c+d\,x\right)}^6} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(5/2)/cos(c + d*x)^6,x)
```

```
[Out] int((a + a*sin(c + d*x))^(5/2)/cos(c + d*x)^6, x)
```

3.138 $\int \sec^7(c+dx)(a+a\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=159

$$\frac{35a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{128\sqrt{2}\,d}-\frac{35a^3}{128d\sqrt{a+a\sin(c+dx)}}+\frac{35a^2\sec^2(c+dx)\sqrt{a+a\sin(c+dx)}}{192d}+$$

[Out] 7/48*a*sec(d*x+c)^4*(a+a*sin(d*x+c))^(3/2)/d+1/6*sec(d*x+c)^6*(a+a*sin(d*x+c))^(5/2)/d+35/256*a^(5/2)*arctanh(1/2*(a+a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)/d-35/128*a^3/d/(a+a*sin(d*x+c))^(1/2)+35/192*a^2*sec(d*x+c)^2*(a+a*sin(d*x+c))^(1/2)/d

Rubi [A]
time = 0.17, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2754, 2746, 53, 65, 212}

$$\frac{35a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{128\sqrt{2}\,d}-\frac{35a^3}{128d\sqrt{a\sin(c+dx)+a}}+\frac{35a^2\sec^2(c+dx)\sqrt{a\sin(c+dx)+a}}{192d}+\frac{\sec^6(c+dx)(a\sin(c+dx)+a)^{5/2}}{6d}+\frac{7a\sec^4(c+dx)(a\sin(c+dx)+a)^{3/2}}{48d}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d*x]^7*(a + a*Sin[c + d*x])^(5/2),x]

[Out] (35*a^(5/2)*ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(128*Sqrt[2]*d) - (35*a^3)/(128*d*Sqrt[a + a*Sin[c + d*x]]) + (35*a^2*Sec[c + d*x]^2*Sqrt[a + a*Sin[c + d*x]])/(192*d) + (7*a*Sec[c + d*x]^4*(a + a*Sin[c + d*x])^(3/2))/(48*d) + (Sec[c + d*x]^6*(a + a*Sin[c + d*x])^(5/2))/(6*d)

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2754

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^m/(a*f*g*(p + 1))), x] + Dist[a*((m + p + 1)/(g^2*(p + 1))), Int[(g*C
os[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e
, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ[
m + 1/2, 2*p]
```

Rubi steps

$$\int \sec^7(c+dx)(a+a\sin(c+dx))^{5/2}\,dx = \frac{\sec^6(c+dx)(a+a\sin(c+dx))^{5/2}}{6d} + \frac{1}{12}(7a)\int \sec^5(c+dx)(a+$$

$$= \frac{7a\sec^4(c+dx)(a+a\sin(c+dx))^{3/2}}{48d} + \frac{\sec^6(c+dx)(a+a\sin(c}{6d}$$

$$= \frac{35a^2\sec^2(c+dx)\sqrt{a+a\sin(c+dx)}}{192d} + \frac{7a\sec^4(c+dx)(a+a\,\mathrm{si}}{48d}$$

$$= \frac{35a^2\sec^2(c+dx)\sqrt{a+a\sin(c+dx)}}{192d} + \frac{7a\sec^4(c+dx)(a+a\,\mathrm{si}}{48d}$$

$$= -\frac{35a^3}{128d\sqrt{a+a\sin(c+dx)}} + \frac{35a^2\sec^2(c+dx)\sqrt{a+a\sin(c+d}}{192d}$$

$$= -\frac{35a^3}{128d\sqrt{a+a\sin(c+dx)}} + \frac{35a^2\sec^2(c+dx)\sqrt{a+a\sin(c+d}}{192d}$$

$$= \frac{35a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{128\sqrt{2}\,d} - \frac{35a^3}{128d\sqrt{a+a\sin(c+dx}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.07, size = 44, normalized size = 0.28

$$-\frac{a^3\,{}_2F_1\left(-\frac{1}{2},4;\frac{1}{2};\frac{1}{2}(1+\sin(c+dx))\right)}{8d\sqrt{a+a\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^7*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] -1/8*(a^3*Hypergeometric2F1[-1/2, 4, 1/2, (1 + Sin[c + d*x])/2])/(d*Sqrt[a
+ a*Sin[c + d*x]])
```

Maple [A]
time = 0.92, size = 113, normalized size = 0.71

method	result
default	$\frac{2a^7\left(-\frac{-\frac{a^2\sqrt{a+a\sin(dx+c)}\,\left(57\left(\cos^2(dx+c)\right)+158\sin(dx+c)-190\right)}{48(a\sin(dx+c)-a)^3}-\frac{35\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+a\sin(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{32\sqrt{a}}}{16a^4}-\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7*(a+a*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*a^7*(-1/16/a^4*(-1/48*a^2*(a+a*sin(d*x+c))^(1/2)*(57*cos(d*x+c)^2+158*sin
(d*x+c)-190)/(a*sin(d*x+c)-a)^3-35/32*2^(1/2)/a^(1/2)*arctanh(1/2*(a+a*sin(
d*x+c))^(1/2)*2^(1/2)/a^(1/2)))-1/16/a^4/(a+a*sin(d*x+c))^(1/2))/d
```

Maxima [A]
time = 0.53, size = 185, normalized size = 1.16

$$-\frac{105\sqrt{2}\,a^{\frac{7}{2}}\log\left(-\frac{\sqrt{2}\,\sqrt{a}-\sqrt{a\sin(dx+c)+a}}{\sqrt{2}\,\sqrt{a}+\sqrt{a\sin(dx+c)+a}}\right)+\frac{4\left(105\,(a\sin(dx+c)+a)^3a^4-560\,(a\sin(dx+c)+a)^2a^5+924\,(a\sin(dx+c)+a)a^6-384\,a^7\right)}{(a\sin(dx+c)+a)^{\frac{7}{2}}-6\,(a\sin(dx+c)+a)^{\frac{5}{2}}a+12\,(a\sin(dx+c)+a)^{\frac{3}{2}}a^2-8\,\sqrt{a\sin(dx+c)+a}\,a^3}}{1536\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -1/1536*(105*sqrt(2)*a^(7/2)*log(-(sqrt(2)*sqrt(a) - sqrt(a*sin(d*x + c) +
a))/(sqrt(2)*sqrt(a) + sqrt(a*sin(d*x + c) + a))) + 4*(105*(a*sin(d*x + c)
+ a)^3*a^4 - 560*(a*sin(d*x + c) + a)^2*a^5 + 924*(a*sin(d*x + c) + a)*a^6
- 384*a^7)/((a*sin(d*x + c) + a)^(7/2) - 6*(a*sin(d*x + c) + a)^(5/2)*a + 1
2*(a*sin(d*x + c) + a)^(3/2)*a^2 - 8*sqrt(a*sin(d*x + c) + a)*a^3))/(a*d)
```

Fricas [A]
time = 0.37, size = 208, normalized size = 1.31

$$\frac{105\left(\sqrt{2}\,a^2\cos\left(dx+c\right)^4+2\,\sqrt{2}\,a^2\cos\left(dx+c\right)^2\sin\left(dx+c\right)-2\,\sqrt{2}\,a^2\cos\left(dx+c\right)^2\right)\sqrt{a}\,\log\left(-\frac{a\sin(dx+c)+2\,\sqrt{2}\,\sqrt{a\sin\left(dx+c\right)+a}\,\sqrt{a}+3\,a}{\sin(dx+c)-1}\right)-4\left(245\,a^2\cos\left(dx+c\right)^2-160\,a^2-7\left(15\,a^2\cos\left(dx+c\right)^2-32\,a^2\right)\sin\left(dx+c\right)\right)\sqrt{a\sin\left(dx+c\right)+a}}{1536\left(d\cos\left(dx+c\right)^4+2\,d\cos\left(dx+c\right)^2\sin\left(dx+c\right)-2\,d\cos\left(dx+c\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/1536*(105*(sqrt(2)*a^2*cos(d*x + c)^4 + 2*sqrt(2)*a^2*cos(d*x + c)^2*sin(
d*x + c) - 2*sqrt(2)*a^2*cos(d*x + c)^2)*sqrt(a)*log(-(a*sin(d*x + c) + 2*s
qrt(2)*sqrt(a*sin(d*x + c) + a)*sqrt(a) + 3*a)/(sin(d*x + c) - 1)) - 4*(245
*a^2*cos(d*x + c)^2 - 160*a^2 - 7*(15*a^2*cos(d*x + c)^2 - 32*a^2)*sin(d*x
+ c))*sqrt(a*sin(d*x + c) + a))/(d*cos(d*x + c)^4 + 2*d*cos(d*x + c)^2*sin(
d*x + c) - 2*d*cos(d*x + c)^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7*(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 5.73, size = 144, normalized size = 0.91

$$-\frac{\sqrt{2}\,a^{\frac{5}{2}}\left(\frac{96}{\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+\frac{2\left(57\,\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-136\,\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+87\,\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^3}-105\,\log\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)+105\,\log\left(-\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{1536\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -1/1536*sqrt(2)*a^(5/2)*(96/cos(-1/4*pi + 1/2*d*x + 1/2*c) + 2*(57*cos(-1/4
*pi + 1/2*d*x + 1/2*c)^5 - 136*cos(-1/4*pi + 1/2*d*x + 1/2*c)^3 + 87*cos(-1
/4*pi + 1/2*d*x + 1/2*c))/(cos(-1/4*pi + 1/2*d*x + 1/2*c)^2 - 1)^3 - 105*lo
g(cos(-1/4*pi + 1/2*d*x + 1/2*c) + 1) + 105*log(-cos(-1/4*pi + 1/2*d*x + 1/
2*c) + 1))*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{5/2}}{{\cos\left(c+d\,x\right)}^7} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(5/2)/cos(c + d*x)^7,x)
```

```
[Out] int((a + a*sin(c + d*x))^(5/2)/cos(c + d*x)^7, x)
```

3.139 $\int \cos^7(c+dx)(a+a\sin(c+dx))^{7/2}\,dx$

Optimal. Leaf size=97

$$\frac{16(a+a\sin(c+dx))^{15/2}}{15a^4d}-\frac{24(a+a\sin(c+dx))^{17/2}}{17a^5d}+\frac{12(a+a\sin(c+dx))^{19/2}}{19a^6d}-\frac{2(a+a\sin(c+dx))^{21/2}}{21a^7d}$$

```
[Out] 16/15*(a+a*sin(d*x+c))^(15/2)/a^4/d-24/17*(a+a*sin(d*x+c))^(17/2)/a^5/d+12/
19*(a+a*sin(d*x+c))^(19/2)/a^6/d-2/21*(a+a*sin(d*x+c))^(21/2)/a^7/d
```

Rubi [A]
time = 0.06, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2746, 45}

$$-\frac{2(a\sin(c+dx)+a)^{21/2}}{21a^7d}+\frac{12(a\sin(c+dx)+a)^{19/2}}{19a^6d}-\frac{24(a\sin(c+dx)+a)^{17/2}}{17a^5d}+\frac{16(a\sin(c+dx)+a)^{15/2}}{15a^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] (16*(a + a*Sin[c + d*x])^(15/2))/(15*a^4*d) - (24*(a + a*Sin[c + d*x])^(17/
2))/(17*a^5*d) + (12*(a + a*Sin[c + d*x])^(19/2))/(19*a^6*d) - (2*(a + a*Si
n[c + d*x])^(21/2))/(21*a^7*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\int \cos^7(c+dx)(a+a\sin(c+dx))^{7/2}\,dx = \frac{\text{Subst}\left(\int (a-x)^3(a+x)^{13/2}\,dx, x, a\sin(c+dx)\right)}{a^7d}$$
$$= \frac{\text{Subst}\left(\int \left(8a^3(a+x)^{13/2} - 12a^2(a+x)^{15/2} + 6a(a+x)^{17/2} - (a\right.}{a^7d}$$
$$= \frac{16(a+a\sin(c+dx))^{15/2}}{15a^4d} - \frac{24(a+a\sin(c+dx))^{17/2}}{17a^5d} + \frac{12(a+a}{}$$

Mathematica [A]
time = 0.38, size = 64, normalized size = 0.66

$$-\frac{2a^3(1+\sin(c+dx))^7\sqrt{a(1+\sin(c+dx))}\,(-3243+7365\sin(c+dx)-5865\sin^2(c+dx)+1615\sin^3(c+dx))}{33915d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] (-2*a^3*(1 + Sin[c + d*x])^7*Sqrt[a*(1 + Sin[c + d*x])]*(-3243 + 7365*Sin[c
 + d*x] - 5865*Sin[c + d*x]^2 + 1615*Sin[c + d*x]^3))/(33915*d)
```

Maple [A]
time = 0.31, size = 57, normalized size = 0.59

method	result	size
default	$\frac{2(a+a\sin(dx+c))^{\frac{15}{2}}\left(1615\left(\cos^2(dx+c)\right)\sin(dx+c)-5865\left(\cos^2(dx+c)\right)-8980\sin(dx+c)+9108\right)}{33915a^4d}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*(a+a*sin(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/33915/a^4*(a+a*sin(d*x+c))^(15/2)*(1615*cos(d*x+c)^2*sin(d*x+c)-5865*cos(
d*x+c)^2-8980*sin(d*x+c)+9108)/d
```

Maxima [A]
time = 0.32, size = 72, normalized size = 0.74

$$-\frac{2\left(1615\,(a\sin(dx+c)+a)^{\frac{21}{2}} - 10710\,(a\sin(dx+c)+a)^{\frac{19}{2}}a + 23940\,(a\sin(dx+c)+a)^{\frac{17}{2}}a^2 - 18088\,(a\sin(dx+c)+a)^{\frac{15}{2}}a^3\right)}{33915\,a^7d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] -2/33915*(1615*(a*sin(d*x + c) + a)^(21/2) - 10710*(a*sin(d*x + c) + a)^(19
/2)*a + 23940*(a*sin(d*x + c) + a)^(17/2)*a^2 - 18088*(a*sin(d*x + c) + a)^
(15/2)*a^3)/(a^7*d)
```

Fricas [A]
time = 0.38, size = 154, normalized size = 1.59

$$\frac{2\,\left(1615\,a^{3}\cos\left(dx+c\right)^{10}-8300\,a^{3}\cos\left(dx+c\right)^{8}+264\,a^{3}\cos\left(dx+c\right)^{6}+448\,a^{3}\cos\left(dx+c\right)^{4}+1024\,a^{3}\cos\left(dx+c\right)^{2}+8192\,a^{3}-8\,\left(680\,a^{3}\cos\left(dx+c\right)^{8}-429\,a^{3}\cos\left(dx+c\right)^{6}-504\,a^{3}\cos\left(dx+c\right)^{4}-640\,a^{3}\cos\left(dx+c\right)^{2}-1024\,a^{3}\right)\sin\left(dx+c\right)\right)\sqrt{a\sin\left(dx+c\right)+a}}{33915\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 2/33915*(1615*a^3*cos(d*x + c)^10 - 8300*a^3*cos(d*x + c)^8 + 264*a^3*cos(d
*x + c)^6 + 448*a^3*cos(d*x + c)^4 + 1024*a^3*cos(d*x + c)^2 + 8192*a^3 - 8
*(680*a^3*cos(d*x + c)^8 - 429*a^3*cos(d*x + c)^6 - 504*a^3*cos(d*x + c)^4
- 640*a^3*cos(d*x + c)^2 - 1024*a^3)*sin(d*x + c))*sqrt(a*sin(d*x + c) + a)
/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*(a+a*sin(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 5.62, size = 140, normalized size = 1.44

$$-\frac{2048\,\sqrt{2}\,\left(1615\,a^{3}\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{21}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)-5355\,a^{3}\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{19}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)+5985\,a^{3}\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{17}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)-2261\,a^{3}\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{15}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\right)\sqrt{a}}{33915\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sin(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] -2048/33915*sqrt(2)*(1615*a^3*cos(-1/4*pi + 1/2*d*x + 1/2*c)^21*sgn(cos(-1/
4*pi + 1/2*d*x + 1/2*c)) - 5355*a^3*cos(-1/4*pi + 1/2*d*x + 1/2*c)^19*sgn(c
os(-1/4*pi + 1/2*d*x + 1/2*c)) + 5985*a^3*cos(-1/4*pi + 1/2*d*x + 1/2*c)^17
*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c)) - 2261*a^3*cos(-1/4*pi + 1/2*d*x + 1/2
*c)^15*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c)))*sqrt(a)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\cos\left(c+d\,x\right)}^7\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{7/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7*(a + a*sin(c + d*x))^(7/2),x)
```

```
[Out] int(cos(c + d*x)^7*(a + a*sin(c + d*x))^(7/2), x)
```

3.140 $\int \cos^6(c+dx)(a+a\sin(c+dx))^{7/2}\,dx$

Optimal. Leaf size=223

$$-\frac{131072a^7\cos^7(c+dx)}{969969d(a+a\sin(c+dx))^{7/2}}-\frac{32768a^6\cos^7(c+dx)}{138567d(a+a\sin(c+dx))^{5/2}}-\frac{12288a^5\cos^7(c+dx)}{46189d(a+a\sin(c+dx))^{3/2}}-\frac{1024a^4}{4199d\sqrt{a}}$$

```
[Out] -131072/969969*a^7*cos(d*x+c)^7/d/(a+a*sin(d*x+c))^(7/2)-32768/138567*a^6*c
os(d*x+c)^7/d/(a+a*sin(d*x+c))^(5/2)-12288/46189*a^5*cos(d*x+c)^7/d/(a+a*si
n(d*x+c))^(3/2)-48/323*a^2*cos(d*x+c)^7*(a+a*sin(d*x+c))^(3/2)/d-2/19*a*cos
(d*x+c)^7*(a+a*sin(d*x+c))^(5/2)/d-1024/4199*a^4*cos(d*x+c)^7/d/(a+a*sin(d*
x+c))^(1/2)-64/323*a^3*cos(d*x+c)^7*(a+a*sin(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.29, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2753, 2752}

$$-\frac{131072a^7\cos^7(c+dx)}{969969d(a\sin(c+dx)+a)^{7/2}}-\frac{32768a^6\cos^7(c+dx)}{138567d(a\sin(c+dx)+a)^{5/2}}-\frac{12288a^5\cos^7(c+dx)}{46189d(a\sin(c+dx)+a)^{3/2}}-\frac{1024a^4\cos^7(c+dx)}{4199d\sqrt{a\sin(c+dx)+a}}-\frac{64a^3\cos^7(c+dx)\sqrt{a\sin(c+dx)+a}}{323d}-\frac{48a^2\cos^7(c+dx)(a\sin(c+dx)+a)^{3/2}}{323d}-\frac{2a\cos^7(c+dx)(a\sin(c+dx)+a)^{5/2}}{19d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] (-131072*a^7*Cos[c + d*x]^7)/(969969*d*(a + a*Sin[c + d*x])^(7/2)) - (32768
*a^6*Cos[c + d*x]^7)/(138567*d*(a + a*Sin[c + d*x])^(5/2)) - (12288*a^5*Cos
[c + d*x]^7)/(46189*d*(a + a*Sin[c + d*x])^(3/2)) - (1024*a^4*Cos[c + d*x]^
7)/(4199*d*Sqrt[a + a*Sin[c + d*x]]) - (64*a^3*Cos[c + d*x]^7*Sqrt[a + a*Si
n[c + d*x]])/(323*d) - (48*a^2*Cos[c + d*x]^7*(a + a*Sin[c + d*x])^(3/2))/(
323*d) - (2*a*Cos[c + d*x]^7*(a + a*Sin[c + d*x])^(5/2))/(19*d)
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2753

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && N
eQ[m + p, 0]
```

Rubi steps

$$\int \cos^6(c+dx)(a+a\sin(c+dx))^{7/2}\,dx = -\frac{2a\cos^7(c+dx)(a+a\sin(c+dx))^{5/2}}{19d} + \frac{1}{19}(24a)\int \cos^6(c+dx)$$

$$= -\frac{48a^2\cos^7(c+dx)(a+a\sin(c+dx))^{3/2}}{323d} - \frac{2a\cos^7(c+dx)(a+a}{19d}$$

$$= -\frac{64a^3\cos^7(c+dx)\sqrt{a+a\sin(c+dx)}}{323d} - \frac{48a^2\cos^7(c+dx)(a+}{323d}$$

$$= -\frac{1024a^4\cos^7(c+dx)}{4199d\sqrt{a+a\sin(c+dx)}} - \frac{64a^3\cos^7(c+dx)\sqrt{a+a\sin(c+}}{323d}$$

$$= -\frac{12288a^5\cos^7(c+dx)}{46189d(a+a\sin(c+dx))^{3/2}} - \frac{1024a^4\cos^7(c+dx)}{4199d\sqrt{a+a\sin(c+dx)}} - \frac{64}{}$$

$$= -\frac{32768a^6\cos^7(c+dx)}{138567d(a+a\sin(c+dx))^{5/2}} - \frac{12288a^5\cos^7(c+dx)}{46189d(a+a\sin(c+dx))^{3/2}} -$$

$$= -\frac{131072a^7\cos^7(c+dx)}{969969d(a+a\sin(c+dx))^{7/2}} - \frac{32768a^6\cos^7(c+dx)}{138567d(a+a\sin(c+dx))^{5/2}}$$

Mathematica [A]
time = 0.34, size = 102, normalized size = 0.46

$$-\frac{2a^3\cos^7(c+dx)\sqrt{a(1+\sin(c+dx))}\,(646739+1778602\sin(c+dx)+2546901\sin^2(c+dx)+2244396\sin^3(c+dx)+1222221\sin^4(c+dx)+378378\sin^5(c+dx)+51051\sin^6(c+dx))}{969969d(1+\sin(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] (-2*a^3*Cos[c + d*x]^7*Sqrt[a*(1 + Sin[c + d*x])]*(646739 + 1778602*Sin[c +
 d*x] + 2546901*Sin[c + d*x]^2 + 2244396*Sin[c + d*x]^3 + 1222221*Sin[c + d
*x]^4 + 378378*Sin[c + d*x]^5 + 51051*Sin[c + d*x]^6))/(969969*d*(1 + Sin[c
 + d*x])^4)
```

Maple [A]
time = 0.40, size = 107, normalized size = 0.48

method	result
default	$-\frac{2(1+\sin(dx+c))a^4(\sin(dx+c)-1)^4(51051(\sin^6(dx+c))+378378(\sin^5(dx+c))+1222221(\sin^4(dx+c))+2244396(\sin^3(dx+c))+25469}{969969\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(a+a*sin(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/969969*(1+sin(d*x+c))*a^4*(sin(d*x+c)-1)^4*(51051*sin(d*x+c)^6+378378*si
n(d*x+c)^5+1222221*sin(d*x+c)^4+2244396*sin(d*x+c)^3+2546901*sin(d*x+c)^2+1
778602*sin(d*x+c)+646739)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(7/2)*cos(d*x + c)^6, x)
```

Fricas [A]
time = 0.36, size = 296, normalized size = 1.33

$$\frac{2\,\left(51051\,a^{3}\cos\left(dx+c\right)^{10}+225225\,a^{3}\cos\left(dx+c\right)^{9}-270270\,a^{3}\cos\left(dx+c\right)^{8}-562716\,a^{3}\cos\left(dx+c\right)^{7}+10752\,a^{3}\cos\left(dx+c\right)^{6}-14336\,a^{3}\cos\left(dx+c\right)^{5}+20480\,a^{3}\cos\left(dx+c\right)^{4}-32768\,a^{3}\cos\left(dx+c\right)^{3}+65536\,a^{3}\cos\left(dx+c\right)^{2}-262144\,a^{3}\cos\left(dx+c\right)-524288\,a^{3}+\left(51051\,a^{3}\cos\left(dx+c\right)^{9}-174174\,a^{3}\cos\left(dx+c\right)^{8}-444444\,a^{3}\cos\left(dx+c\right)^{7}+118272\,a^{3}\cos\left(dx+c\right)^{6}+129024\,a^{3}\cos\left(dx+c\right)^{5}+143360\,a^{3}\cos\left(dx+c\right)^{4}+163840\,a^{3}\cos\left(dx+c\right)^{3}+196608\,a^{3}\cos\left(dx+c\right)^{2}+262144\,a^{3}\cos\left(dx+c\right)+524288\,a^{3}\right)\sin\left(dx+c\right)\right)\sqrt{a\sin\left(dx+c\right)+a}}{969969\,\left(d\cos\left(dx+c\right)+d\sin\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 2/969969*(51051*a^3*cos(d*x + c)^10 + 225225*a^3*cos(d*x + c)^9 - 270270*a^
3*cos(d*x + c)^8 - 562716*a^3*cos(d*x + c)^7 + 10752*a^3*cos(d*x + c)^6 - 1
4336*a^3*cos(d*x + c)^5 + 20480*a^3*cos(d*x + c)^4 - 32768*a^3*cos(d*x + c)
^3 + 65536*a^3*cos(d*x + c)^2 - 262144*a^3*cos(d*x + c) - 524288*a^3 + (510
51*a^3*cos(d*x + c)^9 - 174174*a^3*cos(d*x + c)^8 - 444444*a^3*cos(d*x + c)
^7 + 118272*a^3*cos(d*x + c)^6 + 129024*a^3*cos(d*x + c)^5 + 143360*a^3*cos
(d*x + c)^4 + 163840*a^3*cos(d*x + c)^3 + 196608*a^3*cos(d*x + c)^2 + 26214
4*a^3*cos(d*x + c) + 524288*a^3)*sin(d*x + c))*sqrt(a*sin(d*x + c) + a)/(d*
cos(d*x + c) + d*sin(d*x + c) + d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(a+a*sin(d*x+c))**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 10626 deep
```

Giac [A]
time = 6.41, size = 236, normalized size = 1.06

$$\frac{1024\,\sqrt{2}\,\left(51051\,a^{3}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{19}-342342\,a^{3}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{17}+969969\,a^{3}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{15}-1492260\,a^{3}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}+1322685\,a^{3}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-646646\,a^{3}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+138567\,a^{3}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}\right)\sqrt{a}}{969969\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^6*(a+a*sin(d*x+c))^(7/2),x, algorithm="giac")

[Out] 1024/969969*sqrt(2)*(51051*a^3*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*d*x + 1/2*c)^19 - 342342*a^3*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*d*x + 1/2*c)^17 + 969969*a^3*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*d*x + 1/2*c)^15 - 1492260*a^3*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*d*x + 1/2*c)^13 + 1322685*a^3*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*d*x + 1/2*c)^11 - 646646*a^3*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*d*x + 1/2*c)^9 + 138567*a^3*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*d*x + 1/2*c)^7)*sqrt(a)/d

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(c+d\,x\right)^6\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{7/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^6*(a + a*sin(c + d*x))^(7/2),x)

[Out] int(cos(c + d*x)^6*(a + a*sin(c + d*x))^(7/2), x)

3.141 $\int \cos^5(c+dx)(a+a\sin(c+dx))^{7/2}\,dx$

Optimal. Leaf size=73

$$\frac{8(a+a\sin(c+dx))^{13/2}}{13a^3d}-\frac{8(a+a\sin(c+dx))^{15/2}}{15a^4d}+\frac{2(a+a\sin(c+dx))^{17/2}}{17a^5d}$$

```
[Out] 8/13*(a+a*sin(d*x+c))^(13/2)/a^3/d-8/15*(a+a*sin(d*x+c))^(15/2)/a^4/d+2/17*
(a+a*sin(d*x+c))^(17/2)/a^5/d
```

Rubi [A]

time = 0.05, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2746, 45}

$$\frac{2(a\sin(c+dx)+a)^{17/2}}{17a^5d}-\frac{8(a\sin(c+dx)+a)^{15/2}}{15a^4d}+\frac{8(a\sin(c+dx)+a)^{13/2}}{13a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] (8*(a + a*Sin[c + d*x])^(13/2))/(13*a^3*d) - (8*(a + a*Sin[c + d*x])^(15/2)
)/(15*a^4*d) + (2*(a + a*Sin[c + d*x])^(17/2))/(17*a^5*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\int \cos^5(c+dx)(a+a\sin(c+dx))^{7/2}\,dx=\frac{\text{Subst}\left(\int (a-x)^2(a+x)^{11/2}\,dx,x,a\sin(c+dx)\right)}{a^5d}$$

$$=\frac{\text{Subst}\left(\int \left(4a^2(a+x)^{11/2}-4a(a+x)^{13/2}+(a+x)^{15/2}\right)\,dx,x,a\,\text{s}\right.}{a^5d}$$

$$=\frac{8(a+a\sin(c+dx))^{13/2}}{13a^3d}-\frac{8(a+a\sin(c+dx))^{15/2}}{15a^4d}+\frac{2(a+a\,\text{sin}}{1}$$

Mathematica [A]
time = 0.17, size = 54, normalized size = 0.74

$$\frac{2a^3(1+\sin(c+dx))^6\sqrt{a(1+\sin(c+dx))}\,(331-494\sin(c+dx)+195\sin^2(c+dx))}{3315d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] (2*a^3*(1 + Sin[c + d*x])^6*Sqrt[a*(1 + Sin[c + d*x])]*(331 - 494*Sin[c + d
*x] + 195*Sin[c + d*x]^2))/(3315*d)
```

Maple [A]
time = 0.27, size = 41, normalized size = 0.56

method	result	size
default	$-\frac{2(a+a\sin(dx+c))^{\frac{13}{2}}\left(195\left(\cos^2(dx+c)\right)+494\sin(dx+c)-526\right)}{3315a^3d}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+a*sin(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3315/a^3*(a+a*sin(d*x+c))^(13/2)*(195*cos(d*x+c)^2+494*sin(d*x+c)-526)/d
```

Maxima [A]
time = 0.29, size = 55, normalized size = 0.75

$$\frac{2\left(195\left(a\sin\left(dx+c\right)+a\right)^{\frac{17}{2}}-884\left(a\sin\left(dx+c\right)+a\right)^{\frac{15}{2}}a+1020\left(a\sin\left(dx+c\right)+a\right)^{\frac{13}{2}}a^2\right)}{3315\,a^5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] 2/3315*(195*(a*sin(d*x + c) + a)^(17/2) - 884*(a*sin(d*x + c) + a)^(15/2)*a
 + 1020*(a*sin(d*x + c) + a)^(13/2)*a^2)/(a^5*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(61) = 122$.
time = 0.38, size = 128, normalized size = 1.75

$$\frac{2\left(195\,a^3\cos\left(dx+c\right)^8-1072\,a^3\cos\left(dx+c\right)^6+56\,a^3\cos\left(dx+c\right)^4+128\,a^3\cos\left(dx+c\right)^2+1024\,a^3-4\left(169\,a^3\cos\left(dx+c\right)^6-126\,a^3\cos\left(dx+c\right)^4-160\,a^3\cos\left(dx+c\right)^2-256\,a^3\right)\sin\left(dx+c\right)\right)\sqrt{a\sin\left(dx+c\right)+a}}{3315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 2/3315*(195*a^3*cos(d*x + c)^8 - 1072*a^3*cos(d*x + c)^6 + 56*a^3*cos(d*x +
 c)^4 + 128*a^3*cos(d*x + c)^2 + 1024*a^3 - 4*(169*a^3*cos(d*x + c)^6 - 126
*a^3*cos(d*x + c)^4 - 160*a^3*cos(d*x + c)^2 - 256*a^3)*sin(d*x + c))*sqrt(
a*sin(d*x + c) + a)/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+a*sin(d*x+c))**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8855 deep
```

Giac [A]
time = 7.04, size = 108, normalized size = 1.48

$$\frac{512\,\sqrt{2}\left(195\,a^{3}\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{17}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)-442\,a^{3}\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{15}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)+255\,a^{3}\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\right)\sqrt{a}}{3315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] 512/3315*sqrt(2)*(195*a^3*cos(-1/4*pi + 1/2*d*x + 1/2*c)^17*sgn(cos(-1/4*pi
 + 1/2*d*x + 1/2*c)) - 442*a^3*cos(-1/4*pi + 1/2*d*x + 1/2*c)^15*sgn(cos(-1
/4*pi + 1/2*d*x + 1/2*c)) + 255*a^3*cos(-1/4*pi + 1/2*d*x + 1/2*c)^13*sgn(c
os(-1/4*pi + 1/2*d*x + 1/2*c)))*sqrt(a)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^5\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{7/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(a + a*sin(c + d*x))^(7/2),x)
```

```
[Out] int(cos(c + d*x)^5*(a + a*sin(c + d*x))^(7/2), x)
```

3.142 $\int \cos^4(c+dx)(a+a\sin(c+dx))^{7/2}\,dx$

Optimal. Leaf size=191

$$-\frac{16384a^6\cos^5(c+dx)}{45045d(a+a\sin(c+dx))^{5/2}}-\frac{4096a^5\cos^5(c+dx)}{9009d(a+a\sin(c+dx))^{3/2}}-\frac{512a^4\cos^5(c+dx)}{1287d\sqrt{a+a\sin(c+dx)}}-\frac{128a^3\cos^5(c+dx}{}$$

```
[Out] -16384/45045*a^6*cos(d*x+c)^5/d/(a+a*sin(d*x+c))^(5/2)-4096/9009*a^5*cos(d*
x+c)^5/d/(a+a*sin(d*x+c))^(3/2)-8/39*a^2*cos(d*x+c)^5*(a+a*sin(d*x+c))^(3/2
)/d-2/15*a*cos(d*x+c)^5*(a+a*sin(d*x+c))^(5/2)/d-512/1287*a^4*cos(d*x+c)^5/
d/(a+a*sin(d*x+c))^(1/2)-128/429*a^3*cos(d*x+c)^5*(a+a*sin(d*x+c))^(1/2)/d
```

Rubi [A]

time = 0.25, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2753, 2752}

$$-\frac{16384a^6\cos^5(c+dx)}{45045d(a\sin(c+dx)+a)^{5/2}}-\frac{4096a^5\cos^5(c+dx)}{9009d(a\sin(c+dx)+a)^{3/2}}-\frac{512a^4\cos^5(c+dx)}{1287d\sqrt{a\sin(c+dx)+a}}-\frac{128a^3\cos^5(c+dx)\sqrt{a\sin(c+dx)+a}}{429d}-\frac{8a^2\cos^5(c+dx)(a\sin(c+dx)+a)^{3/2}}{39d}-\frac{2a\cos^5(c+dx)(a\sin(c+dx)+a)^{5/2}}{15d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] (-16384*a^6*Cos[c + d*x]^5)/(45045*d*(a + a*Sin[c + d*x])^(5/2)) - (4096*a^
5*Cos[c + d*x]^5)/(9009*d*(a + a*Sin[c + d*x])^(3/2)) - (512*a^4*Cos[c + d*
x]^5)/(1287*d*Sqrt[a + a*Sin[c + d*x]]) - (128*a^3*Cos[c + d*x]^5*Sqrt[a +
a*Sin[c + d*x]])/(429*d) - (8*a^2*Cos[c + d*x]^5*(a + a*Sin[c + d*x])^(3/2)
)/(39*d) - (2*a*Cos[c + d*x]^5*(a + a*Sin[c + d*x])^(5/2))/(15*d)
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2753

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && N
eQ[m + p, 0]
```

Rubi steps

$$\int \cos^4(c+dx)(a+a\sin(c+dx))^{7/2}\,dx = -\frac{2a\cos^5(c+dx)(a+a\sin(c+dx))^{5/2}}{15d} + \frac{1}{3}(4a)\int \cos^4(c+dx)($$
$$= -\frac{8a^2\cos^5(c+dx)(a+a\sin(c+dx))^{3/2}}{39d} - \frac{2a\cos^5(c+dx)(a+a}{15d}$$
$$= -\frac{128a^3\cos^5(c+dx)\sqrt{a+a\sin(c+dx)}}{429d} - \frac{8a^2\cos^5(c+dx)(a+}{39d}$$
$$= -\frac{512a^4\cos^5(c+dx)}{1287d\sqrt{a+a\sin(c+dx)}} - \frac{128a^3\cos^5(c+dx)\sqrt{a+a\sin(c+}}{429d}$$
$$= -\frac{4096a^5\cos^5(c+dx)}{9009d(a+a\sin(c+dx))^{3/2}} - \frac{512a^4\cos^5(c+dx)}{1287d\sqrt{a+a\sin(c+dx)}} - \frac{12}{}$$
$$= -\frac{16384a^6\cos^5(c+dx)}{45045d(a+a\sin(c+dx))^{5/2}} - \frac{4096a^5\cos^5(c+dx)}{9009d(a+a\sin(c+dx))^{3/2}} -$$

Mathematica [A]
time = 0.15, size = 92, normalized size = 0.48

$$-\frac{2a^3\cos^5(c+dx)\sqrt{a(1+\sin(c+dx))}\,(41735+81815\sin(c+dx)+86870\sin^2(c+dx)+55230\sin^3(c+dx)+19635\sin^4(c+dx)+3003\sin^5(c+dx))}{45045d(1+\sin(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] (-2*a^3*Cos[c + d*x]^5*Sqrt[a*(1 + Sin[c + d*x])]*(41735 + 81815*Sin[c + d*
x] + 86870*Sin[c + d*x]^2 + 55230*Sin[c + d*x]^3 + 19635*Sin[c + d*x]^4 + 3
003*Sin[c + d*x]^5))/(45045*d*(1 + Sin[c + d*x])^3)
```

Maple [A]
time = 0.42, size = 97, normalized size = 0.51

method	result
default	$\frac{2(1+\sin(dx+c))a^4(\sin(dx+c)-1)^3(3003(\sin^5(dx+c))+19635(\sin^4(dx+c))+55230(\sin^3(dx+c))+86870(\sin^2(dx+c))+81815\sin(dx}{45045\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+a*sin(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/45045*(1+sin(d*x+c))*a^4*(sin(d*x+c)-1)^3*(3003*sin(d*x+c)^5+19635*sin(d*
x+c)^4+55230*sin(d*x+c)^3+86870*sin(d*x+c)^2+81815*sin(d*x+c)+41735)/cos(d*
x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(7/2)*cos(d*x + c)^4, x)
```

Fricas [A]
time = 0.37, size = 244, normalized size = 1.28

$$\frac{2\,\left(3003\,a^{3}\cos\left(dx+c\right)^{8}+13629\,a^{3}\cos\left(dx+c\right)^{7}-17346\,a^{3}\cos\left(dx+c\right)^{6}-36932\,a^{3}\cos\left(dx+c\right)^{5}+1280\,a^{3}\cos\left(dx+c\right)^{4}-2048\,a^{3}\cos\left(dx+c\right)^{3}+4096\,a^{3}\cos\left(dx+c\right)^{2}-16384\,a^{3}\cos\left(dx+c\right)-32768\,a^{3}+\left(3003\,a^{3}\cos\left(dx+c\right)^{7}-10626\,a^{3}\cos\left(dx+c\right)^{6}-27972\,a^{3}\cos\left(dx+c\right)^{5}+8960\,a^{3}\cos\left(dx+c\right)^{4}+10240\,a^{3}\cos\left(dx+c\right)^{3}+12288\,a^{3}\cos\left(dx+c\right)^{2}+16384\,a^{3}\cos\left(dx+c\right)+32768\,a^{3}\right)\sin\left(dx+c\right)\right)\sqrt{a\sin\left(dx+c\right)+a}}{45045\,\left(d\cos\left(dx+c\right)+d\sin\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 2/45045*(3003*a^3*cos(d*x + c)^8 + 13629*a^3*cos(d*x + c)^7 - 17346*a^3*cos
(d*x + c)^6 - 36932*a^3*cos(d*x + c)^5 + 1280*a^3*cos(d*x + c)^4 - 2048*a^3
*cos(d*x + c)^3 + 4096*a^3*cos(d*x + c)^2 - 16384*a^3*cos(d*x + c) - 32768*
a^3 + (3003*a^3*cos(d*x + c)^7 - 10626*a^3*cos(d*x + c)^6 - 27972*a^3*cos(d
*x + c)^5 + 8960*a^3*cos(d*x + c)^4 + 10240*a^3*cos(d*x + c)^3 + 12288*a^3*
cos(d*x + c)^2 + 16384*a^3*cos(d*x + c) + 32768*a^3)*sin(d*x + c))*sqrt(a*s
in(d*x + c) + a)/(d*cos(d*x + c) + d*sin(d*x + c) + d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+a*sin(d*x+c))**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 7315 deep
```

Giac [A]
time = 5.68, size = 204, normalized size = 1.07

$$-\frac{256\,\sqrt{2}\left(3003\,a^{3}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{15}-17325\,a^{3}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}+40950\,a^{3}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-50050\,a^{3}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+32175\,a^{3}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-9009\,a^{3}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}\right)\sqrt{a}}{45045\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] -256/45045*sqrt(2)*(3003*a^3*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*p
i + 1/2*d*x + 1/2*c)^15 - 17325*a^3*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin
```

```
(-1/4*pi + 1/2*d*x + 1/2*c)^13 + 40950*a^3*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*
c))*sin(-1/4*pi + 1/2*d*x + 1/2*c)^11 - 50050*a^3*sgn(cos(-1/4*pi + 1/2*d*x
 + 1/2*c))*sin(-1/4*pi + 1/2*d*x + 1/2*c)^9 + 32175*a^3*sgn(cos(-1/4*pi + 1
/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*d*x + 1/2*c)^7 - 9009*a^3*sgn(cos(-1/4*p
i + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*d*x + 1/2*c)^5)*sqrt(a)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+d\,x)^4\,(a+a\,\sin(c+d\,x))^{7/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(a + a*sin(c + d*x))^(7/2),x)
[Out] int(cos(c + d*x)^4*(a + a*sin(c + d*x))^(7/2), x)
```

3.143 $\int \cos^3(c+dx)(a+a\sin(c+dx))^{7/2}\,dx$

Optimal. Leaf size=49

$$\frac{4(a+a\sin(c+dx))^{11/2}}{11a^2d}-\frac{2(a+a\sin(c+dx))^{13/2}}{13a^3d}$$

```
[Out] 4/11*(a+a*sin(d*x+c))^(11/2)/a^2/d-2/13*(a+a*sin(d*x+c))^(13/2)/a^3/d
```

Rubi [A]
time = 0.05, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2746, 45}

$$\frac{4(a\sin(c+dx)+a)^{11/2}}{11a^2d}-\frac{2(a\sin(c+dx)+a)^{13/2}}{13a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] (4*(a + a*Sin[c + d*x])^(11/2))/(11*a^2*d) - (2*(a + a*Sin[c + d*x])^(13/2)
)/(13*a^3*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \cos^3(c+dx)(a+a\sin(c+dx))^{7/2}\,dx &= \frac{\text{Subst}\left(\int (a-x)(a+x)^{9/2}\,dx,x,a\sin(c+dx)\right)}{a^3d}\\ &= \frac{\text{Subst}\left(\int \left(2a(a+x)^{9/2}-(a+x)^{11/2}\right)\,dx,x,a\sin(c+dx)\right)}{a^3d}\\ &= \frac{4(a+a\sin(c+dx))^{11/2}}{11a^2d}-\frac{2(a+a\sin(c+dx))^{13/2}}{13a^3d}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 44, normalized size = 0.90

$$\frac{2\left(26a(a+a\sin(c+dx))^{11/2}-11(a+a\sin(c+dx))^{13/2}\right)}{143a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] (2*(26*a*(a + a*Sin[c + d*x])^(11/2) - 11*(a + a*Sin[c + d*x])^(13/2)))/(14
3*a^3*d)
```

Maple [A]
time = 0.32, size = 31, normalized size = 0.63

method	result	size
default	$-\frac{2(a+a\sin(dx+c))^{\frac{11}{2}}(11\sin(dx+c)-15)}{143a^2d}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*sin(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/143/a^2*(a+a*sin(d*x+c))^(11/2)*(11*sin(d*x+c)-15)/d
```

Maxima [A]
time = 0.28, size = 38, normalized size = 0.78

$$-\frac{2\left(11\left(a\sin\left(dx+c\right)+a\right)^{\frac{13}{2}}-26\left(a\sin\left(dx+c\right)+a\right)^{\frac{11}{2}}a\right)}{143\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] -2/143*(11*(a*sin(d*x + c) + a)^(13/2) - 26*(a*sin(d*x + c) + a)^(11/2)*a)/
(a^3*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 102 vs. 2(41) = 82.
time = 0.35, size = 102, normalized size = 2.08

$$\frac{2\left(11\,a^3\cos\left(dx+c\right)^6-68\,a^3\cos\left(dx+c\right)^4+8\,a^3\cos\left(dx+c\right)^2+64\,a^3-8\left(5\,a^3\cos\left(dx+c\right)^4-5\,a^3\cos\left(dx+c\right)^2-8\,a^3\right)\sin\left(dx+c\right)\right)\sqrt{a\sin\left(dx+c\right)+a}}{143\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 2/143*(11*a^3*cos(d*x + c)^6 - 68*a^3*cos(d*x + c)^4 + 8*a^3*cos(d*x + c)^2
 + 64*a^3 - 8*(5*a^3*cos(d*x + c)^4 - 5*a^3*cos(d*x + c)^2 - 8*a^3)*sin(d*x
 + c))*sqrt(a*sin(d*x + c) + a)/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*sin(d*x+c))**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5985 deep
```

Giac [A]
time = 7.04, size = 76, normalized size = 1.55

$$-\frac{128\,\sqrt{2}\left(11\,a^3\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)-13\,a^3\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\right)\sqrt{a}}{143\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] -128/143*sqrt(2)*(11*a^3*cos(-1/4*pi + 1/2*d*x + 1/2*c)^13*sgn(cos(-1/4*pi
+ 1/2*d*x + 1/2*c)) - 13*a^3*cos(-1/4*pi + 1/2*d*x + 1/2*c)^11*sgn(cos(-1/4
*pi + 1/2*d*x + 1/2*c)))*sqrt(a)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \cos\left(c+d\,x\right)^3\,\left(a+a\,\sin\left(c+d\,x\right)\right)^{7/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a + a*sin(c + d*x))^(7/2),x)
```

```
[Out] int(cos(c + d*x)^3*(a + a*sin(c + d*x))^(7/2), x)
```

3.144 $\int \cos^2(c+dx)(a+a\sin(c+dx))^{7/2}\,dx$

Optimal. Leaf size=159

$$-\frac{4096a^5\cos^3(c+dx)}{3465d(a+a\sin(c+dx))^{3/2}}-\frac{1024a^4\cos^3(c+dx)}{1155d\sqrt{a+a\sin(c+dx)}}-\frac{128a^3\cos^3(c+dx)\sqrt{a+a\sin(c+dx)}}{231d}-32a^2$$

```
[Out] -4096/3465*a^5*cos(d*x+c)^3/d/(a+a*sin(d*x+c))^(3/2)-32/99*a^2*cos(d*x+c)^3
*(a+a*sin(d*x+c))^(3/2)/d-2/11*a*cos(d*x+c)^3*(a+a*sin(d*x+c))^(5/2)/d-1024
/1155*a^4*cos(d*x+c)^3/d/(a+a*sin(d*x+c))^(1/2)-128/231*a^3*cos(d*x+c)^3*(a
+a*sin(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.20, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2753, 2752}

$$-\frac{4096a^5\cos^3(c+dx)}{3465d(a\sin(c+dx)+a)^{3/2}}-\frac{1024a^4\cos^3(c+dx)}{1155d\sqrt{a\sin(c+dx)+a}}-\frac{128a^3\cos^3(c+dx)\sqrt{a\sin(c+dx)+a}}{231d}-\frac{32a^2\cos^3(c+dx)(a\sin(c+dx)+a)^{3/2}}{99d}-\frac{2a\cos^3(c+dx)(a\sin(c+dx)+a)^{5/2}}{11d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] (-4096*a^5*Cos[c + d*x]^3)/(3465*d*(a + a*Sin[c + d*x])^(3/2)) - (1024*a^4*
Cos[c + d*x]^3)/(1155*d*Sqrt[a + a*Sin[c + d*x]]) - (128*a^3*Cos[c + d*x]^3
*Sqrt[a + a*Sin[c + d*x]])/(231*d) - (32*a^2*Cos[c + d*x]^3*(a + a*Sin[c +
d*x])^(3/2))/(99*d) - (2*a*Cos[c + d*x]^3*(a + a*Sin[c + d*x])^(5/2))/(11*d
)
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2753

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && N
eQ[m + p, 0]
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\sin(c+dx))^{7/2}\,dx = -\frac{2a\cos^3(c+dx)(a+a\sin(c+dx))^{5/2}}{11d} + \frac{1}{11}(16a)\int \cos^2(c+dx)$$
$$= -\frac{32a^2\cos^3(c+dx)(a+a\sin(c+dx))^{3/2}}{99d} - \frac{2a\cos^3(c+dx)(a+a}{11d}$$
$$= -\frac{128a^3\cos^3(c+dx)\sqrt{a+a\sin(c+dx)}}{231d} - \frac{32a^2\cos^3(c+dx)(a+}{99d}$$
$$= -\frac{1024a^4\cos^3(c+dx)}{1155d\sqrt{a+a\sin(c+dx)}} - \frac{128a^3\cos^3(c+dx)\sqrt{a+a\sin(c+}}{231d}$$
$$= -\frac{4096a^5\cos^3(c+dx)}{3465d(a+a\sin(c+dx))^{3/2}} - \frac{1024a^4\cos^3(c+dx)}{1155d\sqrt{a+a\sin(c+dx)}} - \frac{128}{}$$

Mathematica [**A**]
time = 0.14, size = 82, normalized size = 0.52

$$-\frac{2a^3\cos^3(c+dx)\sqrt{a(1+\sin(c+dx))}\,(5419+6396\sin(c+dx)+4530\sin^2(c+dx)+1820\sin^3(c+dx)+315\sin^4(c+dx))}{3465d(1+\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] (-2*a^3*Cos[c + d*x]^3*Sqrt[a*(1 + Sin[c + d*x])]*(5419 + 6396*Sin[c + d*x]
 + 4530*Sin[c + d*x]^2 + 1820*Sin[c + d*x]^3 + 315*Sin[c + d*x]^4))/(3465*d
*(1 + Sin[c + d*x])^2)
```

Maple [**A**]
time = 0.42, size = 87, normalized size = 0.55

method	result	size
default	$-\frac{2(1+\sin(dx+c))a^4(\sin(dx+c)-1)^2(315(\sin^4(dx+c))+1820(\sin^3(dx+c))+4530(\sin^2(dx+c))+6396\sin(dx+c)+5419)}{3465\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$	87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*sin(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3465*(1+sin(d*x+c))*a^4*(sin(d*x+c)-1)^2*(315*sin(d*x+c)^4+1820*sin(d*x+
c)^3+4530*sin(d*x+c)^2+6396*sin(d*x+c)+5419)/cos(d*x+c)/(a+a*sin(d*x+c))^(1
/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(7/2)*cos(d*x + c)^2, x)
```

Fricas [A]
time = 0.34, size = 192, normalized size = 1.21

$$\frac{2\left(315\,a^{3}\cos\left(dx+c\right)^{6}+1505\,a^{3}\cos\left(dx+c\right)^{5}-2150\,a^{3}\cos\left(dx+c\right)^{4}-4876\,a^{3}\cos\left(dx+c\right)^{3}+512\,a^{3}\cos\left(dx+c\right)^{2}-2048\,a^{3}\cos\left(dx+c\right)-4096\,a^{3}+\left(315\,a^{3}\cos\left(dx+c\right)^{5}-1190\,a^{3}\cos\left(dx+c\right)^{4}-3340\,a^{3}\cos\left(dx+c\right)^{3}+1536\,a^{3}\cos\left(dx+c\right)^{2}+2048\,a^{3}\cos\left(dx+c\right)+4096\,a^{3}\right)\sin\left(dx+c\right)\right)\sqrt{a\sin\left(dx+c\right)+a}}{3465\left(d\cos\left(dx+c\right)+d\sin\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 2/3465*(315*a^3*cos(d*x + c)^6 + 1505*a^3*cos(d*x + c)^5 - 2150*a^3*cos(d*x
 + c)^4 - 4876*a^3*cos(d*x + c)^3 + 512*a^3*cos(d*x + c)^2 - 2048*a^3*cos(d
*x + c) - 4096*a^3 + (315*a^3*cos(d*x + c)^5 - 1190*a^3*cos(d*x + c)^4 - 33
40*a^3*cos(d*x + c)^3 + 1536*a^3*cos(d*x + c)^2 + 2048*a^3*cos(d*x + c) + 4
096*a^3)*sin(d*x + c))*sqrt(a*sin(d*x + c) + a)/(d*cos(d*x + c) + d*sin(d*x
 + c) + d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*sin(d*x+c))**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4845 deep
```

Giac [A]
time = 6.48, size = 172, normalized size = 1.08

$$\frac{64\,\sqrt{2}\left(315\,a^{3}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-1540\,a^{3}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+2970\,a^{3}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-2772\,a^{3}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+1155\,a^{3}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}\right)\sqrt{a}}{3465\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] 64/3465*sqrt(2)*(315*a^3*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi +
1/2*d*x + 1/2*c)^11 - 1540*a^3*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4
*pi + 1/2*d*x + 1/2*c)^9 + 2970*a^3*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin
(-1/4*pi + 1/2*d*x + 1/2*c)^7 - 2772*a^3*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c)
)*sin(-1/4*pi + 1/2*d*x + 1/2*c)^5 + 1155*a^3*sgn(cos(-1/4*pi + 1/2*d*x + 1
/2*c))*sin(-1/4*pi + 1/2*d*x + 1/2*c)^3)*sqrt(a)/d
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^2\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{7/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a + a*sin(c + d*x))^(7/2),x)
[Out] int(cos(c + d*x)^2*(a + a*sin(c + d*x))^(7/2), x)
```

3.145 $\int \cos(c+dx)(a+a\sin(c+dx))^{7/2}\,dx$

Optimal. Leaf size=24

$$\frac{2(a+a\sin(c+dx))^{9/2}}{9ad}$$

[Out] 2/9*(a+a*sin(d*x+c))^(9/2)/a/d

Rubi [A]
time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2746, 32}

$$\frac{2(a\sin(c+dx)+a)^{9/2}}{9ad}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]*(a + a*Sin[c + d*x])^(7/2),x]

[Out] (2*(a + a*Sin[c + d*x])^(9/2))/(9*a*d)

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a+a\sin(c+dx))^{7/2}\,dx &= \frac{\text{Subst}\left(\int (a+x)^{7/2}\,dx, x, a\sin(c+dx)\right)}{ad}\\ &= \frac{2(a+a\sin(c+dx))^{9/2}}{9ad}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 24, normalized size = 1.00

$$\frac{2(a+a\sin(c+dx))^{9/2}}{9ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] (2*(a + a*Sin[c + d*x])^(9/2))/(9*a*d)
```

Maple [A]
time = 0.05, size = 21, normalized size = 0.88

method	result	size
derivativedivides	$\frac{2(a+a\sin(dx+c))^{\frac{9}{2}}}{9da}$	21
default	$\frac{2(a+a\sin(dx+c))^{\frac{9}{2}}}{9da}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*sin(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/9*(a+a*sin(d*x+c))^(9/2)/d/a
```

Maxima [A]
time = 0.29, size = 20, normalized size = 0.83

$$\frac{2\left(a\sin\left(dx+c\right)+a\right)^{\frac{9}{2}}}{9\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] 2/9*(a*sin(d*x + c) + a)^(9/2)/(a*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(20) = 40$.
time = 0.35, size = 74, normalized size = 3.08

$$\frac{2\left(a^{3}\cos\left(dx+c\right)^{4}-8\,a^{3}\cos\left(dx+c\right)^{2}+8\,a^{3}-4\left(a^{3}\cos\left(dx+c\right)^{2}-2\,a^{3}\right)\sin\left(dx+c\right)\right)\sqrt{a\sin\left(dx+c\right)+a}}{9\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 2/9*(a^3*cos(d*x + c)^4 - 8*a^3*cos(d*x + c)^2 + 8*a^3 - 4*(a^3*cos(d*x + c
)^2 - 2*a^3)*sin(d*x + c))*sqrt(a*sin(d*x + c) + a)/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3876 deep
```

Giac [A]
time = 2.26, size = 38, normalized size = 1.58

$$\frac{32\sqrt{2}\,a^{\frac{7}{2}}\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{9\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] 32/9*sqrt(2)*a^(7/2)*cos(-1/4*pi + 1/2*d*x + 1/2*c)^9*sgn(cos(-1/4*pi + 1/2
*d*x + 1/2*c))/d
```

Mupad [B]
time = 4.84, size = 20, normalized size = 0.83

$$\frac{2\,\left(a\,\left(\sin\left(c+d\,x\right)+1\right)\right)^{9/2}}{9\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + a*sin(c + d*x))^(7/2),x)
```

```
[Out] (2*(a*(sin(c + d*x) + 1))^(9/2))/(9*a*d)
```

3.146 $\int \sec(c+dx)(a+a\sin(c+dx))^{7/2}\,dx$

Optimal. Leaf size=110

$$\frac{8\sqrt{2}\,a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{2}\,\sqrt{a}}\right)}{d}-\frac{8a^3\sqrt{a+a\sin(c+dx)}}{d}-\frac{4a^2(a+a\sin(c+dx))^{3/2}}{3d}-\frac{2a(a+a\sin}{5}$$

```
[Out] -4/3*a^2*(a+a*sin(d*x+c))^(3/2)/d-2/5*a*(a+a*sin(d*x+c))^(5/2)/d+8*a^(7/2)*
arctanh(1/2*(a+a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)/d-8*a^3*(a+a*si
n(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.06, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2746, 52, 65, 212}

$$\frac{8\sqrt{2}\,a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\,\sqrt{a}}\right)}{d}-\frac{8a^3\sqrt{a\sin(c+dx)+a}}{d}-\frac{4a^2(a\sin(c+dx)+a)^{3/2}}{3d}-\frac{2a(a\sin(c+dx)+a)^{5/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] (8*Sqrt[2]*a^(7/2)*ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])])/d -
 (8*a^3*Sqrt[a + a*Sin[c + d*x]])/d - (4*a^2*(a + a*Sin[c + d*x])^(3/2))/(3
*d) - (2*a*(a + a*Sin[c + d*x])^(5/2))/(5*d)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\int \sec(c+dx)(a+a\sin(c+dx))^{7/2}\,dx = \frac{a\text{Subst}\left(\int \frac{(a+x)^{5/2}}{a-x}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= -\frac{2a(a+a\sin(c+dx))^{5/2}}{5d} + \frac{(2a^2)\,\text{Subst}\left(\int \frac{(a+x)^{3/2}}{a-x}\,dx, x, a\sin(c\right.}{d}$$

$$= -\frac{4a^2(a+a\sin(c+dx))^{3/2}}{3d} - \frac{2a(a+a\sin(c+dx))^{5/2}}{5d} + \frac{(4a^3)\,\text{Su}}{}$$

$$= -\frac{8a^3\sqrt{a+a\sin(c+dx)}}{d} - \frac{4a^2(a+a\sin(c+dx))^{3/2}}{3d} - \frac{2a(a+}{}$$

$$= -\frac{8a^3\sqrt{a+a\sin(c+dx)}}{d} - \frac{4a^2(a+a\sin(c+dx))^{3/2}}{3d} - \frac{2a(a+}{}$$

$$= \frac{8\sqrt{2}\,a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d} - \frac{8a^3\sqrt{a+a\sin(c+d}}{d}$$

Mathematica [A]
time = 0.21, size = 85, normalized size = 0.77

$$\frac{120\sqrt{2}\,a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a(1+\sin(c+dx))}}{\sqrt{2}\sqrt{a}}\right) - 2a^3\sqrt{a(1+\sin(c+dx))}\,(73+16\sin(c+dx)+3\sin^2(c+dx))}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sin[c + d*x])^(7/2),x]
```

[Out] (120*Sqrt[2]*a^(7/2)*ArcTanh[Sqrt[a*(1 + Sin[c + d*x])]/(Sqrt[2]*Sqrt[a])] - 2*a^3*Sqrt[a*(1 + Sin[c + d*x])]*(73 + 16*Sin[c + d*x] + 3*Sin[c + d*x]^2))/(15*d)

Maple [A]
time = 0.33, size = 83, normalized size = 0.75

method	result
default	$-\frac{2a\left(\frac{(a+a\sin(dx+c))^{\frac{5}{2}}}{5}+\frac{2a(a+a\sin(dx+c))^{\frac{3}{2}}}{3}+4a^2\sqrt{a+a\sin(dx+c)}-4a^{\frac{5}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+a\sin(dx+c)}\,\sqrt{}}{2\sqrt{a}}\right.\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)*(a+a*sin(d*x+c))^(7/2),x,method=_RETURNVERBOSE)

[Out] -2*a*(1/5*(a+a*sin(d*x+c))^(5/2)+2/3*a*(a+a*sin(d*x+c))^(3/2)+4*a^2*(a+a*sin(d*x+c))^(1/2)-4*a^(5/2)*2^(1/2)*arctanh(1/2*(a+a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2)))/d

Maxima [A]
time = 0.50, size = 115, normalized size = 1.05

$$-\frac{2\left(30\sqrt{2}\,a^{\frac{9}{2}}\log\left(-\frac{\sqrt{2}\,\sqrt{a}-\sqrt{a\sin(dx+c)+a}}{\sqrt{2}\,\sqrt{a}+\sqrt{a\sin(dx+c)+a}}\right)+3\,(a\sin(dx+c)+a)^{\frac{5}{2}}a^2+10\,(a\sin(dx+c)+a)^{\frac{3}{2}}a^3+60\,\sqrt{a\sin(dx+c)+a}\,a^4\right)}{15\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))^(7/2),x, algorithm="maxima")

[Out] -2/15*(30*sqrt(2)*a^(9/2)*log(-(sqrt(2)*sqrt(a) - sqrt(a*sin(d*x + c) + a))/(sqrt(2)*sqrt(a) + sqrt(a*sin(d*x + c) + a))) + 3*(a*sin(d*x + c) + a)^(5/2)*a^2 + 10*(a*sin(d*x + c) + a)^(3/2)*a^3 + 60*sqrt(a*sin(d*x + c) + a)*a^4)/(a*d)

Fricas [A]
time = 0.34, size = 102, normalized size = 0.93

$$\frac{2\left(30\sqrt{2}\,a^{\frac{7}{2}}\log\left(-\frac{a\sin(dx+c)+2\sqrt{2}\,\sqrt{a\sin(dx+c)+a}\,\sqrt{a}+3\,a}{\sin(dx+c)-1}\right)+\left(3\,a^3\cos(dx+c)^2-16\,a^3\sin(dx+c)-76\,a^3\right)\sqrt{a\sin(dx+c)+a}\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))^(7/2),x, algorithm="fricas")

[Out] 2/15*(30*sqrt(2)*a^(7/2)*log(-(a*sin(d*x + c) + 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*sqrt(a) + 3*a)/(sin(d*x + c) - 1)) + (3*a^3*cos(d*x + c)^2 - 16*a^3*sin(d*x + c) - 76*a^3)*sqrt(a*sin(d*x + c) + a))/d

<u>**Sympy**</u> [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

<u>**Giac**</u> [**A**]
time = 3.72, size = 107, normalized size = 0.97

$$-\frac{4\sqrt{2}\left(6\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{5}+10\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{3}+30\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)-15\log\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)+15\log\left(-\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)\right)a^{\frac{7}{2}}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] -4/15*sqrt(2)*(6*cos(-1/4*pi + 1/2*d*x + 1/2*c)^5 + 10*cos(-1/4*pi + 1/2*d*
x + 1/2*c)^3 + 30*cos(-1/4*pi + 1/2*d*x + 1/2*c) - 15*log(cos(-1/4*pi + 1/2
*d*x + 1/2*c) + 1) + 15*log(-cos(-1/4*pi + 1/2*d*x + 1/2*c) + 1))*a^(7/2)*s
gn(cos(-1/4*pi + 1/2*d*x + 1/2*c))/d
```

<u>**Mupad**</u> [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\,\sin(c+d\,x))^{7/2}}{\cos(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(7/2)/cos(c + d*x),x)
```

```
[Out] int((a + a*sin(c + d*x))^(7/2)/cos(c + d*x), x)
```

3.147 $\int \sec^2(c+dx)(a+a\sin(c+dx))^{7/2}\,dx$

Optimal. Leaf size=89

$$\frac{64a^3\sec(c+dx)\sqrt{a+a\sin(c+dx)}}{3d}-\frac{16a^2\sec(c+dx)(a+a\sin(c+dx))^{3/2}}{3d}-\frac{2a\sec(c+dx)(a+a\sin(c+dx))^{5/2}}{3d}$$

```
[Out] -16/3*a^2*sec(d*x+c)*(a+a*sin(d*x+c))^(3/2)/d-2/3*a*sec(d*x+c)*(a+a*sin(d*x
+c))^(5/2)/d+64/3*a^3*sec(d*x+c)*(a+a*sin(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.12, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2753, 2752}

$$\frac{64a^3\sec(c+dx)\sqrt{a\sin(c+dx)+a}}{3d}-\frac{16a^2\sec(c+dx)(a\sin(c+dx)+a)^{3/2}}{3d}-\frac{2a\sec(c+dx)(a\sin(c+dx)+a)^{5/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] (64*a^3*Sec[c + d*x]*Sqrt[a + a*Sin[c + d*x]])/(3*d) - (16*a^2*Sec[c + d*x]
*(a + a*Sin[c + d*x])^(3/2))/(3*d) - (2*a*Sec[c + d*x]*(a + a*Sin[c + d*x])
^(5/2))/(3*d)
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2753

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && N
eQ[m + p, 0]
```

Rubi steps

$$\int \sec^2(c+dx)(a+a\sin(c+dx))^{7/2}\,dx = -\frac{2a\sec(c+dx)(a+a\sin(c+dx))^{5/2}}{3d} + \frac{1}{3}(8a)\int \sec^2(c+dx)(a$$
$$= -\frac{16a^2\sec(c+dx)(a+a\sin(c+dx))^{3/2}}{3d} - \frac{2a\sec(c+dx)(a+a\,\mathrm{s}}{3d}$$
$$= \frac{64a^3\sec(c+dx)\sqrt{a+a\sin(c+dx)}}{3d} - \frac{16a^2\sec(c+dx)(a+a\,\mathrm{si}}{3d}$$

Mathematica [A]
time = 5.30, size = 48, normalized size = 0.54

$$\frac{a^3\sec(c+dx)(45+\cos(2(c+dx))-20\sin(c+dx))\sqrt{a(1+\sin(c+dx))}}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] (a^3*Sec[c + d*x]*(45 + Cos[2*(c + d*x)] - 20*Sin[c + d*x])*Sqrt[a*(1 + Sin
[c + d*x])])/(3*d)
```

Maple [A]
time = 0.34, size = 55, normalized size = 0.62

method	result	size
default	$-\frac{2a^4(1+\sin(dx+c))(\sin^2(dx+c)+10\sin(dx+c)-23)}{3\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+a*sin(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*a^4*(1+sin(d*x+c))*(sin(d*x+c)^2+10*sin(d*x+c)-23)/cos(d*x+c)/(a+a*sin
(d*x+c))^(1/2)/d
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 237 vs. 2(77) = 154.
time = 0.57, size = 237, normalized size = 2.66

$$-\frac{2\left(23\,a^{\frac{7}{2}} - \frac{20\,a^{\frac{7}{2}}\sin(dx+c)}{\cos(dx+c)+1} + \frac{88\,a^{\frac{7}{2}}\sin(dx+c)^2}{(\cos(dx+c)+1)^2} - \frac{60\,a^{\frac{7}{2}}\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{130\,a^{\frac{7}{2}}\sin(dx+c)^4}{(\cos(dx+c)+1)^4} - \frac{60\,a^{\frac{7}{2}}\sin(dx+c)^5}{(\cos(dx+c)+1)^5} + \frac{88\,a^{\frac{7}{2}}\sin(dx+c)^6}{(\cos(dx+c)+1)^6} - \frac{20\,a^{\frac{7}{2}}\sin(dx+c)^7}{(\cos(dx+c)+1)^7} + \frac{23\,a^{\frac{7}{2}}\sin(dx+c)^8}{(\cos(dx+c)+1)^8}\right)}{3\,d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1} - 1\right)\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + 1\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] -2/3*(23*a^(7/2) - 20*a^(7/2)*sin(d*x + c)/(cos(d*x + c) + 1) + 88*a^(7/2)*
sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 60*a^(7/2)*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3 + 130*a^(7/2)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 60*a^(7/2)*si
n(d*x + c)^5/(cos(d*x + c) + 1)^5 + 88*a^(7/2)*sin(d*x + c)^6/(cos(d*x + c)
 + 1)^6 - 20*a^(7/2)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 23*a^(7/2)*sin(d
*x + c)^8/(cos(d*x + c) + 1)^8)/(d*(sin(d*x + c)/(cos(d*x + c) + 1) - 1)*(s
in(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^(7/2))
```

Fricas [**A**]
time = 0.35, size = 54, normalized size = 0.61

$$\frac{2\left(a^{3}\cos\left(dx+c\right)^{2}-10\,a^{3}\sin\left(dx+c\right)+22\,a^{3}\right)\sqrt{a\sin\left(dx+c\right)+a}}{3\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 2/3*(a^3*cos(d*x + c)^2 - 10*a^3*sin(d*x + c) + 22*a^3)*sqrt(a*sin(d*x + c)
 + a)/(d*cos(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+a*sin(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 4.30, size = 105, normalized size = 1.18

$$\frac{4\sqrt{2}\left(a^{3}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{3}-6\,a^{3}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)-\frac{3\,a^{3}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)}\right)\sqrt{a}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] 4/3*sqrt(2)*(a^3*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*d*x
+ 1/2*c)^3 - 6*a^3*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*d*
x + 1/2*c) - 3*a^3*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))/sin(-1/4*pi + 1/2*d*
x + 1/2*c))*sqrt(a)/d
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{7/2}}{{\cos\left(c+d\,x\right)}^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(7/2)/cos(c + d*x)^2,x)
[Out] int((a + a*sin(c + d*x))^(7/2)/cos(c + d*x)^2, x)
```

3.148 $\int \sec^3(c+dx)(a+a\sin(c+dx))^{7/2}\,dx$

Optimal. Leaf size=91

$$-\frac{3\sqrt{2}\,a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{2}\,\sqrt{a}}\right)}{d}+\frac{3a^3\sqrt{a+a\sin(c+dx)}}{d}+\frac{a\sec^2(c+dx)(a+a\sin(c+dx))^{5/2}}{d}$$

[Out] a*sec(d*x+c)^2*(a+a*sin(d*x+c))^(5/2)/d-3*a^(7/2)*arctanh(1/2*(a+a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)/d+3*a^3*(a+a*sin(d*x+c))^(1/2)/d

Rubi [A]
time = 0.10, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2755, 2746, 52, 65, 212}

$$-\frac{3\sqrt{2}\,a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\,\sqrt{a}}\right)}{d}+\frac{3a^3\sqrt{a\sin(c+dx)+a}}{d}+\frac{a\sec^2(c+dx)(a\sin(c+dx)+a)^{5/2}}{d}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d*x]^3*(a + a*Sin[c + d*x])^(7/2),x]

[Out] (-3*Sqrt[2]*a^(7/2)*ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])])/d + (3*a^3*Sqrt[a + a*Sin[c + d*x]])/d + (a*Sec[c + d*x]^2*(a + a*Sin[c + d*x])^(5/2))/d

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2755

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[-2*b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(p + 1))), x] + Dist[b^2*((2*m + p - 1)/(g^2*(p + 1))),
Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[
{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && Inte
gersQ[2*m, 2*p]
```

Rubi steps

$$\int \sec^3(c+dx)(a+a\sin(c+dx))^{7/2}\,dx = \frac{a\sec^2(c+dx)(a+a\sin(c+dx))^{5/2}}{d} - \frac{1}{2}\left(3a^2\right)\int \sec(c+dx)(a+$$

$$= \frac{a\sec^2(c+dx)(a+a\sin(c+dx))^{5/2}}{d} - \frac{\left(3a^3\right)\operatorname{Subst}\left(\int \frac{\sqrt{a+x}}{a-x}\,d\right.}{2d}$$

$$= \frac{3a^3\sqrt{a+a\sin(c+dx)}}{d} + \frac{a\sec^2(c+dx)(a+a\sin(c+dx))^{5/2}}{d} -$$

$$= \frac{3a^3\sqrt{a+a\sin(c+dx)}}{d} + \frac{a\sec^2(c+dx)(a+a\sin(c+dx))^{5/2}}{d} -$$

$$= -\frac{3\sqrt{2}\,a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d} + \frac{3a^3\sqrt{a+a\sin(c-}}{d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.05, size = 42, normalized size = 0.46

$$\frac{a\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};\frac{1}{2}(1+\sin(c+dx))\right)(a+a\sin(c+dx))^{5/2}}{10d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] (a*Hypergeometric2F1[2, 5/2, 7/2, (1 + Sin[c + d*x])/2]*(a + a*Sin[c + d*x]
)^(5/2))/(10*d)
```

Maple [A]
time = 0.66, size = 83, normalized size = 0.91

method	result
default	$\frac{2a^{3}\left(\sqrt{a+a\sin\left(dx+c\right)}+4a\left(-\frac{\sqrt{a+a\sin\left(dx+c\right)}}{4\left(a\sin\left(dx+c\right)-a\right)}-\frac{3\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+a\sin\left(dx+c\right)}\,\sqrt{2}}{2\sqrt{a}}\right)}{8\sqrt{a}}\right)\right)}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+a*sin(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*a^3*((a+a*sin(d*x+c))^(1/2)+4*a*(-1/4*(a+a*sin(d*x+c))^(1/2)/(a*sin(d*x+c
)-a)-3/8*2^(1/2)/a^(1/2)*arctanh(1/2*(a+a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2)
)))/d
```

Maxima [A]
time = 0.49, size = 112, normalized size = 1.23

$$\frac{3\sqrt{2}\,a^{\frac{9}{2}}\log\left(-\frac{\sqrt{2}\,\sqrt{a}-\sqrt{a\sin\left(dx+c\right)+a}}{\sqrt{2}\,\sqrt{a}+\sqrt{a\sin\left(dx+c\right)+a}}\right)+4\,\sqrt{a\sin\left(dx+c\right)+a}\,a^{4}-\frac{4\,\sqrt{a\sin\left(dx+c\right)+a}\,a^{5}}{a\sin\left(dx+c\right)-a}}{2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] 1/2*(3*sqrt(2)*a^(9/2)*log(-(sqrt(2)*sqrt(a) - sqrt(a*sin(d*x + c) + a))/(s
qrt(2)*sqrt(a) + sqrt(a*sin(d*x + c) + a))) + 4*sqrt(a*sin(d*x + c) + a)*a^
4 - 4*sqrt(a*sin(d*x + c) + a)*a^5/(a*sin(d*x + c) - a))/(a*d)
```

Fricas [A]
time = 0.37, size = 116, normalized size = 1.27

$$\frac{3\sqrt{2}\left(a^{3}\sin\left(dx+c\right)-a^{3}\right)\sqrt{a}\,\log\left(-\frac{a\sin\left(dx+c\right)-2\sqrt{2}\,\sqrt{a\sin\left(dx+c\right)+a}\,\sqrt{a}+3\,a}{\sin\left(dx+c\right)-1}\right)+4\left(a^{3}\sin\left(dx+c\right)-2\,a^{3}\right)\sqrt{a\sin\left(dx+c\right)+a}}{2\left(d\sin\left(dx+c\right)-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/2*(3*sqrt(2)*(a^3*sin(d*x + c) - a^3)*sqrt(a)*log(-(a*sin(d*x + c) - 2*sq
rt(2)*sqrt(a*sin(d*x + c) + a)*sqrt(a) + 3*a)/(sin(d*x + c) - 1)) + 4*(a^3*
sin(d*x + c) - 2*a^3)*sqrt(a*sin(d*x + c) + a))/(d*sin(d*x + c) - d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+a*sin(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 3.92, size = 107, normalized size = 1.18

$$-\frac{\sqrt{2}\, a^{\frac{7}{2}} \left(\frac{2 \cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1} - 4\,\cos\left(-\frac{1}{4}\,\pi + \frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 3\,\log\left(\cos\left(-\frac{1}{4}\,\pi + \frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right) - 3\,\log\left(-\cos\left(-\frac{1}{4}\,\pi + \frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right)\right) \mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi + \frac{1}{2}\,dx + \frac{1}{2}\,c\right)\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*a^(7/2)*(2*cos(-1/4*pi + 1/2*d*x + 1/2*c)/(cos(-1/4*pi + 1/2*d
*x + 1/2*c)^2 - 1) - 4*cos(-1/4*pi + 1/2*d*x + 1/2*c) + 3*log(cos(-1/4*pi +
 1/2*d*x + 1/2*c) + 1) - 3*log(-cos(-1/4*pi + 1/2*d*x + 1/2*c) + 1))*sgn(co
s(-1/4*pi + 1/2*d*x + 1/2*c))/d
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+a\,\sin\left(c+d\,x\right)\right)^{7/2}}{{\cos\left(c+d\,x\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(7/2)/cos(c + d*x)^3,x)
```

```
[Out] int((a + a*sin(c + d*x))^(7/2)/cos(c + d*x)^3, x)
```

3.149 $\int \sec^4(c+dx)(a+a\sin(c+dx))^{7/2}\,dx$

Optimal. Leaf size=61

$$-\frac{8a^2\sec^3(c+dx)(a+a\sin(c+dx))^{3/2}}{3d}+\frac{2a\sec^3(c+dx)(a+a\sin(c+dx))^{5/2}}{d}$$

```
[Out] -8/3*a^2*sec(d*x+c)^3*(a+a*sin(d*x+c))^(3/2)/d+2*a*sec(d*x+c)^3*(a+a*sin(d*
x+c))^(5/2)/d
```

Rubi [A]
time = 0.08, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.087$, Rules used = {2753, 2752}

$$\frac{2a\sec^3(c+dx)(a\sin(c+dx)+a)^{5/2}}{d}-\frac{8a^2\sec^3(c+dx)(a\sin(c+dx)+a)^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] (-8*a^2*Sec[c + d*x]^3*(a + a*Sin[c + d*x])^(3/2))/(3*d) + (2*a*Sec[c + d*x
]^3*(a + a*Sin[c + d*x])^(5/2))/d
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2753

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && N
eQ[m + p, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^4(c+dx)(a+a\sin(c+dx))^{7/2}\,dx &= \frac{2a\sec^3(c+dx)(a+a\sin(c+dx))^{5/2}}{d}-(4a)\int \sec^4(c+dx)(a+a\\ &= -\frac{8a^2\sec^3(c+dx)(a+a\sin(c+dx))^{3/2}}{3d}+\frac{2a\sec^3(c+dx)(a+a\,\mathrm{si}}{d}\end{aligned}$$

Mathematica [A]
time = 5.19, size = 82, normalized size = 1.34

$$\frac{2a^3\sqrt{a(1+\sin(c+dx))}\,(-1+3\sin(c+dx))}{3d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] (2*a^3*Sqrt[a*(1 + Sin[c + d*x])]*(-1 + 3*Sin[c + d*x]))/(3*d*(Cos[(c + d*x
)/2] - Sin[(c + d*x)/2])^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

Maple [A]
time = 0.41, size = 57, normalized size = 0.93

method	result	size
default	$-\frac{2a^4(1+\sin(dx+c))(3\sin(dx+c)-1)}{3(\sin(dx+c)-1)\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+a*sin(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*a^4*(1+sin(d*x+c))/(sin(d*x+c)-1)*(3*sin(d*x+c)-1)/cos(d*x+c)/(a+a*sin
(d*x+c))^(1/2)/d
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 320 vs. $2(55) = 110$.
time = 0.59, size = 320, normalized size = 5.25

$$\frac{2\left(a^{\frac{7}{2}}-\frac{6a^{\frac{7}{2}}\sin(dx+c)}{\cos(dx+c)+1}+\frac{5a^{\frac{7}{2}}\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{24a^{\frac{7}{2}}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{10a^{\frac{7}{2}}\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{36a^{\frac{7}{2}}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{10a^{\frac{7}{2}}\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{24a^{\frac{7}{2}}\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{5a^{\frac{7}{2}}\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{6a^{\frac{7}{2}}\sin(dx+c)^9}{(\cos(dx+c)+1)^9}+\frac{a^{\frac{7}{2}}\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}\right)}{3d\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}-\frac{3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-1\right)\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] 2/3*(a^(7/2) - 6*a^(7/2)*sin(d*x + c)/(cos(d*x + c) + 1) + 5*a^(7/2)*sin(d*
x + c)^2/(cos(d*x + c) + 1)^2 - 24*a^(7/2)*sin(d*x + c)^3/(cos(d*x + c) + 1
)^3 + 10*a^(7/2)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 36*a^(7/2)*sin(d*x +
 c)^5/(cos(d*x + c) + 1)^5 + 10*a^(7/2)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6
 - 24*a^(7/2)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 5*a^(7/2)*sin(d*x + c)^
8/(cos(d*x + c) + 1)^8 - 6*a^(7/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 + a^
(7/2)*sin(d*x + c)^10/(cos(d*x + c) + 1)^10)/(d*(3*sin(d*x + c)/(cos(d*x +
c) + 1) - 3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 - 1)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^(7/2))
```

Fricas [A]
time = 0.34, size = 57, normalized size = 0.93

$$-\frac{2\,{\left(3\,a^{3}\sin\left(dx+c\right)-a^{3}\right)}\sqrt{a\sin\left(dx+c\right)+a}}{3\,{\left(d\cos\left(dx+c\right)\sin\left(dx+c\right)-d\cos\left(dx+c\right)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] -2/3*(3*a^3*sin(d*x + c) - a^3)*sqrt(a*sin(d*x + c) + a)/(d*cos(d*x + c)*si
n(d*x + c) - d*cos(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+a*sin(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 5.54, size = 76, normalized size = 1.25

$$\frac{\sqrt{2}\,{\left(3\,a^{3}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a^{3}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\right)}\sqrt{a}}{3\,d\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(2)*(3*a^3*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*d*
x + 1/2*c)^2 - a^3*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c)))*sqrt(a)/(d*sin(-1/4
*pi + 1/2*d*x + 1/2*c)^3)
```

Mupad [B]
time = 8.53, size = 118, normalized size = 1.93

$$-\frac{a^3\,{\mathrm{e}}^{c\,1{}\mathrm{i}+d\,x\,1{}\mathrm{i}}\,\sqrt{a+a\,\left(\frac{{\mathrm{e}}^{-c\,1{}\mathrm{i}-d\,x\,1{}\mathrm{i}}\,1{}\mathrm{i}}{2}-\frac{{\mathrm{e}}^{c\,1{}\mathrm{i}+d\,x\,1{}\mathrm{i}}\,1{}\mathrm{i}}{2}\right)}\,\left(3-3\,{\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+{\mathrm{e}}^{c\,1{}\mathrm{i}+d\,x\,1{}\mathrm{i}}\,2{}\mathrm{i}\right)\,4{}\mathrm{i}}{3\,d\,\left({\mathrm{e}}^{c\,1{}\mathrm{i}+d\,x\,1{}\mathrm{i}}+1{}\mathrm{i}\right)\,{\left(1+{\mathrm{e}}^{c\,1{}\mathrm{i}+d\,x\,1{}\mathrm{i}}\,1{}\mathrm{i}\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(7/2)/cos(c + d*x)^4,x)
```

```
[Out] -(a^3*exp(c*1i + d*x*1i)*(a + a*((exp(- c*1i - d*x*1i)*1i)/2 - (exp(c*1i +
d*x*1i)*1i)/2))^(1/2)*(exp(c*1i + d*x*1i)*2i - 3*exp(c*2i + d*x*2i) + 3)*4i
)/(3*d*(exp(c*1i + d*x*1i) + 1i)*(exp(c*1i + d*x*1i)*1i + 1)^3)
```

3.150 $\int \sec^5(c+dx)(a+a\sin(c+dx))^{7/2}\,dx$

Optimal. Leaf size=106

$$-\frac{a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{8\sqrt{2}\,d}-\frac{a^2\sec^2(c+dx)(a+a\sin(c+dx))^{3/2}}{8d}+\frac{a\sec^4(c+dx)(a+a\sin(c+$$

```
[Out] -1/8*a^2*sec(d*x+c)^2*(a+a*sin(d*x+c))^(3/2)/d+1/2*a*sec(d*x+c)^4*(a+a*sin(
d*x+c))^(5/2)/d-1/16*a^(7/2)*arctanh(1/2*(a+a*sin(d*x+c))^(1/2)*2^(1/2)/a^(
1/2))*2^(1/2)/d
```

Rubi [A]
time = 0.12, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2755, 2754, 2746, 65, 212}

$$-\frac{a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{8\sqrt{2}\,d}-\frac{a^2\sec^2(c+dx)(a\sin(c+dx)+a)^{3/2}}{8d}+\frac{a\sec^4(c+dx)(a\sin(c+dx)+a)^{5/2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] -1/8*(a^(7/2)*ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(Sqrt[2]
*d) - (a^2*Sec[c + d*x]^2*(a + a*Sin[c + d*x])^(3/2))/(8*d) + (a*Sec[c + d*
x]^4*(a + a*Sin[c + d*x])^(5/2))/(2*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
```

```
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2754

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^m/(a*f*g*(p + 1))), x] + Dist[a*((m + p + 1)/(g^2*(p + 1))), Int[(g*C
os[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e
, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ[
m + 1/2, 2*p]
```

Rule 2755

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[-2*b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(p + 1))), x] + Dist[b^2*((2*m + p - 1)/(g^2*(p + 1))),
Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[
{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && Inte
gersQ[2*m, 2*p]
```

Rubi steps

$$\begin{aligned}
\int \sec^5(c+dx)(a+a\sin(c+dx))^{7/2}\,dx &= \frac{a\sec^4(c+dx)(a+a\sin(c+dx))^{5/2}}{2d} - \frac{1}{4}a^2\int \sec^3(c+dx)(a+a\,\mathrm{s} \\
&= -\frac{a^2\sec^2(c+dx)(a+a\sin(c+dx))^{3/2}}{8d} + \frac{a\sec^4(c+dx)(a+a\sin(}{2d} \\
&= -\frac{a^2\sec^2(c+dx)(a+a\sin(c+dx))^{3/2}}{8d} + \frac{a\sec^4(c+dx)(a+a\sin(}{2d} \\
&= -\frac{a^2\sec^2(c+dx)(a+a\sin(c+dx))^{3/2}}{8d} + \frac{a\sec^4(c+dx)(a+a\sin(}{2d} \\
&= -\frac{a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{8\sqrt{2}\,d} - \frac{a^2\sec^2(c+dx)(a+a\sin}{8d}
\end{aligned}$$

Mathematica [A]
time = 0.20, size = 108, normalized size = 1.02

$$\frac{-\sqrt{2}\,a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a(1+\sin(c+dx))}}{\sqrt{2}\sqrt{a}}\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^4+2a^3\sqrt{a(1+\sin(c+dx))}\,(3+\sin(c+dx))}{16d(-1+\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] (-(Sqrt[2]*a^(7/2)*ArcTanh[Sqrt[a*(1 + Sin[c + d*x])]/(Sqrt[2]*Sqrt[a])]*(C
os[(c + d*x)/2] - Sin[(c + d*x)/2])^4) + 2*a^3*Sqrt[a*(1 + Sin[c + d*x])]*(
3 + Sin[c + d*x]))/(16*d*(-1 + Sin[c + d*x])^2)
```

Maple [A]
time = 0.56, size = 75, normalized size = 0.71

method	result	size
default	$-\frac{2a^5\left(-\frac{\sqrt{a+a\sin(dx+c)}\,(3+\sin(dx+c))}{16(a\sin(dx+c)-a)^2}+\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+a\sin(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{32a^{\frac{3}{2}}}\right)}{d}$	75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(a+a*sin(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*a^5*(-1/16*(a+a*sin(d*x+c))^(1/2)*(3+sin(d*x+c))/(a*sin(d*x+c)-a)^2+1/32
/a^(3/2)*2^(1/2)*arctanh(1/2*(a+a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2)))/d
```

Maxima [A]
time = 0.50, size = 132, normalized size = 1.25

$$\frac{\sqrt{2}\,a^{\frac{9}{2}}\log\left(-\frac{\sqrt{2}\,\sqrt{a}-\sqrt{a\sin(dx+c)+a}}{\sqrt{2}\,\sqrt{a}+\sqrt{a\sin(dx+c)+a}}\right)+\frac{4\left((a\sin(dx+c)+a)^{\frac{3}{2}}a^5+2\sqrt{a\sin(dx+c)+a}\,a^6\right)}{(a\sin(dx+c)+a)^2-4\,(a\sin(dx+c)+a)a+4\,a^2}}{32\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] 1/32*(sqrt(2)*a^(9/2)*log(-(sqrt(2)*sqrt(a) - sqrt(a*sin(d*x + c) + a))/(sq
rt(2)*sqrt(a) + sqrt(a*sin(d*x + c) + a))) + 4*((a*sin(d*x + c) + a)^(3/2)*
a^5 + 2*sqrt(a*sin(d*x + c) + a)*a^6)/((a*sin(d*x + c) + a)^2 - 4*(a*sin(d*
x + c) + a)*a + 4*a^2))/(a*d)
```

Fricas [A]
time = 0.35, size = 145, normalized size = 1.37

$$\frac{\left(\sqrt{2}\,a^3\cos(dx+c)^2+2\,\sqrt{2}\,a^3\sin(dx+c)-2\,\sqrt{2}\,a^3\right)\sqrt{a}\,\log\left(-\frac{a\sin(dx+c)-2\,\sqrt{2}\,\sqrt{a\sin(dx+c)+a}\,\sqrt{a}+3\,a}{\sin(dx+c)-1}\right)-4\,(a^3\sin(dx+c)+3\,a^3)\sqrt{a\sin(dx+c)+a}}{32\,\left(d\cos(dx+c)^2+2\,d\sin(dx+c)-2\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/32*((sqrt(2)*a^3*cos(d*x + c)^2 + 2*sqrt(2)*a^3*sin(d*x + c) - 2*sqrt(2)*
a^3)*sqrt(a)*log(-(a*sin(d*x + c) - 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*sqrt
(a) + 3*a)/(sin(d*x + c) - 1)) - 4*(a^3*sin(d*x + c) + 3*a^3)*sqrt(a*sin(d*
x + c) + a))/(d*cos(d*x + c)^2 + 2*d*sin(d*x + c) - 2*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a+a*sin(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 4.97, size = 149, normalized size = 1.41

$$\frac{\sqrt{2}\, a^{\frac{7}{2}} \left(\frac{4 \left(\frac{1}{\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\frac{1}{\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)^{2}-4} - \log\left(\left|\frac{1}{\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)} + \cos\left(-\frac{1}{4}\,\pi + \frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 2\right|\right) + \log\left(\left|\frac{1}{\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)} + \cos\left(-\frac{1}{4}\,\pi + \frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 2\right|\right)\right) \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\,\pi + \frac{1}{2}\,dx + \frac{1}{2}\,c\right)\right)}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] 1/64*sqrt(2)*a^(7/2)*(4*(1/cos(-1/4*pi + 1/2*d*x + 1/2*c) + cos(-1/4*pi + 1
/2*d*x + 1/2*c))/((1/cos(-1/4*pi + 1/2*d*x + 1/2*c) + cos(-1/4*pi + 1/2*d*x
 + 1/2*c))^2 - 4) - log(abs(1/cos(-1/4*pi + 1/2*d*x + 1/2*c) + cos(-1/4*pi
+ 1/2*d*x + 1/2*c) + 2)) + log(abs(1/cos(-1/4*pi + 1/2*d*x + 1/2*c) + cos(-
1/4*pi + 1/2*d*x + 1/2*c) - 2)))*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))/d
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{7/2}}{{\cos\left(c+d\,x\right)}^5} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(7/2)/cos(c + d*x)^5,x)
```

```
[Out] int((a + a*sin(c + d*x))^(7/2)/cos(c + d*x)^5, x)
```

3.151 $\int \sec^6(c+dx)(a+a\sin(c+dx))^{7/2}\,dx$

Optimal. Leaf size=30

$$\frac{2a\sec^5(c+dx)(a+a\sin(c+dx))^{5/2}}{5d}$$

```
[Out] 2/5*a*sec(d*x+c)^5*(a+a*sin(d*x+c))^(5/2)/d
```

Rubi [A]
time = 0.04, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = $\{2752\}$

$$\frac{2a\sec^5(c+dx)(a\sin(c+dx)+a)^{5/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] (2*a*Sec[c + d*x]^5*(a + a*Sin[c + d*x])^(5/2))/(5*d)
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\int \sec^6(c+dx)(a+a\sin(c+dx))^{7/2}\,dx = \frac{2a\sec^5(c+dx)(a+a\sin(c+dx))^{5/2}}{5d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 69 vs. 2(30) = 60.
time = 5.17, size = 69, normalized size = 2.30

$$\frac{2(a(1+\sin(c+dx)))^{7/2}}{5d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^5\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] (2*(a*(1 + Sin[c + d*x]))^(7/2))/(5*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])
^5*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^7)
```

Maple [A]
time = 0.31, size = 47, normalized size = 1.57

method	result	size
default	$\frac{2a^{4}\left(1+\sin\left(dx+c\right)\right)}{5\left(\sin\left(dx+c\right)-1\right)^{2}\cos\left(dx+c\right)\sqrt{a+a\sin\left(dx+c\right)}\,d}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(a+a*sin(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*a^4*(1+sin(d*x+c))/(sin(d*x+c)-1)^2/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 270 vs. 2(26) = 52.
time = 0.56, size = 270, normalized size = 9.00

$$-\frac{2\left(a^{\frac{7}{2}}+\frac{6\,a^{\frac{7}{2}}\sin\left(dx+c\right)^{2}}{\left(\cos\left(dx+c\right)+1\right)^{2}}+\frac{15\,a^{\frac{7}{2}}\sin\left(dx+c\right)^{4}}{\left(\cos\left(dx+c\right)+1\right)^{4}}+\frac{20\,a^{\frac{7}{2}}\sin\left(dx+c\right)^{6}}{\left(\cos\left(dx+c\right)+1\right)^{6}}+\frac{15\,a^{\frac{7}{2}}\sin\left(dx+c\right)^{8}}{\left(\cos\left(dx+c\right)+1\right)^{8}}+\frac{6\,a^{\frac{7}{2}}\sin\left(dx+c\right)^{10}}{\left(\cos\left(dx+c\right)+1\right)^{10}}+\frac{a^{\frac{7}{2}}\sin\left(dx+c\right)^{12}}{\left(\cos\left(dx+c\right)+1\right)^{12}}\right)}{5\,d\left(\frac{5\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}-\frac{10\sin\left(dx+c\right)^{2}}{\left(\cos\left(dx+c\right)+1\right)^{2}}+\frac{10\sin\left(dx+c\right)^{3}}{\left(\cos\left(dx+c\right)+1\right)^{3}}-\frac{5\sin\left(dx+c\right)^{4}}{\left(\cos\left(dx+c\right)+1\right)^{4}}+\frac{\sin\left(dx+c\right)^{5}}{\left(\cos\left(dx+c\right)+1\right)^{5}}-1\right)\left(\frac{\sin\left(dx+c\right)^{2}}{\left(\cos\left(dx+c\right)+1\right)^{2}}+1\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+a*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] -2/5*(a^(7/2) + 6*a^(7/2)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 15*a^(7/2)*
sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 20*a^(7/2)*sin(d*x + c)^6/(cos(d*x +
c) + 1)^6 + 15*a^(7/2)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 6*a^(7/2)*sin(
d*x + c)^10/(cos(d*x + c) + 1)^10 + a^(7/2)*sin(d*x + c)^12/(cos(d*x + c) +
 1)^12)/(d*(5*sin(d*x + c)/(cos(d*x + c) + 1) - 10*sin(d*x + c)^2/(cos(d*x
+ c) + 1)^2 + 10*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 5*sin(d*x + c)^4/(co
s(d*x + c) + 1)^4 + sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 1)*(sin(d*x + c)^
2/(cos(d*x + c) + 1)^2 + 1)^(7/2))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(26) = 52.
time = 0.37, size = 54, normalized size = 1.80

$$-\frac{2\sqrt{a\sin\left(dx+c\right)+a}\,a^{3}}{5\left(d\cos\left(dx+c\right)^{3}+2\,d\cos\left(dx+c\right)\sin\left(dx+c\right)-2\,d\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+a*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] -2/5*sqrt(a*sin(d*x + c) + a)*a^3/(d*cos(d*x + c)^3 + 2*d*cos(d*x + c)*sin(
d*x + c) - 2*d*cos(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(a+a*sin(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 3.85, size = 38, normalized size = 1.27

$$-\frac{\sqrt{2}\,a^{\frac{7}{2}}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{20\,d\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+a*sin(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] -1/20*sqrt(2)*a^(7/2)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))/(d*sin(-1/4*pi +
1/2*d*x + 1/2*c)^5)
```

Mupad [**B**]
time = 8.43, size = 86, normalized size = 2.87

$$-\frac{16\,a^3\,\mathrm{e}^{c\,3\mathrm{i}+d\,x\,3\mathrm{i}}\,\sqrt{a+a\,\left(\frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}}{5\,d\,{\left(\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}-\mathrm{i}\right)}^5\,\left(\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}+1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(7/2)/cos(c + d*x)^6,x)
```

```
[Out] -(16*a^3*exp(c*3i + d*x*3i)*(a + a*((exp(- c*1i - d*x*1i)*1i)/2 - (exp(c*1i
 + d*x*1i)*1i)/2))^(1/2))/(5*d*(exp(c*1i + d*x*1i) - 1i)^5*(exp(c*1i + d*x*
1i) + 1i))
```

3.152 $\int \sec^7(c+dx)(a+a\sin(c+dx))^{7/2}\,dx$

Optimal. Leaf size=135

$$\frac{5a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{64\sqrt{2}\,d}+\frac{5a^2\sec^2(c+dx)(a+a\sin(c+dx))^{3/2}}{64d}+\frac{5a\sec^4(c+dx)(a+a\sin(c+}{48d}$$

```
[Out] 5/64*a^2*sec(d*x+c)^2*(a+a*sin(d*x+c))^(3/2)/d+5/48*a*sec(d*x+c)^4*(a+a*sin
(d*x+c))^(5/2)/d+1/6*sec(d*x+c)^6*(a+a*sin(d*x+c))^(7/2)/d+5/128*a^(7/2)*ar
ctanh(1/2*(a+a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)/d
```

Rubi [A]
time = 0.17, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2754, 2746, 65, 212}

$$\frac{5a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{64\sqrt{2}\,d}+\frac{5a^2\sec^2(c+dx)(a\sin(c+dx)+a)^{3/2}}{64d}+\frac{\sec^6(c+dx)(a\sin(c+dx)+a)^{7/2}}{6d}+\frac{5a\sec^4(c+dx)(a\sin(c+dx)+a)^{5/2}}{48d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^7*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] (5*a^(7/2)*ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(64*Sqrt[2]
*d) + (5*a^2*Sec[c + d*x]^2*(a + a*Sin[c + d*x])^(3/2))/(64*d) + (5*a*Sec[c
 + d*x]^4*(a + a*Sin[c + d*x])^(5/2))/(48*d) + (Sec[c + d*x]^6*(a + a*Sin[c
 + d*x])^(7/2))/(6*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
```

```
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2754

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^m/(a*f*g*(p + 1))), x] + Dist[a*((m + p + 1)/(g^2*(p + 1))), Int[(g*C
os[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e
, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ[
m + 1/2, 2*p]
```

Rubi steps

$$\int \sec^7(c+dx)(a+a\sin(c+dx))^{7/2}\,dx = \frac{\sec^6(c+dx)(a+a\sin(c+dx))^{7/2}}{6d}+\frac{1}{12}(5a)\int \sec^5(c+dx)(a+$$

$$= \frac{5a\sec^4(c+dx)(a+a\sin(c+dx))^{5/2}}{48d}+\frac{\sec^6(c+dx)(a+a\sin(c}{6d}$$

$$= \frac{5a^2\sec^2(c+dx)(a+a\sin(c+dx))^{3/2}}{64d}+\frac{5a\sec^4(c+dx)(a+a\,\mathrm{si}}{48d}$$

$$= \frac{5a^2\sec^2(c+dx)(a+a\sin(c+dx))^{3/2}}{64d}+\frac{5a\sec^4(c+dx)(a+a\,\mathrm{si}}{48d}$$

$$= \frac{5a^2\sec^2(c+dx)(a+a\sin(c+dx))^{3/2}}{64d}+\frac{5a\sec^4(c+dx)(a+a\,\mathrm{si}}{48d}$$

$$= \frac{5a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{64\sqrt{2}\,d}+\frac{5a^2\sec^2(c+dx)(a+a\,\mathrm{s}}{64d}$$

Mathematica [A]
time = 0.36, size = 120, normalized size = 0.89

$$-\frac{15\sqrt{2}\,a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a(1+\sin(c+dx))}}{\sqrt{2}\sqrt{a}}\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^6+2a^3\sqrt{a(1+\sin(c+dx))}\,(67-50\sin(c+dx)+15\sin^2(c+dx))}{384d(-1+\sin(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^7*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] -1/384*(15*Sqrt[2]*a^(7/2)*ArcTanh[Sqrt[a*(1 + Sin[c + d*x])]/(Sqrt[2]*Sqrt
[a])]*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^6 + 2*a^3*Sqrt[a*(1 + Sin[c + d
*x])]*(67 - 50*Sin[c + d*x] + 15*Sin[c + d*x]^2))/(d*(-1 + Sin[c + d*x])^3)
```

Maple [A]
time = 0.83, size = 144, normalized size = 1.07

method	result
default	$\frac{2a^7\left(-\frac{\sqrt{a+a\sin(dx+c)}}{12a(a\sin(dx+c)-a)^3}-\frac{5\left(-\frac{\sqrt{a+a\sin(dx+c)}}{8a(a\sin(dx+c)-a)^2}-\frac{3\left(-\frac{\sqrt{a+a\sin(dx+c)}}{4a(a\sin(dx+c)-a)}+\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+}}{}\right.}{8}\right.}{8a}\right.}{12a}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7*(a+a*sin(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*a^7*(-1/12*(a+a*sin(d*x+c))^(1/2)/a/(a*sin(d*x+c)-a)^3-5/12/a*(-1/8*(a+a*
sin(d*x+c))^(1/2)/a/(a*sin(d*x+c)-a)^2-3/8/a*(-1/4*(a+a*sin(d*x+c))^(1/2)/a
/(a*sin(d*x+c)-a)+1/8/a^(3/2)*2^(1/2)*arctanh(1/2*(a+a*sin(d*x+c))^(1/2)*2^
(1/2)/a^(1/2)))))/d
```

Maxima [A]
time = 0.51, size = 168, normalized size = 1.24

$$-\frac{15\sqrt{2}\,a^{\frac{9}{2}}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{a\sin(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{a\sin(dx+c)+a}}\right)+\frac{4\left(15\,(a\sin(dx+c)+a)^{\frac{5}{2}}a^5-80\,(a\sin(dx+c)+a)^{\frac{3}{2}}a^6+132\,\sqrt{a\sin(dx+c)+a}\,a^7\right)}{(a\sin(dx+c)+a)^3-6\,(a\sin(dx+c)+a)^2a+12\,(a\sin(dx+c)+a)a^2-8\,a^3}}{768\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+a*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] -1/768*(15*sqrt(2)*a^(9/2)*log(-(sqrt(2)*sqrt(a) - sqrt(a*sin(d*x + c) + a)
)/(sqrt(2)*sqrt(a) + sqrt(a*sin(d*x + c) + a))) + 4*(15*(a*sin(d*x + c) + a
)^(5/2)*a^5 - 80*(a*sin(d*x + c) + a)^(3/2)*a^6 + 132*sqrt(a*sin(d*x + c) +
 a)*a^7)/((a*sin(d*x + c) + a)^3 - 6*(a*sin(d*x + c) + a)^2*a + 12*(a*sin(d
*x + c) + a)*a^2 - 8*a^3))/(a*d)
```

Fricas [A]
time = 0.36, size = 193, normalized size = 1.43

$$\frac{15 \left(3 \sqrt{2} a^{3} \cos \left(d x+c\right)^{2}-4 \sqrt{2} a^{3}-\left(\sqrt{2} a^{3} \cos \left(d x+c\right)^{2}-4 \sqrt{2} a^{3}\right) \sin \left(d x+c\right)\right) \sqrt{a} \log \left(-\frac{a \sin \left(d x+c\right)+2 \sqrt{2} \sqrt{a \sin \left(d x+c\right)+a} \sqrt{a}+3 a}{\sin \left(d x+c\right)-1}\right)+4 \left(15 a^{3} \cos \left(d x+c\right)^{2}+50 a^{3} \sin \left(d x+c\right)-82 a^{3}\right) \sqrt{a \sin \left(d x+c\right)+a}}{768 \left(3 d \cos \left(d x+c\right)^{2}-\left(d \cos \left(d x+c\right)^{2}-4 d\right) \sin \left(d x+c\right)-4 d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+a*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/768*(15*(3*sqrt(2)*a^3*cos(d*x + c)^2 - 4*sqrt(2)*a^3 - (sqrt(2)*a^3*cos(
d*x + c)^2 - 4*sqrt(2)*a^3)*sin(d*x + c))*sqrt(a)*log(-(a*sin(d*x + c) + 2*
sqrt(2)*sqrt(a*sin(d*x + c) + a)*sqrt(a) + 3*a)/(sin(d*x + c) - 1)) + 4*(15
*a^3*cos(d*x + c)^2 + 50*a^3*sin(d*x + c) - 82*a^3)*sqrt(a*sin(d*x + c) + a
))/(3*d*cos(d*x + c)^2 - (d*cos(d*x + c)^2 - 4*d)*sin(d*x + c) - 4*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7*(a+a*sin(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 4.84, size = 128, normalized size = 0.95

$$-\frac{\sqrt{2} a^{\frac{7}{2}} \left(\frac{2 \left(15 \cos \left(-\frac{1}{4} \pi+\frac{1}{2} d x+\frac{1}{2} c\right)^{5}-40 \cos \left(-\frac{1}{4} \pi+\frac{1}{2} d x+\frac{1}{2} c\right)^{3}+33 \cos \left(-\frac{1}{4} \pi+\frac{1}{2} d x+\frac{1}{2} c\right)\right)}{\left(\cos \left(-\frac{1}{4} \pi+\frac{1}{2} d x+\frac{1}{2} c\right)^{2}-1\right)^{3}}-15 \log \left(\cos \left(-\frac{1}{4} \pi+\frac{1}{2} d x+\frac{1}{2} c\right)+1\right)+15 \log \left(-\cos \left(-\frac{1}{4} \pi+\frac{1}{2} d x+\frac{1}{2} c\right)+1\right)\right) \mathrm{sgn}\left(\cos \left(-\frac{1}{4} \pi+\frac{1}{2} d x+\frac{1}{2} c\right)\right)}{768 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+a*sin(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] -1/768*sqrt(2)*a^(7/2)*(2*(15*cos(-1/4*pi + 1/2*d*x + 1/2*c)^5 - 40*cos(-1/
4*pi + 1/2*d*x + 1/2*c)^3 + 33*cos(-1/4*pi + 1/2*d*x + 1/2*c))/(cos(-1/4*pi
 + 1/2*d*x + 1/2*c)^2 - 1)^3 - 15*log(cos(-1/4*pi + 1/2*d*x + 1/2*c) + 1) +
 15*log(-cos(-1/4*pi + 1/2*d*x + 1/2*c) + 1))*sgn(cos(-1/4*pi + 1/2*d*x + 1
/2*c))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{7/2}}{{\cos\left(c+d\,x\right)}^7} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(7/2)/cos(c + d*x)^7,x)
```

```
[Out] int((a + a*sin(c + d*x))^(7/2)/cos(c + d*x)^7, x)
```

3.153 $\int \sec^8(c+dx)(a+a\sin(c+dx))^{7/2}\,dx$

Optimal. Leaf size=171

$$-\frac{a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(c+dx)}{\sqrt{2}\,\sqrt{a+a\sin(c+dx)}}\right)}{8\sqrt{2}\,d}+\frac{a^3\sec(c+dx)\sqrt{a+a\sin(c+dx)}}{8d}+\frac{a^2\sec^3(c+dx)(a+a\sin(c\cdots}{12d}$$

[Out] 1/12*a^2*sec(d*x+c)^3*(a+a*sin(d*x+c))^(3/2)/d+1/10*a*sec(d*x+c)^5*(a+a*sin(d*x+c))^(5/2)/d+1/7*sec(d*x+c)^7*(a+a*sin(d*x+c))^(7/2)/d-1/16*a^(7/2)*arctanh(1/2*cos(d*x+c)*a^(1/2)*2^(1/2)/(a+a*sin(d*x+c))^(1/2))/d*2^(1/2)+1/8*a^3*sec(d*x+c)*(a+a*sin(d*x+c))^(1/2)/d

Rubi [A]

time = 0.19, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {2754, 2728, 212}

$$-\frac{a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(c+dx)}{\sqrt{2}\,\sqrt{a\sin(c+dx)+a}}\right)}{8\sqrt{2}\,d}+\frac{a^3\sec(c+dx)\sqrt{a\sin(c+dx)+a}}{8d}+\frac{a^2\sec^3(c+dx)(a\sin(c+dx)+a)^{3/2}}{12d}+\frac{\sec^7(c+dx)(a\sin(c+dx)+a)^{7/2}}{7d}+\frac{a\sec^5(c+dx)(a\sin(c+dx)+a)^{5/2}}{10d}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d*x]^8*(a + a*Sin[c + d*x])^(7/2),x]

[Out] -1/8*(a^(7/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/(Sqrt[2]*d) + (a^3*Sec[c + d*x]*Sqrt[a + a*Sin[c + d*x]])/(8*d) + (a^2*Sec[c + d*x]^3*(a + a*Sin[c + d*x])^(3/2))/(12*d) + (a*Sec[c + d*x]^5*(a + a*Sin[c + d*x])^(5/2))/(10*d) + (Sec[c + d*x]^7*(a + a*Sin[c + d*x])^(7/2))/(7*d)

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^m/(a*f*g*(p + 1))), x] + Dist[a*((m + p + 1)/(g^2*(p + 1))), Int[(g*C
```

```
os[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e
, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ[
m + 1/2, 2*p]
```

Rubi steps

$$\int \sec^8(c+dx)(a+a\sin(c+dx))^{7/2}\,dx = \frac{\sec^7(c+dx)(a+a\sin(c+dx))^{7/2}}{7d}+\frac{1}{2}a\int \sec^6(c+dx)(a+a\sin$$

$$=\frac{a\sec^5(c+dx)(a+a\sin(c+dx))^{5/2}}{10d}+\frac{\sec^7(c+dx)(a+a\sin(c+}{7d}$$

$$=\frac{a^2\sec^3(c+dx)(a+a\sin(c+dx))^{3/2}}{12d}+\frac{a\sec^5(c+dx)(a+a\sin(}{10d}$$

$$=\frac{a^3\sec(c+dx)\sqrt{a+a\sin(c+dx)}}{8d}+\frac{a^2\sec^3(c+dx)(a+a\sin(c}{12d}$$

$$=\frac{a^3\sec(c+dx)\sqrt{a+a\sin(c+dx)}}{8d}+\frac{a^2\sec^3(c+dx)(a+a\sin(c}{12d}$$

$$=-\frac{a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{8\sqrt{2}d}+\frac{a^3\sec(c+dx)\sqrt{a-}}{8d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 5.32, size = 139, normalized size = 0.81

$$\frac{(a(1+\sin(c+dx)))^{7/2}\left((105+105i)(-1)^{3/4}\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(c+dx)\right)\right)\right)+\frac{2286-770\cos(2(c+dx))-2471\sin(c+dx)+105\sin(3(c+dx))}{4\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^7}\right)}{840d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^8*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] ((a*(1 + Sin[c + d*x]))^(7/2)*((105 + 105*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)
*(-1)^(3/4)*(-1 + Tan[(c + d*x)/4])] + (2286 - 770*Cos[2*(c + d*x)] - 2471*
Sin[c + d*x] + 105*Sin[3*(c + d*x)])/(4*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2
])^7)))/(840*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^7)
```

Maple [A]
time = 0.47, size = 139, normalized size = 0.81

method	result

default	$\frac{(1+\sin(dx+c))\left(-210a^{\frac{15}{2}}\sin(dx+c)(\cos^2(dx+c))+770a^{\frac{15}{2}}(\cos^2(dx+c))+1288a^{\frac{15}{2}}\sin(dx+c)-1528a^{\frac{15}{2}}+105\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a}}{}\right.\right.}{1680a^{\frac{7}{2}}(\sin(dx+c)-1)^3\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^8*(a+a*sin(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/1680/a^(7/2)*(1+sin(d*x+c))/(sin(d*x+c)-1)^3*(-210*a^(15/2)*sin(d*x+c)*co
s(d*x+c)^2+770*a^(15/2)*cos(d*x+c)^2+1288*a^(15/2)*sin(d*x+c)-1528*a^(15/2)
+105*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^4*(a-a*s
in(d*x+c))^(7/2))/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+a*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 312 vs. $2(144) = 288$.
time = 0.39, size = 312, normalized size = 1.82

$$\frac{105\left(3\sqrt{2}a^3\cos(dx+c)^3-4\sqrt{2}a^3\cos(dx+c)-\left(\sqrt{2}a^3\cos(dx+c)^3-4\sqrt{2}a^3\cos(dx+c)\right)\sin(dx+c)\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{a\sin(dx+c)+a}\left(\sqrt{2}\cos(dx+c)-\sqrt{2}\sin(dx+c)+\sqrt{2}\right)\sqrt{a}+3a\cos(dx+c)-(a\cos(dx+c)-2a)\sin(dx+c)+2a}{\cos(dx+c)^2-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2}\right)+4\left(385a^3\cos(dx+c)^2-764a^3-7\left(15a^3\cos(dx+c)^2-92a^3\right)\sin(dx+c)\right)\sqrt{a\sin(dx+c)+a}}{3360\left(3d\cos(dx+c)^3-4d\cos(dx+c)-\left(d\cos(dx+c)^3-4d\cos(dx+c)\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+a*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/3360*(105*(3*sqrt(2)*a^3*cos(d*x + c)^3 - 4*sqrt(2)*a^3*cos(d*x + c) - (s
qrt(2)*a^3*cos(d*x + c)^3 - 4*sqrt(2)*a^3*cos(d*x + c))*sin(d*x + c))*sqrt(
a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(a*sin(d*x + c) + a)*(sqrt(2)*cos(d*x + c
) - sqrt(2)*sin(d*x + c) + sqrt(2))*sqrt(a) + 3*a*cos(d*x + c) - (a*cos(d*x
 + c) - 2*a)*sin(d*x + c) + 2*a)/(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d
*x + c) - cos(d*x + c) - 2)) + 4*(385*a^3*cos(d*x + c)^2 - 764*a^3 - 7*(15*
a^3*cos(d*x + c)^2 - 92*a^3)*sin(d*x + c))*sqrt(a*sin(d*x + c) + a))/(3*d*c
os(d*x + c)^3 - 4*d*cos(d*x + c) - (d*cos(d*x + c)^3 - 4*d*cos(d*x + c))*si
n(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**8*(a+a*sin(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 3.16, size = 127, normalized size = 0.74

$$-\frac{\sqrt{2}\, a^{\frac{7}{2}} \left(\frac{2 \left(105 \sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{6}+35 \sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{4}+21 \sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+15\right)}{\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{7}} - 105 \log\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right) + 105 \log\left(-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)\right) \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{3360\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+a*sin(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] -1/3360*sqrt(2)*a^(7/2)*(2*(105*sin(-1/4*pi + 1/2*d*x + 1/2*c)^6 + 35*sin(-
1/4*pi + 1/2*d*x + 1/2*c)^4 + 21*sin(-1/4*pi + 1/2*d*x + 1/2*c)^2 + 15)/sin
(-1/4*pi + 1/2*d*x + 1/2*c)^7 - 105*log(sin(-1/4*pi + 1/2*d*x + 1/2*c) + 1)
 + 105*log(-sin(-1/4*pi + 1/2*d*x + 1/2*c) + 1))*sgn(cos(-1/4*pi + 1/2*d*x
+ 1/2*c))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{7/2}}{{\cos\left(c+d\,x\right)}^8} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(7/2)/cos(c + d*x)^8,x)
```

```
[Out] int((a + a*sin(c + d*x))^(7/2)/cos(c + d*x)^8, x)
```

3.154 $\int \sec^9(c+dx)(a+a\sin(c+dx))^{7/2}\,dx$

Optimal. Leaf size=191

$$\frac{315a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{2048\sqrt{2}\,d}-\frac{315a^4}{2048d\sqrt{a+a\sin(c+dx)}}+\frac{105a^3\sec^2(c+dx)\sqrt{a+a\sin(c+dx)}}{1024d}$$

[Out] 21/256*a^2*sec(d*x+c)^4*(a+a*sin(d*x+c))^(3/2)/d+3/32*a*sec(d*x+c)^6*(a+a*sin(d*x+c))^(5/2)/d+1/8*sec(d*x+c)^8*(a+a*sin(d*x+c))^(7/2)/d+315/4096*a^(7/2)*arctanh(1/2*(a+a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)/d-315/2048*a^4/d/(a+a*sin(d*x+c))^(1/2)+105/1024*a^3*sec(d*x+c)^2*(a+a*sin(d*x+c))^(1/2)/d

Rubi [A]
time = 0.21, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2754, 2746, 53, 65, 212}

$$\frac{315a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{2048\sqrt{2}\,d}-\frac{315a^4}{2048d\sqrt{a\sin(c+dx)+a}}+\frac{105a^3\sec^2(c+dx)\sqrt{a\sin(c+dx)+a}}{1024d}+\frac{21a^2\sec^4(c+dx)(a\sin(c+dx)+a)^{3/2}}{256d}+\frac{\sec^8(c+dx)(a\sin(c+dx)+a)^{7/2}}{8d}+\frac{3a\sec^6(c+dx)(a\sin(c+dx)+a)^{5/2}}{32d}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d*x]^9*(a + a*Sin[c + d*x])^(7/2),x]

[Out] (315*a^(7/2)*ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(2048*Sqrt[2]*d) - (315*a^4)/(2048*d*Sqrt[a + a*Sin[c + d*x]]) + (105*a^3*Sec[c + d*x]^2*Sqrt[a + a*Sin[c + d*x]])/(1024*d) + (21*a^2*Sec[c + d*x]^4*(a + a*Sin[c + d*x])^(3/2))/(256*d) + (3*a*Sec[c + d*x]^6*(a + a*Sin[c + d*x])^(5/2))/(32*d) + (Sec[c + d*x]^8*(a + a*Sin[c + d*x])^(7/2))/(8*d)

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2754

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^m/(a*f*g*(p + 1))), x] + Dist[a*((m + p + 1)/(g^2*(p + 1))), Int[(g*C
os[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e
, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ[
m + 1/2, 2*p]
```

Rubi steps

$$\int \sec^9(c+dx)(a+a\sin(c+dx))^{7/2}\,dx = \frac{\sec^8(c+dx)(a+a\sin(c+dx))^{7/2}}{8d} + \frac{1}{16}(9a)\int \sec^7(c+dx)(a+a$$

$$= \frac{3a\sec^6(c+dx)(a+a\sin(c+dx))^{5/2}}{32d} + \frac{\sec^8(c+dx)(a+a\sin(c+}{8d}$$

$$= \frac{21a^2\sec^4(c+dx)(a+a\sin(c+dx))^{3/2}}{256d} + \frac{3a\sec^6(c+dx)(a+a\sin}{32d}$$

$$= \frac{105a^3\sec^2(c+dx)\sqrt{a+a\sin(c+dx)}}{1024d} + \frac{21a^2\sec^4(c+dx)(a+a}{256d}$$

$$= \frac{105a^3\sec^2(c+dx)\sqrt{a+a\sin(c+dx)}}{1024d} + \frac{21a^2\sec^4(c+dx)(a+a}{256d}$$

$$= -\frac{315a^4}{2048d\sqrt{a+a\sin(c+dx)}} + \frac{105a^3\sec^2(c+dx)\sqrt{a+a\sin(c+}}{1024d}$$

$$= -\frac{315a^4}{2048d\sqrt{a+a\sin(c+dx)}} + \frac{105a^3\sec^2(c+dx)\sqrt{a+a\sin(c+}}{1024d}$$

$$= \frac{315a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{2048\sqrt{2}\,d} - \frac{315a^4}{2048d\sqrt{a+a\sin(c+a}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.07, size = 44, normalized size = 0.23

$$-\frac{a^4\,{}_2F_1\left(-\frac{1}{2},5;\frac{1}{2};\frac{1}{2}(1+\sin(c+dx))\right)}{16d\sqrt{a+a\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^9*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] -1/16*(a^4*Hypergeometric2F1[-1/2, 5, 1/2, (1 + Sin[c + d*x])/2])/(d*Sqrt[a
 + a*Sin[c + d*x]])
```

Maple [A]
time = 0.92, size = 129, normalized size = 0.68

method	result

default	$-\frac{2a^9\left(\frac{-\frac{\sqrt{a+a\sin(dx+c)}\, a^3\left(187\left(\cos^2(dx+c)\right)\sin(dx+c)-725\left(\cos^2(dx+c)\right)-1236\sin(dx+c)+1364\right)}{128(a\sin(dx+c)-a)^4}-\frac{315\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{a}}{}\right.}{}}{32a^5}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^9*(a+a*sin(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*a^9*(1/32/a^5*(-1/128*(a+a*sin(d*x+c))^(1/2)*a^3*(187*cos(d*x+c)^2*sin(d
*x+c)-725*cos(d*x+c)^2-1236*sin(d*x+c)+1364)/(a*sin(d*x+c)-a)^4-315/256*2^(
1/2)/a^(1/2)*arctanh(1/2*(a+a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2)))+1/32/a^5/
(a+a*sin(d*x+c))^(1/2))/d
```

Maxima [A]
time = 0.53, size = 219, normalized size = 1.15

$$-\frac{315\sqrt{2}\,a^{\frac{9}{2}}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{a\sin(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{a\sin(dx+c)+a}}\right)+\frac{4\left(315\,(a\sin(dx+c)+a)^4a^5-2310\,(a\sin(dx+c)+a)^3a^6+6132\,(a\sin(dx+c)+a)^2a^7-6696\,(a\sin(dx+c)+a)a^8+2048\,a^9\right)}{(a\sin(dx+c)+a)^{\frac{9}{2}}-8\,(a\sin(dx+c)+a)^{\frac{7}{2}}a+24\,(a\sin(dx+c)+a)^{\frac{5}{2}}a^2-32\,(a\sin(dx+c)+a)^{\frac{3}{2}}a^3+16\,\sqrt{a\sin(dx+c)+a}\,a^4}}{8192\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*(a+a*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] -1/8192*(315*sqrt(2)*a^(9/2)*log(-(sqrt(2)*sqrt(a) - sqrt(a*sin(d*x + c) +
a))/(sqrt(2)*sqrt(a) + sqrt(a*sin(d*x + c) + a))) + 4*(315*(a*sin(d*x + c)
+ a)^4*a^5 - 2310*(a*sin(d*x + c) + a)^3*a^6 + 6132*(a*sin(d*x + c) + a)^2*
a^7 - 6696*(a*sin(d*x + c) + a)*a^8 + 2048*a^9)/((a*sin(d*x + c) + a)^(9/2)
 - 8*(a*sin(d*x + c) + a)^(7/2)*a + 24*(a*sin(d*x + c) + a)^(5/2)*a^2 - 32*
(a*sin(d*x + c) + a)^(3/2)*a^3 + 16*sqrt(a*sin(d*x + c) + a)*a^4))/(a*d)
```

Fricas [A]
time = 0.37, size = 254, normalized size = 1.33

$$\frac{315\left(3\sqrt{2}\,a^3\cos(dx+c)^4-4\sqrt{2}\,a^3\cos(dx+c)^2-\left(\sqrt{2}\,a^3\cos(dx+c)^4-4\sqrt{2}\,a^3\cos(dx+c)^2\right)\sin(dx+c)\right)\sqrt{a}\,\log\left(-\frac{a\sin(dx+c)+2\sqrt{2}\sqrt{a\sin(dx+c)+a}\sqrt{a}+3a}{\sin(dx+c)-1}\right)+4\left(315\,a^3\cos(dx+c)^4-1722\,a^3\cos(dx+c)^2+896\,a^3+6\left(175\,a^3\cos(dx+c)^2-192\,a^3\right)\sin(dx+c)\right)\sqrt{a\sin(dx+c)+a}}{8192\left(3\,d\cos(dx+c)^4-4\,d\cos(dx+c)^2-\left(d\cos(dx+c)^4-4\,d\cos(dx+c)^2\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*(a+a*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/8192*(315*(3*sqrt(2)*a^3*cos(d*x + c)^4 - 4*sqrt(2)*a^3*cos(d*x + c)^2 -
(sqrt(2)*a^3*cos(d*x + c)^4 - 4*sqrt(2)*a^3*cos(d*x + c)^2)*sin(d*x + c))*s
qrt(a)*log(-(a*sin(d*x + c) + 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*sqrt(a) +
3*a)/(sin(d*x + c) - 1)) + 4*(315*a^3*cos(d*x + c)^4 - 1722*a^3*cos(d*x + c
```

```
)^2 + 896*a^3 + 6*(175*a^3*cos(d*x + c)^2 - 192*a^3)*sin(d*x + c))*sqrt(a*s
in(d*x + c) + a))/(3*d*cos(d*x + c)^4 - 4*d*cos(d*x + c)^2 - (d*cos(d*x + c
)^4 - 4*d*cos(d*x + c)^2)*sin(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**9*(a+a*sin(d*x+c))**(7/2),x)
[Out] Timed out
```

Giac [A]
time = 5.35, size = 160, normalized size = 0.84

$$-\frac{\sqrt{2}\,a^{\frac{7}{2}}\left(\frac{256}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)}+\frac{2\left(187\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{7}-643\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{5}+765\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{3}-325\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-1\right)^{4}}-315\log\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)+315\log\left(-\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)\right)\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{8192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*(a+a*sin(d*x+c))^(7/2),x, algorithm="giac")
[Out] -1/8192*sqrt(2)*a^(7/2)*(256/cos(-1/4*pi + 1/2*d*x + 1/2*c) + 2*(187*cos(-1
/4*pi + 1/2*d*x + 1/2*c)^7 - 643*cos(-1/4*pi + 1/2*d*x + 1/2*c)^5 + 765*cos
(-1/4*pi + 1/2*d*x + 1/2*c)^3 - 325*cos(-1/4*pi + 1/2*d*x + 1/2*c))/(cos(-1
/4*pi + 1/2*d*x + 1/2*c)^2 - 1)^4 - 315*log(cos(-1/4*pi + 1/2*d*x + 1/2*c)
+ 1) + 315*log(-cos(-1/4*pi + 1/2*d*x + 1/2*c) + 1))*sgn(cos(-1/4*pi + 1/2*
d*x + 1/2*c))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{7/2}}{{\cos\left(c+d\,x\right)}^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(7/2)/cos(c + d*x)^9,x)
[Out] int((a + a*sin(c + d*x))^(7/2)/cos(c + d*x)^9, x)
```

3.155 $\int \sec^{10}(c+dx)(a+a\sin(c+dx))^{7/2}\,dx$

Optimal. Leaf size=233

$$-\frac{11a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{64\sqrt{2}\,d}-\frac{11a^5\cos(c+dx)}{64d(a+a\sin(c+dx))^{3/2}}+\frac{11a^4\sec(c+dx)}{48d\sqrt{a+a\sin(c+dx)}}+\frac{11a^3\,\mathrm{se}}{}$$

```
[Out] -11/64*a^5*cos(d*x+c)/d/(a+a*sin(d*x+c))^(3/2)+11/140*a^2*sec(d*x+c)^5*(a+a
*sin(d*x+c))^(3/2)/d+11/126*a*sec(d*x+c)^7*(a+a*sin(d*x+c))^(5/2)/d+1/9*sec
(d*x+c)^9*(a+a*sin(d*x+c))^(7/2)/d-11/128*a^(7/2)*arctanh(1/2*cos(d*x+c)*a^
(1/2)*2^(1/2)/(a+a*sin(d*x+c))^(1/2))/d*2^(1/2)+11/48*a^4*sec(d*x+c)/d/(a+a
*sin(d*x+c))^(1/2)+11/120*a^3*sec(d*x+c)^3*(a+a*sin(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.25, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2754, 2766, 2729, 2728, 212}

$$-\frac{11a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{64\sqrt{2}\,d}-\frac{11a^5\cos(c+dx)}{64d(a\sin(c+dx)+a)^{3/2}}+\frac{11a^4\sec(c+dx)}{48d\sqrt{a\sin(c+dx)+a}}+\frac{11a^3\sec^3(c+dx)\sqrt{a\sin(c+dx)+a}}{120d}+\frac{11a^2\sec^5(c+dx)(a\sin(c+dx)+a)^{3/2}}{140d}+\frac{\sec^9(c+dx)(a\sin(c+dx)+a)^{7/2}}{9d}+\frac{11a\sec^7(c+dx)(a\sin(c+dx)+a)^{5/2}}{126d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^10*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] (-11*a^(7/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x
]]])])/(64*Sqrt[2]*d) - (11*a^5*Cos[c + d*x])/(64*d*(a + a*Sin[c + d*x])^(3/
2)) + (11*a^4*Sec[c + d*x])/(48*d*Sqrt[a + a*Sin[c + d*x]]) + (11*a^3*Sec[c
 + d*x]^3*Sqrt[a + a*Sin[c + d*x]])/(120*d) + (11*a^2*Sec[c + d*x]^5*(a + a
*Sin[c + d*x])^(3/2))/(140*d) + (11*a*Sec[c + d*x]^7*(a + a*Sin[c + d*x])^(
5/2))/(126*d) + (Sec[c + d*x]^9*(a + a*Sin[c + d*x])^(7/2))/(9*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
```

```
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2754

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^m/(a*f*g*(p + 1))), x] + Dist[a*((m + p + 1)/(g^2*(p + 1))), Int[(g*C
os[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e
, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ[
m + 1/2, 2*p]
```

Rule 2766

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(a*f*g*(p + 1)*S
qrt[a + b*Sin[e + f*x]])), x] + Dist[a*((2*p + 1)/(2*g^2*(p + 1))), Int[(g*
Cos[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e,
 f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\int \sec^{10}(c+dx)(a+a\sin(c+dx))^{7/2}\,dx = \frac{\sec^9(c+dx)(a+a\sin(c+dx))^{7/2}}{9d} + \frac{1}{18}(11a)\int \sec^8(c+dx)(a+$$

$$= \frac{11a\sec^7(c+dx)(a+a\sin(c+dx))^{5/2}}{126d} + \frac{\sec^9(c+dx)(a+a\sin(c}{9d}$$

$$= \frac{11a^2\sec^5(c+dx)(a+a\sin(c+dx))^{3/2}}{140d} + \frac{11a\sec^7(c+dx)(a+a}{126d}$$

$$= \frac{11a^3\sec^3(c+dx)\sqrt{a+a\sin(c+dx)}}{120d} + \frac{11a^2\sec^5(c+dx)(a+a}{140d}$$

$$= \frac{11a^4\sec(c+dx)}{48d\sqrt{a+a\sin(c+dx)}} + \frac{11a^3\sec^3(c+dx)\sqrt{a+a\sin(c+dx)}}{120d}$$

$$= -\frac{11a^5\cos(c+dx)}{64d(a+a\sin(c+dx))^{3/2}} + \frac{11a^4\sec(c+dx)}{48d\sqrt{a+a\sin(c+dx)}} + \frac{11a^3\sec}{}$$

$$= -\frac{11a^5\cos(c+dx)}{64d(a+a\sin(c+dx))^{3/2}} + \frac{11a^4\sec(c+dx)}{48d\sqrt{a+a\sin(c+dx)}} + \frac{11a^3\sec}{}$$

$$= -\frac{11a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{64\sqrt{2}\,d} - \frac{11a^5\cos(c+}{64d(a+a\sin(c+}$$

Mathematica [C] Result contains complex when optimal does not.
time = 5.43, size = 388, normalized size = 1.67

$$\frac{\left(630\sin\left(\frac{1}{2}(c+dx)\right)-315\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+(3465+3465i)(-1)^{3/4}\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(c+dx)\right)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2+\frac{1120\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^9}+\frac{1440\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^7}+\frac{1512\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^5}+\frac{1680\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3}+\frac{3150\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)}\right)(a(1+\sin(c+dx)))^{7/2}}{20160d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^10*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] ((630*Sin[(c + d*x)/2] - 315*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) + (3465
+ 3465*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/4])
]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + (1120*(Cos[(c + d*x)/2] + Sin[(
c + d*x)/2])^2)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^9 + (1440*(Cos[(c + d
*x)/2] + Sin[(c + d*x)/2])^2)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^7 + (15
12*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2)/(Cos[(c + d*x)/2] - Sin[(c + d*
x)/2])^5 + (1680*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2)/(Cos[(c + d*x)/2]
 - Sin[(c + d*x)/2])^3 + (3150*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2)/(Co
s[(c + d*x)/2] - Sin[(c + d*x)/2]))*(a*(1 + Sin[c + d*x]))^(7/2))/(20160*d*
(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^9)
```

Maple [A]
time = 0.56, size = 205, normalized size = 0.88

method	result
default	$-\frac{-6930a^{\frac{11}{2}}\sin(dx+c)\left(\cos^4(dx+c)\right)+42504a^{\frac{11}{2}}\sin(dx+c)\left(\cos^2(dx+c)\right)+385\left(9(a-a\sin(dx+c))^{\frac{9}{2}}\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{a-a\sin}}{2}\right.\right.}{4032}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^10*(a+a*sin(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/40320/a^(3/2)*(-6930*a^(11/2)*sin(d*x+c)*cos(d*x+c)^4+42504*a^(11/2)*sin
(d*x+c)*cos(d*x+c)^2+385*(9*(a-a*sin(d*x+c))^(9/2)*2^(1/2)*arctanh(1/2*(a-a
*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a-32*a^(11/2))*sin(d*x+c)+25410*a^(11/2
)*cos(d*x+c)^4-50424*a^(11/2)*cos(d*x+c)^2+3465*(a-a*sin(d*x+c))^(9/2)*2^(1
/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a+7840*a^(11/2))/(s
in(d*x+c)-1)^4/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10*(a+a*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

[Out] Timed out

Fricas [A]
time = 0.38, size = 346, normalized size = 1.48

$$\frac{3465 \left(3 \sqrt{2} a^{3} \cos \left(dx+c\right)^{5}-4 \sqrt{2} a^{3} \cos \left(dx+c\right)^{3}-\left(\sqrt{2} a^{3} \cos \left(dx+c\right)^{5}-4 \sqrt{2} a^{3} \cos \left(dx+c\right)^{3}\right) \sin \left(dx+c\right)\right) \sqrt{a} \log \left(-\frac{a \cos(dx+c)^{2}-2 \sqrt{a \sin \left(dx+c\right)+a} \left(\sqrt{2} \cos(dx+c)-\sqrt{2} \sin(dx+c)+\sqrt{2}\right) \sqrt{a}+3 a \cos(dx+c)-(a \cos(dx+c)-2 a) \sin(dx+c)+2 a}{\cos(dx+c)^{2}-(\cos(dx+c)+2) \sin(dx+c)-\cos(dx+c)-2}\right)+4 \left(12705 a^{3} \cos \left(dx+c\right)^{4}-25212 a^{3} \cos \left(dx+c\right)^{2}+3920 a^{3}-77 \left(45 a^{3} \cos \left(dx+c\right)^{4}-276 a^{3} \cos \left(dx+c\right)^{2}+80 a^{3}\right) \sin \left(dx+c\right)\right) \sqrt{a \sin \left(dx+c\right)+a}}{80640 \left(3 d \cos \left(dx+c\right)^{5}-4 d \cos \left(dx+c\right)^{3}-\left(d \cos \left(dx+c\right)^{5}-4 d \cos \left(dx+c\right)^{3}\right) \sin \left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10*(a+a*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/80640*(3465*(3*sqrt(2)*a^3*cos(d*x + c)^5 - 4*sqrt(2)*a^3*cos(d*x + c)^3
- (sqrt(2)*a^3*cos(d*x + c)^5 - 4*sqrt(2)*a^3*cos(d*x + c)^3)*sin(d*x + c))
*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(a*sin(d*x + c) + a)*(sqrt(2)*cos(d
*x + c) - sqrt(2)*sin(d*x + c) + sqrt(2))*sqrt(a) + 3*a*cos(d*x + c) - (a*c
os(d*x + c) - 2*a)*sin(d*x + c) + 2*a)/(cos(d*x + c)^2 - (cos(d*x + c) + 2)
*sin(d*x + c) - cos(d*x + c) - 2)) + 4*(12705*a^3*cos(d*x + c)^4 - 25212*a^
3*cos(d*x + c)^2 + 3920*a^3 - 77*(45*a^3*cos(d*x + c)^4 - 276*a^3*cos(d*x +
 c)^2 + 80*a^3)*sin(d*x + c))*sqrt(a*sin(d*x + c) + a))/(3*d*cos(d*x + c)^5
 - 4*d*cos(d*x + c)^3 - (d*cos(d*x + c)^5 - 4*d*cos(d*x + c)^3)*sin(d*x + c
))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**10*(a+a*sin(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 6.65, size = 175, normalized size = 0.75

$$-\frac{\sqrt{2} a^{\frac{7}{2}} \left(\frac{630 \sin\left(-\frac{1}{4} \pi+\frac{1}{2} dx+\frac{1}{2} c\right)}{\sin\left(-\frac{1}{4} \pi+\frac{1}{2} dx+\frac{1}{2} c\right)^{2}-1}+\frac{4 \left(1575 \sin\left(-\frac{1}{4} \pi+\frac{1}{2} dx+\frac{1}{2} c\right)^{8}+420 \sin\left(-\frac{1}{4} \pi+\frac{1}{2} dx+\frac{1}{2} c\right)^{6}+189 \sin\left(-\frac{1}{4} \pi+\frac{1}{2} dx+\frac{1}{2} c\right)^{4}+90 \sin\left(-\frac{1}{4} \pi+\frac{1}{2} dx+\frac{1}{2} c\right)^{2}+35\right)}{\sin\left(-\frac{1}{4} \pi+\frac{1}{2} dx+\frac{1}{2} c\right)^{9}}-3465 \log \left(\sin \left(-\frac{1}{4} \pi+\frac{1}{2} dx+\frac{1}{2} c\right)+1\right)+3465 \log \left(-\sin \left(-\frac{1}{4} \pi+\frac{1}{2} dx+\frac{1}{2} c\right)+1\right)\right) \mathrm{sgn}\left(\cos \left(-\frac{1}{4} \pi+\frac{1}{2} dx+\frac{1}{2} c\right)\right)}{80640 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10*(a+a*sin(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] -1/80640*sqrt(2)*a^(7/2)*(630*sin(-1/4*pi + 1/2*d*x + 1/2*c)/(sin(-1/4*pi +
 1/2*d*x + 1/2*c)^2 - 1) + 4*(1575*sin(-1/4*pi + 1/2*d*x + 1/2*c)^8 + 420*s
in(-1/4*pi + 1/2*d*x + 1/2*c)^6 + 189*sin(-1/4*pi + 1/2*d*x + 1/2*c)^4 + 90
*sin(-1/4*pi + 1/2*d*x + 1/2*c)^2 + 35)/sin(-1/4*pi + 1/2*d*x + 1/2*c)^9 -
3465*log(sin(-1/4*pi + 1/2*d*x + 1/2*c) + 1) + 3465*log(-sin(-1/4*pi + 1/2*
d*x + 1/2*c) + 1))*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + a\,\sin(c + d\,x))^{7/2}}{\cos(c + d\,x)^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(7/2)/cos(c + d*x)^10,x)
[Out] int((a + a*sin(c + d*x))^(7/2)/cos(c + d*x)^10, x)
```

3.156 $\int \frac{\cos^7(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=97

$$\frac{16(a+a\sin(c+dx))^{7/2}}{7a^4d}-\frac{8(a+a\sin(c+dx))^{9/2}}{3a^5d}+\frac{12(a+a\sin(c+dx))^{11/2}}{11a^6d}-\frac{2(a+a\sin(c+dx))^{13/2}}{13a^7d}$$

```
[Out] 16/7*(a+a*sin(d*x+c))^(7/2)/a^4/d-8/3*(a+a*sin(d*x+c))^(9/2)/a^5/d+12/11*(a
+a*sin(d*x+c))^(11/2)/a^6/d-2/13*(a+a*sin(d*x+c))^(13/2)/a^7/d
```

Rubi [A]
time = 0.05, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$,
Rules used = {2746, 45}

$$-\frac{2(a\sin(c+dx)+a)^{13/2}}{13a^7d}+\frac{12(a\sin(c+dx)+a)^{11/2}}{11a^6d}-\frac{8(a\sin(c+dx)+a)^{9/2}}{3a^5d}+\frac{16(a\sin(c+dx)+a)^{7/2}}{7a^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (16*(a + a*Sin[c + d*x])^(7/2))/(7*a^4*d) - (8*(a + a*Sin[c + d*x])^(9/2))/
(3*a^5*d) + (12*(a + a*Sin[c + d*x])^(11/2))/(11*a^6*d) - (2*(a + a*Sin[c +
 d*x])^(13/2))/(13*a^7*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\int \frac{\cos^7(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx = \frac{\text{Subst}\left(\int (a-x)^3(a+x)^{5/2}\,dx, x, a\sin(c+dx)\right)}{a^7 d}$$
$$= \frac{\text{Subst}\left(\int \left(8a^3(a+x)^{5/2} - 12a^2(a+x)^{7/2} + 6a(a+x)^{9/2} - (a+x)^{11/2}\right)\,dx, x,$$
$$= \frac{16(a+a\sin(c+dx))^{7/2}}{7a^4 d} - \frac{8(a+a\sin(c+dx))^{9/2}}{3a^5 d} + \frac{12(a+a\sin(c+dx))^{11/}}{11a^6 d}$$

Mathematica [A]
time = 0.18, size = 61, normalized size = 0.63

$$-\frac{2(1+\sin(c+dx))^4\left(-835+1421\sin(c+dx)-945\sin^2(c+dx)+231\sin^3(c+dx)\right)}{3003d\sqrt{a(1+\sin(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-2*(1 + Sin[c + d*x])^4*(-835 + 1421*Sin[c + d*x] - 945*Sin[c + d*x]^2 + 2
31*Sin[c + d*x]^3))/(3003*d*Sqrt[a*(1 + Sin[c + d*x])])
```

Maple [A]
time = 0.39, size = 57, normalized size = 0.59

method	result	size
default	$\frac{2(a+a\sin(dx+c))^{\frac{7}{2}}\left(231\left(\cos^2(dx+c)\right)\sin(dx+c)-945\left(\cos^2(dx+c)\right)-1652\sin(dx+c)+1780\right)}{3003a^4 d}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7/(a+a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3003/a^4*(a+a*sin(d*x+c))^(7/2)*(231*cos(d*x+c)^2*sin(d*x+c)-945*cos(d*x+
c)^2-1652*sin(d*x+c)+1780)/d
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 281 vs. 2(81) = 162.
time = 0.29, size = 281, normalized size = 2.90

$$\frac{2\left(15015\sqrt{a\sin(dx+c)+a} - \frac{3003\left(3(a\sin(dx+c)+a)^{\frac{5}{2}}-10(a\sin(dx+c)+a)^{\frac{3}{2}}a+15\sqrt{a\sin(dx+c)+a}\,a^2\right)}{a^2} + \frac{143\left(35(a\sin(dx+c)+a)^{\frac{9}{2}}-180(a\sin(dx+c)+a)^{\frac{7}{2}}a+378(a\sin(dx+c)+a)^{\frac{5}{2}}a^2-420(a\sin(dx+c)+a)^{\frac{3}{2}}a^3+315\sqrt{a\sin(dx+c)+a}\,a^4\right)}{a^4} - \frac{5\left(231(a\sin(dx+c)+a)^{\frac{13}{2}}-1638(a\sin(dx+c)+a)^{\frac{11}{2}}a+5005(a\sin(dx+c)+a)^{\frac{9}{2}}a^2-8580(a\sin(dx+c)+a)^{\frac{7}{2}}a^3+9009(a\sin(dx+c)+a)^{\frac{5}{2}}a^4-6006(a\sin(dx+c)+a)^{\frac{3}{2}}a^5+3003\sqrt{a\sin(dx+c)+a}\,a^6\right)}{a^6}\right)}{15015ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/15015*(15015*sqrt(a*sin(d*x + c) + a) - 3003*(3*(a*sin(d*x + c) + a)^(5/2
) - 10*(a*sin(d*x + c) + a)^(3/2)*a + 15*sqrt(a*sin(d*x + c) + a)*a^2)/a^2
+ 143*(35*(a*sin(d*x + c) + a)^(9/2) - 180*(a*sin(d*x + c) + a)^(7/2)*a + 3
78*(a*sin(d*x + c) + a)^(5/2)*a^2 - 420*(a*sin(d*x + c) + a)^(3/2)*a^3 + 31
5*sqrt(a*sin(d*x + c) + a)*a^4)/a^4 - 5*(231*(a*sin(d*x + c) + a)^(13/2) -
1638*(a*sin(d*x + c) + a)^(11/2)*a + 5005*(a*sin(d*x + c) + a)^(9/2)*a^2 -
8580*(a*sin(d*x + c) + a)^(7/2)*a^3 + 9009*(a*sin(d*x + c) + a)^(5/2)*a^4 -
 6006*(a*sin(d*x + c) + a)^(3/2)*a^5 + 3003*sqrt(a*sin(d*x + c) + a)*a^6)/a
^6)/(a*d)
```

Fricas [A]
time = 0.36, size = 82, normalized size = 0.85

$$\frac{2\left(231\cos\left(dx+c\right)^{6}+28\cos\left(dx+c\right)^{4}+64\cos\left(dx+c\right)^{2}+4\left(63\cos\left(dx+c\right)^{4}+80\cos\left(dx+c\right)^{2}+128\right)\sin\left(dx+c\right)+512\right)\sqrt{a\sin\left(dx+c\right)+a}}{3003\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3003*(231*cos(d*x + c)^6 + 28*cos(d*x + c)^4 + 64*cos(d*x + c)^2 + 4*(63*
cos(d*x + c)^4 + 80*cos(d*x + c)^2 + 128)*sin(d*x + c) + 512)*sqrt(a*sin(d*
x + c) + a)/(a*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 6.06, size = 112, normalized size = 1.15

$$-\frac{128\left(231\sqrt{2}\sqrt{a}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{13}-819\sqrt{2}\sqrt{a}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{11}+1001\sqrt{2}\sqrt{a}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{9}-429\sqrt{2}\sqrt{a}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{7}\right)}{3003\,ad\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -128/3003*(231*sqrt(2)*sqrt(a)*cos(-1/4*pi + 1/2*d*x + 1/2*c)^13 - 819*sqrt
(2)*sqrt(a)*cos(-1/4*pi + 1/2*d*x + 1/2*c)^11 + 1001*sqrt(2)*sqrt(a)*cos(-1
/4*pi + 1/2*d*x + 1/2*c)^9 - 429*sqrt(2)*sqrt(a)*cos(-1/4*pi + 1/2*d*x + 1/
2*c)^7)/(a*d*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)^7}{\sqrt{a+a\,\sin(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7/(a + a*sin(c + d*x))^(1/2),x)
[Out] int(cos(c + d*x)^7/(a + a*sin(c + d*x))^(1/2), x)
```

3.157 $\int \frac{\cos^6(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=95

$$-\frac{64a^3\cos^7(c+dx)}{693d(a+a\sin(c+dx))^{7/2}}-\frac{16a^2\cos^7(c+dx)}{99d(a+a\sin(c+dx))^{5/2}}-\frac{2a\cos^7(c+dx)}{11d(a+a\sin(c+dx))^{3/2}}$$

```
[Out] -64/693*a^3*cos(d*x+c)^7/d/(a+a*sin(d*x+c))^(7/2)-16/99*a^2*cos(d*x+c)^7/d/
(a+a*sin(d*x+c))^(5/2)-2/11*a*cos(d*x+c)^7/d/(a+a*sin(d*x+c))^(3/2)
```

Rubi [A]
time = 0.12, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2753, 2752}

$$-\frac{64a^3\cos^7(c+dx)}{693d(a\sin(c+dx)+a)^{7/2}}-\frac{16a^2\cos^7(c+dx)}{99d(a\sin(c+dx)+a)^{5/2}}-\frac{2a\cos^7(c+dx)}{11d(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-64*a^3*Cos[c + d*x]^7)/(693*d*(a + a*Sin[c + d*x])^(7/2)) - (16*a^2*Cos[c
 + d*x]^7)/(99*d*(a + a*Sin[c + d*x])^(5/2)) - (2*a*Cos[c + d*x]^7)/(11*d*(
a + a*Sin[c + d*x])^(3/2))
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2753

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && N
eQ[m + p, 0]
```

Rubi steps

$$\int \frac{\cos^6(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx = -\frac{2a\cos^7(c+dx)}{11d(a+a\sin(c+dx))^{3/2}} + \frac{1}{11}(8a)\int \frac{\cos^6(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$$
$$= -\frac{16a^2\cos^7(c+dx)}{99d(a+a\sin(c+dx))^{5/2}} - \frac{2a\cos^7(c+dx)}{11d(a+a\sin(c+dx))^{3/2}} + \frac{1}{99}(32a^2)\int \frac{\text{c}}{(a+a}$$
$$= -\frac{64a^3\cos^7(c+dx)}{693d(a+a\sin(c+dx))^{7/2}} - \frac{16a^2\cos^7(c+dx)}{99d(a+a\sin(c+dx))^{5/2}} - \frac{2a\cos^7(c+}{11d(a+a\sin(c+}$$

Mathematica [A]
time = 0.16, size = 59, normalized size = 0.62

$$-\frac{2\cos^7(c+dx)\,(151+182\sin(c+dx)+63\sin^2(c+dx))}{693d(1+\sin(c+dx))^3\sqrt{a(1+\sin(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-2*Cos[c + d*x]^7*(151 + 182*Sin[c + d*x] + 63*Sin[c + d*x]^2))/(693*d*(1
+ Sin[c + d*x])^3*Sqrt[a*(1 + Sin[c + d*x])])
```

Maple [A]
time = 0.39, size = 64, normalized size = 0.67

method	result	size
default	$-\frac{2(1+\sin(dx+c))(\sin(dx+c)-1)^4(63(\sin^2(dx+c))+182\sin(dx+c)+151)}{693\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6/(a+a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/693*(1+sin(d*x+c))*(sin(d*x+c)-1)^4*(63*sin(d*x+c)^2+182*sin(d*x+c)+151)
/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^6/sqrt(a*sin(d*x + c) + a), x)
```

Fricas [A]
time = 0.34, size = 155, normalized size = 1.63

$$\frac{2\,{\left(63\,\cos\left(dx+c\right)^{6}-7\,\cos\left(dx+c\right)^{5}+10\,\cos\left(dx+c\right)^{4}-16\,\cos\left(dx+c\right)^{3}+32\,\cos\left(dx+c\right)^{2}+{\left(63\,\cos\left(dx+c\right)^{5}+70\,\cos\left(dx+c\right)^{4}+80\,\cos\left(dx+c\right)^{3}+96\,\cos\left(dx+c\right)^{2}+128\,\cos\left(dx+c\right)+256\right)}\sin\left(dx+c\right)-128\,\cos\left(dx+c\right)-256\right)}\sqrt{a\sin\left(dx+c\right)+a}}{693\,{\left(ad\cos\left(dx+c\right)+ad\sin\left(dx+c\right)+ad\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/693*(63*cos(d*x + c)^6 - 7*cos(d*x + c)^5 + 10*cos(d*x + c)^4 - 16*cos(d*
x + c)^3 + 32*cos(d*x + c)^2 + (63*cos(d*x + c)^5 + 70*cos(d*x + c)^4 + 80*
cos(d*x + c)^3 + 96*cos(d*x + c)^2 + 128*cos(d*x + c) + 256)*sin(d*x + c) -
 128*cos(d*x + c) - 256)*sqrt(a*sin(d*x + c) + a)/(a*d*cos(d*x + c) + a*d*s
in(d*x + c) + a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^{6}\left(c+dx\right)}{\sqrt{a\left(\sin\left(c+dx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(cos(c + d*x)**6/sqrt(a*(sin(c + d*x) + 1)), x)
```

Giac [A]
time = 6.49, size = 84, normalized size = 0.88

$$\frac{64\,\sqrt{2}\left(63\,\sqrt{a}\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-154\,\sqrt{a}\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+99\,\sqrt{a}\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}\right)}{693\,ad\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 64/693*sqrt(2)*(63*sqrt(a)*sin(-1/4*pi + 1/2*d*x + 1/2*c)^11 - 154*sqrt(a)*
sin(-1/4*pi + 1/2*d*x + 1/2*c)^9 + 99*sqrt(a)*sin(-1/4*pi + 1/2*d*x + 1/2*c
)^7)/(a*d*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\cos\left(c+d\,x\right)}^6}{\sqrt{a+a\,\sin\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(a + a*sin(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^6/(a + a*sin(c + d*x))^(1/2), x)
```

3.158 $\int \frac{\cos^5(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=73

$$\frac{8(a+a\sin(c+dx))^{5/2}}{5a^3d}-\frac{8(a+a\sin(c+dx))^{7/2}}{7a^4d}+\frac{2(a+a\sin(c+dx))^{9/2}}{9a^5d}$$

```
[Out] 8/5*(a+a*sin(d*x+c))^(5/2)/a^3/d-8/7*(a+a*sin(d*x+c))^(7/2)/a^4/d+2/9*(a+a*
sin(d*x+c))^(9/2)/a^5/d
```

Rubi [A]
time = 0.05, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = $\{2746, 45\}$

$$\frac{2(a\sin(c+dx)+a)^{9/2}}{9a^5d}-\frac{8(a\sin(c+dx)+a)^{7/2}}{7a^4d}+\frac{8(a\sin(c+dx)+a)^{5/2}}{5a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (8*(a + a*Sin[c + d*x])^(5/2))/(5*a^3*d) - (8*(a + a*Sin[c + d*x])^(7/2))/(
7*a^4*d) + (2*(a + a*Sin[c + d*x])^(9/2))/(9*a^5*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\int \frac{\cos^5(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx = \frac{\text{Subst}\left(\int (a-x)^2(a+x)^{3/2}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \left(4a^2(a+x)^{3/2}-4a(a+x)^{5/2}+(a+x)^{7/2}\right)\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{8(a+a\sin(c+dx))^{5/2}}{5a^3d} - \frac{8(a+a\sin(c+dx))^{7/2}}{7a^4d} + \frac{2(a+a\sin(c+dx))^{9/2}}{9a^5d}$$

Mathematica [A]
time = 0.09, size = 51, normalized size = 0.70

$$\frac{2(1+\sin(c+dx))^3\left(107-110\sin(c+dx)+35\sin^2(c+dx)\right)}{315d\sqrt{a(1+\sin(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/Sqrt[a + a*Sin[c + d*x]],x]
[Out] (2*(1 + Sin[c + d*x])^3*(107 - 110*Sin[c + d*x] + 35*Sin[c + d*x]^2))/(315*
d*Sqrt[a*(1 + Sin[c + d*x])])
```

Maple [A]
time = 0.31, size = 41, normalized size = 0.56

method	result	size
default	$-\frac{2(a+a\sin(dx+c))^{\frac{5}{2}}\left(35\left(\cos^2(dx+c)\right)+110\sin(dx+c)-142\right)}{315a^3d}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a+a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/315/a^3*(a+a*sin(d*x+c))^(5/2)*(35*cos(d*x+c)^2+110*sin(d*x+c)-142)/d
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 160 vs. 2(61) = 122.
time = 0.29, size = 160, normalized size = 2.19

$$\frac{2\left(315\sqrt{a\sin(dx+c)+a}-\frac{42\left(3(a\sin(dx+c)+a)^{\frac{5}{2}}-10(a\sin(dx+c)+a)^{\frac{3}{2}}a+15\sqrt{a\sin(dx+c)+a}\,a^2\right)}{a^2}+\frac{35(a\sin(dx+c)+a)^{\frac{9}{2}}-180(a\sin(dx+c)+a)^{\frac{7}{2}}a+378(a\sin(dx+c)+a)^{\frac{5}{2}}a^2-420(a\sin(dx+c)+a)^{\frac{3}{2}}a^3+315\sqrt{a\sin(dx+c)+a}\,a^4}{a^4}\right)}{315\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
[Out] 2/315*(315*sqrt(a*sin(d*x + c) + a) - 42*(3*(a*sin(d*x + c) + a)^(5/2) - 10
*(a*sin(d*x + c) + a)^(3/2)*a + 15*sqrt(a*sin(d*x + c) + a)*a^2)/a^2 + (35*
```

```
(a*sin(d*x + c) + a)^(9/2) - 180*(a*sin(d*x + c) + a)^(7/2)*a + 378*(a*sin(
d*x + c) + a)^(5/2)*a^2 - 420*(a*sin(d*x + c) + a)^(3/2)*a^3 + 315*sqrt(a*s
in(d*x + c) + a)*a^4)/a^4)/(a*d)
```

Fricas [A]
time = 0.33, size = 62, normalized size = 0.85

$$\frac{2\left(35\cos\left(dx+c\right)^{4}+8\cos\left(dx+c\right)^{2}+8\left(5\cos\left(dx+c\right)^{2}+8\right)\sin\left(dx+c\right)+64\right)\sqrt{a\sin\left(dx+c\right)+a}}{315\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
[Out] 2/315*(35*cos(d*x + c)^4 + 8*cos(d*x + c)^2 + 8*(5*cos(d*x + c)^2 + 8)*sin(
d*x + c) + 64)*sqrt(a*sin(d*x + c) + a)/(a*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a+a*sin(d*x+c))**(1/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3062 deep
```

Giac [A]
time = 6.06, size = 90, normalized size = 1.23

$$\frac{32\left(35\sqrt{2}\sqrt{a}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{9}-90\sqrt{2}\sqrt{a}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{7}+63\sqrt{2}\sqrt{a}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{5}\right)}{315\,ad\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
[Out] 32/315*(35*sqrt(2)*sqrt(a)*cos(-1/4*pi + 1/2*d*x + 1/2*c)^9 - 90*sqrt(2)*sq
rt(a)*cos(-1/4*pi + 1/2*d*x + 1/2*c)^7 + 63*sqrt(2)*sqrt(a)*cos(-1/4*pi + 1
/2*d*x + 1/2*c)^5)/(a*d*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^5}{\sqrt{a+a\,\sin\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(a + a*sin(c + d*x))^(1/2),x)
[Out] int(cos(c + d*x)^5/(a + a*sin(c + d*x))^(1/2), x)
```

3.159 $\int \frac{\cos^4(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=63

$$-\frac{8a^2\cos^5(c+dx)}{35d(a+a\sin(c+dx))^{5/2}}-\frac{2a\cos^5(c+dx)}{7d(a+a\sin(c+dx))^{3/2}}$$

```
[Out] -8/35*a^2*cos(d*x+c)^5/d/(a+a*sin(d*x+c))^(5/2)-2/7*a*cos(d*x+c)^5/d/(a+a*s
in(d*x+c))^(3/2)
```

Rubi [A]
time = 0.08, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2753, 2752}

$$-\frac{8a^2\cos^5(c+dx)}{35d(a\sin(c+dx)+a)^{5/2}}-\frac{2a\cos^5(c+dx)}{7d(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-8*a^2*Cos[c + d*x]^5)/(35*d*(a + a*Sin[c + d*x])^(5/2)) - (2*a*Cos[c + d*
x]^5)/(7*d*(a + a*Sin[c + d*x])^(3/2))
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2753

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && N
eQ[m + p, 0]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx = -\frac{2a\cos^5(c+dx)}{7d(a+a\sin(c+dx))^{3/2}} + \frac{1}{7}(4a)\int \frac{\cos^4(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$$
$$= -\frac{8a^2\cos^5(c+dx)}{35d(a+a\sin(c+dx))^{5/2}} - \frac{2a\cos^5(c+dx)}{7d(a+a\sin(c+dx))^{3/2}}$$

Mathematica [A]
time = 0.04, size = 49, normalized size = 0.78

$$-\frac{2\cos^5(c+dx)(9+5\sin(c+dx))}{35d(1+\sin(c+dx))^2\sqrt{a(1+\sin(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-2*Cos[c + d*x]^5*(9 + 5*Sin[c + d*x]))/(35*d*(1 + Sin[c + d*x])^2*Sqrt[a*
(1 + Sin[c + d*x])])
```

Maple [A]
time = 0.46, size = 54, normalized size = 0.86

method	result	size
default	$\frac{2(1+\sin(dx+c))(\sin(dx+c)-1)^3(5\sin(dx+c)+9)}{35\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/35*(1+sin(d*x+c))*(sin(d*x+c)-1)^3*(5*sin(d*x+c)+9)/cos(d*x+c)/(a+a*sin(d
*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^4/sqrt(a*sin(d*x + c) + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 115 vs. 2(55) = 110.
time = 0.34, size = 115, normalized size = 1.83

$$\frac{2\left(5\cos(dx+c)^4-\cos(dx+c)^3+2\cos(dx+c)^2+\left(5\cos(dx+c)^3+6\cos(dx+c)^2+8\cos(dx+c)+16\right)\sin(dx+c)-8\cos(dx+c)-16\right)\sqrt{a\sin(dx+c)+a}}{35\left(ad\cos(dx+c)+ad\sin(dx+c)+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*cos(d*x + c)^4 - cos(d*x + c)^3 + 2*cos(d*x + c)^2 + (5*cos(d*x + c
)^3 + 6*cos(d*x + c)^2 + 8*cos(d*x + c) + 16)*sin(d*x + c) - 8*cos(d*x + c)
 - 16)*sqrt(a*sin(d*x + c) + a)/(a*d*cos(d*x + c) + a*d*sin(d*x + c) + a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^4(c+dx)}{\sqrt{a(\sin(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(cos(c + d*x)**4/sqrt(a*(sin(c + d*x) + 1)), x)
```

Giac [A]
time = 5.93, size = 65, normalized size = 1.03

$$-\frac{16\sqrt{2}\left(5\sqrt{a}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^7-7\sqrt{a}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^5\right)}{35\,ad\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -16/35*sqrt(2)*(5*sqrt(a)*sin(-1/4*pi + 1/2*d*x + 1/2*c)^7 - 7*sqrt(a)*sin(
-1/4*pi + 1/2*d*x + 1/2*c)^5)/(a*d*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos(c+d\,x)^4}{\sqrt{a+a\,\sin(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(a + a*sin(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^4/(a + a*sin(c + d*x))^(1/2), x)
```

3.160 $\int \frac{\cos^3(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=49

$$\frac{4(a+a\sin(c+dx))^{3/2}}{3a^2d}-\frac{2(a+a\sin(c+dx))^{5/2}}{5a^3d}$$

```
[Out] 4/3*(a+a*sin(d*x+c))^(3/2)/a^2/d-2/5*(a+a*sin(d*x+c))^(5/2)/a^3/d
```

Rubi [A]
time = 0.04, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2746, 45}

$$\frac{4(a\sin(c+dx)+a)^{3/2}}{3a^2d}-\frac{2(a\sin(c+dx)+a)^{5/2}}{5a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (4*(a + a*Sin[c + d*x])^(3/2))/(3*a^2*d) - (2*(a + a*Sin[c + d*x])^(5/2))/(
5*a^3*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx = \frac{\text{Subst}\left(\int (a-x)\sqrt{a+x}\,dx, x, a\sin(c+dx)\right)}{a^3d}$$
$$= \frac{\text{Subst}\left(\int \left(2a\sqrt{a+x}-(a+x)^{3/2}\right)\,dx, x, a\sin(c+dx)\right)}{a^3d}$$
$$= \frac{4(a+a\sin(c+dx))^{3/2}}{3a^2d} - \frac{2(a+a\sin(c+dx))^{5/2}}{5a^3d}$$

Mathematica [A]
time = 0.04, size = 34, normalized size = 0.69

$$-\frac{2(a(1+\sin(c+dx)))^{3/2}(-7+3\sin(c+dx))}{15a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/Sqrt[a + a*Sin[c + d*x]],x]
[Out] (-2*(a*(1 + Sin[c + d*x]))^(3/2)*(-7 + 3*Sin[c + d*x]))/(15*a^2*d)
```

Maple [A]
time = 0.26, size = 31, normalized size = 0.63

method	result	size
default	$-\frac{2(a+a\sin(dx+c))^{\frac{3}{2}}(3\sin(dx+c)-7)}{15a^2d}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/15/a^2*(a+a*sin(d*x+c))^(3/2)*(3*sin(d*x+c)-7)/d
```

Maxima [A]
time = 0.31, size = 75, normalized size = 1.53

$$\frac{2\left(15\sqrt{a\sin(dx+c)+a} - \frac{3(a\sin(dx+c)+a)^{\frac{5}{2}}-10(a\sin(dx+c)+a)^{\frac{3}{2}}a+15\sqrt{a\sin(dx+c)+a}\,a^2}{a^2}\right)}{15\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
[Out] 2/15*(15*sqrt(a*sin(d*x + c) + a) - (3*(a*sin(d*x + c) + a)^(5/2) - 10*(a*s
in(d*x + c) + a)^(3/2)*a + 15*sqrt(a*sin(d*x + c) + a)*a^2)/a^2)/(a*d)
```

Fricas [A]
time = 0.33, size = 40, normalized size = 0.82

$$\frac{2\left(3\cos\left(dx+c\right)^{2}+4\sin\left(dx+c\right)+4\right)\sqrt{a\sin\left(dx+c\right)+a}}{15\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*cos(d*x + c)^2 + 4*sin(d*x + c) + 4)*sqrt(a*sin(d*x + c) + a)/(a*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 5.26, size = 68, normalized size = 1.39

$$-\frac{8\left(3\sqrt{2}\sqrt{a}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{5}-5\sqrt{2}\sqrt{a}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{3}\right)}{15\,ad\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -8/15*(3*sqrt(2)*sqrt(a)*cos(-1/4*pi + 1/2*d*x + 1/2*c)^5 - 5*sqrt(2)*sqrt(
a)*cos(-1/4*pi + 1/2*d*x + 1/2*c)^3)/(a*d*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c
)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos\left(c+d\,x\right)^{3}}{\sqrt{a+a\,\sin\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a + a*sin(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^3/(a + a*sin(c + d*x))^(1/2), x)
```

3.161 $\int \frac{\cos^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=30

$$-\frac{2a\cos^3(c+dx)}{3d(a+a\sin(c+dx))^{3/2}}$$

```
[Out] -2/3*a*cos(d*x+c)^3/d/(a+a*sin(d*x+c))^(3/2)
```

Rubi [A]
time = 0.03, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = $\{2752\}$

$$-\frac{2a\cos^3(c+dx)}{3d(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-2*a*Cos[c + d*x]^3)/(3*d*(a + a*Sin[c + d*x])^(3/2))
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx=-\frac{2a\cos^3(c+dx)}{3d(a+a\sin(c+dx))^{3/2}}$$

Mathematica [A]
time = 0.04, size = 30, normalized size = 1.00

$$-\frac{2a\cos^3(c+dx)}{3d(a(1+\sin(c+dx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-2*a*Cos[c + d*x]^3)/(3*d*(a*(1 + Sin[c + d*x]))^(3/2))
```

Maple [A]
time = 0.34, size = 44, normalized size = 1.47

method	result	size
default	$-\frac{2(1+\sin(dx+c))(\sin(dx+c)-1)^2}{3\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/3*(1+sin(d*x+c))*(sin(d*x+c)-1)^2/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(cos(d*x + c)^2/sqrt(a*sin(d*x + c) + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(26) = 52.
time = 0.33, size = 71, normalized size = 2.37

$$\frac{2\left(\cos\left(dx+c\right)^2+\left(\cos\left(dx+c\right)+2\right)\sin\left(dx+c\right)-\cos\left(dx+c\right)-2\right)\sqrt{a\sin\left(dx+c\right)+a}}{3\left(ad\cos\left(dx+c\right)+ad\sin\left(dx+c\right)+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
[Out] 2/3*(cos(d*x + c)^2 + (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2)*s
qrt(a*sin(d*x + c) + a)/(a*d*cos(d*x + c) + a*d*sin(d*x + c) + a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cos^2\left(c+dx\right)}{\sqrt{a\left(\sin\left(c+dx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(cos(c + d*x)**2/sqrt(a*(sin(c + d*x) + 1)), x)
```

Giac [A]
time = 6.33, size = 40, normalized size = 1.33

$$\frac{4\sqrt{2}\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}}{3\,\sqrt{a}\,d\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 4/3*sqrt(2)*sin(-1/4*pi + 1/2*d*x + 1/2*c)^3/(sqrt(a)*d*sgn(cos(-1/4*pi + 1
/2*d*x + 1/2*c)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\cos\left(c+d\,x\right)^2}{\sqrt{a+a\,\sin\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a + a*sin(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^2/(a + a*sin(c + d*x))^(1/2), x)
```

3.162 $\int \frac{\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=22

$$\frac{2\sqrt{a+a\sin(c+dx)}}{ad}$$

```
[Out] 2*(a+a*sin(d*x+c))^(1/2)/a/d
```

Rubi [A]
time = 0.02, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2746, 32}

$$\frac{2\sqrt{a\sin(c+dx)+a}}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (2*Sqrt[a + a*Sin[c + d*x]])/(a*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+x}}\,dx, x, a\sin(c+dx)\right)}{ad}\\ &= \frac{2\sqrt{a+a\sin(c+dx)}}{ad}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 22, normalized size = 1.00

$$\frac{2\sqrt{a+a\sin(c+dx)}}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/Sqrt[a + a*Sin[c + d*x]],x]
[Out] (2*Sqrt[a + a*Sin[c + d*x]])/(a*d)
```

Maple [A]
time = 0.04, size = 21, normalized size = 0.95

method	result	size
derivativedivides	$\frac{2\sqrt{a+a\sin\left(dx+c\right)}}{da}$	21
default	$\frac{2\sqrt{a+a\sin\left(dx+c\right)}}{da}$	21
risch	$-\frac{i\sqrt{2}\,\mathrm{e}^{-i(dx+c)}\left(\mathrm{e}^{i(dx+c)}+i\right)^{2}}{\sqrt{-a\left(-2-2\sin\left(dx+c\right)\right)}\,d}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*(a+a*sin(d*x+c))^(1/2)/d/a
```

Maxima [A]
time = 0.28, size = 20, normalized size = 0.91

$$\frac{2\sqrt{a\sin\left(dx+c\right)+a}}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
[Out] 2*sqrt(a*sin(d*x + c) + a)/(a*d)
```

Fricas [A]
time = 0.35, size = 20, normalized size = 0.91

$$\frac{2\sqrt{a\sin\left(dx+c\right)+a}}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
[Out] 2*sqrt(a*sin(d*x + c) + a)/(a*d)
```

Sympy [A]
time = 0.46, size = 32, normalized size = 1.45

$$\begin{cases} \frac{2\sqrt{a\sin{(c+dx)}+a}}{ad} & \text{for } d\neq 0 \\ \frac{x\cos{(c)}}{\sqrt{a\sin{(c)}+a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sin(d*x+c))**(1/2),x)
[Out] Piecewise((2*sqrt(a*sin(c + d*x) + a)/(a*d), Ne(d, 0)), (x*cos(c)/sqrt(a*si
n(c) + a), True))
```

Giac [A]
time = 5.00, size = 38, normalized size = 1.73

$$\frac{2\sqrt{2}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a}\,d\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
[Out] 2*sqrt(2)*cos(-1/4*pi + 1/2*d*x + 1/2*c)/(sqrt(a)*d*sgn(cos(-1/4*pi + 1/2*d
*x + 1/2*c)))
```

Mupad [B]
time = 4.82, size = 20, normalized size = 0.91

$$\frac{2\sqrt{a\,\left(\sin\left(c+d\,x\right)+1\right)}}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + a*sin(c + d*x))^(1/2),x)
[Out] (2*(a*(sin(c + d*x) + 1))^(1/2))/(a*d)
```

3.163 $\int \frac{\sec(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=60

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}\sqrt{a}\,d}-\frac{1}{d\sqrt{a+a\sin(c+dx)}}$$

```
[Out] 1/2*arctanh(1/2*(a+a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/d*2^(1/2)/a^(1/2)-1
/d/(a+a*sin(d*x+c))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2746, 53, 65, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}\sqrt{a}\,d}-\frac{1}{d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])]/(Sqrt[2]*Sqrt[a]*d) - 1
/(d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\int \frac{\sec(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx = \frac{a\text{Subst}\left(\int \frac{1}{(a-x)(a+x)^{3/2}}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{1}{d\sqrt{a+a\sin(c+dx)}} + \frac{\text{Subst}\left(\int \frac{1}{(a-x)\sqrt{a+x}}\,dx, x, a\sin(c+dx)\right)}{2d}$$
$$= -\frac{1}{d\sqrt{a+a\sin(c+dx)}} + \frac{\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \sqrt{a+a\sin(c+dx)}\right)}{d}$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}\sqrt{a}\,d} - \frac{1}{d\sqrt{a+a\sin(c+dx)}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.03, size = 39, normalized size = 0.65

$$-\frac{{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{1}{2}(1+\sin(c+dx))\right)}{d\sqrt{a+a\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/Sqrt[a + a*Sin[c + d*x]],x]
[Out] -(Hypergeometric2F1[-1/2, 1, 1/2, (1 + Sin[c + d*x])/2]/(d*Sqrt[a + a*Sin[c
 + d*x]]))
```

Maple [A]
time = 0.26, size = 54, normalized size = 0.90

method	result	size
default	$-\frac{2a\left(\frac{1}{2a\sqrt{a+a\sin(dx+c)}}-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+a\sin(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{4a^{\frac{3}{2}}}\right)}{d}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*a*(1/2/a/(a+a*sin(d*x+c))^(1/2)-1/4/a^(3/2)*2^(1/2)*arctanh(1/2*(a+a*sin
(d*x+c))^(1/2)*2^(1/2)/a^(1/2)))/d
```

Maxima [A]
time = 0.50, size = 78, normalized size = 1.30

$$-\frac{\sqrt{2}\,\sqrt{a}\,\log\left(-\frac{\sqrt{2}\,\sqrt{a}-\sqrt{a\sin(dx+c)+a}}{\sqrt{2}\,\sqrt{a}+\sqrt{a\sin(dx+c)+a}}\right)+\frac{4a}{\sqrt{a\sin(dx+c)+a}}}{4ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*(sqrt(2)*sqrt(a)*log(-(sqrt(2)*sqrt(a) - sqrt(a*sin(d*x + c) + a))/(sq
rt(2)*sqrt(a) + sqrt(a*sin(d*x + c) + a))) + 4*a/sqrt(a*sin(d*x + c) + a))/
(a*d)
```

Fricas [A]
time = 0.38, size = 90, normalized size = 1.50

$$\frac{\frac{\sqrt{2}\,(a\sin(dx+c)+a)\log\left(-\frac{\frac{2\sqrt{2}\,\sqrt{a\sin(dx+c)+a}}{\sqrt{a}}+\sin(dx+c)+3}{\sin(dx+c)-1}\right)}{\sqrt{a}}-4\sqrt{a\sin(dx+c)+a}}{4(ad\sin(dx+c)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*(a*sin(d*x + c) + a)*log(-(2*sqrt(2)*sqrt(a*sin(d*x + c) + a)/
sqrt(a) + sin(d*x + c) + 3)/(sin(d*x + c) - 1))/sqrt(a) - 4*sqrt(a*sin(d*x
+ c) + a))/(a*d*sin(d*x + c) + a*d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(c+dx)}{\sqrt{a(\sin(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sin(d*x+c))**(1/2),x)
[Out] Integral(sec(c + d*x)/sqrt(a*(sin(c + d*x) + 1)), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 123 vs. 2(50) = 100.
time = 4.74, size = 123, normalized size = 2.05

$$\frac{\sqrt{a}\left(\frac{\sqrt{2}\log\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{\sqrt{2}\log\left(-\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{2\sqrt{2}}{a\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
[Out] 1/4*sqrt(a)*(sqrt(2)*log(cos(-1/4*pi + 1/2*d*x + 1/2*c) + 1)/(a*sgn(cos(-1/
4*pi + 1/2*d*x + 1/2*c))) - sqrt(2)*log(-cos(-1/4*pi + 1/2*d*x + 1/2*c) + 1
)/(a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))) - 2*sqrt(2)/(a*cos(-1/4*pi + 1/2*
d*x + 1/2*c)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))))/d
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\cos(c+dx)\,\sqrt{a+a\,\sin(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + a*sin(c + d*x))^(1/2)),x)
[Out] int(1/(cos(c + d*x)*(a + a*sin(c + d*x))^(1/2)), x)
```

3.164 $\int \frac{\sec^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=102

$$-\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{4\sqrt{2}\sqrt{a}\,d}-\frac{3a\cos(c+dx)}{4d(a+a\sin(c+dx))^{3/2}}+\frac{\sec(c+dx)}{d\sqrt{a+a\sin(c+dx)}}$$

```
[Out] -3/4*a*cos(d*x+c)/d/(a+a*sin(d*x+c))^(3/2)-3/8*arctanh(1/2*cos(d*x+c)*a^(1/
2)*2^(1/2)/(a+a*sin(d*x+c))^(1/2))/d*2^(1/2)/a^(1/2)+sec(d*x+c)/d/(a+a*sin(
d*x+c))^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2766, 2729, 2728, 212}

$$-\frac{3a\cos(c+dx)}{4d(a\sin(c+dx)+a)^{3/2}}+\frac{\sec(c+dx)}{d\sqrt{a\sin(c+dx)+a}}-\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{4\sqrt{2}\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-3*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/(4*
Sqrt[2]*Sqrt[a]*d) - (3*a*Cos[c + d*x])/(4*d*(a + a*Sin[c + d*x])^(3/2)) +
Sec[c + d*x]/(d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
```

```
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2766

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(a*f*g*(p + 1)*S
qrt[a + b*Sin[e + f*x]])), x] + Dist[a*((2*p + 1)/(2*g^2*(p + 1))), Int[(g*
Cos[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e,
 f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx = \frac{\sec(c+dx)}{d\sqrt{a+a\sin(c+dx)}} + \frac{1}{2}(3a)\int \frac{1}{(a+a\sin(c+dx))^{3/2}}\,dx$$

$$= -\frac{3a\cos(c+dx)}{4d(a+a\sin(c+dx))^{3/2}} + \frac{\sec(c+dx)}{d\sqrt{a+a\sin(c+dx)}} + \frac{3}{8}\int \frac{1}{\sqrt{a+a\sin(c+dx}}$$

$$= -\frac{3a\cos(c+dx)}{4d(a+a\sin(c+dx))^{3/2}} + \frac{\sec(c+dx)}{d\sqrt{a+a\sin(c+dx)}} - \frac{3\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \right.}{4d}$$

$$= -\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{4\sqrt{2}\sqrt{a}\,d} - \frac{3a\cos(c+dx)}{4d(a+a\sin(c+dx))^{3/2}} + \frac{\sec}{d\sqrt{a+}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.17, size = 118, normalized size = 1.16

$$-\frac{\sec(c+dx)\left(-1-(3+3i)(-1)^{3/4}\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(c+dx)\right)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2-3\sin(c+dx)\right)}{4d\sqrt{a(1+\sin(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -1/4*(Sec[c + d*x]*(-1 - (3 + 3*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4
)*(-1 + Tan[(c + d*x)/4])]*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c +
d*x)/2] + Sin[(c + d*x)/2])^2 - 3*Sin[c + d*x]))/(d*Sqrt[a*(1 + Sin[c + d*x
])])
```

Maple [A]
time = 0.46, size = 130, normalized size = 1.27

method	result

default	$-\frac{\sin(dx+c)\left(3\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)a\sqrt{a-a\sin(dx+c)}-6a^{\frac{3}{2}}\right)+3\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-}}{}\right.}{8a^{\frac{3}{2}}\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*(sin(d*x+c)*(3*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1
/2))*a*(a-a*sin(d*x+c))^(1/2)-6*a^(3/2))+3*2^(1/2)*arctanh(1/2*(a-a*sin(d*x
+c))^(1/2)*2^(1/2)/a^(1/2))*a*(a-a*sin(d*x+c))^(1/2)-2*a^(3/2))/a^(3/2)/cos
(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^2/sqrt(a*sin(d*x + c) + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 200 vs. $2(85) = 170$.
time = 0.38, size = 200, normalized size = 1.96

$$\frac{3\sqrt{2}\left(\cos(dx+c)\sin(dx+c)+\cos(dx+c)\right)\sqrt{a}\,\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a\sin(dx+c)+a}\,\sqrt{a}\,(\cos(dx+c)-\sin(dx+c)+1)+3\,a\cos(dx+c)-(a\cos(dx+c)-2\,a)\sin(dx+c)+2\,a}{\cos(dx+c)^2-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2}\right)+4\sqrt{a\sin(dx+c)+a}\,(3\sin(dx+c)+1)}{16\left(ad\cos(dx+c)\sin(dx+c)+ad\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/16*(3*sqrt(2)*(cos(d*x + c)*sin(d*x + c) + cos(d*x + c))*sqrt(a)*log(-(a*
cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*sqrt(a)*(cos(d*x + c) -
 sin(d*x + c) + 1) + 3*a*cos(d*x + c) - (a*cos(d*x + c) - 2*a)*sin(d*x + c)
 + 2*a)/(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) -
2)) + 4*sqrt(a*sin(d*x + c) + a)*(3*sin(d*x + c) + 1))/(a*d*cos(d*x + c)*si
n(d*x + c) + a*d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(c+dx)}{\sqrt{a(\sin(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**2/sqrt(a*(sin(c + d*x) + 1)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)^2\,\sqrt{a+a\,\sin\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + a*sin(c + d*x))^(1/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^2*(a + a*sin(c + d*x))^(1/2)), x)
```

3.165 $\int \frac{\sec^3(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=116

$$\frac{5\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{8\sqrt{2}\sqrt{a}\,d}-\frac{5a}{12d(a+a\sin(c+dx))^{3/2}}-\frac{5}{8d\sqrt{a+a\sin(c+dx)}}+\frac{\sec^2(c+dx)}{2d\sqrt{a+a\sin(c+dx)}}$$

```
[Out] -5/12*a/d/(a+a*sin(d*x+c))^(3/2)+5/16*arctanh(1/2*(a+a*sin(d*x+c))^(1/2)*2^
(1/2)/a^(1/2))/d*2^(1/2)/a^(1/2)-5/8/d/(a+a*sin(d*x+c))^(1/2)+1/2*sec(d*x+c
)^2/d/(a+a*sin(d*x+c))^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2766, 2746, 53, 65, 212}

$$-\frac{5}{8d\sqrt{a\sin(c+dx)+a}}-\frac{5a}{12d(a\sin(c+dx)+a)^{3/2}}+\frac{5\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{8\sqrt{2}\sqrt{a}\,d}+\frac{\sec^2(c+dx)}{2d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (5*ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(8*Sqrt[2]*Sqrt[a]*
d) - (5*a)/(12*d*(a + a*Sin[c + d*x])^(3/2)) - 5/(8*d*Sqrt[a + a*Sin[c + d*
x]]) + Sec[c + d*x]^2/(2*d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2766

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(a*f*g*(p + 1)*S
qrt[a + b*Sin[e + f*x]])), x] + Dist[a*((2*p + 1)/(2*g^2*(p + 1))), Int[(g*
Cos[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e,
 f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx = \frac{\sec^2(c+dx)}{2d\sqrt{a+a\sin(c+dx)}} + \frac{1}{4}(5a)\int \frac{\sec(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$$

$$= \frac{\sec^2(c+dx)}{2d\sqrt{a+a\sin(c+dx)}} + \frac{(5a^2)\,\text{Subst}\left(\int \frac{1}{(a-x)(a+x)^{5/2}}\,dx, x, a\sin(c+dx)\right)}{4d}$$

$$= -\frac{5a}{12d(a+a\sin(c+dx))^{3/2}} + \frac{\sec^2(c+dx)}{2d\sqrt{a+a\sin(c+dx)}} + \frac{(5a)\text{Subst}\left(\int \frac{1}{(a-x)(a+}\right.}{}$$

$$= -\frac{5a}{12d(a+a\sin(c+dx))^{3/2}} - \frac{5}{8d\sqrt{a+a\sin(c+dx)}} + \frac{\sec^2(c+dx)}{2d\sqrt{a+a\sin(c+dx}}$$

$$= -\frac{5a}{12d(a+a\sin(c+dx))^{3/2}} - \frac{5}{8d\sqrt{a+a\sin(c+dx)}} + \frac{\sec^2(c+dx)}{2d\sqrt{a+a\sin(c+dx}}$$

$$= \frac{5\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{8\sqrt{2}\sqrt{a}\,d} - \frac{5a}{12d(a+a\sin(c+dx))^{3/2}} - \frac{5}{8d\sqrt{a+a\sin}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.04, size = 42, normalized size = 0.36

$$-\frac{a\,{}_2F_1\left(-\frac{3}{2},2;-\frac{1}{2};\frac{1}{2}(1+\sin(c+dx))\right)}{6d(a+a\sin(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -1/6*(a*Hypergeometric2F1[-3/2, 2, -1/2, (1 + Sin[c + d*x])/2])/(d*(a + a*S
in[c + d*x])^(3/2))
```

Maple [A]
time = 0.67, size = 107, normalized size = 0.92

method	result
default	$\frac{2a^3\left(-\frac{\frac{\sqrt{a+a\sin(dx+c)}}{4a\sin(dx+c)-4a}-\frac{5\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+a\sin(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{8\sqrt{a}}}{4a^3}-\frac{1}{4a^3\sqrt{a+a\sin(dx+c)}}-\frac{1}{12a^2(}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*a^3*(-1/4/a^3*(1/4*(a+a*sin(d*x+c))^(1/2)/(a*sin(d*x+c)-a)-5/8*2^(1/2)/a^
(1/2)*arctanh(1/2*(a+a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2)))-1/4/a^3/(a+a*sin
(d*x+c))^(1/2)-1/12/a^2/(a+a*sin(d*x+c))^(3/2))/d
```

Maxima [A]
time = 0.50, size = 132, normalized size = 1.14

$$-\frac{15\,\sqrt{2}\,\sqrt{a}\,\log\left(-\frac{\sqrt{2}\,\sqrt{a}-\sqrt{a\sin(dx+c)+a}}{\sqrt{2}\,\sqrt{a}+\sqrt{a\sin(dx+c)+a}}\right)+\frac{4\left(15\,(a\sin(dx+c)+a)^2a-20\,(a\sin(dx+c)+a)a^2-8\,a^3\right)}{(a\sin(dx+c)+a)^{\frac{5}{2}}-2\,(a\sin(dx+c)+a)^{\frac{3}{2}}a}}{96\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/96*(15*sqrt(2)*sqrt(a)*log(-(sqrt(2)*sqrt(a) - sqrt(a*sin(d*x + c) + a))
/(sqrt(2)*sqrt(a) + sqrt(a*sin(d*x + c) + a))) + 4*(15*(a*sin(d*x + c) + a)
^2*a - 20*(a*sin(d*x + c) + a)*a^2 - 8*a^3)/((a*sin(d*x + c) + a)^(5/2) - 2
*(a*sin(d*x + c) + a)^(3/2)*a))/(a*d)
```

Fricas [A]
time = 0.36, size = 145, normalized size = 1.25

$$\frac{15 \sqrt{2} \left(\cos\left(dx+c\right)^{2} \sin\left(dx+c\right)+\cos\left(dx+c\right)^{2}\right) \sqrt{a} \log\left(-\frac{a \sin(dx+c)+2 \sqrt{2} \sqrt{a \sin\left(dx+c\right)+a} \sqrt{a}+3 a}{\sin(dx+c)-1}\right)-4 \left(15 \cos\left(dx+c\right)^{2}-10 \sin\left(dx+c\right)-2\right) \sqrt{a \sin\left(dx+c\right)+a}}{96 \left(a d \cos\left(dx+c\right)^{2} \sin\left(dx+c\right)+a d \cos\left(dx+c\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/96*(15*sqrt(2)*(cos(d*x + c)^2*sin(d*x + c) + cos(d*x + c)^2)*sqrt(a)*log
(-(a*sin(d*x + c) + 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*sqrt(a) + 3*a)/(sin(
d*x + c) - 1)) - 4*(15*cos(d*x + c)^2 - 10*sin(d*x + c) - 2)*sqrt(a*sin(d*x
 + c) + a))/(a*d*cos(d*x + c)^2*sin(d*x + c) + a*d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{3}\left(c+dx\right)}{\sqrt{a \left(\sin\left(c+dx\right)+1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**3/sqrt(a*(sin(c + d*x) + 1)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 195 vs. 2(93) = 186.
time = 6.64, size = 195, normalized size = 1.68

$$\frac{\sqrt{a} \left(\frac{15 \sqrt{2} \log\left(\cos\left(-\frac{1}{4} \pi+\frac{1}{2} dx+\frac{1}{2} c\right)+1\right)}{a \mathrm{sgn}\left(\cos\left(-\frac{1}{4} \pi+\frac{1}{2} dx+\frac{1}{2} c\right)\right)}-\frac{15 \sqrt{2} \log\left(-\cos\left(-\frac{1}{4} \pi+\frac{1}{2} dx+\frac{1}{2} c\right)+1\right)}{a \mathrm{sgn}\left(\cos\left(-\frac{1}{4} \pi+\frac{1}{2} dx+\frac{1}{2} c\right)\right)}-\frac{6 \sqrt{2} \cos\left(-\frac{1}{4} \pi+\frac{1}{2} dx+\frac{1}{2} c\right)}{\left(\cos\left(-\frac{1}{4} \pi+\frac{1}{2} dx+\frac{1}{2} c\right)^{2}-1\right) a \mathrm{sgn}\left(\cos\left(-\frac{1}{4} \pi+\frac{1}{2} dx+\frac{1}{2} c\right)\right)}-\frac{4 \sqrt{2} \left(6 \cos\left(-\frac{1}{4} \pi+\frac{1}{2} dx+\frac{1}{2} c\right)^{2}+1\right)}{a \cos\left(-\frac{1}{4} \pi+\frac{1}{2} dx+\frac{1}{2} c\right)^{3} \mathrm{sgn}\left(\cos\left(-\frac{1}{4} \pi+\frac{1}{2} dx+\frac{1}{2} c\right)\right)}\right)}{96 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/96*sqrt(a)*(15*sqrt(2)*log(cos(-1/4*pi + 1/2*d*x + 1/2*c) + 1)/(a*sgn(cos
(-1/4*pi + 1/2*d*x + 1/2*c))) - 15*sqrt(2)*log(-cos(-1/4*pi + 1/2*d*x + 1/2
*c) + 1)/(a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))) - 6*sqrt(2)*cos(-1/4*pi +
1/2*d*x + 1/2*c)/((cos(-1/4*pi + 1/2*d*x + 1/2*c)^2 - 1)*a*sgn(cos(-1/4*pi
+ 1/2*d*x + 1/2*c))) - 4*sqrt(2)*(6*cos(-1/4*pi + 1/2*d*x + 1/2*c)^2 + 1)/(
a*cos(-1/4*pi + 1/2*d*x + 1/2*c)^3*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\cos\left(c+d\,x\right)}^{3}\,\sqrt{a+a\,\sin\left(c+d\,x\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + a*sin(c + d*x))^(1/2)),x)
[Out] int(1/(cos(c + d*x)^3*(a + a*sin(c + d*x))^(1/2)), x)
```

3.166 $\int \frac{\sec^4(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=162

$$-\frac{35\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{64\sqrt{2}\sqrt{a}\,d}-\frac{35a\cos(c+dx)}{64d(a+a\sin(c+dx))^{3/2}}-\frac{7a\sec(c+dx)}{24d(a+a\sin(c+dx))^{3/2}}+\frac{35\,\mathrm{se}}{48d\sqrt{a+}}$$

```
[Out] -35/64*a*cos(d*x+c)/d/(a+a*sin(d*x+c))^(3/2)-7/24*a*sec(d*x+c)/d/(a+a*sin(d
*x+c))^(3/2)-35/128*arctanh(1/2*cos(d*x+c)*a^(1/2)*2^(1/2)/(a+a*sin(d*x+c))
^(1/2))/d*2^(1/2)/a^(1/2)+35/48*sec(d*x+c)/d/(a+a*sin(d*x+c))^(1/2)+1/3*sec
(d*x+c)^3/d/(a+a*sin(d*x+c))^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$,
Rules used = {2766, 2760, 2729, 2728, 212}

$$-\frac{35a\cos(c+dx)}{64d(a\sin(c+dx)+a)^{3/2}}+\frac{\sec^3(c+dx)}{3d\sqrt{a\sin(c+dx)+a}}+\frac{35\sec(c+dx)}{48d\sqrt{a\sin(c+dx)+a}}-\frac{7a\sec(c+dx)}{24d(a\sin(c+dx)+a)^{3/2}}-\frac{35\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{64\sqrt{2}\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-35*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/(6
4*Sqrt[2]*Sqrt[a]*d) - (35*a*Cos[c + d*x])/(64*d*(a + a*Sin[c + d*x])^(3/2)
) - (7*a*Sec[c + d*x])/(24*d*(a + a*Sin[c + d*x])^(3/2)) + (35*Sec[c + d*x]
)/(48*d*Sqrt[a + a*Sin[c + d*x]]) + Sec[c + d*x]^3/(3*d*Sqrt[a + a*Sin[c +
d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
```

```
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2760

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*(2*m + p + 1))), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2766

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(a*f*g*(p + 1)*S
qrt[a + b*Sin[e + f*x]])), x] + Dist[a*((2*p + 1)/(2*g^2*(p + 1))), Int[(g*
Cos[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e,
 f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx = \frac{\sec^3(c+dx)}{3d\sqrt{a+a\sin(c+dx)}}+\frac{1}{6}(7a)\int \frac{\sec^2(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$$

$$= -\frac{7a\sec(c+dx)}{24d(a+a\sin(c+dx))^{3/2}}+\frac{\sec^3(c+dx)}{3d\sqrt{a+a\sin(c+dx)}}+\frac{35}{48}\int \frac{\sec^2(c+dx)}{\sqrt{a+a\sin(c+}}$$

$$= -\frac{7a\sec(c+dx)}{24d(a+a\sin(c+dx))^{3/2}}+\frac{35\sec(c+dx)}{48d\sqrt{a+a\sin(c+dx)}}+\frac{\sec^3(c+dx)}{3d\sqrt{a+a\sin(c+dx}}$$

$$= -\frac{35a\cos(c+dx)}{64d(a+a\sin(c+dx))^{3/2}}-\frac{7a\sec(c+dx)}{24d(a+a\sin(c+dx))^{3/2}}+\frac{35\sec(c+dx)}{48d\sqrt{a+a\sin(c+}}$$

$$= -\frac{35a\cos(c+dx)}{64d(a+a\sin(c+dx))^{3/2}}-\frac{7a\sec(c+dx)}{24d(a+a\sin(c+dx))^{3/2}}+\frac{35\sec(c+dx)}{48d\sqrt{a+a\sin(c+}}$$

$$= -\frac{35\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{64\sqrt{2}\sqrt{a}\,d}-\frac{35a\cos(c+dx)}{64d(a+a\sin(c+dx))^{3/2}}-\frac{7}{24d(a}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.40, size = 117, normalized size = 0.72

$$\frac{(420+420i)(-1)^{3/4}\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(c+dx)\right)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+\sec^3(c+dx)(102+70\cos(2(c+dx))+329\sin(c+dx)+105\sin(3(c+dx)))}{768d\sqrt{a(1+\sin(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] ((420 + 420*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x
)/4])]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) + Sec[c + d*x]^3*(102 + 70*Cos
[2*(c + d*x)] + 329*Sin[c + d*x] + 105*Sin[3*(c + d*x)]))/(768*d*Sqrt[a*(1
+ Sin[c + d*x])])
```

Maple [A]
time = 0.49, size = 231, normalized size = 1.43

method	result
default	$\frac{-210a^{\frac{7}{2}}\sin(dx+c)(\cos^2(dx+c))+\left(210\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)a^2(a-a\sin(dx+c))^{\frac{3}{2}}-112a^{\frac{7}{2}}\right)\sin(dx}{384a^{\frac{7}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/384*(-210*a^(7/2)*sin(d*x+c)*cos(d*x+c)^2+(210*2^(1/2)*arctanh(1/2*(a-a*s
in(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^2*(a-a*sin(d*x+c))^(3/2)-112*a^(7/2))*s
in(d*x+c)+(-105*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))
*a^2*(a-a*sin(d*x+c))^(3/2)-70*a^(7/2))*cos(d*x+c)^2+210*2^(1/2)*arctanh(1/
2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^2*(a-a*sin(d*x+c))^(3/2)-16*a^(
7/2))/a^(7/2)/(sin(d*x+c)-1)/(1+sin(d*x+c))/cos(d*x+c)/(a+a*sin(d*x+c))^(1/
2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^4/sqrt(a*sin(d*x + c) + a), x)
```

Fricas [A]
time = 0.36, size = 230, normalized size = 1.42

$$\frac{105\sqrt{2}\left(\cos(dx+c)^3\sin(dx+c)+\cos(dx+c)^3\right)\sqrt{a}\,\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\,\sqrt{a\sin(dx+c)+a}\,\sqrt{a}\,(\cos(dx+c)-\sin(dx+c)+1)+3a\cos(dx+c)-(a\cos(dx+c)-2a)\sin(dx+c)+2a}{\cos(dx+c)^2-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2}\right)+4\left(35\cos(dx+c)^2+7\left(15\cos(dx+c)^2+8\right)\sin(dx+c)+8\right)\sqrt{a\sin(dx+c)+a}}{768\left(ad\cos(dx+c)^3\sin(dx+c)+ad\cos(dx+c)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/768*(105*sqrt(2)*(cos(d*x + c)^3*sin(d*x + c) + cos(d*x + c)^3)*sqrt(a)*l
og(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*sqrt(a)*(cos(d*x
 + c) - sin(d*x + c) + 1) + 3*a*cos(d*x + c) - (a*cos(d*x + c) - 2*a)*sin(d
*x + c) + 2*a)/(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x
+ c) - 2)) + 4*(35*cos(d*x + c)^2 + 7*(15*cos(d*x + c)^2 + 8)*sin(d*x + c)
+ 8)*sqrt(a*sin(d*x + c) + a))/(a*d*cos(d*x + c)^3*sin(d*x + c) + a*d*cos(d
*x + c)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^4(c+dx)}{\sqrt{a(\sin(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**4/sqrt(a*(sin(c + d*x) + 1)), x)
```

Giac [A]
time = 7.05, size = 144, normalized size = 0.89

$$-\frac{\frac{3\sqrt{2}\left(11\sqrt{a}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^3-13\sqrt{a}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^2 a\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}+\frac{8\sqrt{2}\left(9\sqrt{a}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^2+\sqrt{a}\right)}{a\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^3}}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/384*(3*sqrt(2)*(11*sqrt(a)*sin(-1/4*pi + 1/2*d*x + 1/2*c)^3 - 13*sqrt(a)
*sin(-1/4*pi + 1/2*d*x + 1/2*c))/((sin(-1/4*pi + 1/2*d*x + 1/2*c)^2 - 1)^2*
a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))) + 8*sqrt(2)*(9*sqrt(a)*sin(-1/4*pi +
 1/2*d*x + 1/2*c)^2 + sqrt(a))/(a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-
1/4*pi + 1/2*d*x + 1/2*c)^3))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(c+d\,x)^4\,\sqrt{a+a\,\sin(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a + a*sin(c + d*x))^(1/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^4*(a + a*sin(c + d*x))^(1/2)), x)
```

3.167 $\int \frac{\sec^5(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=175

$$\frac{63\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{128\sqrt{2}\sqrt{a}\,d}-\frac{21a}{64d(a+a\sin(c+dx))^{3/2}}-\frac{9a\sec^2(c+dx)}{40d(a+a\sin(c+dx))^{3/2}}-\frac{63}{128d\sqrt{a+a\sin}}$$

```
[Out] -21/64*a/d/(a+a*sin(d*x+c))^(3/2)-9/40*a*sec(d*x+c)^2/d/(a+a*sin(d*x+c))^(3
/2)+63/256*arctanh(1/2*(a+a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/d*2^(1/2)/a^
(1/2)-63/128/d/(a+a*sin(d*x+c))^(1/2)+63/160*sec(d*x+c)^2/d/(a+a*sin(d*x+c)
)^(1/2)+1/4*sec(d*x+c)^4/d/(a+a*sin(d*x+c))^(1/2)
```

Rubi [A]

time = 0.18, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2766, 2760, 2746, 53, 65, 212}

$$-\frac{63}{128d\sqrt{a\sin(c+dx)+a}}-\frac{21a}{64d(a\sin(c+dx)+a)^{3/2}}+\frac{63\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{128\sqrt{2}\sqrt{a}\,d}+\frac{\sec^4(c+dx)}{4d\sqrt{a\sin(c+dx)+a}}+\frac{63\sec^2(c+dx)}{160d\sqrt{a\sin(c+dx)+a}}-\frac{9a\sec^2(c+dx)}{40d(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (63*ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(128*Sqrt[2]*Sqrt[
a]*d) - (21*a)/(64*d*(a + a*Sin[c + d*x])^(3/2)) - (9*a*Sec[c + d*x]^2)/(40
*d*(a + a*Sin[c + d*x])^(3/2)) - 63/(128*d*Sqrt[a + a*Sin[c + d*x]]) + (63*
Sec[c + d*x]^2)/(160*d*Sqrt[a + a*Sin[c + d*x]]) + Sec[c + d*x]^4/(4*d*Sqrt
[a + a*Sin[c + d*x]])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2760

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*(2*m + p + 1))), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2766

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(a*f*g*(p + 1)*S
qrt[a + b*Sin[e + f*x]])), x] + Dist[a*((2*p + 1)/(2*g^2*(p + 1))), Int[(g*
Cos[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e,
 f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx = \frac{\sec^4(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} + \frac{1}{8}(9a)\int \frac{\sec^3(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$$
$$= -\frac{9a\sec^2(c+dx)}{40d(a+a\sin(c+dx))^{3/2}} + \frac{\sec^4(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} + \frac{63}{80}\int \frac{\sec^3(c+d}{\sqrt{a+a\sin(c+}}$$
$$= -\frac{9a\sec^2(c+dx)}{40d(a+a\sin(c+dx))^{3/2}} + \frac{63\sec^2(c+dx)}{160d\sqrt{a+a\sin(c+dx)}} + \frac{\sec^4(c+dx)}{4d\sqrt{a+a\sin(c+}}$$
$$= -\frac{9a\sec^2(c+dx)}{40d(a+a\sin(c+dx))^{3/2}} + \frac{63\sec^2(c+dx)}{160d\sqrt{a+a\sin(c+dx)}} + \frac{\sec^4(c+dx)}{4d\sqrt{a+a\sin(c+}}$$
$$= -\frac{21a}{64d(a+a\sin(c+dx))^{3/2}} - \frac{9a\sec^2(c+dx)}{40d(a+a\sin(c+dx))^{3/2}} + \frac{63\sec^2(c+d}{160d\sqrt{a+a\sin(c}}$$
$$= -\frac{21a}{64d(a+a\sin(c+dx))^{3/2}} - \frac{9a\sec^2(c+dx)}{40d(a+a\sin(c+dx))^{3/2}} - \frac{63}{128d\sqrt{a+a\sin(c}}$$
$$= -\frac{21a}{64d(a+a\sin(c+dx))^{3/2}} - \frac{9a\sec^2(c+dx)}{40d(a+a\sin(c+dx))^{3/2}} - \frac{63}{128d\sqrt{a+a\sin(c}}$$
$$= \frac{63\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{128\sqrt{2}\sqrt{a}\,d} - \frac{21a}{64d(a+a\sin(c+dx))^{3/2}} - \frac{9a\sec}{40d(a+a\,s}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.05, size = 44, normalized size = 0.25

$$-\frac{a^2\,{}_2F_1\left(-\frac{5}{2},3;-\frac{3}{2};\frac{1}{2}(1+\sin(c+dx))\right)}{20d(a+a\sin(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -1/20*(a^2*Hypergeometric2F1[-5/2, 3, -3/2, (1 + Sin[c + d*x])/2])/(d*(a +
a*Sin[c + d*x])^(5/2))
```

Maple [A]
time = 0.82, size = 135, normalized size = 0.77

method	result

default	$-\frac{2a^5\left(\frac{\frac{\sqrt{a+a\sin(dx+c)}\,a(15\sin(dx+c)-19)}{16(a\sin(dx+c)-a)^2}-\frac{63\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+a\sin(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{32\sqrt{a}}}{16a^5}+\frac{3}{16a^5\sqrt{a+a\sin}}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*a^5*(1/16/a^5*(1/16*(a+a*sin(d*x+c))^(1/2)*a*(15*sin(d*x+c)-19)/(a*sin(d
*x+c)-a)^2-63/32*2^(1/2)/a^(1/2)*arctanh(1/2*(a+a*sin(d*x+c))^(1/2)*2^(1/2)
/a^(1/2)))+3/16/a^5/(a+a*sin(d*x+c))^(1/2)+1/16/a^4/(a+a*sin(d*x+c))^(3/2)+
1/40/a^3/(a+a*sin(d*x+c))^(5/2))/d
```

Maxima [A]
time = 0.50, size = 183, normalized size = 1.05

$$-\frac{315\sqrt{2}\sqrt{a}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{a\sin(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{a\sin(dx+c)+a}}\right)+\frac{4\left(315\,(a\sin(dx+c)+a)^4a-1050\,(a\sin(dx+c)+a)^3a^2+672\,(a\sin(dx+c)+a)^2a^3+192\,(a\sin(dx+c)+a)a^4+128\,a^5\right)}{(a\sin(dx+c)+a)^{\frac{9}{2}}-4\,(a\sin(dx+c)+a)^{\frac{7}{2}}a+4\,(a\sin(dx+c)+a)^{\frac{5}{2}}a^2}}{2560\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2560*(315*sqrt(2)*sqrt(a)*log(-(sqrt(2)*sqrt(a) - sqrt(a*sin(d*x + c) +
a))/(sqrt(2)*sqrt(a) + sqrt(a*sin(d*x + c) + a))) + 4*(315*(a*sin(d*x + c)
+ a)^4*a - 1050*(a*sin(d*x + c) + a)^3*a^2 + 672*(a*sin(d*x + c) + a)^2*a^3
 + 192*(a*sin(d*x + c) + a)*a^4 + 128*a^5)/((a*sin(d*x + c) + a)^(9/2) - 4*
(a*sin(d*x + c) + a)^(7/2)*a + 4*(a*sin(d*x + c) + a)^(5/2)*a^2))/(a*d)
```

Fricas [A]
time = 0.37, size = 167, normalized size = 0.95

$$\frac{315\sqrt{2}\left(\cos(dx+c)^4\sin(dx+c)+\cos(dx+c)^4\right)\sqrt{a}\log\left(-\frac{a\sin(dx+c)+2\sqrt{2}\sqrt{a\sin(dx+c)+a}\sqrt{a}+3a}{\sin(dx+c)-1}\right)-4\left(315\cos(dx+c)^4-42\cos(dx+c)^2-6\left(35\cos(dx+c)^2+24\right)\sin(dx+c)-16\right)\sqrt{a\sin(dx+c)+a}}{2560\left(ad\cos(dx+c)^4\sin(dx+c)+ad\cos(dx+c)^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2560*(315*sqrt(2)*(cos(d*x + c)^4*sin(d*x + c) + cos(d*x + c)^4)*sqrt(a)*
log(-(a*sin(d*x + c) + 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*sqrt(a) + 3*a)/(s
in(d*x + c) - 1)) - 4*(315*cos(d*x + c)^4 - 42*cos(d*x + c)^2 - 6*(35*cos(d
*x + c)^2 + 24)*sin(d*x + c) - 16)*sqrt(a*sin(d*x + c) + a))/(a*d*cos(d*x +
 c)^4*sin(d*x + c) + a*d*cos(d*x + c)^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^5(c+dx)}{\sqrt{a(\sin(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**5/sqrt(a*(sin(c + d*x) + 1)), x)
```

Giac [A]
time = 6.37, size = 233, normalized size = 1.33

$$\frac{\sqrt{a}\left(\frac{315\sqrt{2}\log\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{315\sqrt{2}\log\left(-\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{10\left(15\sqrt{2}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^3-17\sqrt{2}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^2a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{16\sqrt{2}\left(30\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^4+5\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)}{a\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^5\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}\right)}{2560\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2560*sqrt(a)*(315*sqrt(2)*log(cos(-1/4*pi + 1/2*d*x + 1/2*c) + 1)/(a*sgn(
cos(-1/4*pi + 1/2*d*x + 1/2*c))) - 315*sqrt(2)*log(-cos(-1/4*pi + 1/2*d*x +
 1/2*c) + 1)/(a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))) - 10*(15*sqrt(2)*cos(-
1/4*pi + 1/2*d*x + 1/2*c)^3 - 17*sqrt(2)*cos(-1/4*pi + 1/2*d*x + 1/2*c))/((
cos(-1/4*pi + 1/2*d*x + 1/2*c)^2 - 1)^2*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c
))) - 16*sqrt(2)*(30*cos(-1/4*pi + 1/2*d*x + 1/2*c)^4 + 5*cos(-1/4*pi + 1/2
*d*x + 1/2*c)^2 + 1)/(a*cos(-1/4*pi + 1/2*d*x + 1/2*c)^5*sgn(cos(-1/4*pi +
1/2*d*x + 1/2*c))))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(c+d\,x)^5\,\sqrt{a+a\,\sin(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^5*(a + a*sin(c + d*x))^(1/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^5*(a + a*sin(c + d*x))^(1/2)), x)
```

3.168 $\int \frac{\sec^6(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=221

$$-\frac{231\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(c+dx)}{\sqrt{2}\,\sqrt{a+a\sin(c+dx)}}\right)}{512\sqrt{2}\,\sqrt{a}\,d}-\frac{231a\cos(c+dx)}{512d(a+a\sin(c+dx))^{3/2}}-\frac{77a\sec(c+dx)}{320d(a+a\sin(c+dx))^{3/2}}-\frac{11a}{60d(a+}$$

```
[Out] -231/512*a*cos(d*x+c)/d/(a+a*sin(d*x+c))^(3/2)-77/320*a*sec(d*x+c)/d/(a+a*s
in(d*x+c))^(3/2)-11/60*a*sec(d*x+c)^3/d/(a+a*sin(d*x+c))^(3/2)-231/1024*arc
tanh(1/2*cos(d*x+c)*a^(1/2)*2^(1/2)/(a+a*sin(d*x+c))^(1/2))/d*2^(1/2)/a^(1/
2)+77/128*sec(d*x+c)/d/(a+a*sin(d*x+c))^(1/2)+11/40*sec(d*x+c)^3/d/(a+a*sin
(d*x+c))^(1/2)+1/5*sec(d*x+c)^5/d/(a+a*sin(d*x+c))^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2766, 2760, 2729, 2728, 212}

$$-\frac{231a\cos(c+dx)}{512d(a\sin(c+dx)+a)^{3/2}}+\frac{\sec^5(c+dx)}{5d\sqrt{a\sin(c+dx)+a}}+\frac{11\sec^3(c+dx)}{40d\sqrt{a\sin(c+dx)+a}}-\frac{11a\sec^3(c+dx)}{60d(a\sin(c+dx)+a)^{3/2}}+\frac{77\sec(c+dx)}{128d\sqrt{a\sin(c+dx)+a}}-\frac{77a\sec(c+dx)}{320d(a\sin(c+dx)+a)^{3/2}}-\frac{231\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(c+dx)}{\sqrt{2}\,\sqrt{a\sin(c+dx)+a}}\right)}{512\sqrt{2}\,\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-231*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/(
512*Sqrt[2]*Sqrt[a]*d) - (231*a*Cos[c + d*x])/(512*d*(a + a*Sin[c + d*x])^(
3/2)) - (77*a*Sec[c + d*x])/(320*d*(a + a*Sin[c + d*x])^(3/2)) - (11*a*Sec[
c + d*x]^3)/(60*d*(a + a*Sin[c + d*x])^(3/2)) + (77*Sec[c + d*x])/(128*d*Sq
rt[a + a*Sin[c + d*x]]) + (11*Sec[c + d*x]^3)/(40*d*Sqrt[a + a*Sin[c + d*x]
]) + Sec[c + d*x]^5/(5*d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2760

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*(2*m + p + 1))), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2766

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(a*f*g*(p + 1)*S
qrt[a + b*Sin[e + f*x]])), x] + Dist[a*((2*p + 1)/(2*g^2*(p + 1))), Int[(g*
Cos[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e,
 f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{\sec^6(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx = \frac{\sec^5(c+dx)}{5d\sqrt{a+a\sin(c+dx)}} + \frac{1}{10}(11a)\int \frac{\sec^4(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$$

$$= -\frac{11a\sec^3(c+dx)}{60d(a+a\sin(c+dx))^{3/2}} + \frac{\sec^5(c+dx)}{5d\sqrt{a+a\sin(c+dx)}} + \frac{33}{40}\int \frac{\sec^4(c+dx)}{\sqrt{a+a\sin(c+}}$$

$$= -\frac{11a\sec^3(c+dx)}{60d(a+a\sin(c+dx))^{3/2}} + \frac{11\sec^3(c+dx)}{40d\sqrt{a+a\sin(c+dx)}} + \frac{\sec^5(c+dx)}{5d\sqrt{a+a\sin(c+dx}}$$

$$= -\frac{77a\sec(c+dx)}{320d(a+a\sin(c+dx))^{3/2}} - \frac{11a\sec^3(c+dx)}{60d(a+a\sin(c+dx))^{3/2}} + \frac{11\sec^3(c+dx}{40d\sqrt{a+a\sin(c+}}$$

$$= -\frac{77a\sec(c+dx)}{320d(a+a\sin(c+dx))^{3/2}} - \frac{11a\sec^3(c+dx)}{60d(a+a\sin(c+dx))^{3/2}} + \frac{77\sec(c+dx}{128d\sqrt{a+a\sin(c}}$$

$$= -\frac{231a\cos(c+dx)}{512d(a+a\sin(c+dx))^{3/2}} - \frac{77a\sec(c+dx)}{320d(a+a\sin(c+dx))^{3/2}} - \frac{11a\sec^3(c+}{60d(a+a\sin(c+}$$

$$= -\frac{231a\cos(c+dx)}{512d(a+a\sin(c+dx))^{3/2}} - \frac{77a\sec(c+dx)}{320d(a+a\sin(c+dx))^{3/2}} - \frac{11a\sec^3(c+}{60d(a+a\sin(c+}$$

$$= -\frac{231\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{512\sqrt{2}\sqrt{a}\,d} - \frac{231a\cos(c+dx)}{512d(a+a\sin(c+dx))^{3/2}} - \frac{}{320d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.45, size = 140, normalized size = 0.63

$$\frac{(3465+3465i)(-1)^{3/4}\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(c+dx)\right)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+\frac{1}{16}\sec^5(c+dx)(11090+11352\cos(2(c+dx))+2310\cos(4(c+dx))+36850\sin(c+dx)+17787\sin(3(c+dx))+3465\sin(5(c+dx)))}{7680d\sqrt{a(1+\sin(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] ((3465 + 3465*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d
*x)/4])]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) + (Sec[c + d*x]^5*(11090 + 1
1352*Cos[2*(c + d*x)] + 2310*Cos[4*(c + d*x)] + 36850*Sin[c + d*x] + 17787*
Sin[3*(c + d*x)] + 3465*Sin[5*(c + d*x)]))/16)/(7680*d*Sqrt[a*(1 + Sin[c +
d*x])])
```

Maple [A]
time = 0.60, size = 308, normalized size = 1.39

method	result

default	$-\frac{-6930a^{\frac{11}{2}}\sin(dx+c)(\cos^4(dx+c))+\left(-3696a^{\frac{11}{2}}-3465\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)a^3(a-a\sin(dx+c))^{\frac{5}{2}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^6/(a+a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/15360*(-6930*a^(11/2)*sin(d*x+c)*cos(d*x+c)^4+(-3696*a^(11/2)-3465*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^3*(a-a*sin(d*x+c))^(5/2))*cos(d*x+c)^2*sin(d*x+c)+(-2816*a^(11/2)+13860*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^3*(a-a*sin(d*x+c))^(5/2))*sin(d*x+c)-2310*a^(11/2)*cos(d*x+c)^4+(-528*a^(11/2)-10395*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^3*(a-a*sin(d*x+c))^(5/2))*cos(d*x+c)^2-256*a^(11/2)+13860*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^3*(a-a*sin(d*x+c))^(5/2))/a^(11/2)/(sin(d*x+c)-1)^2/(1+sin(d*x+c))^2/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^6/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(sec(d*x + c)^6/sqrt(a*sin(d*x + c) + a), x)

Fricas [A]
time = 0.40, size = 250, normalized size = 1.13

$$\frac{3465\sqrt{2}\left(\cos(dx+c)^5\sin(dx+c)+\cos(dx+c)^5\right)\sqrt{a}\,\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a\sin(dx+c)+a}\,\sqrt{a}\,(\cos(dx+c)-\sin(dx+c)+1)+3a\cos(dx+c)-(a\cos(dx+c)-2a)\sin(dx+c)+2a}{\cos(dx+c)^2-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2}\right)+4\left(1155\cos(dx+c)^4+264\cos(dx+c)^2+11\left(315\cos(dx+c)^4+168\cos(dx+c)^2+128\right)\sin(dx+c)+128\right)\sqrt{a\sin(dx+c)+a}}{30720\left(ad\cos(dx+c)^5\sin(dx+c)+ad\cos(dx+c)^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^6/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")

[Out] 1/30720*(3465*sqrt(2)*(cos(d*x + c)^5*sin(d*x + c) + cos(d*x + c)^5)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - sin(d*x + c) + 1) + 3*a*cos(d*x + c) - (a*cos(d*x + c) - 2*a)*sin(d*x + c) + 2*a)/(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2)) + 4*(1155*cos(d*x + c)^4 + 264*cos(d*x + c)^2 + 11*(315*cos(d*x + c)^4 + 168*cos(d*x + c)^2 + 128)*sin(d*x + c) + 128)*sqrt(a*sin(d*x + c) + a))/(a*d*cos(d*x + c)^5*sin(d*x + c) + a*d*cos(d*x + c)^5)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^6(c+dx)}{\sqrt{a(\sin(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**6/sqrt(a*(sin(c + d*x) + 1)), x)
```

Giac [A]
time = 5.30, size = 262, normalized size = 1.19

$$\frac{\frac{3465\,\sqrt{2}\,\log\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{\sqrt{a}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}-\frac{3465\,\sqrt{2}\,\log\left(-\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{\sqrt{a}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}-\frac{10\,\sqrt{2}\left(213\,\sqrt{a}\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-472\,\sqrt{a}\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+267\,\sqrt{a}\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{3}a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}-\frac{32\,\sqrt{2}\left(150\,\sqrt{a}\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+20\,\sqrt{a}\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+3\,\sqrt{a}\right)}{a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}}}{30720\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/30720*(3465*sqrt(2)*log(sin(-1/4*pi + 1/2*d*x + 1/2*c) + 1)/(sqrt(a)*sgn(
cos(-1/4*pi + 1/2*d*x + 1/2*c))) - 3465*sqrt(2)*log(-sin(-1/4*pi + 1/2*d*x
+ 1/2*c) + 1)/(sqrt(a)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))) - 10*sqrt(2)*(2
13*sqrt(a)*sin(-1/4*pi + 1/2*d*x + 1/2*c)^5 - 472*sqrt(a)*sin(-1/4*pi + 1/2
*d*x + 1/2*c)^3 + 267*sqrt(a)*sin(-1/4*pi + 1/2*d*x + 1/2*c))/((sin(-1/4*pi
 + 1/2*d*x + 1/2*c)^2 - 1)^3*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))) - 32*sq
rt(2)*(150*sqrt(a)*sin(-1/4*pi + 1/2*d*x + 1/2*c)^4 + 20*sqrt(a)*sin(-1/4*p
i + 1/2*d*x + 1/2*c)^2 + 3*sqrt(a))/(a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*
sin(-1/4*pi + 1/2*d*x + 1/2*c)^5))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\cos\left(c+d\,x\right)}^6\,\sqrt{a+a\,\sin\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^6*(a + a*sin(c + d*x))^(1/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^6*(a + a*sin(c + d*x))^(1/2)), x)
```

3.169 $\int \frac{\cos^7(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=97

$$\frac{16(a+a\sin(c+dx))^{5/2}}{5a^4d}-\frac{24(a+a\sin(c+dx))^{7/2}}{7a^5d}+\frac{4(a+a\sin(c+dx))^{9/2}}{3a^6d}-\frac{2(a+a\sin(c+dx))^{11/2}}{11a^7d}$$

```
[Out] 16/5*(a+a*sin(d*x+c))^(5/2)/a^4/d-24/7*(a+a*sin(d*x+c))^(7/2)/a^5/d+4/3*(a+
a*sin(d*x+c))^(9/2)/a^6/d-2/11*(a+a*sin(d*x+c))^(11/2)/a^7/d
```

Rubi [A]
time = 0.06, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2746, 45}

$$-\frac{2(a\sin(c+dx)+a)^{11/2}}{11a^7d}+\frac{4(a\sin(c+dx)+a)^{9/2}}{3a^6d}-\frac{24(a\sin(c+dx)+a)^{7/2}}{7a^5d}+\frac{16(a\sin(c+dx)+a)^{5/2}}{5a^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (16*(a + a*Sin[c + d*x])^(5/2))/(5*a^4*d) - (24*(a + a*Sin[c + d*x])^(7/2))
/(7*a^5*d) + (4*(a + a*Sin[c + d*x])^(9/2))/(3*a^6*d) - (2*(a + a*Sin[c + d
*x])^(11/2))/(11*a^7*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\int \frac{\cos^7(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx = \frac{\text{Subst}\left(\int (a-x)^3(a+x)^{3/2}\,dx, x, a\sin(c+dx)\right)}{a^7 d}$$
$$= \frac{\text{Subst}\left(\int \left(8a^3(a+x)^{3/2} - 12a^2(a+x)^{5/2} + 6a(a+x)^{7/2} - (a+x)^{9/2}\right)\,dx, x, a$$
$$= \frac{16(a+a\sin(c+dx))^{5/2}}{5a^4 d} - \frac{24(a+a\sin(c+dx))^{7/2}}{7a^5 d} + \frac{4(a+a\sin(c+dx))^{9/2}}{3a^6 d}$$

Mathematica [A]
time = 0.15, size = 54, normalized size = 0.56

$$-\frac{2(a(1+\sin(c+dx)))^{5/2}\left(-533+755\sin(c+dx)-455\sin^2(c+dx)+105\sin^3(c+dx)\right)}{1155a^4 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*(a*(1 + Sin[c + d*x]))^(5/2)*(-533 + 755*Sin[c + d*x] - 455*Sin[c + d*x
]^2 + 105*Sin[c + d*x]^3))/(1155*a^4*d)
```

Maple [A]
time = 0.32, size = 57, normalized size = 0.59

method	result	size
default	$\frac{2(a+a\sin(dx+c))^{\frac{5}{2}}\left(105\left(\cos^2(dx+c)\right)\sin(dx+c)-455\left(\cos^2(dx+c)\right)-860\sin(dx+c)+988\right)}{1155a^4 d}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7/(a+a*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/1155/a^4*(a+a*sin(d*x+c))^(5/2)*(105*cos(d*x+c)^2*sin(d*x+c)-455*cos(d*x+
c)^2-860*sin(d*x+c)+988)/d
```

Maxima [A]
time = 0.29, size = 72, normalized size = 0.74

$$-\frac{2\left(105\left(a\sin\left(dx+c\right)+a\right)^{\frac{11}{2}}-770\left(a\sin\left(dx+c\right)+a\right)^{\frac{9}{2}}a+1980\left(a\sin\left(dx+c\right)+a\right)^{\frac{7}{2}}a^2-1848\left(a\sin\left(dx+c\right)+a\right)^{\frac{5}{2}}a^3\right)}{1155\,a^7 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -2/1155*(105*(a*sin(d*x + c) + a)^(11/2) - 770*(a*sin(d*x + c) + a)^(9/2)*a
 + 1980*(a*sin(d*x + c) + a)^(7/2)*a^2 - 1848*(a*sin(d*x + c) + a)^(5/2)*a^
3)/(a^7*d)
```

Fricas [**A**]
time = 0.35, size = 72, normalized size = 0.74

$$\frac{2\left(245\cos\left(dx+c\right)^{4}+32\cos\left(dx+c\right)^{2}-\left(105\cos\left(dx+c\right)^{4}-160\cos\left(dx+c\right)^{2}-256\right)\sin\left(dx+c\right)+256\right)\sqrt{a\sin\left(dx+c\right)+a}}{1155\,a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(245*cos(d*x + c)^4 + 32*cos(d*x + c)^2 - (105*cos(d*x + c)^4 - 160*
cos(d*x + c)^2 - 256)*sin(d*x + c) + 256)*sqrt(a*sin(d*x + c) + a)/(a^2*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 3.31, size = 112, normalized size = 1.15

$$-\frac{64\left(105\sqrt{2}\sqrt{a}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{11}-385\sqrt{2}\sqrt{a}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{9}+495\sqrt{2}\sqrt{a}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{7}-231\sqrt{2}\sqrt{a}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{5}\right)}{1155\,a^{2}d\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -64/1155*(105*sqrt(2)*sqrt(a)*cos(-1/4*pi + 1/2*d*x + 1/2*c)^11 - 385*sqrt(
2)*sqrt(a)*cos(-1/4*pi + 1/2*d*x + 1/2*c)^9 + 495*sqrt(2)*sqrt(a)*cos(-1/4*
pi + 1/2*d*x + 1/2*c)^7 - 231*sqrt(2)*sqrt(a)*cos(-1/4*pi + 1/2*d*x + 1/2*c
)^5)/(a^2*d*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c)))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^7}{{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7/(a + a*sin(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^7/(a + a*sin(c + d*x))^(3/2), x)
```

3.170 $\int \frac{\cos^6(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=63

$$-\frac{8a^2\cos^7(c+dx)}{63d(a+a\sin(c+dx))^{7/2}}-\frac{2a\cos^7(c+dx)}{9d(a+a\sin(c+dx))^{5/2}}$$

```
[Out] -8/63*a^2*cos(d*x+c)^7/d/(a+a*sin(d*x+c))^(7/2)-2/9*a*cos(d*x+c)^7/d/(a+a*s
in(d*x+c))^(5/2)
```

Rubi [A]
time = 0.08, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2753, 2752}

$$-\frac{8a^2\cos^7(c+dx)}{63d(a\sin(c+dx)+a)^{7/2}}-\frac{2a\cos^7(c+dx)}{9d(a\sin(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-8*a^2*Cos[c + d*x]^7)/(63*d*(a + a*Sin[c + d*x])^(7/2)) - (2*a*Cos[c + d*
x]^7)/(9*d*(a + a*Sin[c + d*x])^(5/2))
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2753

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && N
eQ[m + p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^6(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx &= -\frac{2a\cos^7(c+dx)}{9d(a+a\sin(c+dx))^{5/2}}+\frac{1}{9}(4a)\int \frac{\cos^6(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx\\ &= -\frac{8a^2\cos^7(c+dx)}{63d(a+a\sin(c+dx))^{7/2}}-\frac{2a\cos^7(c+dx)}{9d(a+a\sin(c+dx))^{5/2}}\end{aligned}$$

Mathematica [A]
time = 0.12, size = 49, normalized size = 0.78

$$-\frac{2\cos^7(c+dx)(11+7\sin(c+dx))}{63d(1+\sin(c+dx))^2(a(1+\sin(c+dx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*Cos[c + d*x]^7*(11 + 7*Sin[c + d*x]))/(63*d*(1 + Sin[c + d*x])^2*(a*(1
+ Sin[c + d*x]))^(3/2))
```

Maple [A]
time = 0.44, size = 57, normalized size = 0.90

method	result	size
default	$-\frac{2(1+\sin(dx+c))(\sin(dx+c)-1)^4(7\sin(dx+c)+11)}{63a\cos(dx+c)\sqrt{a+a\sin(dx+c)}\, d}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6/(a+a*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/63/a*(1+sin(d*x+c))*(sin(d*x+c)-1)^4*(7*sin(d*x+c)+11)/cos(d*x+c)/(a+a*s
in(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^6/(a*sin(d*x + c) + a)^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 142 vs. 2(55) = 110.
time = 0.34, size = 142, normalized size = 2.25

$$\frac{2\left(7\cos(dx+c)^5+17\cos(dx+c)^4-2\cos(dx+c)^3+4\cos(dx+c)^2-\left(7\cos(dx+c)^4-10\cos(dx+c)^3-12\cos(dx+c)^2-16\cos(dx+c)-32\right)\sin(dx+c)-16\cos(dx+c)-32\right)\sqrt{a\sin(dx+c)+a}}{63\left(a^2d\cos(dx+c)+a^2d\sin(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/63*(7*cos(d*x + c)^5 + 17*cos(d*x + c)^4 - 2*cos(d*x + c)^3 + 4*cos(d*x +
 c)^2 - (7*cos(d*x + c)^4 - 10*cos(d*x + c)^3 - 12*cos(d*x + c)^2 - 16*cos(
```

```
d*x + c) - 32)*sin(d*x + c) - 16*cos(d*x + c) - 32)*sqrt(a*sin(d*x + c) + a
)/(a^2*d*cos(d*x + c) + a^2*d*sin(d*x + c) + a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^6(c+dx)}{(a(\sin(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral(cos(c + d*x)**6/(a*(sin(c + d*x) + 1))**(3/2), x)
```

Giac [A]
time = 5.32, size = 68, normalized size = 1.08

$$-\frac{32\left(7\sqrt{2}\sqrt{a}\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-9\sqrt{2}\sqrt{a}\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7\right)}{63\,a^2d\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -32/63*(7*sqrt(2)*sqrt(a)*sin(-1/4*pi + 1/2*d*x + 1/2*c)^9 - 9*sqrt(2)*sqrt
(a)*sin(-1/4*pi + 1/2*d*x + 1/2*c)^7)/(a^2*d*sgn(cos(-1/4*pi + 1/2*d*x + 1/
2*c)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos(c+d\,x)^6}{(a+a\,\sin(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(a + a*sin(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^6/(a + a*sin(c + d*x))^(3/2), x)
```

3.171 $\int \frac{\cos^5(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=73

$$\frac{8(a+a\sin(c+dx))^{3/2}}{3a^3d}-\frac{8(a+a\sin(c+dx))^{5/2}}{5a^4d}+\frac{2(a+a\sin(c+dx))^{7/2}}{7a^5d}$$

```
[Out] 8/3*(a+a*sin(d*x+c))^(3/2)/a^3/d-8/5*(a+a*sin(d*x+c))^(5/2)/a^4/d+2/7*(a+a*
sin(d*x+c))^(7/2)/a^5/d
```

Rubi [A]

time = 0.05, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2746, 45}

$$\frac{2(a\sin(c+dx)+a)^{7/2}}{7a^5d}-\frac{8(a\sin(c+dx)+a)^{5/2}}{5a^4d}+\frac{8(a\sin(c+dx)+a)^{3/2}}{3a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (8*(a + a*Sin[c + d*x])^(3/2))/(3*a^3*d) - (8*(a + a*Sin[c + d*x])^(5/2))/(
5*a^4*d) + (2*(a + a*Sin[c + d*x])^(7/2))/(7*a^5*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\int \frac{\cos^5(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx = \frac{\text{Subst}\left(\int (a-x)^2\sqrt{a+x}\,dx, x, a\sin(c+dx)\right)}{a^5d}$$
$$= \frac{\text{Subst}\left(\int \left(4a^2\sqrt{a+x} - 4a(a+x)^{3/2} + (a+x)^{5/2}\right)\,dx, x, a\sin(c+dx)\right)}{a^5d}$$
$$= \frac{8(a+a\sin(c+dx))^{3/2}}{3a^3d} - \frac{8(a+a\sin(c+dx))^{5/2}}{5a^4d} + \frac{2(a+a\sin(c+dx))^{7/2}}{7a^5d}$$

Mathematica [**A**]
time = 0.06, size = 44, normalized size = 0.60

$$\frac{2(a(1+\sin(c+dx)))^{3/2}\left(71-54\sin(c+dx)+15\sin^2(c+dx)\right)}{105a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (2*(a*(1 + Sin[c + d*x]))^(3/2)*(71 - 54*Sin[c + d*x] + 15*Sin[c + d*x]^2))
/(105*a^3*d)
```

Maple [**A**]
time = 0.30, size = 41, normalized size = 0.56

method	result	size
default	$-\frac{2(a+a\sin(dx+c))^{\frac{3}{2}}\left(15\left(\cos^2(dx+c)\right)+54\sin(dx+c)-86\right)}{105a^3d}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a+a*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/105/a^3*(a+a*sin(d*x+c))^(3/2)*(15*cos(d*x+c)^2+54*sin(d*x+c)-86)/d
```

Maxima [**A**]
time = 0.29, size = 55, normalized size = 0.75

$$\frac{2\left(15\left(a\sin\left(dx+c\right)+a\right)^{\frac{7}{2}} - 84\left(a\sin\left(dx+c\right)+a\right)^{\frac{5}{2}}a + 140\left(a\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}a^2\right)}{105\,a^5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/105*(15*(a*sin(d*x + c) + a)^(7/2) - 84*(a*sin(d*x + c) + a)^(5/2)*a + 14
0*(a*sin(d*x + c) + a)^(3/2)*a^2)/(a^5*d)
```

Fricas [**A**]
time = 0.33, size = 52, normalized size = 0.71

$$\frac{2\left(39\cos\left(dx+c\right)^{2}-\left(15\cos\left(dx+c\right)^{2}-32\right)\sin\left(dx+c\right)+32\right)\sqrt{a\sin\left(dx+c\right)+a}}{105\,a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/105*(39*cos(d*x + c)^2 - (15*cos(d*x + c)^2 - 32)*sin(d*x + c) + 32)*sqrt
(a*sin(d*x + c) + a)/(a^2*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 4.05, size = 90, normalized size = 1.23

$$\frac{16\left(15\sqrt{2}\sqrt{a}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{7}-42\sqrt{2}\sqrt{a}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{5}+35\sqrt{2}\sqrt{a}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{3}\right)}{105\,a^{2}d\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 16/105*(15*sqrt(2)*sqrt(a)*cos(-1/4*pi + 1/2*d*x + 1/2*c)^7 - 42*sqrt(2)*sq
rt(a)*cos(-1/4*pi + 1/2*d*x + 1/2*c)^5 + 35*sqrt(2)*sqrt(a)*cos(-1/4*pi + 1
/2*d*x + 1/2*c)^3)/(a^2*d*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c)))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^{5}}{\left(a+a\,\sin\left(c+d\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(a + a*sin(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^5/(a + a*sin(c + d*x))^(3/2), x)
```

3.172 $\int \frac{\cos^4(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=30

$$-\frac{2a\cos^5(c+dx)}{5d(a+a\sin(c+dx))^{5/2}}$$

```
[Out] -2/5*a*cos(d*x+c)^5/d/(a+a*sin(d*x+c))^(5/2)
```

Rubi [A]
time = 0.04, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {2752}

$$-\frac{2a\cos^5(c+dx)}{5d(a\sin(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*a*Cos[c + d*x]^5)/(5*d*(a + a*Sin[c + d*x])^(5/2))
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx = -\frac{2a\cos^5(c+dx)}{5d(a+a\sin(c+dx))^{5/2}}$$

Mathematica [A]
time = 0.04, size = 42, normalized size = 1.40

$$-\frac{2\cos^5(c+dx)\sqrt{a(1+\sin(c+dx))}}{5a^2d(1+\sin(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*Cos[c + d*x]^5*Sqrt[a*(1 + Sin[c + d*x])])/(5*a^2*d*(1 + Sin[c + d*x])^
3)
```

Maple [A]
time = 0.42, size = 47, normalized size = 1.57

method	result	size
default	$\frac{2(1+\sin(dx+c))(\sin(dx+c)-1)^3}{5a\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+a*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5/a*(1+sin(d*x+c))*(sin(d*x+c)-1)^3/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^4/(a*sin(d*x + c) + a)^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(26) = 52$.
time = 0.34, size = 98, normalized size = 3.27

$$\frac{2\left(\cos\left(dx+c\right)^{3}+3\,\cos\left(dx+c\right)^{2}-\left(\cos\left(dx+c\right)^{2}-2\,\cos\left(dx+c\right)-4\right)\sin\left(dx+c\right)-2\,\cos\left(dx+c\right)-4\right)\sqrt{a\sin\left(dx+c\right)+a}}{5\left(a^{2}d\cos\left(dx+c\right)+a^{2}d\sin\left(dx+c\right)+a^{2}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/5*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 - (cos(d*x + c)^2 - 2*cos(d*x + c) -
 4)*sin(d*x + c) - 2*cos(d*x + c) - 4)*sqrt(a*sin(d*x + c) + a)/(a^2*d*cos(
d*x + c) + a^2*d*sin(d*x + c) + a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cos^{4}\left(c+dx\right)}{\left(a\left(\sin\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+a*sin(d*x+c))**(3/2),x)
[Out] Integral(cos(c + d*x)**4/(a*(sin(c + d*x) + 1))**(3/2), x)
```

Giac [A]
time = 5.32, size = 40, normalized size = 1.33

$$\frac{8\,\sqrt{2}\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}}{5\,a^{\frac{3}{2}}d\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
[Out] 8/5*sqrt(2)*sin(-1/4*pi + 1/2*d*x + 1/2*c)^5/(a^(3/2)*d*sgn(cos(-1/4*pi + 1
/2*d*x + 1/2*c)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\cos\left(c+d\,x\right)^4}{{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(a + a*sin(c + d*x))^(3/2),x)
[Out] int(cos(c + d*x)^4/(a + a*sin(c + d*x))^(3/2), x)
```

3.173 $\int \frac{\cos^3(c+dx)}{(a+a\sin(c+dx))^{3/2}} \, dx$

Optimal. Leaf size=47

$$\frac{4\sqrt{a+a\sin(c+dx)}}{a^2d}-\frac{2(a+a\sin(c+dx))^{3/2}}{3a^3d}$$

[Out] -2/3*(a+a*sin(d*x+c))^(3/2)/a^3/d+4*(a+a*sin(d*x+c))^(1/2)/a^2/d

Rubi [A]
time = 0.05, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2746, 45}

$$\frac{4\sqrt{a\sin(c+dx)+a}}{a^2d}-\frac{2(a\sin(c+dx)+a)^{3/2}}{3a^3d}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^3/(a + a*Sin[c + d*x])^(3/2),x]

[Out] (4*Sqrt[a + a*Sin[c + d*x]])/(a^2*d) - (2*(a + a*Sin[c + d*x])^(3/2))/(3*a^3*d)

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{a-x}{\sqrt{a+x}}\,dx, x, a\sin(c+dx)\right)}{a^3 d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{2a}{\sqrt{a+x}} - \sqrt{a+x}\right)\,dx, x, a\sin(c+dx)\right)}{a^3 d}$$
$$= \frac{4\sqrt{a+a\sin(c+dx)}}{a^2 d} - \frac{2(a+a\sin(c+dx))^{3/2}}{3a^3 d}$$

Mathematica [A]
time = 0.03, size = 32, normalized size = 0.68

$$-\frac{2(-5+\sin(c+dx))\sqrt{a(1+\sin(c+dx))}}{3a^2 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + a*Sin[c + d*x])^(3/2),x]
[Out] (-2*(-5 + Sin[c + d*x])*Sqrt[a*(1 + Sin[c + d*x])])/(3*a^2*d)
```

Maple [A]
time = 0.26, size = 29, normalized size = 0.62

method	result	size
default	$-\frac{2\sqrt{a+a\sin(dx+c)}\,(\sin(dx+c)-5)}{3a^2 d}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
[Out] -2/3/a^2*(a+a*sin(d*x+c))^(1/2)*(sin(d*x+c)-5)/d
```

Maxima [A]
time = 0.30, size = 36, normalized size = 0.77

$$-\frac{2\left((a\sin(dx+c)+a)^{\frac{3}{2}} - 6\sqrt{a\sin(dx+c)+a}\,a\right)}{3\,a^3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
[Out] -2/3*((a*sin(d*x + c) + a)^(3/2) - 6*sqrt(a*sin(d*x + c) + a)*a)/(a^3*d)
```

Fricas [A]
time = 0.35, size = 28, normalized size = 0.60

$$-\frac{2\sqrt{a\sin\left(dx+c\right)+a}\left(\sin\left(dx+c\right)-5\right)}{3\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(a*sin(d*x + c) + a)*(sin(d*x + c) - 5)/(a^2*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 4.72, size = 65, normalized size = 1.38

$$-\frac{4\left(\sqrt{2}\sqrt{a}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^3-3\sqrt{2}\sqrt{a}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{3\,a^2d\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -4/3*(sqrt(2)*sqrt(a)*cos(-1/4*pi + 1/2*d*x + 1/2*c)^3 - 3*sqrt(2)*sqrt(a)*
cos(-1/4*pi + 1/2*d*x + 1/2*c))/(a^2*d*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos\left(c+d\,x\right)^3}{\left(a+a\,\sin\left(c+d\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a + a*sin(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^3/(a + a*sin(c + d*x))^(3/2), x)
```

3.174 $\int \frac{\cos^2(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=76

$$-\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{a^{3/2}d}+\frac{2\cos(c+dx)}{ad\sqrt{a+a\sin(c+dx)}}$$

```
[Out] -2*arctanh(1/2*cos(d*x+c)*a^(1/2)*2^(1/2)/(a+a*sin(d*x+c))^(1/2))*2^(1/2)/a
^(3/2)/d+2*cos(d*x+c)/a/d/(a+a*sin(d*x+c))^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {2758, 2728, 212}

$$\frac{2\cos(c+dx)}{ad\sqrt{a\sin(c+dx)+a}}-\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]
])])/(a^(3/2)*d) + (2*Cos[c + d*x])/(a*d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2758

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(a*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
```

```
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx = \frac{2\cos(c+dx)}{ad\sqrt{a+a\sin(c+dx)}} + \frac{2\int \frac{1}{\sqrt{a+a\sin(c+dx)}}\,dx}{a}$$

$$= \frac{2\cos(c+dx)}{ad\sqrt{a+a\sin(c+dx)}} - \frac{4\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{a\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{ad}$$

$$= -\frac{2\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(c+dx)}{\sqrt{2}\,\sqrt{a+a\sin(c+dx)}}\right)}{a^{3/2}d} + \frac{2\cos(c+dx)}{ad\sqrt{a+a\sin(c+dx)}}$$

Mathematica [A]
time = 0.10, size = 84, normalized size = 1.11

$$\frac{2\cos^3(c+dx)\left(-\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{1-\sin(c+dx)}}{\sqrt{2}}\right)+\sqrt{1-\sin(c+dx)}\right)}{d(1-\sin(c+dx))^{3/2}(a(1+\sin(c+dx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + a*Sin[c + d*x])^(3/2),x]
[Out] (2*Cos[c + d*x]^3*(-(Sqrt[2]*ArcTanh[Sqrt[1 - Sin[c + d*x]]/Sqrt[2]]) + Sqr
t[1 - Sin[c + d*x]]))/(d*(1 - Sin[c + d*x])^(3/2)*(a*(1 + Sin[c + d*x]))^(3
/2))
```

Maple [A]
time = 0.54, size = 95, normalized size = 1.25

method	result
default	$-\frac{2(1+\sin(dx+c))\sqrt{-a\left(\sin\left(dx+c\right)-1\right)}\left(\sqrt{a}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin\left(dx+c\right)}\,\sqrt{2}}{2\sqrt{a}}\right)-\sqrt{a-a\sin}\right.}{a^2\cos(dx+c)\sqrt{a+a\sin\left(dx+c\right)}\,d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(a^(1/2)*2^(1/2)*arctanh(1/2*(a
-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))-(a-a*sin(d*x+c))^(1/2))/a^2/cos(d*x+c
)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^2/(a*sin(d*x + c) + a)^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 196 vs. 2(65) = 130.
time = 0.37, size = 196, normalized size = 2.58

$$\frac{\frac{\sqrt{2}\,(a\cos(dx+c)+a\sin(dx+c)+a)\log\left(-\frac{\cos(dx+c)^2-(\cos(dx+c)-2)\sin(dx+c)-\frac{2\sqrt{2}\sqrt{a\sin(dx+c)+a}\,(\cos(dx+c)-\sin(dx+c)+1)}{\sqrt{a}}+3\cos(dx+c)+2}{\cos(dx+c)^2-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2}\right)}{\sqrt{a}}+2\sqrt{a\sin(dx+c)+a}\,(\cos(dx+c)-\sin(dx+c)+1)}{a^2d\cos(dx+c)+a^2d\sin(dx+c)+a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] (sqrt(2)*(a*cos(d*x + c) + a*sin(d*x + c) + a)*log(-(cos(d*x + c)^2 - (cos(
d*x + c) - 2)*sin(d*x + c) - 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*(cos(d*x +
c) - sin(d*x + c) + 1)/sqrt(a) + 3*cos(d*x + c) + 2)/(cos(d*x + c)^2 - (cos
(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2))/sqrt(a) + 2*sqrt(a*sin(d*x
 + c) + a)*(cos(d*x + c) - sin(d*x + c) + 1))/(a^2*d*cos(d*x + c) + a^2*d*s
in(d*x + c) + a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(c+dx)}{(a(\sin(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral(cos(c + d*x)**2/(a*(sin(c + d*x) + 1))**(3/2), x)
```

Giac [A]
time = 4.22, size = 114, normalized size = 1.50

$$\frac{\sqrt{2}\,\sqrt{a}\left(\frac{\log\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{\log\left(-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{2\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)}{a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] sqrt(2)*sqrt(a)*(log(sin(-1/4*pi + 1/2*d*x + 1/2*c) + 1)/(a^2*sgn(cos(-1/4*
pi + 1/2*d*x + 1/2*c))) - log(-sin(-1/4*pi + 1/2*d*x + 1/2*c) + 1)/(a^2*sgn
(cos(-1/4*pi + 1/2*d*x + 1/2*c))) - 2*sin(-1/4*pi + 1/2*d*x + 1/2*c)/(a^2*s
gn(cos(-1/4*pi + 1/2*d*x + 1/2*c))))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)^2}{\left(a+a\,\sin(c+d\,x)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a + a*sin(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^2/(a + a*sin(c + d*x))^(3/2), x)
```

3.175 $\int \frac{\cos(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=22

$$-\frac{2}{ad\sqrt{a+a\sin(c+dx)}}$$

```
[Out] -2/a/d/(a+a*sin(d*x+c))^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2746, 32}

$$-\frac{2}{ad\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -2/(a*d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{(a+x)^{3/2}}\,dx, x, a\sin(c+dx)\right)}{ad}\\ &= -\frac{2}{ad\sqrt{a+a\sin(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 22, normalized size = 1.00

$$-\frac{2}{ad\sqrt{a+a\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + a*Sin[c + d*x])^(3/2),x]
[Out] -2/(a*d*Sqrt[a + a*Sin[c + d*x]])
```

Maple [A]
time = 0.04, size = 21, normalized size = 0.95

method	result	size
derivativedivides	$-\frac{2}{ad\sqrt{a+a\sin(dx+c)}}$	21
default	$-\frac{2}{ad\sqrt{a+a\sin(dx+c)}}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+a*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
[Out] -2/a/d/(a+a*sin(d*x+c))^(1/2)
```

Maxima [A]
time = 0.30, size = 20, normalized size = 0.91

$$-\frac{2}{\sqrt{a\sin(dx+c)+a}\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
[Out] -2/(sqrt(a*sin(d*x + c) + a)*a*d)
```

Fricas [A]
time = 0.34, size = 33, normalized size = 1.50

$$-\frac{2\sqrt{a\sin(dx+c)+a}}{a^2d\sin(dx+c)+a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
[Out] -2*sqrt(a*sin(d*x + c) + a)/(a^2*d*sin(d*x + c) + a^2*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(19) = 38$.
time = 0.85, size = 46, normalized size = 2.09

$$\begin{cases} \text{NaN} & \text{for } c = \frac{3\pi}{2} \wedge d = 0 \\ \frac{x\cos(c)}{(a\sin(c)+a)^{\frac{3}{2}}} & \text{for } d = 0 \\ \text{NaN} & \text{for } c = -dx + \frac{3\pi}{2} \\ -\frac{2}{ad\sqrt{a\sin(c+dx)+a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Piecewise((nan, Eq(d, 0) & Eq(c, 3*pi/2)), (x*cos(c)/(a*sin(c) + a)**(3/2),
 Eq(d, 0)), (nan, Eq(c, -d*x + 3*pi/2)), (-2/(a*d*sqrt(a*sin(c + d*x) + a))
, True))
```

Giac [A]
time = 3.72, size = 40, normalized size = 1.82

$$-\frac{\sqrt{2}}{a^{\frac{3}{2}}d\cos\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right)\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -sqrt(2)/(a^(3/2)*d*cos(-1/4*pi + 1/2*d*x + 1/2*c)*sgn(cos(-1/4*pi + 1/2*d*
x + 1/2*c)))
```

Mupad [B]
time = 4.88, size = 50, normalized size = 2.27

$$-\frac{4\sqrt{a\,\left(\sin\left(c+d\,x\right)+1\right)}\,\left(\sin\left(c+d\,x\right)+1\right)}{a^2\,d\,\left(2\sin\left(c+d\,x\right)^2+4\,\sin\left(c+d\,x\right)+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + a*sin(c + d*x))^(3/2),x)
```

```
[Out] -(4*(a*(sin(c + d*x) + 1))^(1/2)*(sin(c + d*x) + 1))/(a^2*d*(4*sin(c + d*x)
 + 2*sin(c + d*x)^2 + 2))
```

3.176 $\int \frac{\sec(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=89

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{1}{3d(a+a\sin(c+dx))^{3/2}}-\frac{1}{2ad\sqrt{a+a\sin(c+dx)}}$$

[Out] -1/3/d/(a+a*sin(d*x+c))^(3/2)+1/4*arctanh(1/2*(a+a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/a^(3/2)/d*2^(1/2)-1/2/a/d/(a+a*sin(d*x+c))^(1/2)

Rubi [A]
time = 0.06, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2746, 53, 65, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{1}{2ad\sqrt{a\sin(c+dx)+a}}-\frac{1}{3d(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d*x]/(a + a*Sin[c + d*x])^(3/2),x]

[Out] ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])]/(2*Sqrt[2]*a^(3/2)*d) - 1/(3*d*(a + a*Sin[c + d*x])^(3/2)) - 1/(2*a*d*Sqrt[a + a*Sin[c + d*x]])

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx &= \frac{a\text{Subst}\left(\int \frac{1}{(a-x)(a+x)^{5/2}}\,dx, x, a\sin(c+dx)\right)}{d}\\ &= -\frac{1}{3d(a+a\sin(c+dx))^{3/2}}+\frac{\text{Subst}\left(\int \frac{1}{(a-x)(a+x)^{3/2}}\,dx, x, a\sin(c+dx)\right)}{2d}\\ &= -\frac{1}{3d(a+a\sin(c+dx))^{3/2}}-\frac{1}{2ad\sqrt{a+a\sin(c+dx)}}+\frac{\text{Subst}\left(\int \frac{1}{(a-x)\sqrt{a+}}\right.}{4}\\ &= -\frac{1}{3d(a+a\sin(c+dx))^{3/2}}-\frac{1}{2ad\sqrt{a+a\sin(c+dx)}}+\frac{\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x,\right.}{2}\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{1}{3d(a+a\sin(c+dx))^{3/2}}-\frac{1}{2ad\sqrt{a+a\sin}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.04, size = 41, normalized size = 0.46

$$-\frac{{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{1}{2}(1+\sin(c+dx))\right)}{3d(a+a\sin(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -1/3*Hypergeometric2F1[-3/2, 1, -1/2, (1 + Sin[c + d*x])/2]/(d*(a + a*Sin[c
 + d*x])^(3/2))
```

Maple [A]
time = 0.31, size = 71, normalized size = 0.80

method	result	size
default	$-\frac{a\left(-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+a\sin\left(dx+c\right)}\,\sqrt{2}}{2\sqrt{a}}\right)}{4a^{\frac{5}{2}}}+\frac{1}{2a^{2}\sqrt{a+a\sin\left(dx+c\right)}}+\frac{1}{3a\left(a+a\sin\left(dx+c\right)\right)^{\frac{3}{2}}}\right)}{d}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+a*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -a*(-1/4/a^(5/2)*2^(1/2)*arctanh(1/2*(a+a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2)
)+1/2/a^2/(a+a*sin(d*x+c))^(1/2)+1/3/a/(a+a*sin(d*x+c))^(3/2))/d
```

Maxima [A]
time = 0.54, size = 91, normalized size = 1.02

$$-\frac{\frac{3\sqrt{2}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{a\sin\left(dx+c\right)+a}}{\sqrt{2}\sqrt{a}+\sqrt{a\sin\left(dx+c\right)+a}}\right)}{\sqrt{a}}+\frac{4\left(3\,a\sin\left(dx+c\right)+5\,a\right)}{\left(a\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}}}{24\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/24*(3*sqrt(2)*log(-(sqrt(2)*sqrt(a) - sqrt(a*sin(d*x + c) + a))/(sqrt(2)
*sqrt(a) + sqrt(a*sin(d*x + c) + a)))/sqrt(a) + 4*(3*a*sin(d*x + c) + 5*a)/
(a*sin(d*x + c) + a)^(3/2))/(a*d)
```

Fricas [A]
time = 0.38, size = 132, normalized size = 1.48

$$\frac{3\sqrt{2}\left(\cos\left(dx+c\right)^{2}-2\,\sin\left(dx+c\right)-2\right)\sqrt{a}\,\log\left(-\frac{a\sin\left(dx+c\right)+2\sqrt{2}\,\sqrt{a\sin\left(dx+c\right)+a}\,\sqrt{a}+3\,a}{\sin\left(dx+c\right)-1}\right)+4\,\sqrt{a\sin\left(dx+c\right)+a}\,\left(3\,\sin\left(dx+c\right)+5\right)}{24\,\left(a^{2}d\cos\left(dx+c\right)^{2}-2\,a^{2}d\sin\left(dx+c\right)-2\,a^{2}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/24*(3*sqrt(2)*(cos(d*x + c)^2 - 2*sin(d*x + c) - 2)*sqrt(a)*log(-(a*sin(d
*x + c) + 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*sqrt(a) + 3*a)/(sin(d*x + c) -
 1)) + 4*sqrt(a*sin(d*x + c) + a)*(3*sin(d*x + c) + 5))/(a^2*d*cos(d*x + c)
^2 - 2*a^2*d*sin(d*x + c) - 2*a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(c+dx)}{(a(\sin(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral(sec(c + d*x)/(a*(sin(c + d*x) + 1))**(3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 142 vs. 2(70) = 140.
time = 6.22, size = 142, normalized size = 1.60

$$\frac{\sqrt{a}\left(\frac{3\sqrt{2}\log(\cos(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c)+1)}{a^2\mathrm{sgn}(\cos(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c))}-\frac{3\sqrt{2}\log(-\cos(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c)+1)}{a^2\mathrm{sgn}(\cos(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c))}-\frac{2\sqrt{2}\left(3\cos(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c)^2+1\right)}{a^2\cos(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c)^3\mathrm{sgn}(\cos(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c))}\right)}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/24*sqrt(a)*(3*sqrt(2)*log(cos(-1/4*pi + 1/2*d*x + 1/2*c) + 1)/(a^2*sgn(co
s(-1/4*pi + 1/2*d*x + 1/2*c))) - 3*sqrt(2)*log(-cos(-1/4*pi + 1/2*d*x + 1/2
*c) + 1)/(a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))) - 2*sqrt(2)*(3*cos(-1/4*
pi + 1/2*d*x + 1/2*c)^2 + 1)/(a^2*cos(-1/4*pi + 1/2*d*x + 1/2*c)^3*sgn(cos(
-1/4*pi + 1/2*d*x + 1/2*c))))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(c+d\,x)\,\left(a+a\,\sin(c+d\,x)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + a*sin(c + d*x))^(3/2)),x)
```

```
[Out] int(1/(cos(c + d*x)*(a + a*sin(c + d*x))^(3/2)), x)
```

3.177 $\int \frac{\sec^2(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=134

$$-\frac{15\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{32\sqrt{2}\,a^{3/2}d}-\frac{15\cos(c+dx)}{32d(a+a\sin(c+dx))^{3/2}}-\frac{\sec(c+dx)}{4d(a+a\sin(c+dx))^{3/2}}+\frac{5\sec(c+dx)}{8ad\sqrt{a+a\sin(c+dx)}}$$

```
[Out] -15/32*cos(d*x+c)/d/(a+a*sin(d*x+c))^(3/2)-1/4*sec(d*x+c)/d/(a+a*sin(d*x+c)
)^(3/2)-15/64*arctanh(1/2*cos(d*x+c)*a^(1/2)*2^(1/2)/(a+a*sin(d*x+c))^(1/2)
)*2^(1/2)/a^(3/2)/d+5/8*sec(d*x+c)/a/d/(a+a*sin(d*x+c))^(1/2)
```

Rubi [A]

time = 0.11, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2760, 2766, 2729, 2728, 212}

$$-\frac{15\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{32\sqrt{2}\,a^{3/2}d}-\frac{15\cos(c+dx)}{32d(a\sin(c+dx)+a)^{3/2}}+\frac{5\sec(c+dx)}{8ad\sqrt{a\sin(c+dx)+a}}-\frac{\sec(c+dx)}{4d(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-15*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/(3
2*Sqrt[2]*a^(3/2)*d) - (15*Cos[c + d*x])/(32*d*(a + a*Sin[c + d*x])^(3/2))
- Sec[c + d*x]/(4*d*(a + a*Sin[c + d*x])^(3/2)) + (5*Sec[c + d*x])/(8*a*d*S
qrt[a + a*Sin[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
```

```
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2760

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*(2*m + p + 1))), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2766

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(a*f*g*(p + 1)*S
qrt[a + b*Sin[e + f*x]])), x] + Dist[a*((2*p + 1)/(2*g^2*(p + 1))), Int[(g*
Cos[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e,
 f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^2(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx &= -\frac{\sec(c+dx)}{4d(a+a\sin(c+dx))^{3/2}}+\frac{5\int \frac{\sec^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx}{8a}\\
&= -\frac{\sec(c+dx)}{4d(a+a\sin(c+dx))^{3/2}}+\frac{5\sec(c+dx)}{8ad\sqrt{a+a\sin(c+dx)}}+\frac{15}{16}\int \frac{1}{(a+a\sin(c+
\\
&= -\frac{15\cos(c+dx)}{32d(a+a\sin(c+dx))^{3/2}}-\frac{\sec(c+dx)}{4d(a+a\sin(c+dx))^{3/2}}+\frac{5\sec(c+dx)}{8ad\sqrt{a+a\sin(c+
\\
&= -\frac{15\cos(c+dx)}{32d(a+a\sin(c+dx))^{3/2}}-\frac{\sec(c+dx)}{4d(a+a\sin(c+dx))^{3/2}}+\frac{5\sec(c+dx)}{8ad\sqrt{a+a\sin(c+
\\
&= -\frac{15\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{32\sqrt{2}\,a^{3/2}d}-\frac{15\cos(c+dx)}{32d(a+a\sin(c+dx))^{3/2}}-\frac{}{4d(a
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.18, size = 224, normalized size = 1.67

$$\frac{-4+\frac{8\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)}+14\sin\left(\frac{1}{2}(c+dx)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)-7\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2+(15+15i)(-1)^{3/4}\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(c+dx)\right)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3+\frac{8\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)}}{32d(a(1+\sin(c+dx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-4 + (8*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) + 14*Sin[(
c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) - 7*(Cos[(c + d*x)/2] + S
in[(c + d*x)/2])^2 + (15 + 15*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*
(-1 + Tan[(c + d*x)/4])]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3 + (8*(Cos[
(c + d*x)/2] + Sin[(c + d*x)/2])^3)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]))/
(32*d*(a*(1 + Sin[c + d*x]))^(3/2))
```

Maple [A]
time = 0.56, size = 202, normalized size = 1.51

method	result
default	$-\frac{\sin(dx+c)\left(30\sqrt{a-a\sin\left(dx+c\right)}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin\left(dx+c\right)}\,\sqrt{2}}{2\sqrt{a}}\right)a^{2}-40a^{\frac{5}{2}}\right)+\left(-15\sqrt{a-a\,\mathrm{s}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/64/a^(7/2)*(sin(d*x+c)*(30*(a-a*sin(d*x+c))^(1/2)*2^(1/2)*arctanh(1/2*(a
-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^2-40*a^(5/2))+(-15*(a-a*sin(d*x+c))
^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^2+30*a
^(5/2))*cos(d*x+c)^2+30*(a-a*sin(d*x+c))^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin
(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^2-24*a^(5/2))/(1+sin(d*x+c))/cos(d*x+c)/(
a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^2/(a*sin(d*x + c) + a)^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(111) = 222$.
time = 0.37, size = 240, normalized size = 1.79

$$\frac{15\sqrt{2}\left(\cos\left(dx+c\right)^{3}-2\cos\left(dx+c\right)\sin\left(dx+c\right)-2\cos\left(dx+c\right)\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^{2}-2\sqrt{2}\sqrt{a\sin\left(dx+c\right)+a}\sqrt{a}\left(\cos(dx+c)-\sin(dx+c)+1\right)+3a\cos(dx+c)-\left(a\cos(dx+c)-2a\right)\sin(dx+c)+2a}{\cos(dx+c)^{2}-\left(\cos(dx+c)+2\right)\sin(dx+c)-\cos(dx+c)-2}\right)+4\left(15\cos\left(dx+c\right)^{2}-20\sin\left(dx+c\right)-12\right)\sqrt{a\sin\left(dx+c\right)+a}}{128\left(a^{2}d\cos\left(dx+c\right)^{3}-2a^{2}d\cos\left(dx+c\right)\sin\left(dx+c\right)-2a^{2}d\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/128*(15*sqrt(2)*(cos(d*x + c)^3 - 2*cos(d*x + c)*sin(d*x + c) - 2*cos(d*x
 + c))*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*
sqrt(a)*(cos(d*x + c) - sin(d*x + c) + 1) + 3*a*cos(d*x + c) - (a*cos(d*x +
 c) - 2*a)*sin(d*x + c) + 2*a)/(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x
 + c) - cos(d*x + c) - 2)) + 4*(15*cos(d*x + c)^2 - 20*sin(d*x + c) - 12)*s
qrt(a*sin(d*x + c) + a))/(a^2*d*cos(d*x + c)^3 - 2*a^2*d*cos(d*x + c)*sin(d
*x + c) - 2*a^2*d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(c+dx)}{(a(\sin(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral(sec(c + d*x)**2/(a*(sin(c + d*x) + 1))**(3/2), x)
```

Giac [A]
time = 8.81, size = 199, normalized size = 1.49

$$\frac{\sqrt{a}\left(\frac{15\sqrt{2}\log\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{15\sqrt{2}\log\left(-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{16\sqrt{2}}{a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)}-\frac{2\left(7\sqrt{2}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^3-9\sqrt{2}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^2a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}\right)}{128\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/128*sqrt(a)*(15*sqrt(2)*log(sin(-1/4*pi + 1/2*d*x + 1/2*c) + 1)/(a^2*sgn(
cos(-1/4*pi + 1/2*d*x + 1/2*c))) - 15*sqrt(2)*log(-sin(-1/4*pi + 1/2*d*x +
1/2*c) + 1)/(a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))) - 16*sqrt(2)/(a^2*sgn
(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*d*x + 1/2*c)) - 2*(7*sqr
t(2)*sin(-1/4*pi + 1/2*d*x + 1/2*c)^3 - 9*sqrt(2)*sin(-1/4*pi + 1/2*d*x + 1
/2*c))/((sin(-1/4*pi + 1/2*d*x + 1/2*c)^2 - 1)^2*a^2*sgn(cos(-1/4*pi + 1/2*
d*x + 1/2*c))))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(c+d\,x)^2\,(a+a\,\sin(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + a*sin(c + d*x))^(3/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^2*(a + a*sin(c + d*x))^(3/2)), x)
```

3.178 $\int \frac{\sec^3(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=150

$$\frac{7\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{16\sqrt{2}\,a^{3/2}d}-\frac{7}{24d(a+a\sin(c+dx))^{3/2}}-\frac{\sec^2(c+dx)}{5d(a+a\sin(c+dx))^{3/2}}-\frac{7}{16ad\sqrt{a+a\sin(c}$$

```
[Out] -7/24/d/(a+a*sin(d*x+c))^(3/2)-1/5*sec(d*x+c)^2/d/(a+a*sin(d*x+c))^(3/2)+7/
32*arctanh(1/2*(a+a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/a^(3/2)/d*2^(1/2)-7/
16/a/d/(a+a*sin(d*x+c))^(1/2)+7/20*sec(d*x+c)^2/a/d/(a+a*sin(d*x+c))^(1/2)
```

Rubi [A]

time = 0.14, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2760, 2766, 2746, 53, 65, 212}

$$\frac{7\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{16\sqrt{2}\,a^{3/2}d}-\frac{7}{16ad\sqrt{a\sin(c+dx)+a}}-\frac{7}{24d(a\sin(c+dx)+a)^{3/2}}+\frac{7\sec^2(c+dx)}{20ad\sqrt{a\sin(c+dx)+a}}-\frac{\sec^2(c+dx)}{5d(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (7*ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(16*Sqrt[2]*a^(3/2)
*d) - 7/(24*d*(a + a*Sin[c + d*x])^(3/2)) - Sec[c + d*x]^2/(5*d*(a + a*Sin[
c + d*x])^(3/2)) - 7/(16*a*d*Sqrt[a + a*Sin[c + d*x]]) + (7*Sec[c + d*x]^2)
/(20*a*d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2760

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*(2*m + p + 1))), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2766

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(a*f*g*(p + 1)*S
qrt[a + b*Sin[e + f*x]])), x] + Dist[a*((2*p + 1)/(2*g^2*(p + 1))), Int[(g*
Cos[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e,
 f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx = -\frac{\sec^2(c+dx)}{5d(a+a\sin(c+dx))^{3/2}} + \frac{7\int \frac{\sec^3(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx}{10a}$$

$$= -\frac{\sec^2(c+dx)}{5d(a+a\sin(c+dx))^{3/2}} + \frac{7\sec^2(c+dx)}{20ad\sqrt{a+a\sin(c+dx)}} + \frac{7}{8}\int \frac{\sec(c+ …}{(a+a\sin(c+ …}$$

$$= -\frac{\sec^2(c+dx)}{5d(a+a\sin(c+dx))^{3/2}} + \frac{7\sec^2(c+dx)}{20ad\sqrt{a+a\sin(c+dx)}} + \frac{(7a)\mathrm{Subst}\left(\int \frac{…}{(a-x) …}\right. …}{…}$$

$$= -\frac{7}{24d(a+a\sin(c+dx))^{3/2}} - \frac{\sec^2(c+dx)}{5d(a+a\sin(c+dx))^{3/2}} + \frac{7\sec^2(c+dx …}{20ad\sqrt{a+a\sin(c …}}$$

$$= -\frac{7}{24d(a+a\sin(c+dx))^{3/2}} - \frac{\sec^2(c+dx)}{5d(a+a\sin(c+dx))^{3/2}} - \frac{7}{16ad\sqrt{a+a\sin(c …}}$$

$$= -\frac{7}{24d(a+a\sin(c+dx))^{3/2}} - \frac{\sec^2(c+dx)}{5d(a+a\sin(c+dx))^{3/2}} - \frac{7}{16ad\sqrt{a+a\sin(c …}}$$

$$= \frac{7\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{16\sqrt{2}\,a^{3/2}d} - \frac{7}{24d(a+a\sin(c+dx))^{3/2}} - \frac{\sec^2(…}{5d(a+a\sin …}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.05, size = 42, normalized size = 0.28

$$-\frac{a\,{}_2F_1\left(-\frac{5}{2},2;-\frac{3}{2};\frac{1}{2}(1+\sin(c+dx))\right)}{10d(a+a\sin(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -1/10*(a*Hypergeometric2F1[-5/2, 2, -3/2, (1 + Sin[c + d*x])/2])/(d*(a + a*
Sin[c + d*x])^(5/2))
```

Maple [A]
time = 0.62, size = 124, normalized size = 0.83

method	result

default	$\frac{2a^3\left(-\frac{\frac{\sqrt{a+a\sin(dx+c)}}{2a\sin(dx+c)-2a}-\frac{7\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+a\sin(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{4\sqrt{a}}}{16a^4}-\frac{3}{16a^4\sqrt{a+a\sin(dx+c)}}-\frac{}{12a^3}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*a^3*(-1/16/a^4*(1/2*(a+a*sin(d*x+c))^(1/2)/(a*sin(d*x+c)-a)-7/4*2^(1/2)/a
^(1/2)*arctanh(1/2*(a+a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2)))-3/16/a^4/(a+a*s
in(d*x+c))^(1/2)-1/12/a^3/(a+a*sin(d*x+c))^(3/2)-1/20/a^2/(a+a*sin(d*x+c))^
(5/2))/d
```

Maxima [A]
time = 0.52, size = 146, normalized size = 0.97

$$-\frac{\frac{105\sqrt{2}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{a\sin(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{a\sin(dx+c)+a}}\right)}{\sqrt{a}}+\frac{4\left(105\,(a\sin(dx+c)+a)^3-140\,(a\sin(dx+c)+a)^2a-56\,(a\sin(dx+c)+a)a^2-48\,a^3\right)}{(a\sin(dx+c)+a)^{\frac{7}{2}}-2\,(a\sin(dx+c)+a)^{\frac{5}{2}}a}}{960\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/960*(105*sqrt(2)*log(-(sqrt(2)*sqrt(a) - sqrt(a*sin(d*x + c) + a))/(sqrt
(2)*sqrt(a) + sqrt(a*sin(d*x + c) + a)))/sqrt(a) + 4*(105*(a*sin(d*x + c) +
 a)^3 - 140*(a*sin(d*x + c) + a)^2*a - 56*(a*sin(d*x + c) + a)*a^2 - 48*a^3
)/((a*sin(d*x + c) + a)^(7/2) - 2*(a*sin(d*x + c) + a)^(5/2)*a))/(a*d)
```

Fricas [A]
time = 0.38, size = 187, normalized size = 1.25

$$\frac{105\sqrt{2}\left(\cos(dx+c)^4-2\cos(dx+c)^2\sin(dx+c)-2\cos(dx+c)^2\right)\sqrt{a}\log\left(-\frac{a\sin(dx+c)+2\sqrt{2}\sqrt{a\sin(dx+c)+a}\sqrt{a}+3a}{\sin(dx+c)-1}\right)+4\left(175\cos(dx+c)^2+21\left(5\cos(dx+c)^2-4\right)\sin(dx+c)-36\right)\sqrt{a\sin(dx+c)+a}}{960\left(a^2d\cos(dx+c)^4-2a^2d\cos(dx+c)^2\sin(dx+c)-2a^2d\cos(dx+c)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/960*(105*sqrt(2)*(cos(d*x + c)^4 - 2*cos(d*x + c)^2*sin(d*x + c) - 2*cos(
d*x + c)^2)*sqrt(a)*log(-(a*sin(d*x + c) + 2*sqrt(2)*sqrt(a*sin(d*x + c) +
a)*sqrt(a) + 3*a)/(sin(d*x + c) - 1)) + 4*(175*cos(d*x + c)^2 + 21*(5*cos(d
*x + c)^2 - 4)*sin(d*x + c) - 36)*sqrt(a*sin(d*x + c) + a))/(a^2*d*cos(d*x
+ c)^4 - 2*a^2*d*cos(d*x + c)^2*sin(d*x + c) - 2*a^2*d*cos(d*x + c)^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(c+dx)}{(a(\sin(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+a*sin(d*x+c))**(3/2),x)
[Out] Integral(sec(c + d*x)**3/(a*(sin(c + d*x) + 1))**(3/2), x)
```

Giac [**A**]
time = 6.87, size = 211, normalized size = 1.41

$$\frac{\sqrt{a}\left(\frac{105\sqrt{2}\log\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{105\sqrt{2}\log\left(-\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{30\sqrt{2}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{4\sqrt{2}\left(45\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^4+10\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^2+3\right)}{a^2\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^5\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}\right)}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
[Out] 1/960*sqrt(a)*(105*sqrt(2)*log(cos(-1/4*pi + 1/2*d*x + 1/2*c) + 1)/(a^2*sgn
(cos(-1/4*pi + 1/2*d*x + 1/2*c))) - 105*sqrt(2)*log(-cos(-1/4*pi + 1/2*d*x
+ 1/2*c) + 1)/(a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))) - 30*sqrt(2)*cos(-1
/4*pi + 1/2*d*x + 1/2*c)/((cos(-1/4*pi + 1/2*d*x + 1/2*c)^2 - 1)*a^2*sgn(co
s(-1/4*pi + 1/2*d*x + 1/2*c))) - 4*sqrt(2)*(45*cos(-1/4*pi + 1/2*d*x + 1/2*
c)^4 + 10*cos(-1/4*pi + 1/2*d*x + 1/2*c)^2 + 3)/(a^2*cos(-1/4*pi + 1/2*d*x
+ 1/2*c)^5*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))))/d
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(c+d\,x)^3\,(a+a\,\sin(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + a*sin(c + d*x))^(3/2)),x)
[Out] int(1/(cos(c + d*x)^3*(a + a*sin(c + d*x))^(3/2)), x)
```

3.179 $\int \frac{\sec^4(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=195

$$-\frac{105\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{256\sqrt{2}\,a^{3/2}d}-\frac{105\cos(c+dx)}{256d(a+a\sin(c+dx))^{3/2}}-\frac{7\sec(c+dx)}{32d(a+a\sin(c+dx))^{3/2}}-\frac{\sec}{6d(a+a}$$

```
[Out] -105/256*cos(d*x+c)/d/(a+a*sin(d*x+c))^(3/2)-7/32*sec(d*x+c)/d/(a+a*sin(d*x
+c))^(3/2)-1/6*sec(d*x+c)^3/d/(a+a*sin(d*x+c))^(3/2)-105/512*arctanh(1/2*co
s(d*x+c)*a^(1/2)*2^(1/2)/(a+a*sin(d*x+c))^(1/2))*2^(1/2)/a^(3/2)/d+35/64*se
c(d*x+c)/a/d/(a+a*sin(d*x+c))^(1/2)+1/4*sec(d*x+c)^3/a/d/(a+a*sin(d*x+c))^(
1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$,
Rules used = {2760, 2766, 2729, 2728, 212}

$$-\frac{105\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{256\sqrt{2}\,a^{3/2}d}-\frac{105\cos(c+dx)}{256d(a\sin(c+dx)+a)^{3/2}}+\frac{\sec^3(c+dx)}{4ad\sqrt{a\sin(c+dx)+a}}-\frac{\sec^3(c+dx)}{6d(a\sin(c+dx)+a)^{3/2}}+\frac{35\sec(c+dx)}{64ad\sqrt{a\sin(c+dx)+a}}-\frac{7\sec(c+dx)}{32d(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-105*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/(
256*Sqrt[2]*a^(3/2)*d) - (105*Cos[c + d*x])/(256*d*(a + a*Sin[c + d*x])^(3/
2)) - (7*Sec[c + d*x])/(32*d*(a + a*Sin[c + d*x])^(3/2)) - Sec[c + d*x]^3/(
6*d*(a + a*Sin[c + d*x])^(3/2)) + (35*Sec[c + d*x])/(64*a*d*Sqrt[a + a*Sin[
c + d*x]]) + Sec[c + d*x]^3/(4*a*d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
```

```
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2760

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*(2*m + p + 1))), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2766

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(a*f*g*(p + 1)*S
qrt[a + b*Sin[e + f*x]])), x] + Dist[a*((2*p + 1)/(2*g^2*(p + 1))), Int[(g*
Cos[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e,
 f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx = -\frac{\sec^3(c+dx)}{6d(a+a\sin(c+dx))^{3/2}} + \frac{3\int \frac{\sec^4(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx}{4a}$$

$$= -\frac{\sec^3(c+dx)}{6d(a+a\sin(c+dx))^{3/2}} + \frac{\sec^3(c+dx)}{4ad\sqrt{a+a\sin(c+dx)}} + \frac{7}{8}\int \frac{\sec^2(c+d}{(a+a\sin(c+}$$

$$= -\frac{7\sec(c+dx)}{32d(a+a\sin(c+dx))^{3/2}} - \frac{\sec^3(c+dx)}{6d(a+a\sin(c+dx))^{3/2}} + \frac{\sec^3(c+dx)}{4ad\sqrt{a+a\sin(c+}}$$

$$= -\frac{7\sec(c+dx)}{32d(a+a\sin(c+dx))^{3/2}} - \frac{\sec^3(c+dx)}{6d(a+a\sin(c+dx))^{3/2}} + \frac{35\sec(c+dx}{64ad\sqrt{a+a\sin(c}}$$

$$= -\frac{105\cos(c+dx)}{256d(a+a\sin(c+dx))^{3/2}} - \frac{7\sec(c+dx)}{32d(a+a\sin(c+dx))^{3/2}} - \frac{\sec^3(c+d}{6d(a+a\sin(c+}$$

$$= -\frac{105\cos(c+dx)}{256d(a+a\sin(c+dx))^{3/2}} - \frac{7\sec(c+dx)}{32d(a+a\sin(c+dx))^{3/2}} - \frac{\sec^3(c+d}{6d(a+a\sin(c+}$$

$$= -\frac{105\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{256\sqrt{2}\,a^{3/2}d} - \frac{105\cos(c+dx)}{256d(a+a\sin(c+dx))^{3/2}} - 32$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.23, size = 334, normalized size = 1.71

$$\frac{-68+\frac{64\sin\left(\frac{1}{2}(c+dx)\right)}{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3}-\frac{32}{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}+\frac{136\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)}+246\sin\left(\frac{1}{2}(c+dx)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)-123\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2+(315+315i)(-1)^{3/4}\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(c+dx)\right)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3+\frac{32\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3}+\frac{192\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)}}{768d(a(1+\sin(c+dx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-68 + (64*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3 - 32/(
Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + (136*Sin[(c + d*x)/2])/(Cos[(c + d
*x)/2] + Sin[(c + d*x)/2]) + 246*Sin[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(
c + d*x)/2]) - 123*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + (315 + 315*I)*
(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/4])]*(Cos[(c
+ d*x)/2] + Sin[(c + d*x)/2])^3 + (32*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])
^3)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3 + (192*(Cos[(c + d*x)/2] + Sin[
(c + d*x)/2])^3)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]))/(768*d*(a*(1 + Sin[
c + d*x]))^(3/2))
```

Maple [A]
time = 0.62, size = 289, normalized size = 1.48

method	result
default	$\frac{\left(-840a^{\frac{9}{2}}-315(a-a\sin(dx+c))^{\frac{3}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)a^3\right)\sin(dx+c)\left(\cos^2(dx+c)\right)+\left(-384a^{\frac{9}{2}}+126\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+a*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/1536/a^(11/2)*((-840*a^(9/2)-315*(a-a*sin(d*x+c))^(3/2)*2^(1/2)*arctanh(1
/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^3)*sin(d*x+c)*cos(d*x+c)^2+(-3
84*a^(9/2)+1260*(a-a*sin(d*x+c))^(3/2)*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))
^(1/2)*2^(1/2)/a^(1/2))*a^3)*sin(d*x+c)+630*a^(9/2)*cos(d*x+c)^4+(-504*a^(9
/2)-945*(a-a*sin(d*x+c))^(3/2)*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2
^(1/2)/a^(1/2))*a^3)*cos(d*x+c)^2-128*a^(9/2)+1260*(a-a*sin(d*x+c))^(3/2)*2
^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^3)/(sin(d*x+c)
-1)/(1+sin(d*x+c))^2/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^4/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")

[Out] integrate(sec(d*x + c)^4/(a*sin(d*x + c) + a)^(3/2), x)

Fricas [A]
time = 0.40, size = 270, normalized size = 1.38

$$\frac{315\sqrt{2}\left(\cos\left(dx+c\right)^{5}-2\cos\left(dx+c\right)^{3}\sin\left(dx+c\right)-2\cos\left(dx+c\right)^{3}\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^{2}-2\sqrt{2}\sqrt{a\sin\left(dx+c\right)+a}\sqrt{a}\left(\cos(dx+c)-\sin(dx+c)+1\right)+3a\cos(dx+c)-\left(a\cos(dx+c)-2a\right)\sin(dx+c)+2a}{\cos(dx+c)^{2}-\left(\cos(dx+c)+2\right)\sin(dx+c)-\cos(dx+c)-2}\right)+4\left(315\cos\left(dx+c\right)^{4}-252\cos\left(dx+c\right)^{2}-12\left(35\cos\left(dx+c\right)^{2}+16\right)\sin\left(dx+c\right)-64\right)\sqrt{a\sin\left(dx+c\right)+a}}{3072\left(a^{2}d\cos\left(dx+c\right)^{5}-2a^{2}d\cos\left(dx+c\right)^{3}\sin\left(dx+c\right)-2a^{2}d\cos\left(dx+c\right)^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^4/(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")

[Out] 1/3072*(315*sqrt(2)*(cos(d*x + c)^5 - 2*cos(d*x + c)^3*sin(d*x + c) - 2*cos(d*x + c)^3)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - sin(d*x + c) + 1) + 3*a*cos(d*x + c) - (a*cos(d*x + c) - 2*a)*sin(d*x + c) + 2*a)/(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2)) + 4*(315*cos(d*x + c)^4 - 252*cos(d*x + c)^2 - 12*(35*cos(d*x + c)^2 + 16)*sin(d*x + c) - 64)*sqrt(a*sin(d*x + c) + a))/(a^2*d*cos(d*x + c)^5 - 2*a^2*d*cos(d*x + c)^3*sin(d*x + c) - 2*a^2*d*cos(d*x + c)^3)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{4}\left(c+dx\right)}{\left(a\left(\sin\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**4/(a+a*sin(d*x+c))**(3/2),x)

[Out] Integral(sec(c + d*x)**4/(a*(sin(c + d*x) + 1))**(3/2), x)

Giac [A]
time = 6.82, size = 220, normalized size = 1.13

$$\frac{\sqrt{a}\left(\frac{315\sqrt{2}\log\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{315\sqrt{2}\log\left(-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{2\left(315\sqrt{2}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{8}-840\sqrt{2}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{6}+693\sqrt{2}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{4}-144\sqrt{2}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-16\sqrt{2}\right)}{\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{3}-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)^{3}a^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}\right)}{3072\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^4/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")

[Out] 1/3072*sqrt(a)*(315*sqrt(2)*log(sin(-1/4*pi + 1/2*d*x + 1/2*c) + 1)/(a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))) - 315*sqrt(2)*log(-sin(-1/4*pi + 1/2*d*x + 1/2*c) + 1)/(a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))) - 2*(315*sqrt(2)*sin(-1/4*pi + 1/2*d*x + 1/2*c)^8 - 840*sqrt(2)*sin(-1/4*pi + 1/2*d*x + 1/2*c)^6 + 693*sqrt(2)*sin(-1/4*pi + 1/2*d*x + 1/2*c)^4 - 144*sqrt(2)*sin(-1/4*pi + 1/2*d*x + 1/2*c)^2 - 16*sqrt(2))/((sin(-1/4*pi + 1/2*d*x + 1/2*c)^3 - s

```
in(-1/4*pi + 1/2*d*x + 1/2*c))^3*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))))/
d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(c+dx)^4 (a+a\sin(c+dx))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a + a*sin(c + d*x))^(3/2)),x)
[Out] int(1/(cos(c + d*x)^4*(a + a*sin(c + d*x))^(3/2)), x)
```

3.180 $\int \frac{\sec^5(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=211

$$\frac{99\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{256\sqrt{2}\,a^{3/2}d}-\frac{33}{128d(a+a\sin(c+dx))^{3/2}}-\frac{99\sec^2(c+dx)}{560d(a+a\sin(c+dx))^{3/2}}-\frac{\sec^4(c-}{7d(a+a\sin(}$$

```
[Out] -33/128/d/(a+a*sin(d*x+c))^(3/2)-99/560*sec(d*x+c)^2/d/(a+a*sin(d*x+c))^(3/
2)-1/7*sec(d*x+c)^4/d/(a+a*sin(d*x+c))^(3/2)+99/512*arctanh(1/2*(a+a*sin(d*
x+c))^(1/2)*2^(1/2)/a^(1/2))/a^(3/2)/d*2^(1/2)-99/256/a/d/(a+a*sin(d*x+c))^
(1/2)+99/320*sec(d*x+c)^2/a/d/(a+a*sin(d*x+c))^(1/2)+11/56*sec(d*x+c)^4/a/d
/(a+a*sin(d*x+c))^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2760, 2766, 2746, 53, 65, 212}

$$\frac{99\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{256\sqrt{2}\,a^{3/2}d}-\frac{99}{256ad\sqrt{a\sin(c+dx)+a}}-\frac{33}{128d(a\sin(c+dx)+a)^{3/2}}+\frac{11\sec^4(c+dx)}{56ad\sqrt{a\sin(c+dx)+a}}-\frac{\sec^4(c+dx)}{7d(a\sin(c+dx)+a)^{3/2}}+\frac{99\sec^2(c+dx)}{320ad\sqrt{a\sin(c+dx)+a}}-\frac{99\sec^2(c+dx)}{560d(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (99*ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(256*Sqrt[2]*a^(3/
2)*d) - 33/(128*d*(a + a*Sin[c + d*x])^(3/2)) - (99*Sec[c + d*x]^2)/(560*d*
(a + a*Sin[c + d*x])^(3/2)) - Sec[c + d*x]^4/(7*d*(a + a*Sin[c + d*x])^(3/2
)) - 99/(256*a*d*Sqrt[a + a*Sin[c + d*x]]) + (99*Sec[c + d*x]^2)/(320*a*d*S
qrt[a + a*Sin[c + d*x]]) + (11*Sec[c + d*x]^4)/(56*a*d*Sqrt[a + a*Sin[c + d
*x]])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2760

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*(2*m + p + 1))), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2766

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(a*f*g*(p + 1)*S
qrt[a + b*Sin[e + f*x]])), x] + Dist[a*((2*p + 1)/(2*g^2*(p + 1))), Int[(g*
Cos[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e,
 f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx = -\frac{\sec^4(c+dx)}{7d(a+a\sin(c+dx))^{3/2}} + \frac{11\int \frac{\sec^5(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx}{14a}$$

$$= -\frac{\sec^4(c+dx)}{7d(a+a\sin(c+dx))^{3/2}} + \frac{11\sec^4(c+dx)}{56ad\sqrt{a+a\sin(c+dx)}} + \frac{99}{112}\int \frac{\sec^3(c \cdots}{(a+a\sin(\cdots}$$

$$= -\frac{99\sec^2(c+dx)}{560d(a+a\sin(c+dx))^{3/2}} - \frac{\sec^4(c+dx)}{7d(a+a\sin(c+dx))^{3/2}} + \frac{11\sec^4(c+\cdots}{56ad\sqrt{a+a\sin(\cdots}}$$

$$= -\frac{99\sec^2(c+dx)}{560d(a+a\sin(c+dx))^{3/2}} - \frac{\sec^4(c+dx)}{7d(a+a\sin(c+dx))^{3/2}} + \frac{99\sec^2(c+\cdots}{320ad\sqrt{a+a\sin\cdots}}$$

$$= -\frac{99\sec^2(c+dx)}{560d(a+a\sin(c+dx))^{3/2}} - \frac{\sec^4(c+dx)}{7d(a+a\sin(c+dx))^{3/2}} + \frac{99\sec^2(c+\cdots}{320ad\sqrt{a+a\sin\cdots}}$$

$$= -\frac{33}{128d(a+a\sin(c+dx))^{3/2}} - \frac{99\sec^2(c+dx)}{560d(a+a\sin(c+dx))^{3/2}} - \frac{\sec^4(c+\cdots}{7d(a+a\sin(c\cdots}$$

$$= -\frac{33}{128d(a+a\sin(c+dx))^{3/2}} - \frac{99\sec^2(c+dx)}{560d(a+a\sin(c+dx))^{3/2}} - \frac{\sec^4(c+\cdots}{7d(a+a\sin(c\cdots}$$

$$= -\frac{33}{128d(a+a\sin(c+dx))^{3/2}} - \frac{99\sec^2(c+dx)}{560d(a+a\sin(c+dx))^{3/2}} - \frac{\sec^4(c+\cdots}{7d(a+a\sin(c\cdots}$$

$$= \frac{99\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{256\sqrt{2}\,a^{3/2}d} - \frac{33}{128d(a+a\sin(c+dx))^{3/2}} - \frac{99\,\cdots}{560d(a+\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.06, size = 44, normalized size = 0.21

$$-\frac{a^2\,{}_2F_1\left(-\frac{7}{2},3;-\frac{5}{2};\frac{1}{2}(1+\sin(c+dx))\right)}{28d(a+a\sin(c+dx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -1/28*(a^2*Hypergeometric2F1[-7/2, 3, -5/2, (1 + Sin[c + d*x])/2])/(d*(a + a*Sin[c + d*x])^(7/2))
```

Maple [A]
time = 0.85, size = 152, normalized size = 0.72

method	result
default	$-\frac{2a^5\left(\frac{\frac{\sqrt{a+a\sin(dx+c)}\,a(19\sin(dx+c)-23)}{16(a\sin(dx+c)-a)^2}-\frac{99\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+a\sin(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{32\sqrt{a}}}{32a^6}+\frac{5}{32a^6\sqrt{a+a\sin}}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+a*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*a^5*(1/32/a^6*(1/16*(a+a*sin(d*x+c))^(1/2)*a*(19*sin(d*x+c)-23)/(a*sin(d
*x+c)-a)^2-99/32*2^(1/2)/a^(1/2)*arctanh(1/2*(a+a*sin(d*x+c))^(1/2)*2^(1/2)
/a^(1/2)))+5/32/a^6/(a+a*sin(d*x+c))^(1/2)+1/16/a^5/(a+a*sin(d*x+c))^(3/2)+
3/80/a^4/(a+a*sin(d*x+c))^(5/2)+1/56/a^3/(a+a*sin(d*x+c))^(7/2))/d
```

Maxima [A]
time = 0.55, size = 197, normalized size = 0.93

$$-\frac{\frac{3465\sqrt{2}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{a\sin(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{a\sin(dx+c)+a}}\right)}{\sqrt{a}}+\frac{4\left(3465\,(a\sin(dx+c)+a)^5-11550\,(a\sin(dx+c)+a)^4a+7392\,(a\sin(dx+c)+a)^3a^2+2112\,(a\sin(dx+c)+a)^2a^3+1408\,(a\sin(dx+c)+a)a^4+1280\,a^5\right)}{(a\sin(dx+c)+a)^{\frac{11}{2}}-4\,(a\sin(dx+c)+a)^{\frac{9}{2}}a+4\,(a\sin(dx+c)+a)^{\frac{7}{2}}a^2}}{35840\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/35840*(3465*sqrt(2)*log(-(sqrt(2)*sqrt(a) - sqrt(a*sin(d*x + c) + a))/(s
qrt(2)*sqrt(a) + sqrt(a*sin(d*x + c) + a)))/sqrt(a) + 4*(3465*(a*sin(d*x +
c) + a)^5 - 11550*(a*sin(d*x + c) + a)^4*a + 7392*(a*sin(d*x + c) + a)^3*a^
2 + 2112*(a*sin(d*x + c) + a)^2*a^3 + 1408*(a*sin(d*x + c) + a)*a^4 + 1280*
a^5)/((a*sin(d*x + c) + a)^(11/2) - 4*(a*sin(d*x + c) + a)^(9/2)*a + 4*(a*s
in(d*x + c) + a)^(7/2)*a^2))/(a*d)
```

Fricas [A]
time = 0.39, size = 207, normalized size = 0.98

$$\frac{3465\sqrt{2}\left(\cos(dx+c)^6-2\cos(dx+c)^4\sin(dx+c)-2\cos(dx+c)^4\right)\sqrt{a}\log\left(-\frac{a\sin(dx+c)+2\sqrt{2}\sqrt{a\sin(dx+c)+a}\,\sqrt{a}+3a}{\sin(dx+c)-1}\right)+4\left(5775\cos(dx+c)^4-1188\cos(dx+c)^2+11\left(315\cos(dx+c)^4-252\cos(dx+c)^2-160\right)\sin(dx+c)-480\right)\sqrt{a\sin(dx+c)+a}}{35840\left(a^2d\cos(dx+c)^6-2a^2d\cos(dx+c)^4\sin(dx+c)-2a^2d\cos(dx+c)^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/35840*(3465*sqrt(2)*(cos(d*x + c)^6 - 2*cos(d*x + c)^4*sin(d*x + c) - 2*c
os(d*x + c)^4)*sqrt(a)*log(-(a*sin(d*x + c) + 2*sqrt(2)*sqrt(a*sin(d*x + c)
 + a)*sqrt(a) + 3*a)/(sin(d*x + c) - 1)) + 4*(5775*cos(d*x + c)^4 - 1188*co
```

```
s(d*x + c)^2 + 11*(315*cos(d*x + c)^4 - 252*cos(d*x + c)^2 - 160)*sin(d*x +
 c) - 480)*sqrt(a*sin(d*x + c) + a))/(a^2*d*cos(d*x + c)^6 - 2*a^2*d*cos(d*
x + c)^4*sin(d*x + c) - 2*a^2*d*cos(d*x + c)^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^5(c+dx)}{(a(\sin(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+a*sin(d*x+c))**(3/2),x)
[Out] Integral(sec(c + d*x)**5/(a*(sin(c + d*x) + 1))**(3/2), x)
```

Giac [A]
time = 4.24, size = 249, normalized size = 1.18

$$\frac{\sqrt{a}\left(\frac{3465\sqrt{2}\log\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^2\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{3465\sqrt{2}\log\left(-\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^2\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{70\left(19\sqrt{2}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^3-21\sqrt{2}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^2a^2\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{16\sqrt{2}\left(350\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^6+70\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^4+21\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^2+5\right)}{a^2\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^7\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}\right)}{35840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
[Out] 1/35840*sqrt(a)*(3465*sqrt(2)*log(cos(-1/4*pi + 1/2*d*x + 1/2*c) + 1)/(a^2*
sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))) - 3465*sqrt(2)*log(-cos(-1/4*pi + 1/2*
d*x + 1/2*c) + 1)/(a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))) - 70*(19*sqrt(2
)*cos(-1/4*pi + 1/2*d*x + 1/2*c)^3 - 21*sqrt(2)*cos(-1/4*pi + 1/2*d*x + 1/2
*c))/((cos(-1/4*pi + 1/2*d*x + 1/2*c)^2 - 1)^2*a^2*sgn(cos(-1/4*pi + 1/2*d*
x + 1/2*c))) - 16*sqrt(2)*(350*cos(-1/4*pi + 1/2*d*x + 1/2*c)^6 + 70*cos(-1
/4*pi + 1/2*d*x + 1/2*c)^4 + 21*cos(-1/4*pi + 1/2*d*x + 1/2*c)^2 + 5)/(a^2*
cos(-1/4*pi + 1/2*d*x + 1/2*c)^7*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos(c+d\,x)^5\,(a+a\,\sin(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^5*(a + a*sin(c + d*x))^(3/2)),x)
[Out] int(1/(cos(c + d*x)^5*(a + a*sin(c + d*x))^(3/2)), x)
```

3.181 $\int \frac{\sec^6(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=256

$$-\frac{3003\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{8192\sqrt{2}\,a^{3/2}d}-\frac{3003\cos(c+dx)}{8192d(a+a\sin(c+dx))^{3/2}}-\frac{1001\sec(c+dx)}{5120d(a+a\sin(c+dx))^{3/2}}-960\ldots$$

```
[Out] -3003/8192*cos(d*x+c)/d/(a+a*sin(d*x+c))^(3/2)-1001/5120*sec(d*x+c)/d/(a+a*
sin(d*x+c))^(3/2)-143/960*sec(d*x+c)^3/d/(a+a*sin(d*x+c))^(3/2)-1/8*sec(d*x
+c)^5/d/(a+a*sin(d*x+c))^(3/2)-3003/16384*arctanh(1/2*cos(d*x+c)*a^(1/2)*2^
(1/2)/(a+a*sin(d*x+c))^(1/2))*2^(1/2)/a^(3/2)/d+1001/2048*sec(d*x+c)/a/d/(a
+a*sin(d*x+c))^(1/2)+143/640*sec(d*x+c)^3/a/d/(a+a*sin(d*x+c))^(1/2)+13/80*
sec(d*x+c)^5/a/d/(a+a*sin(d*x+c))^(1/2)
```

Rubi [A]
time = 0.30, antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2760, 2766, 2729, 2728, 212}

$$-\frac{3003\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{8192\sqrt{2}\,a^{3/2}d}-\frac{3003\cos(c+dx)}{8192d(a\sin(c+dx)+a)^{3/2}}+\frac{13\sec^5(c+dx)}{80ad\sqrt{a\sin(c+dx)+a}}-\frac{\sec^5(c+dx)}{8d(a\sin(c+dx)+a)^{3/2}}+\frac{143\sec^3(c+dx)}{640ad\sqrt{a\sin(c+dx)+a}}-\frac{143\sec^3(c+dx)}{960d(a\sin(c+dx)+a)^{3/2}}+\frac{1001\sec(c+dx)}{2048ad\sqrt{a\sin(c+dx)+a}}-\frac{1001\sec(c+dx)}{5120d(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-3003*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/
(8192*Sqrt[2]*a^(3/2)*d) - (3003*Cos[c + d*x])/(8192*d*(a + a*Sin[c + d*x])
^(3/2)) - (1001*Sec[c + d*x])/(5120*d*(a + a*Sin[c + d*x])^(3/2)) - (143*Se
c[c + d*x]^3)/(960*d*(a + a*Sin[c + d*x])^(3/2)) - Sec[c + d*x]^5/(8*d*(a +
 a*Sin[c + d*x])^(3/2)) + (1001*Sec[c + d*x])/(2048*a*d*Sqrt[a + a*Sin[c +
d*x]]) + (143*Sec[c + d*x]^3)/(640*a*d*Sqrt[a + a*Sin[c + d*x]]) + (13*Sec[
c + d*x]^5)/(80*a*d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2760

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*(2*m + p + 1))), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2766

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(a*f*g*(p + 1)*S
qrt[a + b*Sin[e + f*x]])), x] + Dist[a*((2*p + 1)/(2*g^2*(p + 1))), Int[(g*
Cos[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e,
 f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{\sec^6(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx = -\frac{\sec^5(c+dx)}{8d(a+a\sin(c+dx))^{3/2}} + \frac{13\int \frac{\sec^6(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx}{16a}$$

$$= -\frac{\sec^5(c+dx)}{8d(a+a\sin(c+dx))^{3/2}} + \frac{13\sec^5(c+dx)}{80ad\sqrt{a+a\sin(c+dx)}} + \frac{143}{160}\int \frac{\sec^4(c+ \cdots}{(a+a\sin(c \cdots}$$

$$= -\frac{143\sec^3(c+dx)}{960d(a+a\sin(c+dx))^{3/2}} - \frac{\sec^5(c+dx)}{8d(a+a\sin(c+dx))^{3/2}} + \frac{13\sec^5(c+dx \cdots}{80ad\sqrt{a+a\sin(c \cdots}}$$

$$= -\frac{143\sec^3(c+dx)}{960d(a+a\sin(c+dx))^{3/2}} - \frac{\sec^5(c+dx)}{8d(a+a\sin(c+dx))^{3/2}} + \frac{143\sec^3(c+ \cdots}{640ad\sqrt{a+a\sin(\cdots}}$$

$$= -\frac{1001\sec(c+dx)}{5120d(a+a\sin(c+dx))^{3/2}} - \frac{143\sec^3(c+dx)}{960d(a+a\sin(c+dx))^{3/2}} - \frac{\sec^5(c+ \cdots}{8d(a+a\sin(c \cdots}$$

$$= -\frac{1001\sec(c+dx)}{5120d(a+a\sin(c+dx))^{3/2}} - \frac{143\sec^3(c+dx)}{960d(a+a\sin(c+dx))^{3/2}} - \frac{\sec^5(c+ \cdots}{8d(a+a\sin(c \cdots}$$

$$= -\frac{3003\cos(c+dx)}{8192d(a+a\sin(c+dx))^{3/2}} - \frac{1001\sec(c+dx)}{5120d(a+a\sin(c+dx))^{3/2}} - \frac{143\sec^3(\cdots}{960d(a+a\sin \cdots}$$

$$= -\frac{3003\cos(c+dx)}{8192d(a+a\sin(c+dx))^{3/2}} - \frac{1001\sec(c+dx)}{5120d(a+a\sin(c+dx))^{3/2}} - \frac{143\sec^3(\cdots}{960d(a+a\sin \cdots}$$

$$= -\frac{3003\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{8192\sqrt{2}\,a^{3/2}d} - \frac{3003\cos(c+dx)}{8192d(a+a\sin(c+dx))^{3/2}} - \cdots$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.91, size = 444, normalized size = 1.73

$$\frac{-8860 + \frac{3840\sin(\frac{1}{2}(c+dx))}{(\cos(\frac{1}{2}(c+dx))+\sin(\frac{1}{2}(c+dx)))^5} - \frac{1920}{(\cos(\frac{1}{2}(c+dx))+\sin(\frac{1}{2}(c+dx)))^4} + \frac{9920\sin(\frac{1}{2}(c+dx))}{(\cos(\frac{1}{2}(c+dx))+\sin(\frac{1}{2}(c+dx)))^3} - \frac{4960}{(\cos(\frac{1}{2}(c+dx))+\sin(\frac{1}{2}(c+dx)))^2} + \frac{17720\sin(\frac{1}{2}(c+dx))}{\cos(\frac{1}{2}(c+dx))+\sin(\frac{1}{2}(c+dx))} + 32490\sin(\frac{1}{2}(c+dx))\left(\cos(\frac{1}{2}(c+dx))+\sin(\frac{1}{2}(c+dx))\right) - 16245\left(\cos(\frac{1}{2}(c+dx))+\sin(\frac{1}{2}(c+dx))\right)^2 + (45045+45045i)(-1)^{3/4}\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(c+dx)\right)\right)\right)\left(\cos(\frac{1}{2}(c+dx))+\sin(\frac{1}{2}(c+dx))\right)^3 + \frac{1536(\cos(\frac{1}{2}(c+dx))+\sin(\frac{1}{2}(c+dx)))^3}{(\cos(\frac{1}{2}(c+dx))-\sin(\frac{1}{2}(c+dx)))^5} + \frac{6400(\cos(\frac{1}{2}(c+dx))+\sin(\frac{1}{2}(c+dx)))^3}{(\cos(\frac{1}{2}(c+dx))-\sin(\frac{1}{2}(c+dx)))^3} + \frac{28800(\cos(\frac{1}{2}(c+dx))+\sin(\frac{1}{2}(c+dx)))^3}{\cos(\frac{1}{2}(c+dx))-\sin(\frac{1}{2}(c+dx))}}{122880d(a(1+\sin(c+dx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-8860 + (3840*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5 -
1920/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4 + (9920*Sin[(c + d*x)/2])/(Cos
[(c + d*x)/2] + Sin[(c + d*x)/2])^3 - 4960/(Cos[(c + d*x)/2] + Sin[(c + d*x
)/2])^2 + (17720*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) +
32490*Sin[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) - 16245*(Cos[(
```

```
c + d*x)/2] + Sin[(c + d*x)/2])^2 + (45045 + 45045*I)*(-1)^(3/4)*ArcTanh[(1
/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/4])]*(Cos[(c + d*x)/2] + Sin[(c +
d*x)/2])^3 + (1536*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3)/(Cos[(c + d*x)/
2] - Sin[(c + d*x)/2])^5 + (6400*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3)/(
Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3 + (28800*(Cos[(c + d*x)/2] + Sin[(c
+ d*x)/2])^3)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]))/(122880*d*(a*(1 + Sin[
c + d*x]))^(3/2))
```

Maple [A]
time = 0.69, size = 367, normalized size = 1.43

method	result
default	$-\frac{-120120a^{\frac{13}{2}}\sin(dx+c)\left(\cos^{4}(dx+c)\right)+\left(-54912a^{\frac{13}{2}}-180180(a-a\sin(dx+c))^{\frac{5}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6/(a+a*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/245760/a^(15/2)*(-120120*a^(13/2)*sin(d*x+c)*cos(d*x+c)^4+(-54912*a^(13/
2)-180180*(a-a*sin(d*x+c))^(5/2)*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)
*2^(1/2)/a^(1/2))*a^4)*cos(d*x+c)^2*sin(d*x+c)+(-39936*a^(13/2)+360360*(a-a
*sin(d*x+c))^(5/2)*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/
2))*a^4)*sin(d*x+c)+90090*a^(13/2)*cos(d*x+c)^6+9009*(-8*a^(13/2)+5*(a-a*si
n(d*x+c))^(5/2)*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))
*a^4)*cos(d*x+c)^4+(-18304*a^(13/2)-360360*(a-a*sin(d*x+c))^(5/2)*2^(1/2)*a
rctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^4)*cos(d*x+c)^2-9216*a
^(13/2)+360360*(a-a*sin(d*x+c))^(5/2)*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^
(1/2)*2^(1/2)/a^(1/2))*a^4)/(sin(d*x+c)-1)^2/(1+sin(d*x+c))^3/cos(d*x+c)/(a
+a*sin(d*x+c))^(1/2)/d
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A]
time = 0.41, size = 290, normalized size = 1.13

$$\frac{45045\sqrt{2}\left(\cos(dx+c)^{7}-2\cos(dx+c)^{5}\sin(dx+c)-2\cos(dx+c)^{5}\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^{2}-2\sqrt{2}\sqrt{a\sin(dx+c)+a}\sqrt{a}(\cos(dx+c)-\sin(dx+c)+1)+3a\cos(dx+c)-(a\cos(dx+c)-2a)\sin(dx+c)+2a}{\cos(dx+c)^{2}-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2}\right)+4\left(45045\cos(dx+c)^{6}-36036\cos(dx+c)^{4}-9152\cos(dx+c)^{2}-156\left(385\cos(dx+c)^{4}+176\cos(dx+c)^{2}+128\right)\sin(dx+c)-4608\right)\sqrt{a\sin(dx+c)+a}}{491520\left(a^{2}d\cos(dx+c)^{7}-2a^{2}d\cos(dx+c)^{5}\sin(dx+c)-2a^{2}d\cos(dx+c)^{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/491520*(45045*sqrt(2)*(cos(d*x + c)^7 - 2*cos(d*x + c)^5*sin(d*x + c) - 2
*cos(d*x + c)^5)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*sin(d*x
+ c) + a)*sqrt(a)*(cos(d*x + c) - sin(d*x + c) + 1) + 3*a*cos(d*x + c) - (a
*cos(d*x + c) - 2*a)*sin(d*x + c) + 2*a)/(cos(d*x + c)^2 - (cos(d*x + c) +
2)*sin(d*x + c) - cos(d*x + c) - 2)) + 4*(45045*cos(d*x + c)^6 - 36036*cos(
d*x + c)^4 - 9152*cos(d*x + c)^2 - 156*(385*cos(d*x + c)^4 + 176*cos(d*x +
c)^2 + 128)*sin(d*x + c) - 4608)*sqrt(a*sin(d*x + c) + a))/(a^2*d*cos(d*x +
 c)^7 - 2*a^2*d*cos(d*x + c)^5*sin(d*x + c) - 2*a^2*d*cos(d*x + c)^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^6(c+dx)}{(a(\sin(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral(sec(c + d*x)**6/(a*(sin(c + d*x) + 1))**(3/2), x)
```

Giac [A]
time = 4.03, size = 271, normalized size = 1.06

$$\frac{\sqrt{a}\left(\frac{45045\sqrt{2}\log\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{45045\sqrt{2}\log\left(-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{10\left(3249\sqrt{2}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^7-10633\sqrt{2}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^5+11767\sqrt{2}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^3-4431\sqrt{2}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^4a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{256\sqrt{2}\left(225\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^4+25\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^2+3\right)}{a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^5}\right)}{491520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/491520*sqrt(a)*(45045*sqrt(2)*log(sin(-1/4*pi + 1/2*d*x + 1/2*c) + 1)/(a^
2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))) - 45045*sqrt(2)*log(-sin(-1/4*pi + 1
/2*d*x + 1/2*c) + 1)/(a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))) - 10*(3249*s
qrt(2)*sin(-1/4*pi + 1/2*d*x + 1/2*c)^7 - 10633*sqrt(2)*sin(-1/4*pi + 1/2*d
*x + 1/2*c)^5 + 11767*sqrt(2)*sin(-1/4*pi + 1/2*d*x + 1/2*c)^3 - 4431*sqrt(
2)*sin(-1/4*pi + 1/2*d*x + 1/2*c))/((sin(-1/4*pi + 1/2*d*x + 1/2*c)^2 - 1)^
4*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))) - 256*sqrt(2)*(225*sin(-1/4*pi +
 1/2*d*x + 1/2*c)^4 + 25*sin(-1/4*pi + 1/2*d*x + 1/2*c)^2 + 3)/(a^2*sgn(cos
(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*d*x + 1/2*c)^5))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos(c+d\,x)^6\,(a+a\,\sin(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^6*(a + a*sin(c + d*x))^(3/2)),x)
[Out] int(1/(cos(c + d*x)^6*(a + a*sin(c + d*x))^(3/2)), x)
```

3.182 $\int \frac{\cos^{10}(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=95

$$-\frac{64a^3\cos^{11}(c+dx)}{2145d(a+a\sin(c+dx))^{11/2}}-\frac{16a^2\cos^{11}(c+dx)}{195d(a+a\sin(c+dx))^{9/2}}-\frac{2a\cos^{11}(c+dx)}{15d(a+a\sin(c+dx))^{7/2}}$$

```
[Out] -64/2145*a^3*cos(d*x+c)^11/d/(a+a*sin(d*x+c))^(11/2)-16/195*a^2*cos(d*x+c)^
11/d/(a+a*sin(d*x+c))^(9/2)-2/15*a*cos(d*x+c)^11/d/(a+a*sin(d*x+c))^(7/2)
```

Rubi [A]
time = 0.12, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2753, 2752}

$$-\frac{64a^3\cos^{11}(c+dx)}{2145d(a\sin(c+dx)+a)^{11/2}}-\frac{16a^2\cos^{11}(c+dx)}{195d(a\sin(c+dx)+a)^{9/2}}-\frac{2a\cos^{11}(c+dx)}{15d(a\sin(c+dx)+a)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^10/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-64*a^3*Cos[c + d*x]^11)/(2145*d*(a + a*Sin[c + d*x])^(11/2)) - (16*a^2*Co
s[c + d*x]^11)/(195*d*(a + a*Sin[c + d*x])^(9/2)) - (2*a*Cos[c + d*x]^11)/(
15*d*(a + a*Sin[c + d*x])^(7/2))
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2753

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && N
eQ[m + p, 0]
```

Rubi steps

$$\int \frac{\cos^{10}(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx = -\frac{2a\cos^{11}(c+dx)}{15d(a+a\sin(c+dx))^{7/2}} + \frac{1}{15}(8a)\int \frac{\cos^{10}(c+dx)}{(a+a\sin(c+dx))^{7/2}}\,dx$$
$$= -\frac{16a^2\cos^{11}(c+dx)}{195d(a+a\sin(c+dx))^{9/2}} - \frac{2a\cos^{11}(c+dx)}{15d(a+a\sin(c+dx))^{7/2}} + \frac{1}{195}(32a^2)\int \frac{}{(a}$$
$$= -\frac{64a^3\cos^{11}(c+dx)}{2145d(a+a\sin(c+dx))^{11/2}} - \frac{16a^2\cos^{11}(c+dx)}{195d(a+a\sin(c+dx))^{9/2}} - \frac{2a\cos^{11}(}{15d(a+a\sin}$$

Mathematica [A]
time = 0.41, size = 59, normalized size = 0.62

$$-\frac{2\cos^{11}(c+dx)\,(263+374\sin(c+dx)+143\sin^2(c+dx))}{2145d(1+\sin(c+dx))^3(a(1+\sin(c+dx)))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^10/(a + a*Sin[c + d*x])^(5/2),x]
[Out] (-2*Cos[c + d*x]^11*(263 + 374*Sin[c + d*x] + 143*Sin[c + d*x]^2))/(2145*d*
(1 + Sin[c + d*x])^3*(a*(1 + Sin[c + d*x]))^(5/2))
```

Maple [A]
time = 0.43, size = 67, normalized size = 0.71

method	result	size
default	$-\frac{2(1+\sin(dx+c))(\sin(dx+c)-1)^6\,(143(\sin^2(dx+c))+374\sin(dx+c)+263)}{2145a^2\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^10/(a+a*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
[Out] -2/2145/a^2*(1+sin(d*x+c))*(sin(d*x+c)-1)^6*(143*sin(d*x+c)^2+374*sin(d*x+c
)+263)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^10/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate(cos(d*x + c)^10/(a*sin(d*x + c) + a)^(5/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(83) = 166$.
time = 0.36, size = 201, normalized size = 2.12

$$-\frac{2\,\left(143\,\cos\left(dx+c\right)^{8}-341\,\cos\left(dx+c\right)^{7}-736\,\cos\left(dx+c\right)^{6}+28\,\cos\left(dx+c\right)^{5}-40\,\cos\left(dx+c\right)^{4}+64\,\cos\left(dx+c\right)^{3}-128\,\cos\left(dx+c\right)^{2}+\left(143\,\cos\left(dx+c\right)^{7}+484\,\cos\left(dx+c\right)^{6}-252\,\cos\left(dx+c\right)^{5}-280\,\cos\left(dx+c\right)^{4}-320\,\cos\left(dx+c\right)^{3}-384\,\cos\left(dx+c\right)^{2}-512\,\cos\left(dx+c\right)-1024\right)\sin\left(dx+c\right)+512\,\cos\left(dx+c\right)+1024\right)\sqrt{a\sin\left(dx+c\right)+a}}{2145\,\left(a^{3}d\cos\left(dx+c\right)+a^{3}d\sin\left(dx+c\right)+a^{3}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^10/(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/2145*(143*cos(d*x + c)^8 - 341*cos(d*x + c)^7 - 736*cos(d*x + c)^6 + 28*
cos(d*x + c)^5 - 40*cos(d*x + c)^4 + 64*cos(d*x + c)^3 - 128*cos(d*x + c)^2
 + (143*cos(d*x + c)^7 + 484*cos(d*x + c)^6 - 252*cos(d*x + c)^5 - 280*cos(
d*x + c)^4 - 320*cos(d*x + c)^3 - 384*cos(d*x + c)^2 - 512*cos(d*x + c) - 1
024)*sin(d*x + c) + 512*cos(d*x + c) + 1024)*sqrt(a*sin(d*x + c) + a)/(a^3*
d*cos(d*x + c) + a^3*d*sin(d*x + c) + a^3*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**10/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4962 deep
```

Giac [A]
time = 3.05, size = 90, normalized size = 0.95

$$\frac{256\,\left(143\,\sqrt{2}\,\sqrt{a}\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{15}-330\,\sqrt{2}\,\sqrt{a}\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}+195\,\sqrt{2}\,\sqrt{a}\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}\right)}{2145\,a^{3}d\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^10/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 256/2145*(143*sqrt(2)*sqrt(a)*sin(-1/4*pi + 1/2*d*x + 1/2*c)^15 - 330*sqrt(
2)*sqrt(a)*sin(-1/4*pi + 1/2*d*x + 1/2*c)^13 + 195*sqrt(2)*sqrt(a)*sin(-1/4
*pi + 1/2*d*x + 1/2*c)^11)/(a^3*d*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\cos\left(c+d\,x\right)}^{10}}{{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^10/(a + a*sin(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^10/(a + a*sin(c + d*x))^(5/2), x)
```

3.183 $\int \frac{\cos^9(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=121

$$\frac{32(a+a\sin(c+dx))^{5/2}}{5a^5d}-\frac{64(a+a\sin(c+dx))^{7/2}}{7a^6d}+\frac{16(a+a\sin(c+dx))^{9/2}}{3a^7d}-\frac{16(a+a\sin(c+dx))^{11/2}}{11a^8d}+2$$

```
[Out] 32/5*(a+a*sin(d*x+c))^(5/2)/a^5/d-64/7*(a+a*sin(d*x+c))^(7/2)/a^6/d+16/3*(a
+a*sin(d*x+c))^(9/2)/a^7/d-16/11*(a+a*sin(d*x+c))^(11/2)/a^8/d+2/13*(a+a*si
n(d*x+c))^(13/2)/a^9/d
```

Rubi [A]
time = 0.06, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2746, 45}

$$\frac{2(a\sin(c+dx)+a)^{13/2}}{13a^9d}-\frac{16(a\sin(c+dx)+a)^{11/2}}{11a^8d}+\frac{16(a\sin(c+dx)+a)^{9/2}}{3a^7d}-\frac{64(a\sin(c+dx)+a)^{7/2}}{7a^6d}+\frac{32(a\sin(c+dx)+a)^{5/2}}{5a^5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^9/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (32*(a + a*Sin[c + d*x])^(5/2))/(5*a^5*d) - (64*(a + a*Sin[c + d*x])^(7/2))
/(7*a^6*d) + (16*(a + a*Sin[c + d*x])^(9/2))/(3*a^7*d) - (16*(a + a*Sin[c +
 d*x])^(11/2))/(11*a^8*d) + (2*(a + a*Sin[c + d*x])^(13/2))/(13*a^9*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\int \frac{\cos^9(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx = \frac{\operatorname{Subst}\left(\int (a-x)^4(a+x)^{3/2}\,dx, x, a\sin(c+dx)\right)}{a^9 d}$$
$$= \frac{\operatorname{Subst}\left(\int \left(16a^4(a+x)^{3/2} - 32a^3(a+x)^{5/2} + 24a^2(a+x)^{7/2} - 8a(a+x)^{9/2} + \right.\right.}{a^9 d}$$
$$= \frac{32(a+a\sin(c+dx))^{5/2}}{5a^5 d} - \frac{64(a+a\sin(c+dx))^{7/2}}{7a^6 d} + \frac{16(a+a\sin(c+dx))^{9/2}}{3a^7 d}$$

Mathematica [A]
time = 0.18, size = 64, normalized size = 0.53

$$\frac{2(a(1+\sin(c+dx)))^{5/2}\left(9683-16700\sin(c+dx)+14210\sin^2(c+dx)-6300\sin^3(c+dx)+1155\sin^4(c+dx)\right)}{15015a^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^9/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (2*(a*(1 + Sin[c + d*x]))^(5/2)*(9683 - 16700*Sin[c + d*x] + 14210*Sin[c +
d*x]^2 - 6300*Sin[c + d*x]^3 + 1155*Sin[c + d*x]^4))/(15015*a^5*d)
```

Maple [A]
time = 0.35, size = 67, normalized size = 0.55

method	result	size
default	$\frac{2(a+a\sin(dx+c))^{\frac{5}{2}}\left(1155\left(\cos^4(dx+c)\right)+6300\left(\cos^2(dx+c)\right)\sin(dx+c)-16520\left(\cos^2(dx+c)\right)-23000\sin(dx+c)+25048\right)}{15015a^5d}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^9/(a+a*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15015/a^5*(a+a*sin(d*x+c))^(5/2)*(1155*cos(d*x+c)^4+6300*cos(d*x+c)^2*sin
(d*x+c)-16520*cos(d*x+c)^2-23000*sin(d*x+c)+25048)/d
```

Maxima [A]
time = 0.30, size = 89, normalized size = 0.74

$$\frac{2\left(1155\,(a\sin(dx+c)+a)^{\frac{13}{2}} - 10920\,(a\sin(dx+c)+a)^{\frac{11}{2}}a + 40040\,(a\sin(dx+c)+a)^{\frac{9}{2}}a^2 - 68640\,(a\sin(dx+c)+a)^{\frac{7}{2}}a^3 + 48048\,(a\sin(dx+c)+a)^{\frac{5}{2}}a^4\right)}{15015\,a^9d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^9/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 2/15015*(1155*(a*sin(d*x + c) + a)^(13/2) - 10920*(a*sin(d*x + c) + a)^(11/
2)*a + 40040*(a*sin(d*x + c) + a)^(9/2)*a^2 - 68640*(a*sin(d*x + c) + a)^(7
/2)*a^3 + 48048*(a*sin(d*x + c) + a)^(5/2)*a^4)/(a^9*d)
```

Fricas [A]
time = 0.37, size = 82, normalized size = 0.68

$$-\frac{2\left(1155\cos\left(dx+c\right)^{6}-6230\cos\left(dx+c\right)^{4}-512\cos\left(dx+c\right)^{2}+2\left(1995\cos\left(dx+c\right)^{4}-1280\cos\left(dx+c\right)^{2}-2048\right)\sin\left(dx+c\right)-4096\right)\sqrt{a\sin\left(dx+c\right)+a}}{15015\,a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^9/(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/15015*(1155*cos(d*x + c)^6 - 6230*cos(d*x + c)^4 - 512*cos(d*x + c)^2 +
2*(1995*cos(d*x + c)^4 - 1280*cos(d*x + c)^2 - 2048)*sin(d*x + c) - 4096)*s
qrt(a*sin(d*x + c) + a)/(a^3*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**9/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4062 deep
```

Giac [A]
time = 3.48, size = 134, normalized size = 1.11

$$\frac{128\left(1155\sqrt{2}\sqrt{a}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{13}-5460\sqrt{2}\sqrt{a}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{11}+10010\sqrt{2}\sqrt{a}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{9}-8580\sqrt{2}\sqrt{a}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{7}+3003\sqrt{2}\sqrt{a}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{5}\right)}{15015\,a^{3}d\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^9/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 128/15015*(1155*sqrt(2)*sqrt(a)*cos(-1/4*pi + 1/2*d*x + 1/2*c)^13 - 5460*sq
rt(2)*sqrt(a)*cos(-1/4*pi + 1/2*d*x + 1/2*c)^11 + 10010*sqrt(2)*sqrt(a)*cos
(-1/4*pi + 1/2*d*x + 1/2*c)^9 - 8580*sqrt(2)*sqrt(a)*cos(-1/4*pi + 1/2*d*x
+ 1/2*c)^7 + 3003*sqrt(2)*sqrt(a)*cos(-1/4*pi + 1/2*d*x + 1/2*c)^5)/(a^3*d*
sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^9}{\left(a+a\,\sin\left(c+d\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^9/(a + a*sin(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^9/(a + a*sin(c + d*x))^(5/2), x)
```

3.184 $\int \frac{\cos^8(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=63

$$-\frac{8a^2\cos^9(c+dx)}{99d(a+a\sin(c+dx))^{9/2}}-\frac{2a\cos^9(c+dx)}{11d(a+a\sin(c+dx))^{7/2}}$$

```
[Out] -8/99*a^2*cos(d*x+c)^9/d/(a+a*sin(d*x+c))^(9/2)-2/11*a*cos(d*x+c)^9/d/(a+a*
sin(d*x+c))^(7/2)
```

Rubi [A]
time = 0.08, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2753, 2752}

$$-\frac{8a^2\cos^9(c+dx)}{99d(a\sin(c+dx)+a)^{9/2}}-\frac{2a\cos^9(c+dx)}{11d(a\sin(c+dx)+a)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^8/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-8*a^2*Cos[c + d*x]^9)/(99*d*(a + a*Sin[c + d*x])^(9/2)) - (2*a*Cos[c + d*
x]^9)/(11*d*(a + a*Sin[c + d*x])^(7/2))
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2753

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && N
eQ[m + p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^8(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx &= -\frac{2a\cos^9(c+dx)}{11d(a+a\sin(c+dx))^{7/2}}+\frac{1}{11}(4a)\int \frac{\cos^8(c+dx)}{(a+a\sin(c+dx))^{7/2}}\,dx\\ &= -\frac{8a^2\cos^9(c+dx)}{99d(a+a\sin(c+dx))^{9/2}}-\frac{2a\cos^9(c+dx)}{11d(a+a\sin(c+dx))^{7/2}}\end{aligned}$$

Mathematica [**A**]
time = 0.21, size = 49, normalized size = 0.78

$$-\frac{2\cos^9(c+dx)(13+9\sin(c+dx))}{99d(1+\sin(c+dx))^2(a(1+\sin(c+dx)))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^8/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*Cos[c + d*x]^9*(13 + 9*Sin[c + d*x]))/(99*d*(1 + Sin[c + d*x])^2*(a*(1
+ Sin[c + d*x]))^(5/2))
```

Maple [**A**]
time = 0.55, size = 57, normalized size = 0.90

method	result	size
default	$\frac{2(1+\sin(dx+c))(\sin(dx+c)-1)^5(9\sin(dx+c)+13)}{99a^2\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8/(a+a*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/99/a^2*(1+sin(d*x+c))*(sin(d*x+c)-1)^5*(9*sin(d*x+c)+13)/cos(d*x+c)/(a+a*
sin(d*x+c))^(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^8/(a*sin(d*x + c) + a)^(5/2), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 161 vs. $2(55) = 110$.
time = 0.36, size = 161, normalized size = 2.56

$$-\frac{2\left(9\cos\left(dx+c\right)^6-23\cos\left(dx+c\right)^5-52\cos\left(dx+c\right)^4+4\cos\left(dx+c\right)^3-8\cos\left(dx+c\right)^2+\left(9\cos\left(dx+c\right)^5+32\cos\left(dx+c\right)^4-20\cos\left(dx+c\right)^3-24\cos\left(dx+c\right)^2-32\cos\left(dx+c\right)-64\right)\sin\left(dx+c\right)+32\cos\left(dx+c\right)+64\right)\sqrt{a\sin\left(dx+c\right)+a}}{99\left(a^3d\cos\left(dx+c\right)+a^3d\sin\left(dx+c\right)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8/(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/99*(9*cos(d*x + c)^6 - 23*cos(d*x + c)^5 - 52*cos(d*x + c)^4 + 4*cos(d*x
 + c)^3 - 8*cos(d*x + c)^2 + (9*cos(d*x + c)^5 + 32*cos(d*x + c)^4 - 20*cos
```

```
(d*x + c)^3 - 24*cos(d*x + c)^2 - 32*cos(d*x + c) - 64)*sin(d*x + c) + 32*c
os(d*x + c) + 64)*sqrt(a*sin(d*x + c) + a)/(a^3*d*cos(d*x + c) + a^3*d*sin(
d*x + c) + a^3*d)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8/(a+a*sin(d*x+c))**(5/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3278 deep
```

Giac [**A**]
time = 4.45, size = 68, normalized size = 1.08

$$-\frac{64\left(9\sqrt{2}\sqrt{a}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{11}-11\sqrt{2}\sqrt{a}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{9}\right)}{99\,a^{3}d\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
[Out] -64/99*(9*sqrt(2)*sqrt(a)*sin(-1/4*pi + 1/2*d*x + 1/2*c)^11 - 11*sqrt(2)*sq
rt(a)*sin(-1/4*pi + 1/2*d*x + 1/2*c)^9)/(a^3*d*sgn(cos(-1/4*pi + 1/2*d*x +
1/2*c)))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos\left(c+d\,x\right)^8}{{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^8/(a + a*sin(c + d*x))^(5/2),x)
[Out] int(cos(c + d*x)^8/(a + a*sin(c + d*x))^(5/2), x)
```

3.185 $\int \frac{\cos^7(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=97

$$\frac{16(a+a\sin(c+dx))^{3/2}}{3a^4d}-\frac{24(a+a\sin(c+dx))^{5/2}}{5a^5d}+\frac{12(a+a\sin(c+dx))^{7/2}}{7a^6d}-\frac{2(a+a\sin(c+dx))^{9/2}}{9a^7d}$$

```
[Out] 16/3*(a+a*sin(d*x+c))^(3/2)/a^4/d-24/5*(a+a*sin(d*x+c))^(5/2)/a^5/d+12/7*(a
+a*sin(d*x+c))^(7/2)/a^6/d-2/9*(a+a*sin(d*x+c))^(9/2)/a^7/d
```

Rubi [A]
time = 0.06, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2746, 45}

$$-\frac{2(a\sin(c+dx)+a)^{9/2}}{9a^7d}+\frac{12(a\sin(c+dx)+a)^{7/2}}{7a^6d}-\frac{24(a\sin(c+dx)+a)^{5/2}}{5a^5d}+\frac{16(a\sin(c+dx)+a)^{3/2}}{3a^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (16*(a + a*Sin[c + d*x])^(3/2))/(3*a^4*d) - (24*(a + a*Sin[c + d*x])^(5/2))
/(5*a^5*d) + (12*(a + a*Sin[c + d*x])^(7/2))/(7*a^6*d) - (2*(a + a*Sin[c +
d*x])^(9/2))/(9*a^7*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\int \frac{\cos^7(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx = \frac{\text{Subst}\left(\int (a-x)^3\sqrt{a+x}\,dx, x, a\sin(c+dx)\right)}{a^7 d}$$
$$= \frac{\text{Subst}\left(\int \left(8a^3\sqrt{a+x} - 12a^2(a+x)^{3/2} + 6a(a+x)^{5/2} - (a+x)^{7/2}\right)dx, x, a\,\text{s}\right.}{a^7 d}$$
$$= \frac{16(a+a\sin(c+dx))^{3/2}}{3a^4 d} - \frac{24(a+a\sin(c+dx))^{5/2}}{5a^5 d} + \frac{12(a+a\sin(c+dx))^{7/}}{7a^6 d}$$

Mathematica [A]
time = 0.11, size = 54, normalized size = 0.56

$$-\frac{2(a(1+\sin(c+dx)))^{3/2}\left(-319+321\sin(c+dx)-165\sin^2(c+dx)+35\sin^3(c+dx)\right)}{315a^4 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*(a*(1 + Sin[c + d*x]))^(3/2)*(-319 + 321*Sin[c + d*x] - 165*Sin[c + d*x
]^2 + 35*Sin[c + d*x]^3))/(315*a^4*d)
```

Maple [A]
time = 0.34, size = 57, normalized size = 0.59

method	result	size
default	$\frac{2(a+a\sin(dx+c))^{\frac{3}{2}}\left(35\left(\cos^2(dx+c)\right)\sin(dx+c)-165\left(\cos^2(dx+c)\right)-356\sin(dx+c)+484\right)}{315a^4 d}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7/(a+a*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/315/a^4*(a+a*sin(d*x+c))^(3/2)*(35*cos(d*x+c)^2*sin(d*x+c)-165*cos(d*x+c)
^2-356*sin(d*x+c)+484)/d
```

Maxima [A]
time = 0.29, size = 72, normalized size = 0.74

$$-\frac{2\left(35\left(a\sin\left(dx+c\right)+a\right)^{\frac{9}{2}}-270\left(a\sin\left(dx+c\right)+a\right)^{\frac{7}{2}}a+756\left(a\sin\left(dx+c\right)+a\right)^{\frac{5}{2}}a^2-840\left(a\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}a^3\right)}{315\,a^7 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -2/315*(35*(a*sin(d*x + c) + a)^(9/2) - 270*(a*sin(d*x + c) + a)^(7/2)*a +
756*(a*sin(d*x + c) + a)^(5/2)*a^2 - 840*(a*sin(d*x + c) + a)^(3/2)*a^3)/(a
^7*d)
```

Fricas [A]
time = 0.34, size = 62, normalized size = 0.64

$$-\frac{2\left(35\cos\left(dx+c\right)^{4}-226\cos\left(dx+c\right)^{2}+2\left(65\cos\left(dx+c\right)^{2}-64\right)\sin\left(dx+c\right)-128\right)\sqrt{a\sin\left(dx+c\right)+a}}{315\,a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/315*(35*cos(d*x + c)^4 - 226*cos(d*x + c)^2 + 2*(65*cos(d*x + c)^2 - 64)
*sin(d*x + c) - 128)*sqrt(a*sin(d*x + c) + a)/(a^3*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 7.42, size = 112, normalized size = 1.15

$$-\frac{32\left(35\sqrt{2}\sqrt{a}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{9}-135\sqrt{2}\sqrt{a}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{7}+189\sqrt{2}\sqrt{a}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{5}-105\sqrt{2}\sqrt{a}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{3}\right)}{315\,a^{3}d\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -32/315*(35*sqrt(2)*sqrt(a)*cos(-1/4*pi + 1/2*d*x + 1/2*c)^9 - 135*sqrt(2)*
sqrt(a)*cos(-1/4*pi + 1/2*d*x + 1/2*c)^7 + 189*sqrt(2)*sqrt(a)*cos(-1/4*pi
+ 1/2*d*x + 1/2*c)^5 - 105*sqrt(2)*sqrt(a)*cos(-1/4*pi + 1/2*d*x + 1/2*c)^3
)/(a^3*d*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^{7}}{\left(a+a\,\sin\left(c+d\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7/(a + a*sin(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^7/(a + a*sin(c + d*x))^(5/2), x)
```

3.186 $\int \frac{\cos^6(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=30

$$-\frac{2a\cos^7(c+dx)}{7d(a+a\sin(c+dx))^{7/2}}$$

```
[Out] -2/7*a*cos(d*x+c)^7/d/(a+a*sin(d*x+c))^(7/2)
```

Rubi [A]
time = 0.04, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {2752}

$$-\frac{2a\cos^7(c+dx)}{7d(a\sin(c+dx)+a)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*a*Cos[c + d*x]^7)/(7*d*(a + a*Sin[c + d*x])^(7/2))
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\int \frac{\cos^6(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx = -\frac{2a\cos^7(c+dx)}{7d(a+a\sin(c+dx))^{7/2}}$$

Mathematica [A]
time = 0.07, size = 42, normalized size = 1.40

$$-\frac{2\cos^7(c+dx)\sqrt{a(1+\sin(c+dx))}}{7a^3d(1+\sin(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*Cos[c + d*x]^7*Sqrt[a*(1 + Sin[c + d*x])])/(7*a^3*d*(1 + Sin[c + d*x])^
4)
```

Maple [A]
time = 0.38, size = 47, normalized size = 1.57

method	result	size
default	$-\frac{2(1+\sin(dx+c))(\sin(dx+c)-1)^4}{7a^2\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6/(a+a*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/7/a^2*(1+sin(d*x+c))*(sin(d*x+c)-1)^4/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/
d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^6/(a*sin(d*x + c) + a)^(5/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(26) = 52$.
time = 0.36, size = 117, normalized size = 3.90

$$-\frac{2\left(\cos\left(dx+c\right)^{4}-3\cos\left(dx+c\right)^{3}-8\cos\left(dx+c\right)^{2}+\left(\cos\left(dx+c\right)^{3}+4\cos\left(dx+c\right)^{2}-4\cos\left(dx+c\right)-8\right)\sin\left(dx+c\right)+4\cos\left(dx+c\right)+8\right)\sqrt{a\sin\left(dx+c\right)+a}}{7\left(a^{3}d\cos\left(dx+c\right)+a^{3}d\sin\left(dx+c\right)+a^{3}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/7*(cos(d*x + c)^4 - 3*cos(d*x + c)^3 - 8*cos(d*x + c)^2 + (cos(d*x + c)^
3 + 4*cos(d*x + c)^2 - 4*cos(d*x + c) - 8)*sin(d*x + c) + 4*cos(d*x + c) +
8)*sqrt(a*sin(d*x + c) + a)/(a^3*d*cos(d*x + c) + a^3*d*sin(d*x + c) + a^3*
d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 4.79, size = 40, normalized size = 1.33

$$\frac{16\sqrt{2}\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}}{7\,a^{\frac{5}{2}}d\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 16/7*sqrt(2)*sin(-1/4*pi + 1/2*d*x + 1/2*c)^7/(a^(5/2)*d*sgn(cos(-1/4*pi +
1/2*d*x + 1/2*c)))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\cos\left(c+d\,x\right)^6}{\left(a+a\,\sin\left(c+d\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(a + a*sin(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^6/(a + a*sin(c + d*x))^(5/2), x)
```

3.187 $\int \frac{\cos^5(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=71

$$\frac{8\sqrt{a+a\sin(c+dx)}}{a^3d}-\frac{8(a+a\sin(c+dx))^{3/2}}{3a^4d}+\frac{2(a+a\sin(c+dx))^{5/2}}{5a^5d}$$

```
[Out] -8/3*(a+a*sin(d*x+c))^(3/2)/a^4/d+2/5*(a+a*sin(d*x+c))^(5/2)/a^5/d+8*(a+a*s
in(d*x+c))^(1/2)/a^3/d
```

Rubi [A]
time = 0.05, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2746, 45}

$$\frac{2(a\sin(c+dx)+a)^{5/2}}{5a^5d}-\frac{8(a\sin(c+dx)+a)^{3/2}}{3a^4d}+\frac{8\sqrt{a\sin(c+dx)+a}}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (8*Sqrt[a + a*Sin[c + d*x]])/(a^3*d) - (8*(a + a*Sin[c + d*x])^(3/2))/(3*a^
4*d) + (2*(a + a*Sin[c + d*x])^(5/2))/(5*a^5*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\int \frac{\cos^5(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^2}{\sqrt{a+x}}\,dx, x, a\sin(c+dx)\right)}{a^5d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{4a^2}{\sqrt{a+x}} - 4a\sqrt{a+x} + (a+x)^{3/2}\right)dx, x, a\sin(c+dx)\right)}{a^5d}$$
$$= \frac{8\sqrt{a+a\sin(c+dx)}}{a^3d} - \frac{8(a+a\sin(c+dx))^{3/2}}{3a^4d} + \frac{2(a+a\sin(c+dx))^{5/2}}{5a^5d}$$

Mathematica [A]
time = 0.05, size = 44, normalized size = 0.62

$$\frac{2\sqrt{a(1+\sin(c+dx))}\,(43-14\sin(c+dx)+3\sin^2(c+dx))}{15a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/(a + a*Sin[c + d*x])^(5/2),x]
[Out] (2*Sqrt[a*(1 + Sin[c + d*x])]*(43 - 14*Sin[c + d*x] + 3*Sin[c + d*x]^2))/(1
5*a^3*d)
```

Maple [A]
time = 0.30, size = 41, normalized size = 0.58

method	result	size
default	$-\frac{2\sqrt{a+a\sin(dx+c)}\,(3(\cos^2(dx+c))+14\sin(dx+c)-46)}{15a^3d}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a+a*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
[Out] -2/15/a^3*(a+a*sin(d*x+c))^(1/2)*(3*cos(d*x+c)^2+14*sin(d*x+c)-46)/d
```

Maxima [A]
time = 0.30, size = 55, normalized size = 0.77

$$\frac{2\left(3\,(a\sin(dx+c)+a)^{\frac{5}{2}} - 20\,(a\sin(dx+c)+a)^{\frac{3}{2}}a + 60\,\sqrt{a\sin(dx+c)+a}\,a^2\right)}{15\,a^5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 2/15*(3*(a*sin(d*x + c) + a)^(5/2) - 20*(a*sin(d*x + c) + a)^(3/2)*a + 60*s
qrt(a*sin(d*x + c) + a)*a^2)/(a^5*d)
```

Fricas [A]
time = 0.34, size = 40, normalized size = 0.56

$$-\frac{2\left(3\cos\left(dx+c\right)^{2}+14\sin\left(dx+c\right)-46\right)\sqrt{a\sin\left(dx+c\right)+a}}{15\,a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
[Out] -2/15*(3*cos(d*x + c)^2 + 14*sin(d*x + c) - 46)*sqrt(a*sin(d*x + c) + a)/(a
^3*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a+a*sin(d*x+c))**(5/2),x)
[Out] Timed out
```

Giac [A]
time = 4.83, size = 88, normalized size = 1.24

$$\frac{8\left(3\sqrt{2}\sqrt{a}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{5}-10\sqrt{2}\sqrt{a}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{3}+15\sqrt{2}\sqrt{a}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{15\,a^{3}d\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
[Out] 8/15*(3*sqrt(2)*sqrt(a)*cos(-1/4*pi + 1/2*d*x + 1/2*c)^5 - 10*sqrt(2)*sqrt(
a)*cos(-1/4*pi + 1/2*d*x + 1/2*c)^3 + 15*sqrt(2)*sqrt(a)*cos(-1/4*pi + 1/2*
d*x + 1/2*c))/(a^3*d*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^{5}}{\left(a+a\,\sin\left(c+d\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(a + a*sin(c + d*x))^(5/2),x)
[Out] int(cos(c + d*x)^5/(a + a*sin(c + d*x))^(5/2), x)
```

3.188 $\int \frac{\cos^4(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=108

$$-\frac{4\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(c+dx)}{\sqrt{2}\,\sqrt{a+a\sin(c+dx)}}\right)}{a^{5/2}d}+\frac{2\cos^3(c+dx)}{3ad(a+a\sin(c+dx))^{3/2}}+\frac{4\cos(c+dx)}{a^2d\sqrt{a+a\sin(c+dx)}}$$

```
[Out] 2/3*cos(d*x+c)^3/a/d/(a+a*sin(d*x+c))^(3/2)-4*arctanh(1/2*cos(d*x+c)*a^(1/2
)*2^(1/2)/(a+a*sin(d*x+c))^(1/2))*2^(1/2)/a^(5/2)/d+4*cos(d*x+c)/a^2/d/(a+a
*sin(d*x+c))^(1/2)
```

Rubi [A]

time = 0.10, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {2758, 2728, 212}

$$-\frac{4\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(c+dx)}{\sqrt{2}\,\sqrt{a\sin(c+dx)+a}}\right)}{a^{5/2}d}+\frac{4\cos(c+dx)}{a^2d\sqrt{a\sin(c+dx)+a}}+\frac{2\cos^3(c+dx)}{3ad(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-4*Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]
])])/(a^(5/2)*d) + (2*Cos[c + d*x]^3)/(3*a*d*(a + a*Sin[c + d*x])^(3/2)) +
(4*Cos[c + d*x])/(a^2*d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2758

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(a*(m + p))), Int[(g*Cos[
```

```
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^4(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx &= \frac{2\cos^3(c+dx)}{3ad(a+a\sin(c+dx))^{3/2}}+\frac{2\int \frac{\cos^2(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx}{a}\\ &= \frac{2\cos^3(c+dx)}{3ad(a+a\sin(c+dx))^{3/2}}+\frac{4\cos(c+dx)}{a^2d\sqrt{a+a\sin(c+dx)}}+\frac{4\int \frac{1}{\sqrt{a+a\sin(c+}}}{a^2}\\ &= \frac{2\cos^3(c+dx)}{3ad(a+a\sin(c+dx))^{3/2}}+\frac{4\cos(c+dx)}{a^2d\sqrt{a+a\sin(c+dx)}}-\frac{8\mathrm{Subst}\left(\int \frac{1}{2a-x^2}\,dx,\right.}{}\\ &= -\frac{4\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{a^{5/2}d}+\frac{2\cos^3(c+dx)}{3ad(a+a\sin(c+dx))^{3/2}}+\end{aligned}$$

Mathematica [A]
time = 0.12, size = 96, normalized size = 0.89

$$-\frac{2\cos(c+dx)\left(6\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{1-\sin(c+dx)}}{\sqrt{2}}\right)+\sqrt{1-\sin(c+dx)}\,(-7+\sin(c+dx))\right)}{3a^2d\sqrt{1-\sin(c+dx)}\,\sqrt{a(1+\sin(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*Cos[c + d*x]*(6*Sqrt[2]*ArcTanh[Sqrt[1 - Sin[c + d*x]]/Sqrt[2]] + Sqrt[
1 - Sin[c + d*x]]*(-7 + Sin[c + d*x])))/(3*a^2*d*Sqrt[1 - Sin[c + d*x]]*Sqr
t[a*(1 + Sin[c + d*x])])
```

Maple [A]
time = 0.45, size = 112, normalized size = 1.04

method	result

default	$-\frac{2(1+\sin(dx+c))\sqrt{-a\left(\sin\left(dx+c\right)-1\right)}\,\left(6a^{\frac{3}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin\left(dx+c\right)}\,\sqrt{2}}{2\sqrt{a}}\right)-(a-a\sin(dx+c))^{\frac{3}{2}}-$
	$3a^4\cos(dx+c)\sqrt{a+a\sin\left(dx+c\right)}\,d$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+a*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(6*a^(3/2)*2^(1/2)*arctanh(1/
2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))-(a-a*sin(d*x+c))^(3/2)-6*a*(a-a*s
in(d*x+c))^(1/2))/a^4/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^4/(a*sin(d*x + c) + a)^(5/2), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 215 vs. $2(93) = 186$.
time = 0.36, size = 215, normalized size = 1.99

$$\frac{2\left(\frac{3\sqrt{2}\,(a\cos(dx+c)+a\sin(dx+c)+a)\log\left(-\frac{\cos(dx+c)^2-(\cos(dx+c)-2)\sin(dx+c)-\frac{2\sqrt{2}\,\sqrt{a\sin\left(dx+c\right)+a}\,(\cos(dx+c)-\sin(dx+c)+1)}{\sqrt{a}}+3\cos(dx+c)+2}{\cos(dx+c)^2-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2}\right)}{\sqrt{a}}-\left(\cos\left(dx+c\right)^2+\left(\cos\left(dx+c\right)+8\right)\sin\left(dx+c\right)-7\cos\left(dx+c\right)-8\right)\sqrt{a\sin\left(dx+c\right)+a}\right)}{3\left(a^3d\cos\left(dx+c\right)+a^3d\sin\left(dx+c\right)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(3*sqrt(2)*(a*cos(d*x + c) + a*sin(d*x + c) + a)*log(-(cos(d*x + c)^2 -
 (cos(d*x + c) - 2)*sin(d*x + c) - 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*(cos(
d*x + c) - sin(d*x + c) + 1)/sqrt(a) + 3*cos(d*x + c) + 2)/(cos(d*x + c)^2
- (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2))/sqrt(a) - (cos(d*x +
 c)^2 + (cos(d*x + c) + 8)*sin(d*x + c) - 7*cos(d*x + c) - 8)*sqrt(a*sin(d*
x + c) + a))/(a^3*d*cos(d*x + c) + a^3*d*sin(d*x + c) + a^3*d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cos^4\left(c+dx\right)}{\left(a\left(\sin\left(c+dx\right)+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Integral(cos(c + d*x)**4/(a*(sin(c + d*x) + 1))**(5/2), x)
```

Giac [A]
time = 3.51, size = 140, normalized size = 1.30

$$\frac{2\sqrt{2}\sqrt{a}\left(\frac{3\log\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{3\log\left(-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{2\left(a^6\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^3+3a^6\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{a^9\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}\right)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(2)*sqrt(a)*(3*log(sin(-1/4*pi + 1/2*d*x + 1/2*c) + 1)/(a^3*sgn(cos
(-1/4*pi + 1/2*d*x + 1/2*c))) - 3*log(-sin(-1/4*pi + 1/2*d*x + 1/2*c) + 1)/
(a^3*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))) - 2*(a^6*sin(-1/4*pi + 1/2*d*x +
1/2*c)^3 + 3*a^6*sin(-1/4*pi + 1/2*d*x + 1/2*c))/(a^9*sgn(cos(-1/4*pi + 1/2
*d*x + 1/2*c))))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^4}{\left(a+a\,\sin\left(c+d\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(a + a*sin(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^4/(a + a*sin(c + d*x))^(5/2), x)
```

3.189 $\int \frac{\cos^3(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=45

$$-\frac{4}{a^2d\sqrt{a+a\sin(c+dx)}}-\frac{2\sqrt{a+a\sin(c+dx)}}{a^3d}$$

```
[Out] -4/a^2/d/(a+a*sin(d*x+c))^(1/2)-2*(a+a*sin(d*x+c))^(1/2)/a^3/d
```

Rubi [A]
time = 0.05, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2746, 45}

$$-\frac{2\sqrt{a\sin(c+dx)+a}}{a^3d}-\frac{4}{a^2d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] -4/(a^2*d*Sqrt[a + a*Sin[c + d*x]]) - (2*Sqrt[a + a*Sin[c + d*x]])/(a^3*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{a-x}{(a+x)^{3/2}}\,dx, x, a\sin(c+dx)\right)}{a^3d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{2a}{(a+x)^{3/2}} - \frac{1}{\sqrt{a+x}}\right)dx, x, a\sin(c+dx)\right)}{a^3d}$$
$$= -\frac{4}{a^2d\sqrt{a+a\sin(c+dx)}} - \frac{2\sqrt{a+a\sin(c+dx)}}{a^3d}$$

Mathematica [A]
time = 0.04, size = 30, normalized size = 0.67

$$-\frac{2(3+\sin(c+dx))}{a^2d\sqrt{a(1+\sin(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + a*Sin[c + d*x])^(5/2),x]
[Out] (-2*(3 + Sin[c + d*x]))/(a^2*d*Sqrt[a*(1 + Sin[c + d*x])])
```

Maple [A]
time = 0.25, size = 29, normalized size = 0.64

method	result	size
default	$-\frac{2(3+\sin(dx+c))}{a^2\sqrt{a+a\sin(dx+c)}\,d}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+a*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
[Out] -2/a^2/(a+a*sin(d*x+c))^(1/2)*(3+sin(d*x+c))/d
```

Maxima [A]
time = 0.27, size = 42, normalized size = 0.93

$$-\frac{2\left(\frac{\sqrt{a\sin(dx+c)+a}}{a^2} + \frac{2}{\sqrt{a\sin(dx+c)+a}\,a}\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -2*(sqrt(a*sin(d*x + c) + a)/a^2 + 2/(sqrt(a*sin(d*x + c) + a)*a))/(a*d)
```

Fricas [A]
time = 0.35, size = 41, normalized size = 0.91

$$-\frac{2\sqrt{a\sin(dx+c)+a}\,(\sin(dx+c)+3)}{a^3d\sin(dx+c)+a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(a*sin(d*x + c) + a)*(sin(d*x + c) + 3)/(a^3*d*sin(d*x + c) + a^3*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 267 vs. 2(41) = 82.
time = 2.83, size = 267, normalized size = 5.93

$$\begin{cases} \mathrm{NaN} & \text{for } \left(c=\frac{3\pi}{2}\vee c=-dx+\frac{3\pi}{2}\right)\wedge\left(c=-dx+\frac{3\pi}{2}\vee d=0\right) \\ \frac{x\cos^3(c)}{(a\sin(c)+a)^{\frac{5}{2}}} & \text{for } d=0 \\ -\frac{8\sqrt{a\sin(c+dx)+a}\sin^2(c+dx)}{3a^3d\sin^2(c+dx)+6a^3d\sin(c+dx)+3a^3d}-\frac{24\sqrt{a\sin(c+dx)+a}\sin(c+dx)}{3a^3d\sin^2(c+dx)+6a^3d\sin(c+dx)+3a^3d}-\frac{2\sqrt{a\sin(c+dx)+a}\cos^2(c+dx)}{3a^3d\sin^2(c+dx)+6a^3d\sin(c+dx)+3a^3d}-\frac{16\sqrt{a\sin(c+dx)+a}}{3a^3d\sin^2(c+dx)+6a^3d\sin(c+dx)+3a^3d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Piecewise((nan, (Eq(d, 0) | Eq(c, -d*x + 3*pi/2)) & (Eq(c, 3*pi/2) | Eq(c,
-d*x + 3*pi/2))), (x*cos(c)**3/(a*sin(c) + a)**(5/2), Eq(d, 0)), (-8*sqrt(a
*sin(c + d*x) + a)*sin(c + d*x)**2/(3*a**3*d*sin(c + d*x)**2 + 6*a**3*d*sin
(c + d*x) + 3*a**3*d) - 24*sqrt(a*sin(c + d*x) + a)*sin(c + d*x)/(3*a**3*d*
sin(c + d*x)**2 + 6*a**3*d*sin(c + d*x) + 3*a**3*d) - 2*sqrt(a*sin(c + d*x)
 + a)*cos(c + d*x)**2/(3*a**3*d*sin(c + d*x)**2 + 6*a**3*d*sin(c + d*x) + 3
*a**3*d) - 16*sqrt(a*sin(c + d*x) + a)/(3*a**3*d*sin(c + d*x)**2 + 6*a**3*d
*sin(c + d*x) + 3*a**3*d), True))
```

Giac [A]
time = 6.80, size = 64, normalized size = 1.42

$$-\frac{2\left(\sqrt{2}\sqrt{a}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+\frac{\sqrt{2}\sqrt{a}}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)}\right)}{a^3d\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -2*(sqrt(2)*sqrt(a)*cos(-1/4*pi + 1/2*d*x + 1/2*c) + sqrt(2)*sqrt(a)/cos(-1
/4*pi + 1/2*d*x + 1/2*c))/(a^3*d*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos (c+d\,x)^3}{\left(a+a\,\sin (c+d\,x)\right)^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a + a*sin(c + d*x))^(5/2),x)
[Out] int(cos(c + d*x)^3/(a + a*sin(c + d*x))^(5/2), x)
```

3.190 $\int \frac{\cos^2(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=75

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{2}\,a^{5/2}d}-\frac{\cos(c+dx)}{ad(a+a\sin(c+dx))^{3/2}}$$

```
[Out] -cos(d*x+c)/a/d/(a+a*sin(d*x+c))^(3/2)+1/2*arctanh(1/2*cos(d*x+c)*a^(1/2)*2
^(1/2)/(a+a*sin(d*x+c))^(1/2))*2^(1/2)/a^(5/2)/d
```

Rubi [A]
time = 0.06, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {2759, 2728, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{\sqrt{2}\,a^{5/2}d}-\frac{\cos(c+dx)}{ad(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])]/(Sqrt[2]
*a^(5/2)*d) - Cos[c + d*x]/(a*d*(a + a*Sin[c + d*x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
```

```
NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx &= -\frac{\cos(c+dx)}{ad(a+a\sin(c+dx))^{3/2}}-\frac{\int \frac{1}{\sqrt{a+a\sin(c+dx)}}\,dx}{2a^2}\\ &= -\frac{\cos(c+dx)}{ad(a+a\sin(c+dx))^{3/2}}+\frac{\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx,x,\frac{a\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{a^2d}\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{2}\,a^{5/2}d}-\frac{\cos(c+dx)}{ad(a+a\sin(c+dx))^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.16, size = 100, normalized size = 1.33

$$\frac{\sec(c+dx)\left(\tanh^{-1}\left(\frac{\sqrt{1-\sin(c+dx)}}{\sqrt{2}}\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2\sqrt{2-2\sin(c+dx)}+2(-1+\sin(c+dx))\right)}{2a^2d\sqrt{a(1+\sin(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (Sec[c + d*x]*(ArcTanh[Sqrt[1 - Sin[c + d*x]]/Sqrt[2]]*(Cos[(c + d*x)/2] +
Sin[(c + d*x)/2])^2*Sqrt[2 - 2*Sin[c + d*x]] + 2*(-1 + Sin[c + d*x])))/(2*a
^2*d*Sqrt[a*(1 + Sin[c + d*x])])
```

Maple [A]
time = 0.47, size = 123, normalized size = 1.64

method	result
default	$-\frac{\left(-\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)a\sin(dx+c)-\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)a+2\right.}{2a^{\frac{7}{2}}\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+a*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a^(7/2)*(-2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*
a*sin(d*x+c)-2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a+
```

```
2*(a-a*sin(d*x+c))^(1/2)*a^(1/2))*(-a*(sin(d*x+c)-1))^(1/2)/cos(d*x+c)/(a+a
*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^2/(a*sin(d*x + c) + a)^(5/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 252 vs. 2(65) = 130.
time = 0.38, size = 252, normalized size = 3.36

$$\frac{\frac{\sqrt{2}\left(a\cos(dx+c)^2-a\cos(dx+c)-(a\cos(dx+c)+2a)\sin(dx+c)-2a\right)\log\left(-\frac{\cos(dx+c)^2-(\cos(dx+c)-2)\sin(dx+c)+\frac{2\sqrt{2}\sqrt{a\sin(dx+c)+a}(\cos(dx+c)-\sin(dx+c)+1)}{\sqrt{a}}+3\cos(dx+c)+2}{\cos(dx+c)^2-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2}\right)}{\sqrt{a}}+4\sqrt{a\sin(dx+c)+a}\,(\cos(dx+c)-\sin(dx+c)+1)}{4\left(a^3d\cos(dx+c)^2-a^3d\cos(dx+c)-2a^3d-(a^3d\cos(dx+c)+2a^3d)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*(a*cos(d*x + c)^2 - a*cos(d*x + c) - (a*cos(d*x + c) + 2*a)*si
n(d*x + c) - 2*a)*log(-(cos(d*x + c)^2 - (cos(d*x + c) - 2)*sin(d*x + c) +
2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*(cos(d*x + c) - sin(d*x + c) + 1)/sqrt(a
) + 3*cos(d*x + c) + 2)/(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c) -
 cos(d*x + c) - 2))/sqrt(a) + 4*sqrt(a*sin(d*x + c) + a)*(cos(d*x + c) - si
n(d*x + c) + 1))/(a^3*d*cos(d*x + c)^2 - a^3*d*cos(d*x + c) - 2*a^3*d - (a^
3*d*cos(d*x + c) + 2*a^3*d)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(c+dx)}{(a(\sin(c+dx)+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Integral(cos(c + d*x)**2/(a*(sin(c + d*x) + 1))**(5/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 139 vs. 2(65) = 130.
time = 6.43, size = 139, normalized size = 1.85

$$-\frac{\sqrt{a}\left(\frac{\sqrt{2}\log\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{\sqrt{2}\log\left(-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}+\frac{2\sqrt{2}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)a^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(a)*(sqrt(2)*log(sin(-1/4*pi + 1/2*d*x + 1/2*c) + 1)/(a^3*sgn(cos(
-1/4*pi + 1/2*d*x + 1/2*c))) - sqrt(2)*log(-sin(-1/4*pi + 1/2*d*x + 1/2*c)
+ 1)/(a^3*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))) + 2*sqrt(2)*sin(-1/4*pi + 1/
2*d*x + 1/2*c)/((sin(-1/4*pi + 1/2*d*x + 1/2*c)^2 - 1)*a^3*sgn(cos(-1/4*pi
+ 1/2*d*x + 1/2*c))))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)^2}{(a+a\,\sin(c+d\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a + a*sin(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^2/(a + a*sin(c + d*x))^(5/2), x)
```

3.191 $\int \frac{\cos(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=24

$$-\frac{2}{3ad(a+a\sin(c+dx))^{3/2}}$$

[Out] -2/3/a/d/(a+a*sin(d*x+c))^(3/2)

Rubi [A]
time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2746, 32}

$$-\frac{2}{3ad(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]/(a + a*Sin[c + d*x])^(5/2),x]

[Out] -2/(3*a*d*(a + a*Sin[c + d*x])^(3/2))

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{(a+x)^{5/2}}\,dx, x, a\sin(c+dx)\right)}{ad}\\ &= -\frac{2}{3ad(a+a\sin(c+dx))^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 24, normalized size = 1.00

$$-\frac{2}{3ad(a+a\sin(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] -2/(3*a*d*(a + a*Sin[c + d*x])^(3/2))
```

Maple [A]
time = 0.04, size = 21, normalized size = 0.88

method	result	size
derivativedivides	$-\frac{2}{3ad\left(a+a\sin\left(dx+c\right)\right)^{\frac{3}{2}}}$	21
default	$-\frac{2}{3ad\left(a+a\sin\left(dx+c\right)\right)^{\frac{3}{2}}}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+a*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/a/d/(a+a*sin(d*x+c))^(3/2)
```

Maxima [A]
time = 0.29, size = 20, normalized size = 0.83

$$-\frac{2}{3\left(a\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3/((a*sin(d*x + c) + a)^(3/2)*a*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(20) = 40$.
time = 0.37, size = 48, normalized size = 2.00

$$\frac{2\sqrt{a\sin\left(dx+c\right)+a}}{3\left(a^{3}d\cos\left(dx+c\right)^{2}-2a^{3}d\sin\left(dx+c\right)-2a^{3}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(a*sin(d*x + c) + a)/(a^3*d*cos(d*x + c)^2 - 2*a^3*d*sin(d*x + c) -
 2*a^3*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(20) = 40$.

time = 2.86, size = 65, normalized size = 2.71

$$\begin{cases} -\frac{2}{3a^2d\sqrt{a\sin(c+dx)+a}\sin(c+dx)+3a^2d\sqrt{a\sin(c+dx)+a}} & \text{for } d \neq 0 \\ \frac{x\cos(c)}{(a\sin(c)+a)^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Piecewise((-2/(3*a**2*d*sqrt(a*sin(c + d*x) + a)*sin(c + d*x) + 3*a**2*d*sq
rt(a*sin(c + d*x) + a)), Ne(d, 0)), (x*cos(c)/(a*sin(c) + a)**(5/2), True))
```

Giac [A]
time = 6.13, size = 40, normalized size = 1.67

$$-\frac{\sqrt{2}}{6\,a^{\frac{5}{2}}d\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(2)/(a^(5/2)*d*cos(-1/4*pi + 1/2*d*x + 1/2*c)^3*sgn(cos(-1/4*pi +
1/2*d*x + 1/2*c)))
```

Mupad [B]
time = 7.52, size = 72, normalized size = 3.00

$$\frac{8\,\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,\sqrt{a+a\,\left(\frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}}{3\,a^3\,d\,(-1+\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}\,1\mathrm{i})^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + a*sin(c + d*x))^(5/2),x)
```

```
[Out] (8*exp(c*2i + d*x*2i)*(a + a*((exp(- c*1i - d*x*1i)*1i)/2 - (exp(c*1i + d*x
*1i)*1i)/2))^(1/2))/(3*a^3*d*(exp(c*1i + d*x*1i)*1i - 1)^4)
```

3.192 $\int \frac{\sec(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=113

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}\,a^{5/2}d}-\frac{1}{5d(a+a\sin(c+dx))^{5/2}}-\frac{1}{6ad(a+a\sin(c+dx))^{3/2}}-\frac{1}{4a^2d\sqrt{a+a\sin(c+dx)}}$$

```
[Out] -1/5/d/(a+a*sin(d*x+c))^(5/2)-1/6/a/d/(a+a*sin(d*x+c))^(3/2)+1/8*arctanh(1/
2*(a+a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/a^(5/2)/d*2^(1/2)-1/4/a^2/d/(a+a*
sin(d*x+c))^(1/2)
```

Rubi [A]

time = 0.06, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2746, 53, 65, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}\,a^{5/2}d}-\frac{1}{4a^2d\sqrt{a\sin(c+dx)+a}}-\frac{1}{6ad(a\sin(c+dx)+a)^{3/2}}-\frac{1}{5d(a\sin(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])]/(4*Sqrt[2]*a^(5/2)*d) -
 1/(5*d*(a + a*Sin[c + d*x])^(5/2)) - 1/(6*a*d*(a + a*Sin[c + d*x])^(3/2))
- 1/(4*a^2*d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx = \frac{a\text{Subst}\left(\int \frac{1}{(a-x)(a+x)^{7/2}}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= -\frac{1}{5d(a+a\sin(c+dx))^{5/2}} + \frac{\text{Subst}\left(\int \frac{1}{(a-x)(a+x)^{5/2}}\,dx, x, a\sin(c+dx)\right)}{2d}$$

$$= -\frac{1}{5d(a+a\sin(c+dx))^{5/2}} - \frac{1}{6ad(a+a\sin(c+dx))^{3/2}} + \frac{\text{Subst}\left(\int \frac{1}{(a-x)(a+x)^{3/}}\right.}{4}$$

$$= -\frac{1}{5d(a+a\sin(c+dx))^{5/2}} - \frac{1}{6ad(a+a\sin(c+dx))^{3/2}} - \frac{1}{4a^2d\sqrt{a+a\sin(c+}}$$

$$= -\frac{1}{5d(a+a\sin(c+dx))^{5/2}} - \frac{1}{6ad(a+a\sin(c+dx))^{3/2}} - \frac{1}{4a^2d\sqrt{a+a\sin(c+}}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}\,a^{5/2}d} - \frac{1}{5d(a+a\sin(c+dx))^{5/2}} - \frac{1}{6ad(a+a\sin(c}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.06, size = 41, normalized size = 0.36

$$-\frac{{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};\frac{1}{2}(1+\sin(c+dx))\right)}{5d(a+a\sin(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] -1/5*Hypergeometric2F1[-5/2, 1, -3/2, (1 + Sin[c + d*x])/2]/(d*(a + a*Sin[c
 + d*x])^(5/2))
```

Maple [A]
time = 0.36, size = 88, normalized size = 0.78

method	result
default	$-\frac{2a\left(-\frac{\sqrt{2}\,\mathrm{arctanh}\left(\frac{\sqrt{a+a\sin(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{16a^{\frac{7}{2}}}+\frac{1}{8a^{3}\sqrt{a+a\sin(dx+c)}}+\frac{1}{12a^{2}(a+a\sin(dx+c))^{\frac{3}{2}}}+\frac{1}{10a(a+a\sin(\ldots}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+a*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*a*(-1/16/a^(7/2)*2^(1/2)*arctanh(1/2*(a+a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1
/2))+1/8/a^3/(a+a*sin(d*x+c))^(1/2)+1/12/a^2/(a+a*sin(d*x+c))^(3/2)+1/10/a/
(a+a*sin(d*x+c))^(5/2))/d
```

Maxima [A]
time = 0.51, size = 114, normalized size = 1.01

$$-\frac{\frac{15\sqrt{2}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{a\sin(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{a\sin(dx+c)+a}}\right)}{a^{\frac{3}{2}}}+\frac{4\left(15\,(a\sin(dx+c)+a)^{2}+10\,(a\sin(dx+c)+a)a+12\,a^{2}\right)}{(a\sin(dx+c)+a)^{\frac{5}{2}}a}}{240\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -1/240*(15*sqrt(2)*log(-(sqrt(2)*sqrt(a) - sqrt(a*sin(d*x + c) + a))/(sqrt(
2)*sqrt(a) + sqrt(a*sin(d*x + c) + a)))/a^(3/2) + 4*(15*(a*sin(d*x + c) + a
)^2 + 10*(a*sin(d*x + c) + a)*a + 12*a^2)/((a*sin(d*x + c) + a)^(5/2)*a))/(
a*d)
```

Fricas [A]
time = 0.36, size = 169, normalized size = 1.50

$$\frac{15\sqrt{2}\left(3\cos(dx+c)^{2}+\left(\cos(dx+c)^{2}-4\right)\sin(dx+c)-4\right)\sqrt{a}\log\left(-\frac{a\sin(dx+c)+2\sqrt{2}\sqrt{a\sin(dx+c)+a}\sqrt{a}+3a}{\sin(dx+c)-1}\right)-4\left(15\cos(dx+c)^{2}-40\sin(dx+c)-52\right)\sqrt{a\sin(dx+c)+a}}{240\left(3a^{3}d\cos(dx+c)^{2}-4a^{3}d+\left(a^{3}d\cos(dx+c)^{2}-4a^{3}d\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/240*(15*sqrt(2)*(3*cos(d*x + c)^2 + (cos(d*x + c)^2 - 4)*sin(d*x + c) - 4
)*sqrt(a)*log(-(a*sin(d*x + c) + 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*sqrt(a)
```

```
 + 3*a)/(sin(d*x + c) - 1)) - 4*(15*cos(d*x + c)^2 - 40*sin(d*x + c) - 52)*
sqrt(a*sin(d*x + c) + a))/(3*a^3*d*cos(d*x + c)^2 - 4*a^3*d + (a^3*d*cos(d*
x + c)^2 - 4*a^3*d)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(c+dx)}{(a(\sin(c+dx)+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Integral(sec(c + d*x)/(a*(sin(c + d*x) + 1))**(5/2), x)
```

Giac [A]
time = 5.39, size = 158, normalized size = 1.40

$$\frac{\sqrt{a}\left(\frac{15\sqrt{2}\log\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{15\sqrt{2}\log\left(-\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{2\sqrt{2}\left(15\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^4+5\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^2+3\right)}{a^3\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^5\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}\right)}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 1/240*sqrt(a)*(15*sqrt(2)*log(cos(-1/4*pi + 1/2*d*x + 1/2*c) + 1)/(a^3*sgn(
cos(-1/4*pi + 1/2*d*x + 1/2*c))) - 15*sqrt(2)*log(-cos(-1/4*pi + 1/2*d*x +
1/2*c) + 1)/(a^3*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))) - 2*sqrt(2)*(15*cos(-
1/4*pi + 1/2*d*x + 1/2*c)^4 + 5*cos(-1/4*pi + 1/2*d*x + 1/2*c)^2 + 3)/(a^3*
cos(-1/4*pi + 1/2*d*x + 1/2*c)^5*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(c+d\,x)\,\left(a+a\,\sin(c+d\,x)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + a*sin(c + d*x))^(5/2)),x)
```

```
[Out] int(1/(cos(c + d*x)*(a + a*sin(c + d*x))^(5/2)), x)
```

3.193 $\int \frac{\sec^2(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=167

$$-\frac{35\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(c+dx)}{\sqrt{2}\,\sqrt{a+a\sin(c+dx)}}\right)}{128\sqrt{2}\,a^{5/2}d}-\frac{\sec(c+dx)}{6d(a+a\sin(c+dx))^{5/2}}-\frac{35\cos(c+dx)}{128ad(a+a\sin(c+dx))^{3/2}}-\frac{7}{48ad(a}$$

```
[Out] -1/6*sec(d*x+c)/d/(a+a*sin(d*x+c))^(5/2)-35/128*cos(d*x+c)/a/d/(a+a*sin(d*x
+c))^(3/2)-7/48*sec(d*x+c)/a/d/(a+a*sin(d*x+c))^(3/2)-35/256*arctanh(1/2*co
s(d*x+c)*a^(1/2)*2^(1/2)/(a+a*sin(d*x+c))^(1/2))*2^(1/2)/a^(5/2)/d+35/96*se
c(d*x+c)/a^2/d/(a+a*sin(d*x+c))^(1/2)
```

Rubi [A]

time = 0.16, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2760, 2766, 2729, 2728, 212}

$$-\frac{35\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(c+dx)}{\sqrt{2}\,\sqrt{a\sin(c+dx)+a}}\right)}{128\sqrt{2}\,a^{5/2}d}+\frac{35\sec(c+dx)}{96a^2d\sqrt{a\sin(c+dx)+a}}-\frac{35\cos(c+dx)}{128ad(a\sin(c+dx)+a)^{3/2}}-\frac{7\sec(c+dx)}{48ad(a\sin(c+dx)+a)^{3/2}}-\frac{\sec(c+dx)}{6d(a\sin(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-35*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/(1
28*Sqrt[2]*a^(5/2)*d) - Sec[c + d*x]/(6*d*(a + a*Sin[c + d*x])^(5/2)) - (35
*Cos[c + d*x])/(128*a*d*(a + a*Sin[c + d*x])^(3/2)) - (7*Sec[c + d*x])/(48*
a*d*(a + a*Sin[c + d*x])^(3/2)) + (35*Sec[c + d*x])/(96*a^2*d*Sqrt[a + a*Si
n[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
```

```
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2760

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*(2*m + p + 1))), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2766

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(a*f*g*(p + 1)*S
qrt[a + b*Sin[e + f*x]])), x] + Dist[a*((2*p + 1)/(2*g^2*(p + 1))), Int[(g*
Cos[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e,
 f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx = -\frac{\sec(c+dx)}{6d(a+a\sin(c+dx))^{5/2}} + \frac{7\int \frac{\sec^2(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx}{12a}$$

$$= -\frac{\sec(c+dx)}{6d(a+a\sin(c+dx))^{5/2}} - \frac{7\sec(c+dx)}{48ad(a+a\sin(c+dx))^{3/2}} + \frac{35\int \frac{\sec^2(c+dx}{\sqrt{a+a\sin(c}}}{96a^2}$$

$$= -\frac{\sec(c+dx)}{6d(a+a\sin(c+dx))^{5/2}} - \frac{7\sec(c+dx)}{48ad(a+a\sin(c+dx))^{3/2}} + \frac{35\sec(c+dx}{96a^2d\sqrt{a+a\sin(c}}$$

$$= -\frac{\sec(c+dx)}{6d(a+a\sin(c+dx))^{5/2}} - \frac{35\cos(c+dx)}{128ad(a+a\sin(c+dx))^{3/2}} - \frac{7\sec(c+d}{48ad(a+a\sin(c-}$$

$$= -\frac{\sec(c+dx)}{6d(a+a\sin(c+dx))^{5/2}} - \frac{35\cos(c+dx)}{128ad(a+a\sin(c+dx))^{3/2}} - \frac{7\sec(c+d}{48ad(a+a\sin(c-}$$

$$= -\frac{35\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{128\sqrt{2}\,a^{5/2}d} - \frac{\sec(c+dx)}{6d(a+a\sin(c+dx))^{5/2}} - \frac{}{128ad(}$$

Mathematica [C] Result contains complex when optimal does not.

time = 0.27, size = 284, normalized size = 1.70

$$\frac{-32+\frac{64\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)}+88\sin\left(\frac{1}{2}(c+dx)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)-44\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2+114\sin\left(\frac{1}{2}(c+dx)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3-57\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^4+(105+105i)(-1)^{3/4}\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(c+dx)\right)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^5+\frac{48\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^5}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)}}{384d(a(1+\sin(c+dx)))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-32 + (64*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) + 88*Sin
[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) - 44*(Cos[(c + d*x)/2]
+ Sin[(c + d*x)/2])^2 + 114*Sin[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c + d
*x)/2])^3 - 57*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4 + (105 + 105*I)*(-1)
^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/4])]*(Cos[(c + d*
x)/2] + Sin[(c + d*x)/2])^5 + (48*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5)/
(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]))/(384*d*(a*(1 + Sin[c + d*x]))^(5/2))
```

Maple [A]
time = 0.64, size = 266, normalized size = 1.59

method	result
default	$-\frac{\left(210a^{\frac{7}{2}}-105\sqrt{a-a\sin(dx+c)}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)a^3\right)\sin(dx+c)\left(\cos^2(dx+c)\right)+\left(\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+a*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/768/a^(11/2)*((210*a^(7/2)-105*(a-a*sin(d*x+c))^(1/2)*2^(1/2)*arctanh(1/
2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^3)*sin(d*x+c)*cos(d*x+c)^2+(-44
8*a^(7/2)+420*(a-a*sin(d*x+c))^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(
1/2)*2^(1/2)/a^(1/2))*a^3)*sin(d*x+c)+(490*a^(7/2)-315*(a-a*sin(d*x+c))^(1/
2)*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^3)*cos(d*x
+c)^2-320*a^(7/2)+420*(a-a*sin(d*x+c))^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(d
*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^3)/(1+sin(d*x+c))^2/cos(d*x+c)/(a+a*sin(d*x
+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^2/(a*sin(d*x + c) + a)^(5/2), x)
```

Fricas [A]
time = 0.37, size = 280, normalized size = 1.68

$$\frac{105 \sqrt{2} \left(3 \cos\left(dx+c\right)^{3} + \left(\cos\left(dx+c\right)^{3} - 4 \cos\left(dx+c\right)\right) \sin\left(dx+c\right) - 4 \cos\left(dx+c\right)\right) \sqrt{a} \log\left(-\frac{a \cos(dx+c)^{2} - 2 \sqrt{2} \sqrt{a \sin\left(dx+c\right)+a} \sqrt{a} (\cos(dx+c) - \sin(dx+c) + 1) + 3 a \cos(dx+c) - (a \cos(dx+c) - 2 a) \sin(dx+c) + 2 a}{\cos(dx+c)^{2} - (\cos(dx+c)+2) \sin(dx+c) - \cos(dx+c) - 2}\right) + 4 \left(245 \cos\left(dx+c\right)^{2} + 7 \left(15 \cos\left(dx+c\right)^{2} - 32\right) \sin\left(dx+c\right) - 160\right) \sqrt{a \sin\left(dx+c\right)+a}}{1536 \left(3 a^{3} d \cos\left(dx+c\right)^{3} - 4 a^{3} d \cos\left(dx+c\right) + \left(a^{3} d \cos\left(dx+c\right)^{3} - 4 a^{3} d \cos\left(dx+c\right)\right) \sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/1536*(105*sqrt(2)*(3*cos(d*x + c)^3 + (cos(d*x + c)^3 - 4*cos(d*x + c))*s
in(d*x + c) - 4*cos(d*x + c))*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sq
rt(a*sin(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - sin(d*x + c) + 1) + 3*a*cos(
d*x + c) - (a*cos(d*x + c) - 2*a)*sin(d*x + c) + 2*a)/(cos(d*x + c)^2 - (co
s(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2)) + 4*(245*cos(d*x + c)^2 +
 7*(15*cos(d*x + c)^2 - 32)*sin(d*x + c) - 160)*sqrt(a*sin(d*x + c) + a))/(
3*a^3*d*cos(d*x + c)^3 - 4*a^3*d*cos(d*x + c) + (a^3*d*cos(d*x + c)^3 - 4*a
^3*d*cos(d*x + c))*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{2}\left(c+dx\right)}{\left(a \left(\sin\left(c+dx\right)+1\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Integral(sec(c + d*x)**2/(a*(sin(c + d*x) + 1))**(5/2), x)
```

Giac [A]
time = 5.53, size = 218, normalized size = 1.31

$$\frac{\sqrt{a} \left(\frac{105 \sqrt{2} \log(\sin(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c) + 1)}{a^{3} \mathrm{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c))} - \frac{105 \sqrt{2} \log(-\sin(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c) + 1)}{a^{3} \mathrm{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c))} - \frac{96 \sqrt{2}}{a^{3} \mathrm{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c)) \sin(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c)} - \frac{2 \left(57 \sqrt{2} \sin(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c)^{5} - 136 \sqrt{2} \sin(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c)^{3} + 87 \sqrt{2} \sin(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c)\right)}{\left(\sin(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c)^{2} - 1\right)^{3} a^{3} \mathrm{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c))}\right)}{1536 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 1/1536*sqrt(a)*(105*sqrt(2)*log(sin(-1/4*pi + 1/2*d*x + 1/2*c) + 1)/(a^3*sg
n(cos(-1/4*pi + 1/2*d*x + 1/2*c))) - 105*sqrt(2)*log(-sin(-1/4*pi + 1/2*d*x
 + 1/2*c) + 1)/(a^3*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))) - 96*sqrt(2)/(a^3*
sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*d*x + 1/2*c)) - 2*(57
*sqrt(2)*sin(-1/4*pi + 1/2*d*x + 1/2*c)^5 - 136*sqrt(2)*sin(-1/4*pi + 1/2*d
*x + 1/2*c)^3 + 87*sqrt(2)*sin(-1/4*pi + 1/2*d*x + 1/2*c))/((sin(-1/4*pi +
1/2*d*x + 1/2*c)^2 - 1)^3*a^3*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(c+d\,x)^2\,(a+a\,\sin(c+d\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + a*sin(c + d*x))^(5/2)),x)
[Out] int(1/(cos(c + d*x)^2*(a + a*sin(c + d*x))^(5/2)), x)
```

3.194 $\int \frac{\sec^3(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=185

$$\frac{9\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{32\sqrt{2}\,a^{5/2}d}-\frac{\sec^2(c+dx)}{7d(a+a\sin(c+dx))^{5/2}}-\frac{3}{16ad(a+a\sin(c+dx))^{3/2}}-\frac{9\sec^2(c+d}{70ad(a+a\sin(c+}$$

```
[Out] -1/7*sec(d*x+c)^2/d/(a+a*sin(d*x+c))^(5/2)-3/16/a/d/(a+a*sin(d*x+c))^(3/2)-
9/70*sec(d*x+c)^2/a/d/(a+a*sin(d*x+c))^(3/2)+9/64*arctanh(1/2*(a+a*sin(d*x+
c))^(1/2)*2^(1/2)/a^(1/2))/a^(5/2)/d*2^(1/2)-9/32/a^2/d/(a+a*sin(d*x+c))^(1
/2)+9/40*sec(d*x+c)^2/a^2/d/(a+a*sin(d*x+c))^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2760, 2766, 2746, 53, 65, 212}

$$\frac{9\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{2}\sqrt{a}}\right)}{32\sqrt{2}\,a^{5/2}d}-\frac{9}{32a^2d\sqrt{a\sin(c+dx)+a}}+\frac{9\sec^2(c+dx)}{40a^2d\sqrt{a\sin(c+dx)+a}}-\frac{3}{16ad(a\sin(c+dx)+a)^{3/2}}-\frac{9\sec^2(c+dx)}{70ad(a\sin(c+dx)+a)^{3/2}}-\frac{\sec^2(c+dx)}{7d(a\sin(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (9*ArcTanh[Sqrt[a + a*Sin[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(32*Sqrt[2]*a^(5/2)
*d) - Sec[c + d*x]^2/(7*d*(a + a*Sin[c + d*x])^(5/2)) - 3/(16*a*d*(a + a*Si
n[c + d*x])^(3/2)) - (9*Sec[c + d*x]^2)/(70*a*d*(a + a*Sin[c + d*x])^(3/2))
 - 9/(32*a^2*d*Sqrt[a + a*Sin[c + d*x]]) + (9*Sec[c + d*x]^2)/(40*a^2*d*Sqr
t[a + a*Sin[c + d*x]])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2760

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*(2*m + p + 1))), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2766

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(a*f*g*(p + 1)*S
qrt[a + b*Sin[e + f*x]])), x] + Dist[a*((2*p + 1)/(2*g^2*(p + 1))), Int[(g*
Cos[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e,
 f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx = -\frac{\sec^2(c+dx)}{7d(a+a\sin(c+dx))^{5/2}} + \frac{9\int \frac{\sec^3(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx}{14a}$$

$$= -\frac{\sec^2(c+dx)}{7d(a+a\sin(c+dx))^{5/2}} - \frac{9\sec^2(c+dx)}{70ad(a+a\sin(c+dx))^{3/2}} + \frac{9\int \frac{\sec^3(c+dx)}{\sqrt{a+a\sin(c+}}}{20a^2}$$

$$= -\frac{\sec^2(c+dx)}{7d(a+a\sin(c+dx))^{5/2}} - \frac{9\sec^2(c+dx)}{70ad(a+a\sin(c+dx))^{3/2}} + \frac{9\sec^2(c+dx}{40a^2d\sqrt{a+a\sin(c}}$$

$$= -\frac{\sec^2(c+dx)}{7d(a+a\sin(c+dx))^{5/2}} - \frac{9\sec^2(c+dx)}{70ad(a+a\sin(c+dx))^{3/2}} + \frac{9\sec^2(c+dx}{40a^2d\sqrt{a+a\sin(c}}$$

$$= -\frac{\sec^2(c+dx)}{7d(a+a\sin(c+dx))^{5/2}} - \frac{3}{16ad(a+a\sin(c+dx))^{3/2}} - \frac{9\sec^2(c+dx}{70ad(a+a\sin(c+}$$

$$= -\frac{\sec^2(c+dx)}{7d(a+a\sin(c+dx))^{5/2}} - \frac{3}{16ad(a+a\sin(c+dx))^{3/2}} - \frac{9\sec^2(c+dx}{70ad(a+a\sin(c+}$$

$$= -\frac{\sec^2(c+dx)}{7d(a+a\sin(c+dx))^{5/2}} - \frac{3}{16ad(a+a\sin(c+dx))^{3/2}} - \frac{9\sec^2(c+dx}{70ad(a+a\sin(c+}$$

$$= \frac{9\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{32\sqrt{2}\,a^{5/2}d} - \frac{\sec^2(c+dx)}{7d(a+a\sin(c+dx))^{5/2}} - \frac{3}{16ad(a+a\sin}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.05, size = 42, normalized size = 0.23

$$-\frac{a\,{}_2F_1\left(-\frac{7}{2},2;-\frac{5}{2};\frac{1}{2}(1+\sin(c+dx))\right)}{14d(a+a\sin(c+dx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] -1/14*(a*Hypergeometric2F1[-7/2, 2, -5/2, (1 + Sin[c + d*x])/2])/(d*(a + a*
Sin[c + d*x])^(7/2))
```

Maple [A]
time = 0.69, size = 141, normalized size = 0.76

method	result
default	$\frac{2a^{3}\left(-\frac{\frac{\sqrt{a+a\sin\left(dx+c\right)}}{4a\sin\left(dx+c\right)-4a}-\frac{9\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+a\sin\left(dx+c\right)}\,\sqrt{2}}{2\sqrt{a}}\right)}{8\sqrt{a}}}{16a^{5}}-\frac{1}{8a^{5}\sqrt{a+a\sin\left(dx+c\right)}}-\frac{}{16a}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+a*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*a^3*(-1/16/a^5*(1/4*(a+a*sin(d*x+c))^(1/2)/(a*sin(d*x+c)-a)-9/8*2^(1/2)/a
^(1/2)*arctanh(1/2*(a+a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2)))-1/8/a^5/(a+a*si
n(d*x+c))^(1/2)-1/16/a^4/(a+a*sin(d*x+c))^(3/2)-1/20/a^3/(a+a*sin(d*x+c))^(
5/2)-1/28/a^2/(a+a*sin(d*x+c))^(7/2))/d
```

Maxima [A]
time = 0.52, size = 167, normalized size = 0.90

$$-\frac{\frac{4\left(315\left(a\sin\left(dx+c\right)+a\right)^{4}-420\left(a\sin\left(dx+c\right)+a\right)^{3}a-168\left(a\sin\left(dx+c\right)+a\right)^{2}a^{2}-144\left(a\sin\left(dx+c\right)+a\right)a^{3}-160\,a^{4}\right)}{\left(a\sin\left(dx+c\right)+a\right)^{\frac{9}{2}}a-2\left(a\sin\left(dx+c\right)+a\right)^{\frac{7}{2}}a^{2}}+\frac{315\sqrt{2}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{a\sin\left(dx+c\right)+a}}{\sqrt{2}\sqrt{a}+\sqrt{a\sin\left(dx+c\right)+a}}\right)}{a^{\frac{3}{2}}}}{4480\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -1/4480*(4*(315*(a*sin(d*x + c) + a)^4 - 420*(a*sin(d*x + c) + a)^3*a - 168
*(a*sin(d*x + c) + a)^2*a^2 - 144*(a*sin(d*x + c) + a)*a^3 - 160*a^4)/((a*s
in(d*x + c) + a)^(9/2)*a - 2*(a*sin(d*x + c) + a)^(7/2)*a^2) + 315*sqrt(2)*
log(-(sqrt(2)*sqrt(a) - sqrt(a*sin(d*x + c) + a))/(sqrt(2)*sqrt(a) + sqrt(a
*sin(d*x + c) + a)))/a^(3/2))/(a*d)
```

Fricas [A]
time = 0.37, size = 225, normalized size = 1.22

$$\frac{315\sqrt{2}\left(3\cos\left(dx+c\right)^{4}-4\cos\left(dx+c\right)^{2}+\left(\cos\left(dx+c\right)^{4}-4\cos\left(dx+c\right)^{2}\right)\sin\left(dx+c\right)\right)\sqrt{a}\log\left(-\frac{a\sin\left(dx+c\right)+2\sqrt{2}\sqrt{a\sin\left(dx+c\right)+a}\sqrt{a}+3a}{\sin\left(dx+c\right)-1}\right)-4\left(315\cos\left(dx+c\right)^{4}-1092\cos\left(dx+c\right)^{2}-120\left(7\cos\left(dx+c\right)^{2}-3\right)\sin\left(dx+c\right)+200\right)\sqrt{a\sin\left(dx+c\right)+a}}{4480\left(3a^{3}d\cos\left(dx+c\right)^{4}-4a^{3}d\cos\left(dx+c\right)^{2}+\left(a^{3}d\cos\left(dx+c\right)^{4}-4a^{3}d\cos\left(dx+c\right)^{2}\right)\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/4480*(315*sqrt(2)*(3*cos(d*x + c)^4 - 4*cos(d*x + c)^2 + (cos(d*x + c)^4
- 4*cos(d*x + c)^2)*sin(d*x + c))*sqrt(a)*log(-(a*sin(d*x + c) + 2*sqrt(2)*
sqrt(a*sin(d*x + c) + a)*sqrt(a) + 3*a)/(sin(d*x + c) - 1)) - 4*(315*cos(d*
```

```
x + c)^4 - 1092*cos(d*x + c)^2 - 120*(7*cos(d*x + c)^2 - 3)*sin(d*x + c) +
200)*sqrt(a*sin(d*x + c) + a))/(3*a^3*d*cos(d*x + c)^4 - 4*a^3*d*cos(d*x +
c)^2 + (a^3*d*cos(d*x + c)^4 - 4*a^3*d*cos(d*x + c)^2)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3 (c + dx)}{(a (\sin (c + dx) + 1))^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Integral(sec(c + d*x)**3/(a*(sin(c + d*x) + 1))**(5/2), x)
```

Giac [A]
time = 4.88, size = 227, normalized size = 1.23

$$\frac{\sqrt{a} \left(\frac{315 \sqrt{2} \log\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right) + 1\right)}{a^3 \mathrm{sgn}\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right)\right)} - \frac{315 \sqrt{2} \log\left(-\cos\left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right) + 1\right)}{a^3 \mathrm{sgn}\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right)\right)} - \frac{70 \sqrt{2} \cos\left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right)}{\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right)^2 - 1\right) a^3 \mathrm{sgn}\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right)\right)} - \frac{4 \sqrt{2} \left(140 \cos\left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right)^6 + 35 \cos\left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right)^4 + 14 \cos\left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right)^2 + 5\right)}{a^3 \cos\left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right)^7 \mathrm{sgn}\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right)\right)}\right)}{4480 \, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 1/4480*sqrt(a)*(315*sqrt(2)*log(cos(-1/4*pi + 1/2*d*x + 1/2*c) + 1)/(a^3*sg
n(cos(-1/4*pi + 1/2*d*x + 1/2*c))) - 315*sqrt(2)*log(-cos(-1/4*pi + 1/2*d*x
 + 1/2*c) + 1)/(a^3*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))) - 70*sqrt(2)*cos(-
1/4*pi + 1/2*d*x + 1/2*c)/((cos(-1/4*pi + 1/2*d*x + 1/2*c)^2 - 1)*a^3*sgn(c
os(-1/4*pi + 1/2*d*x + 1/2*c))) - 4*sqrt(2)*(140*cos(-1/4*pi + 1/2*d*x + 1/
2*c)^6 + 35*cos(-1/4*pi + 1/2*d*x + 1/2*c)^4 + 14*cos(-1/4*pi + 1/2*d*x + 1
/2*c)^2 + 5)/(a^3*cos(-1/4*pi + 1/2*d*x + 1/2*c)^7*sgn(cos(-1/4*pi + 1/2*d*
x + 1/2*c))))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\cos (c + d\,x)}^3\,{\left(a + a\,\sin (c + d\,x)\right)}^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + a*sin(c + d*x))^(5/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^3*(a + a*sin(c + d*x))^(5/2)), x)
```

3.195 $\int \frac{\sec^4(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=233

$$-\frac{1155\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{4096\sqrt{2}\,a^{5/2}d}-\frac{\sec^3(c+dx)}{8d(a+a\sin(c+dx))^{5/2}}-\frac{1155\cos(c+dx)}{4096ad(a+a\sin(c+dx))^{3/2}}-512a$$

```
[Out] -1/8*sec(d*x+c)^3/d/(a+a*sin(d*x+c))^(5/2)-1155/4096*cos(d*x+c)/a/d/(a+a*si
n(d*x+c))^(3/2)-77/512*sec(d*x+c)/a/d/(a+a*sin(d*x+c))^(3/2)-11/96*sec(d*x+
c)^3/a/d/(a+a*sin(d*x+c))^(3/2)-1155/8192*arctanh(1/2*cos(d*x+c)*a^(1/2)*2^
(1/2)/(a+a*sin(d*x+c))^(1/2))*2^(1/2)/a^(5/2)/d+385/1024*sec(d*x+c)/a^2/d/(
a+a*sin(d*x+c))^(1/2)+11/64*sec(d*x+c)^3/a^2/d/(a+a*sin(d*x+c))^(1/2)
```

Rubi [A]
time = 0.26, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2760, 2766, 2729, 2728, 212}

$$-\frac{1155\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{4096\sqrt{2}\,a^{5/2}d}+\frac{11\sec^3(c+dx)}{64a^2d\sqrt{a\sin(c+dx)+a}}+\frac{385\sec(c+dx)}{1024a^2d\sqrt{a\sin(c+dx)+a}}-\frac{1155\cos(c+dx)}{4096ad(a\sin(c+dx)+a)^{3/2}}-\frac{11\sec^3(c+dx)}{96ad(a\sin(c+dx)+a)^{3/2}}-\frac{\sec^3(c+dx)}{8d(a\sin(c+dx)+a)^{5/2}}-\frac{77\sec(c+dx)}{512ad(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-1155*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/
(4096*Sqrt[2]*a^(5/2)*d) - Sec[c + d*x]^3/(8*d*(a + a*Sin[c + d*x])^(5/2))
- (1155*Cos[c + d*x])/(4096*a*d*(a + a*Sin[c + d*x])^(3/2)) - (77*Sec[c + d
*x])/(512*a*d*(a + a*Sin[c + d*x])^(3/2)) - (11*Sec[c + d*x]^3)/(96*a*d*(a
+ a*Sin[c + d*x])^(3/2)) + (385*Sec[c + d*x])/(1024*a^2*d*Sqrt[a + a*Sin[c
+ d*x]]) + (11*Sec[c + d*x]^3)/(64*a^2*d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2760

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*(2*m + p + 1))), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2766

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(a*f*g*(p + 1)*S
qrt[a + b*Sin[e + f*x]])), x] + Dist[a*((2*p + 1)/(2*g^2*(p + 1))), Int[(g*
Cos[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e,
 f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx = -\frac{\sec^3(c+dx)}{8d(a+a\sin(c+dx))^{5/2}} + \frac{11\int \frac{\sec^4(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx}{16a}$$

$$= -\frac{\sec^3(c+dx)}{8d(a+a\sin(c+dx))^{5/2}} - \frac{11\sec^3(c+dx)}{96ad(a+a\sin(c+dx))^{3/2}} + \frac{33\int \frac{\sec^4(c+}{\sqrt{a+a\sin(}}}{64a^2}$$

$$= -\frac{\sec^3(c+dx)}{8d(a+a\sin(c+dx))^{5/2}} - \frac{11\sec^3(c+dx)}{96ad(a+a\sin(c+dx))^{3/2}} + \frac{11\sec^3(c+}{64a^2d\sqrt{a+a\sin}}$$

$$= -\frac{\sec^3(c+dx)}{8d(a+a\sin(c+dx))^{5/2}} - \frac{77\sec(c+dx)}{512ad(a+a\sin(c+dx))^{3/2}} - \frac{11\sec^3(c+}{96ad(a+a\sin(c}$$

$$= -\frac{\sec^3(c+dx)}{8d(a+a\sin(c+dx))^{5/2}} - \frac{77\sec(c+dx)}{512ad(a+a\sin(c+dx))^{3/2}} - \frac{11\sec^3(c+}{96ad(a+a\sin(c}$$

$$= -\frac{\sec^3(c+dx)}{8d(a+a\sin(c+dx))^{5/2}} - \frac{1155\cos(c+dx)}{4096ad(a+a\sin(c+dx))^{3/2}} - \frac{77\sec(c}{512ad(a+a\sin}$$

$$= -\frac{\sec^3(c+dx)}{8d(a+a\sin(c+dx))^{5/2}} - \frac{1155\cos(c+dx)}{4096ad(a+a\sin(c+dx))^{3/2}} - \frac{77\sec(c}{512ad(a+a\sin}$$

$$= -\frac{1155\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{4096\sqrt{2}\,a^{5/2}d} - \frac{\sec^3(c+dx)}{8d(a+a\sin(c+dx))^{5/2}} - \frac{}{409}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.34, size = 394, normalized size = 1.69

$$\frac{-736+\frac{768\sin\left(\frac{1}{2}(c+dx)\right)}{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3}-\frac{384}{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}+\frac{1472\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)}+2072\sin\left(\frac{1}{2}(c+dx)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)-1036\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2+3090\sin\left(\frac{1}{2}(c+dx)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3-1545\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^4+(3465+3465i)(-1)^{3/4}\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(c+dx)\right)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^5+\frac{256\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^5}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3}+\frac{1920\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^5}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)}}{12288d(a(1+\sin(c+dx)))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-736 + (768*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3 - 38
4/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + (1472*Sin[(c + d*x)/2])/(Cos[(c
 + d*x)/2] + Sin[(c + d*x)/2]) + 2072*Sin[(c + d*x)/2]*(Cos[(c + d*x)/2] +
Sin[(c + d*x)/2]) - 1036*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + 3090*Sin
[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3 - 1545*(Cos[(c + d*x)
/2] + Sin[(c + d*x)/2])^4 + (3465 + 3465*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*
(-1)^(3/4)*(-1 + Tan[(c + d*x)/4])]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5
 + (256*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5)/(Cos[(c + d*x)/2] - Sin[(c
```

```
 + d*x)/2])^3 + (1920*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5)/(Cos[(c + d*
x)/2] - Sin[(c + d*x)/2]))/(12288*d*(a*(1 + Sin[c + d*x]))^(5/2))
```

Maple [A]
time = 0.60, size = 355, normalized size = 1.52

method	result
default	$\frac{6930a^{\frac{11}{2}}\sin(dx+c)(\cos^4(dx+c))-924\left(16a^{\frac{11}{2}}+15(a-a\sin(dx+c))^{\frac{3}{2}}\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}\sqrt{2}}{2\sqrt{a}}\right)a^4\right)(\cos^2(}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+a*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24576/a^(15/2)*(6930*a^(11/2)*sin(d*x+c)*cos(d*x+c)^4-924*(16*a^(11/2)+15
*(a-a*sin(d*x+c))^(3/2)*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/
a^(1/2))*a^4)*cos(d*x+c)^2*sin(d*x+c)+(-5632*a^(11/2)+27720*(a-a*sin(d*x+c)
)^(3/2)*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^4)*si
n(d*x+c)+(16170*a^(11/2)+3465*(a-a*sin(d*x+c))^(3/2)*2^(1/2)*arctanh(1/2*(a
-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^4)*cos(d*x+c)^4-1320*(8*a^(11/2)+21
*(a-a*sin(d*x+c))^(3/2)*2^(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/
a^(1/2))*a^4)*cos(d*x+c)^2-2560*a^(11/2)+27720*(a-a*sin(d*x+c))^(3/2)*2^(1/
2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^4)/(sin(d*x+c)-1)/
(1+sin(d*x+c))^3/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^4/(a*sin(d*x + c) + a)^(5/2), x)
```

Fricas [A]
time = 0.41, size = 308, normalized size = 1.32

$$\frac{3465\sqrt{2}\left(3\cos(dx+c)^5-4\cos(dx+c)^3+\left(\cos(dx+c)^5-4\cos(dx+c)^3\right)\sin(dx+c)\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a\sin(dx+c)+a}\sqrt{a}(\cos(dx+c)-\sin(dx+c)+1)+3a\cos(dx+c)-(a\cos(dx+c)-2a)\sin(dx+c)+2a}{\cos(dx+c)^2-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2}\right)+4\left(8085\cos(dx+c)^4-5280\cos(dx+c)^2+11\left(315\cos(dx+c)^4-672\cos(dx+c)^2-256\right)\sin(dx+c)-1280\right)\sqrt{a\sin(dx+c)+a}}{49152\left(3a^3d\cos(dx+c)^5-4a^3d\cos(dx+c)^3+\left(a^3d\cos(dx+c)^5-4a^3d\cos(dx+c)^3\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/49152*(3465*sqrt(2)*(3*cos(d*x + c)^5 - 4*cos(d*x + c)^3 + (cos(d*x + c)^
5 - 4*cos(d*x + c)^3)*sin(d*x + c))*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt
```

```
(2)*sqrt(a*sin(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - sin(d*x + c) + 1) + 3*
a*cos(d*x + c) - (a*cos(d*x + c) - 2*a)*sin(d*x + c) + 2*a)/(cos(d*x + c)^2
 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2)) + 4*(8085*cos(d*x +
 c)^4 - 5280*cos(d*x + c)^2 + 11*(315*cos(d*x + c)^4 - 672*cos(d*x + c)^2 -
 256)*sin(d*x + c) - 1280)*sqrt(a*sin(d*x + c) + a))/(3*a^3*d*cos(d*x + c)^
5 - 4*a^3*d*cos(d*x + c)^3 + (a^3*d*cos(d*x + c)^5 - 4*a^3*d*cos(d*x + c)^3
)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^4(c+dx)}{(a(\sin(c+dx)+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+a*sin(d*x+c))**(5/2),x)
[Out] Integral(sec(c + d*x)**4/(a*(sin(c + d*x) + 1))**(5/2), x)
```

Giac [A]
time = 5.93, size = 255, normalized size = 1.09

$$\frac{\sqrt{a}\left(\frac{3465\sqrt{2}\log\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{3465\sqrt{2}\log\left(-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{256\sqrt{2}\left(15\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)}{a^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^3}-\frac{2\left(1545\sqrt{2}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^7-5153\sqrt{2}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^5+5855\sqrt{2}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^3-2295\sqrt{2}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^4a^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}\right)}{49152\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
[Out] 1/49152*sqrt(a)*(3465*sqrt(2)*log(sin(-1/4*pi + 1/2*d*x + 1/2*c) + 1)/(a^3*
sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))) - 3465*sqrt(2)*log(-sin(-1/4*pi + 1/2*
d*x + 1/2*c) + 1)/(a^3*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))) - 256*sqrt(2)*(
15*sin(-1/4*pi + 1/2*d*x + 1/2*c)^2 + 1)/(a^3*sgn(cos(-1/4*pi + 1/2*d*x + 1
/2*c))*sin(-1/4*pi + 1/2*d*x + 1/2*c)^3) - 2*(1545*sqrt(2)*sin(-1/4*pi + 1/
2*d*x + 1/2*c)^7 - 5153*sqrt(2)*sin(-1/4*pi + 1/2*d*x + 1/2*c)^5 + 5855*sqr
t(2)*sin(-1/4*pi + 1/2*d*x + 1/2*c)^3 - 2295*sqrt(2)*sin(-1/4*pi + 1/2*d*x
+ 1/2*c))/((sin(-1/4*pi + 1/2*d*x + 1/2*c)^2 - 1)^4*a^3*sgn(cos(-1/4*pi + 1
/2*d*x + 1/2*c))))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos(c+d\,x)^4\,\left(a+a\,\sin(c+d\,x)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a + a*sin(c + d*x))^(5/2)),x)
[Out] int(1/(cos(c + d*x)^4*(a + a*sin(c + d*x))^(5/2)), x)
```

3.196 $\int (e\cos(c+dx))^{7/2}(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=124

$$-\frac{2a(e\cos(c+dx))^{9/2}}{9de}+\frac{10ae^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{e\cos(c+dx)}}+\frac{10ae^3\sqrt{e\cos(c+dx)}\,\sin(c+dx)}{21d}+\frac{2ae(e\cos$$

```
[Out] -2/9*a*(e*cos(d*x+c))^(9/2)/d/e+2/7*a*e*(e*cos(d*x+c))^(5/2)*sin(d*x+c)/d+1
0/21*a*e^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/
2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(e*cos(d*x+c))^(1/2)+10/21*a*e^3*s
in(d*x+c)*(e*cos(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.07, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2748, 2715, 2721, 2720}

$$\frac{10ae^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{e\cos(c+dx)}}+\frac{10ae^3\sin(c+dx)\sqrt{e\cos(c+dx)}}{21d}-\frac{2a(e\cos(c+dx))^{9/2}}{9de}+\frac{2ae\sin(c+dx)(e\cos(c+dx))^{5/2}}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(7/2)*(a + a*Sin[c + d*x]),x]
```

```
[Out] (-2*a*(e*Cos[c + d*x])^(9/2))/(9*d*e) + (10*a*e^4*Sqrt[Cos[c + d*x]]*Ellipt
icF[(c + d*x)/2, 2])/(21*d*Sqrt[e*Cos[c + d*x]]) + (10*a*e^3*Sqrt[e*Cos[c +
 d*x]]*Sin[c + d*x])/(21*d) + (2*a*e*(e*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(
7*d)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\int (e\cos(c+dx))^{7/2}(a+a\sin(c+dx))\,dx = -\frac{2a(e\cos(c+dx))^{9/2}}{9de} + a\int (e\cos(c+dx))^{7/2}\,dx$$
$$= -\frac{2a(e\cos(c+dx))^{9/2}}{9de} + \frac{2ae(e\cos(c+dx))^{5/2}\sin(c+dx)}{7d} + \frac{1}{7}$$
$$= -\frac{2a(e\cos(c+dx))^{9/2}}{9de} + \frac{10ae^3\sqrt{e\cos(c+dx)}\,\sin(c+dx)}{21d} +$$
$$= -\frac{2a(e\cos(c+dx))^{9/2}}{9de} + \frac{10ae^3\sqrt{e\cos(c+dx)}\,\sin(c+dx)}{21d} +$$
$$= -\frac{2a(e\cos(c+dx))^{9/2}}{9de} + \frac{10ae^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{e\cos(c+dx)}}$$

Mathematica [A]
time = 0.40, size = 98, normalized size = 0.79

$$-\frac{ae^3\sqrt{e\cos(c+dx)}\left(-120F\left(\frac{1}{2}(c+dx)\middle|2\right)+\sqrt{\cos(c+dx)}\,(21+28\cos(2(c+dx))+7\cos(4(c+dx))-138\sin(c+dx)-18\sin(3(c+dx)))\right)}{252d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(7/2)*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/252*(a*e^3*Sqrt[e*Cos[c + d*x]]*(-120*EllipticF[(c + d*x)/2, 2] + Sqrt[C
os[c + d*x]]*(21 + 28*Cos[2*(c + d*x)] + 7*Cos[4*(c + d*x)] - 138*Sin[c + d
*x] - 18*Sin[3*(c + d*x)])))/(d*Sqrt[Cos[c + d*x]])
```

Maple [A]
time = 1.69, size = 249, normalized size = 2.01

method	result

default	$-\frac{2a\,e^{4}\left(-224\left(\sin^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+144\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+560\left(\sin^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-216\left(\sin^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-560\left(\sin^{7}\left(\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(7/2)*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/63/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*a*e^4*(-224*si
n(1/2*d*x+1/2*c)^11+144*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+560*sin(1/2
*d*x+1/2*c)^9-216*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-560*sin(1/2*d*x+1
/2*c)^7+168*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+280*sin(1/2*d*x+1/2*c)^
5+15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))-48*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-70
*sin(1/2*d*x+1/2*c)^3+7*sin(1/2*d*x+1/2*c))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] e^(7/2)*integrate((a*sin(d*x + c) + a)*cos(d*x + c)^(7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 109, normalized size = 0.88

$$\frac{-15i\sqrt{2}\,ae^{\frac{7}{2}}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+15i\sqrt{2}\,ae^{\frac{7}{2}}\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-2\left(7\,a\cos(dx+c)^{4}e^{\frac{7}{2}}-3\left(3\,a\cos(dx+c)^{2}e^{\frac{7}{2}}+5\,ae^{\frac{7}{2}}\right)\sin(dx+c)\right)\sqrt{\cos(dx+c)}}{63\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/63*(-15*I*sqrt(2)*a*e^(7/2)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*s
in(d*x + c)) + 15*I*sqrt(2)*a*e^(7/2)*weierstrassPInverse(-4, 0, cos(d*x +
c) - I*sin(d*x + c)) - 2*(7*a*cos(d*x + c)^4*e^(7/2) - 3*(3*a*cos(d*x + c)^
2*e^(7/2) + 5*a*e^(7/2))*sin(d*x + c))*sqrt(cos(d*x + c)))/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(7/2)*(a+a*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4846 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)*cos(d*x + c)^(7/2)*e^(7/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (e\,\cos\left(c+d\,x\right))^{7/2}\,\left(a+a\,\sin\left(c+d\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(7/2)*(a + a*sin(c + d*x)),x)
```

```
[Out] int((e*cos(c + d*x))^(7/2)*(a + a*sin(c + d*x)), x)
```

3.197 $\int (e\cos(c+dx))^{5/2}(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=95

$$-\frac{2a(e\cos(c+dx))^{7/2}}{7de}+\frac{6ae^2\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}}+\frac{2ae(e\cos(c+dx))^{3/2}\sin(c+dx)}{5d}$$

```
[Out] -2/7*a*(e*cos(d*x+c))^(7/2)/d/e+2/5*a*e*(e*cos(d*x+c))^(3/2)*sin(d*x+c)/d+6
/5*a*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*
d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2748, 2715, 2721, 2719}

$$\frac{6ae^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}-\frac{2a(e\cos(c+dx))^{7/2}}{7de}+\frac{2ae\sin(c+dx)(e\cos(c+dx))^{3/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(5/2)*(a + a*Sin[c + d*x]),x]
```

```
[Out] (-2*a*(e*Cos[c + d*x])^(7/2))/(7*d*e) + (6*a*e^2*Sqrt[e*Cos[c + d*x]]*Ellip
ticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]) + (2*a*e*(e*Cos[c + d*x])^(3
/2)*Sin[c + d*x])/(5*d)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\begin{aligned}\int (e\cos(c+dx))^{5/2}(a+a\sin(c+dx))\,dx &= -\frac{2a(e\cos(c+dx))^{7/2}}{7de}+a\int (e\cos(c+dx))^{5/2}\,dx\\ &= -\frac{2a(e\cos(c+dx))^{7/2}}{7de}+\frac{2ae(e\cos(c+dx))^{3/2}\sin(c+dx)}{5d}+\frac{1}{5}\cdots\\ &= -\frac{2a(e\cos(c+dx))^{7/2}}{7de}+\frac{2ae(e\cos(c+dx))^{3/2}\sin(c+dx)}{5d}+\frac{(\cdots}{\cdots}\\ &= -\frac{2a(e\cos(c+dx))^{7/2}}{7de}+\frac{6ae^2\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}}\cdots\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.79, size = 264, normalized size = 2.78

$$\frac{ae^3\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\left(-154\cos(dx)-182\cos(2c+dx)+14\cos(2c+3dx)-14\cos(4c+3dx)-30\sin(c)+168\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2idx}(\cos(c)+i\sin(c))^2\right)(\cos(dx)-i\sin(dx))\sqrt{1+\cos(2(c+dx))+i\sin(2(c+dx))}+56\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2idx}(\cos(c)+i\sin(c))^2\right)(\cos(dx)+i\sin(dx))\sqrt{1+\cos(2(c+dx))+i\sin(2(c+dx))}+20\sin(c+2dx)-20\sin(3c+2dx)+5\sin(3c+4dx)-5\sin(5c+4dx)\right)}{560d\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(5/2)*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*e^3*Csc[c/2]*Sec[c/2]*(-154*Cos[d*x] - 182*Cos[2*c + d*x] + 14*Cos[2*c +
 3*d*x] - 14*Cos[4*c + 3*d*x] - 30*Sin[c] + 168*Hypergeometric2F1[-1/4, 1/2
, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*(Cos[d*x] - I*Sin[d*x])*Sqrt
[1 + Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)]] + 56*Hypergeometric2F1[1/2, 3/4
, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*(Cos[d*x] + I*Sin[d*x])*Sqrt
[1 + Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)]] + 20*Sin[c + 2*d*x] - 20*Sin[3*
c + 2*d*x] + 5*Sin[3*c + 4*d*x] - 5*Sin[5*c + 4*d*x]))/(560*d*Sqrt[e*Cos[c
+ d*x]])
```

Maple [A]
time = 1.77, size = 214, normalized size = 2.25

method	result

default	$\frac{2a\,e^{3}\left(-80\left(\sin^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+56\left(\sin^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+160\left(\sin^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-56\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-120\left(\sin^{5}\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}{35\sin\left(\frac{dx}{2}+\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(5/2)*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/35/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*a*e^3*(-80*sin(
1/2*d*x+1/2*c)^9+56*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+160*sin(1/2*d*x
+1/2*c)^7-56*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-120*sin(1/2*d*x+1/2*c)
^5+21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))+14*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+4
0*sin(1/2*d*x+1/2*c)^3-5*sin(1/2*d*x+1/2*c))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] e^(5/2)*integrate((a*sin(d*x + c) + a)*cos(d*x + c)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 105, normalized size = 1.11

$$\frac{21i\sqrt{2}\,ae^{\frac{5}{2}}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)))-21i\sqrt{2}\,ae^{\frac{5}{2}}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c)))-2\left(5\,a\cos(dx+c)^{3}e^{\frac{5}{2}}-7\,a\cos(dx+c)\,e^{\frac{5}{2}}\sin(dx+c)\right)\sqrt{\cos(dx+c)}}{35\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/35*(21*I*sqrt(2)*a*e^(5/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
 0, cos(d*x + c) + I*sin(d*x + c))) - 21*I*sqrt(2)*a*e^(5/2)*weierstrassZet
a(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(5*
a*cos(d*x + c)^3*e^(5/2) - 7*a*cos(d*x + c)*e^(5/2)*sin(d*x + c))*sqrt(cos(
d*x + c)))/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(5/2)*(a+a*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4849 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)*cos(d*x + c)^(5/2)*e^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (e\,\cos\left(c+d\,x\right))^{5/2}\,\left(a+a\,\sin\left(c+d\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(5/2)*(a + a*sin(c + d*x)),x)
```

```
[Out] int((e*cos(c + d*x))^(5/2)*(a + a*sin(c + d*x)), x)
```

3.198 $\int (e\cos(c+dx))^{3/2}(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=95

$$-\frac{2a(e\cos(c+dx))^{5/2}}{5de}+\frac{2ae^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{e\cos(c+dx)}}+\frac{2ae\sqrt{e\cos(c+dx)}\,\sin(c+dx)}{3d}$$

```
[Out] -2/5*a*(e*cos(d*x+c))^(5/2)/d/e+2/3*a*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(
1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(e*
cos(d*x+c))^(1/2)+2/3*a*e*sin(d*x+c)*(e*cos(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.05, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2748, 2715, 2721, 2720}

$$\frac{2ae^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{e\cos(c+dx)}}-\frac{2a(e\cos(c+dx))^{5/2}}{5de}+\frac{2ae\sin(c+dx)\sqrt{e\cos(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x]),x]
```

```
[Out] (-2*a*(e*Cos[c + d*x])^(5/2))/(5*d*e) + (2*a*e^2*Sqrt[Cos[c + d*x]]*Ellipti
cF[(c + d*x)/2, 2])/(3*d*Sqrt[e*Cos[c + d*x]]) + (2*a*e*Sqrt[e*Cos[c + d*x]
]*Sin[c + d*x])/(3*d)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\int (e\cos(c+dx))^{3/2}(a+a\sin(c+dx))\,dx = -\frac{2a(e\cos(c+dx))^{5/2}}{5de} + a\int (e\cos(c+dx))^{3/2}\,dx$$
$$= -\frac{2a(e\cos(c+dx))^{5/2}}{5de} + \frac{2ae\sqrt{e\cos(c+dx)}\,\sin(c+dx)}{3d} + \frac{1}{3}($$
$$= -\frac{2a(e\cos(c+dx))^{5/2}}{5de} + \frac{2ae\sqrt{e\cos(c+dx)}\,\sin(c+dx)}{3d} + \frac{(a}{}$$
$$= -\frac{2a(e\cos(c+dx))^{5/2}}{5de} + \frac{2ae^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{e\cos(c+dx)}}$$

Mathematica [A]
time = 0.25, size = 75, normalized size = 0.79

$$-\frac{a(e\cos(c+dx))^{3/2}\left(-10F\left(\frac{1}{2}(c+dx)\middle|2\right)+\sqrt{\cos(c+dx)}\,(3+3\cos(2(c+dx))-10\sin(c+dx))\right)}{15d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x]),x]
[Out] -1/15*(a*(e*Cos[c + d*x])^(3/2)*(-10*EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c
 + d*x]]*(3 + 3*Cos[2*(c + d*x)] - 10*Sin[c + d*x])))/(d*Cos[c + d*x]^(3/2)
)
```

Maple [A]
time = 1.84, size = 179, normalized size = 1.88

method	result
default	$-\frac{2a\,e^2\left(-24\left(\sin^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+20\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+36\left(\sin^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+5\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}\right.}{15\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^2\left(\frac{dx}{2}\right.\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/15/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*a*e^2*(-24*sin
(1/2*d*x+1/2*c)^7+20*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+36*sin(1/2*d*x
+1/2*c)^5+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))-10*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/
2*c)-18*sin(1/2*d*x+1/2*c)^3+3*sin(1/2*d*x+1/2*c))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] e^(3/2)*integrate((a*sin(d*x + c) + a)*cos(d*x + c)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 93, normalized size = 0.98

$$\frac{-5i\sqrt{2}\,ae^{\frac{3}{2}}\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}\,ae^{\frac{3}{2}}\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-2\left(3a\cos(dx+c)^{2}e^{\frac{3}{2}}-5ae^{\frac{3}{2}}\sin(dx+c)\right)\sqrt{\cos(dx+c)}}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/15*(-5*I*sqrt(2)*a*e^(3/2)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*si
n(d*x + c)) + 5*I*sqrt(2)*a*e^(3/2)*weierstrassPInverse(-4, 0, cos(d*x + c)
 - I*sin(d*x + c)) - 2*(3*a*cos(d*x + c)^2*e^(3/2) - 5*a*e^(3/2)*sin(d*x +
c))*sqrt(cos(d*x + c)))/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(3/2)*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)*cos(d*x + c)^(3/2)*e^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (e\,\cos(c+d\,x))^{3/2}\,(a+a\,\sin(c+d\,x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(3/2)*(a + a*sin(c + d*x)),x)
```

```
[Out] int((e*cos(c + d*x))^(3/2)*(a + a*sin(c + d*x)), x)
```

3.199 $\int \sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=63

$$-\frac{2a(e\cos(c+dx))^{3/2}}{3de}+\frac{2a\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}}$$

```
[Out] -2/3*a*(e*cos(d*x+c))^(3/2)/d/e+2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*
x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/d/cos(d
*x+c)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {2748, 2721, 2719}

$$\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{2a(e\cos(c+dx))^{3/2}}{3de}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x]),x]
```

```
[Out] (-2*a*(e*Cos[c + d*x])^(3/2))/(3*d*e) + (2*a*Sqrt[e*Cos[c + d*x]]*EllipticE
[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\int \sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))\,dx = -\frac{2a(e\cos(c+dx))^{3/2}}{3de} + a\int \sqrt{e\cos(c+dx)}\,dx$$
$$= -\frac{2a(e\cos(c+dx))^{3/2}}{3de} + \frac{\left(a\sqrt{e\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}}{\sqrt{\cos(c+dx)}}$$
$$= -\frac{2a(e\cos(c+dx))^{3/2}}{3de} + \frac{2a\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.63, size = 260, normalized size = 4.13

$$\frac{a\sqrt{e\cos(c+dx)}\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)(\cos(dx)+i\sin(dx))\left(-6\cos(dx)-6\cos(2c+dx)-2\sin(c)+6\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2idx}(\cos(c)+i\sin(c))^2\right)(\cos(dx)-i\sin(dx))\sqrt{1+\cos(2(c+dx))+i\sin(2(c+dx))}+2\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2idx}(\cos(c)+i\sin(c))^2\right)(\cos(dx)+i\sin(dx))\sqrt{1+\cos(2(c+dx))+i\sin(2(c+dx))}+\sin(c+2dx)-\sin(3c+2dx)\right)}{6d\left((1+e^{2idx})\cos(c)+i\left(-1+e^{2idx}\right)\sin(c)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Sqrt[e*Cos[c + d*x]]*Csc[c/2]*Sec[c/2]*(Cos[d*x] + I*Sin[d*x])*(-6*Cos[d
*x] - 6*Cos[2*c + d*x] - 2*Sin[c] + 6*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E
^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*(Cos[d*x] - I*Sin[d*x])*Sqrt[1 + Cos[2
*(c + d*x)] + I*Sin[2*(c + d*x)]] + 2*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^
((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*(Cos[d*x] + I*Sin[d*x])*Sqrt[1 + Cos[2*
(c + d*x)] + I*Sin[2*(c + d*x)]] + Sin[c + 2*d*x] - Sin[3*c + 2*d*x]))/(6*d
*((1 + E^((2*I)*d*x))*Cos[c] + I*(-1 + E^((2*I)*d*x))*Sin[c]))
```

Maple [A]
time = 1.89, size = 120, normalized size = 1.90

method	result
default	$\frac{2ae\left(-4\left(\sin^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)+4\left(\sin^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right.\right.}{3\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)e+e}\,d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))*(e*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*a*e*(-4*sin(1/2*
d*x+1/2*c)^5+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+4*sin(1/2*d*x+1/2*c)^3-sin(1/2*d*x+
1/2*c))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))*(e*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] e^(1/2)*integrate((a*sin(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 77, normalized size = 1.22

$$-\frac{2\,a\cos\left(dx+c\right)^{\frac{3}{2}}e^{\frac{1}{2}}-3i\sqrt{2}\,ae^{\frac{1}{2}}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)))+3i\sqrt{2}\,ae^{\frac{1}{2}}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)))}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))*(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*a*cos(d*x + c)^(3/2)*e^(1/2) - 3*I*sqrt(2)*a*e^(1/2)*weierstrassZet
a(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*I*s
qrt(2)*a*e^(1/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x
+ c) - I*sin(d*x + c))))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\sqrt{e\cos\left(c+dx\right)}\,dx+\int\sqrt{e\cos\left(c+dx\right)}\,\sin\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))*(e*cos(d*x+c))**(1/2),x)
```

```
[Out] a*(Integral(sqrt(e*cos(c + d*x)), x) + Integral(sqrt(e*cos(c + d*x))*sin(c
+ d*x), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))*(e*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)*sqrt(cos(d*x + c))*e^(1/2), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{e \cos (c+d x)}\,(a+a \sin (c+d x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(1/2)*(a + a*sin(c + d*x)),x)
[Out] int((e*cos(c + d*x))^(1/2)*(a + a*sin(c + d*x)), x)
```

3.200 $\int \frac{a+a\sin(c+dx)}{\sqrt{e\cos(c+dx)}}\,dx$

Optimal. Leaf size=61

$$-\frac{2a\sqrt{e\cos(c+dx)}}{de}+\frac{2a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{e\cos(c+dx)}}$$

```
[Out] 2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1
/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(e*cos(d*x+c))^(1/2)-2*a*(e*cos(d*x+c))^(
1/2)/d/e
```

Rubi [A]
time = 0.04, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2748, 2721, 2720}

$$\frac{2a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{e\cos(c+dx)}}-\frac{2a\sqrt{e\cos(c+dx)}}{de}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])/Sqrt[e*Cos[c + d*x]],x]
```

```
[Out] (-2*a*Sqrt[e*Cos[c + d*x]])/(d*e) + (2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c +
d*x)/2, 2])/(d*Sqrt[e*Cos[c + d*x]])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+a\sin(c+dx)}{\sqrt{e\cos(c+dx)}}\,dx &= -\frac{2a\sqrt{e\cos(c+dx)}}{de}+a\int \frac{1}{\sqrt{e\cos(c+dx)}}\,dx \\
&= -\frac{2a\sqrt{e\cos(c+dx)}}{de}+\frac{\left(a\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{\sqrt{e\cos(c+dx)}} \\
&= -\frac{2a\sqrt{e\cos(c+dx)}}{de}+\frac{2a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{d\sqrt{e\cos(c+dx)}}
\end{aligned}
$$

Mathematica [A]
time = 14.17, size = 48, normalized size = 0.79

$$
-\frac{2a\left(\cos(c+dx)-\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)\right)}{d\sqrt{e\cos(c+dx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])/Sqrt[e*Cos[c + d*x]],x]
```

```
[Out] (-2*a*(Cos[c + d*x] - Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]))/(d*Sqr
t[e*Cos[c + d*x]])
```

Maple [A]
time = 0.99, size = 103, normalized size = 1.69

method	result
default	$-\frac{2a\left(\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-2\left(\sin^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)e+e}\,d}$
risch	$-\frac{\left(1+e^{2i(dx+c)}\right)a\sqrt{2}\,e^{-i(dx+c)}}{d\sqrt{e\left(1+e^{2i(dx+c)}\right)e^{-i(dx+c)}}}+\frac{2\sqrt{-i\left(e^{i(dx+c)}+i\right)}\,\sqrt{i\left(e^{i(dx+c)}-i\right)}\,\sqrt{ie^{i(dx+c)}}\,\mathrm{EllipticF}\left(\sqrt{-i\cdots}\right.}{d\sqrt{e^{3i(dx+c)}e+e^{i(dx+c)}e}\,\sqrt{e(1\cdots}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))/(e*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*a*((sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))-2*sin(1/2*d*x+1/2*c)^3+sin(1/2*d*x+1/2*c))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))/(e*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] e^(-1/2)*integrate((a*sin(d*x + c) + a)/sqrt(cos(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 66, normalized size = 1.08

$$\frac{\left(-i\sqrt{2}\,a\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+i\,\sqrt{2}\,a\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right))-2\,a\sqrt{\cos\left(dx+c\right)}\right)e^{\left(-\frac{1}{2}\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))/(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] (-I*sqrt(2)*a*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + I
*sqrt(2)*a*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 2*a*
sqrt(cos(d*x + c)))*e^(-1/2)/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\frac{1}{\sqrt{e\cos\left(c+dx\right)}}\,dx+\int\frac{\sin\left(c+dx\right)}{\sqrt{e\cos\left(c+dx\right)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))/(e*cos(d*x+c))**(1/2),x)
```

```
[Out] a*(Integral(1/sqrt(e*cos(c + d*x)), x) + Integral(sin(c + d*x)/sqrt(e*cos(c
 + d*x)), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))/(e*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)*e^(-1/2)/sqrt(cos(d*x + c)), x)
```

Mupad [B]
time = 0.55, size = 45, normalized size = 0.74

$$-\frac{2\,a\,\sqrt{\cos\left(c+d\,x\right)}\,\left(\sqrt{\cos\left(c+d\,x\right)}-\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)\right)}{d\,\sqrt{e\,\cos\left(c+d\,x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))/(e*cos(c + d*x))^(1/2),x)
```

```
[Out] -(2*a*cos(c + d*x)^(1/2)*(cos(c + d*x)^(1/2) - ellipticF(c/2 + (d*x)/2, 2))
)/(d*(e*cos(c + d*x))^(1/2))
```

3.201 $\int \frac{a+a\sin(c+dx)}{(e\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=91

$$\frac{2a}{de\sqrt{e\cos(c+dx)}}-\frac{2a\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{de^2\sqrt{\cos(c+dx)}}+\frac{2a\sin(c+dx)}{de\sqrt{e\cos(c+dx)}}$$

```
[Out] 2*a/d/e/(e*cos(d*x+c))^(1/2)+2*a*sin(d*x+c)/d/e/(e*cos(d*x+c))^(1/2)-2*a*(c
os(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),
2^(1/2))*(e*cos(d*x+c))^(1/2)/d/e^2/cos(d*x+c)^(1/2)
```

Rubi [A]

time = 0.05, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2748, 2716, 2721, 2719}

$$-\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{de^2\sqrt{\cos(c+dx)}}+\frac{2a}{de\sqrt{e\cos(c+dx)}}+\frac{2a\sin(c+dx)}{de\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])/(e*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*a)/(d*e*Sqrt[e*Cos[c + d*x]]) - (2*a*Sqrt[e*Cos[c + d*x]]*EllipticE[(c +
 d*x)/2, 2])/(d*e^2*Sqrt[Cos[c + d*x]]) + (2*a*Sin[c + d*x])/(d*e*Sqrt[e*Co
s[c + d*x]])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+a\sin(c+dx)}{(e\cos(c+dx))^{3/2}}\,dx &= \frac{2a}{de\sqrt{e\cos(c+dx)}}+a\int \frac{1}{(e\cos(c+dx))^{3/2}}\,dx\\ &= \frac{2a}{de\sqrt{e\cos(c+dx)}}+\frac{2a\sin(c+dx)}{de\sqrt{e\cos(c+dx)}}-\frac{a\int \sqrt{e\cos(c+dx)}\,dx}{e^2}\\ &= \frac{2a}{de\sqrt{e\cos(c+dx)}}+\frac{2a\sin(c+dx)}{de\sqrt{e\cos(c+dx)}}-\frac{\left(a\sqrt{e\cos(c+dx)}\right)\int \sqrt{\cos(c+dx}}{e^2\sqrt{\cos(c+dx)}}\\ &= \frac{2a}{de\sqrt{e\cos(c+dx)}}-\frac{2a\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{de^2\sqrt{\cos(c+dx)}}+\frac{2a\sin(c+dx)}{de\sqrt{e\cos(c+dx)}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.64, size = 188, normalized size = 2.07

$$-\frac{a\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\left(-6(\cos(dx)+\sin(c))+3\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2idx}(\cos(c)+i\sin(c))^2\right)(\cos(dx)-i\sin(dx))\sqrt{1+\cos(2(c+dx))+i\sin(2(c+dx))}+{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2idx}(\cos(c)+i\sin(c))^2\right)(\cos(dx)+i\sin(dx))\sqrt{1+\cos(2(c+dx))+i\sin(2(c+dx))}\right)}{6de\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])/(e*Cos[c + d*x])^(3/2),x]
```

```
[Out] -1/6*(a*Csc[c/2]*Sec[c/2]*(-6*(Cos[d*x] + Sin[c]) + 3*Hypergeometric2F1[-1/
4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*(Cos[d*x] - I*Sin[d*x]
)*Sqrt[1 + Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)]] + Hypergeometric2F1[1/2,
3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*(Cos[d*x] + I*Sin[d*x])*S
qrt[1 + Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)]]))/(d*e*Sqrt[e*Cos[c + d*x]])
```

Maple [A]
time = 1.82, size = 117, normalized size = 1.29

method	result
default	$-\frac{2\left(\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-\sin\left(\right.\right.}{e\sqrt{-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)e+e}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))/(e*cos(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/e/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/sin(1/2*d*x+1/2*c)*((sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))-2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-sin(1/2*d*x+1/2*c))*a/
d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))/(e*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] e^(-3/2)*integrate((a*sin(d*x + c) + a)/cos(d*x + c)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 165, normalized size = 1.81

$$-\frac{\left(i\sqrt{2}\,a\cos\left(dx+c\right)-i\sqrt{2}\,a\sin\left(dx+c\right)+i\sqrt{2}\,a\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)))+\left(-i\sqrt{2}\,a\cos\left(dx+c\right)+i\sqrt{2}\,a\sin\left(dx+c\right)-i\sqrt{2}\,a\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)))-2\left(a\cos\left(dx+c\right)+a\sin\left(dx+c\right)+a\right)\sqrt{\cos\left(dx+c\right)}}{d\cos\left(dx+c\right)e^{\frac{3}{2}}-de^{\frac{3}{2}}\sin\left(dx+c\right)+de^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))/(e*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -((I*sqrt(2)*a*cos(d*x + c) - I*sqrt(2)*a*sin(d*x + c) + I*sqrt(2)*a)*weier
strassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))
) + (-I*sqrt(2)*a*cos(d*x + c) + I*sqrt(2)*a*sin(d*x + c) - I*sqrt(2)*a)*we
ierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x +
c))) - 2*(a*cos(d*x + c) + a*sin(d*x + c) + a)*sqrt(cos(d*x + c)))/(d*cos(d
*x + c)*e^(3/2) - d*e^(3/2)*sin(d*x + c) + d*e^(3/2))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\frac{1}{\left(e\cos\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx+\int\frac{\sin\left(c+dx\right)}{\left(e\cos\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))/(e*cos(d*x+c))**(3/2),x)
```

```
[Out] a*(Integral((e*cos(c + d*x))**(-3/2), x) + Integral(sin(c + d*x)/(e*cos(c +
 d*x))**(3/2), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))/(e*cos(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((a*sin(d*x + c) + a)*e^(-3/2)/cos(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+a\,\sin\left(c+d\,x\right)}{{\left(e\,\cos\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))/(e*cos(c + d*x))^(3/2),x)
[Out] int((a + a*sin(c + d*x))/(e*cos(c + d*x))^(3/2), x)
```

3.202 $\int \frac{a+a\sin(c+dx)}{(e\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=97

$$\frac{2a}{3de(e\cos(c+dx))^{3/2}}+\frac{2a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3de^2\sqrt{e\cos(c+dx)}}+\frac{2a\sin(c+dx)}{3de(e\cos(c+dx))^{3/2}}$$

```
[Out] 2/3*a/d/e/(e*cos(d*x+c))^(3/2)+2/3*a*sin(d*x+c)/d/e/(e*cos(d*x+c))^(3/2)+2/
3*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1
/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/e^2/(e*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2748, 2716, 2721, 2720}

$$\frac{2a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3de^2\sqrt{e\cos(c+dx)}}+\frac{2a}{3de(e\cos(c+dx))^{3/2}}+\frac{2a\sin(c+dx)}{3de(e\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])/(e*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*a)/(3*d*e*(e*Cos[c + d*x])^(3/2)) + (2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c
 + d*x)/2, 2])/(3*d*e^2*Sqrt[e*Cos[c + d*x]]) + (2*a*Sin[c + d*x])/(3*d*e*(
e*Cos[c + d*x])^(3/2))
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\int \frac{a+a\sin(c+dx)}{(e\cos(c+dx))^{5/2}}\,dx = \frac{2a}{3de(e\cos(c+dx))^{3/2}} + a\int \frac{1}{(e\cos(c+dx))^{5/2}}\,dx$$
$$= \frac{2a}{3de(e\cos(c+dx))^{3/2}} + \frac{2a\sin(c+dx)}{3de(e\cos(c+dx))^{3/2}} + \frac{a\int \frac{1}{\sqrt{e\cos(c+dx)}}\,dx}{3e^2}$$
$$= \frac{2a}{3de(e\cos(c+dx))^{3/2}} + \frac{2a\sin(c+dx)}{3de(e\cos(c+dx))^{3/2}} + \frac{\left(a\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+}}}{3e^2\sqrt{e\cos(c+dx)}}$$
$$= \frac{2a}{3de(e\cos(c+dx))^{3/2}} + \frac{2a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3de^2\sqrt{e\cos(c+dx)}} + \frac{2a\sin(c+dx)}{3de(e\cos(c+dx))^{3}}$$

Mathematica [A]
time = 0.29, size = 86, normalized size = 0.89

$$\frac{2a\left(\cos(c+dx)-\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)(-1+\sin(c+dx))\right)}{3de^2\sqrt{e\cos(c+dx)}\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])/(e*Cos[c + d*x])^(5/2),x]
[Out] (2*a*(Cos[c + d*x] - Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*(-1 + Sin
[c + d*x])))/(3*d*e^2*Sqrt[e*Cos[c + d*x]]*(Cos[(c + d*x)/2] - Sin[(c + d*x
)/2])^2)
```

Maple [A]
time = 2.64, size = 189, normalized size = 1.95

method	result
default	$-\frac{2\left(2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ \mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\right.}{3\left(2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\,(}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))/(e*cos(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/(2*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2
*e+e)^(1/2)/e^2*(2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))+2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+sin(1/2*d*x+1/2*c)
)*a/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))/(e*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] e^(-5/2)*integrate((a*sin(d*x + c) + a)/cos(d*x + c)^(5/2), x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 108, normalized size = 1.11

$$\frac{\left(-i\sqrt{2}\,a\sin\left(dx+c\right)+i\sqrt{2}\,a\right)\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+\left(i\sqrt{2}\,a\sin\left(dx+c\right)-i\sqrt{2}\,a\right)\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right))-2\,a\sqrt{\cos\left(dx+c\right)}}{3\left(de^{\frac{5}{2}}\sin\left(dx+c\right)-de^{\frac{5}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))/(e*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*((-I*sqrt(2)*a*sin(d*x + c) + I*sqrt(2)*a)*weierstrassPInverse(-4, 0, c
os(d*x + c) + I*sin(d*x + c)) + (I*sqrt(2)*a*sin(d*x + c) - I*sqrt(2)*a)*we
ierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 2*a*sqrt(cos(d*x
+ c)))/(d*e^(5/2)*sin(d*x + c) - d*e^(5/2))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))/(e*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))/(e*cos(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((a*sin(d*x + c) + a)*e^(-5/2)/cos(d*x + c)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+a\,\sin\left(c+d\,x\right)}{{\left(e\,\cos\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))/(e*cos(c + d*x))^(5/2),x)
[Out] int((a + a*sin(c + d*x))/(e*cos(c + d*x))^(5/2), x)
```

3.203 $\int \frac{a+a\sin(c+dx)}{(e\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=126

$$\frac{2a}{5de(e\cos(c+dx))^{5/2}}-\frac{6a\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5de^4\sqrt{\cos(c+dx)}}+\frac{2a\sin(c+dx)}{5de(e\cos(c+dx))^{5/2}}+\frac{6a\sin(c+dx)}{5de^3\sqrt{e\cos(c+dx)}}$$

```
[Out] 2/5*a/d/e/(e*cos(d*x+c))^(5/2)+2/5*a*sin(d*x+c)/d/e/(e*cos(d*x+c))^(5/2)+6/
5*a*sin(d*x+c)/d/e^3/(e*cos(d*x+c))^(1/2)-6/5*a*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(
1/2)/d/e^4/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2748, 2716, 2721, 2719}

$$-\frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5de^4\sqrt{\cos(c+dx)}}+\frac{6a\sin(c+dx)}{5de^3\sqrt{e\cos(c+dx)}}+\frac{2a}{5de(e\cos(c+dx))^{5/2}}+\frac{2a\sin(c+dx)}{5de(e\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])/(e*Cos[c + d*x])^(7/2),x]
```

```
[Out] (2*a)/(5*d*e*(e*Cos[c + d*x])^(5/2)) - (6*a*Sqrt[e*Cos[c + d*x]]*EllipticE[
(c + d*x)/2, 2])/(5*d*e^4*Sqrt[Cos[c + d*x]]) + (2*a*Sin[c + d*x])/(5*d*e*(
e*Cos[c + d*x])^(5/2)) + (6*a*Sin[c + d*x])/(5*d*e^3*Sqrt[e*Cos[c + d*x]])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{a+a\sin(c+dx)}{(e\cos(c+dx))^{7/2}}\,dx &= \frac{2a}{5de(e\cos(c+dx))^{5/2}}+a\int \frac{1}{(e\cos(c+dx))^{7/2}}\,dx\\
&= \frac{2a}{5de(e\cos(c+dx))^{5/2}}+\frac{2a\sin(c+dx)}{5de(e\cos(c+dx))^{5/2}}+\frac{(3a)\int \frac{1}{(e\cos(c+dx))^{3/2}}\,dx}{5e^2}\\
&= \frac{2a}{5de(e\cos(c+dx))^{5/2}}+\frac{2a\sin(c+dx)}{5de(e\cos(c+dx))^{5/2}}+\frac{6a\sin(c+dx)}{5de^3\sqrt{e\cos(c+dx)}}-\frac{(3a)\int}{}\\
&= \frac{2a}{5de(e\cos(c+dx))^{5/2}}+\frac{2a\sin(c+dx)}{5de(e\cos(c+dx))^{5/2}}+\frac{6a\sin(c+dx)}{5de^3\sqrt{e\cos(c+dx)}}-\frac{\left(3a\sqrt{e}\right.}{}\\
&= \frac{2a}{5de(e\cos(c+dx))^{5/2}}-\frac{6a\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5de^4\sqrt{\cos(c+dx)}}+\frac{2a\sin(c+dx)}{5de(e\cos(c+dx)}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.82, size = 144, normalized size = 1.14

$$\frac{2ae^{i(c+dx)}\left(i-6e^{i(c+dx)}-3ie^{2i(c+dx)}+i\left(-i+e^{i(c+dx)}\right)^2\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)\right)}{5de^3\left(-i+e^{i(c+dx)}\right)^2\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])/(e*Cos[c + d*x])^(7/2),x]
```

```
[Out] (2*a*E^(I*(c + d*x))*(I - 6*E^(I*(c + d*x)) - (3*I)*E^((2*I)*(c + d*x)) + I
*(-I + E^(I*(c + d*x)))^2*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1
/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]))/(5*d*e^3*(-I + E^(I*(c + d*x)))^2*Sqr
t[e*Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 303 vs. $2(134) = 268$.
time = 4.08, size = 304, normalized size = 2.41

method	result
default	$-\frac{2\left(12\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-24\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\mathrm{co}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))/(e*cos(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)/(
-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/e^3*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2
*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-sin(1/2
*d*x+1/2*c))*a/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))/(e*cos(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] e^(-7/2)*integrate((a*sin(d*x + c) + a)/cos(d*x + c)^(7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 175, normalized size = 1.39

$$-\frac{3\left(i\sqrt{2}\,a\cos\left(dx+c\right)\sin\left(dx+c\right)-i\sqrt{2}\,a\cos\left(dx+c\right)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)))+3\left(-i\sqrt{2}\,a\cos\left(dx+c\right)\sin\left(dx+c\right)+i\sqrt{2}\,a\cos\left(dx+c\right)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)))+2\left(3\,a\cos\left(dx+c\right)^2+3\,a\sin\left(dx+c\right)-2\,a\right)\sqrt{\cos\left(dx+c\right)}}{5\left(d\cos\left(dx+c\right)e^{\frac{7}{2}}\sin\left(dx+c\right)-d\cos\left(dx+c\right)e^{\frac{7}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))/(e*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] -1/5*(3*(I*sqrt(2)*a*cos(d*x + c)*sin(d*x + c) - I*sqrt(2)*a*cos(d*x + c))*
weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x
+ c))) + 3*(-I*sqrt(2)*a*cos(d*x + c)*sin(d*x + c) + I*sqrt(2)*a*cos(d*x +
c))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(
d*x + c))) + 2*(3*a*cos(d*x + c)^2 + 3*a*sin(d*x + c) - 2*a)*sqrt(cos(d*x +
 c)))/(d*cos(d*x + c)*e^(7/2)*sin(d*x + c) - d*cos(d*x + c)*e^(7/2))
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))/(e*cos(d*x+c))**(7/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 5990 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))/(e*cos(d*x+c))^(7/2),x, algorithm="giac")
[Out] integrate((a*sin(d*x + c) + a)*e^(-7/2)/cos(d*x + c)^(7/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+a\,\sin\left(c+d\,x\right)}{{\left(e\,\cos\left(c+d\,x\right)\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))/(e*cos(c + d*x))^(7/2),x)
[Out] int((a + a*sin(c + d*x))/(e*cos(c + d*x))^(7/2), x)
```

3.204 $\int (e\cos(c+dx))^{7/2}(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=168

$$-\frac{26a^2(e\cos(c+dx))^{9/2}}{99de}+\frac{130a^2e^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d\sqrt{e\cos(c+dx)}}+\frac{130a^2e^3\sqrt{e\cos(c+dx)}\,\sin(c+dx)}{231d}+\frac{26a}{}$$

```
[Out] -26/99*a^2*(e*cos(d*x+c))^(9/2)/d/e+26/77*a^2*e*(e*cos(d*x+c))^(5/2)*sin(d*
x+c)/d-2/11*(e*cos(d*x+c))^(9/2)*(a^2+a^2*sin(d*x+c))/d/e+130/231*a^2*e^4*(
cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c)
,2^(1/2))*cos(d*x+c)^(1/2)/d/(e*cos(d*x+c))^(1/2)+130/231*a^2*e^3*sin(d*x+c
)*(e*cos(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.11, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2757, 2748, 2715, 2721, 2720}

$$\frac{130a^2e^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d\sqrt{e\cos(c+dx)}}+\frac{130a^2e^3\sin(c+dx)\sqrt{e\cos(c+dx)}}{231d}-\frac{26a^2(e\cos(c+dx))^{9/2}}{99de}-\frac{2\left(a^2\sin(c+dx)+a^2\right)(e\cos(c+dx))^{9/2}}{11de}+\frac{26a^2e\sin(c+dx)(e\cos(c+dx))^{5/2}}{77d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(7/2)*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-26*a^2*(e*Cos[c + d*x])^(9/2))/(99*d*e) + (130*a^2*e^4*Sqrt[Cos[c + d*x]]
*EllipticF[(c + d*x)/2, 2])/(231*d*Sqrt[e*Cos[c + d*x]]) + (130*a^2*e^3*Sqr
t[e*Cos[c + d*x]]*Sin[c + d*x])/(231*d) + (26*a^2*e*(e*Cos[c + d*x])^(5/2)*
Sin[c + d*x])/(77*d) - (2*(e*Cos[c + d*x])^(9/2)*(a^2 + a^2*Sin[c + d*x]))/
(11*d*e)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2757

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ
[2*m, 2*p]
```

Rubi steps

$$\begin{aligned}
\int (e\cos(c+dx))^{7/2}(a+a\sin(c+dx))^2\,dx &= -\frac{2(e\cos(c+dx))^{9/2}\left(a^2+a^2\sin(c+dx)\right)}{11de}+\frac{1}{11}(13a)\int (e\cos \\
&= -\frac{26a^2(e\cos(c+dx))^{9/2}}{99de}-\frac{2(e\cos(c+dx))^{9/2}\left(a^2+a^2\sin(c+\right.}{11de} \\
&= -\frac{26a^2(e\cos(c+dx))^{9/2}}{99de}+\frac{26a^2e(e\cos(c+dx))^{5/2}\sin(c+dx}{77d} \\
&= -\frac{26a^2(e\cos(c+dx))^{9/2}}{99de}+\frac{130a^2e^3\sqrt{e\cos(c+dx)}\,\sin(c+dx}{231d} \\
&= -\frac{26a^2(e\cos(c+dx))^{9/2}}{99de}+\frac{130a^2e^3\sqrt{e\cos(c+dx)}\,\sin(c+dx}{231d} \\
&= -\frac{26a^2(e\cos(c+dx))^{9/2}}{99de}+\frac{130a^2e^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx\right.}{231d\sqrt{e\cos(c+dx)}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.07, size = 66, normalized size = 0.39

$$-\frac{32\sqrt[4]{2}\,a^2(e\cos(c+dx))^{9/2}\,{}_2F_1\left(-\frac{13}{4},\frac{9}{4};\frac{13}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{9de(1+\sin(c+dx))^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(7/2)*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-32*2^(1/4)*a^2*(e*Cos[c + d*x])^(9/2)*Hypergeometric2F1[-13/4, 9/4, 13/4,
 (1 - Sin[c + d*x])/2])/(9*d*e*(1 + Sin[c + d*x])^(9/4))
```

Maple [A]
time = 2.16, size = 295, normalized size = 1.76

method	result
default	$-\frac{2a^2e^4\left(-4032\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+10080\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-4928\left(\sin^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-8208\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{8}\left(\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(7/2)*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/693/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*a^2*e^4*(-403
2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12+10080*sin(1/2*d*x+1/2*c)^10*cos(
1/2*d*x+1/2*c)-4928*sin(1/2*d*x+1/2*c)^11-8208*cos(1/2*d*x+1/2*c)*sin(1/2*d
*x+1/2*c)^8+12320*sin(1/2*d*x+1/2*c)^9+2232*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*
x+1/2*c)-12320*sin(1/2*d*x+1/2*c)^7+924*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/
2*c)+6160*sin(1/2*d*x+1/2*c)^5+195*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-498*sin(1/2*d*x
+1/2*c)^2*cos(1/2*d*x+1/2*c)-1540*sin(1/2*d*x+1/2*c)^3+154*sin(1/2*d*x+1/2*
c))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] e^(7/2)*integrate((a*sin(d*x + c) + a)^2*cos(d*x + c)^(7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 134, normalized size = 0.80

$$\frac{-195i\sqrt{2}\,a^2e^{\frac{7}{2}}\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+195i\sqrt{2}\,a^2e^{\frac{7}{2}}\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-2\left(154\,a^2\cos(dx+c)^4e^{\frac{7}{2}}+3\left(21\,a^2\cos(dx+c)^4e^{\frac{7}{2}}-39\,a^2\cos(dx+c)^2e^{\frac{7}{2}}-65\,a^2e^{\frac{7}{2}}\right)\sin(dx+c)\right)\sqrt{\cos(dx+c)}}{693\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/693*(-195*I*sqrt(2)*a^2*e^(7/2)*weierstrassPInverse(-4, 0, cos(d*x + c) +
 I*sin(d*x + c)) + 195*I*sqrt(2)*a^2*e^(7/2)*weierstrassPInverse(-4, 0, cos
```

```
(d*x + c) - I*sin(d*x + c)) - 2*(154*a^2*cos(d*x + c)^4*e^(7/2) + 3*(21*a^2
*cos(d*x + c)^4*e^(7/2) - 39*a^2*cos(d*x + c)^2*e^(7/2) - 65*a^2*e^(7/2))*s
in(d*x + c))*sqrt(cos(d*x + c)))/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(7/2)*(a+a*sin(d*x+c))**2,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 5986 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)*(a+a*sin(d*x+c))^2,x, algorithm="giac")
[Out] integrate((a*sin(d*x + c) + a)^2*cos(d*x + c)^(7/2)*e^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (e\,\cos\left(c+d\,x\right))^{7/2}\,\left(a+a\,\sin\left(c+d\,x\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(7/2)*(a + a*sin(c + d*x))^2,x)
[Out] int((e*cos(c + d*x))^(7/2)*(a + a*sin(c + d*x))^2, x)
```

3.205 $\int (e\cos(c+dx))^{5/2}(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=137

$$-\frac{22a^2(e\cos(c+dx))^{7/2}}{63de}+\frac{22a^2e^2\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d\sqrt{\cos(c+dx)}}+\frac{22a^2e(e\cos(c+dx))^{3/2}\sin(c+dx)}{45d}-\frac{2(e$$

```
[Out] -22/63*a^2*(e*cos(d*x+c))^(7/2)/d/e+22/45*a^2*e*(e*cos(d*x+c))^(3/2)*sin(d*
x+c)/d-2/9*(e*cos(d*x+c))^(7/2)*(a^2+a^2*sin(d*x+c))/d/e+22/15*a^2*e^2*(cos
(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^
(1/2))*(e*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2757, 2748, 2715, 2721, 2719}

$$\frac{22a^2e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{15d\sqrt{\cos(c+dx)}}-\frac{22a^2(e\cos(c+dx))^{7/2}}{63de}-\frac{2(a^2\sin(c+dx)+a^2)(e\cos(c+dx))^{7/2}}{9de}+\frac{22a^2e\sin(c+dx)(e\cos(c+dx))^{3/2}}{45d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(5/2)*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-22*a^2*(e*Cos[c + d*x])^(7/2))/(63*d*e) + (22*a^2*e^2*Sqrt[e*Cos[c + d*x]
]*EllipticE[(c + d*x)/2, 2])/(15*d*Sqrt[Cos[c + d*x]]) + (22*a^2*e*(e*Cos[c
 + d*x])^(3/2)*Sin[c + d*x])/(45*d) - (2*(e*Cos[c + d*x])^(7/2)*(a^2 + a^2*
Sin[c + d*x]))/(9*d*e)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2757

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ
[2*m, 2*p]
```

Rubi steps

$$\begin{aligned}\int (e\cos(c+dx))^{5/2}(a+a\sin(c+dx))^2\,dx &= -\frac{2(e\cos(c+dx))^{7/2}\left(a^2+a^2\sin(c+dx)\right)}{9de}+\frac{1}{9}(11a)\int (e\cos(\\
&= -\frac{22a^2(e\cos(c+dx))^{7/2}}{63de}-\frac{2(e\cos(c+dx))^{7/2}\left(a^2+a^2\sin(c+\right.}{9de}\\
&= -\frac{22a^2(e\cos(c+dx))^{7/2}}{63de}+\frac{22a^2e(e\cos(c+dx))^{3/2}\sin(c+dx}{45d}\\
&= -\frac{22a^2(e\cos(c+dx))^{7/2}}{63de}+\frac{22a^2e(e\cos(c+dx))^{3/2}\sin(c+dx}{45d}\\
&= -\frac{22a^2(e\cos(c+dx))^{7/2}}{63de}+\frac{22a^2e^2\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+d\right.}{15d\sqrt{\cos(c+dx)}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.07, size = 66, normalized size = 0.48

$$-\frac{16\ 2^{3/4}a^2(e\cos(c+dx))^{7/2}\,{}_2F_1\left(-\frac{11}{4},\frac{7}{4};\frac{11}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{7de(1+\sin(c+dx))^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(5/2)*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-16*2^(3/4)*a^2*(e*Cos[c + d*x])^(7/2)*Hypergeometric2F1[-11/4, 7/4, 11/4,
 (1 - Sin[c + d*x])/2])/(7*d*e*(1 + Sin[c + d*x])^(7/4))
```

Maple [A]
time = 2.09, size = 260, normalized size = 1.90

method	result
default	$\frac{2a^2e^3\left(-1120\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+2240\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1440\left(\sin^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1064\left(\sin^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(5/2)*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/315/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*a^2*e^3*(-1120
*sin(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+2240*cos(1/2*d*x+1/2*c)*sin(1/2*d
*x+1/2*c)^8-1440*sin(1/2*d*x+1/2*c)^9-1064*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x
+1/2*c)+2880*sin(1/2*d*x+1/2*c)^7-56*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c
)-2160*sin(1/2*d*x+1/2*c)^5+231*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+84*sin(1/2*d*x+1/2
*c)^2*cos(1/2*d*x+1/2*c)+720*sin(1/2*d*x+1/2*c)^3-90*sin(1/2*d*x+1/2*c))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] e^(5/2)*integrate((a*sin(d*x + c) + a)^2*cos(d*x + c)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 131, normalized size = 0.96

$$\frac{231i\sqrt{2}\,a^2e^{\frac{5}{2}}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-231i\sqrt{2}\,a^2e^{\frac{5}{2}}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))-2\left(90\,a^2\cos(dx+c)^3e^{\frac{5}{2}}+7\left(5\,a^2\cos(dx+c)^3e^{\frac{5}{2}}-11\,a^2\cos(dx+c)\,e^{\frac{5}{2}}\right)\sin(dx+c)\right)\sqrt{\cos(dx+c)}}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/315*(231*I*sqrt(2)*a^2*e^(5/2)*weierstrassZeta(-4, 0, weierstrassPInverse
(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 231*I*sqrt(2)*a^2*e^(5/2)*weierst
rassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)))
- 2*(90*a^2*cos(d*x + c)^3*e^(5/2) + 7*(5*a^2*cos(d*x + c)^3*e^(5/2) - 11*a
^2*cos(d*x + c)*e^(5/2))*sin(d*x + c))*sqrt(cos(d*x + c)))/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(5/2)*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^2*cos(d*x + c)^(5/2)*e^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (e\cos(c+dx))^{5/2}(a+a\sin(c+dx))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(5/2)*(a + a*sin(c + d*x))^2,x)
```

```
[Out] int((e*cos(c + d*x))^(5/2)*(a + a*sin(c + d*x))^2, x)
```

3.206 $\int (e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=137

$$-\frac{18a^2(e\cos(c+dx))^{5/2}}{35de}+\frac{6a^2e^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7d\sqrt{e\cos(c+dx)}}+\frac{6a^2e\sqrt{e\cos(c+dx)}\,\sin(c+dx)}{7d}-\frac{2(e\cos(c$$

```
[Out] -18/35*a^2*(e*cos(d*x+c))^(5/2)/d/e-2/7*(e*cos(d*x+c))^(5/2)*(a^2+a^2*sin(d
*x+c))/d/e+6/7*a^2*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(e*cos(d*x+c))^(1/2)+6
/7*a^2*e*sin(d*x+c)*(e*cos(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.09, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2757, 2748, 2715, 2721, 2720}

$$\frac{6a^2e^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7d\sqrt{e\cos(c+dx)}}-\frac{18a^2(e\cos(c+dx))^{5/2}}{35de}-\frac{2(a^2\sin(c+dx)+a^2)\,(e\cos(c+dx))^{5/2}}{7de}+\frac{6a^2e\sin(c+dx)\sqrt{e\cos(c+dx)}}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-18*a^2*(e*Cos[c + d*x])^(5/2))/(35*d*e) + (6*a^2*e^2*Sqrt[Cos[c + d*x]]*E
llipticF[(c + d*x)/2, 2])/(7*d*Sqrt[e*Cos[c + d*x]]) + (6*a^2*e*Sqrt[e*Cos[
c + d*x]]*Sin[c + d*x])/(7*d) - (2*(e*Cos[c + d*x])^(5/2)*(a^2 + a^2*Sin[c
+ d*x]))/(7*d*e)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2757

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ
[2*m, 2*p]
```

Rubi steps

$$\int (e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^2\,dx = -\frac{2(e\cos(c+dx))^{5/2}\left(a^2+a^2\sin(c+dx)\right)}{7de}+\frac{1}{7}(9a)\int (e\cos(c$$

$$= -\frac{18a^2(e\cos(c+dx))^{5/2}}{35de}-\frac{2(e\cos(c+dx))^{5/2}\left(a^2+a^2\sin(c+\right.}{7de}$$

$$= -\frac{18a^2(e\cos(c+dx))^{5/2}}{35de}+\frac{6a^2e\sqrt{e\cos(c+dx)}\sin(c+dx)}{7d}$$

$$= -\frac{18a^2(e\cos(c+dx))^{5/2}}{35de}+\frac{6a^2e\sqrt{e\cos(c+dx)}\sin(c+dx)}{7d}$$

$$= -\frac{18a^2(e\cos(c+dx))^{5/2}}{35de}+\frac{6a^2e^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\right|}{7d\sqrt{e\cos(c+dx)}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.05, size = 66, normalized size = 0.48

$$-\frac{16\sqrt[4]{2}\,a^2(e\cos(c+dx))^{5/2}\,{}_2F_1\left(-\frac{9}{4},\frac{5}{4};\frac{9}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{5de(1+\sin(c+dx))^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-16*2^(1/4)*a^2*(e*Cos[c + d*x])^(5/2)*Hypergeometric2F1[-9/4, 5/4, 9/4, (
1 - Sin[c + d*x])/2])/(5*d*e*(1 + Sin[c + d*x])^(5/4))
```

Maple [A]
time = 1.74, size = 203, normalized size = 1.48

method	result
default	$-\frac{2a^{2}e^{2}\left(-80\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+120\left(\sin^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-112\left(\sin^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+168\left(\sin^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+15\sqrt{\frac{1}{2}-}\right.}{35\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/35/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*a^2*e^2*(-80*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+120*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x
+1/2*c)-112*sin(1/2*d*x+1/2*c)^7+168*sin(1/2*d*x+1/2*c)^5+15*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))-20*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-84*sin(1/2*d*x+1/2*c)
^3+14*sin(1/2*d*x+1/2*c))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] e^(3/2)*integrate((a*sin(d*x + c) + a)^2*cos(d*x + c)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 118, normalized size = 0.86

$$\frac{-15i\sqrt{2}\,a^{2}e^{\frac{3}{2}}\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+15i\sqrt{2}\,a^{2}e^{\frac{3}{2}}\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-2\left(14\,a^{2}\cos(dx+c)^{2}e^{\frac{3}{2}}+5\left(a^{2}\cos(dx+c)^{2}e^{\frac{3}{2}}-3\,a^{2}e^{\frac{3}{2}}\right)\sin(dx+c)\right)\sqrt{\cos(dx+c)}}{35\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/35*(-15*I*sqrt(2)*a^2*e^(3/2)*weierstrassPInverse(-4, 0, cos(d*x + c) + I
*sin(d*x + c)) + 15*I*sqrt(2)*a^2*e^(3/2)*weierstrassPInverse(-4, 0, cos(d*
x + c) - I*sin(d*x + c)) - 2*(14*a^2*cos(d*x + c)^2*e^(3/2) + 5*(a^2*cos(d*
x + c)^2*e^(3/2) - 3*a^2*e^(3/2))*sin(d*x + c))*sqrt(cos(d*x + c)))/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(3/2)*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3064 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^2*cos(d*x + c)^(3/2)*e^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (e\,\cos\left(c+d\,x\right))^{3/2}\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(3/2)*(a + a*sin(c + d*x))^2,x)
```

```
[Out] int((e*cos(c + d*x))^(3/2)*(a + a*sin(c + d*x))^2, x)
```

3.207 $\int \sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=105

$$-\frac{14a^2(e\cos(c+dx))^{3/2}}{15de}+\frac{14a^2\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}}-\frac{2(e\cos(c+dx))^{3/2}\,(a^2+a^2\sin(c+dx))}{5de}$$

```
[Out] -14/15*a^2*(e*cos(d*x+c))^(3/2)/d/e-2/5*(e*cos(d*x+c))^(3/2)*(a^2+a^2*sin(d
*x+c))/d/e+14/5*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipti
cE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2757, 2748, 2721, 2719}

$$-\frac{14a^2(e\cos(c+dx))^{3/2}}{15de}-\frac{2(a^2\sin(c+dx)+a^2)\,(e\cos(c+dx))^{3/2}}{5de}+\frac{14a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-14*a^2*(e*Cos[c + d*x])^(3/2))/(15*d*e) + (14*a^2*Sqrt[e*Cos[c + d*x]]*El
lipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]) - (2*(e*Cos[c + d*x])^(3/
2)*(a^2 + a^2*Sin[c + d*x]))/(5*d*e)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2757

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ
[2*m, 2*p]
```

Rubi steps

$$\int \sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^2\,dx = -\frac{2(e\cos(c+dx))^{3/2}\,(a^2+a^2\sin(c+dx))}{5de} + \frac{1}{5}(7a)\int \sqrt{e\cos(}$$

$$= -\frac{14a^2(e\cos(c+dx))^{3/2}}{15de} - \frac{2(e\cos(c+dx))^{3/2}\,(a^2+a^2\sin(c+}{5de}$$

$$= -\frac{14a^2(e\cos(c+dx))^{3/2}}{15de} - \frac{2(e\cos(c+dx))^{3/2}\,(a^2+a^2\sin(c+}{5de}$$

$$= -\frac{14a^2(e\cos(c+dx))^{3/2}}{15de} + \frac{14a^2\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\right|}{5d\sqrt{\cos(c+dx)}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.03, size = 66, normalized size = 0.63

$$-\frac{8\;2^{3/4}a^2(e\cos(c+dx))^{3/2}\,{}_2F_1\left(-\frac{7}{4},\frac{3}{4};\frac{7}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{3de(1+\sin(c+dx))^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-8*2^(3/4)*a^2*(e*Cos[c + d*x])^(3/2)*Hypergeometric2F1[-7/4, 3/4, 7/4, (1
 - Sin[c + d*x])/2])/(3*d*e*(1 + Sin[c + d*x])^(3/4))
```

Maple [A]
time = 1.80, size = 188, normalized size = 1.79

method	result
default	$\frac{2a^2e\left(-24\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+24\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-40\left(\sin^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+21\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\,(\sin^2}\right.}{15\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\,(\sin^2(}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^2*(e*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*a^2*e*(-24*sin(
1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2
*c)-40*sin(1/2*d*x+1/2*c)^5+21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-6*sin(1/2*d*x+1/2*c
)^2*cos(1/2*d*x+1/2*c)+40*sin(1/2*d*x+1/2*c)^3-10*sin(1/2*d*x+1/2*c))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2*(e*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] e^(1/2)*integrate((a*sin(d*x + c) + a)^2*sqrt(cos(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 111, normalized size = 1.06

$$\frac{21i\sqrt{2}\,a^2e^{\frac{1}{2}}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-21i\sqrt{2}\,a^2e^{\frac{1}{2}}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))-2\left(3\,a^2\cos(dx+c)\,e^{\frac{1}{2}}\sin(dx+c)+10\,a^2\cos(dx+c)\,e^{\frac{1}{2}}\right)\sqrt{\cos(dx+c)}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2*(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(21*I*sqrt(2)*a^2*e^(1/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-
4, 0, cos(d*x + c) + I*sin(d*x + c))) - 21*I*sqrt(2)*a^2*e^(1/2)*weierstras
sZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2
*(3*a^2*cos(d*x + c)*e^(1/2)*sin(d*x + c) + 10*a^2*cos(d*x + c)*e^(1/2))*sq
rt(cos(d*x + c)))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int\sqrt{e\cos(c+dx)}\,dx+\int 2\sqrt{e\cos(c+dx)}\sin(c+dx)\,dx+\int\sqrt{e\cos(c+dx)}\sin^2(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**2*(e*cos(d*x+c))**(1/2),x)
```

```
[Out] a**2*(Integral(sqrt(e*cos(c + d*x)), x) + Integral(2*sqrt(e*cos(c + d*x))*s
in(c + d*x), x) + Integral(sqrt(e*cos(c + d*x))*sin(c + d*x)**2, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2*(e*cos(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate((a*sin(d*x + c) + a)^2*sqrt(cos(d*x + c))*e^(1/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{e\,\cos\left(c+d\,x\right)}\,\left(a+a\,\sin\left(c+d\,x\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(1/2)*(a + a*sin(c + d*x))^2,x)
[Out] int((e*cos(c + d*x))^(1/2)*(a + a*sin(c + d*x))^2, x)
```

3.208 $\int \frac{(a+a\sin(c+dx))^2}{\sqrt{e\cos(c+dx)}}\,dx$

Optimal. Leaf size=105

$$-\frac{10a^2\sqrt{e\cos(c+dx)}}{3de}+\frac{10a^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{e\cos(c+dx)}}-\frac{2\sqrt{e\cos(c+dx)}\,(a^2+a^2\sin(c+dx))}{3de}$$

```
[Out] 10/3*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*
d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(e*cos(d*x+c))^(1/2)-10/3*a^2*(e*cos
(d*x+c))^(1/2)/d/e-2/3*(a^2+a^2*sin(d*x+c))*(e*cos(d*x+c))^(1/2)/d/e
```

Rubi [A]
time = 0.07, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2757, 2748, 2721, 2720}

$$-\frac{10a^2\sqrt{e\cos(c+dx)}}{3de}-\frac{2(a^2\sin(c+dx)+a^2)\sqrt{e\cos(c+dx)}}{3de}+\frac{10a^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^2/Sqrt[e*Cos[c + d*x]],x]
```

```
[Out] (-10*a^2*Sqrt[e*Cos[c + d*x]])/(3*d*e) + (10*a^2*Sqrt[Cos[c + d*x]]*Ellipti
cF[(c + d*x)/2, 2])/(3*d*Sqrt[e*Cos[c + d*x]]) - (2*Sqrt[e*Cos[c + d*x]]*(a
^2 + a^2*Sin[c + d*x]))/(3*d*e)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2757

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ
[2*m, 2*p]
```

Rubi steps

$$\int \frac{(a+a\sin(c+dx))^2}{\sqrt{e\cos(c+dx)}}\,dx = -\frac{2\sqrt{e\cos(c+dx)}\,(a^2+a^2\sin(c+dx))}{3de}+\frac{1}{3}(5a)\int \frac{a+a\sin(c+dx)}{\sqrt{e\cos(c+dx)}}\,dx$$
$$= -\frac{10a^2\sqrt{e\cos(c+dx)}}{3de}-\frac{2\sqrt{e\cos(c+dx)}\,(a^2+a^2\sin(c+dx))}{3de}+\frac{1}{3}(5a^2)\int$$
$$= -\frac{10a^2\sqrt{e\cos(c+dx)}}{3de}-\frac{2\sqrt{e\cos(c+dx)}\,(a^2+a^2\sin(c+dx))}{3de}+\frac{\left(5a^2\sqrt{\cos\right.}}{}$$
$$= -\frac{10a^2\sqrt{e\cos(c+dx)}}{3de}+\frac{10a^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{3d\sqrt{e\cos(c+dx)}}-\frac{2\sqrt{e\cos(c+}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.02, size = 64, normalized size = 0.61

$$-\frac{8\sqrt[4]{2}\,a^2\sqrt{e\cos(c+dx)}\;{}_2F_1\left(-\frac{5}{4},\frac{1}{4};\frac{5}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{de\sqrt[4]{1+\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^2/Sqrt[e*Cos[c + d*x]],x]
[Out] (-8*2^(1/4)*a^2*Sqrt[e*Cos[c + d*x]]*Hypergeometric2F1[-5/4, 1/4, 5/4, (1 -
 Sin[c + d*x])/2])/(d*e*(1 + Sin[c + d*x])^(1/4))
```

Maple [A]
time = 1.41, size = 152, normalized size = 1.45

method	result
default	$-\frac{2a^2\left(-4\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+5\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)+\right.}{3\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)e+e}\;d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^2/(e*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*a^2*(-4*sin(1/2
*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*sin(1/2*d*x+1
/2*c)^2*cos(1/2*d*x+1/2*c)-12*sin(1/2*d*x+1/2*c)^3+6*sin(1/2*d*x+1/2*c))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2/(e*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] e^(-1/2)*integrate((a*sin(d*x + c) + a)^2/sqrt(cos(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 86, normalized size = 0.82

$$\frac{\left(-5i\sqrt{2}\,a^2\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)) + 5i\sqrt{2}\,a^2\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)) - 2\,(a^2\sin(dx+c)+6\,a^2)\sqrt{\cos(dx+c)}\right)e^{\left(-\frac{1}{2}\right)}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2/(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(-5*I*sqrt(2)*a^2*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x +
 c)) + 5*I*sqrt(2)*a^2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x
+ c)) - 2*(a^2*sin(d*x + c) + 6*a^2)*sqrt(cos(d*x + c)))*e^(-1/2)/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^{2}\left(\int\frac{1}{\sqrt{e\cos(c+dx)}}\,dx+\int\frac{2\sin(c+dx)}{\sqrt{e\cos(c+dx)}}\,dx+\int\frac{\sin^{2}(c+dx)}{\sqrt{e\cos(c+dx)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**2/(e*cos(d*x+c))**(1/2),x)
```

```
[Out] a**2*(Integral(1/sqrt(e*cos(c + d*x)), x) + Integral(2*sin(c + d*x)/sqrt(e*
cos(c + d*x)), x) + Integral(sin(c + d*x)**2/sqrt(e*cos(c + d*x)), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2/(e*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^2*e^(-1/2)/sqrt(cos(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+a\,\sin\left(c+d\,x\right)\right)}^2}{\sqrt{e\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^2/(e*cos(c + d*x))^(1/2),x)
```

```
[Out] int((a + a*sin(c + d*x))^2/(e*cos(c + d*x))^(1/2), x)
```

3.209 $\int \frac{(a+a\sin(c+dx))^2}{(e\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=85

$$-\frac{6a^2\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{de^2\sqrt{\cos(c+dx)}}+\frac{4a^4(e\cos(c+dx))^{3/2}}{de^3\left(a^2-a^2\sin(c+dx)\right)}$$

```
[Out] 4*a^4*(e*cos(d*x+c))^(3/2)/d/e^3/(a^2-a^2*sin(d*x+c))-6*a^2*(cos(1/2*d*x+1/
2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*c
os(d*x+c))^(1/2)/d/e^2/cos(d*x+c)^(1/2)
```

Rubi [A]

time = 0.10, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2749, 2759, 2721, 2719}

$$\frac{4a^4(e\cos(c+dx))^{3/2}}{de^3\left(a^2-a^2\sin(c+dx)\right)}-\frac{6a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{de^2\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^2/(e*Cos[c + d*x])^(3/2),x]
```

```
[Out] (-6*a^2*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*e^2*Sqrt[Cos[c +
 d*x]]) + (4*a^4*(e*Cos[c + d*x])^(3/2))/(d*e^3*(a^2 - a^2*Sin[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2749

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+a\sin(c+dx))^2}{(e\cos(c+dx))^{3/2}}\,dx &= \frac{a^4\int \frac{(e\cos(c+dx))^{5/2}}{(a-a\sin(c+dx))^2}\,dx}{e^4}\\ &= \frac{4a^4(e\cos(c+dx))^{3/2}}{de^3\left(a^2-a^2\sin(c+dx)\right)}-\frac{(3a^2)\int \sqrt{e\cos(c+dx)}\,dx}{e^2}\\ &= \frac{4a^4(e\cos(c+dx))^{3/2}}{de^3\left(a^2-a^2\sin(c+dx)\right)}-\frac{\left(3a^2\sqrt{e\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx}{e^2\sqrt{\cos(c+dx)}}\\ &= -\frac{6a^2\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{de^2\sqrt{\cos(c+dx)}}+\frac{4a^4(e\cos(c+dx))^{3/2}}{de^3\left(a^2-a^2\sin(c+dx)\right)}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.04, size = 64, normalized size = 0.75

$$\frac{4\,2^{3/4}a^2\,{}_2F_1\left(-\frac{3}{4},-\frac{1}{4};\frac{3}{4};\frac{1}{2}(1-\sin(c+dx))\right)\sqrt[4]{1+\sin(c+dx)}}{de\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^2/(e*Cos[c + d*x])^(3/2),x]
```

```
[Out] (4*2^(3/4)*a^2*Hypergeometric2F1[-3/4, -1/4, 3/4, (1 - Sin[c + d*x])/2]*(1
+ Sin[c + d*x])^(1/4))/(d*e*Sqrt[e*Cos[c + d*x]])
```

Maple [A]
time = 2.44, size = 120, normalized size = 1.41

method	result
default	$-\frac{2\left(3\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ \mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-4\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\sin\right.}{e\sqrt{-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)e+e}\ \sin\left(\frac{dx}{2}+\frac{c}{2}\right)d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^2/(e*cos(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/e/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/sin(1/2*d*x+1/2*c)*(3*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2
*c),2^(1/2))-4*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2*sin(1/2*d*x+1/2*c)
)*a^2/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2/(e*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] e^(-3/2)*integrate((a*sin(d*x + c) + a)^2/cos(d*x + c)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 184, normalized size = 2.16

$$\frac{3\left(-i\sqrt{2}a^2\cos\left(dx+c\right)+i\sqrt{2}a^2\sin\left(dx+c\right)-i\sqrt{2}a^2\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)))+3\left(i\sqrt{2}a^2\cos\left(dx+c\right)-i\sqrt{2}a^2\sin\left(dx+c\right)+i\sqrt{2}a^2\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)))+4\left(a^2\cos\left(dx+c\right)+a^2\sin\left(dx+c\right)+a^2\right)\sqrt{\cos\left(dx+c\right)}}{d\cos\left(dx+c\right)e^{\frac{3}{2}}-de^{\frac{3}{2}}\sin\left(dx+c\right)+de^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2/(e*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] (3*(-I*sqrt(2)*a^2*cos(d*x + c) + I*sqrt(2)*a^2*sin(d*x + c) - I*sqrt(2)*a^
2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d
*x + c))) + 3*(I*sqrt(2)*a^2*cos(d*x + c) - I*sqrt(2)*a^2*sin(d*x + c) + I*
sqrt(2)*a^2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c)
 - I*sin(d*x + c))) + 4*(a^2*cos(d*x + c) + a^2*sin(d*x + c) + a^2)*sqrt(co
s(d*x + c)))/(d*cos(d*x + c)*e^(3/2) - d*e^(3/2)*sin(d*x + c) + d*e^(3/2))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**2/(e*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2/(e*cos(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((a*sin(d*x + c) + a)^2*e^(-3/2)/cos(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\sin(c+dx))^2}{(e\cos(c+dx))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^2/(e*cos(c + d*x))^(3/2),x)
[Out] int((a + a*sin(c + d*x))^2/(e*cos(c + d*x))^(3/2), x)
```

3.210 $\int \frac{(a+a\sin(c+dx))^2}{(e\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=89

$$-\frac{2a^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3de^2\sqrt{e\cos(c+dx)}}+\frac{4a^4\sqrt{e\cos(c+dx)}}{3de^3\left(a^2-a^2\sin(c+dx)\right)}$$

```
[Out] -2/3*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*
d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/e^2/(e*cos(d*x+c))^(1/2)+4/3*a^4*(e*
cos(d*x+c))^(1/2)/d/e^3/(a^2-a^2*sin(d*x+c))
```

Rubi [A]
time = 0.10, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2749, 2759, 2721, 2720}

$$\frac{4a^4\sqrt{e\cos(c+dx)}}{3de^3\left(a^2-a^2\sin(c+dx)\right)}-\frac{2a^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3de^2\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^2/(e*Cos[c + d*x])^(5/2),x]
```

```
[Out] (-2*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*e^2*Sqrt[e*Cos[c
 + d*x]]) + (4*a^4*Sqrt[e*Cos[c + d*x]])/(3*d*e^3*(a^2 - a^2*Sin[c + d*x]))
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2749

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+a\sin(c+dx))^2}{(e\cos(c+dx))^{5/2}}\,dx &= \frac{a^4\int \frac{(e\cos(c+dx))^{3/2}}{(a-a\sin(c+dx))^2}\,dx}{e^4} \\
&= \frac{4a^4\sqrt{e\cos(c+dx)}}{3de^3\left(a^2-a^2\sin(c+dx)\right)} - \frac{a^2\int \frac{1}{\sqrt{e\cos(c+dx)}}\,dx}{3e^2} \\
&= \frac{4a^4\sqrt{e\cos(c+dx)}}{3de^3\left(a^2-a^2\sin(c+dx)\right)} - \frac{\left(a^2\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3e^2\sqrt{e\cos(c+dx)}} \\
&= -\frac{2a^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3de^2\sqrt{e\cos(c+dx)}} + \frac{4a^4\sqrt{e\cos(c+dx)}}{3de^3\left(a^2-a^2\sin(c+dx)\right)}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.04, size = 66, normalized size = 0.74

$$\frac{4\sqrt[4]{2}\,a^2\,{}_2F_1\left(-\frac{3}{4},-\frac{1}{4};\frac{1}{4};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{3/4}}{3de(e\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^2/(e*Cos[c + d*x])^(5/2),x]
```

```
[Out] (4*2^(1/4)*a^2*Hypergeometric2F1[-3/4, -1/4, 1/4, (1 - Sin[c + d*x])/2]*(1
+ Sin[c + d*x])^(3/4))/(3*d*e*(e*Cos[c + d*x])^(3/2))
```

Maple [A]
time = 2.74, size = 193, normalized size = 2.17

method	result

default	$\frac{2\left(2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{}}{3\left(2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\,(\mathrm{si}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^2/(e*cos(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/(2*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*
e+e)^(1/2)/e^2*(2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))-4*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2*sin(1/2*d*x+1/2*c
))*a^2/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2/(e*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] e^(-5/2)*integrate((a*sin(d*x + c) + a)^2/cos(d*x + c)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 120, normalized size = 1.35

$$-\frac{4\,a^2\sqrt{\cos(dx+c)}-\left(i\sqrt{2}\,a^2\sin(dx+c)-i\sqrt{2}\,a^2\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-\left(-i\sqrt{2}\,a^2\sin(dx+c)+i\sqrt{2}\,a^2\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))}{3\left(de^{\frac{5}{2}}\sin(dx+c)-de^{\frac{5}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2/(e*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(4*a^2*sqrt(cos(d*x + c)) - (I*sqrt(2)*a^2*sin(d*x + c) - I*sqrt(2)*a^
2)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) - (-I*sqrt(2)*
a^2*sin(d*x + c) + I*sqrt(2)*a^2)*weierstrassPInverse(-4, 0, cos(d*x + c) -
 I*sin(d*x + c)))/(d*e^(5/2)*sin(d*x + c) - d*e^(5/2))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**2/(e*cos(d*x+c))**(5/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3881 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2/(e*cos(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((a*sin(d*x + c) + a)^2*e^(-5/2)/cos(d*x + c)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + a \sin (c + d\,x))^2}{(e \cos (c + d\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^2/(e*cos(c + d*x))^(5/2),x)
[Out] int((a + a*sin(c + d*x))^2/(e*cos(c + d*x))^(5/2), x)
```

3.211 $\int \frac{(a+a\sin(c+dx))^2}{(e\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=127

$$-\frac{2a^2\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5de^4\sqrt{\cos(c+dx)}}+\frac{2a^4(e\cos(c+dx))^{3/2}}{5de^5(a-a\sin(c+dx))^2}+\frac{2a^4(e\cos(c+dx))^{3/2}}{5de^5\left(a^2-a^2\sin(c+dx)\right)}$$

```
[Out] 2/5*a^4*(e*cos(d*x+c))^(3/2)/d/e^5/(a-a*sin(d*x+c))^2+2/5*a^4*(e*cos(d*x+c)
)^(3/2)/d/e^5/(a^2-a^2*sin(d*x+c))-2/5*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos
(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/
d/e^4/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2749, 2760, 2762, 2721, 2719}

$$\frac{2a^4(e\cos(c+dx))^{3/2}}{5de^5(a-a\sin(c+dx))^2}-\frac{2a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5de^4\sqrt{\cos(c+dx)}}+\frac{2a^4(e\cos(c+dx))^{3/2}}{5de^5\left(a^2-a^2\sin(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^2/(e*Cos[c + d*x])^(7/2),x]
```

```
[Out] (-2*a^2*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*e^4*Sqrt[Cos[c
 + d*x]]) + (2*a^4*(e*Cos[c + d*x])^(3/2))/(5*d*e^5*(a - a*Sin[c + d*x])^2)
 + (2*a^4*(e*Cos[c + d*x])^(3/2))/(5*d*e^5*(a^2 - a^2*Sin[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2749

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2760

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*(2*m + p + 1))), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2762

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[b*((g*Cos[e + f*x])^(p + 1)/(a*f*g*(p - 1)*(a + b*S
in[e + f*x]))), x] + Dist[p/(a*(p - 1)), Int[(g*Cos[e + f*x])^p, x], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] &&  !GeQ[p, 1] && Integer
Q[2*p]
```

Rubi steps

$$\int \frac{(a+a\sin(c+dx))^2}{(e\cos(c+dx))^{7/2}}\,dx = \frac{a^4\int \frac{\sqrt{e\cos(c+dx)}}{(a-a\sin(c+dx))^2}\,dx}{e^4}$$
$$= \frac{2a^4(e\cos(c+dx))^{3/2}}{5de^5(a-a\sin(c+dx))^2} + \frac{a^3\int \frac{\sqrt{e\cos(c+dx)}}{a-a\sin(c+dx)}\,dx}{5e^4}$$
$$= \frac{2a^4(e\cos(c+dx))^{3/2}}{5de^5(a-a\sin(c+dx))^2} + \frac{2a^3(e\cos(c+dx))^{3/2}}{5de^5(a-a\sin(c+dx))} - \frac{a^2\int \sqrt{e\cos(c+dx)}\,dx}{5e^4}$$
$$= \frac{2a^4(e\cos(c+dx))^{3/2}}{5de^5(a-a\sin(c+dx))^2} + \frac{2a^3(e\cos(c+dx))^{3/2}}{5de^5(a-a\sin(c+dx))} - \frac{\left(a^2\sqrt{e\cos(c+dx)}\right)\int}{5e^4\sqrt{\cos(c}}$$
$$= -\frac{2a^2\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5de^4\sqrt{\cos(c+dx)}} + \frac{2a^4(e\cos(c+dx))^{3/2}}{5de^5(a-a\sin(c+dx))^2} + \frac{2a^3(e\,\mathrm{co}}{5de^5(a-}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.05, size = 66, normalized size = 0.52

$$\frac{2\ 2^{3/4}a^2\,{}_2F_1\left(-\frac{5}{4},\frac{1}{4};-\frac{1}{4};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{5/4}}{5de(e\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^2/(e*Cos[c + d*x])^(7/2),x]
```

```
[Out] (2*2^(3/4)*a^2*Hypergeometric2F1[-5/4, 1/4, -1/4, (1 - Sin[c + d*x])/2]*(1
+ Sin[c + d*x])^(5/4))/(5*d*e*(e*Cos[c + d*x])^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 304 vs. $2(139) = 278$.
time = 4.41, size = 305, normalized size = 2.40

method	result
default	$-\frac{2\left(4\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-8\left(\sin^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos(\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^2/(e*cos(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)/(
-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/e^3*(4*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*
d*x+1/2*c)^4-8*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-4*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))-6*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2*sin(1/2*d*x
+1/2*c))*a^2/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2/(e*cos(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] e^(-7/2)*integrate((a*sin(d*x + c) + a)^2/cos(d*x + c)^(7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 300, normalized size = 2.36

$$\frac{\left(-i\sqrt{2}\,a^{2}\cos(dx+c)^{2}+i\sqrt{2}\,a^{2}\cos(dx+c)+2i\sqrt{2}\,a^{2}+\left(-i\sqrt{2}\,a^{2}\cos(dx+c)-2i\sqrt{2}\,a^{2}\right)\sin(dx+c)\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))\right)+\left(i\sqrt{2}\,a^{2}\cos(dx+c)^{2}-i\sqrt{2}\,a^{2}\cos(dx+c)-2i\sqrt{2}\,a^{2}+\left(i\sqrt{2}\,a^{2}\cos(dx+c)+2i\sqrt{2}\,a^{2}\right)\sin(dx+c)\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))\right)-2\left(a^{2}\cos(dx+c)^{2}+2a^{2}\cos(dx+c)+a^{2}-\left(a^{2}\cos(dx+c)-a^{2}\right)\sin(dx+c)\right)\sqrt{\cos(dx+c)}}{5\left(d\cos(dx+c)^{2}e^{\frac{7}{2}}-d\cos(dx+c)e^{\frac{7}{2}}-2de^{\frac{7}{2}}+\left(d\cos(dx+c)e^{\frac{7}{2}}+2de^{\frac{7}{2}}\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2/(e*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/5*((-I*sqrt(2)*a^2*cos(d*x + c)^2 + I*sqrt(2)*a^2*cos(d*x + c) + 2*I*sqrt
(2)*a^2 + (-I*sqrt(2)*a^2*cos(d*x + c) - 2*I*sqrt(2)*a^2)*sin(d*x + c))*wei
```

```
erstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c
))) + (I*sqrt(2)*a^2*cos(d*x + c)^2 - I*sqrt(2)*a^2*cos(d*x + c) - 2*I*sqrt
(2)*a^2 + (I*sqrt(2)*a^2*cos(d*x + c) + 2*I*sqrt(2)*a^2)*sin(d*x + c))*weie
rstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)
)) - 2*(a^2*cos(d*x + c)^2 + 2*a^2*cos(d*x + c) + a^2 - (a^2*cos(d*x + c) -
 a^2)*sin(d*x + c))*sqrt(cos(d*x + c)))/(d*cos(d*x + c)^2*e^(7/2) - d*cos(d
*x + c)*e^(7/2) - 2*d*e^(7/2) + (d*cos(d*x + c)*e^(7/2) + 2*d*e^(7/2))*sin(
d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**2/(e*cos(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2/(e*cos(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^2*e^(-7/2)/cos(d*x + c)^(7/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\,\sin\left(c+d\,x\right))^2}{(e\,\cos\left(c+d\,x\right))^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^2/(e*cos(c + d*x))^(7/2),x)
```

```
[Out] int((a + a*sin(c + d*x))^2/(e*cos(c + d*x))^(7/2), x)
```

3.212 $\int \frac{(a+a\sin(c+dx))^2}{(e\cos(c+dx))^{9/2}}\,dx$

Optimal. Leaf size=114

$$\frac{2a^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7de^4\sqrt{e\cos(c+dx)}}+\frac{2a^2\sin(c+dx)}{7de^3(e\cos(c+dx))^{3/2}}+\frac{4(a^2+a^2\sin(c+dx))}{7de(e\cos(c+dx))^{7/2}}$$

```
[Out] 2/7*a^2*sin(d*x+c)/d/e^3/(e*cos(d*x+c))^(3/2)+4/7*(a^2+a^2*sin(d*x+c))/d/e/
(e*cos(d*x+c))^(7/2)+2/7*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/e^4/(e*cos(d*x+c
))^(1/2)
```

Rubi [A]

time = 0.07, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2755, 2716, 2721, 2720}

$$\frac{2a^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7de^4\sqrt{e\cos(c+dx)}}+\frac{2a^2\sin(c+dx)}{7de^3(e\cos(c+dx))^{3/2}}+\frac{4(a^2\sin(c+dx)+a^2)}{7de(e\cos(c+dx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^2/(e*Cos[c + d*x])^(9/2),x]
```

```
[Out] (2*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(7*d*e^4*Sqrt[e*Cos[c
+ d*x]]) + (2*a^2*Sin[c + d*x])/(7*d*e^3*(e*Cos[c + d*x])^(3/2)) + (4*(a^2
+ a^2*Sin[c + d*x]))/(7*d*e*(e*Cos[c + d*x])^(7/2))
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2755

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[-2*b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(p + 1))), x] + Dist[b^2*((2*m + p - 1)/(g^2*(p + 1))),
Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[
{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && Inte
gersQ[2*m, 2*p]
```

Rubi steps

$$\int \frac{(a+a\sin(c+dx))^2}{(e\cos(c+dx))^{9/2}}\,dx = \frac{4(a^2+a^2\sin(c+dx))}{7de(e\cos(c+dx))^{7/2}} + \frac{(3a^2)\int \frac{1}{(e\cos(c+dx))^{5/2}}\,dx}{7e^2}$$

$$= \frac{2a^2\sin(c+dx)}{7de^3(e\cos(c+dx))^{3/2}} + \frac{4(a^2+a^2\sin(c+dx))}{7de(e\cos(c+dx))^{7/2}} + \frac{a^2\int \frac{1}{\sqrt{e\cos(c+dx)}}\,dx}{7e^4}$$

$$= \frac{2a^2\sin(c+dx)}{7de^3(e\cos(c+dx))^{3/2}} + \frac{4(a^2+a^2\sin(c+dx))}{7de(e\cos(c+dx))^{7/2}} + \frac{\left(a^2\sqrt{\cos(c+dx)}\right)\int \frac{}{\sqrt{\mathrm{co}}}}{7e^4\sqrt{e\cos(c+d}}$$

$$= \frac{2a^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{7de^4\sqrt{e\cos(c+dx)}} + \frac{2a^2\sin(c+dx)}{7de^3(e\cos(c+dx))^{3/2}} + \frac{4(a^2+a^2\sin(c}{7de(e\cos(c+}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.06, size = 66, normalized size = 0.58

$$\frac{2\sqrt[4]{2}\,a^2\,{}_2F_1\left(-\frac{7}{4},\frac{3}{4};-\frac{3}{4};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{7/4}}{7de(e\cos(c+dx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^2/(e*Cos[c + d*x])^(9/2),x]
```

```
[Out] (2*2^(1/4)*a^2*Hypergeometric2F1[-7/4, 3/4, -3/4, (1 - Sin[c + d*x])/2]*(1
+ Sin[c + d*x])^(7/4))/(7*d*e*(e*Cos[c + d*x])^(7/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 374 vs. $2(126) = 252$.
time = 4.53, size = 375, normalized size = 3.29

method	result

default	$-\frac{2\left(8\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-12\sqrt{\frac{1}{2}-\frac{\cos(dx+c}{2}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^2/(e*cos(d*x+c))^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/7/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2
-1)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/e^4*(8*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^6-12*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*
x+1/2*c)^4+8*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+6*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))*sin(1/2*d*x+1/2*c)^2-8*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))+6*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+2*sin(1/2*d*x+1
/2*c))*a^2/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2/(e*cos(d*x+c))^(9/2),x, algorithm="maxima")
```

```
[Out] e^(-9/2)*integrate((a*sin(d*x + c) + a)^2/cos(d*x + c)^(9/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 176, normalized size = 1.54

$$\frac{\left(-i\sqrt{2}\,a^2\cos(dx+c)^2-2i\sqrt{2}\,a^2\sin(dx+c)+2i\sqrt{2}\,a^2\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+\left(i\sqrt{2}\,a^2\cos(dx+c)^2+2i\sqrt{2}\,a^2\sin(dx+c)-2i\sqrt{2}\,a^2\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+2\left(a^2\sin(dx+c)-2a^2\right)\sqrt{\cos(dx+c)}}{7\left(d\cos(dx+c)^2e^{\frac{9}{2}}+2de^{\frac{9}{2}}\sin(dx+c)-2de^{\frac{9}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2/(e*cos(d*x+c))^(9/2),x, algorithm="fricas")
```

```
[Out] 1/7*((-I*sqrt(2)*a^2*cos(d*x + c)^2 - 2*I*sqrt(2)*a^2*sin(d*x + c) + 2*I*sq
rt(2)*a^2)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + (I*s
qrt(2)*a^2*cos(d*x + c)^2 + 2*I*sqrt(2)*a^2*sin(d*x + c) - 2*I*sqrt(2)*a^2)
*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*(a^2*sin(d*x
 + c) - 2*a^2)*sqrt(cos(d*x + c)))/(d*cos(d*x + c)^2*e^(9/2) + 2*d*e^(9/2)*
sin(d*x + c) - 2*d*e^(9/2))
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**2/(e*cos(d*x+c))**(9/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 4847 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2/(e*cos(d*x+c))^(9/2),x, algorithm="giac")
[Out] integrate((a*sin(d*x + c) + a)^2*e^(-9/2)/cos(d*x + c)^(9/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\,\sin(c+d\,x))^2}{(e\,\cos(c+d\,x))^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^2/(e*cos(c + d*x))^(9/2),x)
[Out] int((a + a*sin(c + d*x))^2/(e*cos(c + d*x))^(9/2), x)
```

3.213 $\int \frac{(a+a\sin(c+dx))^2}{(e\cos(c+dx))^{11/2}}\,dx$

Optimal. Leaf size=145

$$-\frac{2a^2\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{3de^6\sqrt{\cos(c+dx)}}+\frac{2a^2\sin(c+dx)}{9de^3(e\cos(c+dx))^{5/2}}+\frac{2a^2\sin(c+dx)}{3de^5\sqrt{e\cos(c+dx)}}+\frac{4(a^2+a^2\sin(c+dx))}{9de(e\cos(c+dx))^{9/2}}$$

```
[Out] 2/9*a^2*sin(d*x+c)/d/e^3/(e*cos(d*x+c))^(5/2)+4/9*(a^2+a^2*sin(d*x+c))/d/e/
(e*cos(d*x+c))^(9/2)+2/3*a^2*sin(d*x+c)/d/e^5/(e*cos(d*x+c))^(1/2)-2/3*a^2*
(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c
),2^(1/2))*(e*cos(d*x+c))^(1/2)/d/e^6/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2755, 2716, 2721, 2719}

$$-\frac{2a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{3de^6\sqrt{\cos(c+dx)}}+\frac{2a^2\sin(c+dx)}{3de^5\sqrt{e\cos(c+dx)}}+\frac{2a^2\sin(c+dx)}{9de^3(e\cos(c+dx))^{5/2}}+\frac{4(a^2\sin(c+dx)+a^2)}{9de(e\cos(c+dx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^2/(e*Cos[c + d*x])^(11/2),x]
```

```
[Out] (-2*a^2*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(3*d*e^6*Sqrt[Cos[c
 + d*x]]) + (2*a^2*Sin[c + d*x])/(9*d*e^3*(e*Cos[c + d*x])^(5/2)) + (2*a^2*
Sin[c + d*x])/(3*d*e^5*Sqrt[e*Cos[c + d*x]]) + (4*(a^2 + a^2*Sin[c + d*x]))
/(9*d*e*(e*Cos[c + d*x])^(9/2))
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2755

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[-2*b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(p + 1))), x] + Dist[b^2*((2*m + p - 1)/(g^2*(p + 1))),
Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[
{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && Inte
gersQ[2*m, 2*p]
```

Rubi steps

$$\int \frac{(a+a\sin(c+dx))^2}{(e\cos(c+dx))^{11/2}}\,dx = \frac{4(a^2+a^2\sin(c+dx))}{9de(e\cos(c+dx))^{9/2}} + \frac{(5a^2)\int \frac{1}{(e\cos(c+dx))^{7/2}}\,dx}{9e^2}$$

$$= \frac{2a^2\sin(c+dx)}{9de^3(e\cos(c+dx))^{5/2}} + \frac{4(a^2+a^2\sin(c+dx))}{9de(e\cos(c+dx))^{9/2}} + \frac{a^2\int \frac{1}{(e\cos(c+dx))^{3/2}}\,dx}{3e^4}$$

$$= \frac{2a^2\sin(c+dx)}{9de^3(e\cos(c+dx))^{5/2}} + \frac{2a^2\sin(c+dx)}{3de^5\sqrt{e\cos(c+dx)}} + \frac{4(a^2+a^2\sin(c+dx))}{9de(e\cos(c+dx))^{9/2}} - \frac{a^2 \ldots}{\ldots}$$

$$= \frac{2a^2\sin(c+dx)}{9de^3(e\cos(c+dx))^{5/2}} + \frac{2a^2\sin(c+dx)}{3de^5\sqrt{e\cos(c+dx)}} + \frac{4(a^2+a^2\sin(c+dx))}{9de(e\cos(c+dx))^{9/2}} - \frac{(a \ldots}{\ldots}$$

$$= -\frac{2a^2\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{3de^6\sqrt{\cos(c+dx)}} + \frac{2a^2\sin(c+dx)}{9de^3(e\cos(c+dx))^{5/2}} + \frac{2a^2\sin(c \ldots}{3de^5\sqrt{e\cos \ldots}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.10, size = 66, normalized size = 0.46

$$\frac{2^{3/4}a^2\,{}_2F_1\left(-\frac{9}{4},\frac{5}{4};-\frac{5}{4};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{9/4}}{9de(e\cos(c+dx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^2/(e*Cos[c + d*x])^(11/2),x]
```

```
[Out] (2^(3/4)*a^2*Hypergeometric2F1[-9/4, 5/4, -5/4, (1 - Sin[c + d*x])/2]*(1 +
Sin[c + d*x])^(9/4))/(9*d*e*(e*Cos[c + d*x])^(9/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 487 vs. 2(153) = 306.
time = 6.98, size = 488, normalized size = 3.37

method	result
default	$-\frac{2\left(48\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-96\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\mathrm{c}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^2/(e*cos(d*x+c))^(11/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/9/(16*sin(1/2*d*x+1/2*c)^8-32*sin(1/2*d*x+1/2*c)^6+24*sin(1/2*d*x+1/2*c)
^4-8*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+
e)^(1/2)/e^5*(48*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^8-96*sin(1/2*d
*x+1/2*c)^10*cos(1/2*d*x+1/2*c)-96*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2
*c)^6+192*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+72*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*sin(1/2*d*x+1/2*c)^4-152*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-24*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+56*sin(1/2*d*x+1/2*c)^4*cos(1/2*d
*x+1/2*c)+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*E
llipticE(cos(1/2*d*x+1/2*c),2^(1/2))-12*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/
2*c)-2*sin(1/2*d*x+1/2*c))*a^2/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2/(e*cos(d*x+c))^(11/2),x, algorithm="maxima")
```

```
[Out] e^(-11/2)*integrate((a*sin(d*x + c) + a)^2/cos(d*x + c)^(11/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 250, normalized size = 1.72

$$-\frac{3\left(i\sqrt{2}\,a^2\cos(dx+c)^3+2i\sqrt{2}\,a^2\cos(dx+c)\sin(dx+c)-2i\sqrt{2}\,a^2\cos(dx+c)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+3\left(-i\sqrt{2}\,a^2\cos(dx+c)^3-2i\sqrt{2}\,a^2\cos(dx+c)\sin(dx+c)+2i\sqrt{2}\,a^2\cos(dx+c)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+2\left(6a^2\cos(dx+c)^2-4a^2-\left(3a^2\cos(dx+c)^2-5a^2\right)\sin(dx+c)\right)\sqrt{\cos(dx+c)}}{9\left(d\cos(dx+c)^3e^{\frac{11}{2}}+2d\cos(dx+c)e^{\frac{11}{2}}\sin(dx+c)-2d\cos(dx+c)e^{\frac{11}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2/(e*cos(d*x+c))^(11/2),x, algorithm="fricas")
```

```
[Out] -1/9*(3*(I*sqrt(2)*a^2*cos(d*x + c)^3 + 2*I*sqrt(2)*a^2*cos(d*x + c)*sin(d*
x + c) - 2*I*sqrt(2)*a^2*cos(d*x + c))*weierstrassZeta(-4, 0, weierstrassPI
```

```
nverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*(-I*sqrt(2)*a^2*cos(d*x +
 c)^3 - 2*I*sqrt(2)*a^2*cos(d*x + c)*sin(d*x + c) + 2*I*sqrt(2)*a^2*cos(d*x
 + c))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*s
in(d*x + c))) + 2*(6*a^2*cos(d*x + c)^2 - 4*a^2 - (3*a^2*cos(d*x + c)^2 - 5
*a^2)*sin(d*x + c))*sqrt(cos(d*x + c)))/(d*cos(d*x + c)^3*e^(11/2) + 2*d*co
s(d*x + c)*e^(11/2)*sin(d*x + c) - 2*d*cos(d*x + c)*e^(11/2))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**2/(e*cos(d*x+c))**(11/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2/(e*cos(d*x+c))^(11/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^2*e^(-11/2)/cos(d*x + c)^(11/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\,\sin\left(c+d\,x\right))^2}{(e\,\cos\left(c+d\,x\right))^{11/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^2/(e*cos(c + d*x))^(11/2),x)
```

```
[Out] int((a + a*sin(c + d*x))^2/(e*cos(c + d*x))^(11/2), x)
```

3.214 $\int (e\cos(c+dx))^{7/2}(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=203

$$-\frac{34a^3(e\cos(c+dx))^{9/2}}{99de}+\frac{170a^3e^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d\sqrt{e\cos(c+dx)}}+\frac{170a^3e^3\sqrt{e\cos(c+dx)}\,\sin(c+dx)}{231d}+\frac{34a}{}$$

```
[Out] -34/99*a^3*(e*cos(d*x+c))^(9/2)/d/e+34/77*a^3*e*(e*cos(d*x+c))^(5/2)*sin(d*
x+c)/d-2/13*a*(e*cos(d*x+c))^(9/2)*(a+a*sin(d*x+c))^2/d/e-34/143*(e*cos(d*x
+c))^(9/2)*(a^3+a^3*sin(d*x+c))/d/e+170/231*a^3*e^4*(cos(1/2*d*x+1/2*c)^2)^
(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(
1/2)/d/(e*cos(d*x+c))^(1/2)+170/231*a^3*e^3*sin(d*x+c)*(e*cos(d*x+c))^(1/2)
/d
```

Rubi [A]
time = 0.15, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2757, 2748, 2715, 2721, 2720}

$$\frac{170a^3e^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d\sqrt{e\cos(c+dx)}}+\frac{170a^3e^3\sin(c+dx)\sqrt{e\cos(c+dx)}}{231d}-\frac{34a^3(e\cos(c+dx))^{9/2}}{99de}-\frac{34(a^3\sin(c+dx)+a^3)\,(e\cos(c+dx))^{9/2}}{143de}+\frac{34a^3e\sin(c+dx)(e\cos(c+dx))^{5/2}}{77d}-\frac{2a(a\sin(c+dx)+a)^2(e\cos(c+dx))^{9/2}}{13de}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(7/2)*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-34*a^3*(e*Cos[c + d*x])^(9/2))/(99*d*e) + (170*a^3*e^4*Sqrt[Cos[c + d*x]]
*EllipticF[(c + d*x)/2, 2])/(231*d*Sqrt[e*Cos[c + d*x]]) + (170*a^3*e^3*Sqr
t[e*Cos[c + d*x]]*Sin[c + d*x])/(231*d) + (34*a^3*e*(e*Cos[c + d*x])^(5/2)*
Sin[c + d*x])/(77*d) - (2*a*(e*Cos[c + d*x])^(9/2)*(a + a*Sin[c + d*x])^2)/
(13*d*e) - (34*(e*Cos[c + d*x])^(9/2)*(a^3 + a^3*Sin[c + d*x]))/(143*d*e)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
```

```
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2757

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ
[2*m, 2*p]
```

Rubi steps

$$\int (e\cos(c+dx))^{7/2}(a+a\sin(c+dx))^3\,dx = -\frac{2a(e\cos(c+dx))^{9/2}(a+a\sin(c+dx))^2}{13de}+\frac{1}{13}(17a)\int (e\cos$$

$$= -\frac{2a(e\cos(c+dx))^{9/2}(a+a\sin(c+dx))^2}{13de}-\frac{34(e\cos(c+dx)}{}$$

$$= -\frac{34a^3(e\cos(c+dx))^{9/2}}{99de}-\frac{2a(e\cos(c+dx))^{9/2}(a+a\sin(c+}{13de}$$

$$= -\frac{34a^3(e\cos(c+dx))^{9/2}}{99de}+\frac{34a^3e(e\cos(c+dx))^{5/2}\sin(c+dx}{77d}$$

$$= -\frac{34a^3(e\cos(c+dx))^{9/2}}{99de}+\frac{170a^3e^3\sqrt{e\cos(c+dx)}\,\sin(c+d}{231d}$$

$$= -\frac{34a^3(e\cos(c+dx))^{9/2}}{99de}+\frac{170a^3e^3\sqrt{e\cos(c+dx)}\,\sin(c+d}{231d}$$

$$= -\frac{34a^3(e\cos(c+dx))^{9/2}}{99de}+\frac{170a^3e^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+d\right.}{231d\sqrt{e\cos(c+dx)}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.05, size = 66, normalized size = 0.33

$$-\frac{64\sqrt[4]{2}\,a^3(e\cos(c+dx))^{9/2}\,{}_2F_1\left(-\frac{17}{4},\frac{9}{4};\frac{13}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{9de(1+\sin(c+dx))^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(7/2)*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-64*2^(1/4)*a^3*(e*Cos[c + d*x])^(9/2)*Hypergeometric2F1[-17/4, 9/4, 13/4,
 (1 - Sin[c + d*x])/2])/(9*d*e*(1 + Sin[c + d*x])^(9/4))
```

Maple [A]
time = 2.40, size = 321, normalized size = 1.58

method	result
default	$-\frac{2a^3e^4\left(88704\left(\sin^{15}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-157248\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-310464\left(\sin^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+393120\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(7/2)*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/9009/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*a^3*e^4*(887
04*sin(1/2*d*x+1/2*c)^15-157248*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12-31
0464*sin(1/2*d*x+1/2*c)^13+393120*sin(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+
337568*sin(1/2*d*x+1/2*c)^11-361296*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8
-67760*sin(1/2*d*x+1/2*c)^9+148824*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-
126280*sin(1/2*d*x+1/2*c)^7-12012*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+1
01948*sin(1/2*d*x+1/2*c)^5+3315*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-5694*sin(1/2*d*x+1
/2*c)^2*cos(1/2*d*x+1/2*c)-30338*sin(1/2*d*x+1/2*c)^3+3311*sin(1/2*d*x+1/2*
c))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] e^(7/2)*integrate((a*sin(d*x + c) + a)^3*cos(d*x + c)^(7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 149, normalized size = 0.73

$$\frac{-3315i\sqrt{2}\,a^3e^{\frac{7}{2}}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+3315i\sqrt{2}\,a^3e^{\frac{7}{2}}\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c))+2\left(693\,a^3\cos(dx+c)^6e^{\frac{7}{2}}-4004\,a^3\cos(dx+c)^4e^{\frac{7}{2}}-39\left(63\,a^3\cos(dx+c)^4e^{\frac{7}{2}}-51\,a^3\cos(dx+c)^2e^{\frac{7}{2}}-85\,a^3e^{\frac{7}{2}}\right)\sin(dx+c)\right)\sqrt{\cos(dx+c)}}{9009\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/9009*(-3315*I*sqrt(2)*a^3*e^(7/2)*weierstrassPInverse(-4, 0, cos(d*x + c)
 + I*sin(d*x + c)) + 3315*I*sqrt(2)*a^3*e^(7/2)*weierstrassPInverse(-4, 0,
cos(d*x + c) - I*sin(d*x + c)) + 2*(693*a^3*cos(d*x + c)^6*e^(7/2) - 4004*a
^3*cos(d*x + c)^4*e^(7/2) - 39*(63*a^3*cos(d*x + c)^4*e^(7/2) - 51*a^3*cos(
d*x + c)^2*e^(7/2) - 85*a^3*e^(7/2))*sin(d*x + c))*sqrt(cos(d*x + c)))/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(7/2)*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 7316 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^3*cos(d*x + c)^(7/2)*e^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (e\cos(c+dx))^{7/2}\,(a+a\sin(c+dx))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(7/2)*(a + a*sin(c + d*x))^3,x)
```

```
[Out] int((e*cos(c + d*x))^(7/2)*(a + a*sin(c + d*x))^3, x)
```

3.215 $\int (e\cos(c+dx))^{5/2}(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=170

$$-\frac{10a^3(e\cos(c+dx))^{7/2}}{21de}+\frac{2a^3e^2\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}}+\frac{2a^3e(e\cos(c+dx))^{3/2}\sin(c+dx)}{3d}-\frac{2a(e\,c$$

```
[Out] -10/21*a^3*(e*cos(d*x+c))^(7/2)/d/e+2/3*a^3*e*(e*cos(d*x+c))^(3/2)*sin(d*x+
c)/d-2/11*a*(e*cos(d*x+c))^(7/2)*(a+a*sin(d*x+c))^2/d/e-10/33*(e*cos(d*x+c)
)^(7/2)*(a^3+a^3*sin(d*x+c))/d/e+2*a^3*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos
(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/
d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2757, 2748, 2715, 2721, 2719}

$$\frac{2a^3e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{10a^3(e\cos(c+dx))^{7/2}}{21de}-\frac{10\left(a^3\sin(c+dx)+a^3\right)(e\cos(c+dx))^{7/2}}{33de}+\frac{2a^3e\sin(c+dx)(e\cos(c+dx))^{3/2}}{3d}-\frac{2a(a\sin(c+dx)+a)^2(e\cos(c+dx))^{7/2}}{11de}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(5/2)*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-10*a^3*(e*Cos[c + d*x])^(7/2))/(21*d*e) + (2*a^3*e^2*Sqrt[e*Cos[c + d*x]]
*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]) + (2*a^3*e*(e*Cos[c + d*
x])^(3/2)*Sin[c + d*x])/(3*d) - (2*a*(e*Cos[c + d*x])^(7/2)*(a + a*Sin[c +
d*x])^2)/(11*d*e) - (10*(e*Cos[c + d*x])^(7/2)*(a^3 + a^3*Sin[c + d*x]))/(3
3*d*e)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2757

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ
[2*m, 2*p]
```

Rubi steps

$$\int (e\cos(c+dx))^{5/2}(a+a\sin(c+dx))^3\,dx = -\frac{2a(e\cos(c+dx))^{7/2}(a+a\sin(c+dx))^2}{11de} + \frac{1}{11}(15a)\int (e\cos$$
$$= -\frac{2a(e\cos(c+dx))^{7/2}(a+a\sin(c+dx))^2}{11de} - \frac{10(e\cos(c+dx)}{}$$
$$= -\frac{10a^3(e\cos(c+dx))^{7/2}}{21de} - \frac{2a(e\cos(c+dx))^{7/2}(a+a\sin(c+}{11de}$$
$$= -\frac{10a^3(e\cos(c+dx))^{7/2}}{21de} + \frac{2a^3e(e\cos(c+dx))^{3/2}\sin(c+dx)}{3d}$$
$$= -\frac{10a^3(e\cos(c+dx))^{7/2}}{21de} + \frac{2a^3e(e\cos(c+dx))^{3/2}\sin(c+dx)}{3d}$$
$$= -\frac{10a^3(e\cos(c+dx))^{7/2}}{21de} + \frac{2a^3e^2\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx\right.}{d\sqrt{\cos(c+dx)}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.06, size = 66, normalized size = 0.39

$$-\frac{32\ 2^{3/4}a^3(e\cos(c+dx))^{7/2}\,{}_2F_1\left(-\frac{15}{4},\frac{7}{4};\frac{11}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{7de(1+\sin(c+dx))^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(5/2)*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-32*2^(3/4)*a^3*(e*Cos[c + d*x])^(7/2)*Hypergeometric2F1[-15/4, 7/4, 11/4,
 (1 - Sin[c + d*x])/2])/(7*d*e*(1 + Sin[c + d*x])^(7/4))
```

Maple [A]
time = 2.44, size = 264, normalized size = 1.55

method	result
default	$\frac{2a^3e^3\left(1344\left(\sin^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2464\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-4032\left(\sin^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4928\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2928\left(\mathrm{s}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(5/2)*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/231/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*a^3*e^3*(1344*
sin(1/2*d*x+1/2*c)^13-2464*sin(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)-4032*si
n(1/2*d*x+1/2*c)^11+4928*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+2928*sin(1
/2*d*x+1/2*c)^9-3080*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+864*sin(1/2*d*
x+1/2*c)^7+616*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-1908*sin(1/2*d*x+1/2
*c)^5+231*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))+804*sin(1/2*d*x+1/2*c)^3-111*sin(1/2*d*x
+1/2*c))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] e^(5/2)*integrate((a*sin(d*x + c) + a)^3*cos(d*x + c)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 145, normalized size = 0.85

$$\frac{231i\sqrt{2}\,a^3e^{\frac{5}{2}}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-231i\sqrt{2}\,a^3e^{\frac{5}{2}}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+2\left(21a^3\cos(dx+c)^5e^{\frac{5}{2}}-132a^3\cos(dx+c)^3e^{\frac{5}{2}}-77\left(a^3\cos(dx+c)^3e^{\frac{5}{2}}-a^3\cos(dx+c)e^{\frac{5}{2}}\right)\sin(dx+c)\right)\sqrt{\cos(dx+c)}}{231\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/231*(231*I*sqrt(2)*a^3*e^(5/2)*weierstrassZeta(-4, 0, weierstrassPInverse
(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 231*I*sqrt(2)*a^3*e^(5/2)*weierst
rassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)))
+ 2*(21*a^3*cos(d*x + c)^5*e^(5/2) - 132*a^3*cos(d*x + c)^3*e^(5/2) - 77*(a
```

```
^3*cos(d*x + c)^3*e^(5/2) - a^3*cos(d*x + c)*e^(5/2))*sin(d*x + c))*sqrt(co
s(d*x + c)))/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(5/2)*(a+a*sin(d*x+c))**3,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3061 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+a*sin(d*x+c))^3,x, algorithm="giac")
[Out] integrate((a*sin(d*x + c) + a)^3*cos(d*x + c)^(5/2)*e^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (e \cos(c+d\,x))^{5/2}\,(a+a\,\sin(c+d\,x))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(5/2)*(a + a*sin(c + d*x))^3,x)
[Out] int((e*cos(c + d*x))^(5/2)*(a + a*sin(c + d*x))^3, x)
```

3.216 $\int (e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=172

$$-\frac{26a^3(e\cos(c+dx))^{5/2}}{35de}+\frac{26a^3e^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{e\cos(c+dx)}}+\frac{26a^3e\sqrt{e\cos(c+dx)}\,\sin(c+dx)}{21d}-\frac{2a(e\,\mathrm{co}}{}$$

```
[Out] -26/35*a^3*(e*cos(d*x+c))^(5/2)/d/e-2/9*a*(e*cos(d*x+c))^(5/2)*(a+a*sin(d*x
+c))^2/d/e-26/63*(e*cos(d*x+c))^(5/2)*(a^3+a^3*sin(d*x+c))/d/e+26/21*a^3*e^
2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2
*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(e*cos(d*x+c))^(1/2)+26/21*a^3*e*sin(d*x+c)
*(e*cos(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.13, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2757, 2748, 2715, 2721, 2720}

$$\frac{26a^3e^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{e\cos(c+dx)}}-\frac{26a^3(e\cos(c+dx))^{5/2}}{35de}+\frac{26a^3e\sin(c+dx)\sqrt{e\cos(c+dx)}}{21d}-\frac{26\left(a^3\sin(c+dx)+a^3\right)(e\cos(c+dx))^{5/2}}{63de}-\frac{2a(a\sin(c+dx)+a)^2(e\cos(c+dx))^{5/2}}{9de}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-26*a^3*(e*Cos[c + d*x])^(5/2))/(35*d*e) + (26*a^3*e^2*Sqrt[Cos[c + d*x]]*
EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[e*Cos[c + d*x]]) + (26*a^3*e*Sqrt[e*C
os[c + d*x]]*Sin[c + d*x])/(21*d) - (2*a*(e*Cos[c + d*x])^(5/2)*(a + a*Sin[
c + d*x])^2)/(9*d*e) - (26*(e*Cos[c + d*x])^(5/2)*(a^3 + a^3*Sin[c + d*x]))
/(63*d*e)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2757

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ
[2*m, 2*p]
```

Rubi steps

$$\begin{aligned}\int (e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^3\,dx &= -\frac{2a(e\cos(c+dx))^{5/2}(a+a\sin(c+dx))^2}{9de}+\frac{1}{9}(13a)\int (e\cos(\\ &= -\frac{2a(e\cos(c+dx))^{5/2}(a+a\sin(c+dx))^2}{9de}-\frac{26(e\cos(c+dx)}{}\\ &= -\frac{26a^3(e\cos(c+dx))^{5/2}}{35de}-\frac{2a(e\cos(c+dx))^{5/2}(a+a\sin(c+}{9de}\\ &= -\frac{26a^3(e\cos(c+dx))^{5/2}}{35de}+\frac{26a^3e\sqrt{e\cos(c+dx)}\,\sin(c+dx)}{21d}\\ &= -\frac{26a^3(e\cos(c+dx))^{5/2}}{35de}+\frac{26a^3e\sqrt{e\cos(c+dx)}\,\sin(c+dx)}{21d}\\ &= -\frac{26a^3(e\cos(c+dx))^{5/2}}{35de}+\frac{26a^3e^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\right.}{21d\sqrt{e\cos(c+dx)}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.04, size = 66, normalized size = 0.38

$$-\frac{32\sqrt[4]{2}\,a^3(e\cos(c+dx))^{5/2}\,{}_2F_1\left(-\frac{13}{4},\frac{5}{4};\frac{9}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{5de(1+\sin(c+dx))^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-32*2^(1/4)*a^3*(e*Cos[c + d*x])^(5/2)*Hypergeometric2F1[-13/4, 5/4, 9/4,
(1 - Sin[c + d*x])/2])/(5*d*e*(1 + Sin[c + d*x])^(5/4))
```

Maple [A]
time = 2.00, size = 251, normalized size = 1.46

method	result
default	$-\frac{2a^3e^2\left(1120\left(\sin^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2160\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2800\left(\sin^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3240\left(\sin^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+784\left(\mathrm{si}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/315/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*a^3*e^2*(1120
*sin(1/2*d*x+1/2*c)^11-2160*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-2800*si
n(1/2*d*x+1/2*c)^9+3240*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+784*sin(1/2
*d*x+1/2*c)^7-840*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+1624*sin(1/2*d*x+
1/2*c)^5+195*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-120*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+
1/2*c)-1162*sin(1/2*d*x+1/2*c)^3+217*sin(1/2*d*x+1/2*c))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] e^(3/2)*integrate((a*sin(d*x + c) + a)^3*cos(d*x + c)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 134, normalized size = 0.78

$$\frac{-195i\sqrt{2}\,a^3e^{\frac{3}{2}}\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)+195i\sqrt{2}\,a^3e^{\frac{3}{2}}\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)+2\left(35\,a^3\cos\left(dx+c\right)^4e^{\frac{3}{2}}-252\,a^3\cos\left(dx+c\right)^2e^{\frac{3}{2}}-15\left(9\,a^3\cos\left(dx+c\right)^2e^{\frac{3}{2}}-13\,a^3e^{\frac{3}{2}}\right)\sin\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)}}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/315*(-195*I*sqrt(2)*a^3*e^(3/2)*weierstrassPInverse(-4, 0, cos(d*x + c) +
 I*sin(d*x + c)) + 195*I*sqrt(2)*a^3*e^(3/2)*weierstrassPInverse(-4, 0, cos
(d*x + c) - I*sin(d*x + c)) + 2*(35*a^3*cos(d*x + c)^4*e^(3/2) - 252*a^3*co
```

```
s(d*x + c)^2*e^(3/2) - 15*(9*a^3*cos(d*x + c)^2*e^(3/2) - 13*a^3*e^(3/2))*s
in(d*x + c))*sqrt(cos(d*x + c)))/d
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(3/2)*(a+a*sin(d*x+c))**3,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 4849 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^3,x, algorithm="giac")
[Out] integrate((a*sin(d*x + c) + a)^3*cos(d*x + c)^(3/2)*e^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (e\,\cos\left(c+d\,x\right))^{3/2}\,\left(a+a\,\sin\left(c+d\,x\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(3/2)*(a + a*sin(c + d*x))^3,x)
[Out] int((e*cos(c + d*x))^(3/2)*(a + a*sin(c + d*x))^3, x)
```

3.217 $\int \sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=140

$$-\frac{22a^3(e\cos(c+dx))^{3/2}}{15de}+\frac{22a^3\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}}-\frac{2a(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^2}{7de}-2$$

```
[Out] -22/15*a^3*(e*cos(d*x+c))^(3/2)/d/e-2/7*a*(e*cos(d*x+c))^(3/2)*(a+a*sin(d*x
+c))^2/d/e-22/35*(e*cos(d*x+c))^(3/2)*(a^3+a^3*sin(d*x+c))/d/e+22/5*a^3*(co
s(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2
^(1/2))*(e*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2757, 2748, 2721, 2719}

$$-\frac{22a^3(e\cos(c+dx))^{3/2}}{15de}-\frac{22\left(a^3\sin(c+dx)+a^3\right)(e\cos(c+dx))^{3/2}}{35de}+\frac{22a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}-\frac{2a(a\sin(c+dx)+a)^2(e\cos(c+dx))^{3/2}}{7de}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-22*a^3*(e*Cos[c + d*x])^(3/2))/(15*d*e) + (22*a^3*Sqrt[e*Cos[c + d*x]]*El
lipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]) - (2*a*(e*Cos[c + d*x])^(
3/2)*(a + a*Sin[c + d*x])^2)/(7*d*e) - (22*(e*Cos[c + d*x])^(3/2)*(a^3 + a^
3*Sin[c + d*x]))/(35*d*e)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2757

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ
[2*m, 2*p]
```

Rubi steps

$$\int \sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^3\,dx = -\frac{2a(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^2}{7de} + \frac{1}{7}(11a)\int \sqrt{e\cos}$$

$$= -\frac{2a(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^2}{7de} - \frac{22(e\cos(c+dx))}{}$$

$$= -\frac{22a^3(e\cos(c+dx))^{3/2}}{15de} - \frac{2a(e\cos(c+dx))^{3/2}(a+a\sin(c+}{7de}$$

$$= -\frac{22a^3(e\cos(c+dx))^{3/2}}{15de} - \frac{2a(e\cos(c+dx))^{3/2}(a+a\sin(c+}{7de}$$

$$= -\frac{22a^3(e\cos(c+dx))^{3/2}}{15de} + \frac{22a^3\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\right|}{5d\sqrt{\cos(c+dx)}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.03, size = 66, normalized size = 0.47

$$-\frac{16\ 2^{3/4}a^3(e\cos(c+dx))^{3/2}\,{}_2F_1\left(-\frac{11}{4},\frac{3}{4};\frac{7}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{3de(1+\sin(c+dx))^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-16*2^(3/4)*a^3*(e*Cos[c + d*x])^(3/2)*Hypergeometric2F1[-11/4, 3/4, 7/4,
(1 - Sin[c + d*x])/2])/(3*d*e*(1 + Sin[c + d*x])^(3/4))
```

Maple [A]
time = 1.88, size = 214, normalized size = 1.53

method	result

default	$\frac{2a^3e\left(240\left(\sin^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-504\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-480\left(\sin^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+504\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-200\left(\sin^5\left(\frac{dx}{2}+\right.\right.\right.}{105\sin\left(\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^3*(e*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/105/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*a^3*e*(240*sin
(1/2*d*x+1/2*c)^9-504*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-480*sin(1/2*d
*x+1/2*c)^7+504*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-200*sin(1/2*d*x+1/2
*c)^5+231*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))-126*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2
*c)+440*sin(1/2*d*x+1/2*c)^3-125*sin(1/2*d*x+1/2*c))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3*(e*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] e^(1/2)*integrate((a*sin(d*x + c) + a)^3*sqrt(cos(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 126, normalized size = 0.90

$$\frac{231i\sqrt{2}\,a^3e^{\frac{1}{2}}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-231i\sqrt{2}\,a^3e^{\frac{1}{2}}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+2\left(15\,a^3\cos(dx+c)^3e^{\frac{1}{2}}-63\,a^3\cos(dx+c)\,e^{\frac{1}{2}}\sin(dx+c)-140\,a^3\cos(dx+c)\,e^{\frac{1}{2}}\right)\sqrt{\cos(dx+c)}}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3*(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/105*(231*I*sqrt(2)*a^3*e^(1/2)*weierstrassZeta(-4, 0, weierstrassPInverse
(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 231*I*sqrt(2)*a^3*e^(1/2)*weierst
rassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)))
+ 2*(15*a^3*cos(d*x + c)^3*e^(1/2) - 63*a^3*cos(d*x + c)*e^(1/2)*sin(d*x +
c) - 140*a^3*cos(d*x + c)*e^(1/2))*sqrt(cos(d*x + c)))/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**3*(e*cos(d*x+c))**(1/2),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3*(e*cos(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate((a*sin(d*x + c) + a)^3*sqrt(cos(d*x + c))*e^(1/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{e\,\cos\left(c+d\,x\right)}\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(1/2)*(a + a*sin(c + d*x))^3,x)
[Out] int((e*cos(c + d*x))^(1/2)*(a + a*sin(c + d*x))^3, x)
```

3.218 $\int \frac{(a+a\sin(c+dx))^3}{\sqrt{e\cos(c+dx)}}\,dx$

Optimal. Leaf size=136

$$-\frac{6a^3\sqrt{e\cos(c+dx)}}{de}+\frac{6a^3\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{e\cos(c+dx)}}-\frac{2a\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^2}{5de}-\frac{6\sqrt{e\,\mathrm{co}}}{}$$

```
[Out] 6*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x
+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(e*cos(d*x+c))^(1/2)-6*a^3*(e*cos(d*x+c
))^(1/2)/d/e-2/5*a*(a+a*sin(d*x+c))^2*(e*cos(d*x+c))^(1/2)/d/e-6/5*(a^3+a^3
*sin(d*x+c))*(e*cos(d*x+c))^(1/2)/d/e
```

Rubi [A]
time = 0.11, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2757, 2748, 2721, 2720}

$$-\frac{6a^3\sqrt{e\cos(c+dx)}}{de}-\frac{6(a^3\sin(c+dx)+a^3)\sqrt{e\cos(c+dx)}}{5de}+\frac{6a^3\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{e\cos(c+dx)}}-\frac{2a(a\sin(c+dx)+a)^2\sqrt{e\cos(c+dx)}}{5de}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^3/Sqrt[e*Cos[c + d*x]],x]
```

```
[Out] (-6*a^3*Sqrt[e*Cos[c + d*x]])/(d*e) + (6*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(
c + d*x)/2, 2])/(d*Sqrt[e*Cos[c + d*x]]) - (2*a*Sqrt[e*Cos[c + d*x]]*(a + a
*Sin[c + d*x])^2)/(5*d*e) - (6*Sqrt[e*Cos[c + d*x]]*(a^3 + a^3*Sin[c + d*x]
))/(5*d*e)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2757

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ
[2*m, 2*p]
```

Rubi steps

$$\int \frac{(a+a\sin(c+dx))^3}{\sqrt{e\cos(c+dx)}}\,dx = -\frac{2a\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^2}{5de} + \frac{1}{5}(9a)\int \frac{(a+a\sin(c+dx))^2}{\sqrt{e\cos(c+dx)}}\,dx$$

$$= -\frac{2a\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^2}{5de} - \frac{6\sqrt{e\cos(c+dx)}\,(a^3+a^3\sin(c+}{5de}$$

$$= -\frac{6a^3\sqrt{e\cos(c+dx)}}{de} - \frac{2a\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^2}{5de} - \frac{6\sqrt{e\cos(c}{}$$

$$= -\frac{6a^3\sqrt{e\cos(c+dx)}}{de} - \frac{2a\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^2}{5de} - \frac{6\sqrt{e\cos(c}{}$$

$$= -\frac{6a^3\sqrt{e\cos(c+dx)}}{de} + \frac{6a^3\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{d\sqrt{e\cos(c+dx)}} - \frac{2a\sqrt{e\cos(c+}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.02, size = 64, normalized size = 0.47

$$-\frac{16\sqrt[4]{2}\,a^3\sqrt{e\cos(c+dx)}\,{}_2F_1\left(-\frac{9}{4},\frac{1}{4};\frac{5}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{de\sqrt[4]{1+\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^3/Sqrt[e*Cos[c + d*x]],x]
```

```
[Out] (-16*2^(1/4)*a^3*Sqrt[e*Cos[c + d*x]]*Hypergeometric2F1[-9/4, 1/4, 5/4, (1
- Sin[c + d*x])/2])/(d*e*(1 + Sin[c + d*x])^(1/4))
```

Maple [A]
time = 2.02, size = 178, normalized size = 1.31

method	result

default	$-\frac{2a^3\left(8\left(\sin^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-20\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-12\left(\sin^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+15\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-}}{5\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^2\left(\frac{dx}{2}+\right.\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^3/(e*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*a^3*(8*sin(1/2*
d*x+1/2*c)^7-20*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-12*sin(1/2*d*x+1/2*
c)^5+15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))+10*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)
-34*sin(1/2*d*x+1/2*c)^3+19*sin(1/2*d*x+1/2*c))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3/(e*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] e^(-1/2)*integrate((a*sin(d*x + c) + a)^3/sqrt(cos(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 99, normalized size = 0.73

$$\frac{\left(-15i\sqrt{2}\,a^3\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+15i\sqrt{2}\,a^3\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+2\left(a^3\cos(dx+c)^2-5a^3\sin(dx+c)-20a^3\right)\sqrt{\cos(dx+c)}\right)e^{\left(-\frac{1}{2}\right)}}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3/(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/5*(-15*I*sqrt(2)*a^3*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x
+ c)) + 15*I*sqrt(2)*a^3*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*
x + c)) + 2*(a^3*cos(d*x + c)^2 - 5*a^3*sin(d*x + c) - 20*a^3)*sqrt(cos(d*x
 + c)))*e^(-1/2)/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**3/(e*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3/(e*cos(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate((a*sin(d*x + c) + a)^3*e^(-1/2)/sqrt(cos(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+a\,\sin\left(c+d\,x\right)\right)}^3}{\sqrt{e\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^3/(e*cos(c + d*x))^(1/2),x)
[Out] int((a + a*sin(c + d*x))^3/(e*cos(c + d*x))^(1/2), x)
```

3.219 $\int \frac{(a+a\sin(c+dx))^3}{(e\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=106

$$\frac{14a^3(e\cos(c+dx))^{3/2}}{3de^3}-\frac{14a^3\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{de^2\sqrt{\cos(c+dx)}}+\frac{4a^5(e\cos(c+dx))^{7/2}}{de^5(a-a\sin(c+dx))^2}$$

```
[Out] 14/3*a^3*(e*cos(d*x+c))^(3/2)/d/e^3+4*a^5*(e*cos(d*x+c))^(7/2)/d/e^5/(a-a*s
in(d*x+c))^2-14*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipti
cE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/d/e^2/cos(d*x+c)^(1/2)
```

Rubi [A]

time = 0.14, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2749, 2759, 2761, 2721, 2719}

$$\frac{4a^5(e\cos(c+dx))^{7/2}}{de^5(a-a\sin(c+dx))^2}+\frac{14a^3(e\cos(c+dx))^{3/2}}{3de^3}-\frac{14a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{de^2\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^3/(e*Cos[c + d*x])^(3/2),x]
```

```
[Out] (14*a^3*(e*Cos[c + d*x])^(3/2))/(3*d*e^3) - (14*a^3*Sqrt[e*Cos[c + d*x]]*El
lipticE[(c + d*x)/2, 2])/(d*e^2*Sqrt[Cos[c + d*x]]) + (4*a^5*(e*Cos[c + d*x
])^(7/2))/(d*e^5*(a - a*Sin[c + d*x])^2)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2749

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{(a+a\sin(c+dx))^3}{(e\cos(c+dx))^{3/2}}\,dx = \frac{a^6\int \frac{(e\cos(c+dx))^{9/2}}{(a-a\sin(c+dx))^3}\,dx}{e^6}$$

$$= \frac{4a^5(e\cos(c+dx))^{7/2}}{de^5(a-a\sin(c+dx))^2} - \frac{(7a^4)\int \frac{(e\cos(c+dx))^{5/2}}{a-a\sin(c+dx)}\,dx}{e^4}$$

$$= \frac{14a^3(e\cos(c+dx))^{3/2}}{3de^3} + \frac{4a^5(e\cos(c+dx))^{7/2}}{de^5(a-a\sin(c+dx))^2} - \frac{(7a^3)\int \sqrt{e\cos(c+dx)}\,d}{e^2}$$

$$= \frac{14a^3(e\cos(c+dx))^{3/2}}{3de^3} + \frac{4a^5(e\cos(c+dx))^{7/2}}{de^5(a-a\sin(c+dx))^2} - \frac{\left(7a^3\sqrt{e\cos(c+dx)}\right)\int \sqrt{}}{e^2\sqrt{\cos(c+}}$$

$$= \frac{14a^3(e\cos(c+dx))^{3/2}}{3de^3} - \frac{14a^3\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{de^2\sqrt{\cos(c+dx)}} + \frac{4a^5(e\cos(c}{de^5(a-a\sin}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.03, size = 64, normalized size = 0.60

$$\frac{8\ 2^{3/4}a^3\,{}_2F_1\left(-\frac{7}{4},-\frac{1}{4};\frac{3}{4};\frac{1}{2}(1-\sin(c+dx))\right)\sqrt[4]{1+\sin(c+dx)}}{de\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^3/(e*Cos[c + d*x])^(3/2),x]
```

```
[Out] (8*2^(3/4)*a^3*Hypergeometric2F1[-7/4, -1/4, 3/4, (1 - Sin[c + d*x])/2]*(1
+ Sin[c + d*x])^(1/4))/(d*e*Sqrt[e*Cos[c + d*x]])
```

Maple [A]
time = 2.94, size = 146, normalized size = 1.38

method	result
default	$-\frac{2\left(-4\left(\sin^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+21\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-24\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}{3e\sqrt{-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)e+e}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^3/(e*cos(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
[Out] -2/3/e/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/sin(1/2*d*x+1/2*c)*(-4*sin(1/2*d
*x+1/2*c)^5+21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-24*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x
+1/2*c)+4*sin(1/2*d*x+1/2*c)^3-13*sin(1/2*d*x+1/2*c))*a^3/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3/(e*cos(d*x+c))^(3/2),x, algorithm="maxima")
[Out] e^(-3/2)*integrate((a*sin(d*x + c) + a)^3/cos(d*x + c)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 214, normalized size = 2.02

$$\frac{21\left(-i\sqrt{2}\,a^3\cos(dx+c)+i\sqrt{2}\,a^3\sin(dx+c)-i\sqrt{2}\,a^3\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)))+21\left(i\sqrt{2}\,a^3\cos(dx+c)-i\sqrt{2}\,a^3\sin(dx+c)+i\sqrt{2}\,a^3\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c)))+2\left(a^3\cos(dx+c)^2+13\,a^3\cos(dx+c)+12\,a^3-\left(a^3\cos(dx+c)-12\,a^3\right)\sin(dx+c)\right)\sqrt{\cos(dx+c)}}{3\left(d\cos(dx+c)\,e^{\frac{3}{2}}-de^{\frac{3}{2}}\sin(dx+c)+de^{\frac{3}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3/(e*cos(d*x+c))^(3/2),x, algorithm="fricas")
[Out] 1/3*(21*(-I*sqrt(2)*a^3*cos(d*x + c) + I*sqrt(2)*a^3*sin(d*x + c) - I*sqrt(
2)*a^3)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*
sin(d*x + c))) + 21*(I*sqrt(2)*a^3*cos(d*x + c) - I*sqrt(2)*a^3*sin(d*x + c
) + I*sqrt(2)*a^3)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*
x + c) - I*sin(d*x + c))) + 2*(a^3*cos(d*x + c)^2 + 13*a^3*cos(d*x + c) + 1
2*a^3 - (a^3*cos(d*x + c) - 12*a^3)*sin(d*x + c))*sqrt(cos(d*x + c)))/(d*co
s(d*x + c)*e^(3/2) - d*e^(3/2)*sin(d*x + c) + d*e^(3/2))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**3/(e*cos(d*x+c))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3065 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3/(e*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^3*e^(-3/2)/cos(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\,\sin(c+d\,x))^3}{(e\,\cos(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^3/(e*cos(c + d*x))^(3/2),x)
```

```
[Out] int((a + a*sin(c + d*x))^3/(e*cos(c + d*x))^(3/2), x)
```

3.220 $\int \frac{(a+a\sin(c+dx))^3}{(e\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=110

$$\frac{10a^3\sqrt{e\cos(c+dx)}}{3de^3}-\frac{10a^3\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3de^2\sqrt{e\cos(c+dx)}}+\frac{4a^5(e\cos(c+dx))^{5/2}}{3de^5(a-a\sin(c+dx))^2}$$

```
[Out] 4/3*a^5*(e*cos(d*x+c))^(5/2)/d/e^5/(a-a*sin(d*x+c))^2-10/3*a^3*(cos(1/2*d*x
+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*c
os(d*x+c)^(1/2)/d/e^2/(e*cos(d*x+c))^(1/2)+10/3*a^3*(e*cos(d*x+c))^(1/2)/d/
e^3
```

Rubi [A]
time = 0.14, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2749, 2759, 2761, 2721, 2720}

$$\frac{4a^5(e\cos(c+dx))^{5/2}}{3de^5(a-a\sin(c+dx))^2}+\frac{10a^3\sqrt{e\cos(c+dx)}}{3de^3}-\frac{10a^3\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3de^2\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^3/(e*Cos[c + d*x])^(5/2),x]
```

```
[Out] (10*a^3*Sqrt[e*Cos[c + d*x]])/(3*d*e^3) - (10*a^3*Sqrt[Cos[c + d*x]]*Ellipt
icF[(c + d*x)/2, 2])/(3*d*e^2*Sqrt[e*Cos[c + d*x]]) + (4*a^5*(e*Cos[c + d*x
])^(5/2))/(3*d*e^5*(a - a*Sin[c + d*x])^2)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2749

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{(a+a\sin(c+dx))^3}{(e\cos(c+dx))^{5/2}}\,dx = \frac{a^6\int \frac{(e\cos(c+dx))^{7/2}}{(a-a\sin(c+dx))^3}\,dx}{e^6}$$

$$= \frac{4a^5(e\cos(c+dx))^{5/2}}{3de^5(a-a\sin(c+dx))^2} - \frac{(5a^4)\int \frac{(e\cos(c+dx))^{3/2}}{a-a\sin(c+dx)}\,dx}{3e^4}$$

$$= \frac{10a^3\sqrt{e\cos(c+dx)}}{3de^3} + \frac{4a^5(e\cos(c+dx))^{5/2}}{3de^5(a-a\sin(c+dx))^2} - \frac{(5a^3)\int \frac{1}{\sqrt{e\cos(c+dx)}}\,d$$

$$= \frac{10a^3\sqrt{e\cos(c+dx)}}{3de^3} + \frac{4a^5(e\cos(c+dx))^{5/2}}{3de^5(a-a\sin(c+dx))^2} - \frac{\left(5a^3\sqrt{\cos(c+dx)}\right)\int -}{3e^2\sqrt{e\cos(c+}}$$

$$= \frac{10a^3\sqrt{e\cos(c+dx)}}{3de^3} - \frac{10a^3\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3de^2\sqrt{e\cos(c+dx)}} + \frac{4a^5(e\cos(c+}{3de^5(a-a\sin}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.03, size = 66, normalized size = 0.60

$$\frac{8\sqrt[4]{2}\,a^3\,{}_2F_1\left(-\frac{5}{4},-\frac{3}{4};\frac{1}{4};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{3/4}}{3de(e\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^3/(e*Cos[c + d*x])^(5/2),x]
```

[Out] (8*2^(1/4)*a^3*Hypergeometric2F1[-5/4, -3/4, 1/4, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^(3/4))/(3*d*e*(e*Cos[c + d*x])^(3/2))

Maple [A]
time = 3.30, size = 219, normalized size = 1.99

method	result
default	$\frac{2\left(10\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ \mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-12\left(\sin^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-5\sqrt{}\right.}{3\left(2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\sin}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(d*x+c))^3/(e*cos(d*x+c))^(5/2),x,method=_RETURNVERBOSE)

[Out] 2/3/(2*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/e^2*(10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-12*sin(1/2*d*x+1/2*c)^5-5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+12*sin(1/2*d*x+1/2*c)^3-7*sin(1/2*d*x+1/2*c))*a^3/d

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(d*x+c))^3/(e*cos(d*x+c))^(5/2),x, algorithm="maxima")

[Out] e^(-5/2)*integrate((a*sin(d*x + c) + a)^3/cos(d*x + c)^(5/2), x)

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 134, normalized size = 1.22

$$-\frac{5\left(-i\sqrt{2}\,a^3\sin(dx+c)+i\sqrt{2}\,a^3\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5\left(i\sqrt{2}\,a^3\sin(dx+c)-i\sqrt{2}\,a^3\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-2\left(3a^3\sin(dx+c)-7a^3\right)\sqrt{\cos(dx+c)}}{3\left(de^{\frac{5}{2}}\sin(dx+c)-de^{\frac{5}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(d*x+c))^3/(e*cos(d*x+c))^(5/2),x, algorithm="fricas")

[Out] -1/3*(5*(-I*sqrt(2)*a^3*sin(d*x + c) + I*sqrt(2)*a^3)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*(I*sqrt(2)*a^3*sin(d*x + c) - I*sqrt(2)*a^3)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 2*(3*a^3*sin(d*x + c) - 7*a^3)*sqrt(cos(d*x + c)))/(d*e^(5/2)*sin(d*x + c) - d*e^(5/2))

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**3/(e*cos(d*x+c))**(5/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 5990 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3/(e*cos(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((a*sin(d*x + c) + a)^3*e^(-5/2)/cos(d*x + c)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\,\sin\left(c+d\,x\right))^3}{(e\,\cos\left(c+d\,x\right))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^3/(e*cos(c + d*x))^(5/2),x)
[Out] int((a + a*sin(c + d*x))^3/(e*cos(c + d*x))^(5/2), x)
```

3.221 $\int \frac{(a+a\sin(c+dx))^3}{(e\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=127

$$\frac{6a^3\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5de^4\sqrt{\cos(c+dx)}}+\frac{4a^5(e\cos(c+dx))^{3/2}}{5de^5(a-a\sin(c+dx))^2}-\frac{6a^6(e\cos(c+dx))^{3/2}}{5de^5\left(a^3-a^3\sin(c+dx)\right)}$$

```
[Out] 4/5*a^5*(e*cos(d*x+c))^(3/2)/d/e^5/(a-a*sin(d*x+c))^2-6/5*a^6*(e*cos(d*x+c)
)^(3/2)/d/e^5/(a^3-a^3*sin(d*x+c))+6/5*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos
(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/
d/e^4/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2749, 2759, 2762, 2721, 2719}

$$\frac{4a^5(e\cos(c+dx))^{3/2}}{5de^5(a-a\sin(c+dx))^2}+\frac{6a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5de^4\sqrt{\cos(c+dx)}}-\frac{6a^6(e\cos(c+dx))^{3/2}}{5de^5\left(a^3-a^3\sin(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^3/(e*Cos[c + d*x])^(7/2),x]
```

```
[Out] (6*a^3*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*e^4*Sqrt[Cos[c
+ d*x]]) + (4*a^5*(e*Cos[c + d*x])^(3/2))/(5*d*e^5*(a - a*Sin[c + d*x])^2)
- (6*a^6*(e*Cos[c + d*x])^(3/2))/(5*d*e^5*(a^3 - a^3*Sin[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2749

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2762

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[b*((g*Cos[e + f*x])^(p + 1)/(a*f*g*(p - 1)*(a + b*S
in[e + f*x]))), x] + Dist[p/(a*(p - 1)), Int[(g*Cos[e + f*x])^p, x], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] &&  !GeQ[p, 1] && Integer
Q[2*p]
```

Rubi steps

$$\int \frac{(a+a\sin(c+dx))^3}{(e\cos(c+dx))^{7/2}}\,dx = \frac{a^6\int \frac{(e\cos(c+dx))^{5/2}}{(a-a\sin(c+dx))^3}\,dx}{e^6}$$

$$= \frac{4a^5(e\cos(c+dx))^{3/2}}{5de^5(a-a\sin(c+dx))^2} - \frac{(3a^4)\int \frac{\sqrt{e\cos(c+dx)}}{a-a\sin(c+dx)}\,dx}{5e^4}$$

$$= \frac{4a^5(e\cos(c+dx))^{3/2}}{5de^5(a-a\sin(c+dx))^2} - \frac{6a^4(e\cos(c+dx))^{3/2}}{5de^5(a-a\sin(c+dx))} + \frac{(3a^3)\int \sqrt{e\cos(c+dx)}$$

$$= \frac{4a^5(e\cos(c+dx))^{3/2}}{5de^5(a-a\sin(c+dx))^2} - \frac{6a^4(e\cos(c+dx))^{3/2}}{5de^5(a-a\sin(c+dx))} + \frac{\left(3a^3\sqrt{e\cos(c+dx)}\right)}{5e^4\sqrt{\cos(c}$$

$$= \frac{6a^3\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5de^4\sqrt{\cos(c+dx)}} + \frac{4a^5(e\cos(c+dx))^{3/2}}{5de^5(a-a\sin(c+dx))^2} - \frac{6a^4(e\cos(}{5de^5(a-a}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.04, size = 66, normalized size = 0.52

$$\frac{4\ 2^{3/4}a^3\,{}_2F_1\left(-\frac{5}{4},-\frac{3}{4};-\frac{1}{4};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{5/4}}{5de(e\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^3/(e*Cos[c + d*x])^(7/2),x]
```

```
[Out] (4*2^(3/4)*a^3*Hypergeometric2F1[-5/4, -3/4, -1/4, (1 - Sin[c + d*x])/2]*(1
 + Sin[c + d*x])^(5/4))/(5*d*e*(e*Cos[c + d*x])^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 331 vs. $2(139) = 278$.
time = 4.99, size = 332, normalized size = 2.61

method	result
default	$\frac{2\left(12\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-24\left(\sin^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^3/(e*cos(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)/(-
2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/e^3*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*
d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-20
*sin(1/2*d*x+1/2*c)^5+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*sin(1/2*d*x+1/2*c)^2*cos
(1/2*d*x+1/2*c)+20*sin(1/2*d*x+1/2*c)^3-sin(1/2*d*x+1/2*c))*a^3/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3/(e*cos(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] e^(-7/2)*integrate((a*sin(d*x + c) + a)^3/cos(d*x + c)^(7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 305, normalized size = 2.40

$$-\frac{3\left(-i\sqrt{2}a^{3}\cos(dx+c)^{2}+i\sqrt{2}a^{3}\cos(dx+c)+2i\sqrt{2}a^{3}+\left(-i\sqrt{2}a^{3}\cos(dx+c)-2i\sqrt{2}a^{3}\right)\sin(dx+c)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+3\left(i\sqrt{2}a^{3}\cos(dx+c)^{2}-i\sqrt{2}a^{3}\cos(dx+c)-2i\sqrt{2}a^{3}+\left(i\sqrt{2}a^{3}\cos(dx+c)+2i\sqrt{2}a^{3}\right)\sin(dx+c)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))-2\left(3a^{3}\cos(dx+c)^{2}+a^{3}\cos(dx+c)-2a^{3}-(3a^{3}\cos(dx+c)+2a^{3})\sin(dx+c)\right)\sqrt{\cos(dx+c)}}{5\left(d\cos(dx+c)^{2}e^{\frac{7}{2}}-d\cos(dx+c)e^{\frac{7}{2}}-2de^{\frac{7}{2}}+\left(d\cos(dx+c)e^{\frac{7}{2}}+2de^{\frac{7}{2}}\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3/(e*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] -1/5*(3*(-I*sqrt(2)*a^3*cos(d*x + c)^2 + I*sqrt(2)*a^3*cos(d*x + c) + 2*I*s
qrt(2)*a^3 + (-I*sqrt(2)*a^3*cos(d*x + c) - 2*I*sqrt(2)*a^3)*sin(d*x + c))*
```

```
weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x
+ c))) + 3*(I*sqrt(2)*a^3*cos(d*x + c)^2 - I*sqrt(2)*a^3*cos(d*x + c) - 2*I
*sqrt(2)*a^3 + (I*sqrt(2)*a^3*cos(d*x + c) + 2*I*sqrt(2)*a^3)*sin(d*x + c))
*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x
 + c))) - 2*(3*a^3*cos(d*x + c)^2 + a^3*cos(d*x + c) - 2*a^3 - (3*a^3*cos(d
*x + c) + 2*a^3)*sin(d*x + c))*sqrt(cos(d*x + c)))/(d*cos(d*x + c)^2*e^(7/2
) - d*cos(d*x + c)*e^(7/2) - 2*d*e^(7/2) + (d*cos(d*x + c)*e^(7/2) + 2*d*e^
(7/2))*sin(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**3/(e*cos(d*x+c))**(7/2),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3/(e*cos(d*x+c))^(7/2),x, algorithm="giac")
[Out] integrate((a*sin(d*x + c) + a)^3*e^(-7/2)/cos(d*x + c)^(7/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\,\sin\left(c+d\,x\right))^3}{(e\,\cos\left(c+d\,x\right))^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^3/(e*cos(c + d*x))^(7/2),x)
[Out] int((a + a*sin(c + d*x))^3/(e*cos(c + d*x))^(7/2), x)
```

3.222 $\int \frac{(a+a\sin(c+dx))^3}{(e\cos(c+dx))^{9/2}}\,dx$

Optimal. Leaf size=127

$$-\frac{2a^3\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21de^4\sqrt{e\cos(c+dx)}}+\frac{4a^5\sqrt{e\cos(c+dx)}}{7de^5(a-a\sin(c+dx))^2}-\frac{2a^6\sqrt{e\cos(c+dx)}}{21de^5\left(a^3-a^3\sin(c+dx)\right)}$$

```
[Out] -2/21*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2
*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/e^4/(e*cos(d*x+c))^(1/2)+4/7*a^5*(e
*cos(d*x+c))^(1/2)/d/e^5/(a-a*sin(d*x+c))^2-2/21*a^6*(e*cos(d*x+c))^(1/2)/d
/e^5/(a^3-a^3*sin(d*x+c))
```

Rubi [A]
time = 0.14, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2749, 2759, 2762, 2721, 2720}

$$\frac{4a^5\sqrt{e\cos(c+dx)}}{7de^5(a-a\sin(c+dx))^2}-\frac{2a^3\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21de^4\sqrt{e\cos(c+dx)}}-\frac{2a^6\sqrt{e\cos(c+dx)}}{21de^5\left(a^3-a^3\sin(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^3/(e*Cos[c + d*x])^(9/2),x]
```

```
[Out] (-2*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*e^4*Sqrt[e*Cos[
c + d*x]]) + (4*a^5*Sqrt[e*Cos[c + d*x]])/(7*d*e^5*(a - a*Sin[c + d*x])^2)
- (2*a^6*Sqrt[e*Cos[c + d*x]])/(21*d*e^5*(a^3 - a^3*Sin[c + d*x]))
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2749

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2762

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[b*((g*Cos[e + f*x])^(p + 1)/(a*f*g*(p - 1)*(a + b*S
in[e + f*x]))), x] + Dist[p/(a*(p - 1)), Int[(g*Cos[e + f*x])^p, x], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] &&  !GeQ[p, 1] && Integer
Q[2*p]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+a\sin(c+dx))^3}{(e\cos(c+dx))^{9/2}}\,dx &= \frac{a^6\int \frac{(e\cos(c+dx))^{3/2}}{(a-a\sin(c+dx))^3}\,dx}{e^6} \\
&= \frac{4a^5\sqrt{e\cos(c+dx)}}{7de^5(a-a\sin(c+dx))^2} - \frac{a^4\int \frac{1}{\sqrt{e\cos(c+dx)}\,(a-a\sin(c+dx))}\,dx}{7e^4} \\
&= \frac{4a^5\sqrt{e\cos(c+dx)}}{7de^5(a-a\sin(c+dx))^2} - \frac{2a^4\sqrt{e\cos(c+dx)}}{21de^5(a-a\sin(c+dx))} - \frac{a^3\int \frac{1}{\sqrt{e\cos(c+dx)}}\,d}{21e^4} \\
&= \frac{4a^5\sqrt{e\cos(c+dx)}}{7de^5(a-a\sin(c+dx))^2} - \frac{2a^4\sqrt{e\cos(c+dx)}}{21de^5(a-a\sin(c+dx))} - \frac{\left(a^3\sqrt{\cos(c+dx)}\right)\int}{21e^4\sqrt{e\cos(}} \\
&= -\frac{2a^3\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{21de^4\sqrt{e\cos(c+dx)}} + \frac{4a^5\sqrt{e\cos(c+dx)}}{7de^5(a-a\sin(c+dx))^2} - \frac{2a^4\sqrt{e\,c}}{21de^5(a-}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.05, size = 66, normalized size = 0.52

$$\frac{4\sqrt[4]{2}\,a^3\,{}_2F_1\left(-\frac{7}{4},-\frac{1}{4};-\frac{3}{4};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{7/4}}{7de(e\cos(c+dx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^3/(e*Cos[c + d*x])^(9/2),x]
```

```
[Out] (4*2^(1/4)*a^3*Hypergeometric2F1[-7/4, -1/4, -3/4, (1 - Sin[c + d*x])/2]*(1
 + Sin[c + d*x])^(7/4))/(7*d*e*(e*Cos[c + d*x])^(7/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 400 vs. $2(139) = 278$.
time = 5.43, size = 401, normalized size = 3.16

method	result
default	$\frac{2\left(8\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\;\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-12\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^3/(e*cos(d*x+c))^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/21/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2
-1)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/e^4*(8*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^6-12*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*
x+1/2*c)^4+8*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+6*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))*sin(1/2*d*x+1/2*c)^2-8*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+28*sin(
1/2*d*x+1/2*c)^5-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-22*sin(1/2*d*x+1/2*c)^2*cos(1/2*d
*x+1/2*c)-28*sin(1/2*d*x+1/2*c)^3-5*sin(1/2*d*x+1/2*c))*a^3/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3/(e*cos(d*x+c))^(9/2),x, algorithm="maxima")
```

```
[Out] e^(-9/2)*integrate((a*sin(d*x + c) + a)^3/cos(d*x + c)^(9/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 176, normalized size = 1.39

$$\frac{\left(i\sqrt{2}\,a^3\cos(dx+c)^2+2i\sqrt{2}\,a^3\sin(dx+c)-2i\sqrt{2}\,a^3\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+\left(-i\sqrt{2}\,a^3\cos(dx+c)^2-2i\sqrt{2}\,a^3\sin(dx+c)+2i\sqrt{2}\,a^3\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-2\left(a^3\sin(dx+c)+5a^3\right)\sqrt{\cos(dx+c)}}{21\left(d\cos(dx+c)^2e^{\frac{9}{2}}+2de^{\frac{9}{2}}\sin(dx+c)-2de^{\frac{9}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3/(e*cos(d*x+c))^(9/2),x, algorithm="fricas")
```

```
[Out] 1/21*((I*sqrt(2)*a^3*cos(d*x + c)^2 + 2*I*sqrt(2)*a^3*sin(d*x + c) - 2*I*sq
rt(2)*a^3)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + (-I*
sqrt(2)*a^3*cos(d*x + c)^2 - 2*I*sqrt(2)*a^3*sin(d*x + c) + 2*I*sqrt(2)*a^3
)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 2*(a^3*sin(d*
x + c) + 5*a^3)*sqrt(cos(d*x + c)))/(d*cos(d*x + c)^2*e^(9/2) + 2*d*e^(9/2)
*sin(d*x + c) - 2*d*e^(9/2))
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**3/(e*cos(d*x+c))**(9/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4847 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3/(e*cos(d*x+c))^(9/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^3*e^(-9/2)/cos(d*x + c)^(9/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\,\sin(c+d\,x))^3}{(e\,\cos(c+d\,x))^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^3/(e*cos(c + d*x))^(9/2),x)
```

```
[Out] int((a + a*sin(c + d*x))^3/(e*cos(c + d*x))^(9/2), x)
```

3.223 $\int \frac{(a+a\sin(c+dx))^3}{(e\cos(c+dx))^{11/2}}\,dx$

Optimal. Leaf size=165

$$-\frac{2a^3\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15de^6\sqrt{\cos(c+dx)}}+\frac{2a^6(e\cos(c+dx))^{3/2}}{9de^7(a-a\sin(c+dx))^3}+\frac{2a^5(e\cos(c+dx))^{3/2}}{15de^7(a-a\sin(c+dx))^2}+\frac{2a^6(e\cos(c$$

```
[Out] 2/9*a^6*(e*cos(d*x+c))^(3/2)/d/e^7/(a-a*sin(d*x+c))^3+2/15*a^5*(e*cos(d*x+c
))^(3/2)/d/e^7/(a-a*sin(d*x+c))^2+2/15*a^6*(e*cos(d*x+c))^(3/2)/d/e^7/(a^3-
a^3*sin(d*x+c))-2/15*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*El
lipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/d/e^6/cos(d*x+c)^(
1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2749, 2760, 2762, 2721, 2719}

$$\frac{2a^6(e\cos(c+dx))^{3/2}}{9de^7(a-a\sin(c+dx))^3}+\frac{2a^5(e\cos(c+dx))^{3/2}}{15de^7(a-a\sin(c+dx))^2}-\frac{2a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{15de^6\sqrt{\cos(c+dx)}}+\frac{2a^6(e\cos(c+dx))^{3/2}}{15de^7\left(a^3-a^3\sin(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^3/(e*Cos[c + d*x])^(11/2),x]
```

```
[Out] (-2*a^3*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*d*e^6*Sqrt[Cos[
c + d*x]]) + (2*a^6*(e*Cos[c + d*x])^(3/2))/(9*d*e^7*(a - a*Sin[c + d*x])^3
) + (2*a^5*(e*Cos[c + d*x])^(3/2))/(15*d*e^7*(a - a*Sin[c + d*x])^2) + (2*a
^6*(e*Cos[c + d*x])^(3/2))/(15*d*e^7*(a^3 - a^3*Sin[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2749

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
```

```
0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2760

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*(2*m + p + 1))), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2762

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[b*((g*Cos[e + f*x])^(p + 1)/(a*f*g*(p - 1)*(a + b*S
in[e + f*x]))), x] + Dist[p/(a*(p - 1)), Int[(g*Cos[e + f*x])^p, x], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] &&  !GeQ[p, 1] && Integer
Q[2*p]
```

Rubi steps

$$\int \frac{(a+a\sin(c+dx))^3}{(e\cos(c+dx))^{11/2}}\,dx = \frac{a^6\int \frac{\sqrt{e\cos(c+dx)}}{(a-a\sin(c+dx))^3}\,dx}{e^6}$$

$$= \frac{2a^6(e\cos(c+dx))^{3/2}}{9de^7(a-a\sin(c+dx))^3} + \frac{a^5\int \frac{\sqrt{e\cos(c+dx)}}{(a-a\sin(c+dx))^2}\,dx}{3e^6}$$

$$= \frac{2a^6(e\cos(c+dx))^{3/2}}{9de^7(a-a\sin(c+dx))^3} + \frac{2a^5(e\cos(c+dx))^{3/2}}{15de^7(a-a\sin(c+dx))^2} + \frac{a^4\int \frac{\sqrt{e\cos(c+dx)}}{a-a\sin(c+dx)}}{15e^6}$$

$$= \frac{2a^6(e\cos(c+dx))^{3/2}}{9de^7(a-a\sin(c+dx))^3} + \frac{2a^5(e\cos(c+dx))^{3/2}}{15de^7(a-a\sin(c+dx))^2} + \frac{2a^4(e\cos(c+dx))^{3/}}{15de^7(a-a\sin(c+d}$$

$$= \frac{2a^6(e\cos(c+dx))^{3/2}}{9de^7(a-a\sin(c+dx))^3} + \frac{2a^5(e\cos(c+dx))^{3/2}}{15de^7(a-a\sin(c+dx))^2} + \frac{2a^4(e\cos(c+dx))^{3/}}{15de^7(a-a\sin(c+d}$$

$$= -\frac{2a^3\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{15de^6\sqrt{\cos(c+dx)}} + \frac{2a^6(e\cos(c+dx))^{3/2}}{9de^7(a-a\sin(c+dx))^3} + \frac{2a^5(e\,c}{15de^7(a\,-}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 0.11, size = 66, normalized size = 0.40

$$\frac{2\ 2^{3/4}a^3\ {}_2F_1\left(-\frac{9}{4},\frac{1}{4};-\frac{5}{4};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{9/4}}{9de(e\cos(c+dx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^3/(e*Cos[c + d*x])^(11/2),x]
```

```
[Out] (2*2^(3/4)*a^3*Hypergeometric2F1[-9/4, 1/4, -5/4, (1 - Sin[c + d*x])/2]*(1
+ Sin[c + d*x])^(9/4))/(9*d*e*(e*Cos[c + d*x])^(9/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 513 vs. 2(173) = 346.
time = 7.64, size = 514, normalized size = 3.12

method	result
default	$-\frac{2\left(48\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-96\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^3/(e*cos(d*x+c))^(11/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/45/(16*sin(1/2*d*x+1/2*c)^8-32*sin(1/2*d*x+1/2*c)^6+24*sin(1/2*d*x+1/2*c
)^4-8*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e
+e)^(1/2)/e^5*(48*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^8-96*sin(1/2*
d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)-96*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/
2*c)^6+192*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+72*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*sin(1/2*d*x+1/2*c)^4-152*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-24*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+56*sin(1/2*d*x+1/2*c)^4*cos(1/2*
d*x+1/2*c)+36*sin(1/2*d*x+1/2*c)^5+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-48*sin(1/2*d*
x+1/2*c)^2*cos(1/2*d*x+1/2*c)-36*sin(1/2*d*x+1/2*c)^3-11*sin(1/2*d*x+1/2*c)
)*a^3/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3/(e*cos(d*x+c))^(11/2),x, algorithm="maxima")
```

```
[Out] e^(-11/2)*integrate((a*sin(d*x + c) + a)^3/cos(d*x + c)^(11/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 422, normalized size = 2.56

$$\frac{3\left(-i\sqrt{2}a^3\cos(dx+c)^3-3i\sqrt{2}a^3\cos(dx+c)^2+2i\sqrt{2}a^3\cos(dx+c)+4i\sqrt{2}a^3+\left(i\sqrt{2}a^3\cos(dx+c)^2-2i\sqrt{2}a^3\cos(dx+c)-4i\sqrt{2}a^3\right)\sin(dx+c)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+3\left(i\sqrt{2}a^3\cos(dx+c)^3+3i\sqrt{2}a^3\cos(dx+c)^2-2i\sqrt{2}a^3\cos(dx+c)-4i\sqrt{2}a^3+\left(-i\sqrt{2}a^3\cos(dx+c)^2+2i\sqrt{2}a^3\cos(dx+c)+4i\sqrt{2}a^3\right)\sin(dx+c)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+2\left(3a^3\cos(dx+c)^3-6a^3\cos(dx+c)^2-14a^3\cos(dx+c)-5a^3+\left(3a^3\cos(dx+c)^2+9a^3\cos(dx+c)-5a^3\right)\sin(dx+c)\right)\sqrt{\cos(dx+c)}}{45\left(d\cos(dx+c)^3e^{\frac{11}{2}}+3d\cos(dx+c)^2e^{\frac{11}{2}}-2d\cos(dx+c)e^{\frac{11}{2}}-4de^{\frac{11}{2}}-\left(d\cos(dx+c)^2e^{\frac{11}{2}}-2d\cos(dx+c)e^{\frac{11}{2}}-4de^{\frac{11}{2}}\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3/(e*cos(d*x+c))^(11/2),x, algorithm="fricas")
```

```
[Out] 1/45*(3*(-I*sqrt(2)*a^3*cos(d*x + c)^3 - 3*I*sqrt(2)*a^3*cos(d*x + c)^2 + 2
*I*sqrt(2)*a^3*cos(d*x + c) + 4*I*sqrt(2)*a^3 + (I*sqrt(2)*a^3*cos(d*x + c)
^2 - 2*I*sqrt(2)*a^3*cos(d*x + c) - 4*I*sqrt(2)*a^3)*sin(d*x + c))*weierstr
assZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) +
 3*(I*sqrt(2)*a^3*cos(d*x + c)^3 + 3*I*sqrt(2)*a^3*cos(d*x + c)^2 - 2*I*sqr
t(2)*a^3*cos(d*x + c) - 4*I*sqrt(2)*a^3 + (-I*sqrt(2)*a^3*cos(d*x + c)^2 +
2*I*sqrt(2)*a^3*cos(d*x + c) + 4*I*sqrt(2)*a^3)*sin(d*x + c))*weierstrassZe
ta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(3
*a^3*cos(d*x + c)^3 - 6*a^3*cos(d*x + c)^2 - 14*a^3*cos(d*x + c) - 5*a^3 +
(3*a^3*cos(d*x + c)^2 + 9*a^3*cos(d*x + c) - 5*a^3)*sin(d*x + c))*sqrt(cos(
d*x + c)))/(d*cos(d*x + c)^3*e^(11/2) + 3*d*cos(d*x + c)^2*e^(11/2) - 2*d*c
os(d*x + c)*e^(11/2) - 4*d*e^(11/2) - (d*cos(d*x + c)^2*e^(11/2) - 2*d*cos(
d*x + c)*e^(11/2) - 4*d*e^(11/2))*sin(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**3/(e*cos(d*x+c))**(11/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3/(e*cos(d*x+c))^(11/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^3*e^(-11/2)/cos(d*x + c)^(11/2), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\sin(c+dx))^3}{(e\cos(c+dx))^{11/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^3/(e*cos(c + d*x))^(11/2),x)
[Out] int((a + a*sin(c + d*x))^3/(e*cos(c + d*x))^(11/2), x)
```

3.224 $\int (e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^4\,dx$

Optimal. Leaf size=210

$$-\frac{442a^4(e\cos(c+dx))^{5/2}}{385de}+\frac{442a^4e^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d\sqrt{e\cos(c+dx)}}+\frac{442a^4e\sqrt{e\cos(c+dx)}\,\sin(c+dx)}{231d}-2$$

```
[Out] -442/385*a^4*(e*cos(d*x+c))^(5/2)/d/e-2/11*a*(e*cos(d*x+c))^(5/2)*(a+a*sin(
d*x+c))^3/d/e-34/99*(e*cos(d*x+c))^(5/2)*(a^2+a^2*sin(d*x+c))^2/d/e-442/693
*(e*cos(d*x+c))^(5/2)*(a^4+a^4*sin(d*x+c))/d/e+442/231*a^4*e^2*(cos(1/2*d*x
+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*c
os(d*x+c)^(1/2)/d/(e*cos(d*x+c))^(1/2)+442/231*a^4*e*sin(d*x+c)*(e*cos(d*x+
c))^(1/2)/d
```

Rubi [A]
time = 0.17, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2757, 2748, 2715, 2721, 2720}

$$\frac{442a^4e^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d\sqrt{e\cos(c+dx)}}-\frac{442a^4(e\cos(c+dx))^{5/2}}{385de}+\frac{442a^4e\sin(c+dx)\sqrt{e\cos(c+dx)}}{231d}-\frac{442(a^4\sin(c+dx)+a^4)(e\cos(c+dx))^{5/2}}{693de}-\frac{34(a^2\sin(c+dx)+a^2)^2(e\cos(c+dx))^{5/2}}{99de}-\frac{2a(a\sin(c+dx)+a)^3(e\cos(c+dx))^{5/2}}{11de}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (-442*a^4*(e*Cos[c + d*x])^(5/2))/(385*d*e) + (442*a^4*e^2*Sqrt[Cos[c + d*x
]]*EllipticF[(c + d*x)/2, 2])/(231*d*Sqrt[e*Cos[c + d*x]]) + (442*a^4*e*Sqr
t[e*Cos[c + d*x]]*Sin[c + d*x])/(231*d) - (2*a*(e*Cos[c + d*x])^(5/2)*(a +
a*Sin[c + d*x])^3)/(11*d*e) - (34*(e*Cos[c + d*x])^(5/2)*(a^2 + a^2*Sin[c +
 d*x])^2)/(99*d*e) - (442*(e*Cos[c + d*x])^(5/2)*(a^4 + a^4*Sin[c + d*x]))/
(693*d*e)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
```

```
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2757

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ
[2*m, 2*p]
```

Rubi steps

$$\int (e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^4\,dx = -\frac{2a(e\cos(c+dx))^{5/2}(a+a\sin(c+dx))^3}{11de}+\frac{1}{11}(17a)\int (e\cos(c$$

$$= -\frac{2a(e\cos(c+dx))^{5/2}(a+a\sin(c+dx))^3}{11de}-\frac{34(e\cos(c+dx))^5}{}$$

$$= -\frac{2a(e\cos(c+dx))^{5/2}(a+a\sin(c+dx))^3}{11de}-\frac{34(e\cos(c+dx))^5}{}$$

$$= -\frac{442a^4(e\cos(c+dx))^{5/2}}{385de}-\frac{2a(e\cos(c+dx))^{5/2}(a+a\sin(c+}{11de}$$

$$= -\frac{442a^4(e\cos(c+dx))^{5/2}}{385de}+\frac{442a^4e\sqrt{e\cos(c+dx)}\,\sin(c+dx}{231d}$$

$$= -\frac{442a^4(e\cos(c+dx))^{5/2}}{385de}+\frac{442a^4e\sqrt{e\cos(c+dx)}\,\sin(c+dx}{231d}$$

$$= -\frac{442a^4(e\cos(c+dx))^{5/2}}{385de}+\frac{442a^4e^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx\right.}{231d\sqrt{e\cos(c+dx)}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.10, size = 66, normalized size = 0.31

$$-\frac{64\sqrt[4]{2}\,a^4(e\cos(c+dx))^{5/2}\,{}_2F_1\left(-\frac{17}{4},\frac{5}{4};\frac{9}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{5de(1+\sin(c+dx))^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (-64*2^(1/4)*a^4*(e*Cos[c + d*x])^(5/2)*Hypergeometric2F1[-17/4, 5/4, 9/4,
(1 - Sin[c + d*x])/2])/(5*d*e*(1 + Sin[c + d*x])^(5/4))
```

Maple [A]
time = 2.15, size = 295, normalized size = 1.40

method	result
default	$-\frac{2a^4e^2\left(20160\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-50400\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+49280\left(\sin^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-6480\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] -2/3465/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*a^4*e^2*(201
60*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12-50400*sin(1/2*d*x+1/2*c)^10*cos
(1/2*d*x+1/2*c)+49280*sin(1/2*d*x+1/2*c)^11-6480*cos(1/2*d*x+1/2*c)*sin(1/2
*d*x+1/2*c)^8-123200*sin(1/2*d*x+1/2*c)^9+60120*sin(1/2*d*x+1/2*c)^6*cos(1/
2*d*x+1/2*c)+78848*sin(1/2*d*x+1/2*c)^7-23100*sin(1/2*d*x+1/2*c)^4*cos(1/2*
d*x+1/2*c)+4928*sin(1/2*d*x+1/2*c)^5+3315*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-150*sin(
1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-17864*sin(1/2*d*x+1/2*c)^3+4004*sin(1/2
*d*x+1/2*c))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] e^(3/2)*integrate((a*sin(d*x + c) + a)^4*cos(d*x + c)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 149, normalized size = 0.71

$$\frac{-3315i\sqrt{2}\,a^4e^{\frac{3}{2}}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+3315i\sqrt{2}\,a^4e^{\frac{3}{2}}\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+2\left(1540\,a^4\cos(dx+c)^4e^{\frac{3}{2}}-5544\,a^4\cos(dx+c)^2e^{\frac{3}{2}}+15\left(21\,a^4\cos(dx+c)^4e^{\frac{3}{2}}-237\,a^4\cos(dx+c)^2e^{\frac{3}{2}}+221\,a^4e^{\frac{3}{2}}\right)\sin(dx+c)\right)\sqrt{\cos(dx+c)}}{3465\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/3465*(-3315*I*sqrt(2)*a^4*e^(3/2)*weierstrassPInverse(-4, 0, cos(d*x + c)
 + I*sin(d*x + c)) + 3315*I*sqrt(2)*a^4*e^(3/2)*weierstrassPInverse(-4, 0,
cos(d*x + c) - I*sin(d*x + c)) + 2*(1540*a^4*cos(d*x + c)^4*e^(3/2) - 5544*
a^4*cos(d*x + c)^2*e^(3/2) + 15*(21*a^4*cos(d*x + c)^4*e^(3/2) - 237*a^4*co
s(d*x + c)^2*e^(3/2) + 221*a^4*e^(3/2))*sin(d*x + c))*sqrt(cos(d*x + c)))/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(3/2)*(a+a*sin(d*x+c))**4,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 7319 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^4*cos(d*x + c)^(3/2)*e^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (e\,\cos\left(c+d\,x\right))^{3/2}\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(3/2)*(a + a*sin(c + d*x))^4,x)
```

```
[Out] int((e*cos(c + d*x))^(3/2)*(a + a*sin(c + d*x))^4, x)
```

3.225 $\int \sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^4\,dx$

Optimal. Leaf size=178

$$-\frac{22a^4(e\cos(c+dx))^{3/2}}{9de}+\frac{22a^4\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{\cos(c+dx)}}-\frac{2a(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^3}{9de}-$$

```
[Out] -22/9*a^4*(e*cos(d*x+c))^(3/2)/d/e-2/9*a*(e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+
c))^3/d/e-10/21*(e*cos(d*x+c))^(3/2)*(a^2+a^2*sin(d*x+c))^2/d/e-22/21*(e*co
s(d*x+c))^(3/2)*(a^4+a^4*sin(d*x+c))/d/e+22/3*a^4*(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))
^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2757, 2748, 2721, 2719}

$$-\frac{22a^4(e\cos(c+dx))^{3/2}}{9de}-\frac{22(a^4\sin(c+dx)+a^4)\,(e\cos(c+dx))^{3/2}}{21de}+\frac{22a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{3d\sqrt{\cos(c+dx)}}-\frac{10(a^2\sin(c+dx)+a^2)^2\,(e\cos(c+dx))^{3/2}}{21de}-\frac{2a(a\sin(c+dx)+a)^3(e\cos(c+dx))^{3/2}}{9de}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (-22*a^4*(e*Cos[c + d*x])^(3/2))/(9*d*e) + (22*a^4*Sqrt[e*Cos[c + d*x]]*Ell
ipticE[(c + d*x)/2, 2])/(3*d*Sqrt[Cos[c + d*x]]) - (2*a*(e*Cos[c + d*x])^(3
/2)*(a + a*Sin[c + d*x])^3)/(9*d*e) - (10*(e*Cos[c + d*x])^(3/2)*(a^2 + a^2
*Sin[c + d*x])^2)/(21*d*e) - (22*(e*Cos[c + d*x])^(3/2)*(a^4 + a^4*Sin[c +
d*x]))/(21*d*e)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2757

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ
[2*m, 2*p]
```

Rubi steps

$$\int \sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^4\,dx = -\frac{2a(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^3}{9de}+\frac{1}{3}(5a)\int \sqrt{e\cos(c$$
$$= -\frac{2a(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^3}{9de}-\frac{10(e\cos(c+dx))^{3/}}{}$$
$$= -\frac{2a(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^3}{9de}-\frac{10(e\cos(c+dx))^{3/}}{}$$
$$= -\frac{22a^4(e\cos(c+dx))^{3/2}}{9de}-\frac{2a(e\cos(c+dx))^{3/2}(a+a\sin(c+d}{9de}$$
$$= -\frac{22a^4(e\cos(c+dx))^{3/2}}{9de}-\frac{2a(e\cos(c+dx))^{3/2}(a+a\sin(c+d}{9de}$$
$$= -\frac{22a^4(e\cos(c+dx))^{3/2}}{9de}+\frac{22a^4\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right.}{3d\sqrt{\cos(c+dx)}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.06, size = 66, normalized size = 0.37

$$-\frac{32\ 2^{3/4}a^4(e\cos(c+dx))^{3/2}\,{}_2F_1\left(-\frac{15}{4},\frac{3}{4};\frac{7}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{3de(1+\sin(c+dx))^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (-32*2^(3/4)*a^4*(e*Cos[c + d*x])^(3/2)*Hypergeometric2F1[-15/4, 3/4, 7/4,
(1 - Sin[c + d*x])/2])/(3*d*e*(1 + Sin[c + d*x])^(3/4))
```

Maple [A]
time = 2.10, size = 258, normalized size = 1.45

method	result
default	$\frac{2a^4e\left(224\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-448\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+576\left(\sin^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-392\left(\sin^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^4*(e*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/63/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*a^4*e*(224*sin(
1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)-448*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2
*c)^8+576*sin(1/2*d*x+1/2*c)^9-392*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-
1152*sin(1/2*d*x+1/2*c)^7+616*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+192*s
in(1/2*d*x+1/2*c)^5+231*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-168*sin(1/2*d*x+1/2*c)^2*c
os(1/2*d*x+1/2*c)+384*sin(1/2*d*x+1/2*c)^3-132*sin(1/2*d*x+1/2*c))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4*(e*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] e^(1/2)*integrate((a*sin(d*x + c) + a)^4*sqrt(cos(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 143, normalized size = 0.80

$$\frac{231i\sqrt{2}\,a^4e^{\frac{1}{2}}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)))-231i\sqrt{2}\,a^4e^{\frac{1}{2}}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c)))+2\left(36\,a^4\cos(dx+c)^3e^{\frac{1}{2}}-168\,a^4\cos(dx+c)\,e^{\frac{1}{2}}+7\left(a^4\cos(dx+c)^3e^{\frac{1}{2}}-13\,a^4\cos(dx+c)\,e^{\frac{1}{2}}\right)\sin(dx+c)\right)\sqrt{\cos(dx+c)}}{63\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4*(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/63*(231*I*sqrt(2)*a^4*e^(1/2)*weierstrassZeta(-4, 0, weierstrassPInverse(
-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 231*I*sqrt(2)*a^4*e^(1/2)*weierstr
assZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) +
 2*(36*a^4*cos(d*x + c)^3*e^(1/2) - 168*a^4*cos(d*x + c)*e^(1/2) + 7*(a^4*c
os(d*x + c)^3*e^(1/2) - 13*a^4*cos(d*x + c)*e^(1/2))*sin(d*x + c))*sqrt(cos
(d*x + c)))/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**4*(e*cos(d*x+c))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3064 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4*(e*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^4*sqrt(cos(d*x + c))*e^(1/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{e\,\cos\left(c+d\,x\right)}\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(1/2)*(a + a*sin(c + d*x))^4,x)
```

```
[Out] int((e*cos(c + d*x))^(1/2)*(a + a*sin(c + d*x))^4, x)
```

3.226 $\int \frac{(a+a\sin(c+dx))^4}{\sqrt{e\cos(c+dx)}}\,dx$

Optimal. Leaf size=178

$$-\frac{78a^4\sqrt{e\cos(c+dx)}}{7de}+\frac{78a^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7d\sqrt{e\cos(c+dx)}}-\frac{2a\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^3}{7de}-26$$

```
[Out] 78/7*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*
d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(e*cos(d*x+c))^(1/2)-78/7*a^4*(e*cos
(d*x+c))^(1/2)/d/e-2/7*a*(a+a*sin(d*x+c))^3*(e*cos(d*x+c))^(1/2)/d/e-26/35*
(a^2+a^2*sin(d*x+c))^2*(e*cos(d*x+c))^(1/2)/d/e-78/35*(a^4+a^4*sin(d*x+c))*
(e*cos(d*x+c))^(1/2)/d/e
```

Rubi [A]
time = 0.14, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2757, 2748, 2721, 2720}

$$-\frac{78a^4\sqrt{e\cos(c+dx)}}{7de}-\frac{78(a^4\sin(c+dx)+a^4)\sqrt{e\cos(c+dx)}}{35de}+\frac{78a^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7d\sqrt{e\cos(c+dx)}}-\frac{26(a^2\sin(c+dx)+a^2)^2\sqrt{e\cos(c+dx)}}{35de}-\frac{2a(a\sin(c+dx)+a)^3\sqrt{e\cos(c+dx)}}{7de}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^4/Sqrt[e*Cos[c + d*x]],x]
```

```
[Out] (-78*a^4*Sqrt[e*Cos[c + d*x]])/(7*d*e) + (78*a^4*Sqrt[Cos[c + d*x]]*Ellipti
cF[(c + d*x)/2, 2])/(7*d*Sqrt[e*Cos[c + d*x]]) - (2*a*Sqrt[e*Cos[c + d*x]]*
(a + a*Sin[c + d*x])^3)/(7*d*e) - (26*Sqrt[e*Cos[c + d*x]]*(a^2 + a^2*Sin[c
 + d*x])^2)/(35*d*e) - (78*Sqrt[e*Cos[c + d*x]]*(a^4 + a^4*Sin[c + d*x]))/(
35*d*e)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
```

```
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2757

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ
[2*m, 2*p]
```

Rubi steps

$$\int \frac{(a+a\sin(c+dx))^4}{\sqrt{e\cos(c+dx)}}\,dx = -\frac{2a\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^3}{7de}+\frac{1}{7}(13a)\int \frac{(a+a\sin(c+dx))^3}{\sqrt{e\cos(c+dx)}}\,dx$$

$$= -\frac{2a\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^3}{7de}-\frac{26\sqrt{e\cos(c+dx)}\,(a^2+a^2\sin(c+$$

$$= -\frac{2a\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^3}{7de}-\frac{26\sqrt{e\cos(c+dx)}\,(a^2+a^2\sin(c+$$

$$= -\frac{78a^4\sqrt{e\cos(c+dx)}}{7de}-\frac{2a\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^3}{7de}-\frac{26\sqrt{e\cos(c$$

$$= -\frac{78a^4\sqrt{e\cos(c+dx)}}{7de}-\frac{2a\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^3}{7de}-\frac{26\sqrt{e\cos(c$$

$$= -\frac{78a^4\sqrt{e\cos(c+dx)}}{7de}+\frac{78a^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{7d\sqrt{e\cos(c+dx)}}-\frac{2a\sqrt{e\cos(c+$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.05, size = 64, normalized size = 0.36

$$-\frac{32\sqrt[4]{2}\,a^4\sqrt{e\cos(c+dx)}\;{}_2F_1\left(-\frac{13}{4},\frac{1}{4};\frac{5}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{de\sqrt[4]{1+\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^4/Sqrt[e*Cos[c + d*x]],x]
```

```
[Out] (-32*2^(1/4)*a^4*Sqrt[e*Cos[c + d*x]]*Hypergeometric2F1[-13/4, 1/4, 5/4, (1
 - Sin[c + d*x])/2])/(d*e*(1 + Sin[c + d*x])^(1/4))
```

Maple [A]
time = 2.42, size = 222, normalized size = 1.25

method	result
default	$-\frac{2a^4\left(80\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-120\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+224\left(\sin^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-280\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-3\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/35/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*a^4*(80*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-120*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*
c)+224*sin(1/2*d*x+1/2*c)^7-280*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-336
*sin(1/2*d*x+1/2*c)^5+195*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+160*sin(1/2*d*x+1/2*c)^2
*cos(1/2*d*x+1/2*c)-392*sin(1/2*d*x+1/2*c)^3+252*sin(1/2*d*x+1/2*c))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] e^(-1/2)*integrate((a*sin(d*x + c) + a)^4/sqrt(cos(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 115, normalized size = 0.65

$$\frac{\left(-195i\sqrt{2}\,a^4\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+195i\sqrt{2}\,a^4\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+2\left(28\,a^4\cos(dx+c)^2-280\,a^4+5\left(a^4\cos(dx+c)^2-17\,a^4\right)\sin(dx+c)\right)\sqrt{\cos(dx+c)}\right)e^{\left(-\frac{1}{2}\right)}}{35\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/35*(-195*I*sqrt(2)*a^4*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*
x + c)) + 195*I*sqrt(2)*a^4*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin
(d*x + c)) + 2*(28*a^4*cos(d*x + c)^2 - 280*a^4 + 5*(a^4*cos(d*x + c)^2 - 1
7*a^4)*sin(d*x + c))*sqrt(cos(d*x + c)))*e^(-1/2)/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**4/(e*cos(d*x+c))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3065 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^4*e^(-1/2)/sqrt(cos(d*x + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\,\sin\left(c+d\,x\right))^4}{\sqrt{e\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^4/(e*cos(c + d*x))^(1/2),x)
```

```
[Out] int((a + a*sin(c + d*x))^4/(e*cos(c + d*x))^(1/2), x)
```

3.227 $\int \frac{(a+a\sin(c+dx))^4}{(e\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=156

$$-\frac{154a^4\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5de^2\sqrt{\cos(c+dx)}}-\frac{154a^4(e\cos(c+dx))^{3/2}\sin(c+dx)}{15de^3}+\frac{4a^7(e\cos(c+dx))^{11/2}}{de^7(a-a\sin(c+dx))^3}+\frac{3a}{}$$

```
[Out] -154/15*a^4*(e*cos(d*x+c))^(3/2)*sin(d*x+c)/d/e^3+4*a^7*(e*cos(d*x+c))^(11/
2)/d/e^7/(a-a*sin(d*x+c))^3+44/3*a^8*(e*cos(d*x+c))^(7/2)/d/e^5/(a^4-a^4*si
n(d*x+c))-154/5*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipti
cE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/d/e^2/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2749, 2759, 2715, 2721, 2719}

$$\frac{4a^7(e\cos(c+dx))^{11/2}}{de^7(a-a\sin(c+dx))^3}-\frac{154a^4\sin(c+dx)(e\cos(c+dx))^{3/2}}{15de^3}-\frac{154a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5de^2\sqrt{\cos(c+dx)}}+\frac{44a^8(e\cos(c+dx))^{7/2}}{3de^5\left(a^4-a^4\sin(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^4/(e*Cos[c + d*x])^(3/2),x]
```

```
[Out] (-154*a^4*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*e^2*Sqrt[Cos
[c + d*x]]) - (154*a^4*(e*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(15*d*e^3) + (4
*a^7*(e*Cos[c + d*x])^(11/2))/(d*e^7*(a - a*Sin[c + d*x])^3) + (44*a^8*(e*C
os[c + d*x])^(7/2))/(3*d*e^5*(a^4 - a^4*Sin[c + d*x]))
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2749

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+a\sin(c+dx))^4}{(e\cos(c+dx))^{3/2}}\,dx &= \frac{a^8\int \frac{(e\cos(c+dx))^{13/2}}{(a-a\sin(c+dx))^4}\,dx}{e^8}\\
&= \frac{4a^7(e\cos(c+dx))^{11/2}}{de^7(a-a\sin(c+dx))^3} - \frac{(11a^6)\int \frac{(e\cos(c+dx))^{9/2}}{(a-a\sin(c+dx))^2}\,dx}{e^6}\\
&= \frac{4a^7(e\cos(c+dx))^{11/2}}{de^7(a-a\sin(c+dx))^3} + \frac{44a^6(e\cos(c+dx))^{7/2}}{3de^5\left(a^2-a^2\sin(c+dx)\right)} - \frac{(77a^4)\int(e\cos(c+dx))^5
\end{aligned}
$$

$$
= -\frac{154a^4(e\cos(c+dx))^{3/2}\sin(c+dx)}{15de^3} + \frac{4a^7(e\cos(c+dx))^{11/2}}{de^7(a-a\sin(c+dx))^3} + \frac{44a^6(e\cos(}{3de^5\left(a^2-a^2\right.}
$$

$$
= -\frac{154a^4(e\cos(c+dx))^{3/2}\sin(c+dx)}{15de^3} + \frac{4a^7(e\cos(c+dx))^{11/2}}{de^7(a-a\sin(c+dx))^3} + \frac{44a^6(e\cos(}{3de^5\left(a^2-a^2\right.}
$$

$$
= -\frac{154a^4\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5de^2\sqrt{\cos(c+dx)}} - \frac{154a^4(e\cos(c+dx))^{3/2}\sin(c+dx)}{15de^3}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.07, size = 64, normalized size = 0.41

$$
\frac{16\ 2^{3/4}a^4\,{}_2F_1\left(-\frac{11}{4},-\frac{1}{4};\frac{3}{4};\frac{1}{2}(1-\sin(c+dx))\right)\sqrt[4]{1+\sin(c+dx)}}{de\sqrt{e\cos(c+dx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^4/(e*Cos[c + d*x])^(3/2),x]
```

```
[Out] (16*2^(3/4)*a^4*Hypergeometric2F1[-11/4, -1/4, 3/4, (1 - Sin[c + d*x])/2]*(
1 + Sin[c + d*x])^(1/4))/(d*e*Sqrt[e*Cos[c + d*x]])
```

Maple [A]
time = 3.06, size = 190, normalized size = 1.22

method	result
default	$-\frac{2\left(-24\left(\sin^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+24\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-80\left(\sin^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+231\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^{2}\right.}\right.}{15e\sqrt{-2\left(\sin^{2}\left(\frac{dx}{2}+\right.\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/15/e/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/sin(1/2*d*x+1/2*c)*(-24*sin(1/2
*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)
-80*sin(1/2*d*x+1/2*c)^5+231*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-246*sin(1/2*d*x+1/2*c
)^2*cos(1/2*d*x+1/2*c)+80*sin(1/2*d*x+1/2*c)^3-140*sin(1/2*d*x+1/2*c))*a^4/
d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] e^(-3/2)*integrate((a*sin(d*x + c) + a)^4/cos(d*x + c)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 241, normalized size = 1.54

$$\frac{231\left(-i\sqrt{2}\,a^{4}\cos\left(dx+c\right)+i\sqrt{2}\,a^{4}\sin\left(dx+c\right)-i\sqrt{2}\,a^{4}\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)))+231\left(i\sqrt{2}\,a^{4}\cos\left(dx+c\right)-i\sqrt{2}\,a^{4}\sin\left(dx+c\right)+i\sqrt{2}\,a^{4}\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)))+2\left(3\,a^{4}\cos\left(dx+c\right)^{3}+20\,a^{4}\cos\left(dx+c\right)^{2}+137\,a^{4}\cos\left(dx+c\right)+120\,a^{4}+\left(3\,a^{4}\cos\left(dx+c\right)^{2}-17\,a^{4}\cos\left(dx+c\right)+120\,a^{4}\right)\sin\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)}}{15\left(d\cos\left(dx+c\right)e^{\frac{3}{2}}-de^{\frac{3}{2}}\sin\left(dx+c\right)+de^{\frac{3}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/15*(231*(-I*sqrt(2)*a^4*cos(d*x + c) + I*sqrt(2)*a^4*sin(d*x + c) - I*sqr
t(2)*a^4)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) +
I*sin(d*x + c))) + 231*(I*sqrt(2)*a^4*cos(d*x + c) - I*sqrt(2)*a^4*sin(d*x
+ c) + I*sqrt(2)*a^4)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos
```

```
(d*x + c) - I*sin(d*x + c))) + 2*(3*a^4*cos(d*x + c)^3 + 20*a^4*cos(d*x + c
)^2 + 137*a^4*cos(d*x + c) + 120*a^4 + (3*a^4*cos(d*x + c)^2 - 17*a^4*cos(d
*x + c) + 120*a^4)*sin(d*x + c))*sqrt(cos(d*x + c)))/(d*cos(d*x + c)*e^(3/2
) - d*e^(3/2)*sin(d*x + c) + d*e^(3/2))
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**4/(e*cos(d*x+c))**(3/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 4850 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((a*sin(d*x + c) + a)^4*e^(-3/2)/cos(d*x + c)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\,\sin(c+d\,x))^4}{(e\,\cos(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^4/(e*cos(c + d*x))^(3/2),x)
[Out] int((a + a*sin(c + d*x))^4/(e*cos(c + d*x))^(3/2), x)
```

3.228 $\int \frac{(a+a\sin(c+dx))^4}{(e\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=152

$$-\frac{10a^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{de^2\sqrt{e\cos(c+dx)}}-\frac{10a^4\sqrt{e\cos(c+dx)}\,\sin(c+dx)}{de^3}+\frac{4a^7(e\cos(c+dx))^{9/2}}{3de^7(a-a\sin(c+dx))^3}+\frac{12a^8}{de^5\,(a}$$

```
[Out] 4/3*a^7*(e*cos(d*x+c))^(9/2)/d/e^7/(a-a*sin(d*x+c))^3+12*a^8*(e*cos(d*x+c))
^(5/2)/d/e^5/(a^4-a^4*sin(d*x+c))-10*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1
/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/e^2/
(e*cos(d*x+c))^(1/2)-10*a^4*sin(d*x+c)*(e*cos(d*x+c))^(1/2)/d/e^3
```

Rubi [A]
time = 0.15, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2749, 2759, 2715, 2721, 2720}

$$\frac{4a^7(e\cos(c+dx))^{9/2}}{3de^7(a-a\sin(c+dx))^3}-\frac{10a^4\sin(c+dx)\sqrt{e\cos(c+dx)}}{de^3}-\frac{10a^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{de^2\sqrt{e\cos(c+dx)}}+\frac{12a^8(e\cos(c+dx))^{5/2}}{de^5\left(a^4-a^4\sin(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^4/(e*Cos[c + d*x])^(5/2),x]
```

```
[Out] (-10*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*e^2*Sqrt[e*Cos[c
+ d*x]]) - (10*a^4*Sqrt[e*Cos[c + d*x]]*Sin[c + d*x])/(d*e^3) + (4*a^7*(e*C
os[c + d*x])^(9/2))/(3*d*e^7*(a - a*Sin[c + d*x])^3) + (12*a^8*(e*Cos[c + d
*x])^(5/2))/(d*e^5*(a^4 - a^4*Sin[c + d*x]))
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2749

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+a\sin(c+dx))^4}{(e\cos(c+dx))^{5/2}}\,dx &= \frac{a^8\int \frac{(e\cos(c+dx))^{11/2}}{(a-a\sin(c+dx))^4}\,dx}{e^8}\\
&= \frac{4a^7(e\cos(c+dx))^{9/2}}{3de^7(a-a\sin(c+dx))^3} - \frac{(3a^6)\int \frac{(e\cos(c+dx))^{7/2}}{(a-a\sin(c+dx))^2}\,dx}{e^6}\\
&= \frac{4a^7(e\cos(c+dx))^{9/2}}{3de^7(a-a\sin(c+dx))^3} + \frac{12a^6(e\cos(c+dx))^{5/2}}{de^5\left(a^2-a^2\sin(c+dx)\right)} - \frac{(15a^4)\int(e\cos(c+dx))^3}{e^4}\\
&= -\frac{10a^4\sqrt{e\cos(c+dx)}\,\sin(c+dx)}{de^3} + \frac{4a^7(e\cos(c+dx))^{9/2}}{3de^7(a-a\sin(c+dx))^3} + \frac{12a^6(e\cos(c}{de^5\left(a^2-a^2\sin\right.}\\
&= -\frac{10a^4\sqrt{e\cos(c+dx)}\,\sin(c+dx)}{de^3} + \frac{4a^7(e\cos(c+dx))^{9/2}}{3de^7(a-a\sin(c+dx))^3} + \frac{12a^6(e\cos(c}{de^5\left(a^2-a^2\sin\right.}\\
&= -\frac{10a^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{de^2\sqrt{e\cos(c+dx)}} - \frac{10a^4\sqrt{e\cos(c+dx)}\,\sin(c+dx)}{de^3} + \frac{4}{3d}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.06, size = 66, normalized size = 0.43

$$\frac{16\sqrt[4]{2}\,a^4\,{}_2F_1\left(-\frac{9}{4},-\frac{3}{4};\frac{1}{4};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{3/4}}{3de(e\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^4/(e*Cos[c + d*x])^(5/2),x]
```

```
[Out] (16*2^(1/4)*a^4*Hypergeometric2F1[-9/4, -3/4, 1/4, (1 - Sin[c + d*x])/2]*(1
 + Sin[c + d*x])^(3/4))/(3*d*e*(e*Cos[c + d*x])^(3/2))
```

Maple [A]
time = 3.56, size = 263, normalized size = 1.73

method	result
default	$\frac{2\left(-8\left(\sin^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+30\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\sin^{2}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/(2*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*
e+e)^(1/2)/e^2*(-8*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+30*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-
48*sin(1/2*d*x+1/2*c)^5-15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-18*sin(1/2*d*x+1/2*c)^2
*cos(1/2*d*x+1/2*c)+48*sin(1/2*d*x+1/2*c)^3-20*sin(1/2*d*x+1/2*c))*a^4/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] e^(-5/2)*integrate((a*sin(d*x + c) + a)^4/cos(d*x + c)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 146, normalized size = 0.96

$$-\frac{15\left(-i\sqrt{2}\,a^{4}\sin\left(dx+c\right)+i\sqrt{2}\,a^{4}\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)+15\left(i\sqrt{2}\,a^{4}\sin\left(dx+c\right)-i\sqrt{2}\,a^{4}\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)+2\left(a^{4}\cos\left(dx+c\right)^{2}-11\,a^{4}\sin\left(dx+c\right)+19\,a^{4}\right)\sqrt{\cos\left(dx+c\right)}}{3\left(de^{\frac{5}{2}}\sin\left(dx+c\right)-de^{\frac{5}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(15*(-I*sqrt(2)*a^4*sin(d*x + c) + I*sqrt(2)*a^4)*weierstrassPInverse(
-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 15*(I*sqrt(2)*a^4*sin(d*x + c) - I*
sqrt(2)*a^4)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*
```

```
(a^4*cos(d*x + c)^2 - 11*a^4*sin(d*x + c) + 19*a^4)*sqrt(cos(d*x + c)))/(d*
e^(5/2)*sin(d*x + c) - d*e^(5/2))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**4/(e*cos(d*x+c))**(5/2),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((a*sin(d*x + c) + a)^4*e^(-5/2)/cos(d*x + c)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + a\,\sin(c + d\,x))^4}{(e\,\cos(c + d\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^4/(e*cos(c + d*x))^(5/2),x)
[Out] int((a + a*sin(c + d*x))^4/(e*cos(c + d*x))^(5/2), x)
```

3.229 $\int \frac{(a+a\sin(c+dx))^4}{(e\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=127

$$\frac{42a^4\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5de^4\sqrt{\cos(c+dx)}}+\frac{4a^7(e\cos(c+dx))^{7/2}}{5de^7(a-a\sin(c+dx))^3}-\frac{28a^8(e\cos(c+dx))^{3/2}}{5de^5\left(a^4-a^4\sin(c+dx)\right)}$$

```
[Out] 4/5*a^7*(e*cos(d*x+c))^(7/2)/d/e^7/(a-a*sin(d*x+c))^3-28/5*a^8*(e*cos(d*x+c
))^(3/2)/d/e^5/(a^4-a^4*sin(d*x+c))+42/5*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/c
os(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2
)/d/e^4/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2749, 2759, 2721, 2719}

$$\frac{4a^7(e\cos(c+dx))^{7/2}}{5de^7(a-a\sin(c+dx))^3}+\frac{42a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5de^4\sqrt{\cos(c+dx)}}-\frac{28a^8(e\cos(c+dx))^{3/2}}{5de^5\left(a^4-a^4\sin(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^4/(e*Cos[c + d*x])^(7/2),x]
```

```
[Out] (42*a^4*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*e^4*Sqrt[Cos[c
 + d*x]]) + (4*a^7*(e*Cos[c + d*x])^(7/2))/(5*d*e^7*(a - a*Sin[c + d*x])^3)
 - (28*a^8*(e*Cos[c + d*x])^(3/2))/(5*d*e^5*(a^4 - a^4*Sin[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2749

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+a\sin(c+dx))^4}{(e\cos(c+dx))^{7/2}}\,dx &= \frac{a^8\int \frac{(e\cos(c+dx))^{9/2}}{(a-a\sin(c+dx))^4}\,dx}{e^8}\\ &= \frac{4a^7(e\cos(c+dx))^{7/2}}{5de^7(a-a\sin(c+dx))^3}-\frac{(7a^6)\int \frac{(e\cos(c+dx))^{5/2}}{(a-a\sin(c+dx))^2}\,dx}{5e^6}\\ &= \frac{4a^7(e\cos(c+dx))^{7/2}}{5de^7(a-a\sin(c+dx))^3}-\frac{28a^6(e\cos(c+dx))^{3/2}}{5de^5\left(a^2-a^2\sin(c+dx)\right)}+\frac{(21a^4)\int \sqrt{e\cos(c+d\ldots}}{5e^4}\\ &= \frac{4a^7(e\cos(c+dx))^{7/2}}{5de^7(a-a\sin(c+dx))^3}-\frac{28a^6(e\cos(c+dx))^{3/2}}{5de^5\left(a^2-a^2\sin(c+dx)\right)}+\frac{\left(21a^4\sqrt{e\cos(c+dx)}\ldots\right.}{5e^4\sqrt{\cos(\ldots}}\\ &= \frac{42a^4\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5de^4\sqrt{\cos(c+dx)}}+\frac{4a^7(e\cos(c+dx))^{7/2}}{5de^7(a-a\sin(c+dx))^3}-\frac{28a^6(e\,\mathrm{co}\ldots}{5de^5\,(a^2-\ldots}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.08, size = 66, normalized size = 0.52

$$\frac{8\,2^{3/4}a^4\,{}_2F_1\left(-\frac{7}{4},-\frac{5}{4};-\frac{1}{4};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{5/4}}{5de(e\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^4/(e*Cos[c + d*x])^(7/2),x]
[Out] (8*2^(3/4)*a^4*Hypergeometric2F1[-7/4, -5/4, -1/4, (1 - Sin[c + d*x])/2]*(1
 + Sin[c + d*x])^(5/4))/(5*d*e*(e*Cos[c + d*x])^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 331 vs. 2(139) = 278.
time = 5.54, size = 332, normalized size = 2.61

method	result
default	$\frac{2\left(84\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-128\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)c\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)/(-
2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/e^3*(84*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*
d*x+1/2*c)^4-128*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-84*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+128*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-
80*sin(1/2*d*x+1/2*c)^5+21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-16*sin(1/2*d*x+1/2*c)^2
*cos(1/2*d*x+1/2*c)+80*sin(1/2*d*x+1/2*c)^3-12*sin(1/2*d*x+1/2*c))*a^4/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] e^(-7/2)*integrate((a*sin(d*x + c) + a)^4/cos(d*x + c)^(7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 304, normalized size = 2.39

$$-\frac{21\left(-i\sqrt{2}\,a^4\cos\left(dx+c\right)^2+i\sqrt{2}\,a^4\cos\left(dx+c\right)+2i\sqrt{2}\,a^4+\left(-i\sqrt{2}\,a^4\cos\left(dx+c\right)-2i\sqrt{2}\,a^4\right)\sin\left(dx+c\right)\right){\rm weierstrassZeta}\left(-4,0,{\rm weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)\right)+21\left(i\sqrt{2}\,a^4\cos\left(dx+c\right)^2-i\sqrt{2}\,a^4\cos\left(dx+c\right)-2i\sqrt{2}\,a^4+\left(i\sqrt{2}\,a^4\cos\left(dx+c\right)+2i\sqrt{2}\,a^4\right)\sin\left(dx+c\right)\right){\rm weierstrassZeta}\left(-4,0,{\rm weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)\right)-8\left(4\,a^4\cos\left(dx+c\right)^2+3\,a^4\cos\left(dx+c\right)-a^4-\left(4\,a^4\cos\left(dx+c\right)+a^4\right)\sin\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)}}{5\left(d\cos\left(dx+c\right)^2e^{\frac{7}{2}}-d\cos\left(dx+c\right)e^{\frac{7}{2}}-2\,de^{\frac{7}{2}}+\left(d\cos\left(dx+c\right)e^{\frac{7}{2}}+2\,de^{\frac{7}{2}}\right)\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] -1/5*(21*(-I*sqrt(2)*a^4*cos(d*x + c)^2 + I*sqrt(2)*a^4*cos(d*x + c) + 2*I*
sqrt(2)*a^4 + (-I*sqrt(2)*a^4*cos(d*x + c) - 2*I*sqrt(2)*a^4)*sin(d*x + c))
*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x
 + c))) + 21*(I*sqrt(2)*a^4*cos(d*x + c)^2 - I*sqrt(2)*a^4*cos(d*x + c) - 2
*I*sqrt(2)*a^4 + (I*sqrt(2)*a^4*cos(d*x + c) + 2*I*sqrt(2)*a^4)*sin(d*x + c
))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d
*x + c))) - 8*(4*a^4*cos(d*x + c)^2 + 3*a^4*cos(d*x + c) - a^4 - (4*a^4*cos
```

```
(d*x + c) + a^4)*sin(d*x + c))*sqrt(cos(d*x + c)))/(d*cos(d*x + c)^2*e^(7/2
) - d*cos(d*x + c)*e^(7/2) - 2*d*e^(7/2) + (d*cos(d*x + c)*e^(7/2) + 2*d*e^
(7/2))*sin(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**4/(e*cos(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^4*e^(-7/2)/cos(d*x + c)^(7/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\,\sin(c+d\,x))^4}{(e\,\cos(c+d\,x))^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^4/(e*cos(c + d*x))^(7/2),x)
```

```
[Out] int((a + a*sin(c + d*x))^4/(e*cos(c + d*x))^(7/2), x)
```

3.230 $\int \frac{(a+a\sin(c+dx))^4}{(e\cos(c+dx))^{9/2}}\,dx$

Optimal. Leaf size=127

$$\frac{10a^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21de^4\sqrt{e\cos(c+dx)}}+\frac{4a^7(e\cos(c+dx))^{5/2}}{7de^7(a-a\sin(c+dx))^3}-\frac{20a^8\sqrt{e\cos(c+dx)}}{21de^5\left(a^4-a^4\sin(c+dx)\right)}$$

```
[Out] 4/7*a^7*(e*cos(d*x+c))^(5/2)/d/e^7/(a-a*sin(d*x+c))^3+10/21*a^4*(cos(1/2*d*
x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*
cos(d*x+c)^(1/2)/d/e^4/(e*cos(d*x+c))^(1/2)-20/21*a^8*(e*cos(d*x+c))^(1/2)/
d/e^5/(a^4-a^4*sin(d*x+c))
```

Rubi [A]
time = 0.14, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2749, 2759, 2721, 2720}

$$\frac{4a^7(e\cos(c+dx))^{5/2}}{7de^7(a-a\sin(c+dx))^3}+\frac{10a^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21de^4\sqrt{e\cos(c+dx)}}-\frac{20a^8\sqrt{e\cos(c+dx)}}{21de^5\left(a^4-a^4\sin(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^4/(e*Cos[c + d*x])^(9/2),x]
```

```
[Out] (10*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*e^4*Sqrt[e*Cos[
c + d*x]]) + (4*a^7*(e*Cos[c + d*x])^(5/2))/(7*d*e^7*(a - a*Sin[c + d*x])^3
) - (20*a^8*Sqrt[e*Cos[c + d*x]])/(21*d*e^5*(a^4 - a^4*Sin[c + d*x]))
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2749

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+a\sin(c+dx))^4}{(e\cos(c+dx))^{9/2}}\,dx &= \frac{a^8\int \frac{(e\cos(c+dx))^{7/2}}{(a-a\sin(c+dx))^4}\,dx}{e^8}\\ &= \frac{4a^7(e\cos(c+dx))^{5/2}}{7de^7(a-a\sin(c+dx))^3}-\frac{(5a^6)\int \frac{(e\cos(c+dx))^{3/2}}{(a-a\sin(c+dx))^2}\,dx}{7e^6}\\ &= \frac{4a^7(e\cos(c+dx))^{5/2}}{7de^7(a-a\sin(c+dx))^3}-\frac{20a^6\sqrt{e\cos(c+dx)}}{21de^5\,(a^2-a^2\sin(c+dx))}+\frac{(5a^4)\int \frac{1}{\sqrt{e\cos(c+d}}}{21e^4}\\ &= \frac{4a^7(e\cos(c+dx))^{5/2}}{7de^7(a-a\sin(c+dx))^3}-\frac{20a^6\sqrt{e\cos(c+dx)}}{21de^5\,(a^2-a^2\sin(c+dx))}+\frac{\left(5a^4\sqrt{\cos(c+dx)}\right)}{21e^4\sqrt{e\cos}}\\ &= \frac{10a^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21de^4\sqrt{e\cos(c+dx)}}+\frac{4a^7(e\cos(c+dx))^{5/2}}{7de^7(a-a\sin(c+dx))^3}-\frac{20a^6\sqrt{e\,c}}{21de^5\,(a^2-}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.09, size = 66, normalized size = 0.52

$$\frac{8\sqrt[4]{2}\,a^4\,{}_2F_1\left(-\frac{7}{4},-\frac{5}{4};-\frac{3}{4};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{7/4}}{7de(e\cos(c+dx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^4/(e*Cos[c + d*x])^(9/2),x]
```

```
[Out] (8*2^(1/4)*a^4*Hypergeometric2F1[-7/4, -5/4, -3/4, (1 - Sin[c + d*x])/2]*(1
 + Sin[c + d*x])^(7/4))/(7*d*e*(e*Cos[c + d*x])^(7/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 400 vs. $2(139) = 278$.
time = 6.11, size = 401, normalized size = 3.16

method	result
default	$-\frac{2\left(40\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-60\sqrt{\frac{1}{2}-\frac{\cos(dx}{2}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/21/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^
2-1)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/e^4*(40*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^6-60*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*
d*x+1/2*c)^4-128*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+30*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2))*sin(1/2*d*x+1/2*c)^2+128*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-
112*sin(1/2*d*x+1/2*c)^5-5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+16*sin(1/2*d*x+1/2*c)^2
*cos(1/2*d*x+1/2*c)+112*sin(1/2*d*x+1/2*c)^3-4*sin(1/2*d*x+1/2*c))*a^4/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(9/2),x, algorithm="maxima")
```

```
[Out] e^(-9/2)*integrate((a*sin(d*x + c) + a)^4/cos(d*x + c)^(9/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 179, normalized size = 1.41

$$-\frac{5\left(i\sqrt{2}\,a^4\cos(dx+c)^2+2i\sqrt{2}\,a^4\sin(dx+c)-2i\sqrt{2}\,a^4\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5\left(-i\sqrt{2}\,a^4\cos(dx+c)^2-2i\sqrt{2}\,a^4\sin(dx+c)+2i\sqrt{2}\,a^4\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+8\left(4a^4\sin(dx+c)-a^4\right)\sqrt{\cos(dx+c)}}{21\left(d\cos(dx+c)^2e^{\frac{9}{2}}+2de^{\frac{9}{2}}\sin(dx+c)-2de^{\frac{9}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(9/2),x, algorithm="fricas")
```

```
[Out] -1/21*(5*(I*sqrt(2)*a^4*cos(d*x + c)^2 + 2*I*sqrt(2)*a^4*sin(d*x + c) - 2*I
*sqrt(2)*a^4)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5
*(-I*sqrt(2)*a^4*cos(d*x + c)^2 - 2*I*sqrt(2)*a^4*sin(d*x + c) + 2*I*sqrt(2
)*a^4)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 8*(4*a^4
*sin(d*x + c) - a^4)*sqrt(cos(d*x + c)))/(d*cos(d*x + c)^2*e^(9/2) + 2*d*e^
(9/2)*sin(d*x + c) - 2*d*e^(9/2))
```

<u>**Sympy**</u> [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**4/(e*cos(d*x+c))**(9/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 4847 deep
```

<u>**Giac**</u> [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(9/2),x, algorithm="giac")
[Out] integrate((a*sin(d*x + c) + a)^4*e^(-9/2)/cos(d*x + c)^(9/2), x)
```

<u>**Mupad**</u> [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\sin(c+dx))^4}{(e\cos(c+dx))^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^4/(e*cos(c + d*x))^(9/2),x)
[Out] int((a + a*sin(c + d*x))^4/(e*cos(c + d*x))^(9/2), x)
```

3.231 $\int \frac{(a+a\sin(c+dx))^4}{(e\cos(c+dx))^{11/2}}\,dx$

Optimal. Leaf size=169

$$\frac{2a^4\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15de^6\sqrt{\cos(c+dx)}}+\frac{4a^7(e\cos(c+dx))^{3/2}}{9de^7(a-a\sin(c+dx))^3}-\frac{2a^8(e\cos(c+dx))^{3/2}}{15de^7\left(a^2-a^2\sin(c+dx)\right)^2}-\frac{2a^8(e\cos}{15de^7\left(a^4-\right.}$$

```
[Out] 4/9*a^7*(e*cos(d*x+c))^(3/2)/d/e^7/(a-a*sin(d*x+c))^3-2/15*a^8*(e*cos(d*x+c
))^(3/2)/d/e^7/(a^2-a^2*sin(d*x+c))^2-2/15*a^8*(e*cos(d*x+c))^(3/2)/d/e^7/(
a^4-a^4*sin(d*x+c))+2/15*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/d/e^6/cos(d*x+
c)^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2749, 2759, 2760, 2762, 2721, 2719}

$$\frac{4a^7(e\cos(c+dx))^{3/2}}{9de^7(a-a\sin(c+dx))^3}+\frac{2a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{15de^6\sqrt{\cos(c+dx)}}-\frac{2a^8(e\cos(c+dx))^{3/2}}{15de^7\left(a^4-a^4\sin(c+dx)\right)}-\frac{2a^8(e\cos(c+dx))^{3/2}}{15de^7\left(a^2-a^2\sin(c+dx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^4/(e*Cos[c + d*x])^(11/2),x]
```

```
[Out] (2*a^4*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*d*e^6*Sqrt[Cos[c
 + d*x]]) + (4*a^7*(e*Cos[c + d*x])^(3/2))/(9*d*e^7*(a - a*Sin[c + d*x])^3)
 - (2*a^8*(e*Cos[c + d*x])^(3/2))/(15*d*e^7*(a^2 - a^2*Sin[c + d*x])^2) - (
2*a^8*(e*Cos[c + d*x])^(3/2))/(15*d*e^7*(a^4 - a^4*Sin[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2749

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
```

```
0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2760

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*(2*m + p + 1))), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2762

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[b*((g*Cos[e + f*x])^(p + 1)/(a*f*g*(p - 1)*(a + b*S
in[e + f*x]))), x] + Dist[p/(a*(p - 1)), Int[(g*Cos[e + f*x])^p, x], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] &&  !GeQ[p, 1] && Integer
Q[2*p]
```

Rubi steps

$$\int \frac{(a+a\sin(c+dx))^4}{(e\cos(c+dx))^{11/2}}\,dx = \frac{a^8\int \frac{(e\cos(c+dx))^{5/2}}{(a-a\sin(c+dx))^4}\,dx}{e^8}$$

$$= \frac{4a^7(e\cos(c+dx))^{3/2}}{9de^7(a-a\sin(c+dx))^3} - \frac{a^6\int \frac{\sqrt{e\cos(c+dx)}}{(a-a\sin(c+dx))^2}\,dx}{3e^6}$$

$$= \frac{4a^7(e\cos(c+dx))^{3/2}}{9de^7(a-a\sin(c+dx))^3} - \frac{2a^6(e\cos(c+dx))^{3/2}}{15de^7(a-a\sin(c+dx))^2} - \frac{a^5\int \frac{\sqrt{e\cos(c+dx)}}{a-a\sin(c+dx)}}{15e^6}$$

$$= \frac{4a^7(e\cos(c+dx))^{3/2}}{9de^7(a-a\sin(c+dx))^3} - \frac{2a^6(e\cos(c+dx))^{3/2}}{15de^7(a-a\sin(c+dx))^2} - \frac{2a^5(e\cos(c+dx))^{3/}}{15de^7(a-a\sin(c+d}$$

$$= \frac{4a^7(e\cos(c+dx))^{3/2}}{9de^7(a-a\sin(c+dx))^3} - \frac{2a^6(e\cos(c+dx))^{3/2}}{15de^7(a-a\sin(c+dx))^2} - \frac{2a^5(e\cos(c+dx))^{3/}}{15de^7(a-a\sin(c+d}$$

$$= \frac{2a^4\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15de^6\sqrt{\cos(c+dx)}} + \frac{4a^7(e\cos(c+dx))^{3/2}}{9de^7(a-a\sin(c+dx))^3} - \frac{2a^6(e\cos}{15de^7(a-}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.13, size = 66, normalized size = 0.39

$$\frac{4\ 2^{3/4}a^4\,{}_2F_1\left(-\frac{9}{4},-\frac{3}{4};-\frac{5}{4};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{9/4}}{9de(e\cos(c+dx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^4/(e*Cos[c + d*x])^(11/2),x]
[Out] (4*2^(3/4)*a^4*Hypergeometric2F1[-9/4, -3/4, -5/4, (1 - Sin[c + d*x])/2]*(1
 + Sin[c + d*x])^(9/4))/(9*d*e*(e*Cos[c + d*x])^(9/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 513 vs. 2(177) = 354.
time = 7.48, size = 514, normalized size = 3.04

method	result
default	$\frac{2\left(48\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-96\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\mathrm{c}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(11/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/45/(16*sin(1/2*d*x+1/2*c)^8-32*sin(1/2*d*x+1/2*c)^6+24*sin(1/2*d*x+1/2*c)
^4-8*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+
e)^(1/2)/e^5*(48*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^8-96*sin(1/2*d
*x+1/2*c)^10*cos(1/2*d*x+1/2*c)-96*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2
*c)^6+192*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+72*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*sin(1/2*d*x+1/2*c)^4-272*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-24*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+176*sin(1/2*d*x+1/2*c)^4*cos(1/2*
d*x+1/2*c)-144*sin(1/2*d*x+1/2*c)^5+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+42*sin(1/2*d
*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+144*sin(1/2*d*x+1/2*c)^3+4*sin(1/2*d*x+1/2*c
))*a^4/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(11/2),x, algorithm="maxima")
```

```
[Out] e^(-11/2)*integrate((a*sin(d*x + c) + a)^4/cos(d*x + c)^(11/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 421, normalized size = 2.49

$$\frac{3\left(i\sqrt{2}a^4\cos(dx+c)^3+3i\sqrt{2}a^4\cos(dx+c)^2-2i\sqrt{2}a^4\cos(dx+c)-4i\sqrt{2}a^4+\left(-i\sqrt{2}a^4\cos(dx+c)^2+2i\sqrt{2}a^4\cos(dx+c)+4i\sqrt{2}a^4\right)\sin(dx+c)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+3\left(-i\sqrt{2}a^4\cos(dx+c)^3-3i\sqrt{2}a^4\cos(dx+c)^2+2i\sqrt{2}a^4\cos(dx+c)+4i\sqrt{2}a^4+\left(i\sqrt{2}a^4\cos(dx+c)^2-2i\sqrt{2}a^4\cos(dx+c)-4i\sqrt{2}a^4\right)\sin(dx+c)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))-2\left(3a^4\cos(dx+c)^3-6a^4\cos(dx+c)^2+a^4\cos(dx+c)+10a^4+\left(3a^4\cos(dx+c)^2+9a^4\cos(dx+c)+10a^4\right)\sin(dx+c)\right)\sqrt{\cos(dx+c)}}{45\left(d\cos(dx+c)^3e^{\frac{11}{2}}+3d\cos(dx+c)^2e^{\frac{11}{2}}-2d\cos(dx+c)e^{\frac{11}{2}}-4de^{\frac{11}{2}}-\left(d\cos(dx+c)^2e^{\frac{11}{2}}-2d\cos(dx+c)e^{\frac{11}{2}}-4de^{\frac{11}{2}}\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(11/2),x, algorithm="fricas")
```

```
[Out] 1/45*(3*(I*sqrt(2)*a^4*cos(d*x + c)^3 + 3*I*sqrt(2)*a^4*cos(d*x + c)^2 - 2*
I*sqrt(2)*a^4*cos(d*x + c) - 4*I*sqrt(2)*a^4 + (-I*sqrt(2)*a^4*cos(d*x + c)
^2 + 2*I*sqrt(2)*a^4*cos(d*x + c) + 4*I*sqrt(2)*a^4)*sin(d*x + c))*weierstr
assZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) +
 3*(-I*sqrt(2)*a^4*cos(d*x + c)^3 - 3*I*sqrt(2)*a^4*cos(d*x + c)^2 + 2*I*sq
rt(2)*a^4*cos(d*x + c) + 4*I*sqrt(2)*a^4 + (I*sqrt(2)*a^4*cos(d*x + c)^2 -
2*I*sqrt(2)*a^4*cos(d*x + c) - 4*I*sqrt(2)*a^4)*sin(d*x + c))*weierstrassZe
ta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(3
*a^4*cos(d*x + c)^3 - 6*a^4*cos(d*x + c)^2 + a^4*cos(d*x + c) + 10*a^4 + (3
*a^4*cos(d*x + c)^2 + 9*a^4*cos(d*x + c) + 10*a^4)*sin(d*x + c))*sqrt(cos(d
```

```
*x + c)))/(d*cos(d*x + c)^3*e^(11/2) + 3*d*cos(d*x + c)^2*e^(11/2) - 2*d*co
s(d*x + c)*e^(11/2) - 4*d*e^(11/2) - (d*cos(d*x + c)^2*e^(11/2) - 2*d*cos(d
*x + c)*e^(11/2) - 4*d*e^(11/2))*sin(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**4/(e*cos(d*x+c))**(11/2),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(11/2),x, algorithm="giac")
[Out] integrate((a*sin(d*x + c) + a)^4*e^(-11/2)/cos(d*x + c)^(11/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\,\sin(c+d\,x))^4}{(e\,\cos(c+d\,x))^{11/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^4/(e*cos(c + d*x))^(11/2),x)
[Out] int((a + a*sin(c + d*x))^4/(e*cos(c + d*x))^(11/2), x)
```

3.232 $\int \frac{(a+a\sin(c+dx))^4}{(e\cos(c+dx))^{13/2}}\,dx$

Optimal. Leaf size=169

$$-\frac{2a^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{77de^6\sqrt{e\cos(c+dx)}}+\frac{4a^7\sqrt{e\cos(c+dx)}}{11de^7(a-a\sin(c+dx))^3}-\frac{2a^8\sqrt{e\cos(c+dx)}}{77de^7\left(a^2-a^2\sin(c+dx)\right)^2}-\frac{2a^8\sqrt{e\,c}}{77de^7\left(a^4-\right.}$$

```
[Out] -2/77*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2
*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/e^6/(e*cos(d*x+c))^(1/2)+4/11*a^7*(
e*cos(d*x+c))^(1/2)/d/e^7/(a-a*sin(d*x+c))^3-2/77*a^8*(e*cos(d*x+c))^(1/2)/
d/e^7/(a^2-a^2*sin(d*x+c))^2-2/77*a^8*(e*cos(d*x+c))^(1/2)/d/e^7/(a^4-a^4*s
in(d*x+c))
```

Rubi [A]
time = 0.18, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2749, 2759, 2760, 2762, 2721, 2720}

$$\frac{4a^7\sqrt{e\cos(c+dx)}}{11de^7(a-a\sin(c+dx))^3}-\frac{2a^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{77de^6\sqrt{e\cos(c+dx)}}-\frac{2a^8\sqrt{e\cos(c+dx)}}{77de^7\left(a^4-a^4\sin(c+dx)\right)}-\frac{2a^8\sqrt{e\cos(c+dx)}}{77de^7\left(a^2-a^2\sin(c+dx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^4/(e*Cos[c + d*x])^(13/2),x]
```

```
[Out] (-2*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(77*d*e^6*Sqrt[e*Cos[
c + d*x]]) + (4*a^7*Sqrt[e*Cos[c + d*x]])/(11*d*e^7*(a - a*Sin[c + d*x])^3)
 - (2*a^8*Sqrt[e*Cos[c + d*x]])/(77*d*e^7*(a^2 - a^2*Sin[c + d*x])^2) - (2*
a^8*Sqrt[e*Cos[c + d*x]])/(77*d*e^7*(a^4 - a^4*Sin[c + d*x]))
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2749

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
```

```
0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2760

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*(2*m + p + 1))), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2762

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[b*((g*Cos[e + f*x])^(p + 1)/(a*f*g*(p - 1)*(a + b*S
in[e + f*x]))), x] + Dist[p/(a*(p - 1)), Int[(g*Cos[e + f*x])^p, x], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] &&  !GeQ[p, 1] && Integer
Q[2*p]
```

Rubi steps

$$\int \frac{(a+a\sin(c+dx))^4}{(e\cos(c+dx))^{13/2}}\,dx = \frac{a^8 \int \frac{(e\cos(c+dx))^{3/2}}{(a-a\sin(c+dx))^4}\,dx}{e^8}$$

$$= \frac{4a^7\sqrt{e\cos(c+dx)}}{11de^7(a-a\sin(c+dx))^3} - \frac{a^6 \int \frac{1}{\sqrt{e\cos(c+dx)}\,(a-a\sin(c+dx))^2}\,dx}{11e^6}$$

$$= \frac{4a^7\sqrt{e\cos(c+dx)}}{11de^7(a-a\sin(c+dx))^3} - \frac{2a^6\sqrt{e\cos(c+dx)}}{77de^7(a-a\sin(c+dx))^2} - \frac{(3a^5)\int \frac{}{\sqrt{e\cos(c+d}}}{77e^6}$$

$$= \frac{4a^7\sqrt{e\cos(c+dx)}}{11de^7(a-a\sin(c+dx))^3} - \frac{2a^6\sqrt{e\cos(c+dx)}}{77de^7(a-a\sin(c+dx))^2} - \frac{2a^5\sqrt{e\cos(c+dx)}}{77de^7(a-a\sin(c+d}$$

$$= \frac{4a^7\sqrt{e\cos(c+dx)}}{11de^7(a-a\sin(c+dx))^3} - \frac{2a^6\sqrt{e\cos(c+dx)}}{77de^7(a-a\sin(c+dx))^2} - \frac{2a^5\sqrt{e\cos(c+dx)}}{77de^7(a-a\sin(c+d}$$

$$= -\frac{2a^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{77de^6\sqrt{e\cos(c+dx)}} + \frac{4a^7\sqrt{e\cos(c+dx)}}{11de^7(a-a\sin(c+dx))^3} - \frac{2a^6\sqrt{e}}{77de^7(a-}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.21, size = 66, normalized size = 0.39

$$\frac{4\sqrt[4]{2}\,a^4\,{}_2F_1\left(-\frac{11}{4},-\frac{1}{4};-\frac{7}{4};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{11/4}}{11de(e\cos(c+dx))^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^4/(e*Cos[c + d*x])^(13/2),x]
[Out] (4*2^(1/4)*a^4*Hypergeometric2F1[-11/4, -1/4, -7/4, (1 - Sin[c + d*x])/2]*(
1 + Sin[c + d*x])^(11/4))/(11*d*e*(e*Cos[c + d*x])^(11/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 582 vs. 2(177) = 354.
time = 8.45, size = 583, normalized size = 3.45

method	result
default	$2\left(32\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-80\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\right.\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(13/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/77/(32*sin(1/2*d*x+1/2*c)^10-80*sin(1/2*d*x+1/2*c)^8+80*sin(1/2*d*x+1/2*c
)^6-40*sin(1/2*d*x+1/2*c)^4+10*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)/(
-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/e^6*(32*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2
*d*x+1/2*c)^10-80*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^8+32*sin(1/2*
d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+80*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/
2*c)^6-64*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-40*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))*sin(1/2*d*x+1/2*c)^4+176*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+10*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-144*sin(1/2*d*x+1/2*c)^4*cos(1/2*
d*x+1/2*c)+176*sin(1/2*d*x+1/2*c)^5-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-78*sin(1/2*d*x
+1/2*c)^2*cos(1/2*d*x+1/2*c)-176*sin(1/2*d*x+1/2*c)^3-12*sin(1/2*d*x+1/2*c)
)*a^4/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(13/2),x, algorithm="maxima")
```

```
[Out] e^(-13/2)*integrate((a*sin(d*x + c) + a)^4/cos(d*x + c)^(13/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 241, normalized size = 1.43

$$-\frac{\left(-3i\sqrt{2}\,a^4\cos\left(dx+c\right)^2+4i\sqrt{2}\,a^4+\left(i\sqrt{2}\,a^4\cos\left(dx+c\right)^2-4i\sqrt{2}\,a^4\right)\sin\left(dx+c\right)\right)\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right))+\left(3i\sqrt{2}\,a^4\cos\left(dx+c\right)^2-4i\sqrt{2}\,a^4+\left(-i\sqrt{2}\,a^4\cos\left(dx+c\right)^2+4i\sqrt{2}\,a^4\right)\sin\left(dx+c\right)\right)\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right))+2\left(a^4\cos\left(dx+c\right)^2+3\,a^4\sin\left(dx+c\right)+11\,a^4\right)\sqrt{\cos\left(dx+c\right)}}{77\left(3\,d\cos\left(dx+c\right)^2e^{\frac{13}{2}}-4\,de^{\frac{13}{2}}-\left(d\cos\left(dx+c\right)^2e^{\frac{13}{2}}-4\,de^{\frac{13}{2}}\right)\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(13/2),x, algorithm="fricas")
```

```
[Out] -1/77*((-3*I*sqrt(2)*a^4*cos(d*x + c)^2 + 4*I*sqrt(2)*a^4 + (I*sqrt(2)*a^4*
cos(d*x + c)^2 - 4*I*sqrt(2)*a^4)*sin(d*x + c))*weierstrassPInverse(-4, 0,
cos(d*x + c) + I*sin(d*x + c)) + (3*I*sqrt(2)*a^4*cos(d*x + c)^2 - 4*I*sqrt
(2)*a^4 + (-I*sqrt(2)*a^4*cos(d*x + c)^2 + 4*I*sqrt(2)*a^4)*sin(d*x + c))*w
eierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*(a^4*cos(d*x +
 c)^2 + 3*a^4*sin(d*x + c) + 11*a^4)*sqrt(cos(d*x + c)))/(3*d*cos(d*x + c)^
2*e^(13/2) - 4*d*e^(13/2) - (d*cos(d*x + c)^2*e^(13/2) - 4*d*e^(13/2))*sin(
d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**4/(e*cos(d*x+c))**(13/2),x)
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(13/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\,\sin\left(c+d\,x\right))^4}{(e\,\cos\left(c+d\,x\right))^{13/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^4/(e*cos(c + d*x))^(13/2),x)
[Out] int((a + a*sin(c + d*x))^4/(e*cos(c + d*x))^(13/2), x)
```

3.233 $\int \frac{(e\cos(c+dx))^{11/2}}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=132

$$\frac{2e(e\cos(c+dx))^{9/2}}{9ad}+\frac{10e^6\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21ad\sqrt{e\cos(c+dx)}}+\frac{10e^5\sqrt{e\cos(c+dx)}\,\sin(c+dx)}{21ad}+\frac{2e^3(e\cos(c+$$

```
[Out] 2/9*e*(e*cos(d*x+c))^(9/2)/a/d+2/7*e^3*(e*cos(d*x+c))^(5/2)*sin(d*x+c)/a/d+
10/21*e^6*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2
*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/a/d/(e*cos(d*x+c))^(1/2)+10/21*e^5*si
n(d*x+c)*(e*cos(d*x+c))^(1/2)/a/d
```

Rubi [A]
time = 0.08, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2761, 2715, 2721, 2720}

$$\frac{10e^6\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21ad\sqrt{e\cos(c+dx)}}+\frac{10e^5\sin(c+dx)\sqrt{e\cos(c+dx)}}{21ad}+\frac{2e^3\sin(c+dx)(e\cos(c+dx))^{5/2}}{7ad}+\frac{2e(e\cos(c+dx))^{9/2}}{9ad}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(11/2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (2*e*(e*Cos[c + d*x])^(9/2))/(9*a*d) + (10*e^6*Sqrt[Cos[c + d*x]]*EllipticF
[(c + d*x)/2, 2])/(21*a*d*Sqrt[e*Cos[c + d*x]]) + (10*e^5*Sqrt[e*Cos[c + d*
x]]*Sin[c + d*x])/(21*a*d) + (2*e^3*(e*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7
*a*d)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{11/2}}{a+a\sin(c+dx)}\,dx = \frac{2e(e\cos(c+dx))^{9/2}}{9ad} + \frac{e^2\int(e\cos(c+dx))^{7/2}\,dx}{a}$$

$$= \frac{2e(e\cos(c+dx))^{9/2}}{9ad} + \frac{2e^3(e\cos(c+dx))^{5/2}\sin(c+dx)}{7ad} + \frac{(5e^4)\int(e\cos(c+dx))$$

$$= \frac{2e(e\cos(c+dx))^{9/2}}{9ad} + \frac{10e^5\sqrt{e\cos(c+dx)}\,\sin(c+dx)}{21ad} + \frac{2e^3(e\cos(c+dx))^{5/2}\,s}{7ad}$$

$$= \frac{2e(e\cos(c+dx))^{9/2}}{9ad} + \frac{10e^5\sqrt{e\cos(c+dx)}\,\sin(c+dx)}{21ad} + \frac{2e^3(e\cos(c+dx))^{5/2}\,s}{7ad}$$

$$= \frac{2e(e\cos(c+dx))^{9/2}}{9ad} + \frac{10e^6\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{21ad\sqrt{e\cos(c+dx)}} + \frac{10e^5\sqrt{e\cos(c+dx}}{21ad}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.14, size = 66, normalized size = 0.50

$$-\frac{8\sqrt[4]{2}\,(e\cos(c+dx))^{13/2}\,{}_2F_1\left(-\frac{5}{4},\frac{13}{4};\frac{17}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{13ade(1+\sin(c+dx))^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(11/2)/(a + a*Sin[c + d*x]),x]
[Out] (-8*2^(1/4)*(e*Cos[c + d*x])^(13/2)*Hypergeometric2F1[-5/4, 13/4, 17/4, (1
- Sin[c + d*x])/2])/(13*a*d*e*(1 + Sin[c + d*x])^(13/4))
```

Maple [A]
time = 2.35, size = 251, normalized size = 1.90

method	result

default	$-\frac{2e^{6}\left(224\left(\sin^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+144\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-560\left(\sin^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-216\left(\sin^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+560\left(\sin^{7}\left(\frac{d}{}\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(11/2)/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/63/a/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^6*(224*sin
(1/2*d*x+1/2*c)^11+144*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-560*sin(1/2*
d*x+1/2*c)^9-216*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+560*sin(1/2*d*x+1/
2*c)^7+168*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-280*sin(1/2*d*x+1/2*c)^5
+15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))-48*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+70*
sin(1/2*d*x+1/2*c)^3-7*sin(1/2*d*x+1/2*c))/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(11/2)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] e^(11/2)*integrate(cos(d*x + c)^(11/2)/(a*sin(d*x + c) + a), x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 107, normalized size = 0.81

$$\frac{-15i\sqrt{2}\,e^{\frac{11}{2}}\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+15i\sqrt{2}\,e^{\frac{11}{2}}\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+2\left(7\cos(dx+c)^{4}e^{\frac{11}{2}}+3\left(3\cos(dx+c)^{2}e^{\frac{11}{2}}+5e^{\frac{11}{2}}\right)\sin(dx+c)\right)\sqrt{\cos(dx+c)}}{63\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(11/2)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/63*(-15*I*sqrt(2)*e^(11/2)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*si
n(d*x + c)) + 15*I*sqrt(2)*e^(11/2)*weierstrassPInverse(-4, 0, cos(d*x + c)
 - I*sin(d*x + c)) + 2*(7*cos(d*x + c)^4*e^(11/2) + 3*(3*cos(d*x + c)^2*e^(
11/2) + 5*e^(11/2))*sin(d*x + c))*sqrt(cos(d*x + c)))/(a*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(11/2)/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(11/2)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(11/2)*e^(11/2)/(a*sin(d*x + c) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,\cos\left(c+d\,x\right))^{11/2}}{a+a\,\sin\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(11/2)/(a + a*sin(c + d*x)),x)
```

```
[Out] int((e*cos(c + d*x))^(11/2)/(a + a*sin(c + d*x)), x)
```

3.234 $\int \frac{(e\cos(c+dx))^{9/2}}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=101

$$\frac{2e(e\cos(c+dx))^{7/2}}{7ad}+\frac{6e^4\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad\sqrt{\cos(c+dx)}}+\frac{2e^3(e\cos(c+dx))^{3/2}\sin(c+dx)}{5ad}$$

```
[Out] 2/7*e*(e*cos(d*x+c))^(7/2)/a/d+2/5*e^3*(e*cos(d*x+c))^(3/2)*sin(d*x+c)/a/d+
6/5*e^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d
*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/a/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2761, 2715, 2721, 2719}

$$\frac{6e^4E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5ad\sqrt{\cos(c+dx)}}+\frac{2e^3\sin(c+dx)(e\cos(c+dx))^{3/2}}{5ad}+\frac{2e(e\cos(c+dx))^{7/2}}{7ad}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(9/2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (2*e*(e*Cos[c + d*x])^(7/2))/(7*a*d) + (6*e^4*Sqrt[e*Cos[c + d*x]]*Elliptic
E[(c + d*x)/2, 2])/(5*a*d*Sqrt[Cos[c + d*x]]) + (2*e^3*(e*Cos[c + d*x])^(3/
2)*Sin[c + d*x])/(5*a*d)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{(e\cos(c+dx))^{9/2}}{a+a\sin(c+dx)}\,dx &= \frac{2e(e\cos(c+dx))^{7/2}}{7ad}+\frac{e^2\int (e\cos(c+dx))^{5/2}\,dx}{a}\\ &= \frac{2e(e\cos(c+dx))^{7/2}}{7ad}+\frac{2e^3(e\cos(c+dx))^{3/2}\sin(c+dx)}{5ad}+\frac{(3e^4)\int \sqrt{e\cos(c+dx)}}{5a}\\ &= \frac{2e(e\cos(c+dx))^{7/2}}{7ad}+\frac{2e^3(e\cos(c+dx))^{3/2}\sin(c+dx)}{5ad}+\frac{\left(3e^4\sqrt{e\cos(c+dx)}\right)}{5a\sqrt{\cos(c}}\\ &= \frac{2e(e\cos(c+dx))^{7/2}}{7ad}+\frac{6e^4\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad\sqrt{\cos(c+dx)}}+\frac{2e^3(e\cos(c+dx))^{3}}{5ad}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.11, size = 66, normalized size = 0.65

$$-\frac{4\,2^{3/4}(e\cos(c+dx))^{11/2}\,{}_2F_1\left(-\frac{3}{4},\frac{11}{4};\frac{15}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{11ade(1+\sin(c+dx))^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(9/2)/(a + a*Sin[c + d*x]),x]
[Out] (-4*2^(3/4)*(e*Cos[c + d*x])^(11/2)*Hypergeometric2F1[-3/4, 11/4, 15/4, (1
- Sin[c + d*x])/2])/(11*a*d*e*(1 + Sin[c + d*x])^(11/4))
```

Maple [A]
time = 2.48, size = 216, normalized size = 2.14

method	result
default	$\frac{2e^5\left(80\left(\sin^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+56\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-160\left(\sin^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-56\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+120\left(\sin^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\right.}{35a\sin\left(\frac{dx}{2}+\frac{c}{2}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(9/2)/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/35/a/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^5*(80*sin(1
/2*d*x+1/2*c)^9+56*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-160*sin(1/2*d*x+
1/2*c)^7-56*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+120*sin(1/2*d*x+1/2*c)^
5+21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))+14*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-40
*sin(1/2*d*x+1/2*c)^3+5*sin(1/2*d*x+1/2*c))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(9/2)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] e^(9/2)*integrate(cos(d*x + c)^(9/2)/(a*sin(d*x + c) + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 104, normalized size = 1.03

$$\frac{21i\sqrt{2}\,e^{\frac{9}{2}}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)))-21i\sqrt{2}\,e^{\frac{9}{2}}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c)))+2\left(5\cos(dx+c)^3e^{\frac{9}{2}}+7\cos(dx+c)e^{\frac{9}{2}}\sin(dx+c)\right)\sqrt{\cos(dx+c)}}{35ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(9/2)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/35*(21*I*sqrt(2)*e^(9/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0
, cos(d*x + c) + I*sin(d*x + c))) - 21*I*sqrt(2)*e^(9/2)*weierstrassZeta(-4
, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(5*cos(
d*x + c)^3*e^(9/2) + 7*cos(d*x + c)*e^(9/2)*sin(d*x + c))*sqrt(cos(d*x + c)
))/(a*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(9/2)/(a+a*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8857 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(9/2)/(a+a*sin(d*x+c)),x, algorithm="giac")
[Out] integrate(cos(d*x + c)^(9/2)*e^(9/2)/(a*sin(d*x + c) + a), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(e\,\cos\left(c+d\,x\right)\right)}^{9/2}}{a+a\,\sin\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(9/2)/(a + a*sin(c + d*x)),x)
[Out] int((e*cos(c + d*x))^(9/2)/(a + a*sin(c + d*x)), x)
```

3.235 $\int \frac{(e\cos(c+dx))^{7/2}}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=101

$$\frac{2e(e\cos(c+dx))^{5/2}}{5ad}+\frac{2e^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad\sqrt{e\cos(c+dx)}}+\frac{2e^3\sqrt{e\cos(c+dx)}\,\sin(c+dx)}{3ad}$$

```
[Out] 2/5*e*(e*cos(d*x+c))^(5/2)/a/d+2/3*e^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2
*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/a/d/(e*c
os(d*x+c))^(1/2)+2/3*e^3*sin(d*x+c)*(e*cos(d*x+c))^(1/2)/a/d
```

Rubi [A]
time = 0.07, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2761, 2715, 2721, 2720}

$$\frac{2e^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad\sqrt{e\cos(c+dx)}}+\frac{2e^3\sin(c+dx)\sqrt{e\cos(c+dx)}}{3ad}+\frac{2e(e\cos(c+dx))^{5/2}}{5ad}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(7/2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (2*e*(e*Cos[c + d*x])^(5/2))/(5*a*d) + (2*e^4*Sqrt[Cos[c + d*x]]*EllipticF[
(c + d*x)/2, 2])/(3*a*d*Sqrt[e*Cos[c + d*x]]) + (2*e^3*Sqrt[e*Cos[c + d*x]]
*Sin[c + d*x])/(3*a*d)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{7/2}}{a+a\sin(c+dx)}\,dx = \frac{2e(e\cos(c+dx))^{5/2}}{5ad}+\frac{e^2\int(e\cos(c+dx))^{3/2}\,dx}{a}$$
$$= \frac{2e(e\cos(c+dx))^{5/2}}{5ad}+\frac{2e^3\sqrt{e\cos(c+dx)}\,\sin(c+dx)}{3ad}+\frac{e^4\int\frac{1}{\sqrt{e\cos(c+dx)}}\,dx}{3a}$$
$$= \frac{2e(e\cos(c+dx))^{5/2}}{5ad}+\frac{2e^3\sqrt{e\cos(c+dx)}\,\sin(c+dx)}{3ad}+\frac{\left(e^4\sqrt{\cos(c+dx)}\right)\int\frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3a\sqrt{e\cos(c+dx)}}$$
$$= \frac{2e(e\cos(c+dx))^{5/2}}{5ad}+\frac{2e^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad\sqrt{e\cos(c+dx)}}+\frac{2e^3\sqrt{e\cos(c+dx)}\,\sin(c+dx)}{3ad}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.08, size = 66, normalized size = 0.65

$$-\frac{4\sqrt[4]{2}\,(e\cos(c+dx))^{9/2}\,{}_2F_1\left(-\frac{1}{4},\frac{9}{4};\frac{13}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{9ade(1+\sin(c+dx))^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(7/2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-4*2^(1/4)*(e*Cos[c + d*x])^(9/2)*Hypergeometric2F1[-1/4, 9/4, 13/4, (1 -
Sin[c + d*x])/2])/(9*a*d*e*(1 + Sin[c + d*x])^(9/4))
```

Maple [A]
time = 2.12, size = 181, normalized size = 1.79

method	result
default	$-\frac{2e^4\left(24\left(\sin^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+20\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-36\left(\sin^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+5\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-}}{15a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^2\left(\frac{dx}{2}+\right.\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/15/a/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^4*(24*sin(
1/2*d*x+1/2*c)^7+20*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-36*sin(1/2*d*x+
1/2*c)^5+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))-10*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2
*c)+18*sin(1/2*d*x+1/2*c)^3-3*sin(1/2*d*x+1/2*c))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] e^(7/2)*integrate(cos(d*x + c)^(7/2)/(a*sin(d*x + c) + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 92, normalized size = 0.91

$$\frac{-5i\sqrt{2}\,e^{\frac{7}{2}}\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+5i\sqrt{2}\,e^{\frac{7}{2}}\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c))+2\left(3\,\cos(dx+c)^{2}e^{\frac{7}{2}}+5\,e^{\frac{7}{2}}\sin(dx+c)\right)\sqrt{\cos(dx+c)}}{15\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/15*(-5*I*sqrt(2)*e^(7/2)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(
d*x + c)) + 5*I*sqrt(2)*e^(7/2)*weierstrassPInverse(-4, 0, cos(d*x + c) - I
*sin(d*x + c)) + 2*(3*cos(d*x + c)^2*e^(7/2) + 5*e^(7/2)*sin(d*x + c))*sqrt
(cos(d*x + c)))/(a*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(7/2)/(a+a*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3878 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c)),x, algorithm="giac")
[Out] integrate(cos(d*x + c)^(7/2)*e^(7/2)/(a*sin(d*x + c) + a), x)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,\cos\left(c+d\,x\right))^{7/2}}{a+a\,\sin\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(7/2)/(a + a*sin(c + d*x)),x)
[Out] int((e*cos(c + d*x))^(7/2)/(a + a*sin(c + d*x)), x)
```

3.236 $\int \frac{(e\cos(c+dx))^{5/2}}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=68

$$\frac{2e(e\cos(c+dx))^{3/2}}{3ad}+\frac{2e^2\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad\sqrt{\cos(c+dx)}}$$

```
[Out] 2/3*e*(e*cos(d*x+c))^(3/2)/a/d+2*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/a/d/co
s(d*x+c)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}=0.120$, Rules used = {2761, 2721, 2719}

$$\frac{2e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{ad\sqrt{\cos(c+dx)}}+\frac{2e(e\cos(c+dx))^{3/2}}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(5/2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (2*e*(e*Cos[c + d*x])^(3/2))/(3*a*d) + (2*e^2*Sqrt[e*Cos[c + d*x]]*Elliptic
E[(c + d*x)/2, 2])/(a*d*Sqrt[Cos[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{5/2}}{a+a\sin(c+dx)}\,dx = \frac{2e(e\cos(c+dx))^{3/2}}{3ad} + \frac{e^2\int\sqrt{e\cos(c+dx)}\,dx}{a}$$
$$= \frac{2e(e\cos(c+dx))^{3/2}}{3ad} + \frac{\left(e^2\sqrt{e\cos(c+dx)}\right)\int\sqrt{\cos(c+dx)}\,dx}{a\sqrt{\cos(c+dx)}}$$
$$= \frac{2e(e\cos(c+dx))^{3/2}}{3ad} + \frac{2e^2\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad\sqrt{\cos(c+dx)}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.10, size = 66, normalized size = 0.97

$$-\frac{2\,2^{3/4}(e\cos(c+dx))^{7/2}\,{}_2F_1\left(\frac{1}{4},\frac{7}{4};\frac{11}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{7ade(1+\sin(c+dx))^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(5/2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-2*2^(3/4)*(e*Cos[c + d*x])^(7/2)*Hypergeometric2F1[1/4, 7/4, 11/4, (1 - S
in[c + d*x])/2])/(7*a*d*e*(1 + Sin[c + d*x])^(7/4))
```

Maple [A]
time = 2.06, size = 122, normalized size = 1.79

method	result
default	$\frac{2e^3\left(4\left(\sin^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-4\left(\sin^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\right.}{3a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)e+e}\,d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/a/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^3*(4*sin(1/2
*d*x+1/2*c)^5+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-4*sin(1/2*d*x+1/2*c)^3+sin(1/2*d*x
+1/2*c))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] e^(5/2)*integrate(cos(d*x + c)^(5/2)/(a*sin(d*x + c) + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 77, normalized size = 1.13

$$\frac{2 \cos \left(dx+c\right)^{\frac{3}{2}} e^{\frac{5}{2}}+3 i \sqrt{2}\, e^{\frac{5}{2}} \mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos \left(dx+c\right)+i \sin \left(dx+c\right)))-3 i \sqrt{2}\, e^{\frac{5}{2}} \mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos \left(dx+c\right)-i \sin \left(dx+c\right)))}{3\, ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/3*(2*cos(d*x + c)^(3/2)*e^(5/2) + 3*I*sqrt(2)*e^(5/2)*weierstrassZeta(-4,
 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*I*sqrt(2
)*e^(5/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) -
I*sin(d*x + c))))/(a*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(5/2)/(a+a*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4849 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(5/2)*e^(5/2)/(a*sin(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(e\,\cos \left(c+d\,x\right)\right)}^{5/2}}{a+a\,\sin \left(c+d\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(5/2)/(a + a*sin(c + d*x)),x)
```

```
[Out] int((e*cos(c + d*x))^(5/2)/(a + a*sin(c + d*x)), x)
```

3.237 $\int \frac{(e\cos(c+dx))^{3/2}}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=66

$$\frac{2e\sqrt{e\cos(c+dx)}}{ad}+\frac{2e^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{ad\sqrt{e\cos(c+dx)}}$$

```
[Out] 2*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x
+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/a/d/(e*cos(d*x+c))^(1/2)+2*e*(e*cos(d*x+c
))^(1/2)/a/d
```

Rubi [A]
time = 0.05, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2761, 2721, 2720}

$$\frac{2e^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{ad\sqrt{e\cos(c+dx)}}+\frac{2e\sqrt{e\cos(c+dx)}}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(3/2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (2*e*Sqrt[e*Cos[c + d*x]])/(a*d) + (2*e^2*Sqrt[Cos[c + d*x]]*EllipticF[(c +
 d*x)/2, 2])/(a*d*Sqrt[e*Cos[c + d*x]])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{3/2}}{a+a\sin(c+dx)}\,dx = \frac{2e\sqrt{e\cos(c+dx)}}{ad} + \frac{e^2\int \frac{1}{\sqrt{e\cos(c+dx)}}\,dx}{a}$$

$$= \frac{2e\sqrt{e\cos(c+dx)}}{ad} + \frac{\left(e^2\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{a\sqrt{e\cos(c+dx)}}$$

$$= \frac{2e\sqrt{e\cos(c+dx)}}{ad} + \frac{2e^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{ad\sqrt{e\cos(c+dx)}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.07, size = 66, normalized size = 1.00

$$-\frac{2\sqrt[4]{2}\,(e\cos(c+dx))^{5/2}\,{}_2F_1\left(\frac{3}{4},\frac{5}{4};\frac{9}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{5ade(1+\sin(c+dx))^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(3/2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-2*2^(1/4)*(e*Cos[c + d*x])^(5/2)*Hypergeometric2F1[3/4, 5/4, 9/4, (1 - Si
n[c + d*x])/2])/(5*a*d*e*(1 + Sin[c + d*x])^(5/4))
```

Maple [A]
time = 1.53, size = 110, normalized size = 1.67

method	result
default	$-\frac{2e^2\left(\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)+2\left(\sin^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)e+e}\,d}$
risch	$\frac{\sqrt{2}\,e\sqrt{e\left(1+\mathrm{e}^{2i(dx+c)}\right)\mathrm{e}^{-i(dx+c)}}}{da}+\frac{2\sqrt{-i\left(\mathrm{e}^{i(dx+c)}+i\right)}\,\sqrt{i\left(\mathrm{e}^{i(dx+c)}-i\right)}\,\sqrt{i\mathrm{e}^{i(dx+c)}}\,\mathrm{EllipticF}\left(\sqrt{-\cdots}\right.}{d\sqrt{\mathrm{e}^{3i(dx+\cdots}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/a/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^2*((sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))+2*sin(1/2*d*x+1/2*c)^3-sin(1/2*d*x+1/2*c))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c)),x, algorithm="maxima")
[Out] e^(3/2)*integrate(cos(d*x + c)^(3/2)/(a*sin(d*x + c) + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 70, normalized size = 1.06

$$\frac{-i\sqrt{2}\,e^{\frac{3}{2}}\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+i\sqrt{2}\,e^{\frac{3}{2}}\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right))+2\sqrt{\cos\left(dx+c\right)}\,e^{\frac{3}{2}}}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c)),x, algorithm="fricas")
[Out] (-I*sqrt(2)*e^(3/2)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c
)) + I*sqrt(2)*e^(3/2)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x
+ c)) + 2*sqrt(cos(d*x + c))*e^(3/2))/(a*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(3/2)/(a+a*sin(d*x+c)),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c)),x, algorithm="giac")
[Out] integrate(cos(d*x + c)^(3/2)*e^(3/2)/(a*sin(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(e\,\cos\left(c+d\,x\right)\right)^{3/2}}{a+a\,\sin\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(3/2)/(a + a*sin(c + d*x)),x)
[Out] int((e*cos(c + d*x))^(3/2)/(a + a*sin(c + d*x)), x)
```

3.238 $\int \frac{\sqrt{e\cos(c+dx)}}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=74

$$-\frac{2\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad\sqrt{\cos(c+dx)}}-\frac{2(e\cos(c+dx))^{3/2}}{de(a+a\sin(c+dx))}$$

```
[Out] -2*(e*cos(d*x+c))^(3/2)/d/e/(a+a*sin(d*x+c))-2*(cos(1/2*d*x+1/2*c)^2)^(1/2)
/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1
/2)/a/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2762, 2721, 2719}

$$-\frac{2(e\cos(c+dx))^{3/2}}{de(a\sin(c+dx)+a)}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{ad\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cos[c + d*x]]/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-2*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(a*d*Sqrt[Cos[c + d*x]]
) - (2*(e*Cos[c + d*x])^(3/2))/(d*e*(a + a*Sin[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2762

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[b*((g*Cos[e + f*x])^(p + 1)/(a*f*g*(p - 1)*(a + b*S
in[e + f*x]))), x] + Dist[p/(a*(p - 1)), Int[(g*Cos[e + f*x])^p, x], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] &&  !GeQ[p, 1] && Integer
Q[2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{e\cos(c+dx)}}{a+a\sin(c+dx)}\,dx &= -\frac{2(e\cos(c+dx))^{3/2}}{de(a+a\sin(c+dx))}-\frac{\int \sqrt{e\cos(c+dx)}\,dx}{a}\\ &= -\frac{2(e\cos(c+dx))^{3/2}}{de(a+a\sin(c+dx))}-\frac{\sqrt{e\cos(c+dx)}\int \sqrt{\cos(c+dx)}\,dx}{a\sqrt{\cos(c+dx)}}\\ &= -\frac{2\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad\sqrt{\cos(c+dx)}}-\frac{2(e\cos(c+dx))^{3/2}}{de(a+a\sin(c+dx))}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.04, size = 66, normalized size = 0.89

$$-\frac{2^{3/4}(e\cos(c+dx))^{3/2}\,{}_2F_1\left(\frac{3}{4},\frac{5}{4};\frac{7}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{3ade(1+\sin(c+dx))^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Cos[c + d*x]]/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/3*(2^(3/4)*(e*Cos[c + d*x])^(3/2)*Hypergeometric2F1[3/4, 5/4, 7/4, (1 -
Sin[c + d*x])/2])/(a*d*e*(1 + Sin[c + d*x])^(3/4))
```

Maple [A]
time = 3.06, size = 115, normalized size = 1.55

method	result
default	$-\frac{2\left(\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sin\left(\frac{dx}{2}\right.\right.}{\sqrt{-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)e+e}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)ad}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/sin(1/2*d*x+1/2*c)/a*((sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))-2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+sin(1/2*d*x+1/2*c))*e/
d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] e^(1/2)*integrate(sqrt(cos(d*x + c))/(a*sin(d*x + c) + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 170, normalized size = 2.30

$$\frac{\left(-i\sqrt{2}\cos\left(dx+c\right)e^{\frac{1}{2}}-i\sqrt{2}e^{\frac{1}{2}}\sin\left(dx+c\right)-i\sqrt{2}e^{\frac{1}{2}}\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)))+\left(i\sqrt{2}\cos\left(dx+c\right)e^{\frac{1}{2}}+i\sqrt{2}e^{\frac{1}{2}}\sin\left(dx+c\right)+i\sqrt{2}e^{\frac{1}{2}}\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)))-2\left(\cos\left(dx+c\right)e^{\frac{1}{2}}-e^{\frac{1}{2}}\sin\left(dx+c\right)+e^{\frac{1}{2}}\right)\sqrt{\cos\left(dx+c\right)}}{ad\cos\left(dx+c\right)+ad\sin\left(dx+c\right)+ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] ((-I*sqrt(2)*cos(d*x + c)*e^(1/2) - I*sqrt(2)*e^(1/2)*sin(d*x + c) - I*sqrt
(2)*e^(1/2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c)
 + I*sin(d*x + c))) + (I*sqrt(2)*cos(d*x + c)*e^(1/2) + I*sqrt(2)*e^(1/2)*s
in(d*x + c) + I*sqrt(2)*e^(1/2))*weierstrassZeta(-4, 0, weierstrassPInverse
(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(cos(d*x + c)*e^(1/2) - e^(1/2)
*sin(d*x + c) + e^(1/2))*sqrt(cos(d*x + c)))/(a*d*cos(d*x + c) + a*d*sin(d*
x + c) + a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{e\cos\left(c+dx\right)}}{\sin\left(c+dx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(1/2)/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(sqrt(e*cos(c + d*x))/(sin(c + d*x) + 1), x)/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(d*x + c))*e^(1/2)/(a*sin(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{e\,\cos\left(c+d\,x\right)}}{a+a\,\sin\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(1/2)/(a + a*sin(c + d*x)),x)
[Out] int((e*cos(c + d*x))^(1/2)/(a + a*sin(c + d*x)), x)
```

3.239 $\int \frac{1}{\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))}\,dx$

Optimal. Leaf size=78

$$\frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad\sqrt{e\cos(c+dx)}}-\frac{2\sqrt{e\cos(c+dx)}}{3de(a+a\sin(c+dx))}$$

```
[Out] 2/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1
/2*c),2^(1/2))*cos(d*x+c)^(1/2)/a/d/(e*cos(d*x+c))^(1/2)-2/3*(e*cos(d*x+c))
^(1/2)/d/e/(a+a*sin(d*x+c))
```

Rubi [A]
time = 0.06, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2762, 2721, 2720}

$$\frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad\sqrt{e\cos(c+dx)}}-\frac{2\sqrt{e\cos(c+dx)}}{3de(a\sin(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*a*d*Sqrt[e*Cos[c + d*x]
]) - (2*Sqrt[e*Cos[c + d*x]])/(3*d*e*(a + a*Sin[c + d*x]))
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2762

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[b*((g*Cos[e + f*x])^(p + 1)/(a*f*g*(p - 1)*(a + b*S
in[e + f*x]))), x] + Dist[p/(a*(p - 1)), Int[(g*Cos[e + f*x])^p, x], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] &&  !GeQ[p, 1] && Integer
Q[2*p]
```

Rubi steps

$$\int \frac{1}{\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))}\,dx = -\frac{2\sqrt{e\cos(c+dx)}}{3de(a+a\sin(c+dx))} + \frac{\int \frac{1}{\sqrt{e\cos(c+dx)}}\,dx}{3a}$$

$$= -\frac{2\sqrt{e\cos(c+dx)}}{3de(a+a\sin(c+dx))} + \frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3a\sqrt{e\cos(c+dx)}}$$

$$= \frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad\sqrt{e\cos(c+dx)}} - \frac{2\sqrt{e\cos(c+dx)}}{3de(a+a\sin(c+dx))}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.04, size = 64, normalized size = 0.82

$$-\frac{\sqrt[4]{2}\,\sqrt{e\cos(c+dx)}\,{}_2F_1\left(\frac{1}{4},\frac{7}{4};\frac{5}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{ade\sqrt[4]{1+\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])),x]
```

```
[Out] -((2^(1/4)*Sqrt[e*Cos[c + d*x]]*Hypergeometric2F1[1/4, 7/4, 5/4, (1 - Sin[c
 + d*x])/2])/(a*d*e*(1 + Sin[c + d*x])^(1/4)))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 189 vs. 2(94) = 188.
time = 3.86, size = 190, normalized size = 2.44

method	result
default	$-\frac{2\left(2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\right.}{3\left(2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(d*x+c))/(e*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/(2*sin(1/2*d*x+1/2*c)^2-1)/a/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)
^2*e+e)^(1/2)*(2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))+2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-sin(1/2*d*x+1/2*c))/
d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))/(e*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] e^(-1/2)*integrate(1/((a*sin(d*x + c) + a)*sqrt(cos(d*x + c))), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 104, normalized size = 1.33

$$\frac{\left(-i\sqrt{2}\sin\left(dx+c\right)-i\sqrt{2}\right)\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right))+\left(i\sqrt{2}\sin\left(dx+c\right)+i\sqrt{2}\right)\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right))-2\sqrt{\cos\left(dx+c\right)}}{3\left(ade^{\frac{1}{2}}\sin\left(dx+c\right)+ade^{\frac{1}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))/(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*((-I*sqrt(2)*sin(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d
*x + c) + I*sin(d*x + c)) + (I*sqrt(2)*sin(d*x + c) + I*sqrt(2))*weierstras
sPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 2*sqrt(cos(d*x + c)))/(a*
d*e^(1/2)*sin(d*x + c) + a*d*e^(1/2))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\sqrt{e\cos\left(c+dx\right)}\sin\left(c+dx\right)+\sqrt{e\cos\left(c+dx\right)}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))/(e*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(e*cos(c + d*x))*sin(c + d*x) + sqrt(e*cos(c + d*x))), x)/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))/(e*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(e^(-1/2)/((a*sin(d*x + c) + a)*sqrt(cos(d*x + c))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{e\,\cos\left(c+d\,x\right)}\,\left(a+a\,\sin\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cos(c + d*x))^(1/2)*(a + a*sin(c + d*x))),x)
[Out] int(1/((e*cos(c + d*x))^(1/2)*(a + a*sin(c + d*x))), x)
```

3.240 $\int \frac{1}{(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))}\,dx$

Optimal. Leaf size=112

$$-\frac{6\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ade^2\sqrt{\cos(c+dx)}}+\frac{6\sin(c+dx)}{5ade\sqrt{e\cos(c+dx)}}-\frac{2}{5de\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))}$$

```
[Out] 6/5*sin(d*x+c)/a/d/e/(e*cos(d*x+c))^(1/2)-2/5/d/e/(a+a*sin(d*x+c))/(e*cos(d
*x+c))^(1/2)-6/5*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(
sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/a/d/e^2/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2762, 2716, 2721, 2719}

$$-\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5ade^2\sqrt{\cos(c+dx)}}+\frac{6\sin(c+dx)}{5ade\sqrt{e\cos(c+dx)}}-\frac{2}{5de(a\sin(c+dx)+a)\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x])),x]
```

```
[Out] (-6*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*a*d*e^2*Sqrt[Cos[c +
 d*x]]) + (6*Sin[c + d*x])/(5*a*d*e*Sqrt[e*Cos[c + d*x]]) - 2/(5*d*e*Sqrt[e
*Cos[c + d*x]]*(a + a*Sin[c + d*x]))
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2762

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[b*((g*Cos[e + f*x])^(p + 1)/(a*f*g*(p - 1)*(a + b*S
in[e + f*x]))), x] + Dist[p/(a*(p - 1)), Int[(g*Cos[e + f*x])^p, x], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] &&  !GeQ[p, 1] && Integer
Q[2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))}\,dx &= -\frac{2}{5de\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))}+\frac{3\int \frac{1}{(e\cos(c+dx))^{3/2}}\,dx}{5a}\\ &= \frac{6\sin(c+dx)}{5ade\sqrt{e\cos(c+dx)}}-\frac{2}{5de\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))}\\ &= \frac{6\sin(c+dx)}{5ade\sqrt{e\cos(c+dx)}}-\frac{2}{5de\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))}\\ &= -\frac{6\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ade^2\sqrt{\cos(c+dx)}}+\frac{6\sin(c+dx)}{5ade\sqrt{e\cos(c+dx)}}-\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.06, size = 63, normalized size = 0.56

$$\frac{{}_2F_1\left(-\frac{1}{4},\frac{9}{4};\frac{3}{4};\frac{1}{2}(1-\sin(c+dx))\right)\sqrt[4]{1+\sin(c+dx)}}{\sqrt[4]{2}\,ade\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x])),x]
[Out] (Hypergeometric2F1[-1/4, 9/4, 3/4, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])
^(1/4))/(2^(1/4)*a*d*e*Sqrt[e*Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 303 vs. 2(124) = 248.
time = 5.84, size = 304, normalized size = 2.71

method	result
default	$-\frac{2\left(12\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-24\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\mathrm{co}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/a/sin(1/2*d*x+1/2*c)
/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/e*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2
*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+sin(1/2
*d*x+1/2*c))/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] e^(-3/2)*integrate(1/((a*sin(d*x + c) + a)*cos(d*x + c)^(3/2)), x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 168, normalized size = 1.50

$$-\frac{3\left(i\sqrt{2}\cos\left(dx+c\right)\sin\left(dx+c\right)+i\sqrt{2}\cos\left(dx+c\right)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)))+3\left(-i\sqrt{2}\cos\left(dx+c\right)\sin\left(dx+c\right)-i\sqrt{2}\cos\left(dx+c\right)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)))+2\left(3\cos\left(dx+c\right)^{2}-3\sin\left(dx+c\right)-2\right)\sqrt{\cos\left(dx+c\right)}}{5\left(ad\cos\left(dx+c\right)e^{\frac{3}{2}}\sin\left(dx+c\right)+ad\cos\left(dx+c\right)e^{\frac{3}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/5*(3*(I*sqrt(2)*cos(d*x + c)*sin(d*x + c) + I*sqrt(2)*cos(d*x + c))*weie
rstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)
)) + 3*(-I*sqrt(2)*cos(d*x + c)*sin(d*x + c) - I*sqrt(2)*cos(d*x + c))*weie
rstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)
)) + 2*(3*cos(d*x + c)^2 - 3*sin(d*x + c) - 2)*sqrt(cos(d*x + c)))/(a*d*cos
(d*x + c)*e^(3/2)*sin(d*x + c) + a*d*cos(d*x + c)*e^(3/2))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\left(e\cos\left(c+dx\right)\right)^{\frac{3}{2}}\sin\left(c+dx\right)+\left(e\cos\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(3/2)/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(1/((e*cos(c + d*x))**(3/2)*sin(c + d*x) + (e*cos(c + d*x))**(3/2))
, x)/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(e^(-3/2)/((a*sin(d*x + c) + a)*cos(d*x + c)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(e\cos(c+d\,x))^{3/2}\,(a+a\sin(c+d\,x))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cos(c + d*x))^(3/2)*(a + a*sin(c + d*x))),x)
```

```
[Out] int(1/((e*cos(c + d*x))^(3/2)*(a + a*sin(c + d*x))), x)
```

3.241 $\int \frac{1}{(e\cos(c+dx))^{5/2}(a+a\sin(c+dx))}\,dx$

Optimal. Leaf size=112

$$\frac{10\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21ade^2\sqrt{e\cos(c+dx)}}+\frac{10\sin(c+dx)}{21ade(e\cos(c+dx))^{3/2}}-\frac{2}{7de(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))}$$

```
[Out] 10/21*sin(d*x+c)/a/d/e/(e*cos(d*x+c))^(3/2)-2/7/d/e/(e*cos(d*x+c))^(3/2)/(a
+a*sin(d*x+c))+10/21*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/a/d/e^2/(e*cos(d*x+c))^(1/
2)
```

Rubi [A]

time = 0.07, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2762, 2716, 2721, 2720}

$$\frac{10\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21ade^2\sqrt{e\cos(c+dx)}}+\frac{10\sin(c+dx)}{21ade(e\cos(c+dx))^{3/2}}-\frac{2}{7de(a\sin(c+dx)+a)(e\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(5/2)*(a + a*Sin[c + d*x])),x]
```

```
[Out] (10*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*a*d*e^2*Sqrt[e*Cos[c
+ d*x]]) + (10*Sin[c + d*x])/(21*a*d*e*(e*Cos[c + d*x])^(3/2)) - 2/(7*d*e*(
e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x]))
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2762

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[b*((g*Cos[e + f*x])^(p + 1)/(a*f*g*(p - 1)*(a + b*S
in[e + f*x]))), x] + Dist[p/(a*(p - 1)), Int[(g*Cos[e + f*x])^p, x], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] &&  !GeQ[p, 1] && Integer
Q[2*p]
```

Rubi steps

$$\int \frac{1}{(e\cos(c+dx))^{5/2}(a+a\sin(c+dx))}\,dx = -\frac{2}{7de(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))}+\frac{5\int \frac{1}{(e\cos(c+dx))^{5/2}}\,d}{7a}$$

$$=\frac{10\sin(c+dx)}{21ade(e\cos(c+dx))^{3/2}}-\frac{2}{7de(e\cos(c+dx))^{3/2}(a+a\sin(c+d}$$

$$=\frac{10\sin(c+dx)}{21ade(e\cos(c+dx))^{3/2}}-\frac{2}{7de(e\cos(c+dx))^{3/2}(a+a\sin(c+d}$$

$$=\frac{10\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21ade^2\sqrt{e\cos(c+dx)}}+\frac{10\sin(c+dx)}{21ade(e\cos(c+dx))^{3/2}}-$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.07, size = 66, normalized size = 0.59

$$\frac{{}_2F_1\left(-\frac{3}{4},\frac{11}{4};\frac{1}{4};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{3/4}}{3\ 2^{3/4}ade(e\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Cos[c + d*x])^(5/2)*(a + a*Sin[c + d*x])),x]
```

```
[Out] (Hypergeometric2F1[-3/4, 11/4, 1/4, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x]
)^(3/4))/(3*2^(3/4)*a*d*e*(e*Cos[c + d*x])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 374 vs. 2(124) = 248.
time = 6.18, size = 375, normalized size = 3.35

method	result
default	$-\frac{2\left(40\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ \text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-60\sqrt{\frac{1}{2}-\frac{\cos(dx+}{2}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/21/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^
2-1)/a/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/e^2*(40*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^6-60*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/
2*d*x+1/2*c)^4+40*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+30*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))*sin(1/2*d*x+1/2*c)^2-40*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-
5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))+16*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-3*sin
(1/2*d*x+1/2*c))/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] e^(-5/2)*integrate(1/((a*sin(d*x + c) + a)*cos(d*x + c)^(5/2)), x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 174, normalized size = 1.55

$$-\frac{5\left(i\sqrt{2}\cos(dx+c)^2\sin(dx+c)+i\sqrt{2}\cos(dx+c)^2\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5\left(-i\sqrt{2}\cos(dx+c)^2\sin(dx+c)-i\sqrt{2}\cos(dx+c)^2\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+2\left(5\cos(dx+c)^2-5\sin(dx+c)-2\right)\sqrt{\cos(dx+c)}}{21\left(ad\cos(dx+c)^2e^{\frac{5}{2}}\sin(dx+c)+ad\cos(dx+c)^2e^{\frac{5}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/21*(5*(I*sqrt(2)*cos(d*x + c)^2*sin(d*x + c) + I*sqrt(2)*cos(d*x + c)^2)
*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*(-I*sqrt(2)*
cos(d*x + c)^2*sin(d*x + c) - I*sqrt(2)*cos(d*x + c)^2)*weierstrassPInverse
(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*(5*cos(d*x + c)^2 - 5*sin(d*x +
c) - 2)*sqrt(cos(d*x + c)))/(a*d*cos(d*x + c)^2*e^(5/2)*sin(d*x + c) + a*d*
cos(d*x + c)^2*e^(5/2))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(5/2)/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(e^(-5/2)/((a*sin(d*x + c) + a)*cos(d*x + c)^(5/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(e\,\cos\left(c+d\,x\right))^{5/2}\,\left(a+a\,\sin\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cos(c + d*x))^(5/2)*(a + a*sin(c + d*x))),x)
```

```
[Out] int(1/((e*cos(c + d*x))^(5/2)*(a + a*sin(c + d*x))), x)
```

3.242 $\int \frac{1}{(e\cos(c+dx))^{7/2}(a+a\sin(c+dx))}\,dx$

Optimal. Leaf size=143

$$-\frac{14\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15ade^4\sqrt{\cos(c+dx)}}+\frac{14\sin(c+dx)}{45ade(e\cos(c+dx))^{5/2}}+\frac{14\sin(c+dx)}{15ade^3\sqrt{e\cos(c+dx)}}-\frac{}{9de(e\cos(c+dx}$$

```
[Out] 14/45*sin(d*x+c)/a/d/e/(e*cos(d*x+c))^(5/2)-2/9/d/e/(e*cos(d*x+c))^(5/2)/(a
+a*sin(d*x+c))+14/15*sin(d*x+c)/a/d/e^3/(e*cos(d*x+c))^(1/2)-14/15*(cos(1/2
*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2
))*(e*cos(d*x+c))^(1/2)/a/d/e^4/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2762, 2716, 2721, 2719}

$$-\frac{14E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{15ade^4\sqrt{\cos(c+dx)}}+\frac{14\sin(c+dx)}{15ade^3\sqrt{e\cos(c+dx)}}+\frac{14\sin(c+dx)}{45ade(e\cos(c+dx))^{5/2}}-\frac{2}{9de(a\sin(c+dx)+a)(e\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(7/2)*(a + a*Sin[c + d*x])),x]
```

```
[Out] (-14*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*a*d*e^4*Sqrt[Cos[c
 + d*x]]) + (14*Sin[c + d*x])/(45*a*d*e*(e*Cos[c + d*x])^(5/2)) + (14*Sin[c
 + d*x])/(15*a*d*e^3*Sqrt[e*Cos[c + d*x]]) - 2/(9*d*e*(e*Cos[c + d*x])^(5/2
)*(a + a*Sin[c + d*x]))
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2762

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[b*((g*Cos[e + f*x])^(p + 1)/(a*f*g*(p - 1)*(a + b*S
in[e + f*x]))), x] + Dist[p/(a*(p - 1)), Int[(g*Cos[e + f*x])^p, x], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] &&  !GeQ[p, 1] && Integer
Q[2*p]
```

Rubi steps

$$\int \frac{1}{(e\cos(c+dx))^{7/2}(a+a\sin(c+dx))}\,dx = -\frac{2}{9de(e\cos(c+dx))^{5/2}(a+a\sin(c+dx))} + \frac{7\int \frac{1}{(e\cos(c+dx))^{7/2}}\,d}{9a}$$

$$= \frac{14\sin(c+dx)}{45ade(e\cos(c+dx))^{5/2}} - \frac{2}{9de(e\cos(c+dx))^{5/2}(a+a\sin(c+$$

$$= \frac{14\sin(c+dx)}{45ade(e\cos(c+dx))^{5/2}} + \frac{14\sin(c+dx)}{15ade^3\sqrt{e\cos(c+dx)}} - \frac{}{9de(e\cos}$$

$$= \frac{14\sin(c+dx)}{45ade(e\cos(c+dx))^{5/2}} + \frac{14\sin(c+dx)}{15ade^3\sqrt{e\cos(c+dx)}} - \frac{}{9de(e\cos}$$

$$= -\frac{14\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15ade^4\sqrt{\cos(c+dx)}} + \frac{14\sin(c+dx)}{45ade(e\cos(c+dx))^{5/}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.08, size = 66, normalized size = 0.46

$$\frac{{}_2F_1\left(-\frac{5}{4},\frac{13}{4};-\frac{1}{4};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{5/4}}{10\sqrt[4]{2}\,ade(e\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Cos[c + d*x])^(7/2)*(a + a*Sin[c + d*x])),x]
```

```
[Out] (Hypergeometric2F1[-5/4, 13/4, -1/4, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x
])^(5/4))/(10*2^(1/4)*a*d*e*(e*Cos[c + d*x])^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 487 vs. 2(151) = 302.
time = 8.60, size = 488, normalized size = 3.41

method	result

default	$-\frac{2\left(336\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-672\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/45/(16*sin(1/2*d*x+1/2*c)^8-32*sin(1/2*d*x+1/2*c)^6+24*sin(1/2*d*x+1/2*c
)^4-8*sin(1/2*d*x+1/2*c)^2+1)/a/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2
*e+e)^(1/2)/e^3*(336*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^8-672*sin(
1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)-672*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d
*x+1/2*c)^6+1344*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+504*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-1064*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c
)-168*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+392*sin(1/2*d*x+1/2*c)^
4*cos(1/2*d*x+1/2*c)+21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-66*sin(1/2*d*x+1/2*c)^2*co
s(1/2*d*x+1/2*c)+5*sin(1/2*d*x+1/2*c))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] e^(-7/2)*integrate(1/((a*sin(d*x + c) + a)*cos(d*x + c)^(7/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 202, normalized size = 1.41

$$-\frac{21\left(i\sqrt{2}\cos\left(dx+c\right)^3\sin\left(dx+c\right)+i\sqrt{2}\cos\left(dx+c\right)^3\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)))+21\left(-i\sqrt{2}\cos\left(dx+c\right)^3\sin\left(dx+c\right)-i\sqrt{2}\cos\left(dx+c\right)^3\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)))+2\left(21\cos\left(dx+c\right)^4-14\cos\left(dx+c\right)^2-7\left(3\cos\left(dx+c\right)^2+1\right)\sin\left(dx+c\right)-2\right)\sqrt{\cos\left(dx+c\right)}}{45\left(ad\cos\left(dx+c\right)^3e^{\frac{7}{2}}\sin\left(dx+c\right)+ad\cos\left(dx+c\right)^3e^{\frac{7}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/45*(21*(I*sqrt(2)*cos(d*x + c)^3*sin(d*x + c) + I*sqrt(2)*cos(d*x + c)^3
)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*
x + c))) + 21*(-I*sqrt(2)*cos(d*x + c)^3*sin(d*x + c) - I*sqrt(2)*cos(d*x +
```

```
c)^3)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*s
in(d*x + c))) + 2*(21*cos(d*x + c)^4 - 14*cos(d*x + c)^2 - 7*(3*cos(d*x + c
)^2 + 1)*sin(d*x + c) - 2)*sqrt(cos(d*x + c)))/(a*d*cos(d*x + c)^3*e^(7/2)*
sin(d*x + c) + a*d*cos(d*x + c)^3*e^(7/2))
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(7/2)/(a+a*sin(d*x+c)),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 5990 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c)),x, algorithm="giac")
[Out] integrate(e^(-7/2)/((a*sin(d*x + c) + a)*cos(d*x + c)^(7/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(e\,\cos\left(c+d\,x\right)\right)}^{7/2}\,\left(a+a\,\sin\left(c+d\,x\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cos(c + d*x))^(7/2)*(a + a*sin(c + d*x))),x)
[Out] int(1/((e*cos(c + d*x))^(7/2)*(a + a*sin(c + d*x))), x)
```

3.243 $\int \frac{(e\cos(c+dx))^{11/2}}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=145

$$\frac{6e^6\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{7a^2d\sqrt{e\cos(c+dx)}}+\frac{6e^5\sqrt{e\cos(c+dx)}\,\sin(c+dx)}{7a^2d}+\frac{18e^3(e\cos(c+dx))^{5/2}\sin(c+dx)}{35a^2d}+\frac{}{5}$$

```
[Out] 18/35*e^3*(e*cos(d*x+c))^(5/2)*sin(d*x+c)/a^2/d+4/5*e*(e*cos(d*x+c))^(9/2)/
d/(a^2+a^2*sin(d*x+c))+6/7*e^6*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2
*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/a^2/d/(e*cos(d*x
+c))^(1/2)+6/7*e^5*sin(d*x+c)*(e*cos(d*x+c))^(1/2)/a^2/d
```

Rubi [A]
time = 0.08, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2759, 2715, 2721, 2720}

$$\frac{6e^6\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{7a^2d\sqrt{e\cos(c+dx)}}+\frac{6e^5\sin(c+dx)\sqrt{e\cos(c+dx)}}{7a^2d}+\frac{18e^3\sin(c+dx)(e\cos(c+dx))^{5/2}}{35a^2d}+\frac{4e(e\cos(c+dx))^{9/2}}{5d\,(a^2\sin(c+dx)+a^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(11/2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (6*e^6*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(7*a^2*d*Sqrt[e*Cos[c
 + d*x]]) + (6*e^5*Sqrt[e*Cos[c + d*x]]*Sin[c + d*x])/(7*a^2*d) + (18*e^3*(e
*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(35*a^2*d) + (4*e*(e*Cos[c + d*x])^(9/2)
)/(5*d*(a^2 + a^2*Sin[c + d*x]))
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{11/2}}{(a+a\sin(c+dx))^2}\,dx = \frac{4e(e\cos(c+dx))^{9/2}}{5d\,(a^2+a^2\sin(c+dx))} + \frac{(9e^2)\int (e\cos(c+dx))^{7/2}\,dx}{5a^2}$$

$$= \frac{18e^3(e\cos(c+dx))^{5/2}\sin(c+dx)}{35a^2d} + \frac{4e(e\cos(c+dx))^{9/2}}{5d\,(a^2+a^2\sin(c+dx))} + \frac{(9e^4)\int (e\cos(c\cdots}{7a^2\cdots}$$

$$= \frac{6e^5\sqrt{e\cos(c+dx)}\,\sin(c+dx)}{7a^2d} + \frac{18e^3(e\cos(c+dx))^{5/2}\sin(c+dx)}{35a^2d} + \frac{4e(e\,c\cdots}{5d\,(a^2+\cdots}$$

$$= \frac{6e^5\sqrt{e\cos(c+dx)}\,\sin(c+dx)}{7a^2d} + \frac{18e^3(e\cos(c+dx))^{5/2}\sin(c+dx)}{35a^2d} + \frac{4e(e\,c\cdots}{5d\,(a^2+\cdots}$$

$$= \frac{6e^6\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{7a^2d\sqrt{e\cos(c+dx)}} + \frac{6e^5\sqrt{e\cos(c+dx)}\,\sin(c+dx)}{7a^2d} + \frac{18e^3(e\cdots}{\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.14, size = 66, normalized size = 0.46

$$-\frac{4\sqrt[4]{2}\,(e\cos(c+dx))^{13/2}\,{}_2F_1\left(-\frac{1}{4},\frac{13}{4};\frac{17}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{13a^2de(1+\sin(c+dx))^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(11/2)/(a + a*Sin[c + d*x])^2,x]
[Out] (-4*2^(1/4)*(e*Cos[c + d*x])^(13/2)*Hypergeometric2F1[-1/4, 13/4, 17/4, (1
- Sin[c + d*x])/2])/(13*a^2*d*e*(1 + Sin[c + d*x])^(13/4))
```

Maple [A]
time = 2.43, size = 203, normalized size = 1.40

method	result

default	$-\frac{2e^{6}\left(-80\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+120\left(\sin^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+112\left(\sin^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-168\left(\sin^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+15\sqrt{\frac{1}{2}-}\right.}{35a^{2}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(11/2)/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/35/a^2/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^6*(-80*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+120*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x
+1/2*c)+112*sin(1/2*d*x+1/2*c)^7-168*sin(1/2*d*x+1/2*c)^5+15*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))-20*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+84*sin(1/2*d*x+1/2*c)
^3-14*sin(1/2*d*x+1/2*c))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(11/2)/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] e^(11/2)*integrate(cos(d*x + c)^(11/2)/(a*sin(d*x + c) + a)^2, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 106, normalized size = 0.73

$$\frac{-15i\sqrt{2}\,e^{\frac{11}{2}}\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+15i\sqrt{2}\,e^{\frac{11}{2}}\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c))+2\left(14\,\cos(dx+c)^{2}\,e^{\frac{11}{2}}-5\left(\cos(dx+c)^{2}\,e^{\frac{11}{2}}-3\,e^{\frac{11}{2}}\right)\sin(dx+c)\right)\sqrt{\cos(dx+c)}}{35\,a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(11/2)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/35*(-15*I*sqrt(2)*e^(11/2)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*si
n(d*x + c)) + 15*I*sqrt(2)*e^(11/2)*weierstrassPInverse(-4, 0, cos(d*x + c)
 - I*sin(d*x + c)) + 2*(14*cos(d*x + c)^2*e^(11/2) - 5*(cos(d*x + c)^2*e^(1
1/2) - 3*e^(11/2))*sin(d*x + c))*sqrt(cos(d*x + c)))/(a^2*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(11/2)/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(11/2)/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(11/2)*e^(11/2)/(a*sin(d*x + c) + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(e\,\cos\left(c+d\,x\right)\right)^{11/2}}{\left(a+a\,\sin\left(c+d\,x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(11/2)/(a + a*sin(c + d*x))^2,x)
```

```
[Out] int((e*cos(c + d*x))^(11/2)/(a + a*sin(c + d*x))^2, x)
```

3.244 $\int \frac{(e\cos(c+dx))^{9/2}}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=114

$$\frac{14e^4\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d\sqrt{\cos(c+dx)}}+\frac{14e^3(e\cos(c+dx))^{3/2}\sin(c+dx)}{15a^2d}+\frac{4e(e\cos(c+dx))^{7/2}}{3d\,(a^2+a^2\sin(c+dx))}$$

```
[Out] 14/15*e^3*(e*cos(d*x+c))^(3/2)*sin(d*x+c)/a^2/d+4/3*e*(e*cos(d*x+c))^(7/2)/
d/(a^2+a^2*sin(d*x+c))+14/5*e^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/
2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/a^2/d/cos(d
*x+c)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2759, 2715, 2721, 2719}

$$\frac{14e^4E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5a^2d\sqrt{\cos(c+dx)}}+\frac{14e^3\sin(c+dx)(e\cos(c+dx))^{3/2}}{15a^2d}+\frac{4e(e\cos(c+dx))^{7/2}}{3d\,(a^2\sin(c+dx)+a^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(9/2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (14*e^4*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*a^2*d*Sqrt[Cos[c
 + d*x]]) + (14*e^3*(e*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(15*a^2*d) + (4*e*
(e*Cos[c + d*x])^(7/2))/(3*d*(a^2 + a^2*Sin[c + d*x]))
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{9/2}}{(a+a\sin(c+dx))^2}\,dx = \frac{4e(e\cos(c+dx))^{7/2}}{3d\,(a^2+a^2\sin(c+dx))} + \frac{(7e^2)\int (e\cos(c+dx))^{5/2}\,dx}{3a^2}$$

$$= \frac{14e^3(e\cos(c+dx))^{3/2}\sin(c+dx)}{15a^2d} + \frac{4e(e\cos(c+dx))^{7/2}}{3d\,(a^2+a^2\sin(c+dx))} + \frac{(7e^4)\int \sqrt{e\cos(}}{5a^2}$$

$$= \frac{14e^3(e\cos(c+dx))^{3/2}\sin(c+dx)}{15a^2d} + \frac{4e(e\cos(c+dx))^{7/2}}{3d\,(a^2+a^2\sin(c+dx))} + \frac{\left(7e^4\sqrt{e\cos(c+}\right.}{5a^2}$$

$$= \frac{14e^4\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d\sqrt{\cos(c+dx)}} + \frac{14e^3(e\cos(c+dx))^{3/2}\sin(c+dx)}{15a^2d} + \frac{}{3a}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.09, size = 66, normalized size = 0.58

$$-\frac{2\,2^{3/4}(e\cos(c+dx))^{11/2}\,{}_2F_1\left(\frac{1}{4},\frac{11}{4};\frac{15}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{11a^2de(1+\sin(c+dx))^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(9/2)/(a + a*Sin[c + d*x])^2,x]
[Out] (-2*2^(3/4)*(e*Cos[c + d*x])^(11/2)*Hypergeometric2F1[1/4, 11/4, 15/4, (1 -
 Sin[c + d*x])/2])/(11*a^2*d*e*(1 + Sin[c + d*x])^(11/4))
```

Maple [A]
time = 2.82, size = 190, normalized size = 1.67

method	result

default	$\frac{2e^5\left(-24\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+24\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+40\left(\sin^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+21\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\right.\right.}}{15a^2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^2\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(9/2)/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/15/a^2/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^5*(-24*si
n(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1
/2*c)+40*sin(1/2*d*x+1/2*c)^5+21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-6*sin(1/2*d*x+1/2
*c)^2*cos(1/2*d*x+1/2*c)-40*sin(1/2*d*x+1/2*c)^3+10*sin(1/2*d*x+1/2*c))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(9/2)/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] e^(9/2)*integrate(cos(d*x + c)^(9/2)/(a*sin(d*x + c) + a)^2, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 102, normalized size = 0.89

$$\frac{21i\sqrt{2}\,e^{\frac{9}{2}}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-21i\sqrt{2}\,e^{\frac{9}{2}}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))-2\left(3\cos(dx+c)e^{\frac{9}{2}}\sin(dx+c)-10\cos(dx+c)e^{\frac{9}{2}}\right)\sqrt{\cos(dx+c)}}{15a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(9/2)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/15*(21*I*sqrt(2)*e^(9/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0
, cos(d*x + c) + I*sin(d*x + c))) - 21*I*sqrt(2)*e^(9/2)*weierstrassZeta(-4
, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(3*cos(
d*x + c)*e^(9/2)*sin(d*x + c) - 10*cos(d*x + c)*e^(9/2))*sqrt(cos(d*x + c))
)/(a^2*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(9/2)/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8857 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(9/2)/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(9/2)*e^(9/2)/(a*sin(d*x + c) + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(e\,\cos\left(c+d\,x\right)\right)}^{9/2}}{{\left(a+a\,\sin\left(c+d\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(9/2)/(a + a*sin(c + d*x))^2,x)
```

```
[Out] int((e*cos(c + d*x))^(9/2)/(a + a*sin(c + d*x))^2, x)
```

3.245 $\int \frac{(e\cos(c+dx))^{7/2}}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=112

$$\frac{10e^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d\sqrt{e\cos(c+dx)}}+\frac{10e^3\sqrt{e\cos(c+dx)}\,\sin(c+dx)}{3a^2d}+\frac{4e(e\cos(c+dx))^{5/2}}{d\left(a^2+a^2\sin(c+dx)\right)}$$

```
[Out] 4*e*(e*cos(d*x+c))^(5/2)/d/(a^2+a^2*sin(d*x+c))+10/3*e^4*(cos(1/2*d*x+1/2*c
)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x
+c)^(1/2)/a^2/d/(e*cos(d*x+c))^(1/2)+10/3*e^3*sin(d*x+c)*(e*cos(d*x+c))^(1/
2)/a^2/d
```

Rubi [A]
time = 0.07, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2759, 2715, 2721, 2720}

$$\frac{10e^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d\sqrt{e\cos(c+dx)}}+\frac{10e^3\sin(c+dx)\sqrt{e\cos(c+dx)}}{3a^2d}+\frac{4e(e\cos(c+dx))^{5/2}}{d\left(a^2\sin(c+dx)+a^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(7/2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (10*e^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*a^2*d*Sqrt[e*Cos[c
 + d*x]]) + (10*e^3*Sqrt[e*Cos[c + d*x]]*Sin[c + d*x])/(3*a^2*d) + (4*e*(e*
Cos[c + d*x])^(5/2))/(d*(a^2 + a^2*Sin[c + d*x]))
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{7/2}}{(a+a\sin(c+dx))^2}\,dx = \frac{4e(e\cos(c+dx))^{5/2}}{d\,(a^2+a^2\sin(c+dx))} + \frac{(5e^2)\int(e\cos(c+dx))^{3/2}\,dx}{a^2}$$

$$= \frac{10e^3\sqrt{e\cos(c+dx)}\,\sin(c+dx)}{3a^2d} + \frac{4e(e\cos(c+dx))^{5/2}}{d\,(a^2+a^2\sin(c+dx))} + \frac{(5e^4)\int\frac{1}{\sqrt{e\cos(c-}}}{3a^2}$$

$$= \frac{10e^3\sqrt{e\cos(c+dx)}\,\sin(c+dx)}{3a^2d} + \frac{4e(e\cos(c+dx))^{5/2}}{d\,(a^2+a^2\sin(c+dx))} + \frac{\left(5e^4\sqrt{\cos(c+dx}\right.}{3a^2\sqrt{e}}$$

$$= \frac{10e^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d\sqrt{e\cos(c+dx)}} + \frac{10e^3\sqrt{e\cos(c+dx)}\,\sin(c+dx)}{3a^2d} + \frac{4e(}{d\,(a^2}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.07, size = 66, normalized size = 0.59

$$-\frac{2\sqrt[4]{2}\,(e\cos(c+dx))^{9/2}\,{}_2F_1\left(\frac{3}{4},\frac{9}{4};\frac{13}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{9a^2de(1+\sin(c+dx))^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(7/2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-2*2^(1/4)*(e*Cos[c + d*x])^(9/2)*Hypergeometric2F1[3/4, 9/4, 13/4, (1 - S
in[c + d*x])/2])/(9*a^2*d*e*(1 + Sin[c + d*x])^(9/4))
```

Maple [A]
time = 1.94, size = 155, normalized size = 1.38

method	result

default	$-\frac{2e^4\left(-4\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+5\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)+\right.}{3a^2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)e+e}\,d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/a^2/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^4*(-4*sin
(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*sin(1/2*d
*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+12*sin(1/2*d*x+1/2*c)^3-6*sin(1/2*d*x+1/2*c)
)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] e^(7/2)*integrate(cos(d*x + c)^(7/2)/(a*sin(d*x + c) + a)^2, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 83, normalized size = 0.74

$$\frac{-5i\sqrt{2}\,e^{\frac{7}{2}}\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}\,e^{\frac{7}{2}}\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-2\left(e^{\frac{7}{2}}\sin(dx+c)-6e^{\frac{7}{2}}\right)\sqrt{\cos(dx+c)}}{3a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(-5*I*sqrt(2)*e^(7/2)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d
*x + c)) + 5*I*sqrt(2)*e^(7/2)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*
sin(d*x + c)) - 2*(e^(7/2)*sin(d*x + c) - 6*e^(7/2))*sqrt(cos(d*x + c)))/(a
^2*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(7/2)/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3878 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^2,x, algorithm="giac")
[Out] integrate(cos(d*x + c)^(7/2)*e^(7/2)/(a*sin(d*x + c) + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,\cos\left(c+d\,x\right))^{7/2}}{(a+a\,\sin\left(c+d\,x\right))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(7/2)/(a + a*sin(c + d*x))^2,x)
[Out] int((e*cos(c + d*x))^(7/2)/(a + a*sin(c + d*x))^2, x)
```

3.246 $\int \frac{(e\cos(c+dx))^{5/2}}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=79

$$-\frac{6e^2\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d\sqrt{\cos(c+dx)}}-\frac{4e(e\cos(c+dx))^{3/2}}{d\,(a^2+a^2\sin(c+dx))}$$

```
[Out] -4*e*(e*cos(d*x+c))^(3/2)/d/(a^2+a^2*sin(d*x+c))-6*e^2*(cos(1/2*d*x+1/2*c)^
2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*
x+c))^(1/2)/a^2/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2759, 2721, 2719}

$$-\frac{6e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{a^2d\sqrt{\cos(c+dx)}}-\frac{4e(e\cos(c+dx))^{3/2}}{d\,(a^2\sin(c+dx)+a^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(5/2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-6*e^2*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(a^2*d*Sqrt[Cos[c +
 d*x]]) - (4*e*(e*Cos[c + d*x])^(3/2))/(d*(a^2 + a^2*Sin[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(e\cos(c+dx))^{5/2}}{(a+a\sin(c+dx))^2}\,dx &= -\frac{4e(e\cos(c+dx))^{3/2}}{d\left(a^2+a^2\sin(c+dx)\right)}-\frac{(3e^2)\int\sqrt{e\cos(c+dx)}\,dx}{a^2}\\
&= -\frac{4e(e\cos(c+dx))^{3/2}}{d\left(a^2+a^2\sin(c+dx)\right)}-\frac{\left(3e^2\sqrt{e\cos(c+dx)}\right)\int\sqrt{\cos(c+dx)}\,dx}{a^2\sqrt{\cos(c+dx)}}\\
&= -\frac{6e^2\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{a^2d\sqrt{\cos(c+dx)}}-\frac{4e(e\cos(c+dx))^{3/2}}{d\left(a^2+a^2\sin(c+dx)\right)}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.08, size = 66, normalized size = 0.84

$$
-\frac{2^{3/4}(e\cos(c+dx))^{7/2}\,{}_2F_1\left(\frac{5}{4},\frac{7}{4};\frac{11}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{7a^2de(1+\sin(c+dx))^{7/4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(5/2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/7*(2^(3/4)*(e*Cos[c + d*x])^(7/2)*Hypergeometric2F1[5/4, 7/4, 11/4, (1 -
 Sin[c + d*x])/2])/(a^2*d*e*(1 + Sin[c + d*x])^(7/4))
```

Maple [A]
time = 3.32, size = 120, normalized size = 1.52

method	result
default	$-\frac{2\left(3\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\;\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-4\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+2\sin(\right.}{\sqrt{-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)e+e}\;\sin\left(\frac{dx}{2}+\frac{c}{2}\right)a^2d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/sin(1/2*d*x+1/2*c)/a^2*(3*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))-4*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+2*sin(1/2*d*x+1/2*
c))*e^3/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] e^(5/2)*integrate(cos(d*x + c)^(5/2)/(a*sin(d*x + c) + a)^2, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 179, normalized size = 2.27

$$-\frac{3\left(i\sqrt{2}\cos\left(dx+c\right)e^{\frac{5}{2}}+i\sqrt{2}e^{\frac{5}{2}}\sin\left(dx+c\right)+i\sqrt{2}e^{\frac{5}{2}}\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)))+3\left(-i\sqrt{2}\cos\left(dx+c\right)e^{\frac{5}{2}}-i\sqrt{2}e^{\frac{5}{2}}\sin\left(dx+c\right)-i\sqrt{2}e^{\frac{5}{2}}\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)))+4\left(\cos\left(dx+c\right)e^{\frac{5}{2}}-e^{\frac{5}{2}}\sin\left(dx+c\right)+e^{\frac{5}{2}}\right)\sqrt{\cos\left(dx+c\right)}}{a^{2}d\cos\left(dx+c\right)+a^{2}d\sin\left(dx+c\right)+a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -(3*(I*sqrt(2)*cos(d*x + c)*e^(5/2) + I*sqrt(2)*e^(5/2)*sin(d*x + c) + I*sq
rt(2)*e^(5/2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x +
c) + I*sin(d*x + c))) + 3*(-I*sqrt(2)*cos(d*x + c)*e^(5/2) - I*sqrt(2)*e^(5
/2)*sin(d*x + c) - I*sqrt(2)*e^(5/2))*weierstrassZeta(-4, 0, weierstrassPIn
verse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 4*(cos(d*x + c)*e^(5/2) - e^
(5/2)*sin(d*x + c) + e^(5/2))*sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c) + a^2
*d*sin(d*x + c) + a^2*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(5/2)/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(5/2)*e^(5/2)/(a*sin(d*x + c) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(e\,\cos\left(c+d\,x\right)\right)}^{5/2}}{{\left(a+a\,\sin\left(c+d\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(5/2)/(a + a*sin(c + d*x))^2,x)
```

```
[Out] int((e*cos(c + d*x))^(5/2)/(a + a*sin(c + d*x))^2, x)
```

3.247 $\int \frac{(e\cos(c+dx))^{3/2}}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=83

$$-\frac{2e^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d\sqrt{e\cos(c+dx)}}-\frac{4e\sqrt{e\cos(c+dx)}}{3d\left(a^2+a^2\sin(c+dx)\right)}$$

```
[Out] -2/3*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*
d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/a^2/d/(e*cos(d*x+c))^(1/2)-4/3*e*(e*co
s(d*x+c))^(1/2)/d/(a^2+a^2*sin(d*x+c))
```

Rubi [A]
time = 0.06, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2759, 2721, 2720}

$$-\frac{2e^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d\sqrt{e\cos(c+dx)}}-\frac{4e\sqrt{e\cos(c+dx)}}{3d\left(a^2\sin(c+dx)+a^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(3/2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-2*e^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*a^2*d*Sqrt[e*Cos[c
 + d*x]]) - (4*e*Sqrt[e*Cos[c + d*x]])/(3*d*(a^2 + a^2*Sin[c + d*x]))
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(e\cos(c+dx))^{3/2}}{(a+a\sin(c+dx))^2}\,dx &= -\frac{4e\sqrt{e\cos(c+dx)}}{3d\left(a^2+a^2\sin(c+dx)\right)}-\frac{e^2\int\frac{1}{\sqrt{e\cos(c+dx)}}\,dx}{3a^2}\\
&= -\frac{4e\sqrt{e\cos(c+dx)}}{3d\left(a^2+a^2\sin(c+dx)\right)}-\frac{\left(e^2\sqrt{\cos(c+dx)}\right)\int\frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3a^2\sqrt{e\cos(c+dx)}}\\
&= -\frac{2e^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{3a^2d\sqrt{e\cos(c+dx)}}-\frac{4e\sqrt{e\cos(c+dx)}}{3d\left(a^2+a^2\sin(c+dx)\right)}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.08, size = 66, normalized size = 0.80

$$
-\frac{\sqrt[4]{2}\,(e\cos(c+dx))^{5/2}\,{}_2F_1\left(\frac{5}{4},\frac{7}{4};\frac{9}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{5a^2de(1+\sin(c+dx))^{5/4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(3/2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/5*(2^(1/4)*(e*Cos[c + d*x])^(5/2)*Hypergeometric2F1[5/4, 7/4, 9/4, (1 - Sin[c + d*x])/2])/(a^2*d*e*(1 + Sin[c + d*x])^(5/4))
```

Maple [A]
time = 4.04, size = 193, normalized size = 2.33

method	result
default	$\frac{2\left(2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}}{3\left(2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)a^2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/(2*sin(1/2*d*x+1/2*c)^2-1)/a^2/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c
)^2*e+e)^(1/2)*(2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))-4*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+2*sin(1/2*d*x+1/2*c
))*e^2/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] e^(3/2)*integrate(cos(d*x + c)^(3/2)/(a*sin(d*x + c) + a)^2, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 114, normalized size = 1.37

$$\frac{\left(i\sqrt{2}\,e^{\frac{3}{2}}\sin\left(dx+c\right)+i\sqrt{2}\,e^{\frac{3}{2}}\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)+\left(-i\sqrt{2}\,e^{\frac{3}{2}}\sin\left(dx+c\right)-i\sqrt{2}\,e^{\frac{3}{2}}\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)-4\sqrt{\cos\left(dx+c\right)}\,e^{\frac{3}{2}}}{3\left(a^{2}d\sin\left(dx+c\right)+a^{2}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/3*((I*sqrt(2)*e^(3/2)*sin(d*x + c) + I*sqrt(2)*e^(3/2))*weierstrassPInver
se(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + (-I*sqrt(2)*e^(3/2)*sin(d*x + c)
 - I*sqrt(2)*e^(3/2))*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x +
 c)) - 4*sqrt(cos(d*x + c))*e^(3/2))/(a^2*d*sin(d*x + c) + a^2*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(3/2)/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3064 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(3/2)*e^(3/2)/(a*sin(d*x + c) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(e\,\cos\left(c+d\,x\right)\right)}^{3/2}}{{\left(a+a\,\sin\left(c+d\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(3/2)/(a + a*sin(c + d*x))^2,x)
[Out] int((e*cos(c + d*x))^(3/2)/(a + a*sin(c + d*x))^2, x)
```

3.248 $\int \frac{\sqrt{e\cos(c+dx)}}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=116

$$-\frac{2\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d\sqrt{\cos(c+dx)}}-\frac{2(e\cos(c+dx))^{3/2}}{5de(a+a\sin(c+dx))^2}-\frac{2(e\cos(c+dx))^{3/2}}{5de\,(a^2+a^2\sin(c+dx))}$$

```
[Out] -2/5*(e*cos(d*x+c))^(3/2)/d/e/(a+a*sin(d*x+c))^2-2/5*(e*cos(d*x+c))^(3/2)/d
/e/(a^2+a^2*sin(d*x+c))-2/5*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/a^2/d/cos(d*x+c
)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2760, 2762, 2721, 2719}

$$-\frac{2(e\cos(c+dx))^{3/2}}{5de\,(a^2\sin(c+dx)+a^2)}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5a^2d\sqrt{\cos(c+dx)}}-\frac{2(e\cos(c+dx))^{3/2}}{5de(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cos[c + d*x]]/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-2*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*a^2*d*Sqrt[Cos[c + d
*x]]) - (2*(e*Cos[c + d*x])^(3/2))/(5*d*e*(a + a*Sin[c + d*x])^2) - (2*(e*C
os[c + d*x])^(3/2))/(5*d*e*(a^2 + a^2*Sin[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2760

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*(2*m + p + 1))), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
```

```
IntegersQ[2*m, 2*p]
```

Rule 2762

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[b*((g*Cos[e + f*x])^(p + 1)/(a*f*g*(p - 1)*(a + b*S
in[e + f*x]))), x] + Dist[p/(a*(p - 1)), Int[(g*Cos[e + f*x])^p, x], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] &&  !GeQ[p, 1] && Integer
Q[2*p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{e\cos(c+dx)}}{(a+a\sin(c+dx))^2}\,dx &= -\frac{2(e\cos(c+dx))^{3/2}}{5de(a+a\sin(c+dx))^2}+\frac{\int\frac{\sqrt{e\cos(c+dx)}}{a+a\sin(c+dx)}\,dx}{5a}\\
&= -\frac{2(e\cos(c+dx))^{3/2}}{5de(a+a\sin(c+dx))^2}-\frac{2(e\cos(c+dx))^{3/2}}{5de\left(a^2+a^2\sin(c+dx)\right)}-\frac{\int\sqrt{e\cos(c+dx)}\,dx}{5a^2}\\
&= -\frac{2(e\cos(c+dx))^{3/2}}{5de(a+a\sin(c+dx))^2}-\frac{2(e\cos(c+dx))^{3/2}}{5de\left(a^2+a^2\sin(c+dx)\right)}-\frac{\sqrt{e\cos(c+dx)}\int\sqrt{\ldots}}{5a^2\sqrt{\cos(c\ldots}}\\
&= -\frac{2\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d\sqrt{\cos(c+dx)}}-\frac{2(e\cos(c+dx))^{3/2}}{5de(a+a\sin(c+dx))^2}-\frac{2(e\cos(c\ldots}{5de\left(a^2+a^2\ldots\right.}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.04, size = 66, normalized size = 0.57

$$-\frac{(e\cos(c+dx))^{3/2}\,{}_2F_1\left(\frac{3}{4},\frac{9}{4};\frac{7}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{3\sqrt[4]{2}\,a^2de(1+\sin(c+dx))^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Cos[c + d*x]]/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/3*((e*Cos[c + d*x])^(3/2)*Hypergeometric2F1[3/4, 9/4, 7/4, (1 - Sin[c +
d*x])/2])/(2^(1/4)*a^2*d*e*(1 + Sin[c + d*x])^(3/4))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 302 vs. 2(128) = 256.
time = 5.88, size = 303, normalized size = 2.61

method	result

default	$-\frac{2\left(4\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-8\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos(\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/5/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/a^2/sin(1/2*d*x+1/2*
c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*(4*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*
d*x+1/2*c)^4-8*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-4*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))-6*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+2*sin(1/2*d*x
+1/2*c))*e/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] e^(1/2)*integrate(sqrt(cos(d*x + c))/(a*sin(d*x + c) + a)^2, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 290, normalized size = 2.50

$$-\frac{\left(i\sqrt{2}\cos(dx+c)^2e^{\frac{1}{2}}-i\sqrt{2}\cos(dx+c)e^{\frac{1}{2}}+\left(-i\sqrt{2}\cos(dx+c)e^{\frac{1}{2}}-2i\sqrt{2}e^{\frac{1}{2}}\right)\sin(dx+c)-2i\sqrt{2}e^{\frac{1}{2}}\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+\left(-i\sqrt{2}\cos(dx+c)^2e^{\frac{1}{2}}+i\sqrt{2}\cos(dx+c)e^{\frac{1}{2}}+\left(i\sqrt{2}\cos(dx+c)e^{\frac{1}{2}}+2i\sqrt{2}e^{\frac{1}{2}}\right)\sin(dx+c)+2i\sqrt{2}e^{\frac{1}{2}}\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))-2\left(\cos(dx+c)^2e^{\frac{1}{2}}+2\cos(dx+c)e^{\frac{1}{2}}+\left(\cos(dx+c)e^{\frac{1}{2}}-e^{\frac{1}{2}}\right)\sin(dx+c)+e^{\frac{1}{2}}\right)\sqrt{\cos(dx+c)}}{5\left(a^2d\cos(dx+c)^2-a^2d\cos(dx+c)-2a^2d-\left(a^2d\cos(dx+c)+2a^2d\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/5*((I*sqrt(2)*cos(d*x + c)^2*e^(1/2) - I*sqrt(2)*cos(d*x + c)*e^(1/2) +
(-I*sqrt(2)*cos(d*x + c)*e^(1/2) - 2*I*sqrt(2)*e^(1/2))*sin(d*x + c) - 2*I*
sqrt(2)*e^(1/2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x
+ c) + I*sin(d*x + c))) + (-I*sqrt(2)*cos(d*x + c)^2*e^(1/2) + I*sqrt(2)*co
s(d*x + c)*e^(1/2) + (I*sqrt(2)*cos(d*x + c)*e^(1/2) + 2*I*sqrt(2)*e^(1/2))
*sin(d*x + c) + 2*I*sqrt(2)*e^(1/2))*weierstrassZeta(-4, 0, weierstrassPInv
erse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(cos(d*x + c)^2*e^(1/2) + 2
*cos(d*x + c)*e^(1/2) + (cos(d*x + c)*e^(1/2) - e^(1/2))*sin(d*x + c) + e^(
```

```
1/2))*sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^2 - a^2*d*cos(d*x + c) - 2*a^
2*d - (a^2*d*cos(d*x + c) + 2*a^2*d)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{e\cos(c+dx)}}{\sin^2(c+dx)+2\sin(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(1/2)/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Integral(sqrt(e*cos(c + d*x))/(sin(c + d*x)**2 + 2*sin(c + d*x) + 1), x)/a*
*2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(d*x + c))*e^(1/2)/(a*sin(d*x + c) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{e\,\cos(c+d\,x)}}{{(a+a\,\sin(c+d\,x))}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(1/2)/(a + a*sin(c + d*x))^2,x)
```

```
[Out] int((e*cos(c + d*x))^(1/2)/(a + a*sin(c + d*x))^2, x)
```

3.249 $\int \frac{1}{\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=116

$$\frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7a^2d\sqrt{e\cos(c+dx)}}-\frac{2\sqrt{e\cos(c+dx)}}{7de(a+a\sin(c+dx))^2}-\frac{2\sqrt{e\cos(c+dx)}}{7de\left(a^2+a^2\sin(c+dx)\right)}$$

```
[Out] 2/7*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1
/2*c),2^(1/2))*cos(d*x+c)^(1/2)/a^2/d/(e*cos(d*x+c))^(1/2)-2/7*(e*cos(d*x+c
))^(1/2)/d/e/(a+a*sin(d*x+c))^2-2/7*(e*cos(d*x+c))^(1/2)/d/e/(a^2+a^2*sin(d
*x+c))
```

Rubi [A]
time = 0.09, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2760, 2762, 2721, 2720}

$$-\frac{2\sqrt{e\cos(c+dx)}}{7de\left(a^2\sin(c+dx)+a^2\right)}+\frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7a^2d\sqrt{e\cos(c+dx)}}-\frac{2\sqrt{e\cos(c+dx)}}{7de(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])^2),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(7*a^2*d*Sqrt[e*Cos[c + d*
x]]) - (2*Sqrt[e*Cos[c + d*x]])/(7*d*e*(a + a*Sin[c + d*x])^2) - (2*Sqrt[e*
Cos[c + d*x]])/(7*d*e*(a^2 + a^2*Sin[c + d*x]))
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2760

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*(2*m + p + 1))), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
```

```
IntegersQ[2*m, 2*p]
```

Rule 2762

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[b*((g*Cos[e + f*x])^(p + 1)/(a*f*g*(p - 1)*(a + b*S
in[e + f*x]))), x] + Dist[p/(a*(p - 1)), Int[(g*Cos[e + f*x])^p, x], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] &&  !GeQ[p, 1] && Integer
Q[2*p]
```

Rubi steps

$$\int \frac{1}{\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^2}dx = -\frac{2\sqrt{e\cos(c+dx)}}{7de(a+a\sin(c+dx))^2} + \frac{3\int \frac{1}{\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))}}{7a}$$

$$= -\frac{2\sqrt{e\cos(c+dx)}}{7de(a+a\sin(c+dx))^2} - \frac{2\sqrt{e\cos(c+dx)}}{7de\left(a^2+a^2\sin(c+dx)\right)} + \frac{\int \frac{}{\sqrt{}}}{}$$

$$= -\frac{2\sqrt{e\cos(c+dx)}}{7de(a+a\sin(c+dx))^2} - \frac{2\sqrt{e\cos(c+dx)}}{7de\left(a^2+a^2\sin(c+dx)\right)} + \frac{\sqrt{\cos}}{}$$

$$= \frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7a^2d\sqrt{e\cos(c+dx)}} - \frac{2\sqrt{e\cos(c+dx)}}{7de(a+a\sin(c+dx))^2} -$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.04, size = 64, normalized size = 0.55

$$-\frac{\sqrt{e\cos(c+dx)}\ {}_2F_1\left(\frac{1}{4},\frac{11}{4};\frac{5}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{2^{3/4}a^2de\sqrt[4]{1+\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])^2),x]
```

```
[Out] -((Sqrt[e*Cos[c + d*x]]*Hypergeometric2F1[1/4, 11/4, 5/4, (1 - Sin[c + d*x]
)/2])/(2^(3/4)*a^2*d*e*(1 + Sin[c + d*x])^(1/4)))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 371 vs. 2(128) = 256.
time = 6.60, size = 372, normalized size = 3.21

method	result
default	$-\frac{2\left(8\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ \mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-12\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(d*x+c))^2/(e*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/7/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2
-1)/a^2/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*(8*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^6-12*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*
x+1/2*c)^4+8*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+6*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))*sin(1/2*d*x+1/2*c)^2-8*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))+6*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2*sin(1/2*d*x+1
/2*c))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))^2/(e*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] e^(-1/2)*integrate(1/((a*sin(d*x + c) + a)^2*sqrt(cos(d*x + c))), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 159, normalized size = 1.37

$$-\frac{\left(i\sqrt{2}\cos\left(dx+c\right)^2-2i\sqrt{2}\sin\left(dx+c\right)-2i\sqrt{2}\right)\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right))+\left(-i\sqrt{2}\cos\left(dx+c\right)^2+2i\sqrt{2}\sin\left(dx+c\right)+2i\sqrt{2}\right)\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right))-2\left(\sin\left(dx+c\right)+2\right)\sqrt{\cos\left(dx+c\right)}}{7\left(a^2d\cos\left(dx+c\right)^2e^{\frac{1}{2}}-2a^2de^{\frac{1}{2}}\sin\left(dx+c\right)-2a^2de^{\frac{1}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))^2/(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/7*((I*sqrt(2)*cos(d*x + c)^2 - 2*I*sqrt(2)*sin(d*x + c) - 2*I*sqrt(2))*w
eierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + (-I*sqrt(2)*cos(
d*x + c)^2 + 2*I*sqrt(2)*sin(d*x + c) + 2*I*sqrt(2))*weierstrassPInverse(-4
, 0, cos(d*x + c) - I*sin(d*x + c)) - 2*(sin(d*x + c) + 2)*sqrt(cos(d*x + c
```

```
)))/(a^2*d*cos(d*x + c)^2*e^(1/2) - 2*a^2*d*e^(1/2)*sin(d*x + c) - 2*a^2*d*
e^(1/2))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\sqrt{e\cos(c+dx)}\sin^2(c+dx)+2\sqrt{e\cos(c+dx)}\sin(c+dx)+\sqrt{e\cos(c+dx)}}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))**2/(e*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(e*cos(c + d*x))*sin(c + d*x)**2 + 2*sqrt(e*cos(c + d*x))*s
in(c + d*x) + sqrt(e*cos(c + d*x))), x)/a**2
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))^2/(e*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(e^(-1/2)/((a*sin(d*x + c) + a)^2*sqrt(cos(d*x + c))), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{e\cos(c+d\,x)}\,{\left(a+a\sin(c+d\,x)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cos(c + d*x))^(1/2)*(a + a*sin(c + d*x))^2),x)
```

```
[Out] int(1/((e*cos(c + d*x))^(1/2)*(a + a*sin(c + d*x))^2), x)
```

3.250 $\int \frac{1}{(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=150

$$-\frac{2\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2de^2\sqrt{\cos(c+dx)}}+\frac{2\sin(c+dx)}{3a^2de\sqrt{e\cos(c+dx)}}-\frac{2}{9de\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^2}-\frac{}{9de}$$

```
[Out] 2/3*sin(d*x+c)/a^2/d/e/(e*cos(d*x+c))^(1/2)-2/9/d/e/(a+a*sin(d*x+c))^2/(e*c
os(d*x+c))^(1/2)-2/9/d/e/(a^2+a^2*sin(d*x+c))/(e*cos(d*x+c))^(1/2)-2/3*(cos
(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^
(1/2))*(e*cos(d*x+c))^(1/2)/a^2/d/e^2/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2760, 2762, 2716, 2721, 2719}

$$-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{3a^2de^2\sqrt{\cos(c+dx)}}+\frac{2\sin(c+dx)}{3a^2de\sqrt{e\cos(c+dx)}}-\frac{2}{9de\left(a^2\sin(c+dx)+a^2\right)\sqrt{e\cos(c+dx)}}-\frac{2}{9de(a\sin(c+dx)+a)^2\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x])^2),x]
```

```
[Out] (-2*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(3*a^2*d*e^2*Sqrt[Cos[c
 + d*x]]) + (2*Sin[c + d*x])/(3*a^2*d*e*Sqrt[e*Cos[c + d*x]]) - 2/(9*d*e*Sq
rt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])^2) - 2/(9*d*e*Sqrt[e*Cos[c + d*x]]*
(a^2 + a^2*Sin[c + d*x]))
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2760

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*(2*m + p + 1))), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2762

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[b*((g*Cos[e + f*x])^(p + 1)/(a*f*g*(p - 1)*(a + b*S
in[e + f*x]))), x] + Dist[p/(a*(p - 1)), Int[(g*Cos[e + f*x])^p, x], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] &&  !GeQ[p, 1] && Integer
Q[2*p]
```

Rubi steps

$$\int \frac{1}{(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^2}\,dx = -\frac{2}{9de\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^2} + \frac{5\int \frac{1}{(e\cos(c+dx))^{3/}}}{9}$$

$$= -\frac{2}{9de\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^2} - \frac{}{9de\sqrt{e\cos(c+}}$$

$$= \frac{2\sin(c+dx)}{3a^2de\sqrt{e\cos(c+dx)}} - \frac{2}{9de\sqrt{e\cos(c+dx)}\,(a+a\sin(c+}$$

$$= \frac{2\sin(c+dx)}{3a^2de\sqrt{e\cos(c+dx)}} - \frac{2}{9de\sqrt{e\cos(c+dx)}\,(a+a\sin(c+}$$

$$= -\frac{2\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2de^2\sqrt{\cos(c+dx)}} + \frac{2\sin(c+dx)}{3a^2de\sqrt{e\cos(c+dx)}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.06, size = 66, normalized size = 0.44

$$\frac{{}_2F_1\left(-\frac{1}{4},\frac{13}{4};\frac{3}{4};\frac{1}{2}(1-\sin(c+dx))\right)\sqrt[4]{1+\sin(c+dx)}}{2\sqrt[4]{2}\,a^2de\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x])^2),x]
```

[Out] (Hypergeometric2F1[-1/4, 13/4, 3/4, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x]
)^(1/4))/(2*2^(1/4)*a^2*d*e*Sqrt[e*Cos[c + d*x]])

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 487 vs. $2(158) = 316$.
time = 8.80, size = 488, normalized size = 3.25

method	result
default	$-\frac{2\left(48\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\left(\sin^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-96\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\mathrm{c}}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)

[Out] -2/9/(16*sin(1/2*d*x+1/2*c)^8-32*sin(1/2*d*x+1/2*c)^6+24*sin(1/2*d*x+1/2*c)
^4-8*sin(1/2*d*x+1/2*c)^2+1)/a^2/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^
2*e+e)^(1/2)/e*(48*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2
c)^2-1)^(1/2)(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^8-96*sin(1/2
*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)-96*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1
/2*c)^6+192*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+72*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*sin(1/2*d*x+1/2*c)^4-152*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-24*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+56*sin(1/2*d*x+1/2*c)^4*cos(1/2
*d*x+1/2*c)+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-12*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+
1/2*c)+2*sin(1/2*d*x+1/2*c))/d

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^2,x, algorithm="maxima")

[Out] e^(-3/2)*integrate(1/((a*sin(d*x + c) + a)^2*cos(d*x + c)^(3/2)), x)

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 226, normalized size = 1.51

$$\frac{3\left(-i\sqrt{2}\cos(dx+c)^{3}+2i\sqrt{2}\cos(dx+c)\sin(dx+c)+2i\sqrt{2}\cos(dx+c)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+3\left(i\sqrt{2}\cos(dx+c)^{3}-2i\sqrt{2}\cos(dx+c)\sin(dx+c)-2i\sqrt{2}\cos(dx+c)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+2\left(6\cos(dx+c)^{2}+\left(3\cos(dx+c)^{2}-5\right)\sin(dx+c)-4\right)\sqrt{\cos(dx+c)}}{9\left(a^{2}d\cos(dx+c)^{3}e^{\frac{3}{2}}-2a^{2}d\cos(dx+c)e^{\frac{3}{2}}\sin(dx+c)-2a^{2}d\cos(dx+c)e^{\frac{3}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/9*(3*(-I*sqrt(2)*cos(d*x + c)^3 + 2*I*sqrt(2)*cos(d*x + c)*sin(d*x + c) +
 2*I*sqrt(2)*cos(d*x + c))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0
, cos(d*x + c) + I*sin(d*x + c))) + 3*(I*sqrt(2)*cos(d*x + c)^3 - 2*I*sqrt(
2)*cos(d*x + c)*sin(d*x + c) - 2*I*sqrt(2)*cos(d*x + c))*weierstrassZeta(-4
, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(6*cos(
d*x + c)^2 + (3*cos(d*x + c)^2 - 5)*sin(d*x + c) - 4)*sqrt(cos(d*x + c)))/(
a^2*d*cos(d*x + c)^3*e^(3/2) - 2*a^2*d*cos(d*x + c)*e^(3/2)*sin(d*x + c) -
2*a^2*d*cos(d*x + c)*e^(3/2))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(3/2)/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(e^(-3/2)/((a*sin(d*x + c) + a)^2*cos(d*x + c)^(3/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(e\,\cos\left(c+d\,x\right)\right)}^{3/2}\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cos(c + d*x))^(3/2)*(a + a*sin(c + d*x))^2),x)
```

```
[Out] int(1/((e*cos(c + d*x))^(3/2)*(a + a*sin(c + d*x))^2), x)
```

3.251 $\int \frac{1}{(e\cos(c+dx))^{5/2}(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=150

$$\frac{10\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{33a^2de^2\sqrt{e\cos(c+dx)}}+\frac{10\sin(c+dx)}{33a^2de(e\cos(c+dx))^{3/2}}-\frac{2}{11de(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^2}-\frac{}{11}$$

```
[Out] 10/33*sin(d*x+c)/a^2/d/e/(e*cos(d*x+c))^(3/2)-2/11/d/e/(e*cos(d*x+c))^(3/2)
/(a+a*sin(d*x+c))^2-2/11/d/e/(e*cos(d*x+c))^(3/2)/(a^2+a^2*sin(d*x+c))+10/3
3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2
*c),2^(1/2))*cos(d*x+c)^(1/2)/a^2/d/e^2/(e*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2760, 2762, 2716, 2721, 2720}

$$\frac{10\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{33a^2de^2\sqrt{e\cos(c+dx)}}+\frac{10\sin(c+dx)}{33a^2de(e\cos(c+dx))^{3/2}}-\frac{2}{11de\left(a^2\sin(c+dx)+a^2\right)(e\cos(c+dx))^{3/2}}-\frac{2}{11de(a\sin(c+dx)+a)^2(e\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(5/2)*(a + a*Sin[c + d*x])^2),x]
```

```
[Out] (10*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(33*a^2*d*e^2*Sqrt[e*Cos[
c + d*x]]) + (10*Sin[c + d*x])/(33*a^2*d*e*(e*Cos[c + d*x])^(3/2)) - 2/(11*
d*e*(e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x])^2) - 2/(11*d*e*(e*Cos[c + d
*x])^(3/2)*(a^2 + a^2*Sin[c + d*x]))
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2760

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*(2*m + p + 1))), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2762

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[b*((g*Cos[e + f*x])^(p + 1)/(a*f*g*(p - 1)*(a + b*S
in[e + f*x]))), x] + Dist[p/(a*(p - 1)), Int[(g*Cos[e + f*x])^p, x], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] &&  !GeQ[p, 1] && Integer
Q[2*p]
```

Rubi steps

$$\int \frac{1}{(e\cos(c+dx))^{5/2}(a+a\sin(c+dx))^2}\,dx = -\frac{2}{11de(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^2} + \frac{7\int \frac{}{(e\cos(c+dx)}}{}$$

$$= -\frac{2}{11de(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^2} - \frac{}{11de(e\cos(c}$$

$$= \frac{10\sin(c+dx)}{33a^2de(e\cos(c+dx))^{3/2}} - \frac{2}{11de(e\cos(c+dx))^{3/2}(a+a\sin(}$$

$$= \frac{10\sin(c+dx)}{33a^2de(e\cos(c+dx))^{3/2}} - \frac{2}{11de(e\cos(c+dx))^{3/2}(a+a\sin(}$$

$$= \frac{10\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{33a^2de^2\sqrt{e\cos(c+dx)}} + \frac{10\sin(c+dx)}{33a^2de(e\cos(c+dx))^{3/}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.08, size = 66, normalized size = 0.44

$$\frac{{}_2F_1\left(-\frac{3}{4},\frac{15}{4};\frac{1}{4};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{3/4}}{6\ 2^{3/4}a^2de(e\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Cos[c + d*x])^(5/2)*(a + a*Sin[c + d*x])^2),x]
```

```
[Out] (Hypergeometric2F1[-3/4, 15/4, 1/4, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x]
)^(3/4))/(6*2^(3/4)*a^2*d*e*(e*Cos[c + d*x])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 556 vs. $2(158) = 316$.
time = 10.11, size = 557, normalized size = 3.71

method	result
default	$-\frac{2\left(160\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-400\,\mathrm{EllipticF}\left(\cos\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/33/(32*sin(1/2*d*x+1/2*c)^10-80*sin(1/2*d*x+1/2*c)^8+80*sin(1/2*d*x+1/2*
c)^6-40*sin(1/2*d*x+1/2*c)^4+10*sin(1/2*d*x+1/2*c)^2-1)/a^2/sin(1/2*d*x+1/2
*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/e^2*(160*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*s
in(1/2*d*x+1/2*c)^10-400*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^8+160*
sin(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+400*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1
/2*d*x+1/2*c)^6-320*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-200*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4+264*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2
*c)+50*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-104*sin(1/2*d*x+1/2*c)
^4*cos(1/2*d*x+1/2*c)-5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+28*sin(1/2*d*x+1/2*c)^2*co
s(1/2*d*x+1/2*c)-6*sin(1/2*d*x+1/2*c))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] e^(-5/2)*integrate(1/((a*sin(d*x + c) + a)^2*cos(d*x + c)^(5/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 232, normalized size = 1.55

$$\frac{5\left(-i\sqrt{2}\cos(dx+c)^{4}+2i\sqrt{2}\cos(dx+c)^{2}\sin(dx+c)+2i\sqrt{2}\cos(dx+c)^{2}\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5\left(i\sqrt{2}\cos(dx+c)^{4}-2i\sqrt{2}\cos(dx+c)^{2}\sin(dx+c)-2i\sqrt{2}\cos(dx+c)^{2}\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+2\left(10\cos(dx+c)^{2}+\left(5\cos(dx+c)^{2}-7\right)\sin(dx+c)-4\right)\sqrt{\cos(dx+c)}}{33\left(a^{2}d\cos(dx+c)^{4}e^{\frac{5}{2}}-2\,a^{2}d\cos(dx+c)^{2}e^{\frac{5}{2}}\sin(dx+c)-2\,a^{2}d\cos(dx+c)^{2}e^{\frac{5}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/33*(5*(-I*sqrt(2)*cos(d*x + c)^4 + 2*I*sqrt(2)*cos(d*x + c)^2*sin(d*x + c
) + 2*I*sqrt(2)*cos(d*x + c)^2)*weierstrassPInverse(-4, 0, cos(d*x + c) + I
*sin(d*x + c)) + 5*(I*sqrt(2)*cos(d*x + c)^4 - 2*I*sqrt(2)*cos(d*x + c)^2*s
in(d*x + c) - 2*I*sqrt(2)*cos(d*x + c)^2)*weierstrassPInverse(-4, 0, cos(d*
x + c) - I*sin(d*x + c)) + 2*(10*cos(d*x + c)^2 + (5*cos(d*x + c)^2 - 7)*si
n(d*x + c) - 4)*sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^4*e^(5/2) - 2*a^2*d
*cos(d*x + c)^2*e^(5/2)*sin(d*x + c) - 2*a^2*d*cos(d*x + c)^2*e^(5/2))
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(5/2)/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3881 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(e^(-5/2)/((a*sin(d*x + c) + a)^2*cos(d*x + c)^(5/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(e\,\cos\left(c+d\,x\right))^{5/2}\,\left(a+a\,\sin\left(c+d\,x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cos(c + d*x))^(5/2)*(a + a*sin(c + d*x))^2),x)
```

```
[Out] int(1/((e*cos(c + d*x))^(5/2)*(a + a*sin(c + d*x))^2), x)
```

3.252 $\int \frac{1}{(e\cos(c+dx))^{7/2}(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=181

$$-\frac{42\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{65a^2de^4\sqrt{\cos(c+dx)}}+\frac{14\sin(c+dx)}{65a^2de(e\cos(c+dx))^{5/2}}+\frac{42\sin(c+dx)}{65a^2de^3\sqrt{e\cos(c+dx)}}-\frac{}{13de(e\cos(c+d}$$

```
[Out] 14/65*sin(d*x+c)/a^2/d/e/(e*cos(d*x+c))^(5/2)-2/13/d/e/(e*cos(d*x+c))^(5/2)
/(a+a*sin(d*x+c))^2-2/13/d/e/(e*cos(d*x+c))^(5/2)/(a^2+a^2*sin(d*x+c))+42/6
5*sin(d*x+c)/a^2/d/e^3/(e*cos(d*x+c))^(1/2)-42/65*(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))
^(1/2)/a^2/d/e^4/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2760, 2762, 2716, 2721, 2719}

$$-\frac{42E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{65a^2de^4\sqrt{\cos(c+dx)}}+\frac{42\sin(c+dx)}{65a^2de^3\sqrt{e\cos(c+dx)}}+\frac{14\sin(c+dx)}{65a^2de(e\cos(c+dx))^{5/2}}-\frac{2}{13de\left(a^2\sin(c+dx)+a^2\right)(e\cos(c+dx))^{5/2}}-\frac{2}{13de(a\sin(c+dx)+a)^2(e\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(7/2)*(a + a*Sin[c + d*x])^2),x]
```

```
[Out] (-42*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(65*a^2*d*e^4*Sqrt[Cos
[c + d*x]]) + (14*Sin[c + d*x])/(65*a^2*d*e*(e*Cos[c + d*x])^(5/2)) + (42*S
in[c + d*x])/(65*a^2*d*e^3*Sqrt[e*Cos[c + d*x]]) - 2/(13*d*e*(e*Cos[c + d*x
])^(5/2)*(a + a*Sin[c + d*x])^2) - 2/(13*d*e*(e*Cos[c + d*x])^(5/2)*(a^2 +
a^2*Sin[c + d*x]))
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
```

```
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2760

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*(2*m + p + 1))), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2762

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[b*((g*Cos[e + f*x])^(p + 1)/(a*f*g*(p - 1)*(a + b*S
in[e + f*x]))), x] + Dist[p/(a*(p - 1)), Int[(g*Cos[e + f*x])^p, x], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] &&  !GeQ[p, 1] && Integer
Q[2*p]
```

Rubi steps

$$\int \frac{1}{(e\cos(c+dx))^{7/2}(a+a\sin(c+dx))^2}\,dx = -\frac{2}{13de(e\cos(c+dx))^{5/2}(a+a\sin(c+dx))^2} + \frac{9\int \frac{}{(e\cos(c+dx)}}{}$$

$$= -\frac{2}{13de(e\cos(c+dx))^{5/2}(a+a\sin(c+dx))^2} - \frac{}{13de(e\cos(c}$$

$$= \frac{14\sin(c+dx)}{65a^2de(e\cos(c+dx))^{5/2}} - \frac{2}{13de(e\cos(c+dx))^{5/2}(a+a\sin(}$$

$$= \frac{14\sin(c+dx)}{65a^2de(e\cos(c+dx))^{5/2}} + \frac{42\sin(c+dx)}{65a^2de^3\sqrt{e\cos(c+dx)}} - \frac{}{13de(}$$

$$= \frac{14\sin(c+dx)}{65a^2de(e\cos(c+dx))^{5/2}} + \frac{42\sin(c+dx)}{65a^2de^3\sqrt{e\cos(c+dx)}} - \frac{}{13de(}$$

$$= -\frac{42\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{65a^2de^4\sqrt{\cos(c+dx)}} + \frac{14\sin(c+dx)}{65a^2de(e\cos(c+dx}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.11, size = 66, normalized size = 0.36

$$\frac{{}_2F_1\left(-\frac{5}{4},\frac{17}{4};-\frac{1}{4};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{5/4}}{20\sqrt[4]{2}\,a^2de(e\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Cos[c + d*x])^(7/2)*(a + a*Sin[c + d*x])^2),x]
```

```
[Out] (Hypergeometric2F1[-5/4, 17/4, -1/4, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x
])^(5/4))/(20*2^(1/4)*a^2*d*e*(e*Cos[c + d*x])^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 669 vs. $2(185) = 370$.
time = 12.48, size = 670, normalized size = 3.70

method	result
default	$-\frac{2\left(1344\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\left(\sin^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2688\left(\sin^{14}\left(\frac{dx}{2}+\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/65/(64*sin(1/2*d*x+1/2*c)^12-192*sin(1/2*d*x+1/2*c)^10+240*sin(1/2*d*x+1
/2*c)^8-160*sin(1/2*d*x+1/2*c)^6+60*sin(1/2*d*x+1/2*c)^4-12*sin(1/2*d*x+1/2
*c)^2+1)/a^2/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/e^3*(13
44*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^12-2688*sin(1/2*d*x+1/2*c)^1
4*cos(1/2*d*x+1/2*c)-4032*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^10+80
64*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12+5040*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*s
in(1/2*d*x+1/2*c)^8-10304*sin(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)-3360*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^6+7168*cos(1/2*d*x+1/2*c)*sin(1/2*
d*x+1/2*c)^8+1260*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-2896*sin(1/
2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-252*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+
1/2*c)^2+656*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+21*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2))-86*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+10*sin(1/2*d*x+1/2*c))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] e^(-7/2)*integrate(1/((a*sin(d*x + c) + a)^2*cos(d*x + c)^(7/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 258, normalized size = 1.43

$$\frac{21 \left(-i \sqrt{2} \cos(dx+c)^5 + 2i \sqrt{2} \cos(dx+c)^3 \sin(dx+c) + 2i \sqrt{2} \cos(dx+c)^3\right) \mathrm{weierstrassZeta}(-4, 0, \mathrm{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c))) + 21 \left(i \sqrt{2} \cos(dx+c)^5 - 2i \sqrt{2} \cos(dx+c)^3 \sin(dx+c) - 2i \sqrt{2} \cos(dx+c)^3\right) \mathrm{weierstrassZeta}(-4, 0, \mathrm{weierstrassPInverse}(-4, 0, \cos(dx+c) - i \sin(dx+c))) + 2 \left(42 \cos(dx+c)^4 - 28 \cos(dx+c)^2 + (21 \cos(dx+c)^4 - 35 \cos(dx+c)^2 - 9) \sin(dx+c) - 4\right) \sqrt{\cos(dx+c)}}{65 \left(a^2 d \cos(dx+c)^5 e^{\frac{7}{2}} - 2 a^2 d \cos(dx+c)^3 e^{\frac{7}{2}} \sin(dx+c) - 2 a^2 d \cos(dx+c)^3 e^{\frac{7}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/65*(21*(-I*sqrt(2)*cos(d*x + c)^5 + 2*I*sqrt(2)*cos(d*x + c)^3*sin(d*x +
c) + 2*I*sqrt(2)*cos(d*x + c)^3)*weierstrassZeta(-4, 0, weierstrassPInverse
(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 21*(I*sqrt(2)*cos(d*x + c)^5 - 2*
I*sqrt(2)*cos(d*x + c)^3*sin(d*x + c) - 2*I*sqrt(2)*cos(d*x + c)^3)*weierst
rassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)))
+ 2*(42*cos(d*x + c)^4 - 28*cos(d*x + c)^2 + (21*cos(d*x + c)^4 - 35*cos(d*
x + c)^2 - 9)*sin(d*x + c) - 4)*sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^5*e
^(7/2) - 2*a^2*d*cos(d*x + c)^3*e^(7/2)*sin(d*x + c) - 2*a^2*d*cos(d*x + c)
^3*e^(7/2))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(7/2)/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3063 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(e^(-7/2)/((a*sin(d*x + c) + a)^2*cos(d*x + c)^(7/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(e\,\cos\left(c+d\,x\right)\right)}^{7/2}\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cos(c + d*x))^(7/2)*(a + a*sin(c + d*x))^2),x)
```

```
[Out] int(1/((e*cos(c + d*x))^(7/2)*(a + a*sin(c + d*x))^2), x)
```

3.253 $\int \frac{(e\cos(c+dx))^{15/2}}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=169

$$\frac{26e^3(e\cos(c+dx))^{9/2}}{45a^3d}+\frac{26e^8\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21a^3d\sqrt{e\cos(c+dx)}}+\frac{26e^7\sqrt{e\cos(c+dx)}\,\sin(c+dx)}{21a^3d}+\frac{26e^5(e\cos(c$$

```
[Out] 26/45*e^3*(e*cos(d*x+c))^(9/2)/a^3/d+26/35*e^5*(e*cos(d*x+c))^(5/2)*sin(d*x
+c)/a^3/d+4/5*e*(e*cos(d*x+c))^(13/2)/a/d/(a+a*sin(d*x+c))^2+26/21*e^8*(cos
(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^
(1/2))*cos(d*x+c)^(1/2)/a^3/d/(e*cos(d*x+c))^(1/2)+26/21*e^7*sin(d*x+c)*(e*
cos(d*x+c))^(1/2)/a^3/d
```

Rubi [A]
time = 0.12, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$,
Rules used = {2759, 2761, 2715, 2721, 2720}

$$\frac{26e^8\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21a^3d\sqrt{e\cos(c+dx)}}+\frac{26e^7\sin(c+dx)\sqrt{e\cos(c+dx)}}{21a^3d}+\frac{26e^5\sin(c+dx)(e\cos(c+dx))^{5/2}}{35a^3d}+\frac{26e^3(e\cos(c+dx))^{9/2}}{45a^3d}+\frac{4e(e\cos(c+dx))^{13/2}}{5ad(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(15/2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (26*e^3*(e*Cos[c + d*x])^(9/2))/(45*a^3*d) + (26*e^8*Sqrt[Cos[c + d*x]]*Ell
ipticF[(c + d*x)/2, 2])/(21*a^3*d*Sqrt[e*Cos[c + d*x]]) + (26*e^7*Sqrt[e*Co
s[c + d*x]]*Sin[c + d*x])/(21*a^3*d) + (26*e^5*(e*Cos[c + d*x])^(5/2)*Sin[c
 + d*x])/(35*a^3*d) + (4*e*(e*Cos[c + d*x])^(13/2))/(5*a*d*(a + a*Sin[c + d
*x])^2)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
```

```
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{15/2}}{(a+a\sin(c+dx))^3}\,dx = \frac{4e(e\cos(c+dx))^{13/2}}{5ad(a+a\sin(c+dx))^2}+\frac{(13e^2)\int \frac{(e\cos(c+dx))^{11/2}}{a+a\sin(c+dx)}\,dx}{5a^2}$$
$$= \frac{26e^3(e\cos(c+dx))^{9/2}}{45a^3d}+\frac{4e(e\cos(c+dx))^{13/2}}{5ad(a+a\sin(c+dx))^2}+\frac{(13e^4)\int (e\cos(c+dx))^{7/2}}{5a^3}$$
$$= \frac{26e^3(e\cos(c+dx))^{9/2}}{45a^3d}+\frac{26e^5(e\cos(c+dx))^{5/2}\sin(c+dx)}{35a^3d}+\frac{4e(e\cos(c+d}{5ad(a+a\sin(c}$$
$$= \frac{26e^3(e\cos(c+dx))^{9/2}}{45a^3d}+\frac{26e^7\sqrt{e\cos(c+dx)}\,\sin(c+dx)}{21a^3d}+\frac{26e^5(e\cos(c+dx}{35a}$$
$$= \frac{26e^3(e\cos(c+dx))^{9/2}}{45a^3d}+\frac{26e^7\sqrt{e\cos(c+dx)}\,\sin(c+dx)}{21a^3d}+\frac{26e^5(e\cos(c+dx}{35a}$$
$$= \frac{26e^3(e\cos(c+dx))^{9/2}}{45a^3d}+\frac{26e^8\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21a^3d\sqrt{e\cos(c+dx)}}+\frac{26e^7\sqrt{e\cos(c}{2}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.37, size = 66, normalized size = 0.39

$$-\frac{4\sqrt[4]{2}\,(e\cos(c+dx))^{17/2}\,{}_2F_1\left(-\frac{1}{4},\frac{17}{4};\frac{21}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{17a^3de(1+\sin(c+dx))^{17/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(15/2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-4*2^(1/4)*(e*Cos[c + d*x])^(17/2)*Hypergeometric2F1[-1/4, 17/4, 21/4, (1
- Sin[c + d*x])/2])/(17*a^3*d*e*(1 + Sin[c + d*x])^(17/4))
```

Maple [A]
time = 2.78, size = 251, normalized size = 1.49

method	result
default	$-\frac{2e^{8}\left(-1120\left(\sin^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2160\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2800\left(\sin^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3240\left(\sin^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-784\left(\sin\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(15/2)/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/315/a^3/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^8*(-112
0*sin(1/2*d*x+1/2*c)^11-2160*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+2800*s
in(1/2*d*x+1/2*c)^9+3240*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-784*sin(1/
2*d*x+1/2*c)^7-840*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-1624*sin(1/2*d*x
+1/2*c)^5+195*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-120*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x
+1/2*c)+1162*sin(1/2*d*x+1/2*c)^3-217*sin(1/2*d*x+1/2*c))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(15/2)/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] e^(15/2)*integrate(cos(d*x + c)^(15/2)/(a*sin(d*x + c) + a)^3, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 119, normalized size = 0.70

$$\frac{-195i\sqrt{2}\,e^{\frac{15}{2}}\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+195i\sqrt{2}\,e^{\frac{15}{2}}\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-2\left(35\cos(dx+c)^{4}e^{\frac{15}{2}}-252\cos(dx+c)^{2}e^{\frac{15}{2}}+15\left(9\cos(dx+c)^{2}e^{\frac{15}{2}}-13e^{\frac{15}{2}}\right)\sin(dx+c)\right)\sqrt{\cos(dx+c)}}{315\,a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(15/2)/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/315*(-195*I*sqrt(2)*e^(15/2)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*
sin(d*x + c)) + 195*I*sqrt(2)*e^(15/2)*weierstrassPInverse(-4, 0, cos(d*x +
```

```
 c) - I*sin(d*x + c)) - 2*(35*cos(d*x + c)^4*e^(15/2) - 252*cos(d*x + c)^2*
e^(15/2) + 15*(9*cos(d*x + c)^2*e^(15/2) - 13*e^(15/2))*sin(d*x + c))*sqrt(
cos(d*x + c)))/(a^3*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(15/2)/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(15/2)/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(15/2)*e^(15/2)/(a*sin(d*x + c) + a)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,\cos\left(c+d\,x\right))^{15/2}}{(a+a\,\sin\left(c+d\,x\right))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(15/2)/(a + a*sin(c + d*x))^3,x)
```

```
[Out] int((e*cos(c + d*x))^(15/2)/(a + a*sin(c + d*x))^3, x)
```

3.254 $\int \frac{(e\cos(c+dx))^{13/2}}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=138

$$\frac{22e^3(e\cos(c+dx))^{7/2}}{21a^3d}+\frac{22e^6\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^3d\sqrt{\cos(c+dx)}}+\frac{22e^5(e\cos(c+dx))^{3/2}\sin(c+dx)}{15a^3d}+\frac{4e(e\cos}{3ad(a+}$$

```
[Out] 22/21*e^3*(e*cos(d*x+c))^(7/2)/a^3/d+22/15*e^5*(e*cos(d*x+c))^(3/2)*sin(d*x
+c)/a^3/d+4/3*e*(e*cos(d*x+c))^(11/2)/a/d/(a+a*sin(d*x+c))^2+22/5*e^6*(cos(
1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(
1/2))*(e*cos(d*x+c))^(1/2)/a^3/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2759, 2761, 2715, 2721, 2719}

$$\frac{22e^6E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5a^3d\sqrt{\cos(c+dx)}}+\frac{22e^5\sin(c+dx)(e\cos(c+dx))^{3/2}}{15a^3d}+\frac{22e^3(e\cos(c+dx))^{7/2}}{21a^3d}+\frac{4e(e\cos(c+dx))^{11/2}}{3ad(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(13/2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (22*e^3*(e*Cos[c + d*x])^(7/2))/(21*a^3*d) + (22*e^6*Sqrt[e*Cos[c + d*x]]*E
llipticE[(c + d*x)/2, 2])/(5*a^3*d*Sqrt[Cos[c + d*x]]) + (22*e^5*(e*Cos[c +
 d*x])^(3/2)*Sin[c + d*x])/(15*a^3*d) + (4*e*(e*Cos[c + d*x])^(11/2))/(3*a*
d*(a + a*Sin[c + d*x])^2)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{13/2}}{(a+a\sin(c+dx))^3}\,dx = \frac{4e(e\cos(c+dx))^{11/2}}{3ad(a+a\sin(c+dx))^2} + \frac{(11e^2)\int \frac{(e\cos(c+dx))^{9/2}}{a+a\sin(c+dx)}\,dx}{3a^2}$$

$$= \frac{22e^3(e\cos(c+dx))^{7/2}}{21a^3d} + \frac{4e(e\cos(c+dx))^{11/2}}{3ad(a+a\sin(c+dx))^2} + \frac{(11e^4)\int(e\cos(c+dx))^{5/2}}{3a^3}$$

$$= \frac{22e^3(e\cos(c+dx))^{7/2}}{21a^3d} + \frac{22e^5(e\cos(c+dx))^{3/2}\sin(c+dx)}{15a^3d} + \frac{4e(e\cos(c+d}{3ad(a+a\sin(c}$$

$$= \frac{22e^3(e\cos(c+dx))^{7/2}}{21a^3d} + \frac{22e^5(e\cos(c+dx))^{3/2}\sin(c+dx)}{15a^3d} + \frac{4e(e\cos(c+d}{3ad(a+a\sin(c}$$

$$= \frac{22e^3(e\cos(c+dx))^{7/2}}{21a^3d} + \frac{22e^6\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^3d\sqrt{\cos(c+dx)}} + \frac{22e^5(e\cos(c}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.22, size = 66, normalized size = 0.48

$$-\frac{2\,2^{3/4}(e\cos(c+dx))^{15/2}\,{}_2F_1\left(\frac{1}{4},\frac{15}{4};\frac{19}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{15a^3de(1+\sin(c+dx))^{15/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(13/2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-2*2^(3/4)*(e*Cos[c + d*x])^(15/2)*Hypergeometric2F1[1/4, 15/4, 19/4, (1 -
 Sin[c + d*x])/2])/(15*a^3*d*e*(1 + Sin[c + d*x])^(15/4))
```

Maple [A]
time = 2.81, size = 216, normalized size = 1.57

method	result
default	$\frac{2e^{7}\left(-240\left(\sin^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-504\left(\sin^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+480\left(\sin^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+504\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+200\left(\sin^{5}\left(\frac{dx}{2}+\right.\right.\right.}{105a^{3}\sin}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(13/2)/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/105/a^3/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^7*(-240*
sin(1/2*d*x+1/2*c)^9-504*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+480*sin(1/
2*d*x+1/2*c)^7+504*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+200*sin(1/2*d*x+
1/2*c)^5+231*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-126*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+
1/2*c)-440*sin(1/2*d*x+1/2*c)^3+125*sin(1/2*d*x+1/2*c))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(13/2)/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] e^(13/2)*integrate(cos(d*x + c)^(13/2)/(a*sin(d*x + c) + a)^3, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 114, normalized size = 0.83

$$\frac{231i\sqrt{2}\,e^{\frac{13}{2}}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-231i\sqrt{2}\,e^{\frac{13}{2}}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))-2\left(15\cos(dx+c)^{3}e^{\frac{13}{2}}+63\cos(dx+c)e^{\frac{13}{2}}\sin(dx+c)-140\cos(dx+c)e^{\frac{13}{2}}\right)\sqrt{\cos(dx+c)}}{105\,a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(13/2)/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/105*(231*I*sqrt(2)*e^(13/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4
, 0, cos(d*x + c) + I*sin(d*x + c))) - 231*I*sqrt(2)*e^(13/2)*weierstrassZe
ta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(1
5*cos(d*x + c)^3*e^(13/2) + 63*cos(d*x + c)*e^(13/2)*sin(d*x + c) - 140*cos
(d*x + c)*e^(13/2))*sqrt(cos(d*x + c)))/(a^3*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(13/2)/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(13/2)/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(13/2)*e^(13/2)/(a*sin(d*x + c) + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,\cos(c+d\,x))^{13/2}}{(a+a\,\sin(c+d\,x))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(13/2)/(a + a*sin(c + d*x))^3,x)
```

```
[Out] int((e*cos(c + d*x))^(13/2)/(a + a*sin(c + d*x))^3, x)
```

3.255 $\int \frac{(e\cos(c+dx))^{11/2}}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=132

$$\frac{18e^3(e\cos(c+dx))^{5/2}}{5a^3d}+\frac{6e^6\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^3d\sqrt{e\cos(c+dx)}}+\frac{6e^5\sqrt{e\cos(c+dx)}\,\sin(c+dx)}{a^3d}+\frac{4e(e\cos(c+\dots}{ad(a+a\sin(c\dots}$$

```
[Out] 18/5*e^3*(e*cos(d*x+c))^(5/2)/a^3/d+4*e*(e*cos(d*x+c))^(9/2)/a/d/(a+a*sin(d
*x+c))^2+6*e^6*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(si
n(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/a^3/d/(e*cos(d*x+c))^(1/2)+6*e^5
*sin(d*x+c)*(e*cos(d*x+c))^(1/2)/a^3/d
```

Rubi [A]
time = 0.11, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2759, 2761, 2715, 2721, 2720}

$$\frac{6e^6\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^3d\sqrt{e\cos(c+dx)}}+\frac{6e^5\sin(c+dx)\sqrt{e\cos(c+dx)}}{a^3d}+\frac{18e^3(e\cos(c+dx))^{5/2}}{5a^3d}+\frac{4e(e\cos(c+dx))^{9/2}}{ad(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(11/2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (18*e^3*(e*Cos[c + d*x])^(5/2))/(5*a^3*d) + (6*e^6*Sqrt[Cos[c + d*x]]*Ellip
ticF[(c + d*x)/2, 2])/(a^3*d*Sqrt[e*Cos[c + d*x]]) + (6*e^5*Sqrt[e*Cos[c +
d*x]]*Sin[c + d*x])/(a^3*d) + (4*e*(e*Cos[c + d*x])^(9/2))/(a*d*(a + a*Sin[
c + d*x])^2)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{11/2}}{(a+a\sin(c+dx))^3}\,dx = \frac{4e(e\cos(c+dx))^{9/2}}{ad(a+a\sin(c+dx))^2} + \frac{(9e^2)\int \frac{(e\cos(c+dx))^{7/2}}{a+a\sin(c+dx)}\,dx}{a^2}$$

$$= \frac{18e^3(e\cos(c+dx))^{5/2}}{5a^3d} + \frac{4e(e\cos(c+dx))^{9/2}}{ad(a+a\sin(c+dx))^2} + \frac{(9e^4)\int (e\cos(c+dx))^{3/2}\,dx}{a^3}$$

$$= \frac{18e^3(e\cos(c+dx))^{5/2}}{5a^3d} + \frac{6e^5\sqrt{e\cos(c+dx)}\,\sin(c+dx)}{a^3d} + \frac{4e(e\cos(c+dx))}{ad(a+a\sin(c+d}$$

$$= \frac{18e^3(e\cos(c+dx))^{5/2}}{5a^3d} + \frac{6e^5\sqrt{e\cos(c+dx)}\,\sin(c+dx)}{a^3d} + \frac{4e(e\cos(c+dx))}{ad(a+a\sin(c+d}$$

$$= \frac{18e^3(e\cos(c+dx))^{5/2}}{5a^3d} + \frac{6e^6\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{a^3d\sqrt{e\cos(c+dx)}} + \frac{6e^5\sqrt{e\cos(c+}}{a}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.14, size = 66, normalized size = 0.50

$$-\frac{2\sqrt[4]{2}\,(e\cos(c+dx))^{13/2}\,{}_2F_1\left(\frac{3}{4},\frac{13}{4};\frac{17}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{13a^3de(1+\sin(c+dx))^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(11/2)/(a + a*Sin[c + d*x])^3,x]
```

[Out] (-2*2^(1/4)*(e*Cos[c + d*x])^(13/2)*Hypergeometric2F1[3/4, 13/4, 17/4, (1 - Sin[c + d*x])/2])/(13*a^3*d*e*(1 + Sin[c + d*x])^(13/4))

Maple [A]
time = 2.62, size = 181, normalized size = 1.37

method	result
default	$-\frac{2e^{6}\left(-8\left(\sin^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-20\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+12\left(\sin^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+15\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}{5a^{3}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^{2}\left(\frac{dx}{2}+\right.\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*cos(d*x+c))^(11/2)/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)

[Out] -2/5/a^3/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^6*(-8*sin(1/2*d*x+1/2*c)^7-20*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+12*sin(1/2*d*x+1/2*c)^5+15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+10*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+34*sin(1/2*d*x+1/2*c)^3-19*sin(1/2*d*x+1/2*c))/d

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*cos(d*x+c))^(11/2)/(a+a*sin(d*x+c))^3,x, algorithm="maxima")

[Out] e^(11/2)*integrate(cos(d*x + c)^(11/2)/(a*sin(d*x + c) + a)^3, x)

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 95, normalized size = 0.72

$$\frac{-15i\sqrt{2}\,e^{\frac{11}{2}}\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+15i\sqrt{2}\,e^{\frac{11}{2}}\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-2\left(\cos(dx+c)^{2}e^{\frac{11}{2}}+5e^{\frac{11}{2}}\sin(dx+c)-20e^{\frac{11}{2}}\right)\sqrt{\cos(dx+c)}}{5a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*cos(d*x+c))^(11/2)/(a+a*sin(d*x+c))^3,x, algorithm="fricas")

[Out] 1/5*(-15*I*sqrt(2)*e^(11/2)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 15*I*sqrt(2)*e^(11/2)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 2*(cos(d*x + c)^2*e^(11/2) + 5*e^(11/2)*sin(d*x + c) - 20*e^(11/2))*sqrt(cos(d*x + c)))/(a^3*d)

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(11/2)/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(11/2)/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(11/2)*e^(11/2)/(a*sin(d*x + c) + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,\cos(c+d\,x))^{11/2}}{(a+a\,\sin(c+d\,x))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(11/2)/(a + a*sin(c + d*x))^3,x)
```

```
[Out] int((e*cos(c + d*x))^(11/2)/(a + a*sin(c + d*x))^3, x)
```

3.256 $\int \frac{(e\cos(c+dx))^{9/2}}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=103

$$-\frac{14e^3(e\cos(c+dx))^{3/2}}{3a^3d}-\frac{14e^4\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^3d\sqrt{\cos(c+dx)}}-\frac{4e(e\cos(c+dx))^{7/2}}{ad(a+a\sin(c+dx))^2}$$

```
[Out] -14/3*e^3*(e*cos(d*x+c))^(3/2)/a^3/d-4*e*(e*cos(d*x+c))^(7/2)/a/d/(a+a*sin(
d*x+c))^2-14*e^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(
sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/a^3/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2759, 2761, 2721, 2719}

$$-\frac{14e^4E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{a^3d\sqrt{\cos(c+dx)}}-\frac{14e^3(e\cos(c+dx))^{3/2}}{3a^3d}-\frac{4e(e\cos(c+dx))^{7/2}}{ad(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(9/2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-14*e^3*(e*Cos[c + d*x])^(3/2))/(3*a^3*d) - (14*e^4*Sqrt[e*Cos[c + d*x]]*E
llipticE[(c + d*x)/2, 2])/(a^3*d*Sqrt[Cos[c + d*x]]) - (4*e*(e*Cos[c + d*x]
)^(7/2))/(a*d*(a + a*Sin[c + d*x])^2)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{9/2}}{(a+a\sin(c+dx))^3}\,dx = -\frac{4e(e\cos(c+dx))^{7/2}}{ad(a+a\sin(c+dx))^2} - \frac{(7e^2)\int \frac{(e\cos(c+dx))^{5/2}}{a+a\sin(c+dx)}\,dx}{a^2}$$
$$= -\frac{14e^3(e\cos(c+dx))^{3/2}}{3a^3d} - \frac{4e(e\cos(c+dx))^{7/2}}{ad(a+a\sin(c+dx))^2} - \frac{(7e^4)\int \sqrt{e\cos(c+dx)}\,d}{a^3}$$
$$= -\frac{14e^3(e\cos(c+dx))^{3/2}}{3a^3d} - \frac{4e(e\cos(c+dx))^{7/2}}{ad(a+a\sin(c+dx))^2} - \frac{\left(7e^4\sqrt{e\cos(c+dx)}\right)\int}{a^3\sqrt{\cos(c+}}$$
$$= -\frac{14e^3(e\cos(c+dx))^{3/2}}{3a^3d} - \frac{14e^4\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^3d\sqrt{\cos(c+dx)}} - \frac{4e(e\cos(}{ad(a+a\,\mathrm{s}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.11, size = 66, normalized size = 0.64

$$-\frac{2^{3/4}(e\cos(c+dx))^{11/2}\,{}_2F_1\left(\frac{5}{4},\frac{11}{4};\frac{15}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{11a^3de(1+\sin(c+dx))^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(9/2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/11*(2^(3/4)*(e*Cos[c + d*x])^(11/2)*Hypergeometric2F1[5/4, 11/4, 15/4, (
1 - Sin[c + d*x])/2])/(a^3*d*e*(1 + Sin[c + d*x])^(11/4))
```

Maple [A]
time = 3.67, size = 146, normalized size = 1.42

method	result
default	$-\frac{2\left(4\left(\sin^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+21\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-24\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}{3\sqrt{-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)e+e}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)a^3d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(9/2)/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/sin(1/2*d*x+1/2*c)/a^3*(4*sin(1/2*
d*x+1/2*c)^5+21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-24*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*
x+1/2*c)-4*sin(1/2*d*x+1/2*c)^3+13*sin(1/2*d*x+1/2*c))*e^5/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(9/2)/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] e^(9/2)*integrate(cos(d*x + c)^(9/2)/(a*sin(d*x + c) + a)^3, x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 204, normalized size = 1.98

$$-\frac{21\left(i\sqrt{2}\cos\left(dx+c\right)e^{\frac{9}{2}}+i\sqrt{2}e^{\frac{9}{2}}\sin\left(dx+c\right)+i\sqrt{2}e^{\frac{9}{2}}\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)))+21\left(-i\sqrt{2}\cos\left(dx+c\right)e^{\frac{9}{2}}-i\sqrt{2}e^{\frac{9}{2}}\sin\left(dx+c\right)-i\sqrt{2}e^{\frac{9}{2}}\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)))+2\left(\cos\left(dx+c\right)^{2}e^{\frac{9}{2}}+13\cos\left(dx+c\right)e^{\frac{9}{2}}+\left(\cos\left(dx+c\right)e^{\frac{9}{2}}-12e^{\frac{9}{2}}\right)\sin\left(dx+c\right)+12e^{\frac{9}{2}}\right)\sqrt{\cos\left(dx+c\right)}}{3\left(a^{3}d\cos\left(dx+c\right)+a^{3}d\sin\left(dx+c\right)+a^{3}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(9/2)/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/3*(21*(I*sqrt(2)*cos(d*x + c)*e^(9/2) + I*sqrt(2)*e^(9/2)*sin(d*x + c) +
 I*sqrt(2)*e^(9/2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d
*x + c) + I*sin(d*x + c))) + 21*(-I*sqrt(2)*cos(d*x + c)*e^(9/2) - I*sqrt(2
)*e^(9/2)*sin(d*x + c) - I*sqrt(2)*e^(9/2))*weierstrassZeta(-4, 0, weierstr
assPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(cos(d*x + c)^2*e^(9
/2) + 13*cos(d*x + c)*e^(9/2) + (cos(d*x + c)*e^(9/2) - 12*e^(9/2))*sin(d*x
 + c) + 12*e^(9/2))*sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c) + a^3*d*sin(d*x
 + c) + a^3*d)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(9/2)/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8857 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(9/2)/(a+a*sin(d*x+c))^3,x, algorithm="giac")
[Out] integrate(cos(d*x + c)^(9/2)*e^(9/2)/(a*sin(d*x + c) + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,\cos(c+d\,x))^{9/2}}{(a+a\,\sin(c+d\,x))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(9/2)/(a + a*sin(c + d*x))^3,x)
[Out] int((e*cos(c + d*x))^(9/2)/(a + a*sin(c + d*x))^3, x)
```

3.257 $\int \frac{(e\cos(c+dx))^{7/2}}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=107

$$-\frac{10e^3\sqrt{e\cos(c+dx)}}{3a^3d}-\frac{10e^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^3d\sqrt{e\cos(c+dx)}}-\frac{4e(e\cos(c+dx))^{5/2}}{3ad(a+a\sin(c+dx))^2}$$

```
[Out] -4/3*e*(e*cos(d*x+c))^(5/2)/a/d/(a+a*sin(d*x+c))^2-10/3*e^4*(cos(1/2*d*x+1/
2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(
d*x+c)^(1/2)/a^3/d/(e*cos(d*x+c))^(1/2)-10/3*e^3*(e*cos(d*x+c))^(1/2)/a^3/d
```

Rubi [A]
time = 0.09, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2759, 2761, 2721, 2720}

$$-\frac{10e^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^3d\sqrt{e\cos(c+dx)}}-\frac{10e^3\sqrt{e\cos(c+dx)}}{3a^3d}-\frac{4e(e\cos(c+dx))^{5/2}}{3ad(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(7/2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-10*e^3*Sqrt[e*Cos[c + d*x]])/(3*a^3*d) - (10*e^4*Sqrt[Cos[c + d*x]]*Ellip
ticF[(c + d*x)/2, 2])/(3*a^3*d*Sqrt[e*Cos[c + d*x]]) - (4*e*(e*Cos[c + d*x]
)^(5/2))/(3*a*d*(a + a*Sin[c + d*x])^2)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{7/2}}{(a+a\sin(c+dx))^3}\,dx = -\frac{4e(e\cos(c+dx))^{5/2}}{3ad(a+a\sin(c+dx))^2} - \frac{(5e^2)\int \frac{(e\cos(c+dx))^{3/2}}{a+a\sin(c+dx)}\,dx}{3a^2}$$
$$= -\frac{10e^3\sqrt{e\cos(c+dx)}}{3a^3d} - \frac{4e(e\cos(c+dx))^{5/2}}{3ad(a+a\sin(c+dx))^2} - \frac{(5e^4)\int \frac{1}{\sqrt{e\cos(c+dx)}}\,d}{3a^3}$$
$$= -\frac{10e^3\sqrt{e\cos(c+dx)}}{3a^3d} - \frac{4e(e\cos(c+dx))^{5/2}}{3ad(a+a\sin(c+dx))^2} - \frac{\left(5e^4\sqrt{\cos(c+dx)}\right)\int -}{3a^3\sqrt{e\cos(c+}}$$
$$= -\frac{10e^3\sqrt{e\cos(c+dx)}}{3a^3d} - \frac{10e^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^3d\sqrt{e\cos(c+dx)}} - \frac{4e(e\cos(c-}{3ad(a+a\sin}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.08, size = 66, normalized size = 0.62

$$-\frac{\sqrt[4]{2}\,(e\cos(c+dx))^{9/2}\,{}_2F_1\left(\frac{7}{4},\frac{9}{4};\frac{13}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{9a^3de(1+\sin(c+dx))^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(7/2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/9*(2^(1/4)*(e*Cos[c + d*x])^(9/2)*Hypergeometric2F1[7/4, 9/4, 13/4, (1 -
 Sin[c + d*x])/2])/(a^3*d*e*(1 + Sin[c + d*x])^(9/4))
```

Maple [A]
time = 4.32, size = 219, normalized size = 2.05

method	result
default	$\frac{2\left(10\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+12\left(\sin^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-5\right.}{3\left(2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/(2*sin(1/2*d*x+1/2*c)^2-1)/a^3/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c
)^2*e+e)^(1/2)*(10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+12*sin(1/2
*d*x+1/2*c)^5-5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x
+1/2*c)-12*sin(1/2*d*x+1/2*c)^3+7*sin(1/2*d*x+1/2*c))*e^4/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] e^(7/2)*integrate(cos(d*x + c)^(7/2)/(a*sin(d*x + c) + a)^3, x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 129, normalized size = 1.21

$$-\frac{5\left(-i\sqrt{2}\,e^{\frac{7}{2}}\sin\left(dx+c\right)-i\sqrt{2}\,e^{\frac{7}{2}}\right)\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+5\left(i\sqrt{2}\,e^{\frac{7}{2}}\sin\left(dx+c\right)+i\sqrt{2}\,e^{\frac{7}{2}}\right)\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right))+2\left(3\,e^{\frac{7}{2}}\sin\left(dx+c\right)+7\,e^{\frac{7}{2}}\right)\sqrt{\cos\left(dx+c\right)}}{3\left(a^{3}d\sin\left(dx+c\right)+a^{3}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/3*(5*(-I*sqrt(2)*e^(7/2)*sin(d*x + c) - I*sqrt(2)*e^(7/2))*weierstrassPI
nverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*(I*sqrt(2)*e^(7/2)*sin(d*x
 + c) + I*sqrt(2)*e^(7/2))*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(
d*x + c)) + 2*(3*e^(7/2)*sin(d*x + c) + 7*e^(7/2))*sqrt(cos(d*x + c)))/(a^3
*d*sin(d*x + c) + a^3*d)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(7/2)/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3878 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^3,x, algorithm="giac")
[Out] integrate(cos(d*x + c)^(7/2)*e^(7/2)/(a*sin(d*x + c) + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\cos(c+dx))^{7/2}}{(a+a\sin(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(7/2)/(a + a*sin(c + d*x))^3,x)
[Out] int((e*cos(c + d*x))^(7/2)/(a + a*sin(c + d*x))^3, x)
```

3.258 $\int \frac{(e\cos(c+dx))^{5/2}}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=118

$$\frac{6e^2\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^3d\sqrt{\cos(c+dx)}}-\frac{4e(e\cos(c+dx))^{3/2}}{5ad(a+a\sin(c+dx))^2}+\frac{6e(e\cos(c+dx))^{3/2}}{5d\left(a^3+a^3\sin(c+dx)\right)}$$

```
[Out] -4/5*e*(e*cos(d*x+c))^(3/2)/a/d/(a+a*sin(d*x+c))^2+6/5*e*(e*cos(d*x+c))^(3/
2)/d/(a^3+a^3*sin(d*x+c))+6/5*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+
1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/a^3/d/cos
(d*x+c)^(1/2)
```

Rubi [A]

time = 0.09, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2759, 2762, 2721, 2719}

$$\frac{6e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5a^3d\sqrt{\cos(c+dx)}}+\frac{6e(e\cos(c+dx))^{3/2}}{5d\left(a^3\sin(c+dx)+a^3\right)}-\frac{4e(e\cos(c+dx))^{3/2}}{5ad(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(5/2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (6*e^2*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*a^3*d*Sqrt[Cos[c
+ d*x]]) - (4*e*(e*Cos[c + d*x])^(3/2))/(5*a*d*(a + a*Sin[c + d*x])^2) + (6
*e*(e*Cos[c + d*x])^(3/2))/(5*d*(a^3 + a^3*Sin[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
```

```
NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2762

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[b*((g*Cos[e + f*x])^(p + 1)/(a*f*g*(p - 1)*(a + b*S
in[e + f*x]))), x] + Dist[p/(a*(p - 1)), Int[(g*Cos[e + f*x])^p, x], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] &&  !GeQ[p, 1] && Integer
Q[2*p]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{5/2}}{(a+a\sin(c+dx))^3}\,dx = -\frac{4e(e\cos(c+dx))^{3/2}}{5ad(a+a\sin(c+dx))^2} - \frac{(3e^2)\int \frac{\sqrt{e\cos(c+dx)}}{a+a\sin(c+dx)}\,dx}{5a^2}$$
$$= -\frac{4e(e\cos(c+dx))^{3/2}}{5ad(a+a\sin(c+dx))^2} + \frac{6e(e\cos(c+dx))^{3/2}}{5d\,(a^3+a^3\sin(c+dx))} + \frac{(3e^2)\int \sqrt{e\cos(c+dx}}{5a^3}$$
$$= -\frac{4e(e\cos(c+dx))^{3/2}}{5ad(a+a\sin(c+dx))^2} + \frac{6e(e\cos(c+dx))^{3/2}}{5d\,(a^3+a^3\sin(c+dx))} + \frac{\left(3e^2\sqrt{e\cos(c+dx)}\right)}{5a^3\sqrt{\cos(}}$$
$$= \frac{6e^2\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5a^3d\sqrt{\cos(c+dx)}} - \frac{4e(e\cos(c+dx))^{3/2}}{5ad(a+a\sin(c+dx))^2} + \frac{6e(e\cos(c}{5d\,(a^3+a^3 s}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.07, size = 66, normalized size = 0.56

$$-\frac{(e\cos(c+dx))^{7/2}\,{}_2F_1\left(\frac{7}{4},\frac{9}{4};\frac{11}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{7\sqrt[4]{2}\,a^3de(1+\sin(c+dx))^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(5/2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/7*((e*Cos[c + d*x])^(7/2)*Hypergeometric2F1[7/4, 9/4, 11/4, (1 - Sin[c +
 d*x])/2])/(2^(1/4)*a^3*d*e*(1 + Sin[c + d*x])^(7/4))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 329 vs. 2(130) = 260.
time = 6.09, size = 330, normalized size = 2.80

method	result

default	$\frac{2\left(12\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-24\left(\sin^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos(\cdots}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/5/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/a^3/sin(1/2*d*x+1/2*c
)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*
d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+20
*sin(1/2*d*x+1/2*c)^5+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*sin(1/2*d*x+1/2*c)^2*cos
(1/2*d*x+1/2*c)-20*sin(1/2*d*x+1/2*c)^3+sin(1/2*d*x+1/2*c))*e^3/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] e^(5/2)*integrate(cos(d*x + c)^(5/2)/(a*sin(d*x + c) + a)^3, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 295, normalized size = 2.50

$$\frac{3\left(i\sqrt{2}\cos(dx+c)^{2}e^{\frac{5}{2}}-i\sqrt{2}\cos(dx+c)e^{\frac{5}{2}}+\left(-i\sqrt{2}\cos(dx+c)e^{\frac{5}{2}}-2i\sqrt{2}e^{\frac{5}{2}}\right)\sin(dx+c)-2i\sqrt{2}e^{\frac{5}{2}}\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+3\left(-i\sqrt{2}\cos(dx+c)^{2}e^{\frac{5}{2}}+i\sqrt{2}\cos(dx+c)e^{\frac{5}{2}}+\left(i\sqrt{2}\cos(dx+c)e^{\frac{5}{2}}+2i\sqrt{2}e^{\frac{5}{2}}\right)\sin(dx+c)+2i\sqrt{2}e^{\frac{5}{2}}\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))-2\left(3\cos(dx+c)^{2}e^{\frac{5}{2}}+\cos(dx+c)e^{\frac{5}{2}}+\left(3\cos(dx+c)e^{\frac{5}{2}}+2e^{\frac{5}{2}}\right)\sin(dx+c)-2e^{\frac{5}{2}}\right)\sqrt{\cos(dx+c)}}{5\left(a^{3}d\cos(dx+c)^{2}-a^{3}d\cos(dx+c)-2a^{3}d-(a^{3}d\cos(dx+c)+2a^{3}d)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/5*(3*(I*sqrt(2)*cos(d*x + c)^2*e^(5/2) - I*sqrt(2)*cos(d*x + c)*e^(5/2) +
 (-I*sqrt(2)*cos(d*x + c)*e^(5/2) - 2*I*sqrt(2)*e^(5/2))*sin(d*x + c) - 2*I
*sqrt(2)*e^(5/2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x
 + c) + I*sin(d*x + c))) + 3*(-I*sqrt(2)*cos(d*x + c)^2*e^(5/2) + I*sqrt(2)
*cos(d*x + c)*e^(5/2) + (I*sqrt(2)*cos(d*x + c)*e^(5/2) + 2*I*sqrt(2)*e^(5/
2))*sin(d*x + c) + 2*I*sqrt(2)*e^(5/2))*weierstrassZeta(-4, 0, weierstrassP
Inverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(3*cos(d*x + c)^2*e^(5/2
) + cos(d*x + c)*e^(5/2) + (3*cos(d*x + c)*e^(5/2) + 2*e^(5/2))*sin(d*x + c
```

```
) - 2*e^(5/2))*sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^2 - a^3*d*cos(d*x +
c) - 2*a^3*d - (a^3*d*cos(d*x + c) + 2*a^3*d)*sin(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(5/2)/(a+a*sin(d*x+c))**3,x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^3,x, algorithm="giac")
[Out] integrate(cos(d*x + c)^(5/2)*e^(5/2)/(a*sin(d*x + c) + a)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(e\,\cos\left(c+d\,x\right)\right)}^{5/2}}{{\left(a+a\,\sin\left(c+d\,x\right)\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(5/2)/(a + a*sin(c + d*x))^3,x)
[Out] int((e*cos(c + d*x))^(5/2)/(a + a*sin(c + d*x))^3, x)
```

3.259 $\int \frac{(e\cos(c+dx))^{3/2}}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=118

$$-\frac{2e^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21a^3d\sqrt{e\cos(c+dx)}}-\frac{4e\sqrt{e\cos(c+dx)}}{7ad(a+a\sin(c+dx))^2}+\frac{2e\sqrt{e\cos(c+dx)}}{21d\left(a^3+a^3\sin(c+dx)\right)}$$

```
[Out] -2/21*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2
*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/a^3/d/(e*cos(d*x+c))^(1/2)-4/7*e*(e*c
os(d*x+c))^(1/2)/a/d/(a+a*sin(d*x+c))^2+2/21*e*(e*cos(d*x+c))^(1/2)/d/(a^3+
a^3*sin(d*x+c))
```

Rubi [A]
time = 0.09, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2759, 2762, 2721, 2720}

$$-\frac{2e^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21a^3d\sqrt{e\cos(c+dx)}}+\frac{2e\sqrt{e\cos(c+dx)}}{21d\left(a^3\sin(c+dx)+a^3\right)}-\frac{4e\sqrt{e\cos(c+dx)}}{7ad(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(3/2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-2*e^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*a^3*d*Sqrt[e*Cos[
c + d*x]]) - (4*e*Sqrt[e*Cos[c + d*x]])/(7*a*d*(a + a*Sin[c + d*x])^2) + (2
*e*Sqrt[e*Cos[c + d*x]])/(21*d*(a^3 + a^3*Sin[c + d*x]))
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
```

```
NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2762

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[b*((g*Cos[e + f*x])^(p + 1)/(a*f*g*(p - 1)*(a + b*S
in[e + f*x]))), x] + Dist[p/(a*(p - 1)), Int[(g*Cos[e + f*x])^p, x], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] &&  !GeQ[p, 1] && Integer
Q[2*p]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{3/2}}{(a+a\sin(c+dx))^3}\,dx = -\frac{4e\sqrt{e\cos(c+dx)}}{7ad(a+a\sin(c+dx))^2} - \frac{e^2\int \frac{1}{\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))}\,dx}{7a^2}$$

$$= -\frac{4e\sqrt{e\cos(c+dx)}}{7ad(a+a\sin(c+dx))^2} + \frac{2e\sqrt{e\cos(c+dx)}}{21d\,(a^3+a^3\sin(c+dx))} - \frac{e^2\int \frac{1}{\sqrt{e\cos(c+dx)}}}{21a^3}$$

$$= -\frac{4e\sqrt{e\cos(c+dx)}}{7ad(a+a\sin(c+dx))^2} + \frac{2e\sqrt{e\cos(c+dx)}}{21d\,(a^3+a^3\sin(c+dx))} - \frac{\left(e^2\sqrt{\cos(c+dx)}\right)\int}{21a^3\sqrt{e\cos}}$$

$$= -\frac{2e^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21a^3d\sqrt{e\cos(c+dx)}} - \frac{4e\sqrt{e\cos(c+dx)}}{7ad(a+a\sin(c+dx))^2} + \frac{2e\sqrt{e\cos}}{21d\,(a^3+a}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.07, size = 66, normalized size = 0.56

$$-\frac{(e\cos(c+dx))^{5/2}\,{}_2F_1\left(\frac{5}{4},\frac{11}{4};\frac{9}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{5\;2^{3/4}a^3de(1+\sin(c+dx))^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(3/2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/5*((e*Cos[c + d*x])^(5/2)*Hypergeometric2F1[5/4, 11/4, 9/4, (1 - Sin[c +
 d*x])/2])/(2^(3/4)*a^3*d*e*(1 + Sin[c + d*x])^(5/4))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 400 vs. 2(130) = 260.
time = 6.82, size = 401, normalized size = 3.40

method	result
default	$\frac{2\left(8\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ \text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-12\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)

[Out] 2/21/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/a^3/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*(8*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^6-12*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4+8*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+6*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-8*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-28*sin(1/2*d*x+1/2*c)^5-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-22*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+28*sin(1/2*d*x+1/2*c)^3+5*sin(1/2*d*x+1/2*c))*e^2/d

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^3,x, algorithm="maxima")

[Out] e^(3/2)*integrate(cos(d*x + c)^(3/2)/(a*sin(d*x + c) + a)^3, x)

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 171, normalized size = 1.45

$$-\frac{\left(-i\sqrt{2}\cos\left(dx+c\right)^2 e^{\frac{3}{2}}+2i\sqrt{2}e^{\frac{3}{2}}\sin\left(dx+c\right)+2i\sqrt{2}e^{\frac{3}{2}}\right)\text{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)+\left(i\sqrt{2}\cos\left(dx+c\right)^2 e^{\frac{3}{2}}-2i\sqrt{2}e^{\frac{3}{2}}\sin\left(dx+c\right)-2i\sqrt{2}e^{\frac{3}{2}}\right)\text{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)+2\left(e^{\frac{3}{2}}\sin\left(dx+c\right)-5e^{\frac{3}{2}}\right)\sqrt{\cos\left(dx+c\right)}}{21\left(a^3d\cos\left(dx+c\right)^2-2a^3d\sin\left(dx+c\right)-2a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^3,x, algorithm="fricas")

[Out] -1/21*((-I*sqrt(2)*cos(d*x + c)^2*e^(3/2) + 2*I*sqrt(2)*e^(3/2)*sin(d*x + c) + 2*I*sqrt(2)*e^(3/2))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + (I*sqrt(2)*cos(d*x + c)^2*e^(3/2) - 2*I*sqrt(2)*e^(3/2)*sin(d*x + c) - 2*I*sqrt(2)*e^(3/2))*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*(e^(3/2)*sin(d*x + c) - 5*e^(3/2))*sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^2 - 2*a^3*d*sin(d*x + c) - 2*a^3*d)

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(3/2)/(a+a*sin(d*x+c))**3,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 4849 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^3,x, algorithm="giac")
[Out] integrate(cos(d*x + c)^(3/2)*e^(3/2)/(a*sin(d*x + c) + a)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,\cos\left(c+d\,x\right))^{3/2}}{\left(a+a\,\sin\left(c+d\,x\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(3/2)/(a + a*sin(c + d*x))^3,x)
[Out] int((e*cos(c + d*x))^(3/2)/(a + a*sin(c + d*x))^3, x)
```

3.260 $\int \frac{\sqrt{e\cos(c+dx)}}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=153

$$-\frac{2\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15a^3d\sqrt{\cos(c+dx)}}-\frac{2(e\cos(c+dx))^{3/2}}{9de(a+a\sin(c+dx))^3}-\frac{2(e\cos(c+dx))^{3/2}}{15ade(a+a\sin(c+dx))^2}-\frac{2(e\cos(c+d}{15de\,(a^3+a^3\sin}$$

```
[Out] -2/9*(e*cos(d*x+c))^(3/2)/d/e/(a+a*sin(d*x+c))^3-2/15*(e*cos(d*x+c))^(3/2)/
a/d/e/(a+a*sin(d*x+c))^2-2/15*(e*cos(d*x+c))^(3/2)/d/e/(a^3+a^3*sin(d*x+c))
-2/15*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x
+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/a^3/d/cos(d*x+c)^(1/2)
```

Rubi [A]

time = 0.12, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2760, 2762, 2721, 2719}

$$-\frac{2(e\cos(c+dx))^{3/2}}{15de\,(a^3\sin(c+dx)+a^3)}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{15a^3d\sqrt{\cos(c+dx)}}-\frac{2(e\cos(c+dx))^{3/2}}{15ade(a\sin(c+dx)+a)^2}-\frac{2(e\cos(c+dx))^{3/2}}{9de(a\sin(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cos[c + d*x]]/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-2*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*a^3*d*Sqrt[Cos[c +
d*x]]) - (2*(e*Cos[c + d*x])^(3/2))/(9*d*e*(a + a*Sin[c + d*x])^3) - (2*(e*
Cos[c + d*x])^(3/2))/(15*a*d*e*(a + a*Sin[c + d*x])^2) - (2*(e*Cos[c + d*x]
)^(3/2))/(15*d*e*(a^3 + a^3*Sin[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2760

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*(2*m + p + 1))), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
```

```
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2762

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[b*((g*Cos[e + f*x])^(p + 1)/(a*f*g*(p - 1)*(a + b*S
in[e + f*x]))), x] + Dist[p/(a*(p - 1)), Int[(g*Cos[e + f*x])^p, x], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] &&  !GeQ[p, 1] && Integer
Q[2*p]
```

Rubi steps

$$\int \frac{\sqrt{e\cos(c+dx)}}{(a+a\sin(c+dx))^3}\,dx = -\frac{2(e\cos(c+dx))^{3/2}}{9de(a+a\sin(c+dx))^3}+\frac{\int \frac{\sqrt{e\cos(c+dx)}}{(a+a\sin(c+dx))^2}\,dx}{3a}$$

$$= -\frac{2(e\cos(c+dx))^{3/2}}{9de(a+a\sin(c+dx))^3}-\frac{2(e\cos(c+dx))^{3/2}}{15ade(a+a\sin(c+dx))^2}+\frac{\int \frac{\sqrt{e\cos(c+dx)}}{a+a\sin(c+dx)}\,d}{15a^2}$$

$$= -\frac{2(e\cos(c+dx))^{3/2}}{9de(a+a\sin(c+dx))^3}-\frac{2(e\cos(c+dx))^{3/2}}{15ade(a+a\sin(c+dx))^2}-\frac{2(e\cos(c+dx))^3}{15de\,(a^3+a^3\sin(c+}$$

$$= -\frac{2(e\cos(c+dx))^{3/2}}{9de(a+a\sin(c+dx))^3}-\frac{2(e\cos(c+dx))^{3/2}}{15ade(a+a\sin(c+dx))^2}-\frac{2(e\cos(c+dx))^3}{15de\,(a^3+a^3\sin(c+}$$

$$= -\frac{2\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15a^3d\sqrt{\cos(c+dx)}}-\frac{2(e\cos(c+dx))^{3/2}}{9de(a+a\sin(c+dx))^3}-\frac{2(e\cos(c}{15ade(a+a}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.04, size = 66, normalized size = 0.43

$$-\frac{(e\cos(c+dx))^{3/2}\,{}_2F_1\left(\frac{3}{4},\frac{13}{4};\frac{7}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{6\sqrt[4]{2}\,a^3de(1+\sin(c+dx))^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Cos[c + d*x]]/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/6*((e*Cos[c + d*x])^(3/2)*Hypergeometric2F1[3/4, 13/4, 7/4, (1 - Sin[c +
 d*x])/2])/(2^(1/4)*a^3*d*e*(1 + Sin[c + d*x])^(3/4))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 511 vs. 2(161) = 322.
time = 9.18, size = 512, normalized size = 3.35

method	result
default	$-\frac{2\left(48\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-96\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\ldots\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/45/(16*sin(1/2*d*x+1/2*c)^8-32*sin(1/2*d*x+1/2*c)^6+24*sin(1/2*d*x+1/2*c
)^4-8*sin(1/2*d*x+1/2*c)^2+1)/a^3/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)
^2*e+e)^(1/2)*(48*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^8-96*sin(1/2*
d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)-96*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/
2*c)^6+192*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+72*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*sin(1/2*d*x+1/2*c)^4-152*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-24*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+56*sin(1/2*d*x+1/2*c)^4*cos(1/2*
d*x+1/2*c)-36*sin(1/2*d*x+1/2*c)^5+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-48*sin(1/2*d*
x+1/2*c)^2*cos(1/2*d*x+1/2*c)+36*sin(1/2*d*x+1/2*c)^3+11*sin(1/2*d*x+1/2*c)
)*e/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] e^(1/2)*integrate(sqrt(cos(d*x + c))/(a*sin(d*x + c) + a)^3, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 408, normalized size = 2.67

$$-\frac{3\left(i\sqrt{2}\cos(dx+c)^3e^{\frac{1}{2}}+3i\sqrt{2}\cos(dx+c)^2e^{\frac{1}{2}}-2i\sqrt{2}\cos(dx+c)e^{\frac{1}{2}}+\left(i\sqrt{2}\cos(dx+c)^2e^{\frac{1}{2}}-2i\sqrt{2}\cos(dx+c)e^{\frac{1}{2}}-4i\sqrt{2}e^{\frac{1}{2}}\right)\sin(dx+c)-4i\sqrt{2}e^{\frac{1}{2}}\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+3\left(-i\sqrt{2}\cos(dx+c)^3e^{\frac{1}{2}}-3i\sqrt{2}\cos(dx+c)^2e^{\frac{1}{2}}+2i\sqrt{2}\cos(dx+c)e^{\frac{1}{2}}+\left(-i\sqrt{2}\cos(dx+c)^2e^{\frac{1}{2}}+2i\sqrt{2}\cos(dx+c)e^{\frac{1}{2}}+4i\sqrt{2}e^{\frac{1}{2}}\right)\sin(dx+c)+4i\sqrt{2}e^{\frac{1}{2}}\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+2\left(3\cos(dx+c)^3e^{\frac{1}{2}}-6\cos(dx+c)^2e^{\frac{1}{2}}-14\cos(dx+c)e^{\frac{1}{2}}-\left(3\cos(dx+c)^2e^{\frac{1}{2}}+9\cos(dx+c)e^{\frac{1}{2}}-5e^{\frac{1}{2}}\right)\sin(dx+c)-5e^{\frac{1}{2}}\right)\sqrt{\cos(dx+c)}}{45\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2-2a^3d\cos(dx+c)-4a^3d+\left(a^3d\cos(dx+c)^2-2a^3d\cos(dx+c)-4a^3d\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/45*(3*(I*sqrt(2)*cos(d*x + c)^3*e^(1/2) + 3*I*sqrt(2)*cos(d*x + c)^2*e^(
1/2) - 2*I*sqrt(2)*cos(d*x + c)*e^(1/2) + (I*sqrt(2)*cos(d*x + c)^2*e^(1/2)
```

```
 - 2*I*sqrt(2)*cos(d*x + c)*e^(1/2) - 4*I*sqrt(2)*e^(1/2))*sin(d*x + c) - 4
*I*sqrt(2)*e^(1/2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d
*x + c) + I*sin(d*x + c))) + 3*(-I*sqrt(2)*cos(d*x + c)^3*e^(1/2) - 3*I*sqr
t(2)*cos(d*x + c)^2*e^(1/2) + 2*I*sqrt(2)*cos(d*x + c)*e^(1/2) + (-I*sqrt(2
)*cos(d*x + c)^2*e^(1/2) + 2*I*sqrt(2)*cos(d*x + c)*e^(1/2) + 4*I*sqrt(2)*e
^(1/2))*sin(d*x + c) + 4*I*sqrt(2)*e^(1/2))*weierstrassZeta(-4, 0, weierstr
assPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(3*cos(d*x + c)^3*e^
(1/2) - 6*cos(d*x + c)^2*e^(1/2) - 14*cos(d*x + c)*e^(1/2) - (3*cos(d*x + c
)^2*e^(1/2) + 9*cos(d*x + c)*e^(1/2) - 5*e^(1/2))*sin(d*x + c) - 5*e^(1/2))
*sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 - 2*a^3
*d*cos(d*x + c) - 4*a^3*d + (a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c) -
4*a^3*d)*sin(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(1/2)/(a+a*sin(d*x+c))**3,x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c))^3,x, algorithm="giac")
[Out] integrate(sqrt(cos(d*x + c))*e^(1/2)/(a*sin(d*x + c) + a)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{e\,\cos\left(c+d\,x\right)}}{{\left(a+a\,\sin\left(c+d\,x\right)\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(1/2)/(a + a*sin(c + d*x))^3,x)
[Out] int((e*cos(c + d*x))^(1/2)/(a + a*sin(c + d*x))^3, x)
```

3.261 $\int \frac{1}{\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=153

$$\frac{10\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{77a^3d\sqrt{e\cos(c+dx)}}-\frac{2\sqrt{e\cos(c+dx)}}{11de(a+a\sin(c+dx))^3}-\frac{10\sqrt{e\cos(c+dx)}}{77ade(a+a\sin(c+dx))^2}-\frac{10\sqrt{e\cos(c+dx)}}{77de\left(a^3+a^3\sin(c+dx)\right)}$$

```
[Out] 10/77*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x
+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/a^3/d/(e*cos(d*x+c))^(1/2)-2/11*(e*cos(d*
x+c))^(1/2)/d/e/(a+a*sin(d*x+c))^3-10/77*(e*cos(d*x+c))^(1/2)/a/d/e/(a+a*si
n(d*x+c))^2-10/77*(e*cos(d*x+c))^(1/2)/d/e/(a^3+a^3*sin(d*x+c))
```

Rubi [A]
time = 0.13, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2760, 2762, 2721, 2720}

$$-\frac{10\sqrt{e\cos(c+dx)}}{77de\left(a^3\sin(c+dx)+a^3\right)}+\frac{10\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{77a^3d\sqrt{e\cos(c+dx)}}-\frac{10\sqrt{e\cos(c+dx)}}{77ade(a\sin(c+dx)+a)^2}-\frac{2\sqrt{e\cos(c+dx)}}{11de(a\sin(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])^3),x]
```

```
[Out] (10*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(77*a^3*d*Sqrt[e*Cos[c +
d*x]]) - (2*Sqrt[e*Cos[c + d*x]])/(11*d*e*(a + a*Sin[c + d*x])^3) - (10*Sqr
t[e*Cos[c + d*x]])/(77*a*d*e*(a + a*Sin[c + d*x])^2) - (10*Sqrt[e*Cos[c + d
*x]])/(77*d*e*(a^3 + a^3*Sin[c + d*x]))
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2760

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*(2*m + p + 1))), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
```

```
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2762

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[b*((g*Cos[e + f*x])^(p + 1)/(a*f*g*(p - 1)*(a + b*S
in[e + f*x]))), x] + Dist[p/(a*(p - 1)), Int[(g*Cos[e + f*x])^p, x], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] &&  !GeQ[p, 1] && Integer
Q[2*p]
```

Rubi steps

$$\int \frac{1}{\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^3}\,dx = -\frac{2\sqrt{e\cos(c+dx)}}{11de(a+a\sin(c+dx))^3} + \frac{5\int \frac{1}{\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))}}{11a}$$

$$= -\frac{2\sqrt{e\cos(c+dx)}}{11de(a+a\sin(c+dx))^3} - \frac{10\sqrt{e\cos(c+dx)}}{77ade(a+a\sin(c+dx))^2} + \frac{15}{}$$

$$= -\frac{2\sqrt{e\cos(c+dx)}}{11de(a+a\sin(c+dx))^3} - \frac{10\sqrt{e\cos(c+dx)}}{77ade(a+a\sin(c+dx))^2} - \frac{}{77}$$

$$= -\frac{2\sqrt{e\cos(c+dx)}}{11de(a+a\sin(c+dx))^3} - \frac{10\sqrt{e\cos(c+dx)}}{77ade(a+a\sin(c+dx))^2} - \frac{}{77}$$

$$= \frac{10\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{77a^3d\sqrt{e\cos(c+dx)}} - \frac{2\sqrt{e\cos(c+dx)}}{11de(a+a\sin(c+dx))^3}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.05, size = 66, normalized size = 0.43

$$-\frac{\sqrt{e\cos(c+dx)}\,{}_2F_1\left(\frac{1}{4},\frac{15}{4};\frac{5}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{2\,2^{3/4}a^3de\sqrt[4]{1+\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])^3),x]
```

```
[Out] -1/2*(Sqrt[e*Cos[c + d*x]]*Hypergeometric2F1[1/4, 15/4, 5/4, (1 - Sin[c + d
*x])/2])/(2^(3/4)*a^3*d*e*(1 + Sin[c + d*x])^(1/4))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 579 vs. $2(161) = 322$.
time = 10.26, size = 580, normalized size = 3.79

method	result
default	$-\frac{2\left(160\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-400\,\mathrm{EllipticF}\left(\cos\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(d*x+c))^3/(e*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/77/(32*sin(1/2*d*x+1/2*c)^10-80*sin(1/2*d*x+1/2*c)^8+80*sin(1/2*d*x+1/2*
c)^6-40*sin(1/2*d*x+1/2*c)^4+10*sin(1/2*d*x+1/2*c)^2-1)/a^3/sin(1/2*d*x+1/2
*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*(160*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1
/2*d*x+1/2*c)^10-400*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^8+160*sin(
1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+400*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d
*x+1/2*c)^6-320*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-200*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2))*sin(1/2*d*x+1/2*c)^4+264*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+
50*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-104*sin(1/2*d*x+1/2*c)^4*c
os(1/2*d*x+1/2*c)+44*sin(1/2*d*x+1/2*c)^5-5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+72*sin
(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-44*sin(1/2*d*x+1/2*c)^3-17*sin(1/2*d*x
+1/2*c))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))^3/(e*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] e^(-1/2)*integrate(1/((a*sin(d*x + c) + a)^3*sqrt(cos(d*x + c))), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 221, normalized size = 1.44

$$-\frac{5\left(3i\sqrt{2}\cos(dx+c)^2+\left(i\sqrt{2}\cos(dx+c)^2-4i\sqrt{2}\right)\sin(dx+c)-4i\sqrt{2}\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5\left(-3i\sqrt{2}\cos(dx+c)^2+\left(-i\sqrt{2}\cos(dx+c)^2+4i\sqrt{2}\right)\sin(dx+c)+4i\sqrt{2}\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+2\left(5\cos(dx+c)^2-15\sin(dx+c)-22\right)\sqrt{\cos(dx+c)}}{77\left(3a^3d\cos(dx+c)^2e^{\frac{1}{2}}-4a^3de^{\frac{1}{2}}+\left(a^3d\cos(dx+c)^2e^{\frac{1}{2}}-4a^3de^{\frac{1}{2}}\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sin(d*x+c))^3/(e*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] -1/77*(5*(3*I*sqrt(2)*cos(d*x + c)^2 + (I*sqrt(2)*cos(d*x + c)^2 - 4*I*sqrt(2))*sin(d*x + c) - 4*I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*(-3*I*sqrt(2)*cos(d*x + c)^2 + (-I*sqrt(2)*cos(d*x + c)^2 + 4*I*sqrt(2))*sin(d*x + c) + 4*I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*(5*cos(d*x + c)^2 - 15*sin(d*x + c) - 22)*sqrt(cos(d*x + c)))/(3*a^3*d*cos(d*x + c)^2*e^(1/2) - 4*a^3*d*e^(1/2) + (a^3*d*cos(d*x + c)^2*e^(1/2) - 4*a^3*d*e^(1/2))*sin(d*x + c))

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sin(d*x+c))**3/(e*cos(d*x+c))**(1/2),x)

[Out] Timed out

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sin(d*x+c))^3/(e*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(e^(-1/2)/((a*sin(d*x + c) + a)^3*sqrt(cos(d*x + c))), x)

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{e\,\cos\left(c+d\,x\right)}\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((e*cos(c + d*x))^(1/2)*(a + a*sin(c + d*x))^3),x)

[Out] int(1/((e*cos(c + d*x))^(1/2)*(a + a*sin(c + d*x))^3), x)

3.262 $\int \frac{1}{(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=187

$$-\frac{14\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{39a^3de^2\sqrt{\cos(c+dx)}}+\frac{14\sin(c+dx)}{39a^3de\sqrt{e\cos(c+dx)}}-\frac{2}{13de\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^3}-$$

```
[Out] 14/39*sin(d*x+c)/a^3/d/e/(e*cos(d*x+c))^(1/2)-2/13/d/e/(a+a*sin(d*x+c))^3/(
e*cos(d*x+c))^(1/2)-14/117/a/d/e/(a+a*sin(d*x+c))^2/(e*cos(d*x+c))^(1/2)-14
/117/d/e/(a^3+a^3*sin(d*x+c))/(e*cos(d*x+c))^(1/2)-14/39*(cos(1/2*d*x+1/2*c
)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(
d*x+c))^(1/2)/a^3/d/e^2/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2760, 2762, 2716, 2721, 2719}

$$-\frac{14E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{39a^3de^2\sqrt{\cos(c+dx)}}+\frac{14\sin(c+dx)}{39a^3de\sqrt{e\cos(c+dx)}}-\frac{14}{117de\left(a^3\sin(c+dx)+a^3\right)\sqrt{e\cos(c+dx)}}-\frac{14}{117ade(a\sin(c+dx)+a)^2\sqrt{e\cos(c+dx)}}-\frac{2}{13de(a\sin(c+dx)+a)^3\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x])^3),x]
```

```
[Out] (-14*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(39*a^3*d*e^2*Sqrt[Cos
[c + d*x]]) + (14*Sin[c + d*x])/(39*a^3*d*e*Sqrt[e*Cos[c + d*x]]) - 2/(13*d
*e*Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])^3) - 14/(117*a*d*e*Sqrt[e*Cos[
c + d*x]]*(a + a*Sin[c + d*x])^2) - 14/(117*d*e*Sqrt[e*Cos[c + d*x]]*(a^3 +
 a^3*Sin[c + d*x]))
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
```

```
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2760

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*(2*m + p + 1))), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2762

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[b*((g*Cos[e + f*x])^(p + 1)/(a*f*g*(p - 1)*(a + b*S
in[e + f*x]))), x] + Dist[p/(a*(p - 1)), Int[(g*Cos[e + f*x])^p, x], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] &&  !GeQ[p, 1] && Integer
Q[2*p]
```

Rubi steps

$$\int \frac{1}{(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^3}\,dx = -\frac{2}{13de\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^3} + \frac{7\int \frac{}{(e\cos(c+dx))^3}}{}$$

$$= -\frac{2}{13de\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^3} - \frac{}{117ade\sqrt{e\cos}}$$

$$= -\frac{2}{13de\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^3} - \frac{}{117ade\sqrt{e\cos}}$$

$$= \frac{14\sin(c+dx)}{39a^3de\sqrt{e\cos(c+dx)}} - \frac{2}{13de\sqrt{e\cos(c+dx)}\,(a+a\sin(c}$$

$$= \frac{14\sin(c+dx)}{39a^3de\sqrt{e\cos(c+dx)}} - \frac{2}{13de\sqrt{e\cos(c+dx)}\,(a+a\sin(c}$$

$$= -\frac{14\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{39a^3de^2\sqrt{\cos(c+dx)}} + \frac{14\sin(c+dx)}{39a^3de\sqrt{e\cos(c+d}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.07, size = 66, normalized size = 0.35

$$\frac{{}_2F_1\left(-\frac{1}{4},\frac{17}{4};\frac{3}{4};\frac{1}{2}(1-\sin(c+dx))\right)\sqrt[4]{1+\sin(c+dx)}}{4\sqrt[4]{2}\,a^3de\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x])^3),x]
```

```
[Out] (Hypergeometric2F1[-1/4, 17/4, 3/4, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x]
)^(1/4))/(4*2^(1/4)*a^3*d*e*Sqrt[e*Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 695 vs. $2(191) = 382$.
time = 12.74, size = 696, normalized size = 3.72

method	result	size
default	Expression too large to display	696

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/117/(64*sin(1/2*d*x+1/2*c)^12-192*sin(1/2*d*x+1/2*c)^10+240*sin(1/2*d*x+
1/2*c)^8-160*sin(1/2*d*x+1/2*c)^6+60*sin(1/2*d*x+1/2*c)^4-12*sin(1/2*d*x+1/
2*c)^2+1)/a^3/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/e*(134
4*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^12-2688*sin(1/2*d*x+1/2*c)^14
*cos(1/2*d*x+1/2*c)-4032*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^10+806
4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12+5040*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*si
n(1/2*d*x+1/2*c)^8-10304*sin(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)-3360*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^6+7168*cos(1/2*d*x+1/2*c)*sin(1/2*d
*x+1/2*c)^8+1260*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-2896*sin(1/2
*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-252*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1
/2*c)^2+656*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-52*sin(1/2*d*x+1/2*c)^5
+21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2))-138*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+52
*sin(1/2*d*x+1/2*c)^3+23*sin(1/2*d*x+1/2*c))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] e^(-3/2)*integrate(1/((a*sin(d*x + c) + a)^3*cos(d*x + c)^(3/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 283, normalized size = 1.51

$$-\frac{21\left(3i\sqrt{2}\cos\left(dx+c\right)^{3}+\left(i\sqrt{2}\cos\left(dx+c\right)^{3}-4i\sqrt{2}\cos\left(dx+c\right)\right)\sin\left(dx+c\right)-4i\sqrt{2}\cos\left(dx+c\right)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)))+21\left(-3i\sqrt{2}\cos\left(dx+c\right)^{3}+\left(-i\sqrt{2}\cos\left(dx+c\right)^{3}+4i\sqrt{2}\cos\left(dx+c\right)\right)\sin\left(dx+c\right)+4i\sqrt{2}\cos\left(dx+c\right)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)))+2\left(21\cos\left(dx+c\right)^{4}-98\cos\left(dx+c\right)^{2}-63\left(\cos\left(dx+c\right)^{2}-1\right)\sin\left(dx+c\right)+54\right)\sqrt{\cos\left(dx+c\right)}}{117\left(3a^{3}d\cos\left(dx+c\right)^{3}e^{\frac{3}{2}}-4a^{3}d\cos\left(dx+c\right)e^{\frac{3}{2}}+\left(a^{3}d\cos\left(dx+c\right)^{3}e^{\frac{3}{2}}-4a^{3}d\cos\left(dx+c\right)e^{\frac{3}{2}}\right)\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
[Out] -1/117*(21*(3*I*sqrt(2)*cos(d*x + c)^3 + (I*sqrt(2)*cos(d*x + c)^3 - 4*I*sq
rt(2)*cos(d*x + c))*sin(d*x + c) - 4*I*sqrt(2)*cos(d*x + c))*weierstrassZet
a(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 21*(-
3*I*sqrt(2)*cos(d*x + c)^3 + (-I*sqrt(2)*cos(d*x + c)^3 + 4*I*sqrt(2)*cos(d
*x + c))*sin(d*x + c) + 4*I*sqrt(2)*cos(d*x + c))*weierstrassZeta(-4, 0, we
ierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(21*cos(d*x +
c)^4 - 98*cos(d*x + c)^2 - 63*(cos(d*x + c)^2 - 1)*sin(d*x + c) + 54)*sqrt(
cos(d*x + c)))/(3*a^3*d*cos(d*x + c)^3*e^(3/2) - 4*a^3*d*cos(d*x + c)*e^(3/
2) + (a^3*d*cos(d*x + c)^3*e^(3/2) - 4*a^3*d*cos(d*x + c)*e^(3/2))*sin(d*x
+ c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(3/2)/(a+a*sin(d*x+c))**3,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3065 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^3,x, algorithm="giac")
[Out] integrate(e^(-3/2)/((a*sin(d*x + c) + a)^3*cos(d*x + c)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(e\,\cos\left(c+d\,x\right)\right)}^{3/2}\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cos(c + d*x))^(3/2)*(a + a*sin(c + d*x))^3),x)
[Out] int(1/((e*cos(c + d*x))^(3/2)*(a + a*sin(c + d*x))^3), x)
```

3.263 $\int \frac{(e\cos(c+dx))^{15/2}}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=180

$$\frac{78e^8\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7a^4d\sqrt{e\cos(c+dx)}}+\frac{78e^7\sqrt{e\cos(c+dx)}\,\sin(c+dx)}{7a^4d}+\frac{234e^5(e\cos(c+dx))^{5/2}\sin(c+dx)}{35a^4d}+$$

```
[Out] 234/35*e^5*(e*cos(d*x+c))^(5/2)*sin(d*x+c)/a^4/d+4*e*(e*cos(d*x+c))^(13/2)/
a/d/(a+a*sin(d*x+c))^3+52/5*e^3*(e*cos(d*x+c))^(9/2)/d/(a^4+a^4*sin(d*x+c))
+78/7*e^8*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2
*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/a^4/d/(e*cos(d*x+c))^(1/2)+78/7*e^7*s
in(d*x+c)*(e*cos(d*x+c))^(1/2)/a^4/d
```

Rubi [A]
time = 0.13, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2759, 2715, 2721, 2720}

$$\frac{78e^8\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7a^4d\sqrt{e\cos(c+dx)}}+\frac{78e^7\sin(c+dx)\sqrt{e\cos(c+dx)}}{7a^4d}+\frac{234e^5\sin(c+dx)(e\cos(c+dx))^{5/2}}{35a^4d}+\frac{52e^3(e\cos(c+dx))^{9/2}}{5d\left(a^4\sin(c+dx)+a^4\right)}+\frac{4e(e\cos(c+dx))^{13/2}}{ad(a\sin(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(15/2)/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (78*e^8*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(7*a^4*d*Sqrt[e*Cos[c
 + d*x]]) + (78*e^7*Sqrt[e*Cos[c + d*x]]*Sin[c + d*x])/(7*a^4*d) + (234*e^5
*(e*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(35*a^4*d) + (4*e*(e*Cos[c + d*x])^(1
3/2))/(a*d*(a + a*Sin[c + d*x])^3) + (52*e^3*(e*Cos[c + d*x])^(9/2))/(5*d*(
a^4 + a^4*Sin[c + d*x]))
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
```

```
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{(e\cos(c+dx))^{15/2}}{(a+a\sin(c+dx))^4}\,dx &= \frac{4e(e\cos(c+dx))^{13/2}}{ad(a+a\sin(c+dx))^3}+\frac{(13e^2)\int \frac{(e\cos(c+dx))^{11/2}}{(a+a\sin(c+dx))^2}\,dx}{a^2}\\ &= \frac{4e(e\cos(c+dx))^{13/2}}{ad(a+a\sin(c+dx))^3}+\frac{52e^3(e\cos(c+dx))^{9/2}}{5d\,(a^4+a^4\sin(c+dx))}+\frac{(117e^4)\int(e\cos(c+dx))^{7}}{5a^4}\\ &= \frac{234e^5(e\cos(c+dx))^{5/2}\sin(c+dx)}{35a^4d}+\frac{4e(e\cos(c+dx))^{13/2}}{ad(a+a\sin(c+dx))^3}+\frac{52e^3(e\cos(c+}{5d\,(a^4+a^4\sin}\\ &= \frac{78e^7\sqrt{e\cos(c+dx)}\,\sin(c+dx)}{7a^4d}+\frac{234e^5(e\cos(c+dx))^{5/2}\sin(c+dx)}{35a^4d}+\frac{4e(}{ad(a}\\ &= \frac{78e^7\sqrt{e\cos(c+dx)}\,\sin(c+dx)}{7a^4d}+\frac{234e^5(e\cos(c+dx))^{5/2}\sin(c+dx)}{35a^4d}+\frac{4e(}{ad(a}\\ &= \frac{78e^8\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7a^4d\sqrt{e\cos(c+dx)}}+\frac{78e^7\sqrt{e\cos(c+dx)}\,\sin(c+dx)}{7a^4d}+\frac{234}{}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.30, size = 66, normalized size = 0.37

$$-\frac{2\sqrt[4]{2}\,(e\cos(c+dx))^{17/2}\,{}_2F_1\left(\frac{3}{4},\frac{17}{4};\frac{21}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{17a^4de(1+\sin(c+dx))^{17/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(15/2)/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (-2*2^(1/4)*(e*Cos[c + d*x])^(17/2)*Hypergeometric2F1[3/4, 17/4, 21/4, (1 -
 Sin[c + d*x])/2])/(17*a^4*d*e*(1 + Sin[c + d*x])^(17/4))
```

Maple [A]
time = 2.56, size = 225, normalized size = 1.25

method	result
default	$-\frac{2e^{8}\left(80\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-120\left(\sin^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-224\left(\sin^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-280\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+336}{35}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(15/2)/(a+a*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] -2/35/a^4/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^8*(80*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-120*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+
1/2*c)-224*sin(1/2*d*x+1/2*c)^7-280*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)
+336*sin(1/2*d*x+1/2*c)^5+195*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+160*sin(1/2*d*x+1/2*
c)^2*cos(1/2*d*x+1/2*c)+392*sin(1/2*d*x+1/2*c)^3-252*sin(1/2*d*x+1/2*c))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(15/2)/(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] e^(15/2)*integrate(cos(d*x + c)^(15/2)/(a*sin(d*x + c) + a)^4, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 110, normalized size = 0.61

$$\frac{-195i\sqrt{2}\,e^{\frac{15}{2}}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+195i\sqrt{2}\,e^{\frac{15}{2}}\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-2\left(28\cos(dx+c)^{2}e^{\frac{15}{2}}-5\left(\cos(dx+c)^{2}e^{\frac{15}{2}}-17e^{\frac{15}{2}}\right)\sin(dx+c)-280e^{\frac{15}{2}}\right)\sqrt{\cos(dx+c)}}{35a^{4}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(15/2)/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/35*(-195*I*sqrt(2)*e^(15/2)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*s
in(d*x + c)) + 195*I*sqrt(2)*e^(15/2)*weierstrassPInverse(-4, 0, cos(d*x +
c) - I*sin(d*x + c)) - 2*(28*cos(d*x + c)^2*e^(15/2) - 5*(cos(d*x + c)^2*e^
(15/2) - 17*e^(15/2))*sin(d*x + c) - 280*e^(15/2))*sqrt(cos(d*x + c)))/(a^4
*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(15/2)/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(15/2)/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(15/2)*e^(15/2)/(a*sin(d*x + c) + a)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(e\,\cos\left(c+d\,x\right)\right)^{15/2}}{\left(a+a\,\sin\left(c+d\,x\right)\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(15/2)/(a + a*sin(c + d*x))^4,x)
```

```
[Out] int((e*cos(c + d*x))^(15/2)/(a + a*sin(c + d*x))^4, x)
```

3.264 $\int \frac{(e\cos(c+dx))^{13/2}}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=149

$$-\frac{154e^6\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^4d\sqrt{\cos(c+dx)}}-\frac{154e^5(e\cos(c+dx))^{3/2}\sin(c+dx)}{15a^4d}-\frac{4e(e\cos(c+dx))^{11/2}}{ad(a+a\sin(c+dx))^3}-\frac{44e}{3d\,(}$$

```
[Out] -154/15*e^5*(e*cos(d*x+c))^(3/2)*sin(d*x+c)/a^4/d-4*e*(e*cos(d*x+c))^(11/2)
/a/d/(a+a*sin(d*x+c))^3-44/3*e^3*(e*cos(d*x+c))^(7/2)/d/(a^4+a^4*sin(d*x+c)
)-154/5*e^6*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1
/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/a^4/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2759, 2715, 2721, 2719}

$$-\frac{154e^6E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5a^4d\sqrt{\cos(c+dx)}}-\frac{154e^5\sin(c+dx)(e\cos(c+dx))^{3/2}}{15a^4d}-\frac{44e^3(e\cos(c+dx))^{7/2}}{3d\left(a^4\sin(c+dx)+a^4\right)}-\frac{4e(e\cos(c+dx))^{11/2}}{ad(a\sin(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(13/2)/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (-154*e^6*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*a^4*d*Sqrt[Cos
[c + d*x]]) - (154*e^5*(e*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(15*a^4*d) - (4
*e*(e*Cos[c + d*x])^(11/2))/(a*d*(a + a*Sin[c + d*x])^3) - (44*e^3*(e*Cos[c
 + d*x])^(7/2))/(3*d*(a^4 + a^4*Sin[c + d*x]))
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{(e\cos(c+dx))^{13/2}}{(a+a\sin(c+dx))^4}\,dx &= -\frac{4e(e\cos(c+dx))^{11/2}}{ad(a+a\sin(c+dx))^3}-\frac{(11e^2)\int \frac{(e\cos(c+dx))^{9/2}}{(a+a\sin(c+dx))^2}\,dx}{a^2}\\ &= -\frac{4e(e\cos(c+dx))^{11/2}}{ad(a+a\sin(c+dx))^3}-\frac{44e^3(e\cos(c+dx))^{7/2}}{3d\,(a^4+a^4\sin(c+dx))}-\frac{(77e^4)\int(e\cos(c+dx))^{5}}{3a^4}\\ &= -\frac{154e^5(e\cos(c+dx))^{3/2}\sin(c+dx)}{15a^4d}-\frac{4e(e\cos(c+dx))^{11/2}}{ad(a+a\sin(c+dx))^3}-\frac{44e^3(e\cos(}{3d\,(a^4+a^4\mathrm{s}}\\ &= -\frac{154e^5(e\cos(c+dx))^{3/2}\sin(c+dx)}{15a^4d}-\frac{4e(e\cos(c+dx))^{11/2}}{ad(a+a\sin(c+dx))^3}-\frac{44e^3(e\cos(}{3d\,(a^4+a^4\mathrm{s}}\\ &= -\frac{154e^6\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^4d\sqrt{\cos(c+dx)}}-\frac{154e^5(e\cos(c+dx))^{3/2}\sin(c+dx}{15a^4d}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.20, size = 66, normalized size = 0.44

$$-\frac{2^{3/4}(e\cos(c+dx))^{15/2}\,{}_2F_1\left(\frac{5}{4},\frac{15}{4};\frac{19}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{15a^4de(1+\sin(c+dx))^{15/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(13/2)/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] -1/15*(2^(3/4)*(e*Cos[c + d*x])^(15/2)*Hypergeometric2F1[5/4, 15/4, 19/4, (
1 - Sin[c + d*x])/2])/(a^4*d*e*(1 + Sin[c + d*x])^(15/4))
```

Maple [A]
time = 3.49, size = 190, normalized size = 1.28

method	result

default	$-\frac{2\left(-24\left(\sin^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+24\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+80\left(\sin^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+231\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^{2}\left(\right.\right.}}{15\sqrt{-2\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(13/2)/(a+a*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] -2/15/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/sin(1/2*d*x+1/2*c)/a^4*(-24*sin(1
/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*
c)+80*sin(1/2*d*x+1/2*c)^5+231*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-246*sin(1/2*d*x+1/2
*c)^2*cos(1/2*d*x+1/2*c)-80*sin(1/2*d*x+1/2*c)^3+140*sin(1/2*d*x+1/2*c))*e^
7/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(13/2)/(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] e^(13/2)*integrate(cos(d*x + c)^(13/2)/(a*sin(d*x + c) + a)^4, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 231, normalized size = 1.55

$$-\frac{231\left(i\sqrt{2}\cos(dx+c)e^{\frac{13}{2}}+i\sqrt{2}e^{\frac{13}{2}}\sin(dx+c)+i\sqrt{2}e^{\frac{13}{2}}\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+231\left(-i\sqrt{2}\cos(dx+c)e^{\frac{13}{2}}-i\sqrt{2}e^{\frac{13}{2}}\sin(dx+c)-i\sqrt{2}e^{\frac{13}{2}}\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+2\left(3\cos(dx+c)^{3}e^{\frac{13}{2}}+20\cos(dx+c)^{2}e^{\frac{13}{2}}+137\cos(dx+c)e^{\frac{13}{2}}-\left(3\cos(dx+c)^{2}e^{\frac{13}{2}}-17\cos(dx+c)e^{\frac{13}{2}}+120e^{\frac{13}{2}}\right)\sin(dx+c)+120e^{\frac{13}{2}}\right)\sqrt{\cos(dx+c)}}{15\left(a^{4}d\cos(dx+c)+a^{4}d\sin(dx+c)+a^{4}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(13/2)/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/15*(231*(I*sqrt(2)*cos(d*x + c)*e^(13/2) + I*sqrt(2)*e^(13/2)*sin(d*x +
c) + I*sqrt(2)*e^(13/2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0,
cos(d*x + c) + I*sin(d*x + c))) + 231*(-I*sqrt(2)*cos(d*x + c)*e^(13/2) - I
*sqrt(2)*e^(13/2)*sin(d*x + c) - I*sqrt(2)*e^(13/2))*weierstrassZeta(-4, 0,
 weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(3*cos(d*x
+ c)^3*e^(13/2) + 20*cos(d*x + c)^2*e^(13/2) + 137*cos(d*x + c)*e^(13/2) -
(3*cos(d*x + c)^2*e^(13/2) - 17*cos(d*x + c)*e^(13/2) + 120*e^(13/2))*sin(d
*x + c) + 120*e^(13/2))*sqrt(cos(d*x + c)))/(a^4*d*cos(d*x + c) + a^4*d*sin
(d*x + c) + a^4*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(13/2)/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(13/2)/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(13/2)*e^(13/2)/(a*sin(d*x + c) + a)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,\cos\left(c+d\,x\right))^{13/2}}{(a+a\,\sin\left(c+d\,x\right))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(13/2)/(a + a*sin(c + d*x))^4,x)
```

```
[Out] int((e*cos(c + d*x))^(13/2)/(a + a*sin(c + d*x))^4, x)
```

3.265 $\int \frac{(e\cos(c+dx))^{11/2}}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=145

$$-\frac{10e^6\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^4d\sqrt{e\cos(c+dx)}}-\frac{10e^5\sqrt{e\cos(c+dx)}\,\sin(c+dx)}{a^4d}-\frac{4e(e\cos(c+dx))^{9/2}}{3ad(a+a\sin(c+dx))^3}-\frac{12e^3(e\cos(c+dx))^{5/2}}{d\,(a^4+a^4\sin(c+dx))}$$

```
[Out] -4/3*e*(e*cos(d*x+c))^(9/2)/a/d/(a+a*sin(d*x+c))^3-12*e^3*(e*cos(d*x+c))^(5
/2)/d/(a^4+a^4*sin(d*x+c))-10*e^6*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+
1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/a^4/d/(e*cos(
d*x+c))^(1/2)-10*e^5*sin(d*x+c)*(e*cos(d*x+c))^(1/2)/a^4/d
```

Rubi [A]
time = 0.11, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2759, 2715, 2721, 2720}

$$-\frac{10e^6\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^4d\sqrt{e\cos(c+dx)}}-\frac{10e^5\sin(c+dx)\sqrt{e\cos(c+dx)}}{a^4d}-\frac{12e^3(e\cos(c+dx))^{5/2}}{d\,(a^4\sin(c+dx)+a^4)}-\frac{4e(e\cos(c+dx))^{9/2}}{3ad(a\sin(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(11/2)/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (-10*e^6*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(a^4*d*Sqrt[e*Cos[c
+ d*x]]) - (10*e^5*Sqrt[e*Cos[c + d*x]]*Sin[c + d*x])/(a^4*d) - (4*e*(e*Cos
[c + d*x])^(9/2))/(3*a*d*(a + a*Sin[c + d*x])^3) - (12*e^3*(e*Cos[c + d*x])
^(5/2))/(d*(a^4 + a^4*Sin[c + d*x]))
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{11/2}}{(a+a\sin(c+dx))^4}\,dx = -\frac{4e(e\cos(c+dx))^{9/2}}{3ad(a+a\sin(c+dx))^3} - \frac{(3e^2)\int \frac{(e\cos(c+dx))^{7/2}}{(a+a\sin(c+dx))^2}\,dx}{a^2}$$

$$= -\frac{4e(e\cos(c+dx))^{9/2}}{3ad(a+a\sin(c+dx))^3} - \frac{12e^3(e\cos(c+dx))^{5/2}}{d\,(a^4+a^4\sin(c+dx))} - \frac{(15e^4)\int (e\cos(c+dx))^{3}}{a^4}$$

$$= -\frac{10e^5\sqrt{e\cos(c+dx)}\,\sin(c+dx)}{a^4d} - \frac{4e(e\cos(c+dx))^{9/2}}{3ad(a+a\sin(c+dx))^3} - \frac{12e^3(e\cos(c-}{d\,(a^4+a^4\sin}$$

$$= -\frac{10e^5\sqrt{e\cos(c+dx)}\,\sin(c+dx)}{a^4d} - \frac{4e(e\cos(c+dx))^{9/2}}{3ad(a+a\sin(c+dx))^3} - \frac{12e^3(e\cos(c-}{d\,(a^4+a^4\sin}$$

$$= -\frac{10e^6\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^4d\sqrt{e\cos(c+dx)}} - \frac{10e^5\sqrt{e\cos(c+dx)}\,\sin(c+dx)}{a^4d} - \frac{}{3}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.13, size = 66, normalized size = 0.46

$$-\frac{\sqrt[4]{2}\,(e\cos(c+dx))^{13/2}\,{}_2F_1\left(\frac{7}{4},\frac{13}{4};\frac{17}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{13a^4de(1+\sin(c+dx))^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(11/2)/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] -1/13*(2^(1/4)*(e*Cos[c + d*x])^(13/2)*Hypergeometric2F1[7/4, 13/4, 17/4, (
1 - Sin[c + d*x])/2])/(a^4*d*e*(1 + Sin[c + d*x])^(13/4))
```

Maple [A]
time = 4.63, size = 263, normalized size = 1.81

method	result

default	$\frac{2\left(-8\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+30\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\sin^2\left(\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(11/2)/(a+a*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/(2*sin(1/2*d*x+1/2*c)^2-1)/a^4/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c
)^2*e+e)^(1/2)*(-8*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+30*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+
48*sin(1/2*d*x+1/2*c)^5-15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-18*sin(1/2*d*x+1/2*c)^2
*cos(1/2*d*x+1/2*c)-48*sin(1/2*d*x+1/2*c)^3+20*sin(1/2*d*x+1/2*c))*e^6/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(11/2)/(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] e^(11/2)*integrate(cos(d*x + c)^(11/2)/(a*sin(d*x + c) + a)^4, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 140, normalized size = 0.97

$$-\frac{15\left(-i\sqrt{2}\,e^{\frac{11}{2}}\sin\left(dx+c\right)-i\sqrt{2}\,e^{\frac{11}{2}}\right)\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right))+15\left(i\sqrt{2}\,e^{\frac{11}{2}}\sin\left(dx+c\right)+i\sqrt{2}\,e^{\frac{11}{2}}\right)\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right))+2\left(\cos\left(dx+c\right)^2e^{\frac{11}{2}}+11\,e^{\frac{11}{2}}\sin\left(dx+c\right)+19\,e^{\frac{11}{2}}\right)\sqrt{\cos\left(dx+c\right)}}{3\left(a^4d\sin\left(dx+c\right)+a^4d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(11/2)/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/3*(15*(-I*sqrt(2)*e^(11/2)*sin(d*x + c) - I*sqrt(2)*e^(11/2))*weierstras
sPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 15*(I*sqrt(2)*e^(11/2)*si
n(d*x + c) + I*sqrt(2)*e^(11/2))*weierstrassPInverse(-4, 0, cos(d*x + c) -
I*sin(d*x + c)) + 2*(cos(d*x + c)^2*e^(11/2) + 11*e^(11/2)*sin(d*x + c) + 1
9*e^(11/2))*sqrt(cos(d*x + c)))/(a^4*d*sin(d*x + c) + a^4*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(11/2)/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(11/2)/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(11/2)*e^(11/2)/(a*sin(d*x + c) + a)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(e\,\cos\left(c+d\,x\right)\right)}^{11/2}}{{\left(a+a\,\sin\left(c+d\,x\right)\right)}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(11/2)/(a + a*sin(c + d*x))^4,x)
```

```
[Out] int((e*cos(c + d*x))^(11/2)/(a + a*sin(c + d*x))^4, x)
```

3.266 $\int \frac{(e\cos(c+dx))^{9/2}}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=120

$$\frac{42e^4\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^4d\sqrt{\cos(c+dx)}}-\frac{4e(e\cos(c+dx))^{7/2}}{5ad(a+a\sin(c+dx))^3}+\frac{28e^3(e\cos(c+dx))^{3/2}}{5d\,(a^4+a^4\sin(c+dx))}$$

```
[Out] -4/5*e*(e*cos(d*x+c))^(7/2)/a/d/(a+a*sin(d*x+c))^3+28/5*e^3*(e*cos(d*x+c))^
(3/2)/d/(a^4+a^4*sin(d*x+c))+42/5*e^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*
d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/a^4/d
/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2759, 2721, 2719}

$$\frac{42e^4E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5a^4d\sqrt{\cos(c+dx)}}+\frac{28e^3(e\cos(c+dx))^{3/2}}{5d\,(a^4\sin(c+dx)+a^4)}-\frac{4e(e\cos(c+dx))^{7/2}}{5ad(a\sin(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(9/2)/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (42*e^4*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*a^4*d*Sqrt[Cos[c
 + d*x]]) - (4*e*(e*Cos[c + d*x])^(7/2))/(5*a*d*(a + a*Sin[c + d*x])^3) + (
28*e^3*(e*Cos[c + d*x])^(3/2))/(5*d*(a^4 + a^4*Sin[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
```

```
NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{9/2}}{(a+a\sin(c+dx))^4}\,dx = -\frac{4e(e\cos(c+dx))^{7/2}}{5ad(a+a\sin(c+dx))^3} - \frac{(7e^2)\int \frac{(e\cos(c+dx))^{5/2}}{(a+a\sin(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{4e(e\cos(c+dx))^{7/2}}{5ad(a+a\sin(c+dx))^3} + \frac{28e^3(e\cos(c+dx))^{3/2}}{5d\,(a^4+a^4\sin(c+dx))} + \frac{(21e^4)\int \sqrt{e\cos(c+d}}{5a^4}$$

$$= -\frac{4e(e\cos(c+dx))^{7/2}}{5ad(a+a\sin(c+dx))^3} + \frac{28e^3(e\cos(c+dx))^{3/2}}{5d\,(a^4+a^4\sin(c+dx))} + \frac{\left(21e^4\sqrt{e\cos(c+dx)}\right.}{5a^4\sqrt{\cos(}}$$

$$= \frac{42e^4\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5a^4d\sqrt{\cos(c+dx)}} - \frac{4e(e\cos(c+dx))^{7/2}}{5ad(a+a\sin(c+dx))^3} + \frac{28e^3(e\cos}{5d\,(a^4+a^4}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.09, size = 66, normalized size = 0.55

$$-\frac{(e\cos(c+dx))^{11/2}\,{}_2F_1\left(\frac{9}{4},\frac{11}{4};\frac{15}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{11\sqrt[4]{2}\,a^4de(1+\sin(c+dx))^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(9/2)/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] -1/11*((e*Cos[c + d*x])^(11/2)*Hypergeometric2F1[9/4, 11/4, 15/4, (1 - Sin[
c + d*x])/2])/(2^(1/4)*a^4*d*e*(1 + Sin[c + d*x])^(11/4))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 331 vs. 2(132) = 264.
time = 6.74, size = 332, normalized size = 2.77

method	result
default	$\frac{2\left(84\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-128\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\mathrm{c}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(9/2)/(a+a*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 2/5/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/a^4/sin(1/2*d*x+1/2*c
)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*(84*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*
d*x+1/2*c)^4-128*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-84*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+128*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+
80*sin(1/2*d*x+1/2*c)^5+21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-16*sin(1/2*d*x+1/2*c)^2
*cos(1/2*d*x+1/2*c)-80*sin(1/2*d*x+1/2*c)^3+12*sin(1/2*d*x+1/2*c))*e^5/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(9/2)/(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] e^(9/2)*integrate(cos(d*x + c)^(9/2)/(a*sin(d*x + c) + a)^4, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 294, normalized size = 2.45

$$\frac{21 \left(i \sqrt{2} \cos\left(dx+c\right)^{2} e^{\frac{9}{2}} - i \sqrt{2} \cos\left(dx+c\right) e^{\frac{9}{2}} + \left(-i \sqrt{2} \cos\left(dx+c\right) e^{\frac{9}{2}} - 2i \sqrt{2} e^{\frac{9}{2}}\right) \sin\left(dx+c\right) - 2i \sqrt{2} e^{\frac{9}{2}}\right) \mathrm{weierstrassZeta}\left(-4, 0, \mathrm{weierstrassPInverse}\left(-4, 0, \cos\left(dx+c\right) + i \sin\left(dx+c\right)\right)\right) + 21 \left(-i \sqrt{2} \cos\left(dx+c\right)^{2} e^{\frac{9}{2}} + i \sqrt{2} \cos\left(dx+c\right) e^{\frac{9}{2}} + \left(i \sqrt{2} \cos\left(dx+c\right) e^{\frac{9}{2}} + 2i \sqrt{2} e^{\frac{9}{2}}\right) \sin\left(dx+c\right) + 2i \sqrt{2} e^{\frac{9}{2}}\right) \mathrm{weierstrassZeta}\left(-4, 0, \mathrm{weierstrassPInverse}\left(-4, 0, \cos\left(dx+c\right) - i \sin\left(dx+c\right)\right)\right) - 8 \left(4 \cos\left(dx+c\right)^{2} e^{\frac{9}{2}} + 3 \cos\left(dx+c\right) e^{\frac{9}{2}} + \left(4 \cos\left(dx+c\right) e^{\frac{9}{2}} + e^{\frac{9}{2}}\right) \sin\left(dx+c\right) - e^{\frac{9}{2}}\right) \sqrt{\cos\left(dx+c\right)}}{5 \left(a^{4} d \cos\left(dx+c\right)^{2} - a^{4} d \cos\left(dx+c\right) - 2 a^{4} d - \left(a^{4} d \cos\left(dx+c\right) + 2 a^{4} d\right) \sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(9/2)/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/5*(21*(I*sqrt(2)*cos(d*x + c)^2*e^(9/2) - I*sqrt(2)*cos(d*x + c)*e^(9/2)
+ (-I*sqrt(2)*cos(d*x + c)*e^(9/2) - 2*I*sqrt(2)*e^(9/2))*sin(d*x + c) - 2*
I*sqrt(2)*e^(9/2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*
x + c) + I*sin(d*x + c))) + 21*(-I*sqrt(2)*cos(d*x + c)^2*e^(9/2) + I*sqrt(
2)*cos(d*x + c)*e^(9/2) + (I*sqrt(2)*cos(d*x + c)*e^(9/2) + 2*I*sqrt(2)*e^(
9/2))*sin(d*x + c) + 2*I*sqrt(2)*e^(9/2))*weierstrassZeta(-4, 0, weierstras
sPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 8*(4*cos(d*x + c)^2*e^(9
/2) + 3*cos(d*x + c)*e^(9/2) + (4*cos(d*x + c)*e^(9/2) + e^(9/2))*sin(d*x +
 c) - e^(9/2))*sqrt(cos(d*x + c)))/(a^4*d*cos(d*x + c)^2 - a^4*d*cos(d*x +
c) - 2*a^4*d - (a^4*d*cos(d*x + c) + 2*a^4*d)*sin(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(9/2)/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8857 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(9/2)/(a+a*sin(d*x+c))^4,x, algorithm="giac")
[Out] integrate(cos(d*x + c)^(9/2)*e^(9/2)/(a*sin(d*x + c) + a)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,\cos\left(c+d\,x\right))^{9/2}}{(a+a\,\sin\left(c+d\,x\right))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(9/2)/(a + a*sin(c + d*x))^4,x)
[Out] int((e*cos(c + d*x))^(9/2)/(a + a*sin(c + d*x))^4, x)
```

3.267 $\int \frac{(e\cos(c+dx))^{7/2}}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=120

$$\frac{10e^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21a^4d\sqrt{e\cos(c+dx)}}-\frac{4e(e\cos(c+dx))^{5/2}}{7ad(a+a\sin(c+dx))^3}+\frac{20e^3\sqrt{e\cos(c+dx)}}{21d\,(a^4+a^4\sin(c+dx))}$$

```
[Out] -4/7*e*(e*cos(d*x+c))^(5/2)/a/d/(a+a*sin(d*x+c))^3+10/21*e^4*(cos(1/2*d*x+1
/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos
(d*x+c)^(1/2)/a^4/d/(e*cos(d*x+c))^(1/2)+20/21*e^3*(e*cos(d*x+c))^(1/2)/d/(
a^4+a^4*sin(d*x+c))
```

Rubi [A]
time = 0.10, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2759, 2721, 2720}

$$\frac{10e^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21a^4d\sqrt{e\cos(c+dx)}}+\frac{20e^3\sqrt{e\cos(c+dx)}}{21d\,(a^4\sin(c+dx)+a^4)}-\frac{4e(e\cos(c+dx))^{5/2}}{7ad(a\sin(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(7/2)/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (10*e^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*a^4*d*Sqrt[e*Cos[
c + d*x]]) - (4*e*(e*Cos[c + d*x])^(5/2))/(7*a*d*(a + a*Sin[c + d*x])^3) +
(20*e^3*Sqrt[e*Cos[c + d*x]])/(21*d*(a^4 + a^4*Sin[c + d*x]))
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
```

```
NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{7/2}}{(a+a\sin(c+dx))^4}\,dx = -\frac{4e(e\cos(c+dx))^{5/2}}{7ad(a+a\sin(c+dx))^3} - \frac{(5e^2)\int \frac{(e\cos(c+dx))^{3/2}}{(a+a\sin(c+dx))^2}\,dx}{7a^2}$$

$$= -\frac{4e(e\cos(c+dx))^{5/2}}{7ad(a+a\sin(c+dx))^3} + \frac{20e^3\sqrt{e\cos(c+dx)}}{21d\,(a^4+a^4\sin(c+dx))} + \frac{(5e^4)\int \frac{1}{\sqrt{e\cos(c+d}}}{21a^4}$$

$$= -\frac{4e(e\cos(c+dx))^{5/2}}{7ad(a+a\sin(c+dx))^3} + \frac{20e^3\sqrt{e\cos(c+dx)}}{21d\,(a^4+a^4\sin(c+dx))} + \frac{\left(5e^4\sqrt{\cos(c+dx)}\right)}{21a^4\sqrt{e\cos}}$$

$$= \frac{10e^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21a^4d\sqrt{e\cos(c+dx)}} - \frac{4e(e\cos(c+dx))^{5/2}}{7ad(a+a\sin(c+dx))^3} + \frac{20e^3\sqrt{e\cos}}{21d\,(a^4+a^4}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.08, size = 66, normalized size = 0.55

$$-\frac{(e\cos(c+dx))^{9/2}\,{}_2F_1\left(\frac{9}{4},\frac{11}{4};\frac{13}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{9\;2^{3/4}a^4de(1+\sin(c+dx))^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(7/2)/(a + a*Sin[c + d*x])^4,x]
[Out] -1/9*((e*Cos[c + d*x])^(9/2)*Hypergeometric2F1[9/4, 11/4, 13/4, (1 - Sin[c
+ d*x])/2])/(2^(3/4)*a^4*d*e*(1 + Sin[c + d*x])^(9/4))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 400 vs. 2(132) = 264.
time = 7.54, size = 401, normalized size = 3.34

method	result
default	$-\frac{2\left(40\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-60\sqrt{\frac{1}{2}-\frac{\cos(dx}{2}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] -2/21/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^
2-1)/a^4/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*(40*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^6-60*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*
d*x+1/2*c)^4-128*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+30*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2))*sin(1/2*d*x+1/2*c)^2+128*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+
112*sin(1/2*d*x+1/2*c)^5-5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+16*sin(1/2*d*x+1/2*c)^2
*cos(1/2*d*x+1/2*c)-112*sin(1/2*d*x+1/2*c)^3+4*sin(1/2*d*x+1/2*c))*e^4/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] e^(7/2)*integrate(cos(d*x + c)^(7/2)/(a*sin(d*x + c) + a)^4, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 172, normalized size = 1.43

$$\frac{5\left(-i\sqrt{2}\cos\left(dx+c\right)^{2}e^{\frac{7}{2}}+2i\sqrt{2}e^{\frac{7}{2}}\sin\left(dx+c\right)+2i\sqrt{2}e^{\frac{7}{2}}\right)\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right))+5\left(i\sqrt{2}\cos\left(dx+c\right)^{2}e^{\frac{7}{2}}-2i\sqrt{2}e^{\frac{7}{2}}\sin\left(dx+c\right)-2i\sqrt{2}e^{\frac{7}{2}}\right)\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right))-8\left(4e^{\frac{7}{2}}\sin\left(dx+c\right)+e^{\frac{7}{2}}\right)\sqrt{\cos\left(dx+c\right)}}{21\left(a^{4}d\cos\left(dx+c\right)^{2}-2a^{4}d\sin\left(dx+c\right)-2a^{4}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/21*(5*(-I*sqrt(2)*cos(d*x + c)^2*e^(7/2) + 2*I*sqrt(2)*e^(7/2)*sin(d*x +
c) + 2*I*sqrt(2)*e^(7/2))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d
*x + c)) + 5*(I*sqrt(2)*cos(d*x + c)^2*e^(7/2) - 2*I*sqrt(2)*e^(7/2)*sin(d*
x + c) - 2*I*sqrt(2)*e^(7/2))*weierstrassPInverse(-4, 0, cos(d*x + c) - I*s
in(d*x + c)) - 8*(4*e^(7/2)*sin(d*x + c) + e^(7/2))*sqrt(cos(d*x + c)))/(a^
4*d*cos(d*x + c)^2 - 2*a^4*d*sin(d*x + c) - 2*a^4*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(7/2)/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3878 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(7/2)*e^(7/2)/(a*sin(d*x + c) + a)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(e\,\cos\left(c+d\,x\right)\right)^{7/2}}{\left(a+a\,\sin\left(c+d\,x\right)\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(7/2)/(a + a*sin(c + d*x))^4,x)
```

```
[Out] int((e*cos(c + d*x))^(7/2)/(a + a*sin(c + d*x))^4, x)
```

3.268 $\int \frac{(e\cos(c+dx))^{5/2}}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=154

$$\frac{2e^2\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15a^4d\sqrt{\cos(c+dx)}}-\frac{4e(e\cos(c+dx))^{3/2}}{9ad(a+a\sin(c+dx))^3}+\frac{2e(e\cos(c+dx))^{3/2}}{15d\left(a^2+a^2\sin(c+dx)\right)^2}+\frac{2e(e\cos(c+d}{15d\left(a^4+a^4\sin(\right.}$$

```
[Out] -4/9*e*(e*cos(d*x+c))^(3/2)/a/d/(a+a*sin(d*x+c))^3+2/15*e*(e*cos(d*x+c))^(3
/2)/d/(a^2+a^2*sin(d*x+c))^2+2/15*e*(e*cos(d*x+c))^(3/2)/d/(a^4+a^4*sin(d*x
+c))+2/15*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin
(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/a^4/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2759, 2760, 2762, 2721, 2719}

$$\frac{2e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{15a^4d\sqrt{\cos(c+dx)}}+\frac{2e(e\cos(c+dx))^{3/2}}{15d\left(a^4\sin(c+dx)+a^4\right)}+\frac{2e(e\cos(c+dx))^{3/2}}{15d\left(a^2\sin(c+dx)+a^2\right)^2}-\frac{4e(e\cos(c+dx))^{3/2}}{9ad(a\sin(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(5/2)/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (2*e^2*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*a^4*d*Sqrt[Cos[c
 + d*x]]) - (4*e*(e*Cos[c + d*x])^(3/2))/(9*a*d*(a + a*Sin[c + d*x])^3) + (
2*e*(e*Cos[c + d*x])^(3/2))/(15*d*(a^2 + a^2*Sin[c + d*x])^2) + (2*e*(e*Cos
[c + d*x])^(3/2))/(15*d*(a^4 + a^4*Sin[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
```

```
NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2760

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*(2*m + p + 1))), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2762

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[b*((g*Cos[e + f*x])^(p + 1)/(a*f*g*(p - 1)*(a + b*S
in[e + f*x]))), x] + Dist[p/(a*(p - 1)), Int[(g*Cos[e + f*x])^p, x], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] &&  !GeQ[p, 1] && Integer
Q[2*p]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{5/2}}{(a+a\sin(c+dx))^4}\,dx = -\frac{4e(e\cos(c+dx))^{3/2}}{9ad(a+a\sin(c+dx))^3} - \frac{e^2\int \frac{\sqrt{e\cos(c+dx)}}{(a+a\sin(c+dx))^2}\,dx}{3a^2}$$

$$= -\frac{4e(e\cos(c+dx))^{3/2}}{9ad(a+a\sin(c+dx))^3} + \frac{2e(e\cos(c+dx))^{3/2}}{15d\left(a^2+a^2\sin(c+dx)\right)^2} - \frac{e^2\int \frac{\sqrt{e\cos(c+dx)}}{a+a\sin(c+dx)}}{15a^3}$$

$$= -\frac{4e(e\cos(c+dx))^{3/2}}{9ad(a+a\sin(c+dx))^3} + \frac{2e(e\cos(c+dx))^{3/2}}{15d\left(a^2+a^2\sin(c+dx)\right)^2} + \frac{2e(e\cos(c+dx))^{3}}{15d\left(a^4+a^4\sin(c+\right.}$$

$$= -\frac{4e(e\cos(c+dx))^{3/2}}{9ad(a+a\sin(c+dx))^3} + \frac{2e(e\cos(c+dx))^{3/2}}{15d\left(a^2+a^2\sin(c+dx)\right)^2} + \frac{2e(e\cos(c+dx))^{3}}{15d\left(a^4+a^4\sin(c+\right.}$$

$$= \frac{2e^2\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15a^4d\sqrt{\cos(c+dx)}} - \frac{4e(e\cos(c+dx))^{3/2}}{9ad(a+a\sin(c+dx))^3} + \frac{2e(e\cos(}{15d\left(a^2+a^{2}\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.09, size = 66, normalized size = 0.43

$$-\frac{(e\cos(c+dx))^{7/2}\,{}_2F_1\left(\frac{7}{4},\frac{13}{4};\frac{11}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{14\sqrt[4]{2}\,a^4de(1+\sin(c+dx))^{7/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(e*Cos[c + d*x])^(5/2)/(a + a*Sin[c + d*x])^4,x]

[Out] -1/14*((e*Cos[c + d*x])^(7/2)*Hypergeometric2F1[7/4, 13/4, 11/4, (1 - Sin[c + d*x])/2])/(2^(1/4)*a^4*d*e*(1 + Sin[c + d*x])^(7/4))

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 513 vs. $2(162) = 324$.
time = 10.38, size = 514, normalized size = 3.34

method	result
default	$\frac{2\left(48\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-96\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^4,x,method=_RETURNVERBOSE)

[Out] 2/45/(16*sin(1/2*d*x+1/2*c)^8-32*sin(1/2*d*x+1/2*c)^6+24*sin(1/2*d*x+1/2*c)^4-8*sin(1/2*d*x+1/2*c)^2+1)/a^4/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*(48*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^8-96*sin(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)-96*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^6+192*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+72*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-272*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-24*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+176*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+144*sin(1/2*d*x+1/2*c)^5+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+42*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-144*sin(1/2*d*x+1/2*c)^3-4*sin(1/2*d*x+1/2*c))*e^3/d

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^4,x, algorithm="maxima")

[Out] e^(5/2)*integrate(cos(d*x + c)^(5/2)/(a*sin(d*x + c) + a)^4, x)

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 407, normalized size = 2.64

$$-\frac{3\left(-i\sqrt{2}\cos\left(dx+c\right)^{3}e^{\frac{5}{2}}-3i\sqrt{2}\cos\left(dx+c\right)^{2}e^{\frac{5}{2}}+2i\sqrt{2}\cos\left(dx+c\right)e^{\frac{5}{2}}+\left(-i\sqrt{2}\cos\left(dx+c\right)^{2}e^{\frac{5}{2}}+2i\sqrt{2}\cos\left(dx+c\right)e^{\frac{5}{2}}+4i\sqrt{2}e^{\frac{5}{2}}\right)\sin\left(dx+c\right)+4i\sqrt{2}e^{\frac{5}{2}}\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)\right)+3\left(i\sqrt{2}\cos\left(dx+c\right)^{3}e^{\frac{5}{2}}+3i\sqrt{2}\cos\left(dx+c\right)^{2}e^{\frac{5}{2}}-2i\sqrt{2}\cos\left(dx+c\right)e^{\frac{5}{2}}+\left(i\sqrt{2}\cos\left(dx+c\right)^{2}e^{\frac{5}{2}}-2i\sqrt{2}\cos\left(dx+c\right)e^{\frac{5}{2}}-4i\sqrt{2}e^{\frac{5}{2}}\right)\sin\left(dx+c\right)-4i\sqrt{2}e^{\frac{5}{2}}\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)\right)-2\left(3\cos\left(dx+c\right)^{3}e^{\frac{5}{2}}-6\cos\left(dx+c\right)^{2}e^{\frac{5}{2}}+\cos\left(dx+c\right)e^{\frac{5}{2}}-\left(3\cos\left(dx+c\right)^{2}e^{\frac{5}{2}}+9\cos\left(dx+c\right)e^{\frac{5}{2}}+10e^{\frac{5}{2}}\right)\sin\left(dx+c\right)+10e^{\frac{5}{2}}\right)\sqrt{\cos\left(dx+c\right)}}{45\left(a^{4}d\cos\left(dx+c\right)^{3}+3a^{4}d\cos\left(dx+c\right)^{2}-2a^{4}d\cos\left(dx+c\right)-4a^{4}d+\left(a^{4}d\cos\left(dx+c\right)^{2}-2a^{4}d\cos\left(dx+c\right)-4a^{4}d\right)\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/45*(3*(-I*sqrt(2)*cos(d*x + c)^3*e^(5/2) - 3*I*sqrt(2)*cos(d*x + c)^2*e^
(5/2) + 2*I*sqrt(2)*cos(d*x + c)*e^(5/2) + (-I*sqrt(2)*cos(d*x + c)^2*e^(5/
2) + 2*I*sqrt(2)*cos(d*x + c)*e^(5/2) + 4*I*sqrt(2)*e^(5/2))*sin(d*x + c) +
 4*I*sqrt(2)*e^(5/2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos
(d*x + c) + I*sin(d*x + c))) + 3*(I*sqrt(2)*cos(d*x + c)^3*e^(5/2) + 3*I*sq
rt(2)*cos(d*x + c)^2*e^(5/2) - 2*I*sqrt(2)*cos(d*x + c)*e^(5/2) + (I*sqrt(2
)*cos(d*x + c)^2*e^(5/2) - 2*I*sqrt(2)*cos(d*x + c)*e^(5/2) - 4*I*sqrt(2)*e
^(5/2))*sin(d*x + c) - 4*I*sqrt(2)*e^(5/2))*weierstrassZeta(-4, 0, weierstr
assPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(3*cos(d*x + c)^3*e^
(5/2) - 6*cos(d*x + c)^2*e^(5/2) + cos(d*x + c)*e^(5/2) - (3*cos(d*x + c)^2
*e^(5/2) + 9*cos(d*x + c)*e^(5/2) + 10*e^(5/2))*sin(d*x + c) + 10*e^(5/2))*
sqrt(cos(d*x + c)))/(a^4*d*cos(d*x + c)^3 + 3*a^4*d*cos(d*x + c)^2 - 2*a^4*
d*cos(d*x + c) - 4*a^4*d + (a^4*d*cos(d*x + c)^2 - 2*a^4*d*cos(d*x + c) - 4
*a^4*d)*sin(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(5/2)/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(5/2)*e^(5/2)/(a*sin(d*x + c) + a)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(e\,\cos\left(c+d\,x\right)\right)}^{5/2}}{{\left(a+a\,\sin\left(c+d\,x\right)\right)}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(5/2)/(a + a*sin(c + d*x))^4,x)
[Out] int((e*cos(c + d*x))^(5/2)/(a + a*sin(c + d*x))^4, x)
```

3.269 $\int \frac{(e\cos(c+dx))^{3/2}}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=154

$$-\frac{2e^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{77a^4d\sqrt{e\cos(c+dx)}}-\frac{4e\sqrt{e\cos(c+dx)}}{11ad(a+a\sin(c+dx))^3}+\frac{2e\sqrt{e\cos(c+dx)}}{77d\left(a^2+a^2\sin(c+dx)\right)^2}+\frac{2e\sqrt{e\cos(}{77d\left(a^4+a^4\,\mathrm{s}\right.}$$

```
[Out] -2/77*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2
*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/a^4/d/(e*cos(d*x+c))^(1/2)-4/11*e*(e*
cos(d*x+c))^(1/2)/a/d/(a+a*sin(d*x+c))^3+2/77*e*(e*cos(d*x+c))^(1/2)/d/(a^2
+a^2*sin(d*x+c))^2+2/77*e*(e*cos(d*x+c))^(1/2)/d/(a^4+a^4*sin(d*x+c))
```

Rubi [A]
time = 0.13, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2759, 2760, 2762, 2721, 2720}

$$-\frac{2e^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{77a^4d\sqrt{e\cos(c+dx)}}+\frac{2e\sqrt{e\cos(c+dx)}}{77d\left(a^4\sin(c+dx)+a^4\right)}+\frac{2e\sqrt{e\cos(c+dx)}}{77d\left(a^2\sin(c+dx)+a^2\right)^2}-\frac{4e\sqrt{e\cos(c+dx)}}{11ad(a\sin(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(3/2)/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (-2*e^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(77*a^4*d*Sqrt[e*Cos[
c + d*x]]) - (4*e*Sqrt[e*Cos[c + d*x]])/(11*a*d*(a + a*Sin[c + d*x])^3) + (
2*e*Sqrt[e*Cos[c + d*x]])/(77*d*(a^2 + a^2*Sin[c + d*x])^2) + (2*e*Sqrt[e*C
os[c + d*x]])/(77*d*(a^4 + a^4*Sin[c + d*x]))
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
```

```
NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2760

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*(2*m + p + 1))), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2762

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[b*((g*Cos[e + f*x])^(p + 1)/(a*f*g*(p - 1)*(a + b*S
in[e + f*x]))), x] + Dist[p/(a*(p - 1)), Int[(g*Cos[e + f*x])^p, x], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] &&  !GeQ[p, 1] && Integer
Q[2*p]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{3/2}}{(a+a\sin(c+dx))^4}\,dx = -\frac{4e\sqrt{e\cos(c+dx)}}{11ad(a+a\sin(c+dx))^3} - \frac{e^2\int \frac{1}{\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^2}\,dx}{11a^2}$$

$$= -\frac{4e\sqrt{e\cos(c+dx)}}{11ad(a+a\sin(c+dx))^3} + \frac{2e\sqrt{e\cos(c+dx)}}{77d\left(a^2+a^2\sin(c+dx)\right)^2} - \frac{(3e^2)\int \frac{}{\sqrt{e\cos(c+}}}{77}$$

$$= -\frac{4e\sqrt{e\cos(c+dx)}}{11ad(a+a\sin(c+dx))^3} + \frac{2e\sqrt{e\cos(c+dx)}}{77d\left(a^2+a^2\sin(c+dx)\right)^2} + \frac{2e\sqrt{e\cos(c+dx)}}{77d\left(a^4+a^4\sin(c+\right.}$$

$$= -\frac{4e\sqrt{e\cos(c+dx)}}{11ad(a+a\sin(c+dx))^3} + \frac{2e\sqrt{e\cos(c+dx)}}{77d\left(a^2+a^2\sin(c+dx)\right)^2} + \frac{2e\sqrt{e\cos(c+dx)}}{77d\left(a^4+a^4\sin(c+\right.}$$

$$= -\frac{2e^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{77a^4d\sqrt{e\cos(c+dx)}} - \frac{4e\sqrt{e\cos(c+dx)}}{11ad(a+a\sin(c+dx))^3} + \frac{2e\sqrt{e\cos}}{77d\left(a^2+a^2\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.08, size = 66, normalized size = 0.43

$$-\frac{(e\cos(c+dx))^{5/2}\,{}_2F_1\left(\frac{5}{4},\frac{15}{4};\frac{9}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{10\ 2^{3/4}a^4de(1+\sin(c+dx))^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(3/2)/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] -1/10*((e*Cos[c + d*x])^(5/2)*Hypergeometric2F1[5/4, 15/4, 9/4, (1 - Sin[c
+ d*x])/2])/(2^(3/4)*a^4*d*e*(1 + Sin[c + d*x])^(5/4))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 582 vs. $2(162) = 324$.
time = 11.23, size = 583, normalized size = 3.79

method	result
default	$\frac{2\left(32\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-80\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 2/77/(32*sin(1/2*d*x+1/2*c)^10-80*sin(1/2*d*x+1/2*c)^8+80*sin(1/2*d*x+1/2*c
)^6-40*sin(1/2*d*x+1/2*c)^4+10*sin(1/2*d*x+1/2*c)^2-1)/a^4/sin(1/2*d*x+1/2*
c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*(32*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2
*d*x+1/2*c)^10-80*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^8+32*sin(1/2*
d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+80*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/
2*c)^6-64*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-40*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))*sin(1/2*d*x+1/2*c)^4+176*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+10*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-144*sin(1/2*d*x+1/2*c)^4*cos(1/2*
d*x+1/2*c)-176*sin(1/2*d*x+1/2*c)^5-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-78*sin(1/2*d*x
+1/2*c)^2*cos(1/2*d*x+1/2*c)+176*sin(1/2*d*x+1/2*c)^3+12*sin(1/2*d*x+1/2*c)
)*e^2/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] e^(3/2)*integrate(cos(d*x + c)^(3/2)/(a*sin(d*x + c) + a)^4, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 233, normalized size = 1.51

$$\frac{\left(3i\sqrt{2}\cos\left(dx+c\right)^{2}e^{\frac{3}{2}}+\left(i\sqrt{2}\cos\left(dx+c\right)^{2}e^{\frac{3}{2}}-4i\sqrt{2}e^{\frac{3}{2}}\right)\sin\left(dx+c\right)-4i\sqrt{2}e^{\frac{3}{2}}\right)\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right))+\left(-3i\sqrt{2}\cos\left(dx+c\right)^{2}e^{\frac{3}{2}}+\left(-i\sqrt{2}\cos\left(dx+c\right)^{2}e^{\frac{3}{2}}+4i\sqrt{2}e^{\frac{3}{2}}\right)\sin\left(dx+c\right)+4i\sqrt{2}e^{\frac{3}{2}}\right)\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right))+2\left(\cos\left(dx+c\right)^{2}e^{\frac{3}{2}}-3e^{\frac{3}{2}}\sin\left(dx+c\right)+11e^{\frac{3}{2}}\right)\sqrt{\cos\left(dx+c\right)}}{77\left(3a^{4}d\cos\left(dx+c\right)^{2}-4a^{4}d+\left(a^{4}d\cos\left(dx+c\right)^{2}-4a^{4}d\right)\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/77*((3*I*sqrt(2)*cos(d*x + c)^2*e^(3/2) + (I*sqrt(2)*cos(d*x + c)^2*e^(3/
2) - 4*I*sqrt(2)*e^(3/2))*sin(d*x + c) - 4*I*sqrt(2)*e^(3/2))*weierstrassPI
nverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + (-3*I*sqrt(2)*cos(d*x + c)^2
*e^(3/2) + (-I*sqrt(2)*cos(d*x + c)^2*e^(3/2) + 4*I*sqrt(2)*e^(3/2))*sin(d*
x + c) + 4*I*sqrt(2)*e^(3/2))*weierstrassPInverse(-4, 0, cos(d*x + c) - I*s
in(d*x + c)) + 2*(cos(d*x + c)^2*e^(3/2) - 3*e^(3/2)*sin(d*x + c) + 11*e^(3
/2))*sqrt(cos(d*x + c)))/(3*a^4*d*cos(d*x + c)^2 - 4*a^4*d + (a^4*d*cos(d*x
 + c)^2 - 4*a^4*d)*sin(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(3/2)/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 7319 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(3/2)*e^(3/2)/(a*sin(d*x + c) + a)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(e\,\cos\left(c+d\,x\right)\right)}^{3/2}}{{\left(a+a\,\sin\left(c+d\,x\right)\right)}^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(3/2)/(a + a*sin(c + d*x))^4,x)
```

```
[Out] int((e*cos(c + d*x))^(3/2)/(a + a*sin(c + d*x))^4, x)
```

3.270 $\int \frac{\sqrt{e\cos(c+dx)}}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=191

$$-\frac{2\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{39a^4d\sqrt{\cos(c+dx)}}-\frac{2(e\cos(c+dx))^{3/2}}{13de(a+a\sin(c+dx))^4}-\frac{10(e\cos(c+dx))^{3/2}}{117ade(a+a\sin(c+dx))^3}-\frac{2(e\cos(c}{39de\,(a^2+a^2}$$

```
[Out] -2/13*(e*cos(d*x+c))^(3/2)/d/e/(a+a*sin(d*x+c))^4-10/117*(e*cos(d*x+c))^(3/
2)/a/d/e/(a+a*sin(d*x+c))^3-2/39*(e*cos(d*x+c))^(3/2)/d/e/(a^2+a^2*sin(d*x+
c))^2-2/39*(e*cos(d*x+c))^(3/2)/d/e/(a^4+a^4*sin(d*x+c))-2/39*(cos(1/2*d*x+
1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e
*cos(d*x+c))^(1/2)/a^4/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2760, 2762, 2721, 2719}

$$-\frac{2(e\cos(c+dx))^{3/2}}{39de\,(a^4\sin(c+dx)+a^4)}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{39a^4d\sqrt{\cos(c+dx)}}-\frac{2(e\cos(c+dx))^{3/2}}{39de\,(a^2\sin(c+dx)+a^2)^2}-\frac{10(e\cos(c+dx))^{3/2}}{117ade(a\sin(c+dx)+a)^3}-\frac{2(e\cos(c+dx))^{3/2}}{13de(a\sin(c+dx)+a)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cos[c + d*x]]/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (-2*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(39*a^4*d*Sqrt[Cos[c +
d*x]]) - (2*(e*Cos[c + d*x])^(3/2))/(13*d*e*(a + a*Sin[c + d*x])^4) - (10*(
e*Cos[c + d*x])^(3/2))/(117*a*d*e*(a + a*Sin[c + d*x])^3) - (2*(e*Cos[c + d
*x])^(3/2))/(39*d*e*(a^2 + a^2*Sin[c + d*x])^2) - (2*(e*Cos[c + d*x])^(3/2)
)/(39*d*e*(a^4 + a^4*Sin[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2760

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*(2*m + p + 1))), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
```

```
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2762

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[b*((g*Cos[e + f*x])^(p + 1)/(a*f*g*(p - 1)*(a + b*S
in[e + f*x]))), x] + Dist[p/(a*(p - 1)), Int[(g*Cos[e + f*x])^p, x], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] &&  !GeQ[p, 1] && Integer
Q[2*p]
```

Rubi steps

$$\int \frac{\sqrt{e\cos(c+dx)}}{(a+a\sin(c+dx))^4}\,dx = -\frac{2(e\cos(c+dx))^{3/2}}{13de(a+a\sin(c+dx))^4} + \frac{5\int \frac{\sqrt{e\cos(c+dx)}}{(a+a\sin(c+dx))^3}\,dx}{13a}$$

$$= -\frac{2(e\cos(c+dx))^{3/2}}{13de(a+a\sin(c+dx))^4} - \frac{10(e\cos(c+dx))^{3/2}}{117ade(a+a\sin(c+dx))^3} + \frac{5\int \frac{\sqrt{e\cos(c+dx)}}{(a+a\sin(c+dx))^2}}{39a^2}$$

$$= -\frac{2(e\cos(c+dx))^{3/2}}{13de(a+a\sin(c+dx))^4} - \frac{10(e\cos(c+dx))^{3/2}}{117ade(a+a\sin(c+dx))^3} - \frac{2(e\cos(c+dx)}{39de\,(a^2+a^2\sin(c}$$

$$= -\frac{2(e\cos(c+dx))^{3/2}}{13de(a+a\sin(c+dx))^4} - \frac{10(e\cos(c+dx))^{3/2}}{117ade(a+a\sin(c+dx))^3} - \frac{2(e\cos(c+dx)}{39de\,(a^2+a^2\sin(c}$$

$$= -\frac{2(e\cos(c+dx))^{3/2}}{13de(a+a\sin(c+dx))^4} - \frac{10(e\cos(c+dx))^{3/2}}{117ade(a+a\sin(c+dx))^3} - \frac{2(e\cos(c+dx)}{39de\,(a^2+a^2\sin(c}$$

$$= -\frac{2\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{39a^4d\sqrt{\cos(c+dx)}} - \frac{2(e\cos(c+dx))^{3/2}}{13de(a+a\sin(c+dx))^4} - \frac{10(e\cos(}{117ade(a+}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.05, size = 66, normalized size = 0.35

$$-\frac{(e\cos(c+dx))^{3/2}\,{}_2F_1\left(\frac{3}{4},\frac{17}{4};\frac{7}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{12\sqrt[4]{2}\,a^4de(1+\sin(c+dx))^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Cos[c + d*x]]/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] -1/12*((e*Cos[c + d*x])^(3/2)*Hypergeometric2F1[3/4, 17/4, 7/4, (1 - Sin[c
+ d*x])/2])/(2^(1/4)*a^4*d*e*(1 + Sin[c + d*x])^(3/4))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 693 vs. $2(195) = 390$.
time = 14.62, size = 694, normalized size = 3.63

method	result
default	$-\frac{2\left(192\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ \text{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ \left(\sin^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-384\left(\sin^{14}\left(\frac{dx}{2}+\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] -2/117/(64*sin(1/2*d*x+1/2*c)^12-192*sin(1/2*d*x+1/2*c)^10+240*sin(1/2*d*x+
1/2*c)^8-160*sin(1/2*d*x+1/2*c)^6+60*sin(1/2*d*x+1/2*c)^4-12*sin(1/2*d*x+1/
2*c)^2+1)/a^4/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*(192*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^12-384*sin(1/2*d*x+1/2*c)^14*cos
(1/2*d*x+1/2*c)-576*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^10+1152*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12+720*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*
d*x+1/2*c)^8-1472*sin(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)-480*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^6+1024*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c
)^8+180*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-280*sin(1/2*d*x+1/2*c
)^6*cos(1/2*d*x+1/2*c)-36*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2-40*
sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-208*sin(1/2*d*x+1/2*c)^5+3*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2))-120*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+208*sin(1/2*d*
x+1/2*c)^3+20*sin(1/2*d*x+1/2*c))*e/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] e^(1/2)*integrate(sqrt(cos(d*x + c))/(a*sin(d*x + c) + a)^4, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 520, normalized size = 2.72

$$\frac{3\left(-i\sqrt{2}\cos\left(dx+c\right)^{4}e^{\frac{1}{2}}+3i\sqrt{2}\cos\left(dx+c\right)^{3}e^{\frac{1}{2}}+8i\sqrt{2}\cos\left(dx+c\right)^{2}e^{\frac{1}{2}}-4i\sqrt{2}\cos\left(dx+c\right)e^{\frac{1}{2}}+\left(i\sqrt{2}\cos\left(dx+c\right)^{3}e^{\frac{1}{2}}+4i\sqrt{2}\cos\left(dx+c\right)^{2}e^{\frac{1}{2}}-4i\sqrt{2}\cos\left(dx+c\right)e^{\frac{1}{2}}-8i\sqrt{2}e^{\frac{1}{2}}\right)\sin\left(dx+c\right)-8i\sqrt{2}e^{\frac{1}{2}}\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)\right)+3\left(i\sqrt{2}\cos\left(dx+c\right)^{4}e^{\frac{1}{2}}-3i\sqrt{2}\cos\left(dx+c\right)^{3}e^{\frac{1}{2}}-8i\sqrt{2}\cos\left(dx+c\right)^{2}e^{\frac{1}{2}}+4i\sqrt{2}\cos\left(dx+c\right)e^{\frac{1}{2}}+\left(-i\sqrt{2}\cos\left(dx+c\right)^{3}e^{\frac{1}{2}}-4i\sqrt{2}\cos\left(dx+c\right)^{2}e^{\frac{1}{2}}+4i\sqrt{2}\cos\left(dx+c\right)e^{\frac{1}{2}}+8i\sqrt{2}e^{\frac{1}{2}}\right)\sin\left(dx+c\right)+8i\sqrt{2}e^{\frac{1}{2}}\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)\right)+2\left(3\cos\left(dx+c\right)^{4}e^{\frac{1}{2}}+12\cos\left(dx+c\right)^{3}e^{\frac{1}{2}}-14\cos\left(dx+c\right)^{2}e^{\frac{1}{2}}-32\cos\left(dx+c\right)e^{\frac{1}{2}}+\left(3\cos\left(dx+c\right)^{3}e^{\frac{1}{2}}-9\cos\left(dx+c\right)^{2}e^{\frac{1}{2}}-23\cos\left(dx+c\right)e^{\frac{1}{2}}+9e^{\frac{1}{2}}\right)\sin\left(dx+c\right)-9e^{\frac{1}{2}}\right)\sqrt{\cos\left(dx+c\right)}}{117\left(a^{4}d\cos\left(dx+c\right)^{4}-3a^{4}d\cos\left(dx+c\right)^{3}-8a^{4}d\cos\left(dx+c\right)^{2}+4a^{4}d\cos\left(dx+c\right)+8a^{4}d-\left(a^{4}d\cos\left(dx+c\right)^{3}+4a^{4}d\cos\left(dx+c\right)^{2}-4a^{4}d\cos\left(dx+c\right)-8a^{4}d\right)\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/117*(3*(-I*sqrt(2)*cos(d*x + c)^4*e^(1/2) + 3*I*sqrt(2)*cos(d*x + c)^3*e^
(1/2) + 8*I*sqrt(2)*cos(d*x + c)^2*e^(1/2) - 4*I*sqrt(2)*cos(d*x + c)*e^(1/
2) + (I*sqrt(2)*cos(d*x + c)^3*e^(1/2) + 4*I*sqrt(2)*cos(d*x + c)^2*e^(1/2)
 - 4*I*sqrt(2)*cos(d*x + c)*e^(1/2) - 8*I*sqrt(2)*e^(1/2))*sin(d*x + c) - 8
*I*sqrt(2)*e^(1/2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d
*x + c) + I*sin(d*x + c))) + 3*(I*sqrt(2)*cos(d*x + c)^4*e^(1/2) - 3*I*sqrt
(2)*cos(d*x + c)^3*e^(1/2) - 8*I*sqrt(2)*cos(d*x + c)^2*e^(1/2) + 4*I*sqrt(
2)*cos(d*x + c)*e^(1/2) + (-I*sqrt(2)*cos(d*x + c)^3*e^(1/2) - 4*I*sqrt(2)*
cos(d*x + c)^2*e^(1/2) + 4*I*sqrt(2)*cos(d*x + c)*e^(1/2) + 8*I*sqrt(2)*e^(
1/2))*sin(d*x + c) + 8*I*sqrt(2)*e^(1/2))*weierstrassZeta(-4, 0, weierstras
sPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(3*cos(d*x + c)^4*e^(1
/2) + 12*cos(d*x + c)^3*e^(1/2) - 14*cos(d*x + c)^2*e^(1/2) - 32*cos(d*x +
c)*e^(1/2) + (3*cos(d*x + c)^3*e^(1/2) - 9*cos(d*x + c)^2*e^(1/2) - 23*cos(
d*x + c)*e^(1/2) + 9*e^(1/2))*sin(d*x + c) - 9*e^(1/2))*sqrt(cos(d*x + c)))
/(a^4*d*cos(d*x + c)^4 - 3*a^4*d*cos(d*x + c)^3 - 8*a^4*d*cos(d*x + c)^2 +
4*a^4*d*cos(d*x + c) + 8*a^4*d - (a^4*d*cos(d*x + c)^3 + 4*a^4*d*cos(d*x +
c)^2 - 4*a^4*d*cos(d*x + c) - 8*a^4*d)*sin(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(1/2)/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3064 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(d*x + c))*e^(1/2)/(a*sin(d*x + c) + a)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{e \cos(c + d x)}}{(a + a \sin(c + d x))^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(1/2)/(a + a*sin(c + d*x))^4,x)
[Out] int((e*cos(c + d*x))^(1/2)/(a + a*sin(c + d*x))^4, x)
```

3.271 $\int \frac{1}{\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=191

$$\frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{33a^4d\sqrt{e\cos(c+dx)}}-\frac{2\sqrt{e\cos(c+dx)}}{15de(a+a\sin(c+dx))^4}-\frac{14\sqrt{e\cos(c+dx)}}{165ade(a+a\sin(c+dx))^3}-\frac{2\sqrt{e\cos(c+}}{33de\,(a^2+a^2\sin(}$$

```
[Out] 2/33*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+
1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/a^4/d/(e*cos(d*x+c))^(1/2)-2/15*(e*cos(d*x
+c))^(1/2)/d/e/(a+a*sin(d*x+c))^4-14/165*(e*cos(d*x+c))^(1/2)/a/d/e/(a+a*si
n(d*x+c))^3-2/33*(e*cos(d*x+c))^(1/2)/d/e/(a^2+a^2*sin(d*x+c))^2-2/33*(e*co
s(d*x+c))^(1/2)/d/e/(a^4+a^4*sin(d*x+c))
```

Rubi [A]

time = 0.17, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2760, 2762, 2721, 2720}

$$-\frac{2\sqrt{e\cos(c+dx)}}{33de\left(a^4\sin(c+dx)+a^4\right)}+\frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{33a^4d\sqrt{e\cos(c+dx)}}-\frac{2\sqrt{e\cos(c+dx)}}{33de\left(a^2\sin(c+dx)+a^2\right)^2}-\frac{14\sqrt{e\cos(c+dx)}}{165ade(a\sin(c+dx)+a)^3}-\frac{2\sqrt{e\cos(c+dx)}}{15de(a\sin(c+dx)+a)^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])^4),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(33*a^4*d*Sqrt[e*Cos[c + d
*x]]) - (2*Sqrt[e*Cos[c + d*x]])/(15*d*e*(a + a*Sin[c + d*x])^4) - (14*Sqrt
[e*Cos[c + d*x]])/(165*a*d*e*(a + a*Sin[c + d*x])^3) - (2*Sqrt[e*Cos[c + d*
x]])/(33*d*e*(a^2 + a^2*Sin[c + d*x])^2) - (2*Sqrt[e*Cos[c + d*x]])/(33*d*e
*(a^4 + a^4*Sin[c + d*x]))
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2760

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*(2*m + p + 1))), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
```

```
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2762

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[b*((g*Cos[e + f*x])^(p + 1)/(a*f*g*(p - 1)*(a + b*S
in[e + f*x]))), x] + Dist[p/(a*(p - 1)), Int[(g*Cos[e + f*x])^p, x], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] &&  !GeQ[p, 1] && Integer
Q[2*p]
```

Rubi steps

$$\int \frac{1}{\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^4}\,dx = -\frac{2\sqrt{e\cos(c+dx)}}{15de(a+a\sin(c+dx))^4}+\frac{7\int \frac{1}{\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))}}{15a}$$

$$= -\frac{2\sqrt{e\cos(c+dx)}}{15de(a+a\sin(c+dx))^4}-\frac{14\sqrt{e\cos(c+dx)}}{165ade(a+a\sin(c+dx))^3}+\frac{7}{}$$

$$= -\frac{2\sqrt{e\cos(c+dx)}}{15de(a+a\sin(c+dx))^4}-\frac{14\sqrt{e\cos(c+dx)}}{165ade(a+a\sin(c+dx))^3}-\frac{}{33}$$

$$= -\frac{2\sqrt{e\cos(c+dx)}}{15de(a+a\sin(c+dx))^4}-\frac{14\sqrt{e\cos(c+dx)}}{165ade(a+a\sin(c+dx))^3}-\frac{}{33}$$

$$= -\frac{2\sqrt{e\cos(c+dx)}}{15de(a+a\sin(c+dx))^4}-\frac{14\sqrt{e\cos(c+dx)}}{165ade(a+a\sin(c+dx))^3}-\frac{}{33}$$

$$= \frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{33a^4d\sqrt{e\cos(c+dx)}}-\frac{2\sqrt{e\cos(c+dx)}}{15de(a+a\sin(c+dx))^4}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.06, size = 66, normalized size = 0.35

$$-\frac{\sqrt{e\cos(c+dx)}\,{}_2F_1\left(\frac{1}{4},\frac{19}{4};\frac{5}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{4\ 2^{3/4}a^4de\sqrt[4]{1+\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])^4),x]
```

```
[Out] -1/4*(Sqrt[e*Cos[c + d*x]]*Hypergeometric2F1[1/4, 19/4, 5/4, (1 - Sin[c + d
*x])/2])/(2^(3/4)*a^4*d*e*(1 + Sin[c + d*x])^(1/4))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 761 vs. $2(195) = 390$.
time = 15.40, size = 762, normalized size = 3.99

method	result	size
default	Expression too large to display	762

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/165/(128*sin(1/2*d*x+1/2*c)^14-448*sin(1/2*d*x+1/2*c)^12+672*sin(1/2*d*x
+1/2*c)^10-560*sin(1/2*d*x+1/2*c)^8+280*sin(1/2*d*x+1/2*c)^6-84*sin(1/2*d*x
+1/2*c)^4+14*sin(1/2*d*x+1/2*c)^2-1)/a^4/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x
+1/2*c)^2*e+e)^(1/2)*(640*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^14-22
40*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^12+640*sin(1/2*d*x+1/2*c)^14
*cos(1/2*d*x+1/2*c)+3360*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^10-192
0*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12-2800*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*si
n(1/2*d*x+1/2*c)^8+2496*sin(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+1400*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^6-1792*cos(1/2*d*x+1/2*c)*sin(1/2*d*
x+1/2*c)^8-420*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4+616*sin(1/2*d*
x+1/2*c)^6*cos(1/2*d*x+1/2*c)+70*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c
)^2-40*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+240*sin(1/2*d*x+1/2*c)^5-5*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))+160*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-240*sin
(1/2*d*x+1/2*c)^3-28*sin(1/2*d*x+1/2*c))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] e^(-1/2)*integrate(1/((a*sin(d*x + c) + a)^4*sqrt(cos(d*x + c))), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 276, normalized size = 1.45

$$\frac{5\left(-i\sqrt{2}\cos\left(dx+c\right)^{4}+8i\sqrt{2}\cos\left(dx+c\right)^{2}+4\left(i\sqrt{2}\cos\left(dx+c\right)^{2}-2i\sqrt{2}\right)\sin\left(dx+c\right)-8i\sqrt{2}\right)\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right))+5\left(i\sqrt{2}\cos\left(dx+c\right)^{4}-8i\sqrt{2}\cos\left(dx+c\right)^{2}+4\left(-i\sqrt{2}\cos\left(dx+c\right)^{2}+2i\sqrt{2}\right)\sin\left(dx+c\right)+8i\sqrt{2}\right)\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right))+2\left(20\cos\left(dx+c\right)^{2}+\left(5\cos\left(dx+c\right)^{2}-37\right)\sin\left(dx+c\right)-48\right)\sqrt{\cos\left(dx+c\right)}}{165\left(a^{4}d\cos\left(dx+c\right)^{4}e^{\frac{1}{2}}-8a^{4}d\cos\left(dx+c\right)^{2}e^{\frac{1}{2}}+8a^{4}de^{\frac{1}{2}}-4\left(a^{4}d\cos\left(dx+c\right)^{2}e^{\frac{1}{2}}-2a^{4}de^{\frac{1}{2}}\right)\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/165*(5*(-I*sqrt(2)*cos(d*x + c)^4 + 8*I*sqrt(2)*cos(d*x + c)^2 + 4*(I*sqr
t(2)*cos(d*x + c)^2 - 2*I*sqrt(2))*sin(d*x + c) - 8*I*sqrt(2))*weierstrassP
Inverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*(I*sqrt(2)*cos(d*x + c)^4
 - 8*I*sqrt(2)*cos(d*x + c)^2 + 4*(-I*sqrt(2)*cos(d*x + c)^2 + 2*I*sqrt(2))
*sin(d*x + c) + 8*I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) - I*si
n(d*x + c)) + 2*(20*cos(d*x + c)^2 + (5*cos(d*x + c)^2 - 37)*sin(d*x + c) -
 48)*sqrt(cos(d*x + c)))/(a^4*d*cos(d*x + c)^4*e^(1/2) - 8*a^4*d*cos(d*x +
c)^2*e^(1/2) + 8*a^4*d*e^(1/2) - 4*(a^4*d*cos(d*x + c)^2*e^(1/2) - 2*a^4*d*
e^(1/2))*sin(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))**4/(e*cos(d*x+c))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3065 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))^4/(e*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(e^(-1/2)/((a*sin(d*x + c) + a)^4*sqrt(cos(d*x + c))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{e\,\cos\left(c+d\,x\right)}\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cos(c + d*x))^(1/2)*(a + a*sin(c + d*x))^4),x)
```

```
[Out] int(1/((e*cos(c + d*x))^(1/2)*(a + a*sin(c + d*x))^4), x)
```

3.272 $\int \frac{1}{(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=225

$$-\frac{42\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{221a^4de^2\sqrt{\cos(c+dx)}}+\frac{42\sin(c+dx)}{221a^4de\sqrt{e\cos(c+dx)}}-\frac{2}{17de\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^4}-$$

```
[Out] 42/221*sin(d*x+c)/a^4/d/e/(e*cos(d*x+c))^(1/2)-2/17/d/e/(a+a*sin(d*x+c))^4/
(e*cos(d*x+c))^(1/2)-18/221/a/d/e/(a+a*sin(d*x+c))^3/(e*cos(d*x+c))^(1/2)-1
4/221/d/e/(a^2+a^2*sin(d*x+c))^2/(e*cos(d*x+c))^(1/2)-14/221/d/e/(a^4+a^4*s
in(d*x+c))/(e*cos(d*x+c))^(1/2)-42/221*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2
*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/a^4/
d/e^2/cos(d*x+c)^(1/2)
```

Rubi [A]

time = 0.21, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2760, 2762, 2716, 2721, 2719}

$$-\frac{42E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{221a^4de^2\sqrt{\cos(c+dx)}}+\frac{42\sin(c+dx)}{221a^4de\sqrt{e\cos(c+dx)}}-\frac{14}{221de\left(a^4\sin(c+dx)+a^4\right)\sqrt{e\cos(c+dx)}}-\frac{14}{221de\left(a^2\sin(c+dx)+a^2\right)^2\sqrt{e\cos(c+dx)}}-\frac{18}{221ade(a\sin(c+dx)+a)^3\sqrt{e\cos(c+dx)}}-\frac{2}{17de(a\sin(c+dx)+a)^4\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x])^4),x]
```

```
[Out] (-42*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(221*a^4*d*e^2*Sqrt[Co
s[c + d*x]]) + (42*Sin[c + d*x])/(221*a^4*d*e*Sqrt[e*Cos[c + d*x]]) - 2/(17
*d*e*Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])^4) - 18/(221*a*d*e*Sqrt[e*Co
s[c + d*x]]*(a + a*Sin[c + d*x])^3) - 14/(221*d*e*Sqrt[e*Cos[c + d*x]]*(a^2
 + a^2*Sin[c + d*x])^2) - 14/(221*d*e*Sqrt[e*Cos[c + d*x]]*(a^4 + a^4*Sin[c
 + d*x]))
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
```

```
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2760

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*(2*m + p + 1))), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2762

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[b*((g*Cos[e + f*x])^(p + 1)/(a*f*g*(p - 1)*(a + b*S
in[e + f*x]))), x] + Dist[p/(a*(p - 1)), Int[(g*Cos[e + f*x])^p, x], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] &&  !GeQ[p, 1] && Integer
Q[2*p]
```

Rubi steps

$$\int \frac{1}{(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^4}\,dx = -\frac{2}{17de\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^4} + \frac{9\int \frac{}{(e\cos(c+dx))^{3}}}{}$$

$$= -\frac{2}{17de\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^4} - \frac{}{221ade\sqrt{e\cos}}$$

$$= -\frac{2}{17de\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^4} - \frac{}{221ade\sqrt{e\cos}}$$

$$= -\frac{2}{17de\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^4} - \frac{}{221ade\sqrt{e\cos}}$$

$$= \frac{42\sin(c+dx)}{221a^4de\sqrt{e\cos(c+dx)}} - \frac{2}{17de\sqrt{e\cos(c+dx)}\,(a+a\sin(c}$$

$$= \frac{42\sin(c+dx)}{221a^4de\sqrt{e\cos(c+dx)}} - \frac{2}{17de\sqrt{e\cos(c+dx)}\,(a+a\sin(c}$$

$$= -\frac{42\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{221a^4de^2\sqrt{\cos(c+dx)}} + \frac{42\sin(c+dx)}{221a^4de\sqrt{e\cos(c+}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 0.09, size = 66, normalized size = 0.29

$$\frac{{}_2F_1\left(-\frac{1}{4},\frac{21}{4};\frac{3}{4};\frac{1}{2}(1-\sin(c+dx))\right)\sqrt[4]{1+\sin(c+dx)}}{8\sqrt[4]{2}\,a^4de\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x])^4),x]
```

```
[Out] (Hypergeometric2F1[-1/4, 21/4, 3/4, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x]
)^(1/4))/(8*2^(1/4)*a^4*d*e*Sqrt[e*Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 877 vs. $2(225) = 450$.
time = 19.17, size = 878, normalized size = 3.90

method	result	size
default	Expression too large to display	878

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] -2/221/(256*sin(1/2*d*x+1/2*c)^16-1024*sin(1/2*d*x+1/2*c)^14+1792*sin(1/2*d
*x+1/2*c)^12-1792*sin(1/2*d*x+1/2*c)^10+1120*sin(1/2*d*x+1/2*c)^8-448*sin(1
/2*d*x+1/2*c)^6+112*sin(1/2*d*x+1/2*c)^4-16*sin(1/2*d*x+1/2*c)^2+1)/a^4/sin
(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/e*(5376*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2))*sin(1/2*d*x+1/2*c)^16-10752*sin(1/2*d*x+1/2*c)^18*cos(1/2*d*x+1/2
*c)-21504*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^14+43008*sin(1/2*d*x+
1/2*c)^16*cos(1/2*d*x+1/2*c)+37632*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2
*c)^12-76160*sin(1/2*d*x+1/2*c)^14*cos(1/2*d*x+1/2*c)-37632*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*sin(1/2*d*x+1/2*c)^10+77952*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)
^12+23520*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^8-50560*sin(1/2*d*x+1
/2*c)^10*cos(1/2*d*x+1/2*c)-9408*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c
)^6+21376*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+2352*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*sin(1/2*d*x+1/2*c)^4-5656*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-336*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+792*sin(1/2*d*x+1/2*c)^4*cos(
1/2*d*x+1/2*c)-272*sin(1/2*d*x+1/2*c)^5+21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*
```

```
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-242*sin
(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+272*sin(1/2*d*x+1/2*c)^3+36*sin(1/2*d*
x+1/2*c))/d
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^4,x, algorithm="maxima")
[Out] Timed out
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.15, size = 338, normalized size = 1.50

$$\frac{21 \left(-i \sqrt{2} \cos\left(dx+c\right)^{5}+8 i \sqrt{2} \cos\left(dx+c\right)^{3}+4 \left(i \sqrt{2} \cos\left(dx+c\right)^{3}-2 i \sqrt{2} \cos\left(dx+c\right)\right) \sin\left(dx+c\right)-8 i \sqrt{2} \cos\left(dx+c\right)\right) \mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i \sin\left(dx+c\right)\right)\right)+21 \left(i \sqrt{2} \cos\left(dx+c\right)^{5}-8 i \sqrt{2} \cos\left(dx+c\right)^{3}+4 \left(-i \sqrt{2} \cos\left(dx+c\right)^{3}+2 i \sqrt{2} \cos\left(dx+c\right)\right) \sin\left(dx+c\right)+8 i \sqrt{2} \cos\left(dx+c\right)\right) \mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i \sin\left(dx+c\right)\right)\right)+2 \left(84 \cos\left(dx+c\right)^{4}-224 \cos\left(dx+c\right)^{2}+\left(21 \cos\left(dx+c\right)^{4}-161 \cos\left(dx+c\right)^{2}+117\right) \sin\left(dx+c\right)+104\right) \sqrt{\cos\left(dx+c\right)}}{221 \left(a^{4} d \cos\left(dx+c\right)^{5} e^{\frac{3}{2}}-8 a^{4} d \cos\left(dx+c\right)^{3} e^{\frac{3}{2}}+8 a^{4} d \cos\left(dx+c\right) e^{\frac{3}{2}}-4 \left(a^{4} d \cos\left(dx+c\right)^{3} e^{\frac{3}{2}}-2 a^{4} d \cos\left(dx+c\right) e^{\frac{3}{2}}\right) \sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
[Out] 1/221*(21*(-I*sqrt(2)*cos(d*x + c)^5 + 8*I*sqrt(2)*cos(d*x + c)^3 + 4*(I*sq
rt(2)*cos(d*x + c)^3 - 2*I*sqrt(2)*cos(d*x + c))*sin(d*x + c) - 8*I*sqrt(2)
*cos(d*x + c))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x +
c) + I*sin(d*x + c))) + 21*(I*sqrt(2)*cos(d*x + c)^5 - 8*I*sqrt(2)*cos(d*x
+ c)^3 + 4*(-I*sqrt(2)*cos(d*x + c)^3 + 2*I*sqrt(2)*cos(d*x + c))*sin(d*x +
 c) + 8*I*sqrt(2)*cos(d*x + c))*weierstrassZeta(-4, 0, weierstrassPInverse(
-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(84*cos(d*x + c)^4 - 224*cos(d*x
 + c)^2 + (21*cos(d*x + c)^4 - 161*cos(d*x + c)^2 + 117)*sin(d*x + c) + 104
)*sqrt(cos(d*x + c)))/(a^4*d*cos(d*x + c)^5*e^(3/2) - 8*a^4*d*cos(d*x + c)^
3*e^(3/2) + 8*a^4*d*cos(d*x + c)*e^(3/2) - 4*(a^4*d*cos(d*x + c)^3*e^(3/2)
- 2*a^4*d*cos(d*x + c)*e^(3/2))*sin(d*x + c))
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(3/2)/(a+a*sin(d*x+c))**4,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 4850 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^4,x, algorithm="giac")
[Out] integrate(e^(-3/2)/((a*sin(d*x + c) + a)^4*cos(d*x + c)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(e\,\cos\left(c+d\,x\right)\right)^{3/2}\,\left(a+a\,\sin\left(c+d\,x\right)\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cos(c + d*x))^(3/2)*(a + a*sin(c + d*x))^4),x)
[Out] int(1/((e*cos(c + d*x))^(3/2)*(a + a*sin(c + d*x))^4), x)
```

3.273 $\int (e\cos(c+dx))^{3/2}\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=236

$$-\frac{a(e\cos(c+dx))^{5/2}}{2de\sqrt{a+a\sin(c+dx)}}+\frac{3e\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}{4d}-\frac{3e^{3/2}\sinh^{-1}\left(\frac{\sqrt{e\cos(c+dx)}}{\sqrt{e}}\right)\sqrt{1+}}{4d(1+\cos(c+dx)}$$

```
[Out] -1/2*a*(e*cos(d*x+c))^(5/2)/d/e/(a+a*sin(d*x+c))^(1/2)+3/4*e*(e*cos(d*x+c))
^(1/2)*(a+a*sin(d*x+c))^(1/2)/d-3/4*e^(3/2)*arcsinh((e*cos(d*x+c))^(1/2)/e^
(1/2))*(1+cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^(1/2)/d/(1+cos(d*x+c)+sin(d*x+
c))+3/4*e^(3/2)*arctan(sin(d*x+c)*e^(1/2)/(e*cos(d*x+c))^(1/2)/(1+cos(d*x+c
))^(1/2))*(1+cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^(1/2)/d/(1+cos(d*x+c)+sin(d
*x+c))
```

Rubi [A]

time = 0.25, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {2757, 2764, 2756, 2854, 209, 2912, 65, 221}

$$\frac{3e^{3/2}\sqrt{\cos(c+dx)+1}\sqrt{a\sin(c+dx)+a}\,\text{ArcTan}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\sqrt{e\cos(c+dx)}}\right)}{4d(\sin(c+dx)+\cos(c+dx)+1)}-\frac{3e^{3/2}\sqrt{\cos(c+dx)+1}\sqrt{a\sin(c+dx)+a}\sinh^{-1}\left(\frac{\sqrt{e\cos(c+dx)}}{\sqrt{e}}\right)}{4d(\sin(c+dx)+\cos(c+dx)+1)}-\frac{a(e\cos(c+dx))^{5/2}}{2de\sqrt{a\sin(c+dx)+a}}+\frac{3e\sqrt{a\sin(c+dx)+a}\sqrt{e\cos(c+dx)}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(3/2)*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -1/2*(a*(e*Cos[c + d*x])^(5/2))/(d*e*Sqrt[a + a*Sin[c + d*x]]) + (3*e*Sqrt[
e*Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(4*d) - (3*e^(3/2)*ArcSinh[Sqrt[e
*Cos[c + d*x]]/Sqrt[e]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(4
*d*(1 + Cos[c + d*x] + Sin[c + d*x])) + (3*e^(3/2)*ArcTan[(Sqrt[e]*Sin[c +
d*x])/(Sqrt[e*Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]])]*Sqrt[1 + Cos[c + d*x]]
*Sqrt[a + a*Sin[c + d*x]])/(4*d*(1 + Cos[c + d*x] + Sin[c + d*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 2756

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[cos[(e_.) + (f_.)*(x_)]
*(g_.)], x_Symbol] :> Dist[a*Sqrt[1 + Cos[e + f*x]]*(Sqrt[a + b*Sin[e + f*x
]]/(a + a*Cos[e + f*x] + b*Sin[e + f*x])), Int[Sqrt[1 + Cos[e + f*x]]/Sqrt[
g*Cos[e + f*x]], x], x] + Dist[b*Sqrt[1 + Cos[e + f*x]]*(Sqrt[a + b*Sin[e +
 f*x]]/(a + a*Cos[e + f*x] + b*Sin[e + f*x])), Int[Sin[e + f*x]/(Sqrt[g*Cos
[e + f*x]]*Sqrt[1 + Cos[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, g}, x] &&
EqQ[a^2 - b^2, 0]
```

Rule 2757

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ
[2*m, 2*p]
```

Rule 2764

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]], x_Symbol] :> Simp[g*Sqrt[g*Cos[e + f*x]]*(Sqrt[a + b*Sin[e + f*x
]]/(b*f)), x] + Dist[g^2/(2*a), Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[g*Cos[e +
 f*x]], x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2854

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b + d*x^2), x], x
, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]))], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (e\cos(c+dx))^{3/2}\sqrt{a+a\sin(c+dx)}\,dx = -\frac{a(e\cos(c+dx))^{5/2}}{2de\sqrt{a+a\sin(c+dx)}} + \frac{1}{4}(3a)\int \frac{(e\cos(c+dx))^{3/2}}{\sqrt{a+a\sin(c+dx)}}\,d$$

$$= -\frac{a(e\cos(c+dx))^{5/2}}{2de\sqrt{a+a\sin(c+dx)}} + \frac{3e\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c}{4d}$$

$$= -\frac{a(e\cos(c+dx))^{5/2}}{2de\sqrt{a+a\sin(c+dx)}} + \frac{3e\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c}{4d}$$

$$= -\frac{a(e\cos(c+dx))^{5/2}}{2de\sqrt{a+a\sin(c+dx)}} + \frac{3e\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c}{4d}$$

$$= -\frac{a(e\cos(c+dx))^{5/2}}{2de\sqrt{a+a\sin(c+dx)}} + \frac{3e\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c}{4d}$$

$$= -\frac{a(e\cos(c+dx))^{5/2}}{2de\sqrt{a+a\sin(c+dx)}} + \frac{3e\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c}{4d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.99, size = 269, normalized size = 1.14

$$-\frac{iee^{-i(c+dx)}\sqrt{e\cos(c+dx)}\left(-i\sqrt{1+e^{2i(c+dx)}}-2e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}+2ie^{2i(c+dx)}\sqrt{1+e^{2i(c+dx)}}+e^{3i(c+dx)}\sqrt{1+e^{2i(c+dx)}}-3de^{2i(c+dx)}x+3e^{2i(c+dx)}\sinh^{-1}\left(e^{i(c+dx)}\right)-3ie^{2i(c+dx)}\log\left(1+\sqrt{1+e^{2i(c+dx)}}\right)\right)\sqrt{a(1+\sin(c+dx))}}{4d\left(i+e^{i(c+dx)}\right)\sqrt{1+e^{2i(c+dx)}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(3/2)*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] ((-1/4*I)*e*Sqrt[e*Cos[c + d*x]]*((-I)*Sqrt[1 + E^((2*I)*(c + d*x))] - 2*E^
(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))] + (2*I)*E^((2*I)*(c + d*x))*Sqr
t[1 + E^((2*I)*(c + d*x))] + E^((3*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x
))] - 3*d*E^((2*I)*(c + d*x))*x + 3*E^((2*I)*(c + d*x))*ArcSinh[E^(I*(c + d
*x))] - (3*I)*E^((2*I)*(c + d*x))*Log[1 + Sqrt[1 + E^((2*I)*(c + d*x))]])*S
qrt[a*(1 + Sin[c + d*x])])/(d*E^(I*(c + d*x))*(I + E^(I*(c + d*x)))*Sqrt[1
+ E^((2*I)*(c + d*x))])
```

Maple [A]
time = 6.41, size = 241, normalized size = 1.02

method	result
default	$\frac{\left(3\sqrt{2}\,\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2}}{2}\right)\sin(dx+c)-3\sqrt{2}\,\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(}{1+\cos}}}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/d*(3*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*sin(d*x+c)-3*2^(1/2)*(-2*cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x
+c)/cos(d*x+c)*2^(1/2))*sin(d*x+c)-4*cos(d*x+c)^2*sin(d*x+c)-4*cos(d*x+c)^3
+6*cos(d*x+c)*sin(d*x+c)-2*cos(d*x+c)^2+6*cos(d*x+c))*(e*cos(d*x+c))^(3/2)*
(a*(1+sin(d*x+c)))^(1/2)/(sin(d*x+c)-cos(d*x+c)+1)/cos(d*x+c)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima"
)
```

```
[Out] e^(3/2)*integrate(sqrt(a*sin(d*x + c) + a)*cos(d*x + c)^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3208 vs. $2(181) = 362$.
time = 191.68, size = 3208, normalized size = 13.59

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas"
)
```

```
[Out] 1/32*(12*sqrt(2)*d*(a^2/d^4)^(1/4)*arctan(-1/4*(2*sqrt(1/2)*((sqrt(2)*d^3*c
os(d*x + c)^6 + 5*sqrt(2)*d^3*cos(d*x + c)^5 - 8*sqrt(2)*d^3*cos(d*x + c)^4
 - 20*sqrt(2)*d^3*cos(d*x + c)^3 + 8*sqrt(2)*d^3*cos(d*x + c)^2 + 16*sqrt(2
)*d^3*cos(d*x + c) + (sqrt(2)*d^3*cos(d*x + c)^5 - 4*sqrt(2)*d^3*cos(d*x +
c)^4 - 12*sqrt(2)*d^3*cos(d*x + c)^3 + 8*sqrt(2)*d^3*cos(d*x + c)^2 + 16*sq
rt(2)*d^3*cos(d*x + c))*sin(d*x + c))*(a^2/d^4)^(3/4)*e^(9/2) + (sqrt(2)*a*
d*cos(d*x + c)^6*e^3 - 3*sqrt(2)*a*d*cos(d*x + c)^5*e^3 - 8*sqrt(2)*a*d*cos
```

```
(d*x + c)^4*e^3 + 4*sqrt(2)*a*d*cos(d*x + c)^3*e^3 + 8*sqrt(2)*a*d*cos(d*x
+ c)^2*e^3 - (sqrt(2)*a*d*cos(d*x + c)^5*e^3 + 4*sqrt(2)*a*d*cos(d*x + c)^4
*e^3 - 4*sqrt(2)*a*d*cos(d*x + c)^3*e^3 - 8*sqrt(2)*a*d*cos(d*x + c)^2*e^3)
*sin(d*x + c))*(a^2/d^4)^(1/4)*e^(3/2) - (a*cos(d*x + c)^4*e^(9/2) - 3*a*co
s(d*x + c)^3*e^(9/2) - 8*a*cos(d*x + c)^2*e^(9/2) + 4*a*cos(d*x + c)*e^(9/2
) + (2*d^2*cos(d*x + c)^5*e^(3/2) - 5*d^2*cos(d*x + c)^4*e^(3/2) - 19*d^2*c
os(d*x + c)^3*e^(3/2) + 20*d^2*cos(d*x + c)*e^(3/2) + 8*d^2*e^(3/2) - (2*d^
2*cos(d*x + c)^4*e^(3/2) + 9*d^2*cos(d*x + c)^3*e^(3/2) - 4*d^2*cos(d*x + c
)^2*e^(3/2) - 20*d^2*cos(d*x + c)*e^(3/2) - 8*d^2*e^(3/2))*sin(d*x + c))*sq
rt(a^2/d^4)*e^3 + 8*a*e^(9/2) - (a*cos(d*x + c)^3*e^(9/2) + 4*a*cos(d*x + c
)^2*e^(9/2) - 4*a*cos(d*x + c)*e^(9/2) - 8*a*e^(9/2))*sin(d*x + c))*sqrt(a*
sin(d*x + c) + a)*sqrt(cos(d*x + c)))*sqrt((2*a^3*cos(d*x + c)*e^9*sin(d*x
+ c) + 2*a^3*cos(d*x + c)*e^9 + (a^2*d^2*e^6*sin(d*x + c) + a^2*d^2*e^6)*sq
rt(a^2/d^4)*e^3 + (sqrt(2)*a*d^3*(a^2/d^4)^(3/4)*cos(d*x + c)*e^9 + (sqrt(2
)*a^2*d*e^(15/2)*sin(d*x + c) + sqrt(2)*a^2*d*e^(15/2))*(a^2/d^4)^(1/4)*e^(
3/2))*sqrt(a*sin(d*x + c) + a)*sqrt(cos(d*x + c)))/(sin(d*x + c) + 1)) - ((
7*sqrt(2)*a*d^3*cos(d*x + c)^4*e^(9/2) + 3*sqrt(2)*a*d^3*cos(d*x + c)^3*e^(
9/2) - 16*sqrt(2)*a*d^3*cos(d*x + c)^2*e^(9/2) - 4*sqrt(2)*a*d^3*cos(d*x +
c)*e^(9/2) + 8*sqrt(2)*a*d^3*e^(9/2) + (2*sqrt(2)*a*d^3*cos(d*x + c)^4*e^(9
/2) + sqrt(2)*a*d^3*cos(d*x + c)^3*e^(9/2) - 12*sqrt(2)*a*d^3*cos(d*x + c)^
2*e^(9/2) - 4*sqrt(2)*a*d^3*cos(d*x + c)*e^(9/2) + 8*sqrt(2)*a*d^3*e^(9/2))
*sin(d*x + c))*(a^2/d^4)^(3/4)*e^(9/2) + (2*sqrt(2)*a^2*d*cos(d*x + c)^5*e^
(15/2) + sqrt(2)*a^2*d*cos(d*x + c)^4*e^(15/2) - 13*sqrt(2)*a^2*d*cos(d*x +
 c)^3*e^(15/2) - 8*sqrt(2)*a^2*d*cos(d*x + c)^2*e^(15/2) + 12*sqrt(2)*a^2*d
*cos(d*x + c)*e^(15/2) + 8*sqrt(2)*a^2*d*e^(15/2) - (7*sqrt(2)*a^2*d*cos(d*
x + c)^3*e^(15/2) + 4*sqrt(2)*a^2*d*cos(d*x + c)^2*e^(15/2) - 12*sqrt(2)*a^
2*d*cos(d*x + c)*e^(15/2) - 8*sqrt(2)*a^2*d*e^(15/2))*sin(d*x + c))*(a^2/d^
4)^(1/4)*e^(3/2))*sqrt(a*sin(d*x + c) + a)*sqrt(cos(d*x + c)))/(a^3*cos(d*x
 + c)^6*e^9 + a^3*cos(d*x + c)^5*e^9 - 8*a^3*cos(d*x + c)^4*e^9 - 8*a^3*cos
(d*x + c)^3*e^9 + 8*a^3*cos(d*x + c)^2*e^9 + 8*a^3*cos(d*x + c)*e^9 - 4*(a^
3*cos(d*x + c)^4*e^9 + a^3*cos(d*x + c)^3*e^9 - 2*a^3*cos(d*x + c)^2*e^9 -
2*a^3*cos(d*x + c)*e^9)*sin(d*x + c)))*e^(3/2) - 12*sqrt(2)*d*(a^2/d^4)^(1/
4)*arctan(1/4*(2*sqrt(1/2)*((sqrt(2)*d^3*cos(d*x + c)^6 + 5*sqrt(2)*d^3*cos
(d*x + c)^5 - 8*sqrt(2)*d^3*cos(d*x + c)^4 - 20*sqrt(2)*d^3*cos(d*x + c)^3
+ 8*sqrt(2)*d^3*cos(d*x + c)^2 + 16*sqrt(2)*d^3*cos(d*x + c) + (sqrt(2)*d^3
*cos(d*x + c)^5 - 4*sqrt(2)*d^3*cos(d*x + c)^4 - 12*sqrt(2)*d^3*cos(d*x + c
)^3 + 8*sqrt(2)*d^3*cos(d*x + c)^2 + 16*sqrt(2)*d^3*cos(d*x + c))*sin(d*x +
 c))*(a^2/d^4)^(3/4)*e^(9/2) + (sqrt(2)*a*d*cos(d*x + c)^6*e^3 - 3*sqrt(2)*
a*d*cos(d*x + c)^5*e^3 - 8*sqrt(2)*a*d*cos(d*x + c)^4*e^3 + 4*sqrt(2)*a*d*c
os(d*x + c)^3*e^3 + 8*sqrt(2)*a*d*cos(d*x + c)^2*e^3 - (sqrt(2)*a*d*cos(d*x
 + c)^5*e^3 + 4*sqrt(2)*a*d*cos(d*x + c)^4*e^3 - 4*sqrt(2)*a*d*cos(d*x + c)
^3*e^3 - 8*sqrt(2)*a*d*cos(d*x + c)^2*e^3)*sin(d*x + c))*(a^2/d^4)^(1/4)*e^
(3/2) + (a*cos(d*x + c)^4*e^(9/2) - 3*a*cos(d*x + c)^3*e^(9/2) - 8*a*cos(d*
x + c)^2*e^(9/2) + 4*a*cos(d*x + c)*e^(9/2) + (2*d^2*cos(d*x + c)^5*e^(3/2)
 - 5*d^2*cos(d*x + c)^4*e^(3/2) - 19*d^2*cos(d*x + c)^3*e^(3/2) + 20*d^2*co
```

```
s(d*x + c)*e^(3/2) + 8*d^2*e^(3/2) - (2*d^2*cos(d*x + c)^4*e^(3/2) + 9*d^2*
cos(d*x + c)^3*e^(3/2) - 4*d^2*cos(d*x + c)^2*e^(3/2) - 20*d^2*cos(d*x + c)
*e^(3/2) - 8*d^2*e^(3/2))*sin(d*x + c))*sqrt(a^2/d^4)*e^3 + 8*a*e^(9/2) - (
a*cos(d*x + c)^3*e^(9/2) + 4*a*cos(d*x + c)^2*e^(9/2) - 4*a*cos(d*x + c)*e^
(9/2) - 8*a*e^(9/2))*sin(d*x + c))*sqrt(a*sin(d*x + c) + a)*sqrt(cos(d*x +
c)))*sqrt((2*a^3*cos(d*x + c)*e^9*sin(d*x + c) + 2*a^3*cos(d*x + c)*e^9 + (
a^2*d^2*e^6*sin(d*x + c) + a^2*d^2*e^6)*sqrt(a^2/d^4)*e^3 - (sqrt(2)*a*d^3*
(a^2/d^4)^(3/4)*cos(d*x + c)*e^9 + (sqrt(2)*a^2*d*e^(15/2)*sin(d*x + c) + s
qrt(2)*a^2*d*e^(15/2))*(a^2/d^4)^(1/4)*e^(3/2))*sqrt(a*sin(d*x + c) + a)*sq
rt(cos(d*x + c)))/(sin(d*x + c) + 1)) - ((7*sqrt(2)*a*d^3*cos(d*x + c)^4*e^
(9/2) + 3*sqrt(2)*a*d^3*cos(d*x + c)^3*e^(9/2) - 16*sqrt(2)*a*d^3*cos(d*x +
 c)^2*e^(9/2) - 4*sqrt(2)*a*d^3*cos(d*x + c)*e^(9/2) + 8*sqrt(2)*a*d^3*e^(9
/2) + (2*sqrt(2)*a*d^3*cos(d*x + c)^4*e^(9/2) + sqrt(2)*a*d^3*cos(d*x + c)^
3*e^(9/2) - 12*sqrt(2)*a*d^3*cos(d*x + c)^2*e^(9/2) - 4*sqrt(2)*a*d^3*cos(d
*x + c)*e^(9/2) + 8*sqrt(2)*a*d^3*e^(9/2))*sin(...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a(\sin(c+dx)+1)}\,(e\cos(c+dx))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(3/2)*(a+a*sin(d*x+c))**(1/2),x)
[Out] Integral(sqrt(a*(sin(c + d*x) + 1))*(e*cos(c + d*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a*sin(d*x + c) + a)*cos(d*x + c)^(3/2)*e^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (e\,\cos(c+d\,x))^{3/2}\,\sqrt{a+a\,\sin(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(3/2)*(a + a*sin(c + d*x))^(1/2),x)
[Out] int((e*cos(c + d*x))^(3/2)*(a + a*sin(c + d*x))^(1/2), x)
```

3.274 $\int \sqrt{e\cos(c+dx)}\,\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=194

$$-\frac{a(e\cos(c+dx))^{3/2}}{de\sqrt{a+a\sin(c+dx)}}+\frac{\sqrt{e}\,\sinh^{-1}\left(\frac{\sqrt{e\cos(c+dx)}}{\sqrt{e}}\right)\sqrt{1+\cos(c+dx)}\,\sqrt{a+a\sin(c+dx)}}{d(1+\cos(c+dx)+\sin(c+dx))}+\frac{\sqrt{e}\,\mathrm{ta}}{}$$

```
[Out] -a*(e*cos(d*x+c))^(3/2)/d/e/(a+a*sin(d*x+c))^(1/2)+arcsinh((e*cos(d*x+c))^(
1/2)/e^(1/2))*e^(1/2)*(1+cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^(1/2)/d/(1+cos(
d*x+c)+sin(d*x+c))+arctan(sin(d*x+c)*e^(1/2)/(e*cos(d*x+c))^(1/2)/(1+cos(d*
x+c))^(1/2))*e^(1/2)*(1+cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^(1/2)/d/(1+cos(d
*x+c)+sin(d*x+c))
```

Rubi [A]
time = 0.18, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2757, 2763, 2854, 209, 2912, 65, 221}

$$\frac{\sqrt{e}\,\sqrt{\cos(c+dx)+1}\,\sqrt{a\sin(c+dx)+a}\,\mathrm{ArcTan}\left(\frac{\sqrt{e}\,\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\,\sqrt{e\cos(c+dx)}}\right)}{d(\sin(c+dx)+\cos(c+dx)+1)}-\frac{a(e\cos(c+dx))^{3/2}}{de\sqrt{a\sin(c+dx)+a}}+\frac{\sqrt{e}\,\sqrt{\cos(c+dx)+1}\,\sqrt{a\sin(c+dx)+a}\,\sinh^{-1}\left(\frac{\sqrt{e\cos(c+dx)}}{\sqrt{e}}\right)}{d(\sin(c+dx)+\cos(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -((a*(e*Cos[c + d*x])^(3/2))/(d*e*Sqrt[a + a*Sin[c + d*x]])) + (Sqrt[e]*Arc
Sinh[Sqrt[e*Cos[c + d*x]]/Sqrt[e]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c
+ d*x]])/(d*(1 + Cos[c + d*x] + Sin[c + d*x])) + (Sqrt[e]*ArcTan[(Sqrt[e]*S
in[c + d*x])/(Sqrt[e*Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]])]*Sqrt[1 + Cos[c
+ d*x]]*Sqrt[a + a*Sin[c + d*x]])/(d*(1 + Cos[c + d*x] + Sin[c + d*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 2757

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ
[2*m, 2*p]
```

Rule 2763

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[g*Sqrt[1 + Cos[e + f*x]]*(Sqrt[a + b*Sin[e + f*x
]]/(a + a*Cos[e + f*x] + b*Sin[e + f*x])), Int[Sqrt[1 + Cos[e + f*x]]/Sqrt[
g*Cos[e + f*x]], x], x] - Dist[g*Sqrt[1 + Cos[e + f*x]]*(Sqrt[a + b*Sin[e +
 f*x]]/(b + b*Cos[e + f*x] + a*Sin[e + f*x])), Int[Sin[e + f*x]/(Sqrt[g*Cos
[e + f*x]]*Sqrt[1 + Cos[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, g}, x] &&
EqQ[a^2 - b^2, 0]
```

Rule 2854

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b + d*x^2), x], x
, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]))], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \sqrt{e\cos(c+dx)}\,\sqrt{a+a\sin(c+dx)}\,dx = -\frac{a(e\cos(c+dx))^{3/2}}{de\sqrt{a+a\sin(c+dx)}} + \frac{1}{2}a\int \frac{\sqrt{e\cos(c+dx)}}{\sqrt{a+a\sin(c+dx)}}\,dx$$

$$= -\frac{a(e\cos(c+dx))^{3/2}}{de\sqrt{a+a\sin(c+dx)}} + \frac{\left(ae\sqrt{1+\cos(c+dx)}\,\sqrt{a+a\sin\right.}{2(a+a\cos(c+a}$$

$$= -\frac{a(e\cos(c+dx))^{3/2}}{de\sqrt{a+a\sin(c+dx)}} + \frac{\left(ae\sqrt{1+\cos(c+dx)}\,\sqrt{a+a\sin\right.}{2d(a+}$$

$$= -\frac{a(e\cos(c+dx))^{3/2}}{de\sqrt{a+a\sin(c+dx)}} + \frac{\sqrt{e}\,\tan^{-1}\left(\frac{\sqrt{e}\,\sin(c+}{\sqrt{e\cos(c+dx)}\,\sqrt{1+}\right.}{d(1+}$$

$$= -\frac{a(e\cos(c+dx))^{3/2}}{de\sqrt{a+a\sin(c+dx)}} + \frac{\sqrt{e}\,\sinh^{-1}\left(\frac{\sqrt{e\cos(c+dx)}}{\sqrt{e}}\right)\sqrt{}}{d(1+\cos(c+}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.74, size = 195, normalized size = 1.01

$$-\frac{i\sqrt{e\cos(c+dx)}\left(-i\sqrt{1+e^{2i(c+dx)}}+e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}+ide^{i(c+dx)}x+ie^{i(c+dx)}\sinh^{-1}\left(e^{i(c+dx)}\right)-e^{i(c+dx)}\log\left(1+\sqrt{1+e^{2i(c+dx)}}\right)\right)\sqrt{a(1+\sin(c+dx))}}{d\left(i+e^{i(c+dx)}\right)\sqrt{1+e^{2i(c+dx)}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] ((-I)*Sqrt[e*Cos[c + d*x]]*((-I)*Sqrt[1 + E^((2*I)*(c + d*x))] + E^(I*(c +
d*x))*Sqrt[1 + E^((2*I)*(c + d*x))] + I*d*E^(I*(c + d*x))*x + I*E^(I*(c + d
*x))*ArcSinh[E^(I*(c + d*x))] - E^(I*(c + d*x))*Log[1 + Sqrt[1 + E^((2*I)*(
c + d*x))]])*Sqrt[a*(1 + Sin[c + d*x])])/(d*(I + E^(I*(c + d*x)))*Sqrt[1 +
E^((2*I)*(c + d*x))])
```

Maple [A]
time = 0.17, size = 213, normalized size = 1.10

method	result

default	$-\frac{\left(\sqrt{2}\,\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2}}{2}\right)\sin(dx+c)+\sqrt{2}\,\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\arctan\mathrm{h}\left(\frac{\sqrt{-\frac{2\cos(}{1+\cos}}}{}\right.\right.}{2d(\sin(dx+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d*(2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*sin(d*x+c)+2^(1/2)*(-2*cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)
/cos(d*x+c)*2^(1/2))*sin(d*x+c)+2*cos(d*x+c)*sin(d*x+c)+2*cos(d*x+c)^2-2*co
s(d*x+c))*(e*cos(d*x+c))^(1/2)*(a*(1+sin(d*x+c)))^(1/2)/(sin(d*x+c)-cos(d*x
+c)+1)/cos(d*x+c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima"
)
```

```
[Out] e^(1/2)*integrate(sqrt(a*sin(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3259 vs. $2(152) = 304$.
time = 199.18, size = 3259, normalized size = 16.80

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas"
)
```

```
[Out] -1/8*(4*(sqrt(2)*d*sin(d*x + c) + sqrt(2)*d)*(a^2/d^4)^(1/4)*arctan(-1/4*(2
*sqrt(1/2)*((sqrt(2)*d^3*cos(d*x + c)^6 - 3*sqrt(2)*d^3*cos(d*x + c)^5 - 8*
sqrt(2)*d^3*cos(d*x + c)^4 + 4*sqrt(2)*d^3*cos(d*x + c)^3 + 8*sqrt(2)*d^3*c
os(d*x + c)^2 - (sqrt(2)*d^3*cos(d*x + c)^5 + 4*sqrt(2)*d^3*cos(d*x + c)^4
- 4*sqrt(2)*d^3*cos(d*x + c)^3 - 8*sqrt(2)*d^3*cos(d*x + c)^2)*sin(d*x + c)
)*(a^2/d^4)^(3/4)*e^(3/2) + (sqrt(2)*a*d*cos(d*x + c)^6*e + 5*sqrt(2)*a*d*c
os(d*x + c)^5*e - 8*sqrt(2)*a*d*cos(d*x + c)^4*e - 20*sqrt(2)*a*d*cos(d*x +
 c)^3*e + 8*sqrt(2)*a*d*cos(d*x + c)^2*e + 16*sqrt(2)*a*d*cos(d*x + c)*e +
```

(sqrt(2)*a*d*cos(d*x + c)^5*e - 4*sqrt(2)*a*d*cos(d*x + c)^4*e - 12*sqrt(2)
*a*d*cos(d*x + c)^3*e + 8*sqrt(2)*a*d*cos(d*x + c)^2*e + 16*sqrt(2)*a*d*cos
(d*x + c)*e)*sin(d*x + c))*(a^2/d^4)^(1/4)*e^(1/2) - (a*cos(d*x + c)^4*e^(3
/2) - 3*a*cos(d*x + c)^3*e^(3/2) - 8*a*cos(d*x + c)^2*e^(3/2) + 4*a*cos(d*x
 + c)*e^(3/2) + (2*d^2*cos(d*x + c)^5*e^(1/2) - 5*d^2*cos(d*x + c)^4*e^(1/2
) - 19*d^2*cos(d*x + c)^3*e^(1/2) + 20*d^2*cos(d*x + c)*e^(1/2) + 8*d^2*e^(
1/2) - (2*d^2*cos(d*x + c)^4*e^(1/2) + 9*d^2*cos(d*x + c)^3*e^(1/2) - 4*d^2
*cos(d*x + c)^2*e^(1/2) - 20*d^2*cos(d*x + c)*e^(1/2) - 8*d^2*e^(1/2))*sin(
d*x + c))*sqrt(a^2/d^4)*e + 8*a*e^(3/2) - (a*cos(d*x + c)^3*e^(3/2) + 4*a*c
os(d*x + c)^2*e^(3/2) - 4*a*cos(d*x + c)*e^(3/2) - 8*a*e^(3/2))*sin(d*x + c
))*sqrt(a*sin(d*x + c) + a)*sqrt(cos(d*x + c)))*sqrt((2*a^3*cos(d*x + c)*e^
3*sin(d*x + c) + 2*a^3*cos(d*x + c)*e^3 + (a^2*d^2*e^2*sin(d*x + c) + a^2*d
^2*e^2)*sqrt(a^2/d^4)*e + (sqrt(2)*a^2*d*(a^2/d^4)^(1/4)*cos(d*x + c)*e^3 +
 (sqrt(2)*a*d^3*e^(3/2)*sin(d*x + c) + sqrt(2)*a*d^3*e^(3/2))*(a^2/d^4)^(3/
4)*e^(3/2))*sqrt(a*sin(d*x + c) + a)*sqrt(cos(d*x + c)))/(sin(d*x + c) + 1)
) + ((2*sqrt(2)*a*d^3*cos(d*x + c)^5*e^(3/2) + sqrt(2)*a*d^3*cos(d*x + c)^4
*e^(3/2) - 13*sqrt(2)*a*d^3*cos(d*x + c)^3*e^(3/2) - 8*sqrt(2)*a*d^3*cos(d*
x + c)^2*e^(3/2) + 12*sqrt(2)*a*d^3*cos(d*x + c)*e^(3/2) + 8*sqrt(2)*a*d^3*
e^(3/2) - (7*sqrt(2)*a*d^3*cos(d*x + c)^3*e^(3/2) + 4*sqrt(2)*a*d^3*cos(d*x
 + c)^2*e^(3/2) - 12*sqrt(2)*a*d^3*cos(d*x + c)*e^(3/2) - 8*sqrt(2)*a*d^3*e
^(3/2))*sin(d*x + c))*(a^2/d^4)^(3/4)*e^(3/2) + (7*sqrt(2)*a^2*d*cos(d*x +
c)^4*e^(5/2) + 3*sqrt(2)*a^2*d*cos(d*x + c)^3*e^(5/2) - 16*sqrt(2)*a^2*d*co
s(d*x + c)^2*e^(5/2) - 4*sqrt(2)*a^2*d*cos(d*x + c)*e^(5/2) + 8*sqrt(2)*a^2
*d*e^(5/2) + (2*sqrt(2)*a^2*d*cos(d*x + c)^4*e^(5/2) + sqrt(2)*a^2*d*cos(d*
x + c)^3*e^(5/2) - 12*sqrt(2)*a^2*d*cos(d*x + c)^2*e^(5/2) - 4*sqrt(2)*a^2*
d*cos(d*x + c)*e^(5/2) + 8*sqrt(2)*a^2*d*e^(5/2))*sin(d*x + c))*(a^2/d^4)^(
1/4)*e^(1/2))*sqrt(a*sin(d*x + c) + a)*sqrt(cos(d*x + c)))/(a^3*cos(d*x + c
)^6*e^3 + a^3*cos(d*x + c)^5*e^3 - 8*a^3*cos(d*x + c)^4*e^3 - 8*a^3*cos(d*x
 + c)^3*e^3 + 8*a^3*cos(d*x + c)^2*e^3 + 8*a^3*cos(d*x + c)*e^3 - 4*(a^3*co
s(d*x + c)^4*e^3 + a^3*cos(d*x + c)^3*e^3 - 2*a^3*cos(d*x + c)^2*e^3 - 2*a^
3*cos(d*x + c)*e^3)*sin(d*x + c)))*e^(1/2) - 4*(sqrt(2)*d*sin(d*x + c) + sq
rt(2)*d)*(a^2/d^4)^(1/4)*arctan(1/4*(2*sqrt(1/2)*((sqrt(2)*d^3*cos(d*x + c)
^6 - 3*sqrt(2)*d^3*cos(d*x + c)^5 - 8*sqrt(2)*d^3*cos(d*x + c)^4 + 4*sqrt(2
)*d^3*cos(d*x + c)^3 + 8*sqrt(2)*d^3*cos(d*x + c)^2 - (sqrt(2)*d^3*cos(d*x
+ c)^5 + 4*sqrt(2)*d^3*cos(d*x + c)^4 - 4*sqrt(2)*d^3*cos(d*x + c)^3 - 8*sq
rt(2)*d^3*cos(d*x + c)^2)*sin(d*x + c))*(a^2/d^4)^(3/4)*e^(3/2) + (sqrt(2)*
a*d*cos(d*x + c)^6*e + 5*sqrt(2)*a*d*cos(d*x + c)^5*e - 8*sqrt(2)*a*d*cos(d
*x + c)^4*e - 20*sqrt(2)*a*d*cos(d*x + c)^3*e + 8*sqrt(2)*a*d*cos(d*x + c)^
2*e + 16*sqrt(2)*a*d*cos(d*x + c)*e + (sqrt(2)*a*d*cos(d*x + c)^5*e - 4*sqr
t(2)*a*d*cos(d*x + c)^4*e - 12*sqrt(2)*a*d*cos(d*x + c)^3*e + 8*sqrt(2)*a*d
*cos(d*x + c)^2*e + 16*sqrt(2)*a*d*cos(d*x + c)*e)*sin(d*x + c))*(a^2/d^4)^
(1/4)*e^(1/2) + (a*cos(d*x + c)^4*e^(3/2) - 3*a*cos(d*x + c)^3*e^(3/2) - 8*
a*cos(d*x + c)^2*e^(3/2) + 4*a*cos(d*x + c)*e^(3/2) + (2*d^2*cos(d*x + c)^5
*e^(1/2) - 5*d^2*cos(d*x + c)^4*e^(1/2) - 19*d^2*cos(d*x + c)^3*e^(1/2) + 2
0*d^2*cos(d*x + c)*e^(1/2) + 8*d^2*e^(1/2) - (2*d^2*cos(d*x + c)^4*e^(1/2)

```
+ 9*d^2*cos(d*x + c)^3*e^(1/2) - 4*d^2*cos(d*x + c)^2*e^(1/2) - 20*d^2*cos(
d*x + c)*e^(1/2) - 8*d^2*e^(1/2))*sin(d*x + c))*sqrt(a^2/d^4)*e + 8*a*e^(3/
2) - (a*cos(d*x + c)^3*e^(3/2) + 4*a*cos(d*x + c)^2*e^(3/2) - 4*a*cos(d*x +
 c)*e^(3/2) - 8*a*e^(3/2))*sin(d*x + c))*sqrt(a*sin(d*x + c) + a)*sqrt(cos(
d*x + c)))*sqrt((2*a^3*cos(d*x + c)*e^3*sin(d*x + c) + 2*a^3*cos(d*x + c)*e
^3 + (a^2*d^2*e^2*sin(d*x + c) + a^2*d^2*e^2)*sqrt(a^2/d^4)*e - (sqrt(2)*a^
2*d*(a^2/d^4)^(1/4)*cos(d*x + c)*e^3 + (sqrt(2)*a*d^3*e^(3/2)*sin(d*x + c)
+ sqrt(2)*a*d^3*e^(3/2))*(a^2/d^4)^(3/4)*e^(3/2))*sqrt(a*sin(d*x + c) + a)*
sqrt(cos(d*x + c)))/(sin(d*x + c) + 1)) + ((2*sqrt(2)*a*d^3*cos(d*x + c)^5*
e^(3/2) + sqrt(2)*a*d^3*cos(d*x + c)^4*e^(3/2) - 13*sqrt(2)*a*d^3*cos(d*x +
 c)^3*e^(3/2) - 8*sqrt(2)*a*d^3*cos(d*x + c)^2*e^(3/2) + 12*sqrt(2)*a*d^3*c
os(d*x + c)*e^(3/2) + 8*sqrt(2)*a*d^3*e^(3/2) - (7*sqrt(2)*a*d^3*cos(d*x +
c)^3*e^(3/2) + 4*sqrt(2)*a*d^3*cos(d*x + c)^2*e^(3/2) - 12*sqrt(2)*a*d^3*co
s(d*x + c)*e^(3/2) - 8*sqrt(2)*a*d^3*e^(3/2))*s...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a(\sin(c+dx)+1)}\,\sqrt{e\cos(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(1/2)*(a+a*sin(d*x+c))**(1/2),x)
[Out] Integral(sqrt(a*(sin(c + d*x) + 1))*sqrt(e*cos(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a*sin(d*x + c) + a)*sqrt(cos(d*x + c))*e^(1/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{e\,\cos(c+d\,x)}\,\sqrt{a+a\,\sin(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(1/2)*(a + a*sin(c + d*x))^(1/2),x)
[Out] int((e*cos(c + d*x))^(1/2)*(a + a*sin(c + d*x))^(1/2), x)
```

3.275 $\int \frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{e\cos(c+dx)}}\,dx$

Optimal. Leaf size=161

$$-\frac{2\sinh^{-1}\left(\frac{\sqrt{e\cos(c+dx)}}{\sqrt{e}}\right)\sqrt{1+\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}{d\sqrt{e}\,(1+\cos(c+dx)+\sin(c+dx))}+\frac{2\tan^{-1}\left(\frac{\sqrt{e}\,\sin(c+dx)}{\sqrt{e\cos(c+dx)}\sqrt{1+\cos(c+dx)}}\right)\sqrt{1+\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}{d\sqrt{e}\,(1+\cos(c+dx)+\sin(c+dx))}$$

```
[Out] -2*arcsinh((e*cos(d*x+c))^(1/2)/e^(1/2))*(1+cos(d*x+c))^(1/2)*(a+a*sin(d*x+
c))^(1/2)/d/(1+cos(d*x+c)+sin(d*x+c))/e^(1/2)+2*arctan(sin(d*x+c)*e^(1/2)/(
e*cos(d*x+c))^(1/2)/(1+cos(d*x+c))^(1/2))*(1+cos(d*x+c))^(1/2)*(a+a*sin(d*x
+c))^(1/2)/d/(1+cos(d*x+c)+sin(d*x+c))/e^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2756, 2854, 209, 2912, 65, 221}

$$\frac{2\sqrt{\cos(c+dx)+1}\sqrt{a\sin(c+dx)+a}\,\text{ArcTan}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\sqrt{e\cos(c+dx)}}\right)}{d\sqrt{e}\,(\sin(c+dx)+\cos(c+dx)+1)}-\frac{2\sqrt{\cos(c+dx)+1}\sqrt{a\sin(c+dx)+a}\,\sinh^{-1}\left(\frac{\sqrt{e\cos(c+dx)}}{\sqrt{e}}\right)}{d\sqrt{e}\,(\sin(c+dx)+\cos(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[c + d*x]]/Sqrt[e*Cos[c + d*x]],x]
```

```
[Out] (-2*ArcSinh[Sqrt[e*Cos[c + d*x]]/Sqrt[e]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[a + a
*Sin[c + d*x]])/(d*Sqrt[e]*(1 + Cos[c + d*x] + Sin[c + d*x])) + (2*ArcTan[(
Sqrt[e]*Sin[c + d*x])/(Sqrt[e*Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]])]*Sqrt[1
 + Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(d*Sqrt[e]*(1 + Cos[c + d*x] + S
in[c + d*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 2756

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[cos[(e_.) + (f_.)*(x_)]
*(g_.)], x_Symbol] :> Dist[a*Sqrt[1 + Cos[e + f*x]]*(Sqrt[a + b*Sin[e + f*x
]]/(a + a*Cos[e + f*x] + b*Sin[e + f*x])), Int[Sqrt[1 + Cos[e + f*x]]/Sqrt[
g*Cos[e + f*x]], x], x] + Dist[b*Sqrt[1 + Cos[e + f*x]]*(Sqrt[a + b*Sin[e +
 f*x]]/(a + a*Cos[e + f*x] + b*Sin[e + f*x])), Int[Sin[e + f*x]/(Sqrt[g*Cos
[e + f*x]]*Sqrt[1 + Cos[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, g}, x] &&
EqQ[a^2 - b^2, 0]
```

Rule 2854

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b + d*x^2), x], x
, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]))], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{e\cos(c+dx)}}\,dx = \frac{\left(a\sqrt{1+\cos(c+dx)}\,\sqrt{a+a\sin(c+dx)}\right)\int \frac{\sqrt{1+\cos(c+dx)}}{\sqrt{e\cos(c+dx)}}\,dx}{a+a\cos(c+dx)+a\sin(c+dx)} + \frac{\left(a\sqrt{\right.}{}$$

$$= -\frac{\left(a\sqrt{1+\cos(c+dx)}\,\sqrt{a+a\sin(c+dx)}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{ex}\,\sqrt{1+x}}\,dx, x, \mathrm{c}\right.}{d(a+a\cos(c+dx)+a\sin(c+dx))}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{e}\,\sin(c+dx)}{\sqrt{e\cos(c+dx)}\,\sqrt{1+\cos(c+dx)}}\right)\sqrt{1+\cos(c+dx)}\,\sqrt{a+a\sin}}{d\sqrt{e}\,(1+\cos(c+dx)+\sin(c+dx))}$$

$$= -\frac{2\sinh^{-1}\left(\frac{\sqrt{e\cos(c+dx)}}{\sqrt{e}}\right)\sqrt{1+\cos(c+dx)}\,\sqrt{a+a\sin(c+dx)}}{d\sqrt{e}\,(1+\cos(c+dx)+\sin(c+dx))} + \frac{2\,\mathrm{ta}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.48, size = 108, normalized size = 0.67

$$\frac{\sqrt{1+e^{2i(c+dx)}}\left(dx-\sinh^{-1}\left(e^{i(c+dx)}\right)+i\log\left(1+\sqrt{1+e^{2i(c+dx)}}\right)\right)\sqrt{a(1+\sin(c+dx))}}{d\left(1-ie^{i(c+dx)}\right)\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[c + d*x]]/Sqrt[e*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[1 + E^((2*I)*(c + d*x))]*(d*x - ArcSinh[E^(I*(c + d*x))] + I*Log[1 +
Sqrt[1 + E^((2*I)*(c + d*x))]])*Sqrt[a*(1 + Sin[c + d*x])])/(d*(1 - I*E^(I*
(c + d*x)))*Sqrt[e*Cos[c + d*x]])
```

Maple [A]
time = 0.16, size = 142, normalized size = 0.88

method	result
default	$-\frac{\left(\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2}}{2}\right)-\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\right)\sqrt{a\left(1+\sin\left(dx+c\right)\right)}\,(\cos(}{d\sin(dx+c)\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{e\cos\left(dx+c\right)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^(1/2)/(e*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))-arctanh(1/2*
(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2)))*(a*(1+
sin(d*x+c)))^(1/2)*(cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c)/(-2*cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)/(e*cos(d*x+c))^(1/2)*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(1/2)/(e*cos(d*x+c))^(1/2),x, algorithm="maxima"
)
```

```
[Out] e^(-1/2)*integrate(sqrt(a*sin(d*x + c) + a)/sqrt(cos(d*x + c)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3105 vs. 2(126) = 252.
time = 178.64, size = 3105, normalized size = 19.29

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(1/2)/(e*cos(d*x+c))^(1/2),x, algorithm="fricas"
)
```

```
[Out] sqrt(2)*(a^2/d^4)^(1/4)*arctan(-1/4*(sqrt(2)*((sqrt(2)*d^3*cos(d*x + c)^6*e
^(3/2) + 5*sqrt(2)*d^3*cos(d*x + c)^5*e^(3/2) - 8*sqrt(2)*d^3*cos(d*x + c)^
4*e^(3/2) - 20*sqrt(2)*d^3*cos(d*x + c)^3*e^(3/2) + 8*sqrt(2)*d^3*cos(d*x +
 c)^2*e^(3/2) + 16*sqrt(2)*d^3*cos(d*x + c)*e^(3/2) + (sqrt(2)*d^3*cos(d*x
+ c)^5*e^(3/2) - 4*sqrt(2)*d^3*cos(d*x + c)^4*e^(3/2) - 12*sqrt(2)*d^3*cos(
d*x + c)^3*e^(3/2) + 8*sqrt(2)*d^3*cos(d*x + c)^2*e^(3/2) + 16*sqrt(2)*d^3*
cos(d*x + c)*e^(3/2))*sin(d*x + c))*(a^2/d^4)^(3/4)*e^(-3/2) + (sqrt(2)*a*d
*cos(d*x + c)^6*e^(1/2) - 3*sqrt(2)*a*d*cos(d*x + c)^5*e^(1/2) - 8*sqrt(2)*
a*d*cos(d*x + c)^4*e^(1/2) + 4*sqrt(2)*a*d*cos(d*x + c)^3*e^(1/2) + 8*sqrt(
2)*a*d*cos(d*x + c)^2*e^(1/2) - (sqrt(2)*a*d*cos(d*x + c)^5*e^(1/2) + 4*sqr
t(2)*a*d*cos(d*x + c)^4*e^(1/2) - 4*sqrt(2)*a*d*cos(d*x + c)^3*e^(1/2) - 8*
sqrt(2)*a*d*cos(d*x + c)^2*e^(1/2))*sin(d*x + c))*(a^2/d^4)^(1/4)*e^(-1/2)
- (a*cos(d*x + c)^4 - 3*a*cos(d*x + c)^3 - 8*a*cos(d*x + c)^2 + (2*d^2*cos(
d*x + c)^5*e - 5*d^2*cos(d*x + c)^4*e - 19*d^2*cos(d*x + c)^3*e + 20*d^2*co
s(d*x + c)*e + 8*d^2*e - (2*d^2*cos(d*x + c)^4*e + 9*d^2*cos(d*x + c)^3*e -
 4*d^2*cos(d*x + c)^2*e - 20*d^2*cos(d*x + c)*e - 8*d^2*e)*sin(d*x + c))*sq
rt(a^2/d^4)*e^(-1) + 4*a*cos(d*x + c) - (a*cos(d*x + c)^3 + 4*a*cos(d*x + c
)^2 - 4*a*cos(d*x + c) - 8*a)*sin(d*x + c) + 8*a)*sqrt(a*sin(d*x + c) + a)*
```

sqrt(cos(d*x + c)))*sqrt((2*a^3*cos(d*x + c)*sin(d*x + c) + 2*a^3*cos(d*x +
c) + (a^2*d^2*e*sin(d*x + c) + a^2*d^2*e)*sqrt(a^2/d^4)*e^(-1) + (sqrt(2)*
a*d^3*(a^2/d^4)^(3/4)*cos(d*x + c) + (sqrt(2)*a^2*d*e^(1/2)*sin(d*x + c) +
sqrt(2)*a^2*d*e^(1/2))*(a^2/d^4)^(1/4)*e^(-1/2))*sqrt(a*sin(d*x + c) + a)*s
qrt(cos(d*x + c)))/(sin(d*x + c) + 1)) - ((7*sqrt(2)*a*d^3*cos(d*x + c)^4*e
^(3/2) + 3*sqrt(2)*a*d^3*cos(d*x + c)^3*e^(3/2) - 16*sqrt(2)*a*d^3*cos(d*x
+ c)^2*e^(3/2) - 4*sqrt(2)*a*d^3*cos(d*x + c)*e^(3/2) + 8*sqrt(2)*a*d^3*e^(
3/2) + (2*sqrt(2)*a*d^3*cos(d*x + c)^4*e^(3/2) + sqrt(2)*a*d^3*cos(d*x + c)
^3*e^(3/2) - 12*sqrt(2)*a*d^3*cos(d*x + c)^2*e^(3/2) - 4*sqrt(2)*a*d^3*cos(
d*x + c)*e^(3/2) + 8*sqrt(2)*a*d^3*e^(3/2))*sin(d*x + c))*(a^2/d^4)^(3/4)*e
^(-3/2) + (2*sqrt(2)*a^2*d*cos(d*x + c)^5*e^(1/2) + sqrt(2)*a^2*d*cos(d*x +
c)^4*e^(1/2) - 13*sqrt(2)*a^2*d*cos(d*x + c)^3*e^(1/2) - 8*sqrt(2)*a^2*d*c
os(d*x + c)^2*e^(1/2) + 12*sqrt(2)*a^2*d*cos(d*x + c)*e^(1/2) + 8*sqrt(2)*a
^2*d*e^(1/2) - (7*sqrt(2)*a^2*d*cos(d*x + c)^3*e^(1/2) + 4*sqrt(2)*a^2*d*co
s(d*x + c)^2*e^(1/2) - 12*sqrt(2)*a^2*d*cos(d*x + c)*e^(1/2) - 8*sqrt(2)*a^
2*d*e^(1/2))*sin(d*x + c))*(a^2/d^4)^(1/4)*e^(-1/2))*sqrt(a*sin(d*x + c) +
a)*sqrt(cos(d*x + c)))/(a^3*cos(d*x + c)^6 + a^3*cos(d*x + c)^5 - 8*a^3*cos
(d*x + c)^4 - 8*a^3*cos(d*x + c)^3 + 8*a^3*cos(d*x + c)^2 + 8*a^3*cos(d*x +
c) - 4*(a^3*cos(d*x + c)^4 + a^3*cos(d*x + c)^3 - 2*a^3*cos(d*x + c)^2 - 2
*a^3*cos(d*x + c))*sin(d*x + c)))*e^(-1/2) - sqrt(2)*(a^2/d^4)^(1/4)*arctan
(1/4*(sqrt(2)*((sqrt(2)*d^3*cos(d*x + c)^6*e^(3/2) + 5*sqrt(2)*d^3*cos(d*x
+ c)^5*e^(3/2) - 8*sqrt(2)*d^3*cos(d*x + c)^4*e^(3/2) - 20*sqrt(2)*d^3*cos(
d*x + c)^3*e^(3/2) + 8*sqrt(2)*d^3*cos(d*x + c)^2*e^(3/2) + 16*sqrt(2)*d^3*
cos(d*x + c)*e^(3/2) + (sqrt(2)*d^3*cos(d*x + c)^5*e^(3/2) - 4*sqrt(2)*d^3*
cos(d*x + c)^4*e^(3/2) - 12*sqrt(2)*d^3*cos(d*x + c)^3*e^(3/2) + 8*sqrt(2)*
d^3*cos(d*x + c)^2*e^(3/2) + 16*sqrt(2)*d^3*cos(d*x + c)*e^(3/2))*sin(d*x +
c))*(a^2/d^4)^(3/4)*e^(-3/2) + (sqrt(2)*a*d*cos(d*x + c)^6*e^(1/2) - 3*sqr
t(2)*a*d*cos(d*x + c)^5*e^(1/2) - 8*sqrt(2)*a*d*cos(d*x + c)^4*e^(1/2) + 4*
sqrt(2)*a*d*cos(d*x + c)^3*e^(1/2) + 8*sqrt(2)*a*d*cos(d*x + c)^2*e^(1/2) -
(sqrt(2)*a*d*cos(d*x + c)^5*e^(1/2) + 4*sqrt(2)*a*d*cos(d*x + c)^4*e^(1/2)
- 4*sqrt(2)*a*d*cos(d*x + c)^3*e^(1/2) - 8*sqrt(2)*a*d*cos(d*x + c)^2*e^(1
/2))*sin(d*x + c))*(a^2/d^4)^(1/4)*e^(-1/2) + (a*cos(d*x + c)^4 - 3*a*cos(d
*x + c)^3 - 8*a*cos(d*x + c)^2 + (2*d^2*cos(d*x + c)^5*e - 5*d^2*cos(d*x +
c)^4*e - 19*d^2*cos(d*x + c)^3*e + 20*d^2*cos(d*x + c)*e + 8*d^2*e - (2*d^2
*cos(d*x + c)^4*e + 9*d^2*cos(d*x + c)^3*e - 4*d^2*cos(d*x + c)^2*e - 20*d^
2*cos(d*x + c)*e - 8*d^2*e)*sin(d*x + c))*sqrt(a^2/d^4)*e^(-1) + 4*a*cos(d*
x + c) - (a*cos(d*x + c)^3 + 4*a*cos(d*x + c)^2 - 4*a*cos(d*x + c) - 8*a)*s
in(d*x + c) + 8*a)*sqrt(a*sin(d*x + c) + a)*sqrt(cos(d*x + c)))*sqrt((2*a^3
*cos(d*x + c)*sin(d*x + c) + 2*a^3*cos(d*x + c) + (a^2*d^2*e*sin(d*x + c) +
a^2*d^2*e)*sqrt(a^2/d^4)*e^(-1) - (sqrt(2)*a*d^3*(a^2/d^4)^(3/4)*cos(d*x +
c) + (sqrt(2)*a^2*d*e^(1/2)*sin(d*x + c) + sqrt(2)*a^2*d*e^(1/2))*(a^2/d^4
)^(1/4)*e^(-1/2))*sqrt(a*sin(d*x + c) + a)*sqrt(cos(d*x + c)))/(sin(d*x + c
) + 1)) - ((7*sqrt(2)*a*d^3*cos(d*x + c)^4*e^(3/2) + 3*sqrt(2)*a*d^3*cos(d*
x + c)^3*e^(3/2) - 16*sqrt(2)*a*d^3*cos(d*x + c)^2*e^(3/2) - 4*sqrt(2)*a*d^
3*cos(d*x + c)*e^(3/2) + 8*sqrt(2)*a*d^3*e^(3/2) + (2*sqrt(2)*a*d^3*cos(d*x

```
 + c)^4*e^(3/2) + sqrt(2)*a*d^3*cos(d*x + c)^3*e^(3/2) - 12*sqrt(2)*a*d^3*c
os(d*x + c)^2*e^(3/2) - 4*sqrt(2)*a*d^3*cos(d*x + c)*e^(3/2) + 8*sqrt(2)*a*
d^3*e^(3/2))*sin(d*x + c))*(a^2/d^4)^(3/4)*e^(-3/2) + (2*sqrt(2)*a^2*d*cos(
d*x + c)^5*e^(1/2) + sqrt(2)*a^2*d*cos(d*x + c)...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\left(\sin\left(c+dx\right)+1\right)}}{\sqrt{e\cos\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**(1/2)/(e*cos(d*x+c))**(1/2),x)
[Out] Integral(sqrt(a*(sin(c + d*x) + 1))/sqrt(e*cos(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(1/2)/(e*cos(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a*sin(d*x + c) + a)*e^(-1/2)/sqrt(cos(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+a\,\sin\left(c+d\,x\right)}}{\sqrt{e\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(1/2)/(e*cos(c + d*x))^(1/2),x)
[Out] int((a + a*sin(c + d*x))^(1/2)/(e*cos(c + d*x))^(1/2), x)
```

3.276 $\int \frac{\sqrt{a+a\sin(c+dx)}}{(e\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=34

$$\frac{2\sqrt{a+a\sin(c+dx)}}{de\sqrt{e\cos(c+dx)}}$$

```
[Out] 2*(a+a*sin(d*x+c))^(1/2)/d/e/(e*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$, Rules used = {2750}

$$\frac{2\sqrt{a\sin(c+dx)+a}}{de\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[c + d*x]]/(e*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*Sqrt[a + a*Sin[c + d*x]])/(d*e*Sqrt[e*Cos[c + d*x]])
```

Rule 2750

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sin(c+dx)}}{(e\cos(c+dx))^{3/2}}\,dx = \frac{2\sqrt{a+a\sin(c+dx)}}{de\sqrt{e\cos(c+dx)}}$$

Mathematica [A]
time = 0.12, size = 34, normalized size = 1.00

$$\frac{2\sqrt{a(1+\sin(c+dx))}}{de\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[c + d*x]]/(e*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*Sqrt[a*(1 + Sin[c + d*x])])/(d*e*Sqrt[e*Cos[c + d*x]])
```

Maple [A]
time = 0.18, size = 34, normalized size = 1.00

method	result	size
default	$\frac{2\cos(dx+c)\sqrt{a\left(1+\sin\left(dx+c\right)\right)}}{d(e\cos(dx+c))^{\frac{3}{2}}}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^(1/2)/(e*cos(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
[Out] 2/d*cos(d*x+c)*(a*(1+sin(d*x+c)))^(1/2)/(e*cos(d*x+c))^(3/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 77 vs. 2(27) = 54.
time = 0.55, size = 77, normalized size = 2.26

$$\frac{2\left(\sqrt{a}-\frac{\sqrt{a}\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)e^{\left(-\frac{3}{2}\right)}}{d\sqrt{\frac{\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}+1}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(1/2)/(e*cos(d*x+c))^(3/2),x, algorithm="maxima"
)
[Out] 2*(sqrt(a) - sqrt(a)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*e^(-3/2)/(d*sqrt(
sin(d*x + c)/(cos(d*x + c) + 1) + 1)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)
^(3/2))
```

Fricas [A]
time = 0.35, size = 27, normalized size = 0.79

$$\frac{2\sqrt{a\sin\left(dx+c\right)+a}\,e^{\left(-\frac{3}{2}\right)}}{d\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(1/2)/(e*cos(d*x+c))^(3/2),x, algorithm="fricas"
)
[Out] 2*sqrt(a*sin(d*x + c) + a)*e^(-3/2)/(d*sqrt(cos(d*x + c)))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{a\left(\sin\left(c+dx\right)+1\right)}}{\left(e\cos\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**(1/2)/(e*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral(sqrt(a*(sin(c + d*x) + 1))/(e*cos(c + d*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(1/2)/(e*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*e^(-3/2)/cos(d*x + c)^(3/2), x)
```

Mupad [B]
time = 5.31, size = 30, normalized size = 0.88

$$\frac{2\,\sqrt{a+a\,\sin\left(c+d\,x\right)}}{d\,e\,\sqrt{e\,\cos\left(c+d\,x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(1/2)/(e*cos(c + d*x))^(3/2),x)
```

```
[Out] (2*(a + a*sin(c + d*x))^(1/2))/(d*e*(e*cos(c + d*x))^(1/2))
```

3.277 $\int \frac{\sqrt{a+a\sin(c+dx)}}{(e\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=74

$$-\frac{2\sqrt{a+a\sin(c+dx)}}{de(e\cos(c+dx))^{3/2}}+\frac{4(a+a\sin(c+dx))^{3/2}}{3ade(e\cos(c+dx))^{3/2}}$$

```
[Out] 4/3*(a+a*sin(d*x+c))^(3/2)/a/d/e/(e*cos(d*x+c))^(3/2)-2*(a+a*sin(d*x+c))^(1
/2)/d/e/(e*cos(d*x+c))^(3/2)
```

Rubi [A]
time = 0.10, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2751, 2750}

$$\frac{4(a\sin(c+dx)+a)^{3/2}}{3ade(e\cos(c+dx))^{3/2}}-\frac{2\sqrt{a\sin(c+dx)+a}}{de(e\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[c + d*x]]/(e*Cos[c + d*x])^(5/2),x]
```

```
[Out] (-2*Sqrt[a + a*Sin[c + d*x]])/(d*e*(e*Cos[c + d*x])^(3/2)) + (4*(a + a*Sin[
c + d*x])^(3/2))/(3*a*d*e*(e*Cos[c + d*x])^(3/2))
```

Rule 2750

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sin(c+dx)}}{(e\cos(c+dx))^{5/2}}\,dx = -\frac{2\sqrt{a+a\sin(c+dx)}}{de(e\cos(c+dx))^{3/2}} + \frac{2\int \frac{(a+a\sin(c+dx))^{3/2}}{(e\cos(c+dx))^{5/2}}\,dx}{a}$$
$$= -\frac{2\sqrt{a+a\sin(c+dx)}}{de(e\cos(c+dx))^{3/2}} + \frac{4(a+a\sin(c+dx))^{3/2}}{3ade(e\cos(c+dx))^{3/2}}$$

Mathematica [A]
time = 0.27, size = 46, normalized size = 0.62

$$\frac{2\sqrt{a(1+\sin(c+dx))}\,(-1+2\sin(c+dx))}{3de(e\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[c + d*x]]/(e*Cos[c + d*x])^(5/2),x]
[Out] (2*Sqrt[a*(1 + Sin[c + d*x])]*(-1 + 2*Sin[c + d*x]))/(3*d*e*(e*Cos[c + d*x]
)^(3/2))
```

Maple [A]
time = 0.18, size = 44, normalized size = 0.59

method	result	size
default	$\frac{2(2\sin(dx+c)-1)\cos(dx+c)\sqrt{a\left(1+\sin\left(dx+c\right)\right)}}{3d(e\cos(dx+c))^{\frac{5}{2}}}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^(1/2)/(e*cos(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
[Out] 2/3/d*(2*sin(d*x+c)-1)*cos(d*x+c)*(a*(1+sin(d*x+c)))^(1/2)/(e*cos(d*x+c))^(
5/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 187 vs. 2(58) = 116.
time = 0.56, size = 187, normalized size = 2.53

$$-\frac{2\left(\sqrt{a} - \frac{4\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1} + \frac{4\sqrt{a}\sin(dx+c)^3}{(\cos(dx+c)+1)^3} - \frac{\sqrt{a}\sin(dx+c)^4}{(\cos(dx+c)+1)^4}\right)\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^2 e^{\left(-\frac{5}{2}\right)}}{3\,d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{3}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}\left(\frac{2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(1/2)/(e*cos(d*x+c))^(5/2),x, algorithm="maxima"
)
```

```
[Out] -2/3*(sqrt(a) - 4*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) + 4*sqrt(a)*sin(d
*x + c)^3/(cos(d*x + c) + 1)^3 - sqrt(a)*sin(d*x + c)^4/(cos(d*x + c) + 1)^
4)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^2*e^(-5/2)/(d*(sin(d*x + c)/(c
os(d*x + c) + 1) + 1)^(3/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/2)*(2
*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + sin(d*x + c)^4/(cos(d*x + c) + 1)^4
+ 1))
```

Fricas [A]
time = 0.35, size = 37, normalized size = 0.50

$$\frac{2\,\sqrt{a\sin\left(dx+c\right)+a}\,\left(2\,\sin\left(dx+c\right)-1\right)e^{\left(-\frac{5}{2}\right)}}{3\,d\cos\left(dx+c\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(1/2)/(e*cos(d*x+c))^(5/2),x, algorithm="fricas"
)
```

```
[Out] 2/3*sqrt(a*sin(d*x + c) + a)*(2*sin(d*x + c) - 1)*e^(-5/2)/(d*cos(d*x + c)^
(3/2))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{a\left(\sin\left(c+dx\right)+1\right)}}{\left(e\cos\left(c+dx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**(1/2)/(e*cos(d*x+c))**(5/2),x)
```

```
[Out] Integral(sqrt(a*(sin(c + d*x) + 1))/(e*cos(c + d*x))**(5/2), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(1/2)/(e*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 5.67, size = 61, normalized size = 0.82

$$-\frac{4\,\sqrt{a\,\left(\sin\left(c+d\,x\right)+1\right)}\,\left(\cos\left(c+d\,x\right)-\sin\left(2\,c+2\,d\,x\right)\right)}{3\,d\,e^2\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)\,\sqrt{e\,\cos\left(c+d\,x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(1/2)/(e*cos(c + d*x))^(5/2),x)
```

```
[Out] -(4*(a*(sin(c + d*x) + 1))^(1/2)*(cos(c + d*x) - sin(2*c + 2*d*x)))/(3*d*e^
2*(cos(2*c + 2*d*x) + 1)*(e*cos(c + d*x))^(1/2))
```

3.278 $\int \frac{\sqrt{a+a\sin(c+dx)}}{(e\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=115

$$-\frac{2\sqrt{a+a\sin(c+dx)}}{3de(e\cos(c+dx))^{5/2}}+\frac{8(a+a\sin(c+dx))^{3/2}}{3ade(e\cos(c+dx))^{5/2}}-\frac{16(a+a\sin(c+dx))^{5/2}}{15a^2de(e\cos(c+dx))^{5/2}}$$

```
[Out] 8/3*(a+a*sin(d*x+c))^(3/2)/a/d/e/(e*cos(d*x+c))^(5/2)-16/15*(a+a*sin(d*x+c)
)^(5/2)/a^2/d/e/(e*cos(d*x+c))^(5/2)-2/3*(a+a*sin(d*x+c))^(1/2)/d/e/(e*cos(
d*x+c))^(5/2)
```

Rubi [A]
time = 0.15, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2751, 2750}

$$-\frac{16(a\sin(c+dx)+a)^{5/2}}{15a^2de(e\cos(c+dx))^{5/2}}+\frac{8(a\sin(c+dx)+a)^{3/2}}{3ade(e\cos(c+dx))^{5/2}}-\frac{2\sqrt{a\sin(c+dx)+a}}{3de(e\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[c + d*x]]/(e*Cos[c + d*x])^(7/2),x]
```

```
[Out] (-2*Sqrt[a + a*Sin[c + d*x]])/(3*d*e*(e*Cos[c + d*x])^(5/2)) + (8*(a + a*Si
n[c + d*x])^(3/2))/(3*a*d*e*(e*Cos[c + d*x])^(5/2)) - (16*(a + a*Sin[c + d*
x])^(5/2))/(15*a^2*d*e*(e*Cos[c + d*x])^(5/2))
```

Rule 2750

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sin(c+dx)}}{(e\cos(c+dx))^{7/2}}\,dx = -\frac{2\sqrt{a+a\sin(c+dx)}}{3de(e\cos(c+dx))^{5/2}} + \frac{4\int \frac{(a+a\sin(c+dx))^{3/2}}{(e\cos(c+dx))^{7/2}}\,dx}{3a}$$
$$= -\frac{2\sqrt{a+a\sin(c+dx)}}{3de(e\cos(c+dx))^{5/2}} + \frac{8(a+a\sin(c+dx))^{3/2}}{3ade(e\cos(c+dx))^{5/2}} - \frac{8\int \frac{(a+a\sin(c+dx))^{5/2}}{(e\cos(c+dx))^{7/2}}\,dx}{3a^2}$$
$$= -\frac{2\sqrt{a+a\sin(c+dx)}}{3de(e\cos(c+dx))^{5/2}} + \frac{8(a+a\sin(c+dx))^{3/2}}{3ade(e\cos(c+dx))^{5/2}} - \frac{16(a+a\sin(c+dx))^{5/2}}{15a^2de(e\cos(c+dx))^{5/2}}$$

Mathematica [A]
time = 0.36, size = 56, normalized size = 0.49

$$\frac{2\sqrt{a(1+\sin(c+dx))}\,(3+4\cos(2(c+dx))+4\sin(c+dx))}{15de(e\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[c + d*x]]/(e*Cos[c + d*x])^(7/2),x]
```

```
[Out] (2*Sqrt[a*(1 + Sin[c + d*x])]*(3 + 4*Cos[2*(c + d*x)] + 4*Sin[c + d*x]))/(1
5*d*e*(e*Cos[c + d*x])^(5/2))
```

Maple [A]
time = 0.18, size = 54, normalized size = 0.47

method	result	size
default	$\frac{2\left(8\left(\cos^{2}(dx+c)\right)+4\sin(dx+c)-1\right)\cos(dx+c)\sqrt{a\left(1+\sin\left(dx+c\right)\right)}}{15d\left(e\cos(dx+c)\right)^{\frac{7}{2}}}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^(1/2)/(e*cos(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15/d*(8*cos(d*x+c)^2+4*sin(d*x+c)-1)*cos(d*x+c)*(a*(1+sin(d*x+c)))^(1/2)/
(e*cos(d*x+c))^(7/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 255 vs. 2(88) = 176.
time = 0.55, size = 255, normalized size = 2.22

$$\frac{2\left(7\sqrt{a}+\frac{8\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}-\frac{25\sqrt{a}\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{25\sqrt{a}\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{8\sqrt{a}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{7\sqrt{a}\sin(dx+c)^6}{(\cos(dx+c)+1)^6}\right)\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^3 e^{\left(-\frac{7}{2}\right)}}{15\,d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(\frac{3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(1/2)/(e*cos(d*x+c))^(7/2),x, algorithm="maxima"
)
```

```
[Out] 2/15*(7*sqrt(a) + 8*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 25*sqrt(a)*si
n(d*x + c)^2/(cos(d*x + c) + 1)^2 + 25*sqrt(a)*sin(d*x + c)^4/(cos(d*x + c)
 + 1)^4 - 8*sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 7*sqrt(a)*sin(d*x
 + c)^6/(cos(d*x + c) + 1)^6)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^3*e
^(-7/2)/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/2)*(-sin(d*x + c)/(cos(
d*x + c) + 1) + 1)^(7/2)*(3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*sin(d*x
 + c)^4/(cos(d*x + c) + 1)^4 + sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 1))
```

Fricas [A]
time = 0.34, size = 47, normalized size = 0.41

$$\frac{2\left(8\cos\left(dx+c\right)^{2}+4\sin\left(dx+c\right)-1\right)\sqrt{a\sin\left(dx+c\right)+a}\,e^{\left(-\frac{7}{2}\right)}}{15\,d\cos\left(dx+c\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(1/2)/(e*cos(d*x+c))^(7/2),x, algorithm="fricas"
)
```

```
[Out] 2/15*(8*cos(d*x + c)^2 + 4*sin(d*x + c) - 1)*sqrt(a*sin(d*x + c) + a)*e^(-7
/2)/(d*cos(d*x + c)^(5/2))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**(1/2)/(e*cos(d*x+c))**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(1/2)/(e*cos(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 6.06, size = 97, normalized size = 0.84

$$\frac{8\,\sqrt{a\,\left(\sin\left(c+d\,x\right)+1\right)}\,\left(2\,\sin\left(c+d\,x\right)+7\,\cos\left(2\,c+2\,d\,x\right)+2\,\cos\left(4\,c+4\,d\,x\right)+2\,\sin\left(3\,c+3\,d\,x\right)+5\right)}{15\,d\,e^3\,\sqrt{e\,\cos\left(c+d\,x\right)}\,\left(4\,\cos\left(2\,c+2\,d\,x\right)+\cos\left(4\,c+4\,d\,x\right)+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(1/2)/(e*cos(c + d*x))^(7/2),x)
```

```
[Out] (8*(a*(sin(c + d*x) + 1))^(1/2)*(2*sin(c + d*x) + 7*cos(2*c + 2*d*x) + 2*co
s(4*c + 4*d*x) + 2*sin(3*c + 3*d*x) + 5))/(15*d*e^3*(e*cos(c + d*x))^(1/2)*
(4*cos(2*c + 2*d*x) + cos(4*c + 4*d*x) + 3))
```

3.279 $\int \frac{\sqrt{a+a\sin(c+dx)}}{(e\cos(c+dx))^{9/2}}\,dx$

Optimal. Leaf size=154

$$-\frac{2\sqrt{a+a\sin(c+dx)}}{5de(e\cos(c+dx))^{7/2}}-\frac{12(a+a\sin(c+dx))^{3/2}}{5ade(e\cos(c+dx))^{7/2}}+\frac{16(a+a\sin(c+dx))^{5/2}}{5a^2de(e\cos(c+dx))^{7/2}}-\frac{32(a+a\sin(c+dx))^{7/2}}{35a^3de(e\cos(c+dx))^{7/2}}$$

```
[Out] -12/5*(a+a*sin(d*x+c))^(3/2)/a/d/e/(e*cos(d*x+c))^(7/2)+16/5*(a+a*sin(d*x+c
))^(5/2)/a^2/d/e/(e*cos(d*x+c))^(7/2)-32/35*(a+a*sin(d*x+c))^(7/2)/a^3/d/e/
(e*cos(d*x+c))^(7/2)-2/5*(a+a*sin(d*x+c))^(1/2)/d/e/(e*cos(d*x+c))^(7/2)
```

Rubi [A]
time = 0.20, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2751, 2750}

$$-\frac{32(a\sin(c+dx)+a)^{7/2}}{35a^3de(e\cos(c+dx))^{7/2}}+\frac{16(a\sin(c+dx)+a)^{5/2}}{5a^2de(e\cos(c+dx))^{7/2}}-\frac{12(a\sin(c+dx)+a)^{3/2}}{5ade(e\cos(c+dx))^{7/2}}-\frac{2\sqrt{a\sin(c+dx)+a}}{5de(e\cos(c+dx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[c + d*x]]/(e*Cos[c + d*x])^(9/2),x]
```

```
[Out] (-2*Sqrt[a + a*Sin[c + d*x]])/(5*d*e*(e*Cos[c + d*x])^(7/2)) - (12*(a + a*S
in[c + d*x])^(3/2))/(5*a*d*e*(e*Cos[c + d*x])^(7/2)) + (16*(a + a*Sin[c + d
*x])^(5/2))/(5*a^2*d*e*(e*Cos[c + d*x])^(7/2)) - (32*(a + a*Sin[c + d*x])^(
7/2))/(35*a^3*d*e*(e*Cos[c + d*x])^(7/2))
```

Rule 2750

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sin(c+dx)}}{(e\cos(c+dx))^{9/2}}\,dx = -\frac{2\sqrt{a+a\sin(c+dx)}}{5de(e\cos(c+dx))^{7/2}} + \frac{6\int \frac{(a+a\sin(c+dx))^{3/2}}{(e\cos(c+dx))^{9/2}}\,dx}{5a}$$

$$= -\frac{2\sqrt{a+a\sin(c+dx)}}{5de(e\cos(c+dx))^{7/2}} - \frac{12(a+a\sin(c+dx))^{3/2}}{5ade(e\cos(c+dx))^{7/2}} + \frac{24\int \frac{(a+a\sin(c+dx))^{5/2}}{(e\cos(c+dx))^{9/2}}\,dx}{5a^2}$$

$$= -\frac{2\sqrt{a+a\sin(c+dx)}}{5de(e\cos(c+dx))^{7/2}} - \frac{12(a+a\sin(c+dx))^{3/2}}{5ade(e\cos(c+dx))^{7/2}} + \frac{16(a+a\sin(c+dx))^{5/2}}{5a^2de(e\cos(c+dx))^{7/2}}$$

$$= -\frac{2\sqrt{a+a\sin(c+dx)}}{5de(e\cos(c+dx))^{7/2}} - \frac{12(a+a\sin(c+dx))^{3/2}}{5ade(e\cos(c+dx))^{7/2}} + \frac{16(a+a\sin(c+dx))^{5/2}}{5a^2de(e\cos(c+dx))^{7/2}}$$

Mathematica [A]
time = 0.85, size = 74, normalized size = 0.48

$$\frac{2\sqrt{e\cos(c+dx)}\sec^4(c+dx)\sqrt{a(1+\sin(c+dx))}\,(-5-4\cos(2(c+dx))+10\sin(c+dx)+4\sin(3(c+dx)))}{35de^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[c + d*x]]/(e*Cos[c + d*x])^(9/2),x]
```

```
[Out] (2*Sqrt[e*Cos[c + d*x]]*Sec[c + d*x]^4*Sqrt[a*(1 + Sin[c + d*x])]*(-5 - 4*C
os[2*(c + d*x)] + 10*Sin[c + d*x] + 4*Sin[3*(c + d*x)]))/(35*d*e^5)
```

Maple [A]
time = 0.19, size = 70, normalized size = 0.45

method	result	size
default	$\frac{2(16(\cos^2(dx+c))\sin(dx+c)-8(\cos^2(dx+c))+6\sin(dx+c)-1)\cos(dx+c)\sqrt{a\,(1+\sin\,(dx+c))}}{35d(e\cos(dx+c))^{\frac{9}{2}}}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^(1/2)/(e*cos(d*x+c))^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/35/d*(16*cos(d*x+c)^2*sin(d*x+c)-8*cos(d*x+c)^2+6*sin(d*x+c)-1)*cos(d*x+c
)*(a*(1+sin(d*x+c)))^(1/2)/(e*cos(d*x+c))^(9/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 321 vs. $2(118) = 236$.
time = 0.55, size = 321, normalized size = 2.08

$$-\frac{2\left(9\sqrt{a}-\frac{44\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}-\frac{14\sqrt{a}\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{84\sqrt{a}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{84\sqrt{a}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{14\sqrt{a}\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{44\sqrt{a}\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{9\sqrt{a}\sin(dx+c)^8}{(\cos(dx+c)+1)^8}\right)\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^4 e^{\left(-\frac{9}{2}\right)}}{35\,d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{9}{2}}\left(\frac{4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{4\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(1/2)/(e*cos(d*x+c))^(9/2),x, algorithm="maxima"
)
```

```
[Out] -2/35*(9*sqrt(a) - 44*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 14*sqrt(a)*
sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 84*sqrt(a)*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3 - 84*sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 14*sqrt(a)*sin
(d*x + c)^6/(cos(d*x + c) + 1)^6 + 44*sqrt(a)*sin(d*x + c)^7/(cos(d*x + c)
+ 1)^7 - 9*sqrt(a)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8)*(sin(d*x + c)^2/(co
s(d*x + c) + 1)^2 + 1)^4*e^(-9/2)/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^
(7/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(4*sin(d*x + c)^2/(cos(d
*x + c) + 1)^2 + 6*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*sin(d*x + c)^6/(
cos(d*x + c) + 1)^6 + sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 1))
```

Fricas [A]
time = 0.36, size = 59, normalized size = 0.38

$$-\frac{2\left(8\,\cos\left(dx+c\right)^{2}-2\left(8\,\cos\left(dx+c\right)^{2}+3\right)\sin\left(dx+c\right)+1\right)\sqrt{a\sin\left(dx+c\right)+a}\,e^{\left(-\frac{9}{2}\right)}}{35\,d\cos\left(dx+c\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(1/2)/(e*cos(d*x+c))^(9/2),x, algorithm="fricas"
)
```

```
[Out] -2/35*(8*cos(d*x + c)^2 - 2*(8*cos(d*x + c)^2 + 3)*sin(d*x + c) + 1)*sqrt(a
*sin(d*x + c) + a)*e^(-9/2)/(d*cos(d*x + c)^(7/2))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**(1/2)/(e*cos(d*x+c))**(9/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(1/2)/(e*cos(d*x+c))^(9/2),x, algorithm="giac")
```

[Out] Timed out

Mupad [B]
time = 7.24, size = 129, normalized size = 0.84

$$-\frac{16\sqrt{a\,\left(\sin\left(c+d\,x\right)+1\right)}\,\left(23\,\cos\left(c+d\,x\right)+11\,\cos\left(3\,c+3\,d\,x\right)+2\,\cos\left(5\,c+5\,d\,x\right)-16\,\sin\left(2\,c+2\,d\,x\right)-11\,\sin\left(4\,c+4\,d\,x\right)-2\,\sin\left(6\,c+6\,d\,x\right)\right)}{35\,d\,e^4\,\sqrt{e\,\cos\left(c+d\,x\right)}\,\left(15\,\cos\left(2\,c+2\,d\,x\right)+6\,\cos\left(4\,c+4\,d\,x\right)+\cos\left(6\,c+6\,d\,x\right)+10\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(1/2)/(e*cos(c + d*x))^(9/2),x)
```

```
[Out] -(16*(a*(sin(c + d*x) + 1))^(1/2)*(23*cos(c + d*x) + 11*cos(3*c + 3*d*x) +
2*cos(5*c + 5*d*x) - 16*sin(2*c + 2*d*x) - 11*sin(4*c + 4*d*x) - 2*sin(6*c
+ 6*d*x)))/(35*d*e^4*(e*cos(c + d*x))^(1/2)*(15*cos(2*c + 2*d*x) + 6*cos(4*
c + 4*d*x) + cos(6*c + 6*d*x) + 10))
```

3.280 $\int (e\cos(c+dx))^{5/2}(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=319

$$-\frac{15a^3(e\cos(c+dx))^{7/2}}{32de(a+a\sin(c+dx))^{3/2}}+\frac{15a^2e(e\cos(c+dx))^{3/2}}{64d\sqrt{a+a\sin(c+dx)}}-\frac{3a^2(e\cos(c+dx))^{7/2}}{8de\sqrt{a+a\sin(c+dx)}}-\frac{a(e\cos(c+dx))^{7/2}\sqrt{a+}}{4de}$$

```
[Out] -15/32*a^3*(e*cos(d*x+c))^(7/2)/d/e/(a+a*sin(d*x+c))^(3/2)+15/64*a^2*e*(e*c
os(d*x+c))^(3/2)/d/(a+a*sin(d*x+c))^(1/2)-3/8*a^2*(e*cos(d*x+c))^(7/2)/d/e/
(a+a*sin(d*x+c))^(1/2)-1/4*a*(e*cos(d*x+c))^(7/2)*(a+a*sin(d*x+c))^(1/2)/d/
e+45/64*a*e^(5/2)*arcsinh((e*cos(d*x+c))^(1/2)/e^(1/2))*(1+cos(d*x+c))^(1/2
)*(a+a*sin(d*x+c))^(1/2)/d/(1+cos(d*x+c)+sin(d*x+c))+45/64*a*e^(5/2)*arctan
(sin(d*x+c)*e^(1/2)/(e*cos(d*x+c))^(1/2)/(1+cos(d*x+c))^(1/2))*(1+cos(d*x+c
))^(1/2)*(a+a*sin(d*x+c))^(1/2)/d/(1+cos(d*x+c)+sin(d*x+c))
```

Rubi [A]
time = 0.38, antiderivative size = 319, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2757, 2765, 2758, 2763, 2854, 209, 2912, 65, 221}

$$-\frac{15a^3(e\cos(c+dx))^{7/2}}{32de(a\sin(c+dx)+a)^{3/2}}-\frac{3a^2(e\cos(c+dx))^{7/2}}{8de\sqrt{a\sin(c+dx)+a}}+\frac{15a^2e(e\cos(c+dx))^{3/2}}{64d\sqrt{a\sin(c+dx)+a}}+\frac{45ae^{5/2}\sqrt{\cos(c+dx)+1}\sqrt{a\sin(c+dx)+a}\,\text{ArcTan}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\sqrt{e\cos(c+dx)}}\right)}{64d(\sin(c+dx)+\cos(c+dx)+1)}+\frac{45ae^{5/2}\sqrt{\cos(c+dx)+1}\sqrt{a\sin(c+dx)+a}\sinh^{-1}\left(\frac{\sqrt{e\cos(c+dx)}}{\sqrt{e}}\right)}{64d(\sin(c+dx)+\cos(c+dx)+1)}-\frac{a\sqrt{a\sin(c+dx)+a}(e\cos(c+dx))^{7/2}}{4de}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(5/2)*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-15*a^3*(e*Cos[c + d*x])^(7/2))/(32*d*e*(a + a*Sin[c + d*x])^(3/2)) + (15*
a^2*e*(e*Cos[c + d*x])^(3/2))/(64*d*Sqrt[a + a*Sin[c + d*x]]) - (3*a^2*(e*C
os[c + d*x])^(7/2))/(8*d*e*Sqrt[a + a*Sin[c + d*x]]) - (a*(e*Cos[c + d*x])^
(7/2)*Sqrt[a + a*Sin[c + d*x]])/(4*d*e) + (45*a*e^(5/2)*ArcSinh[Sqrt[e*Cos[
c + d*x]]/Sqrt[e]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(64*d*(
1 + Cos[c + d*x] + Sin[c + d*x])) + (45*a*e^(5/2)*ArcTan[(Sqrt[e]*Sin[c + d
*x])/(Sqrt[e*Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]])]*Sqrt[1 + Cos[c + d*x]]*
Sqrt[a + a*Sin[c + d*x]])/(64*d*(1 + Cos[c + d*x] + Sin[c + d*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 2757

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ
[2*m, 2*p]
```

Rule 2758

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(a*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2763

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[g*Sqrt[1 + Cos[e + f*x]]*(Sqrt[a + b*Sin[e + f*x
]]/(a + a*Cos[e + f*x] + b*Sin[e + f*x])), Int[Sqrt[1 + Cos[e + f*x]]/Sqrt[
g*Cos[e + f*x]], x], x] - Dist[g*Sqrt[1 + Cos[e + f*x]]*(Sqrt[a + b*Sin[e +
 f*x]]/(b + b*Cos[e + f*x] + a*Sin[e + f*x])), Int[Sin[e + f*x]/(Sqrt[g*Cos
[e + f*x]]*Sqrt[1 + Cos[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, g}, x] &&
EqQ[a^2 - b^2, 0]
```

Rule 2765

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> Simp[-2*b*((g*Cos[e + f*x])^(p + 1)/(f*g*(2*p - 1)*(
a + b*Sin[e + f*x])^(3/2))), x] + Dist[2*a*((p - 2)/(2*p - 1)), Int[(g*Cos[
e + f*x])^p/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 2] && IntegerQ[2*p]
```

Rule 2854

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b + d*x^2), x], x
, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]))], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (e\cos(c+dx))^{5/2}(a+a\sin(c+dx))^{3/2}\,dx = -\frac{a(e\cos(c+dx))^{7/2}\sqrt{a+a\sin(c+dx)}}{4de} + \frac{1}{8}(9a)\int (e\cos(c$$

$$= -\frac{3a^2(e\cos(c+dx))^{7/2}}{8de\sqrt{a+a\sin(c+dx)}} - \frac{a(e\cos(c+dx))^{7/2}\sqrt{a+a\sin(c}}{4de}$$

$$= -\frac{15a^3(e\cos(c+dx))^{7/2}}{32de(a+a\sin(c+dx))^{3/2}} - \frac{3a^2(e\cos(c+dx))^{7/2}}{8de\sqrt{a+a\sin(c+dx)}} - a($$

$$= -\frac{15a^3(e\cos(c+dx))^{7/2}}{32de(a+a\sin(c+dx))^{3/2}} + \frac{15a^2e(e\cos(c+dx))^{3/2}}{64d\sqrt{a+a\sin(c+dx)}} - \frac{3}{8d}$$

$$= -\frac{15a^3(e\cos(c+dx))^{7/2}}{32de(a+a\sin(c+dx))^{3/2}} + \frac{15a^2e(e\cos(c+dx))^{3/2}}{64d\sqrt{a+a\sin(c+dx)}} - \frac{3}{8d}$$

$$= -\frac{15a^3(e\cos(c+dx))^{7/2}}{32de(a+a\sin(c+dx))^{3/2}} + \frac{15a^2e(e\cos(c+dx))^{3/2}}{64d\sqrt{a+a\sin(c+dx)}} - \frac{3}{8d}$$

$$= -\frac{15a^3(e\cos(c+dx))^{7/2}}{32de(a+a\sin(c+dx))^{3/2}} + \frac{15a^2e(e\cos(c+dx))^{3/2}}{64d\sqrt{a+a\sin(c+dx)}} - \frac{3}{8d}$$

$$= -\frac{15a^3(e\cos(c+dx))^{7/2}}{32de(a+a\sin(c+dx))^{3/2}} + \frac{15a^2e(e\cos(c+dx))^{3/2}}{64d\sqrt{a+a\sin(c+dx)}} - \frac{3}{8d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order

3 in optimal.
time = 0.18, size = 78, normalized size = 0.24

$$-\frac{16\sqrt[4]{2}\, a(e\cos(c+dx))^{7/2}\, {}_2F_1\left(-\frac{9}{4},\frac{7}{4};\frac{11}{4};\frac{1}{2}(1-\sin(c+dx))\right)\sqrt{a(1+\sin(c+dx))}}{7de(1+\sin(c+dx))^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(5/2)*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-16*2^(1/4)*a*(e*Cos[c + d*x])^(7/2)*Hypergeometric2F1[-9/4, 7/4, 11/4, (1
 - Sin[c + d*x])/2]*Sqrt[a*(1 + Sin[c + d*x])])/(7*d*e*(1 + Sin[c + d*x])^(
9/4))
```

Maple [A]
time = 0.58, size = 314, normalized size = 0.98

method	result
default	$\left(32\sin(dx+c)(\cos^4(dx+c))-32(\cos^5(dx+c))+45\sqrt{2}\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{2}}{2}\right)\sin(dx+c)+45\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(5/2)*(a+a*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/128/d*(32*sin(d*x+c)*cos(d*x+c)^4-32*cos(d*x+c)^5+45*2^(1/2)*(-2*cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^
(1/2))*sin(d*x+c)+45*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1
/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*sin(
d*x+c)+48*sin(d*x+c)*cos(d*x+c)^3+80*cos(d*x+c)^4-60*cos(d*x+c)^2*sin(d*x+c
)+12*cos(d*x+c)^3+90*cos(d*x+c)*sin(d*x+c)+30*cos(d*x+c)^2-90*cos(d*x+c))*(
e*cos(d*x+c))^(5/2)*(a*(1+sin(d*x+c)))^(3/2)/(cos(d*x+c)*sin(d*x+c)+cos(d*x
+c)^2-2*sin(d*x+c)+cos(d*x+c)-2)/cos(d*x+c)^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima"
)
```

```
[Out] e^(5/2)*integrate((a*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^(5/2), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3472 vs. $2(246) = 492$.

time = 200.53, size = 3472, normalized size = 10.88

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas"
)
```

```
[Out] -1/512*(180*(sqrt(2)*d*sin(d*x + c) + sqrt(2)*d)*(a^6/d^4)^(1/4)*arctan(-1/
4*(2*sqrt(1/2)*((sqrt(2)*d^3*cos(d*x + c)^6 - 3*sqrt(2)*d^3*cos(d*x + c)^5
- 8*sqrt(2)*d^3*cos(d*x + c)^4 + 4*sqrt(2)*d^3*cos(d*x + c)^3 + 8*sqrt(2)*d
^3*cos(d*x + c)^2 - (sqrt(2)*d^3*cos(d*x + c)^5 + 4*sqrt(2)*d^3*cos(d*x + c
)^4 - 4*sqrt(2)*d^3*cos(d*x + c)^3 - 8*sqrt(2)*d^3*cos(d*x + c)^2)*sin(d*x
+ c))*(a^6/d^4)^(3/4)*e^(15/2) + (sqrt(2)*a^3*d*cos(d*x + c)^6*e^5 + 5*sqrt
(2)*a^3*d*cos(d*x + c)^5*e^5 - 8*sqrt(2)*a^3*d*cos(d*x + c)^4*e^5 - 20*sqrt
(2)*a^3*d*cos(d*x + c)^3*e^5 + 8*sqrt(2)*a^3*d*cos(d*x + c)^2*e^5 + 16*sqrt
(2)*a^3*d*cos(d*x + c)*e^5 + (sqrt(2)*a^3*d*cos(d*x + c)^5*e^5 - 4*sqrt(2)*
a^3*d*cos(d*x + c)^4*e^5 - 12*sqrt(2)*a^3*d*cos(d*x + c)^3*e^5 + 8*sqrt(2)*
a^3*d*cos(d*x + c)^2*e^5 + 16*sqrt(2)*a^3*d*cos(d*x + c)*e^5)*sin(d*x + c))
*(a^6/d^4)^(1/4)*e^(5/2) - (a^4*cos(d*x + c)^4*e^(15/2) - 3*a^4*cos(d*x + c
)^3*e^(15/2) - 8*a^4*cos(d*x + c)^2*e^(15/2) + 4*a^4*cos(d*x + c)*e^(15/2)
+ 8*a^4*e^(15/2) + (2*a*d^2*cos(d*x + c)^5*e^(5/2) - 5*a*d^2*cos(d*x + c)^4
*e^(5/2) - 19*a*d^2*cos(d*x + c)^3*e^(5/2) + 20*a*d^2*cos(d*x + c)*e^(5/2)
+ 8*a*d^2*e^(5/2) - (2*a*d^2*cos(d*x + c)^4*e^(5/2) + 9*a*d^2*cos(d*x + c)^
3*e^(5/2) - 4*a*d^2*cos(d*x + c)^2*e^(5/2) - 20*a*d^2*cos(d*x + c)*e^(5/2)
- 8*a*d^2*e^(5/2))*sin(d*x + c))*sqrt(a^6/d^4)*e^5 - (a^4*cos(d*x + c)^3*e^
(15/2) + 4*a^4*cos(d*x + c)^2*e^(15/2) - 4*a^4*cos(d*x + c)*e^(15/2) - 8*a^
4*e^(15/2))*sin(d*x + c))*sqrt(a*sin(d*x + c) + a)*sqrt(cos(d*x + c)))*sqrt
((2*a^9*cos(d*x + c)*e^15*sin(d*x + c) + 2*a^9*cos(d*x + c)*e^15 + (a^6*d^2
*e^10*sin(d*x + c) + a^6*d^2*e^10)*sqrt(a^6/d^4)*e^5 + (sqrt(2)*(a^6/d^4)^(
1/4)*a^7*d*cos(d*x + c)*e^15 + (sqrt(2)*a^4*d^3*e^(15/2)*sin(d*x + c) + sqr
t(2)*a^4*d^3*e^(15/2))*(a^6/d^4)^(3/4)*e^(15/2))*sqrt(a*sin(d*x + c) + a)*s
qrt(cos(d*x + c)))/(sin(d*x + c) + 1)) + ((2*sqrt(2)*a^4*d^3*cos(d*x + c)^5
*e^(15/2) + sqrt(2)*a^4*d^3*cos(d*x + c)^4*e^(15/2) - 13*sqrt(2)*a^4*d^3*co
s(d*x + c)^3*e^(15/2) - 8*sqrt(2)*a^4*d^3*cos(d*x + c)^2*e^(15/2) + 12*sqrt
(2)*a^4*d^3*cos(d*x + c)*e^(15/2) + 8*sqrt(2)*a^4*d^3*e^(15/2) - (7*sqrt(2)
*a^4*d^3*cos(d*x + c)^3*e^(15/2) + 4*sqrt(2)*a^4*d^3*cos(d*x + c)^2*e^(15/2
) - 12*sqrt(2)*a^4*d^3*cos(d*x + c)*e^(15/2) - 8*sqrt(2)*a^4*d^3*e^(15/2))*
sin(d*x + c))*(a^6/d^4)^(3/4)*e^(15/2) + (7*sqrt(2)*a^7*d*cos(d*x + c)^4*e^
(25/2) + 3*sqrt(2)*a^7*d*cos(d*x + c)^3*e^(25/2) - 16*sqrt(2)*a^7*d*cos(d*x
 + c)^2*e^(25/2) - 4*sqrt(2)*a^7*d*cos(d*x + c)*e^(25/2) + 8*sqrt(2)*a^7*d*
e^(25/2) + (2*sqrt(2)*a^7*d*cos(d*x + c)^4*e^(25/2) + sqrt(2)*a^7*d*cos(d*x
 + c)^3*e^(25/2) - 12*sqrt(2)*a^7*d*cos(d*x + c)^2*e^(25/2) - 4*sqrt(2)*a^7
```

```
*d*cos(d*x + c)*e^(25/2) + 8*sqrt(2)*a^7*d*e^(25/2))*sin(d*x + c))*(a^6/d^4
)^(1/4)*e^(5/2))*sqrt(a*sin(d*x + c) + a)*sqrt(cos(d*x + c)))/(a^9*cos(d*x
+ c)^6*e^15 + a^9*cos(d*x + c)^5*e^15 - 8*a^9*cos(d*x + c)^4*e^15 - 8*a^9*c
os(d*x + c)^3*e^15 + 8*a^9*cos(d*x + c)^2*e^15 + 8*a^9*cos(d*x + c)*e^15 -
4*(a^9*cos(d*x + c)^4*e^15 + a^9*cos(d*x + c)^3*e^15 - 2*a^9*cos(d*x + c)^2
*e^15 - 2*a^9*cos(d*x + c)*e^15)*sin(d*x + c)))*e^(5/2) - 180*(sqrt(2)*d*si
n(d*x + c) + sqrt(2)*d)*(a^6/d^4)^(1/4)*arctan(1/4*(2*sqrt(1/2)*((sqrt(2)*d
^3*cos(d*x + c)^6 - 3*sqrt(2)*d^3*cos(d*x + c)^5 - 8*sqrt(2)*d^3*cos(d*x +
c)^4 + 4*sqrt(2)*d^3*cos(d*x + c)^3 + 8*sqrt(2)*d^3*cos(d*x + c)^2 - (sqrt(
2)*d^3*cos(d*x + c)^5 + 4*sqrt(2)*d^3*cos(d*x + c)^4 - 4*sqrt(2)*d^3*cos(d*
x + c)^3 - 8*sqrt(2)*d^3*cos(d*x + c)^2)*sin(d*x + c))*(a^6/d^4)^(3/4)*e^(1
5/2) + (sqrt(2)*a^3*d*cos(d*x + c)^6*e^5 + 5*sqrt(2)*a^3*d*cos(d*x + c)^5*e
^5 - 8*sqrt(2)*a^3*d*cos(d*x + c)^4*e^5 - 20*sqrt(2)*a^3*d*cos(d*x + c)^3*e
^5 + 8*sqrt(2)*a^3*d*cos(d*x + c)^2*e^5 + 16*sqrt(2)*a^3*d*cos(d*x + c)*e^5
 + (sqrt(2)*a^3*d*cos(d*x + c)^5*e^5 - 4*sqrt(2)*a^3*d*cos(d*x + c)^4*e^5 -
 12*sqrt(2)*a^3*d*cos(d*x + c)^3*e^5 + 8*sqrt(2)*a^3*d*cos(d*x + c)^2*e^5 +
 16*sqrt(2)*a^3*d*cos(d*x + c)*e^5)*sin(d*x + c))*(a^6/d^4)^(1/4)*e^(5/2) +
 (a^4*cos(d*x + c)^4*e^(15/2) - 3*a^4*cos(d*x + c)^3*e^(15/2) - 8*a^4*cos(d
*x + c)^2*e^(15/2) + 4*a^4*cos(d*x + c)*e^(15/2) + 8*a^4*e^(15/2) + (2*a*d^
2*cos(d*x + c)^5*e^(5/2) - 5*a*d^2*cos(d*x + c)^4*e^(5/2) - 19*a*d^2*cos(d*
x + c)^3*e^(5/2) + 20*a*d^2*cos(d*x + c)*e^(5/2) + 8*a*d^2*e^(5/2) - (2*a*d
^2*cos(d*x + c)^4*e^(5/2) + 9*a*d^2*cos(d*x + c)^3*e^(5/2) - 4*a*d^2*cos(d*
x + c)^2*e^(5/2) - 20*a*d^2*cos(d*x + c)*e^(5/2) - 8*a*d^2*e^(5/2))*sin(d*x
 + c))*sqrt(a^6/d^4)*e^5 - (a^4*cos(d*x + c)^3*e^(15/2) + 4*a^4*cos(d*x + c
)^2*e^(15/2) - 4*a^4*cos(d*x + c)*e^(15/2) - 8*a^4*e^(15/2))*sin(d*x + c))*
sqrt(a*sin(d*x + c) + a)*sqrt(cos(d*x + c)))*sqrt((2*a^9*cos(d*x + c)*e^15*
sin(d*x + c) + 2*a^9*cos(d*x + c)*e^15 + (a^6*d^2*e^10*sin(d*x + c) + a^6*d
^2*e^10)*sqrt(a^6/d^4)*e^5 - (sqrt(2)*(a^6/d^4)^(1/4)*a^7*d*cos(d*x + c)*e^
15 + (sqrt(2)*a^4*d^3*e^(15/2)*sin(d*x + c) + sqrt(2)*a^4*d^3*e^(15/2))*(a^
6/d^4)^(3/4)*e^(15/2))*sqrt(a*sin(d*x + c) + a)*sqrt(cos(d*x + c)))/(sin(d*
x + c) + 1)) + ((2*sqrt(2)*a^4*d^3*cos(d*x + c)^5*e^(15/2) + sqrt(2)*a^4*d^
3*cos(d*x + c)^4*e^(15/2) - 13*sqrt(2)*a^4*d^3*...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(5/2)*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^(5/2)*e^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(e\,\cos\left(c+d\,x\right)\right)}^{5/2}\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(5/2)*(a + a*sin(c + d*x))^(3/2),x)
```

```
[Out] int((e*cos(c + d*x))^(5/2)*(a + a*sin(c + d*x))^(3/2), x)
```

3.281 $\int (e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=278

$$-\frac{7a^2(e\cos(c+dx))^{5/2}}{12de\sqrt{a+a\sin(c+dx)}}+\frac{7ae\sqrt{e\cos(c+dx)}\,\sqrt{a+a\sin(c+dx)}}{8d}-\frac{a(e\cos(c+dx))^{5/2}\sqrt{a+a\sin(c+}{3de}$$

```
[Out] -7/12*a^2*(e*cos(d*x+c))^(5/2)/d/e/(a+a*sin(d*x+c))^(1/2)-1/3*a*(e*cos(d*x+
c))^(5/2)*(a+a*sin(d*x+c))^(1/2)/d/e+7/8*a*e*(e*cos(d*x+c))^(1/2)*(a+a*sin(
d*x+c))^(1/2)/d-7/8*a*e^(3/2)*arcsinh((e*cos(d*x+c))^(1/2)/e^(1/2))*(1+cos(
d*x+c))^(1/2)*(a+a*sin(d*x+c))^(1/2)/d/(1+cos(d*x+c)+sin(d*x+c))+7/8*a*e^(3
/2)*arctan(sin(d*x+c)*e^(1/2)/(e*cos(d*x+c))^(1/2)/(1+cos(d*x+c))^(1/2))*(1
+cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^(1/2)/d/(1+cos(d*x+c)+sin(d*x+c))
```

Rubi [A]
time = 0.30, antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {2757, 2764, 2756, 2854, 209, 2912, 65, 221}

$$-\frac{7a^2(e\cos(c+dx))^{5/2}}{12de\sqrt{a\sin(c+dx)+a}}+\frac{7ae^{3/2}\sqrt{\cos(c+dx)+1}\,\sqrt{a\sin(c+dx)+a}\,\text{ArcTan}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\,\sqrt{e\cos(c+dx)}}\right)}{8d(\sin(c+dx)+\cos(c+dx)+1)}-\frac{7ae^{3/2}\sqrt{\cos(c+dx)+1}\,\sqrt{a\sin(c+dx)+a}\,\sinh^{-1}\left(\frac{\sqrt{e\cos(c+dx)}}{\sqrt{e}}\right)}{8d(\sin(c+dx)+\cos(c+dx)+1)}-\frac{a\sqrt{a\sin(c+dx)+a}\,(e\cos(c+dx))^{5/2}}{3de}+\frac{7ae\sqrt{a\sin(c+dx)+a}\,\sqrt{e\cos(c+dx)}}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-7*a^2*(e*Cos[c + d*x])^(5/2))/(12*d*e*Sqrt[a + a*Sin[c + d*x]]) + (7*a*e*
Sqrt[e*Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(8*d) - (a*(e*Cos[c + d*x])^
(5/2)*Sqrt[a + a*Sin[c + d*x]])/(3*d*e) - (7*a*e^(3/2)*ArcSinh[Sqrt[e*Cos[c
 + d*x]]/Sqrt[e]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(8*d*(1
+ Cos[c + d*x] + Sin[c + d*x])) + (7*a*e^(3/2)*ArcTan[(Sqrt[e]*Sin[c + d*x]
)/(Sqrt[e*Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]])]*Sqrt[1 + Cos[c + d*x]]*Sqr
t[a + a*Sin[c + d*x]])/(8*d*(1 + Cos[c + d*x] + Sin[c + d*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 2756

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[cos[(e_.) + (f_.)*(x_)]
*(g_.)], x_Symbol] :> Dist[a*Sqrt[1 + Cos[e + f*x]]*(Sqrt[a + b*Sin[e + f*x
]]/(a + a*Cos[e + f*x] + b*Sin[e + f*x])), Int[Sqrt[1 + Cos[e + f*x]]/Sqrt[
g*Cos[e + f*x]], x], x] + Dist[b*Sqrt[1 + Cos[e + f*x]]*(Sqrt[a + b*Sin[e +
 f*x]]/(a + a*Cos[e + f*x] + b*Sin[e + f*x])), Int[Sin[e + f*x]/(Sqrt[g*Cos
[e + f*x]]*Sqrt[1 + Cos[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, g}, x] &&
EqQ[a^2 - b^2, 0]
```

Rule 2757

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ
[2*m, 2*p]
```

Rule 2764

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]], x_Symbol] :> Simp[g*Sqrt[g*Cos[e + f*x]]*(Sqrt[a + b*Sin[e + f*x
]]/(b*f)), x] + Dist[g^2/(2*a), Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[g*Cos[e +
 f*x]], x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2854

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b + d*x^2), x], x
, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]))], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^{3/2}\,dx = -\frac{a(e\cos(c+dx))^{5/2}\sqrt{a+a\sin(c+dx)}}{3de} + \frac{1}{6}(7a)\int (e\cos($$

$$= -\frac{7a^2(e\cos(c+dx))^{5/2}}{12de\sqrt{a+a\sin(c+dx)}} - \frac{a(e\cos(c+dx))^{5/2}\sqrt{a+a\sin}}{3de}$$

$$= -\frac{7a^2(e\cos(c+dx))^{5/2}}{12de\sqrt{a+a\sin(c+dx)}} + \frac{7ae\sqrt{e\cos(c+dx)}\sqrt{a+a\,\mathrm{s}}}{8d}$$

$$= -\frac{7a^2(e\cos(c+dx))^{5/2}}{12de\sqrt{a+a\sin(c+dx)}} + \frac{7ae\sqrt{e\cos(c+dx)}\sqrt{a+a\,\mathrm{s}}}{8d}$$

$$= -\frac{7a^2(e\cos(c+dx))^{5/2}}{12de\sqrt{a+a\sin(c+dx)}} + \frac{7ae\sqrt{e\cos(c+dx)}\sqrt{a+a\,\mathrm{s}}}{8d}$$

$$= -\frac{7a^2(e\cos(c+dx))^{5/2}}{12de\sqrt{a+a\sin(c+dx)}} + \frac{7ae\sqrt{e\cos(c+dx)}\sqrt{a+a\,\mathrm{s}}}{8d}$$

$$= -\frac{7a^2(e\cos(c+dx))^{5/2}}{12de\sqrt{a+a\sin(c+dx)}} + \frac{7ae\sqrt{e\cos(c+dx)}\sqrt{a+a\,\mathrm{s}}}{8d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.14, size = 78, normalized size = 0.28

$$-\frac{8\ 2^{3/4}a(e\cos(c+dx))^{5/2}\,{}_2F_1\left(-\frac{7}{4},\frac{5}{4};\frac{9}{4};\frac{1}{2}(1-\sin(c+dx))\right)\sqrt{a(1+\sin(c+dx))}}{5de(1+\sin(c+dx))^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-8*2^(3/4)*a*(e*Cos[c + d*x])^(5/2)*Hypergeometric2F1[-7/4, 5/4, 9/4, (1 -
 Sin[c + d*x])/2]*Sqrt[a*(1 + Sin[c + d*x])])/(5*d*e*(1 + Sin[c + d*x])^(7/
4))
```

Maple [A]
time = 0.21, size = 288, normalized size = 1.04

method	result
default	$-\frac{\left(21\sqrt{2}\,\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2}}{2}\right)\sin(dx+c)-21\sqrt{2}\,\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2}{1}}}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/48/d*(21*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*sin(d*x+c)-21*2^(1/2)*(-2*cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin
(d*x+c)/cos(d*x+c)*2^(1/2))*sin(d*x+c)+16*cos(d*x+c)^4-16*sin(d*x+c)*cos(d*
x+c)^3-44*cos(d*x+c)^3-28*cos(d*x+c)^2*sin(d*x+c)-14*cos(d*x+c)^2+42*cos(d*
x+c)*sin(d*x+c)+42*cos(d*x+c))*(e*cos(d*x+c))^(3/2)*(a*(1+sin(d*x+c)))^(3/2
)/(cos(d*x+c)*sin(d*x+c)+cos(d*x+c)^2-2*sin(d*x+c)+cos(d*x+c)-2)/cos(d*x+c)
^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima"
)
```

```
[Out] e^(3/2)*integrate((a*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3365 vs. $2(214) = 428$.
time = 204.39, size = 3365, normalized size = 12.10

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas"
)
```

```
[Out] 1/192*(84*sqrt(2)*(a^6/d^4)^(1/4)*d*arctan(-1/4*(2*sqrt(1/2)*((sqrt(2)*d^3*
cos(d*x + c)^6 + 5*sqrt(2)*d^3*cos(d*x + c)^5 - 8*sqrt(2)*d^3*cos(d*x + c)^
4 - 20*sqrt(2)*d^3*cos(d*x + c)^3 + 8*sqrt(2)*d^3*cos(d*x + c)^2 + 16*sqrt(
2)*d^3*cos(d*x + c) + (sqrt(2)*d^3*cos(d*x + c)^5 - 4*sqrt(2)*d^3*cos(d*x +
 c)^4 - 12*sqrt(2)*d^3*cos(d*x + c)^3 + 8*sqrt(2)*d^3*cos(d*x + c)^2 + 16*s
```

qrt(2)*d^3*cos(d*x + c))*sin(d*x + c))*(a^6/d^4)^(3/4)*e^(9/2) + (sqrt(2)*a
^3*d*cos(d*x + c)^6*e^3 - 3*sqrt(2)*a^3*d*cos(d*x + c)^5*e^3 - 8*sqrt(2)*a^
3*d*cos(d*x + c)^4*e^3 + 4*sqrt(2)*a^3*d*cos(d*x + c)^3*e^3 + 8*sqrt(2)*a^3
*d*cos(d*x + c)^2*e^3 - (sqrt(2)*a^3*d*cos(d*x + c)^5*e^3 + 4*sqrt(2)*a^3*d
*cos(d*x + c)^4*e^3 - 4*sqrt(2)*a^3*d*cos(d*x + c)^3*e^3 - 8*sqrt(2)*a^3*d*
cos(d*x + c)^2*e^3)*sin(d*x + c))*(a^6/d^4)^(1/4)*e^(3/2) - (a^4*cos(d*x +
c)^4*e^(9/2) - 3*a^4*cos(d*x + c)^3*e^(9/2) - 8*a^4*cos(d*x + c)^2*e^(9/2)
+ 4*a^4*cos(d*x + c)*e^(9/2) + 8*a^4*e^(9/2) + (2*a*d^2*cos(d*x + c)^5*e^(3
/2) - 5*a*d^2*cos(d*x + c)^4*e^(3/2) - 19*a*d^2*cos(d*x + c)^3*e^(3/2) + 20
*a*d^2*cos(d*x + c)*e^(3/2) + 8*a*d^2*e^(3/2) - (2*a*d^2*cos(d*x + c)^4*e^(
3/2) + 9*a*d^2*cos(d*x + c)^3*e^(3/2) - 4*a*d^2*cos(d*x + c)^2*e^(3/2) - 20
*a*d^2*cos(d*x + c)*e^(3/2) - 8*a*d^2*e^(3/2))*sin(d*x + c))*sqrt(a^6/d^4)*
e^3 - (a^4*cos(d*x + c)^3*e^(9/2) + 4*a^4*cos(d*x + c)^2*e^(9/2) - 4*a^4*co
s(d*x + c)*e^(9/2) - 8*a^4*e^(9/2))*sin(d*x + c))*sqrt(a*sin(d*x + c) + a)*
sqrt(cos(d*x + c)))*sqrt((2*a^9*cos(d*x + c)*e^9*sin(d*x + c) + 2*a^9*cos(d
*x + c)*e^9 + (a^6*d^2*e^6*sin(d*x + c) + a^6*d^2*e^6)*sqrt(a^6/d^4)*e^3 +
(sqrt(2)*(a^6/d^4)^(3/4)*a^4*d^3*cos(d*x + c)*e^9 + (sqrt(2)*a^7*d*e^(15/2)
*sin(d*x + c) + sqrt(2)*a^7*d*e^(15/2))*(a^6/d^4)^(1/4)*e^(3/2))*sqrt(a*sin
(d*x + c) + a)*sqrt(cos(d*x + c)))/(sin(d*x + c) + 1)) - ((7*sqrt(2)*a^4*d^
3*cos(d*x + c)^4*e^(9/2) + 3*sqrt(2)*a^4*d^3*cos(d*x + c)^3*e^(9/2) - 16*sq
rt(2)*a^4*d^3*cos(d*x + c)^2*e^(9/2) - 4*sqrt(2)*a^4*d^3*cos(d*x + c)*e^(9/
2) + 8*sqrt(2)*a^4*d^3*e^(9/2) + (2*sqrt(2)*a^4*d^3*cos(d*x + c)^4*e^(9/2)
+ sqrt(2)*a^4*d^3*cos(d*x + c)^3*e^(9/2) - 12*sqrt(2)*a^4*d^3*cos(d*x + c)^
2*e^(9/2) - 4*sqrt(2)*a^4*d^3*cos(d*x + c)*e^(9/2) + 8*sqrt(2)*a^4*d^3*e^(9
/2))*sin(d*x + c))*(a^6/d^4)^(3/4)*e^(9/2) + (2*sqrt(2)*a^7*d*cos(d*x + c)^
5*e^(15/2) + sqrt(2)*a^7*d*cos(d*x + c)^4*e^(15/2) - 13*sqrt(2)*a^7*d*cos(d
*x + c)^3*e^(15/2) - 8*sqrt(2)*a^7*d*cos(d*x + c)^2*e^(15/2) + 12*sqrt(2)*a
^7*d*cos(d*x + c)*e^(15/2) + 8*sqrt(2)*a^7*d*e^(15/2) - (7*sqrt(2)*a^7*d*co
s(d*x + c)^3*e^(15/2) + 4*sqrt(2)*a^7*d*cos(d*x + c)^2*e^(15/2) - 12*sqrt(2
)*a^7*d*cos(d*x + c)*e^(15/2) - 8*sqrt(2)*a^7*d*e^(15/2))*sin(d*x + c))*(a^
6/d^4)^(1/4)*e^(3/2))*sqrt(a*sin(d*x + c) + a)*sqrt(cos(d*x + c)))/(a^9*cos
(d*x + c)^6*e^9 + a^9*cos(d*x + c)^5*e^9 - 8*a^9*cos(d*x + c)^4*e^9 - 8*a^9
*cos(d*x + c)^3*e^9 + 8*a^9*cos(d*x + c)^2*e^9 + 8*a^9*cos(d*x + c)*e^9 - 4
*(a^9*cos(d*x + c)^4*e^9 + a^9*cos(d*x + c)^3*e^9 - 2*a^9*cos(d*x + c)^2*e^
9 - 2*a^9*cos(d*x + c)*e^9)*sin(d*x + c)))*e^(3/2) - 84*sqrt(2)*(a^6/d^4)^(
1/4)*d*arctan(1/4*(2*sqrt(1/2)*((sqrt(2)*d^3*cos(d*x + c)^6 + 5*sqrt(2)*d^3
*cos(d*x + c)^5 - 8*sqrt(2)*d^3*cos(d*x + c)^4 - 20*sqrt(2)*d^3*cos(d*x + c
)^3 + 8*sqrt(2)*d^3*cos(d*x + c)^2 + 16*sqrt(2)*d^3*cos(d*x + c) + (sqrt(2)
*d^3*cos(d*x + c)^5 - 4*sqrt(2)*d^3*cos(d*x + c)^4 - 12*sqrt(2)*d^3*cos(d*x
 + c)^3 + 8*sqrt(2)*d^3*cos(d*x + c)^2 + 16*sqrt(2)*d^3*cos(d*x + c))*sin(d
x + c))(a^6/d^4)^(3/4)*e^(9/2) + (sqrt(2)*a^3*d*cos(d*x + c)^6*e^3 - 3*sq
rt(2)*a^3*d*cos(d*x + c)^5*e^3 - 8*sqrt(2)*a^3*d*cos(d*x + c)^4*e^3 + 4*sqr
t(2)*a^3*d*cos(d*x + c)^3*e^3 + 8*sqrt(2)*a^3*d*cos(d*x + c)^2*e^3 - (sqrt(
2)*a^3*d*cos(d*x + c)^5*e^3 + 4*sqrt(2)*a^3*d*cos(d*x + c)^4*e^3 - 4*sqrt(2
)*a^3*d*cos(d*x + c)^3*e^3 - 8*sqrt(2)*a^3*d*cos(d*x + c)^2*e^3)*sin(d*x +

```
c))*(a^6/d^4)^(1/4)*e^(3/2) + (a^4*cos(d*x + c)^4*e^(9/2) - 3*a^4*cos(d*x +
 c)^3*e^(9/2) - 8*a^4*cos(d*x + c)^2*e^(9/2) + 4*a^4*cos(d*x + c)*e^(9/2) +
 8*a^4*e^(9/2) + (2*a*d^2*cos(d*x + c)^5*e^(3/2) - 5*a*d^2*cos(d*x + c)^4*e
^(3/2) - 19*a*d^2*cos(d*x + c)^3*e^(3/2) + 20*a*d^2*cos(d*x + c)*e^(3/2) +
8*a*d^2*e^(3/2) - (2*a*d^2*cos(d*x + c)^4*e^(3/2) + 9*a*d^2*cos(d*x + c)^3*
e^(3/2) - 4*a*d^2*cos(d*x + c)^2*e^(3/2) - 20*a*d^2*cos(d*x + c)*e^(3/2) -
8*a*d^2*e^(3/2))*sin(d*x + c))*sqrt(a^6/d^4)*e^3 - (a^4*cos(d*x + c)^3*e^(9
/2) + 4*a^4*cos(d*x + c)^2*e^(9/2) - 4*a^4*cos(d*x + c)*e^(9/2) - 8*a^4*e^(
9/2))*sin(d*x + c))*sqrt(a*sin(d*x + c) + a)*sqrt(cos(d*x + c)))*sqrt((2*a^
9*cos(d*x + c)*e^9*sin(d*x + c) + 2*a^9*cos(d*x + c)*e^9 + (a^6*d^2*e^6*sin
(d*x + c) + a^6*d^2*e^6)*sqrt(a^6/d^4)*e^3 - (sqrt(2)*(a^6/d^4)^(3/4)*a^4*d
^3*cos(d*x + c)*e^9 + (sqrt(2)*a^7*d*e^(15/2)*sin(d*x + c) + sqrt(2)*a^7*d*
e^(15/2))*(a^6/d^4)^(1/4)*e^(3/2))*sqrt(a*sin(d*x + c) + a)*sqrt(cos(d*x +
c)))/(sin(d*x + c) + 1)) - ((7*sqrt(2)*a^4*d^3*cos(d*x + c)^4*e^(9/2) + 3*s
qrt(2)*a^4*d^3*cos(d*x + c)^3*e^(9/2) - 16*sqrt(2)*a^4*d^3*cos(d*x + c)^2*e
^(9/2) - 4*sqrt(2)*a^4*d^3*cos(d*x + c)*e^(9/2) + 8*sqrt(2)*a^4*d^3*e^(9/2)
 + (2*sqrt(2)*a^4*d^3*cos(d*x + c)^4*e^(9/2) + ...
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(3/2)*(a+a*sin(d*x+c))**(3/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^(3/2)*e^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (e\cos(c+dx))^{3/2}\,(a+a\sin(c+dx))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(3/2)*(a + a*sin(c + d*x))^(3/2),x)
[Out] int((e*cos(c + d*x))^(3/2)*(a + a*sin(c + d*x))^(3/2), x)
```

3.282 $\int \sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=243

$$-\frac{5a^2(e\cos(c+dx))^{3/2}}{4de\sqrt{a+a\sin(c+dx)}}-\frac{a(e\cos(c+dx))^{3/2}\sqrt{a+a\sin(c+dx)}}{2de}+\frac{5a\sqrt{e}\,\sinh^{-1}\left(\frac{\sqrt{e\cos(c+dx)}}{\sqrt{e}}\right)\sqrt{1}}{4d(1+\cos(c+dx}$$

```
[Out] -5/4*a^2*(e*cos(d*x+c))^(3/2)/d/e/(a+a*sin(d*x+c))^(1/2)-1/2*a*(e*cos(d*x+c
))^(3/2)*(a+a*sin(d*x+c))^(1/2)/d/e+5/4*a*arcsinh((e*cos(d*x+c))^(1/2)/e^(1
/2))*e^(1/2)*(1+cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^(1/2)/d/(1+cos(d*x+c)+si
n(d*x+c))+5/4*a*arctan(sin(d*x+c)*e^(1/2)/(e*cos(d*x+c))^(1/2)/(1+cos(d*x+c
))^(1/2))*e^(1/2)*(1+cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^(1/2)/d/(1+cos(d*x+
c)+sin(d*x+c))
```

Rubi [A]
time = 0.24, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2757, 2763, 2854, 209, 2912, 65, 221}

$$-\frac{5a^2(e\cos(c+dx))^{3/2}}{4de\sqrt{a\sin(c+dx)+a}}+\frac{5a\sqrt{e}\sqrt{\cos(c+dx)+1}\sqrt{a\sin(c+dx)+a}\,\text{ArcTan}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\sqrt{e\cos(c+dx)}}\right)}{4d(\sin(c+dx)+\cos(c+dx)+1)}-\frac{a\sqrt{a\sin(c+dx)+a}\,(e\cos(c+dx))^{3/2}}{2de}+\frac{5a\sqrt{e}\sqrt{\cos(c+dx)+1}\sqrt{a\sin(c+dx)+a}\,\sinh^{-1}\left(\frac{\sqrt{e\cos(c+dx)}}{\sqrt{e}}\right)}{4d(\sin(c+dx)+\cos(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-5*a^2*(e*Cos[c + d*x])^(3/2))/(4*d*e*Sqrt[a + a*Sin[c + d*x]]) - (a*(e*Co
s[c + d*x])^(3/2)*Sqrt[a + a*Sin[c + d*x]])/(2*d*e) + (5*a*Sqrt[e]*ArcSinh[
Sqrt[e*Cos[c + d*x]]/Sqrt[e]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x
]])/(4*d*(1 + Cos[c + d*x] + Sin[c + d*x])) + (5*a*Sqrt[e]*ArcTan[(Sqrt[e]*
Sin[c + d*x])/(Sqrt[e*Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]])]*Sqrt[1 + Cos[c
 + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(4*d*(1 + Cos[c + d*x] + Sin[c + d*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 2757

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ
[2*m, 2*p]
```

Rule 2763

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[g*Sqrt[1 + Cos[e + f*x]]*(Sqrt[a + b*Sin[e + f*x
]]/(a + a*Cos[e + f*x] + b*Sin[e + f*x])), Int[Sqrt[1 + Cos[e + f*x]]/Sqrt[
g*Cos[e + f*x]], x], x] - Dist[g*Sqrt[1 + Cos[e + f*x]]*(Sqrt[a + b*Sin[e +
 f*x]]/(b + b*Cos[e + f*x] + a*Sin[e + f*x])), Int[Sin[e + f*x]/(Sqrt[g*Cos
[e + f*x]]*Sqrt[1 + Cos[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, g}, x] &&
EqQ[a^2 - b^2, 0]
```

Rule 2854

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b + d*x^2), x], x
, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]))], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^{3/2}\,dx = -\frac{a(e\cos(c+dx))^{3/2}\sqrt{a+a\sin(c+dx)}}{2de} + \frac{1}{4}(5a)\int \sqrt{e\cos$$

$$= -\frac{5a^2(e\cos(c+dx))^{3/2}}{4de\sqrt{a+a\sin(c+dx)}} - \frac{a(e\cos(c+dx))^{3/2}\sqrt{a+a\sin(}}{2de}$$

$$= -\frac{5a^2(e\cos(c+dx))^{3/2}}{4de\sqrt{a+a\sin(c+dx)}} - \frac{a(e\cos(c+dx))^{3/2}\sqrt{a+a\sin(}}{2de}$$

$$= -\frac{5a^2(e\cos(c+dx))^{3/2}}{4de\sqrt{a+a\sin(c+dx)}} - \frac{a(e\cos(c+dx))^{3/2}\sqrt{a+a\sin(}}{2de}$$

$$= -\frac{5a^2(e\cos(c+dx))^{3/2}}{4de\sqrt{a+a\sin(c+dx)}} - \frac{a(e\cos(c+dx))^{3/2}\sqrt{a+a\sin(}}{2de}$$

$$= -\frac{5a^2(e\cos(c+dx))^{3/2}}{4de\sqrt{a+a\sin(c+dx)}} - \frac{a(e\cos(c+dx))^{3/2}\sqrt{a+a\sin(}}{2de}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.12, size = 77, normalized size = 0.32

$$-\frac{8\sqrt[4]{2}\,(e\cos(c+dx))^{3/2}\,{}_2F_1\left(-\frac{5}{4},\frac{3}{4};\frac{7}{4};\frac{1}{2}(1-\sin(c+dx))\right)(a(1+\sin(c+dx)))^{3/2}}{3de(1+\sin(c+dx))^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-8*2^(1/4)*(e*Cos[c + d*x])^(3/2)*Hypergeometric2F1[-5/4, 3/4, 7/4, (1 - S
in[c + d*x])/2]*(a*(1 + Sin[c + d*x]))^(3/2))/(3*d*e*(1 + Sin[c + d*x])^(9/
4))
```

Maple [A]
time = 0.20, size = 262, normalized size = 1.08

method	result

default	$\frac{\left(5\sqrt{2}\,\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2}}{2}\right)\sin(dx+c)+5\sqrt{2}\,\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(}{1+\cos}}}{}\right.\right.}{8d(\cos(dx+c)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^(3/2)*(e*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/d*(5*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*sin(d*x+c)+5*2^(1/2)*(-2*cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x
+c)/cos(d*x+c)*2^(1/2))*sin(d*x+c)-4*cos(d*x+c)^3+4*cos(d*x+c)^2*sin(d*x+c)
+14*cos(d*x+c)^2+10*cos(d*x+c)*sin(d*x+c)-10*cos(d*x+c))*(a*(1+sin(d*x+c)))
^(3/2)*(e*cos(d*x+c))^(1/2)/(cos(d*x+c)*sin(d*x+c)+cos(d*x+c)^2-2*sin(d*x+c
)+cos(d*x+c)-2)/cos(d*x+c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(3/2)*(e*cos(d*x+c))^(1/2),x, algorithm="maxima"
)
```

```
[Out] e^(1/2)*integrate((a*sin(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3443 vs. $2(186) = 372$.
time = 195.04, size = 3443, normalized size = 14.17

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(3/2)*(e*cos(d*x+c))^(1/2),x, algorithm="fricas"
)
```

```
[Out] -1/32*(20*(sqrt(2)*d*sin(d*x + c) + sqrt(2)*d)*(a^6/d^4)^(1/4)*arctan(-1/4*
(2*sqrt(1/2)*((sqrt(2)*d^3*cos(d*x + c)^6 - 3*sqrt(2)*d^3*cos(d*x + c)^5 -
8*sqrt(2)*d^3*cos(d*x + c)^4 + 4*sqrt(2)*d^3*cos(d*x + c)^3 + 8*sqrt(2)*d^3
*cos(d*x + c)^2 - (sqrt(2)*d^3*cos(d*x + c)^5 + 4*sqrt(2)*d^3*cos(d*x + c)^
4 - 4*sqrt(2)*d^3*cos(d*x + c)^3 - 8*sqrt(2)*d^3*cos(d*x + c)^2)*sin(d*x +
c))*(a^6/d^4)^(3/4)*e^(3/2) + (sqrt(2)*a^3*d*cos(d*x + c)^6*e + 5*sqrt(2)*a
^3*d*cos(d*x + c)^5*e - 8*sqrt(2)*a^3*d*cos(d*x + c)^4*e - 20*sqrt(2)*a^3*d
```

*cos(d*x + c)^3*e + 8*sqrt(2)*a^3*d*cos(d*x + c)^2*e + 16*sqrt(2)*a^3*d*cos
(d*x + c)*e + (sqrt(2)*a^3*d*cos(d*x + c)^5*e - 4*sqrt(2)*a^3*d*cos(d*x + c
)^4*e - 12*sqrt(2)*a^3*d*cos(d*x + c)^3*e + 8*sqrt(2)*a^3*d*cos(d*x + c)^2*
e + 16*sqrt(2)*a^3*d*cos(d*x + c)*e)*sin(d*x + c))*(a^6/d^4)^(1/4)*e^(1/2)
- (a^4*cos(d*x + c)^4*e^(3/2) - 3*a^4*cos(d*x + c)^3*e^(3/2) - 8*a^4*cos(d*
x + c)^2*e^(3/2) + 4*a^4*cos(d*x + c)*e^(3/2) + 8*a^4*e^(3/2) + (2*a*d^2*co
s(d*x + c)^5*e^(1/2) - 5*a*d^2*cos(d*x + c)^4*e^(1/2) - 19*a*d^2*cos(d*x +
c)^3*e^(1/2) + 20*a*d^2*cos(d*x + c)*e^(1/2) + 8*a*d^2*e^(1/2) - (2*a*d^2*c
os(d*x + c)^4*e^(1/2) + 9*a*d^2*cos(d*x + c)^3*e^(1/2) - 4*a*d^2*cos(d*x +
c)^2*e^(1/2) - 20*a*d^2*cos(d*x + c)*e^(1/2) - 8*a*d^2*e^(1/2))*sin(d*x + c
))*sqrt(a^6/d^4)*e - (a^4*cos(d*x + c)^3*e^(3/2) + 4*a^4*cos(d*x + c)^2*e^(
3/2) - 4*a^4*cos(d*x + c)*e^(3/2) - 8*a^4*e^(3/2))*sin(d*x + c))*sqrt(a*sin
(d*x + c) + a)*sqrt(cos(d*x + c)))*sqrt((2*a^9*cos(d*x + c)*e^3*sin(d*x + c
) + 2*a^9*cos(d*x + c)*e^3 + (a^6*d^2*e^2*sin(d*x + c) + a^6*d^2*e^2)*sqrt(
a^6/d^4)*e + (sqrt(2)*(a^6/d^4)^(1/4)*a^7*d*cos(d*x + c)*e^3 + (sqrt(2)*a^4
*d^3*e^(3/2)*sin(d*x + c) + sqrt(2)*a^4*d^3*e^(3/2))*(a^6/d^4)^(3/4)*e^(3/2
))*sqrt(a*sin(d*x + c) + a)*sqrt(cos(d*x + c)))/(sin(d*x + c) + 1)) + ((2*s
qrt(2)*a^4*d^3*cos(d*x + c)^5*e^(3/2) + sqrt(2)*a^4*d^3*cos(d*x + c)^4*e^(3
/2) - 13*sqrt(2)*a^4*d^3*cos(d*x + c)^3*e^(3/2) - 8*sqrt(2)*a^4*d^3*cos(d*x
 + c)^2*e^(3/2) + 12*sqrt(2)*a^4*d^3*cos(d*x + c)*e^(3/2) + 8*sqrt(2)*a^4*d
^3*e^(3/2) - (7*sqrt(2)*a^4*d^3*cos(d*x + c)^3*e^(3/2) + 4*sqrt(2)*a^4*d^3*
cos(d*x + c)^2*e^(3/2) - 12*sqrt(2)*a^4*d^3*cos(d*x + c)*e^(3/2) - 8*sqrt(2
)*a^4*d^3*e^(3/2))*sin(d*x + c))*(a^6/d^4)^(3/4)*e^(3/2) + (7*sqrt(2)*a^7*d
*cos(d*x + c)^4*e^(5/2) + 3*sqrt(2)*a^7*d*cos(d*x + c)^3*e^(5/2) - 16*sqrt(
2)*a^7*d*cos(d*x + c)^2*e^(5/2) - 4*sqrt(2)*a^7*d*cos(d*x + c)*e^(5/2) + 8*
sqrt(2)*a^7*d*e^(5/2) + (2*sqrt(2)*a^7*d*cos(d*x + c)^4*e^(5/2) + sqrt(2)*a
^7*d*cos(d*x + c)^3*e^(5/2) - 12*sqrt(2)*a^7*d*cos(d*x + c)^2*e^(5/2) - 4*s
qrt(2)*a^7*d*cos(d*x + c)*e^(5/2) + 8*sqrt(2)*a^7*d*e^(5/2))*sin(d*x + c))*
(a^6/d^4)^(1/4)*e^(1/2))*sqrt(a*sin(d*x + c) + a)*sqrt(cos(d*x + c)))/(a^9*
cos(d*x + c)^6*e^3 + a^9*cos(d*x + c)^5*e^3 - 8*a^9*cos(d*x + c)^4*e^3 - 8*
a^9*cos(d*x + c)^3*e^3 + 8*a^9*cos(d*x + c)^2*e^3 + 8*a^9*cos(d*x + c)*e^3
- 4*(a^9*cos(d*x + c)^4*e^3 + a^9*cos(d*x + c)^3*e^3 - 2*a^9*cos(d*x + c)^2
*e^3 - 2*a^9*cos(d*x + c)*e^3)*sin(d*x + c)))*e^(1/2) - 20*(sqrt(2)*d*sin(d
*x + c) + sqrt(2)*d)*(a^6/d^4)^(1/4)*arctan(1/4*(2*sqrt(1/2)*((sqrt(2)*d^3*
cos(d*x + c)^6 - 3*sqrt(2)*d^3*cos(d*x + c)^5 - 8*sqrt(2)*d^3*cos(d*x + c)^
4 + 4*sqrt(2)*d^3*cos(d*x + c)^3 + 8*sqrt(2)*d^3*cos(d*x + c)^2 - (sqrt(2)*
d^3*cos(d*x + c)^5 + 4*sqrt(2)*d^3*cos(d*x + c)^4 - 4*sqrt(2)*d^3*cos(d*x +
 c)^3 - 8*sqrt(2)*d^3*cos(d*x + c)^2)*sin(d*x + c))*(a^6/d^4)^(3/4)*e^(3/2)
 + (sqrt(2)*a^3*d*cos(d*x + c)^6*e + 5*sqrt(2)*a^3*d*cos(d*x + c)^5*e - 8*s
qrt(2)*a^3*d*cos(d*x + c)^4*e - 20*sqrt(2)*a^3*d*cos(d*x + c)^3*e + 8*sqrt(
2)*a^3*d*cos(d*x + c)^2*e + 16*sqrt(2)*a^3*d*cos(d*x + c)*e + (sqrt(2)*a^3*
d*cos(d*x + c)^5*e - 4*sqrt(2)*a^3*d*cos(d*x + c)^4*e - 12*sqrt(2)*a^3*d*co
s(d*x + c)^3*e + 8*sqrt(2)*a^3*d*cos(d*x + c)^2*e + 16*sqrt(2)*a^3*d*cos(d*
x + c)*e)*sin(d*x + c))*(a^6/d^4)^(1/4)*e^(1/2) + (a^4*cos(d*x + c)^4*e^(3/
2) - 3*a^4*cos(d*x + c)^3*e^(3/2) - 8*a^4*cos(d*x + c)^2*e^(3/2) + 4*a^4*co

```
s(d*x + c)*e^(3/2) + 8*a^4*e^(3/2) + (2*a*d^2*cos(d*x + c)^5*e^(1/2) - 5*a*
d^2*cos(d*x + c)^4*e^(1/2) - 19*a*d^2*cos(d*x + c)^3*e^(1/2) + 20*a*d^2*cos
(d*x + c)*e^(1/2) + 8*a*d^2*e^(1/2) - (2*a*d^2*cos(d*x + c)^4*e^(1/2) + 9*a
*d^2*cos(d*x + c)^3*e^(1/2) - 4*a*d^2*cos(d*x + c)^2*e^(1/2) - 20*a*d^2*cos
(d*x + c)*e^(1/2) - 8*a*d^2*e^(1/2))*sin(d*x + c))*sqrt(a^6/d^4)*e - (a^4*c
os(d*x + c)^3*e^(3/2) + 4*a^4*cos(d*x + c)^2*e^(3/2) - 4*a^4*cos(d*x + c)*e
^(3/2) - 8*a^4*e^(3/2))*sin(d*x + c))*sqrt(a*sin(d*x + c) + a)*sqrt(cos(d*x
 + c)))*sqrt((2*a^9*cos(d*x + c)*e^3*sin(d*x + c) + 2*a^9*cos(d*x + c)*e^3
+ (a^6*d^2*e^2*sin(d*x + c) + a^6*d^2*e^2)*sqrt(a^6/d^4)*e - (sqrt(2)*(a^6/
d^4)^(1/4)*a^7*d*cos(d*x + c)*e^3 + (sqrt(2)*a^4*d^3*e^(3/2)*sin(d*x + c) +
 sqrt(2)*a^4*d^3*e^(3/2))*(a^6/d^4)^(3/4)*e^(3/2))*sqrt(a*sin(d*x + c) + a)
*sqrt(cos(d*x + c)))/(sin(d*x + c) + 1)) + ((2*sqrt(2)*a^4*d^3*cos(d*x + c)
^5*e^(3/2) + sqrt(2)*a^4*d^3*cos(d*x + c)^4*e^(3/2) - 13*sqrt(2)*a^4*d^3*co
s(d*x + c)^3*e^(3/2) - 8*sqrt(2)*a^4*d^3*cos(d*x + c)^2*e^(3/2) + 12*sqrt(2
)*a^4*d^3*cos(d*x + c)*e^(3/2) + 8*sqrt(2)*a^4*...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sin(c+dx)+1))^{\frac{3}{2}}\sqrt{e\cos(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**(3/2)*(e*cos(d*x+c))**(1/2),x)
[Out] Integral((a*(sin(c + d*x) + 1))**(3/2)*sqrt(e*cos(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(3/2)*(e*cos(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c))*e^(1/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{e\,\cos(c+d\,x)}\,(a+a\,\sin(c+d\,x))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(1/2)*(a + a*sin(c + d*x))^(3/2),x)
[Out] int((e*cos(c + d*x))^(1/2)*(a + a*sin(c + d*x))^(3/2), x)
```

3.283 $\int \frac{(a+a\sin(c+dx))^{3/2}}{\sqrt{e\cos(c+dx)}}\,dx$

Optimal. Leaf size=198

$$-\frac{a\sqrt{e\cos(c+dx)}\,\sqrt{a+a\sin(c+dx)}}{de}-\frac{3a\sinh^{-1}\left(\frac{\sqrt{e\cos(c+dx)}}{\sqrt{e}}\right)\sqrt{1+\cos(c+dx)}\,\sqrt{a+a\sin(c+}{d\sqrt{e}\,(1+\cos(c+dx)+\sin(c+dx))}$$

```
[Out] -a*(e*cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^(1/2)/d/e-3*a*arcsinh((e*cos(d*x+c
))^(1/2)/e^(1/2))*(1+cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^(1/2)/d/(1+cos(d*x+
c)+sin(d*x+c))/e^(1/2)+3*a*arctan(sin(d*x+c)*e^(1/2)/(e*cos(d*x+c))^(1/2)/(
1+cos(d*x+c))^(1/2))*(1+cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^(1/2)/d/(1+cos(d
*x+c)+sin(d*x+c))/e^(1/2)
```

Rubi [A]

time = 0.19, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2757, 2756, 2854, 209, 2912, 65, 221}

$$\frac{3a\sqrt{\cos(c+dx)+1}\,\sqrt{a\sin(c+dx)+a}\,\text{ArcTan}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\,\sqrt{e\cos(c+dx)}}\right)}{d\sqrt{e}\,(\sin(c+dx)+\cos(c+dx)+1)}-\frac{a\sqrt{a\sin(c+dx)+a}\,\sqrt{e\cos(c+dx)}}{de}-\frac{3a\sqrt{\cos(c+dx)+1}\,\sqrt{a\sin(c+dx)+a}\,\sinh^{-1}\left(\frac{\sqrt{e\cos(c+dx)}}{\sqrt{e}}\right)}{d\sqrt{e}\,(\sin(c+dx)+\cos(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^(3/2)/Sqrt[e*Cos[c + d*x]],x]
```

```
[Out] -((a*Sqrt[e*Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(d*e)) - (3*a*ArcSinh[S
qrt[e*Cos[c + d*x]]/Sqrt[e]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]
])/(d*Sqrt[e]*(1 + Cos[c + d*x] + Sin[c + d*x])) + (3*a*ArcTan[(Sqrt[e]*Sin
[c + d*x])/(Sqrt[e*Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]])]*Sqrt[1 + Cos[c +
d*x]]*Sqrt[a + a*Sin[c + d*x]])/(d*Sqrt[e]*(1 + Cos[c + d*x] + Sin[c + d*x]
))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 2756

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[cos[(e_.) + (f_.)*(x_)]
*(g_.)], x_Symbol] :> Dist[a*Sqrt[1 + Cos[e + f*x]]*(Sqrt[a + b*Sin[e + f*x
]]/(a + a*Cos[e + f*x] + b*Sin[e + f*x])), Int[Sqrt[1 + Cos[e + f*x]]/Sqrt[
g*Cos[e + f*x]], x], x] + Dist[b*Sqrt[1 + Cos[e + f*x]]*(Sqrt[a + b*Sin[e +
 f*x]]/(a + a*Cos[e + f*x] + b*Sin[e + f*x])), Int[Sin[e + f*x]/(Sqrt[g*Cos
[e + f*x]]*Sqrt[1 + Cos[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, g}, x] &&
EqQ[a^2 - b^2, 0]
```

Rule 2757

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ
[2*m, 2*p]
```

Rule 2854

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b + d*x^2), x], x
, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]))], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(a+a\sin(c+dx))^{3/2}}{\sqrt{e\cos(c+dx)}}\,dx = -\frac{a\sqrt{e\cos(c+dx)}\,\sqrt{a+a\sin(c+dx)}}{de} + \frac{1}{2}(3a)\int \frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{e\cos(c+dx)}}\,dx$$

$$= -\frac{a\sqrt{e\cos(c+dx)}\,\sqrt{a+a\sin(c+dx)}}{de} + \frac{\left(3a^2\sqrt{1+\cos(c+dx)}\,\sqrt{a+a\,\mathrm{s}\right.}{2(a+a\cos(c+}$$

$$= -\frac{a\sqrt{e\cos(c+dx)}\,\sqrt{a+a\sin(c+dx)}}{de} - \frac{\left(3a^2\sqrt{1+\cos(c+dx)}\,\sqrt{a+a\,\mathrm{s}\right.}{2d(a+}$$

$$= -\frac{a\sqrt{e\cos(c+dx)}\,\sqrt{a+a\sin(c+dx)}}{de} + \frac{3a^2\tan^{-1}\left(\frac{\sqrt{e}\,\sin(c+}{\sqrt{e\cos(c+dx)}\,\sqrt{1-}\right.}{d\sqrt{e}\,(a-}$$

$$= -\frac{a\sqrt{e\cos(c+dx)}\,\sqrt{a+a\sin(c+dx)}}{de} - \frac{3a^2\sinh^{-1}\left(\frac{\sqrt{e\cos(c+dx)}}{\sqrt{e}}\right)\sqrt{}}{d\sqrt{e}\,(a+a\cos(c}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.11, size = 75, normalized size = 0.38

$$-\frac{4\ 2^{3/4}\sqrt{e\cos(c+dx)}\ {}_2F_1\left(-\frac{3}{4},\frac{1}{4};\frac{5}{4};\frac{1}{2}(1-\sin(c+dx))\right)(a(1+\sin(c+dx)))^{3/2}}{de(1+\sin(c+dx))^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^(3/2)/Sqrt[e*Cos[c + d*x]],x]
```

```
[Out] (-4*2^(3/4)*Sqrt[e*Cos[c + d*x]]*Hypergeometric2F1[-3/4, 1/4, 5/4, (1 - Sin
[c + d*x])/2]*(a*(1 + Sin[c + d*x]))^(3/2))/(d*e*(1 + Sin[c + d*x])^(7/4))
```

Maple [A]
time = 0.18, size = 228, normalized size = 1.15

method	result
default	$-\frac{\left(3\sqrt{2}\,\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2}}{2}\right)\sin(dx+c)-3\sqrt{2}\,\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\operatorname{arctanh}\left(\sqrt{-\frac{2}{1}}\right.\right.}{2d(\cos(dx+c)\sin(dx+c)+\cos^2(dx+c)-2\sin(dx+c)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^(3/2)/(e*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d*(3*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*sin(d*x+c)-3*2^(1/2)*(-2*cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*
x+c)/cos(d*x+c)*2^(1/2))*sin(d*x+c)-2*cos(d*x+c)*sin(d*x+c)+2*cos(d*x+c)^2-
2*cos(d*x+c))*(a*(1+sin(d*x+c)))^(3/2)/(cos(d*x+c)*sin(d*x+c)+cos(d*x+c)^2-
2*sin(d*x+c)+cos(d*x+c)-2)/(e*cos(d*x+c))^(1/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(3/2)/(e*cos(d*x+c))^(1/2),x, algorithm="maxima"
)
```

```
[Out] e^(-1/2)*integrate((a*sin(d*x + c) + a)^(3/2)/sqrt(cos(d*x + c)), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3274 vs. $2(156) = 312$.
time = 183.86, size = 3274, normalized size = 16.54

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(3/2)/(e*cos(d*x+c))^(1/2),x, algorithm="fricas"
)
```

```
[Out] 1/8*(12*sqrt(2)*(a^6/d^4)^(1/4)*d*arctan(-1/4*(2*sqrt(1/2)*((sqrt(2)*d^3*co
s(d*x + c)^6*e^(3/2) + 5*sqrt(2)*d^3*cos(d*x + c)^5*e^(3/2) - 8*sqrt(2)*d^3
*cos(d*x + c)^4*e^(3/2) - 20*sqrt(2)*d^3*cos(d*x + c)^3*e^(3/2) + 8*sqrt(2)
*d^3*cos(d*x + c)^2*e^(3/2) + 16*sqrt(2)*d^3*cos(d*x + c)*e^(3/2) + (sqrt(2
)*d^3*cos(d*x + c)^5*e^(3/2) - 4*sqrt(2)*d^3*cos(d*x + c)^4*e^(3/2) - 12*sq
rt(2)*d^3*cos(d*x + c)^3*e^(3/2) + 8*sqrt(2)*d^3*cos(d*x + c)^2*e^(3/2) + 1
6*sqrt(2)*d^3*cos(d*x + c)*e^(3/2))*sin(d*x + c))*(a^6/d^4)^(3/4)*e^(-3/2)
+ (sqrt(2)*a^3*d*cos(d*x + c)^6*e^(1/2) - 3*sqrt(2)*a^3*d*cos(d*x + c)^5*e^
(1/2) - 8*sqrt(2)*a^3*d*cos(d*x + c)^4*e^(1/2) + 4*sqrt(2)*a^3*d*cos(d*x +
c)^3*e^(1/2) + 8*sqrt(2)*a^3*d*cos(d*x + c)^2*e^(1/2) - (sqrt(2)*a^3*d*cos(
d*x + c)^5*e^(1/2) + 4*sqrt(2)*a^3*d*cos(d*x + c)^4*e^(1/2) - 4*sqrt(2)*a^3
*d*cos(d*x + c)^3*e^(1/2) - 8*sqrt(2)*a^3*d*cos(d*x + c)^2*e^(1/2))*sin(d*x
```

+ c))*(a^6/d^4)^(1/4)*e^(-1/2) - (a^4*cos(d*x + c)^4 - 3*a^4*cos(d*x + c)^3 - 8*a^4*cos(d*x + c)^2 + 4*a^4*cos(d*x + c) + 8*a^4 + (2*a*d^2*cos(d*x + c)^5*e - 5*a*d^2*cos(d*x + c)^4*e - 19*a*d^2*cos(d*x + c)^3*e + 20*a*d^2*cos(d*x + c)*e + 8*a*d^2*e - (2*a*d^2*cos(d*x + c)^4*e + 9*a*d^2*cos(d*x + c)^3*e - 4*a*d^2*cos(d*x + c)^2*e - 20*a*d^2*cos(d*x + c)*e - 8*a*d^2*e)*sin(d*x + c))*sqrt(a^6/d^4)*e^(-1) - (a^4*cos(d*x + c)^3 + 4*a^4*cos(d*x + c)^2 - 4*a^4*cos(d*x + c) - 8*a^4)*sin(d*x + c))*sqrt(a*sin(d*x + c) + a)*sqrt(cos(d*x + c)))*sqrt((2*a^9*cos(d*x + c)*sin(d*x + c) + 2*a^9*cos(d*x + c) + (a^6*d^2*e*sin(d*x + c) + a^6*d^2*e)*sqrt(a^6/d^4)*e^(-1) + (sqrt(2)*(a^6/d^4)^(3/4)*a^4*d^3*cos(d*x + c) + (sqrt(2)*a^7*d*e^(1/2)*sin(d*x + c) + sqrt(2)*a^7*d*e^(1/2))*(a^6/d^4)^(1/4)*e^(-1/2))*sqrt(a*sin(d*x + c) + a)*sqrt(cos(d*x + c)))/(sin(d*x + c) + 1)) - ((7*sqrt(2)*a^4*d^3*cos(d*x + c)^4*e^(3/2) + 3*sqrt(2)*a^4*d^3*cos(d*x + c)^3*e^(3/2) - 16*sqrt(2)*a^4*d^3*cos(d*x + c)^2*e^(3/2) - 4*sqrt(2)*a^4*d^3*cos(d*x + c)*e^(3/2) + 8*sqrt(2)*a^4*d^3*e^(3/2) + (2*sqrt(2)*a^4*d^3*cos(d*x + c)^4*e^(3/2) + sqrt(2)*a^4*d^3*cos(d*x + c)^3*e^(3/2) - 12*sqrt(2)*a^4*d^3*cos(d*x + c)^2*e^(3/2) - 4*sqrt(2)*a^4*d^3*cos(d*x + c)*e^(3/2) + 8*sqrt(2)*a^4*d^3*e^(3/2))*sin(d*x + c))*(a^6/d^4)^(3/4)*e^(-3/2) + (2*sqrt(2)*a^7*d*cos(d*x + c)^5*e^(1/2) + sqrt(2)*a^7*d*cos(d*x + c)^4*e^(1/2) - 13*sqrt(2)*a^7*d*cos(d*x + c)^3*e^(1/2) - 8*sqrt(2)*a^7*d*cos(d*x + c)^2*e^(1/2) + 12*sqrt(2)*a^7*d*cos(d*x + c)*e^(1/2) + 8*sqrt(2)*a^7*d*e^(1/2) - (7*sqrt(2)*a^7*d*cos(d*x + c)^3*e^(1/2) + 4*sqrt(2)*a^7*d*cos(d*x + c)^2*e^(1/2) - 12*sqrt(2)*a^7*d*cos(d*x + c)*e^(1/2) - 8*sqrt(2)*a^7*d*e^(1/2))*sin(d*x + c))*(a^6/d^4)^(1/4)*e^(-1/2))*sqrt(a*sin(d*x + c) + a)*sqrt(cos(d*x + c)))/(a^9*cos(d*x + c)^6 + a^9*cos(d*x + c)^5 - 8*a^9*cos(d*x + c)^4 - 8*a^9*cos(d*x + c)^3 + 8*a^9*cos(d*x + c)^2 + 8*a^9*cos(d*x + c) - 4*(a^9*cos(d*x + c)^4 + a^9*cos(d*x + c)^3 - 2*a^9*cos(d*x + c)^2 - 2*a^9*cos(d*x + c))*sin(d*x + c))) - 12*sqrt(2)*(a^6/d^4)^(1/4)*d*arctan(1/4*(2*sqrt(1/2)*((sqrt(2)*d^3*cos(d*x + c)^6*e^(3/2) + 5*sqrt(2)*d^3*cos(d*x + c)^5*e^(3/2) - 8*sqrt(2)*d^3*cos(d*x + c)^4*e^(3/2) - 20*sqrt(2)*d^3*cos(d*x + c)^3*e^(3/2) + 8*sqrt(2)*d^3*cos(d*x + c)^2*e^(3/2) + 16*sqrt(2)*d^3*cos(d*x + c)*e^(3/2) + (sqrt(2)*d^3*cos(d*x + c)^5*e^(3/2) - 4*sqrt(2)*d^3*cos(d*x + c)^4*e^(3/2) - 12*sqrt(2)*d^3*cos(d*x + c)^3*e^(3/2) + 8*sqrt(2)*d^3*cos(d*x + c)^2*e^(3/2) + 16*sqrt(2)*d^3*cos(d*x + c)*e^(3/2))*sin(d*x + c))*(a^6/d^4)^(3/4)*e^(-3/2) + (sqrt(2)*a^3*d*cos(d*x + c)^6*e^(1/2) - 3*sqrt(2)*a^3*d*cos(d*x + c)^5*e^(1/2) - 8*sqrt(2)*a^3*d*cos(d*x + c)^4*e^(1/2) + 4*sqrt(2)*a^3*d*cos(d*x + c)^3*e^(1/2) + 8*sqrt(2)*a^3*d*cos(d*x + c)^2*e^(1/2) - (sqrt(2)*a^3*d*cos(d*x + c)^5*e^(1/2) + 4*sqrt(2)*a^3*d*cos(d*x + c)^4*e^(1/2) - 4*sqrt(2)*a^3*d*cos(d*x + c)^3*e^(1/2) - 8*sqrt(2)*a^3*d*cos(d*x + c)^2*e^(1/2))*sin(d*x + c))*(a^6/d^4)^(1/4)*e^(-1/2) + (a^4*cos(d*x + c)^4 - 3*a^4*cos(d*x + c)^3 - 8*a^4*cos(d*x + c)^2 + 4*a^4*cos(d*x + c) + 8*a^4 + (2*a*d^2*cos(d*x + c)^5*e - 5*a*d^2*cos(d*x + c)^4*e - 19*a*d^2*cos(d*x + c)^3*e + 20*a*d^2*cos(d*x + c)*e + 8*a*d^2*e - (2*a*d^2*cos(d*x + c)^4*e + 9*a*d^2*cos(d*x + c)^3*e - 4*a*d^2*cos(d*x + c)^2*e - 20*a*d^2*cos(d*x + c)*e - 8*a*d^2*e)*sin(d*x + c))*sqrt(a^6/d^4)*e^(-1) - (a^4*cos(d*x + c)^3 + 4*a^4*cos(d*x + c)^2 - 4*a^4*cos(d*x + c) - 8*a^

```
4)*sin(d*x + c))*sqrt(a*sin(d*x + c) + a)*sqrt(cos(d*x + c)))*sqrt((2*a^9*c
os(d*x + c)*sin(d*x + c) + 2*a^9*cos(d*x + c) + (a^6*d^2*e*sin(d*x + c) + a
^6*d^2*e)*sqrt(a^6/d^4)*e^(-1) - (sqrt(2)*(a^6/d^4)^(3/4)*a^4*d^3*cos(d*x +
 c) + (sqrt(2)*a^7*d*e^(1/2)*sin(d*x + c) + sqrt(2)*a^7*d*e^(1/2))*(a^6/d^4
)^(1/4)*e^(-1/2))*sqrt(a*sin(d*x + c) + a)*sqrt(cos(d*x + c)))/(sin(d*x + c
) + 1)) - ((7*sqrt(2)*a^4*d^3*cos(d*x + c)^4*e^(3/2) + 3*sqrt(2)*a^4*d^3*co
s(d*x + c)^3*e^(3/2) - 16*sqrt(2)*a^4*d^3*cos(d*x + c)^2*e^(3/2) - 4*sqrt(2
)*a^4*d^3*cos(d*x + c)*e^(3/2) + 8*sqrt(2)*a^4*d^3*e^(3/2) + (2*sqrt(2)*a^4
*d^3*cos(d*x + c)^4*e^(3/2) + sqrt(2)*a^4*d^3*cos(d*x + c)^3*e^(3/2) - 12*s
qrt(2)*a^4*d^3*cos(d*x + c)^2*e^(3/2) - 4*sqrt(...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a(\sin(c+dx)+1))^{\frac{3}{2}}}{\sqrt{e\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**(3/2)/(e*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((a*(sin(c + d*x) + 1))**(3/2)/sqrt(e*cos(c + d*x)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(3/2)/(e*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\,\sin(c+d\,x))^{3/2}}{\sqrt{e\,\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(3/2)/(e*cos(c + d*x))^(1/2),x)
```

```
[Out] int((a + a*sin(c + d*x))^(3/2)/(e*cos(c + d*x))^(1/2), x)
```

3.284 $\int \frac{(a+a\sin(c+dx))^{3/2}}{(e\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=210

$$\frac{4a\sqrt{a+a\sin(c+dx)}}{de\sqrt{e\cos(c+dx)}}-\frac{2a^2\sinh^{-1}\left(\frac{\sqrt{e\cos(c+dx)}}{\sqrt{e}}\right)\sqrt{1+\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}{de^{3/2}(a+a\cos(c+dx)+a\sin(c+dx))}-\frac{2a^2\tan^{-1}\cdots}{\cdots}$$

```
[Out] 4*a*(a+a*sin(d*x+c))^(1/2)/d/e/(e*cos(d*x+c))^(1/2)-2*a^2*arcsinh((e*cos(d*
x+c))^(1/2)/e^(1/2))*(1+cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^(1/2)/d/e^(3/2)/
(a+a*cos(d*x+c)+a*sin(d*x+c))-2*a^2*arctan(sin(d*x+c)*e^(1/2)/(e*cos(d*x+c)
)^(1/2)/(1+cos(d*x+c))^(1/2))*(1+cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^(1/2)/d
/e^(3/2)/(a+a*cos(d*x+c)+a*sin(d*x+c))
```

Rubi [A]
time = 0.19, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2755, 2763, 2854, 209, 2912, 65, 221}

$$-\frac{2a^2\sqrt{\cos(c+dx)+1}\sqrt{a\sin(c+dx)+a}\,\text{ArcTan}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\sqrt{e\cos(c+dx)}}\right)}{de^{3/2}(a\sin(c+dx)+a\cos(c+dx)+a)}-\frac{2a^2\sqrt{\cos(c+dx)+1}\sqrt{a\sin(c+dx)+a}\sinh^{-1}\left(\frac{\sqrt{e\cos(c+dx)}}{\sqrt{e}}\right)}{de^{3/2}(a\sin(c+dx)+a\cos(c+dx)+a)}+\frac{4a\sqrt{a\sin(c+dx)+a}}{de\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^(3/2)/(e*Cos[c + d*x])^(3/2),x]
```

```
[Out] (4*a*Sqrt[a + a*Sin[c + d*x]])/(d*e*Sqrt[e*Cos[c + d*x]]) - (2*a^2*ArcSinh[
Sqrt[e*Cos[c + d*x]]/Sqrt[e]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x
]])/(d*e^(3/2)*(a + a*Cos[c + d*x] + a*Sin[c + d*x])) - (2*a^2*ArcTan[(Sqrt
[e]*Sin[c + d*x])/(Sqrt[e*Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]])]*Sqrt[1 + C
os[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(d*e^(3/2)*(a + a*Cos[c + d*x] + a*S
in[c + d*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 2755

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[-2*b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(p + 1))), x] + Dist[b^2*((2*m + p - 1)/(g^2*(p + 1))),
Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[
{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && Inte
gersQ[2*m, 2*p]
```

Rule 2763

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[g*Sqrt[1 + Cos[e + f*x]]*(Sqrt[a + b*Sin[e + f*x
]]/(a + a*Cos[e + f*x] + b*Sin[e + f*x])), Int[Sqrt[1 + Cos[e + f*x]]/Sqrt[
g*Cos[e + f*x]], x], x] - Dist[g*Sqrt[1 + Cos[e + f*x]]*(Sqrt[a + b*Sin[e +
 f*x]]/(b + b*Cos[e + f*x] + a*Sin[e + f*x])), Int[Sin[e + f*x]/(Sqrt[g*Cos
[e + f*x]]*Sqrt[1 + Cos[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, g}, x] &&
EqQ[a^2 - b^2, 0]
```

Rule 2854

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b + d*x^2), x], x
, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]))], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(a+a\sin(c+dx))^{3/2}}{(e\cos(c+dx))^{3/2}}\,dx = \frac{4a\sqrt{a+a\sin(c+dx)}}{de\sqrt{e\cos(c+dx)}} - \frac{a^2\int \frac{\sqrt{e\cos(c+dx)}}{\sqrt{a+a\sin(c+dx)}}\,dx}{e^2}$$

$$= \frac{4a\sqrt{a+a\sin(c+dx)}}{de\sqrt{e\cos(c+dx)}} - \frac{\left(a^2\sqrt{1+\cos(c+dx)}\sqrt{a+a\sin(c+dx)}\right)\int \frac{\sqrt{}}{\sqrt{}}}{e(a+a\cos(c+dx)+a\sin(c+d}$$

$$= \frac{4a\sqrt{a+a\sin(c+dx)}}{de\sqrt{e\cos(c+dx)}} - \frac{\left(a^2\sqrt{1+\cos(c+dx)}\sqrt{a+a\sin(c+dx)}\right)\text{Subs}}{de(a+a\cos(c+dx)+a}$$

$$= \frac{4a\sqrt{a+a\sin(c+dx)}}{de\sqrt{e\cos(c+dx)}} - \frac{2a^2\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{e\cos(c+dx)}\sqrt{1+\cos(c+dx)}}\right)}{de^{3/2}(a+a\cos(c+dx)+}$$

$$= \frac{4a\sqrt{a+a\sin(c+dx)}}{de\sqrt{e\cos(c+dx)}} - \frac{2a^2\sinh^{-1}\left(\frac{\sqrt{e\cos(c+dx)}}{\sqrt{e}}\right)\sqrt{1+\cos(c+dx)}}{de^{3/2}(a+a\cos(c+dx)+a\sin(c+}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.12, size = 75, normalized size = 0.36

$$\frac{4\sqrt[4]{2}\,{}_2F_1\left(-\frac{1}{4},-\frac{1}{4};\frac{3}{4};\frac{1}{2}(1-\sin(c+dx))\right)(a(1+\sin(c+dx)))^{3/2}}{de\sqrt{e\cos(c+dx)}\,(1+\sin(c+dx))^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^(3/2)/(e*Cos[c + d*x])^(3/2),x]
[Out] (4*2^(1/4)*Hypergeometric2F1[-1/4, -1/4, 3/4, (1 - Sin[c + d*x])/2]*(a*(1 +
 Sin[c + d*x]))^(3/2))/(d*e*Sqrt[e*Cos[c + d*x]]*(1 + Sin[c + d*x])^(5/4))
```

Maple [A]
time = 0.17, size = 323, normalized size = 1.54

method	result

default	$-\frac{2(a(1+\sin(dx+c)))^{\frac{3}{2}}(-1+\cos(dx+c))\left(\sqrt{2}\,\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2}}{2}\right)\sin(dx+c)+\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}}{2\cos(}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^(3/2)/(e*cos(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*(a*(1+sin(d*x+c)))^(3/2)*(-1+cos(d*x+c))*(2^(1/2)*arctan(1/2*(-2*cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*sin(d*x+c)+2^(1/2)*arctanh(1/2*(-2*cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*sin(d*x+c)-2^(
1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))-2^(1/2)*arcta
nh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))-
2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+2*(-2*cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*cos(d*x+c)+2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))/sin(d*x+c)
/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/(sin(d*x+c)-cos(d*x+c)+1)/(e*cos(d*x+
c))^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(3/2)/(e*cos(d*x+c))^(3/2),x, algorithm="maxima"
)
```

```
[Out] e^(-3/2)*integrate((a*sin(d*x + c) + a)^(3/2)/cos(d*x + c)^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3313 vs. 2(168) = 336.
time = 176.02, size = 3313, normalized size = 15.78

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(3/2)/(e*cos(d*x+c))^(3/2),x, algorithm="fricas"
)
```

```
[Out] 1/4*(4*sqrt(2)*(a^6/d^4)^(1/4)*d*arctan(-1/4*(sqrt(2)*((sqrt(2)*d^3*cos(d*x
 + c)^6*e^(9/2) - 3*sqrt(2)*d^3*cos(d*x + c)^5*e^(9/2) - 8*sqrt(2)*d^3*cos(
d*x + c)^4*e^(9/2) + 4*sqrt(2)*d^3*cos(d*x + c)^3*e^(9/2) + 8*sqrt(2)*d^3*c
os(d*x + c)^2*e^(9/2) - (sqrt(2)*d^3*cos(d*x + c)^5*e^(9/2) + 4*sqrt(2)*d^3
```

*cos(d*x + c)^4*e^(9/2) - 4*sqrt(2)*d^3*cos(d*x + c)^3*e^(9/2) - 8*sqrt(2)*
d^3*cos(d*x + c)^2*e^(9/2))*sin(d*x + c))*(a^6/d^4)^(3/4)*e^(-9/2) + (sqrt(
2)*a^3*d*cos(d*x + c)^6*e^(3/2) + 5*sqrt(2)*a^3*d*cos(d*x + c)^5*e^(3/2) -
8*sqrt(2)*a^3*d*cos(d*x + c)^4*e^(3/2) - 20*sqrt(2)*a^3*d*cos(d*x + c)^3*e^
(3/2) + 8*sqrt(2)*a^3*d*cos(d*x + c)^2*e^(3/2) + 16*sqrt(2)*a^3*d*cos(d*x +
 c)*e^(3/2) + (sqrt(2)*a^3*d*cos(d*x + c)^5*e^(3/2) - 4*sqrt(2)*a^3*d*cos(d
*x + c)^4*e^(3/2) - 12*sqrt(2)*a^3*d*cos(d*x + c)^3*e^(3/2) + 8*sqrt(2)*a^3
*d*cos(d*x + c)^2*e^(3/2) + 16*sqrt(2)*a^3*d*cos(d*x + c)*e^(3/2))*sin(d*x
+ c))*(a^6/d^4)^(1/4)*e^(-3/2) - (a^4*cos(d*x + c)^4 - 3*a^4*cos(d*x + c)^3
 - 8*a^4*cos(d*x + c)^2 + 4*a^4*cos(d*x + c) + 8*a^4 + (2*a*d^2*cos(d*x + c
)^5*e^3 - 5*a*d^2*cos(d*x + c)^4*e^3 - 19*a*d^2*cos(d*x + c)^3*e^3 + 20*a*d
^2*cos(d*x + c)*e^3 + 8*a*d^2*e^3 - (2*a*d^2*cos(d*x + c)^4*e^3 + 9*a*d^2*c
os(d*x + c)^3*e^3 - 4*a*d^2*cos(d*x + c)^2*e^3 - 20*a*d^2*cos(d*x + c)*e^3
- 8*a*d^2*e^3)*sin(d*x + c))*sqrt(a^6/d^4)*e^(-3) - (a^4*cos(d*x + c)^3 + 4
*a^4*cos(d*x + c)^2 - 4*a^4*cos(d*x + c) - 8*a^4)*sin(d*x + c))*sqrt(a*sin(
d*x + c) + a)*sqrt(cos(d*x + c)))*sqrt((2*a^9*cos(d*x + c)*sin(d*x + c) + 2
*a^9*cos(d*x + c) + (a^6*d^2*e^3*sin(d*x + c) + a^6*d^2*e^3)*sqrt(a^6/d^4)*
e^(-3) + (sqrt(2)*(a^6/d^4)^(1/4)*a^7*d*cos(d*x + c) + (sqrt(2)*a^4*d^3*e^(
9/2)*sin(d*x + c) + sqrt(2)*a^4*d^3*e^(9/2))*(a^6/d^4)^(3/4)*e^(-9/2))*sqrt
(a*sin(d*x + c) + a)*sqrt(cos(d*x + c)))/(sin(d*x + c) + 1)) + ((2*sqrt(2)*
a^4*d^3*cos(d*x + c)^5*e^(9/2) + sqrt(2)*a^4*d^3*cos(d*x + c)^4*e^(9/2) - 1
3*sqrt(2)*a^4*d^3*cos(d*x + c)^3*e^(9/2) - 8*sqrt(2)*a^4*d^3*cos(d*x + c)^2
*e^(9/2) + 12*sqrt(2)*a^4*d^3*cos(d*x + c)*e^(9/2) + 8*sqrt(2)*a^4*d^3*e^(9
/2) - (7*sqrt(2)*a^4*d^3*cos(d*x + c)^3*e^(9/2) + 4*sqrt(2)*a^4*d^3*cos(d*x
 + c)^2*e^(9/2) - 12*sqrt(2)*a^4*d^3*cos(d*x + c)*e^(9/2) - 8*sqrt(2)*a^4*d
^3*e^(9/2))*sin(d*x + c))*(a^6/d^4)^(3/4)*e^(-9/2) + (7*sqrt(2)*a^7*d*cos(d
*x + c)^4*e^(3/2) + 3*sqrt(2)*a^7*d*cos(d*x + c)^3*e^(3/2) - 16*sqrt(2)*a^7
*d*cos(d*x + c)^2*e^(3/2) - 4*sqrt(2)*a^7*d*cos(d*x + c)*e^(3/2) + 8*sqrt(2
)*a^7*d*e^(3/2) + (2*sqrt(2)*a^7*d*cos(d*x + c)^4*e^(3/2) + sqrt(2)*a^7*d*c
os(d*x + c)^3*e^(3/2) - 12*sqrt(2)*a^7*d*cos(d*x + c)^2*e^(3/2) - 4*sqrt(2)
*a^7*d*cos(d*x + c)*e^(3/2) + 8*sqrt(2)*a^7*d*e^(3/2))*sin(d*x + c))*(a^6/d
^4)^(1/4)*e^(-3/2))*sqrt(a*sin(d*x + c) + a)*sqrt(cos(d*x + c)))/(a^9*cos(d
*x + c)^6 + a^9*cos(d*x + c)^5 - 8*a^9*cos(d*x + c)^4 - 8*a^9*cos(d*x + c)^
3 + 8*a^9*cos(d*x + c)^2 + 8*a^9*cos(d*x + c) - 4*(a^9*cos(d*x + c)^4 + a^9
*cos(d*x + c)^3 - 2*a^9*cos(d*x + c)^2 - 2*a^9*cos(d*x + c))*sin(d*x + c)))
*cos(d*x + c) - 4*sqrt(2)*(a^6/d^4)^(1/4)*d*arctan(1/4*(sqrt(2)*((sqrt(2)*d
^3*cos(d*x + c)^6*e^(9/2) - 3*sqrt(2)*d^3*cos(d*x + c)^5*e^(9/2) - 8*sqrt(2
)*d^3*cos(d*x + c)^4*e^(9/2) + 4*sqrt(2)*d^3*cos(d*x + c)^3*e^(9/2) + 8*sqr
t(2)*d^3*cos(d*x + c)^2*e^(9/2) - (sqrt(2)*d^3*cos(d*x + c)^5*e^(9/2) + 4*s
qrt(2)*d^3*cos(d*x + c)^4*e^(9/2) - 4*sqrt(2)*d^3*cos(d*x + c)^3*e^(9/2) -
8*sqrt(2)*d^3*cos(d*x + c)^2*e^(9/2))*sin(d*x + c))*(a^6/d^4)^(3/4)*e^(-9/2
) + (sqrt(2)*a^3*d*cos(d*x + c)^6*e^(3/2) + 5*sqrt(2)*a^3*d*cos(d*x + c)^5*
e^(3/2) - 8*sqrt(2)*a^3*d*cos(d*x + c)^4*e^(3/2) - 20*sqrt(2)*a^3*d*cos(d*x
 + c)^3*e^(3/2) + 8*sqrt(2)*a^3*d*cos(d*x + c)^2*e^(3/2) + 16*sqrt(2)*a^3*d
*cos(d*x + c)*e^(3/2) + (sqrt(2)*a^3*d*cos(d*x + c)^5*e^(3/2) - 4*sqrt(2)*a

```
^3*d*cos(d*x + c)^4*e^(3/2) - 12*sqrt(2)*a^3*d*cos(d*x + c)^3*e^(3/2) + 8*s
qrt(2)*a^3*d*cos(d*x + c)^2*e^(3/2) + 16*sqrt(2)*a^3*d*cos(d*x + c)*e^(3/2)
)*sin(d*x + c))*(a^6/d^4)^(1/4)*e^(-3/2) + (a^4*cos(d*x + c)^4 - 3*a^4*cos(
d*x + c)^3 - 8*a^4*cos(d*x + c)^2 + 4*a^4*cos(d*x + c) + 8*a^4 + (2*a*d^2*c
os(d*x + c)^5*e^3 - 5*a*d^2*cos(d*x + c)^4*e^3 - 19*a*d^2*cos(d*x + c)^3*e^
3 + 20*a*d^2*cos(d*x + c)*e^3 + 8*a*d^2*e^3 - (2*a*d^2*cos(d*x + c)^4*e^3 +
 9*a*d^2*cos(d*x + c)^3*e^3 - 4*a*d^2*cos(d*x + c)^2*e^3 - 20*a*d^2*cos(d*x
 + c)*e^3 - 8*a*d^2*e^3)*sin(d*x + c))*sqrt(a^6/d^4)*e^(-3) - (a^4*cos(d*x
+ c)^3 + 4*a^4*cos(d*x + c)^2 - 4*a^4*cos(d*x + c) - 8*a^4)*sin(d*x + c))*s
qrt(a*sin(d*x + c) + a)*sqrt(cos(d*x + c)))*sqrt((2*a^9*cos(d*x + c)*sin(d*
x + c) + 2*a^9*cos(d*x + c) + (a^6*d^2*e^3*sin(d*x + c) + a^6*d^2*e^3)*sqrt
(a^6/d^4)*e^(-3) - (sqrt(2)*(a^6/d^4)^(1/4)*a^7*d*cos(d*x + c) + (sqrt(2)*a
^4*d^3*e^(9/2)*sin(d*x + c) + sqrt(2)*a^4*d^3*e^(9/2))*(a^6/d^4)^(3/4)*e^(-
9/2))*sqrt(a*sin(d*x + c) + a)*sqrt(cos(d*x + c)))/(sin(d*x + c) + 1)) + ((
2*sqrt(2)*a^4*d^3*cos(d*x + c)^5*e^(9/2) + sqrt(2)*a^4*d^3*cos(d*x + c)^4*e
^(9/2) - 13*sqrt(2)*a^4*d^3*cos(d*x + c)^3*e^(9/2) - 8*sqrt(2)*a^4*d^3*cos(
d*x + c)^2*e^(9/2) + 12*sqrt(2)*a^4*d^3*cos(d*x + c)*e^(9/2) + 8*sqrt(2)*a^
4*d^3*e^(9/2) - (7*sqrt(2)*a^4*d^3*cos(d*x + c)...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a(\sin (c+dx)+1))^{\frac{3}{2}}}{(e\cos (c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**(3/2)/(e*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral((a*(sin(c + d*x) + 1))**(3/2)/(e*cos(c + d*x))**(3/2), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(3/2)/(e*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{3/2}}{{\left(e\,\cos\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(3/2)/(e*cos(c + d*x))^(3/2),x)
[Out] int((a + a*sin(c + d*x))^(3/2)/(e*cos(c + d*x))^(3/2), x)
```

3.285 $\int \frac{(a+a\sin(c+dx))^{3/2}}{(e\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=36

$$\frac{2(a+a\sin(c+dx))^{3/2}}{3de(e\cos(c+dx))^{3/2}}$$

```
[Out] 2/3*(a+a*sin(d*x+c))^(3/2)/d/e/(e*cos(d*x+c))^(3/2)
```

Rubi [A]
time = 0.05, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$, Rules used = {2750}

$$\frac{2(a\sin(c+dx)+a)^{3/2}}{3de(e\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^(3/2)/(e*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*(a + a*Sin[c + d*x])^(3/2))/(3*d*e*(e*Cos[c + d*x])^(3/2))
```

Rule 2750

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(c+dx))^{3/2}}{(e\cos(c+dx))^{5/2}}\,dx = \frac{2(a+a\sin(c+dx))^{3/2}}{3de(e\cos(c+dx))^{3/2}}$$

Mathematica [A]
time = 0.10, size = 36, normalized size = 1.00

$$\frac{2(a(1+\sin(c+dx)))^{3/2}}{3de(e\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^(3/2)/(e*Cos[c + d*x])^(5/2),x]
```

[Out] (2*(a*(1 + Sin[c + d*x]))^(3/2))/(3*d*e*(e*Cos[c + d*x])^(3/2))

Maple [A]
time = 0.14, size = 34, normalized size = 0.94

method	result	size
default	$\frac{2\cos(dx+c)(a(1+\sin(dx+c)))^{\frac{3}{2}}}{3d(e\cos(dx+c))^{\frac{5}{2}}}$	34

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(d*x+c))^(3/2)/(e*cos(d*x+c))^(5/2),x,method=_RETURNVERBOSE)

[Out] 2/3/d*cos(d*x+c)*(a*(1+sin(d*x+c)))^(3/2)/(e*cos(d*x+c))^(5/2)

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 77 vs. 2(27) = 54.
time = 0.54, size = 77, normalized size = 2.14

$$\frac{2\left(a^{\frac{3}{2}}-\frac{a^{\frac{3}{2}}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}\right)\sqrt{\frac{\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}+1}\,e^{\left(-\frac{5}{2}\right)}}{3\,d\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(d*x+c))^(3/2)/(e*cos(d*x+c))^(5/2),x, algorithm="maxima")

[Out] 2/3*(a^(3/2) - a^(3/2)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*sqrt(sin(d*x + c)/(cos(d*x + c) + 1) + 1)*e^(-5/2)/(d*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/2))

Fricas [A]
time = 0.36, size = 41, normalized size = 1.14

$$-\frac{2\,\sqrt{a\sin\left(dx+c\right)+a}\,a\sqrt{\cos\left(dx+c\right)}}{3\,\left(de^{\frac{5}{2}}\sin\left(dx+c\right)-de^{\frac{5}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(d*x+c))^(3/2)/(e*cos(d*x+c))^(5/2),x, algorithm="fricas")

[Out] -2/3*sqrt(a*sin(d*x + c) + a)*a*sqrt(cos(d*x + c))/(d*e^(5/2)*sin(d*x + c) - d*e^(5/2))

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**(3/2)/(e*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(3/2)/(e*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 5.62, size = 47, normalized size = 1.31

$$-\frac{2\,a\,\cos\left(c+d\,x\right)\,\sqrt{a\,\left(\sin\left(c+d\,x\right)+1\right)}}{3\,d\,e^2\,\sqrt{e\,\cos\left(c+d\,x\right)}\,\left(\sin\left(c+d\,x\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(3/2)/(e*cos(c + d*x))^(5/2),x)
```

```
[Out] -(2*a*cos(c + d*x)*(a*(sin(c + d*x) + 1))^(1/2))/(3*d*e^2*(e*cos(c + d*x))^
(1/2)*(sin(c + d*x) - 1))
```

3.286 $\int \frac{(a+a\sin(c+dx))^{3/2}}{(e\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=74

$$\frac{2(a+a\sin(c+dx))^{3/2}}{de(e\cos(c+dx))^{5/2}}-\frac{4(a+a\sin(c+dx))^{5/2}}{5ade(e\cos(c+dx))^{5/2}}$$

```
[Out] 2*(a+a*sin(d*x+c))^(3/2)/d/e/(e*cos(d*x+c))^(5/2)-4/5*(a+a*sin(d*x+c))^(5/2
)/a/d/e/(e*cos(d*x+c))^(5/2)
```

Rubi [A]
time = 0.09, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}=0.074$, Rules used = {2751, 2750}

$$\frac{2(a\sin(c+dx)+a)^{3/2}}{de(e\cos(c+dx))^{5/2}}-\frac{4(a\sin(c+dx)+a)^{5/2}}{5ade(e\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^(3/2)/(e*Cos[c + d*x])^(7/2),x]
```

```
[Out] (2*(a + a*Sin[c + d*x])^(3/2))/(d*e*(e*Cos[c + d*x])^(5/2)) - (4*(a + a*Sin
[c + d*x])^(5/2))/(5*a*d*e*(e*Cos[c + d*x])^(5/2))
```

Rule 2750

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(c+dx))^{3/2}}{(e\cos(c+dx))^{7/2}}\,dx = \frac{2(a+a\sin(c+dx))^{3/2}}{de(e\cos(c+dx))^{5/2}} - \frac{2\int \frac{(a+a\sin(c+dx))^{5/2}}{(e\cos(c+dx))^{7/2}}\,dx}{a}$$
$$= \frac{2(a+a\sin(c+dx))^{3/2}}{de(e\cos(c+dx))^{5/2}} - \frac{4(a+a\sin(c+dx))^{5/2}}{5ade(e\cos(c+dx))^{5/2}}$$

Mathematica [A]
time = 0.13, size = 72, normalized size = 0.97

$$-\frac{2a\sqrt{a(1+\sin(c+dx))}\,(-3+2\sin(c+dx))}{5de^3\sqrt{e\cos(c+dx)}\,\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^(3/2)/(e*Cos[c + d*x])^(7/2),x]
```

```
[Out] (-2*a*Sqrt[a*(1 + Sin[c + d*x])]*(-3 + 2*Sin[c + d*x]))/(5*d*e^3*Sqrt[e*Cos
[c + d*x]]*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2)
```

Maple [A]
time = 0.15, size = 44, normalized size = 0.59

method	result	size
default	$-\frac{2(2\sin(dx+c)-3)\cos(dx+c)(a(1+\sin(dx+c)))^{\frac{3}{2}}}{5d(e\cos(dx+c))^{\frac{7}{2}}}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^(3/2)/(e*cos(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5/d*(2*sin(d*x+c)-3)*cos(d*x+c)*(a*(1+sin(d*x+c)))^(3/2)/(e*cos(d*x+c))^
(7/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 189 vs. 2(58) = 116.
time = 0.54, size = 189, normalized size = 2.55

$$\frac{2\left(3\,a^{\frac{3}{2}}-\frac{4\,a^{\frac{3}{2}}\sin(dx+c)}{\cos(dx+c)+1}+\frac{4\,a^{\frac{3}{2}}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{3\,a^{\frac{3}{2}}\sin(dx+c)^4}{(\cos(dx+c)+1)^4}\right)\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^2 e^{\left(-\frac{7}{2}\right)}}{5\,d\sqrt{\frac{\sin(dx+c)}{\cos(dx+c)+1}+1}\,\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(\frac{2\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(3/2)/(e*cos(d*x+c))^(7/2),x, algorithm="maxima"
)
```

```
[Out] 2/5*(3*a^(3/2) - 4*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) + 4*a^(3/2)*sin(
d*x + c)^3/(cos(d*x + c) + 1)^3 - 3*a^(3/2)*sin(d*x + c)^4/(cos(d*x + c) +
1)^4)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^2*e^(-7/2)/(d*sqrt(sin(d*x
+ c)/(cos(d*x + c) + 1) + 1)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(
2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + sin(d*x + c)^4/(cos(d*x + c) + 1)^4
 + 1))
```

Fricas [A]
time = 0.36, size = 65, normalized size = 0.88

$$\frac{2\,(2\,a\sin\left(dx+c\right)-3\,a)\sqrt{a\sin\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}}{5\left(d\cos\left(dx+c\right)e^{\frac{7}{2}}\sin\left(dx+c\right)-d\cos\left(dx+c\right)e^{\frac{7}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(3/2)/(e*cos(d*x+c))^(7/2),x, algorithm="fricas"
)
```

```
[Out] 2/5*(2*a*sin(d*x + c) - 3*a)*sqrt(a*sin(d*x + c) + a)*sqrt(cos(d*x + c))/(d
*cos(d*x + c)*e^(7/2)*sin(d*x + c) - d*cos(d*x + c)*e^(7/2))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**(3/2)/(e*cos(d*x+c))**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(3/2)/(e*cos(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 5.98, size = 71, normalized size = 0.96

$$\frac{4\,a\,\sqrt{a\,\left(\sin\left(c+d\,x\right)+1\right)}\,\left(5\,\sin\left(c+d\,x\right)+\cos\left(2\,c+2\,d\,x\right)-4\right)}{5\,d\,e^3\,\sqrt{e\,\cos\left(c+d\,x\right)}\,\left(4\,\sin\left(c+d\,x\right)+\cos\left(2\,c+2\,d\,x\right)-3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(3/2)/(e*cos(c + d*x))^(7/2),x)
```

```
[Out] (4*a*(a*(sin(c + d*x) + 1))^(1/2)*(5*sin(c + d*x) + cos(2*c + 2*d*x) - 4))/
(5*d*e^3*(e*cos(c + d*x))^(1/2)*(4*sin(c + d*x) + cos(2*c + 2*d*x) - 3))
```

3.287 $\int \frac{(a+a\sin(c+dx))^{3/2}}{(e\cos(c+dx))^{9/2}}\,dx$

Optimal. Leaf size=113

$$-\frac{2(a+a\sin(c+dx))^{3/2}}{de(e\cos(c+dx))^{7/2}}+\frac{8(a+a\sin(c+dx))^{5/2}}{3ade(e\cos(c+dx))^{7/2}}-\frac{16(a+a\sin(c+dx))^{7/2}}{21a^2de(e\cos(c+dx))^{7/2}}$$

```
[Out] -2*(a+a*sin(d*x+c))^(3/2)/d/e/(e*cos(d*x+c))^(7/2)+8/3*(a+a*sin(d*x+c))^(5/
2)/a/d/e/(e*cos(d*x+c))^(7/2)-16/21*(a+a*sin(d*x+c))^(7/2)/a^2/d/e/(e*cos(d
*x+c))^(7/2)
```

Rubi [A]
time = 0.15, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2751, 2750}

$$-\frac{16(a\sin(c+dx)+a)^{7/2}}{21a^2de(e\cos(c+dx))^{7/2}}+\frac{8(a\sin(c+dx)+a)^{5/2}}{3ade(e\cos(c+dx))^{7/2}}-\frac{2(a\sin(c+dx)+a)^{3/2}}{de(e\cos(c+dx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^(3/2)/(e*Cos[c + d*x])^(9/2),x]
```

```
[Out] (-2*(a + a*Sin[c + d*x])^(3/2))/(d*e*(e*Cos[c + d*x])^(7/2)) + (8*(a + a*Si
n[c + d*x])^(5/2))/(3*a*d*e*(e*Cos[c + d*x])^(7/2)) - (16*(a + a*Sin[c + d*
x])^(7/2))/(21*a^2*d*e*(e*Cos[c + d*x])^(7/2))
```

Rule 2750

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(c+dx))^{3/2}}{(e\cos(c+dx))^{9/2}}\,dx = -\frac{2(a+a\sin(c+dx))^{3/2}}{de(e\cos(c+dx))^{7/2}} + \frac{4\int \frac{(a+a\sin(c+dx))^{5/2}}{(e\cos(c+dx))^{9/2}}\,dx}{a}$$
$$= -\frac{2(a+a\sin(c+dx))^{3/2}}{de(e\cos(c+dx))^{7/2}} + \frac{8(a+a\sin(c+dx))^{5/2}}{3ade(e\cos(c+dx))^{7/2}} - \frac{8\int \frac{(a+a\sin(c+dx))^{7/2}}{(e\cos(c+dx))^{9/2}}\,dx}{3a^2}$$
$$= -\frac{2(a+a\sin(c+dx))^{3/2}}{de(e\cos(c+dx))^{7/2}} + \frac{8(a+a\sin(c+dx))^{5/2}}{3ade(e\cos(c+dx))^{7/2}} - \frac{16(a+a\sin(c+dx))^{7/2}}{21a^2de(e\cos(c+dx))^{7/2}}$$

Mathematica [A]
time = 0.20, size = 105, normalized size = 0.93

$$\frac{2a\sqrt{a(1+\sin(c+dx))}\,(-5+4\cos(2(c+dx))+12\sin(c+dx))}{21de^4\sqrt{e\cos(c+dx)}\,\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^(3/2)/(e*Cos[c + d*x])^(9/2),x]
[Out] (2*a*Sqrt[a*(1 + Sin[c + d*x])]*(-5 + 4*Cos[2*(c + d*x)] + 12*Sin[c + d*x])
)/(21*d*e^4*Sqrt[e*Cos[c + d*x]]*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3*(C
os[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

Maple [A]
time = 0.16, size = 54, normalized size = 0.48

method	result	size
default	$\frac{2\left(8\left(\cos^{2}(dx+c)\right)+12\sin(dx+c)-9\right)\cos(dx+c)\left(a(1+\sin(dx+c))\right)^{\frac{3}{2}}}{21d\left(e\cos(dx+c)\right)^{\frac{9}{2}}}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^(3/2)/(e*cos(d*x+c))^(9/2),x,method=_RETURNVERBOSE)
[Out] 2/21/d*(8*cos(d*x+c)^2+12*sin(d*x+c)-9)*cos(d*x+c)*(a*(1+sin(d*x+c)))^(3/2)
/(e*cos(d*x+c))^(9/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 253 vs. 2(88) = 176.
time = 0.58, size = 253, normalized size = 2.24

$$-\frac{2\left(a^{\frac{3}{2}}-\frac{24a^{\frac{3}{2}}\sin(dx+c)}{\cos(dx+c)+1}+\frac{33a^{\frac{3}{2}}\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{33a^{\frac{3}{2}}\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{24a^{\frac{3}{2}}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{a^{\frac{3}{2}}\sin(dx+c)^6}{(\cos(dx+c)+1)^6}\right)\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^3e^{\left(-\frac{9}{2}\right)}}{21\,d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{3}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{9}{2}}\left(\frac{3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(3/2)/(e*cos(d*x+c))^(9/2),x, algorithm="maxima"
)
```

```
[Out] -2/21*(a^(3/2) - 24*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) + 33*a^(3/2)*si
n(d*x + c)^2/(cos(d*x + c) + 1)^2 - 33*a^(3/2)*sin(d*x + c)^4/(cos(d*x + c)
 + 1)^4 + 24*a^(3/2)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - a^(3/2)*sin(d*x
+ c)^6/(cos(d*x + c) + 1)^6)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^3*e^
(-9/2)/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(3/2)*(-sin(d*x + c)/(cos(d
*x + c) + 1) + 1)^(9/2)*(3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*sin(d*x
+ c)^4/(cos(d*x + c) + 1)^4 + sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 1))
```

Fricas [A]
time = 0.35, size = 80, normalized size = 0.71

$$-\frac{2\left(8\,a\cos\left(dx+c\right)^{2}+12\,a\sin\left(dx+c\right)-9\,a\right)\sqrt{a\sin\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}}{21\left(d\cos\left(dx+c\right)^{2}e^{\frac{9}{2}}\sin\left(dx+c\right)-d\cos\left(dx+c\right)^{2}e^{\frac{9}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(3/2)/(e*cos(d*x+c))^(9/2),x, algorithm="fricas"
)
```

```
[Out] -2/21*(8*a*cos(d*x + c)^2 + 12*a*sin(d*x + c) - 9*a)*sqrt(a*sin(d*x + c) +
a)*sqrt(cos(d*x + c))/(d*cos(d*x + c)^2*e^(9/2)*sin(d*x + c) - d*cos(d*x +
c)^2*e^(9/2))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**(3/2)/(e*cos(d*x+c))**(9/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(3/2)/(e*cos(d*x+c))^(9/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 6.81, size = 116, normalized size = 1.03

$$\frac{8\,a\,\sqrt{a\,\left(\sin\left(c+d\,x\right)+1\right)}\,\left(12\,\cos\left(c+d\,x\right)-10\,\cos\left(3\,c+3\,d\,x\right)-17\,\sin\left(2\,c+2\,d\,x\right)+2\,\sin\left(4\,c+4\,d\,x\right)\right)}{21\,d\,e^4\,\sqrt{e\,\cos\left(c+d\,x\right)}\,\left(4\,\sin\left(c+d\,x\right)-4\,\cos\left(2\,c+2\,d\,x\right)+\cos\left(4\,c+4\,d\,x\right)+4\,\sin\left(3\,c+3\,d\,x\right)-5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(3/2)/(e*cos(c + d*x))^(9/2),x)
```

```
[Out] (8*a*(a*(sin(c + d*x) + 1))^(1/2)*(12*cos(c + d*x) - 10*cos(3*c + 3*d*x) -
17*sin(2*c + 2*d*x) + 2*sin(4*c + 4*d*x)))/(21*d*e^4*(e*cos(c + d*x))^(1/2)
*(4*sin(c + d*x) - 4*cos(2*c + 2*d*x) + cos(4*c + 4*d*x) + 4*sin(3*c + 3*d*
x) - 5))
```

3.288 $\int \frac{(a+a\sin(c+dx))^{3/2}}{(e\cos(c+dx))^{11/2}}\,dx$

Optimal. Leaf size=152

$$-\frac{2(a+a\sin(c+dx))^{3/2}}{3de(e\cos(c+dx))^{9/2}}+\frac{4(a+a\sin(c+dx))^{5/2}}{ade(e\cos(c+dx))^{9/2}}-\frac{16(a+a\sin(c+dx))^{7/2}}{5a^2de(e\cos(c+dx))^{9/2}}+\frac{32(a+a\sin(c+dx))^{9/2}}{45a^3de(e\cos(c+dx))^{9/2}}$$

```
[Out] -2/3*(a+a*sin(d*x+c))^(3/2)/d/e/(e*cos(d*x+c))^(9/2)+4*(a+a*sin(d*x+c))^(5/
2)/a/d/e/(e*cos(d*x+c))^(9/2)-16/5*(a+a*sin(d*x+c))^(7/2)/a^2/d/e/(e*cos(d*
x+c))^(9/2)+32/45*(a+a*sin(d*x+c))^(9/2)/a^3/d/e/(e*cos(d*x+c))^(9/2)
```

Rubi [A]
time = 0.20, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2751, 2750}

$$\frac{32(a\sin(c+dx)+a)^{9/2}}{45a^3de(e\cos(c+dx))^{9/2}}-\frac{16(a\sin(c+dx)+a)^{7/2}}{5a^2de(e\cos(c+dx))^{9/2}}+\frac{4(a\sin(c+dx)+a)^{5/2}}{ade(e\cos(c+dx))^{9/2}}-\frac{2(a\sin(c+dx)+a)^{3/2}}{3de(e\cos(c+dx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^(3/2)/(e*Cos[c + d*x])^(11/2),x]
```

```
[Out] (-2*(a + a*Sin[c + d*x])^(3/2))/(3*d*e*(e*Cos[c + d*x])^(9/2)) + (4*(a + a*
Sin[c + d*x])^(5/2))/(a*d*e*(e*Cos[c + d*x])^(9/2)) - (16*(a + a*Sin[c + d*
x])^(7/2))/(5*a^2*d*e*(e*Cos[c + d*x])^(9/2)) + (32*(a + a*Sin[c + d*x])^(9
/2))/(45*a^3*d*e*(e*Cos[c + d*x])^(9/2))
```

Rule 2750

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(c+dx))^{3/2}}{(e\cos(c+dx))^{11/2}}\,dx = -\frac{2(a+a\sin(c+dx))^{3/2}}{3de(e\cos(c+dx))^{9/2}} + \frac{2\int \frac{(a+a\sin(c+dx))^{5/2}}{(e\cos(c+dx))^{11/2}}\,dx}{a}$$

$$= -\frac{2(a+a\sin(c+dx))^{3/2}}{3de(e\cos(c+dx))^{9/2}} + \frac{4(a+a\sin(c+dx))^{5/2}}{ade(e\cos(c+dx))^{9/2}} - \frac{8\int \frac{(a+a\sin(c+dx))^{7/2}}{(e\cos(c+dx))^{11/2}}\,dx}{a^2}$$

$$= -\frac{2(a+a\sin(c+dx))^{3/2}}{3de(e\cos(c+dx))^{9/2}} + \frac{4(a+a\sin(c+dx))^{5/2}}{ade(e\cos(c+dx))^{9/2}} - \frac{16(a+a\sin(c+dx))^{7/2}}{5a^2de(e\cos(c+dx))^{9/2}}$$

$$= -\frac{2(a+a\sin(c+dx))^{3/2}}{3de(e\cos(c+dx))^{9/2}} + \frac{4(a+a\sin(c+dx))^{5/2}}{ade(e\cos(c+dx))^{9/2}} - \frac{16(a+a\sin(c+dx))^{7/2}}{5a^2de(e\cos(c+dx))^{9/2}}$$

Mathematica [A]
time = 0.25, size = 74, normalized size = 0.49

$$\frac{2\sqrt{e\cos(c+dx)}\,\sec^5(c+dx)(a(1+\sin(c+dx)))^{3/2}(7+12\cos(2(c+dx))+6\sin(c+dx)-4\sin(3(c+dx)))}{45de^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^(3/2)/(e*Cos[c + d*x])^(11/2),x]
[Out] (2*Sqrt[e*Cos[c + d*x]]*Sec[c + d*x]^5*(a*(1 + Sin[c + d*x]))^(3/2)*(7 + 12
*Cos[2*(c + d*x)] + 6*Sin[c + d*x] - 4*Sin[3*(c + d*x)]))/(45*d*e^6)
```

Maple [A]
time = 0.17, size = 70, normalized size = 0.46

method	result	size
default	$-\frac{2\left(16\left(\cos^2(dx+c)\right)\sin(dx+c)-24\left(\cos^2(dx+c)\right)-10\sin(dx+c)+5\right)\cos(dx+c)\left(a(1+\sin(dx+c))\right)^{\frac{3}{2}}}{45d\left(e\cos(dx+c)\right)^{\frac{11}{2}}}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^(3/2)/(e*cos(d*x+c))^(11/2),x,method=_RETURNVERBOSE)
[Out] -2/45/d*(16*cos(d*x+c)^2*sin(d*x+c)-24*cos(d*x+c)^2-10*sin(d*x+c)+5)*cos(d*
x+c)*(a*(1+sin(d*x+c)))^(3/2)/(e*cos(d*x+c))^(11/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 321 vs. 2(118) = 236.
time = 0.56, size = 321, normalized size = 2.11

$$\frac{2\left(19\,a^{\frac{3}{2}} - \frac{12\,a^{\frac{3}{2}}\sin(dx+c)}{\cos(dx+c)+1} - \frac{58\,a^{\frac{3}{2}}\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{116\,a^{\frac{3}{2}}\sin(dx+c)^3}{(\cos(dx+c)+1)^3} - \frac{116\,a^{\frac{3}{2}}\sin(dx+c)^5}{(\cos(dx+c)+1)^5} + \frac{58\,a^{\frac{3}{2}}\sin(dx+c)^6}{(\cos(dx+c)+1)^6} + \frac{12\,a^{\frac{3}{2}}\sin(dx+c)^7}{(\cos(dx+c)+1)^7} - \frac{19\,a^{\frac{3}{2}}\sin(dx+c)^8}{(\cos(dx+c)+1)^8}\right)\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^4 e^{\left(-\frac{11}{2}\right)}}{45\,d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{11}{2}}\left(\frac{4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{4\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(3/2)/(e*cos(d*x+c))^(11/2),x, algorithm="maxima
")
```

```
[Out] 2/45*(19*a^(3/2) - 12*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 58*a^(3/2)*
sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 116*a^(3/2)*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 - 116*a^(3/2)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 58*a^(3/2)*s
in(d*x + c)^6/(cos(d*x + c) + 1)^6 + 12*a^(3/2)*sin(d*x + c)^7/(cos(d*x + c
) + 1)^7 - 19*a^(3/2)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8)*(sin(d*x + c)^2/
(cos(d*x + c) + 1)^2 + 1)^4*e^(-11/2)/(d*(sin(d*x + c)/(cos(d*x + c) + 1) +
 1)^(5/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)*(4*sin(d*x + c)^2/(
cos(d*x + c) + 1)^2 + 6*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*sin(d*x + c
)^6/(cos(d*x + c) + 1)^6 + sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 1))
```

Fricas [**A**]
time = 0.36, size = 94, normalized size = 0.62

$$-\frac{2\left(24\,a\cos\left(dx+c\right)^{2}-2\left(8\,a\cos\left(dx+c\right)^{2}-5\,a\right)\sin\left(dx+c\right)-5\,a\right)\sqrt{a\sin\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}}{45\left(d\cos\left(dx+c\right)^{3}e^{\frac{11}{2}}\sin\left(dx+c\right)-d\cos\left(dx+c\right)^{3}e^{\frac{11}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(3/2)/(e*cos(d*x+c))^(11/2),x, algorithm="fricas
")
```

```
[Out] -2/45*(24*a*cos(d*x + c)^2 - 2*(8*a*cos(d*x + c)^2 - 5*a)*sin(d*x + c) - 5*
a)*sqrt(a*sin(d*x + c) + a)*sqrt(cos(d*x + c))/(d*cos(d*x + c)^3*e^(11/2)*s
in(d*x + c) - d*cos(d*x + c)^3*e^(11/2))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**(3/2)/(e*cos(d*x+c))**(11/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(3/2)/(e*cos(d*x+c))^(11/2),x, algorithm="giac")
```

[Out] Timed out

Mupad [B]
time = 10.96, size = 261, normalized size = 1.72

$$\frac{14\,a\,\sqrt{a+a\,\sin\left(c+d\,x\right)}+12\,a\,\sin\left(c+d\,x\right)\,\sqrt{a+a\,\sin\left(c+d\,x\right)}+24\,a\,\cos\left(2\,c+2\,d\,x\right)\,\sqrt{a+a\,\sin\left(c+d\,x\right)}-8\,a\,\sin\left(3\,c+3\,d\,x\right)\,\sqrt{a+a\,\sin\left(c+d\,x\right)}}{\frac{45\,d\,e^5\,\sqrt{\frac{e\,{\mathrm{e}}^{-c\,1{}\mathrm{i}-d\,x\,1{}\mathrm{i}}}{2}+\frac{e\,{\mathrm{e}}^{c\,1{}\mathrm{i}+d\,x\,1{}\mathrm{i}}}{2}}}{2}+\frac{45\,d\,e^5\,\cos\left(2\,c+2\,d\,x\right)\,\sqrt{\frac{e\,{\mathrm{e}}^{-c\,1{}\mathrm{i}-d\,x\,1{}\mathrm{i}}}{2}+\frac{e\,{\mathrm{e}}^{c\,1{}\mathrm{i}+d\,x\,1{}\mathrm{i}}}{2}}}{2}-\frac{45\,d\,e^5\,\sin\left(3\,c+3\,d\,x\right)\,\sqrt{\frac{e\,{\mathrm{e}}^{-c\,1{}\mathrm{i}-d\,x\,1{}\mathrm{i}}}{2}+\frac{e\,{\mathrm{e}}^{c\,1{}\mathrm{i}+d\,x\,1{}\mathrm{i}}}{2}}}{4}-\frac{45\,d\,e^5\,\sin\left(c+d\,x\right)\,\sqrt{\frac{e\,{\mathrm{e}}^{-c\,1{}\mathrm{i}-d\,x\,1{}\mathrm{i}}}{2}+\frac{e\,{\mathrm{e}}^{c\,1{}\mathrm{i}+d\,x\,1{}\mathrm{i}}}{2}}}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(3/2)/(e*cos(c + d*x))^(11/2),x)
```

```
[Out] (14*a*(a + a*sin(c + d*x))^(1/2) + 12*a*sin(c + d*x)*(a + a*sin(c + d*x))^(
1/2) + 24*a*cos(2*c + 2*d*x)*(a + a*sin(c + d*x))^(1/2) - 8*a*sin(3*c + 3*d
*x)*(a + a*sin(c + d*x))^(1/2))/((45*d*e^5*((e*exp(- c*1i - d*x*1i))/2 + (e
*exp(c*1i + d*x*1i))/2)^(1/2))/2 + (45*d*e^5*cos(2*c + 2*d*x)*((e*exp(- c*1
i - d*x*1i))/2 + (e*exp(c*1i + d*x*1i))/2)^(1/2))/2 - (45*d*e^5*sin(3*c + 3
*d*x)*((e*exp(- c*1i - d*x*1i))/2 + (e*exp(c*1i + d*x*1i))/2)^(1/2))/4 - (4
5*d*e^5*sin(c + d*x)*((e*exp(- c*1i - d*x*1i))/2 + (e*exp(c*1i + d*x*1i))/2
)^(1/2))/4)
```

3.289 $\int (e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=323

$$-\frac{77a^3(e\cos(c+dx))^{5/2}}{96de\sqrt{a+a\sin(c+dx)}}+\frac{77a^2e\sqrt{e\cos(c+dx)}\,\sqrt{a+a\sin(c+dx)}}{64d}-\frac{11a^2(e\cos(c+dx))^{5/2}\sqrt{a+a\sin}}{24de}$$

```
[Out] -1/4*a*(e*cos(d*x+c))^(5/2)*(a+a*sin(d*x+c))^(3/2)/d/e-77/96*a^3*(e*cos(d*x
+c))^(5/2)/d/e/(a+a*sin(d*x+c))^(1/2)-11/24*a^2*(e*cos(d*x+c))^(5/2)*(a+a*s
in(d*x+c))^(1/2)/d/e+77/64*a^2*e*(e*cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^(1/2
)/d-77/64*a^2*e^(3/2)*arcsinh((e*cos(d*x+c))^(1/2)/e^(1/2))*(1+cos(d*x+c))^
(1/2)*(a+a*sin(d*x+c))^(1/2)/d/(1+cos(d*x+c)+sin(d*x+c))+77/64*a^2*e^(3/2)*
arctan(sin(d*x+c)*e^(1/2)/(e*cos(d*x+c))^(1/2)/(1+cos(d*x+c))^(1/2))*(1+cos
(d*x+c))^(1/2)*(a+a*sin(d*x+c))^(1/2)/d/(1+cos(d*x+c)+sin(d*x+c))
```

Rubi [A]
time = 0.36, antiderivative size = 323, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {2757, 2764, 2756, 2854, 209, 2912, 65, 221}

$$-\frac{77a^3(e\cos(c+dx))^{5/2}}{96de\sqrt{a\sin(c+dx)+a}}+\frac{77a^2e^{3/2}\sqrt{\cos(c+dx)+1}\,\sqrt{a\sin(c+dx)+a}\,\text{ArcTan}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\,\sqrt{e\cos(c+dx)}}\right)}{64d(\sin(c+dx)+\cos(c+dx)+1)}-\frac{77a^2e^{3/2}\sqrt{\cos(c+dx)+1}\,\sqrt{a\sin(c+dx)+a}\,\sinh^{-1}\left(\frac{\sqrt{e\cos(c+dx)}}{\sqrt{e}}\right)}{64d(\sin(c+dx)+\cos(c+dx)+1)}-\frac{11a^2\sqrt{a\sin(c+dx)+a}\,(e\cos(c+dx))^{5/2}}{24de}+\frac{77a^2e\sqrt{a\sin(c+dx)+a}\,\sqrt{e\cos(c+dx)}}{64d}-\frac{a(a\sin(c+dx)+a)^{3/2}(e\cos(c+dx))^{5/2}}{4de}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-77*a^3*(e*Cos[c + d*x])^(5/2))/(96*d*e*Sqrt[a + a*Sin[c + d*x]]) + (77*a^
2*e*Sqrt[e*Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(64*d) - (11*a^2*(e*Cos[
c + d*x])^(5/2)*Sqrt[a + a*Sin[c + d*x]])/(24*d*e) - (77*a^2*e^(3/2)*ArcSin
h[Sqrt[e*Cos[c + d*x]]/Sqrt[e]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c + d
*x]])/(64*d*(1 + Cos[c + d*x] + Sin[c + d*x])) + (77*a^2*e^(3/2)*ArcTan[(Sq
rt[e]*Sin[c + d*x])/(Sqrt[e*Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]])]*Sqrt[1 +
 Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(64*d*(1 + Cos[c + d*x] + Sin[c +
d*x])) - (a*(e*Cos[c + d*x])^(5/2)*(a + a*Sin[c + d*x])^(3/2))/(4*d*e)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 2756

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[cos[(e_.) + (f_.)*(x_)]
*(g_.)], x_Symbol] :> Dist[a*Sqrt[1 + Cos[e + f*x]]*(Sqrt[a + b*Sin[e + f*x
]]/(a + a*Cos[e + f*x] + b*Sin[e + f*x])), Int[Sqrt[1 + Cos[e + f*x]]/Sqrt[
g*Cos[e + f*x]], x], x] + Dist[b*Sqrt[1 + Cos[e + f*x]]*(Sqrt[a + b*Sin[e +
 f*x]]/(a + a*Cos[e + f*x] + b*Sin[e + f*x])), Int[Sin[e + f*x]/(Sqrt[g*Cos
[e + f*x]]*Sqrt[1 + Cos[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, g}, x] &&
EqQ[a^2 - b^2, 0]
```

Rule 2757

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ
[2*m, 2*p]
```

Rule 2764

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]], x_Symbol] :> Simp[g*Sqrt[g*Cos[e + f*x]]*(Sqrt[a + b*Sin[e + f*x
]]/(b*f)), x] + Dist[g^2/(2*a), Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[g*Cos[e +
 f*x]], x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2854

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b + d*x^2), x], x
, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]))], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
```

```
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^{5/2}\,dx = -\frac{a(e\cos(c+dx))^{5/2}(a+a\sin(c+dx))^{3/2}}{4de} + \frac{1}{8}(11a)\int (e\,\mathrm{co}$$

$$= -\frac{11a^2(e\cos(c+dx))^{5/2}\sqrt{a+a\sin(c+dx)}}{24de} - \frac{a(e\cos(c+}{}$$

$$= -\frac{77a^3(e\cos(c+dx))^{5/2}}{96de\sqrt{a+a\sin(c+dx)}} - \frac{11a^2(e\cos(c+dx))^{5/2}\sqrt{a+a}}{24de}$$

$$= -\frac{77a^3(e\cos(c+dx))^{5/2}}{96de\sqrt{a+a\sin(c+dx)}} + \frac{77a^2e\sqrt{e\cos(c+dx)}\sqrt{a+a}}{64d}$$

$$= -\frac{77a^3(e\cos(c+dx))^{5/2}}{96de\sqrt{a+a\sin(c+dx)}} + \frac{77a^2e\sqrt{e\cos(c+dx)}\sqrt{a+a}}{64d}$$

$$= -\frac{77a^3(e\cos(c+dx))^{5/2}}{96de\sqrt{a+a\sin(c+dx)}} + \frac{77a^2e\sqrt{e\cos(c+dx)}\sqrt{a+a}}{64d}$$

$$= -\frac{77a^3(e\cos(c+dx))^{5/2}}{96de\sqrt{a+a\sin(c+dx)}} + \frac{77a^2e\sqrt{e\cos(c+dx)}\sqrt{a+a}}{64d}$$

$$= -\frac{77a^3(e\cos(c+dx))^{5/2}}{96de\sqrt{a+a\sin(c+dx)}} + \frac{77a^2e\sqrt{e\cos(c+dx)}\sqrt{a+a}}{64d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.29, size = 77, normalized size = 0.24

$$-\frac{16\ 2^{3/4}(e\cos(c+dx))^{5/2}\,{}_2F_1\left(-\frac{11}{4},\frac{5}{4};\frac{9}{4};\frac{1}{2}(1-\sin(c+dx))\right)(a(1+\sin(c+dx)))^{5/2}}{5de(1+\sin(c+dx))^{15/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-16*2^(3/4)*(e*Cos[c + d*x])^(5/2)*Hypergeometric2F1[-11/4, 5/4, 9/4, (1 -
 Sin[c + d*x])/2]*(a*(1 + Sin[c + d*x]))^(5/2))/(5*d*e*(1 + Sin[c + d*x])^(
15/4))
```

Maple [A]
time = 0.26, size = 344, normalized size = 1.07

method	result
default	$\frac{\left(-96\sin(dx+c)(\cos^4(dx+c))-96(\cos^5(dx+c))+231\sqrt{2}\,\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\sin\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/384/d*(-96*sin(d*x+c)*cos(d*x+c)^4-96*cos(d*x+c)^5+231*2^(1/2)*(-2*cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*sin(d*x+c)/cos(d*x+c)*2^(1/2))*sin(d*x+c)-231*2^(1/2)*(-2*cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*s
in(d*x+c)+368*sin(d*x+c)*cos(d*x+c)^3-272*cos(d*x+c)^4+308*cos(d*x+c)^2*sin
(d*x+c)+676*cos(d*x+c)^3-462*cos(d*x+c)*sin(d*x+c)+154*cos(d*x+c)^2-462*cos
(d*x+c))*(e*cos(d*x+c))^(3/2)*(a*(1+sin(d*x+c)))^(5/2)/(cos(d*x+c)^2*sin(d*
x+c)-cos(d*x+c)^3+2*cos(d*x+c)*sin(d*x+c)+3*cos(d*x+c)^2-4*sin(d*x+c)+2*cos
(d*x+c)-4)/cos(d*x+c)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima"
)
```

```
[Out] e^(3/2)*integrate((a*sin(d*x + c) + a)^(5/2)*cos(d*x + c)^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3429 vs. $2(250) = 500$.
time = 196.67, size = 3429, normalized size = 10.62

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas"
)
```

[Out] 1/1536*(924*sqrt(2)*(a^10/d^4)^(1/4)*d*arctan(-1/4*(2*sqrt(1/2)*((sqrt(2)*d^3*cos(d*x + c)^6 + 5*sqrt(2)*d^3*cos(d*x + c)^5 - 8*sqrt(2)*d^3*cos(d*x + c)^4 - 20*sqrt(2)*d^3*cos(d*x + c)^3 + 8*sqrt(2)*d^3*cos(d*x + c)^2 + 16*sqrt(2)*d^3*cos(d*x + c) + (sqrt(2)*d^3*cos(d*x + c)^5 - 4*sqrt(2)*d^3*cos(d*x + c)^4 - 12*sqrt(2)*d^3*cos(d*x + c)^3 + 8*sqrt(2)*d^3*cos(d*x + c)^2 + 16*sqrt(2)*d^3*cos(d*x + c))*sin(d*x + c))*(a^10/d^4)^(3/4)*e^(9/2) + (sqrt(2)*a^5*d*cos(d*x + c)^6*e^3 - 3*sqrt(2)*a^5*d*cos(d*x + c)^5*e^3 - 8*sqrt(2)*a^5*d*cos(d*x + c)^4*e^3 + 4*sqrt(2)*a^5*d*cos(d*x + c)^3*e^3 + 8*sqrt(2)*a^5*d*cos(d*x + c)^2*e^3 - (sqrt(2)*a^5*d*cos(d*x + c)^5*e^3 + 4*sqrt(2)*a^5*d*cos(d*x + c)^4*e^3 - 4*sqrt(2)*a^5*d*cos(d*x + c)^3*e^3 - 8*sqrt(2)*a^5*d*cos(d*x + c)^2*e^3)*sin(d*x + c))*(a^10/d^4)^(1/4)*e^(3/2) - (a^7*cos(d*x + c)^4*e^(9/2) - 3*a^7*cos(d*x + c)^3*e^(9/2) - 8*a^7*cos(d*x + c)^2*e^(9/2) + 4*a^7*cos(d*x + c)*e^(9/2) + 8*a^7*e^(9/2) + (2*a^2*d^2*cos(d*x + c)^5*e^(3/2) - 5*a^2*d^2*cos(d*x + c)^4*e^(3/2) - 19*a^2*d^2*cos(d*x + c)^3*e^(3/2) + 20*a^2*d^2*cos(d*x + c)*e^(3/2) + 8*a^2*d^2*e^(3/2) - (2*a^2*d^2*cos(d*x + c)^4*e^(3/2) + 9*a^2*d^2*cos(d*x + c)^3*e^(3/2) - 4*a^2*d^2*cos(d*x + c)^2*e^(3/2) - 20*a^2*d^2*cos(d*x + c)*e^(3/2) - 8*a^2*d^2*e^(3/2))*sin(d*x + c))*sqrt(a^10/d^4)*e^3 - (a^7*cos(d*x + c)^3*e^(9/2) + 4*a^7*cos(d*x + c)^2*e^(9/2) - 4*a^7*cos(d*x + c)*e^(9/2) - 8*a^7*e^(9/2))*sin(d*x + c))*sqrt(a*sin(d*x + c) + a)*sqrt(cos(d*x + c)))*sqrt((2*a^15*cos(d*x + c)*e^9*sin(d*x + c) + 2*a^15*cos(d*x + c)*e^9 + (a^10*d^2*e^6*sin(d*x + c) + a^10*d^2*e^6)*sqrt(a^10/d^4)*e^3 + (sqrt(2)*(a^10/d^4)^(3/4)*a^7*d^3*cos(d*x + c)*e^9 + (sqrt(2)*a^12*d*e^(15/2)*sin(d*x + c) + sqrt(2)*a^12*d*e^(15/2))*(a^10/d^4)^(1/4)*e^(3/2))*sqrt(a*sin(d*x + c) + a)*sqrt(cos(d*x + c)))/(sin(d*x + c) + 1)) - ((7*sqrt(2)*a^7*d^3*cos(d*x + c)^4*e^(9/2) + 3*sqrt(2)*a^7*d^3*cos(d*x + c)^3*e^(9/2) - 16*sqrt(2)*a^7*d^3*cos(d*x + c)^2*e^(9/2) - 4*sqrt(2)*a^7*d^3*cos(d*x + c)*e^(9/2) + 8*sqrt(2)*a^7*d^3*e^(9/2) + (2*sqrt(2)*a^7*d^3*cos(d*x + c)^4*e^(9/2) + sqrt(2)*a^7*d^3*cos(d*x + c)^3*e^(9/2) - 12*sqrt(2)*a^7*d^3*cos(d*x + c)^2*e^(9/2) - 4*sqrt(2)*a^7*d^3*cos(d*x + c)*e^(9/2) + 8*sqrt(2)*a^7*d^3*e^(9/2))*sin(d*x + c))*(a^10/d^4)^(3/4)*e^(9/2) + (2*sqrt(2)*a^12*d*cos(d*x + c)^5*e^(15/2) + sqrt(2)*a^12*d*cos(d*x + c)^4*e^(15/2) - 13*sqrt(2)*a^12*d*cos(d*x + c)^3*e^(15/2) - 8*sqrt(2)*a^12*d*cos(d*x + c)^2*e^(15/2) + 12*sqrt(2)*a^12*d*cos(d*x + c)*e^(15/2) + 8*sqrt(2)*a^12*d*e^(15/2) - (7*sqrt(2)*a^12*d*cos(d*x + c)^3*e^(15/2) + 4*sqrt(2)*a^12*d*cos(d*x + c)^2*e^(15/2) - 12*sqrt(2)*a^12*d*cos(d*x + c)*e^(15/2) - 8*sqrt(2)*a^12*d*e^(15/2))*sin(d*x + c))*(a^10/d^4)^(1/4)*e^(3/2))*sqrt(a*sin(d*x + c) + a)*sqrt(cos(d*x + c)))/(a^15*cos(d*x + c)^6*e^9 + a^15*cos(d*x + c)^5*e^9 - 8*a^15*cos(d*x + c)^4*e^9 - 8*a^15*cos(d*x + c)^3*e^9 + 8*a^15*cos(d*x + c)^2*e^9 + 8*a^15*cos(d*x + c)*e^9 - 4*(a^15*cos(d*x + c)^4*e^9 + a^15*cos(d*x + c)^3*e^9 - 2*a^15*cos(d*x + c)^2*e^9 - 2*a^15*cos(d*x + c)*e^9)*sin(d*x + c)))*e^(3/2) - 924*sqrt(2)*(a^10/d^4)^(1/4)*d*arctan(1/4*(2*sqrt(1/2)*((sqrt(2)*d^3*cos(d*x + c)^6 + 5*sqrt(2)*d^3*cos(d*x + c)^5 - 8*sqrt(2)*d^3*cos(d*x + c)^4 - 20*sqrt(2)*d^3*cos(d*x + c)^3 + 8*sqrt(2)*d^3*cos(d*x + c)^2 + 16*sqrt(2)*d^3*cos(d*x + c) + (sqrt(2)*d^3*cos(d*x + c)^5 - 4*sqrt(2)*d^3*cos(d*x + c)^4 - 12*sqrt(2)*d^3*cos(d*x + c)^3 + 8*sqrt

```
(2)*d^3*cos(d*x + c)^2 + 16*sqrt(2)*d^3*cos(d*x + c))*sin(d*x + c))*(a^10/d
^4)^(3/4)*e^(9/2) + (sqrt(2)*a^5*d*cos(d*x + c)^6*e^3 - 3*sqrt(2)*a^5*d*cos
(d*x + c)^5*e^3 - 8*sqrt(2)*a^5*d*cos(d*x + c)^4*e^3 + 4*sqrt(2)*a^5*d*cos(
d*x + c)^3*e^3 + 8*sqrt(2)*a^5*d*cos(d*x + c)^2*e^3 - (sqrt(2)*a^5*d*cos(d*
x + c)^5*e^3 + 4*sqrt(2)*a^5*d*cos(d*x + c)^4*e^3 - 4*sqrt(2)*a^5*d*cos(d*x
 + c)^3*e^3 - 8*sqrt(2)*a^5*d*cos(d*x + c)^2*e^3)*sin(d*x + c))*(a^10/d^4)^
(1/4)*e^(3/2) + (a^7*cos(d*x + c)^4*e^(9/2) - 3*a^7*cos(d*x + c)^3*e^(9/2)
- 8*a^7*cos(d*x + c)^2*e^(9/2) + 4*a^7*cos(d*x + c)*e^(9/2) + 8*a^7*e^(9/2)
 + (2*a^2*d^2*cos(d*x + c)^5*e^(3/2) - 5*a^2*d^2*cos(d*x + c)^4*e^(3/2) - 1
9*a^2*d^2*cos(d*x + c)^3*e^(3/2) + 20*a^2*d^2*cos(d*x + c)*e^(3/2) + 8*a^2*
d^2*e^(3/2) - (2*a^2*d^2*cos(d*x + c)^4*e^(3/2) + 9*a^2*d^2*cos(d*x + c)^3*
e^(3/2) - 4*a^2*d^2*cos(d*x + c)^2*e^(3/2) - 20*a^2*d^2*cos(d*x + c)*e^(3/2
) - 8*a^2*d^2*e^(3/2))*sin(d*x + c))*sqrt(a^10/d^4)*e^3 - (a^7*cos(d*x + c)
^3*e^(9/2) + 4*a^7*cos(d*x + c)^2*e^(9/2) - 4*a^7*cos(d*x + c)*e^(9/2) - 8*
a^7*e^(9/2))*sin(d*x + c))*sqrt(a*sin(d*x + c) + a)*sqrt(cos(d*x + c)))*sqr
t((2*a^15*cos(d*x + c)*e^9*sin(d*x + c) + 2*a^15*cos(d*x + c)*e^9 + (a^10*d
^2*e^6*sin(d*x + c) + a^10*d^2*e^6)*sqrt(a^10/d^4)*e^3 - (sqrt(2)*(a^10/d^4
)^(3/4)*a^7*d^3*cos(d*x + c)*e^9 + (sqrt(2)*a^12*d*e^(15/2)*sin(d*x + c) +
sqrt(2)*a^12*d*e^(15/2))*(a^10/d^4)^(1/4)*e^(3/2))*sqrt(a*sin(d*x + c) + a)
*sqrt(cos(d*x + c)))/(sin(d*x + c) + 1)) - ((7*sqrt(2)*a^7*d^3*cos(d*x + c)
^4*e^(9/2) + 3*sqrt(2)*a^7*d^3*cos(d*x + c)^3*e^(9/2) - 16*sqrt(2)*a^7*d^3*
cos(d*x + c)^2*e^(9/2) - 4*sqrt(2)*a^7*d^3*cos(...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(3/2)*(a+a*sin(d*x+c))**(5/2),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((a*sin(d*x + c) + a)^(5/2)*cos(d*x + c)^(3/2)*e^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (e\,\cos\left(c+d\,x\right))^{3/2}\,\left(a+a\,\sin\left(c+d\,x\right)\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(3/2)*(a + a*sin(c + d*x))^(5/2),x)
```

```
[Out] int((e*cos(c + d*x))^(3/2)*(a + a*sin(c + d*x))^(5/2), x)
```

3.290 $\int \sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=286

$$-\frac{15a^3(e\cos(c+dx))^{3/2}}{8de\sqrt{a+a\sin(c+dx)}}-\frac{3a^2(e\cos(c+dx))^{3/2}\sqrt{a+a\sin(c+dx)}}{4de}+\frac{15a^2\sqrt{e}\,\sinh^{-1}\left(\frac{\sqrt{e\cos(c+dx)}}{\sqrt{e}}\right)}{8d(1+\cos(c+}$$

```
[Out] -1/3*a*(e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^(3/2)/d/e-15/8*a^3*(e*cos(d*x+
c))^(3/2)/d/e/(a+a*sin(d*x+c))^(1/2)-3/4*a^2*(e*cos(d*x+c))^(3/2)*(a+a*sin(
d*x+c))^(1/2)/d/e+15/8*a^2*arcsinh((e*cos(d*x+c))^(1/2)/e^(1/2))*e^(1/2)*(1
+cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^(1/2)/d/(1+cos(d*x+c)+sin(d*x+c))+15/8*
a^2*arctan(sin(d*x+c)*e^(1/2)/(e*cos(d*x+c))^(1/2)/(1+cos(d*x+c))^(1/2))*e^
(1/2)*(1+cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^(1/2)/d/(1+cos(d*x+c)+sin(d*x+c
))
```

Rubi [A]
time = 0.29, antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2757, 2763, 2854, 209, 2912, 65, 221}

$$-\frac{15a^3(e\cos(c+dx))^{3/2}}{8de\sqrt{a\sin(c+dx)+a}}+\frac{15a^2\sqrt{e}\sqrt{\cos(c+dx)+1}\sqrt{a\sin(c+dx)+a}\,\text{ArcTan}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\sqrt{e\cos(c+dx)}}\right)}{8d(\sin(c+dx)+\cos(c+dx)+1)}-\frac{3a^2\sqrt{a\sin(c+dx)+a}(e\cos(c+dx))^{3/2}}{4de}+\frac{15a^2\sqrt{e}\sqrt{\cos(c+dx)+1}\sqrt{a\sin(c+dx)+a}\sinh^{-1}\left(\frac{\sqrt{e\cos(c+dx)}}{\sqrt{e}}\right)}{8d(\sin(c+dx)+\cos(c+dx)+1)}-\frac{a(a\sin(c+dx)+a)^{3/2}(e\cos(c+dx))^{3/2}}{3de}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-15*a^3*(e*Cos[c + d*x])^(3/2))/(8*d*e*Sqrt[a + a*Sin[c + d*x]]) - (3*a^2*
(e*Cos[c + d*x])^(3/2)*Sqrt[a + a*Sin[c + d*x]])/(4*d*e) + (15*a^2*Sqrt[e]*
ArcSinh[Sqrt[e*Cos[c + d*x]]/Sqrt[e]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[a + a*Sin
[c + d*x]])/(8*d*(1 + Cos[c + d*x] + Sin[c + d*x])) + (15*a^2*Sqrt[e]*ArcTa
n[(Sqrt[e]*Sin[c + d*x])/(Sqrt[e*Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]])]*Sqr
t[1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(8*d*(1 + Cos[c + d*x] + Sin[
c + d*x])) - (a*(e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x])^(3/2))/(3*d*e)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 2757

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ
[2*m, 2*p]
```

Rule 2763

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[g*Sqrt[1 + Cos[e + f*x]]*(Sqrt[a + b*Sin[e + f*x
]]/(a + a*Cos[e + f*x] + b*Sin[e + f*x])), Int[Sqrt[1 + Cos[e + f*x]]/Sqrt[
g*Cos[e + f*x]], x], x] - Dist[g*Sqrt[1 + Cos[e + f*x]]*(Sqrt[a + b*Sin[e +
 f*x]]/(b + b*Cos[e + f*x] + a*Sin[e + f*x])), Int[Sin[e + f*x]/(Sqrt[g*Cos
[e + f*x]]*Sqrt[1 + Cos[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, g}, x] &&
EqQ[a^2 - b^2, 0]
```

Rule 2854

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b + d*x^2), x], x
, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]))], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^{5/2}\,dx = -\frac{a(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^{3/2}}{3de} + \frac{1}{2}(3a)\int \sqrt{e\cos \cdots}$$

$$= -\frac{3a^2(e\cos(c+dx))^{3/2}\sqrt{a+a\sin(c+dx)}}{4de} - \frac{a(e\cos(c+dx) \cdots}{\cdots}$$

$$= -\frac{15a^3(e\cos(c+dx))^{3/2}}{8de\sqrt{a+a\sin(c+dx)}} - \frac{3a^2(e\cos(c+dx))^{3/2}\sqrt{a+a\sin(\cdots}}{4de}$$

$$= -\frac{15a^3(e\cos(c+dx))^{3/2}}{8de\sqrt{a+a\sin(c+dx)}} - \frac{3a^2(e\cos(c+dx))^{3/2}\sqrt{a+a\sin(\cdots}}{4de}$$

$$= -\frac{15a^3(e\cos(c+dx))^{3/2}}{8de\sqrt{a+a\sin(c+dx)}} - \frac{3a^2(e\cos(c+dx))^{3/2}\sqrt{a+a\sin(\cdots}}{4de}$$

$$= -\frac{15a^3(e\cos(c+dx))^{3/2}}{8de\sqrt{a+a\sin(c+dx)}} - \frac{3a^2(e\cos(c+dx))^{3/2}\sqrt{a+a\sin(\cdots}}{4de}$$

$$= -\frac{15a^3(e\cos(c+dx))^{3/2}}{8de\sqrt{a+a\sin(c+dx)}} - \frac{3a^2(e\cos(c+dx))^{3/2}\sqrt{a+a\sin(\cdots}}{4de}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.12, size = 78, normalized size = 0.27

$$-\frac{16\sqrt[4]{2}\,a(e\cos(c+dx))^{3/2}\,{}_2F_1\left(-\frac{9}{4},\frac{3}{4};\frac{7}{4};\frac{1}{2}(1-\sin(c+dx))\right)(a(1+\sin(c+dx)))^{3/2}}{3de(1+\sin(c+dx))^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-16*2^(1/4)*a*(e*Cos[c + d*x])^(3/2)*Hypergeometric2F1[-9/4, 3/4, 7/4, (1
- Sin[c + d*x])/2]*(a*(1 + Sin[c + d*x]))^(3/2))/(3*d*e*(1 + Sin[c + d*x])^
(9/4))
```

Maple [A]
time = 0.21, size = 318, normalized size = 1.11

method	result
default	$\frac{\left(45\sqrt{2}\,\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2}}{2}\right)\sin(dx+c)+45\sqrt{2}\,\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\operatorname{arctanh}\left(\sqrt{-\frac{2}{1-}}\right.\right.}{48d((\cos^2(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^(5/2)*(e*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/48/d*(45*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*sin(d*x+c)+45*2^(1/2)*(-2*cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(
d*x+c)/cos(d*x+c)*2^(1/2))*sin(d*x+c)-16*sin(d*x+c)*cos(d*x+c)^3-16*cos(d*x
+c)^4+68*cos(d*x+c)^2*sin(d*x+c)-52*cos(d*x+c)^3+90*cos(d*x+c)*sin(d*x+c)+1
58*cos(d*x+c)^2-90*cos(d*x+c))*(a*(1+sin(d*x+c)))^(5/2)*(e*cos(d*x+c))^(1/2
)/(cos(d*x+c)^2*sin(d*x+c)-cos(d*x+c)^3+2*cos(d*x+c)*sin(d*x+c)+3*cos(d*x+c
)^2-4*sin(d*x+c)+2*cos(d*x+c)-4)/cos(d*x+c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(5/2)*(e*cos(d*x+c))^(1/2),x, algorithm="maxima"
)
```

```
[Out] e^(1/2)*integrate((a*sin(d*x + c) + a)^(5/2)*sqrt(cos(d*x + c)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3502 vs. $2(220) = 440$.
time = 194.07, size = 3502, normalized size = 12.24

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(5/2)*(e*cos(d*x+c))^(1/2),x, algorithm="fricas"
)
```

```
[Out] -1/192*(180*(a^10/d^4)^(1/4)*(sqrt(2)*d*sin(d*x + c) + sqrt(2)*d)*arctan(-1
/4*(2*sqrt(1/2)*((sqrt(2)*d^3*cos(d*x + c)^6 - 3*sqrt(2)*d^3*cos(d*x + c)^5
 - 8*sqrt(2)*d^3*cos(d*x + c)^4 + 4*sqrt(2)*d^3*cos(d*x + c)^3 + 8*sqrt(2)*
d^3*cos(d*x + c)^2 - (sqrt(2)*d^3*cos(d*x + c)^5 + 4*sqrt(2)*d^3*cos(d*x +
c)^4 - 4*sqrt(2)*d^3*cos(d*x + c)^3 - 8*sqrt(2)*d^3*cos(d*x + c)^2)*sin(d*x
```

+ c))*(a^10/d^4)^(3/4)*e^(3/2) + (sqrt(2)*a^5*d*cos(d*x + c)^6*e + 5*sqrt(
2)*a^5*d*cos(d*x + c)^5*e - 8*sqrt(2)*a^5*d*cos(d*x + c)^4*e - 20*sqrt(2)*a
^5*d*cos(d*x + c)^3*e + 8*sqrt(2)*a^5*d*cos(d*x + c)^2*e + 16*sqrt(2)*a^5*d
*cos(d*x + c)*e + (sqrt(2)*a^5*d*cos(d*x + c)^5*e - 4*sqrt(2)*a^5*d*cos(d*x
 + c)^4*e - 12*sqrt(2)*a^5*d*cos(d*x + c)^3*e + 8*sqrt(2)*a^5*d*cos(d*x + c
)^2*e + 16*sqrt(2)*a^5*d*cos(d*x + c)*e)*sin(d*x + c))*(a^10/d^4)^(1/4)*e^(
1/2) - (a^7*cos(d*x + c)^4*e^(3/2) - 3*a^7*cos(d*x + c)^3*e^(3/2) - 8*a^7*c
os(d*x + c)^2*e^(3/2) + 4*a^7*cos(d*x + c)*e^(3/2) + 8*a^7*e^(3/2) + (2*a^2
*d^2*cos(d*x + c)^5*e^(1/2) - 5*a^2*d^2*cos(d*x + c)^4*e^(1/2) - 19*a^2*d^2
*cos(d*x + c)^3*e^(1/2) + 20*a^2*d^2*cos(d*x + c)*e^(1/2) + 8*a^2*d^2*e^(1/
2) - (2*a^2*d^2*cos(d*x + c)^4*e^(1/2) + 9*a^2*d^2*cos(d*x + c)^3*e^(1/2) -
 4*a^2*d^2*cos(d*x + c)^2*e^(1/2) - 20*a^2*d^2*cos(d*x + c)*e^(1/2) - 8*a^2
*d^2*e^(1/2))*sin(d*x + c))*sqrt(a^10/d^4)*e - (a^7*cos(d*x + c)^3*e^(3/2)
+ 4*a^7*cos(d*x + c)^2*e^(3/2) - 4*a^7*cos(d*x + c)*e^(3/2) - 8*a^7*e^(3/2)
)*sin(d*x + c))*sqrt(a*sin(d*x + c) + a)*sqrt(cos(d*x + c)))*sqrt((2*a^15*c
os(d*x + c)*e^3*sin(d*x + c) + 2*a^15*cos(d*x + c)*e^3 + (a^10*d^2*e^2*sin(
d*x + c) + a^10*d^2*e^2)*sqrt(a^10/d^4)*e + (sqrt(2)*(a^10/d^4)^(1/4)*a^12*
d*cos(d*x + c)*e^3 + (sqrt(2)*a^7*d^3*e^(3/2)*sin(d*x + c) + sqrt(2)*a^7*d^
3*e^(3/2))*(a^10/d^4)^(3/4)*e^(3/2))*sqrt(a*sin(d*x + c) + a)*sqrt(cos(d*x
+ c)))/(sin(d*x + c) + 1)) + ((2*sqrt(2)*a^7*d^3*cos(d*x + c)^5*e^(3/2) + s
qrt(2)*a^7*d^3*cos(d*x + c)^4*e^(3/2) - 13*sqrt(2)*a^7*d^3*cos(d*x + c)^3*e
^(3/2) - 8*sqrt(2)*a^7*d^3*cos(d*x + c)^2*e^(3/2) + 12*sqrt(2)*a^7*d^3*cos(
d*x + c)*e^(3/2) + 8*sqrt(2)*a^7*d^3*e^(3/2) - (7*sqrt(2)*a^7*d^3*cos(d*x +
 c)^3*e^(3/2) + 4*sqrt(2)*a^7*d^3*cos(d*x + c)^2*e^(3/2) - 12*sqrt(2)*a^7*d
^3*cos(d*x + c)*e^(3/2) - 8*sqrt(2)*a^7*d^3*e^(3/2))*sin(d*x + c))*(a^10/d^
4)^(3/4)*e^(3/2) + (7*sqrt(2)*a^12*d*cos(d*x + c)^4*e^(5/2) + 3*sqrt(2)*a^1
2*d*cos(d*x + c)^3*e^(5/2) - 16*sqrt(2)*a^12*d*cos(d*x + c)^2*e^(5/2) - 4*s
qrt(2)*a^12*d*cos(d*x + c)*e^(5/2) + 8*sqrt(2)*a^12*d*e^(5/2) + (2*sqrt(2)*
a^12*d*cos(d*x + c)^4*e^(5/2) + sqrt(2)*a^12*d*cos(d*x + c)^3*e^(5/2) - 12*
sqrt(2)*a^12*d*cos(d*x + c)^2*e^(5/2) - 4*sqrt(2)*a^12*d*cos(d*x + c)*e^(5/
2) + 8*sqrt(2)*a^12*d*e^(5/2))*sin(d*x + c))*(a^10/d^4)^(1/4)*e^(1/2))*sqrt
(a*sin(d*x + c) + a)*sqrt(cos(d*x + c)))/(a^15*cos(d*x + c)^6*e^3 + a^15*co
s(d*x + c)^5*e^3 - 8*a^15*cos(d*x + c)^4*e^3 - 8*a^15*cos(d*x + c)^3*e^3 +
8*a^15*cos(d*x + c)^2*e^3 + 8*a^15*cos(d*x + c)*e^3 - 4*(a^15*cos(d*x + c)^
4*e^3 + a^15*cos(d*x + c)^3*e^3 - 2*a^15*cos(d*x + c)^2*e^3 - 2*a^15*cos(d*
x + c)*e^3)*sin(d*x + c)))*e^(1/2) - 180*(a^10/d^4)^(1/4)*(sqrt(2)*d*sin(d*
x + c) + sqrt(2)*d)*arctan(1/4*(2*sqrt(1/2)*((sqrt(2)*d^3*cos(d*x + c)^6 -
3*sqrt(2)*d^3*cos(d*x + c)^5 - 8*sqrt(2)*d^3*cos(d*x + c)^4 + 4*sqrt(2)*d^3
*cos(d*x + c)^3 + 8*sqrt(2)*d^3*cos(d*x + c)^2 - (sqrt(2)*d^3*cos(d*x + c)^
5 + 4*sqrt(2)*d^3*cos(d*x + c)^4 - 4*sqrt(2)*d^3*cos(d*x + c)^3 - 8*sqrt(2)
*d^3*cos(d*x + c)^2)*sin(d*x + c))*(a^10/d^4)^(3/4)*e^(3/2) + (sqrt(2)*a^5*
d*cos(d*x + c)^6*e + 5*sqrt(2)*a^5*d*cos(d*x + c)^5*e - 8*sqrt(2)*a^5*d*cos
(d*x + c)^4*e - 20*sqrt(2)*a^5*d*cos(d*x + c)^3*e + 8*sqrt(2)*a^5*d*cos(d*x
 + c)^2*e + 16*sqrt(2)*a^5*d*cos(d*x + c)*e + (sqrt(2)*a^5*d*cos(d*x + c)^5
*e - 4*sqrt(2)*a^5*d*cos(d*x + c)^4*e - 12*sqrt(2)*a^5*d*cos(d*x + c)^3*e +

```
 8*sqrt(2)*a^5*d*cos(d*x + c)^2*e + 16*sqrt(2)*a^5*d*cos(d*x + c)*e)*sin(d*
x + c))*(a^10/d^4)^(1/4)*e^(1/2) + (a^7*cos(d*x + c)^4*e^(3/2) - 3*a^7*cos(
d*x + c)^3*e^(3/2) - 8*a^7*cos(d*x + c)^2*e^(3/2) + 4*a^7*cos(d*x + c)*e^(3
/2) + 8*a^7*e^(3/2) + (2*a^2*d^2*cos(d*x + c)^5*e^(1/2) - 5*a^2*d^2*cos(d*x
 + c)^4*e^(1/2) - 19*a^2*d^2*cos(d*x + c)^3*e^(1/2) + 20*a^2*d^2*cos(d*x +
c)*e^(1/2) + 8*a^2*d^2*e^(1/2) - (2*a^2*d^2*cos(d*x + c)^4*e^(1/2) + 9*a^2*
d^2*cos(d*x + c)^3*e^(1/2) - 4*a^2*d^2*cos(d*x + c)^2*e^(1/2) - 20*a^2*d^2*
cos(d*x + c)*e^(1/2) - 8*a^2*d^2*e^(1/2))*sin(d*x + c))*sqrt(a^10/d^4)*e -
(a^7*cos(d*x + c)^3*e^(3/2) + 4*a^7*cos(d*x + c)^2*e^(3/2) - 4*a^7*cos(d*x
+ c)*e^(3/2) - 8*a^7*e^(3/2))*sin(d*x + c))*sqrt(a*sin(d*x + c) + a)*sqrt(c
os(d*x + c)))*sqrt((2*a^15*cos(d*x + c)*e^3*sin(d*x + c) + 2*a^15*cos(d*x +
 c)*e^3 + (a^10*d^2*e^2*sin(d*x + c) + a^10*d^2*e^2)*sqrt(a^10/d^4)*e - (sq
rt(2)*(a^10/d^4)^(1/4)*a^12*d*cos(d*x + c)*e^3 + (sqrt(2)*a^7*d^3*e^(3/2)*s
in(d*x + c) + sqrt(2)*a^7*d^3*e^(3/2))*(a^10/d^4)^(3/4)*e^(3/2))*sqrt(a*sin
(d*x + c) + a)*sqrt(cos(d*x + c)))/(sin(d*x + c) + 1)) + ((2*sqrt(2)*a^7*d^
3*cos(d*x + c)^5*e^(3/2) + sqrt(2)*a^7*d^3*cos(d*x + c)^4*e^(3/2) - 13*sqrt
(2)*a^7*d^3*cos(d*x + c)^3*e^(3/2) - 8*sqrt(2)*...
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**(5/2)*(e*cos(d*x+c))**(1/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 6189 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(5/2)*(e*cos(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate((a*sin(d*x + c) + a)^(5/2)*sqrt(cos(d*x + c))*e^(1/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(1/2)*(a + a*sin(c + d*x))^(5/2),x)
[Out] int((e*cos(c + d*x))^(1/2)*(a + a*sin(c + d*x))^(5/2), x)
```

3.291 $\int \frac{(a+a\sin(c+dx))^{5/2}}{\sqrt{e\cos(c+dx)}}\,dx$

Optimal. Leaf size=247

$$-\frac{7a^2\sqrt{e\cos(c+dx)}\,\sqrt{a+a\sin(c+dx)}}{4de}-\frac{21a^2\sinh^{-1}\left(\frac{\sqrt{e\cos(c+dx)}}{\sqrt{e}}\right)\sqrt{1+\cos(c+dx)}\,\sqrt{a+a\sin(c}{4d\sqrt{e}\,(1+\cos(c+dx)+\sin(c+dx))}$$

```
[Out] -1/2*a*(a+a*sin(d*x+c))^(3/2)*(e*cos(d*x+c))^(1/2)/d/e-7/4*a^2*(e*cos(d*x+c
))^(1/2)*(a+a*sin(d*x+c))^(1/2)/d/e-21/4*a^2*arcsinh((e*cos(d*x+c))^(1/2)/e
^(1/2))*(1+cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^(1/2)/d/(1+cos(d*x+c)+sin(d*x
+c))/e^(1/2)+21/4*a^2*arctan(sin(d*x+c)*e^(1/2)/(e*cos(d*x+c))^(1/2)/(1+cos
(d*x+c))^(1/2))*(1+cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^(1/2)/d/(1+cos(d*x+c)
+sin(d*x+c))/e^(1/2)
```

Rubi [A]

time = 0.25, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2757, 2756, 2854, 209, 2912, 65, 221}

$$\frac{21a^2\sqrt{\cos(c+dx)+1}\,\sqrt{a\sin(c+dx)+a}\,\text{ArcTan}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\,\sqrt{e\cos(c+dx)}}\right)}{4d\sqrt{e}\,(\sin(c+dx)+\cos(c+dx)+1)}-\frac{7a^2\sqrt{a\sin(c+dx)+a}\,\sqrt{e\cos(c+dx)}}{4de}-\frac{21a^2\sqrt{\cos(c+dx)+1}\,\sqrt{a\sin(c+dx)+a}\,\sinh^{-1}\left(\frac{\sqrt{e\cos(c+dx)}}{\sqrt{e}}\right)}{4d\sqrt{e}\,(\sin(c+dx)+\cos(c+dx)+1)}-\frac{a(a\sin(c+dx)+a)^{3/2}\sqrt{e\cos(c+dx)}}{2de}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^(5/2)/Sqrt[e*Cos[c + d*x]],x]
```

```
[Out] (-7*a^2*Sqrt[e*Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(4*d*e) - (21*a^2*Ar
cSinh[Sqrt[e*Cos[c + d*x]]/Sqrt[e]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c
 + d*x]])/(4*d*Sqrt[e]*(1 + Cos[c + d*x] + Sin[c + d*x])) + (21*a^2*ArcTan[
(Sqrt[e]*Sin[c + d*x])/(Sqrt[e*Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]])]*Sqrt[
1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(4*d*Sqrt[e]*(1 + Cos[c + d*x]
+ Sin[c + d*x])) - (a*Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])^(3/2))/(2*d
*e)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 2756

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[cos[(e_.) + (f_.)*(x_)]
*(g_.)], x_Symbol] :> Dist[a*Sqrt[1 + Cos[e + f*x]]*(Sqrt[a + b*Sin[e + f*x
]]/(a + a*Cos[e + f*x] + b*Sin[e + f*x])), Int[Sqrt[1 + Cos[e + f*x]]/Sqrt[
g*Cos[e + f*x]], x], x] + Dist[b*Sqrt[1 + Cos[e + f*x]]*(Sqrt[a + b*Sin[e +
 f*x]]/(a + a*Cos[e + f*x] + b*Sin[e + f*x])), Int[Sin[e + f*x]/(Sqrt[g*Cos
[e + f*x]]*Sqrt[1 + Cos[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, g}, x] &&
EqQ[a^2 - b^2, 0]
```

Rule 2757

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ
[2*m, 2*p]
```

Rule 2854

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b + d*x^2), x], x
, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]))], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(a+a\sin(c+dx))^{5/2}}{\sqrt{e\cos(c+dx)}}\,dx = -\frac{a\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^{3/2}}{2de} + \frac{1}{4}(7a)\int \frac{(a+a\sin(c+dx))^{3/2}}{\sqrt{e\cos(c+dx)}}\,dx$$

$$= -\frac{7a^2\sqrt{e\cos(c+dx)}\,\sqrt{a+a\sin(c+dx)}}{4de} - \frac{a\sqrt{e\cos(c+dx)}\,(a+a\sin(c+$$

$$= -\frac{7a^2\sqrt{e\cos(c+dx)}\,\sqrt{a+a\sin(c+dx)}}{4de} - \frac{a\sqrt{e\cos(c+dx)}\,(a+a\sin(c+$$

$$= -\frac{7a^2\sqrt{e\cos(c+dx)}\,\sqrt{a+a\sin(c+dx)}}{4de} - \frac{a\sqrt{e\cos(c+dx)}\,(a+a\sin(c+$$

$$= -\frac{7a^2\sqrt{e\cos(c+dx)}\,\sqrt{a+a\sin(c+dx)}}{4de} - \frac{a\sqrt{e\cos(c+dx)}\,(a+a\sin(c+$$

$$= -\frac{7a^2\sqrt{e\cos(c+dx)}\,\sqrt{a+a\sin(c+dx)}}{4de} - \frac{a\sqrt{e\cos(c+dx)}\,(a+a\sin(c+$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.09, size = 76, normalized size = 0.31

$$-\frac{8\ 2^{3/4}a\sqrt{e\cos(c+dx)}\ {}_2F_1\left(-\frac{7}{4},\frac{1}{4};\frac{5}{4};\frac{1}{2}(1-\sin(c+dx))\right)(a(1+\sin(c+dx)))^{3/2}}{de(1+\sin(c+dx))^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^(5/2)/Sqrt[e*Cos[c + d*x]],x]
```

```
[Out] (-8*2^(3/4)*a*Sqrt[e*Cos[c + d*x]]*Hypergeometric2F1[-7/4, 1/4, 5/4, (1 - S
in[c + d*x])/2]*(a*(1 + Sin[c + d*x]))^(3/2))/(d*e*(1 + Sin[c + d*x])^(7/4)
)
```

Maple [A]
time = 0.23, size = 283, normalized size = 1.15

method	result

default	$\frac{(a(1+\sin(dx+c)))^{\frac{5}{2}}\left(21\sqrt{2}\,\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2}}{2}\right)\sin(dx+c)-21\sqrt{2}\,\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}}{8d(\cos^3(dx+c)-(\cos^2(dx+c))\sin(dx+c)-3(\cos^2(d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/d*(a*(1+sin(d*x+c)))^(5/2)*(21*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*sin(d*x+c)-21
*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*sin(d*x+c)+4*cos(d*x+c)^
3+4*cos(d*x+c)^2*sin(d*x+c)+18*cos(d*x+c)^2-22*cos(d*x+c)*sin(d*x+c)-22*cos
(d*x+c))/(cos(d*x+c)^3-cos(d*x+c)^2*sin(d*x+c)-3*cos(d*x+c)^2-2*cos(d*x+c)*
sin(d*x+c)-2*cos(d*x+c)+4*sin(d*x+c)+4)/(e*cos(d*x+c))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(1/2),x, algorithm="maxima"
)
```

```
[Out] e^(-1/2)*integrate((a*sin(d*x + c) + a)^(5/2)/sqrt(cos(d*x + c)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3330 vs. $2(190) = 380$.
time = 190.06, size = 3330, normalized size = 13.48

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(1/2),x, algorithm="fricas"
)
```

```
[Out] 1/32*(84*sqrt(2)*(a^10/d^4)^(1/4)*d*arctan(-1/4*(2*sqrt(1/2)*((sqrt(2)*d^3*
cos(d*x + c)^6*e^(3/2) + 5*sqrt(2)*d^3*cos(d*x + c)^5*e^(3/2) - 8*sqrt(2)*d
^3*cos(d*x + c)^4*e^(3/2) - 20*sqrt(2)*d^3*cos(d*x + c)^3*e^(3/2) + 8*sqrt(
2)*d^3*cos(d*x + c)^2*e^(3/2) + 16*sqrt(2)*d^3*cos(d*x + c)*e^(3/2) + (sqrt
(2)*d^3*cos(d*x + c)^5*e^(3/2) - 4*sqrt(2)*d^3*cos(d*x + c)^4*e^(3/2) - 12*
sqrt(2)*d^3*cos(d*x + c)^3*e^(3/2) + 8*sqrt(2)*d^3*cos(d*x + c)^2*e^(3/2) +
 16*sqrt(2)*d^3*cos(d*x + c)*e^(3/2))*sin(d*x + c))*(a^10/d^4)^(3/4)*e^(-3/
```

2) + (sqrt(2)*a^5*d*cos(d*x + c)^6*e^(1/2) - 3*sqrt(2)*a^5*d*cos(d*x + c)^5
*e^(1/2) - 8*sqrt(2)*a^5*d*cos(d*x + c)^4*e^(1/2) + 4*sqrt(2)*a^5*d*cos(d*x
 + c)^3*e^(1/2) + 8*sqrt(2)*a^5*d*cos(d*x + c)^2*e^(1/2) - (sqrt(2)*a^5*d*c
os(d*x + c)^5*e^(1/2) + 4*sqrt(2)*a^5*d*cos(d*x + c)^4*e^(1/2) - 4*sqrt(2)*
a^5*d*cos(d*x + c)^3*e^(1/2) - 8*sqrt(2)*a^5*d*cos(d*x + c)^2*e^(1/2))*sin(
d*x + c))*(a^10/d^4)^(1/4)*e^(-1/2) - (a^7*cos(d*x + c)^4 - 3*a^7*cos(d*x +
 c)^3 - 8*a^7*cos(d*x + c)^2 + 4*a^7*cos(d*x + c) + 8*a^7 + (2*a^2*d^2*cos(
d*x + c)^5*e - 5*a^2*d^2*cos(d*x + c)^4*e - 19*a^2*d^2*cos(d*x + c)^3*e + 2
0*a^2*d^2*cos(d*x + c)*e + 8*a^2*d^2*e - (2*a^2*d^2*cos(d*x + c)^4*e + 9*a^
2*d^2*cos(d*x + c)^3*e - 4*a^2*d^2*cos(d*x + c)^2*e - 20*a^2*d^2*cos(d*x +
c)*e - 8*a^2*d^2*e)*sin(d*x + c))*sqrt(a^10/d^4)*e^(-1) - (a^7*cos(d*x + c)
^3 + 4*a^7*cos(d*x + c)^2 - 4*a^7*cos(d*x + c) - 8*a^7)*sin(d*x + c))*sqrt(
a*sin(d*x + c) + a)*sqrt(cos(d*x + c)))*sqrt((2*a^15*cos(d*x + c)*sin(d*x +
 c) + 2*a^15*cos(d*x + c) + (a^10*d^2*e*sin(d*x + c) + a^10*d^2*e)*sqrt(a^1
0/d^4)*e^(-1) + (sqrt(2)*(a^10/d^4)^(3/4)*a^7*d^3*cos(d*x + c) + (sqrt(2)*a
^12*d*e^(1/2)*sin(d*x + c) + sqrt(2)*a^12*d*e^(1/2))*(a^10/d^4)^(1/4)*e^(-1
/2))*sqrt(a*sin(d*x + c) + a)*sqrt(cos(d*x + c)))/(sin(d*x + c) + 1)) - ((7
*sqrt(2)*a^7*d^3*cos(d*x + c)^4*e^(3/2) + 3*sqrt(2)*a^7*d^3*cos(d*x + c)^3*
e^(3/2) - 16*sqrt(2)*a^7*d^3*cos(d*x + c)^2*e^(3/2) - 4*sqrt(2)*a^7*d^3*cos
(d*x + c)*e^(3/2) + 8*sqrt(2)*a^7*d^3*e^(3/2) + (2*sqrt(2)*a^7*d^3*cos(d*x
+ c)^4*e^(3/2) + sqrt(2)*a^7*d^3*cos(d*x + c)^3*e^(3/2) - 12*sqrt(2)*a^7*d^
3*cos(d*x + c)^2*e^(3/2) - 4*sqrt(2)*a^7*d^3*cos(d*x + c)*e^(3/2) + 8*sqrt(
2)*a^7*d^3*e^(3/2))*sin(d*x + c))*(a^10/d^4)^(3/4)*e^(-3/2) + (2*sqrt(2)*a^
12*d*cos(d*x + c)^5*e^(1/2) + sqrt(2)*a^12*d*cos(d*x + c)^4*e^(1/2) - 13*sq
rt(2)*a^12*d*cos(d*x + c)^3*e^(1/2) - 8*sqrt(2)*a^12*d*cos(d*x + c)^2*e^(1/
2) + 12*sqrt(2)*a^12*d*cos(d*x + c)*e^(1/2) + 8*sqrt(2)*a^12*d*e^(1/2) - (7
*sqrt(2)*a^12*d*cos(d*x + c)^3*e^(1/2) + 4*sqrt(2)*a^12*d*cos(d*x + c)^2*e^
(1/2) - 12*sqrt(2)*a^12*d*cos(d*x + c)*e^(1/2) - 8*sqrt(2)*a^12*d*e^(1/2))*
sin(d*x + c))*(a^10/d^4)^(1/4)*e^(-1/2))*sqrt(a*sin(d*x + c) + a)*sqrt(cos(
d*x + c)))/(a^15*cos(d*x + c)^6 + a^15*cos(d*x + c)^5 - 8*a^15*cos(d*x + c)
^4 - 8*a^15*cos(d*x + c)^3 + 8*a^15*cos(d*x + c)^2 + 8*a^15*cos(d*x + c) -
4*(a^15*cos(d*x + c)^4 + a^15*cos(d*x + c)^3 - 2*a^15*cos(d*x + c)^2 - 2*a^
15*cos(d*x + c))*sin(d*x + c))) - 84*sqrt(2)*(a^10/d^4)^(1/4)*d*arctan(1/4*
(2*sqrt(1/2)*((sqrt(2)*d^3*cos(d*x + c)^6*e^(3/2) + 5*sqrt(2)*d^3*cos(d*x +
 c)^5*e^(3/2) - 8*sqrt(2)*d^3*cos(d*x + c)^4*e^(3/2) - 20*sqrt(2)*d^3*cos(d
*x + c)^3*e^(3/2) + 8*sqrt(2)*d^3*cos(d*x + c)^2*e^(3/2) + 16*sqrt(2)*d^3*c
os(d*x + c)*e^(3/2) + (sqrt(2)*d^3*cos(d*x + c)^5*e^(3/2) - 4*sqrt(2)*d^3*c
os(d*x + c)^4*e^(3/2) - 12*sqrt(2)*d^3*cos(d*x + c)^3*e^(3/2) + 8*sqrt(2)*d
^3*cos(d*x + c)^2*e^(3/2) + 16*sqrt(2)*d^3*cos(d*x + c)*e^(3/2))*sin(d*x +
c))*(a^10/d^4)^(3/4)*e^(-3/2) + (sqrt(2)*a^5*d*cos(d*x + c)^6*e^(1/2) - 3*s
qrt(2)*a^5*d*cos(d*x + c)^5*e^(1/2) - 8*sqrt(2)*a^5*d*cos(d*x + c)^4*e^(1/2
) + 4*sqrt(2)*a^5*d*cos(d*x + c)^3*e^(1/2) + 8*sqrt(2)*a^5*d*cos(d*x + c)^2
*e^(1/2) - (sqrt(2)*a^5*d*cos(d*x + c)^5*e^(1/2) + 4*sqrt(2)*a^5*d*cos(d*x
+ c)^4*e^(1/2) - 4*sqrt(2)*a^5*d*cos(d*x + c)^3*e^(1/2) - 8*sqrt(2)*a^5*d*c
os(d*x + c)^2*e^(1/2))*sin(d*x + c))*(a^10/d^4)^(1/4)*e^(-1/2) + (a^7*cos(d

```
*x + c)^4 - 3*a^7*cos(d*x + c)^3 - 8*a^7*cos(d*x + c)^2 + 4*a^7*cos(d*x + c
) + 8*a^7 + (2*a^2*d^2*cos(d*x + c)^5*e - 5*a^2*d^2*cos(d*x + c)^4*e - 19*a
^2*d^2*cos(d*x + c)^3*e + 20*a^2*d^2*cos(d*x + c)*e + 8*a^2*d^2*e - (2*a^2*
d^2*cos(d*x + c)^4*e + 9*a^2*d^2*cos(d*x + c)^3*e - 4*a^2*d^2*cos(d*x + c)^
2*e - 20*a^2*d^2*cos(d*x + c)*e - 8*a^2*d^2*e)*sin(d*x + c))*sqrt(a^10/d^4)
*e^(-1) - (a^7*cos(d*x + c)^3 + 4*a^7*cos(d*x + c)^2 - 4*a^7*cos(d*x + c) -
 8*a^7)*sin(d*x + c))*sqrt(a*sin(d*x + c) + a)*sqrt(cos(d*x + c)))*sqrt((2*
a^15*cos(d*x + c)*sin(d*x + c) + 2*a^15*cos(d*x + c) + (a^10*d^2*e*sin(d*x
+ c) + a^10*d^2*e)*sqrt(a^10/d^4)*e^(-1) - (sqrt(2)*(a^10/d^4)^(3/4)*a^7*d^
3*cos(d*x + c) + (sqrt(2)*a^12*d*e^(1/2)*sin(d*x + c) + sqrt(2)*a^12*d*e^(1
/2))*(a^10/d^4)^(1/4)*e^(-1/2))*sqrt(a*sin(d*x + c) + a)*sqrt(cos(d*x + c))
)/(sin(d*x + c) + 1)) - ((7*sqrt(2)*a^7*d^3*cos(d*x + c)^4*e^(3/2) + 3*sqrt
(2)*a^7*d^3*cos(d*x + c)^3*e^(3/2) - 16*sqrt(2)*a^7*d^3*cos(d*x + c)^2*e^(3
/2) - 4*sqrt(2)*a^7*d^3*cos(d*x + c)*e^(3/2) + 8*sqrt(2)*a^7*d^3*e^(3/2) +
(2*sqrt(2)*a^7*d^3*cos(d*x + c)^4*e^(3/2) + sqr...
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**(5/2)/(e*cos(d*x+c))**(1/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(1/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{5/2}}{\sqrt{e\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(5/2)/(e*cos(c + d*x))^(1/2),x)
[Out] int((a + a*sin(c + d*x))^(5/2)/(e*cos(c + d*x))^(1/2), x)
```

3.292 $\int \frac{(a+a\sin(c+dx))^{5/2}}{(e\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=239

$$\frac{5a^3(e\cos(c+dx))^{3/2}}{de^3\sqrt{a+a\sin(c+dx)}}-\frac{5a^2\sinh^{-1}\left(\frac{\sqrt{e\cos(c+dx)}}{\sqrt{e}}\right)\sqrt{1+\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}{de^{3/2}(1+\cos(c+dx)+\sin(c+dx))}-\frac{5a^2\tan^{-1}$$

```
[Out] 4*a*(a+a*sin(d*x+c))^(3/2)/d/e/(e*cos(d*x+c))^(1/2)+5*a^3*(e*cos(d*x+c))^(3
/2)/d/e^3/(a+a*sin(d*x+c))^(1/2)-5*a^2*arcsinh((e*cos(d*x+c))^(1/2)/e^(1/2)
)*(1+cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^(1/2)/d/e^(3/2)/(1+cos(d*x+c)+sin(d
*x+c))-5*a^2*arctan(sin(d*x+c)*e^(1/2)/(e*cos(d*x+c))^(1/2)/(1+cos(d*x+c))^
(1/2))*(1+cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^(1/2)/d/e^(3/2)/(1+cos(d*x+c)+
sin(d*x+c))
```

Rubi [A]

time = 0.24, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {2755, 2757, 2763, 2854, 209, 2912, 65, 221}

$$\frac{5a^3(e\cos(c+dx))^{3/2}}{de^3\sqrt{a\sin(c+dx)+a}}-\frac{5a^2\sqrt{\cos(c+dx)+1}\sqrt{a\sin(c+dx)+a}\,\text{ArcTan}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\sqrt{e\cos(c+dx)}}\right)}{de^{3/2}(\sin(c+dx)+\cos(c+dx)+1)}-\frac{5a^2\sqrt{\cos(c+dx)+1}\sqrt{a\sin(c+dx)+a}\sinh^{-1}\left(\frac{\sqrt{e\cos(c+dx)}}{\sqrt{e}}\right)}{de^{3/2}(\sin(c+dx)+\cos(c+dx)+1)}+\frac{4a(a\sin(c+dx)+a)^{3/2}}{de\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^(5/2)/(e*Cos[c + d*x])^(3/2),x]
```

```
[Out] (5*a^3*(e*Cos[c + d*x])^(3/2))/(d*e^3*Sqrt[a + a*Sin[c + d*x]]) - (5*a^2*Ar
cSinh[Sqrt[e*Cos[c + d*x]]/Sqrt[e]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c
 + d*x]])/(d*e^(3/2)*(1 + Cos[c + d*x] + Sin[c + d*x])) - (5*a^2*ArcTan[(Sq
rt[e]*Sin[c + d*x])/(Sqrt[e*Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]])]*Sqrt[1 +
 Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(d*e^(3/2)*(1 + Cos[c + d*x] + Sin
[c + d*x])) + (4*a*(a + a*Sin[c + d*x])^(3/2))/(d*e*Sqrt[e*Cos[c + d*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 2755

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[-2*b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(p + 1))), x] + Dist[b^2*((2*m + p - 1)/(g^2*(p + 1))),
Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[
{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && Inte
gersQ[2*m, 2*p]
```

Rule 2757

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ
[2*m, 2*p]
```

Rule 2763

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[g*Sqrt[1 + Cos[e + f*x]]*(Sqrt[a + b*Sin[e + f*x
]]/(a + a*Cos[e + f*x] + b*Sin[e + f*x])), Int[Sqrt[1 + Cos[e + f*x]]/Sqrt[
g*Cos[e + f*x]], x], x] - Dist[g*Sqrt[1 + Cos[e + f*x]]*(Sqrt[a + b*Sin[e +
 f*x]]/(b + b*Cos[e + f*x] + a*Sin[e + f*x])), Int[Sin[e + f*x]/(Sqrt[g*Cos
[e + f*x]]*Sqrt[1 + Cos[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, g}, x] &&
EqQ[a^2 - b^2, 0]
```

Rule 2854

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b + d*x^2), x], x
, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]))], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
```

```
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(a+a\sin(c+dx))^{5/2}}{(e\cos(c+dx))^{3/2}}\,dx = \frac{4a(a+a\sin(c+dx))^{3/2}}{de\sqrt{e\cos(c+dx)}} - \frac{(5a^2)\int \sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}\,dx}{e^2}$$

$$= \frac{5a^3(e\cos(c+dx))^{3/2}}{de^3\sqrt{a+a\sin(c+dx)}} + \frac{4a(a+a\sin(c+dx))^{3/2}}{de\sqrt{e\cos(c+dx)}} - \frac{(5a^3)\int \frac{\sqrt{e\cos(c+d}}{\sqrt{a+a\sin(c+}}}{2e^2}$$

$$= \frac{5a^3(e\cos(c+dx))^{3/2}}{de^3\sqrt{a+a\sin(c+dx)}} + \frac{4a(a+a\sin(c+dx))^{3/2}}{de\sqrt{e\cos(c+dx)}} - \frac{\left(5a^3\sqrt{1+\cos(c+dx)}\right.}{2e(a+}$$

$$= \frac{5a^3(e\cos(c+dx))^{3/2}}{de^3\sqrt{a+a\sin(c+dx)}} + \frac{4a(a+a\sin(c+dx))^{3/2}}{de\sqrt{e\cos(c+dx)}} - \frac{\left(5a^3\sqrt{1+\cos(c+dx)}\right.}{}$$

$$= \frac{5a^3(e\cos(c+dx))^{3/2}}{de^3\sqrt{a+a\sin(c+dx)}} + \frac{4a(a+a\sin(c+dx))^{3/2}}{de\sqrt{e\cos(c+dx)}} - \frac{5a^3\tan^{-1}\left(\frac{}{\sqrt{e\cos(c+}}\right.}{}$$

$$= \frac{5a^3(e\cos(c+dx))^{3/2}}{de^3\sqrt{a+a\sin(c+dx)}} + \frac{4a(a+a\sin(c+dx))^{3/2}}{de\sqrt{e\cos(c+dx)}} - \frac{5a^3\sinh^{-1}\left(\frac{\sqrt{e\cos(c-}}{\sqrt{e}}\right.}{de^{3/2}(a}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.15, size = 75, normalized size = 0.31

$$\frac{8\sqrt[4]{2}\,{}_2F_1\left(-\frac{5}{4},-\frac{1}{4};\frac{3}{4};\frac{1}{2}(1-\sin(c+dx))\right)(a(1+\sin(c+dx)))^{5/2}}{de\sqrt{e\cos(c+dx)}\,(1+\sin(c+dx))^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^(5/2)/(e*Cos[c + d*x])^(3/2),x]
```

```
[Out] (8*2^(1/4)*Hypergeometric2F1[-5/4, -1/4, 3/4, (1 - Sin[c + d*x])/2]*(a*(1 +
 Sin[c + d*x]))^(5/2))/(d*e*Sqrt[e*Cos[c + d*x]]*(1 + Sin[c + d*x])^(9/4))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 442 vs. 2(209) = 418.
time = 0.16, size = 443, normalized size = 1.85

method	result
default	$-\frac{\left(5\sqrt{2}\,\cos(dx+c)\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2}}{2}\right)+5\sqrt{2}\,\cos(dx+c)\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\text{arctanh}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/d*(5*2^(1/2)*cos(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/
2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))+5*2^(1/2)*cos(d*x+c)*(-2*co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))-5*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*sin(d*x+c)
-5*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*sin(d*x+c)+5*2^(1/2)*(
-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*2^(1/2))+5*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/
2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))-4*cos
(d*x+c)*sin(d*x+c)+36*cos(d*x+c))*(a*(1+sin(d*x+c)))^(5/2)/(cos(d*x+c)^2-2*
sin(d*x+c)-2)/(e*cos(d*x+c))^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(3/2),x, algorithm="maxima"
)
```

```
[Out] e^(-3/2)*integrate((a*sin(d*x + c) + a)^(5/2)/cos(d*x + c)^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3371 vs. 2(190) = 380.
time = 179.78, size = 3371, normalized size = 14.10

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(3/2),x, algorithm="fricas"
)

[Out] 1/8*(20*sqrt(2)*(a^10/d^4)^(1/4)*d*arctan(-1/4*(2*sqrt(1/2)*((sqrt(2)*d^3*c
os(d*x + c)^6*e^(9/2) - 3*sqrt(2)*d^3*cos(d*x + c)^5*e^(9/2) - 8*sqrt(2)*d^
3*cos(d*x + c)^4*e^(9/2) + 4*sqrt(2)*d^3*cos(d*x + c)^3*e^(9/2) + 8*sqrt(2)
*d^3*cos(d*x + c)^2*e^(9/2) - (sqrt(2)*d^3*cos(d*x + c)^5*e^(9/2) + 4*sqrt(
2)*d^3*cos(d*x + c)^4*e^(9/2) - 4*sqrt(2)*d^3*cos(d*x + c)^3*e^(9/2) - 8*sq
rt(2)*d^3*cos(d*x + c)^2*e^(9/2))*sin(d*x + c))*(a^10/d^4)^(3/4)*e^(-9/2) +
 (sqrt(2)*a^5*d*cos(d*x + c)^6*e^(3/2) + 5*sqrt(2)*a^5*d*cos(d*x + c)^5*e^(
3/2) - 8*sqrt(2)*a^5*d*cos(d*x + c)^4*e^(3/2) - 20*sqrt(2)*a^5*d*cos(d*x +
c)^3*e^(3/2) + 8*sqrt(2)*a^5*d*cos(d*x + c)^2*e^(3/2) + 16*sqrt(2)*a^5*d*co
s(d*x + c)*e^(3/2) + (sqrt(2)*a^5*d*cos(d*x + c)^5*e^(3/2) - 4*sqrt(2)*a^5*
d*cos(d*x + c)^4*e^(3/2) - 12*sqrt(2)*a^5*d*cos(d*x + c)^3*e^(3/2) + 8*sqrt
(2)*a^5*d*cos(d*x + c)^2*e^(3/2) + 16*sqrt(2)*a^5*d*cos(d*x + c)*e^(3/2))*s
in(d*x + c))*(a^10/d^4)^(1/4)*e^(-3/2) - (a^7*cos(d*x + c)^4 - 3*a^7*cos(d*
x + c)^3 - 8*a^7*cos(d*x + c)^2 + 4*a^7*cos(d*x + c) + 8*a^7 + (2*a^2*d^2*c
os(d*x + c)^5*e^3 - 5*a^2*d^2*cos(d*x + c)^4*e^3 - 19*a^2*d^2*cos(d*x + c)^
3*e^3 + 20*a^2*d^2*cos(d*x + c)*e^3 + 8*a^2*d^2*e^3 - (2*a^2*d^2*cos(d*x +
c)^4*e^3 + 9*a^2*d^2*cos(d*x + c)^3*e^3 - 4*a^2*d^2*cos(d*x + c)^2*e^3 - 20
*a^2*d^2*cos(d*x + c)*e^3 - 8*a^2*d^2*e^3)*sin(d*x + c))*sqrt(a^10/d^4)*e^(
-3) - (a^7*cos(d*x + c)^3 + 4*a^7*cos(d*x + c)^2 - 4*a^7*cos(d*x + c) - 8*a
^7)*sin(d*x + c))*sqrt(a*sin(d*x + c) + a)*sqrt(cos(d*x + c)))*sqrt((2*a^15
*cos(d*x + c)*sin(d*x + c) + 2*a^15*cos(d*x + c) + (a^10*d^2*e^3*sin(d*x +
c) + a^10*d^2*e^3)*sqrt(a^10/d^4)*e^(-3) + (sqrt(2)*(a^10/d^4)^(1/4)*a^12*d
*cos(d*x + c) + (sqrt(2)*a^7*d^3*e^(9/2)*sin(d*x + c) + sqrt(2)*a^7*d^3*e^(
9/2))*(a^10/d^4)^(3/4)*e^(-9/2))*sqrt(a*sin(d*x + c) + a)*sqrt(cos(d*x + c)
))/(sin(d*x + c) + 1)) + ((2*sqrt(2)*a^7*d^3*cos(d*x + c)^5*e^(9/2) + sqrt(
2)*a^7*d^3*cos(d*x + c)^4*e^(9/2) - 13*sqrt(2)*a^7*d^3*cos(d*x + c)^3*e^(9/
2) - 8*sqrt(2)*a^7*d^3*cos(d*x + c)^2*e^(9/2) + 12*sqrt(2)*a^7*d^3*cos(d*x
+ c)*e^(9/2) + 8*sqrt(2)*a^7*d^3*e^(9/2) - (7*sqrt(2)*a^7*d^3*cos(d*x + c)^
3*e^(9/2) + 4*sqrt(2)*a^7*d^3*cos(d*x + c)^2*e^(9/2) - 12*sqrt(2)*a^7*d^3*c
os(d*x + c)*e^(9/2) - 8*sqrt(2)*a^7*d^3*e^(9/2))*sin(d*x + c))*(a^10/d^4)^(
3/4)*e^(-9/2) + (7*sqrt(2)*a^12*d*cos(d*x + c)^4*e^(3/2) + 3*sqrt(2)*a^12*d
*cos(d*x + c)^3*e^(3/2) - 16*sqrt(2)*a^12*d*cos(d*x + c)^2*e^(3/2) - 4*sqrt
(2)*a^12*d*cos(d*x + c)*e^(3/2) + 8*sqrt(2)*a^12*d*e^(3/2) + (2*sqrt(2)*a^1
2*d*cos(d*x + c)^4*e^(3/2) + sqrt(2)*a^12*d*cos(d*x + c)^3*e^(3/2) - 12*sqr
t(2)*a^12*d*cos(d*x + c)^2*e^(3/2) - 4*sqrt(2)*a^12*d*cos(d*x + c)*e^(3/2)
+ 8*sqrt(2)*a^12*d*e^(3/2))*sin(d*x + c))*(a^10/d^4)^(1/4)*e^(-3/2))*sqrt(a
*sin(d*x + c) + a)*sqrt(cos(d*x + c)))/(a^15*cos(d*x + c)^6 + a^15*cos(d*x
+ c)^5 - 8*a^15*cos(d*x + c)^4 - 8*a^15*cos(d*x + c)^3 + 8*a^15*cos(d*x + c
)^2 + 8*a^15*cos(d*x + c) - 4*(a^15*cos(d*x + c)^4 + a^15*cos(d*x + c)^3 -
2*a^15*cos(d*x + c)^2 - 2*a^15*cos(d*x + c))*sin(d*x + c)))*cos(d*x + c) -
20*sqrt(2)*(a^10/d^4)^(1/4)*d*arctan(1/4*(2*sqrt(1/2)*((sqrt(2)*d^3*cos(d*x
 + c)^6*e^(9/2) - 3*sqrt(2)*d^3*cos(d*x + c)^5*e^(9/2) - 8*sqrt(2)*d^3*cos(
```

```
d*x + c)^4*e^(9/2) + 4*sqrt(2)*d^3*cos(d*x + c)^3*e^(9/2) + 8*sqrt(2)*d^3*c
os(d*x + c)^2*e^(9/2) - (sqrt(2)*d^3*cos(d*x + c)^5*e^(9/2) + 4*sqrt(2)*d^3
*cos(d*x + c)^4*e^(9/2) - 4*sqrt(2)*d^3*cos(d*x + c)^3*e^(9/2) - 8*sqrt(2)*
d^3*cos(d*x + c)^2*e^(9/2))*sin(d*x + c))*(a^10/d^4)^(3/4)*e^(-9/2) + (sqrt
(2)*a^5*d*cos(d*x + c)^6*e^(3/2) + 5*sqrt(2)*a^5*d*cos(d*x + c)^5*e^(3/2) -
 8*sqrt(2)*a^5*d*cos(d*x + c)^4*e^(3/2) - 20*sqrt(2)*a^5*d*cos(d*x + c)^3*e
^(3/2) + 8*sqrt(2)*a^5*d*cos(d*x + c)^2*e^(3/2) + 16*sqrt(2)*a^5*d*cos(d*x
+ c)*e^(3/2) + (sqrt(2)*a^5*d*cos(d*x + c)^5*e^(3/2) - 4*sqrt(2)*a^5*d*cos(
d*x + c)^4*e^(3/2) - 12*sqrt(2)*a^5*d*cos(d*x + c)^3*e^(3/2) + 8*sqrt(2)*a^
5*d*cos(d*x + c)^2*e^(3/2) + 16*sqrt(2)*a^5*d*cos(d*x + c)*e^(3/2))*sin(d*x
 + c))*(a^10/d^4)^(1/4)*e^(-3/2) + (a^7*cos(d*x + c)^4 - 3*a^7*cos(d*x + c)
^3 - 8*a^7*cos(d*x + c)^2 + 4*a^7*cos(d*x + c) + 8*a^7 + (2*a^2*d^2*cos(d*x
 + c)^5*e^3 - 5*a^2*d^2*cos(d*x + c)^4*e^3 - 19*a^2*d^2*cos(d*x + c)^3*e^3
+ 20*a^2*d^2*cos(d*x + c)*e^3 + 8*a^2*d^2*e^3 - (2*a^2*d^2*cos(d*x + c)^4*e
^3 + 9*a^2*d^2*cos(d*x + c)^3*e^3 - 4*a^2*d^2*cos(d*x + c)^2*e^3 - 20*a^2*d
^2*cos(d*x + c)*e^3 - 8*a^2*d^2*e^3)*sin(d*x + c))*sqrt(a^10/d^4)*e^(-3) -
(a^7*cos(d*x + c)^3 + 4*a^7*cos(d*x + c)^2 - 4*a^7*cos(d*x + c) - 8*a^7)*si
n(d*x + c))*sqrt(a*sin(d*x + c) + a)*sqrt(cos(d*x + c)))*sqrt((2*a^15*cos(d
*x + c)*sin(d*x + c) + 2*a^15*cos(d*x + c) + (a^10*d^2*e^3*sin(d*x + c) + a
^10*d^2*e^3)*sqrt(a^10/d^4)*e^(-3) - (sqrt(2)*(a^10/d^4)^(1/4)*a^12*d*cos(d
*x + c) + (sqrt(2)*a^7*d^3*e^(9/2)*sin(d*x + c) + sqrt(2)*a^7*d^3*e^(9/2))*
(a^10/d^4)^(3/4)*e^(-9/2))*sqrt(a*sin(d*x + c) + a)*sqrt(cos(d*x + c)))/(si
n(d*x + c) + 1)) + ((2*sqrt(2)*a^7*d^3*cos(d*x + c)^5*e^(9/2) + sqrt(2)*a^7
*d^3*cos(d*x + c)^4*e^(9/2) - 13*sqrt(2)*a^7*d^3*cos(d*x + c)^3*e^(9/2) - 8
*sqrt(2)*a^7*d^3*cos(d*x + c)^2*e^(9/2) + 12*sq...
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**(5/2)/(e*cos(d*x+c))**(3/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(3/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + a\sin(c + d\,x))^{5/2}}{(e\cos(c + d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(5/2)/(e*cos(c + d*x))^(3/2),x)
[Out] int((a + a*sin(c + d*x))^(5/2)/(e*cos(c + d*x))^(3/2), x)
```

3.293 $\int \frac{(a+a\sin(c+dx))^{5/2}}{(e\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=204

$$\frac{2a^2\sinh^{-1}\left(\frac{\sqrt{e\cos(c+dx)}}{\sqrt{e}}\right)\sqrt{1+\cos(c+dx)}\,\sqrt{a+a\sin(c+dx)}}{de^{5/2}(1+\cos(c+dx)+\sin(c+dx))}-\frac{2a^2\tan^{-1}\left(\frac{\sqrt{e}\,\sin(c+d}{\sqrt{e\cos(c+dx)}\,\sqrt{1+}\right.}{de^{5/2}(1}$$

```
[Out] 4/3*a*(a+a*sin(d*x+c))^(3/2)/d/e/(e*cos(d*x+c))^(3/2)+2*a^2*arcsinh((e*cos(
d*x+c))^(1/2)/e^(1/2))*(1+cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^(1/2)/d/e^(5/2
)/(1+cos(d*x+c)+sin(d*x+c))-2*a^2*arctan(sin(d*x+c)*e^(1/2)/(e*cos(d*x+c))^
(1/2)/(1+cos(d*x+c))^(1/2))*(1+cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^(1/2)/d/e
^(5/2)/(1+cos(d*x+c)+sin(d*x+c))
```

Rubi [A]
time = 0.20, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2755, 2756, 2854, 209, 2912, 65, 221}

$$-\frac{2a^2\sqrt{\cos(c+dx)+1}\,\sqrt{a\sin(c+dx)+a}\,\text{ArcTan}\left(\frac{\sqrt{e}\,\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\,\sqrt{e\cos(c+dx)}}\right)}{de^{5/2}(\sin(c+dx)+\cos(c+dx)+1)}+\frac{2a^2\sqrt{\cos(c+dx)+1}\,\sqrt{a\sin(c+dx)+a}\,\sinh^{-1}\left(\frac{\sqrt{e\cos(c+dx)}}{\sqrt{e}}\right)}{de^{5/2}(\sin(c+dx)+\cos(c+dx)+1)}+\frac{4a(a\sin(c+dx)+a)^{3/2}}{3de(e\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^(5/2)/(e*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*a^2*ArcSinh[Sqrt[e*Cos[c + d*x]]/Sqrt[e]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[a
+ a*Sin[c + d*x]])/(d*e^(5/2)*(1 + Cos[c + d*x] + Sin[c + d*x])) - (2*a^2*A
rcTan[(Sqrt[e]*Sin[c + d*x])/(Sqrt[e*Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]])]
*Sqrt[1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(d*e^(5/2)*(1 + Cos[c + d
*x] + Sin[c + d*x])) + (4*a*(a + a*Sin[c + d*x])^(3/2))/(3*d*e*(e*Cos[c + d
*x])^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 2755

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[-2*b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(p + 1))), x] + Dist[b^2*((2*m + p - 1)/(g^2*(p + 1))),
Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[
{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && Inte
gersQ[2*m, 2*p]
```

Rule 2756

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[cos[(e_.) + (f_.)*(x_)]
*(g_.)], x_Symbol] :> Dist[a*Sqrt[1 + Cos[e + f*x]]*(Sqrt[a + b*Sin[e + f*x
]]/(a + a*Cos[e + f*x] + b*Sin[e + f*x])), Int[Sqrt[1 + Cos[e + f*x]]/Sqrt[
g*Cos[e + f*x]], x], x] + Dist[b*Sqrt[1 + Cos[e + f*x]]*(Sqrt[a + b*Sin[e +
 f*x]]/(a + a*Cos[e + f*x] + b*Sin[e + f*x])), Int[Sin[e + f*x]/(Sqrt[g*Cos
[e + f*x]]*Sqrt[1 + Cos[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, g}, x] &&
EqQ[a^2 - b^2, 0]
```

Rule 2854

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b + d*x^2), x], x
, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]))], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(a+a\sin(c+dx))^{5/2}}{(e\cos(c+dx))^{5/2}}\,dx = \frac{4a(a+a\sin(c+dx))^{3/2}}{3de(e\cos(c+dx))^{3/2}} - \frac{a^2\int \frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{e\cos(c+dx)}}\,dx}{e^2}$$

$$= \frac{4a(a+a\sin(c+dx))^{3/2}}{3de(e\cos(c+dx))^{3/2}} - \frac{\left(a^3\sqrt{1+\cos(c+dx)}\,\sqrt{a+a\sin(c+dx)}\right)\int \sqrt{}}{e^2(a+a\cos(c+dx)+a\sin(c+}$$

$$= \frac{4a(a+a\sin(c+dx))^{3/2}}{3de(e\cos(c+dx))^{3/2}} + \frac{\left(a^3\sqrt{1+\cos(c+dx)}\,\sqrt{a+a\sin(c+dx)}\right)\text{Sub}}{de^2(a+a\cos(c+dx)+}$$

$$= \frac{4a(a+a\sin(c+dx))^{3/2}}{3de(e\cos(c+dx))^{3/2}} - \frac{2a^3\tan^{-1}\left(\frac{\sqrt{e}\,\sin(c+dx)}{\sqrt{e\cos(c+dx)}\,\sqrt{1+\cos(c+dx)}}\right)}{de^{5/2}(a+a\cos(c+dx)}$$

$$= \frac{4a(a+a\sin(c+dx))^{3/2}}{3de(e\cos(c+dx))^{3/2}} + \frac{2a^3\sinh^{-1}\left(\frac{\sqrt{e\cos(c+dx)}}{\sqrt{e}}\right)\sqrt{1+\cos(c+dx}}{de^{5/2}(a+a\cos(c+dx)+a\sin(c}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.19, size = 77, normalized size = 0.38

$$\frac{4\ 2^{3/4}\,{}_2F_1\left(-\frac{3}{4},-\frac{3}{4};\frac{1}{4};\frac{1}{2}(1-\sin(c+dx))\right)(a(1+\sin(c+dx)))^{5/2}}{3de(e\cos(c+dx))^{3/2}(1+\sin(c+dx))^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^(5/2)/(e*Cos[c + d*x])^(5/2),x]
```

```
[Out] (4*2^(3/4)*Hypergeometric2F1[-3/4, -3/4, 1/4, (1 - Sin[c + d*x])/2]*(a*(1 +
 Sin[c + d*x]))^(5/2))/(3*d*e*(e*Cos[c + d*x])^(3/2)*(1 + Sin[c + d*x])^(7/
4))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 544 vs. 2(176) = 352.
time = 0.17, size = 545, normalized size = 2.67

method	result

default	$-\frac{\left(3\sqrt{2}\,\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2}}{2}\right)\sin(dx+c)\cos(dx+c)-3\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\sin($

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/d*(3*2^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*
sin(d*x+c)*cos(d*x+c)-3*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*sin(d*x+c)*cos(d*x+c)-3*2^(1/2)*arctan
(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*cos(d*x+c)^2+3*2^(1/2)*a
rctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/
2))*cos(d*x+c)^2-6*2^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
2^(1/2))*sin(d*x+c)+6*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*sin(d*x+c)-3*2^(1/2)*arctan(1/2*(-2*cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*cos(d*x+c)+3*2^(1/2)*arctanh(1/2*(-2*
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*cos(d*x+c)-
4*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)+6*2^(1/2)*arct
an(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))-6*2^(1/2)*arctanh(1/2*
(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2)))*(a*(1+
sin(d*x+c)))^(5/2)/(1+sin(d*x+c))/sin(d*x+c)/(e*cos(d*x+c))^(5/2)/(-2*cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(5/2),x, algorithm="maxima"
)
```

```
[Out] e^(-5/2)*integrate((a*sin(d*x + c) + a)^(5/2)/cos(d*x + c)^(5/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3497 vs. $2(160) = 320$.
time = 178.44, size = 3497, normalized size = 17.14

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(5/2),x, algorithm="fricas"
)
```

[Out] -1/12*(12*(a^10/d^4)^(1/4)*(sqrt(2)*d*cos(d*x + c)^2*e^(5/2) + 2*sqrt(2)*d*e^(5/2)*sin(d*x + c) - 2*sqrt(2)*d*e^(5/2))*arctan(-1/4*(sqrt(2)*((sqrt(2)*d^3*cos(d*x + c)^6*e^(15/2) + 5*sqrt(2)*d^3*cos(d*x + c)^5*e^(15/2) - 8*sqrt(2)*d^3*cos(d*x + c)^4*e^(15/2) - 20*sqrt(2)*d^3*cos(d*x + c)^3*e^(15/2) + 8*sqrt(2)*d^3*cos(d*x + c)^2*e^(15/2) + 16*sqrt(2)*d^3*cos(d*x + c)*e^(15/2) + (sqrt(2)*d^3*cos(d*x + c)^5*e^(15/2) - 4*sqrt(2)*d^3*cos(d*x + c)^4*e^(15/2) - 12*sqrt(2)*d^3*cos(d*x + c)^3*e^(15/2) + 8*sqrt(2)*d^3*cos(d*x + c)^2*e^(15/2) + 16*sqrt(2)*d^3*cos(d*x + c)*e^(15/2))*sin(d*x + c))*(a^10/d^4)^(3/4)*e^(-15/2) + (sqrt(2)*a^5*d*cos(d*x + c)^6*e^(5/2) - 3*sqrt(2)*a^5*d*cos(d*x + c)^5*e^(5/2) - 8*sqrt(2)*a^5*d*cos(d*x + c)^4*e^(5/2) + 4*sqrt(2)*a^5*d*cos(d*x + c)^3*e^(5/2) + 8*sqrt(2)*a^5*d*cos(d*x + c)^2*e^(5/2) - (sqrt(2)*a^5*d*cos(d*x + c)^5*e^(5/2) + 4*sqrt(2)*a^5*d*cos(d*x + c)^4*e^(5/2) - 4*sqrt(2)*a^5*d*cos(d*x + c)^3*e^(5/2) - 8*sqrt(2)*a^5*d*cos(d*x + c)^2*e^(5/2))*sin(d*x + c))*(a^10/d^4)^(1/4)*e^(-5/2) - (a^7*cos(d*x + c)^4 - 3*a^7*cos(d*x + c)^3 - 8*a^7*cos(d*x + c)^2 + 4*a^7*cos(d*x + c) + 8*a^7 + (2*a^2*d^2*cos(d*x + c)^5*e^5 - 5*a^2*d^2*cos(d*x + c)^4*e^5 - 19*a^2*d^2*cos(d*x + c)^3*e^5 + 20*a^2*d^2*cos(d*x + c)*e^5 + 8*a^2*d^2*e^5 - (2*a^2*d^2*cos(d*x + c)^4*e^5 + 9*a^2*d^2*cos(d*x + c)^3*e^5 - 4*a^2*d^2*cos(d*x + c)^2*e^5 - 20*a^2*d^2*cos(d*x + c)*e^5 - 8*a^2*d^2*e^5)*sin(d*x + c))*sqrt(a^10/d^4)*e^(-5) - (a^7*cos(d*x + c)^3 + 4*a^7*cos(d*x + c)^2 - 4*a^7*cos(d*x + c) - 8*a^7)*sin(d*x + c))*sqrt(a*sin(d*x + c) + a)*sqrt(cos(d*x + c)))*sqrt((2*a^15*cos(d*x + c)*sin(d*x + c) + 2*a^15*cos(d*x + c) + (a^10*d^2*e^5*sin(d*x + c) + a^10*d^2*e^5)*sqrt(a^10/d^4)*e^(-5) + (sqrt(2)*(a^10/d^4)^(3/4)*a^7*d^3*cos(d*x + c) + (sqrt(2)*a^12*d*e^(5/2)*sin(d*x + c) + sqrt(2)*a^12*d*e^(5/2))*(a^10/d^4)^(1/4)*e^(-5/2))*sqrt(a*sin(d*x + c) + a)*sqrt(cos(d*x + c)))/(sin(d*x + c) + 1)) - ((7*sqrt(2)*a^7*d^3*cos(d*x + c)^4*e^(15/2) + 3*sqrt(2)*a^7*d^3*cos(d*x + c)^3*e^(15/2) - 16*sqrt(2)*a^7*d^3*cos(d*x + c)^2*e^(15/2) - 4*sqrt(2)*a^7*d^3*cos(d*x + c)*e^(15/2) + 8*sqrt(2)*a^7*d^3*e^(15/2) + (2*sqrt(2)*a^7*d^3*cos(d*x + c)^4*e^(15/2) + sqrt(2)*a^7*d^3*cos(d*x + c)^3*e^(15/2) - 12*sqrt(2)*a^7*d^3*cos(d*x + c)^2*e^(15/2) - 4*sqrt(2)*a^7*d^3*cos(d*x + c)*e^(15/2) + 8*sqrt(2)*a^7*d^3*e^(15/2))*sin(d*x + c))*(a^10/d^4)^(3/4)*e^(-15/2) + (2*sqrt(2)*a^12*d*cos(d*x + c)^5*e^(5/2) + sqrt(2)*a^12*d*cos(d*x + c)^4*e^(5/2) - 13*sqrt(2)*a^12*d*cos(d*x + c)^3*e^(5/2) - 8*sqrt(2)*a^12*d*cos(d*x + c)^2*e^(5/2) + 12*sqrt(2)*a^12*d*cos(d*x + c)*e^(5/2) + 8*sqrt(2)*a^12*d*e^(5/2) - (7*sqrt(2)*a^12*d*cos(d*x + c)^3*e^(5/2) + 4*sqrt(2)*a^12*d*cos(d*x + c)^2*e^(5/2) - 12*sqrt(2)*a^12*d*cos(d*x + c)*e^(5/2) - 8*sqrt(2)*a^12*d*e^(5/2))*sin(d*x + c))*(a^10/d^4)^(1/4)*e^(-5/2))*sqrt(a*sin(d*x + c) + a)*sqrt(cos(d*x + c)))/(a^15*cos(d*x + c)^6 + a^15*cos(d*x + c)^5 - 8*a^15*cos(d*x + c)^4 - 8*a^15*cos(d*x + c)^3 + 8*a^15*cos(d*x + c)^2 + 8*a^15*cos(d*x + c) - 4*(a^15*cos(d*x + c)^4 + a^15*cos(d*x + c)^3 - 2*a^15*cos(d*x + c)^2 - 2*a^15*cos(d*x + c))*sin(d*x + c)))*e^(-5/2) - 12*(a^10/d^4)^(1/4)*(sqrt(2)*d*cos(d*x + c)^2*e^(5/2) + 2*sqrt(2)*d*e^(5/2)*sin(d*x + c) - 2*sqrt(2)*d*e^(5/2))*arctan(1/4*(sqrt(2)*((sqrt(2)*d^3*cos(d*x + c)^6*e^(15/2) + 5*sqrt(2)*d^3*cos(d*x + c)^5*e^(15/2) - 8*sqrt(2)*d^3*cos(d*x + c)^4*e^(15/2) - 20*sqrt(2)*d^3*cos(d*x + c)^3

```
*e^(15/2) + 8*sqrt(2)*d^3*cos(d*x + c)^2*e^(15/2) + 16*sqrt(2)*d^3*cos(d*x
+ c)*e^(15/2) + (sqrt(2)*d^3*cos(d*x + c)^5*e^(15/2) - 4*sqrt(2)*d^3*cos(d*
x + c)^4*e^(15/2) - 12*sqrt(2)*d^3*cos(d*x + c)^3*e^(15/2) + 8*sqrt(2)*d^3*
cos(d*x + c)^2*e^(15/2) + 16*sqrt(2)*d^3*cos(d*x + c)*e^(15/2))*sin(d*x + c
))*(a^10/d^4)^(3/4)*e^(-15/2) + (sqrt(2)*a^5*d*cos(d*x + c)^6*e^(5/2) - 3*s
qrt(2)*a^5*d*cos(d*x + c)^5*e^(5/2) - 8*sqrt(2)*a^5*d*cos(d*x + c)^4*e^(5/2
) + 4*sqrt(2)*a^5*d*cos(d*x + c)^3*e^(5/2) + 8*sqrt(2)*a^5*d*cos(d*x + c)^2
*e^(5/2) - (sqrt(2)*a^5*d*cos(d*x + c)^5*e^(5/2) + 4*sqrt(2)*a^5*d*cos(d*x
+ c)^4*e^(5/2) - 4*sqrt(2)*a^5*d*cos(d*x + c)^3*e^(5/2) - 8*sqrt(2)*a^5*d*c
os(d*x + c)^2*e^(5/2))*sin(d*x + c))*(a^10/d^4)^(1/4)*e^(-5/2) + (a^7*cos(d
*x + c)^4 - 3*a^7*cos(d*x + c)^3 - 8*a^7*cos(d*x + c)^2 + 4*a^7*cos(d*x + c
) + 8*a^7 + (2*a^2*d^2*cos(d*x + c)^5*e^5 - 5*a^2*d^2*cos(d*x + c)^4*e^5 -
19*a^2*d^2*cos(d*x + c)^3*e^5 + 20*a^2*d^2*cos(d*x + c)*e^5 + 8*a^2*d^2*e^5
 - (2*a^2*d^2*cos(d*x + c)^4*e^5 + 9*a^2*d^2*cos(d*x + c)^3*e^5 - 4*a^2*d^2
*cos(d*x + c)^2*e^5 - 20*a^2*d^2*cos(d*x + c)*e^5 - 8*a^2*d^2*e^5)*sin(d*x
+ c))*sqrt(a^10/d^4)*e^(-5) - (a^7*cos(d*x + c)^3 + 4*a^7*cos(d*x + c)^2 -
4*a^7*cos(d*x + c) - 8*a^7)*sin(d*x + c))*sqrt(a*sin(d*x + c) + a)*sqrt(cos
(d*x + c)))*sqrt((2*a^15*cos(d*x + c)*sin(d*x + c) + 2*a^15*cos(d*x + c) +
(a^10*d^2*e^5*sin(d*x + c) + a^10*d^2*e^5)*sqrt(a^10/d^4)*e^(-5) - (sqrt(2)
*(a^10/d^4)^(3/4)*a^7*d^3*cos(d*x + c) + (sqrt(2)*a^12*d*e^(5/2)*sin(d*x +
c) + sqrt(2)*a^12*d*e^(5/2))*(a^10/d^4)^(1/4)*e^(-5/2))*sqrt(a*sin(d*x + c)
 + a)*sqrt(cos(d*x + c)))/(sin(d*x + c) + 1)) -...
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**(5/2)/(e*cos(d*x+c))**(5/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(5/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+a\,\sin(c+d\,x))^{5/2}}{(e\,\cos(c+d\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(5/2)/(e*cos(c + d*x))^(5/2),x)
[Out] int((a + a*sin(c + d*x))^(5/2)/(e*cos(c + d*x))^(5/2), x)
```

3.294 $\int \frac{(a+a\sin(c+dx))^{5/2}}{(e\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=36

$$\frac{2(a+a\sin(c+dx))^{5/2}}{5de(e\cos(c+dx))^{5/2}}$$

```
[Out] 2/5*(a+a*sin(d*x+c))^(5/2)/d/e/(e*cos(d*x+c))^(5/2)
```

Rubi [A]
time = 0.05, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$, Rules used = {2750}

$$\frac{2(a\sin(c+dx)+a)^{5/2}}{5de(e\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^(5/2)/(e*Cos[c + d*x])^(7/2),x]
```

```
[Out] (2*(a + a*Sin[c + d*x])^(5/2))/(5*d*e*(e*Cos[c + d*x])^(5/2))
```

Rule 2750

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(c+dx))^{5/2}}{(e\cos(c+dx))^{7/2}}\,dx = \frac{2(a+a\sin(c+dx))^{5/2}}{5de(e\cos(c+dx))^{5/2}}$$

Mathematica [A]
time = 0.13, size = 36, normalized size = 1.00

$$\frac{2(a(1+\sin(c+dx)))^{5/2}}{5de(e\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^(5/2)/(e*Cos[c + d*x])^(7/2),x]
```

```
[Out] (2*(a*(1 + Sin[c + d*x]))^(5/2))/(5*d*e*(e*Cos[c + d*x])^(5/2))
```

Maple [A]
time = 0.16, size = 34, normalized size = 0.94

method	result	size
default	$\frac{2\cos(dx+c)(a(1+\sin(dx+c)))^{\frac{5}{2}}}{5d(e\cos(dx+c))^{\frac{7}{2}}}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5/d*cos(d*x+c)*(a*(1+sin(d*x+c)))^(5/2)/(e*cos(d*x+c))^(7/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 77 vs. 2(27) = 54.
time = 0.54, size = 77, normalized size = 2.14

$$\frac{2\left(a^{\frac{5}{2}}-\frac{a^{\frac{5}{2}}\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{3}{2}}e^{\left(-\frac{7}{2}\right)}}{5\,d\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(7/2),x, algorithm="maxima"
)
```

```
[Out] 2/5*(a^(5/2) - a^(5/2)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(sin(d*x + c)/(
cos(d*x + c) + 1) + 1)^(3/2)*e^(-7/2)/(d*(-sin(d*x + c)/(cos(d*x + c) + 1)
+ 1)^(7/2))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 100 vs. 2(27) = 54.
time = 0.35, size = 100, normalized size = 2.78

$$-\frac{2\left(a^2\cos(dx+c)+a^2\sin(dx+c)+a^2\right)\sqrt{a\sin(dx+c)+a}\,\sqrt{\cos(dx+c)}}{5\left(d\cos(dx+c)^2e^{\frac{7}{2}}-d\cos(dx+c)\,e^{\frac{7}{2}}-2\,de^{\frac{7}{2}}+\left(d\cos(dx+c)\,e^{\frac{7}{2}}+2\,de^{\frac{7}{2}}\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(7/2),x, algorithm="fricas"
)
```

```
[Out] -2/5*(a^2*cos(d*x + c) + a^2*sin(d*x + c) + a^2)*sqrt(a*sin(d*x + c) + a)*s
qrt(cos(d*x + c))/(d*cos(d*x + c)^2*e^(7/2) - d*cos(d*x + c)*e^(7/2) - 2*d*
e^(7/2) + (d*cos(d*x + c)*e^(7/2) + 2*d*e^(7/2))*sin(d*x + c))
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**(5/2)/(e*cos(d*x+c))**(7/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 8570 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(7/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [**B**]
time = 6.06, size = 65, normalized size = 1.81

$$-\frac{2\,a^2\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)\,\sqrt{a\,\left(\sin\left(c+d\,x\right)+1\right)}}{5\,d\,e^3\,\sqrt{e\,\cos\left(c+d\,x\right)}\,\left(4\,\sin\left(c+d\,x\right)+\cos\left(2\,c+2\,d\,x\right)-3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(5/2)/(e*cos(c + d*x))^(7/2),x)
[Out] -(2*a^2*(cos(2*c + 2*d*x) + 1)*(a*(sin(c + d*x) + 1))^(1/2))/(5*d*e^3*(e*co
s(c + d*x))^(1/2)*(4*sin(c + d*x) + cos(2*c + 2*d*x) - 3))
```

3.295 $\int \frac{(a+a\sin(c+dx))^{5/2}}{(e\cos(c+dx))^{9/2}}\,dx$

Optimal. Leaf size=76

$$\frac{2(a+a\sin(c+dx))^{5/2}}{3de(e\cos(c+dx))^{7/2}}-\frac{4(a+a\sin(c+dx))^{7/2}}{21ade(e\cos(c+dx))^{7/2}}$$

```
[Out] 2/3*(a+a*sin(d*x+c))^(5/2)/d/e/(e*cos(d*x+c))^(7/2)-4/21*(a+a*sin(d*x+c))^(
7/2)/a/d/e/(e*cos(d*x+c))^(7/2)
```

Rubi [A]
time = 0.10, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}=0.074$, Rules used = {2751, 2750}

$$\frac{2(a\sin(c+dx)+a)^{5/2}}{3de(e\cos(c+dx))^{7/2}}-\frac{4(a\sin(c+dx)+a)^{7/2}}{21ade(e\cos(c+dx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^(5/2)/(e*Cos[c + d*x])^(9/2),x]
```

```
[Out] (2*(a + a*Sin[c + d*x])^(5/2))/(3*d*e*(e*Cos[c + d*x])^(7/2)) - (4*(a + a*S
in[c + d*x])^(7/2))/(21*a*d*e*(e*Cos[c + d*x])^(7/2))
```

Rule 2750

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(c+dx))^{5/2}}{(e\cos(c+dx))^{9/2}}\,dx = \frac{2(a+a\sin(c+dx))^{5/2}}{3de(e\cos(c+dx))^{7/2}} - \frac{2\int \frac{(a+a\sin(c+dx))^{7/2}}{(e\cos(c+dx))^{9/2}}\,dx}{3a}$$
$$= \frac{2(a+a\sin(c+dx))^{5/2}}{3de(e\cos(c+dx))^{7/2}} - \frac{4(a+a\sin(c+dx))^{7/2}}{21ade(e\cos(c+dx))^{7/2}}$$

Mathematica [A]
time = 0.16, size = 54, normalized size = 0.71

$$-\frac{2\sqrt{e\cos(c+dx)}\,\sec^4(c+dx)(a(1+\sin(c+dx)))^{5/2}(-5+2\sin(c+dx))}{21de^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^(5/2)/(e*Cos[c + d*x])^(9/2),x]
[Out] (-2*Sqrt[e*Cos[c + d*x]]*Sec[c + d*x]^4*(a*(1 + Sin[c + d*x]))^(5/2)*(-5 +
2*Sin[c + d*x]))/(21*d*e^5)
```

Maple [A]
time = 0.18, size = 44, normalized size = 0.58

method	result	size
default	$-\frac{2(2\sin(dx+c)-5)\cos(dx+c)(a(1+\sin(dx+c)))^{\frac{5}{2}}}{21d(e\cos(dx+c))^{\frac{9}{2}}}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(9/2),x,method=_RETURNVERBOSE)
[Out] -2/21/d*(2*sin(d*x+c)-5)*cos(d*x+c)*(a*(1+sin(d*x+c)))^(5/2)/(e*cos(d*x+c))
^(9/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 189 vs. 2(58) = 116.
time = 0.55, size = 189, normalized size = 2.49

$$\frac{2\left(5a^{\frac{5}{2}} - \frac{4a^{\frac{5}{2}}\sin(dx+c)}{\cos(dx+c)+1} + \frac{4a^{\frac{5}{2}}\sin(dx+c)^3}{(\cos(dx+c)+1)^3} - \frac{5a^{\frac{5}{2}}\sin(dx+c)^4}{(\cos(dx+c)+1)^4}\right)\sqrt{\frac{\sin(dx+c)}{\cos(dx+c)+1}+1}\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^2 e^{\left(-\frac{9}{2}\right)}}{21d\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{9}{2}}\left(\frac{2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(9/2),x, algorithm="maxima"
)
```

```
[Out] 2/21*(5*a^(5/2) - 4*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) + 4*a^(5/2)*sin
(d*x + c)^3/(cos(d*x + c) + 1)^3 - 5*a^(5/2)*sin(d*x + c)^4/(cos(d*x + c) +
 1)^4)*sqrt(sin(d*x + c)/(cos(d*x + c) + 1) + 1)*(sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 + 1)^2*e^(-9/2)/(d*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*
(2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + sin(d*x + c)^4/(cos(d*x + c) + 1)^
4 + 1))
```

Fricas [**A**]
time = 0.35, size = 70, normalized size = 0.92

$$\frac{2\left(2\,a^2\sin\left(dx+c\right)-5\,a^2\right)\sqrt{a\sin\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}}{21\left(d\cos\left(dx+c\right)^2e^{\frac{9}{2}}+2\,de^{\frac{9}{2}}\sin\left(dx+c\right)-2\,de^{\frac{9}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(9/2),x, algorithm="fricas"
)
```

```
[Out] 2/21*(2*a^2*sin(d*x + c) - 5*a^2)*sqrt(a*sin(d*x + c) + a)*sqrt(cos(d*x + c
))/(d*cos(d*x + c)^2*e^(9/2) + 2*d*e^(9/2)*sin(d*x + c) - 2*d*e^(9/2))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**(5/2)/(e*cos(d*x+c))**(9/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(9/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 6.34, size = 96, normalized size = 1.26

$$\frac{4\,a^2\,\sqrt{a\,\left(\sin\left(c+d\,x\right)+1\right)}\,\left(\cos\left(3\,c+3\,d\,x\right)-11\,\cos\left(c+d\,x\right)+7\,\sin\left(2\,c+2\,d\,x\right)\right)}{21\,d\,e^4\,\sqrt{e\,\cos\left(c+d\,x\right)}\,\left(15\,\sin\left(c+d\,x\right)+6\,\cos\left(2\,c+2\,d\,x\right)-\sin\left(3\,c+3\,d\,x\right)-10\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(5/2)/(e*cos(c + d*x))^(9/2),x)
```

```
[Out] (4*a^2*(a*(sin(c + d*x) + 1))^(1/2)*(cos(3*c + 3*d*x) - 11*cos(c + d*x) + 7
*sin(2*c + 2*d*x)))/(21*d*e^4*(e*cos(c + d*x))^(1/2)*(15*sin(c + d*x) + 6*c
os(2*c + 2*d*x) - sin(3*c + 3*d*x) - 10))
```

3.296 $\int \frac{(a+a\sin(c+dx))^{5/2}}{(e\cos(c+dx))^{11/2}}\,dx$

Optimal. Leaf size=113

$$\frac{2(a+a\sin(c+dx))^{5/2}}{de(e\cos(c+dx))^{9/2}}-\frac{8(a+a\sin(c+dx))^{7/2}}{5ade(e\cos(c+dx))^{9/2}}+\frac{16(a+a\sin(c+dx))^{9/2}}{45a^2de(e\cos(c+dx))^{9/2}}$$

```
[Out] 2*(a+a*sin(d*x+c))^(5/2)/d/e/(e*cos(d*x+c))^(9/2)-8/5*(a+a*sin(d*x+c))^(7/2
)/a/d/e/(e*cos(d*x+c))^(9/2)+16/45*(a+a*sin(d*x+c))^(9/2)/a^2/d/e/(e*cos(d*
x+c))^(9/2)
```

Rubi [A]
time = 0.15, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2751, 2750}

$$\frac{16(a\sin(c+dx)+a)^{9/2}}{45a^2de(e\cos(c+dx))^{9/2}}-\frac{8(a\sin(c+dx)+a)^{7/2}}{5ade(e\cos(c+dx))^{9/2}}+\frac{2(a\sin(c+dx)+a)^{5/2}}{de(e\cos(c+dx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^(5/2)/(e*Cos[c + d*x])^(11/2),x]
```

```
[Out] (2*(a + a*Sin[c + d*x])^(5/2))/(d*e*(e*Cos[c + d*x])^(9/2)) - (8*(a + a*Sin
[c + d*x])^(7/2))/(5*a*d*e*(e*Cos[c + d*x])^(9/2)) + (16*(a + a*Sin[c + d*x
])^(9/2))/(45*a^2*d*e*(e*Cos[c + d*x])^(9/2))
```

Rule 2750

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(c+dx))^{5/2}}{(e\cos(c+dx))^{11/2}}\,dx = \frac{2(a+a\sin(c+dx))^{5/2}}{de(e\cos(c+dx))^{9/2}} - \frac{4\int \frac{(a+a\sin(c+dx))^{7/2}}{(e\cos(c+dx))^{11/2}}\,dx}{a}$$
$$= \frac{2(a+a\sin(c+dx))^{5/2}}{de(e\cos(c+dx))^{9/2}} - \frac{8(a+a\sin(c+dx))^{7/2}}{5ade(e\cos(c+dx))^{9/2}} + \frac{8\int \frac{(a+a\sin(c+dx))^{9/2}}{(e\cos(c+dx))^{11/2}}\,dx}{5a^2}$$
$$= \frac{2(a+a\sin(c+dx))^{5/2}}{de(e\cos(c+dx))^{9/2}} - \frac{8(a+a\sin(c+dx))^{7/2}}{5ade(e\cos(c+dx))^{9/2}} + \frac{16(a+a\sin(c+dx))^{9/2}}{45a^2de(e\cos(c+dx))^{9/2}}$$

Mathematica [A]
time = 0.19, size = 64, normalized size = 0.57

$$\frac{2\sqrt{e\cos(c+dx)}\,\sec^5(c+dx)(a(1+\sin(c+dx)))^{5/2}\left(17-20\sin(c+dx)+8\sin^2(c+dx)\right)}{45de^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^(5/2)/(e*Cos[c + d*x])^(11/2),x]
```

```
[Out] (2*Sqrt[e*Cos[c + d*x]]*Sec[c + d*x]^5*(a*(1 + Sin[c + d*x]))^(5/2)*(17 - 2
0*Sin[c + d*x] + 8*Sin[c + d*x]^2))/(45*d*e^6)
```

Maple [A]
time = 0.16, size = 54, normalized size = 0.48

method	result	size
default	$-\frac{2\left(8\left(\cos^2(dx+c)\right)+20\sin(dx+c)-25\right)\cos(dx+c)\left(a(1+\sin(dx+c))\right)^{\frac{5}{2}}}{45d\left(e\cos(dx+c)\right)^{\frac{11}{2}}}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(11/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/45/d*(8*cos(d*x+c)^2+20*sin(d*x+c)-25)*cos(d*x+c)*(a*(1+sin(d*x+c)))^(5/
2)/(e*cos(d*x+c))^(11/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 255 vs. 2(88) = 176.
time = 0.54, size = 255, normalized size = 2.26

$$\frac{2\left(17\,a^{\frac{5}{2}} - \frac{40\,a^{\frac{5}{2}}\sin(dx+c)}{\cos(dx+c)+1} + \frac{49\,a^{\frac{5}{2}}\sin(dx+c)^2}{(\cos(dx+c)+1)^2} - \frac{49\,a^{\frac{5}{2}}\sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{40\,a^{\frac{5}{2}}\sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{17\,a^{\frac{5}{2}}\sin(dx+c)^6}{(\cos(dx+c)+1)^6}\right)\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^3 e^{\left(-\frac{11}{2}\right)}}{45\,d\sqrt{\frac{\sin(dx+c)}{\cos(dx+c)+1}+1}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{11}{2}}\left(\frac{3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(11/2),x, algorithm="maxima
")
```

```
[Out] 2/45*(17*a^(5/2) - 40*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) + 49*a^(5/2)*
sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 49*a^(5/2)*sin(d*x + c)^4/(cos(d*x +
c) + 1)^4 + 40*a^(5/2)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 17*a^(5/2)*sin
(d*x + c)^6/(cos(d*x + c) + 1)^6)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)
^3*e^(-11/2)/(d*sqrt(sin(d*x + c)/(cos(d*x + c) + 1) + 1)*(-sin(d*x + c)/(c
os(d*x + c) + 1) + 1)^(11/2)*(3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*sin
(d*x + c)^4/(cos(d*x + c) + 1)^4 + sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 1)
)
```

Fricas [A]
time = 0.35, size = 95, normalized size = 0.84

$$\frac{2\left(8\,a^2\cos\left(dx+c\right)^2+20\,a^2\sin\left(dx+c\right)-25\,a^2\right)\sqrt{a\sin\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}}{45\left(d\cos\left(dx+c\right)^3e^{\frac{11}{2}}+2\,d\cos\left(dx+c\right)e^{\frac{11}{2}}\sin\left(dx+c\right)-2\,d\cos\left(dx+c\right)e^{\frac{11}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(11/2),x, algorithm="fricas
")
```

```
[Out] 2/45*(8*a^2*cos(d*x + c)^2 + 20*a^2*sin(d*x + c) - 25*a^2)*sqrt(a*sin(d*x +
 c) + a)*sqrt(cos(d*x + c))/(d*cos(d*x + c)^3*e^(11/2) + 2*d*cos(d*x + c)*e
^(11/2)*sin(d*x + c) - 2*d*cos(d*x + c)*e^(11/2))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**(5/2)/(e*cos(d*x+c))**(11/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(11/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 6.69, size = 119, normalized size = 1.05

$$\frac{8\,a^2\,\sqrt{a\,\left(\sin\left(c+d\,x\right)+1\right)}\,\left(2\,\cos\left(4\,c+4\,d\,x\right)-73\,\cos\left(2\,c+2\,d\,x\right)-162\,\sin\left(c+d\,x\right)+18\,\sin\left(3\,c+3\,d\,x\right)+105\right)}{45\,d\,e^5\,\sqrt{e\,\cos\left(c+d\,x\right)}\,\left(\cos\left(4\,c+4\,d\,x\right)-28\,\cos\left(2\,c+2\,d\,x\right)-56\,\sin\left(c+d\,x\right)+8\,\sin\left(3\,c+3\,d\,x\right)+35\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(5/2)/(e*cos(c + d*x))^(11/2),x)
```

```
[Out] (8*a^2*(a*(sin(c + d*x) + 1))^(1/2)*(2*cos(4*c + 4*d*x) - 73*cos(2*c + 2*d*
x) - 162*sin(c + d*x) + 18*sin(3*c + 3*d*x) + 105))/(45*d*e^5*(e*cos(c + d*
x))^(1/2)*(cos(4*c + 4*d*x) - 28*cos(2*c + 2*d*x) - 56*sin(c + d*x) + 8*sin
(3*c + 3*d*x) + 35))
```

3.297 $\int \frac{(a+a\sin(c+dx))^{5/2}}{(e\cos(c+dx))^{13/2}}\,dx$

Optimal. Leaf size=150

$$-\frac{2(a+a\sin(c+dx))^{5/2}}{de(e\cos(c+dx))^{11/2}}+\frac{4(a+a\sin(c+dx))^{7/2}}{ade(e\cos(c+dx))^{11/2}}-\frac{16(a+a\sin(c+dx))^{9/2}}{7a^2de(e\cos(c+dx))^{11/2}}+\frac{32(a+a\sin(c+dx))^{11/2}}{77a^3de(e\cos(c+dx))^{11/2}}$$

```
[Out] -2*(a+a*sin(d*x+c))^(5/2)/d/e/(e*cos(d*x+c))^(11/2)+4*(a+a*sin(d*x+c))^(7/2
)/a/d/e/(e*cos(d*x+c))^(11/2)-16/7*(a+a*sin(d*x+c))^(9/2)/a^2/d/e/(e*cos(d*
x+c))^(11/2)+32/77*(a+a*sin(d*x+c))^(11/2)/a^3/d/e/(e*cos(d*x+c))^(11/2)
```

Rubi [A]
time = 0.20, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2751, 2750}

$$\frac{32(a\sin(c+dx)+a)^{11/2}}{77a^3de(e\cos(c+dx))^{11/2}}-\frac{16(a\sin(c+dx)+a)^{9/2}}{7a^2de(e\cos(c+dx))^{11/2}}+\frac{4(a\sin(c+dx)+a)^{7/2}}{ade(e\cos(c+dx))^{11/2}}-\frac{2(a\sin(c+dx)+a)^{5/2}}{de(e\cos(c+dx))^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^(5/2)/(e*Cos[c + d*x])^(13/2),x]
```

```
[Out] (-2*(a + a*Sin[c + d*x])^(5/2))/(d*e*(e*Cos[c + d*x])^(11/2)) + (4*(a + a*S
in[c + d*x])^(7/2))/(a*d*e*(e*Cos[c + d*x])^(11/2)) - (16*(a + a*Sin[c + d*
x])^(9/2))/(7*a^2*d*e*(e*Cos[c + d*x])^(11/2)) + (32*(a + a*Sin[c + d*x])^(
11/2))/(77*a^3*d*e*(e*Cos[c + d*x])^(11/2))
```

Rule 2750

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+a\sin(c+dx))^{5/2}}{(e\cos(c+dx))^{13/2}}\,dx &= -\frac{2(a+a\sin(c+dx))^{5/2}}{de(e\cos(c+dx))^{11/2}}+\frac{6\int \frac{(a+a\sin(c+dx))^{7/2}}{(e\cos(c+dx))^{13/2}}\,dx}{a} \\
&= -\frac{2(a+a\sin(c+dx))^{5/2}}{de(e\cos(c+dx))^{11/2}}+\frac{4(a+a\sin(c+dx))^{7/2}}{ade(e\cos(c+dx))^{11/2}}-\frac{8\int \frac{(a+a\sin(c+dx))^{9/2}}{(e\cos(c+dx))^{13/2}}\,dx}{a^2} \\
&= -\frac{2(a+a\sin(c+dx))^{5/2}}{de(e\cos(c+dx))^{11/2}}+\frac{4(a+a\sin(c+dx))^{7/2}}{ade(e\cos(c+dx))^{11/2}}-\frac{16(a+a\sin(c+dx))^{9/2}}{7a^2de(e\cos(c+dx))^{11/2}} \\
&= -\frac{2(a+a\sin(c+dx))^{5/2}}{de(e\cos(c+dx))^{11/2}}+\frac{4(a+a\sin(c+dx))^{7/2}}{ade(e\cos(c+dx))^{11/2}}-\frac{16(a+a\sin(c+dx))^{9/2}}{7a^2de(e\cos(c+dx))^{11/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.25, size = 74, normalized size = 0.49

$$
\frac{2\sqrt{e\cos(c+dx)}\,\sec^6(c+dx)(a(1+\sin(c+dx)))^{5/2}\left(5+26\sin(c+dx)-40\sin^2(c+dx)+16\sin^3(c+dx)\right)}{77de^7}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^(5/2)/(e*Cos[c + d*x])^(13/2),x]
```

```
[Out] (2*Sqrt[e*Cos[c + d*x]]*Sec[c + d*x]^6*(a*(1 + Sin[c + d*x]))^(5/2)*(5 + 26
*Sin[c + d*x] - 40*Sin[c + d*x]^2 + 16*Sin[c + d*x]^3))/(77*d*e^7)
```

Maple [A]
time = 0.16, size = 70, normalized size = 0.47

method	result	size
default	$-\frac{2\left(16\left(\cos^2(dx+c)\right)\sin(dx+c)-40\left(\cos^2(dx+c)\right)-42\sin(dx+c)+35\right)\cos(dx+c)\left(a(1+\sin(dx+c))\right)^{\frac{5}{2}}}{77d\left(e\cos(dx+c)\right)^{\frac{13}{2}}}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(13/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/77/d*(16*cos(d*x+c)^2*sin(d*x+c)-40*cos(d*x+c)^2-42*sin(d*x+c)+35)*cos(d
*x+c)*(a*(1+sin(d*x+c)))^(5/2)/(e*cos(d*x+c))^(13/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 321 vs. 2(118) = 236.
time = 0.55, size = 321, normalized size = 2.14

$$
\frac{2\left(5a^{\frac{5}{2}}+\frac{52a^{\frac{5}{2}}\sin(dx+c)}{\cos(dx+c)+1}-\frac{150a^{\frac{5}{2}}\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{180a^{\frac{5}{2}}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{180a^{\frac{5}{2}}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{150a^{\frac{5}{2}}\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{52a^{\frac{5}{2}}\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{5a^{\frac{5}{2}}\sin(dx+c)^8}{(\cos(dx+c)+1)^8}\right)\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^4e^{\left(-\frac{13}{2}\right)}}{77d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{3}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{13}{2}}\left(\frac{4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{4\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+1\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(13/2),x, algorithm="maxima
")
```

```
[Out] 2/77*(5*a^(5/2) + 52*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 150*a^(5/2)*
sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 180*a^(5/2)*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 - 180*a^(5/2)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 150*a^(5/2)*
sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 52*a^(5/2)*sin(d*x + c)^7/(cos(d*x +
c) + 1)^7 - 5*a^(5/2)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8)*(sin(d*x + c)^2/
(cos(d*x + c) + 1)^2 + 1)^4*e^(-13/2)/(d*(sin(d*x + c)/(cos(d*x + c) + 1) +
 1)^(3/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(13/2)*(4*sin(d*x + c)^2/(
cos(d*x + c) + 1)^2 + 6*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*sin(d*x + c
)^6/(cos(d*x + c) + 1)^6 + sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 1))
```

Fricas [A]
time = 0.37, size = 115, normalized size = 0.77

$$-\frac{2\left(40\,a^2\cos\left(dx+c\right)^2-35\,a^2-2\left(8\,a^2\cos\left(dx+c\right)^2-21\,a^2\right)\sin\left(dx+c\right)\right)\sqrt{a\sin\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}}{77\left(d\cos\left(dx+c\right)^4e^{\frac{13}{2}}+2\,d\cos\left(dx+c\right)^2e^{\frac{13}{2}}\sin\left(dx+c\right)-2\,d\cos\left(dx+c\right)^2e^{\frac{13}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(13/2),x, algorithm="fricas
")
```

```
[Out] -2/77*(40*a^2*cos(d*x + c)^2 - 35*a^2 - 2*(8*a^2*cos(d*x + c)^2 - 21*a^2)*s
in(d*x + c))*sqrt(a*sin(d*x + c) + a)*sqrt(cos(d*x + c))/(d*cos(d*x + c)^4*
e^(13/2) + 2*d*cos(d*x + c)^2*e^(13/2)*sin(d*x + c) - 2*d*cos(d*x + c)^2*e^
(13/2))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**(5/2)/(e*cos(d*x+c))**(13/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(13/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 11.17, size = 232, normalized size = 1.55

$$\frac{30\,a^2\,\sqrt{a+a\,\sin\left(c+d\,x\right)}-40\,a^2\,\cos\left(2\,c+2\,d\,x\right)\,\sqrt{a+a\,\sin\left(c+d\,x\right)}+8\,a^2\,\sin\left(3\,c+3\,d\,x\right)\,\sqrt{a+a\,\sin\left(c+d\,x\right)}-76\,a^2\,\sin\left(c+d\,x\right)\,\sqrt{a+a\,\sin\left(c+d\,x\right)}}{\frac{77\,d\,e^6\,\cos(3\,c+3\,d\,x)\,\sqrt{\frac{e\,{\mathrm{e}}^{-c\,1{}\mathrm{i}-d\,x\,1{}\mathrm{i}}}{2}+\frac{e\,{\mathrm{e}}^{c\,1{}\mathrm{i}+d\,x\,1{}\mathrm{i}}}{2}}}{4}+77\,d\,e^6\,\sin\left(2\,c+2\,d\,x\right)\,\sqrt{\frac{e\,{\mathrm{e}}^{-c\,1{}\mathrm{i}-d\,x\,1{}\mathrm{i}}}{2}+\frac{e\,{\mathrm{e}}^{c\,1{}\mathrm{i}+d\,x\,1{}\mathrm{i}}}{2}}-\frac{385\,d\,e^6\,\cos(c+d\,x)\,\sqrt{\frac{e\,{\mathrm{e}}^{-c\,1{}\mathrm{i}-d\,x\,1{}\mathrm{i}}}{2}+\frac{e\,{\mathrm{e}}^{c\,1{}\mathrm{i}+d\,x\,1{}\mathrm{i}}}{2}}}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^(5/2)/(e*cos(c + d*x))^(13/2),x)
```

```
[Out] (30*a^2*(a + a*sin(c + d*x))^(1/2) - 40*a^2*cos(2*c + 2*d*x)*(a + a*sin(c +
 d*x))^(1/2) + 8*a^2*sin(3*c + 3*d*x)*(a + a*sin(c + d*x))^(1/2) - 76*a^2*s
in(c + d*x)*(a + a*sin(c + d*x))^(1/2))/((77*d*e^6*cos(3*c + 3*d*x)*((e*exp
(- c*1i - d*x*1i))/2 + (e*exp(c*1i + d*x*1i))/2)^(1/2))/4 + 77*d*e^6*sin(2*
c + 2*d*x)*((e*exp(- c*1i - d*x*1i))/2 + (e*exp(c*1i + d*x*1i))/2)^(1/2) -
(385*d*e^6*cos(c + d*x)*((e*exp(- c*1i - d*x*1i))/2 + (e*exp(c*1i + d*x*1i)
)/2)^(1/2))/4)
```

3.298 $\int \frac{(e\cos(c+dx))^{5/2}}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=244

$$-\frac{a(e\cos(c+dx))^{7/2}}{2de(a+a\sin(c+dx))^{3/2}}+\frac{e(e\cos(c+dx))^{3/2}}{4d\sqrt{a+a\sin(c+dx)}}+\frac{3e^{5/2}\sinh^{-1}\left(\frac{\sqrt{e\cos(c+dx)}}{\sqrt{e}}\right)\sqrt{1+\cos(c+dx)}\,\sqrt{}}{4d(a+a\cos(c+dx)+a\sin(c+dx}$$

```
[Out] -1/2*a*(e*cos(d*x+c))^(7/2)/d/e/(a+a*sin(d*x+c))^(3/2)+1/4*e*(e*cos(d*x+c))
^(3/2)/d/(a+a*sin(d*x+c))^(1/2)+3/4*e^(5/2)*arcsinh((e*cos(d*x+c))^(1/2)/e^
(1/2))*(1+cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^(1/2)/d/(a+a*cos(d*x+c)+a*sin(
d*x+c))+3/4*e^(5/2)*arctan(sin(d*x+c)*e^(1/2)/(e*cos(d*x+c))^(1/2)/(1+cos(d
*x+c))^(1/2))*(1+cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^(1/2)/d/(a+a*cos(d*x+c)
+a*sin(d*x+c))
```

Rubi [A]
time = 0.25, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {2765, 2758, 2763, 2854, 209, 2912, 65, 221}

$$\frac{3e^{5/2}\sqrt{\cos(c+dx)+1}\,\sqrt{a\sin(c+dx)+a}\,\text{ArcTan}\left(\frac{\sqrt{e}\,\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\,\sqrt{e\cos(c+dx)}}\right)}{4d(a\sin(c+dx)+a\cos(c+dx)+a)}+\frac{3e^{5/2}\sqrt{\cos(c+dx)+1}\,\sqrt{a\sin(c+dx)+a}\,\sinh^{-1}\left(\frac{\sqrt{e\cos(c+dx)}}{\sqrt{e}}\right)}{4d(a\sin(c+dx)+a\cos(c+dx)+a)}-\frac{a(e\cos(c+dx))^{7/2}}{2de(a\sin(c+dx)+a)^{3/2}}+\frac{e(e\cos(c+dx))^{3/2}}{4d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(5/2)/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -1/2*(a*(e*Cos[c + d*x])^(7/2))/(d*e*(a + a*Sin[c + d*x])^(3/2)) + (e*(e*Co
s[c + d*x])^(3/2))/(4*d*Sqrt[a + a*Sin[c + d*x]]) + (3*e^(5/2)*ArcSinh[Sqrt
[e*Cos[c + d*x]]/Sqrt[e]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/
(4*d*(a + a*Cos[c + d*x] + a*Sin[c + d*x])) + (3*e^(5/2)*ArcTan[(Sqrt[e]*Si
n[c + d*x])/(Sqrt[e*Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]])]*Sqrt[1 + Cos[c +
 d*x]]*Sqrt[a + a*Sin[c + d*x]])/(4*d*(a + a*Cos[c + d*x] + a*Sin[c + d*x])
)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 2758

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(a*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2763

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[g*Sqrt[1 + Cos[e + f*x]]*(Sqrt[a + b*Sin[e + f*x
]]/(a + a*Cos[e + f*x] + b*Sin[e + f*x])), Int[Sqrt[1 + Cos[e + f*x]]/Sqrt[
g*Cos[e + f*x]], x], x] - Dist[g*Sqrt[1 + Cos[e + f*x]]*(Sqrt[a + b*Sin[e +
 f*x]]/(b + b*Cos[e + f*x] + a*Sin[e + f*x])), Int[Sin[e + f*x]/(Sqrt[g*Cos
[e + f*x]]*Sqrt[1 + Cos[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, g}, x] &&
EqQ[a^2 - b^2, 0]
```

Rule 2765

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> Simp[-2*b*((g*Cos[e + f*x])^(p + 1)/(f*g*(2*p - 1)*(
a + b*Sin[e + f*x])^(3/2))), x] + Dist[2*a*((p - 2)/(2*p - 1)), Int[(g*Cos[
e + f*x])^p/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 2] && IntegerQ[2*p]
```

Rule 2854

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b + d*x^2), x], x
, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]))], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
```

```
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{5/2}}{\sqrt{a+a\sin(c+dx)}}\,dx = -\frac{a(e\cos(c+dx))^{7/2}}{2de(a+a\sin(c+dx))^{3/2}} + \frac{1}{4}a\int \frac{(e\cos(c+dx))^{5/2}}{(a+a\sin(c+dx))^{3/2}}\,dx$$

$$= -\frac{a(e\cos(c+dx))^{7/2}}{2de(a+a\sin(c+dx))^{3/2}} + \frac{e(e\cos(c+dx))^{3/2}}{4d\sqrt{a+a\sin(c+dx)}} + \frac{1}{8}(3e^2)\int \frac{\sqrt{e\cos(}}{\sqrt{a+a\sin}}$$

$$= -\frac{a(e\cos(c+dx))^{7/2}}{2de(a+a\sin(c+dx))^{3/2}} + \frac{e(e\cos(c+dx))^{3/2}}{4d\sqrt{a+a\sin(c+dx)}} + \frac{\left(3e^3\sqrt{1+\cos(c+d}\right.}{8(a}$$

$$= -\frac{a(e\cos(c+dx))^{7/2}}{2de(a+a\sin(c+dx))^{3/2}} + \frac{e(e\cos(c+dx))^{3/2}}{4d\sqrt{a+a\sin(c+dx)}} + \frac{\left(3e^3\sqrt{1+\cos(c+d}\right.}{}$$

$$= -\frac{a(e\cos(c+dx))^{7/2}}{2de(a+a\sin(c+dx))^{3/2}} + \frac{e(e\cos(c+dx))^{3/2}}{4d\sqrt{a+a\sin(c+dx)}} + \frac{3e^{5/2}\tan^{-1}\left(\frac{}{\sqrt{e\cos}}\right.}{}$$

$$= -\frac{a(e\cos(c+dx))^{7/2}}{2de(a+a\sin(c+dx))^{3/2}} + \frac{e(e\cos(c+dx))^{3/2}}{4d\sqrt{a+a\sin(c+dx)}} + \frac{3e^{5/2}\sinh^{-1}\left(\frac{\sqrt{e\cos}}{}\right.}{4d(}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.17, size = 77, normalized size = 0.32

$$-\frac{4\sqrt[4]{2}\,(e\cos(c+dx))^{7/2}\,{}_2F_1\left(-\frac{1}{4},\frac{7}{4};\frac{11}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{7de(1+\sin(c+dx))^{5/4}\sqrt{a(1+\sin(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(5/2)/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-4*2^(1/4)*(e*Cos[c + d*x])^(7/2)*Hypergeometric2F1[-1/4, 7/4, 11/4, (1 -
Sin[c + d*x])/2])/(7*d*e*(1 + Sin[c + d*x])^(5/4)*Sqrt[a*(1 + Sin[c + d*x])
])
```

Maple [A]
time = 0.16, size = 239, normalized size = 0.98

method	result
default	$-\frac{\left(3\sqrt{2}\,\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2}}{2}\right)\sin(dx+c)+3\sqrt{2}\,\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\text{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}}{}\right.\right.}{8d(\cos(dx+c)-1+\sin(dx}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/d*(3*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*sin(d*x+c)+3*2^(1/2)*(-2*cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*
x+c)/cos(d*x+c)*2^(1/2))*sin(d*x+c)-4*cos(d*x+c)^2*sin(d*x+c)+4*cos(d*x+c)^
3+6*cos(d*x+c)*sin(d*x+c)+2*cos(d*x+c)^2-6*cos(d*x+c))*(e*cos(d*x+c))^(5/2)
/(cos(d*x+c)-1+sin(d*x+c))/cos(d*x+c)^2/(a*(1+sin(d*x+c)))^(1/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima"
)
```

```
[Out] e^(5/2)*integrate(cos(d*x + c)^(5/2)/sqrt(a*sin(d*x + c) + a), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3304 vs. $2(189) = 378$.
time = 189.47, size = 3304, normalized size = 13.54

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas"
)
```

```
[Out] -1/32*(12*(sqrt(2)*a*d*sin(d*x + c) + sqrt(2)*a*d)*(1/(a^2*d^4))^(1/4)*arct
an(-1/4*(2*sqrt(1/2)*((sqrt(2)*a^2*d^3*cos(d*x + c)^6 - 3*sqrt(2)*a^2*d^3*c
os(d*x + c)^5 - 8*sqrt(2)*a^2*d^3*cos(d*x + c)^4 + 4*sqrt(2)*a^2*d^3*cos(d*
x + c)^3 + 8*sqrt(2)*a^2*d^3*cos(d*x + c)^2 - (sqrt(2)*a^2*d^3*cos(d*x + c)
^5 + 4*sqrt(2)*a^2*d^3*cos(d*x + c)^4 - 4*sqrt(2)*a^2*d^3*cos(d*x + c)^3 -
8*sqrt(2)*a^2*d^3*cos(d*x + c)^2)*sin(d*x + c))*(1/(a^2*d^4))^(3/4)*e^(15/2
```

) + (sqrt(2)*a*d*cos(d*x + c)^6*e^5 + 5*sqrt(2)*a*d*cos(d*x + c)^5*e^5 - 8*
sqrt(2)*a*d*cos(d*x + c)^4*e^5 - 20*sqrt(2)*a*d*cos(d*x + c)^3*e^5 + 8*sqrt
(2)*a*d*cos(d*x + c)^2*e^5 + 16*sqrt(2)*a*d*cos(d*x + c)*e^5 + (sqrt(2)*a*d
*cos(d*x + c)^5*e^5 - 4*sqrt(2)*a*d*cos(d*x + c)^4*e^5 - 12*sqrt(2)*a*d*cos
(d*x + c)^3*e^5 + 8*sqrt(2)*a*d*cos(d*x + c)^2*e^5 + 16*sqrt(2)*a*d*cos(d*x
 + c)*e^5)*sin(d*x + c))*(1/(a^2*d^4))^(1/4)*e^(5/2) - (cos(d*x + c)^4*e^(1
5/2) - 3*cos(d*x + c)^3*e^(15/2) - 8*cos(d*x + c)^2*e^(15/2) + (2*a*d^2*cos
(d*x + c)^5*e^(5/2) - 5*a*d^2*cos(d*x + c)^4*e^(5/2) - 19*a*d^2*cos(d*x + c
)^3*e^(5/2) + 20*a*d^2*cos(d*x + c)*e^(5/2) + 8*a*d^2*e^(5/2) - (2*a*d^2*co
s(d*x + c)^4*e^(5/2) + 9*a*d^2*cos(d*x + c)^3*e^(5/2) - 4*a*d^2*cos(d*x + c
)^2*e^(5/2) - 20*a*d^2*cos(d*x + c)*e^(5/2) - 8*a*d^2*e^(5/2))*sin(d*x + c)
)*sqrt(1/(a^2*d^4))*e^5 + 4*cos(d*x + c)*e^(15/2) - (cos(d*x + c)^3*e^(15/2
) + 4*cos(d*x + c)^2*e^(15/2) - 4*cos(d*x + c)*e^(15/2) - 8*e^(15/2))*sin(d
*x + c) + 8*e^(15/2))*sqrt(a*sin(d*x + c) + a)*sqrt(cos(d*x + c)))*sqrt((2*
a*cos(d*x + c)*e^15*sin(d*x + c) + 2*a*cos(d*x + c)*e^15 + (a^2*d^2*e^10*si
n(d*x + c) + a^2*d^2*e^10)*sqrt(1/(a^2*d^4))*e^5 + (sqrt(2)*a*d*(1/(a^2*d^4
))^(1/4)*cos(d*x + c)*e^15 + (sqrt(2)*a^2*d^3*e^(15/2)*sin(d*x + c) + sqrt(
2)*a^2*d^3*e^(15/2))*(1/(a^2*d^4))^(3/4)*e^(15/2))*sqrt(a*sin(d*x + c) + a)
*sqrt(cos(d*x + c)))/(sin(d*x + c) + 1)) + ((2*sqrt(2)*a^2*d^3*cos(d*x + c)
^5*e^(15/2) + sqrt(2)*a^2*d^3*cos(d*x + c)^4*e^(15/2) - 13*sqrt(2)*a^2*d^3*
cos(d*x + c)^3*e^(15/2) - 8*sqrt(2)*a^2*d^3*cos(d*x + c)^2*e^(15/2) + 12*sq
rt(2)*a^2*d^3*cos(d*x + c)*e^(15/2) + 8*sqrt(2)*a^2*d^3*e^(15/2) - (7*sqrt(
2)*a^2*d^3*cos(d*x + c)^3*e^(15/2) + 4*sqrt(2)*a^2*d^3*cos(d*x + c)^2*e^(15
/2) - 12*sqrt(2)*a^2*d^3*cos(d*x + c)*e^(15/2) - 8*sqrt(2)*a^2*d^3*e^(15/2)
)*sin(d*x + c))*(1/(a^2*d^4))^(3/4)*e^(15/2) + (7*sqrt(2)*a*d*cos(d*x + c)^
4*e^(25/2) + 3*sqrt(2)*a*d*cos(d*x + c)^3*e^(25/2) - 16*sqrt(2)*a*d*cos(d*x
 + c)^2*e^(25/2) - 4*sqrt(2)*a*d*cos(d*x + c)*e^(25/2) + 8*sqrt(2)*a*d*e^(2
5/2) + (2*sqrt(2)*a*d*cos(d*x + c)^4*e^(25/2) + sqrt(2)*a*d*cos(d*x + c)^3*
e^(25/2) - 12*sqrt(2)*a*d*cos(d*x + c)^2*e^(25/2) - 4*sqrt(2)*a*d*cos(d*x +
 c)*e^(25/2) + 8*sqrt(2)*a*d*e^(25/2))*sin(d*x + c))*(1/(a^2*d^4))^(1/4)*e^
(5/2))*sqrt(a*sin(d*x + c) + a)*sqrt(cos(d*x + c)))/(a*cos(d*x + c)^6*e^15
+ a*cos(d*x + c)^5*e^15 - 8*a*cos(d*x + c)^4*e^15 - 8*a*cos(d*x + c)^3*e^15
 + 8*a*cos(d*x + c)^2*e^15 + 8*a*cos(d*x + c)*e^15 - 4*(a*cos(d*x + c)^4*e^
15 + a*cos(d*x + c)^3*e^15 - 2*a*cos(d*x + c)^2*e^15 - 2*a*cos(d*x + c)*e^1
5)*sin(d*x + c)))*e^(5/2) - 12*(sqrt(2)*a*d*sin(d*x + c) + sqrt(2)*a*d)*(1/
(a^2*d^4))^(1/4)*arctan(1/4*(2*sqrt(1/2)*((sqrt(2)*a^2*d^3*cos(d*x + c)^6 -
 3*sqrt(2)*a^2*d^3*cos(d*x + c)^5 - 8*sqrt(2)*a^2*d^3*cos(d*x + c)^4 + 4*sq
rt(2)*a^2*d^3*cos(d*x + c)^3 + 8*sqrt(2)*a^2*d^3*cos(d*x + c)^2 - (sqrt(2)*
a^2*d^3*cos(d*x + c)^5 + 4*sqrt(2)*a^2*d^3*cos(d*x + c)^4 - 4*sqrt(2)*a^2*d
^3*cos(d*x + c)^3 - 8*sqrt(2)*a^2*d^3*cos(d*x + c)^2)*sin(d*x + c))*(1/(a^2
*d^4))^(3/4)*e^(15/2) + (sqrt(2)*a*d*cos(d*x + c)^6*e^5 + 5*sqrt(2)*a*d*cos
(d*x + c)^5*e^5 - 8*sqrt(2)*a*d*cos(d*x + c)^4*e^5 - 20*sqrt(2)*a*d*cos(d*x
 + c)^3*e^5 + 8*sqrt(2)*a*d*cos(d*x + c)^2*e^5 + 16*sqrt(2)*a*d*cos(d*x + c
)*e^5 + (sqrt(2)*a*d*cos(d*x + c)^5*e^5 - 4*sqrt(2)*a*d*cos(d*x + c)^4*e^5
- 12*sqrt(2)*a*d*cos(d*x + c)^3*e^5 + 8*sqrt(2)*a*d*cos(d*x + c)^2*e^5 + 16

```
*sqrt(2)*a*d*cos(d*x + c)*e^5)*sin(d*x + c))*(1/(a^2*d^4))^(1/4)*e^(5/2) +
(cos(d*x + c)^4*e^(15/2) - 3*cos(d*x + c)^3*e^(15/2) - 8*cos(d*x + c)^2*e^(
15/2) + (2*a*d^2*cos(d*x + c)^5*e^(5/2) - 5*a*d^2*cos(d*x + c)^4*e^(5/2) -
19*a*d^2*cos(d*x + c)^3*e^(5/2) + 20*a*d^2*cos(d*x + c)*e^(5/2) + 8*a*d^2*e
^(5/2) - (2*a*d^2*cos(d*x + c)^4*e^(5/2) + 9*a*d^2*cos(d*x + c)^3*e^(5/2) -
 4*a*d^2*cos(d*x + c)^2*e^(5/2) - 20*a*d^2*cos(d*x + c)*e^(5/2) - 8*a*d^2*e
^(5/2))*sin(d*x + c))*sqrt(1/(a^2*d^4))*e^5 + 4*cos(d*x + c)*e^(15/2) - (co
s(d*x + c)^3*e^(15/2) + 4*cos(d*x + c)^2*e^(15/2) - 4*cos(d*x + c)*e^(15/2)
 - 8*e^(15/2))*sin(d*x + c) + 8*e^(15/2))*sqrt(a*sin(d*x + c) + a)*sqrt(cos
(d*x + c)))*sqrt((2*a*cos(d*x + c)*e^15*sin(d*x + c) + 2*a*cos(d*x + c)*e^1
5 + (a^2*d^2*e^10*sin(d*x + c) + a^2*d^2*e^10)*sqrt(1/(a^2*d^4))*e^5 - (sqr
t(2)*a*d*(1/(a^2*d^4))^(1/4)*cos(d*x + c)*e^15 + (sqrt(2)*a^2*d^3*e^(15/2)*
sin(d*x + c) + sqrt(2)*a^2*d^3*e^(15/2))*(1/(a^2*d^4))^(3/4)*e^(15/2))*sqrt
(a*sin(d*x + c) + a)*sqrt(cos(d*x + c)))/(sin(d*x + c) + 1)) + ((2*sqrt(2)*
a^2*d^3*cos(d*x + c)^5*e^(15/2) + sqrt(2)*a^2*d^3*cos(d*x + c)^4*e^(15/2) -
 13*sqrt(2)*a^2*d^3*cos(d*x + c)^3*e^(15/2) - 8...
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(5/2)/(a+a*sin(d*x+c))**(1/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,\cos\left(c+d\,x\right))^{5/2}}{\sqrt{a+a\,\sin\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(5/2)/(a + a*sin(c + d*x))^(1/2),x)
[Out] int((e*cos(c + d*x))^(5/2)/(a + a*sin(c + d*x))^(1/2), x)
```

3.299 $\int \frac{(e\cos(c+dx))^{3/2}}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=200

$$\frac{e\sqrt{e\cos(c+dx)}\,\sqrt{a+a\sin(c+dx)}}{ad}-\frac{e^{3/2}\sinh^{-1}\left(\frac{\sqrt{e\cos(c+dx)}}{\sqrt{e}}\right)\sqrt{1+\cos(c+dx)}\,\sqrt{a+a\sin(c+dx)}}{ad(1+\cos(c+dx)+\sin(c+dx))}$$

```
[Out] e*(e*cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^(1/2)/a/d-e^(3/2)*arcsinh((e*cos(d*
x+c))^(1/2)/e^(1/2))*(1+cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^(1/2)/a/d/(1+cos
(d*x+c)+sin(d*x+c))+e^(3/2)*arctan(sin(d*x+c)*e^(1/2)/(e*cos(d*x+c))^(1/2)/
(1+cos(d*x+c))^(1/2))*(1+cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^(1/2)/a/d/(1+co
s(d*x+c)+sin(d*x+c))
```

Rubi [A]

time = 0.19, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2764, 2756, 2854, 209, 2912, 65, 221}

$$\frac{e^{3/2}\sqrt{\cos(c+dx)+1}\,\sqrt{a\sin(c+dx)+a}\,\text{ArcTan}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\,\sqrt{e\cos(c+dx)}}\right)}{ad(\sin(c+dx)+\cos(c+dx)+1)}-\frac{e^{3/2}\sqrt{\cos(c+dx)+1}\,\sqrt{a\sin(c+dx)+a}\,\sinh^{-1}\left(\frac{\sqrt{e\cos(c+dx)}}{\sqrt{e}}\right)}{ad(\sin(c+dx)+\cos(c+dx)+1)}+\frac{e\sqrt{a\sin(c+dx)+a}\,\sqrt{e\cos(c+dx)}}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(3/2)/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (e*Sqrt[e*Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(a*d) - (e^(3/2)*ArcSinh[
Sqrt[e*Cos[c + d*x]]/Sqrt[e]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x
]])/(a*d*(1 + Cos[c + d*x] + Sin[c + d*x])) + (e^(3/2)*ArcTan[(Sqrt[e]*Sin[
c + d*x])/(Sqrt[e*Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]])]*Sqrt[1 + Cos[c + d
*x]]*Sqrt[a + a*Sin[c + d*x]])/(a*d*(1 + Cos[c + d*x] + Sin[c + d*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 2756

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[cos[(e_.) + (f_.)*(x_)]
*(g_.)], x_Symbol] :> Dist[a*Sqrt[1 + Cos[e + f*x]]*(Sqrt[a + b*Sin[e + f*x
]]/(a + a*Cos[e + f*x] + b*Sin[e + f*x])), Int[Sqrt[1 + Cos[e + f*x]]/Sqrt[
g*Cos[e + f*x]], x], x] + Dist[b*Sqrt[1 + Cos[e + f*x]]*(Sqrt[a + b*Sin[e +
 f*x]]/(a + a*Cos[e + f*x] + b*Sin[e + f*x])), Int[Sin[e + f*x]/(Sqrt[g*Cos
[e + f*x]]*Sqrt[1 + Cos[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, g}, x] &&
EqQ[a^2 - b^2, 0]
```

Rule 2764

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]], x_Symbol] :> Simp[g*Sqrt[g*Cos[e + f*x]]*(Sqrt[a + b*Sin[e + f*x
]]/(b*f)), x] + Dist[g^2/(2*a), Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[g*Cos[e +
 f*x]], x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2854

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b + d*x^2), x], x
, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]))], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{3/2}}{\sqrt{a+a\sin(c+dx)}}\,dx = \frac{e\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}{ad} + \frac{e^2\int \frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{e\cos(c+dx)}}\,dx}{2a}$$

$$= \frac{e\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}{ad} + \frac{\left(e^2\sqrt{1+\cos(c+dx)}\sqrt{a+a\sin(c\right.}{2(a+a\cos(c+dx)}$$

$$= \frac{e\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}{ad} - \frac{\left(e^2\sqrt{1+\cos(c+dx)}\sqrt{a+a\sin(c\right.}{2d(a+a\,\mathrm{co}}$$

$$= \frac{e\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}{ad} + \frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{e\cos(c+dx)}\sqrt{1+\mathrm{c}}}\right.}{d(a+a\,\mathrm{co}}$$

$$= \frac{e\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}{ad} - \frac{e^{3/2}\sinh^{-1}\left(\frac{\sqrt{e\cos(c+dx)}}{\sqrt{e}}\right)\sqrt{1}}{d(a+a\cos(c+dx}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.12, size = 77, normalized size = 0.38

$$-\frac{2\,2^{3/4}(e\cos(c+dx))^{5/2}\,{}_2F_1\left(\frac{1}{4},\frac{5}{4};\frac{9}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{5de(1+\sin(c+dx))^{3/4}\sqrt{a(1+\sin(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(3/2)/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-2*2^(3/4)*(e*Cos[c + d*x])^(5/2)*Hypergeometric2F1[1/4, 5/4, 9/4, (1 - Si
n[c + d*x])/2])/(5*d*e*(1 + Sin[c + d*x])^(3/4)*Sqrt[a*(1 + Sin[c + d*x])])
```

Maple [A]
time = 0.16, size = 212, normalized size = 1.06

method	result
default	$\frac{\left(\sqrt{2}\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{2}}{2}\right)\sin(dx+c)-\sqrt{2}\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(\right.}{1+\cos(}}{2d(\cos(dx+c)-1+\sin(dx+c))\sqrt{a\left(1+\sin(\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d*(2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*sin(d*x+c)-2^(1/2)*(-2*cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/
cos(d*x+c)*2^(1/2))*sin(d*x+c)+2*cos(d*x+c)*sin(d*x+c)-2*cos(d*x+c)^2+2*cos
(d*x+c))*(e*cos(d*x+c))^(3/2)/(cos(d*x+c)-1+sin(d*x+c))/(a*(1+sin(d*x+c)))^
(1/2)/cos(d*x+c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima"
)
```

```
[Out] e^(3/2)*integrate(cos(d*x + c)^(3/2)/sqrt(a*sin(d*x + c) + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3207 vs. $2(160) = 320$.
time = 186.73, size = 3207, normalized size = 16.04

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas"
)
```

```
[Out] 1/8*(4*sqrt(2)*a*d*(1/(a^2*d^4))^(1/4)*arctan(-1/4*(2*sqrt(1/2)*((sqrt(2)*a
^2*d^3*cos(d*x + c)^6 + 5*sqrt(2)*a^2*d^3*cos(d*x + c)^5 - 8*sqrt(2)*a^2*d^
3*cos(d*x + c)^4 - 20*sqrt(2)*a^2*d^3*cos(d*x + c)^3 + 8*sqrt(2)*a^2*d^3*co
s(d*x + c)^2 + 16*sqrt(2)*a^2*d^3*cos(d*x + c) + (sqrt(2)*a^2*d^3*cos(d*x +
 c)^5 - 4*sqrt(2)*a^2*d^3*cos(d*x + c)^4 - 12*sqrt(2)*a^2*d^3*cos(d*x + c)^
3 + 8*sqrt(2)*a^2*d^3*cos(d*x + c)^2 + 16*sqrt(2)*a^2*d^3*cos(d*x + c))*sin
(d*x + c))*(1/(a^2*d^4))^(3/4)*e^(9/2) + (sqrt(2)*a*d*cos(d*x + c)^6*e^3 -
3*sqrt(2)*a*d*cos(d*x + c)^5*e^3 - 8*sqrt(2)*a*d*cos(d*x + c)^4*e^3 + 4*sqr
t(2)*a*d*cos(d*x + c)^3*e^3 + 8*sqrt(2)*a*d*cos(d*x + c)^2*e^3 - (sqrt(2)*a
*d*cos(d*x + c)^5*e^3 + 4*sqrt(2)*a*d*cos(d*x + c)^4*e^3 - 4*sqrt(2)*a*d*co
s(d*x + c)^3*e^3 - 8*sqrt(2)*a*d*cos(d*x + c)^2*e^3)*sin(d*x + c))*(1/(a^2*
d^4))^(1/4)*e^(3/2) - (cos(d*x + c)^4*e^(9/2) - 3*cos(d*x + c)^3*e^(9/2) -
```

8*cos(d*x + c)^2*e^(9/2) + (2*a*d^2*cos(d*x + c)^5*e^(3/2) - 5*a*d^2*cos(d*x + c)^4*e^(3/2) - 19*a*d^2*cos(d*x + c)^3*e^(3/2) + 20*a*d^2*cos(d*x + c)*e^(3/2) + 8*a*d^2*e^(3/2) - (2*a*d^2*cos(d*x + c)^4*e^(3/2) + 9*a*d^2*cos(d*x + c)^3*e^(3/2) - 4*a*d^2*cos(d*x + c)^2*e^(3/2) - 20*a*d^2*cos(d*x + c)*e^(3/2) - 8*a*d^2*e^(3/2))*sin(d*x + c))*sqrt(1/(a^2*d^4))*e^3 + 4*cos(d*x + c)*e^(9/2) - (cos(d*x + c)^3*e^(9/2) + 4*cos(d*x + c)^2*e^(9/2) - 4*cos(d*x + c)*e^(9/2) - 8*e^(9/2))*sin(d*x + c) + 8*e^(9/2))*sqrt(a*sin(d*x + c) + a)*sqrt(cos(d*x + c)))*sqrt((2*a*cos(d*x + c)*e^9*sin(d*x + c) + 2*a*cos(d*x + c)*e^9 + (a^2*d^2*e^6*sin(d*x + c) + a^2*d^2*e^6)*sqrt(1/(a^2*d^4))*e^3 + (sqrt(2)*a^2*d^3*(1/(a^2*d^4))^(3/4)*cos(d*x + c)*e^9 + (sqrt(2)*a*d*e^(15/2)*sin(d*x + c) + sqrt(2)*a*d*e^(15/2))*(1/(a^2*d^4))^(1/4)*e^(3/2))*sqrt(a*sin(d*x + c) + a)*sqrt(cos(d*x + c)))/(sin(d*x + c) + 1)) - ((7*sqrt(2)*a^2*d^3*cos(d*x + c)^4*e^(9/2) + 3*sqrt(2)*a^2*d^3*cos(d*x + c)^3*e^(9/2) - 16*sqrt(2)*a^2*d^3*cos(d*x + c)^2*e^(9/2) - 4*sqrt(2)*a^2*d^3*cos(d*x + c)*e^(9/2) + 8*sqrt(2)*a^2*d^3*e^(9/2) + (2*sqrt(2)*a^2*d^3*cos(d*x + c)^4*e^(9/2) + sqrt(2)*a^2*d^3*cos(d*x + c)^3*e^(9/2) - 12*sqrt(2)*a^2*d^3*cos(d*x + c)^2*e^(9/2) - 4*sqrt(2)*a^2*d^3*cos(d*x + c)*e^(9/2) + 8*sqrt(2)*a^2*d^3*e^(9/2))*sin(d*x + c))*(1/(a^2*d^4))^(3/4)*e^(9/2) + (2*sqrt(2)*a*d*cos(d*x + c)^5*e^(15/2) + sqrt(2)*a*d*cos(d*x + c)^4*e^(15/2) - 13*sqrt(2)*a*d*cos(d*x + c)^3*e^(15/2) - 8*sqrt(2)*a*d*cos(d*x + c)^2*e^(15/2) + 12*sqrt(2)*a*d*cos(d*x + c)*e^(15/2) + 8*sqrt(2)*a*d*e^(15/2) - (7*sqrt(2)*a*d*cos(d*x + c)^3*e^(15/2) + 4*sqrt(2)*a*d*cos(d*x + c)^2*e^(15/2) - 12*sqrt(2)*a*d*cos(d*x + c)*e^(15/2) - 8*sqrt(2)*a*d*e^(15/2))*sin(d*x + c))*(1/(a^2*d^4))^(1/4)*e^(3/2))*sqrt(a*sin(d*x + c) + a)*sqrt(cos(d*x + c)))/(a*cos(d*x + c)^6*e^9 + a*cos(d*x + c)^5*e^9 - 8*a*cos(d*x + c)^4*e^9 - 8*a*cos(d*x + c)^3*e^9 + 8*a*cos(d*x + c)^2*e^9 + 8*a*cos(d*x + c)*e^9 - 4*(a*cos(d*x + c)^4*e^9 + a*cos(d*x + c)^3*e^9 - 2*a*cos(d*x + c)^2*e^9 - 2*a*cos(d*x + c)*e^9)*sin(d*x + c)))*e^(3/2) - 4*sqrt(2)*a*d*(1/(a^2*d^4))^(1/4)*arctan(1/4*(2*sqrt(1/2)*((sqrt(2)*a^2*d^3*cos(d*x + c)^6 + 5*sqrt(2)*a^2*d^3*cos(d*x + c)^5 - 8*sqrt(2)*a^2*d^3*cos(d*x + c)^4 - 20*sqrt(2)*a^2*d^3*cos(d*x + c)^3 + 8*sqrt(2)*a^2*d^3*cos(d*x + c)^2 + 16*sqrt(2)*a^2*d^3*cos(d*x + c) + (sqrt(2)*a^2*d^3*cos(d*x + c)^5 - 4*sqrt(2)*a^2*d^3*cos(d*x + c)^4 - 12*sqrt(2)*a^2*d^3*cos(d*x + c)^3 + 8*sqrt(2)*a^2*d^3*cos(d*x + c)^2 + 16*sqrt(2)*a^2*d^3*cos(d*x + c))*sin(d*x + c))*(1/(a^2*d^4))^(3/4)*e^(9/2) + (sqrt(2)*a*d*cos(d*x + c)^6*e^3 - 3*sqrt(2)*a*d*cos(d*x + c)^5*e^3 - 8*sqrt(2)*a*d*cos(d*x + c)^4*e^3 + 4*sqrt(2)*a*d*cos(d*x + c)^3*e^3 + 8*sqrt(2)*a*d*cos(d*x + c)^2*e^3 - (sqrt(2)*a*d*cos(d*x + c)^5*e^3 + 4*sqrt(2)*a*d*cos(d*x + c)^4*e^3 - 4*sqrt(2)*a*d*cos(d*x + c)^3*e^3 - 8*sqrt(2)*a*d*cos(d*x + c)^2*e^3)*sin(d*x + c))*(1/(a^2*d^4))^(1/4)*e^(3/2) + (cos(d*x + c)^4*e^(9/2) - 3*cos(d*x + c)^3*e^(9/2) - 8*cos(d*x + c)^2*e^(9/2) + (2*a*d^2*cos(d*x + c)^5*e^(3/2) - 5*a*d^2*cos(d*x + c)^4*e^(3/2) - 19*a*d^2*cos(d*x + c)^3*e^(3/2) + 20*a*d^2*cos(d*x + c)*e^(3/2) + 8*a*d^2*e^(3/2) - (2*a*d^2*cos(d*x + c)^4*e^(3/2) + 9*a*d^2*cos(d*x + c)^3*e^(3/2) - 4*a*d^2*cos(d*x + c)^2*e^(3/2) - 20*a*d^2*cos(d*x + c)*e^(3/2) - 8*a*d^2*e^(3/2))*sin(d*x + c))*sqrt(1/(a^2*d^4))*e^3 + 4*cos(d*x + c)*e^(9/2) - (cos(d*x + c)^3*e^(9/2) + 4*cos(d*x

```
+ c)^2*e^(9/2) - 4*cos(d*x + c)*e^(9/2) - 8*e^(9/2))*sin(d*x + c) + 8*e^(9/
2))*sqrt(a*sin(d*x + c) + a)*sqrt(cos(d*x + c)))*sqrt((2*a*cos(d*x + c)*e^9
*sin(d*x + c) + 2*a*cos(d*x + c)*e^9 + (a^2*d^2*e^6*sin(d*x + c) + a^2*d^2*
e^6)*sqrt(1/(a^2*d^4))*e^3 - (sqrt(2)*a^2*d^3*(1/(a^2*d^4))^(3/4)*cos(d*x +
 c)*e^9 + (sqrt(2)*a*d*e^(15/2)*sin(d*x + c) + sqrt(2)*a*d*e^(15/2))*(1/(a^
2*d^4))^(1/4)*e^(3/2))*sqrt(a*sin(d*x + c) + a)*sqrt(cos(d*x + c)))/(sin(d*
x + c) + 1)) - ((7*sqrt(2)*a^2*d^3*cos(d*x + c)^4*e^(9/2) + 3*sqrt(2)*a^2*d
^3*cos(d*x + c)^3*e^(9/2) - 16*sqrt(2)*a^2*d^3*cos(d*x + c)^2*e^(9/2) - 4*s
qrt(2)*a^2*d^3*cos(d*x + c)*e^(9/2) + 8*sqrt(2)*a^2*d^3*e^(9/2) + (2*sqrt(2
)*a^2*d^3*cos(d*x + c)^4*e^(9/2) + sqrt(2)*a^2*...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(c+dx))^{\frac{3}{2}}}{\sqrt{a(\sin(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(3/2)/(a+a*sin(d*x+c))**(1/2),x)
[Out] Integral((e*cos(c + d*x))**(3/2)/sqrt(a*(sin(c + d*x) + 1)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\cos(c+d\,x))^{3/2}}{\sqrt{a+a\,\sin(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(3/2)/(a + a*sin(c + d*x))^(1/2),x)
[Out] int((e*cos(c + d*x))^(3/2)/(a + a*sin(c + d*x))^(1/2), x)
```

3.300 $\int \frac{\sqrt{e\cos(c+dx)}}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=169

$$\frac{2\sqrt{e}\,\sinh^{-1}\left(\frac{\sqrt{e\cos(c+dx)}}{\sqrt{e}}\right)\sqrt{1+\cos(c+dx)}\,\sqrt{a+a\sin(c+dx)}}{d(a+a\cos(c+dx)+a\sin(c+dx))}+\frac{2\sqrt{e}\,\tan^{-1}\left(\frac{\sqrt{e}\,\sin}{\sqrt{e\cos(c+dx)}\,\sqrt{}}\right.}{d(a+}$$

```
[Out] 2*arcsinh((e*cos(d*x+c))^(1/2)/e^(1/2))*e^(1/2)*(1+cos(d*x+c))^(1/2)*(a+a*s
in(d*x+c))^(1/2)/d/(a+a*cos(d*x+c)+a*sin(d*x+c))+2*arctan(sin(d*x+c)*e^(1/2
)/(e*cos(d*x+c))^(1/2)/(1+cos(d*x+c))^(1/2))*e^(1/2)*(1+cos(d*x+c))^(1/2)*(
a+a*sin(d*x+c))^(1/2)/d/(a+a*cos(d*x+c)+a*sin(d*x+c))
```

Rubi [A]
time = 0.14, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2763, 2854, 209, 2912, 65, 221}

$$\frac{2\sqrt{e}\,\sqrt{\cos(c+dx)+1}\,\sqrt{a\sin(c+dx)+a}\,\text{ArcTan}\left(\frac{\sqrt{e}\,\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\,\sqrt{e\cos(c+dx)}}\right)}{d(a\sin(c+dx)+a\cos(c+dx)+a)}+\frac{2\sqrt{e}\,\sqrt{\cos(c+dx)+1}\,\sqrt{a\sin(c+dx)+a}\,\sinh^{-1}\left(\frac{\sqrt{e\cos(c+dx)}}{\sqrt{e}}\right)}{d(a\sin(c+dx)+a\cos(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cos[c + d*x]]/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (2*Sqrt[e]*ArcSinh[Sqrt[e*Cos[c + d*x]]/Sqrt[e]]*Sqrt[1 + Cos[c + d*x]]*Sqr
t[a + a*Sin[c + d*x]])/(d*(a + a*Cos[c + d*x] + a*Sin[c + d*x])) + (2*Sqrt[
e]*ArcTan[(Sqrt[e]*Sin[c + d*x])/(Sqrt[e*Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x
]])]*Sqrt[1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(d*(a + a*Cos[c + d*x
] + a*Sin[c + d*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 2763

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[g*Sqrt[1 + Cos[e + f*x]]*(Sqrt[a + b*Sin[e + f*x
]]/(a + a*Cos[e + f*x] + b*Sin[e + f*x])), Int[Sqrt[1 + Cos[e + f*x]]/Sqrt[
g*Cos[e + f*x]], x], x] - Dist[g*Sqrt[1 + Cos[e + f*x]]*(Sqrt[a + b*Sin[e +
 f*x]]/(b + b*Cos[e + f*x] + a*Sin[e + f*x])), Int[Sin[e + f*x]/(Sqrt[g*Cos
[e + f*x]]*Sqrt[1 + Cos[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, g}, x] &&
EqQ[a^2 - b^2, 0]
```

Rule 2854

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b + d*x^2), x], x
, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]))], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\sqrt{e\cos(c+dx)}}{\sqrt{a+a\sin(c+dx)}}\,dx = \frac{\left(e\sqrt{1+\cos(c+dx)}\,\sqrt{a+a\sin(c+dx)}\right)\int \frac{\sqrt{1+\cos(c+dx)}}{\sqrt{e\cos(c+dx)}}\,dx}{a+a\cos(c+dx)+a\sin(c+dx)} - \frac{\left(e\sqrt{1\right.}{}$$

$$= \frac{\left(e\sqrt{1+\cos(c+dx)}\,\sqrt{a+a\sin(c+dx)}\right)\text{Subst}\left(\int \frac{1}{\sqrt{ex}\,\sqrt{1+x}}\,dx, x, \cos\right.}{d(a+a\cos(c+dx)+a\sin(c+dx))}$$

$$= \frac{2\sqrt{e}\,\tan^{-1}\left(\frac{\sqrt{e}\,\sin(c+dx)}{\sqrt{e\cos(c+dx)}\,\sqrt{1+\cos(c+dx)}}\right)\sqrt{1+\cos(c+dx)}\,\sqrt{a+}}{d(a+a\cos(c+dx)+a\sin(c+dx))}$$

$$= \frac{2\sqrt{e}\,\sinh^{-1}\left(\frac{\sqrt{e\cos(c+dx)}}{\sqrt{e}}\right)\sqrt{1+\cos(c+dx)}\,\sqrt{a+a\sin(c+dx)}}{d(a+a\cos(c+dx)+a\sin(c+dx))} + \frac{2}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.08, size = 77, normalized size = 0.46

$$-\frac{2\sqrt[4]{2}\,(e\cos(c+dx))^{3/2}\,{}_2F_1\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{3de\sqrt[4]{1+\sin(c+dx)}\,\sqrt{a(1+\sin(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Cos[c + d*x]]/Sqrt[a + a*Sin[c + d*x]],x]
[Out] (-2*2^(1/4)*(e*Cos[c + d*x])^(3/2)*Hypergeometric2F1[3/4, 3/4, 7/4, (1 - Si
n[c + d*x])/2])/(3*d*e*(1 + Sin[c + d*x])^(1/4)*Sqrt[a*(1 + Sin[c + d*x])])
```

Maple [A]

time = 0.16, size = 142, normalized size = 0.84

method	result
default	$-\frac{\sqrt{e\cos(dx+c)}\,(\cos(dx+c)-1-\sin(dx+c))\left(\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2}}{2}\right)+\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(d}{2\cos(dx+c)}\right.\right.}{d\sqrt{a(1+\sin(dx+c))}\,\sin(dx+c)\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(e*cos(d*x+c))^(1/2)*(cos(d*x+c)-1-sin(d*x+c))*(arctan(1/2*(-2*cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))+arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2)))/(a*(1+sin(d*x+c)))^(1/2)/sin(d*x+c
)/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima"
)
```

```
[Out] e^(1/2)*integrate(sqrt(cos(d*x + c))/sqrt(a*sin(d*x + c) + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3171 vs. $2(134) = 268$.
time = 183.84, size = 3171, normalized size = 18.76

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas"
)
```

```
[Out] -sqrt(2)*(1/(a^2*d^4))^(1/4)*arctan(-1/4*(sqrt(2)*((sqrt(2)*a^2*d^3*cos(d*x
 + c)^6 - 3*sqrt(2)*a^2*d^3*cos(d*x + c)^5 - 8*sqrt(2)*a^2*d^3*cos(d*x + c)
^4 + 4*sqrt(2)*a^2*d^3*cos(d*x + c)^3 + 8*sqrt(2)*a^2*d^3*cos(d*x + c)^2 -
(sqrt(2)*a^2*d^3*cos(d*x + c)^5 + 4*sqrt(2)*a^2*d^3*cos(d*x + c)^4 - 4*sqrt
(2)*a^2*d^3*cos(d*x + c)^3 - 8*sqrt(2)*a^2*d^3*cos(d*x + c)^2)*sin(d*x + c)
)*(1/(a^2*d^4))^(3/4)*e^(3/2) + (sqrt(2)*a*d*cos(d*x + c)^6*e + 5*sqrt(2)*a
*d*cos(d*x + c)^5*e - 8*sqrt(2)*a*d*cos(d*x + c)^4*e - 20*sqrt(2)*a*d*cos(d
*x + c)^3*e + 8*sqrt(2)*a*d*cos(d*x + c)^2*e + 16*sqrt(2)*a*d*cos(d*x + c)*
e + (sqrt(2)*a*d*cos(d*x + c)^5*e - 4*sqrt(2)*a*d*cos(d*x + c)^4*e - 12*sqr
t(2)*a*d*cos(d*x + c)^3*e + 8*sqrt(2)*a*d*cos(d*x + c)^2*e + 16*sqrt(2)*a*d
*cos(d*x + c)*e)*sin(d*x + c))*(1/(a^2*d^4))^(1/4)*e^(1/2) - (cos(d*x + c)^
4*e^(3/2) - 3*cos(d*x + c)^3*e^(3/2) - 8*cos(d*x + c)^2*e^(3/2) + (2*a*d^2*
cos(d*x + c)^5*e^(1/2) - 5*a*d^2*cos(d*x + c)^4*e^(1/2) - 19*a*d^2*cos(d*x
+ c)^3*e^(1/2) + 20*a*d^2*cos(d*x + c)*e^(1/2) + 8*a*d^2*e^(1/2) - (2*a*d^2
*cos(d*x + c)^4*e^(1/2) + 9*a*d^2*cos(d*x + c)^3*e^(1/2) - 4*a*d^2*cos(d*x
+ c)^2*e^(1/2) - 20*a*d^2*cos(d*x + c)*e^(1/2) - 8*a*d^2*e^(1/2))*sin(d*x +
 c))*sqrt(1/(a^2*d^4))*e + 4*cos(d*x + c)*e^(3/2) - (cos(d*x + c)^3*e^(3/2)
 + 4*cos(d*x + c)^2*e^(3/2) - 4*cos(d*x + c)*e^(3/2) - 8*e^(3/2))*sin(d*x +
 c) + 8*e^(3/2))*sqrt(a*sin(d*x + c) + a)*sqrt(cos(d*x + c)))*sqrt((2*a*cos
(d*x + c)*e^3*sin(d*x + c) + 2*a*cos(d*x + c)*e^3 + (a^2*d^2*e^2*sin(d*x +
```

c) + a^2*d^2*e^2)*sqrt(1/(a^2*d^4))*e + (sqrt(2)*a*d*(1/(a^2*d^4))^(1/4)*co
s(d*x + c)*e^3 + (sqrt(2)*a^2*d^3*e^(3/2)*sin(d*x + c) + sqrt(2)*a^2*d^3*e^
(3/2))*(1/(a^2*d^4))^(3/4)*e^(3/2))*sqrt(a*sin(d*x + c) + a)*sqrt(cos(d*x +
 c)))/(sin(d*x + c) + 1)) + ((2*sqrt(2)*a^2*d^3*cos(d*x + c)^5*e^(3/2) + sq
rt(2)*a^2*d^3*cos(d*x + c)^4*e^(3/2) - 13*sqrt(2)*a^2*d^3*cos(d*x + c)^3*e^
(3/2) - 8*sqrt(2)*a^2*d^3*cos(d*x + c)^2*e^(3/2) + 12*sqrt(2)*a^2*d^3*cos(d
*x + c)*e^(3/2) + 8*sqrt(2)*a^2*d^3*e^(3/2) - (7*sqrt(2)*a^2*d^3*cos(d*x +
c)^3*e^(3/2) + 4*sqrt(2)*a^2*d^3*cos(d*x + c)^2*e^(3/2) - 12*sqrt(2)*a^2*d^
3*cos(d*x + c)*e^(3/2) - 8*sqrt(2)*a^2*d^3*e^(3/2))*sin(d*x + c))*(1/(a^2*d
^4))^(3/4)*e^(3/2) + (7*sqrt(2)*a*d*cos(d*x + c)^4*e^(5/2) + 3*sqrt(2)*a*d*
cos(d*x + c)^3*e^(5/2) - 16*sqrt(2)*a*d*cos(d*x + c)^2*e^(5/2) - 4*sqrt(2)*
a*d*cos(d*x + c)*e^(5/2) + 8*sqrt(2)*a*d*e^(5/2) + (2*sqrt(2)*a*d*cos(d*x +
 c)^4*e^(5/2) + sqrt(2)*a*d*cos(d*x + c)^3*e^(5/2) - 12*sqrt(2)*a*d*cos(d*x
 + c)^2*e^(5/2) - 4*sqrt(2)*a*d*cos(d*x + c)*e^(5/2) + 8*sqrt(2)*a*d*e^(5/2
))*sin(d*x + c))*(1/(a^2*d^4))^(1/4)*e^(1/2))*sqrt(a*sin(d*x + c) + a)*sqrt
(cos(d*x + c)))/(a*cos(d*x + c)^6*e^3 + a*cos(d*x + c)^5*e^3 - 8*a*cos(d*x
+ c)^4*e^3 - 8*a*cos(d*x + c)^3*e^3 + 8*a*cos(d*x + c)^2*e^3 + 8*a*cos(d*x
+ c)*e^3 - 4*(a*cos(d*x + c)^4*e^3 + a*cos(d*x + c)^3*e^3 - 2*a*cos(d*x + c
)^2*e^3 - 2*a*cos(d*x + c)*e^3)*sin(d*x + c)))*e^(1/2) + sqrt(2)*(1/(a^2*d^
4))^(1/4)*arctan(1/4*(sqrt(2)*((sqrt(2)*a^2*d^3*cos(d*x + c)^6 - 3*sqrt(2)*
a^2*d^3*cos(d*x + c)^5 - 8*sqrt(2)*a^2*d^3*cos(d*x + c)^4 + 4*sqrt(2)*a^2*d
^3*cos(d*x + c)^3 + 8*sqrt(2)*a^2*d^3*cos(d*x + c)^2 - (sqrt(2)*a^2*d^3*cos
(d*x + c)^5 + 4*sqrt(2)*a^2*d^3*cos(d*x + c)^4 - 4*sqrt(2)*a^2*d^3*cos(d*x
+ c)^3 - 8*sqrt(2)*a^2*d^3*cos(d*x + c)^2)*sin(d*x + c))*(1/(a^2*d^4))^(3/4
)*e^(3/2) + (sqrt(2)*a*d*cos(d*x + c)^6*e + 5*sqrt(2)*a*d*cos(d*x + c)^5*e
- 8*sqrt(2)*a*d*cos(d*x + c)^4*e - 20*sqrt(2)*a*d*cos(d*x + c)^3*e + 8*sqrt
(2)*a*d*cos(d*x + c)^2*e + 16*sqrt(2)*a*d*cos(d*x + c)*e + (sqrt(2)*a*d*cos
(d*x + c)^5*e - 4*sqrt(2)*a*d*cos(d*x + c)^4*e - 12*sqrt(2)*a*d*cos(d*x + c
)^3*e + 8*sqrt(2)*a*d*cos(d*x + c)^2*e + 16*sqrt(2)*a*d*cos(d*x + c)*e)*sin
(d*x + c))*(1/(a^2*d^4))^(1/4)*e^(1/2) + (cos(d*x + c)^4*e^(3/2) - 3*cos(d*
x + c)^3*e^(3/2) - 8*cos(d*x + c)^2*e^(3/2) + (2*a*d^2*cos(d*x + c)^5*e^(1/
2) - 5*a*d^2*cos(d*x + c)^4*e^(1/2) - 19*a*d^2*cos(d*x + c)^3*e^(1/2) + 20*
a*d^2*cos(d*x + c)*e^(1/2) + 8*a*d^2*e^(1/2) - (2*a*d^2*cos(d*x + c)^4*e^(1
/2) + 9*a*d^2*cos(d*x + c)^3*e^(1/2) - 4*a*d^2*cos(d*x + c)^2*e^(1/2) - 20*
a*d^2*cos(d*x + c)*e^(1/2) - 8*a*d^2*e^(1/2))*sin(d*x + c))*sqrt(1/(a^2*d^4
))*e + 4*cos(d*x + c)*e^(3/2) - (cos(d*x + c)^3*e^(3/2) + 4*cos(d*x + c)^2*
e^(3/2) - 4*cos(d*x + c)*e^(3/2) - 8*e^(3/2))*sin(d*x + c) + 8*e^(3/2))*sqr
t(a*sin(d*x + c) + a)*sqrt(cos(d*x + c)))*sqrt((2*a*cos(d*x + c)*e^3*sin(d*
x + c) + 2*a*cos(d*x + c)*e^3 + (a^2*d^2*e^2*sin(d*x + c) + a^2*d^2*e^2)*sq
rt(1/(a^2*d^4))*e - (sqrt(2)*a*d*(1/(a^2*d^4))^(1/4)*cos(d*x + c)*e^3 + (sq
rt(2)*a^2*d^3*e^(3/2)*sin(d*x + c) + sqrt(2)*a^2*d^3*e^(3/2))*(1/(a^2*d^4))
^(3/4)*e^(3/2))*sqrt(a*sin(d*x + c) + a)*sqrt(cos(d*x + c)))/(sin(d*x + c)
+ 1)) + ((2*sqrt(2)*a^2*d^3*cos(d*x + c)^5*e^(3/2) + sqrt(2)*a^2*d^3*cos(d*
x + c)^4*e^(3/2) - 13*sqrt(2)*a^2*d^3*cos(d*x + c)^3*e^(3/2) - 8*sqrt(2)*a^
2*d^3*cos(d*x + c)^2*e^(3/2) + 12*sqrt(2)*a^2*d^3*cos(d*x + c)*e^(3/2) + 8*

```
sqrt(2)*a^2*d^3*e^(3/2) - (7*sqrt(2)*a^2*d^3*cos(d*x + c)^3*e^(3/2) + 4*sqr
t(2)*a^2*d^3*cos(d*x + c)^2*e^(3/2) - 12*sqrt(2...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e\cos\left(c+dx\right)}}{\sqrt{a\left(\sin\left(c+dx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(1/2)/(a+a*sin(d*x+c))**(1/2),x)
[Out] Integral(sqrt(e*cos(c + d*x))/sqrt(a*(sin(c + d*x) + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(cos(d*x + c))*e^(1/2)/sqrt(a*sin(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{e\,\cos\left(c+d\,x\right)}}{\sqrt{a+a\,\sin\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(1/2)/(a + a*sin(c + d*x))^(1/2),x)
[Out] int((e*cos(c + d*x))^(1/2)/(a + a*sin(c + d*x))^(1/2), x)
```

3.301 $\int \frac{1}{\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=34

$$-\frac{2\sqrt{e\cos(c+dx)}}{de\sqrt{a+a\sin(c+dx)}}$$

[Out] -2*(e*cos(d*x+c))^(1/2)/d/e/(a+a*sin(d*x+c))^(1/2)

Rubi [A]
time = 0.04, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$, Rules used = {2750}

$$-\frac{2\sqrt{e\cos(c+dx)}}{de\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[e*Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]]),x]

[Out] (-2*Sqrt[e*Cos[c + d*x]])/(d*e*Sqrt[a + a*Sin[c + d*x]])

Rule 2750

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}\,dx = -\frac{2\sqrt{e\cos(c+dx)}}{de\sqrt{a+a\sin(c+dx)}}$$

Mathematica [A]
time = 0.06, size = 34, normalized size = 1.00

$$-\frac{2\sqrt{e\cos(c+dx)}}{de\sqrt{a(1+\sin(c+dx))}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[e*Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]]),x]

```
[Out] (-2*Sqrt[e*Cos[c + d*x]])/(d*e*Sqrt[a*(1 + Sin[c + d*x])])
```

Maple [A]
time = 0.34, size = 34, normalized size = 1.00

method	result	size
default	$-\frac{2\cos(dx+c)}{d\sqrt{e\cos(dx+c)}\,\sqrt{a\left(1+\sin(dx+c)\right)}}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*cos(d*x+c)/(e*cos(d*x+c))^(1/2)/(a*(1+sin(d*x+c)))^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 122 vs. 2(27) = 54.
time = 0.56, size = 122, normalized size = 3.59

$$-\frac{2\left(\sqrt{a}-\frac{\sqrt{a}\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)e^{\left(-\frac{1}{2}\right)}}{\left(a+\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{3}{2}}\sqrt{-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxim
a")
```

```
[Out] -2*(sqrt(a) - sqrt(a)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(sin(d*x + c)^2/
(cos(d*x + c) + 1)^2 + 1)*e^(-1/2)/((a + a*sin(d*x + c)^2/(cos(d*x + c) + 1
)^2)*d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(3/2)*sqrt(-sin(d*x + c)/(cos(
d*x + c) + 1) + 1))
```

Fricas [A]
time = 0.35, size = 41, normalized size = 1.21

$$-\frac{2\sqrt{a\sin(dx+c)+a}\,\sqrt{\cos(dx+c)}}{ade^{\frac{1}{2}}\sin(dx+c)+ade^{\frac{1}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c))^(1/2),x, algorithm="frica
s")
```

```
[Out] -2*sqrt(a*sin(d*x + c) + a)*sqrt(cos(d*x + c))/(a*d*e^(1/2)*sin(d*x + c) +
a*d*e^(1/2))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\left(\sin\left(c+dx\right)+1\right)}\,\sqrt{e\cos\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(1/2)/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a*(sin(c + d*x) + 1))*sqrt(e*cos(c + d*x))), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

Mupad [B]
time = 5.66, size = 46, normalized size = 1.35

$$-\frac{2\,\cos\left(c+d\,x\right)\,\sqrt{a\,\left(\sin\left(c+d\,x\right)+1\right)}}{a\,d\,\sqrt{e\,\cos\left(c+d\,x\right)}\,\left(\sin\left(c+d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cos(c + d*x))^(1/2)*(a + a*sin(c + d*x))^(1/2)),x)
```

```
[Out] -(2*cos(c + d*x)*(a*(sin(c + d*x) + 1))^(1/2))/(a*d*(e*cos(c + d*x))^(1/2)*
(sin(c + d*x) + 1))
```

3.302 $\int \frac{1}{(e\cos(c+dx))^{3/2}\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=76

$$-\frac{2}{3de\sqrt{e\cos(c+dx)}\,\sqrt{a+a\sin(c+dx)}}+\frac{4\sqrt{a+a\sin(c+dx)}}{3ade\sqrt{e\cos(c+dx)}}$$

```
[Out] -2/3/d/e/(e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c))^(1/2)+4/3*(a+a*sin(d*x+c))^(
1/2)/a/d/e/(e*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2751, 2750}

$$\frac{4\sqrt{a\sin(c+dx)+a}}{3ade\sqrt{e\cos(c+dx)}}-\frac{2}{3de\sqrt{a\sin(c+dx)+a}\,\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(3/2)*Sqrt[a + a*Sin[c + d*x]]),x]
```

```
[Out] -2/(3*d*e*Sqrt[e*Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]]) + (4*Sqrt[a + a*Si
n[c + d*x]])/(3*a*d*e*Sqrt[e*Cos[c + d*x]])
```

Rule 2750

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{1}{(e\cos(c+dx))^{3/2}\sqrt{a+a\sin(c+dx)}}\,dx = -\frac{2}{3de\sqrt{e\cos(c+dx)}\,\sqrt{a+a\sin(c+dx)}} + \frac{2\int \frac{\sqrt{a+a\sin}}{(e\cos(c+}}{3a}$$
$$= -\frac{2}{3de\sqrt{e\cos(c+dx)}\,\sqrt{a+a\sin(c+dx)}} + \frac{4\sqrt{a+a\sin(c}}{3ade\sqrt{e\cos(c}}$$

Mathematica [A]
time = 0.11, size = 46, normalized size = 0.61

$$\frac{2(1+2\sin(c+dx))}{3de\sqrt{e\cos(c+dx)}\,\sqrt{a(1+\sin(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Cos[c + d*x])^(3/2)*Sqrt[a + a*Sin[c + d*x]]),x]
[Out] (2*(1 + 2*Sin[c + d*x]))/(3*d*e*Sqrt[e*Cos[c + d*x]]*Sqrt[a*(1 + Sin[c + d*
x])])
```

Maple [A]
time = 0.14, size = 44, normalized size = 0.58

method	result	size
default	$\frac{2(2\sin(dx+c)+1)\cos(dx+c)}{3d(e\cos(dx+c))^{\frac{3}{2}}\sqrt{a\left(1+\sin\left(dx+c\right)\right)}}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/3/d*(2*sin(d*x+c)+1)*cos(d*x+c)/(e*cos(d*x+c))^(3/2)/(a*(1+sin(d*x+c)))^(
1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 189 vs. 2(58) = 116.
time = 0.54, size = 189, normalized size = 2.49

$$\frac{2\left(\sqrt{a}+\frac{4\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}-\frac{4\sqrt{a}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{\sqrt{a}\sin(dx+c)^4}{(\cos(dx+c)+1)^4}\right)\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^2 e^{\left(-\frac{3}{2}\right)}}{3\left(a+\frac{2a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}\right)d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxim
a")
```

```
[Out] 2/3*(sqrt(a) + 4*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 4*sqrt(a)*sin(d*
x + c)^3/(cos(d*x + c) + 1)^3 - sqrt(a)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4
)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^2*e^(-3/2)/((a + 2*a*sin(d*x +
c)^2/(cos(d*x + c) + 1)^2 + a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4)*d*(sin(d
*x + c)/(cos(d*x + c) + 1) + 1)^(5/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1
)^(3/2))
```

Fricas [A]
time = 0.33, size = 63, normalized size = 0.83

$$\frac{2\sqrt{a\sin\left(dx+c\right)+a}\,\left(2\,\sin\left(dx+c\right)+1\right)\sqrt{\cos\left(dx+c\right)}}{3\left(ad\cos\left(dx+c\right)e^{\frac{3}{2}}\sin\left(dx+c\right)+ad\cos\left(dx+c\right)e^{\frac{3}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^(1/2),x, algorithm="frica
s")
```

```
[Out] 2/3*sqrt(a*sin(d*x + c) + a)*(2*sin(d*x + c) + 1)*sqrt(cos(d*x + c))/(a*d*c
os(d*x + c)*e^(3/2)*sin(d*x + c) + a*d*cos(d*x + c)*e^(3/2))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{a\left(\sin\left(c+dx\right)+1\right)}\,\left(e\cos\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(3/2)/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a*(sin(c + d*x) + 1))*(e*cos(c + d*x))**(3/2)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

Mupad [B]
time = 6.01, size = 77, normalized size = 1.01

$$\frac{4\sqrt{a\,\left(\sin\left(c+d\,x\right)+1\right)}\,\left(3\,\sin\left(c+d\,x\right)-\cos\left(2\,c+2\,d\,x\right)+2\right)}{3\,a\,d\,e\,\sqrt{e\,\cos\left(c+d\,x\right)}\,\left(4\,\sin\left(c+d\,x\right)-\cos\left(2\,c+2\,d\,x\right)+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cos(c + d*x))^(3/2)*(a + a*sin(c + d*x))^(1/2)),x)
```

```
[Out] (4*(a*(sin(c + d*x) + 1))^(1/2)*(3*sin(c + d*x) - cos(2*c + 2*d*x) + 2))/(3
*a*d*e*(e*cos(c + d*x))^(1/2)*(4*sin(c + d*x) - cos(2*c + 2*d*x) + 3))
```

3.303 $\int \frac{1}{(e\cos(c+dx))^{5/2}\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=115

$$-\frac{2}{5de(e\cos(c+dx))^{3/2}\sqrt{a+a\sin(c+dx)}}-\frac{8\sqrt{a+a\sin(c+dx)}}{5ade(e\cos(c+dx))^{3/2}}+\frac{16(a+a\sin(c+dx))^{3/2}}{15a^2de(e\cos(c+dx))^{3/2}}$$

```
[Out] 16/15*(a+a*sin(d*x+c))^(3/2)/a^2/d/e/(e*cos(d*x+c))^(3/2)-2/5/d/e/(e*cos(d*
x+c))^(3/2)/(a+a*sin(d*x+c))^(1/2)-8/5*(a+a*sin(d*x+c))^(1/2)/a/d/e/(e*cos(
d*x+c))^(3/2)
```

Rubi [A]
time = 0.14, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2751, 2750}

$$\frac{16(a\sin(c+dx)+a)^{3/2}}{15a^2de(e\cos(c+dx))^{3/2}}-\frac{8\sqrt{a\sin(c+dx)+a}}{5ade(e\cos(c+dx))^{3/2}}-\frac{2}{5de\sqrt{a\sin(c+dx)+a}\,(e\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(5/2)*Sqrt[a + a*Sin[c + d*x]]),x]
```

```
[Out] -2/(5*d*e*(e*Cos[c + d*x])^(3/2)*Sqrt[a + a*Sin[c + d*x]]) - (8*Sqrt[a + a*
Sin[c + d*x]])/(5*a*d*e*(e*Cos[c + d*x])^(3/2)) + (16*(a + a*Sin[c + d*x])^
(3/2))/(15*a^2*d*e*(e*Cos[c + d*x])^(3/2))
```

Rule 2750

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{1}{(e\cos(c+dx))^{5/2}\sqrt{a+a\sin(c+dx)}}\,dx = -\frac{2}{5de(e\cos(c+dx))^{3/2}\sqrt{a+a\sin(c+dx)}} + \frac{4\int \frac{\sqrt{a+a\,\mathrm{s}}}{(e\cos(c}}{5}$$

$$= -\frac{2}{5de(e\cos(c+dx))^{3/2}\sqrt{a+a\sin(c+dx)}} - \frac{8\sqrt{a+a\sin(}}{5ade(e\cos(c}$$

$$= -\frac{2}{5de(e\cos(c+dx))^{3/2}\sqrt{a+a\sin(c+dx)}} - \frac{8\sqrt{a+a\sin(}}{5ade(e\cos(c}$$

Mathematica [A]
time = 0.11, size = 56, normalized size = 0.49

$$\frac{2(-7+4\sin(c+dx)+8\sin^2(c+dx))}{15de(e\cos(c+dx))^{3/2}\sqrt{a(1+\sin(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Cos[c + d*x])^(5/2)*Sqrt[a + a*Sin[c + d*x]]),x]
[Out] (2*(-7 + 4*Sin[c + d*x] + 8*Sin[c + d*x]^2))/(15*d*e*(e*Cos[c + d*x])^(3/2)
*Sqrt[a*(1 + Sin[c + d*x])])
```

Maple [A]
time = 0.15, size = 54, normalized size = 0.47

method	result	size
default	$-\frac{2\left(8\left(\cos^2(dx+c)\right)-4\sin(dx+c)-1\right)\cos(dx+c)}{15d(e\cos(dx+c))^{\frac{5}{2}}\sqrt{a\left(1+\sin\left(dx+c\right)\right)}}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/15/d*(8*cos(d*x+c)^2-4*sin(d*x+c)-1)*cos(d*x+c)/(e*cos(d*x+c))^(5/2)/(a*
(1+sin(d*x+c)))^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 258 vs. 2(88) = 176.
time = 0.57, size = 258, normalized size = 2.24

$$-\frac{2\left(7\sqrt{a}-\frac{8\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}-\frac{25\sqrt{a}\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{25\sqrt{a}\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{8\sqrt{a}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{7\sqrt{a}\sin(dx+c)^6}{(\cos(dx+c)+1)^6}\right)\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^3 e^{\left(-\frac{5}{2}\right)}}{15\left(a+\frac{3a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}\right)d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxim
a")
```

```
[Out] -2/15*(7*sqrt(a) - 8*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 25*sqrt(a)*s
in(d*x + c)^2/(cos(d*x + c) + 1)^2 + 25*sqrt(a)*sin(d*x + c)^4/(cos(d*x + c
) + 1)^4 + 8*sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 7*sqrt(a)*sin(d*
x + c)^6/(cos(d*x + c) + 1)^6)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^3*
e^(-5/2)/((a + 3*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*a*sin(d*x + c)^4
/(cos(d*x + c) + 1)^4 + a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6)*d*(sin(d*x +
 c)/(cos(d*x + c) + 1) + 1)^(7/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5
/2))
```

Fricas [**A**]
time = 0.32, size = 77, normalized size = 0.67

$$-\frac{2\left(8\cos\left(dx+c\right)^{2}-4\sin\left(dx+c\right)-1\right)\sqrt{a\sin\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}}{15\left(ad\cos\left(dx+c\right)^{2}e^{\frac{5}{2}}\sin\left(dx+c\right)+ad\cos\left(dx+c\right)^{2}e^{\frac{5}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^(1/2),x, algorithm="frica
s")
```

```
[Out] -2/15*(8*cos(d*x + c)^2 - 4*sin(d*x + c) - 1)*sqrt(a*sin(d*x + c) + a)*sqrt
(cos(d*x + c))/(a*d*cos(d*x + c)^2*e^(5/2)*sin(d*x + c) + a*d*cos(d*x + c)^
2*e^(5/2))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{a\left(\sin\left(c+dx\right)+1\right)}\,\left(e\cos\left(c+dx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(5/2)/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a*(sin(c + d*x) + 1))*(e*cos(c + d*x))**(5/2)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

Mupad [B]
time = 6.68, size = 120, normalized size = 1.04

$$-\frac{8\sqrt{a\left(\sin\left(c+d\,x\right)+1\right)}\left(8\,\cos\left(c+d\,x\right)+6\,\cos\left(3\,c+3\,d\,x\right)-\sin\left(2\,c+2\,d\,x\right)+2\,\sin\left(4\,c+4\,d\,x\right)\right)}{15\,a\,d\,e^2\,\sqrt{e\,\cos\left(c+d\,x\right)}\,\left(4\,\sin\left(c+d\,x\right)+4\,\cos\left(2\,c+2\,d\,x\right)-\cos\left(4\,c+4\,d\,x\right)+4\,\sin\left(3\,c+3\,d\,x\right)+5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cos(c + d*x))^(5/2)*(a + a*sin(c + d*x))^(1/2)),x)
```

```
[Out] -(8*(a*(sin(c + d*x) + 1))^(1/2)*(8*cos(c + d*x) + 6*cos(3*c + 3*d*x) - sin
(2*c + 2*d*x) + 2*sin(4*c + 4*d*x)))/(15*a*d*e^2*(e*cos(c + d*x))^(1/2)*(4*
sin(c + d*x) + 4*cos(2*c + 2*d*x) - cos(4*c + 4*d*x) + 4*sin(3*c + 3*d*x) +
 5))
```

3.304 $\int \frac{1}{(e\cos(c+dx))^{7/2}\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=154

$$-\frac{2}{7de(e\cos(c+dx))^{5/2}\sqrt{a+a\sin(c+dx)}}-\frac{4\sqrt{a+a\sin(c+dx)}}{7ade(e\cos(c+dx))^{5/2}}+\frac{16(a+a\sin(c+dx))^{3/2}}{7a^2de(e\cos(c+dx))^{5/2}}-\frac{32(a+a\sin(c+dx))^{5/2}}{35a^3de(e\cos(c+dx))^{5/2}}$$

```
[Out] 16/7*(a+a*sin(d*x+c))^(3/2)/a^2/d/e/(e*cos(d*x+c))^(5/2)-32/35*(a+a*sin(d*x
+c))^(5/2)/a^3/d/e/(e*cos(d*x+c))^(5/2)-2/7/d/e/(e*cos(d*x+c))^(5/2)/(a+a*s
in(d*x+c))^(1/2)-4/7*(a+a*sin(d*x+c))^(1/2)/a/d/e/(e*cos(d*x+c))^(5/2)
```

Rubi [A]
time = 0.20, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2751, 2750}

$$-\frac{32(a\sin(c+dx)+a)^{5/2}}{35a^3de(e\cos(c+dx))^{5/2}}+\frac{16(a\sin(c+dx)+a)^{3/2}}{7a^2de(e\cos(c+dx))^{5/2}}-\frac{4\sqrt{a\sin(c+dx)+a}}{7ade(e\cos(c+dx))^{5/2}}-\frac{2}{7de\sqrt{a\sin(c+dx)+a}\,(e\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(7/2)*Sqrt[a + a*Sin[c + d*x]]),x]
```

```
[Out] -2/(7*d*e*(e*Cos[c + d*x])^(5/2)*Sqrt[a + a*Sin[c + d*x]]) - (4*Sqrt[a + a*
Sin[c + d*x]])/(7*a*d*e*(e*Cos[c + d*x])^(5/2)) + (16*(a + a*Sin[c + d*x])^
(3/2))/(7*a^2*d*e*(e*Cos[c + d*x])^(5/2)) - (32*(a + a*Sin[c + d*x])^(5/2))
/(35*a^3*d*e*(e*Cos[c + d*x])^(5/2))
```

Rule 2750

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{1}{(e\cos(c+dx))^{7/2}\sqrt{a+a\sin(c+dx)}}\,dx = -\frac{2}{7de(e\cos(c+dx))^{5/2}\sqrt{a+a\sin(c+dx)}} + \frac{6\int \frac{\sqrt{a+a\,\mathrm{s}}}{(e\cos(c}}{7}$$

$$= -\frac{2}{7de(e\cos(c+dx))^{5/2}\sqrt{a+a\sin(c+dx)}} - \frac{4\sqrt{a+a\sin(}}{7ade(e\cos(c}$$

$$= -\frac{2}{7de(e\cos(c+dx))^{5/2}\sqrt{a+a\sin(c+dx)}} - \frac{4\sqrt{a+a\sin(}}{7ade(e\cos(c}$$

$$= -\frac{2}{7de(e\cos(c+dx))^{5/2}\sqrt{a+a\sin(c+dx)}} - \frac{4\sqrt{a+a\sin(}}{7ade(e\cos(c}$$

Mathematica [A]
time = 0.17, size = 66, normalized size = 0.43

$$\frac{2(5+4\cos(2(c+dx))+10\sin(c+dx)+4\sin(3(c+dx)))}{35de(e\cos(c+dx))^{5/2}\sqrt{a(1+\sin(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Cos[c + d*x])^(7/2)*Sqrt[a + a*Sin[c + d*x]]),x]
[Out] (2*(5 + 4*Cos[2*(c + d*x)] + 10*Sin[c + d*x] + 4*Sin[3*(c + d*x)]))/(35*d*e
*(e*Cos[c + d*x])^(5/2)*Sqrt[a*(1 + Sin[c + d*x])])
```

Maple [A]
time = 0.16, size = 70, normalized size = 0.45

method	result	size
default	$\frac{2\left(16\left(\cos^2(dx+c)\right)\sin(dx+c)+8\left(\cos^2(dx+c)\right)+6\sin(dx+c)+1\right)\cos(dx+c)}{35d(e\cos(dx+c))^{\frac{7}{2}}\sqrt{a\left(1+\sin\left(dx+c\right)\right)}}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/35/d*(16*cos(d*x+c)^2*sin(d*x+c)+8*cos(d*x+c)^2+6*sin(d*x+c)+1)*cos(d*x+c
)/(e*cos(d*x+c))^(7/2)/(a*(1+sin(d*x+c)))^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 325 vs. 2(118) = 236.
time = 0.59, size = 325, normalized size = 2.11

$$\frac{2\left(9\sqrt{a}+\frac{44\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}-\frac{14\sqrt{a}\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{84\sqrt{a}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{84\sqrt{a}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{14\sqrt{a}\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{44\sqrt{a}\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{9\sqrt{a}\sin(dx+c)^8}{(\cos(dx+c)+1)^8}\right)\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^4e^{\left(-\frac{7}{2}\right)}}{35\left(a+\frac{4a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{4a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}\right)d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{9}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxim
a")
```

```
[Out] 2/35*(9*sqrt(a) + 44*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 14*sqrt(a)*s
in(d*x + c)^2/(cos(d*x + c) + 1)^2 - 84*sqrt(a)*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3 + 84*sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 14*sqrt(a)*sin(
d*x + c)^6/(cos(d*x + c) + 1)^6 - 44*sqrt(a)*sin(d*x + c)^7/(cos(d*x + c) +
 1)^7 - 9*sqrt(a)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8)*(sin(d*x + c)^2/(cos
(d*x + c) + 1)^2 + 1)^4*e^(-7/2)/((a + 4*a*sin(d*x + c)^2/(cos(d*x + c) + 1
)^2 + 6*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*a*sin(d*x + c)^6/(cos(d*x
 + c) + 1)^6 + a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8)*d*(sin(d*x + c)/(cos(
d*x + c) + 1) + 1)^(9/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2))
```

Fricas [A]
time = 0.33, size = 89, normalized size = 0.58

$$\frac{2\left(8\cos\left(dx+c\right)^{2}+2\left(8\cos\left(dx+c\right)^{2}+3\right)\sin\left(dx+c\right)+1\right)\sqrt{a\sin\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}}{35\left(ad\cos\left(dx+c\right)^{3}e^{\frac{7}{2}}\sin\left(dx+c\right)+ad\cos\left(dx+c\right)^{3}e^{\frac{7}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^(1/2),x, algorithm="frica
s")
```

```
[Out] 2/35*(8*cos(d*x + c)^2 + 2*(8*cos(d*x + c)^2 + 3)*sin(d*x + c) + 1)*sqrt(a*
sin(d*x + c) + a)*sqrt(cos(d*x + c))/(a*d*cos(d*x + c)^3*e^(7/2)*sin(d*x +
c) + a*d*cos(d*x + c)^3*e^(7/2))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(7/2)/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8571 deep
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac"
)

[Out] Timed out

Mupad [B]
time = 11.01, size = 261, normalized size = 1.69

$$\frac{20\sin(c+d\,x)\,\sqrt{a+a\sin(c+d\,x)}+10\,\sqrt{a+a\sin(c+d\,x)}+8\cos(2\,c+2\,d\,x)\,\sqrt{a+a\sin(c+d\,x)}+8\sin(3\,c+3\,d\,x)\,\sqrt{a+a\sin(c+d\,x)}}{\frac{35\,a\,d\,e^3\,\sqrt{\frac{e\,\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}}{2}+\frac{e\,\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}}{2}}}{2}+\frac{35\,a\,d\,e^3\,\sin(c+d\,x)\,\sqrt{\frac{e\,\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}}{2}+\frac{e\,\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}}{2}}}{4}+\frac{35\,a\,d\,e^3\,\cos(2\,c+2\,d\,x)\,\sqrt{\frac{e\,\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}}{2}+\frac{e\,\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}}{2}}}{2}+\frac{35\,a\,d\,e^3\,\sin(3\,c+3\,d\,x)\,\sqrt{\frac{e\,\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}}{2}+\frac{e\,\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}}{2}}}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((e*cos(c + d*x))^(7/2)*(a + a*sin(c + d*x))^(1/2)),x)

[Out] (20*sin(c + d*x)*(a + a*sin(c + d*x))^(1/2) + 10*(a + a*sin(c + d*x))^(1/2)
 + 8*cos(2*c + 2*d*x)*(a + a*sin(c + d*x))^(1/2) + 8*sin(3*c + 3*d*x)*(a +
a*sin(c + d*x))^(1/2))/((35*a*d*e^3*((e*exp(- c*1i - d*x*1i))/2 + (e*exp(c*
1i + d*x*1i))/2)^(1/2))/2 + (35*a*d*e^3*sin(c + d*x)*((e*exp(- c*1i - d*x*1
i))/2 + (e*exp(c*1i + d*x*1i))/2)^(1/2))/4 + (35*a*d*e^3*cos(2*c + 2*d*x)*(
(e*exp(- c*1i - d*x*1i))/2 + (e*exp(c*1i + d*x*1i))/2)^(1/2))/2 + (35*a*d*e
^3*sin(3*c + 3*d*x)*((e*exp(- c*1i - d*x*1i))/2 + (e*exp(c*1i + d*x*1i))/2)
^(1/2))/4)

3.305 $\int \frac{(e\cos(c+dx))^{7/2}}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=247

$$\frac{e(e\cos(c+dx))^{5/2}}{2ad\sqrt{a+a\sin(c+dx)}}+\frac{5e^3\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}{4a^2d}-\frac{5e^{7/2}\sinh^{-1}\left(\frac{\sqrt{e\cos(c+dx)}}{\sqrt{e}}\right)\sqrt{1+\cdots}}{4a^2d(1+\cos(c+dx)\cdots}$$

```
[Out] 1/2*e*(e*cos(d*x+c))^(5/2)/a/d/(a+a*sin(d*x+c))^(1/2)+5/4*e^3*(e*cos(d*x+c)
)^(1/2)*(a+a*sin(d*x+c))^(1/2)/a^2/d-5/4*e^(7/2)*arcsinh((e*cos(d*x+c))^(1/
2)/e^(1/2))*(1+cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^(1/2)/a^2/d/(1+cos(d*x+c)
+sin(d*x+c))+5/4*e^(7/2)*arctan(sin(d*x+c)*e^(1/2)/(e*cos(d*x+c))^(1/2)/(1+
cos(d*x+c))^(1/2))*(1+cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^(1/2)/a^2/d/(1+cos
(d*x+c)+sin(d*x+c))
```

Rubi [A]
time = 0.25, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$,
Rules used = {2758, 2764, 2756, 2854, 209, 2912, 65, 221}

$$\frac{5e^{7/2}\sqrt{\cos(c+dx)+1}\sqrt{a\sin(c+dx)+a}\,\text{ArcTan}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\sqrt{e\cos(c+dx)}}\right)}{4a^2d(\sin(c+dx)+\cos(c+dx)+1)}-\frac{5e^{7/2}\sqrt{\cos(c+dx)+1}\sqrt{a\sin(c+dx)+a}\sinh^{-1}\left(\frac{\sqrt{e\cos(c+dx)}}{\sqrt{e}}\right)}{4a^2d(\sin(c+dx)+\cos(c+dx)+1)}+\frac{5e^3\sqrt{a\sin(c+dx)+a}\sqrt{e\cos(c+dx)}}{4a^2d}+\frac{e(e\cos(c+dx))^{5/2}}{2ad\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(7/2)/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (e*(e*Cos[c + d*x])^(5/2))/(2*a*d*Sqrt[a + a*Sin[c + d*x]]) + (5*e^3*Sqrt[e
*Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(4*a^2*d) - (5*e^(7/2)*ArcSinh[Sqr
t[e*Cos[c + d*x]]/Sqrt[e]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])
/(4*a^2*d*(1 + Cos[c + d*x] + Sin[c + d*x])) + (5*e^(7/2)*ArcTan[(Sqrt[e]*S
in[c + d*x])/(Sqrt[e*Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]])]*Sqrt[1 + Cos[c
+ d*x]]*Sqrt[a + a*Sin[c + d*x]])/(4*a^2*d*(1 + Cos[c + d*x] + Sin[c + d*x]
))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 2756

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[cos[(e_.) + (f_.)*(x_)]
*(g_.)], x_Symbol] :> Dist[a*Sqrt[1 + Cos[e + f*x]]*(Sqrt[a + b*Sin[e + f*x
]]/(a + a*Cos[e + f*x] + b*Sin[e + f*x])), Int[Sqrt[1 + Cos[e + f*x]]/Sqrt[
g*Cos[e + f*x]], x], x] + Dist[b*Sqrt[1 + Cos[e + f*x]]*(Sqrt[a + b*Sin[e +
 f*x]]/(a + a*Cos[e + f*x] + b*Sin[e + f*x])), Int[Sin[e + f*x]/(Sqrt[g*Cos
[e + f*x]]*Sqrt[1 + Cos[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, g}, x] &&
EqQ[a^2 - b^2, 0]
```

Rule 2758

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(a*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2764

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]], x_Symbol] :> Simp[g*Sqrt[g*Cos[e + f*x]]*(Sqrt[a + b*Sin[e + f*x
]]/(b*f)), x] + Dist[g^2/(2*a), Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[g*Cos[e +
 f*x]], x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2854

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b + d*x^2), x], x
, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]))], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
```

```
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{7/2}}{(a+a\sin(c+dx))^{3/2}}\,dx = \frac{e(e\cos(c+dx))^{5/2}}{2ad\sqrt{a+a\sin(c+dx)}} + \frac{(5e^2)\int \frac{(e\cos(c+dx))^{3/2}}{\sqrt{a+a\sin(c+dx)}}\,dx}{4a}$$

$$= \frac{e(e\cos(c+dx))^{5/2}}{2ad\sqrt{a+a\sin(c+dx)}} + \frac{5e^3\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}{4a^2d} + \frac{(5e^4)\int}{}$$

$$= \frac{e(e\cos(c+dx))^{5/2}}{2ad\sqrt{a+a\sin(c+dx)}} + \frac{5e^3\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}{4a^2d} + \frac{\left(5e^4\sqrt{}\right.}{}$$

$$= \frac{e(e\cos(c+dx))^{5/2}}{2ad\sqrt{a+a\sin(c+dx)}} + \frac{5e^3\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}{4a^2d} - \frac{\left(5e^4\sqrt{}\right.}{}$$

$$= \frac{e(e\cos(c+dx))^{5/2}}{2ad\sqrt{a+a\sin(c+dx)}} + \frac{5e^3\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}{4a^2d} + \frac{5e^{7/2}\,\mathrm{ta}}{}$$

$$= \frac{e(e\cos(c+dx))^{5/2}}{2ad\sqrt{a+a\sin(c+dx)}} + \frac{5e^3\sqrt{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}{4a^2d} - \frac{5e^{7/2}\,\mathrm{si}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.14, size = 80, normalized size = 0.32

$$-\frac{2\,2^{3/4}(e\cos(c+dx))^{9/2}\,{}_2F_1\left(\frac{1}{4},\frac{9}{4};\frac{13}{4};\frac{1}{2}(1-\sin(c+dx))\right)\sqrt{a(1+\sin(c+dx))}}{9a^2de(1+\sin(c+dx))^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(7/2)/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*2^(3/4)*(e*Cos[c + d*x])^(9/2)*Hypergeometric2F1[1/4, 9/4, 13/4, (1 - S
in[c + d*x])/2]*Sqrt[a*(1 + Sin[c + d*x])])/(9*a^2*d*e*(1 + Sin[c + d*x])^(
11/4))
```

Maple [A]
time = 0.23, size = 266, normalized size = 1.08

method	result
default	$-\frac{(e\cos(dx+c))^{\frac{7}{2}}\left(-5\sqrt{2}\,\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\sin(dx+c)+5\sqrt{2}\,\sqrt{-\frac{2\cos(}{1+\cos}}}{8d(\cos(dx+c)\sin(dx+c)-(\cos^{2}(d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/d*(e*cos(d*x+c))^(7/2)*(-5*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^
(1/2))*sin(d*x+c)+5*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2
*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*sin(d*x+c)+4*cos(d*x+c)^2*si
n(d*x+c)+4*cos(d*x+c)^3+10*cos(d*x+c)*sin(d*x+c)-14*cos(d*x+c)^2+10*cos(d*x
+c))/(cos(d*x+c)*sin(d*x+c)-cos(d*x+c)^2-2*sin(d*x+c)-cos(d*x+c)+2)/(a*(1+s
in(d*x+c)))^(3/2)/cos(d*x+c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima"
)
```

```
[Out] e^(7/2)*integrate(cos(d*x + c)^(7/2)/(a*sin(d*x + c) + a)^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3361 vs. 2(192) = 384.
time = 190.26, size = 3361, normalized size = 13.61

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas"
)
```

```
[Out] 1/32*(20*sqrt(2)*a^2*d*(1/(a^6*d^4))^(1/4)*arctan(-1/4*(2*sqrt(1/2)*((sqrt(
2)*a^5*d^3*cos(d*x + c)^6 + 5*sqrt(2)*a^5*d^3*cos(d*x + c)^5 - 8*sqrt(2)*a^
5*d^3*cos(d*x + c)^4 - 20*sqrt(2)*a^5*d^3*cos(d*x + c)^3 + 8*sqrt(2)*a^5*d^
3*cos(d*x + c)^2 + 16*sqrt(2)*a^5*d^3*cos(d*x + c) + (sqrt(2)*a^5*d^3*cos(d
```

*x + c)^5 - 4*sqrt(2)*a^5*d^3*cos(d*x + c)^4 - 12*sqrt(2)*a^5*d^3*cos(d*x +
c)^3 + 8*sqrt(2)*a^5*d^3*cos(d*x + c)^2 + 16*sqrt(2)*a^5*d^3*cos(d*x + c))
*sin(d*x + c))*(1/(a^6*d^4))^(3/4)*e^(21/2) + (sqrt(2)*a^2*d*cos(d*x + c)^6
*e^7 - 3*sqrt(2)*a^2*d*cos(d*x + c)^5*e^7 - 8*sqrt(2)*a^2*d*cos(d*x + c)^4*
e^7 + 4*sqrt(2)*a^2*d*cos(d*x + c)^3*e^7 + 8*sqrt(2)*a^2*d*cos(d*x + c)^2*e
^7 - (sqrt(2)*a^2*d*cos(d*x + c)^5*e^7 + 4*sqrt(2)*a^2*d*cos(d*x + c)^4*e^7
- 4*sqrt(2)*a^2*d*cos(d*x + c)^3*e^7 - 8*sqrt(2)*a^2*d*cos(d*x + c)^2*e^7)
*sin(d*x + c))*(1/(a^6*d^4))^(1/4)*e^(7/2) - (cos(d*x + c)^4*e^(21/2) - 3*c
os(d*x + c)^3*e^(21/2) - 8*cos(d*x + c)^2*e^(21/2) + (2*a^3*d^2*cos(d*x + c
)^5*e^(7/2) - 5*a^3*d^2*cos(d*x + c)^4*e^(7/2) - 19*a^3*d^2*cos(d*x + c)^3*
e^(7/2) + 20*a^3*d^2*cos(d*x + c)*e^(7/2) + 8*a^3*d^2*e^(7/2) - (2*a^3*d^2*
cos(d*x + c)^4*e^(7/2) + 9*a^3*d^2*cos(d*x + c)^3*e^(7/2) - 4*a^3*d^2*cos(d
*x + c)^2*e^(7/2) - 20*a^3*d^2*cos(d*x + c)*e^(7/2) - 8*a^3*d^2*e^(7/2))*si
n(d*x + c))*sqrt(1/(a^6*d^4))*e^7 + 4*cos(d*x + c)*e^(21/2) - (cos(d*x + c)
^3*e^(21/2) + 4*cos(d*x + c)^2*e^(21/2) - 4*cos(d*x + c)*e^(21/2) - 8*e^(21
/2))*sin(d*x + c) + 8*e^(21/2))*sqrt(a*sin(d*x + c) + a)*sqrt(cos(d*x + c))
)*sqrt((2*a*cos(d*x + c)*e^21*sin(d*x + c) + 2*a*cos(d*x + c)*e^21 + (a^4*d
^2*e^14*sin(d*x + c) + a^4*d^2*e^14)*sqrt(1/(a^6*d^4))*e^7 + (sqrt(2)*a^5*d
^3*(1/(a^6*d^4))^(3/4)*cos(d*x + c)*e^21 + (sqrt(2)*a^2*d*e^(35/2)*sin(d*x
+ c) + sqrt(2)*a^2*d*e^(35/2))*(1/(a^6*d^4))^(1/4)*e^(7/2))*sqrt(a*sin(d*x
+ c) + a)*sqrt(cos(d*x + c)))/(sin(d*x + c) + 1)) - ((7*sqrt(2)*a^5*d^3*cos
(d*x + c)^4*e^(21/2) + 3*sqrt(2)*a^5*d^3*cos(d*x + c)^3*e^(21/2) - 16*sqrt(
2)*a^5*d^3*cos(d*x + c)^2*e^(21/2) - 4*sqrt(2)*a^5*d^3*cos(d*x + c)*e^(21/2
) + 8*sqrt(2)*a^5*d^3*e^(21/2) + (2*sqrt(2)*a^5*d^3*cos(d*x + c)^4*e^(21/2)
+ sqrt(2)*a^5*d^3*cos(d*x + c)^3*e^(21/2) - 12*sqrt(2)*a^5*d^3*cos(d*x + c
)^2*e^(21/2) - 4*sqrt(2)*a^5*d^3*cos(d*x + c)*e^(21/2) + 8*sqrt(2)*a^5*d^3*
e^(21/2))*sin(d*x + c))*(1/(a^6*d^4))^(3/4)*e^(21/2) + (2*sqrt(2)*a^2*d*cos
(d*x + c)^5*e^(35/2) + sqrt(2)*a^2*d*cos(d*x + c)^4*e^(35/2) - 13*sqrt(2)*a
^2*d*cos(d*x + c)^3*e^(35/2) - 8*sqrt(2)*a^2*d*cos(d*x + c)^2*e^(35/2) + 12
*sqrt(2)*a^2*d*cos(d*x + c)*e^(35/2) + 8*sqrt(2)*a^2*d*e^(35/2) - (7*sqrt(2
)*a^2*d*cos(d*x + c)^3*e^(35/2) + 4*sqrt(2)*a^2*d*cos(d*x + c)^2*e^(35/2) -
12*sqrt(2)*a^2*d*cos(d*x + c)*e^(35/2) - 8*sqrt(2)*a^2*d*e^(35/2))*sin(d*x
+ c))*(1/(a^6*d^4))^(1/4)*e^(7/2))*sqrt(a*sin(d*x + c) + a)*sqrt(cos(d*x +
c)))/(a*cos(d*x + c)^6*e^21 + a*cos(d*x + c)^5*e^21 - 8*a*cos(d*x + c)^4*e
^21 - 8*a*cos(d*x + c)^3*e^21 + 8*a*cos(d*x + c)^2*e^21 + 8*a*cos(d*x + c)*
e^21 - 4*(a*cos(d*x + c)^4*e^21 + a*cos(d*x + c)^3*e^21 - 2*a*cos(d*x + c)^
2*e^21 - 2*a*cos(d*x + c)*e^21)*sin(d*x + c)))*e^(7/2) - 20*sqrt(2)*a^2*d*(
1/(a^6*d^4))^(1/4)*arctan(1/4*(2*sqrt(1/2)*((sqrt(2)*a^5*d^3*cos(d*x + c)^6
+ 5*sqrt(2)*a^5*d^3*cos(d*x + c)^5 - 8*sqrt(2)*a^5*d^3*cos(d*x + c)^4 - 20
*sqrt(2)*a^5*d^3*cos(d*x + c)^3 + 8*sqrt(2)*a^5*d^3*cos(d*x + c)^2 + 16*sqr
t(2)*a^5*d^3*cos(d*x + c) + (sqrt(2)*a^5*d^3*cos(d*x + c)^5 - 4*sqrt(2)*a^5
*d^3*cos(d*x + c)^4 - 12*sqrt(2)*a^5*d^3*cos(d*x + c)^3 + 8*sqrt(2)*a^5*d^3
*cos(d*x + c)^2 + 16*sqrt(2)*a^5*d^3*cos(d*x + c))*sin(d*x + c))*(1/(a^6*d^
4))^(3/4)*e^(21/2) + (sqrt(2)*a^2*d*cos(d*x + c)^6*e^7 - 3*sqrt(2)*a^2*d*co
s(d*x + c)^5*e^7 - 8*sqrt(2)*a^2*d*cos(d*x + c)^4*e^7 + 4*sqrt(2)*a^2*d*cos

```
(d*x + c)^3*e^7 + 8*sqrt(2)*a^2*d*cos(d*x + c)^2*e^7 - (sqrt(2)*a^2*d*cos(d
*x + c)^5*e^7 + 4*sqrt(2)*a^2*d*cos(d*x + c)^4*e^7 - 4*sqrt(2)*a^2*d*cos(d*
x + c)^3*e^7 - 8*sqrt(2)*a^2*d*cos(d*x + c)^2*e^7)*sin(d*x + c))*(1/(a^6*d^
4))^(1/4)*e^(7/2) + (cos(d*x + c)^4*e^(21/2) - 3*cos(d*x + c)^3*e^(21/2) -
8*cos(d*x + c)^2*e^(21/2) + (2*a^3*d^2*cos(d*x + c)^5*e^(7/2) - 5*a^3*d^2*c
os(d*x + c)^4*e^(7/2) - 19*a^3*d^2*cos(d*x + c)^3*e^(7/2) + 20*a^3*d^2*cos(
d*x + c)*e^(7/2) + 8*a^3*d^2*e^(7/2) - (2*a^3*d^2*cos(d*x + c)^4*e^(7/2) +
9*a^3*d^2*cos(d*x + c)^3*e^(7/2) - 4*a^3*d^2*cos(d*x + c)^2*e^(7/2) - 20*a^
3*d^2*cos(d*x + c)*e^(7/2) - 8*a^3*d^2*e^(7/2))*sin(d*x + c))*sqrt(1/(a^6*d
^4))*e^7 + 4*cos(d*x + c)*e^(21/2) - (cos(d*x + c)^3*e^(21/2) + 4*cos(d*x +
 c)^2*e^(21/2) - 4*cos(d*x + c)*e^(21/2) - 8*e^(21/2))*sin(d*x + c) + 8*e^(
21/2))*sqrt(a*sin(d*x + c) + a)*sqrt(cos(d*x + c)))*sqrt((2*a*cos(d*x + c)*
e^21*sin(d*x + c) + 2*a*cos(d*x + c)*e^21 + (a^4*d^2*e^14*sin(d*x + c) + a^
4*d^2*e^14)*sqrt(1/(a^6*d^4))*e^7 - (sqrt(2)*a^5*d^3*(1/(a^6*d^4))^(3/4)*co
s(d*x + c)*e^21 + (sqrt(2)*a^2*d*e^(35/2)*sin(d*x + c) + sqrt(2)*a^2*d*e^(3
5/2))*(1/(a^6*d^4))^(1/4)*e^(7/2))*sqrt(a*sin(d*x + c) + a)*sqrt(cos(d*x +
c)))/(sin(d*x + c) + 1)) - ((7*sqrt(2)*a^5*d^3*cos(d*x + c)^4*e^(21/2) + 3*
sqrt(2)*a^5*d^3*cos(d*x + c)^3*e^(21/2) - 16*sq...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(7/2)/(a+a*sin(d*x+c))**(3/2),x)
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,\cos(c+d\,x))^{7/2}}{(a+a\,\sin(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(7/2)/(a + a*sin(c + d*x))^(3/2),x)
[Out] int((e*cos(c + d*x))^(7/2)/(a + a*sin(c + d*x))^(3/2), x)
```

3.306 $\int \frac{(e\cos(c+dx))^{5/2}}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=215

$$\frac{e(e\cos(c+dx))^{3/2}}{ad\sqrt{a+a\sin(c+dx)}}+\frac{3e^{5/2}\sinh^{-1}\left(\frac{\sqrt{e\cos(c+dx)}}{\sqrt{e}}\right)\sqrt{1+\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}{d\left(a^2+a^2\cos(c+dx)+a^2\sin(c+dx)\right)}+\frac{3e^{5/2}\tan}{}$$

```
[Out] e*(e*cos(d*x+c))^(3/2)/a/d/(a+a*sin(d*x+c))^(1/2)+3*e^(5/2)*arcsinh((e*cos(
d*x+c))^(1/2)/e^(1/2))*(1+cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^(1/2)/d/(a^2+a
^2*cos(d*x+c)+a^2*sin(d*x+c))+3*e^(5/2)*arctan(sin(d*x+c)*e^(1/2)/(e*cos(d*
x+c))^(1/2)/(1+cos(d*x+c))^(1/2))*(1+cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^(1/
2)/d/(a^2+a^2*cos(d*x+c)+a^2*sin(d*x+c))
```

Rubi [A]
time = 0.19, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2758, 2763, 2854, 209, 2912, 65, 221}

$$\frac{3e^{5/2}\sqrt{\cos(c+dx)+1}\sqrt{a\sin(c+dx)+a}\,\text{ArcTan}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\sqrt{e\cos(c+dx)}}\right)}{d\left(a^2\sin(c+dx)+a^2\cos(c+dx)+a^2\right)}+\frac{3e^{5/2}\sqrt{\cos(c+dx)+1}\sqrt{a\sin(c+dx)+a}\sinh^{-1}\left(\frac{\sqrt{e\cos(c+dx)}}{\sqrt{e}}\right)}{d\left(a^2\sin(c+dx)+a^2\cos(c+dx)+a^2\right)}+\frac{e(e\cos(c+dx))^{3/2}}{ad\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(5/2)/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (e*(e*Cos[c + d*x])^(3/2))/(a*d*Sqrt[a + a*Sin[c + d*x]]) + (3*e^(5/2)*ArcS
inh[Sqrt[e*Cos[c + d*x]]/Sqrt[e]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c +
 d*x]])/(d*(a^2 + a^2*Cos[c + d*x] + a^2*Sin[c + d*x])) + (3*e^(5/2)*ArcTan
[(Sqrt[e]*Sin[c + d*x])/(Sqrt[e*Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]])]*Sqrt
[1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(d*(a^2 + a^2*Cos[c + d*x] + a
^2*Sin[c + d*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 2758

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(a*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2763

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[g*Sqrt[1 + Cos[e + f*x]]*(Sqrt[a + b*Sin[e + f*x
]]/(a + a*Cos[e + f*x] + b*Sin[e + f*x])), Int[Sqrt[1 + Cos[e + f*x]]/Sqrt[
g*Cos[e + f*x]], x], x] - Dist[g*Sqrt[1 + Cos[e + f*x]]*(Sqrt[a + b*Sin[e +
 f*x]]/(b + b*Cos[e + f*x] + a*Sin[e + f*x])), Int[Sin[e + f*x]/(Sqrt[g*Cos
[e + f*x]]*Sqrt[1 + Cos[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, g}, x] &&
EqQ[a^2 - b^2, 0]
```

Rule 2854

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b + d*x^2), x], x
, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]))], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{5/2}}{(a+a\sin(c+dx))^{3/2}}\,dx = \frac{e(e\cos(c+dx))^{3/2}}{ad\sqrt{a+a\sin(c+dx)}} + \frac{(3e^2)\int \frac{\sqrt{e\cos(c+dx)}}{\sqrt{a+a\sin(c+dx)}}\,dx}{2a}$$

$$= \frac{e(e\cos(c+dx))^{3/2}}{ad\sqrt{a+a\sin(c+dx)}} + \frac{\left(3e^3\sqrt{1+\cos(c+dx)}\,\sqrt{a+a\sin(c+dx)}\right)\int \frac{\sqrt{1}}{\sqrt{}}}{2a(a+a\cos(c+dx)+a\sin(c+dx}$$

$$= \frac{e(e\cos(c+dx))^{3/2}}{ad\sqrt{a+a\sin(c+dx)}} + \frac{\left(3e^3\sqrt{1+\cos(c+dx)}\,\sqrt{a+a\sin(c+dx)}\right)\text{Subst}}{2ad(a+a\cos(c+dx)+a}$$

$$= \frac{e(e\cos(c+dx))^{3/2}}{ad\sqrt{a+a\sin(c+dx)}} + \frac{3e^{5/2}\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{e\cos(c+dx)}\sqrt{1+\cos(c+dx)}}\right)}{d\left(a^2+a^2\cos(c+dx)+a\right.}$$

$$= \frac{e(e\cos(c+dx))^{3/2}}{ad\sqrt{a+a\sin(c+dx)}} + \frac{3e^{5/2}\sinh^{-1}\left(\frac{\sqrt{e\cos(c+dx)}}{\sqrt{e}}\right)\sqrt{1+\cos(c+dx)}}{d\left(a^2+a^2\cos(c+dx)+a^2\sin(c+\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.15, size = 80, normalized size = 0.37

$$-\frac{2\sqrt[4]{2}\,(e\cos(c+dx))^{7/2}\,{}_2F_1\left(\frac{3}{4},\frac{7}{4};\frac{11}{4};\frac{1}{2}(1-\sin(c+dx))\right)\sqrt{a(1+\sin(c+dx))}}{7a^2de(1+\sin(c+dx))^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(5/2)/(a + a*Sin[c + d*x])^(3/2),x]
[Out] (-2*2^(1/4)*(e*Cos[c + d*x])^(7/2)*Hypergeometric2F1[3/4, 7/4, 11/4, (1 - S
in[c + d*x])/2]*Sqrt[a*(1 + Sin[c + d*x])])/(7*a^2*d*e*(1 + Sin[c + d*x])^(
9/4))
```

Maple [A]
time = 0.16, size = 232, normalized size = 1.08

method	result

default	$\frac{(e\cos(dx+c))^{\frac{5}{2}}\left(3\sqrt{2}\,\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2}}{2}\right)\sin(dx+c)+3\sqrt{2}\,\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\mathrm{arcta}}{2d(\cos(dx+c)\sin(dx+c)-(\cos^2(dx+c))-2\sin(dx+c)-}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d*(e*cos(d*x+c))^(5/2)*(3*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*sin(d*x+c)+3*2^(1/
2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*sin(d*x+c)-2*cos(d*x+c)*sin(d*
x+c)-2*cos(d*x+c)^2+2*cos(d*x+c))/(cos(d*x+c)*sin(d*x+c)-cos(d*x+c)^2-2*sin
(d*x+c)-cos(d*x+c)+2)/(a*(1+sin(d*x+c)))^(3/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima"
)
```

```
[Out] e^(5/2)*integrate(cos(d*x + c)^(5/2)/(a*sin(d*x + c) + a)^(3/2), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3431 vs. $2(175) = 350$.
time = 191.10, size = 3431, normalized size = 15.96

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas"
)
```

```
[Out] -1/8*(12*(sqrt(2)*a^2*d*sin(d*x + c) + sqrt(2)*a^2*d)*(1/(a^6*d^4))^(1/4)*a
rctan(-1/4*(2*sqrt(1/2)*((sqrt(2)*a^5*d^3*cos(d*x + c)^6 - 3*sqrt(2)*a^5*d^
3*cos(d*x + c)^5 - 8*sqrt(2)*a^5*d^3*cos(d*x + c)^4 + 4*sqrt(2)*a^5*d^3*cos
(d*x + c)^3 + 8*sqrt(2)*a^5*d^3*cos(d*x + c)^2 - (sqrt(2)*a^5*d^3*cos(d*x +
 c)^5 + 4*sqrt(2)*a^5*d^3*cos(d*x + c)^4 - 4*sqrt(2)*a^5*d^3*cos(d*x + c)^3
 - 8*sqrt(2)*a^5*d^3*cos(d*x + c)^2)*sin(d*x + c))*(1/(a^6*d^4))^(3/4)*e^(1
5/2) + (sqrt(2)*a^2*d*cos(d*x + c)^6*e^5 + 5*sqrt(2)*a^2*d*cos(d*x + c)^5*e
^5 - 8*sqrt(2)*a^2*d*cos(d*x + c)^4*e^5 - 20*sqrt(2)*a^2*d*cos(d*x + c)^3*e
```

^5 + 8*sqrt(2)*a^2*d*cos(d*x + c)^2*e^5 + 16*sqrt(2)*a^2*d*cos(d*x + c)*e^5 + (sqrt(2)*a^2*d*cos(d*x + c)^5*e^5 - 4*sqrt(2)*a^2*d*cos(d*x + c)^4*e^5 - 12*sqrt(2)*a^2*d*cos(d*x + c)^3*e^5 + 8*sqrt(2)*a^2*d*cos(d*x + c)^2*e^5 + 16*sqrt(2)*a^2*d*cos(d*x + c)*e^5)*sin(d*x + c))*(1/(a^6*d^4))^(1/4)*e^(5/2) - (cos(d*x + c)^4*e^(15/2) - 3*cos(d*x + c)^3*e^(15/2) - 8*cos(d*x + c)^2*e^(15/2) + (2*a^3*d^2*cos(d*x + c)^5*e^(5/2) - 5*a^3*d^2*cos(d*x + c)^4*e^(5/2) - 19*a^3*d^2*cos(d*x + c)^3*e^(5/2) + 20*a^3*d^2*cos(d*x + c)*e^(5/2) + 8*a^3*d^2*e^(5/2) - (2*a^3*d^2*cos(d*x + c)^4*e^(5/2) + 9*a^3*d^2*cos(d*x + c)^3*e^(5/2) - 4*a^3*d^2*cos(d*x + c)^2*e^(5/2) - 20*a^3*d^2*cos(d*x + c)*e^(5/2) - 8*a^3*d^2*e^(5/2))*sin(d*x + c))*sqrt(1/(a^6*d^4))*e^5 + 4*cos(d*x + c)*e^(15/2) - (cos(d*x + c)^3*e^(15/2) + 4*cos(d*x + c)^2*e^(15/2) - 4*cos(d*x + c)*e^(15/2) - 8*e^(15/2))*sin(d*x + c) + 8*e^(15/2))*sqrt(a*sin(d*x + c) + a)*sqrt(cos(d*x + c)))*sqrt((2*a*cos(d*x + c)*e^15*sin(d*x + c) + 2*a*cos(d*x + c)*e^15 + (a^4*d^2*e^10*sin(d*x + c) + a^4*d^2*e^10)*sqrt(1/(a^6*d^4))*e^5 + (sqrt(2)*a^2*d*(1/(a^6*d^4))^(1/4)*cos(d*x + c)*e^15 + (sqrt(2)*a^5*d^3*e^(15/2)*sin(d*x + c) + sqrt(2)*a^5*d^3*e^(15/2))*(1/(a^6*d^4))^(3/4)*e^(15/2))*sqrt(a*sin(d*x + c) + a)*sqrt(cos(d*x + c)))/(sin(d*x + c) + 1)) + ((2*sqrt(2)*a^5*d^3*cos(d*x + c)^5*e^(15/2) + sqrt(2)*a^5*d^3*cos(d*x + c)^4*e^(15/2) - 13*sqrt(2)*a^5*d^3*cos(d*x + c)^3*e^(15/2) - 8*sqrt(2)*a^5*d^3*cos(d*x + c)^2*e^(15/2) + 12*sqrt(2)*a^5*d^3*cos(d*x + c)*e^(15/2) + 8*sqrt(2)*a^5*d^3*e^(15/2) - (7*sqrt(2)*a^5*d^3*cos(d*x + c)^3*e^(15/2) + 4*sqrt(2)*a^5*d^3*cos(d*x + c)^2*e^(15/2) - 12*sqrt(2)*a^5*d^3*cos(d*x + c)*e^(15/2) - 8*sqrt(2)*a^5*d^3*e^(15/2))*sin(d*x + c))*(1/(a^6*d^4))^(3/4)*e^(15/2) + (7*sqrt(2)*a^2*d*cos(d*x + c)^4*e^(25/2) + 3*sqrt(2)*a^2*d*cos(d*x + c)^3*e^(25/2) - 16*sqrt(2)*a^2*d*cos(d*x + c)^2*e^(25/2) - 4*sqrt(2)*a^2*d*cos(d*x + c)*e^(25/2) + 8*sqrt(2)*a^2*d*e^(25/2) + (2*sqrt(2)*a^2*d*cos(d*x + c)^4*e^(25/2) + sqrt(2)*a^2*d*cos(d*x + c)^3*e^(25/2) - 12*sqrt(2)*a^2*d*cos(d*x + c)^2*e^(25/2) - 4*sqrt(2)*a^2*d*cos(d*x + c)*e^(25/2) + 8*sqrt(2)*a^2*d*e^(25/2))*sin(d*x + c))*(1/(a^6*d^4))^(1/4)*e^(5/2))*sqrt(a*sin(d*x + c) + a)*sqrt(cos(d*x + c)))/(a*cos(d*x + c)^6*e^15 + a*cos(d*x + c)^5*e^15 - 8*a*cos(d*x + c)^4*e^15 - 8*a*cos(d*x + c)^3*e^15 + 8*a*cos(d*x + c)^2*e^15 + 8*a*cos(d*x + c)*e^15 - 4*(a*cos(d*x + c)^4*e^15 + a*cos(d*x + c)^3*e^15 - 2*a*cos(d*x + c)^2*e^15 - 2*a*cos(d*x + c)*e^15)*sin(d*x + c)))*e^(5/2) - 12*(sqrt(2)*a^2*d*sin(d*x + c) + sqrt(2)*a^2*d)*(1/(a^6*d^4))^(1/4)*arctan(1/4*(2*sqrt(1/2)*((sqrt(2)*a^5*d^3*cos(d*x + c)^6 - 3*sqrt(2)*a^5*d^3*cos(d*x + c)^5 - 8*sqrt(2)*a^5*d^3*cos(d*x + c)^4 + 4*sqrt(2)*a^5*d^3*cos(d*x + c)^3 + 8*sqrt(2)*a^5*d^3*cos(d*x + c)^2 - (sqrt(2)*a^5*d^3*cos(d*x + c)^5 + 4*sqrt(2)*a^5*d^3*cos(d*x + c)^4 - 4*sqrt(2)*a^5*d^3*cos(d*x + c)^3 - 8*sqrt(2)*a^5*d^3*cos(d*x + c)^2)*sin(d*x + c))*(1/(a^6*d^4))^(3/4)*e^(15/2) + (sqrt(2)*a^2*d*cos(d*x + c)^6*e^5 + 5*sqrt(2)*a^2*d*cos(d*x + c)^5*e^5 - 8*sqrt(2)*a^2*d*cos(d*x + c)^4*e^5 - 20*sqrt(2)*a^2*d*cos(d*x + c)^3*e^5 + 8*sqrt(2)*a^2*d*cos(d*x + c)^2*e^5 + 16*sqrt(2)*a^2*d*cos(d*x + c)*e^5 + (sqrt(2)*a^2*d*cos(d*x + c)^5*e^5 - 4*sqrt(2)*a^2*d*cos(d*x + c)^4*e^5 - 12*sqrt(2)*a^2*d*cos(d*x + c)^3*e^5 + 8*sqrt(2)*a^2*d*cos(d*x + c)^2*e^5 + 16*sqrt(2)*a^2*d*cos(d*x + c)*e^5)*sin(d*x + c))*(1/(a^6*d^4)

```
)^(1/4)*e^(5/2) + (cos(d*x + c)^4*e^(15/2) - 3*cos(d*x + c)^3*e^(15/2) - 8*
cos(d*x + c)^2*e^(15/2) + (2*a^3*d^2*cos(d*x + c)^5*e^(5/2) - 5*a^3*d^2*cos
(d*x + c)^4*e^(5/2) - 19*a^3*d^2*cos(d*x + c)^3*e^(5/2) + 20*a^3*d^2*cos(d*
x + c)*e^(5/2) + 8*a^3*d^2*e^(5/2) - (2*a^3*d^2*cos(d*x + c)^4*e^(5/2) + 9*
a^3*d^2*cos(d*x + c)^3*e^(5/2) - 4*a^3*d^2*cos(d*x + c)^2*e^(5/2) - 20*a^3*
d^2*cos(d*x + c)*e^(5/2) - 8*a^3*d^2*e^(5/2))*sin(d*x + c))*sqrt(1/(a^6*d^4
))*e^5 + 4*cos(d*x + c)*e^(15/2) - (cos(d*x + c)^3*e^(15/2) + 4*cos(d*x + c
)^2*e^(15/2) - 4*cos(d*x + c)*e^(15/2) - 8*e^(15/2))*sin(d*x + c) + 8*e^(15
/2))*sqrt(a*sin(d*x + c) + a)*sqrt(cos(d*x + c)))*sqrt((2*a*cos(d*x + c)*e^
15*sin(d*x + c) + 2*a*cos(d*x + c)*e^15 + (a^4*d^2*e^10*sin(d*x + c) + a^4*
d^2*e^10)*sqrt(1/(a^6*d^4))*e^5 - (sqrt(2)*a^2*d*(1/(a^6*d^4))^(1/4)*cos(d*
x + c)*e^15 + (sqrt(2)*a^5*d^3*e^(15/2)*sin(d*x + c) + sqrt(2)*a^5*d^3*e^(1
5/2))*(1/(a^6*d^4))^(3/4)*e^(15/2))*sqrt(a*sin(d*x + c) + a)*sqrt(cos(d*x +
 c)))/(sin(d*x + c) + 1)) + ((2*sqrt(2)*a^5*d^3...
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(5/2)/(a+a*sin(d*x+c))**(3/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,\cos\left(c+d\,x\right))^{5/2}}{(a+a\,\sin\left(c+d\,x\right))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(5/2)/(a + a*sin(c + d*x))^(3/2),x)
[Out] int((e*cos(c + d*x))^(5/2)/(a + a*sin(c + d*x))^(3/2), x)
```

3.307 $\int \frac{(e\cos(c+dx))^{3/2}}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=236

$$-\frac{2(e\cos(c+dx))^{5/2}}{de(a+a\sin(c+dx))^{3/2}}-\frac{2e\sqrt{e\cos(c+dx)}\,\sqrt{a+a\sin(c+dx)}}{a^2d}+\frac{2e^{3/2}\sinh^{-1}\left(\frac{\sqrt{e\cos(c+dx)}}{\sqrt{e}}\right)\sqrt{1+}}{a^2d(1+\cos(c+dx)}$$

```
[Out] -2*(e*cos(d*x+c))^(5/2)/d/e/(a+a*sin(d*x+c))^(3/2)-2*e*(e*cos(d*x+c))^(1/2)
*(a+a*sin(d*x+c))^(1/2)/a^2/d+2*e^(3/2)*arcsinh((e*cos(d*x+c))^(1/2)/e^(1/2
))*(1+cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^(1/2)/a^2/d/(1+cos(d*x+c)+sin(d*x+
c))-2*e^(3/2)*arctan(sin(d*x+c)*e^(1/2)/(e*cos(d*x+c))^(1/2)/(1+cos(d*x+c))
^(1/2))*(1+cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^(1/2)/a^2/d/(1+cos(d*x+c)+sin
(d*x+c))
```

Rubi [A]

time = 0.24, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {2760, 2764, 2756, 2854, 209, 2912, 65, 221}

$$-\frac{2e^{3/2}\sqrt{\cos(c+dx)+1}\,\sqrt{a\sin(c+dx)+a}\,\text{ArcTan}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\,\sqrt{e\cos(c+dx)}}\right)}{a^2d(\sin(c+dx)+\cos(c+dx)+1)}+\frac{2e^{3/2}\sqrt{\cos(c+dx)+1}\,\sqrt{a\sin(c+dx)+a}\,\sinh^{-1}\left(\frac{\sqrt{e\cos(c+dx)}}{\sqrt{e}}\right)}{a^2d(\sin(c+dx)+\cos(c+dx)+1)}-\frac{2e\sqrt{a\sin(c+dx)+a}\,\sqrt{e\cos(c+dx)}}{a^2d}-\frac{2(e\cos(c+dx))^{5/2}}{de(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(3/2)/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*(e*Cos[c + d*x])^(5/2))/(d*e*(a + a*Sin[c + d*x])^(3/2)) - (2*e*Sqrt[e*
Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(a^2*d) + (2*e^(3/2)*ArcSinh[Sqrt[e
*Cos[c + d*x]]/Sqrt[e]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(a
^2*d*(1 + Cos[c + d*x] + Sin[c + d*x])) - (2*e^(3/2)*ArcTan[(Sqrt[e]*Sin[c
+ d*x])/(Sqrt[e*Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]])]*Sqrt[1 + Cos[c + d*x
]]*Sqrt[a + a*Sin[c + d*x]])/(a^2*d*(1 + Cos[c + d*x] + Sin[c + d*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 2756

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[cos[(e_.) + (f_.)*(x_)]
*(g_.)], x_Symbol] :> Dist[a*Sqrt[1 + Cos[e + f*x]]*(Sqrt[a + b*Sin[e + f*x
]]/(a + a*Cos[e + f*x] + b*Sin[e + f*x])), Int[Sqrt[1 + Cos[e + f*x]]/Sqrt[
g*Cos[e + f*x]], x], x] + Dist[b*Sqrt[1 + Cos[e + f*x]]*(Sqrt[a + b*Sin[e +
 f*x]]/(a + a*Cos[e + f*x] + b*Sin[e + f*x])), Int[Sin[e + f*x]/(Sqrt[g*Cos
[e + f*x]]*Sqrt[1 + Cos[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, g}, x] &&
EqQ[a^2 - b^2, 0]
```

Rule 2760

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*(2*m + p + 1))), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2764

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]], x_Symbol] :> Simp[g*Sqrt[g*Cos[e + f*x]]*(Sqrt[a + b*Sin[e + f*x
]]/(b*f)), x] + Dist[g^2/(2*a), Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[g*Cos[e +
 f*x]], x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2854

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b + d*x^2), x], x
, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]))], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{3/2}}{(a+a\sin(c+dx))^{3/2}}\,dx = -\frac{2(e\cos(c+dx))^{5/2}}{de(a+a\sin(c+dx))^{3/2}} - \frac{2\int \frac{(e\cos(c+dx))^{3/2}}{\sqrt{a+a\sin(c+dx)}}\,dx}{a}$$

$$= -\frac{2(e\cos(c+dx))^{5/2}}{de(a+a\sin(c+dx))^{3/2}} - \frac{2e\sqrt{e\cos(c+dx)}\,\sqrt{a+a\sin(c+dx)}}{a^2d} - \frac{e^2\int \cdots}{\cdots}$$

$$= -\frac{2(e\cos(c+dx))^{5/2}}{de(a+a\sin(c+dx))^{3/2}} - \frac{2e\sqrt{e\cos(c+dx)}\,\sqrt{a+a\sin(c+dx)}}{a^2d} - \frac{\left(e^2\sqrt{\cdots}\right.}{\cdots}$$

$$= -\frac{2(e\cos(c+dx))^{5/2}}{de(a+a\sin(c+dx))^{3/2}} - \frac{2e\sqrt{e\cos(c+dx)}\,\sqrt{a+a\sin(c+dx)}}{a^2d} + \frac{\left(e^2\sqrt{\cdots}\right.}{\cdots}$$

$$= -\frac{2(e\cos(c+dx))^{5/2}}{de(a+a\sin(c+dx))^{3/2}} - \frac{2e\sqrt{e\cos(c+dx)}\,\sqrt{a+a\sin(c+dx)}}{a^2d} - \frac{2e^{3/2}\,\mathrm{t}\cdots}{\cdots}$$

$$= -\frac{2(e\cos(c+dx))^{5/2}}{de(a+a\sin(c+dx))^{3/2}} - \frac{2e\sqrt{e\cos(c+dx)}\,\sqrt{a+a\sin(c+dx)}}{a^2d} + \frac{2e^{3/2}\,\mathrm{s}\cdots}{\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.13, size = 80, normalized size = 0.34

$$-\frac{2^{3/4}(e\cos(c+dx))^{5/2}\,{}_2F_1\left(\frac{5}{4},\frac{5}{4};\frac{9}{4};\frac{1}{2}(1-\sin(c+dx))\right)\sqrt{a(1+\sin(c+dx))}}{5a^2de(1+\sin(c+dx))^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(3/2)/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -1/5*(2^(3/4)*(e*Cos[c + d*x])^(5/2)*Hypergeometric2F1[5/4, 5/4, 9/4, (1 -
Sin[c + d*x])/2]*Sqrt[a*(1 + Sin[c + d*x])])/(a^2*d*e*(1 + Sin[c + d*x])^(7
/4))
```

Maple [A]
time = 0.16, size = 321, normalized size = 1.36

method	result
default	$\frac{2(e\cos(dx+c))^{\frac{3}{2}}(-1+\cos(dx+c))\left(-\sqrt{2}\,\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2}}{2}\right)\sin(dx+c)+\sqrt{2}\,\arctanh\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}}{2\cos(dx}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(e*cos(d*x+c))^(3/2)*(-1+cos(d*x+c))*(-2^(1/2)*arctan(1/2*(-2*cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*sin(d*x+c)+2^(1/2)*arctanh(1/2*(-2*cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*sin(d*x+c)+2*(-2*c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)-2^(1/2)*arctan(1/2*(-2*cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*2^(1/2))+2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))+2*(-2*cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*sin(d*x+c)+2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))/sin(d*x+c)/(-2
*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/(a*(1+sin(d*x+c)))^(3/2)/(cos(d*x+c)-1+si
n(d*x+c))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima"
)
```

```
[Out] e^(3/2)*integrate(cos(d*x + c)^(3/2)/(a*sin(d*x + c) + a)^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3519 vs. $2(189) = 378$.
time = 188.74, size = 3519, normalized size = 14.91

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas"
)
```

```
[Out] -1/4*(4*(sqrt(2)*a^2*d*cos(d*x + c)^2 - 2*sqrt(2)*a^2*d*sin(d*x + c) - 2*sq
rt(2)*a^2*d)*(1/(a^6*d^4))^(1/4)*arctan(-1/4*(sqrt(2)*((sqrt(2)*a^5*d^3*cos
(d*x + c)^6 + 5*sqrt(2)*a^5*d^3*cos(d*x + c)^5 - 8*sqrt(2)*a^5*d^3*cos(d*x
```

+ c)^4 - 20*sqrt(2)*a^5*d^3*cos(d*x + c)^3 + 8*sqrt(2)*a^5*d^3*cos(d*x + c)
^2 + 16*sqrt(2)*a^5*d^3*cos(d*x + c) + (sqrt(2)*a^5*d^3*cos(d*x + c)^5 - 4*
sqrt(2)*a^5*d^3*cos(d*x + c)^4 - 12*sqrt(2)*a^5*d^3*cos(d*x + c)^3 + 8*sqrt
(2)*a^5*d^3*cos(d*x + c)^2 + 16*sqrt(2)*a^5*d^3*cos(d*x + c))*sin(d*x + c))
*(1/(a^6*d^4))^(3/4)*e^(9/2) + (sqrt(2)*a^2*d*cos(d*x + c)^6*e^3 - 3*sqrt(2
)*a^2*d*cos(d*x + c)^5*e^3 - 8*sqrt(2)*a^2*d*cos(d*x + c)^4*e^3 + 4*sqrt(2)
*a^2*d*cos(d*x + c)^3*e^3 + 8*sqrt(2)*a^2*d*cos(d*x + c)^2*e^3 - (sqrt(2)*a
^2*d*cos(d*x + c)^5*e^3 + 4*sqrt(2)*a^2*d*cos(d*x + c)^4*e^3 - 4*sqrt(2)*a^
2*d*cos(d*x + c)^3*e^3 - 8*sqrt(2)*a^2*d*cos(d*x + c)^2*e^3)*sin(d*x + c))*
(1/(a^6*d^4))^(1/4)*e^(3/2) - (cos(d*x + c)^4*e^(9/2) - 3*cos(d*x + c)^3*e^
(9/2) - 8*cos(d*x + c)^2*e^(9/2) + (2*a^3*d^2*cos(d*x + c)^5*e^(3/2) - 5*a^
3*d^2*cos(d*x + c)^4*e^(3/2) - 19*a^3*d^2*cos(d*x + c)^3*e^(3/2) + 20*a^3*d
^2*cos(d*x + c)*e^(3/2) + 8*a^3*d^2*e^(3/2) - (2*a^3*d^2*cos(d*x + c)^4*e^(
3/2) + 9*a^3*d^2*cos(d*x + c)^3*e^(3/2) - 4*a^3*d^2*cos(d*x + c)^2*e^(3/2)
- 20*a^3*d^2*cos(d*x + c)*e^(3/2) - 8*a^3*d^2*e^(3/2))*sin(d*x + c))*sqrt(1
/(a^6*d^4))*e^3 + 4*cos(d*x + c)*e^(9/2) - (cos(d*x + c)^3*e^(9/2) + 4*cos(
d*x + c)^2*e^(9/2) - 4*cos(d*x + c)*e^(9/2) - 8*e^(9/2))*sin(d*x + c) + 8*e
^(9/2))*sqrt(a*sin(d*x + c) + a)*sqrt(cos(d*x + c)))*sqrt((2*a*cos(d*x + c)
*e^9*sin(d*x + c) + 2*a*cos(d*x + c)*e^9 + (a^4*d^2*e^6*sin(d*x + c) + a^4*
d^2*e^6)*sqrt(1/(a^6*d^4))*e^3 + (sqrt(2)*a^5*d^3*(1/(a^6*d^4))^(3/4)*cos(d
*x + c)*e^9 + (sqrt(2)*a^2*d*e^(15/2)*sin(d*x + c) + sqrt(2)*a^2*d*e^(15/2)
)*(1/(a^6*d^4))^(1/4)*e^(3/2))*sqrt(a*sin(d*x + c) + a)*sqrt(cos(d*x + c)))
/(sin(d*x + c) + 1)) - ((7*sqrt(2)*a^5*d^3*cos(d*x + c)^4*e^(9/2) + 3*sqrt(
2)*a^5*d^3*cos(d*x + c)^3*e^(9/2) - 16*sqrt(2)*a^5*d^3*cos(d*x + c)^2*e^(9/
2) - 4*sqrt(2)*a^5*d^3*cos(d*x + c)*e^(9/2) + 8*sqrt(2)*a^5*d^3*e^(9/2) + (
2*sqrt(2)*a^5*d^3*cos(d*x + c)^4*e^(9/2) + sqrt(2)*a^5*d^3*cos(d*x + c)^3*e
^(9/2) - 12*sqrt(2)*a^5*d^3*cos(d*x + c)^2*e^(9/2) - 4*sqrt(2)*a^5*d^3*cos(
d*x + c)*e^(9/2) + 8*sqrt(2)*a^5*d^3*e^(9/2))*sin(d*x + c))*(1/(a^6*d^4))^(
3/4)*e^(9/2) + (2*sqrt(2)*a^2*d*cos(d*x + c)^5*e^(15/2) + sqrt(2)*a^2*d*cos
(d*x + c)^4*e^(15/2) - 13*sqrt(2)*a^2*d*cos(d*x + c)^3*e^(15/2) - 8*sqrt(2)
*a^2*d*cos(d*x + c)^2*e^(15/2) + 12*sqrt(2)*a^2*d*cos(d*x + c)*e^(15/2) + 8
*sqrt(2)*a^2*d*e^(15/2) - (7*sqrt(2)*a^2*d*cos(d*x + c)^3*e^(15/2) + 4*sqrt
(2)*a^2*d*cos(d*x + c)^2*e^(15/2) - 12*sqrt(2)*a^2*d*cos(d*x + c)*e^(15/2)
- 8*sqrt(2)*a^2*d*e^(15/2))*sin(d*x + c))*(1/(a^6*d^4))^(1/4)*e^(3/2))*sqrt
(a*sin(d*x + c) + a)*sqrt(cos(d*x + c)))/(a*cos(d*x + c)^6*e^9 + a*cos(d*x
+ c)^5*e^9 - 8*a*cos(d*x + c)^4*e^9 - 8*a*cos(d*x + c)^3*e^9 + 8*a*cos(d*x
+ c)^2*e^9 + 8*a*cos(d*x + c)*e^9 - 4*(a*cos(d*x + c)^4*e^9 + a*cos(d*x + c
)^3*e^9 - 2*a*cos(d*x + c)^2*e^9 - 2*a*cos(d*x + c)*e^9)*sin(d*x + c)))*e^(
3/2) - 4*(sqrt(2)*a^2*d*cos(d*x + c)^2 - 2*sqrt(2)*a^2*d*sin(d*x + c) - 2*s
qrt(2)*a^2*d)*(1/(a^6*d^4))^(1/4)*arctan(1/4*(sqrt(2)*((sqrt(2)*a^5*d^3*cos
(d*x + c)^6 + 5*sqrt(2)*a^5*d^3*cos(d*x + c)^5 - 8*sqrt(2)*a^5*d^3*cos(d*x
+ c)^4 - 20*sqrt(2)*a^5*d^3*cos(d*x + c)^3 + 8*sqrt(2)*a^5*d^3*cos(d*x + c)
^2 + 16*sqrt(2)*a^5*d^3*cos(d*x + c) + (sqrt(2)*a^5*d^3*cos(d*x + c)^5 - 4*
sqrt(2)*a^5*d^3*cos(d*x + c)^4 - 12*sqrt(2)*a^5*d^3*cos(d*x + c)^3 + 8*sqrt
(2)*a^5*d^3*cos(d*x + c)^2 + 16*sqrt(2)*a^5*d^3*cos(d*x + c))*sin(d*x + c))

```
*(1/(a^6*d^4))^(3/4)*e^(9/2) + (sqrt(2)*a^2*d*cos(d*x + c)^6*e^3 - 3*sqrt(2
)*a^2*d*cos(d*x + c)^5*e^3 - 8*sqrt(2)*a^2*d*cos(d*x + c)^4*e^3 + 4*sqrt(2)
*a^2*d*cos(d*x + c)^3*e^3 + 8*sqrt(2)*a^2*d*cos(d*x + c)^2*e^3 - (sqrt(2)*a
^2*d*cos(d*x + c)^5*e^3 + 4*sqrt(2)*a^2*d*cos(d*x + c)^4*e^3 - 4*sqrt(2)*a^
2*d*cos(d*x + c)^3*e^3 - 8*sqrt(2)*a^2*d*cos(d*x + c)^2*e^3)*sin(d*x + c))*
(1/(a^6*d^4))^(1/4)*e^(3/2) + (cos(d*x + c)^4*e^(9/2) - 3*cos(d*x + c)^3*e^
(9/2) - 8*cos(d*x + c)^2*e^(9/2) + (2*a^3*d^2*cos(d*x + c)^5*e^(3/2) - 5*a^
3*d^2*cos(d*x + c)^4*e^(3/2) - 19*a^3*d^2*cos(d*x + c)^3*e^(3/2) + 20*a^3*d
^2*cos(d*x + c)*e^(3/2) + 8*a^3*d^2*e^(3/2) - (2*a^3*d^2*cos(d*x + c)^4*e^(
3/2) + 9*a^3*d^2*cos(d*x + c)^3*e^(3/2) - 4*a^3*d^2*cos(d*x + c)^2*e^(3/2)
- 20*a^3*d^2*cos(d*x + c)*e^(3/2) - 8*a^3*d^2*e^(3/2))*sin(d*x + c))*sqrt(1
/(a^6*d^4))*e^3 + 4*cos(d*x + c)*e^(9/2) - (cos(d*x + c)^3*e^(9/2) + 4*cos(
d*x + c)^2*e^(9/2) - 4*cos(d*x + c)*e^(9/2) - 8*e^(9/2))*sin(d*x + c) + 8*e
^(9/2))*sqrt(a*sin(d*x + c) + a)*sqrt(cos(d*x + c)))*sqrt((2*a*cos(d*x + c)
*e^9*sin(d*x + c) + 2*a*cos(d*x + c)*e^9 + (a^4*d^2*e^6*sin(d*x + c) + a^4*
d^2*e^6)*sqrt(1/(a^6*d^4))*e^3 - (sqrt(2)*a^5*d^3*(1/(a^6*d^4))^(3/4)*cos(d
*x + c)*e^9 + (sqrt(2)*a^2*d*e^(15/2)*sin(d*x + c) + sqrt(2)*a^2*d*e^(15/2)
)*(1/(a^6*d^4))^(1/4)*e^(3/2))*sqrt(a*sin(d*x + c) + a)*sqrt(cos(d*x + c)))
/(sin(d*x + c) + 1)) - ((7*sqrt(2)*a^5*d^3*cos(...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(c+dx))^{\frac{3}{2}}}{(a(\sin(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(3/2)/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral((e*cos(c + d*x))**(3/2)/(a*(sin(c + d*x) + 1))**(3/2), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,\cos(c+d\,x))^{3/2}}{(a+a\,\sin(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(3/2)/(a + a*sin(c + d*x))^(3/2),x)
```

```
[Out] int((e*cos(c + d*x))^(3/2)/(a + a*sin(c + d*x))^(3/2), x)
```

3.308 $\int \frac{\sqrt{e\cos(c+dx)}}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=36

$$-\frac{2(e\cos(c+dx))^{3/2}}{3de(a+a\sin(c+dx))^{3/2}}$$

```
[Out] -2/3*(e*cos(d*x+c))^(3/2)/d/e/(a+a*sin(d*x+c))^(3/2)
```

Rubi [A]
time = 0.05, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$, Rules used = $\{2750\}$

$$-\frac{2(e\cos(c+dx))^{3/2}}{3de(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cos[c + d*x]]/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*(e*Cos[c + d*x])^(3/2))/(3*d*e*(a + a*Sin[c + d*x])^(3/2))
```

Rule 2750

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rubi steps

$$\int \frac{\sqrt{e\cos(c+dx)}}{(a+a\sin(c+dx))^{3/2}}\,dx = -\frac{2(e\cos(c+dx))^{3/2}}{3de(a+a\sin(c+dx))^{3/2}}$$

Mathematica [A]
time = 0.07, size = 49, normalized size = 1.36

$$-\frac{2(e\cos(c+dx))^{3/2}\sqrt{a(1+\sin(c+dx))}}{3a^2de(1+\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Cos[c + d*x]]/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*(e*Cos[c + d*x])^(3/2)*Sqrt[a*(1 + Sin[c + d*x])])/(3*a^2*d*e*(1 + Sin[
c + d*x])^2)
```

Maple [A]
time = 0.14, size = 34, normalized size = 0.94

method	result	size
default	$-\frac{2\sqrt{e\cos\left(dx+c\right)}\,\cos(dx+c)}{3d(a(1+\sin(dx+c)))^{\frac{3}{2}}}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/d*(e*cos(d*x+c))^(1/2)*cos(d*x+c)/(a*(1+sin(d*x+c)))^(3/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 126 vs. 2(27) = 54.
time = 0.54, size = 126, normalized size = 3.50

$$-\frac{2\left(\sqrt{a}-\frac{\sqrt{a}\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)\sqrt{-\frac{\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}+1}\,\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)e^{\frac{1}{2}}}{3\left(a^2+\frac{a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima"
)
```

```
[Out] -2/3*(sqrt(a) - sqrt(a)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*sqrt(-sin(d*x
+ c)/(cos(d*x + c) + 1) + 1)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)*e^(1
/2)/((a^2 + a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*d*(sin(d*x + c)/(cos(d
*x + c) + 1) + 1)^(5/2))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 104 vs. 2(27) = 54.
time = 0.34, size = 104, normalized size = 2.89

$$\frac{2\left(\cos\left(dx+c\right)e^{\frac{1}{2}}-e^{\frac{1}{2}}\sin\left(dx+c\right)+e^{\frac{1}{2}}\right)\sqrt{a\sin\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}}{3\left(a^2d\cos\left(dx+c\right)^2-a^2d\cos\left(dx+c\right)-2\,a^2d-\left(a^2d\cos\left(dx+c\right)+2\,a^2d\right)\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas"
)
```

```
[Out] 2/3*(cos(d*x + c)*e^(1/2) - e^(1/2)*sin(d*x + c) + e^(1/2))*sqrt(a*sin(d*x
+ c) + a)*sqrt(cos(d*x + c))/(a^2*d*cos(d*x + c)^2 - a^2*d*cos(d*x + c) - 2
*a^2*d - (a^2*d*cos(d*x + c) + 2*a^2*d)*sin(d*x + c))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e\cos{(c+dx)}}}{(a(\sin{(c+dx)}+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(1/2)/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral(sqrt(e*cos(c + d*x))/(a*(sin(c + d*x) + 1))**(3/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 5.99, size = 82, normalized size = 2.28

$$-\frac{4\sqrt{e\cos{(c+d\,x)}}\,\sqrt{a\,(\sin{(c+d\,x)}+1)}\,(2\cos{(c+d\,x)}+\sin{(2\,c+2\,d\,x)})}{3\,a^2\,d\,(15\sin{(c+d\,x)}-6\cos{(2\,c+2\,d\,x)}-\sin{(3\,c+3\,d\,x)}+10)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(1/2)/(a + a*sin(c + d*x))^(3/2),x)
```

```
[Out] -(4*(e*cos(c + d*x))^(1/2)*(a*(sin(c + d*x) + 1))^(1/2)*(2*cos(c + d*x) + s
in(2*c + 2*d*x)))/(3*a^2*d*(15*sin(c + d*x) - 6*cos(2*c + 2*d*x) - sin(3*c
+ 3*d*x) + 10))
```

3.309 $\int \frac{1}{\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=76

$$-\frac{2\sqrt{e\cos(c+dx)}}{5de(a+a\sin(c+dx))^{3/2}}-\frac{4\sqrt{e\cos(c+dx)}}{5ade\sqrt{a+a\sin(c+dx)}}$$

```
[Out] -2/5*(e*cos(d*x+c))^(1/2)/d/e/(a+a*sin(d*x+c))^(3/2)-4/5*(e*cos(d*x+c))^(1/
2)/a/d/e/(a+a*sin(d*x+c))^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2751, 2750}

$$-\frac{4\sqrt{e\cos(c+dx)}}{5ade\sqrt{a\sin(c+dx)+a}}-\frac{2\sqrt{e\cos(c+dx)}}{5de(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])^(3/2)),x]
```

```
[Out] (-2*Sqrt[e*Cos[c + d*x]])/(5*d*e*(a + a*Sin[c + d*x])^(3/2)) - (4*Sqrt[e*Co
s[c + d*x]])/(5*a*d*e*Sqrt[a + a*Sin[c + d*x]])
```

Rule 2750

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] && !ILtQ[p, 0]
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] && !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^{3/2}}\,dx = -\frac{2\sqrt{e\cos(c+dx)}}{5de(a+a\sin(c+dx))^{3/2}} + \frac{2\int \frac{1}{\sqrt{e\cos(c+dx)}\,\sqrt{a+a\sin(c+dx)}}\,dx}{5a}$$
$$= -\frac{2\sqrt{e\cos(c+dx)}}{5de(a+a\sin(c+dx))^{3/2}} - \frac{4\sqrt{e\cos(c+dx)}}{5ade\sqrt{a+a\sin(c+dx)}}$$

Mathematica [A]
time = 0.11, size = 59, normalized size = 0.78

$$-\frac{2\sqrt{e\cos(c+dx)}\,\sqrt{a(1+\sin(c+dx))}\,(3+2\sin(c+dx))}{5a^2de(1+\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])^(3/2)),x]
[Out] (-2*Sqrt[e*Cos[c + d*x]]*Sqrt[a*(1 + Sin[c + d*x])]*(3 + 2*Sin[c + d*x]))/(
5*a^2*d*e*(1 + Sin[c + d*x])^2)
```

Maple [A]
time = 0.14, size = 44, normalized size = 0.58

method	result	size
default	$-\frac{2(2\sin(dx+c)+3)\cos(dx+c)}{5d(a(1+\sin(dx+c)))^{\frac{3}{2}}\sqrt{e\cos(dx+c)}}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(d*x+c))^(3/2)/(e*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/5/d*(2*sin(d*x+c)+3)*cos(d*x+c)/(a*(1+sin(d*x+c)))^(3/2)/(e*cos(d*x+c))^
(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 197 vs. 2(58) = 116.
time = 0.56, size = 197, normalized size = 2.59

$$-\frac{2\left(3\sqrt{a}+\frac{4\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}-\frac{4\sqrt{a}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{3\sqrt{a}\sin(dx+c)^4}{(\cos(dx+c)+1)^4}\right)\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^2 e^{\left(-\frac{1}{2}\right)}}{5\left(a^2+\frac{2a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}\right)d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\sqrt{-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))^(3/2)/(e*cos(d*x+c))^(1/2),x, algorithm="maxim
a")
```

```
[Out] -2/5*(3*sqrt(a) + 4*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 4*sqrt(a)*sin
(d*x + c)^3/(cos(d*x + c) + 1)^3 - 3*sqrt(a)*sin(d*x + c)^4/(cos(d*x + c) +
 1)^4)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^2*e^(-1/2)/((a^2 + 2*a^2*s
in(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^
4)*d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*sqrt(-sin(d*x + c)/(cos(d*
x + c) + 1) + 1))
```

Fricas [A]
time = 0.35, size = 72, normalized size = 0.95

$$\frac{2\,\sqrt{a\sin\left(dx+c\right)+a}\,\left(2\,\sin\left(dx+c\right)+3\right)\sqrt{\cos\left(dx+c\right)}}{5\,\left(a^2d\cos\left(dx+c\right)^2e^{\frac{1}{2}}-2\,a^2de^{\frac{1}{2}}\sin\left(dx+c\right)-2\,a^2de^{\frac{1}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))^(3/2)/(e*cos(d*x+c))^(1/2),x, algorithm="frica
s")
```

```
[Out] 2/5*sqrt(a*sin(d*x + c) + a)*(2*sin(d*x + c) + 3)*sqrt(cos(d*x + c))/(a^2*d
*cos(d*x + c)^2*e^(1/2) - 2*a^2*d*e^(1/2)*sin(d*x + c) - 2*a^2*d*e^(1/2))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(a\left(\sin\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}\sqrt{e\cos\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))**(3/2)/(e*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/((a*(sin(c + d*x) + 1))**(3/2)*sqrt(e*cos(c + d*x))), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))^(3/2)/(e*cos(d*x+c))^(1/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

Mupad [B]
time = 6.38, size = 95, normalized size = 1.25

$$-\frac{4\sqrt{a\,\left(\sin\left(c+d\,x\right)+1\right)}\,\left(7\,\cos\left(c+d\,x\right)-\cos\left(3\,c+3\,d\,x\right)+5\,\sin\left(2\,c+2\,d\,x\right)\right)}{5\,a^2\,d\,\sqrt{e\,\cos\left(c+d\,x\right)}\,\left(15\,\sin\left(c+d\,x\right)-6\,\cos\left(2\,c+2\,d\,x\right)-\sin\left(3\,c+3\,d\,x\right)+10\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cos(c + d*x))^(1/2)*(a + a*sin(c + d*x))^(3/2)),x)
```

```
[Out] -(4*(a*(sin(c + d*x) + 1))^(1/2)*(7*cos(c + d*x) - cos(3*c + 3*d*x) + 5*sin
(2*c + 2*d*x)))/(5*a^2*d*(e*cos(c + d*x))^(1/2)*(15*sin(c + d*x) - 6*cos(2*
c + 2*d*x) - sin(3*c + 3*d*x) + 10))
```

3.310 $\int \frac{1}{(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=115

$$-\frac{2}{7de\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^{3/2}}-\frac{8}{21ade\sqrt{e\cos(c+dx)}\,\sqrt{a+a\sin(c+dx)}}+\frac{16\sqrt{a+a\sin(c+}}{21a^2de\sqrt{e\cos(c+}}$$

```
[Out] -2/7/d/e/(a+a*sin(d*x+c))^(3/2)/(e*cos(d*x+c))^(1/2)-8/21/a/d/e/(e*cos(d*x+
c))^(1/2)/(a+a*sin(d*x+c))^(1/2)+16/21*(a+a*sin(d*x+c))^(1/2)/a^2/d/e/(e*co
s(d*x+c))^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2751, 2750}

$$\frac{16\sqrt{a\sin(c+dx)+a}}{21a^2de\sqrt{e\cos(c+dx)}}-\frac{8}{21ade\sqrt{a\sin(c+dx)+a}\,\sqrt{e\cos(c+dx)}}-\frac{2}{7de(a\sin(c+dx)+a)^{3/2}\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x])^(3/2)),x]
```

```
[Out] -2/(7*d*e*Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])^(3/2)) - 8/(21*a*d*e*Sq
rt[e*Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]]) + (16*Sqrt[a + a*Sin[c + d*x]]
)/(21*a^2*d*e*Sqrt[e*Cos[c + d*x]])
```

Rule 2750

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{1}{(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^{3/2}}\,dx = -\frac{2}{7de\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^{3/2}} + \frac{4\int \frac{}{(e\cos(c+dx}}{}$$

$$= -\frac{2}{7de\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^{3/2}} - \frac{}{21ade\sqrt{e\cos}}$$

$$= -\frac{2}{7de\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^{3/2}} - \frac{}{21ade\sqrt{e\cos}}$$

Mathematica [A]
time = 0.10, size = 56, normalized size = 0.49

$$\frac{2+24\sin(c+dx)+16\sin^2(c+dx)}{21de\sqrt{e\cos(c+dx)}\,(a(1+\sin(c+dx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x])^(3/2)),x]
```

```
[Out] (2 + 24*Sin[c + d*x] + 16*Sin[c + d*x]^2)/(21*d*e*Sqrt[e*Cos[c + d*x]]*(a*(
1 + Sin[c + d*x]))^(3/2))
```

Maple [A]
time = 0.14, size = 54, normalized size = 0.47

method	result	size
default	$\frac{2\left(-8\left(\cos^2(dx+c)\right)+12\sin(dx+c)+9\right)\cos(dx+c)}{21d(e\cos(dx+c))^{\frac{3}{2}}(a(1+\sin(dx+c)))^{\frac{3}{2}}}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/21/d*(-8*cos(d*x+c)^2+12*sin(d*x+c)+9)*cos(d*x+c)/(e*cos(d*x+c))^(3/2)/(a
*(1+sin(d*x+c)))^(3/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 264 vs. 2(88) = 176.
time = 0.55, size = 264, normalized size = 2.30

$$\frac{2\left(\sqrt{a}+\frac{24\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}+\frac{33\sqrt{a}\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{33\sqrt{a}\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{24\sqrt{a}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{\sqrt{a}\sin(dx+c)^6}{(\cos(dx+c)+1)^6}\right)\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^3 e^{\left(-\frac{3}{2}\right)}}{21\left(a^2+\frac{3a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}\right)d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{9}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxim
a")
```

```
[Out] 2/21*(sqrt(a) + 24*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) + 33*sqrt(a)*sin
(d*x + c)^2/(cos(d*x + c) + 1)^2 - 33*sqrt(a)*sin(d*x + c)^4/(cos(d*x + c)
+ 1)^4 - 24*sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - sqrt(a)*sin(d*x +
 c)^6/(cos(d*x + c) + 1)^6)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^3*e^(
-3/2)/((a^2 + 3*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*a^2*sin(d*x + c
)^4/(cos(d*x + c) + 1)^4 + a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6)*d*(sin(
d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) +
1)^(3/2))
```

Fricas [**A**]
time = 0.35, size = 94, normalized size = 0.82

$$\frac{2\left(8\cos\left(dx+c\right)^{2}-12\sin\left(dx+c\right)-9\right)\sqrt{a\sin\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}}{21\left(a^{2}d\cos\left(dx+c\right)^{3}e^{\frac{3}{2}}-2\,a^{2}d\cos\left(dx+c\right)e^{\frac{3}{2}}\sin\left(dx+c\right)-2\,a^{2}d\cos\left(dx+c\right)e^{\frac{3}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^(3/2),x, algorithm="frica
s")
```

```
[Out] 2/21*(8*cos(d*x + c)^2 - 12*sin(d*x + c) - 9)*sqrt(a*sin(d*x + c) + a)*sqrt
(cos(d*x + c))/(a^2*d*cos(d*x + c)^3*e^(3/2) - 2*a^2*d*cos(d*x + c)*e^(3/2)
*sin(d*x + c) - 2*a^2*d*cos(d*x + c)*e^(3/2))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(a\left(\sin\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}\left(e\cos\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(3/2)/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral(1/((a*(sin(c + d*x) + 1))**(3/2)*(e*cos(c + d*x))**(3/2)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

Mupad [B]
time = 6.82, size = 119, normalized size = 1.03

$$\frac{8\,\sqrt{a\,\left(\sin\left(c+d\,x\right)+1\right)}\,\left(70\,\sin\left(c+d\,x\right)-41\,\cos\left(2\,c+2\,d\,x\right)+2\,\cos\left(4\,c+4\,d\,x\right)-14\,\sin\left(3\,c+3\,d\,x\right)+41\right)}{21\,a^2\,d\,e\,\sqrt{e\,\cos\left(c+d\,x\right)}\,\left(56\,\sin\left(c+d\,x\right)-28\,\cos\left(2\,c+2\,d\,x\right)+\cos\left(4\,c+4\,d\,x\right)-8\,\sin\left(3\,c+3\,d\,x\right)+35\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cos(c + d*x))^(3/2)*(a + a*sin(c + d*x))^(3/2)),x)
```

```
[Out] (8*(a*(sin(c + d*x) + 1))^(1/2)*(70*sin(c + d*x) - 41*cos(2*c + 2*d*x) + 2*
cos(4*c + 4*d*x) - 14*sin(3*c + 3*d*x) + 41))/(21*a^2*d*e*(e*cos(c + d*x))^
(1/2)*(56*sin(c + d*x) - 28*cos(2*c + 2*d*x) + cos(4*c + 4*d*x) - 8*sin(3*c
 + 3*d*x) + 35))
```

3.311 $\int \frac{1}{(e\cos(c+dx))^{5/2}(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=154

$$-\frac{2}{9de(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^{3/2}}-\frac{4}{15ade(e\cos(c+dx))^{3/2}\sqrt{a+a\sin(c+dx)}}-\frac{16\sqrt{a+a\sin(c}{15a^2de(e\cos(c-}$$

```
[Out] -2/9/d/e/(e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^(3/2)+32/45*(a+a*sin(d*x+c))
^(3/2)/a^3/d/e/(e*cos(d*x+c))^(3/2)-4/15/a/d/e/(e*cos(d*x+c))^(3/2)/(a+a*si
n(d*x+c))^(1/2)-16/15*(a+a*sin(d*x+c))^(1/2)/a^2/d/e/(e*cos(d*x+c))^(3/2)
```

Rubi [A]
time = 0.20, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2751, 2750}

$$\frac{32(a\sin(c+dx)+a)^{3/2}}{45a^3de(e\cos(c+dx))^{3/2}}-\frac{16\sqrt{a\sin(c+dx)+a}}{15a^2de(e\cos(c+dx))^{3/2}}-\frac{4}{15ade\sqrt{a\sin(c+dx)+a}\,(e\cos(c+dx))^{3/2}}-\frac{2}{9de(a\sin(c+dx)+a)^{3/2}(e\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(5/2)*(a + a*Sin[c + d*x])^(3/2)),x]
```

```
[Out] -2/(9*d*e*(e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x])^(3/2)) - 4/(15*a*d*e*
(e*Cos[c + d*x])^(3/2)*Sqrt[a + a*Sin[c + d*x]]) - (16*Sqrt[a + a*Sin[c + d
*x]])/(15*a^2*d*e*(e*Cos[c + d*x])^(3/2)) + (32*(a + a*Sin[c + d*x])^(3/2))
/(45*a^3*d*e*(e*Cos[c + d*x])^(3/2))
```

Rule 2750

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{1}{(e\cos(c+dx))^{5/2}(a+a\sin(c+dx))^{3/2}}\,dx = -\frac{2}{9de(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^{3/2}} + \frac{2\int \frac{}{(e\cos(c+}}{}$$

$$= -\frac{2}{9de(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^{3/2}} - \frac{}{15ade(e\cos}$$

$$= -\frac{2}{9de(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^{3/2}} - \frac{}{15ade(e\cos}$$

$$= -\frac{2}{9de(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^{3/2}} - \frac{}{15ade(e\cos}$$

Mathematica [**A**]
time = 0.18, size = 66, normalized size = 0.43

$$-\frac{2(7+12\cos(2(c+dx))-6\sin(c+dx)+4\sin(3(c+dx)))}{45de(e\cos(c+dx))^{3/2}(a(1+\sin(c+dx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Cos[c + d*x])^(5/2)*(a + a*Sin[c + d*x])^(3/2)),x]
[Out] (-2*(7 + 12*Cos[2*(c + d*x)] - 6*Sin[c + d*x] + 4*Sin[3*(c + d*x)]))/(45*d*
e*(e*Cos[c + d*x])^(3/2)*(a*(1 + Sin[c + d*x]))^(3/2))
```

Maple [**A**]
time = 0.16, size = 70, normalized size = 0.45

method	result	size
default	$-\frac{2\left(16\left(\cos^{2}(dx+c)\right)\sin(dx+c)+24\left(\cos^{2}(dx+c)\right)-10\sin(dx+c)-5\right)\cos(dx+c)}{45d\left(e\cos(dx+c)\right)^{\frac{5}{2}}\left(a(1+\sin(dx+c))\right)^{\frac{3}{2}}}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
[Out] -2/45/d*(16*cos(d*x+c)^2*sin(d*x+c)+24*cos(d*x+c)^2-10*sin(d*x+c)-5)*cos(d*
x+c)/(e*cos(d*x+c))^(5/2)/(a*(1+sin(d*x+c)))^(3/2)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 335 vs. 2(118) = 236.
time = 0.55, size = 335, normalized size = 2.18

$$-\frac{2\left(19\sqrt{a}+\frac{12\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}-\frac{58\sqrt{a}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{116\sqrt{a}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{116\sqrt{a}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+\frac{58\sqrt{a}\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}-\frac{12\sqrt{a}\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}-\frac{19\sqrt{a}\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}\right)\left(\frac{\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+1\right)^{4}e^{\left(-\frac{5}{2}\right)}}{45\left(a^{2}+\frac{4a^{2}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{6a^{2}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{4a^{2}\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+\frac{a^{2}\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}\right)d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{11}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxim
a")
```

```
[Out] -2/45*(19*sqrt(a) + 12*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 58*sqrt(a)
*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 116*sqrt(a)*sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3 + 116*sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 58*sqrt(a)*
sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 12*sqrt(a)*sin(d*x + c)^7/(cos(d*x +
c) + 1)^7 - 19*sqrt(a)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8)*(sin(d*x + c)^2
/(cos(d*x + c) + 1)^2 + 1)^4*e^(-5/2)/((a^2 + 4*a^2*sin(d*x + c)^2/(cos(d*x
 + c) + 1)^2 + 6*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*a^2*sin(d*x +
c)^6/(cos(d*x + c) + 1)^6 + a^2*sin(d*x + c)^8/(cos(d*x + c) + 1)^8)*d*(sin
(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)*(-sin(d*x + c)/(cos(d*x + c) + 1)
+ 1)^(5/2))
```

Fricas [**A**]
time = 0.36, size = 110, normalized size = 0.71

$$\frac{2\left(24\cos\left(dx+c\right)^{2}+2\left(8\cos\left(dx+c\right)^{2}-5\right)\sin\left(dx+c\right)-5\right)\sqrt{a\sin\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}}{45\left(a^{2}d\cos\left(dx+c\right)^{4}e^{\frac{5}{2}}-2\,a^{2}d\cos\left(dx+c\right)^{2}e^{\frac{5}{2}}\sin\left(dx+c\right)-2\,a^{2}d\cos\left(dx+c\right)^{2}e^{\frac{5}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^(3/2),x, algorithm="frica
s")
```

```
[Out] 2/45*(24*cos(d*x + c)^2 + 2*(8*cos(d*x + c)^2 - 5)*sin(d*x + c) - 5)*sqrt(a
*sin(d*x + c) + a)*sqrt(cos(d*x + c))/(a^2*d*cos(d*x + c)^4*e^(5/2) - 2*a^2
*d*cos(d*x + c)^2*e^(5/2)*sin(d*x + c) - 2*a^2*d*cos(d*x + c)^2*e^(5/2))
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(5/2)/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4371 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

Mupad [B]
time = 11.10, size = 230, normalized size = 1.49

$$-\frac{14\sqrt{a+a\sin(c+dx)}-12\sin(c+dx)\sqrt{a+a\sin(c+dx)}+24\cos(2c+2dx)\sqrt{a+a\sin(c+dx)}+8\sin(3c+3dx)\sqrt{a+a\sin(c+dx)}}{\frac{225a^2de^2\cos(c+dx)\sqrt{\frac{e\,e^{-c\,1i-d\,x\,1i}}{2}+\frac{e\,e^{c\,1i+d\,x\,1i}}{2}}}{4}-\frac{45a^2de^2\cos(3c+3dx)\sqrt{\frac{e\,e^{-c\,1i-d\,x\,1i}}{2}+\frac{e\,e^{c\,1i+d\,x\,1i}}{2}}}{4}+45a^2de^2\sin(2c+2dx)\sqrt{\frac{e\,e^{-c\,1i-d\,x\,1i}}{2}+\frac{e\,e^{c\,1i+d\,x\,1i}}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cos(c + d*x))^(5/2)*(a + a*sin(c + d*x))^(3/2)),x)
```

```
[Out] -(14*(a + a*sin(c + d*x))^(1/2) - 12*sin(c + d*x)*(a + a*sin(c + d*x))^(1/2
) + 24*cos(2*c + 2*d*x)*(a + a*sin(c + d*x))^(1/2) + 8*sin(3*c + 3*d*x)*(a
+ a*sin(c + d*x))^(1/2))/((225*a^2*d*e^2*cos(c + d*x)*((e*exp(- c*1i - d*x*
1i))/2 + (e*exp(c*1i + d*x*1i))/2)^(1/2))/4 - (45*a^2*d*e^2*cos(3*c + 3*d*x
)*((e*exp(- c*1i - d*x*1i))/2 + (e*exp(c*1i + d*x*1i))/2)^(1/2))/4 + 45*a^2
*d*e^2*sin(2*c + 2*d*x)*((e*exp(- c*1i - d*x*1i))/2 + (e*exp(c*1i + d*x*1i)
)/2)^(1/2))
```

3.312 $\int \frac{1}{(e\cos(c+dx))^{7/2}(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=193

$$-\frac{2}{11de(e\cos(c+dx))^{5/2}(a+a\sin(c+dx))^{3/2}}-\frac{16}{77ade(e\cos(c+dx))^{5/2}\sqrt{a+a\sin(c+dx)}}-\frac{32\sqrt{a+a\sin}}{77a^2de(e\cos(c}$$

```
[Out] -2/11/d/e/(e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^(3/2)+128/77*(a+a*sin(d*x+c
))^(3/2)/a^3/d/e/(e*cos(d*x+c))^(5/2)-256/385*(a+a*sin(d*x+c))^(5/2)/a^4/d/
e/(e*cos(d*x+c))^(5/2)-16/77/a/d/e/(e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^(1
/2)-32/77*(a+a*sin(d*x+c))^(1/2)/a^2/d/e/(e*cos(d*x+c))^(5/2)
```

Rubi [A]
time = 0.25, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2751, 2750}

$$-\frac{256(a\sin(c+dx)+a)^{5/2}}{385a^4de(e\cos(c+dx))^{5/2}}+\frac{128(a\sin(c+dx)+a)^{3/2}}{77a^3de(e\cos(c+dx))^{5/2}}-\frac{32\sqrt{a\sin(c+dx)+a}}{77a^2de(e\cos(c+dx))^{5/2}}-\frac{16}{77ade\sqrt{a\sin(c+dx)+a}\,(e\cos(c+dx))^{5/2}}-\frac{2}{11de(a\sin(c+dx)+a)^{3/2}(e\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(7/2)*(a + a*Sin[c + d*x])^(3/2)),x]
```

```
[Out] -2/(11*d*e*(e*Cos[c + d*x])^(5/2)*(a + a*Sin[c + d*x])^(3/2)) - 16/(77*a*d*
e*(e*Cos[c + d*x])^(5/2)*Sqrt[a + a*Sin[c + d*x]]) - (32*Sqrt[a + a*Sin[c +
 d*x]])/(77*a^2*d*e*(e*Cos[c + d*x])^(5/2)) + (128*(a + a*Sin[c + d*x])^(3/
2))/(77*a^3*d*e*(e*Cos[c + d*x])^(5/2)) - (256*(a + a*Sin[c + d*x])^(5/2))/
(385*a^4*d*e*(e*Cos[c + d*x])^(5/2))
```

Rule 2750

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{1}{(e\cos(c+dx))^{7/2}(a+a\sin(c+dx))^{3/2}}\,dx = -\frac{2}{11de(e\cos(c+dx))^{5/2}(a+a\sin(c+dx))^{3/2}} + \frac{8\int \frac{}{(e\cos(c-}}{}$$

$$= -\frac{2}{11de(e\cos(c+dx))^{5/2}(a+a\sin(c+dx))^{3/2}} - \frac{}{77ade(e\cos}$$

$$= -\frac{2}{11de(e\cos(c+dx))^{5/2}(a+a\sin(c+dx))^{3/2}} - \frac{}{77ade(e\cos}$$

$$= -\frac{2}{11de(e\cos(c+dx))^{5/2}(a+a\sin(c+dx))^{3/2}} - \frac{}{77ade(e\cos}$$

$$= -\frac{2}{11de(e\cos(c+dx))^{5/2}(a+a\sin(c+dx))^{3/2}} - \frac{}{77ade(e\cos}$$

Mathematica [A]
time = 0.28, size = 76, normalized size = 0.39

$$\frac{2(45+8\cos(2(c+dx))-16\cos(4(c+dx))+104\sin(c+dx)+48\sin(3(c+dx)))}{385de(e\cos(c+dx))^{5/2}(a(1+\sin(c+dx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Cos[c + d*x])^(7/2)*(a + a*Sin[c + d*x])^(3/2)),x]
```

```
[Out] (2*(45 + 8*Cos[2*(c + d*x)] - 16*Cos[4*(c + d*x)] + 104*Sin[c + d*x] + 48*S
in[3*(c + d*x)]))/(385*d*e*(e*Cos[c + d*x])^(5/2)*(a*(1 + Sin[c + d*x]))^(3
/2))
```

Maple [A]
time = 0.19, size = 80, normalized size = 0.41

method	result	size
default	$-\frac{2\left(128\left(\cos^{4}(dx+c)\right)-192\left(\cos^{2}(dx+c)\right)\sin(dx+c)-144\left(\cos^{2}(dx+c)\right)-56\sin(dx+c)-21\right)\cos(dx+c)}{385d\left(e\cos(dx+c)\right)^{\frac{7}{2}}\left(a(1+\sin(dx+c))\right)^{\frac{3}{2}}}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/385/d*(128*cos(d*x+c)^4-192*cos(d*x+c)^2*sin(d*x+c)-144*cos(d*x+c)^2-56*
sin(d*x+c)-21)*cos(d*x+c)/(e*cos(d*x+c))^(7/2)/(a*(1+sin(d*x+c)))^(3/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 404 vs. $2(148) = 296$.
time = 0.55, size = 404, normalized size = 2.09

$$\frac{2\left(37\sqrt{a} + \frac{496\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1} + \frac{559\sqrt{a}\sin(dx+c)^2}{(\cos(dx+c)+1)^2} - \frac{544\sqrt{a}\sin(dx+c)^3}{(\cos(dx+c)+1)^3} - \frac{1526\sqrt{a}\sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{1526\sqrt{a}\sin(dx+c)^6}{(\cos(dx+c)+1)^6} + \frac{544\sqrt{a}\sin(dx+c)^7}{(\cos(dx+c)+1)^7} - \frac{559\sqrt{a}\sin(dx+c)^8}{(\cos(dx+c)+1)^8} - \frac{496\sqrt{a}\sin(dx+c)^9}{(\cos(dx+c)+1)^9} - \frac{37\sqrt{a}\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}\right)\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^5 e^{\left(-\frac{7}{2}\right)}}{385\left(a^2 + \frac{5a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{10a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{10a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6} + \frac{5a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8} + \frac{a^2\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}\right)d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{13}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxim
a")
```

```
[Out] 2/385*(37*sqrt(a) + 496*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) + 559*sqrt(
a)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 544*sqrt(a)*sin(d*x + c)^3/(cos(d*
x + c) + 1)^3 - 1526*sqrt(a)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 1526*sqr
t(a)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 544*sqrt(a)*sin(d*x + c)^7/(cos(
d*x + c) + 1)^7 - 559*sqrt(a)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 496*sqr
t(a)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 37*sqrt(a)*sin(d*x + c)^10/(cos(
d*x + c) + 1)^10)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^5*e^(-7/2)/((a^
2 + 5*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 10*a^2*sin(d*x + c)^4/(cos(
d*x + c) + 1)^4 + 10*a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 5*a^2*sin(d*
x + c)^8/(cos(d*x + c) + 1)^8 + a^2*sin(d*x + c)^10/(cos(d*x + c) + 1)^10)*
d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(13/2)*(-sin(d*x + c)/(cos(d*x + c)
 + 1) + 1)^(7/2))
```

Fricas [A]
time = 0.35, size = 120, normalized size = 0.62

$$\frac{2\left(128\cos(dx+c)^4 - 144\cos(dx+c)^2 - 8\left(24\cos(dx+c)^2+7\right)\sin(dx+c) - 21\right)\sqrt{a\sin(dx+c)+a}\,\sqrt{\cos(dx+c)}}{385\left(a^2d\cos(dx+c)^5e^{\frac{7}{2}} - 2a^2d\cos(dx+c)^3e^{\frac{7}{2}}\sin(dx+c) - 2a^2d\cos(dx+c)^3e^{\frac{7}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^(3/2),x, algorithm="frica
s")
```

```
[Out] 2/385*(128*cos(d*x + c)^4 - 144*cos(d*x + c)^2 - 8*(24*cos(d*x + c)^2 + 7)*
sin(d*x + c) - 21)*sqrt(a*sin(d*x + c) + a)*sqrt(cos(d*x + c))/(a^2*d*cos(d
*x + c)^5*e^(7/2) - 2*a^2*d*cos(d*x + c)^3*e^(7/2)*sin(d*x + c) - 2*a^2*d*c
os(d*x + c)^3*e^(7/2))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(7/2)/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

Mupad [B]
time = 11.65, size = 413, normalized size = 2.14

$$\frac{\sqrt{a+a\,\sin\left(c+d\,x\right)}\,\left(\frac{288\,{\mathrm{e}}^{c\,4{}\mathrm{i}+d\,x\,4{}\mathrm{i}}}{77\,a^2\,d\,e^3}+\frac{256\,{\mathrm{e}}^{c\,4{}\mathrm{i}+d\,x\,4{}\mathrm{i}}\,\cos\left(2\,c+2\,d\,x\right)}{385\,a^2\,d\,e^3}-\frac{512\,{\mathrm{e}}^{c\,4{}\mathrm{i}+d\,x\,4{}\mathrm{i}}\,\cos\left(4\,c+4\,d\,x\right)}{385\,a^2\,d\,e^3}+\frac{1536\,{\mathrm{e}}^{c\,4{}\mathrm{i}+d\,x\,4{}\mathrm{i}}\,\sin\left(3\,c+3\,d\,x\right)}{385\,a^2\,d\,e^3}+\frac{3328\,{\mathrm{e}}^{c\,4{}\mathrm{i}+d\,x\,4{}\mathrm{i}}\,\sin\left(c+d\,x\right)}{385\,a^2\,d\,e^3}\right)}{10\,{\mathrm{e}}^{c\,4{}\mathrm{i}+d\,x\,4{}\mathrm{i}}\,\sqrt{e\,\left(\frac{{\mathrm{e}}^{-c\,1{}\mathrm{i}-d\,x\,1{}\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{c\,1{}\mathrm{i}+d\,x\,1{}\mathrm{i}}}{2}\right)}+8\,{\mathrm{e}}^{c\,4{}\mathrm{i}+d\,x\,4{}\mathrm{i}}\,\sin\left(c+d\,x\right)\,\sqrt{e\,\left(\frac{{\mathrm{e}}^{-c\,1{}\mathrm{i}-d\,x\,1{}\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{c\,1{}\mathrm{i}+d\,x\,1{}\mathrm{i}}}{2}\right)}+8\,{\mathrm{e}}^{c\,4{}\mathrm{i}+d\,x\,4{}\mathrm{i}}\,\cos\left(2\,c+2\,d\,x\right)\,\sqrt{e\,\left(\frac{{\mathrm{e}}^{-c\,1{}\mathrm{i}-d\,x\,1{}\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{c\,1{}\mathrm{i}+d\,x\,1{}\mathrm{i}}}{2}\right)}-2\,{\mathrm{e}}^{c\,4{}\mathrm{i}+d\,x\,4{}\mathrm{i}}\,\cos\left(4\,c+4\,d\,x\right)\,\sqrt{e\,\left(\frac{{\mathrm{e}}^{-c\,1{}\mathrm{i}-d\,x\,1{}\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{c\,1{}\mathrm{i}+d\,x\,1{}\mathrm{i}}}{2}\right)}+8\,{\mathrm{e}}^{c\,4{}\mathrm{i}+d\,x\,4{}\mathrm{i}}\,\sin\left(3\,c+3\,d\,x\right)\,\sqrt{e\,\left(\frac{{\mathrm{e}}^{-c\,1{}\mathrm{i}-d\,x\,1{}\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{c\,1{}\mathrm{i}+d\,x\,1{}\mathrm{i}}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cos(c + d*x))^(7/2)*(a + a*sin(c + d*x))^(3/2)),x)
```

```
[Out] ((a + a*sin(c + d*x))^(1/2)*((288*exp(c*4i + d*x*4i))/(77*a^2*d*e^3) + (256
*exp(c*4i + d*x*4i)*cos(2*c + 2*d*x))/(385*a^2*d*e^3) - (512*exp(c*4i + d*x
*4i)*cos(4*c + 4*d*x))/(385*a^2*d*e^3) + (1536*exp(c*4i + d*x*4i)*sin(3*c +
 3*d*x))/(385*a^2*d*e^3) + (3328*exp(c*4i + d*x*4i)*sin(c + d*x))/(385*a^2*
d*e^3)))/(10*exp(c*4i + d*x*4i)*(e*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x
*1i)/2))^(1/2) + 8*exp(c*4i + d*x*4i)*sin(c + d*x)*(e*(exp(- c*1i - d*x*1i)
/2 + exp(c*1i + d*x*1i)/2))^(1/2) + 8*exp(c*4i + d*x*4i)*cos(2*c + 2*d*x)*(
e*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2) - 2*exp(c*4i + d*x
*4i)*cos(4*c + 4*d*x)*(e*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(
1/2) + 8*exp(c*4i + d*x*4i)*sin(3*c + 3*d*x)*(e*(exp(- c*1i - d*x*1i)/2 + e
xp(c*1i + d*x*1i)/2))^(1/2))
```

3.313 $\int \frac{(e\cos(c+dx))^{9/2}}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=261

$$\frac{e(e\cos(c+dx))^{7/2}}{2ad(a+a\sin(c+dx))^{3/2}}+\frac{7e^3(e\cos(c+dx))^{3/2}}{4a^2d\sqrt{a+a\sin(c+dx)}}+\frac{21e^{9/2}\sinh^{-1}\left(\frac{\sqrt{e\cos(c+dx)}}{\sqrt{e}}\right)\sqrt{1+\cos(c+dx)}\ \cdots}{4d\left(a^3+a^3\cos(c+dx)+a^3\sin(c+d\cdots\right.}$$

```
[Out] 1/2*e*(e*cos(d*x+c))^(7/2)/a/d/(a+a*sin(d*x+c))^(3/2)+7/4*e^3*(e*cos(d*x+c)
)^(3/2)/a^2/d/(a+a*sin(d*x+c))^(1/2)+21/4*e^(9/2)*arcsinh((e*cos(d*x+c))^(1
/2)/e^(1/2))*(1+cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^(1/2)/d/(a^3+a^3*cos(d*x
+c)+a^3*sin(d*x+c))+21/4*e^(9/2)*arctan(sin(d*x+c)*e^(1/2)/(e*cos(d*x+c))^(
1/2)/(1+cos(d*x+c))^(1/2))*(1+cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^(1/2)/d/(a
^3+a^3*cos(d*x+c)+a^3*sin(d*x+c))
```

Rubi [A]
time = 0.30, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2759, 2765, 2758, 2763, 2854, 209, 2912, 65, 221}

$$\frac{21e^{9/2}\sqrt{\cos(c+dx)+1}\,\sqrt{a\sin(c+dx)+a}\,\text{ArcTan}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\,\sqrt{e\cos(c+dx)}}\right)}{4d\left(a^3\sin(c+dx)+a^3\cos(c+dx)+a^3\right)}+\frac{21e^{9/2}\sqrt{\cos(c+dx)+1}\,\sqrt{a\sin(c+dx)+a}\,\sinh^{-1}\left(\frac{\sqrt{e\cos(c+dx)}}{\sqrt{e}}\right)}{4d\left(a^3\sin(c+dx)+a^3\cos(c+dx)+a^3\right)}+\frac{7e^3(e\cos(c+dx))^{3/2}}{4a^2d\sqrt{a\sin(c+dx)+a}}+\frac{e(e\cos(c+dx))^{7/2}}{2ad(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(9/2)/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (e*(e*Cos[c + d*x])^(7/2))/(2*a*d*(a + a*Sin[c + d*x])^(3/2)) + (7*e^3*(e*C
os[c + d*x])^(3/2))/(4*a^2*d*Sqrt[a + a*Sin[c + d*x]]) + (21*e^(9/2)*ArcSin
h[Sqrt[e*Cos[c + d*x]]/Sqrt[e]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c + d
*x]])/(4*d*(a^3 + a^3*Cos[c + d*x] + a^3*Sin[c + d*x])) + (21*e^(9/2)*ArcTa
n[(Sqrt[e]*Sin[c + d*x])/(Sqrt[e*Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]])]*Sqr
t[1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(4*d*(a^3 + a^3*Cos[c + d*x]
+ a^3*Sin[c + d*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 2758

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(a*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2763

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[g*Sqrt[1 + Cos[e + f*x]]*(Sqrt[a + b*Sin[e + f*x
]]/(a + a*Cos[e + f*x] + b*Sin[e + f*x])), Int[Sqrt[1 + Cos[e + f*x]]/Sqrt[
g*Cos[e + f*x]], x], x] - Dist[g*Sqrt[1 + Cos[e + f*x]]*(Sqrt[a + b*Sin[e +
 f*x]]/(b + b*Cos[e + f*x] + a*Sin[e + f*x])), Int[Sin[e + f*x]/(Sqrt[g*Cos
[e + f*x]]*Sqrt[1 + Cos[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, g}, x] &&
EqQ[a^2 - b^2, 0]
```

Rule 2765

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> Simp[-2*b*((g*Cos[e + f*x])^(p + 1)/(f*g*(2*p - 1)*(
a + b*Sin[e + f*x])^(3/2))), x] + Dist[2*a*((p - 2)/(2*p - 1)), Int[(g*Cos[
e + f*x])^p/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 2] && IntegerQ[2*p]
```

Rule 2854

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b + d*x^2), x], x
, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]))], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{9/2}}{(a+a\sin(c+dx))^{5/2}}\,dx = \frac{4e(e\cos(c+dx))^{7/2}}{ad(a+a\sin(c+dx))^{3/2}} + \frac{(7e^2)\int \frac{(e\cos(c+dx))^{5/2}}{\sqrt{a+a\sin(c+dx)}}\,dx}{a^2}$$

$$= \frac{e(e\cos(c+dx))^{7/2}}{2ad(a+a\sin(c+dx))^{3/2}} + \frac{(7e^2)\int \frac{(e\cos(c+dx))^{5/2}}{(a+a\sin(c+dx))^{3/2}}\,dx}{4a}$$

$$= \frac{e(e\cos(c+dx))^{7/2}}{2ad(a+a\sin(c+dx))^{3/2}} + \frac{7e^3(e\cos(c+dx))^{3/2}}{4a^2d\sqrt{a+a\sin(c+dx)}} + \frac{(21e^4)\int \frac{\sqrt{e\cos(c}}{\sqrt{a+a\sin(}}}{8a^2}$$

$$= \frac{e(e\cos(c+dx))^{7/2}}{2ad(a+a\sin(c+dx))^{3/2}} + \frac{7e^3(e\cos(c+dx))^{3/2}}{4a^2d\sqrt{a+a\sin(c+dx)}} + \frac{\left(21e^5\sqrt{1+\cos(c+}\right.}{8a^2(}$$

$$= \frac{e(e\cos(c+dx))^{7/2}}{2ad(a+a\sin(c+dx))^{3/2}} + \frac{7e^3(e\cos(c+dx))^{3/2}}{4a^2d\sqrt{a+a\sin(c+dx)}} + \frac{\left(21e^5\sqrt{1+\cos(c+}\right.}{}$$

$$= \frac{e(e\cos(c+dx))^{7/2}}{2ad(a+a\sin(c+dx))^{3/2}} + \frac{7e^3(e\cos(c+dx))^{3/2}}{4a^2d\sqrt{a+a\sin(c+dx)}} + \frac{21e^{9/2}\tan^{-1}\left(\frac{}{\sqrt{e\cos}}\right.}{}$$

$$= \frac{e(e\cos(c+dx))^{7/2}}{2ad(a+a\sin(c+dx))^{3/2}} + \frac{7e^3(e\cos(c+dx))^{3/2}}{4a^2d\sqrt{a+a\sin(c+dx)}} + \frac{21e^{9/2}\sinh^{-1}\left(\frac{\sqrt{e\cos}}{}\right.}{4d\,(}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.20, size = 80, normalized size = 0.31

$$-\frac{2\sqrt[4]{2}\,(e\cos(c+dx))^{11/2}\,{}_2F_1\left(\frac{3}{4},\frac{11}{4};\frac{15}{4};\frac{1}{2}(1-\sin(c+dx))\right)\sqrt{a(1+\sin(c+dx))}}{11a^3de(1+\sin(c+dx))^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(9/2)/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*2^(1/4)*(e*Cos[c + d*x])^(11/2)*Hypergeometric2F1[3/4, 11/4, 15/4, (1 -
 Sin[c + d*x])/2]*Sqrt[a*(1 + Sin[c + d*x])])/(11*a^3*d*e*(1 + Sin[c + d*x]
)^(13/4))
```

Maple [A]
time = 0.24, size = 282, normalized size = 1.08

method	result
default	$\frac{(e\cos(dx+c))^{\frac{9}{2}}\left(21\sqrt{2}\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{2}}{2}\right)\sin(dx+c)+21\sqrt{2}\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\mathrm{arc}\right.}{8d((\cos^2(dx+c))\sin(dx+c)+\cos^3(dx+c)+2\cos(dx+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(9/2)/(a+a*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/d*(e*cos(d*x+c))^(9/2)*(21*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*sin(d*x+c)+21*2^(
1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*sin(d*x+c)+4*cos(d*x+c)^2*si
n(d*x+c)-4*cos(d*x+c)^3-22*cos(d*x+c)*sin(d*x+c)-18*cos(d*x+c)^2+22*cos(d*x
+c))/(cos(d*x+c)^2*sin(d*x+c)+cos(d*x+c)^3+2*cos(d*x+c)*sin(d*x+c)-3*cos(d*
x+c)^2-4*sin(d*x+c)-2*cos(d*x+c)+4)/(a*(1+sin(d*x+c)))^(5/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(9/2)/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima"
)
```

```
[Out] e^(9/2)*integrate(cos(d*x + c)^(9/2)/(a*sin(d*x + c) + a)^(5/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3456 vs. 2(206) = 412.
time = 194.48, size = 3456, normalized size = 13.24

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(9/2)/(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas"
)
```

```
[Out] -1/32*(84*(sqrt(2)*a^3*d*sin(d*x + c) + sqrt(2)*a^3*d)*(1/(a^10*d^4))^(1/4)
*arctan(-1/4*(2*sqrt(1/2)*((sqrt(2)*a^8*d^3*cos(d*x + c)^6 - 3*sqrt(2)*a^8*
d^3*cos(d*x + c)^5 - 8*sqrt(2)*a^8*d^3*cos(d*x + c)^4 + 4*sqrt(2)*a^8*d^3*c
os(d*x + c)^3 + 8*sqrt(2)*a^8*d^3*cos(d*x + c)^2 - (sqrt(2)*a^8*d^3*cos(d*x
 + c)^5 + 4*sqrt(2)*a^8*d^3*cos(d*x + c)^4 - 4*sqrt(2)*a^8*d^3*cos(d*x + c)
^3 - 8*sqrt(2)*a^8*d^3*cos(d*x + c)^2)*sin(d*x + c))*(1/(a^10*d^4))^(3/4)*e
^(27/2) + (sqrt(2)*a^3*d*cos(d*x + c)^6*e^9 + 5*sqrt(2)*a^3*d*cos(d*x + c)^
5*e^9 - 8*sqrt(2)*a^3*d*cos(d*x + c)^4*e^9 - 20*sqrt(2)*a^3*d*cos(d*x + c)^
3*e^9 + 8*sqrt(2)*a^3*d*cos(d*x + c)^2*e^9 + 16*sqrt(2)*a^3*d*cos(d*x + c)*
e^9 + (sqrt(2)*a^3*d*cos(d*x + c)^5*e^9 - 4*sqrt(2)*a^3*d*cos(d*x + c)^4*e^
9 - 12*sqrt(2)*a^3*d*cos(d*x + c)^3*e^9 + 8*sqrt(2)*a^3*d*cos(d*x + c)^2*e^
9 + 16*sqrt(2)*a^3*d*cos(d*x + c)*e^9)*sin(d*x + c))*(1/(a^10*d^4))^(1/4)*e
^(9/2) - (cos(d*x + c)^4*e^(27/2) - 3*cos(d*x + c)^3*e^(27/2) - 8*cos(d*x +
 c)^2*e^(27/2) + (2*a^5*d^2*cos(d*x + c)^5*e^(9/2) - 5*a^5*d^2*cos(d*x + c)
^4*e^(9/2) - 19*a^5*d^2*cos(d*x + c)^3*e^(9/2) + 20*a^5*d^2*cos(d*x + c)*e^
(9/2) + 8*a^5*d^2*e^(9/2) - (2*a^5*d^2*cos(d*x + c)^4*e^(9/2) + 9*a^5*d^2*c
os(d*x + c)^3*e^(9/2) - 4*a^5*d^2*cos(d*x + c)^2*e^(9/2) - 20*a^5*d^2*cos(d
*x + c)*e^(9/2) - 8*a^5*d^2*e^(9/2))*sin(d*x + c))*sqrt(1/(a^10*d^4))*e^9 +
 4*cos(d*x + c)*e^(27/2) - (cos(d*x + c)^3*e^(27/2) + 4*cos(d*x + c)^2*e^(2
7/2) - 4*cos(d*x + c)*e^(27/2) - 8*e^(27/2))*sin(d*x + c) + 8*e^(27/2))*sqr
t(a*sin(d*x + c) + a)*sqrt(cos(d*x + c)))*sqrt((2*a*cos(d*x + c)*e^27*sin(d
*x + c) + 2*a*cos(d*x + c)*e^27 + (a^6*d^2*e^18*sin(d*x + c) + a^6*d^2*e^18
)*sqrt(1/(a^10*d^4))*e^9 + (sqrt(2)*a^3*d*(1/(a^10*d^4))^(1/4)*cos(d*x + c)
*e^27 + (sqrt(2)*a^8*d^3*e^(27/2)*sin(d*x + c) + sqrt(2)*a^8*d^3*e^(27/2))*
(1/(a^10*d^4))^(3/4)*e^(27/2))*sqrt(a*sin(d*x + c) + a)*sqrt(cos(d*x + c)))
/(sin(d*x + c) + 1)) + ((2*sqrt(2)*a^8*d^3*cos(d*x + c)^5*e^(27/2) + sqrt(2
)*a^8*d^3*cos(d*x + c)^4*e^(27/2) - 13*sqrt(2)*a^8*d^3*cos(d*x + c)^3*e^(27
/2) - 8*sqrt(2)*a^8*d^3*cos(d*x + c)^2*e^(27/2) + 12*sqrt(2)*a^8*d^3*cos(d*
x + c)*e^(27/2) + 8*sqrt(2)*a^8*d^3*e^(27/2) - (7*sqrt(2)*a^8*d^3*cos(d*x +
 c)^3*e^(27/2) + 4*sqrt(2)*a^8*d^3*cos(d*x + c)^2*e^(27/2) - 12*sqrt(2)*a^8
*d^3*cos(d*x + c)*e^(27/2) - 8*sqrt(2)*a^8*d^3*e^(27/2))*sin(d*x + c))*(1/(
a^10*d^4))^(3/4)*e^(27/2) + (7*sqrt(2)*a^3*d*cos(d*x + c)^4*e^(45/2) + 3*sq
rt(2)*a^3*d*cos(d*x + c)^3*e^(45/2) - 16*sqrt(2)*a^3*d*cos(d*x + c)^2*e^(45
/2) - 4*sqrt(2)*a^3*d*cos(d*x + c)*e^(45/2) + 8*sqrt(2)*a^3*d*e^(45/2) + (2
*sqrt(2)*a^3*d*cos(d*x + c)^4*e^(45/2) + sqrt(2)*a^3*d*cos(d*x + c)^3*e^(45
/2) - 12*sqrt(2)*a^3*d*cos(d*x + c)^2*e^(45/2) - 4*sqrt(2)*a^3*d*cos(d*x +
c)*e^(45/2) + 8*sqrt(2)*a^3*d*e^(45/2))*sin(d*x + c))*(1/(a^10*d^4))^(1/4)*
e^(9/2))*sqrt(a*sin(d*x + c) + a)*sqrt(cos(d*x + c)))/(a*cos(d*x + c)^6*e^2
7 + a*cos(d*x + c)^5*e^27 - 8*a*cos(d*x + c)^4*e^27 - 8*a*cos(d*x + c)^3*e^
27 + 8*a*cos(d*x + c)^2*e^27 + 8*a*cos(d*x + c)*e^27 - 4*(a*cos(d*x + c)^4*
```

```
e^27 + a*cos(d*x + c)^3*e^27 - 2*a*cos(d*x + c)^2*e^27 - 2*a*cos(d*x + c)*e
^27)*sin(d*x + c)))*e^(9/2) - 84*(sqrt(2)*a^3*d*sin(d*x + c) + sqrt(2)*a^3*
d)*(1/(a^10*d^4))^(1/4)*arctan(1/4*(2*sqrt(1/2)*((sqrt(2)*a^8*d^3*cos(d*x +
 c)^6 - 3*sqrt(2)*a^8*d^3*cos(d*x + c)^5 - 8*sqrt(2)*a^8*d^3*cos(d*x + c)^4
 + 4*sqrt(2)*a^8*d^3*cos(d*x + c)^3 + 8*sqrt(2)*a^8*d^3*cos(d*x + c)^2 - (s
qrt(2)*a^8*d^3*cos(d*x + c)^5 + 4*sqrt(2)*a^8*d^3*cos(d*x + c)^4 - 4*sqrt(2
)*a^8*d^3*cos(d*x + c)^3 - 8*sqrt(2)*a^8*d^3*cos(d*x + c)^2)*sin(d*x + c))*
(1/(a^10*d^4))^(3/4)*e^(27/2) + (sqrt(2)*a^3*d*cos(d*x + c)^6*e^9 + 5*sqrt(
2)*a^3*d*cos(d*x + c)^5*e^9 - 8*sqrt(2)*a^3*d*cos(d*x + c)^4*e^9 - 20*sqrt(
2)*a^3*d*cos(d*x + c)^3*e^9 + 8*sqrt(2)*a^3*d*cos(d*x + c)^2*e^9 + 16*sqrt(
2)*a^3*d*cos(d*x + c)*e^9 + (sqrt(2)*a^3*d*cos(d*x + c)^5*e^9 - 4*sqrt(2)*a
^3*d*cos(d*x + c)^4*e^9 - 12*sqrt(2)*a^3*d*cos(d*x + c)^3*e^9 + 8*sqrt(2)*a
^3*d*cos(d*x + c)^2*e^9 + 16*sqrt(2)*a^3*d*cos(d*x + c)*e^9)*sin(d*x + c))*
(1/(a^10*d^4))^(1/4)*e^(9/2) + (cos(d*x + c)^4*e^(27/2) - 3*cos(d*x + c)^3*
e^(27/2) - 8*cos(d*x + c)^2*e^(27/2) + (2*a^5*d^2*cos(d*x + c)^5*e^(9/2) -
5*a^5*d^2*cos(d*x + c)^4*e^(9/2) - 19*a^5*d^2*cos(d*x + c)^3*e^(9/2) + 20*a
^5*d^2*cos(d*x + c)*e^(9/2) + 8*a^5*d^2*e^(9/2) - (2*a^5*d^2*cos(d*x + c)^4
*e^(9/2) + 9*a^5*d^2*cos(d*x + c)^3*e^(9/2) - 4*a^5*d^2*cos(d*x + c)^2*e^(9
/2) - 20*a^5*d^2*cos(d*x + c)*e^(9/2) - 8*a^5*d^2*e^(9/2))*sin(d*x + c))*sq
rt(1/(a^10*d^4))*e^9 + 4*cos(d*x + c)*e^(27/2) - (cos(d*x + c)^3*e^(27/2) +
 4*cos(d*x + c)^2*e^(27/2) - 4*cos(d*x + c)*e^(27/2) - 8*e^(27/2))*sin(d*x
+ c) + 8*e^(27/2))*sqrt(a*sin(d*x + c) + a)*sqrt(cos(d*x + c)))*sqrt((2*a*c
os(d*x + c)*e^27*sin(d*x + c) + 2*a*cos(d*x + c)*e^27 + (a^6*d^2*e^18*sin(d
*x + c) + a^6*d^2*e^18)*sqrt(1/(a^10*d^4))*e^9 - (sqrt(2)*a^3*d*(1/(a^10*d^
4))^(1/4)*cos(d*x + c)*e^27 + (sqrt(2)*a^8*d^3*e^(27/2)*sin(d*x + c) + sqrt
(2)*a^8*d^3*e^(27/2))*(1/(a^10*d^4))^(3/4)*e^(27/2))*sqrt(a*sin(d*x + c) +
a)*sqrt(cos(d*x + c)))/(sin(d*x + c) + 1)) + ((...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(9/2)/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(9/2)/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\cos(c+dx))^{9/2}}{(a+a\sin(c+dx))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(9/2)/(a + a*sin(c + d*x))^(5/2),x)
[Out] int((e*cos(c + d*x))^(9/2)/(a + a*sin(c + d*x))^(5/2), x)
```

3.314 $\int \frac{(e\cos(c+dx))^{7/2}}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=239

$$-\frac{4e(e\cos(c+dx))^{5/2}}{ad(a+a\sin(c+dx))^{3/2}}-\frac{5e^3\sqrt{e\cos(c+dx)}\,\sqrt{a+a\sin(c+dx)}}{a^3d}+\frac{5e^{7/2}\sinh^{-1}\left(\frac{\sqrt{e\cos(c+dx)}}{\sqrt{e}}\right)\sqrt{1}}{a^3d(1+\cos(c+d}$$

```
[Out] -4*e*(e*cos(d*x+c))^(5/2)/a/d/(a+a*sin(d*x+c))^(3/2)-5*e^3*(e*cos(d*x+c))^(
1/2)*(a+a*sin(d*x+c))^(1/2)/a^3/d+5*e^(7/2)*arcsinh((e*cos(d*x+c))^(1/2)/e^
(1/2))*(1+cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^(1/2)/a^3/d/(1+cos(d*x+c)+sin(
d*x+c))-5*e^(7/2)*arctan(sin(d*x+c)*e^(1/2)/(e*cos(d*x+c))^(1/2)/(1+cos(d*x
+c))^(1/2))*(1+cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^(1/2)/a^3/d/(1+cos(d*x+c)
+sin(d*x+c))
```

Rubi [A]
time = 0.25, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {2759, 2764, 2756, 2854, 209, 2912, 65, 221}

$$-\frac{5e^{7/2}\sqrt{\cos(c+dx)+1}\,\sqrt{a\sin(c+dx)+a}\,\text{ArcTan}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\,\sqrt{e\cos(c+dx)}}\right)}{a^3d(\sin(c+dx)+\cos(c+dx)+1)}+\frac{5e^{7/2}\sqrt{\cos(c+dx)+1}\,\sqrt{a\sin(c+dx)+a}\,\sinh^{-1}\left(\frac{\sqrt{e\cos(c+dx)}}{\sqrt{e}}\right)}{a^3d(\sin(c+dx)+\cos(c+dx)+1)}-\frac{5e^3\sqrt{a\sin(c+dx)+a}\,\sqrt{e\cos(c+dx)}}{a^3d}-\frac{4e(e\cos(c+dx))^{5/2}}{ad(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(7/2)/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-4*e*(e*Cos[c + d*x])^(5/2))/(a*d*(a + a*Sin[c + d*x])^(3/2)) - (5*e^3*Sqr
t[e*Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(a^3*d) + (5*e^(7/2)*ArcSinh[Sq
rt[e*Cos[c + d*x]]/Sqrt[e]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]]
)/(a^3*d*(1 + Cos[c + d*x] + Sin[c + d*x])) - (5*e^(7/2)*ArcTan[(Sqrt[e]*Si
n[c + d*x])/(Sqrt[e*Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]])]*Sqrt[1 + Cos[c +
 d*x]]*Sqrt[a + a*Sin[c + d*x]])/(a^3*d*(1 + Cos[c + d*x] + Sin[c + d*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 2756

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[cos[(e_.) + (f_.)*(x_)]
*(g_.)], x_Symbol] :> Dist[a*Sqrt[1 + Cos[e + f*x]]*(Sqrt[a + b*Sin[e + f*x
]]/(a + a*Cos[e + f*x] + b*Sin[e + f*x])), Int[Sqrt[1 + Cos[e + f*x]]/Sqrt[
g*Cos[e + f*x]], x], x] + Dist[b*Sqrt[1 + Cos[e + f*x]]*(Sqrt[a + b*Sin[e +
 f*x]]/(a + a*Cos[e + f*x] + b*Sin[e + f*x])), Int[Sin[e + f*x]/(Sqrt[g*Cos
[e + f*x]]*Sqrt[1 + Cos[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, g}, x] &&
EqQ[a^2 - b^2, 0]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2764

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]], x_Symbol] :> Simp[g*Sqrt[g*Cos[e + f*x]]*(Sqrt[a + b*Sin[e + f*x
]]/(b*f)), x] + Dist[g^2/(2*a), Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[g*Cos[e +
 f*x]], x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2854

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b + d*x^2), x], x
, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]))], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{7/2}}{(a+a\sin(c+dx))^{5/2}}\,dx = -\frac{4e(e\cos(c+dx))^{5/2}}{ad(a+a\sin(c+dx))^{3/2}} - \frac{(5e^2)\int \frac{(e\cos(c+dx))^{3/2}}{\sqrt{a+a\sin(c+dx)}}\,dx}{a^2}$$

$$= -\frac{4e(e\cos(c+dx))^{5/2}}{ad(a+a\sin(c+dx))^{3/2}} - \frac{5e^3\sqrt{e\cos(c+dx)}\,\sqrt{a+a\sin(c+dx)}}{a^3d} - \frac{(5e\ldots}{\ldots}$$

$$= -\frac{4e(e\cos(c+dx))^{5/2}}{ad(a+a\sin(c+dx))^{3/2}} - \frac{5e^3\sqrt{e\cos(c+dx)}\,\sqrt{a+a\sin(c+dx)}}{a^3d} - \frac{\left(5e\ldots\right.}{\ldots}$$

$$= -\frac{4e(e\cos(c+dx))^{5/2}}{ad(a+a\sin(c+dx))^{3/2}} - \frac{5e^3\sqrt{e\cos(c+dx)}\,\sqrt{a+a\sin(c+dx)}}{a^3d} + \frac{\left(5e\ldots\right.}{\ldots}$$

$$= -\frac{4e(e\cos(c+dx))^{5/2}}{ad(a+a\sin(c+dx))^{3/2}} - \frac{5e^3\sqrt{e\cos(c+dx)}\,\sqrt{a+a\sin(c+dx)}}{a^3d} - \frac{5e^{7\ldots}}{\ldots}$$

$$= -\frac{4e(e\cos(c+dx))^{5/2}}{ad(a+a\sin(c+dx))^{3/2}} - \frac{5e^3\sqrt{e\cos(c+dx)}\,\sqrt{a+a\sin(c+dx)}}{a^3d} + \frac{5e^{7\ldots}}{\ldots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.12, size = 80, normalized size = 0.33

$$-\frac{2^{3/4}(e\cos(c+dx))^{9/2}\,{}_2F_1\left(\frac{5}{4},\frac{9}{4};\frac{13}{4};\frac{1}{2}(1-\sin(c+dx))\right)\sqrt{a(1+\sin(c+dx))}}{9a^3de(1+\sin(c+dx))^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(7/2)/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] -1/9*(2^(3/4)*(e*Cos[c + d*x])^(9/2)*Hypergeometric2F1[5/4, 9/4, 13/4, (1 -
 Sin[c + d*x])/2]*Sqrt[a*(1 + Sin[c + d*x])])/(a^3*d*e*(1 + Sin[c + d*x])^(
11/4))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 442 vs. 2(209) = 418.
time = 0.14, size = 443, normalized size = 1.85

method	result
default	$\frac{\left(5\sqrt{2}\,\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2}}{2}\right)\sin(dx+c)-5\sqrt{2}\,\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\arctanh\left(\frac{\sqrt{-\frac{2\cos(}{1+\cos}}}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/d*(5*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*sin(d*x+c)-5*2^(1/2)*(-2*cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x
+c)/cos(d*x+c)*2^(1/2))*sin(d*x+c)+5*2^(1/2)*cos(d*x+c)*(-2*cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))-
5*2^(1/2)*cos(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))+5*2^(1/2)*(-
2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*2^(1/2))-5*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2
*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))+4*cos(
d*x+c)*sin(d*x+c)+36*cos(d*x+c))*(e*cos(d*x+c))^(7/2)/(2*sin(d*x+c)+cos(d*x
+c)^2-2)/(a*(1+sin(d*x+c)))^(5/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima"
)
```

```
[Out] e^(7/2)*integrate(cos(d*x + c)^(7/2)/(a*sin(d*x + c) + a)^(5/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3536 vs. 2(192) = 384.
time = 195.18, size = 3536, normalized size = 14.79

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas"
)
```

```
[Out] -1/8*(20*(sqrt(2)*a^3*d*cos(d*x + c)^2 - 2*sqrt(2)*a^3*d*sin(d*x + c) - 2*s
qrt(2)*a^3*d)*(1/(a^10*d^4))^(1/4)*arctan(-1/4*(2*sqrt(1/2)*((sqrt(2)*a^8*d
```

^3*cos(d*x + c)^6 + 5*sqrt(2)*a^8*d^3*cos(d*x + c)^5 - 8*sqrt(2)*a^8*d^3*co
s(d*x + c)^4 - 20*sqrt(2)*a^8*d^3*cos(d*x + c)^3 + 8*sqrt(2)*a^8*d^3*cos(d*
x + c)^2 + 16*sqrt(2)*a^8*d^3*cos(d*x + c) + (sqrt(2)*a^8*d^3*cos(d*x + c)^
5 - 4*sqrt(2)*a^8*d^3*cos(d*x + c)^4 - 12*sqrt(2)*a^8*d^3*cos(d*x + c)^3 +
8*sqrt(2)*a^8*d^3*cos(d*x + c)^2 + 16*sqrt(2)*a^8*d^3*cos(d*x + c))*sin(d*x
 + c))*(1/(a^10*d^4))^(3/4)*e^(21/2) + (sqrt(2)*a^3*d*cos(d*x + c)^6*e^7 -
3*sqrt(2)*a^3*d*cos(d*x + c)^5*e^7 - 8*sqrt(2)*a^3*d*cos(d*x + c)^4*e^7 + 4
*sqrt(2)*a^3*d*cos(d*x + c)^3*e^7 + 8*sqrt(2)*a^3*d*cos(d*x + c)^2*e^7 - (s
qrt(2)*a^3*d*cos(d*x + c)^5*e^7 + 4*sqrt(2)*a^3*d*cos(d*x + c)^4*e^7 - 4*sq
rt(2)*a^3*d*cos(d*x + c)^3*e^7 - 8*sqrt(2)*a^3*d*cos(d*x + c)^2*e^7)*sin(d*
x + c))*(1/(a^10*d^4))^(1/4)*e^(7/2) - (cos(d*x + c)^4*e^(21/2) - 3*cos(d*x
 + c)^3*e^(21/2) - 8*cos(d*x + c)^2*e^(21/2) + (2*a^5*d^2*cos(d*x + c)^5*e^
(7/2) - 5*a^5*d^2*cos(d*x + c)^4*e^(7/2) - 19*a^5*d^2*cos(d*x + c)^3*e^(7/2
) + 20*a^5*d^2*cos(d*x + c)*e^(7/2) + 8*a^5*d^2*e^(7/2) - (2*a^5*d^2*cos(d*
x + c)^4*e^(7/2) + 9*a^5*d^2*cos(d*x + c)^3*e^(7/2) - 4*a^5*d^2*cos(d*x + c
)^2*e^(7/2) - 20*a^5*d^2*cos(d*x + c)*e^(7/2) - 8*a^5*d^2*e^(7/2))*sin(d*x
+ c))*sqrt(1/(a^10*d^4))*e^7 + 4*cos(d*x + c)*e^(21/2) - (cos(d*x + c)^3*e^
(21/2) + 4*cos(d*x + c)^2*e^(21/2) - 4*cos(d*x + c)*e^(21/2) - 8*e^(21/2))*
sin(d*x + c) + 8*e^(21/2))*sqrt(a*sin(d*x + c) + a)*sqrt(cos(d*x + c))*sqr
t((2*a*cos(d*x + c)*e^21*sin(d*x + c) + 2*a*cos(d*x + c)*e^21 + (a^6*d^2*e^
14*sin(d*x + c) + a^6*d^2*e^14)*sqrt(1/(a^10*d^4))*e^7 + (sqrt(2)*a^8*d^3*(
1/(a^10*d^4))^(3/4)*cos(d*x + c)*e^21 + (sqrt(2)*a^3*d*e^(35/2)*sin(d*x + c
) + sqrt(2)*a^3*d*e^(35/2))*(1/(a^10*d^4))^(1/4)*e^(7/2))*sqrt(a*sin(d*x +
c) + a)*sqrt(cos(d*x + c)))/(sin(d*x + c) + 1)) - ((7*sqrt(2)*a^8*d^3*cos(d
*x + c)^4*e^(21/2) + 3*sqrt(2)*a^8*d^3*cos(d*x + c)^3*e^(21/2) - 16*sqrt(2)
*a^8*d^3*cos(d*x + c)^2*e^(21/2) - 4*sqrt(2)*a^8*d^3*cos(d*x + c)*e^(21/2)
+ 8*sqrt(2)*a^8*d^3*e^(21/2) + (2*sqrt(2)*a^8*d^3*cos(d*x + c)^4*e^(21/2) +
 sqrt(2)*a^8*d^3*cos(d*x + c)^3*e^(21/2) - 12*sqrt(2)*a^8*d^3*cos(d*x + c)^
2*e^(21/2) - 4*sqrt(2)*a^8*d^3*cos(d*x + c)*e^(21/2) + 8*sqrt(2)*a^8*d^3*e^
(21/2))*sin(d*x + c))*(1/(a^10*d^4))^(3/4)*e^(21/2) + (2*sqrt(2)*a^3*d*cos(
d*x + c)^5*e^(35/2) + sqrt(2)*a^3*d*cos(d*x + c)^4*e^(35/2) - 13*sqrt(2)*a^
3*d*cos(d*x + c)^3*e^(35/2) - 8*sqrt(2)*a^3*d*cos(d*x + c)^2*e^(35/2) + 12*
sqrt(2)*a^3*d*cos(d*x + c)*e^(35/2) + 8*sqrt(2)*a^3*d*e^(35/2) - (7*sqrt(2)
*a^3*d*cos(d*x + c)^3*e^(35/2) + 4*sqrt(2)*a^3*d*cos(d*x + c)^2*e^(35/2) -
12*sqrt(2)*a^3*d*cos(d*x + c)*e^(35/2) - 8*sqrt(2)*a^3*d*e^(35/2))*sin(d*x
+ c))*(1/(a^10*d^4))^(1/4)*e^(7/2))*sqrt(a*sin(d*x + c) + a)*sqrt(cos(d*x +
 c)))/(a*cos(d*x + c)^6*e^21 + a*cos(d*x + c)^5*e^21 - 8*a*cos(d*x + c)^4*e
^21 - 8*a*cos(d*x + c)^3*e^21 + 8*a*cos(d*x + c)^2*e^21 + 8*a*cos(d*x + c)*
e^21 - 4*(a*cos(d*x + c)^4*e^21 + a*cos(d*x + c)^3*e^21 - 2*a*cos(d*x + c)^
2*e^21 - 2*a*cos(d*x + c)*e^21)*sin(d*x + c)))*e^(7/2) - 20*(sqrt(2)*a^3*d*
cos(d*x + c)^2 - 2*sqrt(2)*a^3*d*sin(d*x + c) - 2*sqrt(2)*a^3*d)*(1/(a^10*d
^4))^(1/4)*arctan(1/4*(2*sqrt(1/2)*((sqrt(2)*a^8*d^3*cos(d*x + c)^6 + 5*sqr
t(2)*a^8*d^3*cos(d*x + c)^5 - 8*sqrt(2)*a^8*d^3*cos(d*x + c)^4 - 20*sqrt(2)
*a^8*d^3*cos(d*x + c)^3 + 8*sqrt(2)*a^8*d^3*cos(d*x + c)^2 + 16*sqrt(2)*a^8
*d^3*cos(d*x + c) + (sqrt(2)*a^8*d^3*cos(d*x + c)^5 - 4*sqrt(2)*a^8*d^3*cos

```
(d*x + c)^4 - 12*sqrt(2)*a^8*d^3*cos(d*x + c)^3 + 8*sqrt(2)*a^8*d^3*cos(d*x
 + c)^2 + 16*sqrt(2)*a^8*d^3*cos(d*x + c))*sin(d*x + c))*(1/(a^10*d^4))^(3/
4)*e^(21/2) + (sqrt(2)*a^3*d*cos(d*x + c)^6*e^7 - 3*sqrt(2)*a^3*d*cos(d*x +
 c)^5*e^7 - 8*sqrt(2)*a^3*d*cos(d*x + c)^4*e^7 + 4*sqrt(2)*a^3*d*cos(d*x +
c)^3*e^7 + 8*sqrt(2)*a^3*d*cos(d*x + c)^2*e^7 - (sqrt(2)*a^3*d*cos(d*x + c)
^5*e^7 + 4*sqrt(2)*a^3*d*cos(d*x + c)^4*e^7 - 4*sqrt(2)*a^3*d*cos(d*x + c)^
3*e^7 - 8*sqrt(2)*a^3*d*cos(d*x + c)^2*e^7)*sin(d*x + c))*(1/(a^10*d^4))^(1
/4)*e^(7/2) + (cos(d*x + c)^4*e^(21/2) - 3*cos(d*x + c)^3*e^(21/2) - 8*cos(
d*x + c)^2*e^(21/2) + (2*a^5*d^2*cos(d*x + c)^5*e^(7/2) - 5*a^5*d^2*cos(d*x
 + c)^4*e^(7/2) - 19*a^5*d^2*cos(d*x + c)^3*e^(7/2) + 20*a^5*d^2*cos(d*x +
c)*e^(7/2) + 8*a^5*d^2*e^(7/2) - (2*a^5*d^2*cos(d*x + c)^4*e^(7/2) + 9*a^5*
d^2*cos(d*x + c)^3*e^(7/2) - 4*a^5*d^2*cos(d*x + c)^2*e^(7/2) - 20*a^5*d^2*
cos(d*x + c)*e^(7/2) - 8*a^5*d^2*e^(7/2))*sin(d*x + c))*sqrt(1/(a^10*d^4))*
e^7 + 4*cos(d*x + c)*e^(21/2) - (cos(d*x + c)^3*e^(21/2) + 4*cos(d*x + c)^2
*e^(21/2) - 4*cos(d*x + c)*e^(21/2) - 8*e^(21/2))*sin(d*x + c) + 8*e^(21/2)
)*sqrt(a*sin(d*x + c) + a)*sqrt(cos(d*x + c)))*sqrt((2*a*cos(d*x + c)*e^21*
sin(d*x + c) + 2*a*cos(d*x + c)*e^21 + (a^6*d^2*e^14*sin(d*x + c) + a^6*d^2
*e^14)*sqrt(1/(a^10*d^4))*e^7 - (sqrt(2)*a^8*d^3*(1/(a^10*d^4))^(3/4)*cos(d
*x + c)*e^21 + (sqrt(2)*a^3*d*e^(35/2)*sin(d*x + c) + sqrt(2)*a^3*d*e^(35/2
))*(1/(a^10*d^4))^(1/4)*e^(7/2))*sqrt(a*sin(d*x...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(7/2)/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(e\,\cos\left(c+d\,x\right)\right)}^{7/2}}{{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(7/2)/(a + a*sin(c + d*x))^(5/2),x)
[Out] int((e*cos(c + d*x))^(7/2)/(a + a*sin(c + d*x))^(5/2), x)
```

3.315 $\int \frac{(e\cos(c+dx))^{5/2}}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=218

$$-\frac{4e(e\cos(c+dx))^{3/2}}{3ad(a+a\sin(c+dx))^{3/2}}-\frac{2e^{5/2}\sinh^{-1}\left(\frac{\sqrt{e\cos(c+dx)}}{\sqrt{e}}\right)\sqrt{1+\cos(c+dx)}\,\sqrt{a+a\sin(c+dx)}}{d\left(a^3+a^3\cos(c+dx)+a^3\sin(c+dx)\right)}-\frac{2e^{5/2}\cdots}{\cdots}$$

```
[Out] -4/3*e*(e*cos(d*x+c))^(3/2)/a/d/(a+a*sin(d*x+c))^(3/2)-2*e^(5/2)*arcsinh((e
*cos(d*x+c))^(1/2)/e^(1/2))*(1+cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^(1/2)/d/(
a^3+a^3*cos(d*x+c)+a^3*sin(d*x+c))-2*e^(5/2)*arctan(sin(d*x+c)*e^(1/2)/(e*c
os(d*x+c))^(1/2)/(1+cos(d*x+c))^(1/2))*(1+cos(d*x+c))^(1/2)*(a+a*sin(d*x+c)
)^(1/2)/d/(a^3+a^3*cos(d*x+c)+a^3*sin(d*x+c))
```

Rubi [A]
time = 0.20, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2759, 2763, 2854, 209, 2912, 65, 221}

$$-\frac{2e^{5/2}\sqrt{\cos(c+dx)+1}\,\sqrt{a\sin(c+dx)+a}\,\text{ArcTan}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{\cos(c+dx)+1}\,\sqrt{e\cos(c+dx)}}\right)}{d\left(a^3\sin(c+dx)+a^3\cos(c+dx)+a^3\right)}-\frac{2e^{5/2}\sqrt{\cos(c+dx)+1}\,\sqrt{a\sin(c+dx)+a}\,\sinh^{-1}\left(\frac{\sqrt{e\cos(c+dx)}}{\sqrt{e}}\right)}{d\left(a^3\sin(c+dx)+a^3\cos(c+dx)+a^3\right)}-\frac{4e(e\cos(c+dx))^{3/2}}{3ad(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(5/2)/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-4*e*(e*Cos[c + d*x])^(3/2))/(3*a*d*(a + a*Sin[c + d*x])^(3/2)) - (2*e^(5/
2)*ArcSinh[Sqrt[e*Cos[c + d*x]]/Sqrt[e]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[a + a*
Sin[c + d*x]])/(d*(a^3 + a^3*Cos[c + d*x] + a^3*Sin[c + d*x])) - (2*e^(5/2)
*ArcTan[(Sqrt[e]*Sin[c + d*x])/(Sqrt[e*Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]]
)]*Sqrt[1 + Cos[c + d*x]]*Sqrt[a + a*Sin[c + d*x]])/(d*(a^3 + a^3*Cos[c + d
*x] + a^3*Sin[c + d*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2763

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[g*Sqrt[1 + Cos[e + f*x]]*(Sqrt[a + b*Sin[e + f*x
]]/(a + a*Cos[e + f*x] + b*Sin[e + f*x])), Int[Sqrt[1 + Cos[e + f*x]]/Sqrt[
g*Cos[e + f*x]], x], x] - Dist[g*Sqrt[1 + Cos[e + f*x]]*(Sqrt[a + b*Sin[e +
 f*x]]/(b + b*Cos[e + f*x] + a*Sin[e + f*x])), Int[Sin[e + f*x]/(Sqrt[g*Cos
[e + f*x]]*Sqrt[1 + Cos[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, g}, x] &&
EqQ[a^2 - b^2, 0]
```

Rule 2854

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b + d*x^2), x], x
, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]))], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{5/2}}{(a+a\sin(c+dx))^{5/2}}\,dx = -\frac{4e(e\cos(c+dx))^{3/2}}{3ad(a+a\sin(c+dx))^{3/2}} - \frac{e^2\int\frac{\sqrt{e\cos(c+dx)}}{\sqrt{a+a\sin(c+dx)}}\,dx}{a^2}$$

$$= -\frac{4e(e\cos(c+dx))^{3/2}}{3ad(a+a\sin(c+dx))^{3/2}} - \frac{\left(e^3\sqrt{1+\cos(c+dx)}\,\sqrt{a+a\sin(c+dx)}\right)\int}{a^2(a+a\cos(c+dx)+a\sin(c}$$

$$= -\frac{4e(e\cos(c+dx))^{3/2}}{3ad(a+a\sin(c+dx))^{3/2}} - \frac{\left(e^3\sqrt{1+\cos(c+dx)}\,\sqrt{a+a\sin(c+dx)}\right)\text{Su}}{a^2d(a+a\cos(c+dx)}$$

$$= -\frac{4e(e\cos(c+dx))^{3/2}}{3ad(a+a\sin(c+dx))^{3/2}} - \frac{2e^{5/2}\tan^{-1}\left(\frac{\sqrt{e}\sin(c+dx)}{\sqrt{e\cos(c+dx)}\,\sqrt{1+\cos(c+dx}}\right.}{d\left(a^3+a^3\cos(c+dx)\right.}$$

$$= -\frac{4e(e\cos(c+dx))^{3/2}}{3ad(a+a\sin(c+dx))^{3/2}} - \frac{2e^{5/2}\sinh^{-1}\left(\frac{\sqrt{e\cos(c+dx)}}{\sqrt{e}}\right)\sqrt{1+\cos(c+}}{d\left(a^3+a^3\cos(c+dx)+a^3\sin(\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.14, size = 80, normalized size = 0.37

$$-\frac{\sqrt[4]{2}\,(e\cos(c+dx))^{7/2}\,{}_2F_1\left(\frac{7}{4},\frac{7}{4};\frac{11}{4};\frac{1}{2}(1-\sin(c+dx))\right)\sqrt{a(1+\sin(c+dx))}}{7a^3de(1+\sin(c+dx))^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(5/2)/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] -1/7*(2^(1/4)*(e*Cos[c + d*x])^(7/2)*Hypergeometric2F1[7/4, 7/4, 11/4, (1 -
 Sin[c + d*x])/2]*Sqrt[a*(1 + Sin[c + d*x])])/(a^3*d*e*(1 + Sin[c + d*x])^(
9/4))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 544 vs. 2(190) = 380.
time = 0.15, size = 545, normalized size = 2.50

method	result

default	$-\frac{\left(3\sqrt{2}\,\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2}}{2}\right)\sin(dx+c)\cos(dx+c)+3\sqrt{2}\,\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2}}{2}\right)(\cos^2(dx+c)}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/d*(3*2^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*
sin(d*x+c)*cos(d*x+c)+3*2^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*2^(1/2))*cos(d*x+c)^2+3*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*sin(d*x+c)*cos(d*x+c)+3*2^(1/2)*a
rctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/
2))*cos(d*x+c)^2-6*2^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
2^(1/2))*sin(d*x+c)+3*2^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*2^(1/2))*cos(d*x+c)-6*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*sin(d*x+c)+3*2^(1/2)*arctanh(1/2*(-2*
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*cos(d*x+c)-
4*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)-6*2^(1/2)*arct
an(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))-6*2^(1/2)*arctanh(1/2*
(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2)))*(e*cos
(d*x+c))^(5/2)/(sin(d*x+c)-1)/(a*(1+sin(d*x+c)))^(5/2)/(-2*cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)/sin(d*x+c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima"
)
```

```
[Out] e^(5/2)*integrate(cos(d*x + c)^(5/2)/(a*sin(d*x + c) + a)^(5/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3631 vs. $2(176) = 352$.
time = 187.76, size = 3631, normalized size = 16.66

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas"
)
```

```
[Out] 1/12*(12*(3*sqrt(2)*a^3*d*cos(d*x + c)^2 - 4*sqrt(2)*a^3*d + (sqrt(2)*a^3*d
*cos(d*x + c)^2 - 4*sqrt(2)*a^3*d)*sin(d*x + c))*(1/(a^10*d^4))^(1/4)*arcta
n(-1/4*(sqrt(2)*((sqrt(2)*a^8*d^3*cos(d*x + c)^6 - 3*sqrt(2)*a^8*d^3*cos(d*
x + c)^5 - 8*sqrt(2)*a^8*d^3*cos(d*x + c)^4 + 4*sqrt(2)*a^8*d^3*cos(d*x + c
)^3 + 8*sqrt(2)*a^8*d^3*cos(d*x + c)^2 - (sqrt(2)*a^8*d^3*cos(d*x + c)^5 +
4*sqrt(2)*a^8*d^3*cos(d*x + c)^4 - 4*sqrt(2)*a^8*d^3*cos(d*x + c)^3 - 8*sqr
t(2)*a^8*d^3*cos(d*x + c)^2)*sin(d*x + c))*(1/(a^10*d^4))^(3/4)*e^(15/2) +
(sqrt(2)*a^3*d*cos(d*x + c)^6*e^5 + 5*sqrt(2)*a^3*d*cos(d*x + c)^5*e^5 - 8*
sqrt(2)*a^3*d*cos(d*x + c)^4*e^5 - 20*sqrt(2)*a^3*d*cos(d*x + c)^3*e^5 + 8*
sqrt(2)*a^3*d*cos(d*x + c)^2*e^5 + 16*sqrt(2)*a^3*d*cos(d*x + c)*e^5 + (sqr
t(2)*a^3*d*cos(d*x + c)^5*e^5 - 4*sqrt(2)*a^3*d*cos(d*x + c)^4*e^5 - 12*sqr
t(2)*a^3*d*cos(d*x + c)^3*e^5 + 8*sqrt(2)*a^3*d*cos(d*x + c)^2*e^5 + 16*sqr
t(2)*a^3*d*cos(d*x + c)*e^5)*sin(d*x + c))*(1/(a^10*d^4))^(1/4)*e^(5/2) - (
cos(d*x + c)^4*e^(15/2) - 3*cos(d*x + c)^3*e^(15/2) - 8*cos(d*x + c)^2*e^(1
5/2) + (2*a^5*d^2*cos(d*x + c)^5*e^(5/2) - 5*a^5*d^2*cos(d*x + c)^4*e^(5/2)
 - 19*a^5*d^2*cos(d*x + c)^3*e^(5/2) + 20*a^5*d^2*cos(d*x + c)*e^(5/2) + 8*
a^5*d^2*e^(5/2) - (2*a^5*d^2*cos(d*x + c)^4*e^(5/2) + 9*a^5*d^2*cos(d*x + c
)^3*e^(5/2) - 4*a^5*d^2*cos(d*x + c)^2*e^(5/2) - 20*a^5*d^2*cos(d*x + c)*e^
(5/2) - 8*a^5*d^2*e^(5/2))*sin(d*x + c))*sqrt(1/(a^10*d^4))*e^5 + 4*cos(d*x
 + c)*e^(15/2) - (cos(d*x + c)^3*e^(15/2) + 4*cos(d*x + c)^2*e^(15/2) - 4*c
os(d*x + c)*e^(15/2) - 8*e^(15/2))*sin(d*x + c) + 8*e^(15/2))*sqrt(a*sin(d*
x + c) + a)*sqrt(cos(d*x + c)))*sqrt((2*a*cos(d*x + c)*e^15*sin(d*x + c) +
2*a*cos(d*x + c)*e^15 + (a^6*d^2*e^10*sin(d*x + c) + a^6*d^2*e^10)*sqrt(1/(
a^10*d^4))*e^5 + (sqrt(2)*a^3*d*(1/(a^10*d^4))^(1/4)*cos(d*x + c)*e^15 + (s
qrt(2)*a^8*d^3*e^(15/2)*sin(d*x + c) + sqrt(2)*a^8*d^3*e^(15/2))*(1/(a^10*d
^4))^(3/4)*e^(15/2))*sqrt(a*sin(d*x + c) + a)*sqrt(cos(d*x + c)))/(sin(d*x
+ c) + 1)) + ((2*sqrt(2)*a^8*d^3*cos(d*x + c)^5*e^(15/2) + sqrt(2)*a^8*d^3*
cos(d*x + c)^4*e^(15/2) - 13*sqrt(2)*a^8*d^3*cos(d*x + c)^3*e^(15/2) - 8*sq
rt(2)*a^8*d^3*cos(d*x + c)^2*e^(15/2) + 12*sqrt(2)*a^8*d^3*cos(d*x + c)*e^(
15/2) + 8*sqrt(2)*a^8*d^3*e^(15/2) - (7*sqrt(2)*a^8*d^3*cos(d*x + c)^3*e^(1
5/2) + 4*sqrt(2)*a^8*d^3*cos(d*x + c)^2*e^(15/2) - 12*sqrt(2)*a^8*d^3*cos(d
*x + c)*e^(15/2) - 8*sqrt(2)*a^8*d^3*e^(15/2))*sin(d*x + c))*(1/(a^10*d^4))
^(3/4)*e^(15/2) + (7*sqrt(2)*a^3*d*cos(d*x + c)^4*e^(25/2) + 3*sqrt(2)*a^3*
d*cos(d*x + c)^3*e^(25/2) - 16*sqrt(2)*a^3*d*cos(d*x + c)^2*e^(25/2) - 4*sq
rt(2)*a^3*d*cos(d*x + c)*e^(25/2) + 8*sqrt(2)*a^3*d*e^(25/2) + (2*sqrt(2)*a
^3*d*cos(d*x + c)^4*e^(25/2) + sqrt(2)*a^3*d*cos(d*x + c)^3*e^(25/2) - 12*s
qrt(2)*a^3*d*cos(d*x + c)^2*e^(25/2) - 4*sqrt(2)*a^3*d*cos(d*x + c)*e^(25/2
) + 8*sqrt(2)*a^3*d*e^(25/2))*sin(d*x + c))*(1/(a^10*d^4))^(1/4)*e^(5/2))*s
qrt(a*sin(d*x + c) + a)*sqrt(cos(d*x + c)))/(a*cos(d*x + c)^6*e^15 + a*cos(
d*x + c)^5*e^15 - 8*a*cos(d*x + c)^4*e^15 - 8*a*cos(d*x + c)^3*e^15 + 8*a*c
os(d*x + c)^2*e^15 + 8*a*cos(d*x + c)*e^15 - 4*(a*cos(d*x + c)^4*e^15 + a*c
os(d*x + c)^3*e^15 - 2*a*cos(d*x + c)^2*e^15 - 2*a*cos(d*x + c)*e^15)*sin(d
*x + c)))*e^(5/2) - 12*(3*sqrt(2)*a^3*d*cos(d*x + c)^2 - 4*sqrt(2)*a^3*d +
(sqrt(2)*a^3*d*cos(d*x + c)^2 - 4*sqrt(2)*a^3*d)*sin(d*x + c))*(1/(a^10*d^4
))^(1/4)*arctan(1/4*(sqrt(2)*((sqrt(2)*a^8*d^3*cos(d*x + c)^6 - 3*sqrt(2)*a
```

```
^8*d^3*cos(d*x + c)^5 - 8*sqrt(2)*a^8*d^3*cos(d*x + c)^4 + 4*sqrt(2)*a^8*d^
3*cos(d*x + c)^3 + 8*sqrt(2)*a^8*d^3*cos(d*x + c)^2 - (sqrt(2)*a^8*d^3*cos(
d*x + c)^5 + 4*sqrt(2)*a^8*d^3*cos(d*x + c)^4 - 4*sqrt(2)*a^8*d^3*cos(d*x +
 c)^3 - 8*sqrt(2)*a^8*d^3*cos(d*x + c)^2)*sin(d*x + c))*(1/(a^10*d^4))^(3/4
)*e^(15/2) + (sqrt(2)*a^3*d*cos(d*x + c)^6*e^5 + 5*sqrt(2)*a^3*d*cos(d*x +
c)^5*e^5 - 8*sqrt(2)*a^3*d*cos(d*x + c)^4*e^5 - 20*sqrt(2)*a^3*d*cos(d*x +
c)^3*e^5 + 8*sqrt(2)*a^3*d*cos(d*x + c)^2*e^5 + 16*sqrt(2)*a^3*d*cos(d*x +
c)*e^5 + (sqrt(2)*a^3*d*cos(d*x + c)^5*e^5 - 4*sqrt(2)*a^3*d*cos(d*x + c)^4
*e^5 - 12*sqrt(2)*a^3*d*cos(d*x + c)^3*e^5 + 8*sqrt(2)*a^3*d*cos(d*x + c)^2
*e^5 + 16*sqrt(2)*a^3*d*cos(d*x + c)*e^5)*sin(d*x + c))*(1/(a^10*d^4))^(1/4
)*e^(5/2) + (cos(d*x + c)^4*e^(15/2) - 3*cos(d*x + c)^3*e^(15/2) - 8*cos(d*
x + c)^2*e^(15/2) + (2*a^5*d^2*cos(d*x + c)^5*e^(5/2) - 5*a^5*d^2*cos(d*x +
 c)^4*e^(5/2) - 19*a^5*d^2*cos(d*x + c)^3*e^(5/2) + 20*a^5*d^2*cos(d*x + c)
*e^(5/2) + 8*a^5*d^2*e^(5/2) - (2*a^5*d^2*cos(d*x + c)^4*e^(5/2) + 9*a^5*d^
2*cos(d*x + c)^3*e^(5/2) - 4*a^5*d^2*cos(d*x + c)^2*e^(5/2) - 20*a^5*d^2*co
s(d*x + c)*e^(5/2) - 8*a^5*d^2*e^(5/2))*sin(d*x + c))*sqrt(1/(a^10*d^4))*e^
5 + 4*cos(d*x + c)*e^(15/2) - (cos(d*x + c)^3*e^(15/2) + 4*cos(d*x + c)^2*e
^(15/2) - 4*cos(d*x + c)*e^(15/2) - 8*e^(15/2))*sin(d*x + c) + 8*e^(15/2))*
sqrt(a*sin(d*x + c) + a)*sqrt(cos(d*x + c)))*sqrt((2*a*cos(d*x + c)*e^15*si
n(d*x + c) + 2*a*cos(d*x + c)*e^15 + (a^6*d^2*e^10*sin(d*x + c) + a^6*d^2*e
^10)*sqrt(1/(a^10*d^4))*e^5 - (sqrt(2)*a^3*d*(1/(a^10*d^4))^(1/4)*cos(d*x +
 c)*e^15 + (sqrt(2)*a^8*d^3*e^(15/2)*sin(d*x + ...
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(5/2)/(a+a*sin(d*x+c))**(5/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,\cos(c+d\,x))^{5/2}}{(a+a\,\sin(c+d\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(5/2)/(a + a*sin(c + d*x))^(5/2),x)
[Out] int((e*cos(c + d*x))^(5/2)/(a + a*sin(c + d*x))^(5/2), x)
```

3.316 $\int \frac{(e\cos(c+dx))^{3/2}}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=36

$$-\frac{2(e\cos(c+dx))^{5/2}}{5de(a+a\sin(c+dx))^{5/2}}$$

```
[Out] -2/5*(e*cos(d*x+c))^(5/2)/d/e/(a+a*sin(d*x+c))^(5/2)
```

Rubi [A]
time = 0.05, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$, Rules used = {2750}

$$-\frac{2(e\cos(c+dx))^{5/2}}{5de(a\sin(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(3/2)/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*(e*Cos[c + d*x])^(5/2))/(5*d*e*(a + a*Sin[c + d*x])^(5/2))
```

Rule 2750

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{3/2}}{(a+a\sin(c+dx))^{5/2}}\,dx = -\frac{2(e\cos(c+dx))^{5/2}}{5de(a+a\sin(c+dx))^{5/2}}$$

Mathematica [A]
time = 0.11, size = 49, normalized size = 1.36

$$-\frac{2(e\cos(c+dx))^{5/2}\sqrt{a(1+\sin(c+dx))}}{5a^3de(1+\sin(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(3/2)/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*(e*Cos[c + d*x])^(5/2)*Sqrt[a*(1 + Sin[c + d*x])])/(5*a^3*d*e*(1 + Sin[
c + d*x])^3)
```

Maple [A]
time = 0.16, size = 34, normalized size = 0.94

method	result	size
default	$-\frac{2(e\cos(dx+c))^{\frac{3}{2}}\cos(dx+c)}{5d(a(1+\sin(dx+c)))^{\frac{5}{2}}}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5/d*(e*cos(d*x+c))^(3/2)*cos(d*x+c)/(a*(1+sin(d*x+c)))^(5/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 126 vs. 2(27) = 54.
time = 0.55, size = 126, normalized size = 3.50

$$-\frac{2\left(\sqrt{a}-\frac{\sqrt{a}\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{3}{2}}\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)e^{\frac{3}{2}}}{5\left(a^3+\frac{a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima"
)
```

```
[Out] -2/5*(sqrt(a) - sqrt(a)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(-sin(d*x + c)
/(cos(d*x + c) + 1) + 1)^(3/2)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)*e^
(3/2)/((a^3 + a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*d*(sin(d*x + c)/(cos
(d*x + c) + 1) + 1)^(7/2))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 70 vs. 2(27) = 54.
time = 0.36, size = 70, normalized size = 1.94

$$-\frac{2\sqrt{a\sin(dx+c)+a}\left(e^{\frac{3}{2}}\sin(dx+c)-e^{\frac{3}{2}}\right)\sqrt{\cos(dx+c)}}{5\left(a^3d\cos(dx+c)^2-2a^3d\sin(dx+c)-2a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas"
)
```

```
[Out] -2/5*sqrt(a*sin(d*x + c) + a)*(e^(3/2)*sin(d*x + c) - e^(3/2))*sqrt(cos(d*x
 + c))/(a^3*d*cos(d*x + c)^2 - 2*a^3*d*sin(d*x + c) - 2*a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(c+dx))^{\frac{3}{2}}}{(a(\sin(c+dx)+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(3/2)/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Integral((e*cos(c + d*x))**(3/2)/(a*(sin(c + d*x) + 1))**(5/2), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 6.57, size = 102, normalized size = 2.83

$$-\frac{4\,e\,\sqrt{e\,\cos\left(c+d\,x\right)}\,\sqrt{a\,\left(\sin\left(c+d\,x\right)+1\right)}\,\left(\sin\left(c+d\,x\right)+2\,\cos\left(2\,c+2\,d\,x\right)+\sin\left(3\,c+3\,d\,x\right)+2\right)}{5\,a^3\,d\,\left(56\,\sin\left(c+d\,x\right)-28\,\cos\left(2\,c+2\,d\,x\right)+\cos\left(4\,c+4\,d\,x\right)-8\,\sin\left(3\,c+3\,d\,x\right)+35\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(3/2)/(a + a*sin(c + d*x))^(5/2),x)
```

```
[Out] -(4*e*(e*cos(c + d*x))^(1/2)*(a*(sin(c + d*x) + 1))^(1/2)*(sin(c + d*x) + 2
*cos(2*c + 2*d*x) + sin(3*c + 3*d*x) + 2))/(5*a^3*d*(56*sin(c + d*x) - 28*c
os(2*c + 2*d*x) + cos(4*c + 4*d*x) - 8*sin(3*c + 3*d*x) + 35))
```

3.317 $\int \frac{\sqrt{e\cos(c+dx)}}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=76

$$-\frac{2(e\cos(c+dx))^{3/2}}{7de(a+a\sin(c+dx))^{5/2}}-\frac{4(e\cos(c+dx))^{3/2}}{21ade(a+a\sin(c+dx))^{3/2}}$$

```
[Out] -2/7*(e*cos(d*x+c))^(3/2)/d/e/(a+a*sin(d*x+c))^(5/2)-4/21*(e*cos(d*x+c))^(3
/2)/a/d/e/(a+a*sin(d*x+c))^(3/2)
```

Rubi [A]
time = 0.09, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2751, 2750}

$$-\frac{4(e\cos(c+dx))^{3/2}}{21ade(a\sin(c+dx)+a)^{3/2}}-\frac{2(e\cos(c+dx))^{3/2}}{7de(a\sin(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cos[c + d*x]]/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*(e*Cos[c + d*x])^(3/2))/(7*d*e*(a + a*Sin[c + d*x])^(5/2)) - (4*(e*Cos[
c + d*x])^(3/2))/(21*a*d*e*(a + a*Sin[c + d*x])^(3/2))
```

Rule 2750

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{\sqrt{e\cos(c+dx)}}{(a+a\sin(c+dx))^{5/2}}\,dx = -\frac{2(e\cos(c+dx))^{3/2}}{7de(a+a\sin(c+dx))^{5/2}} + \frac{2\int \frac{\sqrt{e\cos(c+dx)}}{(a+a\sin(c+dx))^{3/2}}\,dx}{7a}$$
$$= -\frac{2(e\cos(c+dx))^{3/2}}{7de(a+a\sin(c+dx))^{5/2}} - \frac{4(e\cos(c+dx))^{3/2}}{21ade(a+a\sin(c+dx))^{3/2}}$$

__Mathematica__ [A]
time = 0.10, size = 59, normalized size = 0.78

$$-\frac{2(e\cos(c+dx))^{3/2}\sqrt{a(1+\sin(c+dx))}\,(5+2\sin(c+dx))}{21a^3de(1+\sin(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Cos[c + d*x]]/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*(e*Cos[c + d*x])^(3/2)*Sqrt[a*(1 + Sin[c + d*x])]*(5 + 2*Sin[c + d*x]))
/(21*a^3*d*e*(1 + Sin[c + d*x])^3)
```

__Maple__ [A]
time = 0.18, size = 44, normalized size = 0.58

method	result	size
default	$-\frac{2(2\sin(dx+c)+5)\cos(dx+c)\sqrt{e\cos\left(dx+c\right)}}{21d(a(1+\sin(dx+c)))^{\frac{5}{2}}}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/21/d*(2*sin(d*x+c)+5)*cos(d*x+c)*(e*cos(d*x+c))^(1/2)/(a*(1+sin(d*x+c)))
^(5/2)
```

__Maxima__ [B] Leaf count of result is larger than twice the leaf count of optimal. 197 vs. 2(58) = 116.
time = 0.54, size = 197, normalized size = 2.59

$$-\frac{2\left(5\sqrt{a}+\frac{4\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}-\frac{4\sqrt{a}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{5\sqrt{a}\sin(dx+c)^4}{(\cos(dx+c)+1)^4}\right)\sqrt{-\frac{\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}+1}\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^2 e^{\frac{1}{2}}}{21\left(a^3+\frac{2a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}\right)d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima"
)
```

```
[Out] -2/21*(5*sqrt(a) + 4*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 4*sqrt(a)*si
n(d*x + c)^3/(cos(d*x + c) + 1)^3 - 5*sqrt(a)*sin(d*x + c)^4/(cos(d*x + c)
+ 1)^4)*sqrt(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)*(sin(d*x + c)^2/(cos(d*x
 + c) + 1)^2 + 1)^2*e^(1/2)/((a^3 + 2*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)
^2 + a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4)*d*(sin(d*x + c)/(cos(d*x + c)
 + 1) + 1)^(9/2))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(58) = 116$.
time = 0.35, size = 158, normalized size = 2.08

$$\frac{2\left(2\cos\left(dx+c\right)^{2}e^{\frac{1}{2}}+5\cos\left(dx+c\right)e^{\frac{1}{2}}+\left(2\cos\left(dx+c\right)e^{\frac{1}{2}}-3e^{\frac{1}{2}}\right)\sin\left(dx+c\right)+3e^{\frac{1}{2}}\right)\sqrt{a\sin\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}}{21\left(a^{3}d\cos\left(dx+c\right)^{3}+3a^{3}d\cos\left(dx+c\right)^{2}-2a^{3}d\cos\left(dx+c\right)-4a^{3}d+\left(a^{3}d\cos\left(dx+c\right)^{2}-2a^{3}d\cos\left(dx+c\right)-4a^{3}d\right)\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas"
)
```

```
[Out] 2/21*(2*cos(d*x + c)^2*e^(1/2) + 5*cos(d*x + c)*e^(1/2) + (2*cos(d*x + c)*e
^(1/2) - 3*e^(1/2))*sin(d*x + c) + 3*e^(1/2))*sqrt(a*sin(d*x + c) + a)*sqrt
(cos(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos
(d*x + c) - 4*a^3*d + (a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c) - 4*a^3*
d)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e\cos\left(c+dx\right)}}{\left(a\left(\sin\left(c+dx\right)+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(1/2)/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Integral(sqrt(e*cos(c + d*x))/(a*(sin(c + d*x) + 1))**(5/2), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 7.05, size = 145, normalized size = 1.91

$$-\frac{8\sqrt{a\left(\sin\left(c+d\,x\right)+1\right)}\,\sqrt{-e\left(2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}\,\left(-58\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+18\sin\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)^2+26\,\sin\left(2\,c+2\,d\,x\right)-\sin\left(4\,c+4\,d\,x\right)+20\right)}{21\,a^3\,d\,\left(240\sin\left(c+d\,x\right)^2+210\,\sin\left(c+d\,x\right)-20\sin\left(2\,c+2\,d\,x\right)^2-45\,\sin\left(3\,c+3\,d\,x\right)+\sin\left(5\,c+5\,d\,x\right)+16\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(1/2)/(a + a*sin(c + d*x))^(5/2),x)
```

```
[Out] -(8*(a*(sin(c + d*x) + 1))^(1/2)*(-e*(2*sin(c/2 + (d*x)/2)^2 - 1))^(1/2)*(2
6*sin(2*c + 2*d*x) - sin(4*c + 4*d*x) - 58*sin(c/2 + (d*x)/2)^2 + 18*sin((3
*c)/2 + (3*d*x)/2)^2 + 20))/(21*a^3*d*(210*sin(c + d*x) - 45*sin(3*c + 3*d*
x) + sin(5*c + 5*d*x) - 20*sin(2*c + 2*d*x)^2 + 240*sin(c + d*x)^2 + 16))
```

3.318 $\int \frac{1}{\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=115

$$-\frac{2\sqrt{e\cos(c+dx)}}{9de(a+a\sin(c+dx))^{5/2}}-\frac{8\sqrt{e\cos(c+dx)}}{45ade(a+a\sin(c+dx))^{3/2}}-\frac{16\sqrt{e\cos(c+dx)}}{45a^2de\sqrt{a+a\sin(c+dx)}}$$

```
[Out] -2/9*(e*cos(d*x+c))^(1/2)/d/e/(a+a*sin(d*x+c))^(5/2)-8/45*(e*cos(d*x+c))^(1
/2)/a/d/e/(a+a*sin(d*x+c))^(3/2)-16/45*(e*cos(d*x+c))^(1/2)/a^2/d/e/(a+a*si
n(d*x+c))^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2751, 2750}

$$-\frac{16\sqrt{e\cos(c+dx)}}{45a^2de\sqrt{a\sin(c+dx)+a}}-\frac{8\sqrt{e\cos(c+dx)}}{45ade(a\sin(c+dx)+a)^{3/2}}-\frac{2\sqrt{e\cos(c+dx)}}{9de(a\sin(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])^(5/2)),x]
```

```
[Out] (-2*Sqrt[e*Cos[c + d*x]])/(9*d*e*(a + a*Sin[c + d*x])^(5/2)) - (8*Sqrt[e*Co
s[c + d*x]])/(45*a*d*e*(a + a*Sin[c + d*x])^(3/2)) - (16*Sqrt[e*Cos[c + d*x
]])/(45*a^2*d*e*Sqrt[a + a*Sin[c + d*x]])
```

Rule 2750

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^{5/2}}\,dx = -\frac{2\sqrt{e\cos(c+dx)}}{9de(a+a\sin(c+dx))^{5/2}} + \frac{4\int \frac{1}{\sqrt{e\cos(c+dx)}\,(a+a\sin(c+c}}{9a}$$

$$= -\frac{2\sqrt{e\cos(c+dx)}}{9de(a+a\sin(c+dx))^{5/2}} - \frac{8\sqrt{e\cos(c+dx)}}{45ade(a+a\sin(c+dx))^{3/2}} +$$

$$= -\frac{2\sqrt{e\cos(c+dx)}}{9de(a+a\sin(c+dx))^{5/2}} - \frac{8\sqrt{e\cos(c+dx)}}{45ade(a+a\sin(c+dx))^{3/2}} -$$

Mathematica [A]
time = 0.15, size = 69, normalized size = 0.60

$$-\frac{2\sqrt{e\cos(c+dx)}\,\sqrt{a(1+\sin(c+dx))}\,(17+20\sin(c+dx)+8\sin^2(c+dx))}{45a^3de(1+\sin(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])^(5/2)),x]
```

```
[Out] (-2*Sqrt[e*Cos[c + d*x]]*Sqrt[a*(1 + Sin[c + d*x])]*(17 + 20*Sin[c + d*x] +
 8*Sin[c + d*x]^2))/(45*a^3*d*e*(1 + Sin[c + d*x])^3)
```

Maple [A]
time = 0.17, size = 54, normalized size = 0.47

method	result	size
default	$-\frac{2(-8(\cos^2(dx+c))+20\sin(dx+c)+25)\cos(dx+c)}{45d(a(1+\sin(dx+c)))^{\frac{5}{2}}\sqrt{e\cos(dx+c)}}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/45/d*(-8*cos(d*x+c)^2+20*sin(d*x+c)+25)*cos(d*x+c)/(a*(1+sin(d*x+c)))^(5
/2)/(e*cos(d*x+c))^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 266 vs. 2(88) = 176.
time = 0.55, size = 266, normalized size = 2.31

$$-\frac{2\left(17\sqrt{a}+\frac{40\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}+\frac{49\sqrt{a}\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{49\sqrt{a}\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{40\sqrt{a}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{17\sqrt{a}\sin(dx+c)^6}{(\cos(dx+c)+1)^6}\right)\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^3 e^{\left(-\frac{1}{2}\right)}}{45\left(a^3+\frac{3a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}\right)d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{11}{2}}\sqrt{-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(1/2),x, algorithm="maxim
a")
```

```
[Out] -2/45*(17*sqrt(a) + 40*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) + 49*sqrt(a)
*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 49*sqrt(a)*sin(d*x + c)^4/(cos(d*x +
 c) + 1)^4 - 40*sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 17*sqrt(a)*si
n(d*x + c)^6/(cos(d*x + c) + 1)^6)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1
)^3*e^(-1/2)/((a^3 + 3*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*a^3*sin(
d*x + c)^4/(cos(d*x + c) + 1)^4 + a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6)*
d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)*sqrt(-sin(d*x + c)/(cos(d*x
+ c) + 1) + 1))
```

Fricas [A]
time = 0.36, size = 100, normalized size = 0.87

$$-\frac{2\left(8\cos\left(dx+c\right)^{2}-20\sin\left(dx+c\right)-25\right)\sqrt{a\sin\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}}{45\left(3\,a^{3}d\cos\left(dx+c\right)^{2}e^{\frac{1}{2}}-4\,a^{3}de^{\frac{1}{2}}+\left(a^{3}d\cos\left(dx+c\right)^{2}e^{\frac{1}{2}}-4\,a^{3}de^{\frac{1}{2}}\right)\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(1/2),x, algorithm="frica
s")
```

```
[Out] -2/45*(8*cos(d*x + c)^2 - 20*sin(d*x + c) - 25)*sqrt(a*sin(d*x + c) + a)*sq
rt(cos(d*x + c))/(3*a^3*d*cos(d*x + c)^2*e^(1/2) - 4*a^3*d*e^(1/2) + (a^3*d
*cos(d*x + c)^2*e^(1/2) - 4*a^3*d*e^(1/2))*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\left(\sin\left(c+dx\right)+1\right)\right)^{\frac{5}{2}}\sqrt{e\cos\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))**(5/2)/(e*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/((a*(sin(c + d*x) + 1))**(5/2)*sqrt(e*cos(c + d*x))), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(1/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

Mupad [B]
time = 7.66, size = 137, normalized size = 1.19

$$-\frac{8\sqrt{a\left(\sin\left(c+d\,x\right)+1\right)}\,\left(137\,\cos\left(c+d\,x\right)-71\,\cos\left(3\,c+3\,d\,x\right)+2\,\cos\left(5\,c+5\,d\,x\right)+144\,\sin\left(2\,c+2\,d\,x\right)-18\,\sin\left(4\,c+4\,d\,x\right)\right)}{45\,a^3\,d\,\sqrt{e\,\cos\left(c+d\,x\right)}\,\left(210\,\sin\left(c+d\,x\right)-120\,\cos\left(2\,c+2\,d\,x\right)+10\,\cos\left(4\,c+4\,d\,x\right)-45\,\sin\left(3\,c+3\,d\,x\right)+\sin\left(5\,c+5\,d\,x\right)+126\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cos(c + d*x))^(1/2)*(a + a*sin(c + d*x))^(5/2)),x)
```

```
[Out] -(8*(a*(sin(c + d*x) + 1))^(1/2)*(137*cos(c + d*x) - 71*cos(3*c + 3*d*x) +
2*cos(5*c + 5*d*x) + 144*sin(2*c + 2*d*x) - 18*sin(4*c + 4*d*x)))/(45*a^3*d
*(e*cos(c + d*x))^(1/2)*(210*sin(c + d*x) - 120*cos(2*c + 2*d*x) + 10*cos(4
*c + 4*d*x) - 45*sin(3*c + 3*d*x) + sin(5*c + 5*d*x) + 126))
```

3.319 $\int \frac{1}{(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=154

$$-\frac{2}{11de\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^{5/2}}-\frac{12}{77ade\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^{3/2}}-\frac{}{77a^2de\sqrt{e\cos(c}$$

```
[Out] -2/11/d/e/(a+a*sin(d*x+c))^(5/2)/(e*cos(d*x+c))^(1/2)-12/77/a/d/e/(a+a*sin(
d*x+c))^(3/2)/(e*cos(d*x+c))^(1/2)-16/77/a^2/d/e/(e*cos(d*x+c))^(1/2)/(a+a*
sin(d*x+c))^(1/2)+32/77*(a+a*sin(d*x+c))^(1/2)/a^3/d/e/(e*cos(d*x+c))^(1/2)
```

Rubi [A]

time = 0.20, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2751, 2750}

$$\frac{32\sqrt{a\sin(c+dx)+a}}{77a^3de\sqrt{e\cos(c+dx)}}-\frac{16}{77a^2de\sqrt{a\sin(c+dx)+a}\,\sqrt{e\cos(c+dx)}}-\frac{12}{77ade(a\sin(c+dx)+a)^{3/2}\sqrt{e\cos(c+dx)}}-\frac{2}{11de(a\sin(c+dx)+a)^{5/2}\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x])^(5/2)),x]
```

```
[Out] -2/(11*d*e*Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])^(5/2)) - 12/(77*a*d*e*
Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])^(3/2)) - 16/(77*a^2*d*e*Sqrt[e*Co
s[c + d*x]]*Sqrt[a + a*Sin[c + d*x]]) + (32*Sqrt[a + a*Sin[c + d*x]])/(77*a
^3*d*e*Sqrt[e*Cos[c + d*x]])
```

Rule 2750

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{1}{(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^{5/2}}\,dx = -\frac{2}{11de\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^{5/2}} + \frac{6\int \frac{}{(e\cos(c+}}{}$$

$$= -\frac{2}{11de\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^{5/2}} - \frac{}{77ade\sqrt{e\,c}}$$

$$= -\frac{2}{11de\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^{5/2}} - \frac{}{77ade\sqrt{e\,c}}$$

$$= -\frac{2}{11de\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^{5/2}} - \frac{}{77ade\sqrt{e\,c}}$$

Mathematica [**A**]
time = 0.15, size = 66, normalized size = 0.43

$$\frac{-10+52\sin(c+dx)+80\sin^2(c+dx)+32\sin^3(c+dx)}{77de\sqrt{e\cos(c+dx)}\,(a(1+\sin(c+dx)))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x])^(5/2)),x]
```

```
[Out] (-10 + 52*Sin[c + d*x] + 80*Sin[c + d*x]^2 + 32*Sin[c + d*x]^3)/(77*d*e*Sqr
t[e*Cos[c + d*x]]*(a*(1 + Sin[c + d*x]))^(5/2))
```

Maple [**A**]
time = 0.15, size = 70, normalized size = 0.45

method	result	size
default	$-\frac{2\left(16\left(\cos^2(dx+c)\right)\sin(dx+c)+40\left(\cos^2(dx+c)\right)-42\sin(dx+c)-35\right)\cos(dx+c)}{77d(e\cos(dx+c))^{\frac{3}{2}}(a(1+\sin(dx+c)))^{\frac{5}{2}}}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/77/d*(16*cos(d*x+c)^2*sin(d*x+c)+40*cos(d*x+c)^2-42*sin(d*x+c)-35)*cos(d
*x+c)/(e*cos(d*x+c))^(3/2)/(a*(1+sin(d*x+c)))^(5/2)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 335 vs. 2(118) = 236.
time = 0.56, size = 335, normalized size = 2.18

$$-\frac{2\left(5\sqrt{a}-\frac{52\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}-\frac{150\sqrt{a}\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{180\sqrt{a}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{180\sqrt{a}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{150\sqrt{a}\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{52\sqrt{a}\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{5\sqrt{a}\sin(dx+c)^8}{(\cos(dx+c)+1)^8}\right)\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^4 e^{\left(-\frac{3}{2}\right)}}{77\left(a^3+\frac{4a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{4a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a^3\sin(dx+c)^8}{(\cos(dx+c)+1)^8}\right)d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{13}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxim
a")
```

```
[Out] -2/77*(5*sqrt(a) - 52*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 150*sqrt(a)
*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 180*sqrt(a)*sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3 + 180*sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 150*sqrt(a)
*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 52*sqrt(a)*sin(d*x + c)^7/(cos(d*x +
 c) + 1)^7 - 5*sqrt(a)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8)*(sin(d*x + c)^2
/(cos(d*x + c) + 1)^2 + 1)^4*e^(-3/2)/((a^3 + 4*a^3*sin(d*x + c)^2/(cos(d*x
 + c) + 1)^2 + 6*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*a^3*sin(d*x +
c)^6/(cos(d*x + c) + 1)^6 + a^3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8)*d*(sin
(d*x + c)/(cos(d*x + c) + 1) + 1)^(13/2)*(-sin(d*x + c)/(cos(d*x + c) + 1)
+ 1)^(3/2))
```

Fricas [A]
time = 0.36, size = 124, normalized size = 0.81

$$\frac{2\left(40\cos\left(dx+c\right)^{2}+2\left(8\cos\left(dx+c\right)^{2}-21\right)\sin\left(dx+c\right)-35\right)\sqrt{a\sin\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}}{77\left(3\,a^{3}d\cos\left(dx+c\right)^{3}e^{\frac{3}{2}}-4\,a^{3}d\cos\left(dx+c\right)e^{\frac{3}{2}}+\left(a^{3}d\cos\left(dx+c\right)^{3}e^{\frac{3}{2}}-4\,a^{3}d\cos\left(dx+c\right)e^{\frac{3}{2}}\right)\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^(5/2),x, algorithm="frica
s")
```

```
[Out] 2/77*(40*cos(d*x + c)^2 + 2*(8*cos(d*x + c)^2 - 21)*sin(d*x + c) - 35)*sqrt
(a*sin(d*x + c) + a)*sqrt(cos(d*x + c))/(3*a^3*d*cos(d*x + c)^3*e^(3/2) - 4
*a^3*d*cos(d*x + c)*e^(3/2) + (a^3*d*cos(d*x + c)^3*e^(3/2) - 4*a^3*d*cos(d
*x + c)*e^(3/2))*sin(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(3/2)/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4371 deep
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

Mupad [B]
time = 11.17, size = 261, normalized size = 1.69

$$\frac{76\sin\left(c+d\,x\right)\,\sqrt{a+a\,\sin\left(c+d\,x\right)}+30\,\sqrt{a+a\,\sin\left(c+d\,x\right)}-40\cos\left(2\,c+2\,d\,x\right)\,\sqrt{a+a\,\sin\left(c+d\,x\right)}-8\sin\left(3\,c+3\,d\,x\right)\,\sqrt{a+a\,\sin\left(c+d\,x\right)}}{\frac{385\,a^3\,d\,e\,\sqrt{\frac{e\,{\mathrm{e}}^{-c\,1{}\mathrm{i}-d\,x\,1{}\mathrm{i}}}{2}+\frac{e\,{\mathrm{e}}^{c\,1{}\mathrm{i}+d\,x\,1{}\mathrm{i}}}{2}}}{2}+\frac{1155\,a^3\,d\,e\,\sin\left(c+d\,x\right)\,\sqrt{\frac{e\,{\mathrm{e}}^{-c\,1{}\mathrm{i}-d\,x\,1{}\mathrm{i}}}{2}+\frac{e\,{\mathrm{e}}^{c\,1{}\mathrm{i}+d\,x\,1{}\mathrm{i}}}{2}}}{4}-\frac{231\,a^3\,d\,e\,\cos\left(2\,c+2\,d\,x\right)\,\sqrt{\frac{e\,{\mathrm{e}}^{-c\,1{}\mathrm{i}-d\,x\,1{}\mathrm{i}}}{2}+\frac{e\,{\mathrm{e}}^{c\,1{}\mathrm{i}+d\,x\,1{}\mathrm{i}}}{2}}}{2}-\frac{77\,a^3\,d\,e\,\sin\left(3\,c+3\,d\,x\right)\,\sqrt{\frac{e\,{\mathrm{e}}^{-c\,1{}\mathrm{i}-d\,x\,1{}\mathrm{i}}}{2}+\frac{e\,{\mathrm{e}}^{c\,1{}\mathrm{i}+d\,x\,1{}\mathrm{i}}}{2}}}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cos(c + d*x))^(3/2)*(a + a*sin(c + d*x))^(5/2)),x)
```

```
[Out] (76*sin(c + d*x)*(a + a*sin(c + d*x))^(1/2) + 30*(a + a*sin(c + d*x))^(1/2)
 - 40*cos(2*c + 2*d*x)*(a + a*sin(c + d*x))^(1/2) - 8*sin(3*c + 3*d*x)*(a +
 a*sin(c + d*x))^(1/2))/((385*a^3*d*e*((e*exp(- c*1i - d*x*1i))/2 + (e*exp(
c*1i + d*x*1i))/2)^(1/2))/2 + (1155*a^3*d*e*sin(c + d*x)*((e*exp(- c*1i - d
*x*1i))/2 + (e*exp(c*1i + d*x*1i))/2)^(1/2))/4 - (231*a^3*d*e*cos(2*c + 2*d
*x)*((e*exp(- c*1i - d*x*1i))/2 + (e*exp(c*1i + d*x*1i))/2)^(1/2))/2 - (77*
a^3*d*e*sin(3*c + 3*d*x)*((e*exp(- c*1i - d*x*1i))/2 + (e*exp(c*1i + d*x*1i
))/2)^(1/2))/4)
```

3.320 $\int \frac{1}{(e\cos(c+dx))^{5/2}(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=193

$$-\frac{2}{13de(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^{5/2}}-\frac{16}{117ade(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^{3/2}}-\frac{}{195a^2de(e\,\mathrm{co}}$$

```
[Out] -2/13/d/e/(e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^(5/2)-16/117/a/d/e/(e*cos(d
*x+c))^(3/2)/(a+a*sin(d*x+c))^(3/2)+256/585*(a+a*sin(d*x+c))^(3/2)/a^4/d/e/
(e*cos(d*x+c))^(3/2)-32/195/a^2/d/e/(e*cos(d*x+c))^(3/2)/(a+a*sin(d*x+c))^(
1/2)-128/195*(a+a*sin(d*x+c))^(1/2)/a^3/d/e/(e*cos(d*x+c))^(3/2)
```

Rubi [A]
time = 0.26, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2751, 2750}

$$\frac{256(a\sin(c+dx)+a)^{3/2}}{585a^4de(e\cos(c+dx))^{3/2}}-\frac{128\sqrt{a\sin(c+dx)+a}}{195a^3de(e\cos(c+dx))^{3/2}}-\frac{32}{195a^2de\sqrt{a\sin(c+dx)+a}\,(e\cos(c+dx))^{3/2}}-\frac{16}{117ade(a\sin(c+dx)+a)^{3/2}(e\cos(c+dx))^{3/2}}-\frac{2}{13de(a\sin(c+dx)+a)^{5/2}(e\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(5/2)*(a + a*Sin[c + d*x])^(5/2)),x]
```

```
[Out] -2/(13*d*e*(e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x])^(5/2)) - 16/(117*a*d
*e*(e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x])^(3/2)) - 32/(195*a^2*d*e*(e*
Cos[c + d*x])^(3/2)*Sqrt[a + a*Sin[c + d*x]]) - (128*Sqrt[a + a*Sin[c + d*x
]])/(195*a^3*d*e*(e*Cos[c + d*x])^(3/2)) + (256*(a + a*Sin[c + d*x])^(3/2))
/(585*a^4*d*e*(e*Cos[c + d*x])^(3/2))
```

Rule 2750

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{1}{(e\cos(c+dx))^{5/2}(a+a\sin(c+dx))^{5/2}}\,dx = -\frac{2}{13de(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^{5/2}} + \frac{8\int \frac{}{(e\cos(c-}}{}$$

$$= -\frac{2}{13de(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^{5/2}} - \frac{}{117ade(e}$$

$$= -\frac{2}{13de(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^{5/2}} - \frac{}{117ade(e}$$

$$= -\frac{2}{13de(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^{5/2}} - \frac{}{117ade(e}$$

$$= -\frac{2}{13de(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^{5/2}} - \frac{}{117ade(e}$$

Mathematica [A]
time = 0.28, size = 76, normalized size = 0.39

$$-\frac{2(77+136\cos(2(c+dx))-16\cos(4(c+dx))-40\sin(c+dx)+80\sin(3(c+dx)))}{585de(e\cos(c+dx))^{3/2}(a(1+\sin(c+dx)))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Cos[c + d*x])^(5/2)*(a + a*Sin[c + d*x])^(5/2)),x]
```

```
[Out] (-2*(77 + 136*Cos[2*(c + d*x)] - 16*Cos[4*(c + d*x)] - 40*Sin[c + d*x] + 80
*Sin[3*(c + d*x)]))/(585*d*e*(e*Cos[c + d*x])^(3/2)*(a*(1 + Sin[c + d*x]))^
(5/2))
```

Maple [A]
time = 0.16, size = 80, normalized size = 0.41

method	result	size
default	$\frac{2\left(128\left(\cos^{4}(dx+c)\right)-320\left(\cos^{2}(dx+c)\right)\sin(dx+c)-400\left(\cos^{2}(dx+c)\right)+120\sin(dx+c)+75\right)\cos(dx+c)}{585d\left(e\cos(dx+c)\right)^{\frac{5}{2}}\left(a(1+\sin(dx+c))\right)^{\frac{5}{2}}}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/585/d*(128*cos(d*x+c)^4-320*cos(d*x+c)^2*sin(d*x+c)-400*cos(d*x+c)^2+120*
sin(d*x+c)+75)*cos(d*x+c)/(e*cos(d*x+c))^(5/2)/(a*(1+sin(d*x+c)))^(5/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 404 vs. $2(148) = 296$.
time = 0.55, size = 404, normalized size = 2.09

$$-\frac{2\left(197\sqrt{a}+\frac{400\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}+\frac{15\sqrt{a}\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{1760\sqrt{a}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{2230\sqrt{a}\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{2230\sqrt{a}\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{1760\sqrt{a}\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{15\sqrt{a}\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{400\sqrt{a}\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{197\sqrt{a}\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}\right)\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^5 e^{\left(-\frac{5}{2}\right)}}{585\left(a^3+\frac{5a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{10a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{10a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{5a^3\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{a^3\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}\right)d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{15}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxim
a")
```

```
[Out] -2/585*(197*sqrt(a) + 400*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) + 15*sqrt
(a)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1760*sqrt(a)*sin(d*x + c)^3/(cos(
d*x + c) + 1)^3 - 2230*sqrt(a)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 2230*s
qrt(a)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 1760*sqrt(a)*sin(d*x + c)^7/(c
os(d*x + c) + 1)^7 - 15*sqrt(a)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 400*s
qrt(a)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 197*sqrt(a)*sin(d*x + c)^10/(c
os(d*x + c) + 1)^10)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^5*e^(-5/2)/(
(a^3 + 5*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 10*a^3*sin(d*x + c)^4/(c
os(d*x + c) + 1)^4 + 10*a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 5*a^3*sin
(d*x + c)^8/(cos(d*x + c) + 1)^8 + a^3*sin(d*x + c)^10/(cos(d*x + c) + 1)^1
0)*d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(15/2)*(-sin(d*x + c)/(cos(d*x +
 c) + 1) + 1)^(5/2))
```

Fricas [A]
time = 0.36, size = 138, normalized size = 0.72

$$-\frac{2\left(128\cos(dx+c)^4-400\cos(dx+c)^2-40\left(8\cos(dx+c)^2-3\right)\sin(dx+c)+75\right)\sqrt{a\sin(dx+c)+a}\sqrt{\cos(dx+c)}}{585\left(3a^3d\cos(dx+c)^4e^{\frac{5}{2}}-4a^3d\cos(dx+c)^2e^{\frac{5}{2}}+\left(a^3d\cos(dx+c)^4e^{\frac{5}{2}}-4a^3d\cos(dx+c)^2e^{\frac{5}{2}}\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^(5/2),x, algorithm="frica
s")
```

```
[Out] -2/585*(128*cos(d*x + c)^4 - 400*cos(d*x + c)^2 - 40*(8*cos(d*x + c)^2 - 3)
*sin(d*x + c) + 75)*sqrt(a*sin(d*x + c) + a)*sqrt(cos(d*x + c))/(3*a^3*d*co
s(d*x + c)^4*e^(5/2) - 4*a^3*d*cos(d*x + c)^2*e^(5/2) + (a^3*d*cos(d*x + c)
^4*e^(5/2) - 4*a^3*d*cos(d*x + c)^2*e^(5/2))*sin(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(5/2)/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(5/2)/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

Mupad [B]
time = 11.54, size = 379, normalized size = 1.96

$$-\frac{\sqrt{a+a\sin\left(c+d\,x\right)}\,\left(\frac{\mathrm{e}^{c\,4\mathrm{i}+d\,x\,4\mathrm{i}}\,2464\mathrm{i}}{585\,a^3\,d\,e^2}+\frac{\mathrm{e}^{c\,4\mathrm{i}+d\,x\,4\mathrm{i}}\,\cos\left(2\,c+2\,d\,x\right)\,4352\mathrm{i}}{585\,a^3\,d\,e^2}-\frac{\mathrm{e}^{c\,4\mathrm{i}+d\,x\,4\mathrm{i}}\,\cos\left(4\,c+4\,d\,x\right)\,512\mathrm{i}}{585\,a^3\,d\,e^2}+\frac{\mathrm{e}^{c\,4\mathrm{i}+d\,x\,4\mathrm{i}}\,\sin\left(3\,c+3\,d\,x\right)\,512\mathrm{i}}{117\,a^3\,d\,e^2}-\frac{\mathrm{e}^{c\,4\mathrm{i}+d\,x\,4\mathrm{i}}\,\sin\left(c+d\,x\right)\,256\mathrm{i}}{117\,a^3\,d\,e^2}\right)}{\cos\left(c+d\,x\right)\,\mathrm{e}^{c\,4\mathrm{i}+d\,x\,4\mathrm{i}}\,\sqrt{e\,\left(\frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}}{2}\right)}\,28\mathrm{i}-\mathrm{e}^{c\,4\mathrm{i}+d\,x\,4\mathrm{i}}\,\cos\left(3\,c+3\,d\,x\right)\,\sqrt{e\,\left(\frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}}{2}\right)}\,12\mathrm{i}+\mathrm{e}^{c\,4\mathrm{i}+d\,x\,4\mathrm{i}}\,\sin\left(2\,c+2\,d\,x\right)\,\sqrt{e\,\left(\frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}}{2}\right)}\,28\mathrm{i}-\mathrm{e}^{c\,4\mathrm{i}+d\,x\,4\mathrm{i}}\,\sin\left(4\,c+4\,d\,x\right)\,\sqrt{e\,\left(\frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}}{2}\right)}\,2\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cos(c + d*x))^(5/2)*(a + a*sin(c + d*x))^(5/2)),x)
```

```
[Out] -((a + a*sin(c + d*x))^(1/2)*((exp(c*4i + d*x*4i)*2464i)/(585*a^3*d*e^2) +
(exp(c*4i + d*x*4i)*cos(2*c + 2*d*x)*4352i)/(585*a^3*d*e^2) - (exp(c*4i + d
*x*4i)*cos(4*c + 4*d*x)*512i)/(585*a^3*d*e^2) + (exp(c*4i + d*x*4i)*sin(3*c
 + 3*d*x)*512i)/(117*a^3*d*e^2) - (exp(c*4i + d*x*4i)*sin(c + d*x)*256i)/(1
17*a^3*d*e^2)))/(cos(c + d*x)*exp(c*4i + d*x*4i)*(e*(exp(- c*1i - d*x*1i)/2
 + exp(c*1i + d*x*1i)/2))^(1/2)*28i - exp(c*4i + d*x*4i)*cos(3*c + 3*d*x)*(
e*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*12i + exp(c*4i + d
*x*4i)*sin(2*c + 2*d*x)*(e*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))
^(1/2)*28i - exp(c*4i + d*x*4i)*sin(4*c + 4*d*x)*(e*(exp(- c*1i - d*x*1i)/2
 + exp(c*1i + d*x*1i)/2))^(1/2)*2i)
```

3.321 $\int \frac{(e\cos(c+dx))^{7/3}}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=78

$$-\frac{3\sqrt[6]{2}\,a(e\cos(c+dx))^{10/3}\,{}_2F_1\left(-\frac{1}{6},\frac{5}{3};\frac{8}{3};\frac{1}{2}(1-\sin(c+dx))\right)}{5de\sqrt[6]{1+\sin(c+dx)}\,(a+a\sin(c+dx))^{3/2}}$$

```
[Out] -3/5*2^(1/6)*a*(e*cos(d*x+c))^(10/3)*hypergeom([-1/6, 5/3],[8/3],1/2-1/2*si
n(d*x+c))/d/e/(1+sin(d*x+c))^(1/6)/(a+a*sin(d*x+c))^(3/2)
```

Rubi [A]
time = 0.07, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2768, 72, 71}

$$-\frac{3\sqrt[6]{2}\,a(e\cos(c+dx))^{10/3}\,{}_2F_1\left(-\frac{1}{6},\frac{5}{3};\frac{8}{3};\frac{1}{2}(1-\sin(c+dx))\right)}{5de\sqrt[6]{\sin(c+dx)+1}\,(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(7/3)/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-3*2^(1/6)*a*(e*Cos[c + d*x])^(10/3)*Hypergeometric2F1[-1/6, 5/3, 8/3, (1
- Sin[c + d*x])/2])/(5*d*e*(1 + Sin[c + d*x])^(1/6)*(a + a*Sin[c + d*x])^(3
/2))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
```

```
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{(e\cos(c+dx))^{7/3}}{\sqrt{a+a\sin(c+dx)}}\,dx &= \frac{\left(a^2(e\cos(c+dx))^{10/3}\right)\text{Subst}\left(\int (a-ax)^{2/3}\sqrt[6]{a+ax}\,dx,x,\sin(c+dx)\right)}{de(a-a\sin(c+dx))^{5/3}(a+a\sin(c+dx))^{5/3}}\\
&= \frac{\left(\sqrt[6]{2}\,a^2(e\cos(c+dx))^{10/3}\right)\text{Subst}\left(\int \sqrt[6]{\frac{1}{2}+\frac{x}{2}}\,(a-ax)^{2/3}\,dx,x,\sin(c+dx)\right)}{de(a-a\sin(c+dx))^{5/3}(a+a\sin(c+dx))^{3/2}\sqrt[6]{\frac{a+a\sin(c+dx)}{a}}}\\
&= -\frac{3\sqrt[6]{2}\,a(e\cos(c+dx))^{10/3}\,{}_2F_1\left(-\frac{1}{6},\frac{5}{3};\frac{8}{3};\frac{1}{2}(1-\sin(c+dx))\right)}{5de\sqrt[6]{1+\sin(c+dx)}\,(a+a\sin(c+dx))^{3/2}}
\end{aligned}$$

Mathematica [A]
time = 0.16, size = 77, normalized size = 0.99

$$-\frac{3\sqrt[6]{2}\,(e\cos(c+dx))^{10/3}\,{}_2F_1\left(-\frac{1}{6},\frac{5}{3};\frac{8}{3};\frac{1}{2}(1-\sin(c+dx))\right)}{5de(1+\sin(c+dx))^{7/6}\sqrt{a(1+\sin(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(7/3)/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-3*2^(1/6)*(e*Cos[c + d*x])^(10/3)*Hypergeometric2F1[-1/6, 5/3, 8/3, (1 -
Sin[c + d*x])/2])/(5*d*e*(1 + Sin[c + d*x])^(7/6)*Sqrt[a*(1 + Sin[c + d*x])
])
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^{\frac{7}{3}}}{\sqrt{a+a\sin(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(7/3)/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] int((e*cos(d*x+c))^(7/3)/(a+a*sin(d*x+c))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/3)/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima"
)
[Out] e^(7/3)*integrate(cos(d*x + c)^(7/3)/sqrt(a*sin(d*x + c) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/3)/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas"
)
[Out] integral(cos(d*x + c)^(7/3)*e^(7/3)/sqrt(a*sin(d*x + c) + a), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(7/3)/(a+a*sin(d*x+c))**(1/2),x)
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/3)/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,\cos(c+d\,x))^{7/3}}{\sqrt{a+a\,\sin(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(7/3)/(a + a*sin(c + d*x))^(1/2),x)
[Out] int((e*cos(c + d*x))^(7/3)/(a + a*sin(c + d*x))^(1/2), x)
```

3.322 $\int \frac{(e\cos(c+dx))^{5/3}}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=78

$$-\frac{3a(e\cos(c+dx))^{8/3}\,{}_2F_1\left(\frac{1}{6},\frac{4}{3};\frac{7}{3};\frac{1}{2}(1-\sin(c+dx))\right)\sqrt[6]{1+\sin(c+dx)}}{4\sqrt[6]{2}\,de(a+a\sin(c+dx))^{3/2}}$$

```
[Out] -3/8*a*(e*cos(d*x+c))^(8/3)*hypergeom([1/6, 4/3],[7/3],1/2-1/2*sin(d*x+c))*
(1+sin(d*x+c))^(1/6)*2^(5/6)/d/e/(a+a*sin(d*x+c))^(3/2)
```

Rubi [A]
time = 0.07, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2768, 72, 71}

$$-\frac{3a\sqrt[6]{\sin(c+dx)+1}\,(e\cos(c+dx))^{8/3}\,{}_2F_1\left(\frac{1}{6},\frac{4}{3};\frac{7}{3};\frac{1}{2}(1-\sin(c+dx))\right)}{4\sqrt[6]{2}\,de(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(5/3)/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-3*a*(e*Cos[c + d*x])^(8/3)*Hypergeometric2F1[1/6, 4/3, 7/3, (1 - Sin[c +
d*x])/2]*(1 + Sin[c + d*x])^(1/6))/(4*2^(1/6)*d*e*(a + a*Sin[c + d*x])^(3/2
))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
```

```
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{5/3}}{\sqrt{a+a\sin(c+dx)}}\,dx = \frac{\left(a^2(e\cos(c+dx))^{8/3}\right)\text{Subst}\left(\int \frac{\sqrt[3]{a-ax}}{\sqrt[6]{a+ax}}\,dx, x, \sin(c+dx)\right)}{de(a-a\sin(c+dx))^{4/3}(a+a\sin(c+dx))^{4/3}}$$

$$= \frac{\left(a^2(e\cos(c+dx))^{8/3}\sqrt[6]{\frac{a+a\sin(c+dx)}{a}}\right)\text{Subst}\left(\int \frac{\sqrt[3]{a-ax}}{\sqrt[6]{\frac{1}{2}+\frac{x}{2}}}\,dx, x, \sin(c+\right.}{\sqrt[6]{2}\,de(a-a\sin(c+dx))^{4/3}(a+a\sin(c+dx))^{3/2}}$$

$$= -\frac{3a(e\cos(c+dx))^{8/3}\,{}_2F_1\left(\frac{1}{6},\frac{4}{3};\frac{7}{3};\frac{1}{2}(1-\sin(c+dx))\right)\sqrt[6]{1+\sin(c+dx)}}{4\sqrt[6]{2}\,de(a+a\sin(c+dx))^{3/2}}$$

Mathematica [A]
time = 0.12, size = 77, normalized size = 0.99

$$-\frac{3(e\cos(c+dx))^{8/3}\,{}_2F_1\left(\frac{1}{6},\frac{4}{3};\frac{7}{3};\frac{1}{2}(1-\sin(c+dx))\right)}{4\sqrt[6]{2}\,de(1+\sin(c+dx))^{5/6}\sqrt{a(1+\sin(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(5/3)/Sqrt[a + a*Sin[c + d*x]],x]
[Out] (-3*(e*Cos[c + d*x])^(8/3)*Hypergeometric2F1[1/6, 4/3, 7/3, (1 - Sin[c + d*
x])/2])/(4*2^(1/6)*d*e*(1 + Sin[c + d*x])^(5/6)*Sqrt[a*(1 + Sin[c + d*x])])
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^{\frac{5}{3}}}{\sqrt{a+a\sin(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(5/3)/(a+a*sin(d*x+c))^(1/2),x)
[Out] int((e*cos(d*x+c))^(5/3)/(a+a*sin(d*x+c))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/3)/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima"
)
```

```
[Out] e^(5/3)*integrate(cos(d*x + c)^(5/3)/sqrt(a*sin(d*x + c) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/3)/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(cos(d*x + c)^(5/3)*e^(5/3)/sqrt(a*sin(d*x + c) + a), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(5/3)/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/3)/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Evaluation time: 236.93Unable to divide, perhaps due to r
ounding error%%%{2,[0,2,6,0]%%%}+%%%{6,[0,2,4,0]%%%}+%%%{6,[0,2,2,0]%%%}+%%
%{2,[0,2,
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\cos(c+dx))^{5/3}}{\sqrt{a+a\sin(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(5/3)/(a + a*sin(c + d*x))^(1/2),x)
[Out] int((e*cos(c + d*x))^(5/3)/(a + a*sin(c + d*x))^(1/2), x)
```

3.323 $\int \frac{(e\cos(c+dx))^{2/3}}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=78

$$-\frac{3\sqrt[3]{2}\,a(e\cos(c+dx))^{5/3}\,{}_2F_1\left(\frac{2}{3},\frac{5}{6};\frac{11}{6};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{2/3}}{5de(a+a\sin(c+dx))^{3/2}}$$

```
[Out] -3/5*2^(1/3)*a*(e*cos(d*x+c))^(5/3)*hypergeom([2/3, 5/6],[11/6],1/2-1/2*sin
(d*x+c))*(1+sin(d*x+c))^(2/3)/d/e/(a+a*sin(d*x+c))^(3/2)
```

Rubi [A]
time = 0.07, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2768, 72, 71}

$$-\frac{3\sqrt[3]{2}\,a(\sin(c+dx)+1)^{2/3}(e\cos(c+dx))^{5/3}\,{}_2F_1\left(\frac{2}{3},\frac{5}{6};\frac{11}{6};\frac{1}{2}(1-\sin(c+dx))\right)}{5de(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(2/3)/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-3*2^(1/3)*a*(e*Cos[c + d*x])^(5/3)*Hypergeometric2F1[2/3, 5/6, 11/6, (1 -
 Sin[c + d*x])/2]*(1 + Sin[c + d*x])^(2/3))/(5*d*e*(a + a*Sin[c + d*x])^(3/
2))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
```

```
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{2/3}}{\sqrt{a+a\sin(c+dx)}}\,dx = \frac{\left(a^2(e\cos(c+dx))^{5/3}\right)\text{Subst}\left(\int \frac{1}{\sqrt[6]{a-ax}\,(a+ax)^{2/3}}\,dx,x,\sin(c+dx)\right)}{de(a-a\sin(c+dx))^{5/6}(a+a\sin(c+dx))^{5/6}}$$

$$= \frac{\left(a^2(e\cos(c+dx))^{5/3}\left(\frac{a+a\sin(c+dx)}{a}\right)^{2/3}\right)\text{Subst}\left(\int \frac{1}{\left(\frac{1}{2}+\frac{x}{2}\right)^{2/3}\sqrt[6]{a-ax}}\,dx,x,\sin(\ldots\right)}{2^{2/3}de(a-a\sin(c+dx))^{5/6}(a+a\sin(c+dx))^{3/2}}$$

$$= -\frac{3\sqrt[3]{2}\,a(e\cos(c+dx))^{5/3}\,{}_2F_1\left(\frac{2}{3},\frac{5}{6};\frac{11}{6};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{2/3}}{5de(a+a\sin(c+dx))^{3/2}}$$

Mathematica [**A**]
time = 0.09, size = 77, normalized size = 0.99

$$-\frac{3\sqrt[3]{2}\,(e\cos(c+dx))^{5/3}\,{}_2F_1\left(\frac{2}{3},\frac{5}{6};\frac{11}{6};\frac{1}{2}(1-\sin(c+dx))\right)}{5de\sqrt[3]{1+\sin(c+dx)}\,\sqrt{a(1+\sin(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(2/3)/Sqrt[a + a*Sin[c + d*x]],x]
[Out] (-3*2^(1/3)*(e*Cos[c + d*x])^(5/3)*Hypergeometric2F1[2/3, 5/6, 11/6, (1 - S
in[c + d*x])/2])/(5*d*e*(1 + Sin[c + d*x])^(1/3)*Sqrt[a*(1 + Sin[c + d*x])]
)
```

Maple [**F**]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^{\frac{2}{3}}}{\sqrt{a+a\sin(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(2/3)/(a+a*sin(d*x+c))^(1/2),x)
[Out] int((e*cos(d*x+c))^(2/3)/(a+a*sin(d*x+c))^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(2/3)/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima"
)
```

```
[Out] e^(2/3)*integrate(cos(d*x + c)^(2/3)/sqrt(a*sin(d*x + c) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(2/3)/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(cos(d*x + c)^(2/3)*e^(2/3)/sqrt(a*sin(d*x + c) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(c+dx))^{\frac{2}{3}}}{\sqrt{a(\sin(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(2/3)/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral((e*cos(c + d*x))**(2/3)/sqrt(a*(sin(c + d*x) + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(2/3)/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(2/3)*e^(2/3)/sqrt(a*sin(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\ \cos(c+d\,x))^{2/3}}{\sqrt{a+a\ \sin(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(2/3)/(a + a*sin(c + d*x))^(1/2),x)
```

```
[Out] int((e*cos(c + d*x))^(2/3)/(a + a*sin(c + d*x))^(1/2), x)
```

3.324 $\int \frac{\sqrt[3]{e\cos(c+dx)}}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=78

$$-\frac{3a(e\cos(c+dx))^{4/3}\,{}_2F_1\left(\frac{2}{3},\frac{5}{6};\frac{5}{3};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{5/6}}{2\,2^{5/6}de(a+a\sin(c+dx))^{3/2}}$$

```
[Out] -3/4*a*(e*cos(d*x+c))^(4/3)*hypergeom([2/3, 5/6],[5/3],1/2-1/2*sin(d*x+c))*
(1+sin(d*x+c))^(5/6)*2^(1/6)/d/e/(a+a*sin(d*x+c))^(3/2)
```

Rubi [A]
time = 0.06, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2768, 72, 71}

$$-\frac{3a(\sin(c+dx)+1)^{5/6}(e\cos(c+dx))^{4/3}\,{}_2F_1\left(\frac{2}{3},\frac{5}{6};\frac{5}{3};\frac{1}{2}(1-\sin(c+dx))\right)}{2\,2^{5/6}de(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(1/3)/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-3*a*(e*Cos[c + d*x])^(4/3)*Hypergeometric2F1[2/3, 5/6, 5/3, (1 - Sin[c +
d*x])/2]*(1 + Sin[c + d*x])^(5/6))/(2*2^(5/6)*d*e*(a + a*Sin[c + d*x])^(3/2
))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
```

```
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{\sqrt[3]{e\cos(c+dx)}}{\sqrt{a+a\sin(c+dx)}}\,dx = \frac{\left(a^2(e\cos(c+dx))^{4/3}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a-ax}\,(a+ax)^{5/6}}\,dx, x, \sin(c+dx)\right)}{de(a-a\sin(c+dx))^{2/3}(a+a\sin(c+dx))^{2/3}}$$

$$= \frac{\left(a^2(e\cos(c+dx))^{4/3}\left(\frac{a+a\sin(c+dx)}{a}\right)^{5/6}\right)\text{Subst}\left(\int \frac{1}{\left(\frac{1}{2}+\frac{x}{2}\right)^{5/6}\sqrt[3]{a-ax}}\,dx, x, \sin\right.}{2^{5/6}de(a-a\sin(c+dx))^{2/3}(a+a\sin(c+dx))^{3/2}}$$

$$= -\frac{3a(e\cos(c+dx))^{4/3}\,{}_2F_1\left(\frac{2}{3},\frac{5}{6};\frac{5}{3};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{5/6}}{2\ 2^{5/6}de(a+a\sin(c+dx))^{3/2}}$$

Mathematica [**A**]
time = 0.09, size = 77, normalized size = 0.99

$$-\frac{3(e\cos(c+dx))^{4/3}\,{}_2F_1\left(\frac{2}{3},\frac{5}{6};\frac{5}{3};\frac{1}{2}(1-\sin(c+dx))\right)}{2\ 2^{5/6}de\sqrt[6]{1+\sin(c+dx)}\sqrt{a(1+\sin(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(1/3)/Sqrt[a + a*Sin[c + d*x]],x]
[Out] (-3*(e*Cos[c + d*x])^(4/3)*Hypergeometric2F1[2/3, 5/6, 5/3, (1 - Sin[c + d*
x])/2])/(2*2^(5/6)*d*e*(1 + Sin[c + d*x])^(1/6)*Sqrt[a*(1 + Sin[c + d*x])])
```

Maple [**F**]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^{\frac{1}{3}}}{\sqrt{a+a\sin(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(1/3)/(a+a*sin(d*x+c))^(1/2),x)
[Out] int((e*cos(d*x+c))^(1/3)/(a+a*sin(d*x+c))^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/3)/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima"
)
```

```
[Out] e^(1/3)*integrate(cos(d*x + c)^(1/3)/sqrt(a*sin(d*x + c) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/3)/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(cos(d*x + c)^(1/3)*e^(1/3)/sqrt(a*sin(d*x + c) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{e\cos(c+dx)}}{\sqrt{a(\sin(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(1/3)/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral((e*cos(c + d*x))**(1/3)/sqrt(a*(sin(c + d*x) + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/3)/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(1/3)*e^(1/3)/sqrt(a*sin(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\cos(c+d\,x))^{1/3}}{\sqrt{a+a\sin(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(1/3)/(a + a*sin(c + d*x))^(1/2),x)
```

```
[Out] int((e*cos(c + d*x))^(1/3)/(a + a*sin(c + d*x))^(1/2), x)
```

3.325 $\int \frac{1}{\sqrt[3]{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=77

$$-\frac{3(e\cos(c+dx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{7}{6};\frac{4}{3};\frac{1}{2}(1-\sin(c+dx))\right)\sqrt[6]{1+\sin(c+dx)}}{2\sqrt[6]{2}\,de\sqrt{a+a\sin(c+dx)}}$$

```
[Out] -3/4*(e*cos(d*x+c))^(2/3)*hypergeom([1/3, 7/6],[4/3],1/2-1/2*sin(d*x+c))*(1
+sin(d*x+c))^(1/6)*2^(5/6)/d/e/(a+a*sin(d*x+c))^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2768, 72, 71}

$$-\frac{3\sqrt[6]{\sin(c+dx)+1}\,(e\cos(c+dx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{7}{6};\frac{4}{3};\frac{1}{2}(1-\sin(c+dx))\right)}{2\sqrt[6]{2}\,de\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(1/3)*Sqrt[a + a*Sin[c + d*x]]),x]
```

```
[Out] (-3*(e*Cos[c + d*x])^(2/3)*Hypergeometric2F1[1/3, 7/6, 4/3, (1 - Sin[c + d*
x])/2]*(1 + Sin[c + d*x])^(1/6))/(2*2^(1/6)*d*e*Sqrt[a + a*Sin[c + d*x]])
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
```

```
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{e\cos(c+dx)}\,\sqrt{a+a\sin(c+dx)}}\,dx = \frac{\left(a^2(e\cos(c+dx))^{2/3}\right)\text{Subst}\left(\int \frac{1}{(a-ax)^{2/3}(a+ax)^{7/6}}\,dx,x,\sin(c+d$$
$$\overline{de\sqrt[3]{a-a\sin(c+dx)}\,\sqrt[3]{a+a\sin(c+dx)}}$$
$$= \frac{\left(a(e\cos(c+dx))^{2/3}\sqrt[6]{\frac{a+a\sin(c+dx)}{a}}\right)\text{Subst}\left(\int \frac{1}{\left(\frac{1}{2}+\frac{x}{2}\right)^{7/6}(\cdots}\right.}{2\sqrt[6]{2}\,de\sqrt[3]{a-a\sin(c+dx)}\,\sqrt{a+a\sin(c\cdots}}$$
$$= -\frac{3(e\cos(c+dx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{7}{6};\frac{4}{3};\frac{1}{2}(1-\sin(c+dx))\right)\sqrt[6]{1+\sin\cdots}}{2\sqrt[6]{2}\,de\sqrt{a+a\sin(c+dx)}}$$

Mathematica [A]
time = 0.09, size = 77, normalized size = 1.00

$$-\frac{3(e\cos(c+dx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{7}{6};\frac{4}{3};\frac{1}{2}(1-\sin(c+dx))\right)\sqrt[6]{1+\sin(c+dx)}}{2\sqrt[6]{2}\,de\sqrt{a(1+\sin(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Cos[c + d*x])^(1/3)*Sqrt[a + a*Sin[c + d*x]]),x]
[Out] (-3*(e*Cos[c + d*x])^(2/3)*Hypergeometric2F1[1/3, 7/6, 4/3, (1 - Sin[c + d*
x])/2]*(1 + Sin[c + d*x])^(1/6))/(2*2^(1/6)*d*e*Sqrt[a*(1 + Sin[c + d*x])])
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{1}{(e\cos(dx+c))^{\frac{1}{3}}\sqrt{a+a\sin(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(1/3)/(a+a*sin(d*x+c))^(1/2),x)
[Out] int(1/(e*cos(d*x+c))^(1/3)/(a+a*sin(d*x+c))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(1/3)/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxim
a")
```

```
[Out] e^(-1/3)*integrate(1/(sqrt(a*sin(d*x + c) + a)*cos(d*x + c)^(1/3)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(1/3)/(a+a*sin(d*x+c))^(1/2),x, algorithm="frica
s")
```

```
[Out] integral(sqrt(a*sin(d*x + c) + a)*cos(d*x + c)^(2/3)/(a*cos(d*x + c)*e^(1/3
)*sin(d*x + c) + a*cos(d*x + c)*e^(1/3)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\left(\sin\left(c+dx\right)+1\right)}\,\sqrt[3]{e\cos\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(1/3)/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a*(sin(c + d*x) + 1))*(e*cos(c + d*x))**(1/3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(1/3)/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(e^(-1/3)/(sqrt(a*sin(d*x + c) + a)*cos(d*x + c)^(1/3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(e\,\cos\left(c+d\,x\right)\right)}^{1/3}\,\sqrt{a+a\,\sin\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cos(c + d*x))^(1/3)*(a + a*sin(c + d*x))^(1/2)),x)
```

```
[Out] int(1/((e*cos(c + d*x))^(1/3)*(a + a*sin(c + d*x))^(1/2)), x)
```

3.326 $\int \frac{1}{(e\cos(c+dx))^{4/3}\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=75

$$\frac{3\,{}_2F_1\left(-\frac{1}{6},\frac{5}{3};\frac{5}{6};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{2/3}}{2^{2/3}de\sqrt[3]{e\cos(c+dx)}\sqrt{a+a\sin(c+dx)}}$$

```
[Out] 3/2*hypergeom([-1/6, 5/3],[5/6],1/2-1/2*sin(d*x+c))*(1+sin(d*x+c))^(2/3)*2^
(1/3)/d/e/(e*cos(d*x+c))^(1/3)/(a+a*sin(d*x+c))^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2768, 72, 71}

$$\frac{3(\sin(c+dx)+1)^{2/3}\,{}_2F_1\left(-\frac{1}{6},\frac{5}{3};\frac{5}{6};\frac{1}{2}(1-\sin(c+dx))\right)}{2^{2/3}de\sqrt{a\sin(c+dx)+a}\sqrt[3]{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(4/3)*Sqrt[a + a*Sin[c + d*x]]),x]
```

```
[Out] (3*Hypergeometric2F1[-1/6, 5/3, 5/6, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x
])^(2/3))/(2^(2/3)*d*e*(e*Cos[c + d*x])^(1/3)*Sqrt[a + a*Sin[c + d*x]])
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
```

```
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{(e\cos(c+dx))^{4/3}\sqrt{a+a\sin(c+dx)}}\,dx = \frac{\left(a^2\sqrt[6]{a-a\sin(c+dx)}\,\sqrt[6]{a+a\sin(c+dx)}\right)\text{Subst}\left(\int \frac{}{(a-a}\right.}{de\sqrt[3]{e\cos(c+dx)}}$$
$$= \frac{\left(a\sqrt[6]{a-a\sin(c+dx)}\left(\frac{a+a\sin(c+dx)}{a}\right)^{2/3}\right)\text{Subst}\left(\int \frac{}{\left(\frac{1}{2}+\frac{x}{2}\right)^{5/3}}\right.}{2\,2^{2/3}de\sqrt[3]{e\cos(c+dx)}\,\sqrt{a+a\sin(c}}$$
$$= \frac{3\,{}_2F_1\left(-\frac{1}{6},\frac{5}{3};\frac{5}{6};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{2/3}}{2^{2/3}de\sqrt[3]{e\cos(c+dx)}\,\sqrt{a+a\sin(c+dx)}}$$

Mathematica [A]
time = 0.10, size = 75, normalized size = 1.00

$$\frac{3\,{}_2F_1\left(-\frac{1}{6},\frac{5}{3};\frac{5}{6};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{2/3}}{2^{2/3}de\sqrt[3]{e\cos(c+dx)}\,\sqrt{a(1+\sin(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Cos[c + d*x])^(4/3)*Sqrt[a + a*Sin[c + d*x]]),x]
[Out] (3*Hypergeometric2F1[-1/6, 5/3, 5/6, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x
])^(2/3))/(2^(2/3)*d*e*(e*Cos[c + d*x])^(1/3)*Sqrt[a*(1 + Sin[c + d*x])])
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{1}{(e\cos(dx+c))^{\frac{4}{3}}\sqrt{a+a\sin(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(4/3)/(a+a*sin(d*x+c))^(1/2),x)
[Out] int(1/(e*cos(d*x+c))^(4/3)/(a+a*sin(d*x+c))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(4/3)/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxim
a")
```

```
[Out] e^(-4/3)*integrate(1/(sqrt(a*sin(d*x + c) + a)*cos(d*x + c)^(4/3)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(4/3)/(a+a*sin(d*x+c))^(1/2),x, algorithm="frica
s")
```

```
[Out] integral(sqrt(a*sin(d*x + c) + a)*cos(d*x + c)^(2/3)/(a*cos(d*x + c)^2*e^(4
/3)*sin(d*x + c) + a*cos(d*x + c)^2*e^(4/3)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\left(\sin\left(c+dx\right)+1\right)}\,\left(e\cos\left(c+dx\right)\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(4/3)/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a*(sin(c + d*x) + 1))*(e*cos(c + d*x))**(4/3)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(4/3)/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(e\,\cos\left(c+d\,x\right)\right)}^{4/3}\,\sqrt{a+a\,\sin\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cos(c + d*x))^(4/3)*(a + a*sin(c + d*x))^(1/2)),x)
```

```
[Out] int(1/((e*cos(c + d*x))^(4/3)*(a + a*sin(c + d*x))^(1/2)), x)
```

3.327 $\int (e\cos(c+dx))^p(a+a\sin(c+dx))^8\,dx$

Optimal. Leaf size=95

$$-\frac{2^{\frac{17}{2}+\frac{p}{2}}a^8(e\cos(c+dx))^{1+p}\,{}_2F_1\left(\frac{1}{2}(-15-p),\frac{1+p}{2};\frac{3+p}{2};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{\frac{1}{2}(-1-p)}}{de(1+p)}$$

```
[Out] -2^(17/2+1/2*p)*a^8*(e*cos(d*x+c))^(1+p)*hypergeom([1/2+1/2*p, -15/2-1/2*p]
,[3/2+1/2*p],1/2-1/2*sin(d*x+c))*(1+sin(d*x+c))^(-1/2-1/2*p)/d/e/(1+p)
```

Rubi [A]
time = 0.06, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2767, 71}

$$-\frac{a^8 2^{\frac{p}{2}+\frac{17}{2}}(\sin(c+dx)+1)^{\frac{1}{2}(-p-1)}(e\cos(c+dx))^{p+1}\,{}_2F_1\left(\frac{1}{2}(-p-15),\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{de(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^p*(a + a*Sin[c + d*x])^8,x]
```

```
[Out] -((2^(17/2 + p/2)*a^8*(e*Cos[c + d*x])^(1 + p)*Hypergeometric2F1[(-15 - p)/
2, (1 + p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^((-1 - p)
/2))/(d*e*(1 + p)))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 2767

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^m*((g*Cos[e + f*x])^(p + 1)/(f*g*(1 + Sin[e
 + f*x])^((p + 1)/2)*(1 - Sin[e + f*x])^((p + 1)/2))), Subst[Int[(1 + (b/a)
*x)^(m + (p - 1)/2)*(1 - (b/a)*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int (e\cos(c+dx))^p(a+a\sin(c+dx))^8\,dx = \frac{\left(a^8(e\cos(c+dx))^{1+p}(1-\sin(c+dx))^{\frac{1}{2}(-1-p)}(1+\sin(c+dx))^{\frac{1}{2}(}\right.}{d}$$

$$= -\frac{2^{\frac{17}{2}+\frac{p}{2}}a^8(e\cos(c+dx))^{1+p}\,{}_2F_1\left(\frac{1}{2}(-15-p),\frac{1+p}{2};\frac{3+p}{2};\frac{1}{2}(1-\sin(\right.}{de(1+p)}$$

Mathematica [A]
time = 0.20, size = 94, normalized size = 0.99

$$-\frac{2^{\frac{17+p}{2}}a^8\cos(c+dx)(e\cos(c+dx))^p\,{}_2F_1\left(\frac{1}{2}(-15-p),\frac{1+p}{2};\frac{3+p}{2};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{\frac{1}{2}(-1-p)}}{d(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^p*(a + a*Sin[c + d*x])^8,x]
```

```
[Out] -((2^((17 + p)/2)*a^8*Cos[c + d*x]*(e*Cos[c + d*x])^p*Hypergeometric2F1[(-1
5 - p)/2, (1 + p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^((
-1 - p)/2))/(d*(1 + p)))
```

Maple [F]
time = 0.46, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^p\,(a+a\sin(dx+c))^8\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^8,x)
```

```
[Out] int((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^8,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^8*(cos(d*x + c)*e)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] integral((a^8*cos(d*x + c)^8 - 32*a^8*cos(d*x + c)^6 + 160*a^8*cos(d*x + c)
^4 - 256*a^8*cos(d*x + c)^2 + 128*a^8 - 8*(a^8*cos(d*x + c)^6 - 10*a^8*cos(
d*x + c)^4 + 24*a^8*cos(d*x + c)^2 - 16*a^8)*sin(d*x + c))*(cos(d*x + c)*e)
^p, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**p*(a+a*sin(d*x+c))**8,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^8*(cos(d*x + c)*e)^p, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (e\cos(c+d\,x))^p\,(a+a\,\sin(c+d\,x))^8\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^p*(a + a*sin(c + d*x))^8,x)
```

```
[Out] int((e*cos(c + d*x))^p*(a + a*sin(c + d*x))^8, x)
```

3.328 $\int (e\cos(c+dx))^p(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=95

$$-\frac{2^{\frac{7}{2}+\frac{p}{2}}a^3(e\cos(c+dx))^{1+p}\,{}_2F_1\left(\frac{1}{2}(-5-p),\frac{1+p}{2};\frac{3+p}{2};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{\frac{1}{2}(-1-p)}}{de(1+p)}$$

```
[Out] -2^(7/2+1/2*p)*a^3*(e*cos(d*x+c))^(1+p)*hypergeom([-5/2-1/2*p, 1/2+1/2*p],[
3/2+1/2*p],1/2-1/2*sin(d*x+c))*(1+sin(d*x+c))^(-1/2-1/2*p)/d/e/(1+p)
```

Rubi [A]
time = 0.06, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2767, 71}

$$-\frac{a^3 2^{\frac{p}{2}+\frac{7}{2}}(\sin(c+dx)+1)^{\frac{1}{2}(-p-1)}(e\cos(c+dx))^{p+1}\,{}_2F_1\left(\frac{1}{2}(-p-5),\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{de(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^p*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -((2^(7/2 + p/2)*a^3*(e*Cos[c + d*x])^(1 + p)*Hypergeometric2F1[(-5 - p)/2,
 (1 + p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^((-1 - p)/2
))/(d*e*(1 + p)))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 2767

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^m*((g*Cos[e + f*x])^(p + 1)/(f*g*(1 + Sin[e
 + f*x])^((p + 1)/2)*(1 - Sin[e + f*x])^((p + 1)/2))), Subst[Int[(1 + (b/a)
*x)^(m + (p - 1)/2)*(1 - (b/a)*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int (e\cos(c+dx))^p(a+a\sin(c+dx))^3\,dx = \frac{\left(a^3(e\cos(c+dx))^{1+p}(1-\sin(c+dx))^{\frac{1}{2}(-1-p)}(1+\sin(c+dx))\right.}{}$$

$$= -\frac{2^{\frac{7}{2}+\frac{p}{2}}a^3(e\cos(c+dx))^{1+p}\,{}_2F_1\left(\frac{1}{2}(-5-p),\frac{1+p}{2};\frac{3+p}{2};\frac{1}{2}(1-\sin(\right.}{de(1+p)}$$

Mathematica [A]
time = 0.11, size = 94, normalized size = 0.99

$$-\frac{2^{\frac{7+p}{2}}a^3\cos(c+dx)(e\cos(c+dx))^p\,{}_2F_1\left(\frac{1}{2}(-5-p),\frac{1+p}{2};\frac{3+p}{2};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{\frac{1}{2}(-1-p)}}{d(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^p*(a + a*Sin[c + d*x])^3,x]
[Out] -((2^((7 + p)/2)*a^3*Cos[c + d*x]*(e*Cos[c + d*x])^p*Hypergeometric2F1[(-5
- p)/2, (1 + p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^((-1
 - p)/2))/(d*(1 + p)))
```

Maple [F]
time = 0.56, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^p\,(a+a\sin(dx+c))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^3,x)
[Out] int((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
[Out] integrate((a*sin(d*x + c) + a)^3*(cos(d*x + c)*e)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(-(3*a^3*cos(d*x + c)^2 - 4*a^3 + (a^3*cos(d*x + c)^2 - 4*a^3)*sin(
d*x + c))*(cos(d*x + c)*e)^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int (e\cos (c+dx))^p\,dx+\int 3(e\cos (c+dx))^p\sin (c+dx)\,dx+\int 3(e\cos (c+dx))^p\sin^2 (c+dx)\,dx+\int (e\cos (c+dx))^p\sin^3 (c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**p*(a+a*sin(d*x+c))**3,x)
```

```
[Out] a**3*(Integral((e*cos(c + d*x))**p, x) + Integral(3*(e*cos(c + d*x))**p*sin
(c + d*x), x) + Integral(3*(e*cos(c + d*x))**p*sin(c + d*x)**2, x) + Integr
al((e*cos(c + d*x))**p*sin(c + d*x)**3, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^3*(cos(d*x + c)*e)^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(e\,\cos\left(c+d\,x\right)\right)}^p\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^p*(a + a*sin(c + d*x))^3,x)
```

```
[Out] int((e*cos(c + d*x))^p*(a + a*sin(c + d*x))^3, x)
```

3.329 $\int (e\cos(c+dx))^p(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=95

$$-\frac{2^{\frac{5}{2}+\frac{p}{2}}a^2(e\cos(c+dx))^{1+p}\,{}_2F_1\left(\frac{1}{2}(-3-p),\frac{1+p}{2};\frac{3+p}{2};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{\frac{1}{2}(-1-p)}}{de(1+p)}$$

```
[Out] -2^(5/2+1/2*p)*a^2*(e*cos(d*x+c))^(1+p)*hypergeom([-3/2-1/2*p, 1/2+1/2*p],[
3/2+1/2*p],1/2-1/2*sin(d*x+c))*(1+sin(d*x+c))^(-1/2-1/2*p)/d/e/(1+p)
```

Rubi [A]
time = 0.06, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = $\{2767, 71\}$

$$-\frac{a^2 2^{\frac{p}{2}+\frac{5}{2}}(\sin(c+dx)+1)^{\frac{1}{2}(-p-1)}(e\cos(c+dx))^{p+1}\,{}_2F_1\left(\frac{1}{2}(-p-3),\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{de(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^p*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -((2^(5/2 + p/2)*a^2*(e*Cos[c + d*x])^(1 + p)*Hypergeometric2F1[(-3 - p)/2,
 (1 + p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^((-1 - p)/2
))/(d*e*(1 + p)))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 2767

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^m*((g*Cos[e + f*x])^(p + 1)/(f*g*(1 + Sin[e
 + f*x])^((p + 1)/2)*(1 - Sin[e + f*x])^((p + 1)/2))), Subst[Int[(1 + (b/a)
*x)^(m + (p - 1)/2)*(1 - (b/a)*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int (e\cos(c+dx))^p(a+a\sin(c+dx))^2\,dx = \frac{\left(a^2(e\cos(c+dx))^{1+p}(1-\sin(c+dx))^{\frac{1}{2}(-1-p)}(1+\sin(c+dx))^{\frac{1}{2}(}\right.}{d}$$
$$= -\frac{2^{\frac{5}{2}+\frac{p}{2}}a^2(e\cos(c+dx))^{1+p}\,{}_2F_1\left(\frac{1}{2}(-3-p),\frac{1+p}{2};\frac{3+p}{2};\frac{1}{2}(1-\sin(c\right.}{de(1+p)}$$

Mathematica [A]
time = 0.11, size = 94, normalized size = 0.99

$$-\frac{2^{\frac{5+p}{2}}a^2\cos(c+dx)(e\cos(c+dx))^p\,{}_2F_1\left(\frac{1}{2}(-3-p),\frac{1+p}{2};\frac{3+p}{2};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{\frac{1}{2}(-1-p)}}{d(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^p*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -((2^((5 + p)/2)*a^2*Cos[c + d*x]*(e*Cos[c + d*x])^p*Hypergeometric2F1[(-3
- p)/2, (1 + p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^((-1
 - p)/2))/(d*(1 + p)))
```

Maple [F]
time = 0.54, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^p\,(a+a\sin(dx+c))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^2,x)
```

```
[Out] int((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^2*(cos(d*x + c)*e)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(-(a^2*cos(d*x + c)^2 - 2*a^2*sin(d*x + c) - 2*a^2)*(cos(d*x + c)*e
)^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int (e\cos(c+dx))^p\, dx + \int 2(e\cos(c+dx))^p \sin(c+dx)\, dx + \int (e\cos(c+dx))^p \sin^2(c+dx)\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**p*(a+a*sin(d*x+c))**2,x)
```

```
[Out] a**2*(Integral((e*cos(c + d*x))**p, x) + Integral(2*(e*cos(c + d*x))**p*sin
(c + d*x), x) + Integral((e*cos(c + d*x))**p*sin(c + d*x)**2, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^2*(cos(d*x + c)*e)^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (e\,\cos(c+d\,x))^p\,(a+a\,\sin(c+d\,x))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^p*(a + a*sin(c + d*x))^2,x)
```

```
[Out] int((e*cos(c + d*x))^p*(a + a*sin(c + d*x))^2, x)
```

3.330 $\int (e\cos(c+dx))^p(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=93

$$-\frac{2^{\frac{3}{2}+\frac{p}{2}}a(e\cos(c+dx))^{1+p}\,{}_2F_1\left(\frac{1}{2}(-1-p),\frac{1+p}{2};\frac{3+p}{2};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{\frac{1}{2}(-1-p)}}{de(1+p)}$$

```
[Out] -2^(3/2+1/2*p)*a*(e*cos(d*x+c))^(1+p)*hypergeom([-1/2-1/2*p, 1/2+1/2*p],[3/
2+1/2*p],1/2-1/2*sin(d*x+c))*(1+sin(d*x+c))^(-1/2-1/2*p)/d/e/(1+p)
```

Rubi [A]
time = 0.04, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = $\{2767, 71\}$

$$-\frac{a2^{\frac{p}{2}+\frac{3}{2}}(\sin(c+dx)+1)^{\frac{1}{2}(-p-1)}(e\cos(c+dx))^{p+1}\,{}_2F_1\left(\frac{1}{2}(-p-1),\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{de(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^p*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((2^(3/2 + p/2)*a*(e*Cos[c + d*x])^(1 + p)*Hypergeometric2F1[(-1 - p)/2, (
1 + p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^((-1 - p)/2))
/(d*e*(1 + p)))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 2767

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^m*((g*Cos[e + f*x])^(p + 1)/(f*g*(1 + Sin[e
 + f*x])^((p + 1)/2)*(1 - Sin[e + f*x])^((p + 1)/2))), Subst[Int[(1 + (b/a)
*x)^(m + (p - 1)/2)*(1 - (b/a)*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int (e\cos(c+dx))^p(a+a\sin(c+dx))\,dx = \frac{\left(a(e\cos(c+dx))^{1+p}(1-\sin(c+dx))^{\frac{1}{2}(-1-p)}(1+\sin(c+dx))^{\frac{1}{2}(}\right.}{d}$$
$$= -\frac{2^{\frac{3}{2}+\frac{p}{2}}a(e\cos(c+dx))^{1+p}\,{}_2F_1\left(\frac{1}{2}(-1-p),\frac{1+p}{2};\frac{3+p}{2};\frac{1}{2}(1-\sin(c\right.}{de(1+p)}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.54, size = 266, normalized size = 2.86

$$\frac{2^{-1-p}a\left(1+e^{2i(c+dx)}\right)^{-1-p}\left(e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)\right)^{1+p}\cos^{-p}(c+dx)(e\cos(c+dx))^p\left(-\left((-1+p)p\,{}_2F_1\left(\frac{1}{2}(-1-p),-p;\frac{1-p}{2};-e^{2i(c+dx)}\right)\right)+e^{i(c+dx)}(1+p)\left(e^{i(c+dx)}p\,{}_2F_1\left(\frac{1-p}{2},-p;\frac{3-p}{2};-e^{2i(c+dx)}\right)+2i(-1+p)\,{}_2F_1\left(-p,-\frac{p}{2};1-\frac{p}{2};-e^{2i(c+dx)}\right)\right)\right)(1+\sin(c+dx))}{d(-1+p)p(1+p)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^p*(a + a*Sin[c + d*x]),x]
```

```
[Out] (2^(-1 - p)*a*(1 + E^((2*I)*(c + d*x)))^(-1 - p)*((1 + E^((2*I)*(c + d*x)))
/E^(I*(c + d*x)))^(1 + p)*(e*Cos[c + d*x])^p*(-((-1 + p)*p*Hypergeometric2F
1[(-1 - p)/2, -p, (1 - p)/2, -E^((2*I)*(c + d*x))]) + E^(I*(c + d*x))*(1 +
p)*(E^(I*(c + d*x))*p*Hypergeometric2F1[(1 - p)/2, -p, (3 - p)/2, -E^((2*I)
*(c + d*x))] + (2*I)*(-1 + p)*Hypergeometric2F1[-p, -1/2*p, 1 - p/2, -E^((2
*I)*(c + d*x))]))*(1 + Sin[c + d*x]))/(d*(-1 + p)*p*(1 + p)*Cos[c + d*x]^p*
(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2)
```

Maple [F]
time = 0.23, size = 0, normalized size = 0.00

$$\int \left(e\cos\left(dx+c\right)\right)^{p}\left(a+a\sin\left(dx+c\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^p*(a+a*sin(d*x+c)),x)
```

```
[Out] int((e*cos(d*x+c))^p*(a+a*sin(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)*(cos(d*x + c)*e)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((a*sin(d*x + c) + a)*(cos(d*x + c)*e)^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int (e\cos{(c+dx)})^p\,dx+\int (e\cos{(c+dx)})^p\sin{(c+dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**p*(a+a*sin(d*x+c)),x)
```

```
[Out] a*(Integral((e*cos(c + d*x))**p, x) + Integral((e*cos(c + d*x))**p*sin(c +
d*x), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)*(cos(d*x + c)*e)^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (e\,\cos{(c+d\,x)})^p\,(a+a\,\sin{(c+d\,x)})\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^p*(a + a*sin(c + d*x)),x)
```

```
[Out] int((e*cos(c + d*x))^p*(a + a*sin(c + d*x)), x)
```

3.331 $\int \frac{(e\cos(c+dx))^p}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=95

$$-\frac{2^{-\frac{1}{2}+\frac{p}{2}}(e\cos(c+dx))^{1+p}\,{}_2F_1\left(\frac{3-p}{2},\frac{1+p}{2};\frac{3+p}{2};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{\frac{1}{2}(-1-p)}}{ade(1+p)}$$

```
[Out] -2^(-1/2+1/2*p)*(e*cos(d*x+c))^(1+p)*hypergeom([3/2-1/2*p, 1/2+1/2*p],[3/2+
1/2*p],1/2-1/2*sin(d*x+c))*(1+sin(d*x+c))^(-1/2-1/2*p)/a/d/e/(1+p)
```

Rubi [A]
time = 0.07, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2767, 71}

$$-\frac{2^{\frac{p}{2}-\frac{1}{2}}(\sin(c+dx)+1)^{\frac{1}{2}(-p-1)}(e\cos(c+dx))^{p+1}\,{}_2F_1\left(\frac{3-p}{2},\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{ade(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^p/(a + a*Sin[c + d*x]),x]
```

```
[Out] -((2^(-1/2 + p/2)*(e*Cos[c + d*x])^(1 + p)*Hypergeometric2F1[(3 - p)/2, (1
+ p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^((-1 - p)/2))/(
a*d*e*(1 + p)))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 2767

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^m*((g*Cos[e + f*x])^(p + 1)/(f*g*(1 + Sin[e
 + f*x])^((p + 1)/2)*(1 - Sin[e + f*x])^((p + 1)/2))), Subst[Int[(1 + (b/a)
*x)^(m + (p - 1)/2)*(1 - (b/a)*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^p}{a+a\sin(c+dx)}\,dx = \frac{\left((e\cos(c+dx))^{1+p}(1-\sin(c+dx))^{\frac{1}{2}(-1-p)}(1+\sin(c+dx))^{\frac{1}{2}(-1-p)}\right)\mathrm{Subst}\left(\int(1-\right.}{ade}$$

$$= -\frac{2^{-\frac{1}{2}+\frac{p}{2}}(e\cos(c+dx))^{1+p}\,{}_2F_1\left(\frac{3-p}{2},\frac{1+p}{2};\frac{3+p}{2};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))}{ade(1+p)}$$

Mathematica [A]
time = 0.16, size = 94, normalized size = 0.99

$$-\frac{2^{\frac{1}{2}(-1+p)}\cos(c+dx)(e\cos(c+dx))^p\,{}_2F_1\left(\frac{3-p}{2},\frac{1+p}{2};\frac{3+p}{2};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{\frac{1}{2}(-1-p)}}{ad(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^p/(a + a*Sin[c + d*x]),x]
[Out] -((2^((-1 + p)/2)*Cos[c + d*x]*(e*Cos[c + d*x])^p*Hypergeometric2F1[(3 - p)
/2, (1 + p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^((-1 - p
)/2))/(a*d*(1 + p)))
```

Maple [F]
time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^p}{a+a\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^p/(a+a*sin(d*x+c)),x)
[Out] int((e*cos(d*x+c))^p/(a+a*sin(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+a*sin(d*x+c)),x, algorithm="maxima")
[Out] integrate((cos(d*x + c)*e)^p/(a*sin(d*x + c) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+a*sin(d*x+c)),x, algorithm="fricas")
[Out] integral((cos(d*x + c)*e)^p/(a*sin(d*x + c) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{(e\cos(c+dx))^p}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**p/(a+a*sin(d*x+c)),x)
[Out] Integral((e*cos(c + d*x))**p/(sin(c + d*x) + 1), x)/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+a*sin(d*x+c)),x, algorithm="giac")
[Out] integrate((cos(d*x + c)*e)^p/(a*sin(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\cos(c+dx))^p}{a+a\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^p/(a + a*sin(c + d*x)),x)
[Out] int((e*cos(c + d*x))^p/(a + a*sin(c + d*x)), x)
```

3.332 $\int \frac{(e\cos(c+dx))^p}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=93

$$-\frac{2^{\frac{1}{2}(-3+p)}(e\cos(c+dx))^{1+p}\,{}_2F_1\left(\frac{5-p}{2},\frac{1+p}{2};\frac{3+p}{2};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{\frac{1}{2}(-1-p)}}{a^2de(1+p)}$$

```
[Out] -2^(-3/2+1/2*p)*(e*cos(d*x+c))^(1+p)*hypergeom([5/2-1/2*p, 1/2+1/2*p],[3/2+
1/2*p],1/2-1/2*sin(d*x+c))*(1+sin(d*x+c))^(-1/2-1/2*p)/a^2/d/e/(1+p)
```

Rubi [A]
time = 0.07, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2767, 71}

$$-\frac{2^{\frac{p-3}{2}}(\sin(c+dx)+1)^{\frac{1}{2}(-p-1)}(e\cos(c+dx))^{p+1}\,{}_2F_1\left(\frac{5-p}{2},\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{a^2de(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^p/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -((2^((-3 + p)/2)*(e*Cos[c + d*x])^(1 + p)*Hypergeometric2F1[(5 - p)/2, (1
+ p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^((-1 - p)/2))/(
a^2*d*e*(1 + p)))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 2767

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^m*((g*Cos[e + f*x])^(p + 1)/(f*g*(1 + Sin[e
 + f*x])^((p + 1)/2)*(1 - Sin[e + f*x])^((p + 1)/2))), Subst[Int[(1 + (b/a)
*x)^(m + (p - 1)/2)*(1 - (b/a)*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^p}{(a+a\sin(c+dx))^2}\,dx = \frac{\left((e\cos(c+dx))^{1+p}(1-\sin(c+dx))^{\frac{1}{2}(-1-p)}(1+\sin(c+dx))^{\frac{1}{2}(-1-p)}\right)\text{Subst}\Big(\int}{a^2de}$$
$$= -\frac{2^{\frac{1}{2}(-3+p)}(e\cos(c+dx))^{1+p}\,{}_2F_1\left(\frac{5-p}{2},\frac{1+p}{2};\frac{3+p}{2};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c-}{a^2de(1+p)}$$

Mathematica [A]
time = 0.17, size = 94, normalized size = 1.01

$$-\frac{2^{\frac{1}{2}(-3+p)}\cos(c+dx)(e\cos(c+dx))^p\,{}_2F_1\left(\frac{5-p}{2},\frac{1+p}{2};\frac{3+p}{2};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{\frac{1}{2}(-1-p)}}{a^2d(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^p/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -((2^((-3 + p)/2)*Cos[c + d*x]*(e*Cos[c + d*x])^p*Hypergeometric2F1[(5 - p)
/2, (1 + p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^((-1 - p
)/2))/(a^2*d*(1 + p)))
```

Maple [F]
time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^p}{(a+a\sin(dx+c))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^p/(a+a*sin(d*x+c))^2,x)
```

```
[Out] int((e*cos(d*x+c))^p/(a+a*sin(d*x+c))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((cos(d*x + c)*e)^p/(a*sin(d*x + c) + a)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(-(cos(d*x + c)*e)^p/(a^2*cos(d*x + c)^2 - 2*a^2*sin(d*x + c) - 2*a
^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{(e\cos(c+dx))^p}{\sin^2(c+dx)+2\sin(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**p/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Integral((e*cos(c + d*x))**p/(sin(c + d*x)**2 + 2*sin(c + d*x) + 1), x)/a**
2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((cos(d*x + c)*e)^p/(a*sin(d*x + c) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,\cos(c+d\,x))^p}{(a+a\,\sin(c+d\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^p/(a + a*sin(c + d*x))^2,x)
```

```
[Out] int((e*cos(c + d*x))^p/(a + a*sin(c + d*x))^2, x)
```

3.333 $\int \frac{(e\cos(c+dx))^p}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=93

$$-\frac{2^{\frac{1}{2}(-5+p)}(e\cos(c+dx))^{1+p}\,{}_2F_1\left(\frac{7-p}{2},\frac{1+p}{2};\frac{3+p}{2};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{\frac{1}{2}(-1-p)}}{a^3de(1+p)}$$

```
[Out] -2^(-5/2+1/2*p)*(e*cos(d*x+c))^(1+p)*hypergeom([7/2-1/2*p, 1/2+1/2*p],[3/2+
1/2*p],1/2-1/2*sin(d*x+c))*(1+sin(d*x+c))^(-1/2-1/2*p)/a^3/d/e/(1+p)
```

Rubi [A]
time = 0.07, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2767, 71}

$$-\frac{2^{\frac{p-5}{2}}(\sin(c+dx)+1)^{\frac{1}{2}(-p-1)}(e\cos(c+dx))^{p+1}\,{}_2F_1\left(\frac{7-p}{2},\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{a^3de(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^p/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -((2^((-5 + p)/2)*(e*Cos[c + d*x])^(1 + p)*Hypergeometric2F1[(7 - p)/2, (1
+ p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^((-1 - p)/2))/(
a^3*d*e*(1 + p)))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 2767

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^m*((g*Cos[e + f*x])^(p + 1)/(f*g*(1 + Sin[e
 + f*x])^((p + 1)/2)*(1 - Sin[e + f*x])^((p + 1)/2))), Subst[Int[(1 + (b/a)
*x)^(m + (p - 1)/2)*(1 - (b/a)*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^p}{(a+a\sin(c+dx))^3}\,dx = \frac{\left((e\cos(c+dx))^{1+p}(1-\sin(c+dx))^{\frac{1}{2}(-1-p)}(1+\sin(c+dx))^{\frac{1}{2}(-1-p)}\right)\operatorname{Subst}\left(\int\right.(}{a^3de}$$

$$= -\frac{2^{\frac{1}{2}(-5+p)}(e\cos(c+dx))^{1+p}\,{}_2F_1\left(\frac{7-p}{2},\frac{1+p}{2};\frac{3+p}{2};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+}{a^3de(1+p)}$$

Mathematica [A]
time = 0.17, size = 94, normalized size = 1.01

$$-\frac{2^{\frac{1}{2}(-5+p)}\cos(c+dx)(e\cos(c+dx))^p\,{}_2F_1\left(\frac{7-p}{2},\frac{1+p}{2};\frac{3+p}{2};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{\frac{1}{2}(-1-p)}}{a^3d(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^p/(a + a*Sin[c + d*x])^3,x]
[Out] -((2^((-5 + p)/2)*Cos[c + d*x]*(e*Cos[c + d*x])^p*Hypergeometric2F1[(7 - p)
/2, (1 + p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^((-1 - p
)/2))/(a^3*d*(1 + p)))
```

Maple [F]
time = 0.74, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^p}{(a+a\sin(dx+c))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^p/(a+a*sin(d*x+c))^3,x)
[Out] int((e*cos(d*x+c))^p/(a+a*sin(d*x+c))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
[Out] integrate((cos(d*x + c)*e)^p/(a*sin(d*x + c) + a)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(-(cos(d*x + c)*e)^p/(3*a^3*cos(d*x + c)^2 - 4*a^3 + (a^3*cos(d*x +
 c)^2 - 4*a^3)*sin(d*x + c)), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{(e\cos(c+dx))^p}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**p/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Integral((e*cos(c + d*x))**p/(sin(c + d*x)**3 + 3*sin(c + d*x)**2 + 3*sin(c
 + d*x) + 1), x)/a**3
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((cos(d*x + c)*e)^p/(a*sin(d*x + c) + a)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,\cos(c+d\,x))^p}{(a+a\,\sin(c+d\,x))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^p/(a + a*sin(c + d*x))^3,x)
```

```
[Out] int((e*cos(c + d*x))^p/(a + a*sin(c + d*x))^3, x)
```

3.334 $\int \frac{(e\cos(c+dx))^p}{(a+a\sin(c+dx))^8}\,dx$

Optimal. Leaf size=93

$$-\frac{2^{\frac{1}{2}(-15+p)}(e\cos(c+dx))^{1+p}\,{}_2F_1\left(\frac{17-p}{2},\frac{1+p}{2};\frac{3+p}{2};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{\frac{1}{2}(-1-p)}}{a^8de(1+p)}$$

```
[Out] -2^(-15/2+1/2*p)*(e*cos(d*x+c))^(1+p)*hypergeom([1/2+1/2*p, 17/2-1/2*p],[3/
2+1/2*p],1/2-1/2*sin(d*x+c))*(1+sin(d*x+c))^(-1/2-1/2*p)/a^8/d/e/(1+p)
```

Rubi [A]
time = 0.06, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2767, 71}

$$-\frac{2^{\frac{p-15}{2}}(\sin(c+dx)+1)^{\frac{1}{2}(-p-1)}(e\cos(c+dx))^{p+1}\,{}_2F_1\left(\frac{17-p}{2},\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{a^8de(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^p/(a + a*Sin[c + d*x])^8,x]
```

```
[Out] -((2^((-15 + p)/2)*(e*Cos[c + d*x])^(1 + p)*Hypergeometric2F1[(17 - p)/2, (
1 + p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^((-1 - p)/2))
/(a^8*d*e*(1 + p)))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 2767

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^m*((g*Cos[e + f*x])^(p + 1)/(f*g*(1 + Sin[e
 + f*x])^((p + 1)/2)*(1 - Sin[e + f*x])^((p + 1)/2))), Subst[Int[(1 + (b/a)
*x)^(m + (p - 1)/2)*(1 - (b/a)*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /;
FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^p}{(a+a\sin(c+dx))^8}\,dx = \frac{\left((e\cos(c+dx))^{1+p}(1-\sin(c+dx))^{\frac{1}{2}(-1-p)}(1+\sin(c+dx))^{\frac{1}{2}(-1-p)}\right)\text{Subst}\big(\int$$
$$a^8de$$
$$= -\frac{2^{\frac{1}{2}(-15+p)}(e\cos(c+dx))^{1+p}\,{}_2F_1\left(\frac{17-p}{2},\frac{1+p}{2};\frac{3+p}{2};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c}{a^8de(1+p)}$$

Mathematica [A]
time = 0.20, size = 94, normalized size = 1.01

$$-\frac{2^{\frac{1}{2}(-15+p)}\cos(c+dx)(e\cos(c+dx))^p\,{}_2F_1\left(\frac{17-p}{2},\frac{1+p}{2};\frac{3+p}{2};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{\frac{1}{2}(-1-p)}}{a^8d(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^p/(a + a*Sin[c + d*x])^8,x]
```

```
[Out] -((2^((-15 + p)/2)*Cos[c + d*x]*(e*Cos[c + d*x])^p*Hypergeometric2F1[(17 -
p)/2, (1 + p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^((-1 -
 p)/2))/(a^8*d*(1 + p)))
```

Maple [F]
time = 2.88, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^p}{(a+a\sin(dx+c))^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^p/(a+a*sin(d*x+c))^8,x)
```

```
[Out] int((e*cos(d*x+c))^p/(a+a*sin(d*x+c))^8,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+a*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] integrate((cos(d*x + c)*e)^p/(a*sin(d*x + c) + a)^8, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+a*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] integral((cos(d*x + c)*e)^p/(a^8*cos(d*x + c)^8 - 32*a^8*cos(d*x + c)^6 + 1
60*a^8*cos(d*x + c)^4 - 256*a^8*cos(d*x + c)^2 + 128*a^8 - 8*(a^8*cos(d*x +
 c)^6 - 10*a^8*cos(d*x + c)^4 + 24*a^8*cos(d*x + c)^2 - 16*a^8)*sin(d*x + c
)), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**p/(a+a*sin(d*x+c))**8,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+a*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] integrate((cos(d*x + c)*e)^p/(a*sin(d*x + c) + a)^8, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,\cos\left(c+d\,x\right))^p}{{\left(a+a\,\sin\left(c+d\,x\right)\right)}^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^p/(a + a*sin(c + d*x))^8,x)
```

```
[Out] int((e*cos(c + d*x))^p/(a + a*sin(c + d*x))^8, x)
```

3.335 $\int (e\cos(c+dx))^p(a+a\sin(c+dx))^{7/2}\,dx$

Optimal. Leaf size=103

$$-\frac{2^{4+\frac{p}{2}}a^4(e\cos(c+dx))^{1+p}\,{}_2F_1\left(\frac{1}{2}(-6-p),\frac{1+p}{2};\frac{3+p}{2};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{-p/2}}{de(1+p)\sqrt{a+a\sin(c+dx)}}$$

```
[Out] -2^(4+1/2*p)*a^4*(e*cos(d*x+c))^(1+p)*hypergeom([-3-1/2*p, 1/2+1/2*p],[3/2+
1/2*p],1/2-1/2*sin(d*x+c))/d/e/(1+p)/((1+sin(d*x+c))^(1/2*p))/(a+a*sin(d*x+
c))^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2768, 72, 71}

$$-\frac{a^4 2^{\frac{p}{2}+4}(\sin(c+dx)+1)^{-p/2}(e\cos(c+dx))^{p+1}\,{}_2F_1\left(\frac{1}{2}(-p-6),\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{de(p+1)\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^p*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] -((2^(4 + p/2)*a^4*(e*Cos[c + d*x])^(1 + p)*Hypergeometric2F1[(-6 - p)/2, (
1 + p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2])/(d*e*(1 + p)*(1 + Sin[c + d*x])
^(p/2)*Sqrt[a + a*Sin[c + d*x]]))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
```

```
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int (e\cos(c+dx))^p(a+a\sin(c+dx))^{7/2}\,dx = \frac{\left(a^2(e\cos(c+dx))^{1+p}(a-a\sin(c+dx))^{\frac{1}{2}(-1-p)}(a+a\sin(c+a\right.}{}$$

$$= \frac{\left(2^{3+\frac{p}{2}}a^5(e\cos(c+dx))^{1+p}(a-a\sin(c+dx))^{\frac{1}{2}(-1-p)}(a+a\sin(\right.}{}$$

$$= -\frac{2^{4+\frac{p}{2}}a^4(e\cos(c+dx))^{1+p}\,{}_2F_1\left(\frac{1}{2}(-6-p),\frac{1+p}{2};\frac{3+p}{2};\frac{1}{2}(1-\sin(\right.}{de(1+p)\sqrt{a+a\sin(c+dx)}}$$

Mathematica [A]
time = 0.25, size = 102, normalized size = 0.99

$$-\frac{2^{4+\frac{p}{2}}a^4\cos(c+dx)(e\cos(c+dx))^p\,{}_2F_1\left(-3-\frac{p}{2},\frac{1+p}{2};\frac{3+p}{2};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{-p/2}}{d(1+p)\sqrt{a(1+\sin(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^p*(a + a*Sin[c + d*x])^(7/2),x]
```

```
[Out] -((2^(4 + p/2)*a^4*Cos[c + d*x]*(e*Cos[c + d*x])^p*Hypergeometric2F1[-3 - p
/2, (1 + p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2])/(d*(1 + p)*(1 + Sin[c + d*
x])^(p/2)*Sqrt[a*(1 + Sin[c + d*x])]))
```

Maple [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^p\,(a+a\sin(dx+c))^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^(7/2),x)
```

```
[Out] int((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^(7/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(7/2)*(cos(d*x + c)*e)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] integral(-(3*a^3*cos(d*x + c)^2 - 4*a^3 + (a^3*cos(d*x + c)^2 - 4*a^3)*sin(
d*x + c))*sqrt(a*sin(d*x + c) + a)*(cos(d*x + c)*e)^p, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**p*(a+a*sin(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(e\,\cos\left(c+d\,x\right)\right)}^p\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{7/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^p*(a + a*sin(c + d*x))^(7/2),x)
```

```
[Out] int((e*cos(c + d*x))^p*(a + a*sin(c + d*x))^(7/2), x)
```

3.336 $\int (e\cos(c+dx))^p(a+a\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=103

$$-\frac{2^{3+\frac{p}{2}}a^3(e\cos(c+dx))^{1+p}\,{}_2F_1\left(\frac{1}{2}(-4-p),\frac{1+p}{2};\frac{3+p}{2};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{-p/2}}{de(1+p)\sqrt{a+a\sin(c+dx)}}$$

```
[Out] -2^(3+1/2*p)*a^3*(e*cos(d*x+c))^(1+p)*hypergeom([-2-1/2*p, 1/2+1/2*p],[3/2+
1/2*p],1/2-1/2*sin(d*x+c))/d/e/(1+p)/((1+sin(d*x+c))^(1/2*p))/(a+a*sin(d*x+
c))^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2768, 72, 71}

$$-\frac{a^3 2^{\frac{p}{2}+3}(\sin(c+dx)+1)^{-p/2}(e\cos(c+dx))^{p+1}\,{}_2F_1\left(\frac{1}{2}(-p-4),\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{de(p+1)\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^p*(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] -((2^(3 + p/2)*a^3*(e*Cos[c + d*x])^(1 + p)*Hypergeometric2F1[(-4 - p)/2, (
1 + p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2])/(d*e*(1 + p)*(1 + Sin[c + d*x])
^(p/2)*Sqrt[a + a*Sin[c + d*x]]))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
```

```
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int (e\cos(c+dx))^p(a+a\sin(c+dx))^{5/2}\,dx = \frac{\left(a^2(e\cos(c+dx))^{1+p}(a-a\sin(c+dx))^{\frac{1}{2}(-1-p)}(a+a\sin(c+\right.}{}$$

$$= \frac{\left(2^{2+\frac{p}{2}}a^4(e\cos(c+dx))^{1+p}(a-a\sin(c+dx))^{\frac{1}{2}(-1-p)}(a+a\sin\right.}{}$$

$$= -\frac{2^{3+\frac{p}{2}}a^3(e\cos(c+dx))^{1+p}\,{}_2F_1\left(\frac{1}{2}(-4-p),\frac{1+p}{2};\frac{3+p}{2};\frac{1}{2}(1-\sin\right.}{de(1+p)\sqrt{a+a\sin(c+dx)}}$$

Mathematica [A]
time = 0.20, size = 102, normalized size = 0.99

$$-\frac{2^{3+\frac{p}{2}}a^3\cos(c+dx)(e\cos(c+dx))^p\,{}_2F_1\left(-2-\frac{p}{2},\frac{1+p}{2};\frac{3+p}{2};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{-p/2}}{d(1+p)\sqrt{a(1+\sin(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^p*(a + a*Sin[c + d*x])^(5/2),x]
[Out] -((2^(3 + p/2)*a^3*Cos[c + d*x]*(e*Cos[c + d*x])^p*Hypergeometric2F1[-2 - p
/2, (1 + p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2])/(d*(1 + p)*(1 + Sin[c + d*
x])^(p/2)*Sqrt[a*(1 + Sin[c + d*x])]))
```

Maple [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^p\,(a+a\sin(dx+c))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^(5/2),x)
[Out] int((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((a*sin(d*x + c) + a)^(5/2)*(cos(d*x + c)*e)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
[Out] integral(-(a^2*cos(d*x + c)^2 - 2*a^2*sin(d*x + c) - 2*a^2)*sqrt(a*sin(d*x
+ c) + a)*(cos(d*x + c)*e)^p, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**p*(a+a*sin(d*x+c))**(5/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 6189 deep
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (e\,\cos\left(c+d\,x\right))^p\,\left(a+a\,\sin\left(c+d\,x\right)\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^p*(a + a*sin(c + d*x))^(5/2),x)
[Out] int((e*cos(c + d*x))^p*(a + a*sin(c + d*x))^(5/2), x)
```

3.337 $\int (e\cos(c+dx))^p(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=103

$$-\frac{2^{2+\frac{p}{2}}a^2(e\cos(c+dx))^{1+p}\,{}_2F_1\left(\frac{1}{2}(-2-p),\frac{1+p}{2};\frac{3+p}{2};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{-p/2}}{de(1+p)\sqrt{a+a\sin(c+dx)}}$$

```
[Out] -2^(2+1/2*p)*a^2*(e*cos(d*x+c))^(1+p)*hypergeom([-1-1/2*p, 1/2+1/2*p],[3/2+
1/2*p],1/2-1/2*sin(d*x+c))/d/e/(1+p)/((1+sin(d*x+c))^(1/2*p))/(a+a*sin(d*x+
c))^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2768, 72, 71}

$$-\frac{a^2 2^{\frac{p}{2}+2}(\sin(c+dx)+1)^{-p/2}(e\cos(c+dx))^{p+1}\,{}_2F_1\left(\frac{1}{2}(-p-2),\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{de(p+1)\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^p*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -((2^(2 + p/2)*a^2*(e*Cos[c + d*x])^(1 + p)*Hypergeometric2F1[(-2 - p)/2, (
1 + p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2])/(d*e*(1 + p)*(1 + Sin[c + d*x])
^(p/2)*Sqrt[a + a*Sin[c + d*x]]))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
```

```
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int (e\cos(c+dx))^p(a+a\sin(c+dx))^{3/2}\,dx = \frac{\left(a^2(e\cos(c+dx))^{1+p}(a-a\sin(c+dx))^{\frac{1}{2}(-1-p)}(a+a\sin(c+a\right.}{}$$

$$= \frac{\left(2^{1+\frac{p}{2}}a^3(e\cos(c+dx))^{1+p}(a-a\sin(c+dx))^{\frac{1}{2}(-1-p)}(a+a\sin(\right.}{}$$

$$= -\frac{2^{2+\frac{p}{2}}a^2(e\cos(c+dx))^{1+p}\,{}_2F_1\left(\frac{1}{2}(-2-p),\frac{1+p}{2};\frac{3+p}{2};\frac{1}{2}(1-\sin(\right.}{de(1+p)\sqrt{a+a\sin(c+dx)}}$$

Mathematica [A]
time = 0.20, size = 101, normalized size = 0.98

$$-\frac{2^{2+\frac{p}{2}}\cos(c+dx)(e\cos(c+dx))^p\,{}_2F_1\left(-1-\frac{p}{2},\frac{1+p}{2};\frac{3+p}{2};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{-2-\frac{p}{2}}(a(1+\sin(c+dx)))^{3/2}}{d(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^p*(a + a*Sin[c + d*x])^(3/2),x]
[Out] -((2^(2 + p/2)*Cos[c + d*x]*(e*Cos[c + d*x])^p*Hypergeometric2F1[-1 - p/2,
(1 + p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^(-2 - p/2)*(
a*(1 + Sin[c + d*x]))^(3/2))/(d*(1 + p)))
```

Maple [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^p\,(a+a\sin(dx+c))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^(3/2),x)
[Out] int((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*(cos(d*x + c)*e)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a*sin(d*x + c) + a)^(3/2)*(cos(d*x + c)*e)^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sin (c+dx)+1))^{\frac{3}{2}} (e\cos (c+dx))^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**p*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral((a*(sin(c + d*x) + 1))**(3/2)*(e*cos(c + d*x))**p, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(e\,\cos\left(c+d\,x\right)\right)}^p\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^p*(a + a*sin(c + d*x))^(3/2),x)
```

```
[Out] int((e*cos(c + d*x))^p*(a + a*sin(c + d*x))^(3/2), x)
```

3.338 $\int (e\cos(c+dx))^p\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=97

$$-\frac{2^{1+\frac{p}{2}}a(e\cos(c+dx))^{1+p}\,{}_2F_1\left(-\frac{p}{2},\frac{1+p}{2};\frac{3+p}{2};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{-p/2}}{de(1+p)\sqrt{a+a\sin(c+dx)}}$$

```
[Out] -2^(1+1/2*p)*a*(e*cos(d*x+c))^(1+p)*hypergeom([-1/2*p, 1/2+1/2*p],[3/2+1/2*
p],1/2-1/2*sin(d*x+c))/d/e/(1+p)/((1+sin(d*x+c))^(1/2*p))/(a+a*sin(d*x+c))^
(1/2)
```

Rubi [A]

time = 0.07, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2768, 72, 71}

$$-\frac{a2^{\frac{p}{2}+1}(\sin(c+dx)+1)^{-p/2}(e\cos(c+dx))^{p+1}\,{}_2F_1\left(-\frac{p}{2},\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{de(p+1)\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^p*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -((2^(1 + p/2)*a*(e*Cos[c + d*x])^(1 + p)*Hypergeometric2F1[-1/2*p, (1 + p)
/2, (3 + p)/2, (1 - Sin[c + d*x])/2])/(d*e*(1 + p)*(1 + Sin[c + d*x])^(p/2)
*Sqrt[a + a*Sin[c + d*x]]))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
```

```
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int (e\cos(c+dx))^p\sqrt{a+a\sin(c+dx)}\,dx = \frac{\left(a^2(e\cos(c+dx))^{1+p}(a-a\sin(c+dx))^{\frac{1}{2}(-1-p)}(a+a\sin(c+\right.}{}$$

$$= \frac{\left(2^{p/2}a^2(e\cos(c+dx))^{1+p}(a-a\sin(c+dx))^{\frac{1}{2}(-1-p)}(a+a\sin(\right.}{}$$

$$= -\frac{2^{1+\frac{p}{2}}a(e\cos(c+dx))^{1+p}\,{}_2F_1\left(-\frac{p}{2},\frac{1+p}{2};\frac{3+p}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{de(1+p)\sqrt{a+a\sin(c+dx)}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.94, size = 310, normalized size = 3.20

$$\frac{(1+i)2^{-p}e^{-\frac{1}{2}idx}\cos^{-p}(c+dx)(e\cos(c+dx))^p\left(e^{idx}(1+2p)\,{}_2F_1\left(\frac{1}{4}(1-2p),-p;\frac{1}{4}(5-2p);-e^{2idx}(\cos(c)+i\sin(c))^2\right)\left(\cos\left(\frac{c}{2}\right)+i\sin\left(\frac{c}{2}\right)\right)+(-1+2p)\,{}_2F_1\left(\frac{1}{4}(-1-2p),-p;\frac{1}{4}(3-2p);-e^{2idx}(\cos(c)+i\sin(c))^2\right)\left(i\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\right)\left(e^{-idx}\left(\left(1+e^{2idx}\right)\cos(c)+i\left(-1+e^{2idx}\right)\sin(c)\right)\right)^p\left(1+e^{2idx}\cos(2c)+ie^{2idx}\sin(2c)\right)^{-p}\sqrt{a(1+\sin(c+dx))}}{d(-1+2p)(1+2p)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^p*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] ((1 + I)*(e*Cos[c + d*x])^p*(E^(I*d*x)*(1 + 2*p)*Hypergeometric2F1[(1 - 2*p
)/4, -p, (5 - 2*p)/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*(Cos[c/2] + I
*Sin[c/2]) + (-1 + 2*p)*Hypergeometric2F1[(-1 - 2*p)/4, -p, (3 - 2*p)/4, -(
E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*(I*Cos[c/2] + Sin[c/2]))*(((1 + E^((2
*I)*d*x))*Cos[c] + I*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x))^p*Sqrt[a*(1 +
Sin[c + d*x])])/(2^p*d*E^((I/2)*d*x)*(-1 + 2*p)*(1 + 2*p)*Cos[c + d*x]^p*(1
 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c])^p*(Cos[(c + d*x)/2] +
 Sin[(c + d*x)/2]))
```

Maple [F]
time = 0.15, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^p\sqrt{a+a\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] int((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a*sin(d*x + c) + a)*(cos(d*x + c)*e)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(a*sin(d*x + c) + a)*(cos(d*x + c)*e)^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sin\left(c+dx\right)+1\right)}\left(e\cos\left(c+dx\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**p*(a+a*sin(d*x+c))**(1/2),x)
[Out] Integral(sqrt(a*(sin(c + d*x) + 1))*(e*cos(c + d*x))**p, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(e\,\cos\left(c+d\,x\right)\right)}^p\,\sqrt{a+a\,\sin\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^p*(a + a*sin(c + d*x))^(1/2),x)
[Out] int((e*cos(c + d*x))^p*(a + a*sin(c + d*x))^(1/2), x)
```

3.339 $\int \frac{(e\cos(c+dx))^p}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=101

$$-\frac{2^{p/2}a(e\cos(c+dx))^{1+p}\,{}_2F_1\left(\frac{2-p}{2},\frac{1+p}{2};\frac{3+p}{2};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{1-\frac{p}{2}}}{de(1+p)(a+a\sin(c+dx))^{3/2}}$$

```
[Out] -2^(1/2*p)*a*(e*cos(d*x+c))^(1+p)*hypergeom([1-1/2*p, 1/2+1/2*p],[3/2+1/2*p
],1/2-1/2*sin(d*x+c))*(1+sin(d*x+c))^(1-1/2*p)/d/e/(1+p)/(a+a*sin(d*x+c))^(
3/2)
```

Rubi [A]
time = 0.07, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2768, 72, 71}

$$-\frac{a2^{p/2}(\sin(c+dx)+1)^{1-\frac{p}{2}}(e\cos(c+dx))^{p+1}\,{}_2F_1\left(\frac{2-p}{2},\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{de(p+1)(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^p/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -((2^(p/2)*a*(e*Cos[c + d*x])^(1 + p)*Hypergeometric2F1[(2 - p)/2, (1 + p)/
2, (3 + p)/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^(1 - p/2))/(d*e*(1 +
 p)*(a + a*Sin[c + d*x])^(3/2)))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
```

```
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^p}{\sqrt{a+a\sin(c+dx)}}\,dx = \frac{\left(a^2(e\cos(c+dx))^{1+p}(a-a\sin(c+dx))^{\frac{1}{2}(-1-p)}(a+a\sin(c+dx))^{\frac{1}{2}(-1-p)}\right)\text{Su}}{de}$$

$$= \frac{\left(2^{-1+\frac{p}{2}}a^2(e\cos(c+dx))^{1+p}(a-a\sin(c+dx))^{\frac{1}{2}(-1-p)}(a+a\sin(c+dx))^{-1+\frac{1}{2}}\right.}{}$$

$$= -\frac{2^{p/2}a(e\cos(c+dx))^{1+p}\,{}_2F_1\left(\frac{2-p}{2},\frac{1+p}{2};\frac{3+p}{2};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+d}{de(1+p)(a+a\sin(c+dx))^{3/2}}$$

Mathematica [A]
time = 0.11, size = 97, normalized size = 0.96

$$-\frac{2^{p/2}\cos(c+dx)(e\cos(c+dx))^p\,{}_2F_1\left(1-\frac{p}{2},\frac{1+p}{2};\frac{3+p}{2};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{-p/2}}{d(1+p)\sqrt{a(1+\sin(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^p/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -((2^(p/2)*Cos[c + d*x]*(e*Cos[c + d*x])^p*Hypergeometric2F1[1 - p/2, (1 +
p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2])/(d*(1 + p)*(1 + Sin[c + d*x])^(p/2)
*Sqrt[a*(1 + Sin[c + d*x])]))
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^p}{\sqrt{a+a\sin(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^p/(a+a*sin(d*x+c))^(1/2),x)
```

```
[Out] int((e*cos(d*x+c))^p/(a+a*sin(d*x+c))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate((cos(d*x + c)*e)^p/sqrt(a*sin(d*x + c) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral((cos(d*x + c)*e)^p/sqrt(a*sin(d*x + c) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(c+dx))^p}{\sqrt{a(\sin(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**p/(a+a*sin(d*x+c))**(1/2),x)
[Out] Integral((e*cos(c + d*x))**p/sqrt(a*(sin(c + d*x) + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate((cos(d*x + c)*e)^p/sqrt(a*sin(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,\cos(c+d\,x))^p}{\sqrt{a+a\,\sin(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^p/(a + a*sin(c + d*x))^(1/2),x)
[Out] int((e*cos(c + d*x))^p/(a + a*sin(c + d*x))^(1/2), x)
```

3.340 $\int \frac{(e\cos(c+dx))^p}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=102

$$-\frac{2^{-1+\frac{p}{2}}(e\cos(c+dx))^{1+p}\,{}_2F_1\left(\frac{4-p}{2},\frac{1+p}{2};\frac{3+p}{2};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{1-\frac{p}{2}}}{de(1+p)(a+a\sin(c+dx))^{3/2}}$$

```
[Out] -2^(-1+1/2*p)*(e*cos(d*x+c))^(1+p)*hypergeom([2-1/2*p, 1/2+1/2*p],[3/2+1/2*
p],1/2-1/2*sin(d*x+c))*(1+sin(d*x+c))^(1-1/2*p)/d/e/(1+p)/(a+a*sin(d*x+c))^
(3/2)
```

Rubi [A]
time = 0.08, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2768, 72, 71}

$$-\frac{2^{\frac{p}{2}-1}(\sin(c+dx)+1)^{1-\frac{p}{2}}(e\cos(c+dx))^{p+1}\,{}_2F_1\left(\frac{4-p}{2},\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{de(p+1)(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^p/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -((2^(-1 + p/2)*(e*Cos[c + d*x])^(1 + p)*Hypergeometric2F1[(4 - p)/2, (1 +
p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^(1 - p/2))/(d*e*(
1 + p)*(a + a*Sin[c + d*x])^(3/2)))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
```

```
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^p}{(a+a\sin(c+dx))^{3/2}}\,dx = \frac{\left(a^2(e\cos(c+dx))^{1+p}(a-a\sin(c+dx))^{\frac{1}{2}(-1-p)}(a+a\sin(c+dx))^{\frac{1}{2}(-1-p)}\right)\text{Su}}{de}$$

$$= \frac{\left(2^{-2+\frac{p}{2}}a(e\cos(c+dx))^{1+p}(a-a\sin(c+dx))^{\frac{1}{2}(-1-p)}(a+a\sin(c+dx))^{-1+\frac{1}{2}(\cdot}\right.}{}$$

$$= -\frac{2^{-1+\frac{p}{2}}(e\cos(c+dx))^{1+p}\,{}_2F_1\left(\frac{4-p}{2},\frac{1+p}{2};\frac{3+p}{2};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+}{de(1+p)(a+a\sin(c+dx))^{3/2}}$$

Mathematica [A]
time = 0.17, size = 101, normalized size = 0.99

$$-\frac{2^{-1+\frac{p}{2}}\cos(c+dx)(e\cos(c+dx))^p\,{}_2F_1\left(2-\frac{p}{2},\frac{1+p}{2};\frac{3+p}{2};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{1-\frac{p}{2}}}{d(1+p)(a(1+\sin(c+dx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^p/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -((2^(-1 + p/2)*Cos[c + d*x]*(e*Cos[c + d*x])^p*Hypergeometric2F1[2 - p/2,
(1 + p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^(1 - p/2))/(
d*(1 + p)*(a*(1 + Sin[c + d*x]))^(3/2)))
```

Maple [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^p}{(a+a\sin(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^p/(a+a*sin(d*x+c))^(3/2),x)
```

```
[Out] int((e*cos(d*x+c))^p/(a+a*sin(d*x+c))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((cos(d*x + c)*e)^p/(a*sin(d*x + c) + a)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(a*sin(d*x + c) + a)*(cos(d*x + c)*e)^p/(a^2*cos(d*x + c)^2 -
 2*a^2*sin(d*x + c) - 2*a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(c+dx))^p}{(a(\sin(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**p/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral((e*cos(c + d*x))**p/(a*(sin(c + d*x) + 1))**(3/2), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,\cos(c+d\,x))^p}{(a+a\,\sin(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^p/(a + a*sin(c + d*x))^(3/2),x)
```

```
[Out] int((e*cos(c + d*x))^p/(a + a*sin(c + d*x))^(3/2), x)
```

3.341 $\int \frac{(e\cos(c+dx))^p}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=105

$$-\frac{2^{-2+\frac{p}{2}}(e\cos(c+dx))^{1+p}\,{}_2F_1\left(\frac{6-p}{2},\frac{1+p}{2};\frac{3+p}{2};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{1-\frac{p}{2}}}{ade(1+p)(a+a\sin(c+dx))^{3/2}}$$

```
[Out] -2^(-2+1/2*p)*(e*cos(d*x+c))^(1+p)*hypergeom([3-1/2*p, 1/2+1/2*p],[3/2+1/2*
p],1/2-1/2*sin(d*x+c))*(1+sin(d*x+c))^(1-1/2*p)/a/d/e/(1+p)/(a+a*sin(d*x+c)
)^(3/2)
```

Rubi [A]
time = 0.08, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2768, 72, 71}

$$-\frac{2^{\frac{p}{2}-2}(\sin(c+dx)+1)^{1-\frac{p}{2}}(e\cos(c+dx))^{p+1}\,{}_2F_1\left(\frac{6-p}{2},\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{ade(p+1)(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^p/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] -((2^(-2 + p/2)*(e*Cos[c + d*x])^(1 + p)*Hypergeometric2F1[(6 - p)/2, (1 +
p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^(1 - p/2))/(a*d*e
*(1 + p)*(a + a*Sin[c + d*x])^(3/2)))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
```

```
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^p}{(a+a\sin(c+dx))^{5/2}}\,dx = \frac{\left(a^2(e\cos(c+dx))^{1+p}(a-a\sin(c+dx))^{\frac{1}{2}(-1-p)}(a+a\sin(c+dx))^{\frac{1}{2}(-1-p)}\right) \mathrm{S}}{de}$$

$$= \frac{\left(2^{-3+\frac{p}{2}}(e\cos(c+dx))^{1+p}(a-a\sin(c+dx))^{\frac{1}{2}(-1-p)}(a+a\sin(c+dx))^{-1+\frac{1}{2}(\cdot}\right.}{}$$

$$= -\frac{2^{-2+\frac{p}{2}}(e\cos(c+dx))^{1+p}\,{}_2F_1\left(\frac{6-p}{2},\frac{1+p}{2};\frac{3+p}{2};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+}{ade(1+p)(a+a\sin(c+dx))^{3/2}}$$

Mathematica [A]
time = 0.13, size = 102, normalized size = 0.97

$$-\frac{2^{-2+\frac{p}{2}}\cos(c+dx)(e\cos(c+dx))^p\,{}_2F_1\left(3-\frac{p}{2},\frac{1+p}{2};\frac{3+p}{2};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{-p/2}}{a^2d(1+p)\sqrt{a(1+\sin(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^p/(a + a*Sin[c + d*x])^(5/2),x]
[Out] -((2^(-2 + p/2)*Cos[c + d*x]*(e*Cos[c + d*x])^p*Hypergeometric2F1[3 - p/2,
(1 + p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2])/(a^2*d*(1 + p)*(1 + Sin[c + d*
x])^(p/2)*Sqrt[a*(1 + Sin[c + d*x])]))
```

Maple [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^p}{(a+a\sin(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^p/(a+a*sin(d*x+c))^(5/2),x)
[Out] int((e*cos(d*x+c))^p/(a+a*sin(d*x+c))^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((cos(d*x + c)*e)^p/(a*sin(d*x + c) + a)^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(a*sin(d*x + c) + a)*(cos(d*x + c)*e)^p/(3*a^3*cos(d*x + c)^2
 - 4*a^3 + (a^3*cos(d*x + c)^2 - 4*a^3)*sin(d*x + c)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(c+dx))^p}{(a(\sin(c+dx)+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**p/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Integral((e*cos(c + d*x))**p/(a*(sin(c + d*x) + 1))**(5/2), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{(e\,\cos\left(c+d\,x\right))}^p}{{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^p/(a + a*sin(c + d*x))^(5/2),x)
```

```
[Out] int((e*cos(c + d*x))^p/(a + a*sin(c + d*x))^(5/2), x)
```

3.342 $\int (e\cos(c+dx))^p(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=114

$$-\frac{2^{\frac{1}{2}+m+\frac{p}{2}}a(e\cos(c+dx))^{1+p}\,{}_2F_1\left(\frac{1}{2}(1-2m-p),\frac{1+p}{2};\frac{3+p}{2};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{\frac{1}{2}(1-2m-p)}(a}{de(1+p)}$$

```
[Out] -2^(1/2+m+1/2*p)*a*(e*cos(d*x+c))^(1+p)*hypergeom([1/2+1/2*p, 1/2-m-1/2*p],
[3/2+1/2*p],1/2-1/2*sin(d*x+c))*(1+sin(d*x+c))^(1/2-m-1/2*p)*(a+a*sin(d*x+c
))^(-1+m)/d/e/(1+p)
```

Rubi [A]
time = 0.08, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {2768, 72, 71}

$$-\frac{a2^{m+\frac{p}{2}+\frac{1}{2}}(a\sin(c+dx)+a)^{m-1}(e\cos(c+dx))^{p+1}(\sin(c+dx)+1)^{\frac{1}{2}(-2m-p+1)}\,{}_2F_1\left(\frac{1}{2}(-2m-p+1),\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{de(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^p*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] -((2^(1/2 + m + p/2)*a*(e*Cos[c + d*x])^(1 + p)*Hypergeometric2F1[(1 - 2*m
- p)/2, (1 + p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^((1
- 2*m - p)/2)*(a + a*Sin[c + d*x])^(-1 + m))/(d*e*(1 + p)))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
```

```
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int (e\cos(c+dx))^p(a+a\sin(c+dx))^m\,dx = \frac{\left(a^2(e\cos(c+dx))^{1+p}(a-a\sin(c+dx))^{\frac{1}{2}(-1-p)}(a+a\sin(c+dx$$

$$= \frac{\left(2^{-\frac{1}{2}+m+\frac{p}{2}}a^2(e\cos(c+dx))^{1+p}(a-a\sin(c+dx))^{\frac{1}{2}(-1-p)}(a+a$$

$$= -\frac{2^{\frac{1}{2}+m+\frac{p}{2}}a(e\cos(c+dx))^{1+p}\,{}_2F_1\left(\frac{1}{2}(1-2m-p),\frac{1+p}{2};\frac{3+p}{2};\frac{1}{2}(1-}{de(}$$

Mathematica [A]
time = 0.19, size = 112, normalized size = 0.98

$$-\frac{2^{\frac{1}{2}(1+2m+p)}\cos(c+dx)(e\cos(c+dx))^p\,{}_2F_1\left(\frac{1}{2}(1-2m-p),\frac{1+p}{2};\frac{3+p}{2};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{\frac{1}{2}(-1-2m-p)}(a(1+\sin(c+dx)))^m}{d(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^p*(a + a*Sin[c + d*x])^m,x]
[Out] -((2^((1 + 2*m + p)/2)*Cos[c + d*x]*(e*Cos[c + d*x])^p*Hypergeometric2F1[(1
 - 2*m - p)/2, (1 + p)/2, (3 + p)/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x
])^((-1 - 2*m - p)/2)*(a*(1 + Sin[c + d*x]))^m)/(d*(1 + p)))
```

Maple [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^p\,(a+a\sin(dx+c))^m\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^m,x)
[Out] int((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((cos(d*x + c)*e)^p*(a*sin(d*x + c) + a)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((cos(d*x + c)*e)^p*(a*sin(d*x + c) + a)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sin (c+dx)+1))^m (e \cos (c+dx))^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**p*(a+a*sin(d*x+c))**m,x)
```

```
[Out] Integral((a*(sin(c + d*x) + 1))**m*(e*cos(c + d*x))**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+a*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((cos(d*x + c)*e)^p*(a*sin(d*x + c) + a)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(e\,\cos\left(c+d\,x\right)\right)}^p\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^m \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^p*(a + a*sin(c + d*x))^m,x)
```

```
[Out] int((e*cos(c + d*x))^p*(a + a*sin(c + d*x))^m, x)
```

3.343 $\int \cos^7(c+dx)(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=109

$$\frac{8(a+a\sin(c+dx))^{4+m}}{a^4d(4+m)}-\frac{12(a+a\sin(c+dx))^{5+m}}{a^5d(5+m)}+\frac{6(a+a\sin(c+dx))^{6+m}}{a^6d(6+m)}-\frac{(a+a\sin(c+dx))^{7+m}}{a^7d(7+m)}$$

```
[Out] 8*(a+a*sin(d*x+c))^(4+m)/a^4/d/(4+m)-12*(a+a*sin(d*x+c))^(5+m)/a^5/d/(5+m)+
6*(a+a*sin(d*x+c))^(6+m)/a^6/d/(6+m)-(a+a*sin(d*x+c))^(7+m)/a^7/d/(7+m)
```

Rubi [A]
time = 0.06, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2746, 45}

$$-\frac{(a\sin(c+dx)+a)^{m+7}}{a^7d(m+7)}+\frac{6(a\sin(c+dx)+a)^{m+6}}{a^6d(m+6)}-\frac{12(a\sin(c+dx)+a)^{m+5}}{a^5d(m+5)}+\frac{8(a\sin(c+dx)+a)^{m+4}}{a^4d(m+4)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] (8*(a + a*Sin[c + d*x])^(4 + m))/(a^4*d*(4 + m)) - (12*(a + a*Sin[c + d*x])
^(5 + m))/(a^5*d*(5 + m)) + (6*(a + a*Sin[c + d*x])^(6 + m))/(a^6*d*(6 + m)
) - (a + a*Sin[c + d*x])^(7 + m)/(a^7*d*(7 + m))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\int \cos^7(c+dx)(a+a\sin(c+dx))^m\,dx = \frac{\text{Subst}\left(\int (a-x)^3(a+x)^{3+m}\,dx, x, a\sin(c+dx)\right)}{a^7d}$$
$$= \frac{\text{Subst}\left(\int \left(8a^3(a+x)^{3+m} - 12a^2(a+x)^{4+m} + 6a(a+x)^{5+m} - (a \cdots\right.\right.}{a^7d}$$
$$= \frac{8(a+a\sin(c+dx))^{4+m}}{a^4d(4+m)} - \frac{12(a+a\sin(c+dx))^{5+m}}{a^5d(5+m)} + \frac{6(a+a\,\text{si}\cdots}{a^6d\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.14, size = 796, normalized size = 7.30

$$\frac{(a(1+\sin(c+dx)))^m\left(\frac{6144+1084m+117m^2+5m^3}{16(4+m)(5+m)(6+m)(7+m)} + \frac{(29400+2578m+171m^2+5m^3)\left(-\frac{1}{128}i\cos(c+dx)+\frac{1}{128}\sin(c+dx)\right)}{(4+m)(5+m)(6+m)(7+m)} + \frac{(29400+2578m+171m^2+5m^3)\left(\frac{1}{128}i\cos(c+dx)+\frac{1}{128}\sin(c+dx)\right)}{(4+m)(5+m)(6+m)(7+m)} + \frac{(804m+109m^2+5m^3)\left(\frac{3}{64}\cos(2(c+dx))-\frac{3}{64}i\sin(2(c+dx))\right)}{(4+m)(5+m)(6+m)(7+m)} + \frac{(804m+109m^2+5m^3)\left(\frac{3}{64}\cos(2(c+dx))+\frac{3}{64}i\sin(2(c+dx))\right)}{(4+m)(5+m)(6+m)(7+m)} + \frac{(1960+1070m+93m^2+3m^3)\left(-\frac{3}{128}i\cos(3(c+dx))+\frac{3}{128}\sin(3(c+dx))\right)}{(4+m)(5+m)(6+m)(7+m)} + \frac{(1960+1070m+93m^2+3m^3)\left(\frac{3}{128}i\cos(3(c+dx))+\frac{3}{128}\sin(3(c+dx))\right)}{(4+m)(5+m)(6+m)(7+m)} + \cdots\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] ((a*(1 + Sin[c + d*x]))^m*((6144 + 1084*m + 117*m^2 + 5*m^3)/(16*(4 + m)*(5
 + m)*(6 + m)*(7 + m)) + ((29400 + 2578*m + 171*m^2 + 5*m^3)*((-1/128*I)*Co
s[c + d*x] + Sin[c + d*x]/128))/((4 + m)*(5 + m)*(6 + m)*(7 + m)) + ((29400
 + 2578*m + 171*m^2 + 5*m^3)*((I/128)*Cos[c + d*x] + Sin[c + d*x]/128))/((4
 + m)*(5 + m)*(6 + m)*(7 + m)) + ((804*m + 109*m^2 + 5*m^3)*((3*Cos[2*(c +
d*x)])/64 - ((3*I)/64)*Sin[2*(c + d*x)]))/((4 + m)*(5 + m)*(6 + m)*(7 + m))
 + ((804*m + 109*m^2 + 5*m^3)*((3*Cos[2*(c + d*x)])/64 + ((3*I)/64)*Sin[2*(
c + d*x)]))/((4 + m)*(5 + m)*(6 + m)*(7 + m)) + ((1960 + 1070*m + 93*m^2 +
3*m^3)*(((-3*I)/128)*Cos[3*(c + d*x)] + (3*Sin[3*(c + d*x)])/128))/((4 + m)
*(5 + m)*(6 + m)*(7 + m)) + ((1960 + 1070*m + 93*m^2 + 3*m^3)*(((3*I)/128)*
Cos[3*(c + d*x)] + (3*Sin[3*(c + d*x)])/128))/((4 + m)*(5 + m)*(6 + m)*(7 +
 m)) + ((44*m + 17*m^2 + m^3)*((3*Cos[4*(c + d*x)])/32 - ((3*I)/32)*Sin[4*(
c + d*x)]))/((4 + m)*(5 + m)*(6 + m)*(7 + m)) + ((44*m + 17*m^2 + m^3)*((3*
Cos[4*(c + d*x)])/32 + ((3*I)/32)*Sin[4*(c + d*x)]))/((4 + m)*(5 + m)*(6 +
m)*(7 + m)) + ((294 + 103*m + 5*m^2)*((-1/128*I)*Cos[5*(c + d*x)] + Sin[5*(
c + d*x)]/128))/((5 + m)*(6 + m)*(7 + m)) + ((294 + 103*m + 5*m^2)*((I/128)
*Cos[5*(c + d*x)] + Sin[5*(c + d*x)]/128))/((5 + m)*(6 + m)*(7 + m)) + ((m*
Cos[6*(c + d*x)])/64 - (I/64)*m*Sin[6*(c + d*x)])/((6 + m)*(7 + m)) + ((m*C
os[6*(c + d*x)])/64 + (I/64)*m*Sin[6*(c + d*x)])/((6 + m)*(7 + m)) + ((-1/1
28*I)*Cos[7*(c + d*x)] + Sin[7*(c + d*x)]/128)/(7 + m) + ((I/128)*Cos[7*(c
+ d*x)] + Sin[7*(c + d*x)]/128)/(7 + m)))/d
```

Maple [F]
time = 0.30, size = 0, normalized size = 0.00

$$\int \left(\cos^7(dx+c)\right)(a+a\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*(a+a*sin(d*x+c))^m,x)
```

```
[Out] int(cos(d*x+c)^7*(a+a*sin(d*x+c))^m,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 520 vs. $2(109) = 218$.
time = 0.31, size = 520, normalized size = 4.77

$$-\frac{\frac{\left((m^6+21m^5+175m^4+735m^3+1624m^2+1764m+720)a^m\sin(dx+c)^7+(m^6+15m^5+85m^4+225m^3+274m^2+120m)a^m\sin(dx+c)^6-6(m^5+10m^4+35m^3+50m^2+24m)a^m\sin(dx+c)^5+30(m^4+6m^3+11m^2+6m)a^m\sin(dx+c)^4-120(m^3+3m^2+2m)a^m\sin(dx+c)^3+360(m^2+m)a^m\sin(dx+c)^2-720a^m m\sin(dx+c)+720a^m\right)(\sin(dx+c)+1)^m}{m^7+28m^6+322m^5+1960m^4+6769m^3+13132m^2+13068m+5040}-\frac{3\left((m^4+10m^3+35m^2+50m+24)a^m\sin(dx+c)^5+(m^4+6m^3+11m^2+6m)a^m\sin(dx+c)^4-4(m^3+3m^2+2m)a^m\sin(dx+c)^3+12(m^2+m)a^m\sin(dx+c)^2-24a^m m\sin(dx+c)+24a^m\right)(\sin(dx+c)+1)^m}{m^5+15m^4+85m^3+225m^2+274m+120}+\frac{3\left((m^2+3m+2)a^m\sin(dx+c)^3+(m^2+m)a^m\sin(dx+c)^2-2a^m m\sin(dx+c)+2a^m\right)(\sin(dx+c)+1)^m}{m^3+6m^2+11m+6}-\frac{(a\sin(dx+c)+a)^{m+1}}{a(m+1)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] -((((m^6 + 21*m^5 + 175*m^4 + 735*m^3 + 1624*m^2 + 1764*m + 720)*a^m*sin(d*x
 + c)^7 + (m^6 + 15*m^5 + 85*m^4 + 225*m^3 + 274*m^2 + 120*m)*a^m*sin(d*x +
 c)^6 - 6*(m^5 + 10*m^4 + 35*m^3 + 50*m^2 + 24*m)*a^m*sin(d*x + c)^5 + 30*(
m^4 + 6*m^3 + 11*m^2 + 6*m)*a^m*sin(d*x + c)^4 - 120*(m^3 + 3*m^2 + 2*m)*a^
m*sin(d*x + c)^3 + 360*(m^2 + m)*a^m*sin(d*x + c)^2 - 720*a^m*m*sin(d*x + c
) + 720*a^m)*(sin(d*x + c) + 1)^m/(m^7 + 28*m^6 + 322*m^5 + 1960*m^4 + 6769
*m^3 + 13132*m^2 + 13068*m + 5040) - 3*((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)
*a^m*sin(d*x + c)^5 + (m^4 + 6*m^3 + 11*m^2 + 6*m)*a^m*sin(d*x + c)^4 - 4*(
m^3 + 3*m^2 + 2*m)*a^m*sin(d*x + c)^3 + 12*(m^2 + m)*a^m*sin(d*x + c)^2 - 2
4*a^m*m*sin(d*x + c) + 24*a^m)*(sin(d*x + c) + 1)^m/(m^5 + 15*m^4 + 85*m^3
+ 225*m^2 + 274*m + 120) + 3*((m^2 + 3*m + 2)*a^m*sin(d*x + c)^3 + (m^2 + m
)*a^m*sin(d*x + c)^2 - 2*a^m*m*sin(d*x + c) + 2*a^m)*(sin(d*x + c) + 1)^m/(
m^3 + 6*m^2 + 11*m + 6) - (a*sin(d*x + c) + a)^(m + 1)/(a*(m + 1)))/d
```

Fricas [A]
time = 0.39, size = 153, normalized size = 1.40

$$\frac{\left((m^3+9m^2+20m)\cos(dx+c)^6+12(m^2+3m)\cos(dx+c)^4+96m\cos(dx+c)^2+\left((m^3+15m^2+74m+120)\cos(dx+c)^6+12(m^2+7m+12)\cos(dx+c)^4+96(m+2)\cos(dx+c)^2+384\right)\sin(dx+c)+384\right)(a\sin(dx+c)+a)^m}{dm^4+22dm^3+179dm^2+638dm+840d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] ((m^3 + 9*m^2 + 20*m)*cos(d*x + c)^6 + 12*(m^2 + 3*m)*cos(d*x + c)^4 + 96*m
*cos(d*x + c)^2 + ((m^3 + 15*m^2 + 74*m + 120)*cos(d*x + c)^6 + 12*(m^2 + 7
*m + 12)*cos(d*x + c)^4 + 96*(m + 2)*cos(d*x + c)^2 + 384)*sin(d*x + c) + 3
84)*(a*sin(d*x + c) + a)^m/(d*m^4 + 22*d*m^3 + 179*d*m^2 + 638*d*m + 840*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*(a+a*sin(d*x+c))**m,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 517 vs. 2(109) = 218.
time = 5.11, size = 517, normalized size = 4.74

$$-\frac{(a\sin(dx+c)+a)^7(a\sin(dx+c)+a)^m m^3 - 6(a\sin(dx+c)+a)^6(a\sin(dx+c)+a)^m a m^3 + 12(a\sin(dx+c)+a)^5(a\sin(dx+c)+a)^m a^2 m^3 - 8(a\sin(dx+c)+a)^4(a\sin(dx+c)+a)^m a^3 m^3 + 15(a\sin(dx+c)+a)^7(a\sin(dx+c)+a)^m m^2 - 96(a\sin(dx+c)+a)^6(a\sin(dx+c)+a)^m a m^2 + 204(a\sin(dx+c)+a)^5(a\sin(dx+c)+a)^m a^2 m^2 - 144(a\sin(dx+c)+a)^4(a\sin(dx+c)+a)^m a^3 m^2 + 74(a\sin(dx+c)+a)^7(a\sin(dx+c)+a)^m m - 498(a\sin(dx+c)+a)^6(a\sin(dx+c)+a)^m a m + 1128(a\sin(dx+c)+a)^5(a\sin(dx+c)+a)^m a^2 m - 856(a\sin(dx+c)+a)^4(a\sin(dx+c)+a)^m a^3 m + 120(a\sin(dx+c)+a)^7(a\sin(dx+c)+a)^m - 840(a\sin(dx+c)+a)^6(a\sin(dx+c)+a)^m a + 2016(a\sin(dx+c)+a)^5(a\sin(dx+c)+a)^m a^2 - 1680(a\sin(dx+c)+a)^4(a\sin(dx+c)+a)^m a^3}{(a^6m^4 + 22a^6m^3 + 179a^6m^2 + 638a^6m + 840a^6)ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] -((a*sin(d*x + c) + a)^7*(a*sin(d*x + c) + a)^m*m^3 - 6*(a*sin(d*x + c) + a
)^6*(a*sin(d*x + c) + a)^m*a*m^3 + 12*(a*sin(d*x + c) + a)^5*(a*sin(d*x + c
) + a)^m*a^2*m^3 - 8*(a*sin(d*x + c) + a)^4*(a*sin(d*x + c) + a)^m*a^3*m^3
+ 15*(a*sin(d*x + c) + a)^7*(a*sin(d*x + c) + a)^m*m^2 - 96*(a*sin(d*x + c)
 + a)^6*(a*sin(d*x + c) + a)^m*a*m^2 + 204*(a*sin(d*x + c) + a)^5*(a*sin(d*
x + c) + a)^m*a^2*m^2 - 144*(a*sin(d*x + c) + a)^4*(a*sin(d*x + c) + a)^m*a
^3*m^2 + 74*(a*sin(d*x + c) + a)^7*(a*sin(d*x + c) + a)^m*m - 498*(a*sin(d*
x + c) + a)^6*(a*sin(d*x + c) + a)^m*a*m + 1128*(a*sin(d*x + c) + a)^5*(a*s
in(d*x + c) + a)^m*a^2*m - 856*(a*sin(d*x + c) + a)^4*(a*sin(d*x + c) + a)^
m*a^3*m + 120*(a*sin(d*x + c) + a)^7*(a*sin(d*x + c) + a)^m - 840*(a*sin(d*
x + c) + a)^6*(a*sin(d*x + c) + a)^m*a + 2016*(a*sin(d*x + c) + a)^5*(a*sin
(d*x + c) + a)^m*a^2 - 1680*(a*sin(d*x + c) + a)^4*(a*sin(d*x + c) + a)^m*a
^3)/((a^6*m^4 + 22*a^6*m^3 + 179*a^6*m^2 + 638*a^6*m + 840*a^6)*a*d)
```

Mupad [B]
time = 10.48, size = 555, normalized size = 5.09

$$e^{-c\,7\mathrm{i}-d\,x\,7\mathrm{i}}\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^m\,\left(\frac{e^{c\,7\mathrm{i}+d\,x\,7\mathrm{i}}\,\left(m^3\,40\mathrm{i}+m^2\,936\mathrm{i}+m\,8672\mathrm{i}+49152\mathrm{i}\right)}{128\,d\,\left(m^4\,1\mathrm{i}+m^3\,22\mathrm{i}+m^2\,179\mathrm{i}+m\,638\mathrm{i}+840\mathrm{i}\right)}+\frac{e^{c\,7\mathrm{i}+d\,x\,7\mathrm{i}}\,\cos\left(2\,c+2\,d\,x\right)\,\left(m^3\,30\mathrm{i}+m^2\,654\mathrm{i}+m\,4824\mathrm{i}\right)}{64\,d\,\left(m^4\,1\mathrm{i}+m^3\,22\mathrm{i}+m^2\,179\mathrm{i}+m\,638\mathrm{i}+840\mathrm{i}\right)}+\frac{e^{c\,7\mathrm{i}+d\,x\,7\mathrm{i}}\,\sin\left(5\,c+5\,d\,x\right)\,\left(5\,m^3+123\,m^2+706\,m+1176\right)\,1\mathrm{i}}{64\,d\,\left(m^4\,1\mathrm{i}+m^3\,22\mathrm{i}+m^2\,179\mathrm{i}+m\,638\mathrm{i}+840\mathrm{i}\right)}+\frac{e^{c\,7\mathrm{i}+d\,x\,7\mathrm{i}}\,\sin\left(3\,c+3\,d\,x\right)\,\left(9\,m^3+279\,m^2+3210\,m+5880\right)\,1\mathrm{i}}{64\,d\,\left(m^4\,1\mathrm{i}+m^3\,22\mathrm{i}+m^2\,179\mathrm{i}+m\,638\mathrm{i}+840\mathrm{i}\right)}+\frac{e^{c\,7\mathrm{i}+d\,x\,7\mathrm{i}}\,\sin\left(7\,c+7\,d\,x\right)\,\left(m^3+15\,m^2+74\,m+120\right)\,1\mathrm{i}}{64\,d\,\left(m^4\,1\mathrm{i}+m^3\,22\mathrm{i}+m^2\,179\mathrm{i}+m\,638\mathrm{i}+840\mathrm{i}\right)}+\frac{e^{c\,7\mathrm{i}+d\,x\,7\mathrm{i}}\,\sin\left(c+d\,x\right)\,\left(5\,m^3+171\,m^2+2578\,m+29400\right)\,1\mathrm{i}}{64\,d\,\left(m^4\,1\mathrm{i}+m^3\,22\mathrm{i}+m^2\,179\mathrm{i}+m\,638\mathrm{i}+840\mathrm{i}\right)}+\frac{m\,e^{c\,7\mathrm{i}+d\,x\,7\mathrm{i}}\,\cos\left(6\,c+6\,d\,x\right)\,\left(m^2\,1\mathrm{i}+m\,9\mathrm{i}+20\mathrm{i}\right)}{32\,d\,\left(m^4\,1\mathrm{i}+m^3\,22\mathrm{i}+m^2\,179\mathrm{i}+m\,638\mathrm{i}+840\mathrm{i}\right)}+\frac{3\,m\,e^{c\,7\mathrm{i}+d\,x\,7\mathrm{i}}\,\cos\left(4\,c+4\,d\,x\right)\,\left(m^2\,1\mathrm{i}+m\,17\mathrm{i}+44\mathrm{i}\right)}{16\,d\,\left(m^4\,1\mathrm{i}+m^3\,22\mathrm{i}+m^2\,179\mathrm{i}+m\,638\mathrm{i}+840\mathrm{i}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7*(a + a*sin(c + d*x))^m,x)
```

```
[Out] exp(- c*7i - d*x*7i)*(a + a*sin(c + d*x))^m*((exp(c*7i + d*x*7i)*(m*8672i +
 m^2*936i + m^3*40i + 49152i))/(128*d*(m*638i + m^2*179i + m^3*22i + m^4*1i
 + 840i)) + (exp(c*7i + d*x*7i)*cos(2*c + 2*d*x)*(m*4824i + m^2*654i + m^3*
30i))/(64*d*(m*638i + m^2*179i + m^3*22i + m^4*1i + 840i)) + (exp(c*7i + d*
x*7i)*sin(5*c + 5*d*x)*(706*m + 123*m^2 + 5*m^3 + 1176)*1i)/(64*d*(m*638i +
 m^2*179i + m^3*22i + m^4*1i + 840i)) + (exp(c*7i + d*x*7i)*sin(3*c + 3*d*x
)*(3210*m + 279*m^2 + 9*m^3 + 5880)*1i)/(64*d*(m*638i + m^2*179i + m^3*22i
+ m^4*1i + 840i)) + (exp(c*7i + d*x*7i)*sin(7*c + 7*d*x)*(74*m + 15*m^2 + m
^3 + 120)*1i)/(64*d*(m*638i + m^2*179i + m^3*22i + m^4*1i + 840i)) + (exp(c
```

```
*7i + d*x*7i)*sin(c + d*x)*(2578*m + 171*m^2 + 5*m^3 + 29400)*1i)/(64*d*(m*
638i + m^2*179i + m^3*22i + m^4*1i + 840i)) + (m*exp(c*7i + d*x*7i)*cos(6*c
 + 6*d*x)*(m*9i + m^2*1i + 20i))/(32*d*(m*638i + m^2*179i + m^3*22i + m^4*1
i + 840i)) + (3*m*exp(c*7i + d*x*7i)*cos(4*c + 4*d*x)*(m*17i + m^2*1i + 44i
))/(16*d*(m*638i + m^2*179i + m^3*22i + m^4*1i + 840i)))
```

3.344 $\int \cos^5(c+dx)(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=81

$$\frac{4(a+a\sin(c+dx))^{3+m}}{a^3d(3+m)}-\frac{4(a+a\sin(c+dx))^{4+m}}{a^4d(4+m)}+\frac{(a+a\sin(c+dx))^{5+m}}{a^5d(5+m)}$$

```
[Out] 4*(a+a*sin(d*x+c))^(3+m)/a^3/d/(3+m)-4*(a+a*sin(d*x+c))^(4+m)/a^4/d/(4+m)+(
a+a*sin(d*x+c))^(5+m)/a^5/d/(5+m)
```

Rubi [A]

time = 0.05, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.095$, Rules used = {2746, 45}

$$\frac{(a\sin(c+dx)+a)^{m+5}}{a^5d(m+5)}-\frac{4(a\sin(c+dx)+a)^{m+4}}{a^4d(m+4)}+\frac{4(a\sin(c+dx)+a)^{m+3}}{a^3d(m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] (4*(a + a*Sin[c + d*x])^(3 + m))/(a^3*d*(3 + m)) - (4*(a + a*Sin[c + d*x])^
(4 + m))/(a^4*d*(4 + m)) + (a + a*Sin[c + d*x])^(5 + m)/(a^5*d*(5 + m))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \cos^5(c+dx)(a+a\sin(c+dx))^m\,dx &= \frac{\text{Subst}\left(\int (a-x)^2(a+x)^{2+m}\,dx,x,a\sin(c+dx)\right)}{a^5d}\\ &= \frac{\text{Subst}\left(\int \left(4a^2(a+x)^{2+m}-4a(a+x)^{3+m}+(a+x)^{4+m}\right)\,dx,x,a\sin\right.}{a^5d}\\ &= \frac{4(a+a\sin(c+dx))^{3+m}}{a^3d(3+m)}-\frac{4(a+a\sin(c+dx))^{4+m}}{a^4d(4+m)}+\frac{(a+a\sin(c}{a^5d(5}\end{aligned}$$

Mathematica [A]
time = 1.48, size = 100, normalized size = 1.23

$$-\frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^6(a(1+\sin(c+dx)))^m\left(-76-29m-3m^2+(12+7m+m^2)\cos(2(c+dx))+4(18+9m+m^2)\sin(c+dx)\right)}{2d(3+m)(4+m)(5+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] -1/2*((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6*(a*(1 + Sin[c + d*x]))^m*(-76
 - 29*m - 3*m^2 + (12 + 7*m + m^2)*Cos[2*(c + d*x)] + 4*(18 + 9*m + m^2)*Si
n[c + d*x]))/(d*(3 + m)*(4 + m)*(5 + m))
```

Maple [F]
time = 0.30, size = 0, normalized size = 0.00

$$\int \left(\cos^5(dx+c)\right)(a+a\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+a*sin(d*x+c))^m,x)
```

```
[Out] int(cos(d*x+c)^5*(a+a*sin(d*x+c))^m,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 266 vs. 2(81) = 162.
time = 0.28, size = 266, normalized size = 3.28

$$\frac{\frac{\left((m^4+10m^3+35m^2+50m+24)a^m\sin(dx+c)^5+(m^4+6m^3+11m^2+6m)a^m\sin(dx+c)^4-4(m^3+3m^2+2m)a^m\sin(dx+c)^3+12(m^2+m)a^m\sin(dx+c)^2-24a^m m\sin(dx+c)+24a^m\right)(\sin(dx+c)+1)^m}{m^5+15m^4+85m^3+225m^2+274m+120}-\frac{2\left((m^2+3m+2)a^m\sin(dx+c)^3+(m^2+m)a^m\sin(dx+c)^2-2a^m m\sin(dx+c)+2a^m\right)(\sin(dx+c)+1)^m}{m^3+6m^2+11m+6}+\frac{(a\sin(dx+c)+a)^{m+1}}{a(m+1)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] (((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*a^m*sin(d*x + c)^5 + (m^4 + 6*m^3 + 1
1*m^2 + 6*m)*a^m*sin(d*x + c)^4 - 4*(m^3 + 3*m^2 + 2*m)*a^m*sin(d*x + c)^3
+ 12*(m^2 + m)*a^m*sin(d*x + c)^2 - 24*a^m*m*sin(d*x + c) + 24*a^m)*(sin(d*
x + c) + 1)^m/(m^5 + 15*m^4 + 85*m^3 + 225*m^2 + 274*m + 120) - 2*((m^2 + 3
*m + 2)*a^m*sin(d*x + c)^3 + (m^2 + m)*a^m*sin(d*x + c)^2 - 2*a^m*m*sin(d*x
 + c) + 2*a^m)*(sin(d*x + c) + 1)^m/(m^3 + 6*m^2 + 11*m + 6) + (a*sin(d*x +
 c) + a)^(m + 1)/(a*(m + 1)))/d
```

Fricas [A]
time = 0.38, size = 102, normalized size = 1.26

$$\frac{\left((m^2+3m)\cos(dx+c)^4+8m\cos(dx+c)^2+\left((m^2+7m+12)\cos(dx+c)^4+8(m+2)\cos(dx+c)^2+32\right)\sin(dx+c)+32\right)(a\sin(dx+c)+a)^m}{dm^3+12dm^2+47dm+60d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] ((m^2 + 3*m)*cos(d*x + c)^4 + 8*m*cos(d*x + c)^2 + ((m^2 + 7*m + 12)*cos(d*
x + c)^4 + 8*(m + 2)*cos(d*x + c)^2 + 32)*sin(d*x + c) + 32)*(a*sin(d*x + c
) + a)^m/(d*m^3 + 12*d*m^2 + 47*d*m + 60*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 5534 vs. $2(68) = 136$.
time = 42.01, size = 5534, normalized size = 68.32

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+a*sin(d*x+c))**m,x)
```

```
[Out] Piecewise((x*(a*sin(c) + a)**m*cos(c)**5, Eq(d, 0)), (12*log(sin(c + d*x) +
 1)*sin(c + d*x)**4/(12*a**5*d*sin(c + d*x)**4 + 48*a**5*d*sin(c + d*x)**3
+ 72*a**5*d*sin(c + d*x)**2 + 48*a**5*d*sin(c + d*x) + 12*a**5*d) + 48*log(
sin(c + d*x) + 1)*sin(c + d*x)**3/(12*a**5*d*sin(c + d*x)**4 + 48*a**5*d*si
n(c + d*x)**3 + 72*a**5*d*sin(c + d*x)**2 + 48*a**5*d*sin(c + d*x) + 12*a**
5*d) + 72*log(sin(c + d*x) + 1)*sin(c + d*x)**2/(12*a**5*d*sin(c + d*x)**4
+ 48*a**5*d*sin(c + d*x)**3 + 72*a**5*d*sin(c + d*x)**2 + 48*a**5*d*sin(c +
 d*x) + 12*a**5*d) + 48*log(sin(c + d*x) + 1)*sin(c + d*x)/(12*a**5*d*sin(c
 + d*x)**4 + 48*a**5*d*sin(c + d*x)**3 + 72*a**5*d*sin(c + d*x)**2 + 48*a**
5*d*sin(c + d*x) + 12*a**5*d) + 12*log(sin(c + d*x) + 1)/(12*a**5*d*sin(c +
 d*x)**4 + 48*a**5*d*sin(c + d*x)**3 + 72*a**5*d*sin(c + d*x)**2 + 48*a**5*
d*sin(c + d*x) + 12*a**5*d) + 20*sin(c + d*x)**3/(12*a**5*d*sin(c + d*x)**4
 + 48*a**5*d*sin(c + d*x)**3 + 72*a**5*d*sin(c + d*x)**2 + 48*a**5*d*sin(c
+ d*x) + 12*a**5*d) + 6*sin(c + d*x)**2*cos(c + d*x)**2/(12*a**5*d*sin(c +
d*x)**4 + 48*a**5*d*sin(c + d*x)**3 + 72*a**5*d*sin(c + d*x)**2 + 48*a**5*d
*sin(c + d*x) + 12*a**5*d) + 56*sin(c + d*x)**2/(12*a**5*d*sin(c + d*x)**4
+ 48*a**5*d*sin(c + d*x)**3 + 72*a**5*d*sin(c + d*x)**2 + 48*a**5*d*sin(c +
 d*x) + 12*a**5*d) + 8*sin(c + d*x)*cos(c + d*x)**2/(12*a**5*d*sin(c + d*x)
**4 + 48*a**5*d*sin(c + d*x)**3 + 72*a**5*d*sin(c + d*x)**2 + 48*a**5*d*sin
(c + d*x) + 12*a**5*d) + 52*sin(c + d*x)/(12*a**5*d*sin(c + d*x)**4 + 48*a*
*5*d*sin(c + d*x)**3 + 72*a**5*d*sin(c + d*x)**2 + 48*a**5*d*sin(c + d*x) +
 12*a**5*d) - 3*cos(c + d*x)**4/(12*a**5*d*sin(c + d*x)**4 + 48*a**5*d*sin(
c + d*x)**3 + 72*a**5*d*sin(c + d*x)**2 + 48*a**5*d*sin(c + d*x) + 12*a**5*
d) + 2*cos(c + d*x)**2/(12*a**5*d*sin(c + d*x)**4 + 48*a**5*d*sin(c + d*x)*
*3 + 72*a**5*d*sin(c + d*x)**2 + 48*a**5*d*sin(c + d*x) + 12*a**5*d) + 16/(
12*a**5*d*sin(c + d*x)**4 + 48*a**5*d*sin(c + d*x)**3 + 72*a**5*d*sin(c + d
*x)**2 + 48*a**5*d*sin(c + d*x) + 12*a**5*d), Eq(m, -5)), (-12*log(sin(c +
d*x) + 1)*sin(c + d*x)**3/(3*a**4*d*sin(c + d*x)**3 + 9*a**4*d*sin(c + d*x)
**2 + 9*a**4*d*sin(c + d*x) + 3*a**4*d) - 36*log(sin(c + d*x) + 1)*sin(c +
d*x)**2/(3*a**4*d*sin(c + d*x)**3 + 9*a**4*d*sin(c + d*x)**2 + 9*a**4*d*sin
(c + d*x) + 3*a**4*d) - 36*log(sin(c + d*x) + 1)*sin(c + d*x)/(3*a**4*d*sin
```

```
(c + d*x)**3 + 9*a**4*d*sin(c + d*x)**2 + 9*a**4*d*sin(c + d*x) + 3*a**4*d)
 - 12*log(sin(c + d*x) + 1)/(3*a**4*d*sin(c + d*x)**3 + 9*a**4*d*sin(c + d*
x)**2 + 9*a**4*d*sin(c + d*x) + 3*a**4*d) + 8*sin(c + d*x)**4/(3*a**4*d*sin
(c + d*x)**3 + 9*a**4*d*sin(c + d*x)**2 + 9*a**4*d*sin(c + d*x) + 3*a**4*d)
 + 4*sin(c + d*x)**2*cos(c + d*x)**2/(3*a**4*d*sin(c + d*x)**3 + 9*a**4*d*s
in(c + d*x)**2 + 9*a**4*d*sin(c + d*x) + 3*a**4*d) - 52*sin(c + d*x)**2/(3*
a**4*d*sin(c + d*x)**3 + 9*a**4*d*sin(c + d*x)**2 + 9*a**4*d*sin(c + d*x) +
 3*a**4*d) + 6*sin(c + d*x)*cos(c + d*x)**2/(3*a**4*d*sin(c + d*x)**3 + 9*a
**4*d*sin(c + d*x)**2 + 9*a**4*d*sin(c + d*x) + 3*a**4*d) - 72*sin(c + d*x)
/(3*a**4*d*sin(c + d*x)**3 + 9*a**4*d*sin(c + d*x)**2 + 9*a**4*d*sin(c + d*
x) + 3*a**4*d) - cos(c + d*x)**4/(3*a**4*d*sin(c + d*x)**3 + 9*a**4*d*sin(c
 + d*x)**2 + 9*a**4*d*sin(c + d*x) + 3*a**4*d) + 2*cos(c + d*x)**2/(3*a**4*
d*sin(c + d*x)**3 + 9*a**4*d*sin(c + d*x)**2 + 9*a**4*d*sin(c + d*x) + 3*a*
*4*d) - 28/(3*a**4*d*sin(c + d*x)**3 + 9*a**4*d*sin(c + d*x)**2 + 9*a**4*d*
sin(c + d*x) + 3*a**4*d), Eq(m, -4)), (8*log(tan(c/2 + d*x/2) + 1)*tan(c/2
+ d*x/2)**4/(a**3*d*tan(c/2 + d*x/2)**4 + 2*a**3*d*tan(c/2 + d*x/2)**2 + a*
*3*d) + 16*log(tan(c/2 + d*x/2) + 1)*tan(c/2 + d*x/2)**2/(a**3*d*tan(c/2 +
d*x/2)**4 + 2*a**3*d*tan(c/2 + d*x/2)**2 + a**3*d) + 8*log(tan(c/2 + d*x/2)
 + 1)/(a**3*d*tan(c/2 + d*x/2)**4 + 2*a**3*d*tan(c/2 + d*x/2)**2 + a**3*d)
- 4*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2)**4/(a**3*d*tan(c/2 + d*x/
2)**4 + 2*a**3*d*tan(c/2 + d*x/2)**2 + a**3*d) - 8*log(tan(c/2 + d*x/2)**2
+ 1)*tan(c/2 + d*x/2)**2/(a**3*d*tan(c/2 + d*x/2)**4 + 2*a**3*d*tan(c/2 + d
*x/2)**2 + a**3*d) - 4*log(tan(c/2 + d*x/2)**2 + 1)/(a**3*d*tan(c/2 + d*x/2
)**4 + 2*a**3*d*tan(c/2 + d*x/2)**2 + a**3*d) - 6*tan(c/2 + d*x/2)**3/(a**3
*d*tan(c/2 + d*x/2)**4 + 2*a**3*d*tan(c/2 + d*x/2)**2 + a**3*d) + 2*tan(c/2
 + d*x/2)**2/(a**3*d*tan(c/2 + d*x/2)**4 + 2*a**3*d*tan(c/2 + d*x/2)**2 + a
**3*d) - 6*tan(c/2 + d*x/2)/(a**3*d*tan(c/2 + d*x/2)**4 + 2*a**3*d*tan(c/2
+ d*x/2)**2 + a**3*d), Eq(m, -3)), (6*tan(c/2 + d*x/2)**5/(3*a**2*d*tan(c/2
 + d*x/2)**6 + 9*a**2*d*tan(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2 + d*x/2)**2
+ 3*a**2*d) - 12*tan(c/2 + d*x/2)**4/(3*a**2*d*tan(c/2 + d*x/2)**6 + 9*a**2
*d*tan(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2 + d*x/2)**2 + 3*a**2*d) + 20*tan(
c/2 + d*x/2)**3/(3*a**2*d*tan(c/2 + d*x/2)**6 + 9*a**2*d*tan(c/2 + d*x/2)**
4 + 9*a**2*d*tan(c/2 + d*x/2)**2 + 3*a**2*d) - 12*tan(c/2 + d*x/2)**2/(3*a*
*2*d*tan(c/2 + d*x/2)**6 + 9*a**2*d*tan(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2
+ d*x/2)**2 + 3*a**2*d) + 6*tan(c/2 + d*x/2)/(3*a**2*d*tan(c/2 + d*x/2)**6
+ 9*a**2*d*tan(c/2 + d*x/2)**4 + 9*a**2*d*tan(c...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 294 vs. $2(81) = 162$.
time = 4.38, size = 294, normalized size = 3.63

$$\frac{(a\sin(dx+c)+a)^5(a\sin(dx+c)+a)^m m^2 - 4(a\sin(dx+c)+a)^4(a\sin(dx+c)+a)^m a m^2 + 4(a\sin(dx+c)+a)^3(a\sin(dx+c)+a)^m a^2 m^2 + 7(a\sin(dx+c)+a)^5(a\sin(dx+c)+a)^m m - 32(a\sin(dx+c)+a)^4(a\sin(dx+c)+a)^m a m + 36(a\sin(dx+c)+a)^3(a\sin(dx+c)+a)^m a^2 m + 12(a\sin(dx+c)+a)^5(a\sin(dx+c)+a)^m - 60(a\sin(dx+c)+a)^4(a\sin(dx+c)+a)^m a + 80(a\sin(dx+c)+a)^3(a\sin(dx+c)+a)^m a^2}{(a^4m^3 + 12a^4m^2 + 47a^4m + 60a^4)ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] ((a*sin(d*x + c) + a)^5*(a*sin(d*x + c) + a)^m*m^2 - 4*(a*sin(d*x + c) + a)
^4*(a*sin(d*x + c) + a)^m*a*m^2 + 4*(a*sin(d*x + c) + a)^3*(a*sin(d*x + c)
+ a)^m*a^2*m^2 + 7*(a*sin(d*x + c) + a)^5*(a*sin(d*x + c) + a)^m*m - 32*(a*
sin(d*x + c) + a)^4*(a*sin(d*x + c) + a)^m*a*m + 36*(a*sin(d*x + c) + a)^3*
(a*sin(d*x + c) + a)^m*a^2*m + 12*(a*sin(d*x + c) + a)^5*(a*sin(d*x + c) +
a)^m - 60*(a*sin(d*x + c) + a)^4*(a*sin(d*x + c) + a)^m*a + 80*(a*sin(d*x +
 c) + a)^3*(a*sin(d*x + c) + a)^m*a^2)/((a^4*m^3 + 12*a^4*m^2 + 47*a^4*m +
60*a^4)*a*d)
```

Mupad [B]
time = 1.97, size = 195, normalized size = 2.41

$$\frac{(a\,(\sin\left(c+d\,x\right)+1))^m\,\left(82\,m+600\,\sin\left(c+d\,x\right)+100\,\sin\left(3\,c+3\,d\,x\right)+12\,\sin\left(5\,c+5\,d\,x\right)+46\,m\,\sin\left(c+d\,x\right)+88\,m\,\cos\left(2\,c+2\,d\,x\right)+6\,m\,\cos\left(4\,c+4\,d\,x\right)+53\,m\,\sin\left(3\,c+3\,d\,x\right)+7\,m\,\sin\left(5\,c+5\,d\,x\right)+2\,m^2\,\sin\left(c+d\,x\right)+6\,m^2+8\,m^2\,\cos\left(2\,c+2\,d\,x\right)+2\,m^2\,\cos\left(4\,c+4\,d\,x\right)+3\,m^2\,\sin\left(3\,c+3\,d\,x\right)+m^2\,\sin\left(5\,c+5\,d\,x\right)+512\right)}{16\,d\,\left(m^3+12\,m^2+47\,m+60\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(a + a*sin(c + d*x))^m,x)
```

```
[Out] ((a*(sin(c + d*x) + 1))^m*(82*m + 600*sin(c + d*x) + 100*sin(3*c + 3*d*x) +
 12*sin(5*c + 5*d*x) + 46*m*sin(c + d*x) + 88*m*cos(2*c + 2*d*x) + 6*m*cos(
4*c + 4*d*x) + 53*m*sin(3*c + 3*d*x) + 7*m*sin(5*c + 5*d*x) + 2*m^2*sin(c +
 d*x) + 6*m^2 + 8*m^2*cos(2*c + 2*d*x) + 2*m^2*cos(4*c + 4*d*x) + 3*m^2*sin
(3*c + 3*d*x) + m^2*sin(5*c + 5*d*x) + 512))/(16*d*(47*m + 12*m^2 + m^3 + 6
0))
```

3.345 $\int \cos^3(c+dx)(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=55

$$\frac{2(a+a\sin(c+dx))^{2+m}}{a^2d(2+m)}-\frac{(a+a\sin(c+dx))^{3+m}}{a^3d(3+m)}$$

```
[Out] 2*(a+a*sin(d*x+c))^(2+m)/a^2/d/(2+m)-(a+a*sin(d*x+c))^(3+m)/a^3/d/(3+m)
```

Rubi [A]
time = 0.04, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2746, 45}

$$\frac{2(a\sin(c+dx)+a)^{m+2}}{a^2d(m+2)}-\frac{(a\sin(c+dx)+a)^{m+3}}{a^3d(m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] (2*(a + a*Sin[c + d*x])^(2 + m))/(a^2*d*(2 + m)) - (a + a*Sin[c + d*x])^(3
+ m)/(a^3*d*(3 + m))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \cos^3(c+dx)(a+a\sin(c+dx))^m\,dx&=\frac{\text{Subst}\left(\int (a-x)(a+x)^{1+m}\,dx,x,a\sin(c+dx)\right)}{a^3d}\\&=\frac{\text{Subst}\left(\int \left(2a(a+x)^{1+m}-(a+x)^{2+m}\right)\,dx,x,a\sin(c+dx)\right)}{a^3d}\\&=\frac{2(a+a\sin(c+dx))^{2+m}}{a^2d(2+m)}-\frac{(a+a\sin(c+dx))^{3+m}}{a^3d(3+m)}\end{aligned}$$

Mathematica [A]
time = 0.21, size = 65, normalized size = 1.18

$$-\frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^4(a(1+\sin(c+dx)))^m(-4-m+(2+m)\sin(c+dx))}{d(2+m)(3+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Sin[c + d*x])^m,x]
[Out] -(((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4*(a*(1 + Sin[c + d*x]))^m*(-4 - m
 + (2 + m)*Sin[c + d*x]))/(d*(2 + m)*(3 + m)))
```

Maple [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int \left(\cos^3(dx+c)\right)(a+a\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*sin(d*x+c))^m,x)
[Out] int(cos(d*x+c)^3*(a+a*sin(d*x+c))^m,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(55) = 110$.
time = 0.28, size = 111, normalized size = 2.02

$$-\frac{\frac{\left((m^2+3m+2)a^m\sin(dx+c)^3+(m^2+m)a^m\sin(dx+c)^2-2a^m m\sin(dx+c)+2a^m\right)(\sin(dx+c)+1)^m}{m^3+6m^2+11m+6}-\frac{(a\sin(dx+c)+a)^{m+1}}{a(m+1)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
[Out] -(((m^2 + 3*m + 2)*a^m*sin(d*x + c)^3 + (m^2 + m)*a^m*sin(d*x + c)^2 - 2*a^
m*m*sin(d*x + c) + 2*a^m)*(sin(d*x + c) + 1)^m/(m^3 + 6*m^2 + 11*m + 6) - (
a*sin(d*x + c) + a)^(m + 1)/(a*(m + 1)))/d
```

Fricas [A]
time = 0.39, size = 61, normalized size = 1.11

$$\frac{\left(m\cos(dx+c)^2+\left((m+2)\cos(dx+c)^2+4\right)\sin(dx+c)+4\right)(a\sin(dx+c)+a)^m}{dm^2+5\,dm+6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] (m*cos(d*x + c)^2 + ((m + 2)*cos(d*x + c)^2 + 4)*sin(d*x + c) + 4)*(a*sin(d
*x + c) + a)^m/(d*m^2 + 5*d*m + 6*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1114 vs. $2(44) = 88$.
time = 3.81, size = 1114, normalized size = 20.25

$$\begin{cases} x(a\sin(c)+a)^m\cos^3(c) & \text{for } d=0 \\ -\frac{2\log(\sin(c+dx)+1)\sin^2(c+dx)}{2a^3d\sin^2(c+dx)+4a^3d\sin(c+dx)+2a^3d} - \frac{4\log(\sin(c+dx)+1)\sin(c+dx)}{2a^3d\sin^2(c+dx)+4a^3d\sin(c+dx)+2a^3d} - \frac{2\log(\sin(c+dx)+1)}{2a^3d\sin^2(c+dx)+4a^3d\sin(c+dx)+2a^3d} - \frac{2\sin(c+dx)}{2a^3d\sin^2(c+dx)+4a^3d\sin(c+dx)+2a^3d} - \frac{\cos^2(c+dx)}{2a^3d\sin^2(c+dx)+4a^3d\sin(c+dx)+2a^3d} - \frac{2}{2a^3d\sin^2(c+dx)+4a^3d\sin(c+dx)+2a^3d} & \text{for } m=-3 \\ \frac{2\log(\sin(c+dx)+1)\sin(c+dx)}{a^2d\sin(c+dx)+a^2d} + \frac{2\log(\sin(c+dx)+1)}{a^2d\sin(c+dx)+a^2d} - \frac{2\sin^2(c+dx)}{a^2d\sin(c+dx)+a^2d} - \frac{\cos^2(c+dx)}{a^2d\sin(c+dx)+a^2d} + \frac{2}{a^2d\sin(c+dx)+a^2d} & \text{for } m=-2 \\ \frac{2\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad} - \frac{2\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad} + \frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad} & \text{for } m=-1 \\ \frac{m^2(a\sin(c+dx)+a)^m\sin(c+dx)\cos^2(c+dx)}{dm^3+6dm^2+11dm+6d} + \frac{m^2(a\sin(c+dx)+a)^m\cos^2(c+dx)}{dm^3+6dm^2+11dm+6d} + \frac{2m(a\sin(c+dx)+a)^m\sin^3(c+dx)}{dm^3+6dm^2+11dm+6d} + \frac{4m(a\sin(c+dx)+a)^m\sin^2(c+dx)}{dm^3+6dm^2+11dm+6d} + \frac{5m(a\sin(c+dx)+a)^m\sin(c+dx)\cos^2(c+dx)}{dm^3+6dm^2+11dm+6d} + \frac{2m(a\sin(c+dx)+a)^m\sin(c+dx)}{dm^3+6dm^2+11dm+6d} + \frac{5m(a\sin(c+dx)+a)^m\cos^2(c+dx)}{dm^3+6dm^2+11dm+6d} + \frac{4(a\sin(c+dx)+a)^m\sin^3(c+dx)}{dm^3+6dm^2+11dm+6d} + \frac{6(a\sin(c+dx)+a)^m\sin^2(c+dx)}{dm^3+6dm^2+11dm+6d} + \frac{6(a\sin(c+dx)+a)^m\sin(c+dx)\cos^2(c+dx)}{dm^3+6dm^2+11dm+6d} + \frac{6(a\sin(c+dx)+a)^m\cos^2(c+dx)}{dm^3+6dm^2+11dm+6d} - \frac{2(a\sin(c+dx)+a)^m}{dm^3+6dm^2+11dm+6d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*sin(d*x+c))**m,x)
```

```
[Out] Piecewise((x*(a*sin(c) + a)**m*cos(c)**3, Eq(d, 0)), (-2*log(sin(c + d*x) +
 1)*sin(c + d*x)**2/(2*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x) + 2*a
**3*d) - 4*log(sin(c + d*x) + 1)*sin(c + d*x)/(2*a**3*d*sin(c + d*x)**2 + 4
*a**3*d*sin(c + d*x) + 2*a**3*d) - 2*log(sin(c + d*x) + 1)/(2*a**3*d*sin(c
+ d*x)**2 + 4*a**3*d*sin(c + d*x) + 2*a**3*d) - 2*sin(c + d*x)/(2*a**3*d*si
n(c + d*x)**2 + 4*a**3*d*sin(c + d*x) + 2*a**3*d) - cos(c + d*x)**2/(2*a**3
*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x) + 2*a**3*d) - 2/(2*a**3*d*sin(c
+ d*x)**2 + 4*a**3*d*sin(c + d*x) + 2*a**3*d), Eq(m, -3)), (2*log(sin(c + d
*x) + 1)*sin(c + d*x)/(a**2*d*sin(c + d*x) + a**2*d) + 2*log(sin(c + d*x) +
 1)/(a**2*d*sin(c + d*x) + a**2*d) - 2*sin(c + d*x)**2/(a**2*d*sin(c + d*x)
 + a**2*d) - cos(c + d*x)**2/(a**2*d*sin(c + d*x) + a**2*d) + 2/(a**2*d*sin
(c + d*x) + a**2*d), Eq(m, -2)), (2*tan(c/2 + d*x/2)**3/(a*d*tan(c/2 + d*x/
2)**4 + 2*a*d*tan(c/2 + d*x/2)**2 + a*d) - 2*tan(c/2 + d*x/2)**2/(a*d*tan(c
/2 + d*x/2)**4 + 2*a*d*tan(c/2 + d*x/2)**2 + a*d) + 2*tan(c/2 + d*x/2)/(a*d
*tan(c/2 + d*x/2)**4 + 2*a*d*tan(c/2 + d*x/2)**2 + a*d), Eq(m, -1)), (m**2*
(a*sin(c + d*x) + a)**m*sin(c + d*x)*cos(c + d*x)**2/(d*m**3 + 6*d*m**2 + 1
1*d*m + 6*d) + m**2*(a*sin(c + d*x) + a)**m*cos(c + d*x)**2/(d*m**3 + 6*d*m
**2 + 11*d*m + 6*d) + 2*m*(a*sin(c + d*x) + a)**m*sin(c + d*x)**3/(d*m**3 +
 6*d*m**2 + 11*d*m + 6*d) + 4*m*(a*sin(c + d*x) + a)**m*sin(c + d*x)**2/(d*
m**3 + 6*d*m**2 + 11*d*m + 6*d) + 5*m*(a*sin(c + d*x) + a)**m*sin(c + d*x)*
cos(c + d*x)**2/(d*m**3 + 6*d*m**2 + 11*d*m + 6*d) + 2*m*(a*sin(c + d*x) +
a)**m*sin(c + d*x)/(d*m**3 + 6*d*m**2 + 11*d*m + 6*d) + 5*m*(a*sin(c + d*x)
 + a)**m*cos(c + d*x)**2/(d*m**3 + 6*d*m**2 + 11*d*m + 6*d) + 4*(a*sin(c +
d*x) + a)**m*sin(c + d*x)**3/(d*m**3 + 6*d*m**2 + 11*d*m + 6*d) + 6*(a*sin(
c + d*x) + a)**m*sin(c + d*x)**2/(d*m**3 + 6*d*m**2 + 11*d*m + 6*d) + 6*(a*
sin(c + d*x) + a)**m*sin(c + d*x)*cos(c + d*x)**2/(d*m**3 + 6*d*m**2 + 11*d
*m + 6*d) + 6*(a*sin(c + d*x) + a)**m*cos(c + d*x)**2/(d*m**3 + 6*d*m**2 +
11*d*m + 6*d) - 2*(a*sin(c + d*x) + a)**m/(d*m**3 + 6*d*m**2 + 11*d*m + 6*d
), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(55) = 110$.

time = 5.65, size = 137, normalized size = 2.49

$$-\frac{(a \sin(dx+c)+a)^3 (a \sin(dx+c)+a)^m m - 2 (a \sin(dx+c)+a)^2 (a \sin(dx+c)+a)^m am + 2 (a \sin(dx+c)+a)^3 (a \sin(dx+c)+a)^m - 6 (a \sin(dx+c)+a)^2 (a \sin(dx+c)+a)^m a}{(a^2m^2+5\,a^2m+6\,a^2)ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] -((a*sin(d*x + c) + a)^3*(a*sin(d*x + c) + a)^m*m - 2*(a*sin(d*x + c) + a)^
2*(a*sin(d*x + c) + a)^m*a*m + 2*(a*sin(d*x + c) + a)^3*(a*sin(d*x + c) + a
)^m - 6*(a*sin(d*x + c) + a)^2*(a*sin(d*x + c) + a)^m*a)/((a^2*m^2 + 5*a^2*
m + 6*a^2)*a*d)
```

Mupad [B]
time = 0.73, size = 85, normalized size = 1.55

$$\frac{(a\,(\sin(c+d\,x)+1))^m\,\left(2\,m+18\,\sin(c+d\,x)+2\,\sin(3\,c+3\,d\,x)+m\,\sin(c+d\,x)-2\,m\,\left(2\sin(c+d\,x)^2-1\right)+m\,\sin(3\,c+3\,d\,x)+16\right)}{4\,d\,\left(m^2+5\,m+6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a + a*sin(c + d*x))^m,x)
```

```
[Out] ((a*(sin(c + d*x) + 1))^m*(2*m + 18*sin(c + d*x) + 2*sin(3*c + 3*d*x) + m*s
in(c + d*x) - 2*m*(2*sin(c + d*x)^2 - 1) + m*sin(3*c + 3*d*x) + 16))/(4*d*(
5*m + m^2 + 6))
```

3.346 $\int \cos(c+dx)(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=26

$$\frac{(a+a\sin(c+dx))^{1+m}}{ad(1+m)}$$

[Out] (a+a*sin(d*x+c))^(1+m)/a/d/(1+m)

Rubi [A]
time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2746, 32}

$$\frac{(a\sin(c+dx)+a)^{m+1}}{ad(m+1)}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]*(a + a*Sin[c + d*x])^m,x]

[Out] (a + a*Sin[c + d*x])^(1 + m)/(a*d*(1 + m))

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a+a\sin(c+dx))^m\,dx &= \frac{\text{Subst}\left(\int (a+x)^m\,dx, x, a\sin(c+dx)\right)}{ad}\\ &= \frac{(a+a\sin(c+dx))^{1+m}}{ad(1+m)}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 26, normalized size = 1.00

$$\frac{(a(1+\sin(c+dx)))^{1+m}}{ad(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] (a*(1 + Sin[c + d*x]))^(1 + m)/(a*d*(1 + m))
```

Maple [A]
time = 3.10, size = 27, normalized size = 1.04

method	result	si
derivativedivides	$\frac{(a+a\sin(dx+c))^{1+m}}{ad(1+m)}$	27
default	$\frac{(a+a\sin(dx+c))^{1+m}}{ad(1+m)}$	27
norman	$\frac{\frac{e^{m\ln\left(a+\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)}}{d(1+m)}+\frac{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)e^{m\ln\left(a+\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)}}{d(1+m)}+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)e^{m\ln\left(a+\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)}}{d(1+m)}}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}$	10
risch	Expression too large to display	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*sin(d*x+c))^m,x,method=_RETURNVERBOSE)
```

```
[Out] (a+a*sin(d*x+c))^(1+m)/a/d/(1+m)
```

Maxima [A]
time = 0.28, size = 26, normalized size = 1.00

$$\frac{(a\sin(dx+c)+a)^{m+1}}{ad(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] (a*sin(d*x + c) + a)^(m + 1)/(a*d*(m + 1))
```

Fricas [A]
time = 0.36, size = 28, normalized size = 1.08

$$\frac{(a\sin(dx+c)+a)^m(\sin(dx+c)+1)}{dm+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] (a*sin(d*x + c) + a)^m*(sin(d*x + c) + 1)/(d*m + d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(19) = 38$.
time = 0.46, size = 80, normalized size = 3.08

$$\begin{cases} \frac{x\cos(c)}{a\sin(c)+a} & \text{for } d = 0 \wedge m = -1 \\ x(a\sin(c)+a)^m\cos(c) & \text{for } d = 0 \\ \frac{\log(\sin(c+dx)+1)}{ad} & \text{for } m = -1 \\ \frac{(a\sin(c+dx)+a)^m\sin(c+dx)}{dm+d} + \frac{(a\sin(c+dx)+a)^m}{dm+d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))**m,x)
```

```
[Out] Piecewise((x*cos(c)/(a*sin(c) + a), Eq(d, 0) & Eq(m, -1)), (x*(a*sin(c) + a
)**m*cos(c), Eq(d, 0)), (log(sin(c + d*x) + 1)/(a*d), Eq(m, -1)), ((a*sin(c
 + d*x) + a)**m*sin(c + d*x)/(d*m + d) + (a*sin(c + d*x) + a)**m/(d*m + d),
 True))
```

Giac [A]
time = 5.62, size = 26, normalized size = 1.00

$$\frac{(a\sin(dx+c)+a)^{m+1}}{ad(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] (a*sin(d*x + c) + a)^(m + 1)/(a*d*(m + 1))
```

Mupad [B]
time = 0.22, size = 29, normalized size = 1.12

$$\frac{(a(\sin(c+d\,x)+1))^m\,(\sin(c+d\,x)+1)}{d\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + a*sin(c + d*x))^m,x)
```

```
[Out] ((a*(sin(c + d*x) + 1))^m*(sin(c + d*x) + 1))/(d*(m + 1))
```

3.347 $\int \sec(c+dx)(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=40

$$\frac{{}_2F_1\left(1,m;1+m;\frac{1}{2}(1+\sin(c+dx))\right)(a+a\sin(c+dx))^m}{2dm}$$

```
[Out] 1/2*hypergeom([1, m],[1+m],1/2+1/2*sin(d*x+c))*(a+a*sin(d*x+c))^m/d/m
```

Rubi [A]
time = 0.03, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2746, 70}

$$\frac{(a\sin(c+dx)+a)^m\,{}_2F_1\left(1,m;m+1;\frac{1}{2}(\sin(c+dx)+1)\right)}{2dm}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] (Hypergeometric2F1[1, m, 1 + m, (1 + Sin[c + d*x])/2]*(a + a*Sin[c + d*x])^
m)/(2*d*m)
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \sec(c+dx)(a+a\sin(c+dx))^m\,dx &= \frac{a\text{Subst}\left(\int \frac{(a+x)^{-1+m}}{a-x}\,dx,x,a\sin(c+dx)\right)}{d}\\ &= \frac{{}_2F_1\left(1,m;1+m;\frac{1}{2}(1+\sin(c+dx))\right)(a+a\sin(c+dx))^m}{2dm}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 63, normalized size = 1.58

$$\frac{(a(1+\sin(c+dx)))^m\left(2(1+m)+m\,{}_2F_1\left(1,1+m;2+m;\frac{1}{2}(1+\sin(c+dx))\right)(1+\sin(c+dx))\right)}{4dm(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sin[c + d*x])^m,x]
[Out] ((a*(1 + Sin[c + d*x]))^m*(2*(1 + m) + m*Hypergeometric2F1[1, 1 + m, 2 + m,
 (1 + Sin[c + d*x])/2]*(1 + Sin[c + d*x])))/(4*d*m*(1 + m))
```

Maple [F]
time = 0.16, size = 0, normalized size = 0.00

$$\int \sec\left(dx+c\right)\left(a+a\sin\left(dx+c\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+a*sin(d*x+c))^m,x)
[Out] int(sec(d*x+c)*(a+a*sin(d*x+c))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
[Out] integrate((a*sin(d*x + c) + a)^m*sec(d*x + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
[Out] integral((a*sin(d*x + c) + a)^m*sec(d*x + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sin\left(c+dx\right)+1\right)\right)^m\sec\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))**m,x)
```

```
[Out] Integral((a*(sin(c + d*x) + 1))**m*sec(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^m*sec(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a+a\,\sin\left(c+d\,x\right))^m}{\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^m/cos(c + d*x),x)
```

```
[Out] int((a + a*sin(c + d*x))^m/cos(c + d*x), x)
```

3.348 $\int \sec^3(c+dx)(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=47

$$-\frac{a\,_2F_1\left(2,-1+m;m;\frac{1}{2}(1+\sin(c+dx))\right)(a+a\sin(c+dx))^{-1+m}}{4d(1-m)}$$

[Out] -1/4*a*hypergeom([2, -1+m],[m],1/2+1/2*sin(d*x+c))*(a+a*sin(d*x+c))^(-1+m)/d/(1-m)

Rubi [A]
time = 0.04, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2746, 70}

$$-\frac{a(a\sin(c+dx)+a)^{m-1}\,_2F_1\left(2,m-1;m;\frac{1}{2}(\sin(c+dx)+1)\right)}{4d(1-m)}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d*x]^3*(a + a*Sin[c + d*x])^m,x]

[Out] -1/4*(a*Hypergeometric2F1[2, -1 + m, m, (1 + Sin[c + d*x])/2]*(a + a*Sin[c + d*x])^(-1 + m))/(d*(1 - m))

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \sec^3(c+dx)(a+a\sin(c+dx))^m\,dx &= \frac{a^3\text{Subst}\left(\int \frac{(a+x)^{-2+m}}{(a-x)^2}\,dx,x,a\sin(c+dx)\right)}{d}\\ &= -\frac{a\,_2F_1\left(2,-1+m;m;\frac{1}{2}(1+\sin(c+dx))\right)(a+a\sin(c+dx))^{-1+m}}{4d(1-m)}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 37.80, size = 10814, normalized size = 230.09

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] Result too large to show
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int \left(\sec^3\left(dx+c\right)\right)\left(a+a\sin\left(dx+c\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+a*sin(d*x+c))^m,x)
```

```
[Out] int(sec(d*x+c)^3*(a+a*sin(d*x+c))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^m*sec(d*x + c)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sin(d*x + c) + a)^m*sec(d*x + c)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sin\left(c+dx\right)+1\right)\right)^m\sec^3\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+a*sin(d*x+c))**m,x)
```

```
[Out] Integral((a*(sin(c + d*x) + 1))**m*sec(c + d*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^m*sec(d*x + c)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a+a\,\sin\left(c+d\,x\right))^m}{{\cos\left(c+d\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^m/cos(c + d*x)^3,x)
```

```
[Out] int((a + a*sin(c + d*x))^m/cos(c + d*x)^3, x)
```

3.349 $\int \sec^5(c+dx)(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=51

$$-\frac{a^2\,{}_2F_1\left(3,-2+m;-1+m;\frac{1}{2}(1+\sin(c+dx))\right)(a+a\sin(c+dx))^{-2+m}}{8d(2-m)}$$

```
[Out] -1/8*a^2*hypergeom([3, -2+m],[-1+m],1/2+1/2*sin(d*x+c))*(a+a*sin(d*x+c))^(-
2+m)/d/(2-m)
```

Rubi [A]
time = 0.04, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2746, 70}

$$-\frac{a^2(a\sin(c+dx)+a)^{m-2}\,{}_2F_1\left(3,m-2;m-1;\frac{1}{2}(\sin(c+dx)+1)\right)}{8d(2-m)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] -1/8*(a^2*Hypergeometric2F1[3, -2 + m, -1 + m, (1 + Sin[c + d*x])/2]*(a + a
*Sin[c + d*x])^(-2 + m))/(d*(2 - m))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\int \sec^5(c+dx)(a+a\sin(c+dx))^m\,dx = \frac{a^5\text{Subst}\left(\int \frac{(a+x)^{-3+m}}{(a-x)^3}\,dx,x,a\sin(c+dx)\right)}{d}$$

$$= -\frac{a^2\,{}_2F_1\left(3,-2+m;-1+m;\frac{1}{2}(1+\sin(c+dx))\right)(a+a\sin(c+dx))^{-2+m}}{8d(2-m)}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 47.34, size = 26029, normalized size = 510.37

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^5*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] Result too large to show
```

Maple [F]
time = 0.14, size = 0, normalized size = 0.00

$$\int \left(\sec^{5}\left(dx+c\right)\right) \left(a+a\sin\left(dx+c\right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(a+a*sin(d*x+c))^m,x)
```

```
[Out] int(sec(d*x+c)^5*(a+a*sin(d*x+c))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^m*sec(d*x + c)^5, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sin(d*x + c) + a)^m*sec(d*x + c)^5, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a+a*sin(d*x+c))**m,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^m*sec(d*x + c)^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a + a \sin(c + d x))^m}{\cos(c + d x)^5} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^m/cos(c + d*x)^5,x)
```

```
[Out] int((a + a*sin(c + d*x))^m/cos(c + d*x)^5, x)
```

3.350 $\int \cos^4(c+dx)(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=83

$$-\frac{2^{\frac{5}{2}+m}a^2\cos^5(c+dx)\,{}_2F_1\left(\frac{5}{2},-\frac{3}{2}-m;\frac{7}{2};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{-\frac{1}{2}-m}(a+a\sin(c+dx))^{-2+m}}{5d}$$

```
[Out] -1/5*2^(5/2+m)*a^2*cos(d*x+c)^5*hypergeom([5/2, -3/2-m],[7/2],1/2-1/2*sin(d
*x+c))*(1+sin(d*x+c))^(-1/2-m)*(a+a*sin(d*x+c))^(-2+m)/d
```

Rubi [A]
time = 0.05, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2768, 72, 71}

$$-\frac{a^2 2^{m+\frac{5}{2}}\cos^5(c+dx)(\sin(c+dx)+1)^{-m-\frac{1}{2}}(a\sin(c+dx)+a)^{m-2}\,{}_2F_1\left(\frac{5}{2},-m-\frac{3}{2};\frac{7}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] -1/5*(2^(5/2 + m)*a^2*Cos[c + d*x]^5*Hypergeometric2F1[5/2, -3/2 - m, 7/2,
(1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^(-1/2 - m)*(a + a*Sin[c + d*x])^(-
2 + m))/d
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
```

```
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int \cos^4(c+dx)(a+a\sin(c+dx))^m\,dx = \frac{(a^2\cos^5(c+dx))\,\text{Subst}\left(\int (a-ax)^{3/2}(a+ax)^{\frac{3}{2}+m}\,dx, x, \sin(c+d\right.}{d(a-a\sin(c+dx))^{5/2}(a+a\sin(c+dx))^{5/2}}$$

$$= \frac{\left(2^{\frac{3}{2}+m}a^3\cos^5(c+dx)(a+a\sin(c+dx))^{-2+m}\left(\frac{a+a\sin(c+dx)}{a}\right)^{-\frac{1}{2}-m}\right.}{d(a-a\sin(c+d}$$

$$= -\frac{2^{\frac{5}{2}+m}a^2\cos^5(c+dx)\,{}_2F_1\left(\frac{5}{2},-\frac{3}{2}-m;\frac{7}{2};\frac{1}{2}(1-\sin(c+dx))\right)(1+}{5d}$$

Mathematica [A]
time = 0.12, size = 78, normalized size = 0.94

$$-\frac{2^{\frac{5}{2}+m}\cos^5(c+dx)\,{}_2F_1\left(\frac{5}{2},-\frac{3}{2}-m;\frac{7}{2};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{-\frac{5}{2}-m}(a(1+\sin(c+dx)))^m}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + a*Sin[c + d*x])^m,x]
[Out] -1/5*(2^(5/2 + m)*Cos[c + d*x]^5*Hypergeometric2F1[5/2, -3/2 - m, 7/2, (1 -
 Sin[c + d*x])/2]*(1 + Sin[c + d*x])^(-5/2 - m)*(a*(1 + Sin[c + d*x]))^m)/d
```

Maple [F]
time = 0.26, size = 0, normalized size = 0.00

$$\int \left(\cos^4(dx+c)\right)(a+a\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+a*sin(d*x+c))^m,x)
[Out] int(cos(d*x+c)^4*(a+a*sin(d*x+c))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^m*cos(d*x + c)^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
[Out] integral((a*sin(d*x + c) + a)^m*cos(d*x + c)^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sin(c+dx)+1))^m \cos^4(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+a*sin(d*x+c))**m,x)
[Out] Integral((a*(sin(c + d*x) + 1))**m*cos(c + d*x)**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))^m,x, algorithm="giac")
[Out] integrate((a*sin(d*x + c) + a)^m*cos(d*x + c)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+d\,x)^4\,(a+a\,\sin(c+d\,x))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(a + a*sin(c + d*x))^m,x)
[Out] int(cos(c + d*x)^4*(a + a*sin(c + d*x))^m, x)
```

3.351 $\int \cos^2(c+dx)(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=81

$$-\frac{2^{\frac{3}{2}+m}a\cos^3(c+dx)\,{}_2F_1\left(\frac{3}{2},-\frac{1}{2}-m;\frac{5}{2};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{-\frac{1}{2}-m}(a+a\sin(c+dx))^{-1+m}}{3d}$$

```
[Out] -1/3*2^(3/2+m)*a*cos(d*x+c)^3*hypergeom([3/2, -1/2-m],[5/2],1/2-1/2*sin(d*x
+c))*(1+sin(d*x+c))^(-1/2-m)*(a+a*sin(d*x+c))^(-1+m)/d
```

Rubi [A]
time = 0.05, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2768, 72, 71}

$$-\frac{a2^{m+\frac{3}{2}}\cos^3(c+dx)(\sin(c+dx)+1)^{-m-\frac{1}{2}}(a\sin(c+dx)+a)^{m-1}\,{}_2F_1\left(\frac{3}{2},-m-\frac{1}{2};\frac{5}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] -1/3*(2^(3/2 + m)*a*Cos[c + d*x]^3*Hypergeometric2F1[3/2, -1/2 - m, 5/2, (1
 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^(-1/2 - m)*(a + a*Sin[c + d*x])^(-1
+ m))/d
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
```

```
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\sin(c+dx))^m\,dx = \frac{\left(a^2\cos^3(c+dx)\right)\mathrm{Subst}\left(\int \sqrt{a-ax}\,(a+ax)^{\frac{1}{2}+m}\,dx,x,\sin(c+dx)\right)}{d(a-a\sin(c+dx))^{3/2}(a+a\sin(c+dx))^{3/2}}$$

$$= \frac{\left(2^{\frac{1}{2}+m}a^2\cos^3(c+dx)(a+a\sin(c+dx))^{-1+m}\left(\frac{a+a\sin(c+dx)}{a}\right)^{-\frac{1}{2}-m}\right.}{d(a-a\sin(c+dx)}$$

$$= -\frac{2^{\frac{3}{2}+m}a\cos^3(c+dx)\,{}_2F_1\left(\frac{3}{2},-\frac{1}{2}-m;\frac{5}{2};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin}{3d}$$

Mathematica [A]
time = 0.10, size = 78, normalized size = 0.96

$$-\frac{2^{\frac{3}{2}+m}\cos^3(c+dx)\,{}_2F_1\left(\frac{3}{2},-\frac{1}{2}-m;\frac{5}{2};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{-\frac{3}{2}-m}(a(1+\sin(c+dx)))^m}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Sin[c + d*x])^m,x]
[Out] -1/3*(2^(3/2 + m)*Cos[c + d*x]^3*Hypergeometric2F1[3/2, -1/2 - m, 5/2, (1 -
 Sin[c + d*x])/2]*(1 + Sin[c + d*x])^(-3/2 - m)*(a*(1 + Sin[c + d*x]))^m)/d
```

Maple [F]
time = 0.19, size = 0, normalized size = 0.00

$$\int \left(\cos^2(dx+c)\right)(a+a\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*sin(d*x+c))^m,x)
[Out] int(cos(d*x+c)^2*(a+a*sin(d*x+c))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^m*cos(d*x + c)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
[Out] integral((a*sin(d*x + c) + a)^m*cos(d*x + c)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sin(c+dx)+1))^m \cos^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*sin(d*x+c))**m,x)
[Out] Integral((a*(sin(c + d*x) + 1))**m*cos(c + d*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^m,x, algorithm="giac")
[Out] integrate((a*sin(d*x + c) + a)^m*cos(d*x + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+d\,x)^2\,(a+a\,\sin(c+d\,x))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a + a*sin(c + d*x))^m,x)
[Out] int(cos(c + d*x)^2*(a + a*sin(c + d*x))^m, x)
```

3.352 $\int \sec^2(c+dx)(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=73

$$\frac{2^{-\frac{1}{2}+m}\,{}_2F_1\left(-\frac{1}{2},\frac{3}{2}-m;\frac{1}{2};\frac{1}{2}(1-\sin(c+dx))\right)\sec(c+dx)(1+\sin(c+dx))^{\frac{1}{2}-m}(a+a\sin(c+dx))^m}{d}$$

```
[Out] 2^(-1/2+m)*hypergeom([-1/2, 3/2-m],[1/2],1/2-1/2*sin(d*x+c))*sec(d*x+c)*(1+
sin(d*x+c))^(1/2-m)*(a+a*sin(d*x+c))^m/d
```

Rubi [A]
time = 0.06, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2768, 72, 71}

$$\frac{2^{m-\frac{1}{2}}\sec(c+dx)(\sin(c+dx)+1)^{\frac{1}{2}-m}(a\sin(c+dx)+a)^m\,{}_2F_1\left(-\frac{1}{2},\frac{3}{2}-m;\frac{1}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] (2^(-1/2 + m)*Hypergeometric2F1[-1/2, 3/2 - m, 1/2, (1 - Sin[c + d*x])/2]*S
ec[c + d*x]*(1 + Sin[c + d*x])^(1/2 - m)*(a + a*Sin[c + d*x])^m)/d
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
```

```
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int \sec^2(c+dx)(a+a\sin(c+dx))^m\,dx = \frac{\left(a^2\sec(c+dx)\sqrt{a-a\sin(c+dx)}\,\sqrt{a+a\sin(c+dx)}\right)\text{Subst}\Big(}{d}$$

$$= \frac{\Big(2^{-\frac{3}{2}+m}a\sec(c+dx)\sqrt{a-a\sin(c+dx)}\,(a+a\sin(c+dx))^m\,\Big(\frac{a}{}}{d}$$

$$= \frac{2^{-\frac{1}{2}+m}\,{}_2F_1\left(-\frac{1}{2},\frac{3}{2}-m;\frac{1}{2};\frac{1}{2}(1-\sin(c+dx))\right)\sec(c+dx)(1+\sin(}{d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 6.37, size = 3917, normalized size = 53.66

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] -1/4*((Cos[(-c + Pi/2 - d*x)/4]^2)^(2*m)*Cot[(-c + Pi/2 - d*x)/4]*(a + a*Si
n[c + d*x])^m*(-(AppellF1[-1/2, -2*m, 2*m, 1/2, Tan[(-c + Pi/2 - d*x)/4]^2,
 -Tan[(-c + Pi/2 - d*x)/4]^2]*(Sec[(-c + Pi/2 - d*x)/4]^2)^(2*m)) + (3*Appe
llF1[1/2, -2*m, 2*m, 3/2, Tan[(-c + Pi/2 - d*x)/4]^2, -Tan[(-c + Pi/2 - d*x
)/4]^2]*Tan[(-c + Pi/2 - d*x)/4]^2*(1 - Tan[(-c + Pi/2 - d*x)/4]^2)^(2*m))/
(3*AppellF1[1/2, -2*m, 2*m, 3/2, Tan[(-c + Pi/2 - d*x)/4]^2, -Tan[(-c + Pi/
2 - d*x)/4]^2] - 4*m*(AppellF1[3/2, 1 - 2*m, 2*m, 5/2, Tan[(-c + Pi/2 - d*x
)/4]^2, -Tan[(-c + Pi/2 - d*x)/4]^2] + AppellF1[3/2, -2*m, 1 + 2*m, 5/2, Ta
n[(-c + Pi/2 - d*x)/4]^2, -Tan[(-c + Pi/2 - d*x)/4]^2])*Tan[(-c + Pi/2 - d*
x)/4]^2)))/(d*(Cos[Pi/4 + (c - Pi/2 + d*x)/2] - Sin[Pi/4 + (c - Pi/2 + d*x)
/2])^2*(-1/2*(m*(Cos[(-c + Pi/2 - d*x)/4]^2)^(2*m)*(-(AppellF1[-1/2, -2*m,
2*m, 1/2, Tan[(-c + Pi/2 - d*x)/4]^2, -Tan[(-c + Pi/2 - d*x)/4]^2]*(Sec[(-c
 + Pi/2 - d*x)/4]^2)^(2*m)) + (3*AppellF1[1/2, -2*m, 2*m, 3/2, Tan[(-c + Pi
/2 - d*x)/4]^2, -Tan[(-c + Pi/2 - d*x)/4]^2]*Tan[(-c + Pi/2 - d*x)/4]^2*(1
- Tan[(-c + Pi/2 - d*x)/4]^2)^(2*m))/(3*AppellF1[1/2, -2*m, 2*m, 3/2, Tan[(
-c + Pi/2 - d*x)/4]^2, -Tan[(-c + Pi/2 - d*x)/4]^2] - 4*m*(AppellF1[3/2, 1
- 2*m, 2*m, 5/2, Tan[(-c + Pi/2 - d*x)/4]^2, -Tan[(-c + Pi/2 - d*x)/4]^2] +
 AppellF1[3/2, -2*m, 1 + 2*m, 5/2, Tan[(-c + Pi/2 - d*x)/4]^2, -Tan[(-c + P
i/2 - d*x)/4]^2])*Tan[(-c + Pi/2 - d*x)/4]^2))) - ((Cos[(-c + Pi/2 - d*x)/4
```

]^2)^(2*m)*Csc[(-c + Pi/2 - d*x)/4]^2*(-(AppellF1[-1/2, -2*m, 2*m, 1/2, Tan [(-c + Pi/2 - d*x)/4]^2, -Tan[(-c + Pi/2 - d*x)/4]^2]*(Sec[(-c + Pi/2 - d*x)/4]^2)^(2*m)) + (3*AppellF1[1/2, -2*m, 2*m, 3/2, Tan[(-c + Pi/2 - d*x)/4]^ 2, -Tan[(-c + Pi/2 - d*x)/4]^2]*Tan[(-c + Pi/2 - d*x)/4]^2*(1 - Tan[(-c + P i/2 - d*x)/4]^2)^(2*m))/(3*AppellF1[1/2, -2*m, 2*m, 3/2, Tan[(-c + Pi/2 - d *x)/4]^2, -Tan[(-c + Pi/2 - d*x)/4]^2] - 4*m*(AppellF1[3/2, 1 - 2*m, 2*m, 5 /2, Tan[(-c + Pi/2 - d*x)/4]^2, -Tan[(-c + Pi/2 - d*x)/4]^2] + AppellF1[3/2 , -2*m, 1 + 2*m, 5/2, Tan[(-c + Pi/2 - d*x)/4]^2, -Tan[(-c + Pi/2 - d*x)/4] ^2])*Tan[(-c + Pi/2 - d*x)/4]^2)))/8 + ((Cos[(-c + Pi/2 - d*x)/4]^2)^(2*m)* Cot[(-c + Pi/2 - d*x)/4]*(-(m*AppellF1[-1/2, -2*m, 2*m, 1/2, Tan[(-c + Pi/2 - d*x)/4]^2, -Tan[(-c + Pi/2 - d*x)/4]^2]*(Sec[(-c + Pi/2 - d*x)/4]^2)^(2* m)*Tan[(-c + Pi/2 - d*x)/4]) - (Sec[(-c + Pi/2 - d*x)/4]^2)^(2*m)*(m*Appell F1[1/2, 1 - 2*m, 2*m, 3/2, Tan[(-c + Pi/2 - d*x)/4]^2, -Tan[(-c + Pi/2 - d* x)/4]^2]*Sec[(-c + Pi/2 - d*x)/4]^2*Tan[(-c + Pi/2 - d*x)/4] + m*AppellF1[1 /2, -2*m, 1 + 2*m, 3/2, Tan[(-c + Pi/2 - d*x)/4]^2, -Tan[(-c + Pi/2 - d*x)/ 4]^2]*Sec[(-c + Pi/2 - d*x)/4]^2*Tan[(-c + Pi/2 - d*x)/4]) + (3*AppellF1[1/ 2, -2*m, 2*m, 3/2, Tan[(-c + Pi/2 - d*x)/4]^2, -Tan[(-c + Pi/2 - d*x)/4]^2] *Sec[(-c + Pi/2 - d*x)/4]^2*Tan[(-c + Pi/2 - d*x)/4]*(1 - Tan[(-c + Pi/2 - d*x)/4]^2)^(2*m))/(2*(3*AppellF1[1/2, -2*m, 2*m, 3/2, Tan[(-c + Pi/2 - d*x) /4]^2, -Tan[(-c + Pi/2 - d*x)/4]^2] - 4*m*(AppellF1[3/2, 1 - 2*m, 2*m, 5/2, Tan[(-c + Pi/2 - d*x)/4]^2, -Tan[(-c + Pi/2 - d*x)/4]^2] + AppellF1[3/2, - 2*m, 1 + 2*m, 5/2, Tan[(-c + Pi/2 - d*x)/4]^2, -Tan[(-c + Pi/2 - d*x)/4]^2])*Tan[(-c + Pi/2 - d*x)/4]^2)) + (3*Tan[(-c + Pi/2 - d*x)/4]^2*(-1/3*(m*App ellF1[3/2, 1 - 2*m, 2*m, 5/2, Tan[(-c + Pi/2 - d*x)/4]^2, -Tan[(-c + Pi/2 - d*x)/4]^2]*Sec[(-c + Pi/2 - d*x)/4]^2*Tan[(-c + Pi/2 - d*x)/4]) - (m*Appel lF1[3/2, -2*m, 1 + 2*m, 5/2, Tan[(-c + Pi/2 - d*x)/4]^2, -Tan[(-c + Pi/2 - d*x)/4]^2]*Sec[(-c + Pi/2 - d*x)/4]^2*Tan[(-c + Pi/2 - d*x)/4])/3)*(1 - Tan [(-c + Pi/2 - d*x)/4]^2)^(2*m))/(3*AppellF1[1/2, -2*m, 2*m, 3/2, Tan[(-c + Pi/2 - d*x)/4]^2, -Tan[(-c + Pi/2 - d*x)/4]^2] - 4*m*(AppellF1[3/2, 1 - 2*m , 2*m, 5/2, Tan[(-c + Pi/2 - d*x)/4]^2, -Tan[(-c + Pi/2 - d*x)/4]^2] + Appe llF1[3/2, -2*m, 1 + 2*m, 5/2, Tan[(-c + Pi/2 - d*x)/4]^2, -Tan[(-c + Pi/2 - d*x)/4]^2])*Tan[(-c + Pi/2 - d*x)/4]^2) - (3*m*AppellF1[1/2, -2*m, 2*m, 3/ 2, Tan[(-c + Pi/2 - d*x)/4]^2, -Tan[(-c + Pi/2 - d*x)/4]^2]*Sec[(-c + Pi/2 - d*x)/4]^2*Tan[(-c + Pi/2 - d*x)/4]^3*(1 - Tan[(-c + Pi/2 - d*x)/4]^2)^(-1 + 2*m))/(3*AppellF1[1/2, -2*m, 2*m, 3/2, Tan[(-c + Pi/2 - d*x)/4]^2, -Tan[(-c + Pi/2 - d*x)/4]^2] - 4*m*(AppellF1[3/2, 1 - 2*m, 2*m, 5/2, Tan[(-c + P i/2 - d*x)/4]^2, -Tan[(-c + Pi/2 - d*x)/4]^2] + AppellF1[3/2, -2*m, 1 + 2*m , 5/2, Tan[(-c + Pi/2 - d*x)/4]^2, -Tan[(-c + Pi/2 - d*x)/4]^2])*Tan[(-c + Pi/2 - d*x)/4]^2) - (3*AppellF1[1/2, -2*m, 2*m, 3/2, Tan[(-c + Pi/2 - d*x)/ 4]^2, -Tan[(-c + Pi/2 - d*x)/4]^2]*Tan[(-c + Pi/2 - d*x)/4]^2*(1 - Tan[(-c + Pi/2 - d*x)/4]^2)^(2*m)*(-2*m*(AppellF1[3/2, 1 - 2*m, 2*m, 5/2, Tan[(-c + Pi/2 - d*x)/4]^2, -Tan[(-c + Pi/2 - d*x)/4]^2] + AppellF1[3/2, -2*m, 1 + 2 *m, 5/2, Tan[(-c + Pi/2 - d*x)/4]^2, -Tan[(-c + Pi/2 - d*x)/4]^2])*Sec[(-c + Pi/2 - d*x)/4]^2*Tan[(-c + Pi/2 - d*x)/4] + 3*(-1/3*(m*AppellF1[3/2, 1 - 2*m, 2*m, 5/2, Tan[(-c + Pi/2 - d*x)/4]^2, -Tan[(-c + Pi/2 - d*x)/4]^2]*Sec

```
[(-c + Pi/2 - d*x)/4]^2*Tan[(-c + Pi/2 - d*x)/4]) - (m*AppellF1[3/2, -2*m,
1 + 2*m, 5/2, Tan[(-c + Pi/2 - d*x)/4]^2, -Tan[(-c + Pi/2 - d*x)/4]^2]*Sec[
(-c + Pi/2 - d*x)/4]^2*Tan[(-c + Pi/2 - d*x)/4]...
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int \left(\sec^2\left(dx+c\right)\right)\left(a+a\sin\left(dx+c\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+a*sin(d*x+c))^m,x)
[Out] int(sec(d*x+c)^2*(a+a*sin(d*x+c))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
[Out] integrate((a*sin(d*x + c) + a)^m*sec(d*x + c)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
[Out] integral((a*sin(d*x + c) + a)^m*sec(d*x + c)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sin\left(c+dx\right)+1\right)\right)^m\sec^2\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+a*sin(d*x+c))**m,x)
[Out] Integral((a*(sin(c + d*x) + 1))**m*sec(c + d*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^m*sec(d*x + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\,\sin\left(c+d\,x\right))^m}{\cos\left(c+d\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^m/cos(c + d*x)^2,x)
```

```
[Out] int((a + a*sin(c + d*x))^m/cos(c + d*x)^2, x)
```

3.353 $\int \sec^4(c+dx)(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=83

$$\frac{2^{-\frac{3}{2}+m}\,{}_2F_1\left(-\frac{3}{2},\frac{5}{2}-m;-\frac{1}{2};\frac{1}{2}(1-\sin(c+dx))\right)\sec^3(c+dx)(1+\sin(c+dx))^{\frac{1}{2}-m}(a+a\sin(c+dx))^{1+m}}{3ad}$$

```
[Out] 1/3*2^(-3/2+m)*hypergeom([-3/2, 5/2-m],[-1/2],1/2-1/2*sin(d*x+c))*sec(d*x+c
)^3*(1+sin(d*x+c))^(1/2-m)*(a+a*sin(d*x+c))^(1+m)/a/d
```

Rubi [A]
time = 0.06, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2768, 72, 71}

$$\frac{2^{m-\frac{3}{2}}\sec^3(c+dx)(\sin(c+dx)+1)^{\frac{1}{2}-m}(a\sin(c+dx)+a)^{m+1}\,{}_2F_1\left(-\frac{3}{2},\frac{5}{2}-m;-\frac{1}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] (2^(-3/2 + m)*Hypergeometric2F1[-3/2, 5/2 - m, -1/2, (1 - Sin[c + d*x])/2]*
Sec[c + d*x]^3*(1 + Sin[c + d*x])^(1/2 - m)*(a + a*Sin[c + d*x])^(1 + m))/(
3*a*d)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
```

```
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int \sec^4(c+dx)(a+a\sin(c+dx))^m\,dx = \frac{\left(a^2\sec^3(c+dx)(a-a\sin(c+dx))^{3/2}(a+a\sin(c+dx))^{3/2}\right)\text{Subst}}{d}$$

$$= \frac{\left(2^{-\frac{5}{2}+m}\sec^3(c+dx)(a-a\sin(c+dx))^{3/2}(a+a\sin(c+dx))^{1+m}\right.}{d}$$

$$= \frac{2^{-\frac{3}{2}+m}\,{}_2F_1\left(-\frac{3}{2},\frac{5}{2}-m;-\frac{1}{2};\frac{1}{2}(1-\sin(c+dx))\right)\sec^3(c+dx)(1+\sin}{3ad}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 6.65, size = 9400, normalized size = 113.25

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^4*(a + a*Sin[c + d*x])^m,x]
[Out] Result too large to show
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int \left(\sec^4(dx+c)\right)(a+a\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+a*sin(d*x+c))^m,x)
[Out] int(sec(d*x+c)^4*(a+a*sin(d*x+c))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^m*sec(d*x + c)^4, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
[Out] integral((a*sin(d*x + c) + a)^m*sec(d*x + c)^4, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+a*sin(d*x+c))**m,x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))^m,x, algorithm="giac")
[Out] integrate((a*sin(d*x + c) + a)^m*sec(d*x + c)^4, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\,\sin\left(c+d\,x\right))^m}{{\cos\left(c+d\,x\right)}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^m/cos(c + d*x)^4,x)
[Out] int((a + a*sin(c + d*x))^m/cos(c + d*x)^4, x)
```

3.354 $\int (e\cos(c+dx))^{5/2}(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=88

$$-\frac{2^{\frac{11}{4}+m}a(e\cos(c+dx))^{7/2}\,{}_2F_1\left(\frac{7}{4},-\frac{3}{4}-m;\frac{11}{4};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{-\frac{3}{4}-m}(a+a\sin(c+dx))^{-1+m}}{7de}$$

```
[Out] -1/7*2^(11/4+m)*a*(e*cos(d*x+c))^(7/2)*hypergeom([7/4, -3/4-m],[11/4],1/2-1
/2*sin(d*x+c))*(1+sin(d*x+c))^(-3/4-m)*(a+a*sin(d*x+c))^(-1+m)/d/e
```

Rubi [A]
time = 0.06, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2768, 72, 71}

$$-\frac{a2^{m+\frac{11}{4}}(e\cos(c+dx))^{7/2}(\sin(c+dx)+1)^{-m-\frac{3}{4}}(a\sin(c+dx)+a)^{m-1}\,{}_2F_1\left(\frac{7}{4},-m-\frac{3}{4};\frac{11}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{7de}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(5/2)*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] -1/7*(2^(11/4 + m)*a*(e*Cos[c + d*x])^(7/2)*Hypergeometric2F1[7/4, -3/4 - m
, 11/4, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^(-3/4 - m)*(a + a*Sin[c +
d*x])^(-1 + m))/(d*e)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
```

```
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m]
```

Rubi steps

$$\int (e\cos(c+dx))^{5/2}(a+a\sin(c+dx))^m\,dx = \frac{\left(a^2(e\cos(c+dx))^{7/2}\right)\text{Subst}\left(\int (a-ax)^{3/4}(a+ax)^{\frac{3}{4}+m}\,dx, x\right.}{de(a-a\sin(c+dx))^{7/4}(a+a\sin(c+dx))^{7/4}}$$
$$= \frac{\left(2^{\frac{3}{4}+m}a^2(e\cos(c+dx))^{7/2}(a+a\sin(c+dx))^{-1+m}\left(\frac{a+a\sin(c+}{a}\right.\right.}{de(a-a\sin}$$
$$= -\frac{2^{\frac{11}{4}+m}a(e\cos(c+dx))^{7/2}\,{}_2F_1\left(\frac{7}{4},-\frac{3}{4}-m;\frac{11}{4};\frac{1}{2}(1-\sin(c+\right.}{7de}$$

Mathematica [A]
time = 0.20, size = 85, normalized size = 0.97

$$-\frac{2^{\frac{11}{4}+m}(e\cos(c+dx))^{7/2}\,{}_2F_1\left(\frac{7}{4},-\frac{3}{4}-m;\frac{11}{4};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{-\frac{7}{4}-m}(a(1+\sin(c+dx)))^m}{7de}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(5/2)*(a + a*Sin[c + d*x])^m,x]
[Out] -1/7*(2^(11/4 + m)*(e*Cos[c + d*x])^(7/2)*Hypergeometric2F1[7/4, -3/4 - m,
11/4, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^(-7/4 - m)*(a*(1 + Sin[c + d
*x]))^m)/(d*e)
```

Maple [F]
time = 0.16, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{\frac{5}{2}}(a+a\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(5/2)*(a+a*sin(d*x+c))^m,x)
[Out] int((e*cos(d*x+c))^(5/2)*(a+a*sin(d*x+c))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
[Out] e^(5/2)*integrate((a*sin(d*x + c) + a)^m*cos(d*x + c)^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
[Out] integral((a*sin(d*x + c) + a)^m*cos(d*x + c)^(5/2)*e^(5/2), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(5/2)*(a+a*sin(d*x+c))**m,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 8570 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+a*sin(d*x+c))^m,x, algorithm="giac")
[Out] integrate((a*sin(d*x + c) + a)^m*cos(d*x + c)^(5/2)*e^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (e\,\cos\left(c+d\,x\right))^{5/2}\,\left(a+a\,\sin\left(c+d\,x\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(5/2)*(a + a*sin(c + d*x))^m,x)
[Out] int((e*cos(c + d*x))^(5/2)*(a + a*sin(c + d*x))^m, x)
```

3.355 $\int (e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=88

$$-\frac{2^{\frac{9}{4}+m}a(e\cos(c+dx))^{5/2}\,{}_2F_1\left(\frac{5}{4},-\frac{1}{4}-m;\frac{9}{4};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{-\frac{1}{4}-m}(a+a\sin(c+dx))^{-1+m}}{5de}$$

```
[Out] -1/5*2^(9/4+m)*a*(e*cos(d*x+c))^(5/2)*hypergeom([5/4, -1/4-m],[9/4],1/2-1/2
*sin(d*x+c))*(1+sin(d*x+c))^(-1/4-m)*(a+a*sin(d*x+c))^(-1+m)/d/e
```

Rubi [A]
time = 0.06, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2768, 72, 71}

$$-\frac{a2^{m+\frac{9}{4}}(e\cos(c+dx))^{5/2}(\sin(c+dx)+1)^{-m-\frac{1}{4}}(a\sin(c+dx)+a)^{m-1}\,{}_2F_1\left(\frac{5}{4},-m-\frac{1}{4};\frac{9}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{5de}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] -1/5*(2^(9/4 + m)*a*(e*Cos[c + d*x])^(5/2)*Hypergeometric2F1[5/4, -1/4 - m,
 9/4, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^(-1/4 - m)*(a + a*Sin[c + d*
x])^(-1 + m))/(d*e)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
```

```
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int (e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^m\,dx = \frac{\left(a^2(e\cos(c+dx))^{5/2}\right)\text{Subst}\left(\int \sqrt[4]{a-ax}\,(a+ax)^{\frac{1}{4}+m}\,dx, x, \text{si}\right.}{de(a-a\sin(c+dx))^{5/4}(a+a\sin(c+dx))^{5/4}}$$

$$= \frac{\left(2^{\frac{1}{4}+m}a^2(e\cos(c+dx))^{5/2}(a+a\sin(c+dx))^{-1+m}\left(\frac{a+a\sin(c+d}{a}\right.\right.}{de(a-a\sin(}$$

$$= -\frac{2^{\frac{9}{4}+m}a(e\cos(c+dx))^{5/2}\,{}_2F_1\left(\frac{5}{4},-\frac{1}{4}-m;\frac{9}{4};\frac{1}{2}(1-\sin(c+dx)\right.}{5de}$$

Mathematica [A]
time = 0.14, size = 85, normalized size = 0.97

$$-\frac{2^{\frac{9}{4}+m}(e\cos(c+dx))^{5/2}\,{}_2F_1\left(\frac{5}{4},-\frac{1}{4}-m;\frac{9}{4};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{-\frac{5}{4}-m}(a(1+\sin(c+dx)))^m}{5de}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(3/2)*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] -1/5*(2^(9/4 + m)*(e*Cos[c + d*x])^(5/2)*Hypergeometric2F1[5/4, -1/4 - m, 9
/4, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^(-5/4 - m)*(a*(1 + Sin[c + d*x
]))^m)/(d*e)
```

Maple [F]
time = 0.14, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{\frac{3}{2}}(a+a\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^m,x)
```

```
[Out] int((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] e^(3/2)*integrate((a*sin(d*x + c) + a)^m*cos(d*x + c)^(3/2), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sin(d*x + c) + a)^m*cos(d*x + c)^(3/2)*e^(3/2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(3/2)*(a+a*sin(d*x+c))**m,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+a*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^m*cos(d*x + c)^(3/2)*e^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (e\,\cos\left(c+d\,x\right))^{3/2}\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(3/2)*(a + a*sin(c + d*x))^m,x)
```

```
[Out] int((e*cos(c + d*x))^(3/2)*(a + a*sin(c + d*x))^m, x)
```

3.356 $\int \sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=88

$$-\frac{2^{\frac{7}{4}+m}a(e\cos(c+dx))^{3/2}\,{}_2F_1\left(\frac{3}{4},\frac{1}{4}-m;\frac{7}{4};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{\frac{1}{4}-m}(a+a\sin(c+dx))^{-1+m}}{3de}$$

```
[Out] -1/3*2^(7/4+m)*a*(e*cos(d*x+c))^(3/2)*hypergeom([3/4, 1/4-m],[7/4],1/2-1/2*
sin(d*x+c))*(1+sin(d*x+c))^(1/4-m)*(a+a*sin(d*x+c))^(-1+m)/d/e
```

Rubi [A]
time = 0.06, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2768, 72, 71}

$$-\frac{a2^{m+\frac{7}{4}}(e\cos(c+dx))^{3/2}(\sin(c+dx)+1)^{\frac{1}{4}-m}(a\sin(c+dx)+a)^{m-1}\,{}_2F_1\left(\frac{3}{4},\frac{1}{4}-m;\frac{7}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{3de}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] -1/3*(2^(7/4 + m)*a*(e*Cos[c + d*x])^(3/2)*Hypergeometric2F1[3/4, 1/4 - m,
7/4, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^(1/4 - m)*(a + a*Sin[c + d*x]
)^(-1 + m))/(d*e)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
```

```
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m]
```

Rubi steps

$$\int \sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^m\,dx = \frac{\left(a^2(e\cos(c+dx))^{3/2}\right)\operatorname{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{4}+m}}{\sqrt[4]{a-ax}}\,dx, x, \sin(c+dx)\right)}{de(a-a\sin(c+dx))^{3/4}(a+a\sin(c+dx))^{3/4}}$$

$$= \frac{\left(2^{-\frac{1}{4}+m}a^2(e\cos(c+dx))^{3/2}(a+a\sin(c+dx))^{-1+m}\left(\frac{a+a\sin(c\cdots}{a}\right.\right.}{de(a-a\sin(c+\cdots}$$

$$= -\frac{2^{\frac{7}{4}+m}a(e\cos(c+dx))^{3/2}\,{}_2F_1\left(\frac{3}{4},\frac{1}{4}-m;\frac{7}{4};\frac{1}{2}(1-\sin(c+dx))\cdots\right.}{3de}$$

Mathematica [A]
time = 0.09, size = 85, normalized size = 0.97

$$-\frac{2^{\frac{7}{4}+m}(e\cos(c+dx))^{3/2}\,{}_2F_1\left(\frac{3}{4},\frac{1}{4}-m;\frac{7}{4};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{-\frac{3}{4}-m}(a(1+\sin(c+dx)))^m}{3de}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Cos[c + d*x]]*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] -1/3*(2^(7/4 + m)*(e*Cos[c + d*x])^(3/2)*Hypergeometric2F1[3/4, 1/4 - m, 7/
4, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^(-3/4 - m)*(a*(1 + Sin[c + d*x]
))^m)/(d*e)
```

Maple [F]
time = 0.16, size = 0, normalized size = 0.00

$$\int \sqrt{e\cos(dx+c)}\,(a+a\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^m,x)
```

```
[Out] int((e*cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
[Out] e^(1/2)*integrate((a*sin(d*x + c) + a)^m*sqrt(cos(d*x + c)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
[Out] integral((a*sin(d*x + c) + a)^m*sqrt(cos(d*x + c))*e^(1/2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sin{(c+dx)}+1))^m \sqrt{e\cos{(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(1/2)*(a+a*sin(d*x+c))**m,x)
[Out] Integral((a*(sin(c + d*x) + 1))**m*sqrt(e*cos(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)*(a+a*sin(d*x+c))^m,x, algorithm="giac")
[Out] integrate((a*sin(d*x + c) + a)^m*sqrt(cos(d*x + c))*e^(1/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{e\,\cos{(c+d\,x)}}\,(a+a\,\sin{(c+d\,x)})^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(1/2)*(a + a*sin(c + d*x))^m,x)
[Out] int((e*cos(c + d*x))^(1/2)*(a + a*sin(c + d*x))^m, x)
```

3.357 $\int \frac{(a+a\sin(c+dx))^m}{\sqrt{e\cos(c+dx)}}\,dx$

Optimal. Leaf size=86

$$-\frac{2^{\frac{5}{4}+m}a\sqrt{e\cos(c+dx)}\;{}_2F_1\left(\frac{1}{4},\frac{3}{4}-m;\frac{5}{4};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{\frac{3}{4}-m}(a+a\sin(c+dx))^{-1+m}}{de}$$

```
[Out] -2^(5/4+m)*a*hypergeom([1/4, 3/4-m],[5/4],1/2-1/2*sin(d*x+c))*(1+sin(d*x+c)
)^(3/4-m)*(a+a*sin(d*x+c))^(-1+m)*(e*cos(d*x+c))^(1/2)/d/e
```

Rubi [A]
time = 0.06, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2768, 72, 71}

$$-\frac{a2^{m+\frac{5}{4}}\sqrt{e\cos(c+dx)}\,(\sin(c+dx)+1)^{\frac{3}{4}-m}(a\sin(c+dx)+a)^{m-1}\;{}_2F_1\left(\frac{1}{4},\frac{3}{4}-m;\frac{5}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{de}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^m/Sqrt[e*Cos[c + d*x]],x]
```

```
[Out] -((2^(5/4 + m)*a*Sqrt[e*Cos[c + d*x]]*Hypergeometric2F1[1/4, 3/4 - m, 5/4,
(1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^(3/4 - m)*(a + a*Sin[c + d*x])^(-1
 + m))/(d*e))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
```

```
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{(a+a\sin(c+dx))^m}{\sqrt{e\cos(c+dx)}}\,dx = \frac{\left(a^2\sqrt{e\cos(c+dx)}\right)\text{Subst}\left(\int \frac{(a+ax)^{-\frac{3}{4}+m}}{(a-ax)^{3/4}}\,dx, x, \sin(c+dx)\right)}{de\sqrt[4]{a-a\sin(c+dx)}\,\sqrt[4]{a+a\sin(c+dx)}}$$

$$= \frac{\left(2^{-\frac{3}{4}+m}a^2\sqrt{e\cos(c+dx)}\,(a+a\sin(c+dx))^{-1+m}\left(\frac{a+a\sin(c+dx)}{a}\right)^{\frac{3}{4}-m}\right)\text{Subst}\left(\right.}{de\sqrt[4]{a-a\sin(c+dx)}}$$

$$= -\frac{2^{\frac{5}{4}+m}a\sqrt{e\cos(c+dx)}\;{}_2F_1\left(\frac{1}{4},\frac{3}{4}-m;\frac{5}{4};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))}{de}$$

Mathematica [A]
time = 0.09, size = 83, normalized size = 0.97

$$-\frac{2^{\frac{5}{4}+m}\sqrt{e\cos(c+dx)}\;{}_2F_1\left(\frac{1}{4},\frac{3}{4}-m;\frac{5}{4};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{-\frac{1}{4}-m}(a(1+\sin(c+dx)))^m}{de}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^m/Sqrt[e*Cos[c + d*x]],x]
```

```
[Out] -((2^(5/4 + m)*Sqrt[e*Cos[c + d*x]]*Hypergeometric2F1[1/4, 3/4 - m, 5/4, (1
 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^(-1/4 - m)*(a*(1 + Sin[c + d*x]))^m)
/(d*e))
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{(a+a\sin(dx+c))^m}{\sqrt{e\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^m/(e*cos(d*x+c))^(1/2),x)
```

```
[Out] int((a+a*sin(d*x+c))^m/(e*cos(d*x+c))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^m/(e*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] e^(-1/2)*integrate((a*sin(d*x + c) + a)^m/sqrt(cos(d*x + c)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^m/(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a*sin(d*x + c) + a)^m*e^(-1/2)/sqrt(cos(d*x + c)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a(\sin(c+dx)+1))^m}{\sqrt{e\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**m/(e*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((a*(sin(c + d*x) + 1))**m/sqrt(e*cos(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^m/(e*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^m*e^(-1/2)/sqrt(cos(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\,\sin(c+d\,x))^m}{\sqrt{e\,\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^m/(e*cos(c + d*x))^(1/2),x)
```

```
[Out] int((a + a*sin(c + d*x))^m/(e*cos(c + d*x))^(1/2), x)
```

3.358 $\int \frac{(a+a\sin(c+dx))^m}{(e\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=82

$$\frac{2^{\frac{3}{4}+m}\,{}_2F_1\left(-\frac{1}{4},\frac{5}{4}-m;\frac{3}{4};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{\frac{1}{4}-m}(a+a\sin(c+dx))^m}{de\sqrt{e\cos(c+dx)}}$$

```
[Out] 2^(3/4+m)*hypergeom([-1/4, 5/4-m],[3/4],1/2-1/2*sin(d*x+c))*(1+sin(d*x+c))^
(1/4-m)*(a+a*sin(d*x+c))^m/d/e/(e*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2768, 72, 71}

$$\frac{2^{m+\frac{3}{4}}(\sin(c+dx)+1)^{\frac{1}{4}-m}(a\sin(c+dx)+a)^m\,{}_2F_1\left(-\frac{1}{4},\frac{5}{4}-m;\frac{3}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{de\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^m/(e*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2^(3/4 + m)*Hypergeometric2F1[-1/4, 5/4 - m, 3/4, (1 - Sin[c + d*x])/2]*(1
 + Sin[c + d*x])^(1/4 - m)*(a + a*Sin[c + d*x])^m)/(d*e*Sqrt[e*Cos[c + d*x]
])
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
```

```
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{(a+a\sin(c+dx))^m}{(e\cos(c+dx))^{3/2}}\,dx = \frac{\left(a^2\sqrt[4]{a-a\sin(c+dx)}\,\sqrt[4]{a+a\sin(c+dx)}\right)\text{Subst}\left(\int\frac{(a+ax)^{-\frac{5}{4}+m}}{(a-ax)^{5/4}}\,dx,x,\sin(c$$

$$=\frac{\left(2^{-\frac{5}{4}+m}a\sqrt[4]{a-a\sin(c+dx)}\,(a+a\sin(c+dx))^m\left(\frac{a+a\sin(c+dx)}{a}\right)^{\frac{1}{4}-m}\right)\text{Subst}}{de\sqrt{e\cos(c+dx)}}$$

$$=\frac{2^{\frac{3}{4}+m}\,{}_2F_1\left(-\frac{1}{4},\frac{5}{4}-m;\frac{3}{4};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{\frac{1}{4}-m}(a+a\sin(c}{de\sqrt{e\cos(c+dx)}}$$

Mathematica [A]
time = 0.11, size = 82, normalized size = 1.00

$$\frac{2^{\frac{3}{4}+m}\,{}_2F_1\left(-\frac{1}{4},\frac{5}{4}-m;\frac{3}{4};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{\frac{1}{4}-m}(a(1+\sin(c+dx)))^m}{de\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^m/(e*Cos[c + d*x])^(3/2),x]
[Out] (2^(3/4 + m)*Hypergeometric2F1[-1/4, 5/4 - m, 3/4, (1 - Sin[c + d*x])/2]*(1
 + Sin[c + d*x])^(1/4 - m)*(a*(1 + Sin[c + d*x]))^m)/(d*e*Sqrt[e*Cos[c + d*
x]])
```

Maple [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int\frac{(a+a\sin(dx+c))^m}{(e\cos(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^m/(e*cos(d*x+c))^(3/2),x)
[Out] int((a+a*sin(d*x+c))^m/(e*cos(d*x+c))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^m/(e*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] e^(-3/2)*integrate((a*sin(d*x + c) + a)^m/cos(d*x + c)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^m/(e*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a*sin(d*x + c) + a)^m*e^(-3/2)/cos(d*x + c)^(3/2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a(\sin{(c+dx)}+1))^m}{(e\cos{(c+dx)})^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**m/(e*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral((a*(sin(c + d*x) + 1))**m/(e*cos(c + d*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^m/(e*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^m*e^(-3/2)/cos(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\,\sin{(c+d\,x)})^m}{(e\,\cos{(c+d\,x)})^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^m/(e*cos(c + d*x))^(3/2),x)
```

```
[Out] int((a + a*sin(c + d*x))^m/(e*cos(c + d*x))^(3/2), x)
```

3.359 $\int \frac{(a+a\sin(c+dx))^m}{(e\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=85

$$\frac{2^{\frac{1}{4}+m}\,{}_2F_1\left(-\frac{3}{4},\frac{7}{4}-m;\frac{1}{4};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{\frac{3}{4}-m}(a+a\sin(c+dx))^m}{3de(e\cos(c+dx))^{3/2}}$$

```
[Out] 1/3*2^(1/4+m)*hypergeom([-3/4, 7/4-m],[1/4],1/2-1/2*sin(d*x+c))*(1+sin(d*x+
c))^(3/4-m)*(a+a*sin(d*x+c))^m/d/e/(e*cos(d*x+c))^(3/2)
```

Rubi [A]
time = 0.07, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2768, 72, 71}

$$\frac{2^{m+\frac{1}{4}}(\sin(c+dx)+1)^{\frac{3}{4}-m}(a\sin(c+dx)+a)^m\,{}_2F_1\left(-\frac{3}{4},\frac{7}{4}-m;\frac{1}{4};\frac{1}{2}(1-\sin(c+dx))\right)}{3de(e\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^m/(e*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2^(1/4 + m)*Hypergeometric2F1[-3/4, 7/4 - m, 1/4, (1 - Sin[c + d*x])/2]*(1
 + Sin[c + d*x])^(3/4 - m)*(a + a*Sin[c + d*x])^m)/(3*d*e*(e*Cos[c + d*x])^
(3/2))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
```

```
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{(a+a\sin(c+dx))^m}{(e\cos(c+dx))^{5/2}}\,dx = \frac{\left(a^2(a-a\sin(c+dx))^{3/4}(a+a\sin(c+dx))^{3/4}\right)\mathrm{Subst}\left(\int \frac{(a+ax)^{-\frac{7}{4}+m}}{(a-ax)^{7/4}}\,dx,x,\sin(\right.}{de(e\cos(c+dx))^{3/2}}$$

$$= \frac{\left(2^{-\frac{7}{4}+m}a(a-a\sin(c+dx))^{3/4}(a+a\sin(c+dx))^m\left(\frac{a+a\sin(c+dx)}{a}\right)^{\frac{3}{4}-m}\right)\mathrm{Subst}}{de(e\cos(c+dx))^{3/2}}$$

$$= \frac{2^{\frac{1}{4}+m}\,{}_2F_1\left(-\frac{3}{4},\frac{7}{4}-m;\frac{1}{4};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{\frac{3}{4}-m}(a+a\sin(c+}{3de(e\cos(c+dx))^{3/2}}$$

Mathematica [A]
time = 0.12, size = 85, normalized size = 1.00

$$\frac{2^{\frac{1}{4}+m}\,{}_2F_1\left(-\frac{3}{4},\frac{7}{4}-m;\frac{1}{4};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{\frac{3}{4}-m}(a(1+\sin(c+dx)))^m}{3de(e\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^m/(e*Cos[c + d*x])^(5/2),x]
[Out] (2^(1/4 + m)*Hypergeometric2F1[-3/4, 7/4 - m, 1/4, (1 - Sin[c + d*x])/2]*(1
 + Sin[c + d*x])^(3/4 - m)*(a*(1 + Sin[c + d*x]))^m)/(3*d*e*(e*Cos[c + d*x]
)^(3/2))
```

Maple [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{(a+a\sin(dx+c))^m}{(e\cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^m/(e*cos(d*x+c))^(5/2),x)
[Out] int((a+a*sin(d*x+c))^m/(e*cos(d*x+c))^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^m/(e*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] e^(-5/2)*integrate((a*sin(d*x + c) + a)^m/cos(d*x + c)^(5/2), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^m/(e*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((a*sin(d*x + c) + a)^m*e^(-5/2)/cos(d*x + c)^(5/2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**m/(e*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^m/(e*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^m*e^(-5/2)/cos(d*x + c)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\,\sin\left(c+d\,x\right))^m}{(e\,\cos\left(c+d\,x\right))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^m/(e*cos(c + d*x))^(5/2),x)
```

```
[Out] int((a + a*sin(c + d*x))^m/(e*cos(c + d*x))^(5/2), x)
```

3.360 $\int (e\cos(c+dx))^{-4-m}(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=201

$$-\frac{(e\cos(c+dx))^{-3-m}(a+a\sin(c+dx))^m}{de(3-m)}-\frac{3(e\cos(c+dx))^{-3-m}(a+a\sin(c+dx))^{1+m}}{ade(1-m)(3-m)}+\frac{6(e\cos(c+dx))^{-3}}{a^2de(3}$$

```
[Out] -(e*cos(d*x+c))^(-3-m)*(a+a*sin(d*x+c))^m/d/e/(3-m)-3*(e*cos(d*x+c))^(-3-m)
*(a+a*sin(d*x+c))^(1+m)/a/d/e/(1-m)/(3-m)+6*(e*cos(d*x+c))^(-3-m)*(a+a*sin(
d*x+c))^(2+m)/a^2/d/e/(3-m)/(-m^2+1)-6*(e*cos(d*x+c))^(-3-m)*(a+a*sin(d*x+c
))^(3+m)/a^3/d/e/(m^4-10*m^2+9)
```

Rubi [A]
time = 0.22, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2751, 2750}

$$-\frac{6(a\sin(c+dx)+a)^{m+3}(e\cos(c+dx))^{-m-3}}{a^3de\,(m^4-10m^2+9)}+\frac{6(a\sin(c+dx)+a)^{m+2}(e\cos(c+dx))^{-m-3}}{a^2de(3-m)\,(1-m^2)}-\frac{(a\sin(c+dx)+a)^m(e\cos(c+dx))^{-m-3}}{de(3-m)}-\frac{3(a\sin(c+dx)+a)^{m+1}(e\cos(c+dx))^{-m-3}}{ade(1-m)(3-m)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(-4 - m)*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] -(((e*Cos[c + d*x])^(-3 - m)*(a + a*Sin[c + d*x])^m)/(d*e*(3 - m))) - (3*(e
*Cos[c + d*x])^(-3 - m)*(a + a*Sin[c + d*x])^(1 + m))/(a*d*e*(1 - m)*(3 - m
)) + (6*(e*Cos[c + d*x])^(-3 - m)*(a + a*Sin[c + d*x])^(2 + m))/(a^2*d*e*(3
 - m)*(1 - m^2)) - (6*(e*Cos[c + d*x])^(-3 - m)*(a + a*Sin[c + d*x])^(3 + m
))/(a^3*d*e*(9 - 10*m^2 + m^4))
```

Rule 2750

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int (e\cos(c+dx))^{-4-m}(a+a\sin(c+dx))^m\,dx = -\frac{(e\cos(c+dx))^{-3-m}(a+a\sin(c+dx))^m}{de(3-m)} + \frac{3\int(e\cos(c+}{}$$
$$= -\frac{(e\cos(c+dx))^{-3-m}(a+a\sin(c+dx))^m}{de(3-m)} - \frac{3(e\cos(c+d}{c}$$
$$= -\frac{(e\cos(c+dx))^{-3-m}(a+a\sin(c+dx))^m}{de(3-m)} - \frac{3(e\cos(c+d}{c}$$
$$= -\frac{(e\cos(c+dx))^{-3-m}(a+a\sin(c+dx))^m}{de(3-m)} - \frac{3(e\cos(c+d}{c}$$

Mathematica [A]
time = 0.20, size = 101, normalized size = 0.50

$$\frac{(e\cos(c+dx))^{-m}\sec^3(c+dx)(a(1+\sin(c+dx)))^m\left(m(-7+m^2)-3(-3+m^2)\sin(c+dx)+6m\sin^2(c+dx)-6\sin^3(c+dx)\right)}{de^4(-3+m)(-1+m)(1+m)(3+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(-4 - m)*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] (Sec[c + d*x]^3*(a*(1 + Sin[c + d*x]))^m*(m*(-7 + m^2) - 3*(-3 + m^2)*Sin[c
 + d*x] + 6*m*Sin[c + d*x]^2 - 6*Sin[c + d*x]^3))/(d*e^4*(-3 + m)*(-1 + m)*
(1 + m)*(3 + m)*(e*Cos[c + d*x])^m)
```

Maple [F]
time = 0.22, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{-4-m}(a+a\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(-4-m)*(a+a*sin(d*x+c))^m,x)
```

```
[Out] int((e*cos(d*x+c))^(-4-m)*(a+a*sin(d*x+c))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-4-m)*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((cos(d*x + c)*e)^(-m - 4)*(a*sin(d*x + c) + a)^m, x)
```

Fricas [**A**]
time = 0.36, size = 105, normalized size = 0.52

$$-\frac{\left(6\,m\cos\left(dx+c\right)^{3}-\left(m^{3}-m\right)\cos\left(dx+c\right)-3\left(2\,\cos\left(dx+c\right)^{3}-\left(m^{2}-1\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)\right)\left(\cos\left(dx+c\right)e\right)^{-m-4}\left(a\sin\left(dx+c\right)+a\right)^{m}}{dm^{4}-10\,dm^{2}+9\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-4-m)*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] -(6*m*cos(d*x + c)^3 - (m^3 - m)*cos(d*x + c) - 3*(2*cos(d*x + c)^3 - (m^2
- 1)*cos(d*x + c))*sin(d*x + c))*(cos(d*x + c)*e)^(-m - 4)*(a*sin(d*x + c)
+ a)^m/(d*m^4 - 10*d*m^2 + 9*d)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(-4-m)*(a+a*sin(d*x+c))**m,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5007 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-4-m)*(a+a*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((cos(d*x + c)*e)^(-m - 4)*(a*sin(d*x + c) + a)^m, x)
```

Mupad [**B**]
time = 6.83, size = 137, normalized size = 0.68

$$\frac{2\,{\left(a\,\left(\sin\left(c+d\,x\right)+1\right)\right)}^m\,\left(12\,\sin\left(2\,c+2\,d\,x\right)+3\,\sin\left(4\,c+4\,d\,x\right)-22\,m\,\cos\left(c+d\,x\right)-6\,m\,\cos\left(3\,c+3\,d\,x\right)+4\,m^3\,\cos\left(c+d\,x\right)-6\,m^2\,\sin\left(2\,c+2\,d\,x\right)\right)}{d\,e^4\,{\left(e\,\cos\left(c+d\,x\right)\right)}^m\,\left(4\,\cos\left(2\,c+2\,d\,x\right)+\cos\left(4\,c+4\,d\,x\right)+3\right)\,\left(m^4-10\,m^2+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^m/(e*cos(c + d*x))^(m + 4),x)
```

```
[Out] (2*(a*(sin(c + d*x) + 1))^m*(12*sin(2*c + 2*d*x) + 3*sin(4*c + 4*d*x) - 22*
m*cos(c + d*x) - 6*m*cos(3*c + 3*d*x) + 4*m^3*cos(c + d*x) - 6*m^2*sin(2*c
+ 2*d*x)))/(d*e^4*(e*cos(c + d*x))^m*(4*cos(2*c + 2*d*x) + cos(4*c + 4*d*x)
 + 3)*(m^4 - 10*m^2 + 9))
```

3.361 $\int (e\cos(c+dx))^{-3-m}(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=142

$$-\frac{(e\cos(c+dx))^{-2-m}(a+a\sin(c+dx))^m}{de(2-m)}+\frac{2(e\cos(c+dx))^{-2-m}(a+a\sin(c+dx))^{1+m}}{ade(2-m)m}-\frac{2(e\cos(c+dx))^{-}}{a^2}$$

```
[Out] -(e*cos(d*x+c))^(-2-m)*(a+a*sin(d*x+c))^m/d/e/(2-m)+2*(e*cos(d*x+c))^(-2-m)
*(a+a*sin(d*x+c))^(1+m)/a/d/e/(2-m)/m-2*(e*cos(d*x+c))^(-2-m)*(a+a*sin(d*x+
c))^(2+m)/a^2/d/e/m/(-m^2+4)
```

Rubi [A]

time = 0.15, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2751, 2750}

$$-\frac{2(a\sin(c+dx)+a)^{m+2}(e\cos(c+dx))^{-m-2}}{a^2dem\,(4-m^2)}-\frac{(a\sin(c+dx)+a)^m(e\cos(c+dx))^{-m-2}}{de(2-m)}+\frac{2(a\sin(c+dx)+a)^{m+1}(e\cos(c+dx))^{-m-2}}{ade(2-m)m}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(-3 - m)*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] -(((e*Cos[c + d*x])^(-2 - m)*(a + a*Sin[c + d*x])^m)/(d*e*(2 - m))) + (2*(e
*Cos[c + d*x])^(-2 - m)*(a + a*Sin[c + d*x])^(1 + m))/(a*d*e*(2 - m)*m) - (
2*(e*Cos[c + d*x])^(-2 - m)*(a + a*Sin[c + d*x])^(2 + m))/(a^2*d*e*m*(4 - m
^2))
```

Rule 2750

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int (e\cos(c+dx))^{-3-m}(a+a\sin(c+dx))^m\,dx = -\frac{(e\cos(c+dx))^{-2-m}(a+a\sin(c+dx))^m}{de(2-m)} + \frac{2\int(e\cos(c+

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-3-m)*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] (m^2*cos(d*x + c) - 2*cos(d*x + c)^3 - 2*m*cos(d*x + c)*sin(d*x + c))*(cos(
d*x + c)*e)^(-m - 3)*(a*sin(d*x + c) + a)^m/(d*m^3 - 4*d*m)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(-3-m)*(a+a*sin(d*x+c))**m,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-3-m)*(a+a*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((cos(d*x + c)*e)^(-m - 3)*(a*sin(d*x + c) + a)^m, x)
```

Mupad [B]
time = 6.13, size = 103, normalized size = 0.73

$$-\frac{2\,{\left(a\,\left(\sin\left(c+d\,x\right)+1\right)\right)}^m\,\left(-2\,\cos\left(c+d\,x\right)\,m^2+2\,\sin\left(2\,c+2\,d\,x\right)\,m+3\,\cos\left(c+d\,x\right)+\cos\left(3\,c+3\,d\,x\right)\right)}{d\,e^3\,m\,{\left(e\,\cos\left(c+d\,x\right)\right)}^m\,\left(m^2-4\right)\,\left(3\,\cos\left(c+d\,x\right)+\cos\left(3\,c+3\,d\,x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^m/(e*cos(c + d*x))^(m + 3),x)
```

```
[Out] -(2*(a*(sin(c + d*x) + 1))^m*(3*cos(c + d*x) + cos(3*c + 3*d*x) - 2*m^2*cos
(c + d*x) + 2*m*sin(2*c + 2*d*x)))/(d*e^3*m*(e*cos(c + d*x))^m*(m^2 - 4)*(3
*cos(c + d*x) + cos(3*c + 3*d*x)))
```

3.362 $\int (e\cos(c+dx))^{-2-m}(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=89

$$-\frac{(e\cos(c+dx))^{-1-m}(a+a\sin(c+dx))^m}{de(1-m)}+\frac{(e\cos(c+dx))^{-1-m}(a+a\sin(c+dx))^{1+m}}{ade\,(1-m^2)}$$

```
[Out] -(e*cos(d*x+c))^(-1-m)*(a+a*sin(d*x+c))^m/d/e/(1-m)+(e*cos(d*x+c))^(-1-m)*(
a+a*sin(d*x+c))^(1+m)/a/d/e/(-m^2+1)
```

Rubi [A]
time = 0.09, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}=0.074$, Rules used = {2751, 2750}

$$\frac{(a\sin(c+dx)+a)^{m+1}(e\cos(c+dx))^{-m-1}}{ade\,(1-m^2)}-\frac{(a\sin(c+dx)+a)^m(e\cos(c+dx))^{-m-1}}{de(1-m)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(-2 - m)*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] -(((e*Cos[c + d*x])^(-1 - m)*(a + a*Sin[c + d*x])^m)/(d*e*(1 - m))) + ((e*C
os[c + d*x])^(-1 - m)*(a + a*Sin[c + d*x])^(1 + m))/(a*d*e*(1 - m^2))
```

Rule 2750

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int (e\cos(c+dx))^{-2-m}(a+a\sin(c+dx))^m\,dx &= -\frac{(e\cos(c+dx))^{-1-m}(a+a\sin(c+dx))^m}{de(1-m)}+\frac{\int (e\cos(c+dx\\ &= -\frac{(e\cos(c+dx))^{-1-m}(a+a\sin(c+dx))^m}{de(1-m)}+\frac{(e\cos(c+dx)\end{aligned}$$

Mathematica [A]
time = 0.12, size = 53, normalized size = 0.60

$$\frac{(e\cos(c+dx))^{-1-m}(m-\sin(c+dx))(a(1+\sin(c+dx)))^m}{de(-1+m)(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(-2 - m)*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] ((e*Cos[c + d*x])^(-1 - m)*(m - Sin[c + d*x])*(a*(1 + Sin[c + d*x]))^m)/(d*
e*(-1 + m)*(1 + m))
```

Maple [F]
time = 0.19, size = 0, normalized size = 0.00

$$\int \left(e\cos\left(dx+c\right)\right)^{-2-m}\left(a+a\sin\left(dx+c\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(-2-m)*(a+a*sin(d*x+c))^m,x)
```

```
[Out] int((e*cos(d*x+c))^(-2-m)*(a+a*sin(d*x+c))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-2-m)*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((cos(d*x + c)*e)^(-m - 2)*(a*sin(d*x + c) + a)^m, x)
```

Fricas [A]
time = 0.35, size = 62, normalized size = 0.70

$$\frac{(m\cos\left(dx+c\right)-\cos\left(dx+c\right)\sin\left(dx+c\right))(\cos\left(dx+c\right)e)^{-m-2}\left(a\sin\left(dx+c\right)+a\right)^{m}}{dm^2-d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-2-m)*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] (m*cos(d*x + c) - cos(d*x + c)*sin(d*x + c))*(cos(d*x + c)*e)^(-m - 2)*(a*s
in(d*x + c) + a)^m/(d*m^2 - d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(-2-m)*(a+a*sin(d*x+c))**m,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-2-m)*(a+a*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((cos(d*x + c)*e)^(-m - 2)*(a*sin(d*x + c) + a)^m, x)
```

Mupad [B]
time = 5.60, size = 71, normalized size = 0.80

$$-\frac{\left(\sin\left(2\,c+2\,d\,x\right)-2\,m\,\cos\left(c+d\,x\right)\right)\,{\left(a\,\left(\sin\left(c+d\,x\right)+1\right)\right)}^m}{d\,e^2\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)\,{\left(e\,\cos\left(c+d\,x\right)\right)}^m\,\left(m^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^m/(e*cos(c + d*x))^(m + 2),x)
```

```
[Out] -((sin(2*c + 2*d*x) - 2*m*cos(c + d*x))*(a*(sin(c + d*x) + 1))^m)/(d*e^2*(c
os(2*c + 2*d*x) + 1)*(e*cos(c + d*x))^m*(m^2 - 1))
```

3.363 $\int (e\cos(c+dx))^{-1-m}(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=34

$$\frac{(e\cos(c+dx))^{-m}(a+a\sin(c+dx))^m}{dem}$$

```
[Out] (a+a*sin(d*x+c))^m/d/e/m/((e*cos(d*x+c))^m)
```

Rubi [A]
time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$, Rules used = {2750}

$$\frac{(a\sin(c+dx)+a)^m(e\cos(c+dx))^{-m}}{dem}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(-1 - m)*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] (a + a*Sin[c + d*x])^m/(d*e*m*(e*Cos[c + d*x])^m)
```

Rule 2750

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rubi steps

$$\int (e\cos(c+dx))^{-1-m}(a+a\sin(c+dx))^m\,dx = \frac{(e\cos(c+dx))^{-m}(a+a\sin(c+dx))^m}{dem}$$

Mathematica [A]
time = 0.06, size = 34, normalized size = 1.00

$$\frac{(e\cos(c+dx))^{-m}(a(1+\sin(c+dx)))^m}{dem}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(-1 - m)*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] (a*(1 + Sin[c + d*x]))^m/(d*e*m*(e*Cos[c + d*x])^m)
```

Maple [F]
time = 0.20, size = 0, normalized size = 0.00

$$\int \left(e\cos\left(dx+c\right)\right)^{-1-m} \left(a+a\sin\left(dx+c\right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(-1-m)*(a+a*sin(d*x+c))^m,x)
```

```
[Out] int((e*cos(d*x+c))^(-1-m)*(a+a*sin(d*x+c))^m,x)
```

Maxima [A]
time = 0.49, size = 62, normalized size = 1.82

$$\frac{a^{m} e^{\left(m \log\left(\frac{\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}+1\right)-m \log\left(-\frac{\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}+1\right)-m-1\right)}}{dm}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-1-m)*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] a^m*e^(m*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1) - m*log(-sin(d*x + c)/(co
s(d*x + c) + 1) + 1) - m - 1)/(d*m)
```

Fricas [A]
time = 0.37, size = 40, normalized size = 1.18

$$\frac{\left(\cos\left(dx+c\right)e\right)^{-m-1} \left(a\sin\left(dx+c\right)+a\right)^{m} \cos\left(dx+c\right)}{dm}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-1-m)*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] (cos(d*x + c)*e)^(-m - 1)*(a*sin(d*x + c) + a)^m*cos(d*x + c)/(d*m)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a(\sin\left(c+dx\right)+1)\right)^{m} \left(e\cos\left(c+dx\right)\right)^{-m-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(-1-m)*(a+a*sin(d*x+c))**m,x)
```

```
[Out] Integral((a*(sin(c + d*x) + 1))**m*(e*cos(c + d*x))**(-m - 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-1-m)*(a+a*sin(d*x+c))^m,x, algorithm="giac")
[Out] integrate((cos(d*x + c)*e)^(-m - 1)*(a*sin(d*x + c) + a)^m, x)
```

Mupad [B]
time = 0.29, size = 34, normalized size = 1.00

$$\frac{(a (\sin (c+d x)+1))^m}{d e m (e \cos (c+d x))^m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^m/(e*cos(c + d*x))^(m + 1),x)
[Out] (a*(sin(c + d*x) + 1))^m/(d*e*m*(e*cos(c + d*x))^m)
```

3.364 $\int (e\cos(c+dx))^{-m}(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=115

$$-\frac{2^{\frac{1}{2}+\frac{m}{2}}a(e\cos(c+dx))^{1-m}\,{}_2F_1\left(\frac{1-m}{2},\frac{1-m}{2};\frac{3-m}{2};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{\frac{1-m}{2}}(a+a\sin(c+dx))^{-1+m}}{de(1-m)}$$

```
[Out] -2^(1/2+1/2*m)*a*(e*cos(d*x+c))^(1-m)*hypergeom([1/2-1/2*m, 1/2-1/2*m],[3/2
-1/2*m],1/2-1/2*sin(d*x+c))*(1+sin(d*x+c))^(1/2-1/2*m)*(a+a*sin(d*x+c))^(-1
+m)/d/e/(1-m)
```

Rubi [A]
time = 0.08, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2768, 72, 71}

$$-\frac{a2^{\frac{m}{2}+\frac{1}{2}}(\sin(c+dx)+1)^{\frac{1-m}{2}}(a\sin(c+dx)+a)^{m-1}(e\cos(c+dx))^{1-m}\,{}_2F_1\left(\frac{1-m}{2},\frac{1-m}{2};\frac{3-m}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{de(1-m)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^m/(e*Cos[c + d*x])^m,x]
```

```
[Out] -((2^(1/2 + m/2)*a*(e*Cos[c + d*x])^(1 - m)*Hypergeometric2F1[(1 - m)/2, (1
 - m)/2, (3 - m)/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^((1 - m)/2)*(a
 + a*Sin[c + d*x])^(-1 + m))/(d*e*(1 - m)))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
```

```
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m]
```

Rubi steps

$$\int (e\cos(c+dx))^{-m}(a+a\sin(c+dx))^m\,dx = \frac{\left(a^2(e\cos(c+dx))^{1-m}(a-a\sin(c+dx))^{\frac{1}{2}(-1+m)}(a+a\sin(c\right.}{}$$

$$= \frac{\left(2^{-\frac{1}{2}+\frac{m}{2}}a^2(e\cos(c+dx))^{1-m}(a-a\sin(c+dx))^{\frac{1}{2}(-1+m)}(a+\right.}{}$$

$$= -\frac{2^{\frac{1}{2}+\frac{m}{2}}a(e\cos(c+dx))^{1-m}\,{}_2F_1\left(\frac{1-m}{2},\frac{1-m}{2};\frac{3-m}{2};\frac{1}{2}(1-\sin(c\right.}{de(1-r}$$

Mathematica [A]
time = 0.13, size = 108, normalized size = 0.94

$$\frac{2^{\frac{1+m}{2}}\cos(c+dx)(e\cos(c+dx))^{-m}\,{}_2F_1\left(\frac{1-m}{2},\frac{1-m}{2};\frac{3-m}{2};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{\frac{1}{2}(-1-m)}(a(1+\sin(c+dx)))^m}{d(-1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^m/(e*Cos[c + d*x])^m,x]
[Out] (2^((1 + m)/2)*Cos[c + d*x]*Hypergeometric2F1[(1 - m)/2, (1 - m)/2, (3 - m)
/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^((-1 - m)/2)*(a*(1 + Sin[c + d
*x]))^m)/(d*(-1 + m)*(e*Cos[c + d*x])^m)
```

Maple [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int (a+a\sin(dx+c))^m (e\cos(dx+c))^{-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^m/((e*cos(d*x+c))^m),x)
[Out] int((a+a*sin(d*x+c))^m/((e*cos(d*x+c))^m),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^m/((e*cos(d*x+c))^m),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^m/(cos(d*x + c)*e)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^m/((e*cos(d*x+c))^m),x, algorithm="fricas")
```

```
[Out] integral((a*sin(d*x + c) + a)^m/(cos(d*x + c)*e)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sin(c+dx)+1))^m (e\cos(c+dx))^{-m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**m/((e*cos(d*x+c))**m),x)
```

```
[Out] Integral((a*(sin(c + d*x) + 1))**m/(e*cos(c + d*x))**m, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^m/((e*cos(d*x+c))^m),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^m/(cos(d*x + c)*e)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\,\sin(c+d\,x))^m}{(e\,\cos(c+d\,x))^m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^m/(e*cos(c + d*x))^m,x)
```

```
[Out] int((a + a*sin(c + d*x))^m/(e*cos(c + d*x))^m, x)
```

3.365 $\int (e\cos(c+dx))^{1-m}(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=97

$$\frac{2^{1-\frac{m}{2}}(e\cos(c+dx))^{2-m}\,{}_2F_1\left(\frac{m}{2},\frac{2+m}{2};\frac{4+m}{2};\frac{1}{2}(1+\sin(c+dx))\right)(1-\sin(c+dx))^{-1+\frac{m}{2}}(a+a\sin(c+dx))^m}{de(2+m)}$$

```
[Out] 2^(1-1/2*m)*(e*cos(d*x+c))^(2-m)*hypergeom([1/2*m, 1+1/2*m],[2+1/2*m],1/2+1
/2*sin(d*x+c))*(1-sin(d*x+c))^(-1+1/2*m)*(a+a*sin(d*x+c))^m/d/e/(2+m)
```

Rubi [A]
time = 0.07, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2768, 72, 71}

$$\frac{2^{1-\frac{m}{2}}(1-\sin(c+dx))^{\frac{m}{2}-1}(a\sin(c+dx)+a)^m(e\cos(c+dx))^{2-m}\,{}_2F_1\left(\frac{m}{2},\frac{m+2}{2};\frac{m+4}{2};\frac{1}{2}(\sin(c+dx)+1)\right)}{de(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(1 - m)*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] (2^(1 - m/2)*(e*Cos[c + d*x])^(2 - m)*Hypergeometric2F1[m/2, (2 + m)/2, (4
+ m)/2, (1 + Sin[c + d*x])/2]*(1 - Sin[c + d*x])^(-1 + m/2)*(a + a*Sin[c +
d*x])^m)/(d*e*(2 + m))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
```

```
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int (e\cos(c+dx))^{1-m}(a+a\sin(c+dx))^m\,dx = \frac{\left(a^2(e\cos(c+dx))^{2-m}(a-a\sin(c+dx))^{\frac{1}{2}(-2+m)}(a+a\sin(c\right.}{}$$

$$= \frac{\left(2^{-m/2}a^2(e\cos(c+dx))^{2-m}(a-a\sin(c+dx))^{\frac{1}{2}(-2+m)-\frac{m}{2}}\left(\right.\right.}{}$$

$$= \frac{2^{1-\frac{m}{2}}(e\cos(c+dx))^{2-m}\,{}_2F_1\left(\frac{m}{2},\frac{2+m}{2};\frac{4+m}{2};\frac{1}{2}(1+\sin(c+dx))\right.}{de(2+m)}$$

Mathematica [A]
time = 0.27, size = 97, normalized size = 1.00

$$\frac{2^{1+\frac{m}{2}}(e\cos(c+dx))^{2-m}\,{}_2F_1\left(1-\frac{m}{2},-\frac{m}{2};2-\frac{m}{2};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{-1-\frac{m}{2}}(a(1+\sin(c+dx)))^m}{de(-2+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(1 - m)*(a + a*Sin[c + d*x])^m,x]
[Out] (2^(1 + m/2)*(e*Cos[c + d*x])^(2 - m)*Hypergeometric2F1[1 - m/2, -1/2*m, 2
- m/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^(-1 - m/2)*(a*(1 + Sin[c +
d*x]))^m)/(d*e*(-2 + m))
```

Maple [F]
time = 0.20, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{1-m}(a+a\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(1-m)*(a+a*sin(d*x+c))^m,x)
[Out] int((e*cos(d*x+c))^(1-m)*(a+a*sin(d*x+c))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1-m)*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
[Out] integrate((cos(d*x + c)*e)^(-m + 1)*(a*sin(d*x + c) + a)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1-m)*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
[Out] integral((cos(d*x + c)*e)^(-m + 1)*(a*sin(d*x + c) + a)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sin(c+dx)+1))^m (e\cos(c+dx))^{1-m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(1-m)*(a+a*sin(d*x+c))**m,x)
[Out] Integral((a*(sin(c + d*x) + 1))**m*(e*cos(c + d*x))**(1 - m), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1-m)*(a+a*sin(d*x+c))^m,x, algorithm="giac")
[Out] integrate((cos(d*x + c)*e)^(-m + 1)*(a*sin(d*x + c) + a)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(e\,\cos\left(c+d\,x\right)\right)}^{1-m}\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^m \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(1 - m)*(a + a*sin(c + d*x))^m,x)
[Out] int((e*cos(c + d*x))^(1 - m)*(a + a*sin(c + d*x))^m, x)
```

3.366 $\int (e\cos(c+dx))^{2-m}(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=115

$$-\frac{2^{\frac{3}{2}+\frac{m}{2}}a(e\cos(c+dx))^{3-m}\,{}_2F_1\left(\frac{1}{2}(-1-m),\frac{3-m}{2};\frac{5-m}{2};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{\frac{1}{2}(-1-m)}(a+a\sin(c+dx))^{-1+m}}{de(3-m)}$$

```
[Out] -2^(3/2+1/2*m)*a*(e*cos(d*x+c))^(3-m)*hypergeom([-1/2-1/2*m, 3/2-1/2*m],[5/
2-1/2*m],1/2-1/2*sin(d*x+c))*(1+sin(d*x+c))^(-1/2-1/2*m)*(a+a*sin(d*x+c))^(
-1+m)/d/e/(3-m)
```

Rubi [A]
time = 0.08, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2768, 72, 71}

$$-\frac{a2^{\frac{m}{2}+\frac{3}{2}}(\sin(c+dx)+1)^{\frac{1}{2}(-m-1)}(a\sin(c+dx)+a)^{m-1}(e\cos(c+dx))^{3-m}\,{}_2F_1\left(\frac{1}{2}(-m-1),\frac{3-m}{2};\frac{5-m}{2};\frac{1}{2}(1-\sin(c+dx))\right)}{de(3-m)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(2 - m)*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] -((2^(3/2 + m/2)*a*(e*Cos[c + d*x])^(3 - m)*Hypergeometric2F1[(-1 - m)/2, (
3 - m)/2, (5 - m)/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^((-1 - m)/2)*
(a + a*Sin[c + d*x])^(-1 + m))/(d*e*(3 - m)))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
```

```
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m]
```

Rubi steps

$$\int (e\cos(c+dx))^{2-m}(a+a\sin(c+dx))^m\,dx = \frac{\left(a^2(e\cos(c+dx))^{3-m}(a-a\sin(c+dx))^{\frac{1}{2}(-3+m)}(a+a\sin(\right.}{}$$

$$= \frac{\left(2^{\frac{1}{2}+\frac{m}{2}}a^2(e\cos(c+dx))^{3-m}(a-a\sin(c+dx))^{\frac{1}{2}(-3+m)}(a+\right.}{}$$

$$= -\frac{2^{\frac{3}{2}+\frac{m}{2}}a(e\cos(c+dx))^{3-m}\,{}_2F_1\left(\frac{1}{2}(-1-m),\frac{3-m}{2};\frac{5-m}{2};\frac{1}{2}(1-\right.}{de}$$

Mathematica [A]
time = 0.25, size = 113, normalized size = 0.98

$$\frac{2^{\frac{3+m}{2}}e^2\cos^3(c+dx)(e\cos(c+dx))^{-m}\,{}_2F_1\left(\frac{1}{2}(-1-m),\frac{3-m}{2};\frac{5-m}{2};\frac{1}{2}(1-\sin(c+dx))\right)(1+\sin(c+dx))^{\frac{1}{2}(-3-m)}(a(1+\sin(c+dx)))^m}{d(-3+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(2 - m)*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] (2^((3 + m)/2)*e^2*Cos[c + d*x]^3*Hypergeometric2F1[(-1 - m)/2, (3 - m)/2,
(5 - m)/2, (1 - Sin[c + d*x])/2]*(1 + Sin[c + d*x])^((-3 - m)/2)*(a*(1 + Si
n[c + d*x]))^m)/(d*(-3 + m)*(e*Cos[c + d*x])^m)
```

Maple [F]
time = 0.20, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{2-m}(a+a\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(2-m)*(a+a*sin(d*x+c))^m,x)
```

```
[Out] int((e*cos(d*x+c))^(2-m)*(a+a*sin(d*x+c))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(2-m)*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((cos(d*x + c)*e)^(-m + 2)*(a*sin(d*x + c) + a)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(2-m)*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((cos(d*x + c)*e)^(-m + 2)*(a*sin(d*x + c) + a)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sin{(c+dx)}+1))^m (e\cos{(c+dx)})^{2-m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(2-m)*(a+a*sin(d*x+c))**m,x)
```

```
[Out] Integral((a*(sin(c + d*x) + 1))**m*(e*cos(c + d*x))**(2 - m), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(2-m)*(a+a*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((cos(d*x + c)*e)^(-m + 2)*(a*sin(d*x + c) + a)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(e\,\cos\left(c+d\,x\right)\right)}^{2-m}\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^m \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(2 - m)*(a + a*sin(c + d*x))^m,x)
```

```
[Out] int((e*cos(c + d*x))^(2 - m)*(a + a*sin(c + d*x))^m, x)
```

3.367 $\int (e\cos(c+dx))^{5-2m}(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=150

$$-\frac{8a^3(e\cos(c+dx))^{6-2m}(a+a\sin(c+dx))^{-3+m}}{de(5-m)\left(12-7m+m^2\right)}-\frac{4a^2(e\cos(c+dx))^{6-2m}(a+a\sin(c+dx))^{-2+m}}{de\left(20-9m+m^2\right)}-\frac{a(e\cos$$

```
[Out] -8*a^3*(e*cos(d*x+c))^(6-2*m)*(a+a*sin(d*x+c))^(-3+m)/d/e/(-m^3+12*m^2-47*m
+60)-4*a^2*(e*cos(d*x+c))^(6-2*m)*(a+a*sin(d*x+c))^(-2+m)/d/e/(4-m)/(5-m)-a
*(e*cos(d*x+c))^(6-2*m)*(a+a*sin(d*x+c))^(-1+m)/d/e/(5-m)
```

Rubi [A]
time = 0.17, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2753, 2752}

$$-\frac{8a^3(a\sin(c+dx)+a)^{m-3}(e\cos(c+dx))^{6-2m}}{de(5-m)\left(m^2-7m+12\right)}-\frac{4a^2(a\sin(c+dx)+a)^{m-2}(e\cos(c+dx))^{6-2m}}{de\left(m^2-9m+20\right)}-\frac{a(a\sin(c+dx)+a)^{m-1}(e\cos(c+dx))^{6-2m}}{de(5-m)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(5 - 2*m)*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] (-8*a^3*(e*Cos[c + d*x])^(6 - 2*m)*(a + a*Sin[c + d*x])^(-3 + m))/(d*e*(5 -
 m)*(12 - 7*m + m^2)) - (4*a^2*(e*Cos[c + d*x])^(6 - 2*m)*(a + a*Sin[c + d*
x])^(-2 + m))/(d*e*(20 - 9*m + m^2)) - (a*(e*Cos[c + d*x])^(6 - 2*m)*(a + a
*Sin[c + d*x])^(-1 + m))/(d*e*(5 - m))
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2753

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && N
eQ[m + p, 0]
```

Rubi steps

$$\int (e\cos(c+dx))^{5-2m}(a+a\sin(c+dx))^m\,dx = -\frac{a(e\cos(c+dx))^{6-2m}(a+a\sin(c+dx))^{-1+m}}{de(5-m)} + \frac{(4a)\int(e\cos\ldots}{\ldots}$$
$$= -\frac{4a^2(e\cos(c+dx))^{6-2m}(a+a\sin(c+dx))^{-2+m}}{de\,(20-9m+m^2)} - \frac{a(e\cos(\ldots}{\ldots}$$
$$= -\frac{8a^3(e\cos(c+dx))^{6-2m}(a+a\sin(c+dx))^{-3+m}}{de(3-m)\,(20-9m+m^2)} - \frac{4a^2(e\cos\ldots}{\ldots}$$

Mathematica [A]
time = 0.40, size = 105, normalized size = 0.70

$$\frac{e^5\cos^6(c+dx)(e\cos(c+dx))^{-2m}(a(1+\sin(c+dx)))^m\,(32-11m+m^2+2(18-9m+m^2)\sin(c+dx)+(12-7m+m^2)\sin^2(c+dx))}{d(-5+m)(-4+m)(-3+m)(1+\sin(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(5 - 2*m)*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] (e^5*Cos[c + d*x]^6*(a*(1 + Sin[c + d*x]))^m*(32 - 11*m + m^2 + 2*(18 - 9*m
 + m^2)*Sin[c + d*x] + (12 - 7*m + m^2)*Sin[c + d*x]^2))/(d*(-5 + m)*(-4 +
m)*(-3 + m)*(e*Cos[c + d*x])^(2*m)*(1 + Sin[c + d*x])^3)
```

Maple [F]
time = 0.21, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{5-2m}\,(a+a\sin(dx+c))^m\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(5-2*m)*(a+a*sin(d*x+c))^m,x)
```

```
[Out] int((e*cos(d*x+c))^(5-2*m)*(a+a*sin(d*x+c))^m,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 689 vs. 2(145) = 290.
time = 0.55, size = 689, normalized size = 4.59

$$\frac{\left(\left(m^2-11\,m+32\right)a^m e^5-\frac{2\left(m^2-15\,m+60\right)a^m e^5\sin(dx+c)}{\cos(dx+c)+1}-\frac{\left(3\,m^2-m-160\right)a^m e^5\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{8\left(m^2-7\,m-20\right)a^m e^5\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{2\left(m^2+5\,m+160\right)a^m e^5\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{4\left(3\,m^2-13\,m+116\right)a^m e^5\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{2\left(m^2+5\,m+160\right)a^m e^5\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{8\left(m^2-7\,m-20\right)a^m e^5\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{\left(3\,m^2-m-160\right)a^m e^5\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{2\left(m^2-15\,m+60\right)a^m e^5\sin(dx+c)^9}{(\cos(dx+c)+1)^9}+\frac{\left(m^2-11\,m+32\right)a^m e^5\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}\right)e^{\left(-2\,m\log\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)+m\log\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)\right)}}{\left(m^3e^{(2\,m)}-12\,m^2e^{(2\,m)}+47\,me^{(2\,m)}+\frac{5\left(m^3e^{(2\,m)}-12\,m^2e^{(2\,m)}+47\,me^{(2\,m)}-60\,e^{(2\,m)}\right)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{10\left(m^3e^{(2\,m)}-12\,m^2e^{(2\,m)}+47\,me^{(2\,m)}-60\,e^{(2\,m)}\right)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{10\left(m^3e^{(2\,m)}-12\,m^2e^{(2\,m)}+47\,me^{(2\,m)}-60\,e^{(2\,m)}\right)\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{5\left(m^3e^{(2\,m)}-12\,m^2e^{(2\,m)}+47\,me^{(2\,m)}-60\,e^{(2\,m)}\right)\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{\left(m^3e^{(2\,m)}-12\,m^2e^{(2\,m)}+47\,me^{(2\,m)}-60\,e^{(2\,m)}\right)\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}-60\,e^{(2\,m)}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5-2*m)*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] ((m^2 - 11*m + 32)*a^m*e^5 - 2*(m^2 - 15*m + 60)*a^m*e^5*sin(d*x + c)/(cos(
d*x + c) + 1) - (3*m^2 - m - 160)*a^m*e^5*sin(d*x + c)^2/(cos(d*x + c) + 1)
^2 + 8*(m^2 - 7*m - 20)*a^m*e^5*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 2*(m^
```

```
2 + 5*m + 160)*a^m*e^5*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 4*(3*m^2 - 13*
m + 116)*a^m*e^5*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 2*(m^2 + 5*m + 160)*
a^m*e^5*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 8*(m^2 - 7*m - 20)*a^m*e^5*si
n(d*x + c)^7/(cos(d*x + c) + 1)^7 - (3*m^2 - m - 160)*a^m*e^5*sin(d*x + c)^
8/(cos(d*x + c) + 1)^8 - 2*(m^2 - 15*m + 60)*a^m*e^5*sin(d*x + c)^9/(cos(d*
x + c) + 1)^9 + (m^2 - 11*m + 32)*a^m*e^5*sin(d*x + c)^10/(cos(d*x + c) + 1
)^10)*e^(-2*m*log(-sin(d*x + c)/(cos(d*x + c) + 1) + 1) + m*log(sin(d*x + c
)^2/(cos(d*x + c) + 1)^2 + 1))/((m^3*e^(2*m) - 12*m^2*e^(2*m) + 47*m*e^(2*m
) + 5*(m^3*e^(2*m) - 12*m^2*e^(2*m) + 47*m*e^(2*m) - 60*e^(2*m))*sin(d*x +
c)^2/(cos(d*x + c) + 1)^2 + 10*(m^3*e^(2*m) - 12*m^2*e^(2*m) + 47*m*e^(2*m)
 - 60*e^(2*m))*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 10*(m^3*e^(2*m) - 12*m
^2*e^(2*m) + 47*m*e^(2*m) - 60*e^(2*m))*sin(d*x + c)^6/(cos(d*x + c) + 1)^6
 + 5*(m^3*e^(2*m) - 12*m^2*e^(2*m) + 47*m*e^(2*m) - 60*e^(2*m))*sin(d*x + c
)^8/(cos(d*x + c) + 1)^8 + (m^3*e^(2*m) - 12*m^2*e^(2*m) + 47*m*e^(2*m) - 6
0*e^(2*m))*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 - 60*e^(2*m))*d)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 315 vs. $2(145) = 290$.
time = 0.42, size = 315, normalized size = 2.10

$$-\frac{\left((m^2-7m+12)\cos(dx+c)^3-(m^2-11m+24)\cos(dx+c)^2-2(m^2-9m+22)\cos(dx+c)-\left((m^2-7m+12)\cos(dx+c)^2+2(m^2-9m+18)\cos(dx+c)-8\right)\sin(dx+c)-8\right)(\cos(dx+c)e)^{-2m+5}(a\sin(dx+c)+a)^m}{4dm^3-(dm^3-12dm^2+47dm-60d)\cos(dx+c)^3-48dm^2-3(dm^3-12dm^2+47dm-60d)\cos(dx+c)^2+188dm+2(dm^3-12dm^2+47dm-60d)\cos(dx+c)+\left(4dm^3-48dm^2-(dm^3-12dm^2+47dm-60d)\cos(dx+c)^2+188dm+2(dm^3-12dm^2+47dm-60d)\cos(dx+c)-240d\right)\sin(dx+c)-240d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5-2*m)*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] -((m^2 - 7*m + 12)*cos(d*x + c)^3 - (m^2 - 11*m + 24)*cos(d*x + c)^2 - 2*(m
^2 - 9*m + 22)*cos(d*x + c) - ((m^2 - 7*m + 12)*cos(d*x + c)^2 + 2*(m^2 - 9
*m + 18)*cos(d*x + c) - 8)*sin(d*x + c) - 8)*(cos(d*x + c)*e)^(-2*m + 5)*(a
*sin(d*x + c) + a)^m/(4*d*m^3 - (d*m^3 - 12*d*m^2 + 47*d*m - 60*d)*cos(d*x
+ c)^3 - 48*d*m^2 - 3*(d*m^3 - 12*d*m^2 + 47*d*m - 60*d)*cos(d*x + c)^2 + 1
88*d*m + 2*(d*m^3 - 12*d*m^2 + 47*d*m - 60*d)*cos(d*x + c) + (4*d*m^3 - 48*
d*m^2 - (d*m^3 - 12*d*m^2 + 47*d*m - 60*d)*cos(d*x + c)^2 + 188*d*m + 2*(d*
m^3 - 12*d*m^2 + 47*d*m - 60*d)*cos(d*x + c) - 240*d)*sin(d*x + c) - 240*d)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(5-2*m)*(a+a*sin(d*x+c))**m,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5-2*m)*(a+a*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((cos(d*x + c)*e)^(-2*m + 5)*(a*sin(d*x + c) + a)^m, x)
```

Mupad [B]
time = 13.48, size = 601, normalized size = 4.01

$$-\frac{{\left(a+a\,\sin\left(c+d\,x\right)\right)}^m\,\left(-\frac{{\left(e\,\cos\left(c+d\,x\right)\right)}^{5-2\,m}\,\left(m^2-7\,m+12\right)}{d\,\left(m^3-12\,m^2+47\,m-60\right)}+\frac{{\left(e\,\cos\left(c+d\,x\right)\right)}^{5-2\,m}\,\left(\cos\left(c+d\,x\right)+\sin\left(c+d\,x\right)\,1{}\mathrm{i}\right)\,\left(m^2\,3{}\mathrm{i}-m\,29{}\mathrm{i}+60{}\mathrm{i}\right)}{d\,\left(m^3-12\,m^2+47\,m-60\right)}-\frac{{\left(e\,\cos\left(c+d\,x\right)\right)}^{5-2\,m}\,\left(\cos\left(5\,c+5\,d\,x\right)+\sin\left(5\,c+5\,d\,x\right)\,1{}\mathrm{i}\right)\,\left(m^2\,1{}\mathrm{i}-m\,7{}\mathrm{i}+12{}\mathrm{i}\right)}{d\,\left(m^3-12\,m^2+47\,m-60\right)}+\frac{{\left(e\,\cos\left(c+d\,x\right)\right)}^{5-2\,m}\,\left(\cos\left(4\,c+4\,d\,x\right)+\sin\left(4\,c+4\,d\,x\right)\,1{}\mathrm{i}\right)\,\left(3\,m^2-29\,m+60\right)}{d\,\left(m^3-12\,m^2+47\,m-60\right)}+\frac{{\left(e\,\cos\left(c+d\,x\right)\right)}^{5-2\,m}\,\left(\cos\left(2\,c+2\,d\,x\right)+\sin\left(2\,c+2\,d\,x\right)\,1{}\mathrm{i}\right)\,\left(2\,m^2-22\,m+80\right)}{d\,\left(m^3-12\,m^2+47\,m-60\right)}+\frac{{\left(e\,\cos\left(c+d\,x\right)\right)}^{5-2\,m}\,\left(\cos\left(3\,c+3\,d\,x\right)+\sin\left(3\,c+3\,d\,x\right)\,1{}\mathrm{i}\right)\,\left(m^2\,2{}\mathrm{i}-m\,22{}\mathrm{i}+80{}\mathrm{i}\right)}{d\,\left(m^3-12\,m^2+47\,m-60\right)}\right)}{5\,\cos\left(c+d\,x\right)+\sin\left(c+d\,x\right)\,5{}\mathrm{i}-10\,\cos\left(3\,c+3\,d\,x\right)+\cos\left(5\,c+5\,d\,x\right)-\sin\left(3\,c+3\,d\,x\right)\,10{}\mathrm{i}+\sin\left(5\,c+5\,d\,x\right)\,1{}\mathrm{i}+\frac{m^3\,1{}\mathrm{i}-m^2\,12{}\mathrm{i}+m\,47{}\mathrm{i}-60{}\mathrm{i}}{m^3-12\,m^2+47\,m-60}-\frac{10\,\left(\cos\left(2\,c+2\,d\,x\right)+\sin\left(2\,c+2\,d\,x\right)\,1{}\mathrm{i}\right)\,\left(m^3\,1{}\mathrm{i}-m^2\,12{}\mathrm{i}+m\,47{}\mathrm{i}-60{}\mathrm{i}\right)}{m^3-12\,m^2+47\,m-60}+\frac{5\,\left(\cos\left(4\,c+4\,d\,x\right)+\sin\left(4\,c+4\,d\,x\right)\,1{}\mathrm{i}\right)\,\left(m^3\,1{}\mathrm{i}-m^2\,12{}\mathrm{i}+m\,47{}\mathrm{i}-60{}\mathrm{i}\right)}{m^3-12\,m^2+47\,m-60}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(5 - 2*m)*(a + a*sin(c + d*x))^m,x)
```

```
[Out] -((a + a*sin(c + d*x))^m*(((e*cos(c + d*x))^(5 - 2*m)*(cos(c + d*x) + sin(c
 + d*x)*1i)*(m^2*3i - m*29i + 60i))/(d*(47*m - 12*m^2 + m^3 - 60)) - ((e*co
s(c + d*x))^(5 - 2*m)*(m^2 - 7*m + 12))/(d*(47*m - 12*m^2 + m^3 - 60)) - ((
e*cos(c + d*x))^(5 - 2*m)*(cos(5*c + 5*d*x) + sin(5*c + 5*d*x)*1i)*(m^2*1i
- m*7i + 12i))/(d*(47*m - 12*m^2 + m^3 - 60)) + ((e*cos(c + d*x))^(5 - 2*m)
*(cos(4*c + 4*d*x) + sin(4*c + 4*d*x)*1i)*(3*m^2 - 29*m + 60))/(d*(47*m - 1
2*m^2 + m^3 - 60)) + ((e*cos(c + d*x))^(5 - 2*m)*(cos(2*c + 2*d*x) + sin(2*
c + 2*d*x)*1i)*(2*m^2 - 22*m + 80))/(d*(47*m - 12*m^2 + m^3 - 60)) + ((e*co
s(c + d*x))^(5 - 2*m)*(cos(3*c + 3*d*x) + sin(3*c + 3*d*x)*1i)*(m^2*2i - m*
22i + 80i))/(d*(47*m - 12*m^2 + m^3 - 60))))/(5*cos(c + d*x) + sin(c + d*x)
*5i - 10*cos(3*c + 3*d*x) + cos(5*c + 5*d*x) - sin(3*c + 3*d*x)*10i + sin(5
*c + 5*d*x)*1i + (m*47i - m^2*12i + m^3*1i - 60i)/(47*m - 12*m^2 + m^3 - 60
) - (10*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i)*(m*47i - m^2*12i + m^3*1i
- 60i))/(47*m - 12*m^2 + m^3 - 60) + (5*(cos(4*c + 4*d*x) + sin(4*c + 4*d*x
)*1i)*(m*47i - m^2*12i + m^3*1i - 60i))/(47*m - 12*m^2 + m^3 - 60))
```

3.368 $\int (e\cos(c+dx))^{3-2m}(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=94

$$-\frac{2a^2(e\cos(c+dx))^{4-2m}(a+a\sin(c+dx))^{-2+m}}{de\,(6-5m+m^2)}-\frac{a(e\cos(c+dx))^{4-2m}(a+a\sin(c+dx))^{-1+m}}{de(3-m)}$$

```
[Out] -2*a^2*(e*cos(d*x+c))^(4-2*m)*(a+a*sin(d*x+c))^(-2+m)/d/e/(2-m)/(3-m)-a*(e*
cos(d*x+c))^(4-2*m)*(a+a*sin(d*x+c))^(-1+m)/d/e/(3-m)
```

Rubi [A]

time = 0.10, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2753, 2752}

$$-\frac{2a^2(a\sin(c+dx)+a)^{m-2}(e\cos(c+dx))^{4-2m}}{de\,(m^2-5m+6)}-\frac{a(a\sin(c+dx)+a)^{m-1}(e\cos(c+dx))^{4-2m}}{de(3-m)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(3 - 2*m)*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] (-2*a^2*(e*Cos[c + d*x])^(4 - 2*m)*(a + a*Sin[c + d*x])^(-2 + m))/(d*e*(6 -
 5*m + m^2)) - (a*(e*Cos[c + d*x])^(4 - 2*m)*(a + a*Sin[c + d*x])^(-1 + m))
/(d*e*(3 - m))
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2753

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && N
eQ[m + p, 0]
```

Rubi steps

$$\int (e\cos(c+dx))^{3-2m}(a+a\sin(c+dx))^m\,dx = -\frac{a(e\cos(c+dx))^{4-2m}(a+a\sin(c+dx))^{-1+m}}{de(3-m)}+\frac{(2a)\int(e$$

$$= -\frac{2a^2(e\cos(c+dx))^{4-2m}(a+a\sin(c+dx))^{-2+m}}{de\,(6-5m+m^2)}-\frac{a(e\cos$$

Mathematica [A]
time = 0.22, size = 72, normalized size = 0.77

$$\frac{e^3\cos^4(c+dx)(e\cos(c+dx))^{-2m}(a(1+\sin(c+dx)))^m(-4+m+(-2+m)\sin(c+dx))}{d(-3+m)(-2+m)(1+\sin(c+dx))^2}$$

Antiderivative was successfully verified.

[In] Integrate[(e*Cos[c + d*x])^(3 - 2*m)*(a + a*Sin[c + d*x])^m,x]

[Out] (e^3*Cos[c + d*x]^4*(a*(1 + Sin[c + d*x]))^m*(-4 + m + (-2 + m)*Sin[c + d*x
]))/(d*(-3 + m)*(-2 + m)*(e*Cos[c + d*x])^(2*m)*(1 + Sin[c + d*x])^2)

Maple [F]
time = 0.21, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{3-2m}(a+a\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*cos(d*x+c))^(3-2*m)*(a+a*sin(d*x+c))^m,x)

[Out] int((e*cos(d*x+c))^(3-2*m)*(a+a*sin(d*x+c))^m,x)

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 378 vs. 2(91) = 182.
time = 0.51, size = 378, normalized size = 4.02

$$\frac{\left(a^m(m-4)e^3-\frac{2a^m(m-6)e^3\sin(dx+c)}{\cos(dx+c)+1}-\frac{a^m(m+12)e^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{4a^m(m+2)e^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{a^m(m+12)e^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{2a^m(m-6)e^3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{a^m(m-4)e^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}\right)e^{\left(-2m\log\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)+m\log\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)\right)}}{\left(m^2e^{(2m)}-5me^{(2m)}+\frac{3(m^2e^{(2m)}-5me^{(2m)}+6e^{(2m)})\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3(m^2e^{(2m)}-5me^{(2m)}+6e^{(2m)})\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{(m^2e^{(2m)}-5me^{(2m)}+6e^{(2m)})\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+6e^{(2m)}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*cos(d*x+c))^(3-2*m)*(a+a*sin(d*x+c))^m,x, algorithm="maxima")

[Out] (a^m*(m - 4)*e^3 - 2*a^m*(m - 6)*e^3*sin(d*x + c)/(cos(d*x + c) + 1) - a^m*
(m + 12)*e^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 4*a^m*(m + 2)*e^3*sin(d*
x + c)^3/(cos(d*x + c) + 1)^3 - a^m*(m + 12)*e^3*sin(d*x + c)^4/(cos(d*x +
c) + 1)^4 - 2*a^m*(m - 6)*e^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + a^m*(m
- 4)*e^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6)*e^(-2*m*log(-sin(d*x + c)/(co
s(d*x + c) + 1) + 1) + m*log(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1))/((m^
2*e^(2*m) - 5*m*e^(2*m) + 3*(m^2*e^(2*m) - 5*m*e^(2*m) + 6*e^(2*m))*sin(d*x
 + c)^2/(cos(d*x + c) + 1)^2 + 3*(m^2*e^(2*m) - 5*m*e^(2*m) + 6*e^(2*m))*si
n(d*x + c)^4/(cos(d*x + c) + 1)^4 + (m^2*e^(2*m) - 5*m*e^(2*m) + 6*e^(2*m))
*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 6*e^(2*m))*d)

Fricas [A]
time = 0.36, size = 171, normalized size = 1.82

$$\frac{((m-2)\cos(dx+c)^2+(m-4)\cos(dx+c)+((m-2)\cos(dx+c)+2)\sin(dx+c)-2)(\cos(dx+c)e)^{-2m+3}(a\sin(dx+c)+a)^m}{2dm^2-(dm^2-5dm+6d)\cos(dx+c)^2-10dm+(dm^2-5dm+6d)\cos(dx+c)+(2dm^2-10dm+(dm^2-5dm+6d)\cos(dx+c)+12d)\sin(dx+c)+12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3-2*m)*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] ((m - 2)*cos(d*x + c)^2 + (m - 4)*cos(d*x + c) + ((m - 2)*cos(d*x + c) + 2)
*sin(d*x + c) - 2)*(cos(d*x + c)*e)^(-2*m + 3)*(a*sin(d*x + c) + a)^m/(2*d*
m^2 - (d*m^2 - 5*d*m + 6*d)*cos(d*x + c)^2 - 10*d*m + (d*m^2 - 5*d*m + 6*d)
*cos(d*x + c) + (2*d*m^2 - 10*d*m + (d*m^2 - 5*d*m + 6*d)*cos(d*x + c) + 12
*d)*sin(d*x + c) + 12*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(3-2*m)*(a+a*sin(d*x+c))**m,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3-2*m)*(a+a*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((cos(d*x + c)*e)^(-2*m + 3)*(a*sin(d*x + c) + a)^m, x)
```

Mupad [**B**]
time = 8.77, size = 241, normalized size = 2.56

$$\frac{e^3\,{\left(a\,\left(\sin\left(c+d\,x\right)+1\right)\right)}^m\,\left(14\,m-24\,\sin\left(c+d\,x\right)-36\,\sin\left(3\,c+3\,d\,x\right)-12\,\sin\left(5\,c+5\,d\,x\right)+24\,{\sin\left(2\,c+2\,d\,x\right)}^2-4\,{\sin\left(3\,c+3\,d\,x\right)}^2+8\,m\,\sin\left(c+d\,x\right)-17\,m\,\left(2\,{\sin\left(c+d\,x\right)}^2-1\right)+12\,m\,\sin\left(3\,c+3\,d\,x\right)+4\,m\,\sin\left(5\,c+5\,d\,x\right)-2\,m\,\left(2\,{\sin\left(2\,c+2\,d\,x\right)}^2-1\right)+m\,\left(2\,{\sin\left(3\,c+3\,d\,x\right)}^2-1\right)+132\,{\sin\left(c+d\,x\right)}^2-128\right)}{8\,d\,{\left(-e\,\left(2\,{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)\right)}^{2\,m}\,\left(m^2-5\,m+6\right)\,\left(12\,{\sin\left(c+d\,x\right)}^2+15\,\sin\left(c+d\,x\right)-\sin\left(3\,c+3\,d\,x\right)+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(3 - 2*m)*(a + a*sin(c + d*x))^m,x)
```

```
[Out] (e^3*(a*(sin(c + d*x) + 1))^m*(14*m - 24*sin(c + d*x) - 36*sin(3*c + 3*d*x)
 - 12*sin(5*c + 5*d*x) + 24*sin(2*c + 2*d*x)^2 - 4*sin(3*c + 3*d*x)^2 + 8*m
*sin(c + d*x) - 17*m*(2*sin(c + d*x)^2 - 1) + 12*m*sin(3*c + 3*d*x) + 4*m*s
in(5*c + 5*d*x) - 2*m*(2*sin(2*c + 2*d*x)^2 - 1) + m*(2*sin(3*c + 3*d*x)^2
- 1) + 132*sin(c + d*x)^2 - 128))/(8*d*(-e*(2*sin(c/2 + (d*x)/2)^2 - 1))^(2
*m)*(m^2 - 5*m + 6)*(15*sin(c + d*x) - sin(3*c + 3*d*x) + 12*sin(c + d*x)^2
 + 4))
```

3.369 $\int (e\cos(c+dx))^{1-2m}(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=44

$$-\frac{a(e\cos(c+dx))^{2-2m}(a+a\sin(c+dx))^{-1+m}}{de(1-m)}$$

```
[Out] -a*(e*cos(d*x+c))^(2-2*m)*(a+a*sin(d*x+c))^(-1+m)/d/e/(1-m)
```

Rubi [A]
time = 0.04, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$, Rules used = {2752}

$$-\frac{a(a\sin(c+dx)+a)^{m-1}(e\cos(c+dx))^{2-2m}}{de(1-m)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(1 - 2*m)*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] -((a*(e*Cos[c + d*x])^(2 - 2*m)*(a + a*Sin[c + d*x])^(-1 + m))/(d*e*(1 - m)
))
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\int (e\cos(c+dx))^{1-2m}(a+a\sin(c+dx))^m\,dx = -\frac{a(e\cos(c+dx))^{2-2m}(a+a\sin(c+dx))^{-1+m}}{de(1-m)}$$

Mathematica [A]
time = 0.16, size = 43, normalized size = 0.98

$$-\frac{e(e\cos(c+dx))^{-2m}(-1+\sin(c+dx))(a(1+\sin(c+dx)))^m}{d(-1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(1 - 2*m)*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] -((e*(-1 + Sin[c + d*x])*(a*(1 + Sin[c + d*x]))^m)/(d*(-1 + m)*(e*Cos[c + d
*x])^(2*m)))
```

Maple [F]
time = 0.19, size = 0, normalized size = 0.00

$$\int \left(e \cos \left(dx+c\right)\right)^{1-2m} \left(a+a \sin \left(dx+c\right)\right)^{m} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(1-2*m)*(a+a*sin(d*x+c))^m,x)
```

```
[Out] int((e*cos(d*x+c))^(1-2*m)*(a+a*sin(d*x+c))^m,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 155 vs. $2(41) = 82$.
time = 0.53, size = 155, normalized size = 3.52

$$\frac{\left(a^{m}e-\frac{2\,a^{m}e\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}+\frac{a^{m}e\sin\left(dx+c\right)^{2}}{\left(\cos\left(dx+c\right)+1\right)^{2}}\right)e^{\left(-2\,m\log\left(-\frac{\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}+1\right)+m\log\left(\frac{\sin\left(dx+c\right)^{2}}{\left(\cos\left(dx+c\right)+1\right)^{2}}+1\right)\right)}}{\left(me^{\left(2\,m\right)}+\frac{\left(me^{\left(2\,m\right)}-e^{\left(2\,m\right)}\right)\sin\left(dx+c\right)^{2}}{\left(\cos\left(dx+c\right)+1\right)^{2}}-e^{\left(2\,m\right)}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1-2*m)*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] (a^m*e - 2*a^m*e*sin(d*x + c)/(cos(d*x + c) + 1) + a^m*e*sin(d*x + c)^2/(co
s(d*x + c) + 1)^2)*e^(-2*m*log(-sin(d*x + c)/(cos(d*x + c) + 1) + 1) + m*lo
g(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1))/((m*e^(2*m) + (m*e^(2*m) - e^(2
*m))*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - e^(2*m))*d)
```

Fricas [A]
time = 0.35, size = 81, normalized size = 1.84

$$\frac{\left(\cos\left(dx+c\right)e\right)^{-2\,m+1}\left(a\sin\left(dx+c\right)+a\right)^{m}\left(\cos\left(dx+c\right)-\sin\left(dx+c\right)+1\right)}{dm+\left(dm-d\right)\cos\left(dx+c\right)+\left(dm-d\right)\sin\left(dx+c\right)-d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1-2*m)*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] (cos(d*x + c)*e)^(-2*m + 1)*(a*sin(d*x + c) + a)^m*(cos(d*x + c) - sin(d*x
+ c) + 1)/(d*m + (d*m - d)*cos(d*x + c) + (d*m - d)*sin(d*x + c) - d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sin\left(c+dx\right)+1\right)\right)^{m}\left(e\cos\left(c+dx\right)\right)^{1-2m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(1-2*m)*(a+a*sin(d*x+c))**m,x)
```

```
[Out] Integral((a*(sin(c + d*x) + 1))**m*(e*cos(c + d*x))**(1 - 2*m), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1-2*m)*(a+a*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((cos(d*x + c)*e)^(-2*m + 1)*(a*sin(d*x + c) + a)^m, x)
```

Mupad [B]
time = 5.58, size = 58, normalized size = 1.32

$$\frac{e\left(\cos\left(2\,c+2\,d\,x\right)+1\right)\,{\left(a\left(\sin\left(c+d\,x\right)+1\right)\right)}^m}{2\,d\,{\left(e\,\cos\left(c+d\,x\right)\right)}^{2\,m}\,\left(m-1\right)\,\left(\sin\left(c+d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(1 - 2*m)*(a + a*sin(c + d*x))^m,x)
```

```
[Out] (e*(cos(2*c + 2*d*x) + 1)*(a*(sin(c + d*x) + 1))^m)/(2*d*(e*cos(c + d*x))^(
2*m)*(m - 1)*(sin(c + d*x) + 1))
```

3.370 $\int (e\cos(c+dx))^{-1-2m}(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=61

$$\frac{(e\cos(c+dx))^{-2m}\,{}_2F_1\left(1,-m;1-m;\frac{1}{2}(1-\sin(c+dx))\right)(a+a\sin(c+dx))^m}{2dem}$$

```
[Out] 1/2*hypergeom([1, -m],[1-m],1/2-1/2*sin(d*x+c))*(a+a*sin(d*x+c))^m/d/e/m/((
e*cos(d*x+c))^(2*m))
```

Rubi [A]
time = 0.05, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2768, 7, 70}

$$\frac{(a\sin(c+dx)+a)^m(e\cos(c+dx))^{-2m}\,{}_2F_1\left(1,-m;1-m;\frac{1}{2}(1-\sin(c+dx))\right)}{2dem}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(-1 - 2*m)*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] (Hypergeometric2F1[1, -m, 1 - m, (1 - Sin[c + d*x])/2]*(a + a*Sin[c + d*x])
^m)/(2*d*e*m*(e*Cos[c + d*x])^(2*m))
```

Rule 7

```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> Int[u*Px^Simplify[p], x] /; PolyQ[Px, x]
 &&  !RationalQ[p] && FreeQ[p, x] && RationalQ[Simplify[p]]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int (e\cos(c+dx))^{-1-2m}(a+a\sin(c+dx))^m\,dx = \frac{(a^2(e\cos(c+dx))^{-2m}(a-a\sin(c+dx))^m(a+a\sin(c+dx}{}$$

$$= \frac{(a^2(e\cos(c+dx))^{-2m}(a-a\sin(c+dx))^m(a+a\sin(c+dx}{de}$$

$$= \frac{(e\cos(c+dx))^{-2m}\,{}_2F_1\left(1,-m;1-m;\frac{1}{2}(1-\sin(c+dx))\right)(}{2dem}$$

Mathematica [A]
time = 0.07, size = 61, normalized size = 1.00

$$\frac{(e\cos(c+dx))^{-2m}\,{}_2F_1\left(1,-m;1-m;\frac{1}{2}(1-\sin(c+dx))\right)(a(1+\sin(c+dx)))^m}{2dem}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(-1 - 2*m)*(a + a*Sin[c + d*x])^m,x]
[Out] (Hypergeometric2F1[1, -m, 1 - m, (1 - Sin[c + d*x])/2]*(a*(1 + Sin[c + d*x]
))^m)/(2*d*e*m*(e*Cos[c + d*x])^(2*m))
```

Maple [F]
time = 0.17, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{-1-2m}(a+a\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(-1-2*m)*(a+a*sin(d*x+c))^m,x)
[Out] int((e*cos(d*x+c))^(-1-2*m)*(a+a*sin(d*x+c))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-1-2*m)*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
[Out] integrate((cos(d*x + c)*e)^(-2*m - 1)*(a*sin(d*x + c) + a)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-1-2*m)*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
[Out] integral((cos(d*x + c)*e)^(-2*m - 1)*(a*sin(d*x + c) + a)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sin(c+dx)+1))^m (e\cos(c+dx))^{-2m-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(-1-2*m)*(a+a*sin(d*x+c))**m,x)
[Out] Integral((a*(sin(c + d*x) + 1))**m*(e*cos(c + d*x))**(-2*m - 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-1-2*m)*(a+a*sin(d*x+c))^m,x, algorithm="giac")
[Out] integrate((cos(d*x + c)*e)^(-2*m - 1)*(a*sin(d*x + c) + a)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a+a\,\sin(c+d\,x))^m}{(e\,\cos(c+d\,x))^{2\,m+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^m/(e*cos(c + d*x))^(2*m + 1),x)
[Out] int((a + a*sin(c + d*x))^m/(e*cos(c + d*x))^(2*m + 1), x)
```

3.371 $\int (e\cos (c+dx))^{-3-2m}(a+a\sin (c+dx))^m\,dx$

Optimal. Leaf size=70

$$\frac{(e\cos(c+dx))^{-2(1+m)}\,{}_2F_1\left(2,-1-m;-m;\frac{1}{2}(1-\sin(c+dx))\right)(a+a\sin(c+dx))^{1+m}}{4ade(1+m)}$$

```
[Out] 1/4*hypergeom([2, -1-m],[-m],1/2-1/2*sin(d*x+c))*(a+a*sin(d*x+c))^(1+m)/a/d
/e/(1+m)/((e*cos(d*x+c))^(2+2*m))
```

Rubi [A]
time = 0.05, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2768, 7, 70}

$$\frac{(a\sin(c+dx)+a)^{m+1}(e\cos(c+dx))^{-2(m+1)}\,{}_2F_1\left(2,-m-1;-m;\frac{1}{2}(1-\sin(c+dx))\right)}{4ade(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(-3 - 2*m)*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] (Hypergeometric2F1[2, -1 - m, -m, (1 - Sin[c + d*x])/2]*(a + a*Sin[c + d*x]
)^(1 + m))/(4*a*d*e*(1 + m)*(e*Cos[c + d*x])^(2*(1 + m)))
```

Rule 7

```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> Int[u*Px^Simplify[p], x] /; PolyQ[Px, x]
 &&  !RationalQ[p] && FreeQ[p, x] && RationalQ[Simplify[p]]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int (e\cos(c+dx))^{-3-2m}(a+a\sin(c+dx))^m\,dx = \frac{\left(a^2(e\cos(c+dx))^{-2-2m}(a-a\sin(c+dx))^{\frac{1}{2}(2+2m)}(a+a

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-3-2*m)*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
[Out] integral((cos(d*x + c)*e)^(-2*m - 3)*(a*sin(d*x + c) + a)^m, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(-3-2*m)*(a+a*sin(d*x+c))**m,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-3-2*m)*(a+a*sin(d*x+c))^m,x, algorithm="giac")
[Out] integrate((cos(d*x + c)*e)^(-2*m - 3)*(a*sin(d*x + c) + a)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\,\sin(c+d\,x))^m}{(e\,\cos(c+d\,x))^{2\,m+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^m/(e*cos(c + d*x))^(2*m + 3),x)
[Out] int((a + a*sin(c + d*x))^m/(e*cos(c + d*x))^(2*m + 3), x)
```

3.372 $\int (e\cos(c+dx))^{4-2m}(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=89

$$\frac{2^{\frac{5}{2}-m}(e\cos(c+dx))^{5-2m}\,{}_2F_1\left(\frac{5}{2},\frac{1}{2}(-3+2m);\frac{7}{2};\frac{1}{2}(1+\sin(c+dx))\right)(1-\sin(c+dx))^{-\frac{5}{2}+m}(a+a\sin(c+dx))^m}{5de}$$

```
[Out] 1/5*2^(5/2-m)*(e*cos(d*x+c))^(5-2*m)*hypergeom([5/2, -3/2+m],[7/2],1/2+1/2*
sin(d*x+c))*(1-sin(d*x+c))^(-5/2+m)*(a+a*sin(d*x+c))^m/d/e
```

Rubi [A]
time = 0.07, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {2768, 7, 72, 71}

$$\frac{2^{\frac{5}{2}-m}(1-\sin(c+dx))^{m-\frac{5}{2}}(a\sin(c+dx)+a)^m(e\cos(c+dx))^{5-2m}\,{}_2F_1\left(\frac{5}{2},\frac{1}{2}(2m-3);\frac{7}{2};\frac{1}{2}(\sin(c+dx)+1)\right)}{5de}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(4 - 2*m)*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] (2^(5/2 - m)*(e*Cos[c + d*x])^(5 - 2*m)*Hypergeometric2F1[5/2, (-3 + 2*m)/2
, 7/2, (1 + Sin[c + d*x])/2]*(1 - Sin[c + d*x])^(-5/2 + m)*(a + a*Sin[c + d
*x])^m)/(5*d*e)
```

Rule 7

```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> Int[u*Px^Simplify[p], x] /; PolyQ[Px, x]
 &&  !RationalQ[p] && FreeQ[p, x] && RationalQ[Simplify[p]]
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int (e\cos(c+dx))^{4-2m}(a+a\sin(c+dx))^m\,dx = \frac{\left(a^2(e\cos(c+dx))^{5-2m}(a-a\sin(c+dx))^{\frac{1}{2}(-5+2m)}(a+a\sin\right.}{}$$

$$= \frac{\left(a^2(e\cos(c+dx))^{5-2m}(a-a\sin(c+dx))^{\frac{1}{2}(-5+2m)}(a+a\sin\right.}{}$$

$$= \frac{\left(2^{\frac{3}{2}-m}a^3(e\cos(c+dx))^{5-2m}(a-a\sin(c+dx))^{\frac{1}{2}-m+\frac{1}{2}(-5+2m}\right.}{}$$

$$= \frac{2^{\frac{5}{2}-m}(e\cos(c+dx))^{5-2m}\,{}_2F_1\left(\frac{5}{2},\frac{1}{2}(-3+2m);\frac{7}{2};\frac{1}{2}(1+\sin(c\right.}{5de}$$

Mathematica [A]
time = 0.22, size = 96, normalized size = 1.08

$$\frac{4\sqrt{2}\,e^4\cos^5(c+dx)(e\cos(c+dx))^{-2m}\,{}_2F_1\left(-\frac{3}{2},\frac{5}{2}-m;\frac{7}{2}-m;\frac{1}{2}(1-\sin(c+dx))\right)(a(1+\sin(c+dx)))^m}{d(-5+2m)(1+\sin(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(4 - 2*m)*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] (4*Sqrt[2]*e^4*Cos[c + d*x]^5*Hypergeometric2F1[-3/2, 5/2 - m, 7/2 - m, (1
- Sin[c + d*x])/2]*(a*(1 + Sin[c + d*x]))^m)/(d*(-5 + 2*m)*(e*Cos[c + d*x])
^(2*m)*(1 + Sin[c + d*x])^(5/2))
```

Maple [F]
time = 0.18, size = 0, normalized size = 0.00

$$\int \left(e\cos\left(dx+c\right)\right)^{4-2m}\left(a+a\sin\left(dx+c\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(4-2*m)*(a+a*sin(d*x+c))^m,x)
```

```
[Out] int((e*cos(d*x+c))^(4-2*m)*(a+a*sin(d*x+c))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(4-2*m)*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
[Out] integrate((cos(d*x + c)*e)^(-2*m + 4)*(a*sin(d*x + c) + a)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(4-2*m)*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
[Out] integral((cos(d*x + c)*e)^(-2*m + 4)*(a*sin(d*x + c) + a)^m, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(4-2*m)*(a+a*sin(d*x+c))**m,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(4-2*m)*(a+a*sin(d*x+c))^m,x, algorithm="giac")
[Out] integrate((cos(d*x + c)*e)^(-2*m + 4)*(a*sin(d*x + c) + a)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (e\,\cos\left(c+d\,x\right))^{4-2\,m}\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(4 - 2*m)*(a + a*sin(c + d*x))^m,x)
[Out] int((e*cos(c + d*x))^(4 - 2*m)*(a + a*sin(c + d*x))^m, x)
```

3.373 $\int (e\cos(c+dx))^{2-2m}(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=89

$$\frac{2^{\frac{3}{2}-m}(e\cos(c+dx))^{3-2m}\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}(-1+2m);\frac{5}{2};\frac{1}{2}(1+\sin(c+dx))\right)(1-\sin(c+dx))^{-\frac{3}{2}+m}(a+a\sin(c+dx)}{3de}$$

```
[Out] 1/3*2^(3/2-m)*(e*cos(d*x+c))^(3-2*m)*hypergeom([3/2, -1/2+m],[5/2],1/2+1/2*
sin(d*x+c))*(1-sin(d*x+c))^(-3/2+m)*(a+a*sin(d*x+c))^m/d/e
```

Rubi [A]
time = 0.07, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {2768, 7, 72, 71}

$$\frac{2^{\frac{3}{2}-m}(1-\sin(c+dx))^{m-\frac{3}{2}}(a\sin(c+dx)+a)^m(e\cos(c+dx))^{3-2m}\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}(2m-1);\frac{5}{2};\frac{1}{2}(\sin(c+dx)+1)\right)}{3de}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(2 - 2*m)*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] (2^(3/2 - m)*(e*Cos[c + d*x])^(3 - 2*m)*Hypergeometric2F1[3/2, (-1 + 2*m)/2
, 5/2, (1 + Sin[c + d*x])/2]*(1 - Sin[c + d*x])^(-3/2 + m)*(a + a*Sin[c + d
*x])^m)/(3*d*e)
```

Rule 7

```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> Int[u*Px^Simplify[p], x] /; PolyQ[Px, x]
 &&  !RationalQ[p] && FreeQ[p, x] && RationalQ[Simplify[p]]
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m]
```

Rubi steps

$$\int (e\cos(c+dx))^{2-2m}(a+a\sin(c+dx))^m\,dx = \frac{\left(a^2(e\cos(c+dx))^{3-2m}(a-a\sin(c+dx))^{\frac{1}{2}(-3+2m)}(a+a\,\mathrm{si}\right.}{}$$

$$= \frac{\left(a^2(e\cos(c+dx))^{3-2m}(a-a\sin(c+dx))^{\frac{1}{2}(-3+2m)}(a+a\,\mathrm{si}\right.}{}$$

$$= \frac{\left(2^{\frac{1}{2}-m}a^2(e\cos(c+dx))^{3-2m}(a-a\sin(c+dx))^{\frac{1}{2}-m+\frac{1}{2}(-3+}\right.}{}$$

$$= \frac{2^{\frac{3}{2}-m}(e\cos(c+dx))^{3-2m}\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}(-1+2m);\frac{5}{2};\frac{1}{2}(1+\sin(c\right.}{3de}$$

Mathematica [A]
time = 0.18, size = 96, normalized size = 1.08

$$\frac{2\sqrt{2}\,e^2\cos^3(c+dx)(e\cos(c+dx))^{-2m}\,{}_2F_1\left(-\frac{1}{2},\frac{3}{2}-m;\frac{5}{2}-m;\frac{1}{2}(1-\sin(c+dx))\right)(a(1+\sin(c+dx)))^m}{d(-3+2m)(1+\sin(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(2 - 2*m)*(a + a*Sin[c + d*x])^m,x]
[Out] (2*Sqrt[2]*e^2*Cos[c + d*x]^3*Hypergeometric2F1[-1/2, 3/2 - m, 5/2 - m, (1
- Sin[c + d*x])/2]*(a*(1 + Sin[c + d*x]))^m)/(d*(-3 + 2*m)*(e*Cos[c + d*x])
^(2*m)*(1 + Sin[c + d*x])^(3/2))
```

Maple [F]
time = 0.19, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{2-2m}(a+a\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(2-2*m)*(a+a*sin(d*x+c))^m,x)
[Out] int((e*cos(d*x+c))^(2-2*m)*(a+a*sin(d*x+c))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(2-2*m)*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
[Out] integrate((cos(d*x + c)*e)^(-2*m + 2)*(a*sin(d*x + c) + a)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(2-2*m)*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
[Out] integral((cos(d*x + c)*e)^(-2*m + 2)*(a*sin(d*x + c) + a)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sin (c+dx)+1))^m (e\cos (c+dx))^{2-2m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(2-2*m)*(a+a*sin(d*x+c))**m,x)
[Out] Integral((a*(sin(c + d*x) + 1))**m*(e*cos(c + d*x))**(2 - 2*m), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(2-2*m)*(a+a*sin(d*x+c))^m,x, algorithm="giac")
[Out] integrate((cos(d*x + c)*e)^(-2*m + 2)*(a*sin(d*x + c) + a)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(e\,\cos \left(c+d\,x\right)\right)}^{2-2\,m}\,{\left(a+a\,\sin \left(c+d\,x\right)\right)}^m \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(2 - 2*m)*(a + a*sin(c + d*x))^m,x)
[Out] int((e*cos(c + d*x))^(2 - 2*m)*(a + a*sin(c + d*x))^m, x)
```

3.374 $\int (e\cos(c+dx))^{-2m}(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=86

$$\frac{2^{\frac{1}{2}-m}(e\cos(c+dx))^{1-2m}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1+2m);\frac{3}{2};\frac{1}{2}(1+\sin(c+dx))\right)(1-\sin(c+dx))^{-\frac{1}{2}+m}(a+a\sin(c+dx))}{de}$$

```
[Out] 2^(1/2-m)*(e*cos(d*x+c))^(1-2*m)*hypergeom([1/2, 1/2+m],[3/2],1/2+1/2*sin(d
*x+c))*(1-sin(d*x+c))^(-1/2+m)*(a+a*sin(d*x+c))^m/d/e
```

Rubi [A]
time = 0.06, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2768, 7, 72, 71}

$$\frac{2^{\frac{1}{2}-m}(1-\sin(c+dx))^{m-\frac{1}{2}}(a\sin(c+dx)+a)^m(e\cos(c+dx))^{1-2m}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(2m+1);\frac{3}{2};\frac{1}{2}(\sin(c+dx)+1)\right)}{de}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^m/(e*Cos[c + d*x])^(2*m),x]
```

```
[Out] (2^(1/2 - m)*(e*Cos[c + d*x])^(1 - 2*m)*Hypergeometric2F1[1/2, (1 + 2*m)/2,
 3/2, (1 + Sin[c + d*x])/2]*(1 - Sin[c + d*x])^(-1/2 + m)*(a + a*Sin[c + d*
x])^m)/(d*e)
```

Rule 7

```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> Int[u*Px^Simplify[p], x] /; PolyQ[Px, x]
 &&  !RationalQ[p] && FreeQ[p, x] && RationalQ[Simplify[p]]
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int (e\cos(c+dx))^{-2m}(a+a\sin(c+dx))^m\,dx = \frac{\left(a^2(e\cos(c+dx))^{1-2m}(a-a\sin(c+dx))^{\frac{1}{2}(-1+2m)}(a+a\sin(\right.}{}$$

$$= \frac{\left(a^2(e\cos(c+dx))^{1-2m}(a-a\sin(c+dx))^{\frac{1}{2}(-1+2m)}(a+a\sin(\right.}{d}$$

$$= \frac{\left(2^{-\frac{1}{2}-m}a^2(e\cos(c+dx))^{1-2m}(a-a\sin(c+dx))^{-\frac{1}{2}-m+\frac{1}{2}(-1+}\right.}{}$$

$$= \frac{2^{\frac{1}{2}-m}(e\cos(c+dx))^{1-2m}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1+2m);\frac{3}{2};\frac{1}{2}(1+\sin(c+\right.}{de}$$

Mathematica [A]
time = 0.09, size = 90, normalized size = 1.05

$$\frac{\sqrt{2}\,\cos(c+dx)(e\cos(c+dx))^{-2m}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2}-m;\frac{1}{2}(1-\sin(c+dx))\right)(a(1+\sin(c+dx)))^m}{d(-1+2m)\sqrt{1+\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^m/(e*Cos[c + d*x])^(2*m),x]
```

```
[Out] (Sqrt[2]*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - m, 3/2 - m, (1 - Sin[c +
 d*x])/2]*(a*(1 + Sin[c + d*x]))^m)/(d*(-1 + 2*m)*(e*Cos[c + d*x])^(2*m)*Sq
rt[1 + Sin[c + d*x]])
```

Maple [F]
time = 0.16, size = 0, normalized size = 0.00

$$\int (a+a\sin(dx+c))^m\,(e\cos(dx+c))^{-2m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^m/((e*cos(d*x+c))^(2*m)),x)
```

```
[Out] int((a+a*sin(d*x+c))^m/((e*cos(d*x+c))^(2*m)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^m/((e*cos(d*x+c))^(2*m)),x, algorithm="maxima")
[Out] integrate((a*sin(d*x + c) + a)^m/(cos(d*x + c)*e)^(2*m), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^m/((e*cos(d*x+c))^(2*m)),x, algorithm="fricas")
[Out] integral((a*sin(d*x + c) + a)^m/(cos(d*x + c)*e)^(2*m), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sin (c + dx) + 1))^m (e \cos (c + dx))^{-2m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**m/((e*cos(d*x+c))**(2*m)),x)
[Out] Integral((a*(sin(c + d*x) + 1))**m/(e*cos(c + d*x))**(2*m), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^m/((e*cos(d*x+c))^(2*m)),x, algorithm="giac")
[Out] integrate((a*sin(d*x + c) + a)^m/(cos(d*x + c)*e)^(2*m), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + a\,\sin (c + d\,x))^m}{(e\,\cos (c + d\,x))^{2\,m}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^m/(e*cos(c + d*x))^(2*m),x)
[Out] int((a + a*sin(c + d*x))^m/(e*cos(c + d*x))^(2*m), x)
```

3.375 $\int (e\cos(c+dx))^{-2-2m}(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=87

$$-\frac{2^{-\frac{1}{2}-m}(e\cos(c+dx))^{-1-2m}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2}(3+2m);\frac{1}{2};\frac{1}{2}(1+\sin(c+dx))\right)(1-\sin(c+dx))^{\frac{1}{2}+m}(a+a\sin(c+dx))^m}{de}$$

```
[Out] -2^(-1/2-m)*(e*cos(d*x+c))^(-1-2*m)*hypergeom([-1/2, 3/2+m],[1/2],1/2+1/2*s
in(d*x+c))*(1-sin(d*x+c))^(1/2+m)*(a+a*sin(d*x+c))^m/d/e
```

Rubi [A]
time = 0.07, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {2768, 7, 72, 71}

$$-\frac{2^{-m-\frac{1}{2}}(1-\sin(c+dx))^{m+\frac{1}{2}}(a\sin(c+dx)+a)^m(e\cos(c+dx))^{-2m-1}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2}(2m+3);\frac{1}{2};\frac{1}{2}(\sin(c+dx)+1)\right)}{de}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(-2 - 2*m)*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] -((2^(-1/2 - m)*(e*Cos[c + d*x])^(-1 - 2*m)*Hypergeometric2F1[-1/2, (3 + 2*
m)/2, 1/2, (1 + Sin[c + d*x])/2]*(1 - Sin[c + d*x])^(1/2 + m)*(a + a*Sin[c
+ d*x])^m)/(d*e))
```

Rule 7

```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> Int[u*Px^Simplify[p], x] /; PolyQ[Px, x]
 &&  !RationalQ[p] && FreeQ[p, x] && RationalQ[Simplify[p]]
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m]
```

Rubi steps

$$\int (e\cos(c+dx))^{-2-2m}(a+a\sin(c+dx))^m\,dx = \frac{\left(a^2(e\cos(c+dx))^{-1-2m}(a-a\sin(c+dx))^{\frac{1}{2}(1+2m)}(a+a\ldots\right.}{\ldots}$$

$$= \frac{\left(a^2(e\cos(c+dx))^{-1-2m}(a-a\sin(c+dx))^{\frac{1}{2}(1+2m)}(a+a\ldots\right.}{\ldots}$$

$$= \frac{\left(2^{-\frac{3}{2}-m}a(e\cos(c+dx))^{-1-2m}(a-a\sin(c+dx))^{-\frac{1}{2}-m+\frac{1}{2}}\ldots\right.}{\ldots}$$

$$= -\frac{2^{-\frac{1}{2}-m}(e\cos(c+dx))^{-1-2m}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2}(3+2m);\frac{1}{2};\frac{1}{2}(1+\ldots\right.}{\ldots}$$

Mathematica [A]
time = 0.22, size = 87, normalized size = 1.00

$$\frac{(e\cos(c+dx))^{-1-2m}\,{}_2F_1\left(\frac{3}{2},-\frac{1}{2}-m;\frac{1}{2}-m;\frac{1}{2}(1-\sin(c+dx))\right)\sqrt{1+\sin(c+dx)}\,(a(1+\sin(c+dx)))^m}{\sqrt{2}\,e(d+2dm)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(-2 - 2*m)*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] ((e*Cos[c + d*x])^(-1 - 2*m)*Hypergeometric2F1[3/2, -1/2 - m, 1/2 - m, (1 -
 Sin[c + d*x])/2]*Sqrt[1 + Sin[c + d*x]]*(a*(1 + Sin[c + d*x]))^m)/(Sqrt[2]
*e*(d + 2*d*m))
```

Maple [F]
time = 0.16, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{-2-2m}(a+a\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(-2-2*m)*(a+a*sin(d*x+c))^m,x)
```

```
[Out] int((e*cos(d*x+c))^(-2-2*m)*(a+a*sin(d*x+c))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-2-2*m)*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
[Out] integrate((cos(d*x + c)*e)^(-2*m - 2)*(a*sin(d*x + c) + a)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-2-2*m)*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
[Out] integral((cos(d*x + c)*e)^(-2*m - 2)*(a*sin(d*x + c) + a)^m, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(-2-2*m)*(a+a*sin(d*x+c))**m,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-2-2*m)*(a+a*sin(d*x+c))^m,x, algorithm="giac")
[Out] integrate((cos(d*x + c)*e)^(-2*m - 2)*(a*sin(d*x + c) + a)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + a \sin(c + d x))^m}{(e \cos(c + d x))^{2m+2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^m/(e*cos(c + d*x))^(2*m + 2),x)
[Out] int((a + a*sin(c + d*x))^m/(e*cos(c + d*x))^(2*m + 2), x)
```

3.376 $\int \cos^5(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=60

$$-\frac{b\cos^6(c+dx)}{6d}+\frac{a\sin(c+dx)}{d}-\frac{2a\sin^3(c+dx)}{3d}+\frac{a\sin^5(c+dx)}{5d}$$

```
[Out] -1/6*b*cos(d*x+c)^6/d+a*sin(d*x+c)/d-2/3*a*sin(d*x+c)^3/d+1/5*a*sin(d*x+c)^
5/d
```

Rubi [A]
time = 0.03, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2747, 655, 200}

$$\frac{a\sin^5(c+dx)}{5d}-\frac{2a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}-\frac{b\cos^6(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/6*(b*Cos[c + d*x]^6)/d + (a*Sin[c + d*x])/d - (2*a*Sin[c + d*x]^3)/(3*d)
 + (a*Sin[c + d*x]^5)/(5*d)
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^5(c+dx)(a+b\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int (a+x)\left(b^2-x^2\right)^2\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$
$$= -\frac{b\cos^6(c+dx)}{6d} + \frac{a\text{Subst}\left(\int \left(b^2-x^2\right)^2\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$
$$= -\frac{b\cos^6(c+dx)}{6d} + \frac{a\text{Subst}\left(\int \left(b^4-2b^2x^2+x^4\right)\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$
$$= -\frac{b\cos^6(c+dx)}{6d} + \frac{a\sin(c+dx)}{d} - \frac{2a\sin^3(c+dx)}{3d} + \frac{a\sin^5(c+dx)}{5d}$$

Mathematica [A]
time = 0.02, size = 63, normalized size = 1.05

$$-\frac{b\cos^6(c+dx)}{6d} + \frac{5a\sin(c+dx)}{8d} + \frac{5a\sin(3(c+dx))}{48d} + \frac{a\sin(5(c+dx))}{80d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/6*(b*Cos[c + d*x]^6)/d + (5*a*Sin[c + d*x])/(8*d) + (5*a*Sin[3*(c + d*x)
])/(48*d) + (a*Sin[5*(c + d*x)])/(80*d)
```

Maple [A]
time = 0.28, size = 46, normalized size = 0.77

method	result
derivativedivides	$\frac{-\frac{b\left(\cos^6(dx+c)\right)}{6}+\frac{a\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}}{d}$
default	$\frac{-\frac{b\left(\cos^6(dx+c)\right)}{6}+\frac{a\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}}{d}$
risch	$\frac{5a\sin(dx+c)}{8d}-\frac{b\cos(6dx+6c)}{192d}+\frac{a\sin(5dx+5c)}{80d}-\frac{b\cos(4dx+4c)}{32d}+\frac{5a\sin(3dx+3c)}{48d}-\frac{5b\cos(2dx+2c)}{64d}$
norman	$\frac{\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{14a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{52a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5d}+\frac{52a\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5d}+\frac{14a\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{2a\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2b}{}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/6*b*cos(d*x+c)^6+1/5*a*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+
c))
```

Maxima [A]
time = 0.29, size = 70, normalized size = 1.17

$$\frac{5\,b\sin\left(dx+c\right)^{6}+6\,a\sin\left(dx+c\right)^{5}-15\,b\sin\left(dx+c\right)^{4}-20\,a\sin\left(dx+c\right)^{3}+15\,b\sin\left(dx+c\right)^{2}+30\,a\sin\left(dx+c\right)}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/30*(5*b*sin(d*x + c)^6 + 6*a*sin(d*x + c)^5 - 15*b*sin(d*x + c)^4 - 20*a*
sin(d*x + c)^3 + 15*b*sin(d*x + c)^2 + 30*a*sin(d*x + c))/d
```

Fricas [A]
time = 0.34, size = 51, normalized size = 0.85

$$-\frac{5\,b\cos\left(dx+c\right)^{6}-2\,\left(3\,a\cos\left(dx+c\right)^{4}+4\,a\cos\left(dx+c\right)^{2}+8\,a\right)\sin\left(dx+c\right)}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/30*(5*b*cos(d*x + c)^6 - 2*(3*a*cos(d*x + c)^4 + 4*a*cos(d*x + c)^2 + 8*
a)*sin(d*x + c))/d
```

Sympy [A]
time = 0.40, size = 83, normalized size = 1.38

$$\begin{cases} \frac{8a\sin^{5}\left(c+dx\right)}{15d}+\frac{4a\sin^{3}\left(c+dx\right)\cos^{2}\left(c+dx\right)}{3d}+\frac{a\sin\left(c+dx\right)\cos^{4}\left(c+dx\right)}{d}-\frac{b\cos^{6}\left(c+dx\right)}{6d} & \text{for}\ d\neq 0\\ x(a+b\sin\left(c\right))\cos^{5}\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+b*sin(d*x+c)),x)
```

```
[Out] Piecewise((8*a*sin(c + d*x)**5/(15*d) + 4*a*sin(c + d*x)**3*cos(c + d*x)**2
/(3*d) + a*sin(c + d*x)*cos(c + d*x)**4/d - b*cos(c + d*x)**6/(6*d), Ne(d,
0)), (x*(a + b*sin(c))*cos(c)**5, True))
```

Giac [A]
time = 5.95, size = 88, normalized size = 1.47

$$-\frac{b\cos\left(6\,dx+6\,c\right)}{192\,d}-\frac{b\cos\left(4\,dx+4\,c\right)}{32\,d}-\frac{5\,b\cos\left(2\,dx+2\,c\right)}{64\,d}+\frac{a\sin\left(5\,dx+5\,c\right)}{80\,d}+\frac{5\,a\sin\left(3\,dx+3\,c\right)}{48\,d}+\frac{5\,a\sin\left(dx+c\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/192*b*cos(6*d*x + 6*c)/d - 1/32*b*cos(4*d*x + 4*c)/d - 5/64*b*cos(2*d*x
+ 2*c)/d + 1/80*a*sin(5*d*x + 5*c)/d + 5/48*a*sin(3*d*x + 3*c)/d + 5/8*a*si
n(d*x + c)/d
```

Mupad [B]
time = 0.07, size = 68, normalized size = 1.13

$$\frac{\frac{b\sin(c+d\,x)^6}{6}+\frac{a\sin(c+d\,x)^5}{5}-\frac{b\sin(c+d\,x)^4}{2}-\frac{2\,a\sin(c+d\,x)^3}{3}+\frac{b\sin(c+d\,x)^2}{2}+a\,\sin\left(c+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(a + b*sin(c + d*x)),x)
```

```
[Out] (a*sin(c + d*x) - (2*a*sin(c + d*x)^3)/3 + (a*sin(c + d*x)^5)/5 + (b*sin(c
+ d*x)^2)/2 - (b*sin(c + d*x)^4)/2 + (b*sin(c + d*x)^6)/6)/d
```

3.377 $\int \cos^3(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=44

$$-\frac{b\cos^4(c+dx)}{4d}+\frac{a\sin(c+dx)}{d}-\frac{a\sin^3(c+dx)}{3d}$$

```
[Out] -1/4*b*cos(d*x+c)^4/d+a*sin(d*x+c)/d-1/3*a*sin(d*x+c)^3/d
```

Rubi [A]
time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2747, 655}

$$-\frac{a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}-\frac{b\cos^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/4*(b*Cos[c + d*x]^4)/d + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^3)/(3*d)
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \cos^3(c+dx)(a+b\sin(c+dx))\,dx &= \frac{\text{Subst}\left(\int (a+x)\left(b^2-x^2\right)\,dx,x,b\sin(c+dx)\right)}{b^3d}\\
&= -\frac{b\cos^4(c+dx)}{4d}+\frac{a\text{Subst}\left(\int \left(b^2-x^2\right)\,dx,x,b\sin(c+dx)\right)}{b^3d}\\
&= -\frac{b\cos^4(c+dx)}{4d}+\frac{a\sin(c+dx)}{d}-\frac{a\sin^3(c+dx)}{3d}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 44, normalized size = 1.00

$$-\frac{b\cos^4(c+dx)}{4d}+\frac{a\sin(c+dx)}{d}-\frac{a\sin^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Sin[c + d*x]),x]
[Out] -1/4*(b*Cos[c + d*x]^4)/d + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^3)/(3*d)
```

Maple [A]
time = 0.19, size = 36, normalized size = 0.82

method	result	siz
derivativedivides	$\frac{-\frac{b\left(\cos^4(dx+c)\right)}{4}+\frac{a\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}}{d}$	36
default	$\frac{-\frac{b\left(\cos^4(dx+c)\right)}{4}+\frac{a\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}}{d}$	36
risch	$\frac{3a\sin(dx+c)}{4d}-\frac{b\cos(4dx+4c)}{32d}+\frac{a\sin(3dx+3c)}{12d}-\frac{b\cos(2dx+2c)}{8d}$	59
norman	$\frac{\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{10a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{10a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{2a\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2b\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}$	118

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(-1/4*b*cos(d*x+c)^4+1/3*a*(2+cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A]
time = 0.28, size = 48, normalized size = 1.09

$$-\frac{3\,b\sin\left(dx+c\right)^4+4\,a\sin\left(dx+c\right)^3-6\,b\sin\left(dx+c\right)^2-12\,a\sin\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="maxima")
[Out] -1/12*(3*b*sin(d*x + c)^4 + 4*a*sin(d*x + c)^3 - 6*b*sin(d*x + c)^2 - 12*a*
sin(d*x + c))/d
```

Fricas [A]
time = 0.38, size = 39, normalized size = 0.89

$$-\frac{3\,b\cos\left(dx+c\right)^4-4\,\left(a\cos\left(dx+c\right)^2+2\,a\right)\sin\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/12*(3*b*cos(d*x + c)^4 - 4*(a*cos(d*x + c)^2 + 2*a)*sin(d*x + c))/d
```

Sympy [A]
time = 0.17, size = 60, normalized size = 1.36

$$\begin{cases} \frac{2a\sin^3(c+dx)}{3d} + \frac{a\sin(c+dx)\cos^2(c+dx)}{d} - \frac{b\cos^4(c+dx)}{4d} & \text{for } d \neq 0 \\ x(a+b\sin(c))\cos^3(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*sin(d*x+c)),x)
```

```
[Out] Piecewise((2*a*sin(c + d*x)**3/(3*d) + a*sin(c + d*x)*cos(c + d*x)**2/d - b
*cos(c + d*x)**4/(4*d), Ne(d, 0)), (x*(a + b*sin(c))*cos(c)**3, True))
```

Giac [A]
time = 6.14, size = 48, normalized size = 1.09

$$-\frac{3\,b\sin\left(dx+c\right)^4+4\,a\sin\left(dx+c\right)^3-6\,b\sin\left(dx+c\right)^2-12\,a\sin\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/12*(3*b*sin(d*x + c)^4 + 4*a*sin(d*x + c)^3 - 6*b*sin(d*x + c)^2 - 12*a*
sin(d*x + c))/d
```

Mupad [B]
time = 0.06, size = 46, normalized size = 1.05

$$\frac{-\frac{b\sin(c+d\,x)^4}{4}-\frac{a\sin(c+d\,x)^3}{3}+\frac{b\sin(c+d\,x)^2}{2}+a\,\sin\left(c+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a + b*sin(c + d*x)),x)
```

```
[Out] (a*sin(c + d*x) - (a*sin(c + d*x)^3)/3 + (b*sin(c + d*x)^2)/2 - (b*sin(c +
d*x)^4)/4)/d
```

3.378 $\int \cos(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=22

$$\frac{(a+b\sin(c+dx))^2}{2bd}$$

```
[Out] 1/2*(a+b*sin(d*x+c))^2/b/d
```

Rubi [A]
time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.27, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2747}

$$\frac{a\sin(c+dx)}{d}+\frac{b\sin^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Sin[c + d*x]),x]
```

```
[Out] (a*Sin[c + d*x])/d + (b*Sin[c + d*x]^2)/(2*d)
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a+b\sin(c+dx))\,dx &= \frac{\text{Subst}\left(\int (a+x)\,dx, x, b\sin(c+dx)\right)}{bd}\\ &= \frac{a\sin(c+dx)}{d}+\frac{b\sin^2(c+dx)}{2d}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 39, normalized size = 1.77

$$-\frac{b\cos^2(c+dx)}{2d}+\frac{a\cos(dx)\sin(c)}{d}+\frac{a\cos(c)\sin(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Sin[c + d*x]),x]
```

[Out] -1/2*(b*Cos[c + d*x]^2)/d + (a*Cos[d*x]*Sin[c])/d + (a*Cos[c]*Sin[d*x])/d

Maple [A]
time = 0.10, size = 25, normalized size = 1.14

method	result	size
derivativedivides	$\frac{\frac{\left(\sin^2(dx+c)\right)b}{2}+a\sin(dx+c)}{d}$	25
default	$\frac{\frac{\left(\sin^2(dx+c)\right)b}{2}+a\sin(dx+c)}{d}$	25
risch	$\frac{a\sin(dx+c)}{d}-\frac{b\cos(2dx+2c)}{4d}$	28
norman	$\frac{\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}$	67

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)*(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)

[Out] 1/d*(1/2*sin(d*x+c)^2*b+a*sin(d*x+c))

Maxima [A]
time = 0.29, size = 20, normalized size = 0.91

$$\frac{(b\sin(dx+c)+a)^2}{2\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="maxima")

[Out] 1/2*(b*sin(d*x + c) + a)^2/(b*d)

Fricas [A]
time = 0.34, size = 25, normalized size = 1.14

$$-\frac{b\cos(dx+c)^2-2\,a\sin(dx+c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="fricas")

[Out] -1/2*(b*cos(d*x + c)^2 - 2*a*sin(d*x + c))/d

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(15) = 30$.
time = 0.07, size = 34, normalized size = 1.55

$$\begin{cases}\frac{a\sin(c+dx)}{d}+\frac{b\sin^2(c+dx)}{2d} & \text{for } d\neq 0\\ x(a+b\sin(c))\cos(c) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c)),x)
```

```
[Out] Piecewise((a*sin(c + d*x)/d + b*sin(c + d*x)**2/(2*d), Ne(d, 0)), (x*(a + b
*sin(c))*cos(c), True))
```

Giac [A]
time = 6.46, size = 25, normalized size = 1.14

$$\frac{b \sin{(dx + c)}^2 + 2 a \sin{(dx + c)}}{2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(b*sin(d*x + c)^2 + 2*a*sin(d*x + c))/d
```

Mupad [B]
time = 0.04, size = 23, normalized size = 1.05

$$\frac{\sin{(c + d\,x)}\,\left(2\,a + b\,\sin{(c + d\,x)}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + b*sin(c + d*x)),x)
```

```
[Out] (sin(c + d*x)*(2*a + b*sin(c + d*x)))/(2*d)
```

3.379 $\int \sec(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=43

$$-\frac{(a+b)\log(1-\sin(c+dx))}{2d}+\frac{(a-b)\log(1+\sin(c+dx))}{2d}$$

```
[Out] -1/2*(a+b)*ln(1-sin(d*x+c))/d+1/2*(a-b)*ln(1+sin(d*x+c))/d
```

Rubi [A]
time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2747, 647, 31}

$$\frac{(a-b)\log(\sin(c+dx)+1)}{2d}-\frac{(a+b)\log(1-\sin(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/2*((a + b)*Log[1 - Sin[c + d*x]])/d + ((a - b)*Log[1 + Sin[c + d*x]])/(2
*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
(-a)*c]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec(c+dx)(a+b\sin(c+dx))\,dx = \frac{b\text{Subst}\left(\int \frac{a+x}{b^2-x^2}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= -\frac{(a-b)\text{Subst}\left(\int \frac{1}{-b-x}\,dx, x, b\sin(c+dx)\right)}{2d} + \frac{(a+b)\text{Subst}\left(\int \frac{1}{b-x}\,dx,\right.}{2d}$$
$$= -\frac{(a+b)\log(1-\sin(c+dx))}{2d} + \frac{(a-b)\log(1+\sin(c+dx))}{2d}$$

Mathematica [A]
time = 0.02, size = 26, normalized size = 0.60

$$\frac{a\tanh^{-1}(\sin(c+dx))}{d} - \frac{b\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sin[c + d*x]),x]
[Out] (a*ArcTanh[Sin[c + d*x]])/d - (b*Log[Cos[c + d*x]])/d
```

Maple [A]
time = 0.15, size = 32, normalized size = 0.74

method	result	size
derivativedivides	$\frac{a\ln(\sec(dx+c)+\tan(dx+c))-b\ln(\cos(dx+c))}{d}$	32
default	$\frac{a\ln(\sec(dx+c)+\tan(dx+c))-b\ln(\cos(dx+c))}{d}$	32
norman	$\frac{b\ln\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d} + \frac{(a-b)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d} - \frac{(a+b)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}$	62
risch	$ibx + \frac{2ibc}{d} + \frac{a\ln(e^{i(dx+c)}+i)}{d} - \frac{\ln(e^{i(dx+c)}+i)b}{d} - \frac{a\ln(e^{i(dx+c)}-i)}{d} - \frac{\ln(e^{i(dx+c)}-i)b}{d}$	90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(a*ln(sec(d*x+c)+tan(d*x+c))-b*ln(cos(d*x+c)))
```

Maxima [A]
time = 0.29, size = 35, normalized size = 0.81

$$\frac{(a-b)\log(\sin(dx+c)+1) - (a+b)\log(\sin(dx+c)-1)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*((a - b)*log(sin(d*x + c) + 1) - (a + b)*log(sin(d*x + c) - 1))/d
```

Fricas [A]
time = 0.35, size = 37, normalized size = 0.86

$$\frac{(a-b)\log\left(\sin\left(dx+c\right)+1\right)-(a+b)\log\left(-\sin\left(dx+c\right)+1\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="fricas")
[Out] 1/2*((a - b)*log(sin(d*x + c) + 1) - (a + b)*log(-sin(d*x + c) + 1))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int(a+b\sin\left(c+dx\right))\sec\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c)),x)
[Out] Integral((a + b*sin(c + d*x))*sec(c + d*x), x)
```

Giac [A]
time = 4.04, size = 37, normalized size = 0.86

$$\frac{(a-b)\log\left(\left|\sin\left(dx+c\right)+1\right|\right)-(a+b)\log\left(\left|\sin\left(dx+c\right)-1\right|\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="giac")
[Out] 1/2*((a - b)*log(abs(sin(d*x + c) + 1)) - (a + b)*log(abs(sin(d*x + c) - 1)
))/d
```

Mupad [B]
time = 0.07, size = 54, normalized size = 1.26

$$-\frac{\frac{a\,\ln(\sin(c+d\,x)-1)}{2}-\frac{a\,\ln(\sin(c+d\,x)+1)}{2}+\frac{b\,\ln(\sin(c+d\,x)-1)}{2}+\frac{b\,\ln(\sin(c+d\,x)+1)}{2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))/cos(c + d*x),x)
[Out] -((a*log(sin(c + d*x) - 1))/2 - (a*log(sin(c + d*x) + 1))/2 + (b*log(sin(c
+ d*x) - 1))/2 + (b*log(sin(c + d*x) + 1))/2)/d
```

3.380 $\int \sec^3(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=41

$$\frac{a\tanh^{-1}(\sin(c+dx))}{2d}+\frac{\sec^2(c+dx)(b+a\sin(c+dx))}{2d}$$

```
[Out] 1/2*a*arctanh(sin(d*x+c))/d+1/2*sec(d*x+c)^2*(b+a*sin(d*x+c))/d
```

Rubi [A]
time = 0.03, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2747, 653, 212}

$$\frac{\sec^2(c+dx)(a\sin(c+dx)+b)}{2d}+\frac{a\tanh^{-1}(\sin(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + b*Sin[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/(2*d) + (Sec[c + d*x]^2*(b + a*Sin[c + d*x]))/(2*
d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 653

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)/(2*a*c*(p + 1)))*(a + c*x^2)^(p + 1), x] + Dist[d*((2*p + 3)/(2*a
*(p + 1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^3(c+dx)(a+b\sin(c+dx))\,dx = \frac{b^3\text{Subst}\left(\int \frac{a+x}{(b^2-x^2)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{\sec^2(c+dx)(b+a\sin(c+dx))}{2d} + \frac{(ab)\text{Subst}\left(\int \frac{1}{b^2-x^2}\,dx, x, b\sin(c+\right.}{2d}$$
$$= \frac{a\tanh^{-1}(\sin(c+dx))}{2d} + \frac{\sec^2(c+dx)(b+a\sin(c+dx))}{2d}$$

Mathematica [A]
time = 0.03, size = 52, normalized size = 1.27

$$\frac{a\tanh^{-1}(\sin(c+dx))}{2d} + \frac{b\sec^2(c+dx)}{2d} + \frac{a\sec(c+dx)\tan(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Sin[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/(2*d) + (b*Sec[c + d*x]^2)/(2*d) + (a*Sec[c + d*x
]*Tan[c + d*x])/(2*d)
```

Maple [A]
time = 0.22, size = 50, normalized size = 1.22

method	result
derivativedivides	$\frac{a\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+\frac{b}{2\cos(dx+c)^2}}{d}$
default	$\frac{a\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+\frac{b}{2\cos(dx+c)^2}}{d}$
risch	$\frac{-ia\,e^{3i(dx+c)}+ia\,e^{i(dx+c)}+2b\,e^{2i(dx+c)}}{d\left(1+e^{2i(dx+c)}\right)^2} + \frac{a\ln\left(e^{i(dx+c)}+i\right)}{2d} - \frac{a\ln\left(e^{i(dx+c)}-i\right)}{2d}$
norman	$\frac{\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2b\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)} - \frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+1/2*b/cos(
d*x+c)^2)
```

Maxima [A]
time = 0.36, size = 53, normalized size = 1.29

$$\frac{a\log\left(\sin\left(dx+c\right)+1\right) - a\log\left(\sin\left(dx+c\right)-1\right) - \frac{2\left(a\sin(dx+c)+b\right)}{\sin(dx+c)^2-1}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*(a*log(sin(d*x + c) + 1) - a*log(sin(d*x + c) - 1) - 2*(a*sin(d*x + c)
+ b)/(sin(d*x + c)^2 - 1))/d
```

Fricas [A]
time = 0.36, size = 67, normalized size = 1.63

$$\frac{a\cos\left(dx+c\right)^{2}\log\left(\sin\left(dx+c\right)+1\right)-a\cos\left(dx+c\right)^{2}\log\left(-\sin\left(dx+c\right)+1\right)+2\,a\sin\left(dx+c\right)+2\,b}{4\,d\cos\left(dx+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(a*cos(d*x + c)^2*log(sin(d*x + c) + 1) - a*cos(d*x + c)^2*log(-sin(d*x
 + c) + 1) + 2*a*sin(d*x + c) + 2*b)/(d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sin\left(c+dx\right)\right)\sec^{3}\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((a + b*sin(c + d*x))*sec(c + d*x)**3, x)
```

Giac [A]
time = 3.61, size = 55, normalized size = 1.34

$$\frac{a\log\left(\left|\sin\left(dx+c\right)+1\right|\right)-a\log\left(\left|\sin\left(dx+c\right)-1\right|\right)-\frac{2\,\left(a\sin\left(dx+c\right)+b\right)}{\sin\left(dx+c\right)^{2}-1}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/4*(a*log(abs(sin(d*x + c) + 1)) - a*log(abs(sin(d*x + c) - 1)) - 2*(a*sin
(d*x + c) + b)/(sin(d*x + c)^2 - 1))/d
```

Mupad [B]
time = 0.07, size = 44, normalized size = 1.07

$$\frac{a\,\mathrm{atanh}\left(\sin\left(c+d\,x\right)\right)}{2\,d}-\frac{\frac{b}{2}+\frac{a\,\sin\left(c+d\,x\right)}{2}}{d\,\left({\sin\left(c+d\,x\right)}^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))/cos(c + d*x)^3,x)
```

```
[Out] (a*atanh(sin(c + d*x)))/(2*d) - (b/2 + (a*sin(c + d*x))/2)/(d*(sin(c + d*x)
^2 - 1))
```

3.381 $\int \sec^5(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=61

$$\frac{3a\tanh^{-1}(\sin(c+dx))}{8d}+\frac{\sec^4(c+dx)(b+a\sin(c+dx))}{4d}+\frac{3a\sec(c+dx)\tan(c+dx)}{8d}$$

```
[Out] 3/8*a*arctanh(sin(d*x+c))/d+1/4*sec(d*x+c)^4*(b+a*sin(d*x+c))/d+3/8*a*sec(d
*x+c)*tan(d*x+c)/d
```

Rubi [A]

time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {2747, 653, 205, 212}

$$\frac{\sec^4(c+dx)(a\sin(c+dx)+b)}{4d}+\frac{3a\tanh^{-1}(\sin(c+dx))}{8d}+\frac{3a\tan(c+dx)\sec(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*(a + b*Sin[c + d*x]),x]
```

```
[Out] (3*a*ArcTanh[Sin[c + d*x]])/(8*d) + (Sec[c + d*x]^4*(b + a*Sin[c + d*x]))/(
4*d) + (3*a*Sec[c + d*x]*Tan[c + d*x])/(8*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 653

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)/(2*a*c*(p + 1)))*(a + c*x^2)^(p + 1), x] + Dist[d*((2*p + 3)/(2*a
*(p + 1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^5(c+dx)(a+b\sin(c+dx))\,dx = \frac{b^5\text{Subst}\left(\int \frac{a+x}{(b^2-x^2)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{\sec^4(c+dx)(b+a\sin(c+dx))}{4d} + \frac{(3ab^3)\,\text{Subst}\left(\int \frac{1}{(b^2-x^2)^2}\,dx, x, b\sin\right.}{4d}$$
$$= \frac{\sec^4(c+dx)(b+a\sin(c+dx))}{4d} + \frac{3a\sec(c+dx)\tan(c+dx)}{8d} + \frac{(3a}{}$$
$$= \frac{3a\tanh^{-1}(\sin(c+dx))}{8d} + \frac{\sec^4(c+dx)(b+a\sin(c+dx))}{4d} + \frac{3a\sec(}{}$$

Mathematica [A]
time = 0.17, size = 68, normalized size = 1.11

$$\frac{b\sec^4(c+dx)}{4d} + \frac{a\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{3a\left(\tanh^{-1}(\sin(c+dx)) + \sec(c+dx)\tan(c+dx)\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*(a + b*Sin[c + d*x]),x]
```

```
[Out] (b*Sec[c + d*x]^4)/(4*d) + (a*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (3*a*(Ar
cTanh[Sin[c + d*x]] + Sec[c + d*x]*Tan[c + d*x]))/(8*d)
```

Maple [A]
time = 0.27, size = 63, normalized size = 1.03

method	result
derivativedivides	$\frac{a\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+\frac{b}{4\cos(dx+c)^4}}{d}$
default	$\frac{a\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+\frac{b}{4\cos(dx+c)^4}}{d}$
risch	$\frac{-3ia\,e^{7i(dx+c)}-11ia\,e^{5i(dx+c)}+11ia\,e^{3i(dx+c)}+16b\,e^{4i(dx+c)}+3ia\,e^{i(dx+c)}}{4d\left(1+e^{2i(dx+c)}\right)^4}+\frac{3a\ln\left(e^{i(dx+c)}+i\right)}{8d}-\frac{3a\ln\left(e^{i(dx+c)}-i\right.}{8d}$

norman	$\frac{\frac{2b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2b\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{5a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}+\frac{2a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{3a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}+\frac{2a\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{5a\left(\tan^9\right.}{}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+ta
n(d*x+c)))+1/4*b/cos(d*x+c)^4)
```

Maxima [A]
time = 0.30, size = 78, normalized size = 1.28

$$\frac{3\,a\log\left(\sin\left(dx+c\right)+1\right)-3\,a\log\left(\sin\left(dx+c\right)-1\right)-\frac{2\left(3\,a\sin(dx+c)^3-5\,a\sin(dx+c)-2\,b\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/16*(3*a*log(sin(d*x + c) + 1) - 3*a*log(sin(d*x + c) - 1) - 2*(3*a*sin(d*
x + c)^3 - 5*a*sin(d*x + c) - 2*b)/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1))
/d
```

Fricas [A]
time = 0.35, size = 82, normalized size = 1.34

$$\frac{3\,a\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-3\,a\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+1\right)+2\left(3\,a\cos\left(dx+c\right)^2+2\,a\right)\sin\left(dx+c\right)+4\,b}{16\,d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/16*(3*a*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*a*cos(d*x + c)^4*log(-si
n(d*x + c) + 1) + 2*(3*a*cos(d*x + c)^2 + 2*a)*sin(d*x + c) + 4*b)/(d*cos(d
*x + c)^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sin\left(c+dx\right)\right)\sec^5\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((a + b*sin(c + d*x))*sec(c + d*x)**5, x)
```

Giac [A]
time = 6.63, size = 70, normalized size = 1.15

$$\frac{3\,a\log\left(\left|\sin\left(dx+c\right)+1\right|\right)-3\,a\log\left(\left|\sin\left(dx+c\right)-1\right|\right)-\frac{2\left(3\,a\sin(dx+c)^{3}-5\,a\sin(dx+c)-2\,b\right)}{\left(\sin(dx+c)^{2}-1\right)^{2}}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/16*(3*a*log(abs(sin(d*x + c) + 1)) - 3*a*log(abs(sin(d*x + c) - 1)) - 2*(
3*a*sin(d*x + c)^3 - 5*a*sin(d*x + c) - 2*b)/(sin(d*x + c)^2 - 1)^2)/d
```

Mupad [B]
time = 5.14, size = 64, normalized size = 1.05

$$\frac{3\,a\,\mathrm{atanh}(\sin\left(c+d\,x\right))}{8\,d}+\frac{-\frac{3\,a\sin(c+d\,x)^{3}}{8}+\frac{5\,a\,\sin(c+d\,x)}{8}+\frac{b}{4}}{d\,\left(\sin\left(c+d\,x\right)^{4}-2\sin\left(c+d\,x\right)^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))/cos(c + d*x)^5,x)
```

```
[Out] (3*a*atanh(sin(c + d*x)))/(8*d) + (b/4 + (5*a*sin(c + d*x))/8 - (3*a*sin(c
+ d*x)^3)/8)/(d*(sin(c + d*x)^4 - 2*sin(c + d*x)^2 + 1))
```

3.382 $\int \cos^4(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=65

$$\frac{3ax}{8}-\frac{b\cos^5(c+dx)}{5d}+\frac{3a\cos(c+dx)\sin(c+dx)}{8d}+\frac{a\cos^3(c+dx)\sin(c+dx)}{4d}$$

```
[Out] 3/8*a*x-1/5*b*cos(d*x+c)^5/d+3/8*a*cos(d*x+c)*sin(d*x+c)/d+1/4*a*cos(d*x+c)
^3*sin(d*x+c)/d
```

Rubi [A]
time = 0.03, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2748, 2715, 8}

$$\frac{a\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3a\sin(c+dx)\cos(c+dx)}{8d}+\frac{3ax}{8}-\frac{b\cos^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + b*Sin[c + d*x]),x]
```

```
[Out] (3*a*x)/8 - (b*Cos[c + d*x]^5)/(5*d) + (3*a*Cos[c + d*x]*Sin[c + d*x])/(8*d
) + (a*Cos[c + d*x]^3*Sin[c + d*x])/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\int \cos^4(c+dx)(a+b\sin(c+dx))\,dx = -\frac{b\cos^5(c+dx)}{5d} + a\int \cos^4(c+dx)\,dx$$
$$= -\frac{b\cos^5(c+dx)}{5d} + \frac{a\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{1}{4}(3a)\int \cos^2(c+d$$
$$= -\frac{b\cos^5(c+dx)}{5d} + \frac{3a\cos(c+dx)\sin(c+dx)}{8d} + \frac{a\cos^3(c+dx)\sin(}{4d}$$
$$= \frac{3ax}{8} - \frac{b\cos^5(c+dx)}{5d} + \frac{3a\cos(c+dx)\sin(c+dx)}{8d} + \frac{a\cos^3(c+dx}{4d}$$

Mathematica [A]
time = 0.12, size = 62, normalized size = 0.95

$$\frac{3a(c+dx)}{8d} - \frac{b\cos^5(c+dx)}{5d} + \frac{a\sin(2(c+dx))}{4d} + \frac{a\sin(4(c+dx))}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + b*Sin[c + d*x]),x]
```

```
[Out] (3*a*(c + d*x))/(8*d) - (b*Cos[c + d*x]^5)/(5*d) + (a*Sin[2*(c + d*x)])/(4*
d) + (a*Sin[4*(c + d*x)])/(32*d)
```

Maple [A]
time = 0.22, size = 52, normalized size = 0.80

method	result
derivativedivides	$\frac{-\frac{b\left(\cos^5(dx+c)\right)}{5}+a\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}$
default	$\frac{-\frac{b\left(\cos^5(dx+c)\right)}{5}+a\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}$
risch	$\frac{3ax}{8}-\frac{b\cos(dx+c)}{8d}-\frac{b\cos(5dx+5c)}{80d}+\frac{a\sin(4dx+4c)}{32d}-\frac{b\cos(3dx+3c)}{16d}+\frac{a\sin(2dx+2c)}{4d}$
norman	$\frac{\frac{3ax}{8}-\frac{2b}{5d}+\frac{5a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}+\frac{a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-\frac{a\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-\frac{5a\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{15ax\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8}+\frac{15ax\left(\tan^4\left(\frac{dx}{2}\right.\right.}{4}}{\left(1+\tan^2\left(\frac{dx}{2}+\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/5*b*cos(d*x+c)^5+a*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/
8*d*x+3/8*c))
```

Maxima [A]
time = 0.29, size = 48, normalized size = 0.74

$$-\frac{32\,b\cos\left(dx+c\right)^{5}-5\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right)\right)a}{160\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/160*(32*b*cos(d*x + c)^5 - 5*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2
*d*x + 2*c))*a)/d
```

Fricas [A]
time = 0.34, size = 51, normalized size = 0.78

$$-\frac{8\,b\cos\left(dx+c\right)^{5}-15\,adx-5\,\left(2\,a\cos\left(dx+c\right)^{3}+3\,a\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{40\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/40*(8*b*cos(d*x + c)^5 - 15*a*d*x - 5*(2*a*cos(d*x + c)^3 + 3*a*cos(d*x
+ c))*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(60) = 120$.
time = 0.26, size = 124, normalized size = 1.91

$$\begin{cases}\frac{3ax\sin^{4}\left(c+dx\right)}{8}+\frac{3ax\sin^{2}\left(c+dx\right)\cos^{2}\left(c+dx\right)}{4}+\frac{3ax\cos^{4}\left(c+dx\right)}{8}+\frac{3a\sin^{3}\left(c+dx\right)\cos\left(c+dx\right)}{8d}+\frac{5a\sin\left(c+dx\right)\cos^{3}\left(c+dx\right)}{8d}-\frac{b\cos^{5}\left(c+dx\right)}{5d} & \text{for}\ d\neq 0\\ x\left(a+b\sin\left(c\right)\right)\cos^{4}\left(c\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+b*sin(d*x+c)),x)
```

```
[Out] Piecewise((3*a*x*sin(c + d*x)**4/8 + 3*a*x*sin(c + d*x)**2*cos(c + d*x)**2/
4 + 3*a*x*cos(c + d*x)**4/8 + 3*a*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 5*a*
sin(c + d*x)*cos(c + d*x)**3/(8*d) - b*cos(c + d*x)**5/(5*d), Ne(d, 0)), (x
*(a + b*sin(c))*cos(c)**4, True))
```

Giac [A]
time = 6.65, size = 77, normalized size = 1.18

$$\frac{3}{8}\,ax-\frac{b\cos\left(5\,dx+5\,c\right)}{80\,d}-\frac{b\cos\left(3\,dx+3\,c\right)}{16\,d}-\frac{b\cos\left(dx+c\right)}{8\,d}+\frac{a\sin\left(4\,dx+4\,c\right)}{32\,d}+\frac{a\sin\left(2\,dx+2\,c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 3/8*a*x - 1/80*b*cos(5*d*x + 5*c)/d - 1/16*b*cos(3*d*x + 3*c)/d - 1/8*b*cos
(d*x + c)/d + 1/32*a*sin(4*d*x + 4*c)/d + 1/4*a*sin(2*d*x + 2*c)/d
```

Mupad [B]
time = 8.69, size = 111, normalized size = 1.71

$$\frac{3\,a\,x}{8}-\frac{\frac{5\,a\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^9}{4}+2\,b\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8+\frac{a\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7}{2}+4\,b\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4-\frac{a\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3}{2}-\frac{5\,a\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}+\frac{2\,b}{5}}{d\,{\left({\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(a + b*sin(c + d*x)),x)
```

```
[Out] (3*a*x)/8 - ((2*b)/5 - (5*a*tan(c/2 + (d*x)/2))/4 - (a*tan(c/2 + (d*x)/2)^3
)/2 + (a*tan(c/2 + (d*x)/2)^7)/2 + (5*a*tan(c/2 + (d*x)/2)^9)/4 + 4*b*tan(c
/2 + (d*x)/2)^4 + 2*b*tan(c/2 + (d*x)/2)^8)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^5
)
```

3.383 $\int \cos^2(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=43

$$\frac{ax}{2}-\frac{b\cos^3(c+dx)}{3d}+\frac{a\cos(c+dx)\sin(c+dx)}{2d}$$

```
[Out] 1/2*a*x-1/3*b*cos(d*x+c)^3/d+1/2*a*cos(d*x+c)*sin(d*x+c)/d
```

Rubi [A]

time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2748, 2715, 8}

$$\frac{a\sin(c+dx)\cos(c+dx)}{2d}+\frac{ax}{2}-\frac{b\cos^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Sin[c + d*x]),x]
[Out] (a*x)/2 - (b*Cos[c + d*x]^3)/(3*d) + (a*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\begin{aligned}
\int \cos^2(c+dx)(a+b\sin(c+dx))\,dx &= -\frac{b\cos^3(c+dx)}{3d}+a\int \cos^2(c+dx)\,dx\\
&= -\frac{b\cos^3(c+dx)}{3d}+\frac{a\cos(c+dx)\sin(c+dx)}{2d}+\frac{1}{2}a\int 1\,dx\\
&= \frac{ax}{2}-\frac{b\cos^3(c+dx)}{3d}+\frac{a\cos(c+dx)\sin(c+dx)}{2d}
\end{aligned}$$

Mathematica [A]
time = 0.07, size = 46, normalized size = 1.07

$$\frac{a(c+dx)}{2d}-\frac{b\cos^3(c+dx)}{3d}+\frac{a\sin(2(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Sin[c + d*x]),x]
[Out] (a*(c + d*x))/(2*d) - (b*Cos[c + d*x]^3)/(3*d) + (a*Sin[2*(c + d*x)])/(4*d)
```

Maple [A]
time = 0.14, size = 41, normalized size = 0.95

method	result
derivativedivides	$\frac{-\frac{b\left(\cos^3(dx+c)\right)}{3}+a\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
default	$\frac{-\frac{b\left(\cos^3(dx+c)\right)}{3}+a\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
risch	$\frac{ax}{2}-\frac{b\cos(dx+c)}{4d}-\frac{b\cos(3dx+3c)}{12d}+\frac{a\sin(2dx+2c)}{4d}$
norman	$\frac{\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{ax}{2}-\frac{2b}{3d}-\frac{a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{3ax\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{3ax\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{ax\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-\frac{2b\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(-1/3*b*cos(d*x+c)^3+a*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))
```

Maxima [A]
time = 0.31, size = 37, normalized size = 0.86

$$-\frac{4\,b\cos\left(dx+c\right)^3-3\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)a}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="maxima")
[Out] -1/12*(4*b*cos(d*x + c)^3 - 3*(2*d*x + 2*c + sin(2*d*x + 2*c))*a)/d
```

Fricas [A]
time = 0.35, size = 37, normalized size = 0.86

$$-\frac{2\,b\cos\left(dx+c\right)^3-3\,adx-3\,a\cos\left(dx+c\right)\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/6*(2*b*cos(d*x + c)^3 - 3*a*d*x - 3*a*cos(d*x + c)*sin(d*x + c))/d
```

Sympy [A]
time = 0.11, size = 71, normalized size = 1.65

$$\begin{cases} \frac{ax\sin^2(c+dx)}{2} + \frac{ax\cos^2(c+dx)}{2} + \frac{a\sin(c+dx)\cos(c+dx)}{2d} - \frac{b\cos^3(c+dx)}{3d} & \text{for } d \neq 0 \\ x(a+b\sin(c))\cos^2(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*sin(d*x+c)),x)
```

```
[Out] Piecewise((a*x*sin(c + d*x)**2/2 + a*x*cos(c + d*x)**2/2 + a*sin(c + d*x)*c
os(c + d*x)/(2*d) - b*cos(c + d*x)**3/(3*d), Ne(d, 0)), (x*(a + b*sin(c))*c
os(c)**2, True))
```

Giac [A]
time = 4.72, size = 47, normalized size = 1.09

$$\frac{1}{2}ax - \frac{b\cos(3dx+3c)}{12d} - \frac{b\cos(dx+c)}{4d} + \frac{a\sin(2dx+2c)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*a*x - 1/12*b*cos(3*d*x + 3*c)/d - 1/4*b*cos(d*x + c)/d + 1/4*a*sin(2*d*
x + 2*c)/d
```

Mupad [B]
time = 7.38, size = 68, normalized size = 1.58

$$\frac{a\,x}{2} - \frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5 + 2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 - a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right) + \frac{2\,b}{3}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a + b*sin(c + d*x)),x)
```

```
[Out] (a*x)/2 - ((2*b)/3 - a*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^5 + 2*b*ta
n(c/2 + (d*x)/2)^4)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^3)
```

3.384 $\int \sec^2(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=23

$$\frac{b\sec(c+dx)}{d}+\frac{a\tan(c+dx)}{d}$$

```
[Out] b*sec(d*x+c)/d+a*tan(d*x+c)/d
```

Rubi [A]
time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2748, 3852, 8}

$$\frac{a\tan(c+dx)}{d}+\frac{b\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Sin[c + d*x]),x]
```

```
[Out] (b*Sec[c + d*x])/d + (a*Tan[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^2(c+dx)(a+b\sin(c+dx))\,dx &= \frac{b\sec(c+dx)}{d}+a\int \sec^2(c+dx)\,dx\\ &= \frac{b\sec(c+dx)}{d}-\frac{a\text{Subst}\left(\int 1\,dx,x,-\tan(c+dx)\right)}{d}\\ &= \frac{b\sec(c+dx)}{d}+\frac{a\tan(c+dx)}{d}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 23, normalized size = 1.00

$$\frac{b\sec(c+dx)}{d}+\frac{a\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Sin[c + d*x]),x]
```

```
[Out] (b*Sec[c + d*x])/d + (a*Tan[c + d*x])/d
```

Maple [A]
time = 0.15, size = 24, normalized size = 1.04

method	result	size
derivativedivides	$\frac{a\tan(dx+c)+\frac{b}{\cos(dx+c)}}{d}$	24
default	$\frac{a\tan(dx+c)+\frac{b}{\cos(dx+c)}}{d}$	24
risch	$\frac{2ia+2b\,e^{i(dx+c)}}{d\left(1+e^{2i(dx+c)}\right)}$	35
norman	$\frac{-\frac{2b}{d}-\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{2a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$	88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*tan(d*x+c)+b/cos(d*x+c))
```

Maxima [A]
time = 0.32, size = 23, normalized size = 1.00

$$\frac{a\tan\left(dx+c\right)+\frac{b}{\cos(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] (a*tan(d*x + c) + b/cos(d*x + c))/d
```

Fricas [A]
time = 0.33, size = 22, normalized size = 0.96

$$\frac{a\sin\left(dx+c\right)+b}{d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="fricas")
[Out] (a*sin(d*x + c) + b)/(d*cos(d*x + c))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \sin (c + dx)) \sec^2 (c + dx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*sin(d*x+c)),x)
[Out] Integral((a + b*sin(c + d*x))*sec(c + d*x)**2, x)
```

Giac [**A**]
time = 4.05, size = 33, normalized size = 1.43

$$-\frac{2 \left(a \tan \left(\frac{1}{2} \, dx + \frac{1}{2} \, c\right) + b\right)}{\left(\tan \left(\frac{1}{2} \, dx + \frac{1}{2} \, c\right)^2 - 1\right) d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="giac")
[Out] -2*(a*tan(1/2*d*x + 1/2*c) + b)/((tan(1/2*d*x + 1/2*c)^2 - 1)*d)
```

Mupad [**B**]
time = 5.13, size = 22, normalized size = 0.96

$$\frac{b + a \, \sin (c + d \, x)}{d \, \cos (c + d \, x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))/cos(c + d*x)^2,x)
[Out] (b + a*sin(c + d*x))/(d*cos(c + d*x))
```

3.385 $\int \sec^4(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=44

$$\frac{b\sec^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{a\tan^3(c+dx)}{3d}$$

```
[Out] 1/3*b*sec(d*x+c)^3/d+a*tan(d*x+c)/d+1/3*a*tan(d*x+c)^3/d
```

Rubi [A]
time = 0.03, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2748, 3852}

$$\frac{a\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{b\sec^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + b*Sin[c + d*x]),x]
```

```
[Out] (b*Sec[c + d*x]^3)/(3*d) + (a*Tan[c + d*x])/d + (a*Tan[c + d*x]^3)/(3*d)
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \sec^4(c+dx)(a+b\sin(c+dx))\,dx &= \frac{b\sec^3(c+dx)}{3d}+a\int \sec^4(c+dx)\,dx\\
&= \frac{b\sec^3(c+dx)}{3d}-\frac{a\text{Subst}\left(\int (1+x^2)\,dx,x,-\tan(c+dx)\right)}{d}\\
&= \frac{b\sec^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{a\tan^3(c+dx)}{3d}
\end{aligned}$$

Mathematica [A]
time = 0.09, size = 41, normalized size = 0.93

$$\frac{b\sec^3(c+dx)}{3d}+\frac{a\left(\tan(c+dx)+\frac{1}{3}\tan^3(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + b*Sin[c + d*x]),x]
[Out] (b*Sec[c + d*x]^3)/(3*d) + (a*(Tan[c + d*x] + Tan[c + d*x]^3/3))/d
```

Maple [A]
time = 0.20, size = 38, normalized size = 0.86

method	result
derivativedivides	$\frac{-a\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+\frac{b}{3\cos(dx+c)^3}}{d}$
default	$\frac{-a\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+\frac{b}{3\cos(dx+c)^3}}{d}$
risch	$\frac{4i e^{2i(dx+c)}a+\frac{8b\,e^{3i(dx+c)}}{3}+\frac{4ia}{3}}{d\left(1+e^{2i(dx+c)}\right)^3}$
norman	$\frac{-\frac{2b}{3d}-\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{2a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{2a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{2a\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2b\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(-a*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+1/3*b/cos(d*x+c)^3)
```

Maxima [A]
time = 0.29, size = 35, normalized size = 0.80

$$\frac{\left(\tan\left(dx+c\right)^3+3\,\tan\left(dx+c\right)\right)a+\frac{b}{\cos(dx+c)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="maxima")
[Out] 1/3*((tan(d*x + c)^3 + 3*tan(d*x + c))*a + b/cos(d*x + c)^3)/d
```

Fricas [A]
time = 0.35, size = 35, normalized size = 0.80

$$\frac{\left(2\,a\cos\left(dx+c\right)^2+a\right)\sin\left(dx+c\right)+b}{3\,d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/3*((2*a*cos(d*x + c)^2 + a)*sin(d*x + c) + b)/(d*cos(d*x + c)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sin(c + dx))\sec^4(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((a + b*sin(c + d*x))*sec(c + d*x)**4, x)
```

Giac [A]
time = 4.57, size = 76, normalized size = 1.73

$$-\frac{2\left(3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+3\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b\right)}{3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -2/3*(3*a*tan(1/2*d*x + 1/2*c)^5 + 3*b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2
*d*x + 1/2*c)^3 + 3*a*tan(1/2*d*x + 1/2*c) + b)/((tan(1/2*d*x + 1/2*c)^2 -
1)^3*d)
```

Mupad [B]
time = 5.27, size = 42, normalized size = 0.95

$$\frac{\frac{2\,a\,\sin(c+d\,x)\,\cos(c+d\,x)^2}{3}+\frac{b}{3}+\frac{a\,\sin(c+d\,x)}{3}}{d\cos\left(c+d\,x\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))/cos(c + d*x)^4,x)
```

```
[Out] (b/3 + (a*sin(c + d*x))/3 + (2*a*cos(c + d*x)^2*sin(c + d*x))/3)/(d*cos(c +
 d*x)^3)
```

3.386 $\int \sec^6(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=60

$$\frac{b\sec^5(c+dx)}{5d}+\frac{a\tan(c+dx)}{d}+\frac{2a\tan^3(c+dx)}{3d}+\frac{a\tan^5(c+dx)}{5d}$$

```
[Out] 1/5*b*sec(d*x+c)^5/d+a*tan(d*x+c)/d+2/3*a*tan(d*x+c)^3/d+1/5*a*tan(d*x+c)^5
/d
```

Rubi [A]
time = 0.03, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2748, 3852}

$$\frac{a\tan^5(c+dx)}{5d}+\frac{2a\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{b\sec^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6*(a + b*Sin[c + d*x]),x]
```

```
[Out] (b*Sec[c + d*x]^5)/(5*d) + (a*Tan[c + d*x])/d + (2*a*Tan[c + d*x]^3)/(3*d)
+ (a*Tan[c + d*x]^5)/(5*d)
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^6(c+dx)(a+b\sin(c+dx))\,dx &= \frac{b\sec^5(c+dx)}{5d}+a\int \sec^6(c+dx)\,dx\\ &= \frac{b\sec^5(c+dx)}{5d}-\frac{a\text{Subst}\left(\int \left(1+2x^2+x^4\right)\,dx,x,-\tan(c+dx)\right)}{d}\\ &= \frac{b\sec^5(c+dx)}{5d}+\frac{a\tan(c+dx)}{d}+\frac{2a\tan^3(c+dx)}{3d}+\frac{a\tan^5(c+dx)}{5d}\end{aligned}$$

Mathematica [A]
time = 0.19, size = 53, normalized size = 0.88

$$\frac{b\sec^5(c+dx)}{5d}+\frac{a\left(\tan(c+dx)+\frac{2}{3}\tan^3(c+dx)+\frac{1}{5}\tan^5(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6*(a + b*Sin[c + d*x]),x]
```

```
[Out] (b*Sec[c + d*x]^5)/(5*d) + (a*(Tan[c + d*x] + (2*Tan[c + d*x]^3)/3 + Tan[c
+ d*x]^5/5))/d
```

Maple [A]
time = 0.26, size = 48, normalized size = 0.80

method	result
derivativedivides	$\frac{-a\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c)\right)}{5}-\frac{4\left(\sec^2(dx+c)\right)}{15}\right)\tan(dx+c)+\frac{b}{5\cos(dx+c)^5}}{d}$
default	$\frac{-a\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c)\right)}{5}-\frac{4\left(\sec^2(dx+c)\right)}{15}\right)\tan(dx+c)+\frac{b}{5\cos(dx+c)^5}}{d}$
risch	$\frac{\frac{32ia\,e^{4i(dx+c)}}{3}+\frac{32b\,e^{5i(dx+c)}}{5}+\frac{16ie^{2i(dx+c)}a}{3}+\frac{16ia}{15}}{d\left(1+e^{2i(dx+c)}\right)^5}$
norman	$\frac{-\frac{2b}{5d}-\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{76a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15d}-\frac{76a\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15d}+\frac{2a\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{2a\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^5\left(1+\tan^2\left(\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c)+1/5*b/cos(d*x
+c)^5)
```

Maxima [A]
time = 0.31, size = 48, normalized size = 0.80

$$\frac{\left(3\tan\left(dx+c\right)^5+10\tan\left(dx+c\right)^3+15\tan\left(dx+c\right)\right)a+\frac{3b}{\cos(dx+c)^5}}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/15*((3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a + 3*b/cos(
d*x + c)^5)/d
```

Fricas [A]
time = 0.34, size = 50, normalized size = 0.83

$$\frac{\left(8\,a\cos\left(dx+c\right)^{4}+4\,a\cos\left(dx+c\right)^{2}+3\,a\right)\sin\left(dx+c\right)+3\,b}{15\,d\cos\left(dx+c\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/15*((8*a*cos(d*x + c)^4 + 4*a*cos(d*x + c)^2 + 3*a)*sin(d*x + c) + 3*b)/(
d*cos(d*x + c)^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sin\left(c+dx\right)\right)\sec^{6}\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((a + b*sin(c + d*x))*sec(c + d*x)**6, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 120 vs. 2(54) = 108.
time = 6.15, size = 120, normalized size = 2.00

$$-\frac{2\left(15\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+15\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}-20\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+58\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+30\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-20\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+15\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,b\right)}{15\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{5}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -2/15*(15*a*tan(1/2*d*x + 1/2*c)^9 + 15*b*tan(1/2*d*x + 1/2*c)^8 - 20*a*tan
(1/2*d*x + 1/2*c)^7 + 58*a*tan(1/2*d*x + 1/2*c)^5 + 30*b*tan(1/2*d*x + 1/2*
c)^4 - 20*a*tan(1/2*d*x + 1/2*c)^3 + 15*a*tan(1/2*d*x + 1/2*c) + 3*b)/((tan
(1/2*d*x + 1/2*c)^2 - 1)^5*d)
```

Mupad [B]
time = 5.30, size = 75, normalized size = 1.25

$$\frac{b}{5\,d\cos\left(c+d\,x\right)^{5}}+\frac{8\,a\,\sin\left(c+d\,x\right)}{15\,d\,\cos\left(c+d\,x\right)}+\frac{4\,a\,\sin\left(c+d\,x\right)}{15\,d\cos\left(c+d\,x\right)^{3}}+\frac{a\,\sin\left(c+d\,x\right)}{5\,d\cos\left(c+d\,x\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))/cos(c + d*x)^6,x)
```

```
[Out] b/(5*d*cos(c + d*x)^5) + (8*a*sin(c + d*x))/(15*d*cos(c + d*x)) + (4*a*sin(
c + d*x))/(15*d*cos(c + d*x)^3) + (a*sin(c + d*x))/(5*d*cos(c + d*x)^5)
```

3.387 $\int \cos^5(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=99

$$-\frac{ab\cos^6(c+dx)}{3d}+\frac{a^2\sin(c+dx)}{d}-\frac{(2a^2-b^2)\sin^3(c+dx)}{3d}+\frac{(a^2-2b^2)\sin^5(c+dx)}{5d}+\frac{b^2\sin^7(c+dx)}{7d}$$

```
[Out] -1/3*a*b*cos(d*x+c)^6/d+a^2*sin(d*x+c)/d-1/3*(2*a^2-b^2)*sin(d*x+c)^3/d+1/5
*(a^2-2*b^2)*sin(d*x+c)^5/d+1/7*b^2*sin(d*x+c)^7/d
```

Rubi [A]
time = 0.06, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2747, 710, 1824}

$$\frac{(a^2-2b^2)\sin^5(c+dx)}{5d}-\frac{(2a^2-b^2)\sin^3(c+dx)}{3d}+\frac{a^2\sin(c+dx)}{d}-\frac{ab\cos^6(c+dx)}{3d}+\frac{b^2\sin^7(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -1/3*(a*b*Cos[c + d*x]^6)/d + (a^2*Sin[c + d*x])/d - ((2*a^2 - b^2)*Sin[c +
 d*x]^3)/(3*d) + ((a^2 - 2*b^2)*Sin[c + d*x]^5)/(5*d) + (b^2*Sin[c + d*x]^7
)/(7*d)
```

Rule 710

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*m*d^(m - 1)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Int[((d + e*x)^m -
e*m*d^(m - 1)*x)*(a + c*x^2)^p, x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
 a*e^2, 0] && IGtQ[p, 1] && IGtQ[m, 0] && LeQ[m, p]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^5(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int (a+x)^2\left(b^2-x^2\right)^2\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$
$$= -\frac{ab\cos^6(c+dx)}{3d} + \frac{\text{Subst}\left(\int \left(b^2-x^2\right)^2\left(-2ax+(a+x)^2\right)\,dx, x, b\right.}{b^5 d}$$
$$= -\frac{ab\cos^6(c+dx)}{3d} + \frac{\text{Subst}\left(\int \left(a^2b^4+b^2(-2a^2+b^2)\,x^2+(a^2-2b^2\right.\right.}{b^5 d}$$
$$= -\frac{ab\cos^6(c+dx)}{3d} + \frac{a^2\sin(c+dx)}{d} - \frac{(2a^2-b^2)\sin^3(c+dx)}{3d} + \frac{(a^2}{}$$

Mathematica [A]

time = 0.21, size = 104, normalized size = 1.05

$$\frac{\sin(c+dx)\left(105a^2+105ab\sin(c+dx)+35(-2a^2+b^2)\sin^2(c+dx)-105ab\sin^3(c+dx)+21(a^2-2b^2)\sin^4(c+dx)+35ab\sin^5(c+dx)+15b^2\sin^6(c+dx)\right)}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (Sin[c + d*x]*(105*a^2 + 105*a*b*Sin[c + d*x] + 35*(-2*a^2 + b^2)*Sin[c + d
*x]^2 - 105*a*b*Sin[c + d*x]^3 + 21*(a^2 - 2*b^2)*Sin[c + d*x]^4 + 35*a*b*S
in[c + d*x]^5 + 15*b^2*Sin[c + d*x]^6))/(105*d)
```

Maple [A]

time = 0.46, size = 98, normalized size = 0.99

method	result
derivativedivides	$\frac{b^2\left(-\frac{\sin(dx+c)\left(\cos^6(dx+c)\right)}{7}+\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{35}\right)-\frac{ab\left(\cos^6(dx+c)\right)}{3}+\frac{a^2\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2\right.}{}\right.}{5}}{d}$
default	$\frac{b^2\left(-\frac{\sin(dx+c)\left(\cos^6(dx+c)\right)}{7}+\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{35}\right)-\frac{ab\left(\cos^6(dx+c)\right)}{3}+\frac{a^2\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2\right.}{}\right.}{5}}{d}$
risch	$\frac{5a^2\sin(dx+c)}{8d}+\frac{5b^2\sin(dx+c)}{64d}-\frac{b^2\sin(7dx+7c)}{448d}-\frac{ab\cos(6dx+6c)}{96d}+\frac{a^2\sin(5dx+5c)}{80d}-\frac{3\sin(5dx+5c)b^2}{320d}-\frac{a}{}$
norman	$\frac{\frac{4ab\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{4ab\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{4ab\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{4ab\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2a^2\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{d}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

[Out] 1/d*(b^2*(-1/7*sin(d*x+c)*cos(d*x+c)^6+1/35*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c
)^2)*sin(d*x+c))-1/3*a*b*cos(d*x+c)^6+1/5*a^2*(8/3+cos(d*x+c)^4+4/3*cos(d*x
+c)^2)*sin(d*x+c))

Maxima [A]
time = 0.29, size = 106, normalized size = 1.07

$$\frac{15\,b^2\sin\left(dx+c\right)^7+35\,ab\sin\left(dx+c\right)^6-105\,ab\sin\left(dx+c\right)^4+21\,\left(a^2-2\,b^2\right)\sin\left(dx+c\right)^5+105\,ab\sin\left(dx+c\right)^2-35\,\left(2\,a^2-b^2\right)\sin\left(dx+c\right)^3+105\,a^2\sin\left(dx+c\right)}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="maxima")

[Out] 1/105*(15*b^2*sin(d*x + c)^7 + 35*a*b*sin(d*x + c)^6 - 105*a*b*sin(d*x + c)
^4 + 21*(a^2 - 2*b^2)*sin(d*x + c)^5 + 105*a*b*sin(d*x + c)^2 - 35*(2*a^2 -
 b^2)*sin(d*x + c)^3 + 105*a^2*sin(d*x + c))/d

Fricas [A]
time = 0.38, size = 87, normalized size = 0.88

$$-\frac{35\,ab\cos\left(dx+c\right)^6+\left(15\,b^2\cos\left(dx+c\right)^6-3\,\left(7\,a^2+b^2\right)\cos\left(dx+c\right)^4-4\,\left(7\,a^2+b^2\right)\cos\left(dx+c\right)^2-56\,a^2-8\,b^2\right)\sin\left(dx+c\right)}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="fricas")

[Out] -1/105*(35*a*b*cos(d*x + c)^6 + (15*b^2*cos(d*x + c)^6 - 3*(7*a^2 + b^2)*co
s(d*x + c)^4 - 4*(7*a^2 + b^2)*cos(d*x + c)^2 - 56*a^2 - 8*b^2)*sin(d*x + c
))/d

Sympy [A]
time = 0.60, size = 158, normalized size = 1.60

$$\begin{cases} \frac{8a^2\sin^5\left(c+dx\right)}{15d}+\frac{4a^2\sin^3\left(c+dx\right)\cos^2\left(c+dx\right)}{3d}+\frac{a^2\sin\left(c+dx\right)\cos^4\left(c+dx\right)}{d}-\frac{ab\cos^6\left(c+dx\right)}{3d}+\frac{8b^2\sin^7\left(c+dx\right)}{105d}+\frac{4b^2\sin^5\left(c+dx\right)\cos^2\left(c+dx\right)}{15d}+\frac{b^2\sin^3\left(c+dx\right)\cos^4\left(c+dx\right)}{3d} & \text{for}\ d\neq 0 \\ x\left(a+b\sin\left(c\right)\right)^2\cos^5\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**5*(a+b*sin(d*x+c))**2,x)

[Out] Piecewise((8*a**2*sin(c + d*x)**5/(15*d) + 4*a**2*sin(c + d*x)**3*cos(c + d
*x)**2/(3*d) + a**2*sin(c + d*x)*cos(c + d*x)**4/d - a*b*cos(c + d*x)**6/(3
*d) + 8*b**2*sin(c + d*x)**7/(105*d) + 4*b**2*sin(c + d*x)**5*cos(c + d*x)*
*2/(15*d) + b**2*sin(c + d*x)**3*cos(c + d*x)**4/(3*d), Ne(d, 0)), (x*(a +
b*sin(c))**2*cos(c)**5, True))

Giac [A]
time = 6.36, size = 136, normalized size = 1.37

$$-\frac{ab\cos\left(6\,dx+6\,c\right)}{96\,d}-\frac{ab\cos\left(4\,dx+4\,c\right)}{16\,d}-\frac{5\,ab\cos\left(2\,dx+2\,c\right)}{32\,d}-\frac{b^2\sin\left(7\,dx+7\,c\right)}{448\,d}+\frac{\left(4\,a^2-3\,b^2\right)\sin\left(5\,dx+5\,c\right)}{320\,d}+\frac{\left(20\,a^2-b^2\right)\sin\left(3\,dx+3\,c\right)}{192\,d}+\frac{5\,\left(8\,a^2+b^2\right)\sin\left(dx+c\right)}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/96*a*b*cos(6*d*x + 6*c)/d - 1/16*a*b*cos(4*d*x + 4*c)/d - 5/32*a*b*cos(2
*d*x + 2*c)/d - 1/448*b^2*sin(7*d*x + 7*c)/d + 1/320*(4*a^2 - 3*b^2)*sin(5*
d*x + 5*c)/d + 1/192*(20*a^2 - b^2)*sin(3*d*x + 3*c)/d + 5/64*(8*a^2 + b^2)
*sin(d*x + c)/d
```

Mupad [B]
time = 0.07, size = 104, normalized size = 1.05

$$\frac{a^2\sin\left(c+d\,x\right)-{\sin\left(c+d\,x\right)}^3\,\left(\frac{2\,a^2}{3}-\frac{b^2}{3}\right)+{\sin\left(c+d\,x\right)}^5\,\left(\frac{a^2}{5}-\frac{2\,b^2}{5}\right)+\frac{b^2\,{\sin\left(c+d\,x\right)}^7}{7}+a\,b\,{\sin\left(c+d\,x\right)}^2-a\,b\,{\sin\left(c+d\,x\right)}^4+\frac{a\,b\,{\sin\left(c+d\,x\right)}^6}{3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(a + b*sin(c + d*x))^2,x)
```

```
[Out] (a^2*sin(c + d*x) - sin(c + d*x)^3*((2*a^2)/3 - b^2/3) + sin(c + d*x)^5*(a^
2/5 - (2*b^2)/5) + (b^2*sin(c + d*x)^7)/7 + a*b*sin(c + d*x)^2 - a*b*sin(c
+ d*x)^4 + (a*b*sin(c + d*x)^6)/3)/d
```

3.388 $\int \cos^3(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=77

$$-\frac{(a^2-b^2)(a+b\sin(c+dx))^3}{3b^3d}+\frac{a(a+b\sin(c+dx))^4}{2b^3d}-\frac{(a+b\sin(c+dx))^5}{5b^3d}$$

```
[Out] -1/3*(a^2-b^2)*(a+b*sin(d*x+c))^3/b^3/d+1/2*a*(a+b*sin(d*x+c))^4/b^3/d-1/5*
(a+b*sin(d*x+c))^5/b^3/d
```

Rubi [A]
time = 0.05, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2747, 711}

$$-\frac{(a^2-b^2)(a+b\sin(c+dx))^3}{3b^3d}-\frac{(a+b\sin(c+dx))^5}{5b^3d}+\frac{a(a+b\sin(c+dx))^4}{2b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -1/3*((a^2 - b^2)*(a + b*Sin[c + d*x])^3)/(b^3*d) + (a*(a + b*Sin[c + d*x])
^4)/(2*b^3*d) - (a + b*Sin[c + d*x])^5/(5*b^3*d)
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^3(c+dx)(a+b\sin(c+dx))^2\,dx &= \frac{\text{Subst}\left(\int (a+x)^2\left(b^2-x^2\right)\,dx,x,b\sin(c+dx)\right)}{b^3d}\\ &= \frac{\text{Subst}\left(\int \left(\left(-a^2+b^2\right)(a+x)^2+2a(a+x)^3-(a+x)^4\right)\,dx,x,b\sin(c\right.}{b^3d}\\ &= -\frac{(a^2-b^2)(a+b\sin(c+dx))^3}{3b^3d}+\frac{a(a+b\sin(c+dx))^4}{2b^3d}-\frac{(a+b\sin}{5b}\end{aligned}$$

Mathematica [A]
time = 0.13, size = 56, normalized size = 0.73

$$\frac{(a+b\sin(c+dx))^3\left(-a^2+7b^2+3b^2\cos(2(c+dx))+3ab\sin(c+dx)\right)}{30b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Sin[c + d*x])^2,x]
[Out] ((a + b*Sin[c + d*x])^3*(-a^2 + 7*b^2 + 3*b^2*Cos[2*(c + d*x)] + 3*a*b*Sin[
c + d*x]))/(30*b^3*d)
```

Maple [A]
time = 0.35, size = 78, normalized size = 1.01

method	result
derivativedivides	$\frac{b^2\left(-\frac{\sin(dx+c)\left(\cos^4(dx+c)\right)}{5}+\frac{\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{15}\right)-\frac{ab\left(\cos^4(dx+c)\right)}{2}+\frac{a^2\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}}{d}$
default	$\frac{b^2\left(-\frac{\sin(dx+c)\left(\cos^4(dx+c)\right)}{5}+\frac{\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{15}\right)-\frac{ab\left(\cos^4(dx+c)\right)}{2}+\frac{a^2\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}}{d}$
risch	$\frac{3a^2\sin(dx+c)}{4d}+\frac{b^2\sin(dx+c)}{8d}-\frac{\sin(5dx+5c)b^2}{80d}-\frac{ab\cos(4dx+4c)}{16d}+\frac{a^2\sin(3dx+3c)}{12d}-\frac{\sin(3dx+3c)b^2}{48d}-$ ab co
norman	$\frac{\frac{4ab\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{4ab\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2a^2\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{8\left(2a^2+b^2\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{8\left(2a^2+b^2\right)(\text{t}}{3}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] 1/d*(b^2*(-1/5*sin(d*x+c)*cos(d*x+c)^4+1/15*(2+cos(d*x+c)^2)*sin(d*x+c))-1/
2*a*b*cos(d*x+c)^4+1/3*a^2*(2+cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A]
time = 0.28, size = 73, normalized size = 0.95

$$-\frac{6\,b^2\sin\left(dx+c\right)^5+15\,ab\sin\left(dx+c\right)^4-30\,ab\sin\left(dx+c\right)^2+10\,\left(a^2-b^2\right)\sin\left(dx+c\right)^3-30\,a^2\sin\left(dx+c\right)}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
[Out] -1/30*(6*b^2*sin(d*x + c)^5 + 15*a*b*sin(d*x + c)^4 - 30*a*b*sin(d*x + c)^2
 + 10*(a^2 - b^2)*sin(d*x + c)^3 - 30*a^2*sin(d*x + c))/d
```

Fricas [A]
time = 0.35, size = 69, normalized size = 0.90

$$-\frac{15\,ab\cos\left(dx+c\right)^4+2\,\left(3\,b^2\cos\left(dx+c\right)^4-\left(5\,a^2+b^2\right)\cos\left(dx+c\right)^2-10\,a^2-2\,b^2\right)\sin\left(dx+c\right)}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/30*(15*a*b*cos(d*x + c)^4 + 2*(3*b^2*cos(d*x + c)^4 - (5*a^2 + b^2)*cos(
d*x + c)^2 - 10*a^2 - 2*b^2)*sin(d*x + c))/d
```

Sympy [A]
time = 0.27, size = 107, normalized size = 1.39

$$\begin{cases} \frac{2a^2\sin^3{(c+dx)}}{3d} + \frac{a^2\sin{(c+dx)}\cos^2{(c+dx)}}{d} - \frac{ab\cos^4{(c+dx)}}{2d} + \frac{2b^2\sin^5{(c+dx)}}{15d} + \frac{b^2\sin^3{(c+dx)}\cos^2{(c+dx)}}{3d} & \text{for } d \neq 0 \\ x(a+b\sin{(c)})^2\cos^3{(c)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((2*a**2*sin(c + d*x)**3/(3*d) + a**2*sin(c + d*x)*cos(c + d*x)**2
/d - a*b*cos(c + d*x)**4/(2*d) + 2*b**2*sin(c + d*x)**5/(15*d) + b**2*sin(c
 + d*x)**3*cos(c + d*x)**2/(3*d), Ne(d, 0)), (x*(a + b*sin(c))**2*cos(c)**3
, True))
```

Giac [A]
time = 4.48, size = 80, normalized size = 1.04

$$-\frac{6\,b^2\sin\left(dx+c\right)^5 + 15\,ab\sin\left(dx+c\right)^4 + 10\,a^2\sin\left(dx+c\right)^3 - 10\,b^2\sin\left(dx+c\right)^3 - 30\,ab\sin\left(dx+c\right)^2 - 30\,a^2\sin\left(dx+c\right)}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/30*(6*b^2*sin(d*x + c)^5 + 15*a*b*sin(d*x + c)^4 + 10*a^2*sin(d*x + c)^3
 - 10*b^2*sin(d*x + c)^3 - 30*a*b*sin(d*x + c)^2 - 30*a^2*sin(d*x + c))/d
```

Mupad [B]
time = 0.05, size = 74, normalized size = 0.96

$$-\frac{\sin\left(c+d\,x\right)^3\,\left(\frac{a^2}{3}-\frac{b^2}{3}\right)-a^2\,\sin\left(c+d\,x\right)+\frac{b^2\,\sin\left(c+d\,x\right)^5}{5}-a\,b\,\sin\left(c+d\,x\right)^2+\frac{a\,b\,\sin\left(c+d\,x\right)^4}{2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a + b*sin(c + d*x))^2,x)
```

```
[Out] -(sin(c + d*x)^3*(a^2/3 - b^2/3) - a^2*sin(c + d*x) + (b^2*sin(c + d*x)^5)/
5 - a*b*sin(c + d*x)^2 + (a*b*sin(c + d*x)^4)/2)/d
```

3.389 $\int \cos(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=22

$$\frac{(a+b\sin(c+dx))^3}{3bd}$$

```
[Out] 1/3*(a+b*sin(d*x+c))^3/b/d
```

Rubi [A]
time = 0.02, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2747, 32}

$$\frac{(a+b\sin(c+dx))^3}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (a + b*Sin[c + d*x])^3/(3*b*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a+b\sin(c+dx))^2\,dx &= \frac{\text{Subst}\left(\int (a+x)^2\,dx, x, b\sin(c+dx)\right)}{bd}\\ &= \frac{(a+b\sin(c+dx))^3}{3bd}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 46 vs. 2(22) = 44.
time = 0.02, size = 46, normalized size = 2.09

$$\frac{a^2\sin(c+dx)}{d}+\frac{ab\sin^2(c+dx)}{d}+\frac{b^2\sin^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*Sin[c + d*x])/d + (a*b*Sin[c + d*x]^2)/d + (b^2*Sin[c + d*x]^3)/(3*d)
```

Maple [A]
time = 0.20, size = 21, normalized size = 0.95

method	result	size
derivativedivides	$\frac{(a+b\sin(dx+c))^3}{3bd}$	21
default	$\frac{(a+b\sin(dx+c))^3}{3bd}$	21
risch	$\frac{a^2\sin(dx+c)}{d}+\frac{b^2\sin(dx+c)}{4d}-\frac{\sin(3dx+3c)b^2}{12d}-\frac{ab\cos(2dx+2c)}{2d}$	62
norman	$\frac{\frac{4ab\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{4ab\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2a^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{4\left(3a^2+2b^2\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}$	117

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(a+b*sin(d*x+c))^3/b/d
```

Maxima [A]
time = 0.30, size = 20, normalized size = 0.91

$$\frac{(b\sin(dx+c)+a)^3}{3\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(b*sin(d*x + c) + a)^3/(b*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 48 vs. 2(20) = 40.
time = 0.35, size = 48, normalized size = 2.18

$$-\frac{3\,ab\cos(dx+c)^2+\left(b^2\cos(dx+c)^2-3\,a^2-b^2\right)\sin(dx+c)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(3*a*b*cos(d*x + c)^2 + (b^2*cos(d*x + c)^2 - 3*a^2 - b^2)*sin(d*x + c
))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(15) = 30$.
time = 0.11, size = 53, normalized size = 2.41

$$\begin{cases} \frac{a^2\sin(c+dx)}{d} + \frac{ab\sin^2(c+dx)}{d} + \frac{b^2\sin^3(c+dx)}{3d} & \text{for } d \neq 0 \\ x(a+b\sin(c))^2\cos(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*sin(c + d*x)/d + a*b*sin(c + d*x)**2/d + b**2*sin(c + d*x)*
*3/(3*d), Ne(d, 0)), (x*(a + b*sin(c))**2*cos(c), True))
```

Giac [A]
time = 4.31, size = 20, normalized size = 0.91

$$\frac{(b\sin(dx+c)+a)^3}{3\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/3*(b*sin(d*x + c) + a)^3/(b*d)
```

Mupad [B]
time = 0.06, size = 39, normalized size = 1.77

$$\frac{a^2\sin(c+d\,x) + a\,b\sin(c+d\,x)^2 + \frac{b^2\sin(c+d\,x)^3}{3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + b*sin(c + d*x))^2,x)
```

```
[Out] (a^2*sin(c + d*x) + (b^2*sin(c + d*x)^3)/3 + a*b*sin(c + d*x)^2)/d
```

3.390 $\int \sec(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=61

$$-\frac{(a+b)^2\log(1-\sin(c+dx))}{2d}+\frac{(a-b)^2\log(1+\sin(c+dx))}{2d}-\frac{b^2\sin(c+dx)}{d}$$

```
[Out] -1/2*(a+b)^2*ln(1-sin(d*x+c))/d+1/2*(a-b)^2*ln(1+sin(d*x+c))/d-b^2*sin(d*x+
c)/d
```

Rubi [A]
time = 0.06, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2747, 716, 647, 31}

$$\frac{(a-b)^2\log(\sin(c+dx)+1)}{2d}-\frac{(a+b)^2\log(1-\sin(c+dx))}{2d}-\frac{b^2\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -1/2*((a + b)^2*Log[1 - Sin[c + d*x]])/d + ((a - b)^2*Log[1 + Sin[c + d*x]]
)/(2*d) - (b^2*Sin[c + d*x])/d
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
(-a)*c]
```

Rule 716

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Polyno
mialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
```

```
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{b\text{Subst}\left(\int \frac{(a+x)^2}{b^2-x^2}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{b\text{Subst}\left(\int \left(-1+\frac{a^2+b^2+2ax}{b^2-x^2}\right)\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= -\frac{b^2\sin(c+dx)}{d} + \frac{b\text{Subst}\left(\int \frac{a^2+b^2+2ax}{b^2-x^2}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= -\frac{b^2\sin(c+dx)}{d} - \frac{(a-b)^2\text{Subst}\left(\int \frac{1}{-b-x}\,dx, x, b\sin(c+dx)\right)}{2d} + \frac{(a}{}$$
$$= -\frac{(a+b)^2\log(1-\sin(c+dx))}{2d} + \frac{(a-b)^2\log(1+\sin(c+dx))}{2d} - \frac{b^2}{}$$

Mathematica [A]
time = 0.07, size = 54, normalized size = 0.89

$$\frac{-(a+b)^2\log(1-\sin(c+dx))+(a-b)^2\log(1+\sin(c+dx))-2b^2\sin(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sin[c + d*x])^2,x]
[Out] (-((a + b)^2*Log[1 - Sin[c + d*x]]) + (a - b)^2*Log[1 + Sin[c + d*x]] - 2*b
^2*Sin[c + d*x])/(2*d)
```

Maple [A]
time = 0.28, size = 62, normalized size = 1.02

method	result
derivativedivides	$\frac{a^2\ln(\sec(dx+c)+\tan(dx+c))-2ab\ln(\cos(dx+c))+b^2(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))}{d}$
default	$\frac{a^2\ln(\sec(dx+c)+\tan(dx+c))-2ab\ln(\cos(dx+c))+b^2(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))}{d}$
norman	$\frac{-\frac{2b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{2b^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{(a^2-2ab+b^2)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}-\frac{(a^2+2ab+b^2)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}+$
risch	$2iabx+\frac{ib^2e^{i(dx+c)}}{2d}-\frac{ib^2e^{-i(dx+c)}}{2d}+\frac{4iabc}{d}-\frac{a^2\ln(e^{i(dx+c)}-i)}{d}-\frac{2\ln(e^{i(dx+c)}-i)ab}{d}-\frac{\ln(e^{i(dx+c)}-i)b^2}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*ln(sec(d*x+c)+tan(d*x+c))-2*a*b*ln(cos(d*x+c))+b^2*(-sin(d*x+c)+ln
(sec(d*x+c)+tan(d*x+c))))
```

Maxima [A]
time = 0.28, size = 60, normalized size = 0.98

$$-\frac{2\,b^2\sin\left(dx+c\right)-\left(a^2-2\,ab+b^2\right)\log\left(\sin\left(dx+c\right)+1\right)+\left(a^2+2\,ab+b^2\right)\log\left(\sin\left(dx+c\right)-1\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*b^2*sin(d*x + c) - (a^2 - 2*a*b + b^2)*log(sin(d*x + c) + 1) + (a^2
 + 2*a*b + b^2)*log(sin(d*x + c) - 1))/d
```

Fricas [A]
time = 0.41, size = 62, normalized size = 1.02

$$-\frac{2\,b^2\sin\left(dx+c\right)-\left(a^2-2\,ab+b^2\right)\log\left(\sin\left(dx+c\right)+1\right)+\left(a^2+2\,ab+b^2\right)\log\left(-\sin\left(dx+c\right)+1\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b^2*sin(d*x + c) - (a^2 - 2*a*b + b^2)*log(sin(d*x + c) + 1) + (a^2
 + 2*a*b + b^2)*log(-sin(d*x + c) + 1))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sin\left(c+dx\right)\right)^2\sec\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sin(c + d*x))**2*sec(c + d*x), x)
```

Giac [A]
time = 4.14, size = 62, normalized size = 1.02

$$-\frac{2\,b^2\sin\left(dx+c\right)-\left(a^2-2\,ab+b^2\right)\log\left(\left|\sin\left(dx+c\right)+1\right|\right)+\left(a^2+2\,ab+b^2\right)\log\left(\left|\sin\left(dx+c\right)-1\right|\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*b^2*sin(d*x + c) - (a^2 - 2*a*b + b^2)*log(abs(sin(d*x + c) + 1)) +
 (a^2 + 2*a*b + b^2)*log(abs(sin(d*x + c) - 1)))/d
```

Mupad [B]
time = 5.16, size = 50, normalized size = 0.82

$$-\frac{\frac{\ln(\sin(c+d\,x)-1)\,(a+b)^2}{2}-\frac{\ln(\sin(c+d\,x)+1)\,(a-b)^2}{2}+b^2\,\sin\left(c+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^2/cos(c + d*x),x)
```

```
[Out] -((log(sin(c + d*x) - 1)*(a + b)^2)/2 - (log(sin(c + d*x) + 1)*(a - b)^2)/2
 + b^2*sin(c + d*x))/d
```

3.391 $\int \sec^3(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=59

$$\frac{(a^2-b^2)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{\sec^2(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))}{2d}$$

```
[Out] 1/2*(a^2-b^2)*arctanh(sin(d*x+c))/d+1/2*sec(d*x+c)^2*(b+a*sin(d*x+c))*(a+b*
sin(d*x+c))/d
```

Rubi [A]
time = 0.04, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2747, 737, 212}

$$\frac{(a^2-b^2)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{\sec^2(c+dx)(a\sin(c+dx)+b)(a+b\sin(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((a^2 - b^2)*ArcTanh[Sin[c + d*x]])/(2*d) + (Sec[c + d*x]^2*(b + a*Sin[c +
d*x])*(a + b*Sin[c + d*x]))/(2*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 737

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m - 1)*(a*e - c*d*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] +
 Dist[(2*p + 3)*((c*d^2 + a*e^2)/(2*a*c*(p + 1))), Int[(d + e*x)^(m - 2)*(a
 + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0
] && EqQ[m + 2*p + 2, 0] && LtQ[p, -1]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^3(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{b^3\text{Subst}\left(\int \frac{(a+x)^2}{(b^2-x^2)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{\sec^2(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))}{2d} + \frac{(b(a^2-b^2))\,\text{S}}{}$$
$$= \frac{(a^2-b^2)\tanh^{-1}(\sin(c+dx))}{2d} + \frac{\sec^2(c+dx)(b+a\sin(c+dx))(a}{2d}$$

Mathematica [A]
time = 1.01, size = 113, normalized size = 1.92

$$\frac{(a^2-b^2)^2(\log(1-\sin(c+dx))-\log(1+\sin(c+dx)))+2a^3b\sec^2(c+dx)-2(a^4-b^4)\sec(c+dx)\tan(c+dx)+(-6a^3b+4ab^3)\tan^2(c+dx)}{4(-a^2+b^2)d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((a^2 - b^2)^2*(Log[1 - Sin[c + d*x]] - Log[1 + Sin[c + d*x]]) + 2*a^3*b*Se
c[c + d*x]^2 - 2*(a^4 - b^4)*Sec[c + d*x]*Tan[c + d*x] + (-6*a^3*b + 4*a*b^
3)*Tan[c + d*x]^2)/(4*(-a^2 + b^2)*d)
```

Maple [A]
time = 0.34, size = 99, normalized size = 1.68

method	result
derivativedivides	$\frac{a^2\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+\frac{ab}{\cos(dx+c)^2}+b^2\left(\frac{\sin^3(dx+c)}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$
default	$\frac{a^2\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+\frac{ab}{\cos(dx+c)^2}+b^2\left(\frac{\sin^3(dx+c)}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$
risch	$-\frac{i\left(a^2e^{3i(dx+c)}+b^2e^{3i(dx+c)}-a^2e^{i(dx+c)}-b^2e^{i(dx+c)}+4iab\,e^{2i(dx+c)}\right)}{d\left(1+e^{2i(dx+c)}\right)^2}-\frac{a^2\ln\left(e^{i(dx+c)}-i\right)}{2d}+\frac{\ln\left(e^{i(dx+c)}-i\right)b^2}{2d}+$
norman	$\frac{\frac{(a^2+b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{(a^2+b^2)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{3(a^2+b^2)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{3(a^2+b^2)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{8ab\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+a*b/cos(
d*x+c)^2+b^2*(1/2*sin(d*x+c)^3/cos(d*x+c)^2+1/2*sin(d*x+c)-1/2*ln(sec(d*x+c
)+tan(d*x+c))))
```

Maxima [A]
time = 0.31, size = 78, normalized size = 1.32

$$\frac{(a^2-b^2)\log(\sin(dx+c)+1)-(a^2-b^2)\log(\sin(dx+c)-1)-\frac{2(2ab+(a^2+b^2)\sin(dx+c))}{\sin(dx+c)^2-1}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/4*((a^2 - b^2)*log(sin(d*x + c) + 1) - (a^2 - b^2)*log(sin(d*x + c) - 1)
- 2*(2*a*b + (a^2 + b^2)*sin(d*x + c))/(sin(d*x + c)^2 - 1))/d
```

Fricas [A]
time = 0.36, size = 90, normalized size = 1.53

$$\frac{(a^2-b^2)\cos(dx+c)^2\log(\sin(dx+c)+1)-(a^2-b^2)\cos(dx+c)^2\log(-\sin(dx+c)+1)+4ab+2(a^2+b^2)\sin(dx+c)}{4d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/4*((a^2 - b^2)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (a^2 - b^2)*cos(d*x
 + c)^2*log(-sin(d*x + c) + 1) + 4*a*b + 2*(a^2 + b^2)*sin(d*x + c))/(d*cos
(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sin(c+dx))^2\sec^3(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sin(c + d*x))**2*sec(c + d*x)**3, x)
```

Giac [A]
time = 4.41, size = 86, normalized size = 1.46

$$\frac{(a^2-b^2)\log(|\sin(dx+c)+1|)-(a^2-b^2)\log(|\sin(dx+c)-1|)-\frac{2(a^2\sin(dx+c)+b^2\sin(dx+c)+2ab)}{\sin(dx+c)^2-1}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/4*((a^2 - b^2)*log(abs(sin(d*x + c) + 1)) - (a^2 - b^2)*log(abs(sin(d*x +
 c) - 1)) - 2*(a^2*sin(d*x + c) + b^2*sin(d*x + c) + 2*a*b)/(sin(d*x + c)^2
 - 1))/d
```

Mupad [B]
time = 0.11, size = 62, normalized size = 1.05

$$\frac{\operatorname{atanh}(\sin(c+d\,x))\,\left(\frac{a^2}{2}-\frac{b^2}{2}\right)}{d}-\frac{a\,b+\sin(c+d\,x)\,\left(\frac{a^2}{2}+\frac{b^2}{2}\right)}{d\,\left(\sin(c+d\,x)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^2/cos(c + d*x)^3,x)
```

```
[Out] (atanh(sin(c + d*x))*(a^2/2 - b^2/2))/d - (a*b + sin(c + d*x)*(a^2/2 + b^2/
2))/(d*(sin(c + d*x)^2 - 1))
```

3.392 $\int \sec^5(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=99

$$\frac{(3a^2-b^2)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{\sec^4(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))}{4d}+\frac{\sec^2(c+dx)\left(2ab+(3a^2\right.}{8d}$$

[Out] 1/8*(3*a^2-b^2)*arctanh(sin(d*x+c))/d+1/4*sec(d*x+c)^4*(b+a*sin(d*x+c))*(a+b*sin(d*x+c))/d+1/8*sec(d*x+c)^2*(2*a*b+(3*a^2-b^2)*sin(d*x+c))/d

Rubi [A]
time = 0.06, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2747, 753, 653, 212}

$$\frac{(3a^2-b^2)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{\sec^2(c+dx)\left((3a^2-b^2)\sin(c+dx)+2ab\right)}{8d}+\frac{\sec^4(c+dx)(a\sin(c+dx)+b)(a+b\sin(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((3*a^2 - b^2)*ArcTanh[Sin[c + d*x]])/(8*d) + (Sec[c + d*x]^4*(b + a*Sin[c + d*x])*(a + b*Sin[c + d*x]))/(4*d) + (Sec[c + d*x]^2*(2*a*b + (3*a^2 - b^2)*Sin[c + d*x]))/(8*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 653

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e - c*d*x)/(2*a*c*(p + 1)))*(a + c*x^2)^(p + 1), x] + Dist[d*((2*p + 3)/(2*a*(p + 1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 753

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a*e - c*d*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^5(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{b^5\text{Subst}\left(\int \frac{(a+x)^2}{(b^2-x^2)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^4(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))}{4d} - \frac{b^3\text{Subst}\left(\int \frac{-3}{\cdots}\right.}{\cdots}$$

$$= \frac{\sec^4(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))}{4d} + \frac{\sec^2(c+dx)\,(\cdots}{\cdots}$$

$$= \frac{(3a^2-b^2)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{\sec^4(c+dx)(b+a\sin(c+dx))(\cdots}{4d}$$

Mathematica [A]
time = 0.78, size = 166, normalized size = 1.68

$$\frac{4(-a^2+b^2)\sec^4(c+dx)(b-a\sin(c+dx))(a+b\sin(c+dx))^3+(-3a^2+b^2)\left((a^2-b^2)^2(\log(1-\sin(c+dx))-\log(1+\sin(c+dx)))+2a^3b\sec^2(c+dx)-2(a^4-b^4)\sec(c+dx)\tan(c+dx)+(-6a^3b+4ab^3)\tan^2(c+dx)\right)}{16(a^2-b^2)^2 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (4*(-a^2 + b^2)*Sec[c + d*x]^4*(b - a*Sin[c + d*x])*(a + b*Sin[c + d*x])^3
+ (-3*a^2 + b^2)*((a^2 - b^2)^2*(Log[1 - Sin[c + d*x]] - Log[1 + Sin[c + d*
x]]) + 2*a^3*b*Sec[c + d*x]^2 - 2*(a^4 - b^4)*Sec[c + d*x]*Tan[c + d*x] + (
-6*a^3*b + 4*a*b^3)*Tan[c + d*x]^2))/(16*(a^2 - b^2)^2*d)
```

Maple [A]
time = 0.42, size = 131, normalized size = 1.32

method	result
derivativedivides	$\frac{a^2\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+\frac{ab}{2\cos(dx+c)^4}+b^2\left(\frac{\sin^3(dx+c)}{4\cos(dx+c)^4}+\frac{\sin^3(dx\cdots}{8\cos(dx\cdots}\right.}{d}$
default	$\frac{a^2\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+\frac{ab}{2\cos(dx+c)^4}+b^2\left(\frac{\sin^3(dx+c)}{4\cos(dx+c)^4}+\frac{\sin^3(dx\cdots}{8\cos(dx\cdots}\right.}{d}$

risch	$\frac{i\left(-3a^2 e^{7i(dx+c)}+b^2 e^{7i(dx+c)}-11a^2 e^{5i(dx+c)}-7b^2 e^{5i(dx+c)}+11a^2 e^{3i(dx+c)}+7b^2 e^{3i(dx+c)}-32iab\, e^{4i(dx+c)}+3a^2 e^{i(dx+c)}\right.}{4d\left(1+e^{2i(dx+c)}\right)^4}$
norman	$\frac{\frac{8ab\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{8ab\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{(5a^2+b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}+\frac{(5a^2+b^2)\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{(7a^2+11b^2)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}+}{\left(\tan^2\left(\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^5*(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)

[Out] 1/d*(a^2*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c)))+1/2*a*b/cos(d*x+c)^4+b^2*(1/4*sin(d*x+c)^3/cos(d*x+c)^4+1/8*sin(d*x+c)^3/cos(d*x+c)^2+1/8*sin(d*x+c)-1/8*ln(sec(d*x+c)+tan(d*x+c))))

Maxima [A]
time = 0.29, size = 115, normalized size = 1.16

$$\frac{(3a^2-b^2)\log(\sin(dx+c)+1)-(3a^2-b^2)\log(\sin(dx+c)-1)-\frac{2\left((3a^2-b^2)\sin(dx+c)^3-4ab-(5a^2+b^2)\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="maxima")

[Out] 1/16*((3*a^2 - b^2)*log(sin(d*x + c) + 1) - (3*a^2 - b^2)*log(sin(d*x + c) - 1) - 2*((3*a^2 - b^2)*sin(d*x + c)^3 - 4*a*b - (5*a^2 + b^2)*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1))/d

Fricas [A]
time = 0.36, size = 118, normalized size = 1.19

$$\frac{(3a^2-b^2)\cos(dx+c)^4\log(\sin(dx+c)+1)-(3a^2-b^2)\cos(dx+c)^4\log(-\sin(dx+c)+1)+8ab+2\left((3a^2-b^2)\cos(dx+c)^2+2a^2+2b^2\right)\sin(dx+c)}{16d\cos(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="fricas")

[Out] 1/16*((3*a^2 - b^2)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - (3*a^2 - b^2)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 8*a*b + 2*((3*a^2 - b^2)*cos(d*x + c)^2 + 2*a^2 + 2*b^2)*sin(d*x + c))/(d*cos(d*x + c)^4)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sin(c+dx))^2\sec^5(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**5*(a+b*sin(d*x+c))**2,x)

```
[Out] Integral((a + b*sin(c + d*x))**2*sec(c + d*x)**5, x)
```

Giac [A]
time = 7.98, size = 118, normalized size = 1.19

$$\frac{(3a^2-b^2)\log(|\sin(dx+c)+1|)-(3a^2-b^2)\log(|\sin(dx+c)-1|)-\frac{2\left(3a^2\sin(dx+c)^3-b^2\sin(dx+c)^3-5a^2\sin(dx+c)-b^2\sin(dx+c)-4ab\right)}{\left(\sin(dx+c)^2-1\right)^2}}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/16*((3*a^2 - b^2)*log(abs(sin(d*x + c) + 1)) - (3*a^2 - b^2)*log(abs(sin(
d*x + c) - 1)) - 2*(3*a^2*sin(d*x + c)^3 - b^2*sin(d*x + c)^3 - 5*a^2*sin(d
*x + c) - b^2*sin(d*x + c) - 4*a*b)/(sin(d*x + c)^2 - 1)^2)/d
```

Mupad [B]
time = 5.10, size = 93, normalized size = 0.94

$$\frac{\mathrm{atanh}(\sin(c+d\,x))\,\left(\frac{3\,a^2}{8}-\frac{b^2}{8}\right)}{d}+\frac{\left(\frac{b^2}{8}-\frac{3\,a^2}{8}\right)\,\sin(c+d\,x)^3+\left(\frac{5\,a^2}{8}+\frac{b^2}{8}\right)\,\sin(c+d\,x)+\frac{a\,b}{2}}{d\,\left(\sin(c+d\,x)^4-2\sin(c+d\,x)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^2/cos(c + d*x)^5,x)
```

```
[Out] (atanh(sin(c + d*x))*((3*a^2)/8 - b^2/8))/d + ((a*b)/2 + sin(c + d*x)*((5*a
^2)/8 + b^2/8) - sin(c + d*x)^3*((3*a^2)/8 - b^2/8))/(d*(sin(c + d*x)^4 - 2
*sin(c + d*x)^2 + 1))
```

3.393 $\int \cos^6(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=146

$$\frac{5}{128}\left(8a^2+b^2\right)x-\frac{9ab\cos^7(c+dx)}{56d}+\frac{5(8a^2+b^2)\cos(c+dx)\sin(c+dx)}{128d}+\frac{5(8a^2+b^2)\cos^3(c+dx)\sin(c+dx)}{192d}$$

```
[Out] 5/128*(8*a^2+b^2)*x-9/56*a*b*cos(d*x+c)^7/d+5/128*(8*a^2+b^2)*cos(d*x+c)*si
n(d*x+c)/d+5/192*(8*a^2+b^2)*cos(d*x+c)^3*sin(d*x+c)/d+1/48*(8*a^2+b^2)*cos
(d*x+c)^5*sin(d*x+c)/d-1/8*b*cos(d*x+c)^7*(a+b*sin(d*x+c))/d
```

Rubi [A]
time = 0.09, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2771, 2748, 2715, 8}

$$\frac{(8a^2+b^2)\sin(c+dx)\cos^5(c+dx)}{48d}+\frac{5(8a^2+b^2)\sin(c+dx)\cos^3(c+dx)}{192d}+\frac{5(8a^2+b^2)\sin(c+dx)\cos(c+dx)}{128d}+\frac{5}{128}x(8a^2+b^2)-\frac{9ab\cos^7(c+dx)}{56d}-\frac{b\cos^7(c+dx)(a+b\sin(c+dx))}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (5*(8*a^2 + b^2)*x)/128 - (9*a*b*Cos[c + d*x]^7)/(56*d) + (5*(8*a^2 + b^2)*
Cos[c + d*x]*Sin[c + d*x])/(128*d) + (5*(8*a^2 + b^2)*Cos[c + d*x]^3*Sin[c
+ d*x])/(192*d) + ((8*a^2 + b^2)*Cos[c + d*x]^5*Sin[c + d*x])/(48*d) - (b*C
os[c + d*x]^7*(a + b*Sin[c + d*x]))/(8*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2771

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
```

```
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(
a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rubi steps

$$\begin{aligned}\int \cos^6(c+dx)(a+b\sin(c+dx))^2\,dx &= -\frac{b\cos^7(c+dx)(a+b\sin(c+dx))}{8d}+\frac{1}{8}\int \cos^6(c+dx)\left(8a^2+b^2+\right.\\
&= -\frac{9ab\cos^7(c+dx)}{56d}-\frac{b\cos^7(c+dx)(a+b\sin(c+dx))}{8d}+\frac{1}{8}\left(8a^2+\right.\\
&= -\frac{9ab\cos^7(c+dx)}{56d}+\frac{(8a^2+b^2)\cos^5(c+dx)\sin(c+dx)}{48d}-\frac{b\cos^7(\ldots}{}\\
&= -\frac{9ab\cos^7(c+dx)}{56d}+\frac{5(8a^2+b^2)\cos^3(c+dx)\sin(c+dx)}{192d}+\frac{(8a^2\ldots}{}\\
&= -\frac{9ab\cos^7(c+dx)}{56d}+\frac{5(8a^2+b^2)\cos(c+dx)\sin(c+dx)}{128d}+\frac{5(8a^2\ldots}{}\\
&= \frac{5}{128}\left(8a^2+b^2\right)x-\frac{9ab\cos^7(c+dx)}{56d}+\frac{5(8a^2+b^2)\cos(c+dx)\sin(\ldots}{128d}\end{aligned}$$

Mathematica [A]
time = 0.37, size = 141, normalized size = 0.97

$$\frac{840(8a^2+b^2)(c+dx)-3360ab\cos(c+dx)-2016ab\cos(3(c+dx))-672ab\cos(5(c+dx))-96ab\cos(7(c+dx))+336(15a^2+b^2)\sin(2(c+dx))+168(6a^2-b^2)\sin(4(c+dx))+112(a-b)(a+b)\sin(6(c+dx))-21b^2\sin(8(c+dx))}{21504d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (840*(8*a^2 + b^2)*(c + d*x) - 3360*a*b*Cos[c + d*x] - 2016*a*b*Cos[3*(c +
d*x)] - 672*a*b*Cos[5*(c + d*x)] - 96*a*b*Cos[7*(c + d*x)] + 336*(15*a^2 +
b^2)*Sin[2*(c + d*x)] + 168*(6*a^2 - b^2)*Sin[4*(c + d*x)] + 112*(a - b)*(a
 + b)*Sin[6*(c + d*x)] - 21*b^2*Sin[8*(c + d*x)])/(21504*d)
```

Maple [A]
time = 0.59, size = 128, normalized size = 0.88

method	result
derivativedivides	$\frac{b^2\left(-\frac{\sin(dx+c)\left(\cos^7(dx+c)\right)}{8}+\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{48}+\frac{5dx}{128}+\frac{5c}{128}\right)-\frac{2ab\left(\cos^7(dx+c)\right)}{7}+a^2}{d}$

default	$\frac{b^2\left(-\frac{\sin(dx+c)\left(\cos^7(dx+c)\right)}{8}+\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{48}+\frac{5dx}{128}+\frac{5c}{128}\right)-\frac{2ab\left(\cos^7(dx+c)\right)}{7}+a^2\left(\right.}{d}$
risch	$\frac{5a^2x}{16}+\frac{5b^2x}{128}-\frac{5ab\cos(dx+c)}{32d}-\frac{b^2\sin(8dx+8c)}{1024d}-\frac{ab\cos(7dx+7c)}{224d}+\frac{a^2\sin(6dx+6c)}{192d}-\frac{\sin(6dx+6c)b^2}{192d}-ab\cos$
norman	$\frac{\left(\frac{5a^2}{16}+\frac{5b^2}{128}\right)x\left(\tan^{16}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{\left(488a^2+397b^2\right)\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{192d}-\frac{4ab}{7d}-\frac{12ab\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{20ab\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-4ab\left(\tan\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^2*(-1/8*sin(d*x+c)*cos(d*x+c)^7+1/48*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+
15/8*cos(d*x+c))*sin(d*x+c)+5/128*d*x+5/128*c)-2/7*a*b*cos(d*x+c)^7+a^2*(1/
6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/16*d*x+5/16*
c))
```

Maxima [A]
time = 0.29, size = 114, normalized size = 0.78

$$-\frac{6144\,ab\cos\left(dx+c\right)^7+112\left(4\sin\left(2\,dx+2\,c\right)^3-60\,dx-60\,c-9\sin\left(4\,dx+4\,c\right)-48\sin\left(2\,dx+2\,c\right)\right)a^2-7\left(64\sin\left(2\,dx+2\,c\right)^3+120\,dx+120\,c-3\sin\left(8\,dx+8\,c\right)-24\sin\left(4\,dx+4\,c\right)\right)b^2}{21504\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/21504*(6144*a*b*cos(d*x + c)^7 + 112*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60
*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*a^2 - 7*(64*sin(2*d*x + 2*c)
^3 + 120*d*x + 120*c - 3*sin(8*d*x + 8*c) - 24*sin(4*d*x + 4*c))*b^2)/d
```

Fricas [A]
time = 0.37, size = 108, normalized size = 0.74

$$-\frac{768\,ab\cos\left(dx+c\right)^7-105\left(8\,a^2+b^2\right)dx+7\left(48\,b^2\cos\left(dx+c\right)^7-8\left(8\,a^2+b^2\right)\cos\left(dx+c\right)^5-10\left(8\,a^2+b^2\right)\cos\left(dx+c\right)^3-15\left(8\,a^2+b^2\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{2688\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2688*(768*a*b*cos(d*x + c)^7 - 105*(8*a^2 + b^2)*d*x + 7*(48*b^2*cos(d*x
 + c)^7 - 8*(8*a^2 + b^2)*cos(d*x + c)^5 - 10*(8*a^2 + b^2)*cos(d*x + c)^3
- 15*(8*a^2 + b^2)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 398 vs. $2(141) = 282$.
time = 0.93, size = 398, normalized size = 2.73

$$\begin{cases} \frac{5a^2x\sin^6(c+dx)}{16}+\frac{15a^2x\sin^4(c+dx)\cos^2(c+dx)}{16}+\frac{15a^2x\sin^2(c+dx)\cos^4(c+dx)}{16}+\frac{5a^2x\cos^6(c+dx)}{16}+\frac{5a^2\sin^5(c+dx)\cos(c+dx)}{16d}+\frac{5a^2\sin^3(c+dx)\cos^3(c+dx)}{6d}+\frac{11a^2\sin(c+dx)\cos^5(c+dx)}{16d}-\frac{2ab\cos^7(c+dx)}{7d}+\frac{5b^2x\sin^8(c+dx)}{128}+\frac{5b^2x\sin^6(c+dx)\cos^2(c+dx)}{32}+\frac{15b^2x\sin^4(c+dx)\cos^4(c+dx)}{64}+\frac{5b^2x\sin^2(c+dx)\cos^6(c+dx)}{32}+\frac{5b^2x\cos^8(c+dx)}{128}+\frac{5b^2\sin^7(c+dx)\cos(c+dx)}{128d}+\frac{55b^2\sin^5(c+dx)\cos^3(c+dx)}{384d}+\frac{73b^2\sin^3(c+dx)\cos^5(c+dx)}{384d}-\frac{5b^2\sin(c+dx)\cos^7(c+dx)}{128d} & \text{for}\ d\neq0 \\ x(a+b\sin(c))^2\cos^6(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((5*a**2*x*sin(c + d*x)**6/16 + 15*a**2*x*sin(c + d*x)**4*cos(c +
d*x)**2/16 + 15*a**2*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 5*a**2*x*cos(c
+ d*x)**6/16 + 5*a**2*sin(c + d*x)**5*cos(c + d*x)/(16*d) + 5*a**2*sin(c +
d*x)**3*cos(c + d*x)**3/(6*d) + 11*a**2*sin(c + d*x)*cos(c + d*x)**5/(16*d)
 - 2*a*b*cos(c + d*x)**7/(7*d) + 5*b**2*x*sin(c + d*x)**8/128 + 5*b**2*x*si
n(c + d*x)**6*cos(c + d*x)**2/32 + 15*b**2*x*sin(c + d*x)**4*cos(c + d*x)**
4/64 + 5*b**2*x*sin(c + d*x)**2*cos(c + d*x)**6/32 + 5*b**2*x*cos(c + d*x)*
*8/128 + 5*b**2*sin(c + d*x)**7*cos(c + d*x)/(128*d) + 55*b**2*sin(c + d*x)
**5*cos(c + d*x)**3/(384*d) + 73*b**2*sin(c + d*x)**3*cos(c + d*x)**5/(384*
d) - 5*b**2*sin(c + d*x)*cos(c + d*x)**7/(128*d), Ne(d, 0)), (x*(a + b*sin(
c))**2*cos(c)**6, True))
```

Giac [A]
time = 7.27, size = 162, normalized size = 1.11

$$\frac{5}{128}\left(8\,a^{2}+b^{2}\right)x-\frac{ab\cos\left(7\,dx+7\,c\right)}{224\,d}-\frac{ab\cos\left(5\,dx+5\,c\right)}{32\,d}-\frac{3\,ab\cos\left(3\,dx+3\,c\right)}{32\,d}-\frac{5\,ab\cos\left(dx+c\right)}{32\,d}-\frac{b^{2}\sin\left(8\,dx+8\,c\right)}{1024\,d}+\frac{\left(a^{2}-b^{2}\right)\sin\left(6\,dx+6\,c\right)}{192\,d}+\frac{\left(6\,a^{2}-b^{2}\right)\sin\left(4\,dx+4\,c\right)}{128\,d}+\frac{\left(15\,a^{2}+b^{2}\right)\sin\left(2\,dx+2\,c\right)}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 5/128*(8*a^2 + b^2)*x - 1/224*a*b*cos(7*d*x + 7*c)/d - 1/32*a*b*cos(5*d*x +
 5*c)/d - 3/32*a*b*cos(3*d*x + 3*c)/d - 5/32*a*b*cos(d*x + c)/d - 1/1024*b^
2*sin(8*d*x + 8*c)/d + 1/192*(a^2 - b^2)*sin(6*d*x + 6*c)/d + 1/128*(6*a^2
- b^2)*sin(4*d*x + 4*c)/d + 1/64*(15*a^2 + b^2)*sin(2*d*x + 2*c)/d
```

Mupad [B]
time = 5.47, size = 178, normalized size = 1.22

$$\frac{5\,a^2\,x}{16}+\frac{5\,b^2\,x}{128}+\frac{5\,a^2\,{\cos\left(c+d\,x\right)}^3\,\sin\left(c+d\,x\right)}{24\,d}+\frac{a^2\,{\cos\left(c+d\,x\right)}^5\,\sin\left(c+d\,x\right)}{6\,d}+\frac{5\,b^2\,{\cos\left(c+d\,x\right)}^3\,\sin\left(c+d\,x\right)}{192\,d}+\frac{b^2\,{\cos\left(c+d\,x\right)}^5\,\sin\left(c+d\,x\right)}{48\,d}-\frac{b^2\,{\cos\left(c+d\,x\right)}^7\,\sin\left(c+d\,x\right)}{8\,d}-\frac{2\,a\,b\,{\cos\left(c+d\,x\right)}^7}{7\,d}+\frac{5\,a^2\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{16\,d}+\frac{5\,b^2\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{128\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6*(a + b*sin(c + d*x))^2,x)
```

```
[Out] (5*a^2*x)/16 + (5*b^2*x)/128 + (5*a^2*cos(c + d*x)^3*sin(c + d*x))/(24*d) +
 (a^2*cos(c + d*x)^5*sin(c + d*x))/(6*d) + (5*b^2*cos(c + d*x)^3*sin(c + d*
x))/(192*d) + (b^2*cos(c + d*x)^5*sin(c + d*x))/(48*d) - (b^2*cos(c + d*x)^
7*sin(c + d*x))/(8*d) - (2*a*b*cos(c + d*x)^7)/(7*d) + (5*a^2*cos(c + d*x)*
sin(c + d*x))/(16*d) + (5*b^2*cos(c + d*x)*sin(c + d*x))/(128*d)
```

3.394 $\int \cos^4(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=116

$$\frac{1}{16}\left(6a^2+b^2\right)x-\frac{7ab\cos^5(c+dx)}{30d}+\frac{(6a^2+b^2)\cos(c+dx)\sin(c+dx)}{16d}+\frac{(6a^2+b^2)\cos^3(c+dx)\sin(c+dx)}{24d}-$$

```
[Out] 1/16*(6*a^2+b^2)*x-7/30*a*b*cos(d*x+c)^5/d+1/16*(6*a^2+b^2)*cos(d*x+c)*sin(
d*x+c)/d+1/24*(6*a^2+b^2)*cos(d*x+c)^3*sin(d*x+c)/d-1/6*b*cos(d*x+c)^5*(a+b
*sin(d*x+c))/d
```

Rubi [A]

time = 0.08, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2771, 2748, 2715, 8}

$$\frac{(6a^2+b^2)\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{(6a^2+b^2)\sin(c+dx)\cos(c+dx)}{16d}+\frac{1}{16}x\left(6a^2+b^2\right)-\frac{7ab\cos^5(c+dx)}{30d}-\frac{b\cos^5(c+dx)(a+b\sin(c+dx))}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((6*a^2 + b^2)*x)/16 - (7*a*b*Cos[c + d*x]^5)/(30*d) + ((6*a^2 + b^2)*Cos[c
 + d*x]*Sin[c + d*x])/(16*d) + ((6*a^2 + b^2)*Cos[c + d*x]^3*Sin[c + d*x])/
(24*d) - (b*Cos[c + d*x]^5*(a + b*Sin[c + d*x]))/(6*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2771

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
```

```
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(
a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rubi steps

$$\begin{aligned}\int \cos^4(c+dx)(a+b\sin(c+dx))^2\,dx &= -\frac{b\cos^5(c+dx)(a+b\sin(c+dx))}{6d}+\frac{1}{6}\int \cos^4(c+dx)\left(6a^2+b^2+\right.\\ &= -\frac{7ab\cos^5(c+dx)}{30d}-\frac{b\cos^5(c+dx)(a+b\sin(c+dx))}{6d}+\frac{1}{6}\left(6a^2+\right.\\ &= -\frac{7ab\cos^5(c+dx)}{30d}+\frac{(6a^2+b^2)\cos^3(c+dx)\sin(c+dx)}{24d}-\frac{b\cos^5(\\ &= -\frac{7ab\cos^5(c+dx)}{30d}+\frac{(6a^2+b^2)\cos(c+dx)\sin(c+dx)}{16d}+\frac{(6a^2+b\\ &= \frac{1}{16}\left(6a^2+b^2\right)x-\frac{7ab\cos^5(c+dx)}{30d}+\frac{(6a^2+b^2)\cos(c+dx)\sin(c-}{16d}\end{aligned}$$

Mathematica [A]
time = 0.21, size = 133, normalized size = 1.15

$$\frac{360a^2c+60b^2c+360a^2dx+60b^2dx-240ab\cos(c+dx)-120ab\cos(3(c+dx))-24ab\cos(5(c+dx))+240a^2\sin(2(c+dx))+15b^2\sin(2(c+dx))+30a^2\sin(4(c+dx))-15b^2\sin(4(c+dx))-5b^2\sin(6(c+dx))}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (360*a^2*c + 60*b^2*c + 360*a^2*d*x + 60*b^2*d*x - 240*a*b*Cos[c + d*x] - 1
20*a*b*Cos[3*(c + d*x)] - 24*a*b*Cos[5*(c + d*x)] + 240*a^2*Sin[2*(c + d*x)
] + 15*b^2*Sin[2*(c + d*x)] + 30*a^2*Sin[4*(c + d*x)] - 15*b^2*Sin[4*(c + d
*x)] - 5*b^2*Sin[6*(c + d*x)])/(960*d)
```

Maple [A]
time = 0.42, size = 108, normalized size = 0.93

method	result
derivativedivides	$\frac{b^2\left(-\frac{\sin(dx+c)\left(\cos^5(dx+c)\right)}{6}+\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{24}+\frac{dx}{16}+\frac{c}{16}\right)-\frac{2ab\left(\cos^5(dx+c)\right)}{5}+a^2\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos}{}\right.}{4}\right.}{d}$
default	$\frac{b^2\left(-\frac{\sin(dx+c)\left(\cos^5(dx+c)\right)}{6}+\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{24}+\frac{dx}{16}+\frac{c}{16}\right)-\frac{2ab\left(\cos^5(dx+c)\right)}{5}+a^2\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos}{}\right.}{4}\right.}{d}$

risch	$\frac{3a^2x}{8}+\frac{b^2x}{16}-\frac{ab\cos(dx+c)}{4d}-\frac{\sin(6dx+6c)b^2}{192d}-\frac{ab\cos(5dx+5c)}{40d}+\frac{a^2\sin(4dx+4c)}{32d}-\frac{\sin(4dx+4c)b^2}{64d}-\frac{ab\cos($
norman	$\frac{\left(\frac{3a^2}{8}+\frac{b^2}{16}\right)x+\left(\frac{3a^2}{8}+\frac{b^2}{16}\right)x\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{9a^2}{4}+\frac{3b^2}{8}\right)x\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{9a^2}{4}+\frac{3b^2}{8}\right)x\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{15a^2}{2}+\frac{5}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^2*(-1/6*sin(d*x+c)*cos(d*x+c)^5+1/24*(cos(d*x+c)^3+3/2*cos(d*x+c))*s
in(d*x+c)+1/16*d*x+1/16*c)-2/5*a*b*cos(d*x+c)^5+a^2*(1/4*(cos(d*x+c)^3+3/2*
cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c))
```

Maxima [A]
time = 0.30, size = 88, normalized size = 0.76

$$-\frac{384\,ab\cos\left(dx+c\right)^{5}-30\,\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right)\right)a^{2}-5\,\left(4\,\sin\left(2\,dx+2\,c\right)^{3}+12\,dx+12\,c-3\,\sin\left(4\,dx+4\,c\right)\right)b^{2}}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/960*(384*a*b*cos(d*x + c)^5 - 30*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*s
in(2*d*x + 2*c))*a^2 - 5*(4*sin(2*d*x + 2*c)^3 + 12*d*x + 12*c - 3*sin(4*d*
x + 4*c))*b^2)/d
```

Fricas [A]
time = 0.36, size = 89, normalized size = 0.77

$$-\frac{96\,ab\cos\left(dx+c\right)^{5}-15\,\left(6\,a^{2}+b^{2}\right)dx+5\,\left(8\,b^{2}\cos\left(dx+c\right)^{5}-2\,\left(6\,a^{2}+b^{2}\right)\cos\left(dx+c\right)^{3}-3\,\left(6\,a^{2}+b^{2}\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/240*(96*a*b*cos(d*x + c)^5 - 15*(6*a^2 + b^2)*d*x + 5*(8*b^2*cos(d*x + c
)^5 - 2*(6*a^2 + b^2)*cos(d*x + c)^3 - 3*(6*a^2 + b^2)*cos(d*x + c))*sin(d*
x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 287 vs. $2(107) = 214$.
time = 0.44, size = 287, normalized size = 2.47

$$\begin{cases} \frac{3a^2x\sin^4(c+dx)}{8}+\frac{3a^2x\sin^2(c+dx)\cos^2(c+dx)}{4}+\frac{3a^2x\cos^4(c+dx)}{8}+\frac{3a^2\sin^3(c+dx)\cos(c+dx)}{8d}+\frac{5a^2\sin(c+dx)\cos^3(c+dx)}{8d}-\frac{2ab\cos^5(c+dx)}{5d}+\frac{b^2x\sin^6(c+dx)}{16}+\frac{3b^2x\sin^4(c+dx)\cos^2(c+dx)}{16}+\frac{3b^2x\sin^2(c+dx)\cos^4(c+dx)}{16}+\frac{b^2x\cos^6(c+dx)}{16}+\frac{b^2\sin^5(c+dx)\cos(c+dx)}{16d}+\frac{b^2\sin^3(c+dx)\cos^3(c+dx)}{6d}-\frac{b^2\sin(c+dx)\cos^5(c+dx)}{16d} & \text{for } d \neq 0 \\ x(a+b\sin(c))^2\cos^4(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((3*a**2*x*sin(c + d*x)**4/8 + 3*a**2*x*sin(c + d*x)**2*cos(c + d*
x)**2/4 + 3*a**2*x*cos(c + d*x)**4/8 + 3*a**2*sin(c + d*x)**3*cos(c + d*x)/
(8*d) + 5*a**2*sin(c + d*x)*cos(c + d*x)**3/(8*d) - 2*a*b*cos(c + d*x)**5/(
5*d) + b**2*x*sin(c + d*x)**6/16 + 3*b**2*x*sin(c + d*x)**4*cos(c + d*x)**2
/16 + 3*b**2*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + b**2*x*cos(c + d*x)**6/
16 + b**2*sin(c + d*x)**5*cos(c + d*x)/(16*d) + b**2*sin(c + d*x)**3*cos(c
+ d*x)**3/(6*d) - b**2*sin(c + d*x)*cos(c + d*x)**5/(16*d), Ne(d, 0)), (x*(
a + b*sin(c))**2*cos(c)**4, True))
```

Giac [A]
time = 4.98, size = 123, normalized size = 1.06

$$\frac{1}{16}\left(6\,a^2+b^2\right)x-\frac{ab\cos\left(5\,dx+5\,c\right)}{40\,d}-\frac{ab\cos\left(3\,dx+3\,c\right)}{8\,d}-\frac{ab\cos\left(dx+c\right)}{4\,d}-\frac{b^2\sin\left(6\,dx+6\,c\right)}{192\,d}+\frac{\left(2\,a^2-b^2\right)\sin\left(4\,dx+4\,c\right)}{64\,d}+\frac{\left(16\,a^2+b^2\right)\sin\left(2\,dx+2\,c\right)}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/16*(6*a^2 + b^2)*x - 1/40*a*b*cos(5*d*x + 5*c)/d - 1/8*a*b*cos(3*d*x + 3*
c)/d - 1/4*a*b*cos(d*x + c)/d - 1/192*b^2*sin(6*d*x + 6*c)/d + 1/64*(2*a^2
- b^2)*sin(4*d*x + 4*c)/d + 1/64*(16*a^2 + b^2)*sin(2*d*x + 2*c)/d
```

Mupad [B]
time = 5.31, size = 134, normalized size = 1.16

$$\frac{3\,a^2\,x}{8}+\frac{b^2\,x}{16}+\frac{a^2\,{\cos\left(c+d\,x\right)}^3\,\sin\left(c+d\,x\right)}{4\,d}+\frac{b^2\,{\cos\left(c+d\,x\right)}^3\,\sin\left(c+d\,x\right)}{24\,d}-\frac{b^2\,{\cos\left(c+d\,x\right)}^5\,\sin\left(c+d\,x\right)}{6\,d}-\frac{2\,a\,b\,{\cos\left(c+d\,x\right)}^5}{5\,d}+\frac{3\,a^2\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{8\,d}+\frac{b^2\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(a + b*sin(c + d*x))^2,x)
```

```
[Out] (3*a^2*x)/8 + (b^2*x)/16 + (a^2*cos(c + d*x)^3*sin(c + d*x))/(4*d) + (b^2*c
os(c + d*x)^3*sin(c + d*x))/(24*d) - (b^2*cos(c + d*x)^5*sin(c + d*x))/(6*d
) - (2*a*b*cos(c + d*x)^5)/(5*d) + (3*a^2*cos(c + d*x)*sin(c + d*x))/(8*d)
+ (b^2*cos(c + d*x)*sin(c + d*x))/(16*d)
```

3.395 $\int \cos^2(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=86

$$\frac{1}{8}\left(4a^2+b^2\right)x-\frac{5ab\cos^3(c+dx)}{12d}+\frac{(4a^2+b^2)\cos(c+dx)\sin(c+dx)}{8d}-\frac{b\cos^3(c+dx)(a+b\sin(c+dx))}{4d}$$

```
[Out] 1/8*(4*a^2+b^2)*x-5/12*a*b*cos(d*x+c)^3/d+1/8*(4*a^2+b^2)*cos(d*x+c)*sin(d*
x+c)/d-1/4*b*cos(d*x+c)^3*(a+b*sin(d*x+c))/d
```

Rubi [A]
time = 0.06, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2771, 2748, 2715, 8}

$$\frac{(4a^2+b^2)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}x\left(4a^2+b^2\right)-\frac{5ab\cos^3(c+dx)}{12d}-\frac{b\cos^3(c+dx)(a+b\sin(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((4*a^2 + b^2)*x)/8 - (5*a*b*Cos[c + d*x]^3)/(12*d) + ((4*a^2 + b^2)*Cos[c
+ d*x]*Sin[c + d*x])/(8*d) - (b*Cos[c + d*x]^3*(a + b*Sin[c + d*x]))/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2771

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(
```

```
a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rubi steps

$$\begin{aligned}\int \cos^2(c+dx)(a+b\sin(c+dx))^2\,dx &= -\frac{b\cos^3(c+dx)(a+b\sin(c+dx))}{4d}+\frac{1}{4}\int \cos^2(c+dx)\left(4a^2+b^2+\right.\\ &= -\frac{5ab\cos^3(c+dx)}{12d}-\frac{b\cos^3(c+dx)(a+b\sin(c+dx))}{4d}+\frac{1}{4}\left(4a^2+\right.\\ &= -\frac{5ab\cos^3(c+dx)}{12d}+\frac{(4a^2+b^2)\cos(c+dx)\sin(c+dx)}{8d}-\frac{b\cos^3(c\right.}{}\\ &= \frac{1}{8}\left(4a^2+b^2\right)x-\frac{5ab\cos^3(c+dx)}{12d}+\frac{(4a^2+b^2)\cos(c+dx)\sin(c+}{8d}\end{aligned}$$

Mathematica [A]
time = 0.25, size = 85, normalized size = 0.99

$$\frac{-48ab\cos(c+dx)-16ab\cos(3(c+dx))+3(16a^2c+4b^2c+16a^2dx+4b^2dx+8a^2\sin(2(c+dx))-b^2\sin(4(c+dx)))}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-48*a*b*Cos[c + d*x] - 16*a*b*Cos[3*(c + d*x)] + 3*(16*a^2*c + 4*b^2*c + 1
6*a^2*d*x + 4*b^2*d*x + 8*a^2*Sin[2*(c + d*x)] - b^2*Sin[4*(c + d*x)]))/(96
*d)
```

Maple [A]
time = 0.29, size = 86, normalized size = 1.00

method	result
risch	$\frac{a^2x}{2}+\frac{b^2x}{8}-\frac{ab\cos(dx+c)}{2d}-\frac{\sin(4dx+4c)b^2}{32d}-\frac{ab\cos(3dx+3c)}{6d}+\frac{a^2\sin(2dx+2c)}{4d}$
derivativedivides	$\frac{b^2\left(-\frac{\sin(dx+c)\left(\cos^3(dx+c)\right)}{4}+\frac{\cos(dx+c)\sin(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)-\frac{2ab\left(\cos^3(dx+c)\right)}{3}+a^2\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
default	$\frac{b^2\left(-\frac{\sin(dx+c)\left(\cos^3(dx+c)\right)}{4}+\frac{\cos(dx+c)\sin(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)-\frac{2ab\left(\cos^3(dx+c)\right)}{3}+a^2\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
norman	$\left(\frac{a^2}{2}+\frac{b^2}{8}\right)x+\left(2a^2+\frac{b^2}{2}\right)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(2a^2+\frac{b^2}{2}\right)x\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(3a^2+\frac{3b^2}{4}\right)x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{a^2}{2}+\frac{b^2}{8}\right)x($

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^2*(-1/4*sin(d*x+c)*cos(d*x+c)^3+1/8*cos(d*x+c)*sin(d*x+c)+1/8*d*x+1/
8*c)-2/3*a*b*cos(d*x+c)^3+a^2*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))
```

Maxima [A]
time = 0.28, size = 64, normalized size = 0.74

$$-\frac{64\,ab\cos\left(dx+c\right)^{3}-24\,\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)a^{2}-3\,\left(4\,dx+4\,c-\sin\left(4\,dx+4\,c\right)\right)b^{2}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/96*(64*a*b*cos(d*x + c)^3 - 24*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^2 - 3*
(4*d*x + 4*c - sin(4*d*x + 4*c))*b^2)/d
```

Fricas [A]
time = 0.36, size = 70, normalized size = 0.81

$$-\frac{16\,ab\cos\left(dx+c\right)^{3}-3\,\left(4\,a^{2}+b^{2}\right)dx+3\,\left(2\,b^{2}\cos\left(dx+c\right)^{3}-\left(4\,a^{2}+b^{2}\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/24*(16*a*b*cos(d*x + c)^3 - 3*(4*a^2 + b^2)*d*x + 3*(2*b^2*cos(d*x + c)^
3 - (4*a^2 + b^2)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(76) = 152$.
time = 0.19, size = 180, normalized size = 2.09

$$\begin{cases} \frac{a^{2}x\sin^{2}\left(c+dx\right)}{2}+\frac{a^{2}x\cos^{2}\left(c+dx\right)}{2}+\frac{a^{2}\sin\left(c+dx\right)\cos\left(c+dx\right)}{2d}-\frac{2ab\cos^{3}\left(c+dx\right)}{3d}+\frac{b^{2}x\sin^{4}\left(c+dx\right)}{8}+\frac{b^{2}x\sin^{2}\left(c+dx\right)\cos^{2}\left(c+dx\right)}{4}+\frac{b^{2}x\cos^{4}\left(c+dx\right)}{8}+\frac{b^{2}\sin^{3}\left(c+dx\right)\cos\left(c+dx\right)}{8d}-\frac{b^{2}\sin\left(c+dx\right)\cos^{3}\left(c+dx\right)}{8d} & \text{for}\: d\neq 0\\ x\left(a+b\sin\left(c\right)\right)^{2}\cos^{2}\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*x*sin(c + d*x)**2/2 + a**2*x*cos(c + d*x)**2/2 + a**2*sin(c
 + d*x)*cos(c + d*x)/(2*d) - 2*a*b*cos(c + d*x)**3/(3*d) + b**2*x*sin(c + d
*x)**4/8 + b**2*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + b**2*x*cos(c + d*x)**
4/8 + b**2*sin(c + d*x)**3*cos(c + d*x)/(8*d) - b**2*sin(c + d*x)*cos(c + d
*x)**3/(8*d), Ne(d, 0)), (x*(a + b*sin(c))**2*cos(c)**2, True))
```

Giac [A]
time = 3.81, size = 76, normalized size = 0.88

$$\frac{1}{8}\left(4\,a^2+b^2\right)x-\frac{ab\cos\left(3\,dx+3\,c\right)}{6\,d}-\frac{ab\cos\left(dx+c\right)}{2\,d}-\frac{b^2\sin\left(4\,dx+4\,c\right)}{32\,d}+\frac{a^2\sin\left(2\,dx+2\,c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/8*(4*a^2 + b^2)*x - 1/6*a*b*cos(3*d*x + 3*c)/d - 1/2*a*b*cos(d*x + c)/d -
 1/32*b^2*sin(4*d*x + 4*c)/d + 1/4*a^2*sin(2*d*x + 2*c)/d
```

Mupad [B]
time = 5.38, size = 71, normalized size = 0.83

$$\frac{6\,a^2\,\sin\left(2\,c+2\,d\,x\right)-\frac{3\,b^2\,\sin\left(4\,c+4\,d\,x\right)}{4}-12\,a\,b\,\cos\left(c+d\,x\right)-4\,a\,b\,\cos\left(3\,c+3\,d\,x\right)+12\,a^2\,d\,x+3\,b^2\,d\,x}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a + b*sin(c + d*x))^2,x)
```

```
[Out] (6*a^2*sin(2*c + 2*d*x) - (3*b^2*sin(4*c + 4*d*x))/4 - 12*a*b*cos(c + d*x)
- 4*a*b*cos(3*c + 3*d*x) + 12*a^2*d*x + 3*b^2*d*x)/(24*d)
```

3.396 $\int \sec^2(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=49

$$-b^2x+\frac{ab\cos(c+dx)}{d}+\frac{\sec(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))}{d}$$

```
[Out] -b^2*x+a*b*cos(d*x+c)/d+sec(d*x+c)*(b+a*sin(d*x+c))*(a+b*sin(d*x+c))/d
```

Rubi [A]
time = 0.03, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2770, 2718}

$$\frac{ab\cos(c+dx)}{d}+\frac{\sec(c+dx)(a\sin(c+dx)+b)(a+b\sin(c+dx))}{d}+b^2(-x)$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -(b^2*x) + (a*b*Cos[c + d*x])/d + (Sec[c + d*x]*(b + a*Sin[c + d*x])*(a + b
*Sin[c + d*x]))/d
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2770

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x
])^(m - 1)*((b + a*Sin[e + f*x])/(f*g*(p + 1))), x] + Dist[1/(g^2*(p + 1)),
 Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a
^2*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g}
, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p
] || IntegerQ[m])
```

Rubi steps

$$\int \sec^2(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{\sec(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))}{d}-\int \left(b^2+ab\sin(c\right.$$

$$= -b^2x+\frac{\sec(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))}{d}-(ab)\int \mathrm{s}$$

$$= -b^2x+\frac{ab\cos(c+dx)}{d}+\frac{\sec(c+dx)(b+a\sin(c+dx))(a+b\sin(c}{d}$$

Mathematica [A]
time = 0.06, size = 55, normalized size = 1.12

$$-\frac{b^2\tan^{-1}(\tan(c+dx))}{d}+\frac{2ab\sec(c+dx)}{d}+\frac{a^2\tan(c+dx)}{d}+\frac{b^2\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -((b^2*ArcTan[Tan[c + d*x]])/d) + (2*a*b*Sec[c + d*x])/d + (a^2*Tan[c + d*x
])/d + (b^2*Tan[c + d*x])/d
```

Maple [A]
time = 0.21, size = 46, normalized size = 0.94

method	result
derivativedivides	$\frac{a^2\tan(dx+c)+\frac{2ab}{\cos(dx+c)}+b^2(\tan(dx+c)-dx-c)}{d}$
default	$\frac{a^2\tan(dx+c)+\frac{2ab}{\cos(dx+c)}+b^2(\tan(dx+c)-dx-c)}{d}$
risch	$-b^2x+\frac{2ia^2+2ib^2+4a\,e^{i(dx+c)}b}{d\left(1+e^{2i(dx+c)}\right)}$
norman	$\frac{b^2x+b^2x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{4ab\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{8ab}{d}-b^2x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b^2x\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{2\left(a^2+b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*tan(d*x+c)+2*a*b/cos(d*x+c)+b^2*(tan(d*x+c)-d*x-c))
```

Maxima [A]
time = 0.51, size = 46, normalized size = 0.94

$$-\frac{(dx+c-\tan(dx+c))b^2-a^2\tan(dx+c)-\frac{2ab}{\cos(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -((d*x + c - tan(d*x + c))*b^2 - a^2*tan(d*x + c) - 2*a*b/cos(d*x + c))/d
```

Fricas [A]
time = 0.33, size = 45, normalized size = 0.92

$$-\frac{b^2dx\cos(dx+c)-2ab-(a^2+b^2)\sin(dx+c)}{d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -(b^2*d*x*cos(d*x + c) - 2*a*b - (a^2 + b^2)*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sin{(c + dx)})^2 \sec^2{(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sin(c + d*x))**2*sec(c + d*x)**2, x)
```

Giac [A]
time = 5.14, size = 63, normalized size = 1.29

$$-\frac{(dx+c)b^2 + \frac{2\left(a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\,ab\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -((d*x + c)*b^2 + 2*(a^2*tan(1/2*d*x + 1/2*c) + b^2*tan(1/2*d*x + 1/2*c) +
2*a*b)/(tan(1/2*d*x + 1/2*c)^2 - 1))/d
```

Mupad [B]
time = 5.21, size = 53, normalized size = 1.08

$$-b^2\,x-\frac{4\,a\,b+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,a^2+2\,b^2\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^2/cos(c + d*x)^2,x)
```

```
[Out] - b^2*x - (4*a*b + tan(c/2 + (d*x)/2)*(2*a^2 + 2*b^2))/(d*(tan(c/2 + (d*x)/
2)^2 - 1))
```

3.397 $\int \sec^4(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=75

$$\frac{ab\sec(c+dx)}{3d}+\frac{\sec^3(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))}{3d}+\frac{(2a^2-b^2)\tan(c+dx)}{3d}$$

```
[Out] 1/3*a*b*sec(d*x+c)/d+1/3*sec(d*x+c)^3*(b+a*sin(d*x+c))*(a+b*sin(d*x+c))/d+1
/3*(2*a^2-b^2)*tan(d*x+c)/d
```

Rubi [A]
time = 0.07, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2770, 2748, 3852, 8}

$$\frac{(2a^2-b^2)\tan(c+dx)}{3d}+\frac{ab\sec(c+dx)}{3d}+\frac{\sec^3(c+dx)(a\sin(c+dx)+b)(a+b\sin(c+dx))}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (a*b*Sec[c + d*x])/(3*d) + (Sec[c + d*x]^3*(b + a*Sin[c + d*x])*(a + b*Sin[
c + d*x]))/(3*d) + ((2*a^2 - b^2)*Tan[c + d*x])/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2770

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x
])^(m - 1)*((b + a*Sin[e + f*x])/(f*g*(p + 1))), x] + Dist[1/(g^2*(p + 1)),
 Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a
^2*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g}
, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p
] || IntegerQ[m])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^4(c+dx)(a+b\sin(c+dx))^2\,dx &= \frac{\sec^3(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))}{3d}-\frac{1}{3}\int \sec^2(c+d\\ &= \frac{ab\sec(c+dx)}{3d}+\frac{\sec^3(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))}{3d}\\ &= \frac{ab\sec(c+dx)}{3d}+\frac{\sec^3(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))}{3d}\\ &= \frac{ab\sec(c+dx)}{3d}+\frac{\sec^3(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))}{3d}\end{aligned}$$

Mathematica [A]
time = 0.34, size = 105, normalized size = 1.40

$$\frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(8ab+3(2a^2+b^2)\sin(c+dx)+(2a^2-b^2)\sin(3(c+dx))\right)}{12d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3(-1+\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + b*Sin[c + d*x])^2,x]
[Out] ((Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(8*a*b + 3*(2*a^2 + b^2)*Sin[c + d*x
] + (2*a^2 - b^2)*Sin[3*(c + d*x)]))/(12*d*(Cos[(c + d*x)/2] + Sin[(c + d*x
)/2])^3*(-1 + Sin[c + d*x])^2)
```

Maple [A]
time = 0.34, size = 62, normalized size = 0.83

method	result
derivativedivides	$\frac{-a^2\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+\frac{2ab}{3\cos(dx+c)^3}+\frac{b^2\left(\sin^3(dx+c)\right)}{3\cos(dx+c)^3}}{d}$
default	$\frac{-a^2\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+\frac{2ab}{3\cos(dx+c)^3}+\frac{b^2\left(\sin^3(dx+c)\right)}{3\cos(dx+c)^3}}{d}$
risch	$-\frac{2i\left(8iab\,e^{3i(dx+c)}+3b^2e^{4i(dx+c)}-6a^2e^{2i(dx+c)}-2a^2+b^2\right)}{3d\left(1+e^{2i(dx+c)}\right)^3}$

norman	$\frac{-\frac{4ab}{3d}-\frac{2a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{2a^2\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{8\left(a^2+b^2\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{8\left(a^2+b^2\right)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{4\left(a^2+4b^2\right)\left(\tan^5\left(\frac{dx}{2}\right.\right.}{3d}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3\left(1+\tan^2\left(\frac{dx}{2}+\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^2*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+2/3*a*b/cos(d*x+c)^3+1/3*b^2*s
in(d*x+c)^3/cos(d*x+c)^3)
```

Maxima [**A**]
time = 0.28, size = 51, normalized size = 0.68

$$\frac{b^2\tan\left(dx+c\right)^3+\left(\tan\left(dx+c\right)^3+3\,\tan\left(dx+c\right)\right)a^2+\frac{2\,ab}{\cos\left(dx+c\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(b^2*tan(d*x + c)^3 + (tan(d*x + c)^3 + 3*tan(d*x + c))*a^2 + 2*a*b/cos
(d*x + c)^3)/d
```

Fricas [**A**]
time = 0.32, size = 52, normalized size = 0.69

$$\frac{2\,ab+\left(\left(2\,a^2-b^2\right)\cos\left(dx+c\right)^2+a^2+b^2\right)\sin\left(dx+c\right)}{3\,d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(2*a*b + ((2*a^2 - b^2)*cos(d*x + c)^2 + a^2 + b^2)*sin(d*x + c))/(d*co
s(d*x + c)^3)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sin\left(c+dx\right)\right)^2\sec^4\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sin(c + d*x))**2*sec(c + d*x)**4, x)
```

Giac [A]
time = 4.61, size = 102, normalized size = 1.36

$$-\frac{2\left(3\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+6\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-2\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+4\,b^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+3\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\,ab\right)}{3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -2/3*(3*a^2*tan(1/2*d*x + 1/2*c)^5 + 6*a*b*tan(1/2*d*x + 1/2*c)^4 - 2*a^2*t
an(1/2*d*x + 1/2*c)^3 + 4*b^2*tan(1/2*d*x + 1/2*c)^3 + 3*a^2*tan(1/2*d*x +
1/2*c) + 2*a*b)/((tan(1/2*d*x + 1/2*c)^2 - 1)^3*d)
```

Mupad [B]
time = 5.26, size = 71, normalized size = 0.95

$$\frac{\frac{2\,a\,b}{3}+\frac{a^2\,\sin(c+d\,x)}{3}+\frac{b^2\,\sin(c+d\,x)}{3}+\cos\left(c+d\,x\right)^2\,\left(\frac{2\,a^2\,\sin(c+d\,x)}{3}-\frac{b^2\,\sin(c+d\,x)}{3}\right)}{d\cos\left(c+d\,x\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^2/cos(c + d*x)^4,x)
```

```
[Out] ((2*a*b)/3 + (a^2*sin(c + d*x))/3 + (b^2*sin(c + d*x))/3 + cos(c + d*x)^2*(
(2*a^2*sin(c + d*x))/3 - (b^2*sin(c + d*x))/3))/(d*cos(c + d*x)^3)
```

3.398 $\int \sec^6(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=103

$$\frac{ab\sec^3(c+dx)}{5d}+\frac{\sec^5(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))}{5d}+\frac{(4a^2-b^2)\tan(c+dx)}{5d}+\frac{(4a^2-b^2)\tan^3(c+dx)}{15d}$$

```
[Out] 1/5*a*b*sec(d*x+c)^3/d+1/5*sec(d*x+c)^5*(b+a*sin(d*x+c))*(a+b*sin(d*x+c))/d
+1/5*(4*a^2-b^2)*tan(d*x+c)/d+1/15*(4*a^2-b^2)*tan(d*x+c)^3/d
```

Rubi [A]
time = 0.07, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2770, 2748, 3852}

$$\frac{(4a^2-b^2)\tan^3(c+dx)}{15d}+\frac{(4a^2-b^2)\tan(c+dx)}{5d}+\frac{ab\sec^3(c+dx)}{5d}+\frac{\sec^5(c+dx)(a\sin(c+dx)+b)(a+b\sin(c+dx))}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (a*b*Sec[c + d*x]^3)/(5*d) + (Sec[c + d*x]^5*(b + a*Sin[c + d*x])*(a + b*Si
n[c + d*x]))/(5*d) + ((4*a^2 - b^2)*Tan[c + d*x])/(5*d) + ((4*a^2 - b^2)*Ta
n[c + d*x]^3)/(15*d)
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2770

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x
])^(m - 1)*((b + a*Sin[e + f*x])/(f*g*(p + 1))), x] + Dist[1/(g^2*(p + 1)),
 Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a
^2*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g}
, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p
] || IntegerQ[m])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \sec^6(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{\sec^5(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))}{5d} - \frac{1}{5}\int \sec^4(c+d$$
$$= \frac{ab\sec^3(c+dx)}{5d} + \frac{\sec^5(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))}{5d}$$
$$= \frac{ab\sec^3(c+dx)}{5d} + \frac{\sec^5(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))}{5d}$$
$$= \frac{ab\sec^3(c+dx)}{5d} + \frac{\sec^5(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))}{5d}$$

Mathematica [A]
time = 0.46, size = 84, normalized size = 0.82

$$\frac{\sec^5(c+dx)\left(48ab+20(2a^2+b^2)\sin(c+dx)+5(4a^2-b^2)\sin(3(c+dx))+4a^2\sin(5(c+dx))-b^2\sin(5(c+dx))\right)}{120d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (Sec[c + d*x]^5*(48*a*b + 20*(2*a^2 + b^2)*Sin[c + d*x] + 5*(4*a^2 - b^2)*S
in[3*(c + d*x)] + 4*a^2*Sin[5*(c + d*x)] - b^2*Sin[5*(c + d*x)]))/(120*d)
```

Maple [A]
time = 0.41, size = 92, normalized size = 0.89

method	result
derivativedivides	$\frac{-a^2\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c)\right)}{5}-\frac{4\left(\sec^2(dx+c)\right)}{15}\right)\tan(dx+c)+\frac{2ab}{5\cos(dx+c)^5}+b^2\left(\frac{\sin^3(dx+c)}{5\cos(dx+c)^5}+\frac{2\left(\sin^3(dx+c)\right)}{15\cos(dx+c)^3}\right)}{d}$
default	$\frac{-a^2\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c)\right)}{5}-\frac{4\left(\sec^2(dx+c)\right)}{15}\right)\tan(dx+c)+\frac{2ab}{5\cos(dx+c)^5}+b^2\left(\frac{\sin^3(dx+c)}{5\cos(dx+c)^5}+\frac{2\left(\sin^3(dx+c)\right)}{15\cos(dx+c)^3}\right)}{d}$
risch	$-\frac{4i\left(48iab\,e^{5i(dx+c)}+15b^2e^{6i(dx+c)}-40a^2e^{4i(dx+c)}-5b^2e^{4i(dx+c)}-20a^2e^{2i(dx+c)}+5b^2e^{2i(dx+c)}-4a^2+b^2\right)}{15d\left(1+e^{2i(dx+c)}\right)^5}$
norman	$\frac{-\frac{4ab}{5d}-\frac{2a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{2a^2\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{4\left(a^2+2b^2\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{4\left(a^2+2b^2\right)\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{2\left(11a^2+16b^2\right)\left(\tan^5\right.}{5d}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^2*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c)+2/5*a*b/cos
(d*x+c)^5+b^2*(1/5*sin(d*x+c)^3/cos(d*x+c)^5+2/15*sin(d*x+c)^3/cos(d*x+c)^3
))
```

Maxima [A]
time = 0.28, size = 76, normalized size = 0.74

$$\frac{\left(3\,\tan\left(dx+c\right)^{5}+10\,\tan\left(dx+c\right)^{3}+15\,\tan\left(dx+c\right)\right)a^{2}+\left(3\,\tan\left(dx+c\right)^{5}+5\,\tan\left(dx+c\right)^{3}\right)b^{2}+\frac{6\,ab}{\cos(dx+c)^{5}}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/15*((3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a^2 + (3*tan
(d*x + c)^5 + 5*tan(d*x + c)^3)*b^2 + 6*a*b/cos(d*x + c)^5)/d
```

Fricas [A]
time = 0.35, size = 77, normalized size = 0.75

$$\frac{6\,ab+\left(2\,(4\,a^{2}-b^{2})\cos\left(dx+c\right)^{4}+(4\,a^{2}-b^{2})\cos\left(dx+c\right)^{2}+3\,a^{2}+3\,b^{2}\right)\sin\left(dx+c\right)}{15\,d\cos\left(dx+c\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/15*(6*a*b + (2*(4*a^2 - b^2)*cos(d*x + c)^4 + (4*a^2 - b^2)*cos(d*x + c)^
2 + 3*a^2 + 3*b^2)*sin(d*x + c))/(d*cos(d*x + c)^5)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 5.83, size = 181, normalized size = 1.76

$$-\frac{2\left(15\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+30\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}-20\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+20\,b^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+58\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+8\,b^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+60\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-20\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+20\,b^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+15\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+6\,ab\right)}{15\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{5}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -2/15*(15*a^2*tan(1/2*d*x + 1/2*c)^9 + 30*a*b*tan(1/2*d*x + 1/2*c)^8 - 20*a
^2*tan(1/2*d*x + 1/2*c)^7 + 20*b^2*tan(1/2*d*x + 1/2*c)^7 + 58*a^2*tan(1/2*
d*x + 1/2*c)^5 + 8*b^2*tan(1/2*d*x + 1/2*c)^5 + 60*a*b*tan(1/2*d*x + 1/2*c)
```

```
^4 - 20*a^2*tan(1/2*d*x + 1/2*c)^3 + 20*b^2*tan(1/2*d*x + 1/2*c)^3 + 15*a^2
*tan(1/2*d*x + 1/2*c) + 6*a*b)/((tan(1/2*d*x + 1/2*c)^2 - 1)^5*d)
```

Mupad [B]
time = 5.36, size = 103, normalized size = 1.00

$$\frac{\frac{2\,a\,b}{5}+\frac{a^2\,\sin(c+d\,x)}{5}+\frac{b^2\,\sin(c+d\,x)}{5}+\cos\left(c+d\,x\right)^2\left(\frac{4\,a^2\,\sin(c+d\,x)}{15}-\frac{b^2\,\sin(c+d\,x)}{15}\right)+\cos\left(c+d\,x\right)^4\left(\frac{8\,a^2\,\sin(c+d\,x)}{15}-\frac{2\,b^2\,\sin(c+d\,x)}{15}\right)}{d\cos\left(c+d\,x\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^2/cos(c + d*x)^6,x)
```

```
[Out] ((2*a*b)/5 + (a^2*sin(c + d*x))/5 + (b^2*sin(c + d*x))/5 + cos(c + d*x)^2*(
(4*a^2*sin(c + d*x))/15 - (b^2*sin(c + d*x))/15) + cos(c + d*x)^4*((8*a^2*s
in(c + d*x))/15 - (2*b^2*sin(c + d*x))/15))/(d*cos(c + d*x)^5)
```

3.399 $\int \sec^8(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=129

$$\frac{ab\sec^5(c+dx)}{7d}+\frac{\sec^7(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))}{7d}+\frac{(6a^2-b^2)\tan(c+dx)}{7d}+\frac{2(6a^2-b^2)\tan^3(c+dx)}{21d}$$

[Out] 1/7*a*b*sec(d*x+c)^5/d+1/7*sec(d*x+c)^7*(b+a*sin(d*x+c))*(a+b*sin(d*x+c))/d +1/7*(6*a^2-b^2)*tan(d*x+c)/d+2/21*(6*a^2-b^2)*tan(d*x+c)^3/d+1/35*(6*a^2-b ^2)*tan(d*x+c)^5/d

Rubi [A]
time = 0.08, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2770, 2748, 3852}

$$\frac{(6a^2-b^2)\tan^5(c+dx)}{35d}+\frac{2(6a^2-b^2)\tan^3(c+dx)}{21d}+\frac{(6a^2-b^2)\tan(c+dx)}{7d}+\frac{ab\sec^5(c+dx)}{7d}+\frac{\sec^7(c+dx)(a\sin(c+dx)+b)(a+b\sin(c+dx))}{7d}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d*x]^8*(a + b*Sin[c + d*x])^2,x]

[Out] (a*b*Sec[c + d*x]^5)/(7*d) + (Sec[c + d*x]^7*(b + a*Sin[c + d*x])*(a + b*Si n[c + d*x]))/(7*d) + ((6*a^2 - b^2)*Tan[c + d*x])/(7*d) + (2*(6*a^2 - b^2)* Tan[c + d*x]^3)/(21*d) + ((6*a^2 - b^2)*Tan[c + d*x]^5)/(35*d)

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2770

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x
])^(m - 1)*((b + a*Sin[e + f*x])/(f*g*(p + 1))), x] + Dist[1/(g^2*(p + 1)),
 Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a
^2*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g}
, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p
] || IntegerQ[m])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

d}, x] && IGtQ[n/2, 0]

Rubi steps

$$\int \sec^8(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{\sec^7(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))}{7d} - \frac{1}{7}\int \sec^6(c+d$$
$$= \frac{ab\sec^5(c+dx)}{7d} + \frac{\sec^7(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))}{7d}$$
$$= \frac{ab\sec^5(c+dx)}{7d} + \frac{\sec^7(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))}{7d}$$
$$= \frac{ab\sec^5(c+dx)}{7d} + \frac{\sec^7(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))}{7d}$$

Mathematica [A]
time = 0.84, size = 110, normalized size = 0.85

$$\frac{\sec^7(c+dx)\,(240ab+105(2a^2+b^2)\sin(c+dx)+21(6a^2-b^2)\sin(3(c+dx))+42a^2\sin(5(c+dx))-7b^2\sin(5(c+dx))+6a^2\sin(7(c+dx))-b^2\sin(7(c+dx)))}{840d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^8*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (Sec[c + d*x]^7*(240*a*b + 105*(2*a^2 + b^2)*Sin[c + d*x] + 21*(6*a^2 - b^2
)*Sin[3*(c + d*x)] + 42*a^2*Sin[5*(c + d*x)] - 7*b^2*Sin[5*(c + d*x)] + 6*a
^2*Sin[7*(c + d*x)] - b^2*Sin[7*(c + d*x)]))/(840*d)
```

Maple [A]
time = 0.38, size = 120, normalized size = 0.93

method	result
derivativedivides	$\frac{-a^2\left(-\frac{16}{35}-\frac{\left(\sec^6(dx+c)\right)}{7}-\frac{6\left(\sec^4(dx+c)\right)}{35}-\frac{8\left(\sec^2(dx+c)\right)}{35}\right)\tan(dx+c)+\frac{2ab}{7\cos(dx+c)^7}+b^2\left(\frac{\sin^3(dx+c)}{7\cos(dx+c)^7}+\frac{4\left(\sin^3(dx+c)\right)}{35\cos(dx+c)^5}+\right.}{d}$
default	$\frac{-a^2\left(-\frac{16}{35}-\frac{\left(\sec^6(dx+c)\right)}{7}-\frac{6\left(\sec^4(dx+c)\right)}{35}-\frac{8\left(\sec^2(dx+c)\right)}{35}\right)\tan(dx+c)+\frac{2ab}{7\cos(dx+c)^7}+b^2\left(\frac{\sin^3(dx+c)}{7\cos(dx+c)^7}+\frac{4\left(\sin^3(dx+c)\right)}{35\cos(dx+c)^5}+\right.}{d}$
risch	$-\frac{16i\left(240iab\,e^{7i(dx+c)}+70b^2e^{8i(dx+c)}-210a^2e^{6i(dx+c)}-35b^2e^{6i(dx+c)}-126a^2e^{4i(dx+c)}+21b^2e^{4i(dx+c)}-42a^2e^{2i(dx+c)}+\right.}{105d\left(1+e^{2i(dx+c)}\right)^7}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^8*(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^2*(-16/35-1/7*sec(d*x+c)^6-6/35*sec(d*x+c)^4-8/35*sec(d*x+c)^2)*tan
(d*x+c)+2/7*a*b/cos(d*x+c)^7+b^2*(1/7*sin(d*x+c)^3/cos(d*x+c)^7+4/35*sin(d*
x+c)^3/cos(d*x+c)^5+8/105*sin(d*x+c)^3/cos(d*x+c)^3))
```

Maxima [A]
time = 0.28, size = 97, normalized size = 0.75

$$\frac{3\left(5\tan\left(dx+c\right)^{7}+21\tan\left(dx+c\right)^{5}+35\tan\left(dx+c\right)^{3}+35\tan\left(dx+c\right)\right)a^{2}+\left(15\tan\left(dx+c\right)^{7}+42\tan\left(dx+c\right)^{5}+35\tan\left(dx+c\right)^{3}\right)b^{2}+\frac{30\,ab}{\cos(dx+c)^{7}}}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/105*(3*(5*tan(d*x + c)^7 + 21*tan(d*x + c)^5 + 35*tan(d*x + c)^3 + 35*tan
(d*x + c))*a^2 + (15*tan(d*x + c)^7 + 42*tan(d*x + c)^5 + 35*tan(d*x + c)^3
)*b^2 + 30*a*b/cos(d*x + c)^7)/d
```

Fricas [A]
time = 0.33, size = 99, normalized size = 0.77

$$\frac{30\,ab+\left(8\left(6\,a^{2}-b^{2}\right)\cos\left(dx+c\right)^{6}+4\left(6\,a^{2}-b^{2}\right)\cos\left(dx+c\right)^{4}+3\left(6\,a^{2}-b^{2}\right)\cos\left(dx+c\right)^{2}+15\,a^{2}+15\,b^{2}\right)\sin\left(dx+c\right)}{105\,d\cos\left(dx+c\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/105*(30*a*b + (8*(6*a^2 - b^2)*cos(d*x + c)^6 + 4*(6*a^2 - b^2)*cos(d*x +
 c)^4 + 3*(6*a^2 - b^2)*cos(d*x + c)^2 + 15*a^2 + 15*b^2)*sin(d*x + c))/(d*
cos(d*x + c)^7)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**8*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 260 vs. 2(119) = 238.
time = 4.88, size = 260, normalized size = 2.02

$$-\frac{2\left(105\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}+210\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{12}-210\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+140\,b^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+903\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+112\,b^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+1050\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}-636\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+456\,b^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+903\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+112\,b^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+630\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-210\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+140\,b^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+105\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+30\,ab\right)}{105\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{7}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -2/105*(105*a^2*tan(1/2*d*x + 1/2*c)^13 + 210*a*b*tan(1/2*d*x + 1/2*c)^12 -
 210*a^2*tan(1/2*d*x + 1/2*c)^11 + 140*b^2*tan(1/2*d*x + 1/2*c)^11 + 903*a^
```

```
2*tan(1/2*d*x + 1/2*c)^9 + 112*b^2*tan(1/2*d*x + 1/2*c)^9 + 1050*a*b*tan(1/
2*d*x + 1/2*c)^8 - 636*a^2*tan(1/2*d*x + 1/2*c)^7 + 456*b^2*tan(1/2*d*x + 1
/2*c)^7 + 903*a^2*tan(1/2*d*x + 1/2*c)^5 + 112*b^2*tan(1/2*d*x + 1/2*c)^5 +
 630*a*b*tan(1/2*d*x + 1/2*c)^4 - 210*a^2*tan(1/2*d*x + 1/2*c)^3 + 140*b^2*
tan(1/2*d*x + 1/2*c)^3 + 105*a^2*tan(1/2*d*x + 1/2*c) + 30*a*b)/((tan(1/2*d
*x + 1/2*c)^2 - 1)^7*d)
```

Mupad [B]
time = 5.55, size = 135, normalized size = 1.05

$$\frac{\frac{2\,a\,b}{7}+\frac{a^2\,\sin(c+d\,x)}{7}+\frac{b^2\,\sin(c+d\,x)}{7}+\cos\left(c+d\,x\right)^2\,\left(\frac{6\,a^2\,\sin(c+d\,x)}{35}-\frac{b^2\,\sin(c+d\,x)}{35}\right)+\cos\left(c+d\,x\right)^4\,\left(\frac{8\,a^2\,\sin(c+d\,x)}{35}-\frac{4\,b^2\,\sin(c+d\,x)}{105}\right)+\cos\left(c+d\,x\right)^6\,\left(\frac{16\,a^2\,\sin(c+d\,x)}{35}-\frac{8\,b^2\,\sin(c+d\,x)}{105}\right)}{d\,\cos\left(c+d\,x\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^2/cos(c + d*x)^8,x)
```

```
[Out] ((2*a*b)/7 + (a^2*sin(c + d*x))/7 + (b^2*sin(c + d*x))/7 + cos(c + d*x)^2*(
(6*a^2*sin(c + d*x))/35 - (b^2*sin(c + d*x))/35) + cos(c + d*x)^4*((8*a^2*s
in(c + d*x))/35 - (4*b^2*sin(c + d*x))/105) + cos(c + d*x)^6*((16*a^2*sin(c
 + d*x))/35 - (8*b^2*sin(c + d*x))/105))/(d*cos(c + d*x)^7)
```

3.400 $\int \cos^5(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=144

$$\frac{(a^2-b^2)^2(a+b\sin(c+dx))^4}{4b^5d}-\frac{4a(a^2-b^2)(a+b\sin(c+dx))^5}{5b^5d}+\frac{(3a^2-b^2)(a+b\sin(c+dx))^6}{3b^5d}-\frac{4a(a+b$$

```
[Out] 1/4*(a^2-b^2)^2*(a+b*sin(d*x+c))^4/b^5/d-4/5*a*(a^2-b^2)*(a+b*sin(d*x+c))^5
/b^5/d+1/3*(3*a^2-b^2)*(a+b*sin(d*x+c))^6/b^5/d-4/7*a*(a+b*sin(d*x+c))^7/b^
5/d+1/8*(a+b*sin(d*x+c))^8/b^5/d
```

Rubi [A]
time = 0.09, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2747, 711}

$$\frac{(3a^2-b^2)(a+b\sin(c+dx))^6}{3b^5d}-\frac{4a(a^2-b^2)(a+b\sin(c+dx))^5}{5b^5d}+\frac{(a^2-b^2)^2(a+b\sin(c+dx))^4}{4b^5d}+\frac{(a+b\sin(c+dx))^8}{8b^5d}-\frac{4a(a+b\sin(c+dx))^7}{7b^5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((a^2 - b^2)^2*(a + b*Sin[c + d*x])^4)/(4*b^5*d) - (4*a*(a^2 - b^2)*(a + b*
Sin[c + d*x])^5)/(5*b^5*d) + ((3*a^2 - b^2)*(a + b*Sin[c + d*x])^6)/(3*b^5*
d) - (4*a*(a + b*Sin[c + d*x])^7)/(7*b^5*d) + (a + b*Sin[c + d*x])^8/(8*b^5
*d)
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^5(c+dx)(a+b\sin(c+dx))^3\,dx = \frac{\text{Subst}\left(\int (a+x)^3 (b^2-x^2)^2\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$
$$= \frac{\text{Subst}\left(\int \left((a^2-b^2)^2 (a+x)^3 - 4(a^3-ab^2)(a+x)^4 + 2(3a^2-b^2)(\right.}{b^5 d}$$
$$= \frac{(a^2-b^2)^2 (a+b\sin(c+dx))^4}{4b^5 d} - \frac{4a(a^2-b^2)(a+b\sin(c+dx))^5}{5b^5 d} +$$

Mathematica [A]

time = 0.54, size = 120, normalized size = 0.83

$$\frac{\frac{1}{4}(a^2-b^2)^2 (a+b\sin(c+dx))^4 - \frac{4}{5}a(a-b)(a+b)(a+b\sin(c+dx))^5 + \frac{1}{3}(3a^2-b^2)(a+b\sin(c+dx))^6 - \frac{4}{7}a(a+b\sin(c+dx))^7 + \frac{1}{8}(a+b\sin(c+dx))^8}{b^5 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + b*Sin[c + d*x])^3,x]
[Out] (((a^2 - b^2)^2*(a + b*Sin[c + d*x])^4)/4 - (4*a*(a - b)*(a + b)*(a + b*Sin
[c + d*x])^5)/5 + ((3*a^2 - b^2)*(a + b*Sin[c + d*x])^6)/3 - (4*a*(a + b*Si
n[c + d*x])^7)/7 + (a + b*Sin[c + d*x])^8/8)/(b^5*d)
```

Maple [A]

time = 0.66, size = 135, normalized size = 0.94

method	result
derivativedivides	$\frac{b^3\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^6(dx+c)\right)}{8}-\frac{\left(\cos^6(dx+c)\right)}{24}\right)+3a\,b^2\left(-\frac{\sin(dx+c)\left(\cos^6(dx+c)\right)}{7}+\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx}{35}\right.}{d}$
default	$\frac{b^3\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^6(dx+c)\right)}{8}-\frac{\left(\cos^6(dx+c)\right)}{24}\right)+3a\,b^2\left(-\frac{\sin(dx+c)\left(\cos^6(dx+c)\right)}{7}+\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx}{35}\right.}{d}$
risch	$\frac{5a^3\sin(dx+c)}{8d}+\frac{15a\,b^2\sin(dx+c)}{64d}+\frac{b^3\cos(8dx+8c)}{1024d}-\frac{3a\,b^2\sin(7dx+7c)}{448d}-\frac{b\cos(6dx+6c)a^2}{64d}+\frac{b^3\cos(6dx+6c)}{384d}$
norman	$\frac{2a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2a^3\left(\tan^{15}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{6a^2b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{6a^2b\left(\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{4\left(3a^2b+b^3\right)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{4\left(3a^2b+b\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
[Out] 1/d*(b^3*(-1/8*sin(d*x+c)^2*cos(d*x+c)^6-1/24*cos(d*x+c)^6)+3*a*b^2*(-1/7*s
in(d*x+c)*cos(d*x+c)^6+1/35*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))
```

```
-1/2*a^2*b*cos(d*x+c)^6+1/5*a^3*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x
+c))
```

Maxima [A]
time = 0.27, size = 144, normalized size = 1.00

$$\frac{105\,b^3\sin\left(dx+c\right)^8+360\,ab^2\sin\left(dx+c\right)^7+140\left(3\,a^2b-2\,b^3\right)\sin\left(dx+c\right)^6+168\left(a^3-6\,ab^2\right)\sin\left(dx+c\right)^5+1260\,a^2b\sin\left(dx+c\right)^2-210\left(6\,a^2b-b^3\right)\sin\left(dx+c\right)^4+840\,a^3\sin\left(dx+c\right)-280\left(2\,a^3-3\,ab^2\right)\sin\left(dx+c\right)^3}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/840*(105*b^3*sin(d*x + c)^8 + 360*a*b^2*sin(d*x + c)^7 + 140*(3*a^2*b - 2
*b^3)*sin(d*x + c)^6 + 168*(a^3 - 6*a*b^2)*sin(d*x + c)^5 + 1260*a^2*b*sin(
d*x + c)^2 - 210*(6*a^2*b - b^3)*sin(d*x + c)^4 + 840*a^3*sin(d*x + c) - 28
0*(2*a^3 - 3*a*b^2)*sin(d*x + c)^3)/d
```

Fricas [A]
time = 0.34, size = 117, normalized size = 0.81

$$\frac{105\,b^3\cos\left(dx+c\right)^8-140\left(3\,a^2b+b^3\right)\cos\left(dx+c\right)^6-8\left(45\,ab^2\cos\left(dx+c\right)^6-3\left(7\,a^3+3\,ab^2\right)\cos\left(dx+c\right)^4-56\,a^3-24\,ab^2-4\left(7\,a^3+3\,ab^2\right)\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/840*(105*b^3*cos(d*x + c)^8 - 140*(3*a^2*b + b^3)*cos(d*x + c)^6 - 8*(45*
a*b^2*cos(d*x + c)^6 - 3*(7*a^3 + 3*a*b^2)*cos(d*x + c)^4 - 56*a^3 - 24*a*b
^2 - 4*(7*a^3 + 3*a*b^2)*cos(d*x + c)^2)*sin(d*x + c))/d
```

Sympy [A]
time = 0.90, size = 202, normalized size = 1.40

$$\begin{cases} \frac{8a^3\sin^5(c+dx)}{15d}+\frac{4a^3\sin^3(c+dx)\cos^2(c+dx)}{3d}+\frac{a^3\sin(c+dx)\cos^4(c+dx)}{d}-\frac{a^2b\cos^6(c+dx)}{2d}+\frac{8ab^2\sin^7(c+dx)}{35d}+\frac{4ab^2\sin^5(c+dx)\cos^2(c+dx)}{5d}+\frac{ab^2\sin^3(c+dx)\cos^4(c+dx)}{d}-\frac{b^3\sin^2(c+dx)\cos^6(c+dx)}{6d}-\frac{b^3\cos^8(c+dx)}{24d} & \text{for}\ d\neq 0 \\ x(a+b\sin(c))^3\cos^5(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((8*a**3*sin(c + d*x)**5/(15*d) + 4*a**3*sin(c + d*x)**3*cos(c + d
*x)**2/(3*d) + a**3*sin(c + d*x)*cos(c + d*x)**4/d - a**2*b*cos(c + d*x)**6
/(2*d) + 8*a*b**2*sin(c + d*x)**7/(35*d) + 4*a*b**2*sin(c + d*x)**5*cos(c +
 d*x)**2/(5*d) + a*b**2*sin(c + d*x)**3*cos(c + d*x)**4/d - b**3*sin(c + d*
x)**2*cos(c + d*x)**6/(6*d) - b**3*cos(c + d*x)**8/(24*d), Ne(d, 0)), (x*(a
 + b*sin(c))**3*cos(c)**5, True))
```

Giac [A]
time = 3.66, size = 185, normalized size = 1.28

$$\frac{b^3\cos\left(8\,dx+8\,c\right)}{1024\,d}-\frac{3\,ab^2\sin\left(7\,dx+7\,c\right)}{448\,d}-\frac{\left(6\,a^2b-b^3\right)\cos\left(6\,dx+6\,c\right)}{384\,d}-\frac{\left(24\,a^2b+b^3\right)\cos\left(4\,dx+4\,c\right)}{256\,d}-\frac{3\left(10\,a^2b+b^3\right)\cos\left(2\,dx+2\,c\right)}{128\,d}+\frac{\left(4\,a^3-9\,ab^2\right)\sin\left(5\,dx+5\,c\right)}{320\,d}+\frac{\left(20\,a^3-3\,ab^2\right)\sin\left(3\,dx+3\,c\right)}{192\,d}+\frac{5\left(8\,a^3+3\,ab^2\right)\sin\left(dx+c\right)}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/1024*b^3*cos(8*d*x + 8*c)/d - 3/448*a*b^2*sin(7*d*x + 7*c)/d - 1/384*(6*a
^2*b - b^3)*cos(6*d*x + 6*c)/d - 1/256*(24*a^2*b + b^3)*cos(4*d*x + 4*c)/d
- 3/128*(10*a^2*b + b^3)*cos(2*d*x + 2*c)/d + 1/320*(4*a^3 - 9*a*b^2)*sin(5
*d*x + 5*c)/d + 1/192*(20*a^3 - 3*a*b^2)*sin(3*d*x + 3*c)/d + 5/64*(8*a^3 +
 3*a*b^2)*sin(d*x + c)/d
```

Mupad [B]
time = 0.09, size = 141, normalized size = 0.98

$$\frac{\sin\left(c+d\,x\right)^3\,\left(a\,b^2-\frac{2\,a^3}{3}\right)-\sin\left(c+d\,x\right)^5\,\left(\frac{6\,a\,b^2}{5}-\frac{a^3}{5}\right)+\sin\left(c+d\,x\right)^6\,\left(\frac{a^2\,b}{2}-\frac{b^3}{3}\right)-\sin\left(c+d\,x\right)^4\,\left(\frac{3\,a^2\,b}{2}-\frac{b^3}{4}\right)+a^3\,\sin\left(c+d\,x\right)+\frac{b^3\,\sin(c+d\,x)^8}{8}+\frac{3\,a^2\,b\,\sin(c+d\,x)^2}{2}+\frac{3\,a\,b^2\,\sin(c+d\,x)^7}{7}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(a + b*sin(c + d*x))^3,x)
```

```
[Out] (sin(c + d*x)^3*(a*b^2 - (2*a^3)/3) - sin(c + d*x)^5*((6*a*b^2)/5 - a^3/5)
+ sin(c + d*x)^6*((a^2*b)/2 - b^3/3) - sin(c + d*x)^4*((3*a^2*b)/2 - b^3/4)
 + a^3*sin(c + d*x) + (b^3*sin(c + d*x)^8)/8 + (3*a^2*b*sin(c + d*x)^2)/2 +
 (3*a*b^2*sin(c + d*x)^7)/7)/d
```

3.401 $\int \cos^3(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=77

$$-\frac{(a^2-b^2)(a+b\sin(c+dx))^4}{4b^3d}+\frac{2a(a+b\sin(c+dx))^5}{5b^3d}-\frac{(a+b\sin(c+dx))^6}{6b^3d}$$

```
[Out] -1/4*(a^2-b^2)*(a+b*sin(d*x+c))^4/b^3/d+2/5*a*(a+b*sin(d*x+c))^5/b^3/d-1/6*
(a+b*sin(d*x+c))^6/b^3/d
```

Rubi [A]
time = 0.05, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2747, 711}

$$-\frac{(a^2-b^2)(a+b\sin(c+dx))^4}{4b^3d}-\frac{(a+b\sin(c+dx))^6}{6b^3d}+\frac{2a(a+b\sin(c+dx))^5}{5b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -1/4*((a^2 - b^2)*(a + b*Sin[c + d*x])^4)/(b^3*d) + (2*a*(a + b*Sin[c + d*x
])^5)/(5*b^3*d) - (a + b*Sin[c + d*x])^6/(6*b^3*d)
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^3(c+dx)(a+b\sin(c+dx))^3\,dx &= \frac{\text{Subst}\left(\int (a+x)^3\left(b^2-x^2\right)\,dx,x,b\sin(c+dx)\right)}{b^3d}\\ &= \frac{\text{Subst}\left(\int \left(\left(-a^2+b^2\right)(a+x)^3+2a(a+x)^4-(a+x)^5\right)\,dx,x,b\sin\right.}{b^3d}\\ &= -\frac{(a^2-b^2)(a+b\sin(c+dx))^4}{4b^3d}+\frac{2a(a+b\sin(c+dx))^5}{5b^3d}-\frac{(a+b\right.}{}\end{aligned}$$

Mathematica [A]
time = 0.15, size = 56, normalized size = 0.73

$$\frac{(a+b\sin(c+dx))^4\left(-a^2+10b^2+5b^2\cos(2(c+dx))+4ab\sin(c+dx)\right)}{60b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((a + b*Sin[c + d*x])^4*(-a^2 + 10*b^2 + 5*b^2*Cos[2*(c + d*x)] + 4*a*b*Sin
[c + d*x]))/(60*b^3*d)
```

Maple [A]
time = 0.46, size = 115, normalized size = 1.49

method	result
derivativedivides	$\frac{b^3\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^4(dx+c)\right)}{6}-\frac{\left(\cos^4(dx+c)\right)}{12}\right)+3a\,b^2\left(-\frac{\sin(dx+c)\left(\cos^4(dx+c)\right)}{5}+\frac{\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{15}\right)-\frac{3a^2b\left(\cos^4\right.}{4}}{d}$
default	$\frac{b^3\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^4(dx+c)\right)}{6}-\frac{\left(\cos^4(dx+c)\right)}{12}\right)+3a\,b^2\left(-\frac{\sin(dx+c)\left(\cos^4(dx+c)\right)}{5}+\frac{\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{15}\right)-\frac{3a^2b\left(\cos^4\right.}{4}}{d}$
risch	$\frac{3a^3\sin(dx+c)}{4d}+\frac{3a\,b^2\sin(dx+c)}{8d}+\frac{b^3\cos(6dx+6c)}{192d}-\frac{3\sin(5dx+5c)a\,b^2}{80d}-\frac{3b\cos(4dx+4c)a^2}{32d}+\frac{a^3\sin(3dx+3c)}{12d}$
norman	$\frac{\frac{\left(12a^2b+4b^3\right)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{\left(12a^2b+4b^3\right)\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2a^3\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{6a^2b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^3*(-1/6*sin(d*x+c)^2*cos(d*x+c)^4-1/12*cos(d*x+c)^4)+3*a*b^2*(-1/5*s
in(d*x+c)*cos(d*x+c)^4+1/15*(2+cos(d*x+c)^2)*sin(d*x+c))-3/4*a^2*b*cos(d*x+
c)^4+1/3*a^3*(2+cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A]
time = 0.28, size = 100, normalized size = 1.30

$$-\frac{10\,b^3\sin\left(dx+c\right)^6+36\,ab^2\sin\left(dx+c\right)^5-90\,a^2b\sin\left(dx+c\right)^2+15\,(3\,a^2b-b^3)\sin\left(dx+c\right)^4-60\,a^3\sin\left(dx+c\right)+20\,(a^3-3\,ab^2)\sin\left(dx+c\right)^3}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/60*(10*b^3*sin(d*x + c)^6 + 36*a*b^2*sin(d*x + c)^5 - 90*a^2*b*sin(d*x +
 c)^2 + 15*(3*a^2*b - b^3)*sin(d*x + c)^4 - 60*a^3*sin(d*x + c) + 20*(a^3 -
 3*a*b^2)*sin(d*x + c)^3)/d
```

Fricas [A]
time = 0.37, size = 95, normalized size = 1.23

$$\frac{10 b^3 \cos (dx+c)^6 - 15 (3 a^2 b + b^3) \cos (dx+c)^4 - 4 \left(9 a b^2 \cos (dx+c)^4 - 10 a^3 - 6 a b^2 - (5 a^3 + 3 a b^2) \cos (dx+c)^2\right) \sin (dx+c)}{60 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/60*(10*b^3*cos(d*x + c)^6 - 15*(3*a^2*b + b^3)*cos(d*x + c)^4 - 4*(9*a*b^
2*cos(d*x + c)^4 - 10*a^3 - 6*a*b^2 - (5*a^3 + 3*a*b^2)*cos(d*x + c)^2)*sin
(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(66) = 132$.
time = 0.41, size = 151, normalized size = 1.96

$$\begin{cases} \frac{2a^3 \sin^3 (c+dx)}{3d} + \frac{a^3 \sin (c+dx) \cos^2 (c+dx)}{d} - \frac{3a^2 b \cos^4 (c+dx)}{4d} + \frac{2ab^2 \sin^5 (c+dx)}{5d} + \frac{ab^2 \sin^3 (c+dx) \cos^2 (c+dx)}{d} - \frac{b^3 \sin^2 (c+dx) \cos^4 (c+dx)}{4d} - \frac{b^3 \cos^6 (c+dx)}{12d} & \text{for } d \neq 0 \\ x(a + b \sin (c))^3 \cos^3 (c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((2*a**3*sin(c + d*x)**3/(3*d) + a**3*sin(c + d*x)*cos(c + d*x)**2
/d - 3*a**2*b*cos(c + d*x)**4/(4*d) + 2*a*b**2*sin(c + d*x)**5/(5*d) + a*b*
*2*sin(c + d*x)**3*cos(c + d*x)**2/d - b**3*sin(c + d*x)**2*cos(c + d*x)**4
/(4*d) - b**3*cos(c + d*x)**6/(12*d), Ne(d, 0)), (x*(a + b*sin(c))**3*cos(c
)**3, True))
```

Giac [A]
time = 4.28, size = 112, normalized size = 1.45

$$-\frac{10 b^3 \sin (dx+c)^6 + 36 a b^2 \sin (dx+c)^5 + 45 a^2 b \sin (dx+c)^4 - 15 b^3 \sin (dx+c)^4 + 20 a^3 \sin (dx+c)^3 - 60 a b^2 \sin (dx+c)^3 - 90 a^2 b \sin (dx+c)^2 - 60 a^3 \sin (dx+c)}{60 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/60*(10*b^3*sin(d*x + c)^6 + 36*a*b^2*sin(d*x + c)^5 + 45*a^2*b*sin(d*x +
 c)^4 - 15*b^3*sin(d*x + c)^4 + 20*a^3*sin(d*x + c)^3 - 60*a*b^2*sin(d*x +
c)^3 - 90*a^2*b*sin(d*x + c)^2 - 60*a^3*sin(d*x + c))/d
```

Mupad [B]
time = 5.13, size = 98, normalized size = 1.27

$$\frac{\sin (c+d\,x)^3 \left(a\,b^2 - \frac{a^3}{3}\right) - \sin (c+d\,x)^4 \left(\frac{3\,a^2\,b}{4} - \frac{b^3}{4}\right) + a^3 \sin (c+d\,x) - \frac{b^3 \sin(c+d\,x)^6}{6} + \frac{3\,a^2\,b \sin(c+d\,x)^2}{2} - \frac{3\,a\,b^2 \sin(c+d\,x)^5}{5}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a + b*sin(c + d*x))^3,x)
```

```
[Out] (sin(c + d*x)^3*(a*b^2 - a^3/3) - sin(c + d*x)^4*((3*a^2*b)/4 - b^3/4) + a^
3*sin(c + d*x) - (b^3*sin(c + d*x)^6)/6 + (3*a^2*b*sin(c + d*x)^2)/2 - (3*a
*b^2*sin(c + d*x)^5)/5)/d
```

3.402 $\int \cos(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=22

$$\frac{(a+b\sin(c+dx))^4}{4bd}$$

[Out] 1/4*(a+b*sin(d*x+c))^4/b/d

Rubi [A]
time = 0.02, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2747, 32}

$$\frac{(a+b\sin(c+dx))^4}{4bd}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]*(a + b*Sin[c + d*x])^3,x]

[Out] (a + b*Sin[c + d*x])^4/(4*b*d)

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a+b\sin(c+dx))^3\,dx &= \frac{\text{Subst}\left(\int (a+x)^3\,dx, x, b\sin(c+dx)\right)}{bd}\\ &= \frac{(a+b\sin(c+dx))^4}{4bd}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 67 vs. 2(22) = 44.
time = 0.03, size = 67, normalized size = 3.05

$$\frac{a^3\sin(c+dx)}{d}+\frac{3a^2b\sin^2(c+dx)}{2d}+\frac{ab^2\sin^3(c+dx)}{d}+\frac{b^3\sin^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*Sin[c + d*x])/d + (3*a^2*b*Sin[c + d*x]^2)/(2*d) + (a*b^2*Sin[c + d*x]
^3)/d + (b^3*Sin[c + d*x]^4)/(4*d)
```

Maple [A]
time = 0.23, size = 21, normalized size = 0.95

method	result
derivativedivides	$\frac{(a+b\sin(dx+c))^4}{4bd}$
default	$\frac{(a+b\sin(dx+c))^4}{4bd}$
risch	$\frac{a^3\sin(dx+c)}{d}+\frac{3a\,b^2\sin(dx+c)}{4d}+\frac{b^3\cos(4dx+4c)}{32d}-\frac{\sin(3dx+3c)a\,b^2}{4d}-\frac{3b\cos(2dx+2c)a^2}{4d}-\frac{b^3\cos(2dx+2c)}{8d}$
norman	$\frac{\frac{2a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2a^3\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{6a^2b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{6a^2b\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2\left(6a^2b+2b^3\right)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2a\left(3a^2+4b\right.}{}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(a+b*sin(d*x+c))^4/b/d
```

Maxima [A]
time = 0.27, size = 20, normalized size = 0.91

$$\frac{(b\sin(dx+c)+a)^4}{4\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/4*(b*sin(d*x + c) + a)^4/(b*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(20) = 40.
time = 0.32, size = 71, normalized size = 3.23

$$\frac{b^3\cos(dx+c)^4-2\,(3\,a^2b+b^3)\cos(dx+c)^2-4\,(ab^2\cos(dx+c)^2-a^3-ab^2)\sin(dx+c)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(b^3*cos(d*x + c)^4 - 2*(3*a^2*b + b^3)*cos(d*x + c)^2 - 4*(a*b^2*cos(d
*x + c)^2 - a^3 - a*b^2)*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(15) = 30$.

time = 0.18, size = 73, normalized size = 3.32

$$\begin{cases} \frac{a^3\sin(c+dx)}{d} + \frac{3a^2b\sin^2(c+dx)}{2d} + \frac{ab^2\sin^3(c+dx)}{d} + \frac{b^3\sin^4(c+dx)}{4d} & \text{for } d \neq 0 \\ x(a+b\sin(c))^3\cos(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((a**3*sin(c + d*x)/d + 3*a**2*b*sin(c + d*x)**2/(2*d) + a*b**2*si
n(c + d*x)**3/d + b**3*sin(c + d*x)**4/(4*d), Ne(d, 0)), (x*(a + b*sin(c))*
*3*cos(c), True))
```

Giac [A]

time = 6.92, size = 20, normalized size = 0.91

$$\frac{(b\sin(dx+c)+a)^4}{4\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/4*(b*sin(d*x + c) + a)^4/(b*d)
```

Mupad [B]

time = 0.06, size = 55, normalized size = 2.50

$$\frac{a^3\,\sin(c+d\,x) + \frac{3\,a^2\,b\,\sin(c+d\,x)^2}{2} + a\,b^2\,\sin(c+d\,x)^3 + \frac{b^3\,\sin(c+d\,x)^4}{4}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + b*sin(c + d*x))^3,x)
```

```
[Out] (a^3*sin(c + d*x) + (b^3*sin(c + d*x)^4)/4 + (3*a^2*b*sin(c + d*x)^2)/2 + a
*b^2*sin(c + d*x)^3)/d
```

3.403 $\int \sec(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=80

$$-\frac{(a+b)^3\log(1-\sin(c+dx))}{2d}+\frac{(a-b)^3\log(1+\sin(c+dx))}{2d}-\frac{3ab^2\sin(c+dx)}{d}-\frac{b^3\sin^2(c+dx)}{2d}$$

```
[Out] -1/2*(a+b)^3*ln(1-sin(d*x+c))/d+1/2*(a-b)^3*ln(1+sin(d*x+c))/d-3*a*b^2*sin(
d*x+c)/d-1/2*b^3*sin(d*x+c)^2/d
```

Rubi [A]
time = 0.07, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {2747, 716, 647, 31}

$$-\frac{3ab^2\sin(c+dx)}{d}+\frac{(a-b)^3\log(\sin(c+dx)+1)}{2d}-\frac{(a+b)^3\log(1-\sin(c+dx))}{2d}-\frac{b^3\sin^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -1/2*((a + b)^3*Log[1 - Sin[c + d*x]])/d + ((a - b)^3*Log[1 + Sin[c + d*x]]
)/(2*d) - (3*a*b^2*Sin[c + d*x])/d - (b^3*Sin[c + d*x]^2)/(2*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
(-a)*c]
```

Rule 716

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Polyno
mialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
```

```
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec(c+dx)(a+b\sin(c+dx))^3\,dx = \frac{b\text{Subst}\left(\int \frac{(a+x)^3}{b^2-x^2}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{b\text{Subst}\left(\int \left(-3a-x+\frac{a^3+3ab^2+(3a^2+b^2)x}{b^2-x^2}\right)dx, x, b\sin(c+dx)\right)}{d}$$
$$= -\frac{3ab^2\sin(c+dx)}{d} - \frac{b^3\sin^2(c+dx)}{2d} + \frac{b\text{Subst}\left(\int \frac{a^3+3ab^2+(3a^2+b^2)x}{b^2-x^2}\,d\right.}{d}$$
$$= -\frac{3ab^2\sin(c+dx)}{d} - \frac{b^3\sin^2(c+dx)}{2d} - \frac{(a-b)^3\text{Subst}\left(\int \frac{1}{-b-x}\,dx, x,\right.}{2d}$$
$$= -\frac{(a+b)^3\log(1-\sin(c+dx))}{2d} + \frac{(a-b)^3\log(1+\sin(c+dx))}{2d} - \frac{3a}{}$$

Mathematica [A]
time = 0.12, size = 67, normalized size = 0.84

$$-\frac{(a+b)^3\log(1-\sin(c+dx)) - (a-b)^3\log(1+\sin(c+dx)) + 6ab^2\sin(c+dx) + b^3\sin^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -1/2*((a + b)^3*Log[1 - Sin[c + d*x]] - (a - b)^3*Log[1 + Sin[c + d*x]] + 6
*a*b^2*Sin[c + d*x] + b^3*Sin[c + d*x]^2)/d
```

Maple [A]
time = 0.32, size = 90, normalized size = 1.12

method	result
derivativedivides	$\frac{a^3\ln(\sec(dx+c)+\tan(dx+c))-3a^2b\ln(\cos(dx+c))+3ab^2(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))+b^3\left(-\frac{\left(\sin^2(dx+c)\right.}{2}\right.}{d}$
default	$\frac{a^3\ln(\sec(dx+c)+\tan(dx+c))-3a^2b\ln(\cos(dx+c))+3ab^2(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))+b^3\left(-\frac{\left(\sin^2(dx+c)\right.}{2}\right.}{d}$
norman	$\frac{-\frac{2b^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2b^3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{6ab^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{12ab^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{6ab^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{(a^3-3a^2b}{}$

risch	$-\frac{3ia\,b^2\mathrm{e}^{-i(dx+c)}}{2d}+\frac{3ia\,b^2\mathrm{e}^{i(dx+c)}}{2d}+3ia^2bx+ib^3x+\frac{6ia^2bc}{d}+\frac{2ib^3c}{d}+\frac{a^3\ln(\mathrm{e}^{i(dx+c)}+i)}{d}-\frac{3\ln(\mathrm{e}^{i(dx+c)}\cdots}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*ln(sec(d*x+c)+tan(d*x+c))-3*a^2*b*ln(cos(d*x+c))+3*a*b^2*(-sin(d*x
+c)+ln(sec(d*x+c)+tan(d*x+c)))+b^3*(-1/2*sin(d*x+c)^2-ln(cos(d*x+c))))
```

Maxima [A]
time = 0.28, size = 91, normalized size = 1.14

$$-\frac{b^3\sin(dx+c)^2+6\,ab^2\sin(dx+c)-(a^3-3\,a^2b+3\,ab^2-b^3)\log(\sin(dx+c)+1)+(a^3+3\,a^2b+3\,ab^2+b^3)\log(\sin(dx+c)-1)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*(b^3*sin(d*x + c)^2 + 6*a*b^2*sin(d*x + c) - (a^3 - 3*a^2*b + 3*a*b^2
- b^3)*log(sin(d*x + c) + 1) + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*log(sin(d*x
+ c) - 1))/d
```

Fricas [A]
time = 0.37, size = 93, normalized size = 1.16

$$\frac{b^3\cos(dx+c)^2-6\,ab^2\sin(dx+c)+(a^3-3\,a^2b+3\,ab^2-b^3)\log(\sin(dx+c)+1)-(a^3+3\,a^2b+3\,ab^2+b^3)\log(-\sin(dx+c)+1)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(b^3*cos(d*x + c)^2 - 6*a*b^2*sin(d*x + c) + (a^3 - 3*a^2*b + 3*a*b^2 -
 b^3)*log(sin(d*x + c) + 1) - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*log(-sin(d*x
+ c) + 1))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int(a+b\sin(c+dx))^3\sec(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral((a + b*sin(c + d*x))**3*sec(c + d*x), x)
```

Giac [A]
time = 5.42, size = 93, normalized size = 1.16

$$-\frac{b^3\sin(dx+c)^2+6\,ab^2\sin(dx+c)-(a^3-3\,a^2b+3\,ab^2-b^3)\log(|\sin(dx+c)+1|)+(a^3+3\,a^2b+3\,ab^2+b^3)\log(|\sin(dx+c)-1|)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(b^3*sin(d*x + c)^2 + 6*a*b^2*sin(d*x + c) - (a^3 - 3*a^2*b + 3*a*b^2
- b^3)*log(abs(sin(d*x + c) + 1)) + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*log(abs
(sin(d*x + c) - 1)))/d
```

Mupad [B]
time = 5.13, size = 65, normalized size = 0.81

$$-\frac{\frac{\ln(\sin(c+d\,x)-1)\,(a+b)^3}{2}-\frac{\ln(\sin(c+d\,x)+1)\,(a-b)^3}{2}+\frac{b^3\,\sin(c+d\,x)^2}{2}+3\,a\,b^2\,\sin\left(c+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^3/cos(c + d*x),x)
```

```
[Out] -((log(sin(c + d*x) - 1)*(a + b)^3)/2 - (log(sin(c + d*x) + 1)*(a - b)^3)/2
 + (b^3*sin(c + d*x)^2)/2 + 3*a*b^2*sin(c + d*x))/d
```

3.404 $\int \sec^3(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=111

$$-\frac{(a-2b)(a+b)^2\log(1-\sin(c+dx))}{4d}+\frac{(a-b)^2(a+2b)\log(1+\sin(c+dx))}{4d}+\frac{ab^2\sin(c+dx)}{2d}+\frac{\sec^2(c+dx}{}$$

```
[Out] -1/4*(a-2*b)*(a+b)^2*ln(1-sin(d*x+c))/d+1/4*(a-b)^2*(a+2*b)*ln(1+sin(d*x+c)
)/d+1/2*a*b^2*sin(d*x+c)/d+1/2*sec(d*x+c)^2*(b+a*sin(d*x+c))*(a+b*sin(d*x+c
))^2/d
```

Rubi [A]
time = 0.09, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2747, 753, 788, 647, 31}

$$\frac{ab^2\sin(c+dx)}{2d}+\frac{(a+2b)(a-b)^2\log(\sin(c+dx)+1)}{4d}-\frac{(a-2b)(a+b)^2\log(1-\sin(c+dx))}{4d}+\frac{\sec^2(c+dx)(a\sin(c+dx)+b)(a+b\sin(c+dx))^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -1/4*((a - 2*b)*(a + b)^2*Log[1 - Sin[c + d*x]])/d + ((a - b)^2*(a + 2*b)*L
og[1 + Sin[c + d*x]])/(4*d) + (a*b^2*Sin[c + d*x])/(2*d) + (Sec[c + d*x]^2*
(b + a*Sin[c + d*x])*(a + b*Sin[c + d*x])^2)/(2*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
(-a)*c]
```

Rule 753

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m - 1)*(a*e - c*d*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 788

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[e*g*(x/c), x] + Dist[1/c, Int[(c*d*f - a*e*g + c*(e*f + d*g)*x
)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^3(c+dx)(a+b\sin(c+dx))^3\,dx = \frac{b^3\text{Subst}\left(\int \frac{(a+x)^3}{(b^2-x^2)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^2(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^2}{2d} - \frac{b\text{Subst}\left(\int \frac{(a\cdots}{}\right.}{}$$

$$= \frac{ab^2\sin(c+dx)}{2d} + \frac{\sec^2(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx\cdots}{2d}$$

$$= \frac{ab^2\sin(c+dx)}{2d} + \frac{\sec^2(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx\cdots}{2d}$$

$$= -\frac{(a-2b)(a+b)^2\log(1-\sin(c+dx))}{4d} + \frac{(a-b)^2(a+2b)\log(1+\cdots}{4d}$$

Mathematica [A]
time = 1.38, size = 176, normalized size = 1.59

$$\frac{(a^2-b^2)\left((a-2b)(a+b)^2\log(1-\sin(c+dx))-(a-b)^2(a+2b)\log(1+\sin(c+dx))\right)+2a^4b\sec^2(c+dx)-a(2a^4+4a^2b^2-7b^4+b^4\cos(2(c+dx)))\sec(c+dx)\tan(c+dx)+(-8a^4b+4a^2b^3+2b^5-2ab^4\sin(c+dx))\tan^2(c+dx)}{4(-a^2+b^2)d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((a^2 - b^2)*((a - 2*b)*(a + b)^2*Log[1 - Sin[c + d*x]] - (a - b)^2*(a + 2*
b)*Log[1 + Sin[c + d*x]]) + 2*a^4*b*Sec[c + d*x]^2 - a*(2*a^4 + 4*a^2*b^2 -
 7*b^4 + b^4*Cos[2*(c + d*x)])*Sec[c + d*x]*Tan[c + d*x] + (-8*a^4*b + 4*a^
2*b^3 + 2*b^5 - 2*a*b^4*Sin[c + d*x])*Tan[c + d*x]^2)/(4*(-a^2 + b^2)*d)
```

Maple [A]
time = 0.40, size = 126, normalized size = 1.14

method	result
derivativedivides	$\frac{a^3\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+\frac{3a^2b}{2\cos(dx+c)^2}+3a\,b^2\left(\frac{\sin^3(dx+c)}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right.}{d}$
default	$\frac{a^3\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+\frac{3a^2b}{2\cos(dx+c)^2}+3a\,b^2\left(\frac{\sin^3(dx+c)}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right.}{d}$
risch	$-ib^3x-\frac{2ib^3c}{d}+\frac{-ia^3e^{3i(dx+c)}-3ia\,b^2e^{3i(dx+c)}+ia^3e^{i(dx+c)}+3ia\,b^2e^{i(dx+c)}+6a^2b\,e^{2i(dx+c)}+2b^3e^{2i(dx+c)}}{d\left(1+e^{2i(dx+c)}\right)^2}+\frac{a^3\,\mathrm{l}}{}$
norman	$\frac{\frac{a\left(a^2+3b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{a\left(a^2+3b^2\right)\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{6a^2b+2b^3}{d}-\frac{\left(6a^2b+2b^3\right)\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2\left(15a^2b+5b^3\right)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2\left(\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+3/2*a^2*
b/cos(d*x+c)^2+3*a*b^2*(1/2*sin(d*x+c)^3/cos(d*x+c)^2+1/2*sin(d*x+c)-1/2*ln
(sec(d*x+c)+tan(d*x+c)))+b^3*(1/2*tan(d*x+c)^2+ln(cos(d*x+c))))
```

Maxima [A]
time = 0.28, size = 98, normalized size = 0.88

$$\frac{(a^3-3\,ab^2+2\,b^3)\log\left(\sin\left(dx+c\right)+1\right)-(a^3-3\,ab^2-2\,b^3)\log\left(\sin\left(dx+c\right)-1\right)-\frac{2\,(3\,a^2b+b^3+(a^3+3\,ab^2)\sin(dx+c))}{\sin(dx+c)^2-1}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/4*((a^3 - 3*a*b^2 + 2*b^3)*log(sin(d*x + c) + 1) - (a^3 - 3*a*b^2 - 2*b^3
)*log(sin(d*x + c) - 1) - 2*(3*a^2*b + b^3 + (a^3 + 3*a*b^2)*sin(d*x + c))/
(sin(d*x + c)^2 - 1))/d
```

Fricas [A]
time = 0.37, size = 112, normalized size = 1.01

$$\frac{(a^3-3\,ab^2+2\,b^3)\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-(a^3-3\,ab^2-2\,b^3)\cos\left(dx+c\right)^2\log\left(-\sin\left(dx+c\right)+1\right)+6\,a^2b+2\,b^3+2\,(a^3+3\,ab^2)\sin\left(dx+c\right)}{4\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4*((a^3 - 3*a*b^2 + 2*b^3)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (a^3 -
3*a*b^2 - 2*b^3)*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 6*a^2*b + 2*b^3 +
2*(a^3 + 3*a*b^2)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sin(c + dx))^3 \sec^3(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral((a + b*sin(c + d*x))**3*sec(c + d*x)**3, x)
```

Giac [A]
time = 5.19, size = 114, normalized size = 1.03

$$\frac{(a^3 - 3ab^2 + 2b^3)\log(|\sin(dx + c) + 1|) - (a^3 - 3ab^2 - 2b^3)\log(|\sin(dx + c) - 1|) - \frac{2\left(b^3\sin(dx+c)^2 + a^3\sin(dx+c) + 3ab^2\sin(dx+c) + 3a^2b\right)}{\sin(dx+c)^2 - 1}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/4*((a^3 - 3*a*b^2 + 2*b^3)*log(abs(sin(d*x + c) + 1)) - (a^3 - 3*a*b^2 -
2*b^3)*log(abs(sin(d*x + c) - 1)) - 2*(b^3*sin(d*x + c)^2 + a^3*sin(d*x + c
) + 3*a*b^2*sin(d*x + c) + 3*a^2*b)/(sin(d*x + c)^2 - 1))/d
```

Mupad [B]
time = 5.21, size = 99, normalized size = 0.89

$$\frac{\ln(\sin(c + dx) + 1)\,(a - b)^2\,(a + 2b)}{4d} - \frac{\ln(\sin(c + dx) - 1)\,(a + b)^2\,(a - 2b)}{4d} - \frac{\frac{3a^2b}{2} + \frac{b^3}{2} + \sin(c + dx)\,\left(\frac{a^3}{2} + \frac{3ab^2}{2}\right)}{d\,\left(\sin(c + dx)^2 - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^3/cos(c + d*x)^3,x)
```

```
[Out] (log(sin(c + d*x) + 1)*(a - b)^2*(a + 2*b))/(4*d) - (log(sin(c + d*x) - 1)*
(a + b)^2*(a - 2*b))/(4*d) - ((3*a^2*b)/2 + b^3/2 + sin(c + d*x)*((3*a*b^2)
/2 + a^3/2))/(d*(sin(c + d*x)^2 - 1))
```

3.405 $\int \sec^5(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=94

$$\frac{3a(a^2-b^2)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{3a\sec^2(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))}{8d}+\sec^3(c+dx)(a+b\,\mathrm{s}$$

```
[Out] 3/8*a*(a^2-b^2)*arctanh(sin(d*x+c))/d+3/8*a*sec(d*x+c)^2*(b+a*sin(d*x+c))*(
a+b*sin(d*x+c))/d+1/4*sec(d*x+c)^3*(a+b*sin(d*x+c))^3*tan(d*x+c)/d
```

Rubi [A]
time = 0.06, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2747, 743, 737, 212}

$$\frac{3a(a^2-b^2)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{3a\sec^2(c+dx)(a\sin(c+dx)+b)(a+b\sin(c+dx))}{8d}+\frac{\tan(c+dx)\sec^3(c+dx)(a+b\sin(c+dx))^3}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (3*a*(a^2 - b^2)*ArcTanh[Sin[c + d*x]])/(8*d) + (3*a*Sec[c + d*x]^2*(b + a*
Sin[c + d*x])*(a + b*Sin[c + d*x]))/(8*d) + (Sec[c + d*x]^3*(a + b*Sin[c +
d*x])^3*Tan[c + d*x])/(4*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 737

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m - 1)*(a*e - c*d*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] +
 Dist[(2*p + 3)*((c*d^2 + a*e^2)/(2*a*c*(p + 1))), Int[(d + e*x)^(m - 2)*(a
 + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0
] && EqQ[m + 2*p + 2, 0] && LtQ[p, -1]
```

Rule 743

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^m)*(2*c*x)*((a + c*x^2)^(p + 1)/(4*a*c*(p + 1))), x] - Dist[m*(
(2*c*d)/(4*a*c*(p + 1))), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1), x], x]
 /; FreeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p +
3, 0] && LtQ[p, -1]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^5(c+dx)(a+b\sin(c+dx))^3\,dx = \frac{b^5\text{Subst}\left(\int \frac{(a+x)^3}{(b^2-x^2)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^3(c+dx)(a+b\sin(c+dx))^3\tan(c+dx)}{4d} + \frac{(3ab^3)\,\text{Subst}\left(\int \frac{(a-}{(b^2-}\right.}{}$$

$$= \frac{3a\sec^2(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))}{8d} + \frac{\sec^3(c+d}{}$$

$$= \frac{3a(a^2-b^2)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{3a\sec^2(c+dx)(b+a\sin(c+d}{8d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 318 vs. 2(94) = 188.
time = 3.94, size = 318, normalized size = 3.38

$$\frac{-6a(a^2-b^2)^3(\log(1-\sin(c+dx))-\log(1+\sin(c+dx)))+ab\sec^4(c+dx)\left(-8a^5+8a^3b^2+(18a^4b-11a^2b^3+5b^5)\sin(3(c+dx))\right)+a(8a^6-22a^4b^2+29a^2b^4-3b^6)\sec^3(c+dx)\tan(c+dx)+16a^4b(3a^2-2b^2)\tan^2(c+dx)+8b^3(4a^4-5a^2b^2+b^4)\tan^4(c+dx)+4a\sec(c+dx)\tan(c+dx)\left(3(a^6-5a^4b^2)+4b^2(3a^4-5a^2b^2+2b^4)\tan^2(c+dx)\right)+16a^2b\sec^2(c+dx)\left(-a^4+(2a^4-5a^2b^2+3b^4)\tan^2(c+dx)\right)}{32\left(a^2-b^2\right)^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-6*a*(a^2 - b^2)^3*(Log[1 - Sin[c + d*x]] - Log[1 + Sin[c + d*x]]) + a*b*S
ec[c + d*x]^4*(-8*a^5 + 8*a^3*b^2 + (18*a^4*b - 11*a^2*b^3 + 5*b^5)*Sin[3*(
c + d*x)]) + a*(8*a^6 - 22*a^4*b^2 + 29*a^2*b^4 - 3*b^6)*Sec[c + d*x]^3*Tan
[c + d*x] + 16*a^4*b*(3*a^2 - 2*b^2)*Tan[c + d*x]^2 + 8*b^3*(4*a^4 - 5*a^2*
b^2 + b^4)*Tan[c + d*x]^4 + 4*a*Sec[c + d*x]*Tan[c + d*x]*(3*(a^6 - 5*a^4*b
^2) + 4*b^2*(3*a^4 - 5*a^2*b^2 + 2*b^4)*Tan[c + d*x]^2) + 16*a^2*b*Sec[c +
d*x]^2*(-a^4 + (2*a^4 - 5*a^2*b^2 + 3*b^4)*Tan[c + d*x]^2))/(32*(a^2 - b^2)
^2*d)
```

Maple [A]
time = 0.48, size = 156, normalized size = 1.66

method	result

derivativedivides	$\frac{a^3\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+\frac{3a^2b}{4\cos(dx+c)^4}+3a\,b^2\left(\frac{\sin^3(dx+c)}{4\cos(dx+c)^4}+\frac{\sin^3(d}{8\cos(d}\right.}{d}$
default	$\frac{a^3\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+\frac{3a^2b}{4\cos(dx+c)^4}+3a\,b^2\left(\frac{\sin^3(dx+c)}{4\cos(dx+c)^4}+\frac{\sin^3(d}{8\cos(d}\right.}{d}$
risch	$-\frac{3ia^3e^{7i(dx+c)}-3ia\,b^2e^{7i(dx+c)}+11ia^3e^{5i(dx+c)}+21ia\,b^2e^{5i(dx+c)}+8b^3e^{6i(dx+c)}-11ia^3e^{3i(dx+c)}-21ia\,b^2e^{3i(dx+c)}-48}{4d\left(1+e^{2i(dx+c)}\right)^4}$
norman	$\frac{6a^2b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{6a^2b\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{\left(18a^2b+4b^3\right)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{\left(18a^2b+4b^3\right)\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{a\left(7a^2+33b^2\right)\left(\tan\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+
tan(d*x+c)))+3/4*a^2*b/cos(d*x+c)^4+3*a*b^2*(1/4*sin(d*x+c)^3/cos(d*x+c)^4+
1/8*sin(d*x+c)^3/cos(d*x+c)^2+1/8*sin(d*x+c)-1/8*ln(sec(d*x+c)+tan(d*x+c)))
+1/4*b^3*sin(d*x+c)^4/cos(d*x+c)^4)
```

Maxima [A]
time = 0.29, size = 136, normalized size = 1.45

$$\frac{3\left(a^3-ab^2\right)\log\left(\sin\left(dx+c\right)+1\right)-3\left(a^3-ab^2\right)\log\left(\sin\left(dx+c\right)-1\right)+\frac{2\left(4b^3\sin(dx+c)^2-3\left(a^3-ab^2\right)\sin(dx+c)^3+6a^2b-2b^3+\left(5a^3+3ab^2\right)\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/16*(3*(a^3 - a*b^2)*log(sin(d*x + c) + 1) - 3*(a^3 - a*b^2)*log(sin(d*x +
 c) - 1) + 2*(4*b^3*sin(d*x + c)^2 - 3*(a^3 - a*b^2)*sin(d*x + c)^3 + 6*a^2
*b - 2*b^3 + (5*a^3 + 3*a*b^2)*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x +
c)^2 + 1))/d
```

Fricas [A]
time = 0.35, size = 138, normalized size = 1.47

$$\frac{3\left(a^3-ab^2\right)\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-3\left(a^3-ab^2\right)\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+1\right)-8b^3\cos\left(dx+c\right)^2+12a^2b+4b^3+2\left(2a^3+6ab^2+3\left(a^3-ab^2\right)\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)}{16d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/16*(3*(a^3 - a*b^2)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(a^3 - a*b^2
)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) - 8*b^3*cos(d*x + c)^2 + 12*a^2*b +
 4*b^3 + 2*(2*a^3 + 6*a*b^2 + 3*(a^3 - a*b^2)*cos(d*x + c)^2)*sin(d*x + c))
/(d*cos(d*x + c)^4)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 3.29, size = 139, normalized size = 1.48

$$\frac{3\left(a^{3}-ab^{2}\right)\log\left(\left|\sin\left(dx+c\right)+1\right|\right)-3\left(a^{3}-ab^{2}\right)\log\left(\left|\sin\left(dx+c\right)-1\right|\right)-\frac{2\left(3\,a^{3}\sin(dx+c)^{3}-3\,ab^{2}\sin(dx+c)^{3}-4\,b^{3}\sin(dx+c)^{2}-5\,a^{3}\sin(dx+c)-3\,ab^{2}\sin(dx+c)-6\,a^{2}b+2\,b^{3}\right)}{\left(\sin(dx+c)^{2}-1\right)^{2}}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/16*(3*(a^3 - a*b^2)*log(abs(sin(d*x + c) + 1)) - 3*(a^3 - a*b^2)*log(abs(
sin(d*x + c) - 1)) - 2*(3*a^3*sin(d*x + c)^3 - 3*a*b^2*sin(d*x + c)^3 - 4*b
^3*sin(d*x + c)^2 - 5*a^3*sin(d*x + c) - 3*a*b^2*sin(d*x + c) - 6*a^2*b + 2
*b^3)/(sin(d*x + c)^2 - 1)^2)/d
```

Mupad [**B**]
time = 5.17, size = 114, normalized size = 1.21

$$\frac{\sin\left(c+d\,x\right)^{3}\,\left(\frac{3\,a\,b^{2}}{8}-\frac{3\,a^{3}}{8}\right)+\frac{3\,a^{2}\,b}{4}-\frac{b^{3}}{4}+\sin\left(c+d\,x\right)\,\left(\frac{5\,a^{3}}{8}+\frac{3\,a\,b^{2}}{8}\right)+\frac{b^{3}\,\sin(c+d\,x)^{2}}{2}}{d\,\left(\sin\left(c+d\,x\right)^{4}-2\,\sin\left(c+d\,x\right)^{2}+1\right)}+\frac{3\,a\,\mathrm{atanh}(\sin\left(c+d\,x\right))\,\left(a^{2}-b^{2}\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^3/cos(c + d*x)^5,x)
```

```
[Out] (sin(c + d*x)^3*((3*a*b^2)/8 - (3*a^3)/8) + (3*a^2*b)/4 - b^3/4 + sin(c + d
*x)*((3*a*b^2)/8 + (5*a^3)/8) + (b^3*sin(c + d*x)^2)/2)/(d*(sin(c + d*x)^4
- 2*sin(c + d*x)^2 + 1)) + (3*a*atanh(sin(c + d*x))*(a^2 - b^2))/(8*d)
```

3.406 $\int \cos^4(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=158

$$\frac{3}{16}a\left(2a^2+b^2\right)x-\frac{b(17a^2+4b^2)\cos^5(c+dx)}{70d}+\frac{3a(2a^2+b^2)\cos(c+dx)\sin(c+dx)}{16d}+\frac{a(2a^2+b^2)\cos^3(c+dx)}{8d}$$

```
[Out] 3/16*a*(2*a^2+b^2)*x-1/70*b*(17*a^2+4*b^2)*cos(d*x+c)^5/d+3/16*a*(2*a^2+b^2
)*cos(d*x+c)*sin(d*x+c)/d+1/8*a*(2*a^2+b^2)*cos(d*x+c)^3*sin(d*x+c)/d-3/14*
a*b*cos(d*x+c)^5*(a+b*sin(d*x+c))/d-1/7*b*cos(d*x+c)^5*(a+b*sin(d*x+c))^2/d
```

Rubi [A]
time = 0.15, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2771, 2941, 2748, 2715, 8}

$$-\frac{b(17a^2+4b^2)\cos^5(c+dx)}{70d}+\frac{a(2a^2+b^2)\sin(c+dx)\cos^3(c+dx)}{8d}+\frac{3a(2a^2+b^2)\sin(c+dx)\cos(c+dx)}{16d}+\frac{3}{16}ax\left(2a^2+b^2\right)-\frac{b\cos^5(c+dx)(a+b\sin(c+dx))^2}{7d}-\frac{3ab\cos^5(c+dx)(a+b\sin(c+dx))}{14d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (3*a*(2*a^2 + b^2)*x)/16 - (b*(17*a^2 + 4*b^2)*Cos[c + d*x]^5)/(70*d) + (3*
a*(2*a^2 + b^2)*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (a*(2*a^2 + b^2)*Cos[c
+ d*x]^3*Sin[c + d*x])/(8*d) - (3*a*b*Cos[c + d*x]^5*(a + b*Sin[c + d*x]))/
(14*d) - (b*Cos[c + d*x]^5*(a + b*Sin[c + d*x])^2)/(7*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2771

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
```

```
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(
a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rule 2941

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp
[a*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x],
x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
&&  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Si
mplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\begin{aligned}\int \cos^4(c+dx)(a+b\sin(c+dx))^3\,dx &= -\frac{b\cos^5(c+dx)(a+b\sin(c+dx))^2}{7d}+\frac{1}{7}\int \cos^4(c+dx)(a+b\sin(\\ &= -\frac{3ab\cos^5(c+dx)(a+b\sin(c+dx))}{14d}-\frac{b\cos^5(c+dx)(a+b\sin(c\\ &= -\frac{b(17a^2+4b^2)\cos^5(c+dx)}{70d}-\frac{3ab\cos^5(c+dx)(a+b\sin(c+dx))}{14d}\\ &= -\frac{b(17a^2+4b^2)\cos^5(c+dx)}{70d}+\frac{a(2a^2+b^2)\cos^3(c+dx)\sin(c+dx}{8d}\\ &= -\frac{b(17a^2+4b^2)\cos^5(c+dx)}{70d}+\frac{3a(2a^2+b^2)\cos(c+dx)\sin(c+dx}{16d}\\ &= \frac{3}{16}a\left(2a^2+b^2\right)x-\frac{b(17a^2+4b^2)\cos^5(c+dx)}{70d}+\frac{3a(2a^2+b^2)\cos(}{1}\end{aligned}$$

Mathematica [A]
time = 0.40, size = 182, normalized size = 1.15

$$\frac{840a^3c+420ab^2c+840a^3dx+420ab^2dx-105b(8a^2+b^2)\cos(c+dx)-35(12a^2b+b^3)\cos(3(c+dx))-84a^2b\cos(5(c+dx))+7b^3\cos(5(c+dx))+5b^3\cos(7(c+dx))+560a^3\sin(2(c+dx))+105ab^2\sin(2(c+dx))+70a^3\sin(4(c+dx))-105ab^2\sin(4(c+dx))-35ab^2\sin(6(c+dx))}{2240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (840*a^3*c + 420*a*b^2*c + 840*a^3*d*x + 420*a*b^2*d*x - 105*b*(8*a^2 + b^2
)*Cos[c + d*x] - 35*(12*a^2*b + b^3)*Cos[3*(c + d*x)] - 84*a^2*b*Cos[5*(c +
```

```
d*x)] + 7*b^3*Cos[5*(c + d*x)] + 5*b^3*Cos[7*(c + d*x)] + 560*a^3*Sin[2*(c
 + d*x)] + 105*a*b^2*Sin[2*(c + d*x)] + 70*a^3*Sin[4*(c + d*x)] - 105*a*b^2
*Sin[4*(c + d*x)] - 35*a*b^2*Sin[6*(c + d*x)])/(2240*d)
```

Maple [A]
time = 0.52, size = 145, normalized size = 0.92

method	result
derivativedivides	$\frac{b^3\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^5(dx+c)\right)}{7}-\frac{2\left(\cos^5(dx+c)\right)}{35}\right)+3ab^2\left(-\frac{\sin(dx+c)\left(\cos^5(dx+c)\right)}{6}+\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{24}+\right.}{d}$
default	$\frac{b^3\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^5(dx+c)\right)}{7}-\frac{2\left(\cos^5(dx+c)\right)}{35}\right)+3ab^2\left(-\frac{\sin(dx+c)\left(\cos^5(dx+c)\right)}{6}+\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{24}+\right.}{d}$
risch	$\frac{3a^3x}{8}+\frac{3a\,b^2x}{16}-\frac{3b\cos(dx+c)a^2}{8d}-\frac{3b^3\cos(dx+c)}{64d}+\frac{b^3\cos(7dx+7c)}{448d}-\frac{a\,b^2\sin(6dx+6c)}{64d}-\frac{3b\cos(5dx+5c)a^2}{80d}+$
norman	$\frac{(\frac{3}{8}a^3+\frac{3}{16}a\,b^2)x+(\frac{3}{8}a^3+\frac{3}{16}a\,b^2)x\left(\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(\frac{21}{8}a^3+\frac{21}{16}a\,b^2)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(\frac{21}{8}a^3+\frac{21}{16}a\,b^2)x\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^3*(-1/7*sin(d*x+c)^2*cos(d*x+c)^5-2/35*cos(d*x+c)^5)+3*a*b^2*(-1/6*s
in(d*x+c)*cos(d*x+c)^5+1/24*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+1/16*d
*x+1/16*c)-3/5*a^2*b*cos(d*x+c)^5+a^3*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*si
n(d*x+c)+3/8*d*x+3/8*c))
```

Maxima [A]
time = 0.28, size = 117, normalized size = 0.74

$$-\frac{1344\,a^2b\cos(dx+c)^5-70\,(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin(2\,dx+2\,c))a^3-35\,(4\,\sin(2\,dx+2\,c)^3+12\,dx+12\,c-3\,\sin(4\,dx+4\,c))ab^2-64\,(5\,\cos(dx+c)^7-7\,\cos(dx+c)^5)b^3}{2240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2240*(1344*a^2*b*cos(d*x + c)^5 - 70*(12*d*x + 12*c + sin(4*d*x + 4*c) +
 8*sin(2*d*x + 2*c))*a^3 - 35*(4*sin(2*d*x + 2*c)^3 + 12*d*x + 12*c - 3*sin
(4*d*x + 4*c))*a*b^2 - 64*(5*cos(d*x + c)^7 - 7*cos(d*x + c)^5)*b^3)/d
```

Fricas [A]
time = 0.39, size = 117, normalized size = 0.74

$$\frac{80\,b^3\cos(dx+c)^7-112\,(3\,a^2b+b^3)\cos(dx+c)^5+105\,(2\,a^3+ab^2)dx-35\,(8\,ab^2\cos(dx+c)^5-2\,(2\,a^3+ab^2)\cos(dx+c)^3-3\,(2\,a^3+ab^2)\cos(dx+c))\sin(dx+c)}{560\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*(a+b*sin(d*x+c))^3,x, algorithm="fricas")

[Out] 1/560*(80*b^3*cos(d*x + c)^7 - 112*(3*a^2*b + b^3)*cos(d*x + c)^5 + 105*(2*a^3 + a*b^2)*d*x - 35*(8*a*b^2*cos(d*x + c)^5 - 2*(2*a^3 + a*b^2)*cos(d*x + c)^3 - 3*(2*a^3 + a*b^2)*cos(d*x + c))*sin(d*x + c))/d

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 348 vs. $2(148) = 296$.
time = 0.65, size = 348, normalized size = 2.20

$$\begin{cases} \frac{3a^3x\sin^4(c+dx)}{8} + \frac{3a^3x\sin^2(c+dx)\cos^2(c+dx)}{4} + \frac{3a^3x\cos^4(c+dx)}{8} + \frac{3a^3\sin^3(c+dx)\cos(c+dx)}{8d} + \frac{5a^3\sin(c+dx)\cos^3(c+dx)}{8d} - \frac{3a^2b\cos^5(c+dx)}{5d} + \frac{3ab^2x\sin^6(c+dx)}{16} + \frac{9ab^2x\sin^4(c+dx)\cos^2(c+dx)}{16} + \frac{9ab^2x\sin^2(c+dx)\cos^4(c+dx)}{16} + \frac{3ab^2x\cos^6(c+dx)}{16} + \frac{3ab^2\sin^5(c+dx)\cos(c+dx)}{16d} + \frac{ab^2\sin^3(c+dx)\cos^3(c+dx)}{2d} - \frac{3ab^2\sin(c+dx)\cos^5(c+dx)}{16d} - \frac{b^3\sin^2(c+dx)\cos^5(c+dx)}{5d} - \frac{2b^3\cos^7(c+dx)}{35d} & \text{for}\ d \neq 0 \\ x(a+b\sin(c))^3\cos^4(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**4*(a+b*sin(d*x+c))**3,x)

[Out] Piecewise((3*a**3*x*sin(c + d*x)**4/8 + 3*a**3*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 3*a**3*x*cos(c + d*x)**4/8 + 3*a**3*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 5*a**3*sin(c + d*x)*cos(c + d*x)**3/(8*d) - 3*a**2*b*cos(c + d*x)**5/(5*d) + 3*a*b**2*x*sin(c + d*x)**6/16 + 9*a*b**2*x*sin(c + d*x)**4*cos(c + d*x)**2/16 + 9*a*b**2*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 3*a*b**2*x*cos(c + d*x)**6/16 + 3*a*b**2*sin(c + d*x)**5*cos(c + d*x)/(16*d) + a*b**2*sin(c + d*x)**3*cos(c + d*x)**3/(2*d) - 3*a*b**2*sin(c + d*x)*cos(c + d*x)**5/(16*d) - b**3*sin(c + d*x)**2*cos(c + d*x)**5/(5*d) - 2*b**3*cos(c + d*x)**7/(35*d), Ne(d, 0)), (x*(a + b*sin(c))**3*cos(c)**4, True))

Giac [A]
time = 4.61, size = 173, normalized size = 1.09

$$\frac{b^3\cos(7\,dx+7\,c)}{448\,d} - \frac{ab^2\sin(6\,dx+6\,c)}{64\,d} + \frac{3}{16}\left(2\,a^3+ab^2\right)x - \frac{(12\,a^2b-b^3)\cos(5\,dx+5\,c)}{320\,d} - \frac{(12\,a^2b+b^3)\cos(3\,dx+3\,c)}{64\,d} - \frac{3\,(8\,a^2b+b^3)\cos(dx+c)}{64\,d} + \frac{(2\,a^3-3\,ab^2)\sin(4\,dx+4\,c)}{64\,d} + \frac{(16\,a^3+3\,ab^2)\sin(2\,dx+2\,c)}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*(a+b*sin(d*x+c))^3,x, algorithm="giac")

[Out] 1/448*b^3*cos(7*d*x + 7*c)/d - 1/64*a*b^2*sin(6*d*x + 6*c)/d + 3/16*(2*a^3 + a*b^2)*x - 1/320*(12*a^2*b - b^3)*cos(5*d*x + 5*c)/d - 1/64*(12*a^2*b + b^3)*cos(3*d*x + 3*c)/d - 3/64*(8*a^2*b + b^3)*cos(d*x + c)/d + 1/64*(2*a^3 - 3*a*b^2)*sin(4*d*x + 4*c)/d + 1/64*(16*a^3 + 3*a*b^2)*sin(2*d*x + 2*c)/d

Mupad [B]
time = 6.93, size = 474, normalized size = 3.00

$$\frac{3\,a\,\mathrm{atan}\left(\frac{3\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,a^2+b^2\right)}{8\,\left(\frac{3\,a^3}{4}+\frac{3\,a\,b^2}{8}\right)}\right)\,\left(2\,a^2+b^2\right)}{8\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{3\,a\,b^2}{8}-\frac{5\,a^3}{4}\right)+\frac{6\,a^2\,b}{5}-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(3\,a^3+\frac{11\,a\,b^2}{2}\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{11}\,\left(3\,a^3+\frac{11\,a\,b^2}{2}\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{13}\,\left(\frac{3\,a\,b^2}{8}-\frac{5\,a^3}{4}\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5\,\left(\frac{31\,a\,b^2}{8}-\frac{9\,a^3}{4}\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^9\,\left(\frac{31\,a\,b^2}{8}-\frac{9\,a^3}{4}\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}\,\left(12\,a^2\,b+4\,b^3\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(\frac{12\,a^2\,b}{5}+\frac{4\,b^3}{5}\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8\,\left(18\,a^2\,b-4\,b^3\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6\,\left(24\,a^2\,b+8\,b^3\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(\frac{66\,a^2\,b}{5}-\frac{8\,b^3}{5}\right)+\frac{4\,b^3}{35}+6\,a^2\,b\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{12}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{14}+7\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{12}+21\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}+35\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8+35\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+21\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+7\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}-\frac{3\,a\,\left(2\,a^2+b^2\right)\,\left(\mathrm{atan}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-\frac{d\,x}{2}\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^4*(a + b*sin(c + d*x))^3,x)

```
[Out] (3*a*atan((3*a*tan(c/2 + (d*x)/2)*(2*a^2 + b^2))/(8*((3*a*b^2)/8 + (3*a^3)/
4)))*(2*a^2 + b^2))/(8*d) - (tan(c/2 + (d*x)/2)*((3*a*b^2)/8 - (5*a^3)/4) +
 (6*a^2*b)/5 - tan(c/2 + (d*x)/2)^3*((11*a*b^2)/2 + 3*a^3) + tan(c/2 + (d*x
)/2)^11*((11*a*b^2)/2 + 3*a^3) - tan(c/2 + (d*x)/2)^13*((3*a*b^2)/8 - (5*a^
3)/4) + tan(c/2 + (d*x)/2)^5*((31*a*b^2)/8 - (9*a^3)/4) - tan(c/2 + (d*x)/2
)^9*((31*a*b^2)/8 - (9*a^3)/4) + tan(c/2 + (d*x)/2)^10*(12*a^2*b + 4*b^3) +
 tan(c/2 + (d*x)/2)^2*((12*a^2*b)/5 + (4*b^3)/5) + tan(c/2 + (d*x)/2)^8*(18
*a^2*b - 4*b^3) + tan(c/2 + (d*x)/2)^6*(24*a^2*b + 8*b^3) + tan(c/2 + (d*x)
/2)^4*((66*a^2*b)/5 - (8*b^3)/5) + (4*b^3)/35 + 6*a^2*b*tan(c/2 + (d*x)/2)^
12)/(d*(7*tan(c/2 + (d*x)/2)^2 + 21*tan(c/2 + (d*x)/2)^4 + 35*tan(c/2 + (d*
x)/2)^6 + 35*tan(c/2 + (d*x)/2)^8 + 21*tan(c/2 + (d*x)/2)^10 + 7*tan(c/2 +
(d*x)/2)^12 + tan(c/2 + (d*x)/2)^14 + 1)) - (3*a*(2*a^2 + b^2)*(atan(tan(c/
2 + (d*x)/2)) - (d*x)/2))/(8*d)
```

3.407 $\int \cos^2(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=131

$$\frac{1}{8}a\left(4a^2+3b^2\right)x-\frac{b(27a^2+8b^2)\cos^3(c+dx)}{60d}+\frac{a(4a^2+3b^2)\cos(c+dx)\sin(c+dx)}{8d}-\frac{7ab\cos^3(c+dx)(a+b$$

```
[Out] 1/8*a*(4*a^2+3*b^2)*x-1/60*b*(27*a^2+8*b^2)*cos(d*x+c)^3/d+1/8*a*(4*a^2+3*b
^2)*cos(d*x+c)*sin(d*x+c)/d-7/20*a*b*cos(d*x+c)^3*(a+b*sin(d*x+c))/d-1/5*b*
cos(d*x+c)^3*(a+b*sin(d*x+c))^2/d
```

Rubi [A]
time = 0.14, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2771, 2941, 2748, 2715, 8}

$$-\frac{b(27a^2+8b^2)\cos^3(c+dx)}{60d}+\frac{a(4a^2+3b^2)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}ax\left(4a^2+3b^2\right)-\frac{b\cos^3(c+dx)(a+b\sin(c+dx))^2}{5d}-\frac{7ab\cos^3(c+dx)(a+b\sin(c+dx))}{20d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (a*(4*a^2 + 3*b^2)*x)/8 - (b*(27*a^2 + 8*b^2)*Cos[c + d*x]^3)/(60*d) + (a*(
4*a^2 + 3*b^2)*Cos[c + d*x]*Sin[c + d*x])/(8*d) - (7*a*b*Cos[c + d*x]^3*(a
+ b*Sin[c + d*x]))/(20*d) - (b*Cos[c + d*x]^3*(a + b*Sin[c + d*x])^2)/(5*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2771

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
```

```
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(
a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rule 2941

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp
[a*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x],
x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
&&  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Si
mplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\begin{aligned}\int \cos^2(c+dx)(a+b\sin(c+dx))^3\,dx &= -\frac{b\cos^3(c+dx)(a+b\sin(c+dx))^2}{5d}+\frac{1}{5}\int \cos^2(c+dx)(a+b\sin(c\\ &= -\frac{7ab\cos^3(c+dx)(a+b\sin(c+dx))}{20d}-\frac{b\cos^3(c+dx)(a+b\sin(c+}{5d}\\ &= -\frac{b(27a^2+8b^2)\cos^3(c+dx)}{60d}-\frac{7ab\cos^3(c+dx)(a+b\sin(c+dx))}{20d}\\ &= -\frac{b(27a^2+8b^2)\cos^3(c+dx)}{60d}+\frac{a(4a^2+3b^2)\cos(c+dx)\sin(c+dx)}{8d}\\ &= \frac{1}{8}a\left(4a^2+3b^2\right)x-\frac{b(27a^2+8b^2)\cos^3(c+dx)}{60d}+\frac{a(4a^2+3b^2)\cos(c}{8d}\end{aligned}$$

Mathematica [A]
time = 0.55, size = 107, normalized size = 0.82

$$\frac{-60b(6a^2+b^2)\cos(c+dx)-10(12a^2b+b^3)\cos(3(c+dx))+6b^3\cos(5(c+dx))+15a(4(4a^2+3b^2)(c+dx)+8a^2\sin(2(c+dx))-3b^2\sin(4(c+dx)))}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Sin[c + d*x])^3,x]
[Out] (-60*b*(6*a^2 + b^2)*Cos[c + d*x] - 10*(12*a^2*b + b^3)*Cos[3*(c + d*x)] +
6*b^3*Cos[5*(c + d*x)] + 15*a*(4*(4*a^2 + 3*b^2)*(c + d*x) + 8*a^2*Sin[2*(c
 + d*x)] - 3*b^2*Sin[4*(c + d*x)]))/(480*d)
```

Maple [A]
time = 0.35, size = 123, normalized size = 0.94

method	result
derivativedivides	$\frac{b^3\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^3(dx+c)\right)}{5}-\frac{2\left(\cos^3(dx+c)\right)}{15}\right)+3a\,b^2\left(-\frac{\sin(dx+c)\left(\cos^3(dx+c)\right)}{4}+\frac{\cos(dx+c)\sin(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)-a^2b(}{d}$
default	$\frac{b^3\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^3(dx+c)\right)}{5}-\frac{2\left(\cos^3(dx+c)\right)}{15}\right)+3a\,b^2\left(-\frac{\sin(dx+c)\left(\cos^3(dx+c)\right)}{4}+\frac{\cos(dx+c)\sin(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)-a^2b(}{d}$
risch	$\frac{a^3x}{2}+\frac{3a\,b^2x}{8}-\frac{3b\cos(dx+c)a^2}{4d}-\frac{b^3\cos(dx+c)}{8d}+\frac{b^3\cos(5dx+5c)}{80d}-\frac{3\sin(4dx+4c)a\,b^2}{32d}-\frac{b\cos(3dx+3c)a^2}{4d}-$
norman	$\frac{\left(\frac{1}{2}a^3+\frac{3}{8}a\,b^2\right)x+\left(5a^3+\frac{15}{4}a\,b^2\right)x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(5a^3+\frac{15}{4}a\,b^2\right)x\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{1}{2}a^3+\frac{3}{8}a\,b^2\right)x\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^3*(-1/5*sin(d*x+c)^2*cos(d*x+c)^3-2/15*cos(d*x+c)^3)+3*a*b^2*(-1/4*s
in(d*x+c)*cos(d*x+c)^3+1/8*cos(d*x+c)*sin(d*x+c)+1/8*d*x+1/8*c)-a^2*b*cos(d
*x+c)^3+a^3*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))
```

Maxima [A]

time = 0.28, size = 93, normalized size = 0.71

$$-\frac{480\,a^2b\cos\left(dx+c\right)^3-120\,\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)a^3-45\,\left(4\,dx+4\,c-\sin\left(4\,dx+4\,c\right)\right)ab^2-32\,\left(3\,\cos\left(dx+c\right)^5-5\,\cos\left(dx+c\right)^3\right)b^3}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/480*(480*a^2*b*cos(d*x + c)^3 - 120*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^3
 - 45*(4*d*x + 4*c - sin(4*d*x + 4*c))*a*b^2 - 32*(3*cos(d*x + c)^5 - 5*cos
(d*x + c)^3)*b^3)/d
```

Fricas [A]

time = 0.34, size = 98, normalized size = 0.75

$$\frac{24\,b^3\cos\left(dx+c\right)^5-40\,\left(3\,a^2b+b^3\right)\cos\left(dx+c\right)^3+15\,\left(4\,a^3+3\,ab^2\right)dx-15\,\left(6\,ab^2\cos\left(dx+c\right)^3-\left(4\,a^3+3\,ab^2\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/120*(24*b^3*cos(d*x + c)^5 - 40*(3*a^2*b + b^3)*cos(d*x + c)^3 + 15*(4*a^
3 + 3*a*b^2)*d*x - 15*(6*a*b^2*cos(d*x + c)^3 - (4*a^3 + 3*a*b^2)*cos(d*x +
 c))*sin(d*x + c))/d
```

Sympy [A]

time = 0.30, size = 236, normalized size = 1.80

$$\begin{cases}\frac{a^3x\sin^2\left(c+dx\right)}{2}+\frac{a^3x\cos^2\left(c+dx\right)}{2}+\frac{a^3\sin\left(c+dx\right)\cos\left(c+dx\right)}{2d}-\frac{a^2b\cos^3\left(c+dx\right)}{d}+\frac{3ab^2x\sin^4\left(c+dx\right)}{8}+\frac{3ab^2x\sin^2\left(c+dx\right)\cos^2\left(c+dx\right)}{4}+\frac{3ab^2x\cos^4\left(c+dx\right)}{8}+\frac{3ab^2\sin^3\left(c+dx\right)\cos\left(c+dx\right)}{8d}-\frac{3ab^2\sin\left(c+dx\right)\cos^3\left(c+dx\right)}{8d}-\frac{b^3\sin^2\left(c+dx\right)\cos^3\left(c+dx\right)}{3d}-\frac{2b^3\cos^5\left(c+dx\right)}{15d} & \text{for}\: d\neq 0\\x\left(a+b\sin\left(c\right)\right)^3\cos^2\left(c\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((a**3*x*sin(c + d*x)**2/2 + a**3*x*cos(c + d*x)**2/2 + a**3*sin(c
 + d*x)*cos(c + d*x)/(2*d) - a**2*b*cos(c + d*x)**3/d + 3*a*b**2*x*sin(c +
d*x)**4/8 + 3*a*b**2*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 3*a*b**2*x*cos(c
 + d*x)**4/8 + 3*a*b**2*sin(c + d*x)**3*cos(c + d*x)/(8*d) - 3*a*b**2*sin(c
 + d*x)*cos(c + d*x)**3/(8*d) - b**3*sin(c + d*x)**2*cos(c + d*x)**3/(3*d)
- 2*b**3*cos(c + d*x)**5/(15*d), Ne(d, 0)), (x*(a + b*sin(c))**3*cos(c)**2,
 True))
```

Giac [A]
time = 7.01, size = 113, normalized size = 0.86

$$\frac{b^3\cos(5\,dx+5\,c)}{80\,d}-\frac{3\,ab^2\sin(4\,dx+4\,c)}{32\,d}+\frac{a^3\sin(2\,dx+2\,c)}{4\,d}+\frac{1}{8}\,(4\,a^3+3\,ab^2)x-\frac{(12\,a^2b+b^3)\cos(3\,dx+3\,c)}{48\,d}-\frac{(6\,a^2b+b^3)\cos(dx+c)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/80*b^3*cos(5*d*x + 5*c)/d - 3/32*a*b^2*sin(4*d*x + 4*c)/d + 1/4*a^3*sin(2
*d*x + 2*c)/d + 1/8*(4*a^3 + 3*a*b^2)*x - 1/48*(12*a^2*b + b^3)*cos(3*d*x +
 3*c)/d - 1/8*(6*a^2*b + b^3)*cos(d*x + c)/d
```

Mupad [B]
time = 6.63, size = 356, normalized size = 2.72

$$\frac{a\,\mathrm{atan}\left(\frac{a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,(4\,a^2+3\,b^2)}{4\,\left(a^3+\frac{3\,a\,b^2}{4}\right)}\right)\,(4\,a^2+3\,b^2)}{4\,d}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,\left(\frac{3\,a\,b^2}{4}-a^3\right)+2\,a^2\,b-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\,\left(2\,a^3+\frac{9\,a\,b^2}{2}\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9\,\left(\frac{3\,a\,b^2}{4}-a^3\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7\,\left(2\,a^3+\frac{9\,a\,b^2}{2}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\,\left(4\,a^2\,b+\frac{4\,b^3}{3}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\,\left(8\,a^2\,b-\frac{4\,b^3}{3}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6\,(12\,a^2\,b+4\,b^3)+\frac{4\,b^3}{15}+6\,a^2\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8}{d\,\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8+10\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+10\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}-\frac{a\,(4\,a^2+3\,b^2)\,\left(\mathrm{atan}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)-\frac{dx}{2}\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a + b*sin(c + d*x))^3,x)
```

```
[Out] (a*atan((a*tan(c/2 + (d*x)/2)*(4*a^2 + 3*b^2))/(4*((3*a*b^2)/4 + a^3)))*(4*
a^2 + 3*b^2))/(4*d) - (tan(c/2 + (d*x)/2)*((3*a*b^2)/4 - a^3) + 2*a^2*b - t
an(c/2 + (d*x)/2)^3*((9*a*b^2)/2 + 2*a^3) - tan(c/2 + (d*x)/2)^9*((3*a*b^2)
/4 - a^3) + tan(c/2 + (d*x)/2)^7*((9*a*b^2)/2 + 2*a^3) + tan(c/2 + (d*x)/2)
^2*(4*a^2*b + (4*b^3)/3) + tan(c/2 + (d*x)/2)^4*(8*a^2*b - (4*b^3)/3) + tan
(c/2 + (d*x)/2)^6*(12*a^2*b + 4*b^3) + (4*b^3)/15 + 6*a^2*b*tan(c/2 + (d*x)
/2)^8)/(d*(5*tan(c/2 + (d*x)/2)^2 + 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 +
(d*x)/2)^6 + 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 + 1)) - (a*(4*a
^2 + 3*b^2)*(atan(tan(c/2 + (d*x)/2)) - (d*x)/2))/(4*d)
```

3.408 $\int \sec^2(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=79

$$-3ab^2x+\frac{2b(a^2+b^2)\cos(c+dx)}{d}+\frac{ab^2\cos(c+dx)\sin(c+dx)}{d}+\frac{\sec(c+dx)(b+a\sin(c+dx))(a+b\sin(c+$$

```
[Out] -3*a*b^2*x+2*b*(a^2+b^2)*cos(d*x+c)/d+a*b^2*cos(d*x+c)*sin(d*x+c)/d+sec(d*x
+c)*(b+a*sin(d*x+c))*(a+b*sin(d*x+c))^2/d
```

Rubi [A]

time = 0.05, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2770, 2813}

$$\frac{2b(a^2+b^2)\cos(c+dx)}{d}+\frac{ab^2\sin(c+dx)\cos(c+dx)}{d}-3ab^2x+\frac{\sec(c+dx)(a\sin(c+dx)+b)(a+b\sin(c+dx))^2}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -3*a*b^2*x + (2*b*(a^2 + b^2)*Cos[c + d*x])/d + (a*b^2*Cos[c + d*x]*Sin[c +
 d*x])/d + (Sec[c + d*x]*(b + a*Sin[c + d*x])*(a + b*Sin[c + d*x])^2)/d
```

Rule 2770

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x
])^(m - 1)*((b + a*Sin[e + f*x])/(f*g*(p + 1))), x] + Dist[1/(g^2*(p + 1)),
 Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a
^2*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g}
, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p
] || IntegerQ[m])
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^2(c+dx)(a+b\sin(c+dx))^3\,dx &= \frac{\sec(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^2}{d}-\int (a+b\sin(c\\ &= -3ab^2x+\frac{2b(a^2+b^2)\cos(c+dx)}{d}+\frac{ab^2\cos(c+dx)\sin(c+dx)}{d}+\end{aligned}$$

Mathematica [A]
time = 0.33, size = 68, normalized size = 0.86

$$\frac{-6ab^2(c+dx)+(6a^2b+3b^3+b^3\cos(2(c+dx)))\sec(c+dx)+2a(a^2+3b^2)\tan(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-6*a*b^2*(c + d*x) + (6*a^2*b + 3*b^3 + b^3*Cos[2*(c + d*x)])*Sec[c + d*x]
 + 2*a*(a^2 + 3*b^2)*Tan[c + d*x])/(2*d)
```

Maple [A]
time = 0.23, size = 89, normalized size = 1.13

method	result
derivativedivides	$\frac{a^3\tan(dx+c)+\frac{3a^2b}{\cos(dx+c)}+3a\,b^2(\tan(dx+c)-dx-c)+b^3\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+(2+\sin^2(dx+c))\cos(dx+c)\right)}{d}$
default	$\frac{a^3\tan(dx+c)+\frac{3a^2b}{\cos(dx+c)}+3a\,b^2(\tan(dx+c)-dx-c)+b^3\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+(2+\sin^2(dx+c))\cos(dx+c)\right)}{d}$
risch	$-3a\,b^2x+\frac{b^3e^{i(dx+c)}}{2d}+\frac{b^3e^{-i(dx+c)}}{2d}+\frac{2ia^3+6ia\,b^2+6a^2b\,e^{i(dx+c)}+2b^3e^{i(dx+c)}}{d(1+e^{2i(dx+c)})}$
norman	$\frac{-\frac{6a^2b+4b^3}{d}+3a\,b^2x-\frac{6a^2b\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2\left(9a^2b+4b^3\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+6a\,b^2x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-6a\,b^2x\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*tan(d*x+c)+3*a^2*b/cos(d*x+c)+3*a*b^2*(tan(d*x+c)-d*x-c)+b^3*(sin(
d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d*x+c)))
```

Maxima [A]
time = 0.53, size = 70, normalized size = 0.89

$$-\frac{3\,(dx+c-\tan(dx+c))ab^2-b^3\left(\frac{1}{\cos(dx+c)}+\cos(dx+c)\right)-a^3\tan(dx+c)-\frac{3\,a^2b}{\cos(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -(3*(d*x + c - tan(d*x + c))*a*b^2 - b^3*(1/cos(d*x + c) + cos(d*x + c)) -
a^3*tan(d*x + c) - 3*a^2*b/cos(d*x + c))/d
```

Fricas [A]
time = 0.33, size = 70, normalized size = 0.89

$$-\frac{3\,ab^2dx\cos\left(dx+c\right)-b^3\cos\left(dx+c\right)^2-3\,a^2b-b^3-\left(a^3+3\,ab^2\right)\sin\left(dx+c\right)}{d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -(3*a*b^2*d*x*cos(d*x + c) - b^3*cos(d*x + c)^2 - 3*a^2*b - b^3 - (a^3 + 3*
a*b^2)*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sin\left(c+dx\right)\right)^3\sec^2\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral((a + b*sin(c + d*x))**3*sec(c + d*x)**2, x)
```

Giac [A]
time = 7.31, size = 123, normalized size = 1.56

$$-\frac{3\left(dx+c\right)ab^2+\frac{2\left(a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,a^2b+2\,b^3\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -(3*(d*x + c)*a*b^2 + 2*(a^3*tan(1/2*d*x + 1/2*c)^3 + 3*a*b^2*tan(1/2*d*x +
 1/2*c)^3 + 3*a^2*b*tan(1/2*d*x + 1/2*c)^2 + a^3*tan(1/2*d*x + 1/2*c) + 3*a
*b^2*tan(1/2*d*x + 1/2*c) + 3*a^2*b + 2*b^3)/(tan(1/2*d*x + 1/2*c)^4 - 1))/
d
```

Mupad [B]
time = 5.81, size = 103, normalized size = 1.30

$$-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,a^3+6\,a\,b^2\right)+6\,a^2\,b+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(2\,a^3+6\,a\,b^2\right)+4\,b^3+6\,a^2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-1\right)}-3\,a\,b^2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^3/cos(c + d*x)^2,x)
```

```
[Out] - (tan(c/2 + (d*x)/2)*(6*a*b^2 + 2*a^3) + 6*a^2*b + tan(c/2 + (d*x)/2)^3*(6
*a*b^2 + 2*a^3) + 4*b^3 + 6*a^2*b*tan(c/2 + (d*x)/2)^2)/(d*(tan(c/2 + (d*x)
/2)^4 - 1)) - 3*a*b^2*x
```

3.409 $\int \sec^4(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=84

$$\frac{2b(a^2-b^2)\sec(c+dx)}{3d}+\frac{\sec^3(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^2}{3d}+\frac{2a(a^2-b^2)\tan(c+dx)}{3d}$$

```
[Out] 2/3*b*(a^2-b^2)*sec(d*x+c)/d+1/3*sec(d*x+c)^3*(b+a*sin(d*x+c))*(a+b*sin(d*x
+c))^2/d+2/3*a*(a^2-b^2)*tan(d*x+c)/d
```

Rubi [A]
time = 0.06, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2770, 12, 2748, 3852, 8}

$$\frac{2a(a^2-b^2)\tan(c+dx)}{3d}+\frac{2b(a^2-b^2)\sec(c+dx)}{3d}+\frac{\sec^3(c+dx)(a\sin(c+dx)+b)(a+b\sin(c+dx))^2}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (2*b*(a^2 - b^2)*Sec[c + d*x])/(3*d) + (Sec[c + d*x]^3*(b + a*Sin[c + d*x])
*(a + b*Sin[c + d*x])^2)/(3*d) + (2*a*(a^2 - b^2)*Tan[c + d*x])/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2770

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x
])^(m - 1)*((b + a*Sin[e + f*x])/(f*g*(p + 1))), x] + Dist[1/(g^2*(p + 1)),
 Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a
^2*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g}
```

```
, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p
] || IntegerQ[m])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \sec^4(c+dx)(a+b\sin(c+dx))^3\,dx = \frac{\sec^3(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^2}{3d} - \frac{1}{3}\int \left(-2a^2 +\right.$$

$$= \frac{\sec^3(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^2}{3d} + \frac{1}{3}\left(2\left(a^2-b^2\right)\right.$$

$$= \frac{2b(a^2-b^2)\sec(c+dx)}{3d} + \frac{\sec^3(c+dx)(b+a\sin(c+dx))(a+b\sin}{3d}$$

$$= \frac{2b(a^2-b^2)\sec(c+dx)}{3d} + \frac{\sec^3(c+dx)(b+a\sin(c+dx))(a+b\sin}{3d}$$

$$= \frac{2b(a^2-b^2)\sec(c+dx)}{3d} + \frac{\sec^3(c+dx)(b+a\sin(c+dx))(a+b\sin}{3d}$$

Mathematica [A]
time = 0.43, size = 136, normalized size = 1.62

$$\frac{\sec^3(c+dx)\left(24a^2b-4b^3+(-9a^2b+15b^3)\cos(c+dx)-12b^3\cos(2(c+dx))-3a^2b\cos(3(c+dx))+5b^3\cos(3(c+dx))+12a^3\sin(c+dx)+18ab^2\sin(c+dx)+4a^3\sin(3(c+dx))-6ab^2\sin(3(c+dx))\right)}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]^3*(24*a^2*b - 4*b^3 + (-9*a^2*b + 15*b^3)*Cos[c + d*x] - 12*b
^3*Cos[2*(c + d*x)] - 3*a^2*b*Cos[3*(c + d*x)] + 5*b^3*Cos[3*(c + d*x)] + 1
2*a^3*Sin[c + d*x] + 18*a*b^2*Sin[c + d*x] + 4*a^3*Sin[3*(c + d*x)] - 6*a*b
^2*Sin[3*(c + d*x)]))/(24*d)
```

Maple [A]
time = 0.36, size = 122, normalized size = 1.45

method	result
risch	$-\frac{2\left(9iab^2 e^{4i(dx+c)}+3b^3 e^{5i(dx+c)}-6ia^3 e^{2i(dx+c)}-12a^2b\,e^{3i(dx+c)}+2b^3 e^{3i(dx+c)}-2ia^3+3iab^2+3b^3 e^{i(dx+c)}\right)}{3d\left(1+e^{2i(dx+c)}\right)^3}$

derivativedivides	$\frac{-a^3\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+\frac{a^2b}{\cos(dx+c)^3}+\frac{a\,b^2\left(\sin^3(dx+c)\right)}{\cos(dx+c)^3}+b^3\left(\frac{\sin^4(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^4(dx+c)}{3\cos(dx+c)}-\frac{\left(2+\sin^2(dx+c)\right)\cos(c}{3}\right.}{d}$
default	$\frac{-a^3\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+\frac{a^2b}{\cos(dx+c)^3}+\frac{a\,b^2\left(\sin^3(dx+c)\right)}{\cos(dx+c)^3}+b^3\left(\frac{\sin^4(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^4(dx+c)}{3\cos(dx+c)}-\frac{\left(2+\sin^2(dx+c)\right)\cos(c}{3}\right.}{d}$
norman	$\frac{-\frac{6a^2b-4b^3}{3d}-\frac{2a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{2a^3\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{\left(12a^2b+8b^3\right)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{\left(18a^2b+4b^3\right)\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{4a\left(a^2+6b\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^3*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+a^2*b/cos(d*x+c)^3+a*b^2*sin(d
*x+c)^3/cos(d*x+c)^3+b^3*(1/3*sin(d*x+c)^4/cos(d*x+c)^3-1/3*sin(d*x+c)^4/co
s(d*x+c)-1/3*(2+sin(d*x+c)^2)*cos(d*x+c)))
```

Maxima [A]
time = 0.29, size = 80, normalized size = 0.95

$$\frac{3\,ab^2\tan\left(dx+c\right)^3+\left(\tan\left(dx+c\right)^3+3\,\tan\left(dx+c\right)\right)a^3-\frac{\left(3\,\cos(dx+c)^2-1\right)b^3}{\cos(dx+c)^3}+\frac{3\,a^2b}{\cos(dx+c)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/3*(3*a*b^2*tan(d*x + c)^3 + (tan(d*x + c)^3 + 3*tan(d*x + c))*a^3 - (3*co
s(d*x + c)^2 - 1)*b^3/cos(d*x + c)^3 + 3*a^2*b/cos(d*x + c)^3)/d
```

Fricas [A]
time = 0.33, size = 77, normalized size = 0.92

$$-\frac{3\,b^3\cos\left(dx+c\right)^2-3\,a^2b-b^3-\left(a^3+3\,ab^2+\left(2\,a^3-3\,ab^2\right)\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)}{3\,d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/3*(3*b^3*cos(d*x + c)^2 - 3*a^2*b - b^3 - (a^3 + 3*a*b^2 + (2*a^3 - 3*a*
b^2)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sin\left(c+dx\right)\right)^3\sec^4\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral((a + b*sin(c + d*x))**3*sec(c + d*x)**4, x)
```

Giac [A]
time = 5.74, size = 128, normalized size = 1.52

$$-\frac{2\left(3\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+9\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-2\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+12\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+6\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+3\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,a^2b-2\,b^3\right)}{3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -2/3*(3*a^3*tan(1/2*d*x + 1/2*c)^5 + 9*a^2*b*tan(1/2*d*x + 1/2*c)^4 - 2*a^3
*tan(1/2*d*x + 1/2*c)^3 + 12*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 6*b^3*tan(1/2*d
*x + 1/2*c)^2 + 3*a^3*tan(1/2*d*x + 1/2*c) + 3*a^2*b - 2*b^3)/((tan(1/2*d*x
 + 1/2*c)^2 - 1)^3*d)
```

Mupad [B]
time = 5.25, size = 81, normalized size = 0.96

$$\frac{a^2\,b+\frac{a^3\,\sin(c+d\,x)}{3}+\frac{b^3}{3}-\cos\left(c+d\,x\right)^2\,\left(-\frac{2\,\sin(c+d\,x)\,a^3}{3}+\sin\left(c+d\,x\right)\,a\,b^2+b^3\right)+a\,b^2\,\sin\left(c+d\,x\right)}{d\cos\left(c+d\,x\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^3/cos(c + d*x)^4,x)
```

```
[Out] (a^2*b + (a^3*sin(c + d*x))/3 + b^3/3 - cos(c + d*x)^2*(b^3 - (2*a^3*sin(c
+ d*x))/3 + a*b^2*sin(c + d*x)) + a*b^2*sin(c + d*x))/(d*cos(c + d*x)^3)
```

3.410 $\int \sec^6(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=135

$$\frac{2b(2a^2-b^2)\sec(c+dx)}{15d}+\frac{\sec^5(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^2}{5d}+\frac{2\sec^3(c+dx)(a+b\sin(c+d}{}$$

```
[Out] 2/15*b*(2*a^2-b^2)*sec(d*x+c)/d+1/5*sec(d*x+c)^5*(b+a*sin(d*x+c))*(a+b*sin(
d*x+c))^2/d+2/15*sec(d*x+c)^3*(a+b*sin(d*x+c))*(a*b+(2*a^2-b^2)*sin(d*x+c))
/d+2/15*a*(4*a^2-3*b^2)*tan(d*x+c)/d
```

Rubi [A]

time = 0.13, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2770, 2940, 2748, 3852, 8}

$$\frac{2a(4a^2-3b^2)\tan(c+dx)}{15d}+\frac{2b(2a^2-b^2)\sec(c+dx)}{15d}+\frac{2\sec^3(c+dx)(a+b\sin(c+dx))\left((2a^2-b^2)\sin(c+dx)+ab\right)}{15d}+\frac{\sec^5(c+dx)(a\sin(c+dx)+b)(a+b\sin(c+dx))^2}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (2*b*(2*a^2 - b^2)*Sec[c + d*x])/(15*d) + (Sec[c + d*x]^5*(b + a*Sin[c + d*
x])*(a + b*Sin[c + d*x])^2)/(5*d) + (2*Sec[c + d*x]^3*(a + b*Sin[c + d*x])*
(a*b + (2*a^2 - b^2)*Sin[c + d*x]))/(15*d) + (2*a*(4*a^2 - 3*b^2)*Tan[c + d
*x])/(15*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2770

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x
])^(m - 1)*((b + a*Sin[e + f*x])/(f*g*(p + 1))), x] + Dist[1/(g^2*(p + 1)),
 Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a
^2*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g}
, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p
] || IntegerQ[m])
```

Rule 2940

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(g*
Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x])^m*((d + c*Sin[e + f*x])/(f*g*(p
 + 1))), x] + Dist[1/(g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin
[e + f*x])^(m - 1)*Simp[a*c*(p + 2) + b*d*m + b*c*(m + p + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && GtQ[
m, 0] && LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] &
& SimplerQ[c + d*x, a + b*x])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \sec^6(c+dx)(a+b\sin(c+dx))^3\,dx = \frac{\sec^5(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^2}{5d} - \frac{1}{5}\int \sec^4(c+$$

$$= \frac{\sec^5(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^2}{5d} + \frac{2\sec^3(c+dx$$

$$= \frac{2b(2a^2-b^2)\sec(c+dx)}{15d} + \frac{\sec^5(c+dx)(b+a\sin(c+dx))(a+b\,\mathrm{si}}{5d}$$

$$= \frac{2b(2a^2-b^2)\sec(c+dx)}{15d} + \frac{\sec^5(c+dx)(b+a\sin(c+dx))(a+b\,\mathrm{si}}{5d}$$

$$= \frac{2b(2a^2-b^2)\sec(c+dx)}{15d} + \frac{\sec^5(c+dx)(b+a\sin(c+dx))(a+b\,\mathrm{si}}{5d}$$

Mathematica [A]
time = 0.61, size = 190, normalized size = 1.41

$$\frac{\sec^5(c+dx)\left(1152a^2b+64b^3+(-270a^2b+110b^3)\cos(c+dx)-320b^3\cos(2(c+dx))-135a^2b\cos(3(c+dx))+55b^3\cos(3(c+dx))-27a^2b\cos(5(c+dx))+11b^3\cos(5(c+dx))+640a^3\sin(c+dx)+960ab^2\sin(c+dx)+320a^3\sin(3(c+dx))-240ab^2\sin(3(c+dx))+64a^3\sin(5(c+dx))-48ab^2\sin(5(c+dx))\right)}{1920d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]^5*(1152*a^2*b + 64*b^3 + (-270*a^2*b + 110*b^3)*Cos[c + d*x]
- 320*b^3*Cos[2*(c + d*x)] - 135*a^2*b*Cos[3*(c + d*x)] + 55*b^3*Cos[3*(c +
 d*x)] - 27*a^2*b*Cos[5*(c + d*x)] + 11*b^3*Cos[5*(c + d*x)] + 640*a^3*Sin[
c + d*x] + 960*a*b^2*Sin[c + d*x] + 320*a^3*Sin[3*(c + d*x)] - 240*a*b^2*Si
```

```
n[3*(c + d*x)] + 64*a^3*Sin[5*(c + d*x)] - 48*a*b^2*Sin[5*(c + d*x)]))/(192
0*d)
```

Maple [A]
time = 0.47, size = 173, normalized size = 1.28

method	result
risch	$-\frac{4(45ia\,b^2 e^{6i(dx+c)}+10b^3 e^{7i(dx+c)}-40ia^3 e^{4i(dx+c)}-15ia\,b^2 e^{4i(dx+c)}-72a^2 b\,e^{5i(dx+c)}-4b^3 e^{5i(dx+c)}-20ia^3 e^{2i(dx+c)}+}{15d(1+e^{2i(dx+c)})^5}$
derivativedivides	$\frac{-a^3\left(-\frac{8}{15}-\frac{(\sec^4(dx+c))}{5}-\frac{4(\sec^2(dx+c))}{15}\right)\tan(dx+c)+\frac{3a^2 b}{5\cos(dx+c)^5}+3a\,b^2\left(\frac{\sin^3(dx+c)}{5\cos(dx+c)^5}+\frac{2(\sin^3(dx+c))}{15\cos(dx+c)^3}\right)+b^3\left(\frac{\sin^4(dx}{5\cos(dx}\right.}{d}$
default	$\frac{-a^3\left(-\frac{8}{15}-\frac{(\sec^4(dx+c))}{5}-\frac{4(\sec^2(dx+c))}{15}\right)\tan(dx+c)+\frac{3a^2 b}{5\cos(dx+c)^5}+3a\,b^2\left(\frac{\sin^3(dx+c)}{5\cos(dx+c)^5}+\frac{2(\sin^3(dx+c))}{15\cos(dx+c)^3}\right)+b^3\left(\frac{\sin^4(dx}{5\cos(dx}\right.}{d}$
norman	$-\frac{18a^2b-4b^3}{15d}-\frac{2a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{2a^3\left(\tan^{15}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{6a^2b\left(\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2(9a^2b+2b^3)\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2(27a^2b+4b^3)(}{15}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^3*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c)+3/5*a^2*b/c
os(d*x+c)^5+3*a*b^2*(1/5*sin(d*x+c)^3/cos(d*x+c)^5+2/15*sin(d*x+c)^3/cos(d*
x+c)^3)+b^3*(1/5*sin(d*x+c)^4/cos(d*x+c)^5+1/15*sin(d*x+c)^4/cos(d*x+c)^3-1
/15*sin(d*x+c)^4/cos(d*x+c)-1/15*(2+sin(d*x+c)^2)*cos(d*x+c)))
```

Maxima [A]
time = 0.28, size = 105, normalized size = 0.78

$$\frac{(3\tan(dx+c)^5+10\tan(dx+c)^3+15\tan(dx+c))a^3+3\left(3\tan(dx+c)^5+5\tan(dx+c)^3\right)ab^2-\frac{(5\cos(dx+c)^2-3)b^3}{\cos(dx+c)^5}+\frac{9a^2b}{\cos(dx+c)^5}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/15*((3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a^3 + 3*(3*t
an(d*x + c)^5 + 5*tan(d*x + c)^3)*a*b^2 - (5*cos(d*x + c)^2 - 3)*b^3/cos(d*
x + c)^5 + 9*a^2*b/cos(d*x + c)^5)/d
```

Fricas [A]
time = 0.34, size = 101, normalized size = 0.75

$$-\frac{5\,b^3\cos(dx+c)^2-9\,a^2b-3\,b^3-\left(2\left(4\,a^3-3\,ab^2\right)\cos(dx+c)^4+3\,a^3+9\,ab^2+\left(4\,a^3-3\,ab^2\right)\cos(dx+c)^2\right)\sin(dx+c)}{15\,d\cos(dx+c)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/15*(5*b^3*cos(d*x + c)^2 - 9*a^2*b - 3*b^3 - (2*(4*a^3 - 3*a*b^2)*cos(d*
x + c)^4 + 3*a^3 + 9*a*b^2 + (4*a^3 - 3*a*b^2)*cos(d*x + c)^2)*sin(d*x + c)
)/(d*cos(d*x + c)^5)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [A]
time = 3.64, size = 243, normalized size = 1.80

$$-\frac{2\left(15a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9+45a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^8-20a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+60ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+30b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6+58a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+24ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+90a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+10b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-20a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+60ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+10b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+15a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+9a^2b-2b^3\right)}{15\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -2/15*(15*a^3*tan(1/2*d*x + 1/2*c)^9 + 45*a^2*b*tan(1/2*d*x + 1/2*c)^8 - 20
*a^3*tan(1/2*d*x + 1/2*c)^7 + 60*a*b^2*tan(1/2*d*x + 1/2*c)^7 + 30*b^3*tan(
1/2*d*x + 1/2*c)^6 + 58*a^3*tan(1/2*d*x + 1/2*c)^5 + 24*a*b^2*tan(1/2*d*x +
 1/2*c)^5 + 90*a^2*b*tan(1/2*d*x + 1/2*c)^4 + 10*b^3*tan(1/2*d*x + 1/2*c)^4
 - 20*a^3*tan(1/2*d*x + 1/2*c)^3 + 60*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 10*b^3
*tan(1/2*d*x + 1/2*c)^2 + 15*a^3*tan(1/2*d*x + 1/2*c) + 9*a^2*b - 2*b^3)/((
tan(1/2*d*x + 1/2*c)^2 - 1)^5*d)
```

Mupad [B]
time = 5.41, size = 119, normalized size = 0.88

$$\frac{\cos\left(c+d\,x\right)^4\,\left(\frac{8\,a^3\,\sin\left(c+d\,x\right)}{15}-\frac{2\,a\,b^2\,\sin\left(c+d\,x\right)}{5}\right)-\cos\left(c+d\,x\right)^2\,\left(-\frac{4\,\sin\left(c+d\,x\right)\,a^3}{15}+\frac{\sin\left(c+d\,x\right)\,a\,b^2}{5}+\frac{b^3}{3}\right)+\frac{3\,a^2\,b}{5}+\frac{a^3\,\sin\left(c+d\,x\right)}{5}+\frac{b^3}{5}+\frac{3\,a\,b^2\,\sin\left(c+d\,x\right)}{5}}{d\,\cos\left(c+d\,x\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^3/cos(c + d*x)^6,x)
```

```
[Out] (cos(c + d*x)^4*((8*a^3*sin(c + d*x))/15 - (2*a*b^2*sin(c + d*x))/5) - cos(
c + d*x)^2*(b^3/3 - (4*a^3*sin(c + d*x))/15 + (a*b^2*sin(c + d*x))/5) + (3*
a^2*b)/5 + (a^3*sin(c + d*x))/5 + b^3/5 + (3*a*b^2*sin(c + d*x))/5)/(d*cos(
c + d*x)^5)
```

3.411 $\int \sec^8(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=165

$$\frac{2b(3a^2-b^2)\sec^3(c+dx)}{35d}+\frac{\sec^7(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^2}{7d}+\frac{2\sec^5(c+dx)(a+b\sin(c+$$

```
[Out] 2/35*b*(3*a^2-b^2)*sec(d*x+c)^3/d+1/7*sec(d*x+c)^7*(b+a*sin(d*x+c))*(a+b*si
n(d*x+c))^2/d+2/35*sec(d*x+c)^5*(a+b*sin(d*x+c))*(2*a*b+(3*a^2-b^2)*sin(d*x
+c))/d+12/35*a*(2*a^2-b^2)*tan(d*x+c)/d+4/35*a*(2*a^2-b^2)*tan(d*x+c)^3/d
```

Rubi [A]

time = 0.14, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2770, 2940, 2748, 3852}

$$\frac{4a(2a^2-b^2)\tan^3(c+dx)}{35d}+\frac{12a(2a^2-b^2)\tan(c+dx)}{35d}+\frac{2b(3a^2-b^2)\sec^3(c+dx)}{35d}+\frac{2\sec^5(c+dx)(a+b\sin(c+dx))\left((3a^2-b^2)\sin(c+dx)+2ab\right)}{35d}+\frac{\sec^7(c+dx)(a\sin(c+dx)+b)(a+b\sin(c+dx))^2}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^8*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (2*b*(3*a^2 - b^2)*Sec[c + d*x]^3)/(35*d) + (Sec[c + d*x]^7*(b + a*Sin[c +
d*x])*(a + b*Sin[c + d*x])^2)/(7*d) + (2*Sec[c + d*x]^5*(a + b*Sin[c + d*x]
)*(2*a*b + (3*a^2 - b^2)*Sin[c + d*x]))/(35*d) + (12*a*(2*a^2 - b^2)*Tan[c
+ d*x])/(35*d) + (4*a*(2*a^2 - b^2)*Tan[c + d*x]^3)/(35*d)
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2770

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x
])^(m - 1)*((b + a*Sin[e + f*x])/(f*g*(p + 1))), x] + Dist[1/(g^2*(p + 1)),
 Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a
^2*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g}
, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p
] || IntegerQ[m])
```

Rule 2940

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(g*
```

```
Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x])^m*((d + c*Sin[e + f*x])/(f*g*(p
 + 1))), x] + Dist[1/(g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin
[e + f*x])^(m - 1)*Simp[a*c*(p + 2) + b*d*m + b*c*(m + p + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && GtQ[
m, 0] && LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] &
& SimplerQ[c + d*x, a + b*x])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \sec^8(c+dx)(a+b\sin(c+dx))^3\,dx = \frac{\sec^7(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^2}{7d} - \frac{1}{7}\int \sec^6(c+$$
$$= \frac{\sec^7(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^2}{7d} + \frac{2\sec^5(c+dx}{}$$
$$= \frac{2b(3a^2-b^2)\sec^3(c+dx)}{35d} + \frac{\sec^7(c+dx)(b+a\sin(c+dx))(a+b\,\mathrm{s}}{7d}$$
$$= \frac{2b(3a^2-b^2)\sec^3(c+dx)}{35d} + \frac{\sec^7(c+dx)(b+a\sin(c+dx))(a+b\,\mathrm{s}}{7d}$$
$$= \frac{2b(3a^2-b^2)\sec^3(c+dx)}{35d} + \frac{\sec^7(c+dx)(b+a\sin(c+dx))(a+b\,\mathrm{s}}{7d}$$

Mathematica [A]
time = 0.93, size = 245, normalized size = 1.48

$$\frac{\sec^7(c+dx)\left(15360a^2b+1536b^3+35b(-75a^2+17b^2)\cos(c+dx)-3584b^3\cos(2(c+dx))-1575a^2b\cos(3(c+dx))+357b^3\cos(3(c+dx))-525a^2b\cos(5(c+dx))+119b^3\cos(5(c+dx))-75a^2b\cos(7(c+dx))+17b^3\cos(7(c+dx))+8960a^3\sin(c+dx)+13440ab^2\sin(c+dx)+5376a^3\sin(3(c+dx))-2688ab^2\sin(3(c+dx))+1792a^3\sin(5(c+dx))-896ab^2\sin(5(c+dx))+256a^3\sin(7(c+dx))-128ab^2\sin(7(c+dx))\right)}{35840d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^8*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]^7*(15360*a^2*b + 1536*b^3 + 35*b*(-75*a^2 + 17*b^2)*Cos[c + d
*x] - 3584*b^3*Cos[2*(c + d*x)] - 1575*a^2*b*Cos[3*(c + d*x)] + 357*b^3*Cos
[3*(c + d*x)] - 525*a^2*b*Cos[5*(c + d*x)] + 119*b^3*Cos[5*(c + d*x)] - 75*
a^2*b*Cos[7*(c + d*x)] + 17*b^3*Cos[7*(c + d*x)] + 8960*a^3*Sin[c + d*x] +
13440*a*b^2*Sin[c + d*x] + 5376*a^3*Sin[3*(c + d*x)] - 2688*a*b^2*Sin[3*(c
+ d*x)] + 1792*a^3*Sin[5*(c + d*x)] - 896*a*b^2*Sin[5*(c + d*x)] + 256*a^3*
Sin[7*(c + d*x)] - 128*a*b^2*Sin[7*(c + d*x)]))/(35840*d)
```

Maple [A]
time = 0.45, size = 219, normalized size = 1.33

method	result
risch	$-\frac{16\left(70ia\,b^{2}\mathrm{e}^{8i(dx+c)}+14b^{3}\mathrm{e}^{9i(dx+c)}-70ia^{3}\mathrm{e}^{6i(dx+c)}-35ia\,b^{2}\mathrm{e}^{6i(dx+c)}-120a^{2}b\,\mathrm{e}^{7i(dx+c)}-12b^{3}\mathrm{e}^{7i(dx+c)}-42ia^{3}\mathrm{e}^{4i(dx+}}{35d\left(1+\mathrm{e}^{2i(dx+c)}\right)^{7}}$
derivativedivides	$\frac{-a^{3}\left(-\frac{16}{35}-\frac{\left(\sec^{6}(dx+c)\right)}{7}-\frac{6\left(\sec^{4}(dx+c)\right)}{35}-\frac{8\left(\sec^{2}(dx+c)\right)}{35}\right)\tan(dx+c)+\frac{3a^{2}b}{7\cos(dx+c)^{7}}+3a\,b^{2}\left(\frac{\sin^{3}(dx+c)}{7\cos(dx+c)^{7}}+\frac{4\left(\sin^{3}(dx+c)\right)}{35\cos(dx+c)^{5}}\right.}{d}$
default	$\frac{-a^{3}\left(-\frac{16}{35}-\frac{\left(\sec^{6}(dx+c)\right)}{7}-\frac{6\left(\sec^{4}(dx+c)\right)}{35}-\frac{8\left(\sec^{2}(dx+c)\right)}{35}\right)\tan(dx+c)+\frac{3a^{2}b}{7\cos(dx+c)^{7}}+3a\,b^{2}\left(\frac{\sin^{3}(dx+c)}{7\cos(dx+c)^{7}}+\frac{4\left(\sin^{3}(dx+c)\right)}{35\cos(dx+c)^{5}}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^8*(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^3*(-16/35-1/7*sec(d*x+c)^6-6/35*sec(d*x+c)^4-8/35*sec(d*x+c)^2)*tan
(d*x+c)+3/7*a^2*b/cos(d*x+c)^7+3*a*b^2*(1/7*sin(d*x+c)^3/cos(d*x+c)^7+4/35*
sin(d*x+c)^3/cos(d*x+c)^5+8/105*sin(d*x+c)^3/cos(d*x+c)^3)+b^3*(1/7*sin(d*x
+c)^4/cos(d*x+c)^7+3/35*sin(d*x+c)^4/cos(d*x+c)^5+1/35*sin(d*x+c)^4/cos(d*x
+c)^3-1/35*sin(d*x+c)^4/cos(d*x+c)-1/35*(2+sin(d*x+c)^2)*cos(d*x+c)))
```

Maxima [A]
time = 0.28, size = 124, normalized size = 0.75

$$\frac{\left(5\,\tan\left(dx+c\right)^{7}+21\,\tan\left(dx+c\right)^{5}+35\,\tan\left(dx+c\right)^{3}+35\,\tan\left(dx+c\right)\right)a^{3}+\left(15\,\tan\left(dx+c\right)^{7}+42\,\tan\left(dx+c\right)^{5}+35\,\tan\left(dx+c\right)^{3}\right)ab^{2}-\frac{\left(7\,\cos\left(dx+c\right)^{2}-5\right)b^{3}}{\cos\left(dx+c\right)^{7}}+\frac{15\,a^{2}b}{\cos\left(dx+c\right)^{7}}}{35\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/35*((5*tan(d*x + c)^7 + 21*tan(d*x + c)^5 + 35*tan(d*x + c)^3 + 35*tan(d*
x + c))*a^3 + (15*tan(d*x + c)^7 + 42*tan(d*x + c)^5 + 35*tan(d*x + c)^3)*a
*b^2 - (7*cos(d*x + c)^2 - 5)*b^3/cos(d*x + c)^7 + 15*a^2*b/cos(d*x + c)^7)
/d
```

Fricas [A]
time = 0.39, size = 124, normalized size = 0.75

$$-\frac{7\,b^{3}\cos\left(dx+c\right)^{2}-15\,a^{2}b-5\,b^{3}-\left(8\left(2\,a^{3}-ab^{2}\right)\cos\left(dx+c\right)^{6}+4\left(2\,a^{3}-ab^{2}\right)\cos\left(dx+c\right)^{4}+5\,a^{3}+15\,ab^{2}+3\left(2\,a^{3}-ab^{2}\right)\cos\left(dx+c\right)^{2}\right)\sin\left(dx+c\right)}{35\,d\cos\left(dx+c\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/35*(7*b^3*cos(d*x + c)^2 - 15*a^2*b - 5*b^3 - (8*(2*a^3 - a*b^2)*cos(d*x
 + c)^6 + 4*(2*a^3 - a*b^2)*cos(d*x + c)^4 + 5*a^3 + 15*a*b^2 + 3*(2*a^3 -
a*b^2)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^7)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**8*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 358 vs. 2(155) = 310.
time = 3.34, size = 358, normalized size = 2.17

$$-\frac{2\left(35a^{3}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{13}+105a^{2}b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{12}-70a^{3}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{11}+140ab^{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{11}+70b^{3}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{10}+301a^{3}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{9}+112ab^{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{9}+525a^{2}b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{8}+70b^{3}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{8}-212a^{3}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{7}+456ab^{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{7}+140b^{3}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{6}+301a^{3}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{5}+112ab^{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{5}+315a^{2}b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{4}+28b^{3}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{4}-70a^{3}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}+140ab^{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}+14b^{3}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+35a^{3}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+15a^{2}b-2b^{3}\right)}{35\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-1\right)^{7}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -2/35*(35*a^3*tan(1/2*d*x + 1/2*c)^13 + 105*a^2*b*tan(1/2*d*x + 1/2*c)^12 -
 70*a^3*tan(1/2*d*x + 1/2*c)^11 + 140*a*b^2*tan(1/2*d*x + 1/2*c)^11 + 70*b^
3*tan(1/2*d*x + 1/2*c)^10 + 301*a^3*tan(1/2*d*x + 1/2*c)^9 + 112*a*b^2*tan(
1/2*d*x + 1/2*c)^9 + 525*a^2*b*tan(1/2*d*x + 1/2*c)^8 + 70*b^3*tan(1/2*d*x
+ 1/2*c)^8 - 212*a^3*tan(1/2*d*x + 1/2*c)^7 + 456*a*b^2*tan(1/2*d*x + 1/2*c
)^7 + 140*b^3*tan(1/2*d*x + 1/2*c)^6 + 301*a^3*tan(1/2*d*x + 1/2*c)^5 + 112
*a*b^2*tan(1/2*d*x + 1/2*c)^5 + 315*a^2*b*tan(1/2*d*x + 1/2*c)^4 + 28*b^3*t
an(1/2*d*x + 1/2*c)^4 - 70*a^3*tan(1/2*d*x + 1/2*c)^3 + 140*a*b^2*tan(1/2*d
*x + 1/2*c)^3 + 14*b^3*tan(1/2*d*x + 1/2*c)^2 + 35*a^3*tan(1/2*d*x + 1/2*c)
 + 15*a^2*b - 2*b^3)/((tan(1/2*d*x + 1/2*c)^2 - 1)^7*d)
```

Mupad [**B**]
time = 5.60, size = 152, normalized size = 0.92

$$\frac{\cos\left(c+d\,x\right)^4\,\left(\frac{8\,a^3\,\sin\left(c+d\,x\right)}{35}-\frac{4\,a\,b^2\,\sin\left(c+d\,x\right)}{35}\right)+\cos\left(c+d\,x\right)^6\,\left(\frac{16\,a^3\,\sin\left(c+d\,x\right)}{35}-\frac{8\,a\,b^2\,\sin\left(c+d\,x\right)}{35}\right)-\cos\left(c+d\,x\right)^2\,\left(-\frac{6\,\sin\left(c+d\,x\right)\,a^3}{35}+\frac{3\,\sin\left(c+d\,x\right)\,a\,b^2}{35}+\frac{b^3}{5}\right)+\frac{3\,a^2\,b}{7}+\frac{a^3\,\sin\left(c+d\,x\right)}{7}+\frac{b^3}{7}+\frac{3\,a\,b^2\,\sin\left(c+d\,x\right)}{7}}{d\,\cos\left(c+d\,x\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^3/cos(c + d*x)^8,x)
```

```
[Out] (cos(c + d*x)^4*((8*a^3*sin(c + d*x))/35 - (4*a*b^2*sin(c + d*x))/35) + cos
(c + d*x)^6*((16*a^3*sin(c + d*x))/35 - (8*a*b^2*sin(c + d*x))/35) - cos(c
+ d*x)^2*(b^3/5 - (6*a^3*sin(c + d*x))/35 + (3*a*b^2*sin(c + d*x))/35) + (3
*a^2*b)/7 + (a^3*sin(c + d*x))/7 + b^3/7 + (3*a*b^2*sin(c + d*x))/7)/(d*cos
(c + d*x)^7)
```

3.412 $\int \sec^{10}(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=192

$$\frac{2b(4a^2-b^2)\sec^5(c+dx)}{63d}+\frac{\sec^9(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^2}{9d}+\frac{2\sec^7(c+dx)(a+b\sin(c+$$

```
[Out] 2/63*b*(4*a^2-b^2)*sec(d*x+c)^5/d+1/9*sec(d*x+c)^9*(b+a*sin(d*x+c))*(a+b*si
n(d*x+c))^2/d+2/63*sec(d*x+c)^7*(a+b*sin(d*x+c))*(3*a*b+(4*a^2-b^2)*sin(d*x
+c))/d+2/21*a*(8*a^2-3*b^2)*tan(d*x+c)/d+4/63*a*(8*a^2-3*b^2)*tan(d*x+c)^3/
d+2/105*a*(8*a^2-3*b^2)*tan(d*x+c)^5/d
```

Rubi [A]
time = 0.15, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2770, 2940, 2748, 3852}

$$\frac{2a(8a^2-3b^2)\tan^5(c+dx)}{105d}+\frac{4a(8a^2-3b^2)\tan^3(c+dx)}{63d}+\frac{2a(8a^2-3b^2)\tan(c+dx)}{21d}+\frac{2b(4a^2-b^2)\sec^5(c+dx)}{63d}+\frac{2\sec^7(c+dx)(a+b\sin(c+dx))\left((4a^2-b^2)\sin(c+dx)+3ab\right)}{63d}+\frac{\sec^9(c+dx)(a\sin(c+dx)+b)(a+b\sin(c+dx))^2}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^10*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (2*b*(4*a^2 - b^2)*Sec[c + d*x]^5)/(63*d) + (Sec[c + d*x]^9*(b + a*Sin[c +
d*x])*(a + b*Sin[c + d*x])^2)/(9*d) + (2*Sec[c + d*x]^7*(a + b*Sin[c + d*x]
)*(3*a*b + (4*a^2 - b^2)*Sin[c + d*x]))/(63*d) + (2*a*(8*a^2 - 3*b^2)*Tan[c
 + d*x])/(21*d) + (4*a*(8*a^2 - 3*b^2)*Tan[c + d*x]^3)/(63*d) + (2*a*(8*a^2
 - 3*b^2)*Tan[c + d*x]^5)/(105*d)
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2770

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x
])^(m - 1)*((b + a*Sin[e + f*x])/(f*g*(p + 1))), x] + Dist[1/(g^2*(p + 1)),
 Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a
^2*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g}
, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p
] || IntegerQ[m])
```

Rule 2940

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(g*
Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x])^m*((d + c*Sin[e + f*x])/(f*g*(p
 + 1))), x] + Dist[1/(g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin
[e + f*x])^(m - 1)*Simp[a*c*(p + 2) + b*d*m + b*c*(m + p + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && GtQ[
m, 0] && LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] &
& SimplerQ[c + d*x, a + b*x])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \sec^{10}(c+dx)(a+b\sin(c+dx))^3\,dx = \frac{\sec^9(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^2}{9d} - \frac{1}{9}\int \sec^8(c\ \cdots$$

$$= \frac{\sec^9(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^2}{9d} + \frac{2\sec^7(c+d\ \cdots}{}$$

$$= \frac{2b(4a^2-b^2)\sec^5(c+dx)}{63d} + \frac{\sec^9(c+dx)(b+a\sin(c+dx))(a+b\ \cdots}{9d}$$

$$= \frac{2b(4a^2-b^2)\sec^5(c+dx)}{63d} + \frac{\sec^9(c+dx)(b+a\sin(c+dx))(a+b\ \cdots}{9d}$$

$$= \frac{2b(4a^2-b^2)\sec^5(c+dx)}{63d} + \frac{\sec^9(c+dx)(b+a\sin(c+dx))(a+b\ \cdots}{9d}$$

Mathematica [A]
time = 1.54, size = 299, normalized size = 1.56

$$\frac{\sec^9(c+dx)\left(3440640a^2b+409600b^3+3150b(-147a^2+23b^2)\cos(c+dx)-737280b^3\cos(2(c+dx))-308700a^2b\cos(3(c+dx))+48300b^3\cos(3(c+dx))-132300a^2b\cos(5(c+dx))+20700b^3\cos(5(c+dx))-33075a^2b\cos(7(c+dx))+5175b^3\cos(7(c+dx))-3675a^2b\cos(9(c+dx))+575b^3\cos(9(c+dx))+2064384a^3\sin(c+dx)+3096576ab^2\sin(c+dx)+1376256a^3\sin(3(c+dx))-516096ab^2\sin(3(c+dx))+589824a^3\sin(5(c+dx))-221184ab^2\sin(5(c+dx))+147456a^3\sin(7(c+dx))-55296ab^2\sin(7(c+dx))+16384a^3\sin(9(c+dx))-6144ab^2\sin(9(c+dx))\right)}{10321920d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^10*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]^9*(3440640*a^2*b + 409600*b^3 + 3150*b*(-147*a^2 + 23*b^2)*Co
s[c + d*x] - 737280*b^3*Cos[2*(c + d*x)] - 308700*a^2*b*Cos[3*(c + d*x)] +
48300*b^3*Cos[3*(c + d*x)] - 132300*a^2*b*Cos[5*(c + d*x)] + 20700*b^3*Cos[
5*(c + d*x)] - 33075*a^2*b*Cos[7*(c + d*x)] + 5175*b^3*Cos[7*(c + d*x)] - 3
675*a^2*b*Cos[9*(c + d*x)] + 575*b^3*Cos[9*(c + d*x)] + 2064384*a^3*Sin[c +
```

```
d*x] + 3096576*a*b^2*Sin[c + d*x] + 1376256*a^3*Sin[3*(c + d*x)] - 516096*
a*b^2*Sin[3*(c + d*x)] + 589824*a^3*Sin[5*(c + d*x)] - 221184*a*b^2*Sin[5*(
c + d*x)] + 147456*a^3*Sin[7*(c + d*x)] - 55296*a*b^2*Sin[7*(c + d*x)] + 16
384*a^3*Sin[9*(c + d*x)] - 6144*a*b^2*Sin[9*(c + d*x)]))/(10321920*d)
```

Maple [A]

time = 0.60, size = 265, normalized size = 1.38

method	result
risch	$-\frac{32(945iab^{2}e^{10i(dx+c)}+180b^{3}e^{11i(dx+c)}-1008ia^{3}e^{8i(dx+c)}-567iab^{2}e^{8i(dx+c)}-1680a^{2}be^{9i(dx+c)}-200b^{3}e^{9i(dx+c)}-67}{3}$
derivativedivides	$-a^{3}\left(-\frac{128}{315}-\frac{(\sec^{8}(dx+c))}{9}-\frac{8(\sec^{6}(dx+c))}{63}-\frac{16(\sec^{4}(dx+c))}{105}-\frac{64(\sec^{2}(dx+c))}{315}\right)\tan(dx+c)+\frac{a^{2}b}{3\cos(dx+c)^{9}}+3ab^{2}\left(\frac{\sin^{3}(dx+}{9\cos(dx+}\right.$
default	$-a^{3}\left(-\frac{128}{315}-\frac{(\sec^{8}(dx+c))}{9}-\frac{8(\sec^{6}(dx+c))}{63}-\frac{16(\sec^{4}(dx+c))}{105}-\frac{64(\sec^{2}(dx+c))}{315}\right)\tan(dx+c)+\frac{a^{2}b}{3\cos(dx+c)^{9}}+3ab^{2}\left(\frac{\sin^{3}(dx+}{9\cos(dx+}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^10*(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^3*(-128/315-1/9*sec(d*x+c)^8-8/63*sec(d*x+c)^6-16/105*sec(d*x+c)^4-
64/315*sec(d*x+c)^2)*tan(d*x+c)+1/3*a^2*b/cos(d*x+c)^9+3*a*b^2*(1/9*sin(d*x
+c)^3/cos(d*x+c)^9+2/21*sin(d*x+c)^3/cos(d*x+c)^7+8/105*sin(d*x+c)^3/cos(d*
x+c)^5+16/315*sin(d*x+c)^3/cos(d*x+c)^3)+b^3*(1/9*sin(d*x+c)^4/cos(d*x+c)^9
+5/63*sin(d*x+c)^4/cos(d*x+c)^7+1/21*sin(d*x+c)^4/cos(d*x+c)^5+1/63*sin(d*x
+c)^4/cos(d*x+c)^3-1/63*sin(d*x+c)^4/cos(d*x+c)-1/63*(2+sin(d*x+c)^2)*cos(d
*x+c)))
```

Maxima [A]

time = 0.34, size = 145, normalized size = 0.76

$$\frac{\left(35\,\tan\left(dx+c\right)^{9}+180\,\tan\left(dx+c\right)^{7}+378\,\tan\left(dx+c\right)^{5}+420\,\tan\left(dx+c\right)^{3}+315\,\tan\left(dx+c\right)\right)a^{3}+3\left(35\,\tan\left(dx+c\right)^{9}+135\,\tan\left(dx+c\right)^{7}+189\,\tan\left(dx+c\right)^{5}+105\,\tan\left(dx+c\right)^{3}\right)ab^{2}-\frac{5\left(9\cos(dx+c)^{2}-7\right)b^{3}}{\cos(dx+c)^{9}}+\frac{105\,a^{2}b}{\cos(dx+c)^{9}}}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/315*((35*tan(d*x + c)^9 + 180*tan(d*x + c)^7 + 378*tan(d*x + c)^5 + 420*t
an(d*x + c)^3 + 315*tan(d*x + c))*a^3 + 3*(35*tan(d*x + c)^9 + 135*tan(d*x
+ c)^7 + 189*tan(d*x + c)^5 + 105*tan(d*x + c)^3)*a*b^2 - 5*(9*cos(d*x + c)
^2 - 7)*b^3/cos(d*x + c)^9 + 105*a^2*b/cos(d*x + c)^9)/d
```

Fricas [A]

time = 0.35, size = 146, normalized size = 0.76

$$-\frac{45\,b^{3}\cos\left(dx+c\right)^{2}-105\,a^{2}b-35\,b^{3}-\left(16\left(8\,a^{3}-3\,ab^{2}\right)\cos\left(dx+c\right)^{8}+8\left(8\,a^{3}-3\,ab^{2}\right)\cos\left(dx+c\right)^{6}+6\left(8\,a^{3}-3\,ab^{2}\right)\cos\left(dx+c\right)^{4}+35\,a^{3}+105\,ab^{2}+5\left(8\,a^{3}-3\,ab^{2}\right)\cos\left(dx+c\right)^{2}\right)\sin\left(dx+c\right)}{315\,d\cos\left(dx+c\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/315*(45*b^3*cos(d*x + c)^2 - 105*a^2*b - 35*b^3 - (16*(8*a^3 - 3*a*b^2)*
cos(d*x + c)^8 + 8*(8*a^3 - 3*a*b^2)*cos(d*x + c)^6 + 6*(8*a^3 - 3*a*b^2)*c
os(d*x + c)^4 + 35*a^3 + 105*a*b^2 + 5*(8*a^3 - 3*a*b^2)*cos(d*x + c)^2)*si
n(d*x + c))/(d*cos(d*x + c)^9)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**10*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 473 vs. 2(180) = 360.
time = 5.21, size = 473, normalized size = 2.46

$$-\frac{2\left(315\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{17}+945\,a^{2}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{16}-840\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{15}+1260\,ab^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{15}+630\,b^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{14}+4788\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}+1512\,ab^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}+8820\,a^{2}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{12}+1050\,b^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{12}-5112\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+8532\,ab^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+3150\,b^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}+10658\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+4272\,ab^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+13230\,a^{2}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}+1890\,b^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}-5112\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+8532\,ab^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+1890\,b^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+4788\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+1512\,ab^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+3780\,a^{2}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+270\,b^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-840\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+1260\,ab^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+90\,b^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+315\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+105\,a^{2}b-10\,b^{3}\right)}{315\,\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{9}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -2/315*(315*a^3*tan(1/2*d*x + 1/2*c)^17 + 945*a^2*b*tan(1/2*d*x + 1/2*c)^16
 - 840*a^3*tan(1/2*d*x + 1/2*c)^15 + 1260*a*b^2*tan(1/2*d*x + 1/2*c)^15 + 6
30*b^3*tan(1/2*d*x + 1/2*c)^14 + 4788*a^3*tan(1/2*d*x + 1/2*c)^13 + 1512*a*
b^2*tan(1/2*d*x + 1/2*c)^13 + 8820*a^2*b*tan(1/2*d*x + 1/2*c)^12 + 1050*b^3
*tan(1/2*d*x + 1/2*c)^12 - 5112*a^3*tan(1/2*d*x + 1/2*c)^11 + 8532*a*b^2*ta
n(1/2*d*x + 1/2*c)^11 + 3150*b^3*tan(1/2*d*x + 1/2*c)^10 + 10658*a^3*tan(1/
2*d*x + 1/2*c)^9 + 4272*a*b^2*tan(1/2*d*x + 1/2*c)^9 + 13230*a^2*b*tan(1/2*
d*x + 1/2*c)^8 + 1890*b^3*tan(1/2*d*x + 1/2*c)^8 - 5112*a^3*tan(1/2*d*x + 1
/2*c)^7 + 8532*a*b^2*tan(1/2*d*x + 1/2*c)^7 + 1890*b^3*tan(1/2*d*x + 1/2*c)
^6 + 4788*a^3*tan(1/2*d*x + 1/2*c)^5 + 1512*a*b^2*tan(1/2*d*x + 1/2*c)^5 +
3780*a^2*b*tan(1/2*d*x + 1/2*c)^4 + 270*b^3*tan(1/2*d*x + 1/2*c)^4 - 840*a^
3*tan(1/2*d*x + 1/2*c)^3 + 1260*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 90*b^3*tan(1
/2*d*x + 1/2*c)^2 + 315*a^3*tan(1/2*d*x + 1/2*c) + 105*a^2*b - 10*b^3)/((ta
n(1/2*d*x + 1/2*c)^2 - 1)^9*d)
```

Mupad [**B**]
time = 6.13, size = 275, normalized size = 1.43

$$\frac{b^3}{9\,d\cos\left(c+d\,x\right)^9}-\frac{b^3}{7\,d\cos\left(c+d\,x\right)^7}+\frac{a^2\,b}{3\,d\cos\left(c+d\,x\right)^9}+\frac{128\,a^3\,\sin\left(c+d\,x\right)}{315\,d\,\cos\left(c+d\,x\right)}+\frac{64\,a^3\,\sin\left(c+d\,x\right)}{315\,d\cos\left(c+d\,x\right)^3}+\frac{16\,a^3\,\sin\left(c+d\,x\right)}{105\,d\cos\left(c+d\,x\right)^5}+\frac{8\,a^3\,\sin\left(c+d\,x\right)}{63\,d\cos\left(c+d\,x\right)^7}+\frac{a^3\,\sin\left(c+d\,x\right)}{9\,d\cos\left(c+d\,x\right)^9}-\frac{16\,a\,b^2\,\sin\left(c+d\,x\right)}{105\,d\,\cos\left(c+d\,x\right)}-\frac{8\,a\,b^2\,\sin\left(c+d\,x\right)}{105\,d\cos\left(c+d\,x\right)^3}-\frac{2\,a\,b^2\,\sin\left(c+d\,x\right)}{35\,d\cos\left(c+d\,x\right)^5}-\frac{a\,b^2\,\sin\left(c+d\,x\right)}{21\,d\cos\left(c+d\,x\right)^7}+\frac{a\,b^2\,\sin\left(c+d\,x\right)}{3\,d\cos\left(c+d\,x\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^3/cos(c + d*x)^10,x)
```

```
[Out] b^3/(9*d*cos(c + d*x)^9) - b^3/(7*d*cos(c + d*x)^7) + (a^2*b)/(3*d*cos(c +
d*x)^9) + (128*a^3*sin(c + d*x))/(315*d*cos(c + d*x)) + (64*a^3*sin(c + d*x
))/(315*d*cos(c + d*x)^3) + (16*a^3*sin(c + d*x))/(105*d*cos(c + d*x)^5) +
(8*a^3*sin(c + d*x))/(63*d*cos(c + d*x)^7) + (a^3*sin(c + d*x))/(9*d*cos(c
+ d*x)^9) - (16*a*b^2*sin(c + d*x))/(105*d*cos(c + d*x)) - (8*a*b^2*sin(c +
 d*x))/(105*d*cos(c + d*x)^3) - (2*a*b^2*sin(c + d*x))/(35*d*cos(c + d*x)^5
) - (a*b^2*sin(c + d*x))/(21*d*cos(c + d*x)^7) + (a*b^2*sin(c + d*x))/(3*d*
cos(c + d*x)^9)
```

3.413 $\int \cos^5(c+dx)(a+b\sin(c+dx))^8\,dx$

Optimal. Leaf size=144

$$\frac{(a^2-b^2)^2\,(a+b\sin(c+dx))^9}{9b^5d}-\frac{2a(a^2-b^2)\,(a+b\sin(c+dx))^{10}}{5b^5d}+\frac{2(3a^2-b^2)\,(a+b\sin(c+dx))^{11}}{11b^5d}-\frac{a(a+$$

```
[Out] 1/9*(a^2-b^2)^2*(a+b*sin(d*x+c))^9/b^5/d-2/5*a*(a^2-b^2)*(a+b*sin(d*x+c))^1
0/b^5/d+2/11*(3*a^2-b^2)*(a+b*sin(d*x+c))^11/b^5/d-1/3*a*(a+b*sin(d*x+c))^1
2/b^5/d+1/13*(a+b*sin(d*x+c))^13/b^5/d
```

Rubi [A]
time = 0.16, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2747, 711}

$$\frac{2(3a^2-b^2)\,(a+b\sin(c+dx))^{11}}{11b^5d}-\frac{2a(a^2-b^2)\,(a+b\sin(c+dx))^{10}}{5b^5d}+\frac{(a^2-b^2)^2\,(a+b\sin(c+dx))^9}{9b^5d}+\frac{(a+b\sin(c+dx))^{13}}{13b^5d}-\frac{a(a+b\sin(c+dx))^{12}}{3b^5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + b*Sin[c + d*x])^8,x]
```

```
[Out] ((a^2 - b^2)^2*(a + b*Sin[c + d*x])^9)/(9*b^5*d) - (2*a*(a^2 - b^2)*(a + b*
Sin[c + d*x])^10)/(5*b^5*d) + (2*(3*a^2 - b^2)*(a + b*Sin[c + d*x])^11)/(11
*b^5*d) - (a*(a + b*Sin[c + d*x])^12)/(3*b^5*d) + (a + b*Sin[c + d*x])^13/(
13*b^5*d)
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^5(c+dx)(a+b\sin(c+dx))^8\,dx = \frac{\text{Subst}\left(\int (a+x)^8\left(b^2-x^2\right)^2\,dx, x, b\sin(c+dx)\right)}{b^5d}$$
$$= \frac{\text{Subst}\left(\int \left(\left(a^2-b^2\right)^2(a+x)^8 - 4\left(a^3-ab^2\right)(a+x)^9 + 2\left(3a^2-b^2\right)(\right.}{b^5d}$$
$$= \frac{\left(a^2-b^2\right)^2(a+b\sin(c+dx))^9}{9b^5d} - \frac{2a\left(a^2-b^2\right)(a+b\sin(c+dx))^{10}}{5b^5d} +$$

Mathematica [A]
time = 1.93, size = 120, normalized size = 0.83

$$\frac{\frac{1}{9}\left(a^2-b^2\right)^2(a+b\sin(c+dx))^9 - \frac{2}{5}a(a-b)(a+b)(a+b\sin(c+dx))^{10} + \frac{2}{11}\left(3a^2-b^2\right)(a+b\sin(c+dx))^{11} - \frac{1}{3}a(a+b\sin(c+dx))^{12} + \frac{1}{13}(a+b\sin(c+dx))^{13}}{b^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + b*Sin[c + d*x])^8,x]
```

```
[Out] (((a^2 - b^2)^2*(a + b*Sin[c + d*x])^9)/9 - (2*a*(a - b)*(a + b)*(a + b*Sin
[c + d*x])^10)/5 + (2*(3*a^2 - b^2)*(a + b*Sin[c + d*x])^11)/11 - (a*(a + b
*Sin[c + d*x])^12)/3 + (a + b*Sin[c + d*x])^13/13)/(b^5*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 529 vs. 2(134) = 268.
time = 2.40, size = 530, normalized size = 3.68 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+b*sin(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^8*(-1/13*sin(d*x+c)^7*cos(d*x+c)^6-7/143*sin(d*x+c)^5*cos(d*x+c)^6-3
5/1287*sin(d*x+c)^3*cos(d*x+c)^6-5/429*sin(d*x+c)*cos(d*x+c)^6+1/429*(8/3+c
os(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))+8*a*b^7*(-1/12*sin(d*x+c)^6*cos(d
*x+c)^6-1/20*sin(d*x+c)^4*cos(d*x+c)^6-1/40*sin(d*x+c)^2*cos(d*x+c)^6-1/120
*cos(d*x+c)^6)+28*a^2*b^6*(-1/11*sin(d*x+c)^5*cos(d*x+c)^6-5/99*sin(d*x+c)^
3*cos(d*x+c)^6-5/231*sin(d*x+c)*cos(d*x+c)^6+1/231*(8/3+cos(d*x+c)^4+4/3*co
s(d*x+c)^2)*sin(d*x+c))+56*a^3*b^5*(-1/10*sin(d*x+c)^4*cos(d*x+c)^6-1/20*si
n(d*x+c)^2*cos(d*x+c)^6-1/60*cos(d*x+c)^6)+70*a^4*b^4*(-1/9*sin(d*x+c)^3*co
s(d*x+c)^6-1/21*sin(d*x+c)*cos(d*x+c)^6+1/105*(8/3+cos(d*x+c)^4+4/3*cos(d*x
+c)^2)*sin(d*x+c))+56*a^5*b^3*(-1/8*sin(d*x+c)^2*cos(d*x+c)^6-1/24*cos(d*x+
c)^6)+28*a^6*b^2*(-1/7*sin(d*x+c)*cos(d*x+c)^6+1/35*(8/3+cos(d*x+c)^4+4/3*c
os(d*x+c)^2)*sin(d*x+c))-4/3*a^7*b*cos(d*x+c)^6+1/5*a^8*(8/3+cos(d*x+c)^4+4
/3*cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 311 vs. 2(134) = 268.

time = 0.27, size = 311, normalized size = 2.16

$$\frac{495 b^8 \sin(dx+c)^{13} + 4290 a b^7 \sin(dx+c)^{12} + 1170 (14 a^2 b^6 - b^8) \sin(dx+c)^{11} + 5148 (7 a^3 b^5 - 2 a b^7) \sin(dx+c)^{10} + 25740 a^7 b \sin(dx+c)^2 + 715 (70 a^4 b^4 - 56 a^2 b^6 + b^8) \sin(dx+c)^9 + 6435 a^8 \sin(dx+c) + 6435 (7 a^5 b^3 - 14 a^3 b^5 + a b^7) \sin(dx+c)^8 + 25740 (a^6 b^2 - 5 a^4 b^4 + a^2 b^6) \sin(dx+c)^7 + 8580 (a^7 b - 14 a^5 b^3 + 7 a^3 b^5) \sin(dx+c)^6 + 1287 (a^8 - 56 a^6 b^2 + 70 a^4 b^4) \sin(dx+c)^5 - 12870 (2 a^7 b - 7 a^5 b^3) \sin(dx+c)^4 - 4290 (a^8 - 14 a^6 b^2) \sin(dx+c)^3}{6435 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 1/6435*(495*b^8*sin(d*x + c)^13 + 4290*a*b^7*sin(d*x + c)^12 + 1170*(14*a^2
*b^6 - b^8)*sin(d*x + c)^11 + 5148*(7*a^3*b^5 - 2*a*b^7)*sin(d*x + c)^10 +
25740*a^7*b*sin(d*x + c)^2 + 715*(70*a^4*b^4 - 56*a^2*b^6 + b^8)*sin(d*x +
c)^9 + 6435*a^8*sin(d*x + c) + 6435*(7*a^5*b^3 - 14*a^3*b^5 + a*b^7)*sin(d*
x + c)^8 + 25740*(a^6*b^2 - 5*a^4*b^4 + a^2*b^6)*sin(d*x + c)^7 + 8580*(a^7
*b - 14*a^5*b^3 + 7*a^3*b^5)*sin(d*x + c)^6 + 1287*(a^8 - 56*a^6*b^2 + 70*a
^4*b^4)*sin(d*x + c)^5 - 12870*(2*a^7*b - 7*a^5*b^3)*sin(d*x + c)^4 - 4290*
(a^8 - 14*a^6*b^2)*sin(d*x + c)^3)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 356 vs. $2(134) = 268$.
time = 0.46, size = 356, normalized size = 2.47

$$\frac{4290 a b^7 \cos(dx+c)^{12} - 5148 (7 a^3 b^5 + 3 a b^7) \cos(dx+c)^{10} + 6435 (7 a^5 b^3 + 14 a^3 b^5 + 3 a b^7) \cos(dx+c)^8 - 8580 (a^7 b + 7 a^5 b^3 + 7 a^3 b^5 + a b^7) \cos(dx+c)^6 + (495 b^8 \cos(dx+c)^{12} - 180 (91 a^2 b^6 + 10 b^8) \cos(dx+c)^{10} + 10 (5005 a^4 b^4 + 4186 a^2 b^6 + 229 b^8) \cos(dx+c)^8 + 3432 a^8 + 13728 a^6 b^2 + 11440 a^4 b^4 + 2080 a^2 b^6 + 40 b^8 - 20 (1287 a^6 b^2 + 3575 a^4 b^4 + 1469 a^2 b^6 + 53 b^8) \cos(dx+c)^6 + 3 (429 a^8 + 1716 a^6 b^2 + 1430 a^4 b^4 + 260 a^2 b^6 + 5 b^8) \cos(dx+c)^4 + 4 (429 a^8 + 1716 a^6 b^2 + 1430 a^4 b^4 + 260 a^2 b^6 + 5 b^8) \cos(dx+c)^2) \sin(dx+c)}{6435 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/6435*(4290*a*b^7*cos(d*x + c)^12 - 5148*(7*a^3*b^5 + 3*a*b^7)*cos(d*x + c
)^10 + 6435*(7*a^5*b^3 + 14*a^3*b^5 + 3*a*b^7)*cos(d*x + c)^8 - 8580*(a^7*b
 + 7*a^5*b^3 + 7*a^3*b^5 + a*b^7)*cos(d*x + c)^6 + (495*b^8*cos(d*x + c)^12
 - 180*(91*a^2*b^6 + 10*b^8)*cos(d*x + c)^10 + 10*(5005*a^4*b^4 + 4186*a^2*
b^6 + 229*b^8)*cos(d*x + c)^8 + 3432*a^8 + 13728*a^6*b^2 + 11440*a^4*b^4 +
2080*a^2*b^6 + 40*b^8 - 20*(1287*a^6*b^2 + 3575*a^4*b^4 + 1469*a^2*b^6 + 53
*b^8)*cos(d*x + c)^6 + 3*(429*a^8 + 1716*a^6*b^2 + 1430*a^4*b^4 + 260*a^2*b
^6 + 5*b^8)*cos(d*x + c)^4 + 4*(429*a^8 + 1716*a^6*b^2 + 1430*a^4*b^4 + 260
*a^2*b^6 + 5*b^8)*cos(d*x + c)^2)*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 614 vs. $2(124) = 248$.
time = 4.72, size = 614, normalized size = 4.26

$$\begin{cases} \frac{8a^8\sin^5(c+dx)}{15d} + \frac{4a^8\sin^3(c+dx)\cos^2(c+dx)}{3d} + \frac{a^8\sin(c+dx)\cos^4(c+dx)}{d} - \frac{4a^7b\cos^6(c+dx)}{3d} + \frac{32a^6b^2\sin^7(c+dx)}{15d} + \frac{112a^6b^2\sin^5(c+dx)\cos^2(c+dx)}{15d} + \frac{28a^6b^2\sin^3(c+dx)\cos^4(c+dx)}{3d} - \frac{28a^5b^3\sin^2(c+dx)\cos^6(c+dx)}{3d} - \frac{7a^5b^3\cos^8(c+dx)}{3d} + \frac{16a^4b^4\sin^9(c+dx)}{9d} + \frac{8a^4b^4\sin^7(c+dx)\cos^2(c+dx)}{d} + \frac{14a^4b^4\sin^5(c+dx)\cos^4(c+dx)}{d} - \frac{28a^3b^5\sin^4(c+dx)\cos^6(c+dx)}{3d} - \frac{14a^3b^5\sin^2(c+dx)\cos^8(c+dx)}{3d} - \frac{14a^3b^5\cos^{10}(c+dx)}{15d} + \frac{32a^2b^6\sin^{11}(c+dx)}{99d} + \frac{16a^2b^6\sin^9(c+dx)\cos^2(c+dx)}{9d} + \frac{4a^2b^6\sin^7(c+dx)\cos^4(c+dx)}{d} - \frac{4ab^7\sin^6(c+dx)\cos^6(c+dx)}{3d} - \frac{ab^7\sin^4(c+dx)\cos^8(c+dx)}{d} - \frac{2ab^7\sin^2(c+dx)\cos^{10}(c+dx)}{5d} - \frac{ab^7\cos^{12}(c+dx)}{15d} + \frac{8b^8\sin^{13}(c+dx)}{1287d} + \frac{4b^8\sin^{11}(c+dx)\cos^2(c+dx)}{99d} + \frac{b^8\sin^9(c+dx)\cos^4(c+dx)}{9d} & \text{for } d \neq 0 \\ x(a+b\sin(c))^8\cos^5(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+b*sin(d*x+c))**8,x)
```

```
[Out] Piecewise((8*a**8*sin(c + d*x)**5/(15*d) + 4*a**8*sin(c + d*x)**3*cos(c + d
*x)**2/(3*d) + a**8*sin(c + d*x)*cos(c + d*x)**4/d - 4*a**7*b*cos(c + d*x)*
```

```
*6/(3*d) + 32*a**6*b**2*sin(c + d*x)**7/(15*d) + 112*a**6*b**2*sin(c + d*x)
**5*cos(c + d*x)**2/(15*d) + 28*a**6*b**2*sin(c + d*x)**3*cos(c + d*x)**4/(
3*d) - 28*a**5*b**3*sin(c + d*x)**2*cos(c + d*x)**6/(3*d) - 7*a**5*b**3*cos
(c + d*x)**8/(3*d) + 16*a**4*b**4*sin(c + d*x)**9/(9*d) + 8*a**4*b**4*sin(c
 + d*x)**7*cos(c + d*x)**2/d + 14*a**4*b**4*sin(c + d*x)**5*cos(c + d*x)**4
/d - 28*a**3*b**5*sin(c + d*x)**4*cos(c + d*x)**6/(3*d) - 14*a**3*b**5*sin(
c + d*x)**2*cos(c + d*x)**8/(3*d) - 14*a**3*b**5*cos(c + d*x)**10/(15*d) +
32*a**2*b**6*sin(c + d*x)**11/(99*d) + 16*a**2*b**6*sin(c + d*x)**9*cos(c +
 d*x)**2/(9*d) + 4*a**2*b**6*sin(c + d*x)**7*cos(c + d*x)**4/d - 4*a*b**7*s
in(c + d*x)**6*cos(c + d*x)**6/(3*d) - a*b**7*sin(c + d*x)**4*cos(c + d*x)*
*8/d - 2*a*b**7*sin(c + d*x)**2*cos(c + d*x)**10/(5*d) - a*b**7*cos(c + d*x
)**12/(15*d) + 8*b**8*sin(c + d*x)**13/(1287*d) + 4*b**8*sin(c + d*x)**11*c
os(c + d*x)**2/(99*d) + b**8*sin(c + d*x)**9*cos(c + d*x)**4/(9*d), Ne(d, 0
)), (x*(a + b*sin(c))**8*cos(c)**5, True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 464 vs. $2(134) = 268$.
time = 7.93, size = 464, normalized size = 3.22

$$\frac{ab^7\cos(12\,dx+12\,c)}{3072\,d}+\frac{b^8\sin(13\,dx+13\,c)}{53248\,d}-\frac{(14\,a^3b^5+ab^7)\cos(10\,dx+10\,c)}{1280\,d}+\frac{(28\,a^5b^3-ab^7)\cos(8\,dx+8\,c)}{512\,d}-\frac{(32\,a^7b-112\,a^5b^3-70\,a^3b^5-5\,ab^7)\cos(6\,dx+6\,c)}{768\,d}-\frac{(256\,a^7b+224\,a^5b^3-5\,ab^7)\cos(4\,dx+4\,c)}{1024\,d}-\frac{(80\,a^7b+168\,a^5b^3+70\,a^3b^5+5\,ab^7)\cos(2\,dx+2\,c)}{128\,d}-\frac{(112\,a^2b^6+3\,b^8)\sin(11\,dx+11\,c)}{45056\,d}+\frac{(560\,a^4b^4+56\,a^2b^6-b^8)\sin(9\,dx+9\,c)}{18432\,d}-\frac{(128\,a^6b^2-80\,a^4b^4-40\,a^2b^6-b^8)\sin(7\,dx+7\,c)}{2048\,d}+\frac{(256\,a^8-5376\,a^6b^2-4480\,a^4b^4-560\,a^2b^6-5\,b^8)\sin(5\,dx+5\,c)}{20480\,d}+\frac{(1280\,a^8-1792\,a^6b^2-4480\,a^4b^4-1120\,a^2b^6-25\,b^8)\sin(3\,dx+3\,c)}{12288\,d}+\frac{5\,(128\,a^8+448\,a^6b^2+336\,a^4b^4+56\,a^2b^6+b^8)\sin(dx+c)}{1024\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/3072*a*b^7*cos(12*d*x + 12*c)/d + 1/53248*b^8*sin(13*d*x + 13*c)/d - 1/12
80*(14*a^3*b^5 + a*b^7)*cos(10*d*x + 10*c)/d + 1/512*(28*a^5*b^3 - a*b^7)*c
os(8*d*x + 8*c)/d - 1/768*(32*a^7*b - 112*a^5*b^3 - 70*a^3*b^5 - 5*a*b^7)*c
os(6*d*x + 6*c)/d - 1/1024*(256*a^7*b + 224*a^5*b^3 - 5*a*b^7)*cos(4*d*x +
4*c)/d - 1/128*(80*a^7*b + 168*a^5*b^3 + 70*a^3*b^5 + 5*a*b^7)*cos(2*d*x +
2*c)/d - 1/45056*(112*a^2*b^6 + 3*b^8)*sin(11*d*x + 11*c)/d + 1/18432*(560*
a^4*b^4 + 56*a^2*b^6 - b^8)*sin(9*d*x + 9*c)/d - 1/2048*(128*a^6*b^2 - 80*a
^4*b^4 - 40*a^2*b^6 - b^8)*sin(7*d*x + 7*c)/d + 1/20480*(256*a^8 - 5376*a^6
*b^2 - 4480*a^4*b^4 - 560*a^2*b^6 - 5*b^8)*sin(5*d*x + 5*c)/d + 1/12288*(12
80*a^8 - 1792*a^6*b^2 - 4480*a^4*b^4 - 1120*a^2*b^6 - 25*b^8)*sin(3*d*x + 3
*c)/d + 5/1024*(128*a^8 + 448*a^6*b^2 + 336*a^4*b^4 + 56*a^2*b^6 + b^8)*sin
(d*x + c)/d
```

Mupad [**B**]
time = 5.45, size = 306, normalized size = 2.12

$$\frac{\sin(c+d\,x)^5\left(\frac{a^8}{5}-\frac{56\,a^6\,b^2}{5}+14\,a^4\,b^4\right)+\sin(c+d\,x)^9\left(\frac{70\,a^4\,b^4}{9}-\frac{56\,a^2\,b^6}{9}+\frac{b^8}{9}\right)+a^8\,\sin(c+d\,x)+\frac{b^8\,\sin(c+d\,x)^{13}}{13}-\sin(c+d\,x)^4\,(4\,a^7\,b-14\,a^5\,b^3)-\sin(c+d\,x)^{10}\left(\frac{8\,a\,b^7}{5}-\frac{28\,a^3\,b^5}{5}\right)-\frac{2\,a^6\,\sin(c+d\,x)^3\,(a^2-14\,b^2)}{3}+4\,a^7\,b\,\sin(c+d\,x)^2+\frac{2\,a\,b^7\,\sin(c+d\,x)^{12}}{3}+\frac{2\,b^6\,\sin(c+d\,x)^{11}\,(14\,a^2-b^2)}{11}+\frac{4\,a^3\,b\,\sin(c+d\,x)^6\,(a^4-14\,a^2\,b^2+7\,b^4)}{3}+a\,b^3\,\sin(c+d\,x)^8\,(7\,a^4-14\,a^2\,b^2+b^4)+4\,a^2\,b^2\,\sin(c+d\,x)^7\,(a^4-5\,a^2\,b^2+b^4)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(a + b*sin(c + d*x))^8,x)
```

```
[Out] (sin(c + d*x)^5*(a^8/5 + 14*a^4*b^4 - (56*a^6*b^2)/5) + sin(c + d*x)^9*(b^8
/9 - (56*a^2*b^6)/9 + (70*a^4*b^4)/9) + a^8*sin(c + d*x) + (b^8*sin(c + d*x
)^13)/13 - sin(c + d*x)^4*(4*a^7*b - 14*a^5*b^3) - sin(c + d*x)^10*((8*a*b^
7)/5 - (28*a^3*b^5)/5) - (2*a^6*sin(c + d*x)^3*(a^2 - 14*b^2))/3 + 4*a^7*b*
sin(c + d*x)^2 + (2*a*b^7*sin(c + d*x)^12)/3 + (2*b^6*sin(c + d*x)^11*(14*a
^2 - b^2))/11 + (4*a^3*b*sin(c + d*x)^6*(a^4 + 7*b^4 - 14*a^2*b^2))/3 + a*b
^3*sin(c + d*x)^8*(7*a^4 + b^4 - 14*a^2*b^2) + 4*a^2*b^2*sin(c + d*x)^7*(a^
4 + b^4 - 5*a^2*b^2))/d
```

3.414 $\int \cos^3(c+dx)(a+b\sin(c+dx))^8\,dx$

Optimal. Leaf size=77

$$-\frac{(a^2-b^2)\,(a+b\sin(c+dx))^9}{9b^3d}+\frac{a(a+b\sin(c+dx))^{10}}{5b^3d}-\frac{(a+b\sin(c+dx))^{11}}{11b^3d}$$

```
[Out] -1/9*(a^2-b^2)*(a+b*sin(d*x+c))^9/b^3/d+1/5*a*(a+b*sin(d*x+c))^10/b^3/d-1/1
1*(a+b*sin(d*x+c))^11/b^3/d
```

Rubi [A]
time = 0.11, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.095$, Rules used = {2747, 711}

$$-\frac{(a^2-b^2)\,(a+b\sin(c+dx))^9}{9b^3d}-\frac{(a+b\sin(c+dx))^{11}}{11b^3d}+\frac{a(a+b\sin(c+dx))^{10}}{5b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Sin[c + d*x])^8,x]
```

```
[Out] -1/9*((a^2 - b^2)*(a + b*Sin[c + d*x])^9)/(b^3*d) + (a*(a + b*Sin[c + d*x])
^10)/(5*b^3*d) - (a + b*Sin[c + d*x])^11/(11*b^3*d)
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^3(c+dx)(a+b\sin(c+dx))^8\,dx &= \frac{\text{Subst}\left(\int (a+x)^8\left(b^2-x^2\right)\,dx,x,b\sin(c+dx)\right)}{b^3d}\\ &= \frac{\text{Subst}\left(\int \left(\left(-a^2+b^2\right)(a+x)^8+2a(a+x)^9-(a+x)^{10}\right)\,dx,x,b\sin(\right.}{b^3d}\\ &= -\frac{(a^2-b^2)\,(a+b\sin(c+dx))^9}{9b^3d}+\frac{a(a+b\sin(c+dx))^{10}}{5b^3d}-\frac{(a+b\sin}{1}\end{aligned}$$

Mathematica [A]
time = 0.83, size = 56, normalized size = 0.73

$$\frac{(a+b\sin(c+dx))^9\,(-2a^2+65b^2+45b^2\cos(2(c+dx))+18ab\sin(c+dx))}{990b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Sin[c + d*x])^8,x]
```

```
[Out] ((a + b*Sin[c + d*x])^9*(-2*a^2 + 65*b^2 + 45*b^2*Cos[2*(c + d*x)] + 18*a*b
*Sin[c + d*x]))/(990*b^3*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 479 vs. $2(71) = 142$.
time = 1.41, size = 480, normalized size = 6.23 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*sin(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^8*(-1/11*sin(d*x+c)^7*cos(d*x+c)^4-7/99*sin(d*x+c)^5*cos(d*x+c)^4-5/
99*sin(d*x+c)^3*cos(d*x+c)^4-1/33*sin(d*x+c)*cos(d*x+c)^4+1/99*(2+cos(d*x+c
)^2)*sin(d*x+c))+8*a*b^7*(-1/10*sin(d*x+c)^6*cos(d*x+c)^4-3/40*sin(d*x+c)^4
*cos(d*x+c)^4-1/20*sin(d*x+c)^2*cos(d*x+c)^4-1/40*cos(d*x+c)^4)+28*a^2*b^6*
(-1/9*sin(d*x+c)^5*cos(d*x+c)^4-5/63*sin(d*x+c)^3*cos(d*x+c)^4-1/21*sin(d*x
+c)*cos(d*x+c)^4+1/63*(2+cos(d*x+c)^2)*sin(d*x+c))+56*a^3*b^5*(-1/8*sin(d*x
+c)^4*cos(d*x+c)^4-1/12*sin(d*x+c)^2*cos(d*x+c)^4-1/24*cos(d*x+c)^4)+70*a^4
*b^4*(-1/7*sin(d*x+c)^3*cos(d*x+c)^4-3/35*sin(d*x+c)*cos(d*x+c)^4+1/35*(2+c
os(d*x+c)^2)*sin(d*x+c))+56*a^5*b^3*(-1/6*sin(d*x+c)^2*cos(d*x+c)^4-1/12*co
s(d*x+c)^4)+28*a^6*b^2*(-1/5*sin(d*x+c)*cos(d*x+c)^4+1/15*(2+cos(d*x+c)^2)*
sin(d*x+c))-2*a^7*b*cos(d*x+c)^4+1/3*a^8*(2+cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(71) = 142$.
time = 0.27, size = 233, normalized size = 3.03

$$-\frac{45\,b^8\sin(dx+c)^{11}+396\,ab^7\sin(dx+c)^{10}-1980\,a^7b\sin(dx+c)^2+55\,(28\,a^2b^6-b^8)\sin(dx+c)^9-495\,a^8\sin(dx+c)+495\,(7\,a^3b^5-ab^7)\sin(dx+c)^8+990\,(5\,a^4b^4-2\,a^2b^6)\sin(dx+c)^7+4620\,(a^5b^3-a^3b^5)\sin(dx+c)^6+1386\,(2\,a^6b^2-5\,a^4b^4)\sin(dx+c)^5+990\,(a^7b-7\,a^5b^3)\sin(dx+c)^4+165\,(a^8-28\,a^6b^2)\sin(dx+c)^3}{495\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/495*(45*b^8*sin(d*x + c)^11 + 396*a*b^7*sin(d*x + c)^10 - 1980*a^7*b*sin
(d*x + c)^2 + 55*(28*a^2*b^6 - b^8)*sin(d*x + c)^9 - 495*a^8*sin(d*x + c) +
 495*(7*a^3*b^5 - a*b^7)*sin(d*x + c)^8 + 990*(5*a^4*b^4 - 2*a^2*b^6)*sin(d
*x + c)^7 + 4620*(a^5*b^3 - a^3*b^5)*sin(d*x + c)^6 + 1386*(2*a^6*b^2 - 5*a
^4*b^4)*sin(d*x + c)^5 + 990*(a^7*b - 7*a^5*b^3)*sin(d*x + c)^4 + 165*(a^8
- 28*a^6*b^2)*sin(d*x + c)^3)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 310 vs. $2(71) = 142$.
time = 0.38, size = 310, normalized size = 4.03

$$\frac{396\,ab^{7}\cos\left(dx+c\right)^{10}-495\,\left(7\,a^{3}b^{5}+3\,ab^{7}\right)\cos\left(dx+c\right)^{8}+660\,\left(7\,a^{5}b^{3}+14\,a^{3}b^{5}+3\,ab^{7}\right)\cos\left(dx+c\right)^{6}-990\,\left(a^{7}b+7\,a^{5}b^{3}+7\,a^{3}b^{5}+ab^{7}\right)\cos\left(dx+c\right)^{4}+\left(45\,b^{8}\cos\left(dx+c\right)^{10}-10\,\left(154\,a^{2}b^{6}+17\,b^{8}\right)\cos\left(dx+c\right)^{8}+330\,a^{8}+1848\,a^{6}b^{2}+1980\,a^{4}b^{4}+440\,a^{2}b^{6}+10\,b^{8}+10\,\left(495\,a^{4}b^{4}+418\,a^{2}b^{6}+23\,b^{8}\right)\cos\left(dx+c\right)^{6}-12\,\left(231\,a^{6}b^{2}+660\,a^{4}b^{4}+275\,a^{2}b^{6}+10\,b^{8}\right)\cos\left(dx+c\right)^{4}+\left(165\,a^{8}+924\,a^{6}b^{2}+990\,a^{4}b^{4}+220\,a^{2}b^{6}+5\,b^{8}\right)\cos\left(dx+c\right)^{2}\right)\sin\left(dx+c\right)}{495\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/495*(396*a*b^7*cos(d*x + c)^10 - 495*(7*a^3*b^5 + 3*a*b^7)*cos(d*x + c)^8
 + 660*(7*a^5*b^3 + 14*a^3*b^5 + 3*a*b^7)*cos(d*x + c)^6 - 990*(a^7*b + 7*a
^5*b^3 + 7*a^3*b^5 + a*b^7)*cos(d*x + c)^4 + (45*b^8*cos(d*x + c)^10 - 10*(
154*a^2*b^6 + 17*b^8)*cos(d*x + c)^8 + 330*a^8 + 1848*a^6*b^2 + 1980*a^4*b^
4 + 440*a^2*b^6 + 10*b^8 + 10*(495*a^4*b^4 + 418*a^2*b^6 + 23*b^8)*cos(d*x
+ c)^6 - 12*(231*a^6*b^2 + 660*a^4*b^4 + 275*a^2*b^6 + 10*b^8)*cos(d*x + c)
^4 + (165*a^8 + 924*a^6*b^2 + 990*a^4*b^4 + 220*a^2*b^6 + 5*b^8)*cos(d*x +
c)^2)*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 468 vs. $2(65) = 130$.
time = 2.57, size = 468, normalized size = 6.08

$$\begin{cases} \frac{2a^{8}\sin^{3}(c+dx)}{3d}+\frac{a^{8}\sin(c+dx)\cos^{2}(c+dx)}{d}-\frac{2a^{7}b\cos^{4}(c+dx)}{d}+\frac{56a^{6}b^{2}\sin^{5}(c+dx)}{15d}+\frac{28a^{6}b^{2}\sin^{3}(c+dx)\cos^{2}(c+dx)}{3d}-\frac{14a^{5}b^{3}\sin^{2}(c+dx)\cos^{4}(c+dx)}{d}-\frac{14a^{5}b^{3}\cos^{6}(c+dx)}{3d}+\frac{4a^{4}b^{4}\sin^{7}(c+dx)}{d}+\frac{14a^{4}b^{4}\sin^{5}(c+dx)\cos^{2}(c+dx)}{d}-\frac{14a^{3}b^{5}\sin^{4}(c+dx)\cos^{4}(c+dx)}{d}-\frac{28a^{3}b^{5}\sin^{2}(c+dx)\cos^{6}(c+dx)}{3d}-\frac{7a^{3}b^{5}\cos^{8}(c+dx)}{3d}+\frac{8a^{2}b^{6}\sin^{9}(c+dx)}{9d}+\frac{4a^{2}b^{6}\sin^{7}(c+dx)\cos^{2}(c+dx)}{d}-\frac{2ab^{7}\sin^{6}(c+dx)\cos^{4}(c+dx)}{d}-\frac{2ab^{7}\sin^{4}(c+dx)\cos^{6}(c+dx)}{d}-\frac{ab^{7}\sin^{2}(c+dx)\cos^{8}(c+dx)}{d}-\frac{ab^{7}\cos^{10}(c+dx)}{5d}+\frac{2b^{8}\sin^{11}(c+dx)}{99d}+\frac{b^{8}\sin^{9}(c+dx)\cos^{2}(c+dx)}{9d} & \text{for}\ d\neq 0 \\ x(a+b\sin(c))^{8}\cos^{3}(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*sin(d*x+c))**8,x)
```

```
[Out] Piecewise((2*a**8*sin(c + d*x)**3/(3*d) + a**8*sin(c + d*x)*cos(c + d*x)**2
/d - 2*a**7*b*cos(c + d*x)**4/d + 56*a**6*b**2*sin(c + d*x)**5/(15*d) + 28*
a**6*b**2*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) - 14*a**5*b**3*sin(c + d*x)
**2*cos(c + d*x)**4/d - 14*a**5*b**3*cos(c + d*x)**6/(3*d) + 4*a**4*b**4*si
n(c + d*x)**7/d + 14*a**4*b**4*sin(c + d*x)**5*cos(c + d*x)**2/d - 14*a**3*
b**5*sin(c + d*x)**4*cos(c + d*x)**4/d - 28*a**3*b**5*sin(c + d*x)**2*cos(c
 + d*x)**6/(3*d) - 7*a**3*b**5*cos(c + d*x)**8/(3*d) + 8*a**2*b**6*sin(c +
d*x)**9/(9*d) + 4*a**2*b**6*sin(c + d*x)**7*cos(c + d*x)**2/d - 2*a*b**7*si
n(c + d*x)**6*cos(c + d*x)**4/d - 2*a*b**7*sin(c + d*x)**4*cos(c + d*x)**6/
d - a*b**7*sin(c + d*x)**2*cos(c + d*x)**8/d - a*b**7*cos(c + d*x)**10/(5*d
) + 2*b**8*sin(c + d*x)**11/(99*d) + b**8*sin(c + d*x)**9*cos(c + d*x)**2/(
9*d), Ne(d, 0)), (x*(a + b*sin(c))**8*cos(c)**3, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(71) = 142$.
time = 7.39, size = 272, normalized size = 3.53

$$-\frac{45\,b^{8}\sin\left(dx+c\right)^{11}+396\,ab^{7}\sin\left(dx+c\right)^{10}+1540\,a^{2}b^{6}\sin\left(dx+c\right)^{9}-55\,b^{8}\sin\left(dx+c\right)^{9}+3465\,a^{3}b^{5}\sin\left(dx+c\right)^{8}-495\,ab^{7}\sin\left(dx+c\right)^{8}+4950\,a^{4}b^{4}\sin\left(dx+c\right)^{7}-1980\,a^{2}b^{6}\sin\left(dx+c\right)^{7}+4620\,a^{5}b^{3}\sin\left(dx+c\right)^{6}-4620\,a^{3}b^{5}\sin\left(dx+c\right)^{6}+2772\,a^{6}b^{2}\sin\left(dx+c\right)^{5}-6930\,a^{4}b^{4}\sin\left(dx+c\right)^{5}+990\,a^{7}b\sin\left(dx+c\right)^{4}-6930\,a^{5}b^{3}\sin\left(dx+c\right)^{4}+165\,a^{8}\sin\left(dx+c\right)^{3}-4620\,a^{6}b^{2}\sin\left(dx+c\right)^{3}-1980\,a^{7}b\sin\left(dx+c\right)^{2}-495\,a^{8}\sin\left(dx+c\right)}{495\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^3*(a+b*sin(d*x+c))^8,x, algorithm="giac")

[Out] -1/495*(45*b^8*sin(d*x + c)^11 + 396*a*b^7*sin(d*x + c)^10 + 1540*a^2*b^6*s
in(d*x + c)^9 - 55*b^8*sin(d*x + c)^9 + 3465*a^3*b^5*sin(d*x + c)^8 - 495*a
*b^7*sin(d*x + c)^8 + 4950*a^4*b^4*sin(d*x + c)^7 - 1980*a^2*b^6*sin(d*x +
c)^7 + 4620*a^5*b^3*sin(d*x + c)^6 - 4620*a^3*b^5*sin(d*x + c)^6 + 2772*a^6
*b^2*sin(d*x + c)^5 - 6930*a^4*b^4*sin(d*x + c)^5 + 990*a^7*b*sin(d*x + c)^
4 - 6930*a^5*b^3*sin(d*x + c)^4 + 165*a^8*sin(d*x + c)^3 - 4620*a^6*b^2*sin
(d*x + c)^3 - 1980*a^7*b*sin(d*x + c)^2 - 495*a^8*sin(d*x + c))/d

Mupad [B]
time = 5.37, size = 231, normalized size = 3.00

$$-\frac{\sin\left(c+d\,x\right)^3\,\left(\frac{a^8}{3}-\frac{28\,a^6\,b^2}{3}\right)-\sin\left(c+d\,x\right)^5\,\left(14\,a^4\,b^4-\frac{28\,a^6\,b^2}{5}\right)-\sin\left(c+d\,x\right)^7\,\left(4\,a^2\,b^6-10\,a^4\,b^4\right)-a^8\,\sin\left(c+d\,x\right)-\sin\left(c+d\,x\right)^9\,\left(\frac{b^8}{9}-\frac{28\,a^2\,b^6}{9}\right)+\frac{b^8\,\sin(c+d\,x)^{11}}{11}-4\,a^7\,b\,\sin\left(c+d\,x\right)^2+\frac{4\,a\,b^7\,\sin(c+d\,x)^{10}}{5}+2\,a^5\,b\,\sin\left(c+d\,x\right)^4\,\left(a^2-7\,b^2\right)+a\,b^5\,\sin\left(c+d\,x\right)^8\,\left(7\,a^2-b^2\right)+\frac{28\,a^3\,b^3\,\sin(c+d\,x)^6\,\left(a^2-b^2\right)}{3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^3*(a + b*sin(c + d*x))^8,x)

[Out] -(sin(c + d*x)^3*(a^8/3 - (28*a^6*b^2)/3) - sin(c + d*x)^5*(14*a^4*b^4 - (2
8*a^6*b^2)/5) - sin(c + d*x)^7*(4*a^2*b^6 - 10*a^4*b^4) - a^8*sin(c + d*x)
- sin(c + d*x)^9*(b^8/9 - (28*a^2*b^6)/9) + (b^8*sin(c + d*x)^11)/11 - 4*a^
7*b*sin(c + d*x)^2 + (4*a*b^7*sin(c + d*x)^10)/5 + 2*a^5*b*sin(c + d*x)^4*(
a^2 - 7*b^2) + a*b^5*sin(c + d*x)^8*(7*a^2 - b^2) + (28*a^3*b^3*sin(c + d*x
)^6*(a^2 - b^2))/3)/d

3.415 $\int \cos(c+dx)(a+b\sin(c+dx))^8\,dx$

Optimal. Leaf size=22

$$\frac{(a+b\sin(c+dx))^9}{9bd}$$

[Out] 1/9*(a+b*sin(d*x+c))^9/b/d

Rubi [A]
time = 0.02, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2747, 32}

$$\frac{(a+b\sin(c+dx))^9}{9bd}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]*(a + b*Sin[c + d*x])^8,x]

[Out] (a + b*Sin[c + d*x])^9/(9*b*d)

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a+b\sin(c+dx))^8\,dx &= \frac{\text{Subst}\left(\int (a+x)^8\,dx, x, b\sin(c+dx)\right)}{bd}\\ &= \frac{(a+b\sin(c+dx))^9}{9bd}\end{aligned}$$

Mathematica [A]
time = 0.16, size = 22, normalized size = 1.00

$$\frac{(a+b\sin(c+dx))^9}{9bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Sin[c + d*x])^8,x]
```

```
[Out] (a + b*Sin[c + d*x])^9/(9*b*d)
```

Maple [A]
time = 0.70, size = 21, normalized size = 0.95

method	result
derivativedivides	$\frac{(a+b\sin(dx+c))^9}{9bd}$
default	$\frac{(a+b\sin(dx+c))^9}{9bd}$
risch	$\frac{7\sin(dx+c)b^8}{128d}+\frac{\sin(9dx+9c)b^8}{2304d}+\frac{7a^3b^5\cos(4dx+4c)}{4d}+\frac{a^8\sin(dx+c)}{d}-\frac{\sin(7dx+7c)b^8}{256d}+\frac{\sin(5dx+5c)b^8}{64d}-7$
norman	$\frac{\frac{16a^7b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{16a^7b\left(\tan^{16}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2a^8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2a^8\left(\tan^{17}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{4\left(28a^7b+56a^5b^3\right)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*sin(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*(a+b*sin(d*x+c))^9/b/d
```

Maxima [A]
time = 0.28, size = 20, normalized size = 0.91

$$\frac{(b\sin(dx+c)+a)^9}{9\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 1/9*(b*sin(d*x + c) + a)^9/(b*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 257 vs. $2(20) = 40$.
time = 0.43, size = 257, normalized size = 11.68

$$\frac{9\,ab^7\cos(dx+c)^8-12\,(7\,a^3b^5+3\,ab^7)\cos(dx+c)^6+18\,(7\,a^5b^3+14\,a^3b^5+3\,ab^7)\cos(dx+c)^4-36\,(a^7b+7\,a^5b^3+7\,a^3b^5+ab^7)\cos(dx+c)^2+\left(b^8\cos(dx+c)^8+9\,a^8+84\,a^6b^2+126\,a^4b^4+36\,a^2b^6+b^8-4\,(9\,a^2b^6+b^8)\cos(dx+c)^6+6\,(21\,a^4b^4+18\,a^2b^6+b^8)\cos(dx+c)^4-4\,(21\,a^6b^2+63\,a^4b^4+27\,a^2b^6+b^8)\cos(dx+c)^2\right)\sin(dx+c)}{9\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/9*(9*a*b^7*cos(d*x + c)^8 - 12*(7*a^3*b^5 + 3*a*b^7)*cos(d*x + c)^6 + 18*
(7*a^5*b^3 + 14*a^3*b^5 + 3*a*b^7)*cos(d*x + c)^4 - 36*(a^7*b + 7*a^5*b^3 +
 7*a^3*b^5 + a*b^7)*cos(d*x + c)^2 + (b^8*cos(d*x + c)^8 + 9*a^8 + 84*a^6*b
```

```
^2 + 126*a^4*b^4 + 36*a^2*b^6 + b^8 - 4*(9*a^2*b^6 + b^8)*cos(d*x + c)^6 +
6*(21*a^4*b^4 + 18*a^2*b^6 + b^8)*cos(d*x + c)^4 - 4*(21*a^6*b^2 + 63*a^4*b
^4 + 27*a^2*b^6 + b^8)*cos(d*x + c)^2)*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(15) = 30$.
time = 1.29, size = 168, normalized size = 7.64

$$\begin{cases} \frac{a^8 \sin(c+dx)}{d} + \frac{4a^7 b \sin^2(c+dx)}{d} + \frac{28a^6 b^2 \sin^3(c+dx)}{3d} + \frac{14a^5 b^3 \sin^4(c+dx)}{d} + \frac{14a^4 b^4 \sin^5(c+dx)}{d} + \frac{28a^3 b^5 \sin^6(c+dx)}{3d} + \frac{4a^2 b^6 \sin^7(c+dx)}{d} + \frac{ab^7 \sin^8(c+dx)}{d} + \frac{b^8 \sin^9(c+dx)}{9d} & \text{for } d \neq 0 \\ x(a + b\sin(c))^8 \cos(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))**8,x)
```

```
[Out] Piecewise((a**8*sin(c + d*x)/d + 4*a**7*b*sin(c + d*x)**2/d + 28*a**6*b**2*
sin(c + d*x)**3/(3*d) + 14*a**5*b**3*sin(c + d*x)**4/d + 14*a**4*b**4*sin(c
 + d*x)**5/d + 28*a**3*b**5*sin(c + d*x)**6/(3*d) + 4*a**2*b**6*sin(c + d*x
)**7/d + a*b**7*sin(c + d*x)**8/d + b**8*sin(c + d*x)**9/(9*d), Ne(d, 0)),
(x*(a + b*sin(c))**8*cos(c), True))
```

Giac [A]
time = 4.99, size = 20, normalized size = 0.91

$$\frac{(b\sin(dx+c)+a)^9}{9\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/9*(b*sin(d*x + c) + a)^9/(b*d)
```

Mupad [B]
time = 5.27, size = 135, normalized size = 6.14

$$\frac{a^8\,\sin(c+d\,x)+4\,a^7\,b\,\sin(c+d\,x)^2+\frac{28\,a^6\,b^2\,\sin(c+d\,x)^3}{3}+14\,a^5\,b^3\,\sin(c+d\,x)^4+14\,a^4\,b^4\,\sin(c+d\,x)^5+\frac{28\,a^3\,b^5\,\sin(c+d\,x)^6}{3}+4\,a^2\,b^6\,\sin(c+d\,x)^7+a\,b^7\,\sin(c+d\,x)^8+\frac{b^8\,\sin(c+d\,x)^9}{9}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + b*sin(c + d*x))^8,x)
```

```
[Out] (a^8*sin(c + d*x) + (b^8*sin(c + d*x)^9)/9 + 4*a^7*b*sin(c + d*x)^2 + a*b^7
*sin(c + d*x)^8 + (28*a^6*b^2*sin(c + d*x)^3)/3 + 14*a^5*b^3*sin(c + d*x)^4
 + 14*a^4*b^4*sin(c + d*x)^5 + (28*a^3*b^5*sin(c + d*x)^6)/3 + 4*a^2*b^6*si
n(c + d*x)^7)/d
```

3.416 $\int \sec(c+dx)(a+b\sin(c+dx))^8\,dx$

Optimal. Leaf size=245

$$-\frac{(a+b)^8\log(1-\sin(c+dx))}{2d}+\frac{(a-b)^8\log(1+\sin(c+dx))}{2d}-\frac{b^2(28a^6+70a^4b^2+28a^2b^4+b^6)\sin(c+dx)}{d}$$

```
[Out] -1/2*(a+b)^8*ln(1-sin(d*x+c))/d+1/2*(a-b)^8*ln(1+sin(d*x+c))/d-b^2*(28*a^6+
70*a^4*b^2+28*a^2*b^4+b^6)*sin(d*x+c)/d-4*a*b^3*(7*a^4+7*a^2*b^2+b^4)*sin(d
*x+c)^2/d-1/3*b^4*(70*a^4+28*a^2*b^2+b^4)*sin(d*x+c)^3/d-2*a*b^5*(7*a^2+b^2
)*sin(d*x+c)^4/d-1/5*b^6*(28*a^2+b^2)*sin(d*x+c)^5/d-4/3*a*b^7*sin(d*x+c)^6
/d-1/7*b^8*sin(d*x+c)^7/d
```

Rubi [A]
time = 0.12, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2747, 716, 647, 31}

$$-\frac{b^6(28a^2+b^2)\sin^5(c+dx)}{5d}-\frac{2ab^5(7a^2+b^2)\sin^4(c+dx)}{d}-\frac{b^4(70a^4+28a^2b^2+b^4)\sin^3(c+dx)}{3d}-\frac{4ab^3(7a^4+7a^2b^2+b^4)\sin^2(c+dx)}{d}-\frac{b^2(28a^6+70a^4b^2+28a^2b^4+b^6)\sin(c+dx)}{d}-\frac{4ab^7\sin^6(c+dx)}{3d}+\frac{(a-b)^8\log(\sin(c+dx)+1)}{2d}-\frac{(a+b)^8\log(1-\sin(c+dx))}{2d}-\frac{b^8\sin^7(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sin[c + d*x])^8,x]
```

```
[Out] -1/2*((a + b)^8*Log[1 - Sin[c + d*x]])/d + ((a - b)^8*Log[1 + Sin[c + d*x]]
)/(2*d) - (b^2*(28*a^6 + 70*a^4*b^2 + 28*a^2*b^4 + b^6)*Sin[c + d*x])/d - (
4*a*b^3*(7*a^4 + 7*a^2*b^2 + b^4)*Sin[c + d*x]^2)/d - (b^4*(70*a^4 + 28*a^2
*b^2 + b^4)*Sin[c + d*x]^3)/(3*d) - (2*a*b^5*(7*a^2 + b^2)*Sin[c + d*x]^4)/
d - (b^6*(28*a^2 + b^2)*Sin[c + d*x]^5)/(5*d) - (4*a*b^7*Sin[c + d*x]^6)/(3
*d) - (b^8*Sin[c + d*x]^7)/(7*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
(-a)*c]
```

Rule 716

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Polyno
mialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[
```

```
c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec(c+dx)(a+b\sin(c+dx))^8\,dx = \frac{b\text{Subst}\left(\int \frac{(a+x)^8}{b^2-x^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b\text{Subst}\left(\int \left(-28a^6-70a^4b^2-28a^2b^4-b^6-8a(7a^4+7a^2b^2+b^4)\,x -$$

$$= -\frac{b^2(28a^6+70a^4b^2+28a^2b^4+b^6)\sin(c+dx)}{d} - \frac{4ab^3(7a^4+7a^2b^2+}{d}$$

$$= -\frac{b^2(28a^6+70a^4b^2+28a^2b^4+b^6)\sin(c+dx)}{d} - \frac{4ab^3(7a^4+7a^2b^2+}{d}$$

$$= -\frac{(a+b)^8\log(1-\sin(c+dx))}{2d} + \frac{(a-b)^8\log(1+\sin(c+dx))}{2d} - \frac{b^2(}{}$$

Mathematica [A]
time = 0.24, size = 227, normalized size = 0.93

$$\frac{b\left(-\frac{(a+b)^8\log(1-\sin(c+dx))}{2b}+\frac{(a-b)^8\log(1+\sin(c+dx))}{2b}-b(28a^6+70a^4b^2+28a^2b^4+b^6)\sin(c+dx)-4ab^2(7a^4+7a^2b^2+b^4)\sin^2(c+dx)-\frac{1}{3}b^3(70a^4+28a^2b^2+b^4)\sin^3(c+dx)-2ab^4(7a^2+b^2)\sin^4(c+dx)-\frac{1}{5}b^5(28a^2+b^2)\sin^5(c+dx)-\frac{4}{3}ab^6\sin^6(c+dx)-\frac{1}{7}b^7\sin^7(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sin[c + d*x])^8,x]
```

```
[Out] (b*(-1/2*((a + b)^8*Log[1 - Sin[c + d*x]])/b + ((a - b)^8*Log[1 + Sin[c + d
*x]])/(2*b) - b*(28*a^6 + 70*a^4*b^2 + 28*a^2*b^4 + b^6)*Sin[c + d*x] - 4*a
*b^2*(7*a^4 + 7*a^2*b^2 + b^4)*Sin[c + d*x]^2 - (b^3*(70*a^4 + 28*a^2*b^2 +
 b^4)*Sin[c + d*x]^3)/3 - 2*a*b^4*(7*a^2 + b^2)*Sin[c + d*x]^4 - (b^5*(28*a
^2 + b^2)*Sin[c + d*x]^5)/5 - (4*a*b^6*Sin[c + d*x]^6)/3 - (b^7*Sin[c + d*x
]^7)/7))/d
```

Maple [A]
time = 0.54, size = 329, normalized size = 1.34 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*sin(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^8*ln(sec(d*x+c)+tan(d*x+c))-8*a^7*b*ln(cos(d*x+c))+28*a^6*b^2*(-sin(
d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+56*a^5*b^3*(-1/2*sin(d*x+c)^2-ln(cos(d*x+
c)))+70*a^4*b^4*(-1/3*sin(d*x+c)^3-sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+56
*a^3*b^5*(-1/4*sin(d*x+c)^4-1/2*sin(d*x+c)^2-ln(cos(d*x+c)))+28*a^2*b^6*(-1
/5*sin(d*x+c)^5-1/3*sin(d*x+c)^3-sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+8*a*
b^7*(-1/6*sin(d*x+c)^6-1/4*sin(d*x+c)^4-1/2*sin(d*x+c)^2-ln(cos(d*x+c)))+b^
8*(-1/7*sin(d*x+c)^7-1/5*sin(d*x+c)^5-1/3*sin(d*x+c)^3-sin(d*x+c)+ln(sec(d*
x+c)+tan(d*x+c))))
```

Maxima [A]
time = 0.29, size = 317, normalized size = 1.29

$$-\frac{30\,b^{8}\sin\left(dx+c\right)^{7}+280\,ab^{7}\sin\left(dx+c\right)^{6}+42\,\left(28\,a^{2}b^{6}+b^{8}\right)\sin\left(dx+c\right)^{5}+420\,\left(7\,a^{3}b^{5}+ab^{7}\right)\sin\left(dx+c\right)^{4}+70\,\left(70\,a^{4}b^{4}+28\,a^{2}b^{6}+b^{8}\right)\sin\left(dx+c\right)^{3}+840\,\left(7\,a^{5}b^{3}+7\,a^{3}b^{5}+ab^{7}\right)\sin\left(dx+c\right)^{2}-105\,\left(a^{8}-8\,a^{7}b+28\,a^{6}b^{2}-56\,a^{5}b^{3}+70\,a^{4}b^{4}-56\,a^{3}b^{5}+28\,a^{2}b^{6}-8\,ab^{7}+b^{8}\right)\log\left(\sin\left(dx+c\right)+1\right)+105\,\left(a^{8}+8\,a^{7}b+28\,a^{6}b^{2}+56\,a^{5}b^{3}+70\,a^{4}b^{4}+56\,a^{3}b^{5}+28\,a^{2}b^{6}+8\,ab^{7}+b^{8}\right)\log\left(\sin\left(dx+c\right)-1\right)+210\,\left(28\,a^{6}b^{2}+70\,a^{4}b^{4}+28\,a^{2}b^{6}+b^{8}\right)\sin\left(dx+c\right)}{210\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/210*(30*b^8*sin(d*x + c)^7 + 280*a*b^7*sin(d*x + c)^6 + 42*(28*a^2*b^6 +
 b^8)*sin(d*x + c)^5 + 420*(7*a^3*b^5 + a*b^7)*sin(d*x + c)^4 + 70*(70*a^4*
b^4 + 28*a^2*b^6 + b^8)*sin(d*x + c)^3 + 840*(7*a^5*b^3 + 7*a^3*b^5 + a*b^7
)*sin(d*x + c)^2 - 105*(a^8 - 8*a^7*b + 28*a^6*b^2 - 56*a^5*b^3 + 70*a^4*b^
4 - 56*a^3*b^5 + 28*a^2*b^6 - 8*a*b^7 + b^8)*log(sin(d*x + c) + 1) + 105*(a
^8 + 8*a^7*b + 28*a^6*b^2 + 56*a^5*b^3 + 70*a^4*b^4 + 56*a^3*b^5 + 28*a^2*b
^6 + 8*a*b^7 + b^8)*log(sin(d*x + c) - 1) + 210*(28*a^6*b^2 + 70*a^4*b^4 +
28*a^2*b^6 + b^8)*sin(d*x + c))/d
```

Fricas [A]
time = 0.40, size = 327, normalized size = 1.33

$$\frac{280\,ab^{7}\cos\left(dx+c\right)^{6}-420\,\left(7\,a^{3}b^{5}+3\,ab^{7}\right)\cos\left(dx+c\right)^{4}+840\,\left(7\,a^{5}b^{3}+14\,a^{3}b^{5}+3\,ab^{7}\right)\cos\left(dx+c\right)^{2}+105\,\left(a^{8}-8\,a^{7}b+28\,a^{6}b^{2}-56\,a^{5}b^{3}+70\,a^{4}b^{4}-56\,a^{3}b^{5}+28\,a^{2}b^{6}-8\,ab^{7}+b^{8}\right)\log\left(\sin\left(dx+c\right)+1\right)-105\,\left(a^{8}+8\,a^{7}b+28\,a^{6}b^{2}+56\,a^{5}b^{3}+70\,a^{4}b^{4}+56\,a^{3}b^{5}+28\,a^{2}b^{6}+8\,ab^{7}+b^{8}\right)\log\left(-\sin\left(dx+c\right)+1\right)+2\,\left(15\,b^{8}\cos\left(dx+c\right)^{6}-2940\,a^{6}b^{2}-9800\,a^{4}b^{4}-4508\,a^{2}b^{6}-176\,b^{8}-6\,\left(98\,a^{2}b^{6}+11\,b^{8}\right)\cos\left(dx+c\right)^{4}+2\,\left(1225\,a^{4}b^{4}+1078\,a^{2}b^{6}+61\,b^{8}\right)\cos\left(dx+c\right)^{2}\right)\sin\left(dx+c\right)}{210\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/210*(280*a*b^7*cos(d*x + c)^6 - 420*(7*a^3*b^5 + 3*a*b^7)*cos(d*x + c)^4
+ 840*(7*a^5*b^3 + 14*a^3*b^5 + 3*a*b^7)*cos(d*x + c)^2 + 105*(a^8 - 8*a^7*
b + 28*a^6*b^2 - 56*a^5*b^3 + 70*a^4*b^4 - 56*a^3*b^5 + 28*a^2*b^6 - 8*a*b^
7 + b^8)*log(sin(d*x + c) + 1) - 105*(a^8 + 8*a^7*b + 28*a^6*b^2 + 56*a^5*b
^3 + 70*a^4*b^4 + 56*a^3*b^5 + 28*a^2*b^6 + 8*a*b^7 + b^8)*log(-sin(d*x + c
) + 1) + 2*(15*b^8*cos(d*x + c)^6 - 2940*a^6*b^2 - 9800*a^4*b^4 - 4508*a^2*
b^6 - 176*b^8 - 6*(98*a^2*b^6 + 11*b^8)*cos(d*x + c)^4 + 2*(1225*a^4*b^4 +
1078*a^2*b^6 + 61*b^8)*cos(d*x + c)^2)*sin(d*x + c))/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))**8,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [A]
time = 4.56, size = 378, normalized size = 1.54

$$-\frac{30\,b^8\sin\left(dx+c\right)^7+280\,ab^7\sin\left(dx+c\right)^6+1176\,a^2b^6\sin\left(dx+c\right)^5+42\,b^8\sin\left(dx+c\right)^5+2940\,a^3b^5\sin\left(dx+c\right)^4+420\,ab^7\sin\left(dx+c\right)^4+4900\,a^4b^4\sin\left(dx+c\right)^3+1960\,a^2b^6\sin\left(dx+c\right)^3+70\,b^8\sin\left(dx+c\right)^3+5880\,a^5b^3\sin\left(dx+c\right)^2+5880\,a^3b^5\sin\left(dx+c\right)^2+840\,ab^7\sin\left(dx+c\right)^2+5880\,a^6b^2\sin\left(dx+c\right)+14700\,a^4b^4\sin\left(dx+c\right)+5880\,a^2b^6\sin\left(dx+c\right)+210\,b^8\sin\left(dx+c\right)-105\,\left(a^8-8\,a^7b+28\,a^6b^2-56\,a^5b^3+70\,a^4b^4-56\,a^3b^5+28\,a^2b^6-8\,ab^7+b^8\right)\log\left(\left|\sin\left(dx+c\right)+1\right|\right)+105\,\left(a^8+8\,a^7b+28\,a^6b^2+56\,a^5b^3+70\,a^4b^4+56\,a^3b^5+28\,a^2b^6+8\,ab^7+b^8\right)\log\left(\left|\sin\left(dx+c\right)-1\right|\right)}{210\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -1/210*(30*b^8*sin(d*x + c)^7 + 280*a*b^7*sin(d*x + c)^6 + 1176*a^2*b^6*sin
(d*x + c)^5 + 42*b^8*sin(d*x + c)^5 + 2940*a^3*b^5*sin(d*x + c)^4 + 420*a*b
^7*sin(d*x + c)^4 + 4900*a^4*b^4*sin(d*x + c)^3 + 1960*a^2*b^6*sin(d*x + c)
^3 + 70*b^8*sin(d*x + c)^3 + 5880*a^5*b^3*sin(d*x + c)^2 + 5880*a^3*b^5*sin
(d*x + c)^2 + 840*a*b^7*sin(d*x + c)^2 + 5880*a^6*b^2*sin(d*x + c) + 14700*
a^4*b^4*sin(d*x + c) + 5880*a^2*b^6*sin(d*x + c) + 210*b^8*sin(d*x + c) - 1
05*(a^8 - 8*a^7*b + 28*a^6*b^2 - 56*a^5*b^3 + 70*a^4*b^4 - 56*a^3*b^5 + 28*
a^2*b^6 - 8*a*b^7 + b^8)*log(abs(sin(d*x + c) + 1)) + 105*(a^8 + 8*a^7*b +
28*a^6*b^2 + 56*a^5*b^3 + 70*a^4*b^4 + 56*a^3*b^5 + 28*a^2*b^6 + 8*a*b^7 +
b^8)*log(abs(sin(d*x + c) - 1)))/d
```

Mupad [B]
time = 5.36, size = 212, normalized size = 0.87

$$-\frac{\frac{\ln\left(\sin\left(c+d\,x\right)-1\right)\,{\left(a+b\right)}^8}{2}+{\sin\left(c+d\,x\right)}^3\,\left(\frac{70\,a^4\,b^4}{3}+\frac{28\,a^2\,b^6}{3}+\frac{b^8}{3}\right)-\frac{\ln\left(\sin\left(c+d\,x\right)+1\right)\,{\left(a-b\right)}^8}{2}+{\sin\left(c+d\,x\right)}^5\,\left(\frac{28\,a^2\,b^6}{5}+\frac{b^8}{5}\right)+\sin\left(c+d\,x\right)\,\left(28\,a^6\,b^2+70\,a^4\,b^4+28\,a^2\,b^6+b^8\right)+{\sin\left(c+d\,x\right)}^2\,\left(28\,a^5\,b^3+28\,a^3\,b^5+4\,a\,b^7\right)+\frac{b^8\,{\sin\left(c+d\,x\right)}^7}{7}+{\sin\left(c+d\,x\right)}^4\,\left(14\,a^3\,b^5+2\,a\,b^7\right)+\frac{4\,a\,b^7\,{\sin\left(c+d\,x\right)}^6}{3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^8/cos(c + d*x),x)
```

```
[Out] -((log(sin(c + d*x) - 1)*(a + b)^8)/2 + sin(c + d*x)^3*(b^8/3 + (28*a^2*b^6
)/3 + (70*a^4*b^4)/3) - (log(sin(c + d*x) + 1)*(a - b)^8)/2 + sin(c + d*x)^
5*(b^8/5 + (28*a^2*b^6)/5) + sin(c + d*x)*(b^8 + 28*a^2*b^6 + 70*a^4*b^4 +
28*a^6*b^2) + sin(c + d*x)^2*(4*a*b^7 + 28*a^3*b^5 + 28*a^5*b^3) + (b^8*sin
(c + d*x)^7)/7 + sin(c + d*x)^4*(2*a*b^7 + 14*a^3*b^5) + (4*a*b^7*sin(c + d
*x)^6)/3)/d
```

3.417 $\int \sec^3(c+dx)(a+b\sin(c+dx))^8\,dx$

Optimal. Leaf size=284

$$-\frac{(a-7b)(a+b)^7\log(1-\sin(c+dx))}{4d}+\frac{(a-b)^7(a+7b)\log(1+\sin(c+dx))}{4d}+\frac{7b^2(3a^6+30a^4b^2+20a^2b^4}{2d}$$

```
[Out] -1/4*(a-7*b)*(a+b)^7*ln(1-sin(d*x+c))/d+1/4*(a-b)^7*(a+7*b)*ln(1+sin(d*x+c)
)/d+7/2*b^2*(3*a^6+30*a^4*b^2+20*a^2*b^4+b^6)*sin(d*x+c)/d+1/2*a*b^3*(35*a^
4+112*a^2*b^2+24*b^4)*sin(d*x+c)^2/d+7/6*b^4*(15*a^4+20*a^2*b^2+b^4)*sin(d*
x+c)^3/d+3/2*a*b^5*(7*a^2+4*b^2)*sin(d*x+c)^4/d+7/10*b^6*(5*a^2+b^2)*sin(d*
x+c)^5/d+1/2*a*b^7*sin(d*x+c)^6/d+1/2*sec(d*x+c)^2*(b+a*sin(d*x+c))*(a+b*si
n(d*x+c))^7/d
```

Rubi [A]
time = 0.17, antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2747, 753, 815, 647, 31}

$$\frac{7b^6(5a^2+b^2)\sin^5(c+dx)}{10d}+\frac{3ab^5(7a^2+4b^2)\sin^4(c+dx)}{2d}+\frac{7b^4(15a^4+20a^2b^2+b^4)\sin^3(c+dx)}{6d}+\frac{ab^3(35a^4+112a^2b^2+24b^4)\sin^2(c+dx)}{2d}+\frac{7b^2(3a^6+30a^4b^2+20a^2b^4+b^6)\sin(c+dx)}{2d}+\frac{ab^7\sin^6(c+dx)}{2d}+\frac{(a+7b)(a-b)^7\log(\sin(c+dx)+1)}{4d}-\frac{(a-7b)(a+b)^7\log(1-\sin(c+dx))}{4d}+\frac{\sec^2(c+dx)(a\sin(c+dx)+b)(a+b\sin(c+dx))^7}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + b*Sin[c + d*x])^8,x]
```

```
[Out] -1/4*((a - 7*b)*(a + b)^7*Log[1 - Sin[c + d*x]])/d + ((a - b)^7*(a + 7*b)*L
og[1 + Sin[c + d*x]])/(4*d) + (7*b^2*(3*a^6 + 30*a^4*b^2 + 20*a^2*b^4 + b^6
)*Sin[c + d*x])/(2*d) + (a*b^3*(35*a^4 + 112*a^2*b^2 + 24*b^4)*Sin[c + d*x]
^2)/(2*d) + (7*b^4*(15*a^4 + 20*a^2*b^2 + b^4)*Sin[c + d*x]^3)/(6*d) + (3*a
*b^5*(7*a^2 + 4*b^2)*Sin[c + d*x]^4)/(2*d) + (7*b^6*(5*a^2 + b^2)*Sin[c + d
*x]^5)/(10*d) + (a*b^7*Sin[c + d*x]^6)/(2*d) + (Sec[c + d*x]^2*(b + a*Sin[c
 + d*x])*(a + b*Sin[c + d*x])^7)/(2*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
(-a)*c]
```

Rule 753

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m - 1)*(a*e - c*d*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] +
```

```
Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^3(c+dx)(a+b\sin(c+dx))^8\,dx = \frac{b^3\text{Subst}\left(\int \frac{(a+x)^8}{(b^2-x^2)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^2(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^7}{2d} - \frac{b\text{Subst}\left(\int \frac{(a+x}{}\right.}{}$$

$$= \frac{\sec^2(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^7}{2d} - \frac{b\text{Subst}\left(\int \left(-\right.\right.}{}$$

$$= \frac{7b^2(3a^6+30a^4b^2+20a^2b^4+b^6)\sin(c+dx)}{2d} + \frac{ab^3(35a^4+112a^2b^2+}{2d}$$

$$= \frac{7b^2(3a^6+30a^4b^2+20a^2b^4+b^6)\sin(c+dx)}{2d} + \frac{ab^3(35a^4+112a^2b^2+}{2d}$$

$$= -\frac{(a-7b)(a+b)^7\log(1-\sin(c+dx))}{4d} + \frac{(a-b)^7(a+7b)\log(1+\text{si}}{4d}$$

Mathematica [A]
time = 2.38, size = 366, normalized size = 1.29

$$\frac{\frac{1}{2}b(a^2-b^2)\left((a-7b)(a+b)^7\log(1-\sin(c+dx))-(a-b)^7(a+7b)\log(1+\sin(c+dx))\right)+b^3(-36a^8-182a^6b^2+70a^4b^4+133a^2b^6+7b^8)\sin(c+dx)-4ab^4(21a^6+14a^4b^2-22a^2b^4-6b^6)\sin^2(c+dx)+\frac{7}{3}b^5(-54a^6+10a^4b^2+19a^2b^4+b^6)\sin^3(c+dx)-2ab^6(63a^4-22a^2b^2-6b^4)\sin^4(c+dx)+\frac{7}{5}b^7(-60a^4+19a^2b^2+b^4)\sin^5(c+dx)-4ab^8(9a^2-2b^2)\sin^6(c+dx)+b^9(-9a^2+b^2)\sin^7(c+dx)-ab^{10}\sin^8(c+dx)+b\sec^2(c+dx)(b-a\sin(c+dx))(a+b\sin(c+dx))^9}{2b(-a^2+b^2)d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Sin[c + d*x])^8,x]
```

```
[Out] ((b*(a^2 - b^2)*((a - 7*b)*(a + b)^7*Log[1 - Sin[c + d*x]] - (a - b)^7*(a +
 7*b)*Log[1 + Sin[c + d*x]]))/2 + b^3*(-36*a^8 - 182*a^6*b^2 + 70*a^4*b^4 +
 133*a^2*b^6 + 7*b^8)*Sin[c + d*x] - 4*a*b^4*(21*a^6 + 14*a^4*b^2 - 22*a^2*
b^4 - 6*b^6)*Sin[c + d*x]^2 + (7*b^5*(-54*a^6 + 10*a^4*b^2 + 19*a^2*b^4 + b
^6)*Sin[c + d*x]^3)/3 - 2*a*b^6*(63*a^4 - 22*a^2*b^2 - 6*b^4)*Sin[c + d*x]^
4 + (7*b^7*(-60*a^4 + 19*a^2*b^2 + b^4)*Sin[c + d*x]^5)/5 - 4*a*b^8*(9*a^2
- 2*b^2)*Sin[c + d*x]^6 + b^9*(-9*a^2 + b^2)*Sin[c + d*x]^7 - a*b^10*Sin[c
+ d*x]^8 + b*Sec[c + d*x]^2*(b - a*Sin[c + d*x])*(a + b*Sin[c + d*x])^9)/(2
*b*(-a^2 + b^2)*d)
```

Maple [A]
time = 0.51, size = 459, normalized size = 1.62 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+b*sin(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^8*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+4*a^7*b/
cos(d*x+c)^2+28*a^6*b^2*(1/2*sin(d*x+c)^3/cos(d*x+c)^2+1/2*sin(d*x+c)-1/2*l
n(sec(d*x+c)+tan(d*x+c)))+56*a^5*b^3*(1/2*tan(d*x+c)^2+ln(cos(d*x+c)))+70*a
^4*b^4*(1/2*sin(d*x+c)^5/cos(d*x+c)^2+1/2*sin(d*x+c)^3+3/2*sin(d*x+c)-3/2*l
n(sec(d*x+c)+tan(d*x+c)))+56*a^3*b^5*(1/2*sin(d*x+c)^6/cos(d*x+c)^2+1/2*sin
(d*x+c)^4+sin(d*x+c)^2+2*ln(cos(d*x+c)))+28*a^2*b^6*(1/2*sin(d*x+c)^7/cos(d
*x+c)^2+1/2*sin(d*x+c)^5+5/6*sin(d*x+c)^3+5/2*sin(d*x+c)-5/2*ln(sec(d*x+c)+
tan(d*x+c)))+8*a*b^7*(1/2*sin(d*x+c)^8/cos(d*x+c)^2+1/2*sin(d*x+c)^6+3/4*si
n(d*x+c)^4+3/2*sin(d*x+c)^2+3*ln(cos(d*x+c)))+b^8*(1/2*sin(d*x+c)^9/cos(d*x
+c)^2+1/2*sin(d*x+c)^7+7/10*sin(d*x+c)^5+7/6*sin(d*x+c)^3+7/2*sin(d*x+c)-7/
2*ln(sec(d*x+c)+tan(d*x+c))))
```

Maxima [A]
time = 0.28, size = 323, normalized size = 1.14

$$\frac{12\,b^8\sin\left(dx+c\right)^5+120\,ab^7\sin\left(dx+c\right)^4+40\left(14\,a^2b^6+b^8\right)\sin\left(dx+c\right)^3+240\left(7\,a^3b^5+2\,ab^7\right)\sin\left(dx+c\right)^2+15\left(a^8-28\,a^6b^2+112\,a^5b^3-210\,a^4b^4+224\,a^3b^5-140\,a^2b^6+48\,ab^7-7\,b^8\right)\log\left(\sin\left(dx+c\right)+1\right)-15\left(a^8-28\,a^6b^2-112\,a^5b^3-210\,a^4b^4-224\,a^3b^5-140\,a^2b^6-48\,ab^7-7\,b^8\right)\log\left(\sin\left(dx+c\right)-1\right)+60\left(70\,a^4b^4+56\,a^2b^6+3\,b^8\right)\sin\left(dx+c\right)-\frac{30\left(8\,a^7b+56\,a^5b^3+56\,a^3b^5+8\,ab^7+\left(a^8+28\,a^6b^2+70\,a^4b^4+28\,a^2b^6+b^8\right)\sin\left(dx+c\right)\right)}{\sin\left(dx+c\right)^2-1}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 1/60*(12*b^8*sin(d*x + c)^5 + 120*a*b^7*sin(d*x + c)^4 + 40*(14*a^2*b^6 + b
^8)*sin(d*x + c)^3 + 240*(7*a^3*b^5 + 2*a*b^7)*sin(d*x + c)^2 + 15*(a^8 - 2
8*a^6*b^2 + 112*a^5*b^3 - 210*a^4*b^4 + 224*a^3*b^5 - 140*a^2*b^6 + 48*a*b^
7 - 7*b^8)*log(sin(d*x + c) + 1) - 15*(a^8 - 28*a^6*b^2 - 112*a^5*b^3 - 210
*a^4*b^4 - 224*a^3*b^5 - 140*a^2*b^6 - 48*a*b^7 - 7*b^8)*log(sin(d*x + c) -
 1) + 60*(70*a^4*b^4 + 56*a^2*b^6 + 3*b^8)*sin(d*x + c) - 30*(8*a^7*b + 56*
a^5*b^3 + 56*a^3*b^5 + 8*a*b^7 + (a^8 + 28*a^6*b^2 + 70*a^4*b^4 + 28*a^2*b^
6 + b^8)*sin(d*x + c))/(sin(d*x + c)^2 - 1))/d
```

Fricas [A]
time = 0.41, size = 368, normalized size = 1.30

$$\frac{120\,ab^{7}\cos\left(dx+c\right)^{6}+240\,a^{7}b+1680\,a^{5}b^{3}+1680\,a^{3}b^{5}+240\,ab^{7}-240\,\left(7\,a^{3}b^{5}+3\,ab^{7}\right)\cos\left(dx+c\right)^{4}+15\,\left(a^{8}-28\,a^{6}b^{2}+112\,a^{5}b^{3}-210\,a^{4}b^{4}+224\,a^{3}b^{5}-140\,a^{2}b^{6}+48\,ab^{7}-7\,b^{8}\right)\cos\left(dx+c\right)^{2}\log\left(\sin\left(dx+c\right)+1\right)-15\,\left(a^{8}-28\,a^{6}b^{2}-112\,a^{5}b^{3}-210\,a^{4}b^{4}-224\,a^{3}b^{5}-140\,a^{2}b^{6}-48\,ab^{7}-7\,b^{8}\right)\cos\left(dx+c\right)^{2}\log\left(-\sin\left(dx+c\right)+1\right)+105\,\left(8\,a^{3}b^{5}+3\,ab^{7}\right)\cos\left(dx+c\right)^{2}+2\,\left(6\,b^{8}\cos\left(dx+c\right)^{6}+15\,a^{8}+420\,a^{6}b^{2}+1050\,a^{4}b^{4}+420\,a^{2}b^{6}+15\,b^{8}-8\,\left(35\,a^{2}b^{6}+4\,b^{8}\right)\cos\left(dx+c\right)^{4}+4\,\left(525\,a^{4}b^{4}+490\,a^{2}b^{6}+29\,b^{8}\right)\cos\left(dx+c\right)^{2}\right)\sin\left(dx+c\right)}{60\,d\cos\left(dx+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/60*(120*a*b^7*cos(d*x + c)^6 + 240*a^7*b + 1680*a^5*b^3 + 1680*a^3*b^5 +
240*a*b^7 - 240*(7*a^3*b^5 + 3*a*b^7)*cos(d*x + c)^4 + 15*(a^8 - 28*a^6*b^2
 + 112*a^5*b^3 - 210*a^4*b^4 + 224*a^3*b^5 - 140*a^2*b^6 + 48*a*b^7 - 7*b^8
)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - 15*(a^8 - 28*a^6*b^2 - 112*a^5*b^3
 - 210*a^4*b^4 - 224*a^3*b^5 - 140*a^2*b^6 - 48*a*b^7 - 7*b^8)*cos(d*x + c)
^2*log(-sin(d*x + c) + 1) + 105*(8*a^3*b^5 + 3*a*b^7)*cos(d*x + c)^2 + 2*(6
*b^8*cos(d*x + c)^6 + 15*a^8 + 420*a^6*b^2 + 1050*a^4*b^4 + 420*a^2*b^6 + 1
5*b^8 - 8*(35*a^2*b^6 + 4*b^8)*cos(d*x + c)^4 + 4*(525*a^4*b^4 + 490*a^2*b^
6 + 29*b^8)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+b*sin(d*x+c))**8,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

Giac [A]
time = 6.62, size = 408, normalized size = 1.44

$$\frac{12\,b^{8}\sin\left(dx+c\right)^{5}+120\,ab^{7}\sin\left(dx+c\right)^{4}+560\,a^{2}b^{6}\sin\left(dx+c\right)^{3}+40\,b^{8}\sin\left(dx+c\right)^{3}+1680\,a^{3}b^{5}\sin\left(dx+c\right)^{2}+480\,ab^{7}\sin\left(dx+c\right)^{2}+4200\,a^{4}b^{4}\sin\left(dx+c\right)+3360\,a^{2}b^{6}\sin\left(dx+c\right)+180\,b^{8}\sin\left(dx+c\right)+15\,\left(a^{8}-28\,a^{6}b^{2}+112\,a^{5}b^{3}-210\,a^{4}b^{4}+224\,a^{3}b^{5}-140\,a^{2}b^{6}+48\,ab^{7}-7\,b^{8}\right)\log\left(\left|\sin\left(dx+c\right)+1\right|\right)-15\,\left(a^{8}-28\,a^{6}b^{2}-112\,a^{5}b^{3}-210\,a^{4}b^{4}-224\,a^{3}b^{5}-140\,a^{2}b^{6}-48\,ab^{7}-7\,b^{8}\right)\log\left(\left|\sin\left(dx+c\right)-1\right|\right)-\frac{30\,\left(56\,a^{5}b^{3}\sin(dx+c)^{2}+112\,a^{3}b^{5}\sin(dx+c)^{2}+24\,ab^{7}\sin(dx+c)^{2}+a^{8}\sin(dx+c)+28\,a^{6}b^{2}\sin(dx+c)+70\,a^{4}b^{4}\sin(dx+c)+28\,a^{2}b^{6}\sin(dx+c)+b^{8}\sin(dx+c)+8\,a^{7}b-56\,a^{3}b^{5}-16\,ab^{7}\right)}{\sin(dx+c)^{2}-1}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/60*(12*b^8*sin(d*x + c)^5 + 120*a*b^7*sin(d*x + c)^4 + 560*a^2*b^6*sin(d*
x + c)^3 + 40*b^8*sin(d*x + c)^3 + 1680*a^3*b^5*sin(d*x + c)^2 + 480*a*b^7*
sin(d*x + c)^2 + 4200*a^4*b^4*sin(d*x + c) + 3360*a^2*b^6*sin(d*x + c) + 18
0*b^8*sin(d*x + c) + 15*(a^8 - 28*a^6*b^2 + 112*a^5*b^3 - 210*a^4*b^4 + 224
*a^3*b^5 - 140*a^2*b^6 + 48*a*b^7 - 7*b^8)*log(abs(sin(d*x + c) + 1)) - 15*
(a^8 - 28*a^6*b^2 - 112*a^5*b^3 - 210*a^4*b^4 - 224*a^3*b^5 - 140*a^2*b^6 -
 48*a*b^7 - 7*b^8)*log(abs(sin(d*x + c) - 1)) - 30*(56*a^5*b^3*sin(d*x + c)
^2 + 112*a^3*b^5*sin(d*x + c)^2 + 24*a*b^7*sin(d*x + c)^2 + a^8*sin(d*x + c
) + 28*a^6*b^2*sin(d*x + c) + 70*a^4*b^4*sin(d*x + c) + 28*a^2*b^6*sin(d*x
```

```
+ c) + b^8*sin(d*x + c) + 8*a^7*b - 56*a^3*b^5 - 16*a*b^7)/(sin(d*x + c)^2
- 1))/d
```

Mupad [B]
time = 5.39, size = 257, normalized size = 0.90

$$\frac{\sin\left(c+d\,x\right)^3\,\left(\frac{28\,a^2\,b^6}{3}+\frac{2\,b^8}{3}\right)}{d}+\frac{b^8\,\sin\left(c+d\,x\right)^5}{5\,d}+\frac{\sin\left(c+d\,x\right)^2\,\left(28\,a^3\,b^5+8\,a\,b^7\right)}{d}+\frac{\sin\left(c+d\,x\right)\,\left(70\,a^4\,b^4+56\,a^2\,b^6+3\,b^8\right)}{d}-\frac{\sin\left(c+d\,x\right)\,\left(\frac{a^8}{2}+14\,a^6\,b^2+35\,a^4\,b^4+14\,a^2\,b^6+\frac{b^8}{2}\right)+4\,a\,b^7+4\,a^7\,b+28\,a^3\,b^5+28\,a^5\,b^3}{d\,\left(\sin\left(c+d\,x\right)^2-1\right)}+\frac{2\,a\,b^7\,\sin\left(c+d\,x\right)^4}{d}-\frac{\ln\left(\sin\left(c+d\,x\right)-1\right)\,{\left(a+b\right)}^7\,\left(a-7\,b\right)}{4\,d}+\frac{\ln\left(\sin\left(c+d\,x\right)+1\right)\,{\left(a-b\right)}^7\,\left(a+7\,b\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^8/cos(c + d*x)^3,x)
```

```
[Out] (sin(c + d*x)^3*((2*b^8)/3 + (28*a^2*b^6)/3))/d + (b^8*sin(c + d*x)^5)/(5*d
) + (sin(c + d*x)^2*(8*a*b^7 + 28*a^3*b^5))/d + (sin(c + d*x)*(3*b^8 + 56*a
^2*b^6 + 70*a^4*b^4))/d - (sin(c + d*x)*(a^8/2 + b^8/2 + 14*a^2*b^6 + 35*a^
4*b^4 + 14*a^6*b^2) + 4*a*b^7 + 4*a^7*b + 28*a^3*b^5 + 28*a^5*b^3)/(d*(sin(
c + d*x)^2 - 1)) + (2*a*b^7*sin(c + d*x)^4)/d - (log(sin(c + d*x) - 1)*(a +
 b)^7*(a - 7*b))/(4*d) + (log(sin(c + d*x) + 1)*(a - b)^7*(a + 7*b))/(4*d)
```

3.418 $\int \sec^5(c+dx)(a+b\sin(c+dx))^8\,dx$

Optimal. Leaf size=320

$$-\frac{(a+b)^6\left(3a^2-18ab+35b^2\right)\log(1-\sin(c+dx))}{16d}+\frac{(a-b)^6\left(3a^2+18ab+35b^2\right)\log(1+\sin(c+dx))}{16d}+\frac{5b^2(\cdots}{}$$

```
[Out] -1/16*(a+b)^6*(3*a^2-18*a*b+35*b^2)*ln(1-sin(d*x+c))/d+1/16*(a-b)^6*(3*a^2+
18*a*b+35*b^2)*ln(1+sin(d*x+c))/d+5/8*b^2*(6*a^6-35*a^4*b^2-84*a^2*b^4-7*b^
6)*sin(d*x+c)/d+1/4*a*b^3*(15*a^4-77*a^2*b^2-48*b^4)*sin(d*x+c)^2/d+5/24*b^
4*(9*a^4-42*a^2*b^2-7*b^4)*sin(d*x+c)^3/d-1/8*a*(13-3*a^2/b^2)*b^7*sin(d*x+
c)^4/d+1/4*sec(d*x+c)^4*(b+a*sin(d*x+c))*(a+b*sin(d*x+c))^7/d-1/8*sec(d*x+c
)^2*(a+b*sin(d*x+c))^5*(b*(a^2+7*b^2)-a*(3*a^2-11*b^2)*sin(d*x+c))/d
```

Rubi [A]
time = 0.21, antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$,
Rules used = {2747, 753, 833, 815, 647, 31}

$$-\frac{(a+b)^6\left(3a^2-18ab+35b^2\right)\log(1-\sin(c+dx))}{16d}+\frac{(a-b)^6\left(3a^2+18ab+35b^2\right)\log(\sin(c+dx)+1)}{16d}-\frac{\sec^2(c+dx)(a+b\sin(c+dx))^5\left(b(a^2+7b^2)-a(3a^2-11b^2)\sin(c+dx)\right)}{8d}-\frac{ab^7\left(13-\frac{3a^2}{b^2}\right)\sin^4(c+dx)}{8d}+\frac{5b^4(9a^4-42a^2b^2-7b^4)\sin^3(c+dx)}{24d}+\frac{ab^3(15a^4-77a^2b^2-48b^4)\sin^2(c+dx)}{4d}+\frac{5b^2(6a^6-35a^4b^2-84a^2b^4-7b^6)\sin(c+dx)}{8d}+\frac{\sec^4(c+dx)(a\sin(c+dx)+b)(a+b\sin(c+dx))^7}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*(a + b*Sin[c + d*x])^8,x]
```

```
[Out] -1/16*((a + b)^6*(3*a^2 - 18*a*b + 35*b^2)*Log[1 - Sin[c + d*x]])/d + ((a -
 b)^6*(3*a^2 + 18*a*b + 35*b^2)*Log[1 + Sin[c + d*x]])/(16*d) + (5*b^2*(6*a
^6 - 35*a^4*b^2 - 84*a^2*b^4 - 7*b^6)*Sin[c + d*x])/(8*d) + (a*b^3*(15*a^4
- 77*a^2*b^2 - 48*b^4)*Sin[c + d*x]^2)/(4*d) + (5*b^4*(9*a^4 - 42*a^2*b^2 -
 7*b^4)*Sin[c + d*x]^3)/(24*d) - (a*(13 - (3*a^2)/b^2)*b^7*Sin[c + d*x]^4)/
(8*d) + (Sec[c + d*x]^4*(b + a*Sin[c + d*x])*(a + b*Sin[c + d*x])^7)/(4*d)
- (Sec[c + d*x]^2*(a + b*Sin[c + d*x])^5*(b*(a^2 + 7*b^2) - a*(3*a^2 - 11*b
^2)*Sin[c + d*x]))/(8*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
(-a)*c]
```

Rule 753

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m - 1)*(a*e - c*d*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^5(c+dx)(a+b\sin(c+dx))^8\,dx = \frac{b^5\text{Subst}\left(\int \frac{(a+x)^8}{(b^2-x^2)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^4(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^7}{4d} - \frac{b^3\text{Subst}\left(\int \frac{(a+}{}\right.}{}$$

$$= \frac{\sec^4(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^7}{4d} - \frac{\sec^2(c+dx)(}{}$$

$$= \frac{\sec^4(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^7}{4d} - \frac{\sec^2(c+dx)(}{}$$

$$= \frac{5b^2(6a^6-35a^4b^2-84a^2b^4-7b^6)\sin(c+dx)}{8d} + \frac{ab^3(15a^4-77a^2b^2-}{4}$$

$$= \frac{5b^2(6a^6-35a^4b^2-84a^2b^4-7b^6)\sin(c+dx)}{8d} + \frac{ab^3(15a^4-77a^2b^2-}{4}$$

$$= -\frac{(a+b)^6(3a^2-18ab+35b^2)\log(1-\sin(c+dx))}{16d} + \frac{(a-b)^6(3a^2}{}$$

Mathematica [A]
time = 4.09, size = 514, normalized size = 1.61

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*(a + b*Sin[c + d*x])^8,x]
```

```
[Out] -1/48*(3*(a^2 - b^2)^2*((a + b)^6*(3*a^2 - 18*a*b + 35*b^2)*Log[1 - Sin[c +
 d*x]] - (a - b)^6*(3*a^2 + 18*a*b + 35*b^2)*Log[1 + Sin[c + d*x]]) + 6*b^2
*(-108*a^10 + 234*a^8*b^2 - 28*a^6*b^4 - 595*a^4*b^6 + 350*a^2*b^8 + 35*b^1
0)*Sin[c + d*x] - 24*a*b^3*(63*a^8 - 21*a^6*b^2 + 88*a^4*b^4 - 8*a^2*b^6 -
24*b^8)*Sin[c + d*x]^2 + 14*b^4*(-162*a^8 - 144*a^6*b^2 - 85*a^4*b^4 + 50*a
^2*b^6 + 5*b^8)*Sin[c + d*x]^3 - 12*a*b^5*(189*a^6 + 333*a^4*b^2 - 8*a^2*b^
4 - 24*b^6)*Sin[c + d*x]^4 + 42*b^6*(-36*a^6 - 87*a^4*b^2 + 10*a^2*b^4 + b^
6)*Sin[c + d*x]^5 - 24*a*b^7*(27*a^4 + 79*a^2*b^2 - 8*b^4)*Sin[c + d*x]^6 +
 6*b^8*(-27*a^4 - 90*a^2*b^2 + 5*b^4)*Sin[c + d*x]^7 - 6*a*b^9*(3*a^2 + 11*
b^2)*Sin[c + d*x]^8 + 12*(a^2 - b^2)*Sec[c + d*x]^4*(b - a*Sin[c + d*x])*(a
 + b*Sin[c + d*x])^9 + 6*Sec[c + d*x]^2*(a + b*Sin[c + d*x])^9*(9*a^2*b + 5
*b^3 - a*(3*a^2 + 11*b^2)*Sin[c + d*x]))/((a^2 - b^2)^2*d)
```

Maple [A]
time = 0.60, size = 544, normalized size = 1.70 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(a+b*sin(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^8*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+
tan(d*x+c)))+2*a^7*b/cos(d*x+c)^4+28*a^6*b^2*(1/4*sin(d*x+c)^3/cos(d*x+c)^4
+1/8*sin(d*x+c)^3/cos(d*x+c)^2+1/8*sin(d*x+c)-1/8*ln(sec(d*x+c)+tan(d*x+c))
)+14*a^5*b^3*sin(d*x+c)^4/cos(d*x+c)^4+70*a^4*b^4*(1/4*sin(d*x+c)^5/cos(d*x
+c)^4-1/8*sin(d*x+c)^5/cos(d*x+c)^2-1/8*sin(d*x+c)^3-3/8*sin(d*x+c)+3/8*ln(
sec(d*x+c)+tan(d*x+c)))+56*a^3*b^5*(1/4*tan(d*x+c)^4-1/2*tan(d*x+c)^2-ln(co
s(d*x+c)))+28*a^2*b^6*(1/4*sin(d*x+c)^7/cos(d*x+c)^4-3/8*sin(d*x+c)^7/cos(d
*x+c)^2-3/8*sin(d*x+c)^5-5/8*sin(d*x+c)^3-15/8*sin(d*x+c)+15/8*ln(sec(d*x+c
)+tan(d*x+c)))+8*a*b^7*(1/4*sin(d*x+c)^8/cos(d*x+c)^4-1/2*sin(d*x+c)^8/cos(
d*x+c)^2-1/2*sin(d*x+c)^6-3/4*sin(d*x+c)^4-3/2*sin(d*x+c)^2-3*ln(cos(d*x+c)
))+b^8*(1/4*sin(d*x+c)^9/cos(d*x+c)^4-5/8*sin(d*x+c)^9/cos(d*x+c)^2-5/8*sin
(d*x+c)^7-7/8*sin(d*x+c)^5-35/24*sin(d*x+c)^3-35/8*sin(d*x+c)+35/8*ln(sec(d
*x+c)+tan(d*x+c))))
```

Maxima [A]
time = 0.28, size = 348, normalized size = 1.09

$$-\frac{16\,b^{8}\sin\left(dx+c\right)^{3}+192\,ab^{7}\sin\left(dx+c\right)^{2}-3\left(3\,a^{8}-28\,a^{6}b^{2}+210\,a^{4}b^{4}-448\,a^{3}b^{5}+420\,a^{2}b^{6}-192\,ab^{7}+35\,b^{8}\right)\log\left(\sin\left(dx+c\right)+1\right)+3\left(3\,a^{8}-28\,a^{6}b^{2}+210\,a^{4}b^{4}+448\,a^{3}b^{5}+420\,a^{2}b^{6}+192\,ab^{7}+35\,b^{8}\right)\log\left(\sin\left(dx+c\right)-1\right)+48\left(28\,a^{2}b^{6}+3\,b^{8}\right)\sin\left(dx+c\right)-\frac{6\left(16\,a^{7}b-112\,a^{5}b^{3}-336\,a^{3}b^{5}-80\,ab^{7}-\left(3\,a^{8}-28\,a^{6}b^{2}-350\,a^{4}b^{4}-252\,a^{2}b^{6}-13\,b^{8}\right)\sin(dx+c)^{3}+32\left(7\,a^{5}b^{3}+14\,a^{3}b^{5}+3\,ab^{7}\right)\sin(dx+c)^{2}+\left(5\,a^{8}+28\,a^{6}b^{2}-210\,a^{4}b^{4}-196\,a^{2}b^{6}-11\,b^{8}\right)\sin(dx+c)\right)}{\sin(dx+c)^{4}-2\,\sin(dx+c)^{2}+1}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/48*(16*b^8*sin(d*x + c)^3 + 192*a*b^7*sin(d*x + c)^2 - 3*(3*a^8 - 28*a^6
*b^2 + 210*a^4*b^4 - 448*a^3*b^5 + 420*a^2*b^6 - 192*a*b^7 + 35*b^8)*log(si
n(d*x + c) + 1) + 3*(3*a^8 - 28*a^6*b^2 + 210*a^4*b^4 + 448*a^3*b^5 + 420*a
^2*b^6 + 192*a*b^7 + 35*b^8)*log(sin(d*x + c) - 1) + 48*(28*a^2*b^6 + 3*b^8
)*sin(d*x + c) - 6*(16*a^7*b - 112*a^5*b^3 - 336*a^3*b^5 - 80*a*b^7 - (3*a^
8 - 28*a^6*b^2 - 350*a^4*b^4 - 252*a^2*b^6 - 13*b^8)*sin(d*x + c)^3 + 32*(7
*a^5*b^3 + 14*a^3*b^5 + 3*a*b^7)*sin(d*x + c)^2 + (5*a^8 + 28*a^6*b^2 - 210
*a^4*b^4 - 196*a^2*b^6 - 11*b^8)*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x
+ c)^2 + 1))/d
```

Fricas [A]
time = 0.40, size = 366, normalized size = 1.14

$$\frac{192\,ab^{7}\cos\left(dx+c\right)^{6}-96\,ab^{7}\cos\left(dx+c\right)^{4}+96\,a^{7}b+672\,a^{5}b^{3}+672\,a^{3}b^{5}+96\,ab^{7}+3\left(3\,a^{8}-28\,a^{6}b^{2}+210\,a^{4}b^{4}-448\,a^{3}b^{5}+420\,a^{2}b^{6}-192\,ab^{7}+35\,b^{8}\right)\cos\left(dx+c\right)^{4}\log\left(\sin\left(dx+c\right)+1\right)-3\left(3\,a^{8}-28\,a^{6}b^{2}+210\,a^{4}b^{4}+448\,a^{3}b^{5}+420\,a^{2}b^{6}+192\,ab^{7}+35\,b^{8}\right)\cos\left(dx+c\right)^{4}\log\left(-\sin\left(dx+c\right)+1\right)-192\left(7\,a^{5}b^{3}+14\,a^{3}b^{5}+3\,ab^{7}\right)\cos\left(dx+c\right)^{2}+2\left(8\,b^{8}\cos\left(dx+c\right)^{6}+6\,a^{8}+168\,a^{6}b^{2}+420\,a^{4}b^{4}+168\,a^{2}b^{6}+6\,b^{8}-16\left(42\,a^{2}b^{6}+5\,b^{8}\right)\cos\left(dx+c\right)^{4}+3\left(3\,a^{8}-28\,a^{6}b^{2}-350\,a^{4}b^{4}-252\,a^{2}b^{6}-13\,b^{8}\right)\cos\left(dx+c\right)^{2}\right)\sin\left(dx+c\right)}{48\,d\cos\left(dx+c\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/48*(192*a*b^7*cos(d*x + c)^6 - 96*a*b^7*cos(d*x + c)^4 + 96*a^7*b + 672*a
^5*b^3 + 672*a^3*b^5 + 96*a*b^7 + 3*(3*a^8 - 28*a^6*b^2 + 210*a^4*b^4 - 448
*a^3*b^5 + 420*a^2*b^6 - 192*a*b^7 + 35*b^8)*cos(d*x + c)^4*log(sin(d*x + c
) + 1) - 3*(3*a^8 - 28*a^6*b^2 + 210*a^4*b^4 + 448*a^3*b^5 + 420*a^2*b^6 +
```

```
192*a*b^7 + 35*b^8)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) - 192*(7*a^5*b^3
+ 14*a^3*b^5 + 3*a*b^7)*cos(d*x + c)^2 + 2*(8*b^8*cos(d*x + c)^6 + 6*a^8 +
168*a^6*b^2 + 420*a^4*b^4 + 168*a^2*b^6 + 6*b^8 - 16*(42*a^2*b^6 + 5*b^8)*c
os(d*x + c)^4 + 3*(3*a^8 - 28*a^6*b^2 - 350*a^4*b^4 - 252*a^2*b^6 - 13*b^8)
*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a+b*sin(d*x+c))**8,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 3.62, size = 429, normalized size = 1.34

$$-\frac{16\,b^8\sin\left(dx+c\right)^3+192\,ab^7\sin\left(dx+c\right)^2+1344\,a^2b^6\sin\left(dx+c\right)+144\,b^8\sin\left(dx+c\right)-3\left(3\,a^8-28\,a^6b^2+210\,a^4b^4-448\,a^3b^5+420\,a^2b^6-192\,ab^7+35\,b^8\right)\log\left(\left|\sin\left(dx+c\right)+1\right|\right)+3\left(3\,a^8-28\,a^6b^2+210\,a^4b^4+448\,a^3b^5+420\,a^2b^6+192\,ab^7+35\,b^8\right)\log\left(\left|\sin\left(dx+c\right)-1\right|\right)-\frac{6\left(336\,a^3b^5\sin(dx+c)^4+144\,ab^7\sin(dx+c)^4-3\,a^8\sin(dx+c)^3+28\,a^6b^2\sin(dx+c)^3+350\,a^4b^4\sin(dx+c)^3+252\,a^2b^6\sin(dx+c)^3+13\,b^8\sin(dx+c)^3+224\,a^5b^3\sin(dx+c)^2-224\,a^3b^5\sin(dx+c)^2-192\,ab^7\sin(dx+c)^2+5\,a^8\sin(dx+c)+28\,a^6b^2\sin(dx+c)-210\,a^4b^4\sin(dx+c)-196\,a^2b^6\sin(dx+c)-11\,b^8\sin(dx+c)+16\,a^7b-112\,a^5b^3+64\,ab^7\right)}{\left(\sin(dx+c)^2-1\right)^2}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -1/48*(16*b^8*sin(d*x + c)^3 + 192*a*b^7*sin(d*x + c)^2 + 1344*a^2*b^6*sin(
d*x + c) + 144*b^8*sin(d*x + c) - 3*(3*a^8 - 28*a^6*b^2 + 210*a^4*b^4 - 448
*a^3*b^5 + 420*a^2*b^6 - 192*a*b^7 + 35*b^8)*log(abs(sin(d*x + c) + 1)) + 3
*(3*a^8 - 28*a^6*b^2 + 210*a^4*b^4 + 448*a^3*b^5 + 420*a^2*b^6 + 192*a*b^7
+ 35*b^8)*log(abs(sin(d*x + c) - 1)) - 6*(336*a^3*b^5*sin(d*x + c)^4 + 144*
a*b^7*sin(d*x + c)^4 - 3*a^8*sin(d*x + c)^3 + 28*a^6*b^2*sin(d*x + c)^3 + 3
50*a^4*b^4*sin(d*x + c)^3 + 252*a^2*b^6*sin(d*x + c)^3 + 13*b^8*sin(d*x + c
)^3 + 224*a^5*b^3*sin(d*x + c)^2 - 224*a^3*b^5*sin(d*x + c)^2 - 192*a*b^7*s
in(d*x + c)^2 + 5*a^8*sin(d*x + c) + 28*a^6*b^2*sin(d*x + c) - 210*a^4*b^4*
sin(d*x + c) - 196*a^2*b^6*sin(d*x + c) - 11*b^8*sin(d*x + c) + 16*a^7*b -
112*a^5*b^3 + 64*a*b^7)/(sin(d*x + c)^2 - 1)^2)/d
```

Mupad [**B**]
time = 5.48, size = 305, normalized size = 0.95

$$\frac{\ln\left(\sin\left(c+d\,x\right)+1\right)\,{\left(a-b\right)}^6\,\left(3\,a^2+18\,a\,b+35\,b^2\right)}{16\,d}-\frac{b^8\,{\sin\left(c+d\,x\right)}^3}{3\,d}-\frac{\sin\left(c+d\,x\right)\,\left(28\,a^2\,b^6+3\,b^8\right)}{d}-\frac{\sin\left(c+d\,x\right)\,\left(-\frac{5\,a^8}{8}-\frac{7\,a^6\,b^2}{2}+\frac{105\,a^4\,b^4}{4}+\frac{49\,a^2\,b^6}{2}+\frac{11\,b^8}{8}\right)-{\sin\left(c+d\,x\right)}^3\,\left(-\frac{3\,a^8}{8}+\frac{7\,a^6\,b^2}{2}+\frac{175\,a^4\,b^4}{4}+\frac{63\,a^2\,b^6}{2}+\frac{13\,b^8}{8}\right)+10\,a\,b^7-2\,a^7\,b-{\sin\left(c+d\,x\right)}^2\,\left(28\,a^5\,b^3+56\,a^3\,b^5+12\,a\,b^7\right)+42\,a^3\,b^5+14\,a^5\,b^3}{d\,\left({\sin\left(c+d\,x\right)}^4-2\,{\sin\left(c+d\,x\right)}^2+1\right)}-\frac{4\,a\,b^7\,{\sin\left(c+d\,x\right)}^2}{d}-\frac{\ln\left(\sin\left(c+d\,x\right)-1\right)\,{\left(a+b\right)}^6\,\left(3\,a^2-18\,a\,b+35\,b^2\right)}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^8/cos(c + d*x)^5,x)
```

```
[Out] (log(sin(c + d*x) + 1)*(a - b)^6*(18*a*b + 3*a^2 + 35*b^2))/(16*d) - (b^8*s
in(c + d*x)^3)/(3*d) - (sin(c + d*x)*(3*b^8 + 28*a^2*b^6))/d - (sin(c + d*x
```

```
)*((11*b^8)/8 - (5*a^8)/8 + (49*a^2*b^6)/2 + (105*a^4*b^4)/4 - (7*a^6*b^2)/
2) - sin(c + d*x)^3*((13*b^8)/8 - (3*a^8)/8 + (63*a^2*b^6)/2 + (175*a^4*b^4
)/4 + (7*a^6*b^2)/2) + 10*a*b^7 - 2*a^7*b - sin(c + d*x)^2*(12*a*b^7 + 56*a
^3*b^5 + 28*a^5*b^3) + 42*a^3*b^5 + 14*a^5*b^3)/(d*(sin(c + d*x)^4 - 2*sin(
c + d*x)^2 + 1)) - (4*a*b^7*sin(c + d*x)^2)/d - (log(sin(c + d*x) - 1)*(a +
 b)^6*(3*a^2 - 18*a*b + 35*b^2))/(16*d)
```

3.419 $\int \cos^2(c+dx)(a+b\sin(c+dx))^8\,dx$

Optimal. Leaf size=423

$$\frac{1}{256}\left(128a^8+896a^6b^2+1120a^4b^4+280a^2b^6+7b^8\right)x-\frac{11ab(1792a^6+10536a^4b^2+9588a^2b^4+1289b^6)\cos^3(c}{40320d}$$

```
[Out] 1/256*(128*a^8+896*a^6*b^2+1120*a^4*b^4+280*a^2*b^6+7*b^8)*x-11/40320*a*b*(
1792*a^6+10536*a^4*b^2+9588*a^2*b^4+1289*b^6)*cos(d*x+c)^3/d+1/256*(128*a^8
+896*a^6*b^2+1120*a^4*b^4+280*a^2*b^6+7*b^8)*cos(d*x+c)*sin(d*x+c)/d-1/1344
0*b*(6272*a^6+28088*a^4*b^2+15956*a^2*b^4+735*b^6)*cos(d*x+c)^3*(a+b*sin(d*
x+c))/d-13/3360*a*b*(112*a^4+348*a^2*b^2+101*b^4)*cos(d*x+c)^3*(a+b*sin(d*x
+c))^2/d-1/2016*b*(784*a^4+1500*a^2*b^2+147*b^4)*cos(d*x+c)^3*(a+b*sin(d*x+
c))^3/d-1/336*a*b*(112*a^2+109*b^2)*cos(d*x+c)^3*(a+b*sin(d*x+c))^4/d-1/240
*b*(64*a^2+21*b^2)*cos(d*x+c)^3*(a+b*sin(d*x+c))^5/d-17/90*a*b*cos(d*x+c)^3
*(a+b*sin(d*x+c))^6/d-1/10*b*cos(d*x+c)^3*(a+b*sin(d*x+c))^7/d
```

Rubi [A]
time = 0.80, antiderivative size = 423, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2771, 2941, 2748, 2715, 8}

$$-\frac{b(64a^2+21b^2)\cos^3(c+dx)(a+b\sin(c+dx))^5}{240d}-\frac{ab(112a^2+109b^2)\cos^3(c+dx)(a+b\sin(c+dx))^4}{336d}-\frac{b(784a^4+1500a^2b^2+147b^4)\cos^3(c+dx)(a+b\sin(c+dx))^3}{2016d}-\frac{13ab(112a^4+348a^2b^2+101b^4)\cos^3(c+dx)(a+b\sin(c+dx))^2}{3360d}-\frac{11ab(1792a^6+10536a^4b^2+9588a^2b^4+1289b^6)\cos^3(c+dx)}{40320d}-\frac{b(6272a^6+28088a^4b^2+15956a^2b^4+735b^6)\cos^3(c+dx)(a+b\sin(c+dx))}{13440d}+\frac{(128a^8+896a^6b^2+1120a^4b^4+280a^2b^6+7b^8)\sin(c+dx)\cos(c+dx)}{256d}+\frac{1}{256}x(128a^8+896a^6b^2+1120a^4b^4+280a^2b^6+7b^8)-\frac{b\cos^3(c+dx)(a+b\sin(c+dx))^7}{10d}-\frac{17ab\cos^3(c+dx)(a+b\sin(c+dx))^6}{90d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Sin[c + d*x])^8,x]
```

```
[Out] ((128*a^8 + 896*a^6*b^2 + 1120*a^4*b^4 + 280*a^2*b^6 + 7*b^8)*x)/256 - (11*
a*b*(1792*a^6 + 10536*a^4*b^2 + 9588*a^2*b^4 + 1289*b^6)*Cos[c + d*x]^3)/(4
0320*d) + ((128*a^8 + 896*a^6*b^2 + 1120*a^4*b^4 + 280*a^2*b^6 + 7*b^8)*Cos
[c + d*x]*Sin[c + d*x])/(256*d) - (b*(6272*a^6 + 28088*a^4*b^2 + 15956*a^2*
b^4 + 735*b^6)*Cos[c + d*x]^3*(a + b*Sin[c + d*x]))/(13440*d) - (13*a*b*(11
2*a^4 + 348*a^2*b^2 + 101*b^4)*Cos[c + d*x]^3*(a + b*Sin[c + d*x])^2)/(3360
*d) - (b*(784*a^4 + 1500*a^2*b^2 + 147*b^4)*Cos[c + d*x]^3*(a + b*Sin[c + d
*x])^3)/(2016*d) - (a*b*(112*a^2 + 109*b^2)*Cos[c + d*x]^3*(a + b*Sin[c + d
*x])^4)/(336*d) - (b*(64*a^2 + 21*b^2)*Cos[c + d*x]^3*(a + b*Sin[c + d*x])^
5)/(240*d) - (17*a*b*Cos[c + d*x]^3*(a + b*Sin[c + d*x])^6)/(90*d) - (b*Cos
[c + d*x]^3*(a + b*Sin[c + d*x])^7)/(10*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
```

```
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2771

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(
a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rule 2941

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp
[a*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x],
x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
&&  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Si
mplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int \cos^2(c+dx)(a+b\sin(c+dx))^8\,dx = -\frac{b\cos^3(c+dx)(a+b\sin(c+dx))^7}{10d} + \frac{1}{10}\int \cos^2(c+dx)(a+b\sin($$

$$= -\frac{17ab\cos^3(c+dx)(a+b\sin(c+dx))^6}{90d} - \frac{b\cos^3(c+dx)(a+b\sin(c}{10d}$$

$$= -\frac{b(64a^2+21b^2)\cos^3(c+dx)(a+b\sin(c+dx))^5}{240d} - \frac{17ab\cos^3(c+d}{}$$

$$= -\frac{ab(112a^2+109b^2)\cos^3(c+dx)(a+b\sin(c+dx))^4}{336d} - \frac{b(64a^2+21}{}$$

$$= -\frac{b(784a^4+1500a^2b^2+147b^4)\cos^3(c+dx)(a+b\sin(c+dx))^3}{2016d} - \frac{a}{}$$

$$= -\frac{13ab(112a^4+348a^2b^2+101b^4)\cos^3(c+dx)(a+b\sin(c+dx))^2}{3360d} -$$

$$= -\frac{b(6272a^6+28088a^4b^2+15956a^2b^4+735b^6)\cos^3(c+dx)(a+b\sin}{13440d}$$

$$= -\frac{11ab(1792a^6+10536a^4b^2+9588a^2b^4+1289b^6)\cos^3(c+dx)}{40320d} - \frac{b(}{}$$

$$= -\frac{11ab(1792a^6+10536a^4b^2+9588a^2b^4+1289b^6)\cos^3(c+dx)}{40320d} + \frac{(1}{}$$

$$= \frac{1}{256}\left(128a^8+896a^6b^2+1120a^4b^4+280a^2b^6+7b^8\right)x - \frac{11ab(1792a^6}{}$$

Mathematica [A]
time = 1.02, size = 457, normalized size = 1.08

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Sin[c + d*x])^8,x]
```

```
[Out] (322560*a^8*c + 2257920*a^6*b^2*c + 2822400*a^4*b^4*c + 705600*a^2*b^6*c +
17640*b^8*c + 322560*a^8*d*x + 2257920*a^6*b^2*d*x + 2822400*a^4*b^4*d*x +
705600*a^2*b^6*d*x + 17640*b^8*d*x - 40320*a*b*(32*a^6 + 112*a^4*b^2 + 70*a
^2*b^4 + 7*b^6)*Cos[c + d*x] - 26880*(16*a^7*b + 28*a^5*b^3 + 7*a^3*b^5)*Co
s[3*(c + d*x)] + 451584*a^5*b^3*Cos[5*(c + d*x)] + 338688*a^3*b^5*Cos[5*(c
+ d*x)] + 32256*a*b^7*Cos[5*(c + d*x)] - 80640*a^3*b^5*Cos[7*(c + d*x)] - 1
4400*a*b^7*Cos[7*(c + d*x)] + 2240*a*b^7*Cos[9*(c + d*x)] + 161280*a^8*Sin[
2*(c + d*x)] - 705600*a^4*b^4*Sin[2*(c + d*x)] - 282240*a^2*b^6*Sin[2*(c +
d*x)] - 8820*b^8*Sin[2*(c + d*x)] - 564480*a^6*b^2*Sin[4*(c + d*x)] - 70560
0*a^4*b^4*Sin[4*(c + d*x)] - 141120*a^2*b^6*Sin[4*(c + d*x)] - 2520*b^8*Sin
[4*(c + d*x)] + 235200*a^4*b^4*Sin[6*(c + d*x)] + 94080*a^2*b^6*Sin[6*(c +
```

```
d*x)] + 2730*b^8*Sin[6*(c + d*x)] - 17640*a^2*b^6*Sin[8*(c + d*x)] - 945*b^
8*Sin[8*(c + d*x)] + 126*b^8*Sin[10*(c + d*x)])/(645120*d)
```

Maple [A]
time = 1.06, size = 497, normalized size = 1.17 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*sin(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^8*(-1/10*sin(d*x+c)^7*cos(d*x+c)^3-7/80*sin(d*x+c)^5*cos(d*x+c)^3-7/
96*sin(d*x+c)^3*cos(d*x+c)^3-7/128*sin(d*x+c)*cos(d*x+c)^3+7/256*cos(d*x+c)
*sin(d*x+c)+7/256*d*x+7/256*c)+8*a*b^7*(-1/9*sin(d*x+c)^6*cos(d*x+c)^3-2/21
*sin(d*x+c)^4*cos(d*x+c)^3-8/105*sin(d*x+c)^2*cos(d*x+c)^3-16/315*cos(d*x+c
)^3)+28*a^2*b^6*(-1/8*sin(d*x+c)^5*cos(d*x+c)^3-5/48*sin(d*x+c)^3*cos(d*x+c
)^3-5/64*sin(d*x+c)*cos(d*x+c)^3+5/128*cos(d*x+c)*sin(d*x+c)+5/128*d*x+5/12
8*c)+56*a^3*b^5*(-1/7*sin(d*x+c)^4*cos(d*x+c)^3-4/35*sin(d*x+c)^2*cos(d*x+c
)^3-8/105*cos(d*x+c)^3)+70*a^4*b^4*(-1/6*sin(d*x+c)^3*cos(d*x+c)^3-1/8*sin(
d*x+c)*cos(d*x+c)^3+1/16*cos(d*x+c)*sin(d*x+c)+1/16*d*x+1/16*c)+56*a^5*b^3*
(-1/5*sin(d*x+c)^2*cos(d*x+c)^3-2/15*cos(d*x+c)^3)+28*a^6*b^2*(-1/4*sin(d*x
+c)*cos(d*x+c)^3+1/8*cos(d*x+c)*sin(d*x+c)+1/8*d*x+1/8*c)-8/3*a^7*b*cos(d*x
+c)^3+a^8*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))
```

Maxima [A]
time = 0.28, size = 336, normalized size = 0.79

$$-\frac{1720320\,a^{7}b\cos\left(dx+c\right)^{3}-161280\,\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)a^{8}-564480\,\left(4\,dx+4\,c-\sin\left(4\,dx+4\,c\right)\right)a^{6}b^{2}-2408448\,\left(3\,\cos\left(dx+c\right)^{5}-5\,\cos\left(dx+c\right)^{3}\right)a^{5}b^{3}+235200\,\left(4\,\sin\left(2\,dx+2\,c\right)^{3}-12\,dx-12\,c+3\,\sin\left(4\,dx+4\,c\right)\right)a^{4}b^{4}+344064\,\left(15\,\cos\left(dx+c\right)^{7}-42\,\cos\left(dx+c\right)^{5}+35\,\cos\left(dx+c\right)^{3}\right)a^{3}b^{5}+5880\,\left(64\,\sin\left(2\,dx+2\,c\right)^{3}-120\,dx-120\,c+3\,\sin\left(8\,dx+8\,c\right)+24\,\sin\left(4\,dx+4\,c\right)\right)a^{2}b^{6}-16384\,\left(35\,\cos\left(dx+c\right)^{9}-135\,\cos\left(dx+c\right)^{7}+189\,\cos\left(dx+c\right)^{5}-105\,\cos\left(dx+c\right)^{3}\right)ab^{7}-21\,\left(96\,\sin\left(2\,dx+2\,c\right)^{5}-640\,\sin\left(2\,dx+2\,c\right)^{3}+840\,dx+840\,c-45\,\sin\left(8\,dx+8\,c\right)-120\,\sin\left(4\,dx+4\,c\right)\right)b^{8}}{645120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/645120*(1720320*a^7*b*cos(d*x + c)^3 - 161280*(2*d*x + 2*c + sin(2*d*x +
 2*c))*a^8 - 564480*(4*d*x + 4*c - sin(4*d*x + 4*c))*a^6*b^2 - 2408448*(3*c
os(d*x + c)^5 - 5*cos(d*x + c)^3)*a^5*b^3 + 235200*(4*sin(2*d*x + 2*c)^3 -
12*d*x - 12*c + 3*sin(4*d*x + 4*c))*a^4*b^4 + 344064*(15*cos(d*x + c)^7 - 4
2*cos(d*x + c)^5 + 35*cos(d*x + c)^3)*a^3*b^5 + 5880*(64*sin(2*d*x + 2*c)^3
 - 120*d*x - 120*c + 3*sin(8*d*x + 8*c) + 24*sin(4*d*x + 4*c))*a^2*b^6 - 16
384*(35*cos(d*x + c)^9 - 135*cos(d*x + c)^7 + 189*cos(d*x + c)^5 - 105*cos(
d*x + c)^3)*a*b^7 - 21*(96*sin(2*d*x + 2*c)^5 - 640*sin(2*d*x + 2*c)^3 + 84
0*d*x + 840*c - 45*sin(8*d*x + 8*c) - 120*sin(4*d*x + 4*c))*b^8)/d
```

Fricas [A]
time = 0.40, size = 315, normalized size = 0.74

$$\frac{71680\,ab^{7}\cos\left(dx+c\right)^{9}-92160\,\left(7\,a^{3}b^{5}+3\,ab^{7}\right)\cos\left(dx+c\right)^{7}+129024\,\left(7\,a^{5}b^{3}+14\,a^{3}b^{5}+3\,ab^{7}\right)\cos\left(dx+c\right)^{5}-215040\,\left(a^{7}b+7\,a^{5}b^{3}+7\,a^{3}b^{5}+ab^{7}\right)\cos\left(dx+c\right)^{3}+315\,\left(128\,a^{8}+896\,a^{6}b^{2}+1120\,a^{4}b^{4}+280\,a^{2}b^{6}+7\,b^{8}\right)dx+21\,\left(384\,b^{8}\cos\left(dx+c\right)^{9}-48\,\left(280\,a^{2}b^{6}+31\,b^{8}\right)\cos\left(dx+c\right)^{7}+8\,\left(5600\,a^{4}b^{4}+4760\,a^{2}b^{6}+263\,b^{8}\right)\cos\left(dx+c\right)^{5}-10\,\left(2688\,a^{6}b^{2}+7840\,a^{4}b^{4}+3304\,a^{2}b^{6}+121\,b^{8}\right)\cos\left(dx+c\right)^{3}+15\,\left(128\,a^{8}+896\,a^{6}b^{2}+1120\,a^{4}b^{4}+280\,a^{2}b^{6}+7\,b^{8}\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{80640\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/80640*(71680*a*b^7*cos(d*x + c)^9 - 92160*(7*a^3*b^5 + 3*a*b^7)*cos(d*x +
 c)^7 + 129024*(7*a^5*b^3 + 14*a^3*b^5 + 3*a*b^7)*cos(d*x + c)^5 - 215040*(
a^7*b + 7*a^5*b^3 + 7*a^3*b^5 + a*b^7)*cos(d*x + c)^3 + 315*(128*a^8 + 896*
a^6*b^2 + 1120*a^4*b^4 + 280*a^2*b^6 + 7*b^8)*d*x + 21*(384*b^8*cos(d*x + c
)^9 - 48*(280*a^2*b^6 + 31*b^8)*cos(d*x + c)^7 + 8*(5600*a^4*b^4 + 4760*a^2
*b^6 + 263*b^8)*cos(d*x + c)^5 - 10*(2688*a^6*b^2 + 7840*a^4*b^4 + 3304*a^2
*b^6 + 121*b^8)*cos(d*x + c)^3 + 15*(128*a^8 + 896*a^6*b^2 + 1120*a^4*b^4 +
 280*a^2*b^6 + 7*b^8)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1115 vs. $2(415) = 830$.

time = 1.96, size = 1115, normalized size = 2.64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*sin(d*x+c))**8,x)
```

```
[Out] Piecewise((a**8*x*sin(c + d*x)**2/2 + a**8*x*cos(c + d*x)**2/2 + a**8*sin(c
 + d*x)*cos(c + d*x)/(2*d) - 8*a**7*b*cos(c + d*x)**3/(3*d) + 7*a**6*b**2*x
*sin(c + d*x)**4/2 + 7*a**6*b**2*x*sin(c + d*x)**2*cos(c + d*x)**2 + 7*a**6
*b**2*x*cos(c + d*x)**4/2 + 7*a**6*b**2*sin(c + d*x)**3*cos(c + d*x)/(2*d)
- 7*a**6*b**2*sin(c + d*x)*cos(c + d*x)**3/(2*d) - 56*a**5*b**3*sin(c + d*x
)**2*cos(c + d*x)**3/(3*d) - 112*a**5*b**3*cos(c + d*x)**5/(15*d) + 35*a**4
*b**4*x*sin(c + d*x)**6/8 + 105*a**4*b**4*x*sin(c + d*x)**4*cos(c + d*x)**2
/8 + 105*a**4*b**4*x*sin(c + d*x)**2*cos(c + d*x)**4/8 + 35*a**4*b**4*x*cos
(c + d*x)**6/8 + 35*a**4*b**4*sin(c + d*x)**5*cos(c + d*x)/(8*d) - 35*a**4*
b**4*sin(c + d*x)**3*cos(c + d*x)**3/(3*d) - 35*a**4*b**4*sin(c + d*x)*cos(
c + d*x)**5/(8*d) - 56*a**3*b**5*sin(c + d*x)**4*cos(c + d*x)**3/(3*d) - 22
4*a**3*b**5*sin(c + d*x)**2*cos(c + d*x)**5/(15*d) - 64*a**3*b**5*cos(c + d
*x)**7/(15*d) + 35*a**2*b**6*x*sin(c + d*x)**8/32 + 35*a**2*b**6*x*sin(c +
d*x)**6*cos(c + d*x)**2/8 + 105*a**2*b**6*x*sin(c + d*x)**4*cos(c + d*x)**4
/16 + 35*a**2*b**6*x*sin(c + d*x)**2*cos(c + d*x)**6/8 + 35*a**2*b**6*x*cos
(c + d*x)**8/32 + 35*a**2*b**6*sin(c + d*x)**7*cos(c + d*x)/(32*d) - 511*a*
*2*b**6*sin(c + d*x)**5*cos(c + d*x)**3/(96*d) - 385*a**2*b**6*sin(c + d*x)
**3*cos(c + d*x)**5/(96*d) - 35*a**2*b**6*sin(c + d*x)*cos(c + d*x)**7/(32*
d) - 8*a*b**7*sin(c + d*x)**6*cos(c + d*x)**3/(3*d) - 16*a*b**7*sin(c + d*x
)**4*cos(c + d*x)**5/(5*d) - 64*a*b**7*sin(c + d*x)**2*cos(c + d*x)**7/(35*
d) - 128*a*b**7*cos(c + d*x)**9/(315*d) + 7*b**8*x*sin(c + d*x)**10/256 + 3
5*b**8*x*sin(c + d*x)**8*cos(c + d*x)**2/256 + 35*b**8*x*sin(c + d*x)**6*co
s(c + d*x)**4/128 + 35*b**8*x*sin(c + d*x)**4*cos(c + d*x)**6/128 + 35*b**8
*x*sin(c + d*x)**2*cos(c + d*x)**8/256 + 7*b**8*x*cos(c + d*x)**10/256 + 7*
b**8*sin(c + d*x)**9*cos(c + d*x)/(256*d) - 79*b**8*sin(c + d*x)**7*cos(c +
 d*x)**3/(384*d) - 7*b**8*sin(c + d*x)**5*cos(c + d*x)**5/(30*d) - 49*b**8*
```

```
sin(c + d*x)**3*cos(c + d*x)**7/(384*d) - 7*b**8*sin(c + d*x)*cos(c + d*x)*
*9/(256*d), Ne(d, 0)), (x*(a + b*sin(c))**8*cos(c)**2, True))
```

Giac [A]
time = 6.16, size = 364, normalized size = 0.86

$$\frac{ab^7\cos(9\,dx+9\,c)}{288\,d}+\frac{b^8\sin(10\,dx+10\,c)}{5120\,d}+\frac{1}{256}\,(128\,a^8+896\,a^6b^2+1120\,a^4b^4+280\,a^2b^6+7\,b^8)x-\frac{(28\,a^3b^5+5\,ab^7)\cos(7\,dx+7\,c)}{224\,d}+\frac{(28\,a^5b^3+21\,a^3b^5+2\,ab^7)\cos(5\,dx+5\,c)}{40\,d}-\frac{(16\,a^7b+28\,a^5b^3+7\,a^3b^5)\cos(3\,dx+3\,c)}{24\,d}-\frac{(32\,a^7b+112\,a^5b^3+70\,a^3b^5+7\,ab^7)\cos(dx+c)}{16\,d}-\frac{(56\,a^2b^6+3\,b^8)\sin(8\,dx+8\,c)}{2048\,d}+\frac{(1120\,a^4b^4+448\,a^2b^6+13\,b^8)\sin(6\,dx+6\,c)}{3072\,d}-\frac{(224\,a^6b^2+280\,a^4b^4+56\,a^2b^6+b^8)\sin(4\,dx+4\,c)}{256\,d}+\frac{(128\,a^8-560\,a^4b^4-224\,a^2b^6-7\,b^8)\sin(2\,dx+2\,c)}{512\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/288*a*b^7*cos(9*d*x + 9*c)/d + 1/5120*b^8*sin(10*d*x + 10*c)/d + 1/256*(1
28*a^8 + 896*a^6*b^2 + 1120*a^4*b^4 + 280*a^2*b^6 + 7*b^8)*x - 1/224*(28*a^
3*b^5 + 5*a*b^7)*cos(7*d*x + 7*c)/d + 1/40*(28*a^5*b^3 + 21*a^3*b^5 + 2*a*b
^7)*cos(5*d*x + 5*c)/d - 1/24*(16*a^7*b + 28*a^5*b^3 + 7*a^3*b^5)*cos(3*d*x
 + 3*c)/d - 1/16*(32*a^7*b + 112*a^5*b^3 + 70*a^3*b^5 + 7*a*b^7)*cos(d*x +
c)/d - 1/2048*(56*a^2*b^6 + 3*b^8)*sin(8*d*x + 8*c)/d + 1/3072*(1120*a^4*b^
4 + 448*a^2*b^6 + 13*b^8)*sin(6*d*x + 6*c)/d - 1/256*(224*a^6*b^2 + 280*a^4
*b^4 + 56*a^2*b^6 + b^8)*sin(4*d*x + 4*c)/d + 1/512*(128*a^8 - 560*a^4*b^4
- 224*a^2*b^6 - 7*b^8)*sin(2*d*x + 2*c)/d
```

Mupad [B]
time = 7.34, size = 467, normalized size = 1.10

$$-\frac{\frac{2205\,b^8\sin(2\,c+2\,d\,x)}{2}-20160\,a^8\sin(2\,c+2\,d\,x)+315\,b^8\sin(4\,c+4\,d\,x)-\frac{1365\,b^8\sin(6\,c+6\,d\,x)}{4}+\frac{945\,b^8\sin(8\,c+8\,d\,x)}{8}-\frac{63\,b^8\sin(10\,c+10\,d\,x)}{4}+53760\,a^7\,b\cos(3\,c+3\,d\,x)-4032\,a\,b^7\cos(5\,c+5\,d\,x)+1800\,a\,b^7\cos(7\,c+7\,d\,x)-280\,a\,b^7\cos(9\,c+9\,d\,x)+352800\,a^3\,b^5\cos(c+d\,x)+564480\,a^5\,b^3\cos(c+d\,x)+23520\,a^3\,b^5\cos(3\,c+3\,d\,x)+94080\,a^5\,b^3\cos(3\,c+3\,d\,x)-42336\,a^3\,b^5\cos(5\,c+5\,d\,x)-56448\,a^5\,b^3\cos(5\,c+5\,d\,x)+10080\,a^3\,b^5\cos(7\,c+7\,d\,x)+35280\,a^2\,b^6\sin(2\,c+2\,d\,x)+88200\,a^4\,b^4\sin(2\,c+2\,d\,x)+17640\,a^2\,b^6\sin(4\,c+4\,d\,x)+88200\,a^4\,b^4\sin(4\,c+4\,d\,x)+70560\,a^6\,b^2\sin(4\,c+4\,d\,x)-11760\,a^2\,b^6\sin(6\,c+6\,d\,x)-29400\,a^4\,b^4\sin(6\,c+6\,d\,x)+2205\,a^2\,b^6\sin(8\,c+8\,d\,x)+35280\,a\,b^7\cos(c+d\,x)+161280\,a^7\,b\cos(c+d\,x)-40320\,a^8\,d\,x-2205\,b^8\,d\,x-88200\,a^2\,b^6\,d\,x-352800\,a^4\,b^4\,d\,x-282240\,a^6\,b^2\,d\,x}{80640\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a + b*sin(c + d*x))^8,x)
```

```
[Out] -((2205*b^8*sin(2*c + 2*d*x))/2 - 20160*a^8*sin(2*c + 2*d*x) + 315*b^8*sin(
4*c + 4*d*x) - (1365*b^8*sin(6*c + 6*d*x))/4 + (945*b^8*sin(8*c + 8*d*x))/8
 - (63*b^8*sin(10*c + 10*d*x))/4 + 53760*a^7*b*cos(3*c + 3*d*x) - 4032*a*b^
7*cos(5*c + 5*d*x) + 1800*a*b^7*cos(7*c + 7*d*x) - 280*a*b^7*cos(9*c + 9*d*
x) + 352800*a^3*b^5*cos(c + d*x) + 564480*a^5*b^3*cos(c + d*x) + 23520*a^3*
b^5*cos(3*c + 3*d*x) + 94080*a^5*b^3*cos(3*c + 3*d*x) - 42336*a^3*b^5*cos(5
*c + 5*d*x) - 56448*a^5*b^3*cos(5*c + 5*d*x) + 10080*a^3*b^5*cos(7*c + 7*d*
x) + 35280*a^2*b^6*sin(2*c + 2*d*x) + 88200*a^4*b^4*sin(2*c + 2*d*x) + 1764
0*a^2*b^6*sin(4*c + 4*d*x) + 88200*a^4*b^4*sin(4*c + 4*d*x) + 70560*a^6*b^2
*sin(4*c + 4*d*x) - 11760*a^2*b^6*sin(6*c + 6*d*x) - 29400*a^4*b^4*sin(6*c
+ 6*d*x) + 2205*a^2*b^6*sin(8*c + 8*d*x) + 35280*a*b^7*cos(c + d*x) + 16128
0*a^7*b*cos(c + d*x) - 40320*a^8*d*x - 2205*b^8*d*x - 88200*a^2*b^6*d*x - 3
52800*a^4*b^4*d*x - 282240*a^6*b^2*d*x)/(80640*d)
```

3.420 $\int \sec^2(c+dx)(a+b\sin(c+dx))^8\,dx$

Optimal. Leaf size=349

$$-\frac{7}{16}b^2\left(64a^6+240a^4b^2+120a^2b^4+5b^6\right)x+\frac{ab(40a^6+1664a^4b^2+2789a^2b^4+512b^6)\cos(c+dx)}{20d}+\frac{b^2(80a^6+$$

```
[Out] -7/16*b^2*(64*a^6+240*a^4*b^2+120*a^2*b^4+5*b^6)*x+1/20*a*b*(40*a^6+1664*a^
4*b^2+2789*a^2*b^4+512*b^6)*cos(d*x+c)/d+1/80*b^2*(80*a^6+2248*a^4*b^2+2502
*a^2*b^4+175*b^6)*cos(d*x+c)*sin(d*x+c)/d+1/40*a*b*(40*a^4+624*a^2*b^2+337*
b^4)*cos(d*x+c)*(a+b*sin(d*x+c))^2/d+1/120*b*(120*a^4+992*a^2*b^2+175*b^4)*
cos(d*x+c)*(a+b*sin(d*x+c))^3/d+1/30*a*b*(30*a^2+113*b^2)*cos(d*x+c)*(a+b*s
in(d*x+c))^4/d+1/6*b*(6*a^2+7*b^2)*cos(d*x+c)*(a+b*sin(d*x+c))^5/d+a*b*cos(
d*x+c)*(a+b*sin(d*x+c))^6/d+sec(d*x+c)*(b+a*sin(d*x+c))*(a+b*sin(d*x+c))^7/
d
```

Rubi [A]
time = 0.38, antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2770, 2832, 2813}

$$\frac{b(6a^2+7b^2)\cos(c+dx)(a+b\sin(c+dx))^5}{6d}+\frac{ab(30a^2+113b^2)\cos(c+dx)(a+b\sin(c+dx))^4}{30d}+\frac{b(120a^4+992a^2b^2+175b^4)\cos(c+dx)(a+b\sin(c+dx))^3}{120d}+\frac{ab(40a^4+624a^2b^2+337b^4)\cos(c+dx)(a+b\sin(c+dx))^2}{40d}+\frac{ab(40a^6+1664a^4b^2+2789a^2b^4+512b^6)\cos(c+dx)}{20d}+\frac{b^2(80a^6+2248a^4b^2+2502a^2b^4+175b^6)\sin(c+dx)\cos(c+dx)}{80d}-\frac{7}{16}b^2x\left(64a^6+240a^4b^2+120a^2b^4+5b^6\right)+\frac{ab\cos(c+dx)(a+b\sin(c+dx))^6}{d}+\frac{\sec(c+dx)(a\sin(c+dx)+b)(a+b\sin(c+dx))^7}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Sin[c + d*x])^8,x]
```

```
[Out] (-7*b^2*(64*a^6 + 240*a^4*b^2 + 120*a^2*b^4 + 5*b^6)*x)/16 + (a*b*(40*a^6 +
 1664*a^4*b^2 + 2789*a^2*b^4 + 512*b^6)*Cos[c + d*x])/(20*d) + (b^2*(80*a^6
 + 2248*a^4*b^2 + 2502*a^2*b^4 + 175*b^6)*Cos[c + d*x]*Sin[c + d*x])/(80*d)
 + (a*b*(40*a^4 + 624*a^2*b^2 + 337*b^4)*Cos[c + d*x]*(a + b*Sin[c + d*x])^
2)/(40*d) + (b*(120*a^4 + 992*a^2*b^2 + 175*b^4)*Cos[c + d*x]*(a + b*Sin[c
+ d*x])^3)/(120*d) + (a*b*(30*a^2 + 113*b^2)*Cos[c + d*x]*(a + b*Sin[c + d*
x])^4)/(30*d) + (b*(6*a^2 + 7*b^2)*Cos[c + d*x]*(a + b*Sin[c + d*x])^5)/(6*
d) + (a*b*Cos[c + d*x]*(a + b*Sin[c + d*x])^6)/d + (Sec[c + d*x]*(b + a*Sin
[c + d*x])*(a + b*Sin[c + d*x])^7)/d
```

Rule 2770

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x
])^(m - 1)*((b + a*Sin[e + f*x])/(f*g*(p + 1))), x] + Dist[1/(g^2*(p + 1)),
 Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a
^2*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g}
, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p
] || IntegerQ[m])
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rubi steps

$$\int \sec^2(c+dx)(a+b\sin(c+dx))^8\,dx = \frac{\sec(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^7}{d} - \int (a+b\sin(c$$
$$= \frac{ab\cos(c+dx)(a+b\sin(c+dx))^6}{d} + \frac{\sec(c+dx)(b+a\sin(c+dx)}{d}$$
$$= \frac{b(6a^2+7b^2)\cos(c+dx)(a+b\sin(c+dx))^5}{6d} + \frac{ab\cos(c+dx)(a+}{d}$$
$$= \frac{ab(30a^2+113b^2)\cos(c+dx)(a+b\sin(c+dx))^4}{30d} + \frac{b(6a^2+7b^2)\,\mathrm{co}}{}$$
$$= \frac{b(120a^4+992a^2b^2+175b^4)\cos(c+dx)(a+b\sin(c+dx))^3}{120d} + \frac{ab(3}{}$$
$$= \frac{ab(40a^4+624a^2b^2+337b^4)\cos(c+dx)(a+b\sin(c+dx))^2}{40d} + \frac{b(12}{}$$
$$= -\frac{7}{16}b^2\left(64a^6+240a^4b^2+120a^2b^4+5b^6\right)x + \frac{ab(40a^6+1664a^4b^2+}{}$$

Mathematica [A]
time = 1.12, size = 313, normalized size = 0.90

$$\frac{\sec(c+dx)\left(15360a^7b+161280a^5b^3+201600a^3b^5+33600ab^7-840b^2(64a^6+240a^4b^2+120a^2b^4+5b^6)(c+dx)\cos(c+dx)+1120(48a^5b^3+80a^3b^5+15ab^7)\cos(2(c+dx))-4480a^3b^5\cos(4(c+dx))-1344ab^7\cos(4(c+dx))+96ab^7\cos(6(c+dx))+1920a^8\sin(c+dx)+53760a^6b^2\sin(c+dx)+151200a^4b^4\sin(c+dx)+67200a^2b^6\sin(c+dx)+2625b^8\sin(c+dx)+16800a^4b^4\sin(3(c+dx))+12600a^2b^6\sin(3(c+dx))+630b^8\sin(3(c+dx))-840a^2b^6\sin(5(c+dx))-70b^8\sin(5(c+dx))+5b^8\sin(7(c+dx))\right)}{1920d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Sin[c + d*x])^8,x]
```

```
[Out] (Sec[c + d*x]*(15360*a^7*b + 161280*a^5*b^3 + 201600*a^3*b^5 + 33600*a*b^7
- 840*b^2*(64*a^6 + 240*a^4*b^2 + 120*a^2*b^4 + 5*b^6)*(c + d*x)*Cos[c + d*
```

```
x] + 1120*(48*a^5*b^3 + 80*a^3*b^5 + 15*a*b^7)*Cos[2*(c + d*x)] - 4480*a^3*
b^5*Cos[4*(c + d*x)] - 1344*a*b^7*Cos[4*(c + d*x)] + 96*a*b^7*Cos[6*(c + d*
x)] + 1920*a^8*Sin[c + d*x] + 53760*a^6*b^2*Sin[c + d*x] + 151200*a^4*b^4*S
in[c + d*x] + 67200*a^2*b^6*Sin[c + d*x] + 2625*b^8*Sin[c + d*x] + 16800*a^
4*b^4*Sin[3*(c + d*x)] + 12600*a^2*b^6*Sin[3*(c + d*x)] + 630*b^8*Sin[3*(c
+ d*x)] - 840*a^2*b^6*Sin[5*(c + d*x)] - 70*b^8*Sin[5*(c + d*x)] + 5*b^8*Si
n[7*(c + d*x)]))/(1920*d)
```

Maple [A]
time = 0.29, size = 406, normalized size = 1.16

method	result
derivativedivides	$\frac{a^8\tan(dx+c)+\frac{8a^7b}{\cos(dx+c)}+28a^6b^2(\tan(dx+c)-dx-c)+56a^5b^3\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+(2+\sin^2(dx+c))\cos(dx+c)\right)+70a^4b^4\left(\frac{\sin^5(\ldots}{\cos(\ldots}\right.}{}$
default	$\frac{a^8\tan(dx+c)+\frac{8a^7b}{\cos(dx+c)}+28a^6b^2(\tan(dx+c)-dx-c)+56a^5b^3\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+(2+\sin^2(dx+c))\cos(dx+c)\right)+70a^4b^4\left(\frac{\sin^5(\ldots}{\cos(\ldots}\right.}{}$
risch	$-28x\,a^6b^2-105x\,a^4b^4-\frac{105x\,a^2b^6}{2}-\frac{35b^8x}{16}-\frac{35ie^{2i(dx+c)}a^4b^4}{4d}+\frac{7ie^{-2i(dx+c)}a^2b^6}{d}-\frac{47ie^{2i(dx+c)}b^8}{128d}+\ldots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+b*sin(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^8*tan(d*x+c)+8*a^7*b/cos(d*x+c)+28*a^6*b^2*(tan(d*x+c)-d*x-c)+56*a^5
*b^3*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d*x+c))+70*a^4*b^4*(sin(
d*x+c)^5/cos(d*x+c)+(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)-3/2*d*x-3/2*c)
+56*a^3*b^5*(sin(d*x+c)^6/cos(d*x+c)+(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*co
s(d*x+c))+28*a^2*b^6*(sin(d*x+c)^7/cos(d*x+c)+(sin(d*x+c)^5+5/4*sin(d*x+c)^
3+15/8*sin(d*x+c))*cos(d*x+c)-15/8*d*x-15/8*c)+8*a*b^7*(sin(d*x+c)^8/cos(d*
x+c)+(16/5+sin(d*x+c)^6+6/5*sin(d*x+c)^4+8/5*sin(d*x+c)^2)*cos(d*x+c))+b^8*
(sin(d*x+c)^9/cos(d*x+c)+(sin(d*x+c)^7+7/6*sin(d*x+c)^5+35/24*sin(d*x+c)^3+
35/16*sin(d*x+c))*cos(d*x+c)-35/16*d*x-35/16*c))
```

Maxima [A]
time = 0.51, size = 348, normalized size = 1.00

$$-\frac{6720\,(dx+c-\tan(dx+c))a^6b^2+8400\left(3\,dx+3\,c-\frac{\tan(dx+c)}{\tan(dx+c)^2+1}-2\,\tan(dx+c)\right)a^4b^4+4480\left(\cos(dx+c)^3-\frac{3}{\cos(dx+c)}-6\,\cos(dx+c)\right)a^3b^5+840\left(15\,dx+15\,c-\frac{9\,\tan(dx+c)^3+7\,\tan(dx+c)}{\tan(dx+c)^4+2\,\tan(dx+c)^2+1}-8\,\tan(dx+c)\right)a^2b^6-384\left(\cos(dx+c)^5-5\,\cos(dx+c)^3+\frac{5}{\cos(dx+c)}+15\,\cos(dx+c)\right)ab^7+5\left(105\,dx+105\,c-\frac{87\,\tan(dx+c)^5+136\,\tan(dx+c)^3+57\,\tan(dx+c)}{\tan(dx+c)^6+3\,\tan(dx+c)^4+3\,\tan(dx+c)^2+1}-48\,\tan(dx+c)\right)b^8-13440\,a^5b^3\left(\frac{1}{\cos(dx+c)}+\cos(dx+c)\right)-240\,a^8\tan(dx+c)-\frac{1920\,a^7b}{\cos(dx+c)}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/240*(6720*(d*x + c - tan(d*x + c))*a^6*b^2 + 8400*(3*d*x + 3*c - tan(d*x
 + c)/(tan(d*x + c)^2 + 1) - 2*tan(d*x + c))*a^4*b^4 + 4480*(cos(d*x + c)^3
 - 3/cos(d*x + c) - 6*cos(d*x + c))*a^3*b^5 + 840*(15*d*x + 15*c - (9*tan(d
*x + c)^3 + 7*tan(d*x + c))/(tan(d*x + c)^4 + 2*tan(d*x + c)^2 + 1) - 8*tan
```

```
(d*x + c))*a^2*b^6 - 384*(cos(d*x + c)^5 - 5*cos(d*x + c)^3 + 5/cos(d*x + c
) + 15*cos(d*x + c))*a*b^7 + 5*(105*d*x + 105*c - (87*tan(d*x + c)^5 + 136*
tan(d*x + c)^3 + 57*tan(d*x + c))/(tan(d*x + c)^6 + 3*tan(d*x + c)^4 + 3*ta
n(d*x + c)^2 + 1) - 48*tan(d*x + c))*b^8 - 13440*a^5*b^3*(1/cos(d*x + c) +
cos(d*x + c)) - 240*a^8*tan(d*x + c) - 1920*a^7*b/cos(d*x + c))/d
```

Fricas [A]
time = 0.38, size = 266, normalized size = 0.76

$$\frac{384\,ab^7\cos(dx+c)^6+1920\,a^7b+13440\,a^5b^3+13440\,a^3b^5+1920\,ab^7-640\,(7\,a^3b^5+3\,ab^7)\cos(dx+c)^4-105\,(64\,a^6b^2+240\,a^4b^4+120\,a^2b^6+5\,b^8)dx\cos(dx+c)+1920\,(7\,a^5b^3+14\,a^3b^5+3\,ab^7)\cos(dx+c)^2+5\,(8\,b^8\cos(dx+c)^6+48\,a^8+1344\,a^6b^2+3360\,a^4b^4+1344\,a^2b^6+48\,b^8-2\,(168\,a^2b^6+19\,b^8)\cos(dx+c)^4+3\,(560\,a^4b^4+504\,a^2b^6+29\,b^8)\cos(dx+c)^2)\sin(dx+c)}{240\,d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/240*(384*a*b^7*cos(d*x + c)^6 + 1920*a^7*b + 13440*a^5*b^3 + 13440*a^3*b^
5 + 1920*a*b^7 - 640*(7*a^3*b^5 + 3*a*b^7)*cos(d*x + c)^4 - 105*(64*a^6*b^2
 + 240*a^4*b^4 + 120*a^2*b^6 + 5*b^8)*d*x*cos(d*x + c) + 1920*(7*a^5*b^3 +
14*a^3*b^5 + 3*a*b^7)*cos(d*x + c)^2 + 5*(8*b^8*cos(d*x + c)^6 + 48*a^8 + 1
344*a^6*b^2 + 3360*a^4*b^4 + 1344*a^2*b^6 + 48*b^8 - 2*(168*a^2*b^6 + 19*b^
8)*cos(d*x + c)^4 + 3*(560*a^4*b^4 + 504*a^2*b^6 + 29*b^8)*cos(d*x + c)^2)*
sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*sin(d*x+c))**8,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 799 vs. 2(335) = 670.
time = 6.57, size = 799, normalized size = 2.29

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -1/240*(105*(64*a^6*b^2 + 240*a^4*b^4 + 120*a^2*b^6 + 5*b^8)*(d*x + c) + 48
0*(a^8*tan(1/2*d*x + 1/2*c) + 28*a^6*b^2*tan(1/2*d*x + 1/2*c) + 70*a^4*b^4*
tan(1/2*d*x + 1/2*c) + 28*a^2*b^6*tan(1/2*d*x + 1/2*c) + b^8*tan(1/2*d*x +
1/2*c) + 8*a^7*b + 56*a^5*b^3 + 56*a^3*b^5 + 8*a*b^7)/(tan(1/2*d*x + 1/2*c)
^2 - 1) + 2*(8400*a^4*b^4*tan(1/2*d*x + 1/2*c)^11 + 5880*a^2*b^6*tan(1/2*d*
```

```
x + 1/2*c)^11 + 285*b^8*tan(1/2*d*x + 1/2*c)^11 - 13440*a^5*b^3*tan(1/2*d*x
 + 1/2*c)^10 - 13440*a^3*b^5*tan(1/2*d*x + 1/2*c)^10 - 1920*a*b^7*tan(1/2*d
*x + 1/2*c)^10 + 25200*a^4*b^4*tan(1/2*d*x + 1/2*c)^9 + 24360*a^2*b^6*tan(1
/2*d*x + 1/2*c)^9 + 1295*b^8*tan(1/2*d*x + 1/2*c)^9 - 67200*a^5*b^3*tan(1/2
*d*x + 1/2*c)^8 - 94080*a^3*b^5*tan(1/2*d*x + 1/2*c)^8 - 13440*a*b^7*tan(1/
2*d*x + 1/2*c)^8 + 16800*a^4*b^4*tan(1/2*d*x + 1/2*c)^7 + 18480*a^2*b^6*tan
(1/2*d*x + 1/2*c)^7 + 1650*b^8*tan(1/2*d*x + 1/2*c)^7 - 134400*a^5*b^3*tan(
1/2*d*x + 1/2*c)^6 - 224000*a^3*b^5*tan(1/2*d*x + 1/2*c)^6 - 42240*a*b^7*ta
n(1/2*d*x + 1/2*c)^6 - 16800*a^4*b^4*tan(1/2*d*x + 1/2*c)^5 - 18480*a^2*b^6
*tan(1/2*d*x + 1/2*c)^5 - 1650*b^8*tan(1/2*d*x + 1/2*c)^5 - 134400*a^5*b^3*
tan(1/2*d*x + 1/2*c)^4 - 241920*a^3*b^5*tan(1/2*d*x + 1/2*c)^4 - 49920*a*b^
7*tan(1/2*d*x + 1/2*c)^4 - 25200*a^4*b^4*tan(1/2*d*x + 1/2*c)^3 - 24360*a^2
*b^6*tan(1/2*d*x + 1/2*c)^3 - 1295*b^8*tan(1/2*d*x + 1/2*c)^3 - 67200*a^5*b
^3*tan(1/2*d*x + 1/2*c)^2 - 120960*a^3*b^5*tan(1/2*d*x + 1/2*c)^2 - 23424*a
*b^7*tan(1/2*d*x + 1/2*c)^2 - 8400*a^4*b^4*tan(1/2*d*x + 1/2*c) - 5880*a^2*
b^6*tan(1/2*d*x + 1/2*c) - 285*b^8*tan(1/2*d*x + 1/2*c) - 13440*a^5*b^3 - 2
2400*a^3*b^5 - 4224*a*b^7)/(tan(1/2*d*x + 1/2*c)^2 + 1)^6)/d
```

Mupad [B]
time = 7.73, size = 767, normalized size = 2.20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^8/cos(c + d*x)^2,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^8*(240*a^7*b + (1792*a^3*b^5)/3 + 1120*a^5*b^3) + tan(c
/2 + (d*x)/2)^10*(96*a^7*b + 224*a^5*b^3) + tan(c/2 + (d*x)/2)*(2*a^8 + (35
*b^8)/8 + 105*a^2*b^6 + 210*a^4*b^4 + 56*a^6*b^2) + (256*a*b^7)/5 + 16*a^7*
b + tan(c/2 + (d*x)/2)^2*(256*a*b^7 + 96*a^7*b + (4480*a^3*b^5)/3 + 1120*a^
5*b^3) + tan(c/2 + (d*x)/2)^4*((2304*a*b^7)/5 + 240*a^7*b + 2688*a^3*b^5 +
2240*a^5*b^3) + tan(c/2 + (d*x)/2)^6*(256*a*b^7 + 320*a^7*b + (6272*a^3*b^5
)/3 + 2240*a^5*b^3) + tan(c/2 + (d*x)/2)^13*(2*a^8 + (35*b^8)/8 + 105*a^2*b
^6 + 210*a^4*b^4 + 56*a^6*b^2) + tan(c/2 + (d*x)/2)^3*(12*a^8 + (245*b^8)/1
2 + 490*a^2*b^6 + 980*a^4*b^4 + 336*a^6*b^2) + tan(c/2 + (d*x)/2)^11*(12*a^
8 + (245*b^8)/12 + 490*a^2*b^6 + 980*a^4*b^4 + 336*a^6*b^2) + tan(c/2 + (d*
x)/2)^5*(30*a^8 + (791*b^8)/24 + 791*a^2*b^6 + 2030*a^4*b^4 + 840*a^6*b^2)
+ tan(c/2 + (d*x)/2)^9*(30*a^8 + (791*b^8)/24 + 791*a^2*b^6 + 2030*a^4*b^4
+ 840*a^6*b^2) + tan(c/2 + (d*x)/2)^7*(40*a^8 + (25*b^8)/2 + 812*a^2*b^6 +
2520*a^4*b^4 + 1120*a^6*b^2) + (896*a^3*b^5)/3 + 224*a^5*b^3 + 16*a^7*b*tan
(c/2 + (d*x)/2)^12)/(d*(5*tan(c/2 + (d*x)/2)^2 + 9*tan(c/2 + (d*x)/2)^4 + 5
*tan(c/2 + (d*x)/2)^6 - 5*tan(c/2 + (d*x)/2)^8 - 9*tan(c/2 + (d*x)/2)^10 -
5*tan(c/2 + (d*x)/2)^12 - tan(c/2 + (d*x)/2)^14 + 1)) - (7*b^2*atan((7*b^2*
tan(c/2 + (d*x)/2)*(64*a^6 + 5*b^6 + 120*a^2*b^4 + 240*a^4*b^2))/(35*b^8 +
840*a^2*b^6 + 1680*a^4*b^4 + 448*a^6*b^2))*(64*a^6 + 5*b^6 + 120*a^2*b^4 +
240*a^4*b^2))/(8*d)
```

3.421 $\int \sec^4(c+dx)(a+b\sin(c+dx))^8\,dx$

Optimal. Leaf size=369

$$\frac{35}{8}b^4\left(16a^4+16a^2b^2+b^4\right)x+\frac{ab(8a^6-104a^4b^2-803a^2b^4-256b^6)\cos(c+dx)}{6d}+\frac{b^2(16a^6-200a^4b^2-866a^2$$

```
[Out] 35/8*b^4*(16*a^4+16*a^2*b^2+b^4)*x+1/6*a*b*(8*a^6-104*a^4*b^2-803*a^2*b^4-2
56*b^6)*cos(d*x+c)/d+1/24*b^2*(16*a^6-200*a^4*b^2-866*a^2*b^4-105*b^6)*cos(
d*x+c)*sin(d*x+c)/d+1/12*a*b*(8*a^4-88*a^2*b^2-151*b^4)*cos(d*x+c)*(a+b*sin
(d*x+c))^2/d+1/12*b*(8*a^4-72*a^2*b^2-35*b^4)*cos(d*x+c)*(a+b*sin(d*x+c))^3
/d+1/3*a*b*(2*a^2-13*b^2)*cos(d*x+c)*(a+b*sin(d*x+c))^4/d+1/3*b*(2*a^2-7*b^
2)*cos(d*x+c)*(a+b*sin(d*x+c))^5/d+1/3*sec(d*x+c)^3*(b+a*sin(d*x+c))*(a+b*s
in(d*x+c))^7/d-1/3*sec(d*x+c)*(a+b*sin(d*x+c))^6*(5*a*b-(2*a^2-7*b^2)*sin(d
*x+c))/d
```

Rubi [A]
time = 0.43, antiderivative size = 369, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2770, 2940, 2832, 2813}

$$\frac{b(2a^2-7b^2)\cos(c+dx)(a+b\sin(c+dx))^5}{3d}+\frac{ab(2a^2-13b^2)\cos(c+dx)(a+b\sin(c+dx))^4}{3d}-\frac{\sec(c+dx)\left(5ab-(2a^2-7b^2)\sin(c+dx)\right)(a+b\sin(c+dx))^6}{3d}+\frac{b(8a^4-72a^2b^2-35b^4)\cos(c+dx)(a+b\sin(c+dx))^3}{12d}+\frac{ab(8a^4-88a^2b^2-151b^4)\cos(c+dx)(a+b\sin(c+dx))^2}{12d}+\frac{35}{8}b^4x\left(16a^4+16a^2b^2+b^4\right)+\frac{ab(8a^6-104a^4b^2-803a^2b^4-256b^6)\cos(c+dx)}{6d}+\frac{b^2(16a^6-200a^4b^2-866a^2b^4-105b^6)\sin(c+dx)\cos(c+dx)}{24d}+\frac{\sec^3(c+dx)(a\sin(c+dx)+b)(a+b\sin(c+dx))^7}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + b*Sin[c + d*x])^8,x]
```

```
[Out] (35*b^4*(16*a^4 + 16*a^2*b^2 + b^4)*x)/8 + (a*b*(8*a^6 - 104*a^4*b^2 - 803*
a^2*b^4 - 256*b^6)*Cos[c + d*x])/(6*d) + (b^2*(16*a^6 - 200*a^4*b^2 - 866*a
^2*b^4 - 105*b^6)*Cos[c + d*x]*Sin[c + d*x])/(24*d) + (a*b*(8*a^4 - 88*a^2*
b^2 - 151*b^4)*Cos[c + d*x]*(a + b*Sin[c + d*x])^2)/(12*d) + (b*(8*a^4 - 72
*a^2*b^2 - 35*b^4)*Cos[c + d*x]*(a + b*Sin[c + d*x])^3)/(12*d) + (a*b*(2*a^
2 - 13*b^2)*Cos[c + d*x]*(a + b*Sin[c + d*x])^4)/(3*d) + (b*(2*a^2 - 7*b^2)
*Cos[c + d*x]*(a + b*Sin[c + d*x])^5)/(3*d) + (Sec[c + d*x]^3*(b + a*Sin[c
+ d*x])*(a + b*Sin[c + d*x])^7)/(3*d) - (Sec[c + d*x]*(a + b*Sin[c + d*x])^
6*(5*a*b - (2*a^2 - 7*b^2)*Sin[c + d*x]))/(3*d)
```

Rule 2770

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x
])^(m - 1)*((b + a*Sin[e + f*x])/(f*g*(p + 1))), x] + Dist[1/(g^2*(p + 1)),
 Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a
^2*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g}
, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p
] || IntegerQ[m])
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rule 2940

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(g*
Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x])^m*((d + c*Sin[e + f*x])/(f*g*(p
 + 1))), x] + Dist[1/(g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin
[e + f*x])^(m - 1)*Simp[a*c*(p + 2) + b*d*m + b*c*(m + p + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && GtQ[
m, 0] && LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] &
& SimplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int \sec^4(c+dx)(a+b\sin(c+dx))^8\,dx = \frac{\sec^3(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^7}{3d} - \frac{1}{3}\int \sec^2(c+$$

$$= \frac{\sec^3(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^7}{3d} - \frac{\sec(c+dx)(a}{}$$

$$= \frac{b(2a^2-7b^2)\cos(c+dx)(a+b\sin(c+dx))^5}{3d} + \frac{\sec^3(c+dx)(b+a\sin}{}$$

$$= \frac{ab(2a^2-13b^2)\cos(c+dx)(a+b\sin(c+dx))^4}{3d} + \frac{b(2a^2-7b^2)\cos(c}{}$$

$$= \frac{b(8a^4-72a^2b^2-35b^4)\cos(c+dx)(a+b\sin(c+dx))^3}{12d} + \frac{ab(2a^2-1}{}$$

$$= \frac{ab(8a^4-88a^2b^2-151b^4)\cos(c+dx)(a+b\sin(c+dx))^2}{12d} + \frac{b(8a^4-}{}$$

$$= \frac{35}{8}b^4\left(16a^4+16a^2b^2+b^4\right)x + \frac{ab(8a^6-104a^4b^2-803a^2b^4-256b^6)}{6d}$$

Mathematica [A]
time = 1.15, size = 414, normalized size = 1.12

$$\frac{\sec^3(c+dx)\left(2048a^7b-7168a^5b^3-44800a^3b^5-13440ab^7+40320a^4b^4(c+dx)\cos(c+dx)+40320a^2b^6(c+dx)\cos(c+dx)+2520b^8(c+dx)\cos(c+dx)-21504a^5b^3\cos(2(c+dx))-64512a^3b^5\cos(2(c+dx))-17472ab^7\cos(2(c+dx))+13440a^4b^4(c+dx)\cos(3(c+dx))+13440a^2b^6(c+dx)\cos(3(c+dx))+840b^8(c+dx)\cos(3(c+dx))-5376a^3b^5\cos(4(c+dx))-1920ab^7\cos(4(c+dx))+64ab^7\cos(6(c+dx))+384a^8\sin(c+dx)+5376a^6b^2\sin(c+dx)-6720a^2b^6\sin(c+dx)-525b^8\sin(c+dx)+128a^8\sin(3(c+dx))-1792a^6b^2\sin(3(c+dx))-17920a^4b^4\sin(3(c+dx))-14560a^2b^6\sin(3(c+dx))-847b^8\sin(3(c+dx))-672a^2b^6\sin(5(c+dx))-63b^8\sin(5(c+dx))+3b^8\sin(7(c+dx))\right)}{768d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + b*Sin[c + d*x])^8,x]
```

```
[Out] (Sec[c + d*x]^3*(2048*a^7*b - 7168*a^5*b^3 - 44800*a^3*b^5 - 13440*a*b^7 +
40320*a^4*b^4*(c + d*x)*Cos[c + d*x] + 40320*a^2*b^6*(c + d*x)*Cos[c + d*x]
 + 2520*b^8*(c + d*x)*Cos[c + d*x] - 21504*a^5*b^3*Cos[2*(c + d*x)] - 64512
*a^3*b^5*Cos[2*(c + d*x)] - 17472*a*b^7*Cos[2*(c + d*x)] + 13440*a^4*b^4*(c
 + d*x)*Cos[3*(c + d*x)] + 13440*a^2*b^6*(c + d*x)*Cos[3*(c + d*x)] + 840*b
^8*(c + d*x)*Cos[3*(c + d*x)] - 5376*a^3*b^5*Cos[4*(c + d*x)] - 1920*a*b^7*
Cos[4*(c + d*x)] + 64*a*b^7*Cos[6*(c + d*x)] + 384*a^8*Sin[c + d*x] + 5376*
a^6*b^2*Sin[c + d*x] - 6720*a^2*b^6*Sin[c + d*x] - 525*b^8*Sin[c + d*x] + 1
28*a^8*Sin[3*(c + d*x)] - 1792*a^6*b^2*Sin[3*(c + d*x)] - 17920*a^4*b^4*Sin
[3*(c + d*x)] - 14560*a^2*b^6*Sin[3*(c + d*x)] - 847*b^8*Sin[3*(c + d*x)] -
 672*a^2*b^6*Sin[5*(c + d*x)] - 63*b^8*Sin[5*(c + d*x)] + 3*b^8*Sin[7*(c +
d*x)]))/(768*d)
```

Maple [A]
time = 0.48, size = 495, normalized size = 1.34 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+b*sin(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^8*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+8/3*a^7*b/cos(d*x+c)^3+28/3*a^
6*b^2*sin(d*x+c)^3/cos(d*x+c)^3+56*a^5*b^3*(1/3*sin(d*x+c)^4/cos(d*x+c)^3-1
/3*sin(d*x+c)^4/cos(d*x+c)-1/3*(2+sin(d*x+c)^2)*cos(d*x+c))+70*a^4*b^4*(1/3
*tan(d*x+c)^3-tan(d*x+c)+d*x+c)+56*a^3*b^5*(1/3*sin(d*x+c)^6/cos(d*x+c)^3-s
in(d*x+c)^6/cos(d*x+c)-(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c))+28*a
^2*b^6*(1/3*sin(d*x+c)^7/cos(d*x+c)^3-4/3*sin(d*x+c)^7/cos(d*x+c)-4/3*(sin(
d*x+c)^5+5/4*sin(d*x+c)^3+15/8*sin(d*x+c))*cos(d*x+c)+5/2*d*x+5/2*c)+8*a*b^
7*(1/3*sin(d*x+c)^8/cos(d*x+c)^3-5/3*sin(d*x+c)^8/cos(d*x+c)-5/3*(16/5+sin(
d*x+c)^6+6/5*sin(d*x+c)^4+8/5*sin(d*x+c)^2)*cos(d*x+c))+b^8*(1/3*sin(d*x+c)
^9/cos(d*x+c)^3-2*sin(d*x+c)^9/cos(d*x+c)-2*(sin(d*x+c)^7+7/6*sin(d*x+c)^5+
35/24*sin(d*x+c)^3+35/16*sin(d*x+c))*cos(d*x+c)+35/8*d*x+35/8*c))
```

Maxima [A]
time = 0.49, size = 328, normalized size = 0.89

$$\frac{224\,a^6b^2\tan\left(dx+c\right)^3+8\left(\tan\left(dx+c\right)^3+3\,\tan\left(dx+c\right)\right)a^8+560\left(\tan\left(dx+c\right)^3+3\,dx+3\,c-3\,\tan\left(dx+c\right)\right)a^4b^4+112\left(2\,\tan\left(dx+c\right)^3+15\,dx+15\,c-\frac{3\,\tan(dx+c)}{\tan(dx+c)^2+1}-12\,\tan\left(dx+c\right)\right)a^2b^6+64\left(\cos\left(dx+c\right)^3-\frac{9\,\cos(dx+c)^2-1}{\cos(dx+c)^3}-9\,\cos\left(dx+c\right)\right)ab^7+\left(8\,\tan\left(dx+c\right)^3+105\,dx+105\,c-\frac{3\left(13\,\tan(dx+c)^3+11\,\tan(dx+c)\right)}{\tan(dx+c)^4+2\,\tan(dx+c)^2+1}-72\,\tan\left(dx+c\right)\right)b^8-448\,a^3b^5\left(\frac{6\,\cos(dx+c)^2-1}{\cos(dx+c)^3}+3\,\cos\left(dx+c\right)\right)-\frac{448\left(3\,\cos(dx+c)^2-1\right)a^5b^3}{\cos(dx+c)^3}+\frac{64\,a^7b}{\cos(dx+c)^3}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 1/24*(224*a^6*b^2*tan(d*x + c)^3 + 8*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^8
+ 560*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c))*a^4*b^4 + 112*(2*tan(
d*x + c)^3 + 15*d*x + 15*c - 3*tan(d*x + c)/(tan(d*x + c)^2 + 1) - 12*tan(d
*x + c))*a^2*b^6 + 64*(cos(d*x + c)^3 - (9*cos(d*x + c)^2 - 1)/cos(d*x + c)
^3 - 9*cos(d*x + c))*a*b^7 + (8*tan(d*x + c)^3 + 105*d*x + 105*c - 3*(13*ta
n(d*x + c)^3 + 11*tan(d*x + c))/(tan(d*x + c)^4 + 2*tan(d*x + c)^2 + 1) - 7
2*tan(d*x + c))*b^8 - 448*a^3*b^5*((6*cos(d*x + c)^2 - 1)/cos(d*x + c)^3 +
3*cos(d*x + c)) - 448*(3*cos(d*x + c)^2 - 1)*a^5*b^3/cos(d*x + c)^3 + 64*a^
7*b/cos(d*x + c)^3)/d
```

<u>Fricas</u> **[A]**
time = 0.38, size = 268, normalized size = 0.73

$$\frac{64\,ab^{7}\cos\left(dx+c\right)^{6}+64\,a^{7}b+448\,a^{5}b^{3}+448\,a^{3}b^{5}+64\,ab^{7}+105\,\left(16\,a^{4}b^{4}+16\,a^{2}b^{6}+b^{8}\right)dx\cos\left(dx+c\right)^{3}-192\,\left(7\,a^{3}b^{5}+3\,ab^{7}\right)\cos\left(dx+c\right)^{4}-192\,\left(7\,a^{5}b^{3}+14\,a^{3}b^{5}+3\,ab^{7}\right)\cos\left(dx+c\right)^{2}+\left(6\,b^{8}\cos\left(dx+c\right)^{6}+8\,a^{8}+224\,a^{6}b^{2}+560\,a^{4}b^{4}+224\,a^{2}b^{6}+8\,b^{8}-3\,\left(112\,a^{2}b^{6}+13\,b^{8}\right)\cos\left(dx+c\right)^{4}+16\,\left(a^{8}-14\,a^{6}b^{2}-140\,a^{4}b^{4}-98\,a^{2}b^{6}-5\,b^{8}\right)\cos\left(dx+c\right)^{2}\right)\sin\left(dx+c\right)}{24\,d\cos\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/24*(64*a*b^7*cos(d*x + c)^6 + 64*a^7*b + 448*a^5*b^3 + 448*a^3*b^5 + 64*a
*b^7 + 105*(16*a^4*b^4 + 16*a^2*b^6 + b^8)*d*x*cos(d*x + c)^3 - 192*(7*a^3*
b^5 + 3*a*b^7)*cos(d*x + c)^4 - 192*(7*a^5*b^3 + 14*a^3*b^5 + 3*a*b^7)*cos(
d*x + c)^2 + (6*b^8*cos(d*x + c)^6 + 8*a^8 + 224*a^6*b^2 + 560*a^4*b^4 + 22
4*a^2*b^6 + 8*b^8 - 3*(112*a^2*b^6 + 13*b^8)*cos(d*x + c)^4 + 16*(a^8 - 14*
a^6*b^2 - 140*a^4*b^4 - 98*a^2*b^6 - 5*b^8)*cos(d*x + c)^2)*sin(d*x + c))/(
d*cos(d*x + c)^3)
```

<u>Sympy</u> **[F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+b*sin(d*x+c))**8,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8570 deep
```

<u>Giac</u> **[A]**
time = 4.87, size = 684, normalized size = 1.85

$$\frac{105\,\left(16\,a^{4}b^{4}+16\,a^{2}b^{6}+b^{8}\right)\left(dx+c\right)-\frac{16\,\left(3\,a^{8}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-210\,a^{4}b^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-168\,a^{2}b^{6}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-9\,b^{8}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+24\,a^{7}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-168\,a^{3}b^{5}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-48\,ab^{7}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-2\,a^{8}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+112\,a^{6}b^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+700\,a^{4}b^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+448\,a^{2}b^{6}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+22\,b^{8}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+336\,a^{5}b^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+672\,a^{3}b^{5}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+144\,ab^{7}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+3\,a^{8}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-210\,a^{4}b^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-168\,a^{2}b^{6}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-9\,b^{8}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+8\,a^{7}b-112\,a^{5}b^{3}-280\,a^{3}b^{5}-64\,ab^{7}\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{3}}+\frac{2\,\left(336\,a^{2}b^{6}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+33\,b^{8}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-1344\,a^{3}b^{5}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-384\,ab^{7}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+336\,a^{2}b^{6}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+57\,b^{8}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-4032\,a^{3}b^{5}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-1536\,ab^{7}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-336\,a^{2}b^{6}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-57\,b^{8}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-4032\,a^{3}b^{5}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1664\,ab^{7}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-336\,a^{2}b^{6}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-33\,b^{8}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1344\,a^{3}b^{5}-512\,ab^{7}\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{4}}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/24*(105*(16*a^4*b^4 + 16*a^2*b^6 + b^8)*(d*x + c) - 16*(3*a^8*tan(1/2*d*x
 + 1/2*c)^5 - 210*a^4*b^4*tan(1/2*d*x + 1/2*c)^5 - 168*a^2*b^6*tan(1/2*d*x
```

```
+ 1/2*c)^5 - 9*b^8*tan(1/2*d*x + 1/2*c)^5 + 24*a^7*b*tan(1/2*d*x + 1/2*c)^4
 - 168*a^3*b^5*tan(1/2*d*x + 1/2*c)^4 - 48*a*b^7*tan(1/2*d*x + 1/2*c)^4 - 2
*a^8*tan(1/2*d*x + 1/2*c)^3 + 112*a^6*b^2*tan(1/2*d*x + 1/2*c)^3 + 700*a^4*
b^4*tan(1/2*d*x + 1/2*c)^3 + 448*a^2*b^6*tan(1/2*d*x + 1/2*c)^3 + 22*b^8*ta
n(1/2*d*x + 1/2*c)^3 + 336*a^5*b^3*tan(1/2*d*x + 1/2*c)^2 + 672*a^3*b^5*tan
(1/2*d*x + 1/2*c)^2 + 144*a*b^7*tan(1/2*d*x + 1/2*c)^2 + 3*a^8*tan(1/2*d*x
+ 1/2*c) - 210*a^4*b^4*tan(1/2*d*x + 1/2*c) - 168*a^2*b^6*tan(1/2*d*x + 1/2
*c) - 9*b^8*tan(1/2*d*x + 1/2*c) + 8*a^7*b - 112*a^5*b^3 - 280*a^3*b^5 - 64
*a*b^7)/(tan(1/2*d*x + 1/2*c)^2 - 1)^3 + 2*(336*a^2*b^6*tan(1/2*d*x + 1/2*c
)^7 + 33*b^8*tan(1/2*d*x + 1/2*c)^7 - 1344*a^3*b^5*tan(1/2*d*x + 1/2*c)^6 -
 384*a*b^7*tan(1/2*d*x + 1/2*c)^6 + 336*a^2*b^6*tan(1/2*d*x + 1/2*c)^5 + 57
*b^8*tan(1/2*d*x + 1/2*c)^5 - 4032*a^3*b^5*tan(1/2*d*x + 1/2*c)^4 - 1536*a*
b^7*tan(1/2*d*x + 1/2*c)^4 - 336*a^2*b^6*tan(1/2*d*x + 1/2*c)^3 - 57*b^8*ta
n(1/2*d*x + 1/2*c)^3 - 4032*a^3*b^5*tan(1/2*d*x + 1/2*c)^2 - 1664*a*b^7*tan
(1/2*d*x + 1/2*c)^2 - 336*a^2*b^6*tan(1/2*d*x + 1/2*c) - 33*b^8*tan(1/2*d*x
 + 1/2*c) - 1344*a^3*b^5 - 512*a*b^7)/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

Mupad [B]
time = 7.82, size = 726, normalized size = 1.97

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^8/cos(c + d*x)^4,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^8*((304*a^7*b)/3 + (1792*a^3*b^5)/3 + (2464*a^5*b^3)/3)
 + tan(c/2 + (d*x)/2)^10*(64*a^7*b + 224*a^5*b^3) - (256*a*b^7)/3 + (16*a^7
*b)/3 - tan(c/2 + (d*x)/2)*((35*b^8)/4 - 2*a^8 + 140*a^2*b^6 + 140*a^4*b^4)
 - tan(c/2 + (d*x)/2)^2*((256*a*b^7)/3 - (64*a^7*b)/3 + (896*a^3*b^5)/3 + (
224*a^5*b^3)/3) + tan(c/2 + (d*x)/2)^4*(256*a*b^7 + 48*a^7*b + 896*a^3*b^5
+ 448*a^5*b^3) + tan(c/2 + (d*x)/2)^6*(256*a*b^7 + (256*a^7*b)/3 + (4480*a^
3*b^5)/3 + (3136*a^5*b^3)/3) - tan(c/2 + (d*x)/2)^3*((35*b^8)/6 - (20*a^8)/
3 + (280*a^2*b^6)/3 + (280*a^4*b^4)/3 - (224*a^6*b^2)/3) - tan(c/2 + (d*x)/
2)^11*((35*b^8)/6 - (20*a^8)/3 + (280*a^2*b^6)/3 + (280*a^4*b^4)/3 - (224*a
^6*b^2)/3) + tan(c/2 + (d*x)/2)^7*(8*a^8 + 17*b^8 + 784*a^2*b^6 + 1680*a^4*
b^4 + 448*a^6*b^2) + tan(c/2 + (d*x)/2)^5*((26*a^8)/3 + (329*b^8)/12 + (131
6*a^2*b^6)/3 + (2660*a^4*b^4)/3 + (896*a^6*b^2)/3) + tan(c/2 + (d*x)/2)^9*(
(26*a^8)/3 + (329*b^8)/12 + (1316*a^2*b^6)/3 + (2660*a^4*b^4)/3 + (896*a^6*
b^2)/3) - (896*a^3*b^5)/3 - (224*a^5*b^3)/3 - tan(c/2 + (d*x)/2)^13*((35*b^
8)/4 - 2*a^8 + 140*a^2*b^6 + 140*a^4*b^4) + 16*a^7*b*tan(c/2 + (d*x)/2)^12)
/(d*(tan(c/2 + (d*x)/2)^2 - 3*tan(c/2 + (d*x)/2)^4 - 3*tan(c/2 + (d*x)/2)^6
 + 3*tan(c/2 + (d*x)/2)^8 + 3*tan(c/2 + (d*x)/2)^10 - tan(c/2 + (d*x)/2)^12
 - tan(c/2 + (d*x)/2)^14 + 1)) + (35*b^4*atan((35*b^4*tan(c/2 + (d*x)/2)*(1
6*a^4 + b^4 + 16*a^2*b^2))/(35*b^8 + 560*a^2*b^6 + 560*a^4*b^4))*(16*a^4 +
b^4 + 16*a^2*b^2))/(4*d)
```

3.422 $\int \sec^6(c+dx)(a+b\sin(c+dx))^8\,dx$

Optimal. Leaf size=381

$$-\frac{7}{2}b^6\left(8a^2+b^2\right)x+\frac{2ab(8a^6-48a^4b^2+163a^2b^4+192b^6)\cos(c+dx)}{15d}+\frac{b^2(16a^6-88a^4b^2+282a^2b^4+105b^6)\cos(c+dx)}{30d}$$

```
[Out] -7/2*b^6*(8*a^2+b^2)*x+2/15*a*b*(8*a^6-48*a^4*b^2+163*a^2*b^4+192*b^6)*cos(
d*x+c)/d+1/30*b^2*(16*a^6-88*a^4*b^2+282*a^2*b^4+105*b^6)*cos(d*x+c)*sin(d*
x+c)/d+1/15*a*b*(8*a^4-32*a^2*b^2+87*b^4)*cos(d*x+c)*(a+b*sin(d*x+c))^2/d+1
/15*b*(8*a^4-16*a^2*b^2+35*b^4)*cos(d*x+c)*(a+b*sin(d*x+c))^3/d+4/15*a*b*(2
*a^2+b^2)*cos(d*x+c)*(a+b*sin(d*x+c))^4/d+1/5*sec(d*x+c)^5*(b+a*sin(d*x+c))
*(a+b*sin(d*x+c))^7/d-1/15*sec(d*x+c)^3*(a+b*sin(d*x+c))^6*(3*a*b-(4*a^2-7*
b^2)*sin(d*x+c))/d-4/15*sec(d*x+c)*(a+b*sin(d*x+c))^5*(b*(4*a^2-7*b^2)-a*(2
*a^2+b^2)*sin(d*x+c))/d
```

Rubi [A]
time = 0.48, antiderivative size = 381, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2770, 2940, 2832, 2813}

$$\frac{4ab(2a^2+b^2)\cos(c+dx)(a+b\sin(c+dx))^4}{15d}-\frac{\sec^3(c+dx)\left(3ab-(4a^2-7b^2)\sin(c+dx)\right)(a+b\sin(c+dx))^6}{15d}-\frac{4\sec(c+dx)\left(b(4a^2-7b^2)-a(2a^2+b^2)\sin(c+dx)\right)(a+b\sin(c+dx))^5}{15d}-\frac{7}{2}b^6x\left(8a^2+b^2\right)+\frac{b(8a^4-16a^2b^2+35b^4)\cos(c+dx)(a+b\sin(c+dx))^3}{15d}+\frac{ab(8a^4-32a^2b^2+87b^4)\cos(c+dx)(a+b\sin(c+dx))^2}{15d}+\frac{2ab(8a^6-48a^4b^2+163a^2b^4+192b^6)\cos(c+dx)}{15d}+\frac{b^2(16a^6-88a^4b^2+282a^2b^4+105b^6)\sin(c+dx)\cos(c+dx)}{30d}+\frac{\sec^5(c+dx)(a\sin(c+dx)+b)(a+b\sin(c+dx))^7}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6*(a + b*Sin[c + d*x])^8,x]
```

```
[Out] (-7*b^6*(8*a^2 + b^2)*x)/2 + (2*a*b*(8*a^6 - 48*a^4*b^2 + 163*a^2*b^4 + 192
*b^6)*Cos[c + d*x])/(15*d) + (b^2*(16*a^6 - 88*a^4*b^2 + 282*a^2*b^4 + 105*
b^6)*Cos[c + d*x]*Sin[c + d*x])/(30*d) + (a*b*(8*a^4 - 32*a^2*b^2 + 87*b^4)
*Cos[c + d*x]*(a + b*Sin[c + d*x])^2)/(15*d) + (b*(8*a^4 - 16*a^2*b^2 + 35*
b^4)*Cos[c + d*x]*(a + b*Sin[c + d*x])^3)/(15*d) + (4*a*b*(2*a^2 + b^2)*Cos
[c + d*x]*(a + b*Sin[c + d*x])^4)/(15*d) + (Sec[c + d*x]^5*(b + a*Sin[c + d
*x])*(a + b*Sin[c + d*x])^7)/(5*d) - (Sec[c + d*x]^3*(a + b*Sin[c + d*x])^6
*(3*a*b - (4*a^2 - 7*b^2)*Sin[c + d*x]))/(15*d) - (4*Sec[c + d*x]*(a + b*Si
n[c + d*x])^5*(b*(4*a^2 - 7*b^2) - a*(2*a^2 + b^2)*Sin[c + d*x]))/(15*d)
```

Rule 2770

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x
])^(m - 1)*((b + a*Sin[e + f*x])/(f*g*(p + 1))), x] + Dist[1/(g^2*(p + 1)),
 Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a
^2*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g}
, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p
] || IntegerQ[m])
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rule 2940

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(g*
Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x])^m*((d + c*Sin[e + f*x])/(f*g*(p
 + 1))), x] + Dist[1/(g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin
[e + f*x])^(m - 1)*Simp[a*c*(p + 2) + b*d*m + b*c*(m + p + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && GtQ[
m, 0] && LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] &
& SimplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int \sec^6(c+dx)(a+b\sin(c+dx))^8\,dx = \frac{\sec^5(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^7}{5d} - \frac{1}{5}\int \sec^4(c+$$

$$= \frac{\sec^5(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^7}{5d} - \frac{\sec^3(c+dx)}{}$$

$$= \frac{\sec^5(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^7}{5d} - \frac{\sec^3(c+dx)}{}$$

$$= \frac{4ab(2a^2+b^2)\cos(c+dx)(a+b\sin(c+dx))^4}{15d} + \frac{\sec^5(c+dx)(b+a}{}$$

$$= \frac{b(8a^4-16a^2b^2+35b^4)\cos(c+dx)(a+b\sin(c+dx))^3}{15d} + \frac{4ab(2a^2}{}$$

$$= \frac{ab(8a^4-32a^2b^2+87b^4)\cos(c+dx)(a+b\sin(c+dx))^2}{15d} + \frac{b(8a^4-}{}$$

$$= -\frac{7}{2}b^6\left(8a^2+b^2\right)x + \frac{2ab(8a^6-48a^4b^2+163a^2b^4+192b^6)\cos(c+}{15d}$$

Mathematica [A]
time = 1.31, size = 472, normalized size = 1.24

$$\frac{\sec^5(c+dx)\left(3072a^7b+3584a^5b^3+25984a^3b^5+17472ab^7-33600a^2b^6(c+dx)\cos(c+dx)-4200b^8(c+dx)\cos(c+dx)-17920a^5b^3\cos(2(c+dx))+17920a^3b^5\cos(2(c+dx))+22560ab^7\cos(2(c+dx))-16800a^2b^6(c+dx)\cos(3(c+dx))-2100b^8(c+dx)\cos(3(c+dx))+13440a^3b^5\cos(4(c+dx))+8640ab^7\cos(4(c+dx))-3360a^2b^6(c+dx)\cos(5(c+dx))-420b^8(c+dx)\cos(5(c+dx))+480ab^7\cos(6(c+dx))+640a^8\sin(c+dx)+8960a^6b^2\sin(c+dx)+16800a^4b^4\sin(c+dx)+11200a^2b^6\sin(c+dx)+875b^8\sin(c+dx)+320a^8\sin(3(c+dx))-2240a^6b^2\sin(3(c+dx))-8400a^4b^4\sin(3(c+dx))+5600a^2b^6\sin(3(c+dx))+1015b^8\sin(3(c+dx))+64a^8\sin(5(c+dx))-448a^6b^2\sin(5(c+dx))+1680a^4b^4\sin(5(c+dx))+5152a^2b^6\sin(5(c+dx))+539b^8\sin(5(c+dx))+15b^8\sin(7(c+dx))\right)}{1920d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6*(a + b*Sin[c + d*x])^8,x]
```

```
[Out] (Sec[c + d*x]^5*(3072*a^7*b + 3584*a^5*b^3 + 25984*a^3*b^5 + 17472*a*b^7 -
33600*a^2*b^6*(c + d*x)*Cos[c + d*x] - 4200*b^8*(c + d*x)*Cos[c + d*x] - 17
920*a^5*b^3*Cos[2*(c + d*x)] + 17920*a^3*b^5*Cos[2*(c + d*x)] + 22560*a*b^7
*Cos[2*(c + d*x)] - 16800*a^2*b^6*(c + d*x)*Cos[3*(c + d*x)] - 2100*b^8*(c
+ d*x)*Cos[3*(c + d*x)] + 13440*a^3*b^5*Cos[4*(c + d*x)] + 8640*a*b^7*Cos[4
*(c + d*x)] - 3360*a^2*b^6*(c + d*x)*Cos[5*(c + d*x)] - 420*b^8*(c + d*x)*C
os[5*(c + d*x)] + 480*a*b^7*Cos[6*(c + d*x)] + 640*a^8*Sin[c + d*x] + 8960*
a^6*b^2*Sin[c + d*x] + 16800*a^4*b^4*Sin[c + d*x] + 11200*a^2*b^6*Sin[c + d
*x] + 875*b^8*Sin[c + d*x] + 320*a^8*Sin[3*(c + d*x)] - 2240*a^6*b^2*Sin[3*
(c + d*x)] - 8400*a^4*b^4*Sin[3*(c + d*x)] + 5600*a^2*b^6*Sin[3*(c + d*x)]
+ 1015*b^8*Sin[3*(c + d*x)] + 64*a^8*Sin[5*(c + d*x)] - 448*a^6*b^2*Sin[5*(
c + d*x)] + 1680*a^4*b^4*Sin[5*(c + d*x)] + 5152*a^2*b^6*Sin[5*(c + d*x)] +
 539*b^8*Sin[5*(c + d*x)] + 15*b^8*Sin[7*(c + d*x)]))/(1920*d)
```

Maple [A]
time = 0.56, size = 544, normalized size = 1.43 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(a+b*sin(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^8*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c)+8/5*a^7*b/c
os(d*x+c)^5+28*a^6*b^2*(1/5*sin(d*x+c)^3/cos(d*x+c)^5+2/15*sin(d*x+c)^3/cos
(d*x+c)^3)+56*a^5*b^3*(1/5*sin(d*x+c)^4/cos(d*x+c)^5+1/15*sin(d*x+c)^4/cos(
d*x+c)^3-1/15*sin(d*x+c)^4/cos(d*x+c)-1/15*(2+sin(d*x+c)^2)*cos(d*x+c))+14*
a^4*b^4*sin(d*x+c)^5/cos(d*x+c)^5+56*a^3*b^5*(1/5*sin(d*x+c)^6/cos(d*x+c)^5
-1/15*sin(d*x+c)^6/cos(d*x+c)^3+1/5*sin(d*x+c)^6/cos(d*x+c)+1/5*(8/3+sin(d*
x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c))+28*a^2*b^6*(1/5*tan(d*x+c)^5-1/3*tan(d
*x+c)^3+tan(d*x+c)-d*x-c)+8*a*b^7*(1/5*sin(d*x+c)^8/cos(d*x+c)^5-1/5*sin(d*
x+c)^8/cos(d*x+c)^3+sin(d*x+c)^8/cos(d*x+c)+(16/5+sin(d*x+c)^6+6/5*sin(d*x+
c)^4+8/5*sin(d*x+c)^2)*cos(d*x+c))+b^8*(1/5*sin(d*x+c)^9/cos(d*x+c)^5-4/15*
sin(d*x+c)^9/cos(d*x+c)^3+8/5*sin(d*x+c)^9/cos(d*x+c)+8/5*(sin(d*x+c)^7+7/6
*sin(d*x+c)^5+35/24*sin(d*x+c)^3+35/16*sin(d*x+c))*cos(d*x+c)-7/2*d*x-7/2*c
))
```

Maxima [A]
time = 0.53, size = 315, normalized size = 0.83

$$\frac{420\,a^4b^4\tan\left(dx+c\right)^5+2\left(3\,\tan\left(dx+c\right)^5+10\,\tan\left(dx+c\right)^3+15\,\tan\left(dx+c\right)\right)a^8+56\left(3\,\tan\left(dx+c\right)^5+5\,\tan\left(dx+c\right)^3\right)a^6b^2+56\left(3\,\tan\left(dx+c\right)^5-5\,\tan\left(dx+c\right)^3-15\,dx-15\,c+15\,\tan\left(dx+c\right)\right)a^2b^6+\left(6\,\tan\left(dx+c\right)^5-20\,\tan\left(dx+c\right)^3-105\,dx-105\,c+\frac{15\,\tan(dx+c)}{\tan(dx+c)^2+1}+90\,\tan\left(dx+c\right)\right)b^8+48\,ab^7\left(\frac{15\,\cos(dx+c)^4-5\,\cos(dx+c)^2+1}{\cos(dx+c)^5}+5\,\cos\left(dx+c\right)\right)-\frac{112\left(5\,\cos(dx+c)^2-3\right)a^5b^3}{\cos(dx+c)^5}+\frac{112\left(15\,\cos(dx+c)^4-10\,\cos(dx+c)^2+3\right)a^3b^5}{\cos(dx+c)^5}+\frac{48\,a^7b}{\cos(dx+c)^5}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^6*(a+b*sin(d*x+c))^8,x, algorithm="maxima")

[Out] 1/30*(420*a^4*b^4*tan(d*x + c)^5 + 2*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a^8 + 56*(3*tan(d*x + c)^5 + 5*tan(d*x + c)^3)*a^6*b^2 + 56*(3*tan(d*x + c)^5 - 5*tan(d*x + c)^3 - 15*d*x - 15*c + 15*tan(d*x + c))*a^2*b^6 + (6*tan(d*x + c)^5 - 20*tan(d*x + c)^3 - 105*d*x - 105*c + 15*tan(d*x + c)/(tan(d*x + c)^2 + 1) + 90*tan(d*x + c))*b^8 + 48*a*b^7*((15*cos(d*x + c)^4 - 5*cos(d*x + c)^2 + 1)/cos(d*x + c)^5 + 5*cos(d*x + c)) - 112*(5*cos(d*x + c)^2 - 3)*a^5*b^3/cos(d*x + c)^5 + 112*(15*cos(d*x + c)^4 - 10*cos(d*x + c)^2 + 3)*a^3*b^5/cos(d*x + c)^5 + 48*a^7*b/cos(d*x + c)^5)/d

Fricas [**A**]
time = 0.43, size = 281, normalized size = 0.74

$$\frac{240\,ab^{7}\cos\left(dx+c\right)^{6}+48\,a^{7}b+336\,a^{5}b^{3}+336\,a^{3}b^{5}+48\,ab^{7}-105\,(8\,a^{2}b^{6}+b^{8})dx\cos\left(dx+c\right)^{5}+240\,(7\,a^{3}b^{5}+3\,ab^{7})\cos\left(dx+c\right)^{4}-80\,(7\,a^{5}b^{3}+14\,a^{3}b^{5}+3\,ab^{7})\cos\left(dx+c\right)^{2}+\left(15\,b^{8}\cos\left(dx+c\right)^{6}+6\,a^{8}+168\,a^{6}b^{2}+420\,a^{4}b^{4}+168\,a^{2}b^{6}+6\,b^{8}+4\,(4\,a^{8}-28\,a^{6}b^{2}+105\,a^{4}b^{4}+322\,a^{2}b^{6}+29\,b^{8})\cos\left(dx+c\right)^{4}+8\,(a^{8}-7\,a^{6}b^{2}-105\,a^{4}b^{4}-77\,a^{2}b^{6}-4\,b^{8})\cos\left(dx+c\right)^{2}\right)\sin\left(dx+c\right)}{30\,d\cos\left(dx+c\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^6*(a+b*sin(d*x+c))^8,x, algorithm="fricas")

[Out] 1/30*(240*a*b^7*cos(d*x + c)^6 + 48*a^7*b + 336*a^5*b^3 + 336*a^3*b^5 + 48*a*b^7 - 105*(8*a^2*b^6 + b^8)*d*x*cos(d*x + c)^5 + 240*(7*a^3*b^5 + 3*a*b^7)*cos(d*x + c)^4 - 80*(7*a^5*b^3 + 14*a^3*b^5 + 3*a*b^7)*cos(d*x + c)^2 + (15*b^8*cos(d*x + c)^6 + 6*a^8 + 168*a^6*b^2 + 420*a^4*b^4 + 168*a^2*b^6 + 6*b^8 + 4*(4*a^8 - 28*a^6*b^2 + 105*a^4*b^4 + 322*a^2*b^6 + 29*b^8)*cos(d*x + c)^4 + 8*(a^8 - 7*a^6*b^2 - 105*a^4*b^4 - 77*a^2*b^6 - 4*b^8)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^5)

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**6*(a+b*sin(d*x+c))**8,x)

[Out] Timed out

Giac [**A**]
time = 5.86, size = 663, normalized size = 1.74

$$-\frac{105\,(8\,a^{2}b^{6}+b^{8})(dx+c)+\frac{30\left(b^{8}\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^{3}-16\,ab^{7}\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^{2}-b^{8}\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)-16\,ab^{7}\right)}{\left(\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^{2}+1\right)^{2}}+\frac{4\left(15\,a^{8}\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^{9}+420\,a^{4}b^{4}\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^{9}+45\,b^{8}\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^{9}+120\,a^{7}b\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^{8}+120\,ab^{7}\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^{8}-20\,a^{8}\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^{7}+\cdots\right)}{\cdots}}{30\,d}$$

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^6*(a+b*sin(d*x+c))^8,x, algorithm="giac")

[Out] -1/30*(105*(8*a^2*b^6 + b^8)*(d*x + c) + 30*(b^8*tan(1/2*d*x + 1/2*c)^3 - 16*a*b^7*tan(1/2*d*x + 1/2*c)^2 - b^8*tan(1/2*d*x + 1/2*c) - 16*a*b^7)/(tan(1/2*d*x + 1/2*c)^2 + 1)^2 + 4*(15*a^8*tan(1/2*d*x + 1/2*c)^9 + 420*a^2*b^6*tan(1/2*d*x + 1/2*c)^9 + 45*b^8*tan(1/2*d*x + 1/2*c)^9 + 120*a^7*b*tan(1/2*d*x + 1/2*c)^8 + 120*a*b^7*tan(1/2*d*x + 1/2*c)^8 - 20*a^8*tan(1/2*d*x + 1/2*c)^7 + 560*a^6*b^2*tan(1/2*d*x + 1/2*c)^7 - 2240*a^2*b^6*tan(1/2*d*x + 1/2*c)^7 - 220*b^8*tan(1/2*d*x + 1/2*c)^7 + 1680*a^5*b^3*tan(1/2*d*x + 1/2*c)^6 - 720*a*b^7*tan(1/2*d*x + 1/2*c)^6 + 58*a^8*tan(1/2*d*x + 1/2*c)^5 + 224*a^6*b^2*tan(1/2*d*x + 1/2*c)^5 + 3360*a^4*b^4*tan(1/2*d*x + 1/2*c)^5 + 4984*a^2*b^6*tan(1/2*d*x + 1/2*c)^5 + 398*b^8*tan(1/2*d*x + 1/2*c)^5 + 240*a^7*b*tan(1/2*d*x + 1/2*c)^4 + 560*a^5*b^3*tan(1/2*d*x + 1/2*c)^4 + 4480*a^3*b^5*tan(1/2*d*x + 1/2*c)^4 + 1920*a*b^7*tan(1/2*d*x + 1/2*c)^4 - 20*a^8*tan(1/2*d*x + 1/2*c)^3 + 560*a^6*b^2*tan(1/2*d*x + 1/2*c)^3 - 2240*a^2*b^6*tan(1/2*d*x + 1/2*c)^3 - 220*b^8*tan(1/2*d*x + 1/2*c)^3 + 560*a^5*b^3*tan(1/2*d*x + 1/2*c)^2 - 2240*a^3*b^5*tan(1/2*d*x + 1/2*c)^2 - 1200*a*b^7*tan(1/2*d*x + 1/2*c)^2 + 15*a^8*tan(1/2*d*x + 1/2*c) + 420*a^2*b^6*tan(1/2*d*x + 1/2*c) + 45*b^8*tan(1/2*d*x + 1/2*c) + 24*a^7*b - 112*a^5*b^3 + 448*a^3*b^5 + 264*a*b^7)/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d

Mupad [B]
time = 7.60, size = 665, normalized size = 1.75

$$-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}\,\left(2\,a^8+56\,a^2\,b^6+7\,b^8\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\,\left(48\,a^7\,b+\frac{1568\,a^5\,b^3}{3}+\frac{1792\,a^3\,b^5}{3}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}\,\left(32\,a^7\,b+224\,a^5\,b^3\right)+\frac{256\,a\,b^7}{5}+\frac{16\,a^7\,b}{5}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,a^8+56\,a^2\,b^6+7\,b^8\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,\left(64\,a^7\,b+448\,a^5\,b^3+896\,a^3\,b^5+256\,a\,b^7\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(-\frac{32\,a^7\,b}{5}-\frac{224\,a^5\,b^3}{5}+\frac{896\,a^3\,b^5}{5}+\frac{768\,a\,b^7}{5}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(\frac{176\,a^7\,b}{5}+\frac{3136\,a^5\,b^3}{15}+\frac{896\,a^3\,b^5}{15}+\frac{256\,a\,b^7}{5}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\left(\frac{22\,a^8}{5}+\frac{896\,a^6\,b^2}{5}+448\,a^4\,b^4+\frac{616\,a^2\,b^6}{5}+\frac{77\,b^8}{5}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9\,\left(\frac{22\,a^8}{5}+\frac{896\,a^6\,b^2}{5}+448\,a^4\,b^4+\frac{616\,a^2\,b^6}{5}+\frac{77\,b^8}{5}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\,\left(\frac{152\,a^8}{15}+\frac{3136\,a^6\,b^2}{15}+896\,a^4\,b^4+\frac{10976\,a^2\,b^6}{15}+\frac{412\,b^8}{15}\right)+\frac{896\,a^3\,b^5}{15}-\frac{224\,a^5\,b^3}{15}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(\frac{4\,a^8}{3}+\frac{224\,a^6\,b^2}{3}-\frac{560\,a^2\,b^6}{3}-\frac{70\,b^8}{3}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}\,\left(\frac{4\,a^8}{3}+\frac{224\,a^6\,b^2}{3}-\frac{560\,a^2\,b^6}{3}-\frac{70\,b^8}{3}\right)+16\,a^7\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{14}-3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{12}+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}+5\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8-5\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}-\frac{7\,b^6\,\mathrm{atan}\left(\frac{7\,b^6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(8\,a^2+b^2\right)}{56\,a^2\,b^6+7\,b^8}\right)\,\left(8\,a^2+b^2\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*sin(c + d*x))^8/cos(c + d*x)^6,x)

[Out] - (tan(c/2 + (d*x)/2)^13*(2*a^8 + 7*b^8 + 56*a^2*b^6) + tan(c/2 + (d*x)/2)^8*(48*a^7*b + (1792*a^3*b^5)/3 + (1568*a^5*b^3)/3) + tan(c/2 + (d*x)/2)^10*(32*a^7*b + 224*a^5*b^3) + (256*a*b^7)/5 + (16*a^7*b)/5 + tan(c/2 + (d*x)/2)*(2*a^8 + 7*b^8 + 56*a^2*b^6) + tan(c/2 + (d*x)/2)^6*(256*a*b^7 + 64*a^7*b + 896*a^3*b^5 + 448*a^5*b^3) - tan(c/2 + (d*x)/2)^2*((768*a*b^7)/5 - (32*a^7*b)/5 + (896*a^3*b^5)/5 - (224*a^5*b^3)/5) + tan(c/2 + (d*x)/2)^4*((256*a*b^7)/5 + (176*a^7*b)/5 + (896*a^3*b^5)/15 + (3136*a^5*b^3)/15) + tan(c/2 + (d*x)/2)^5*((22*a^8)/5 + (77*b^8)/5 + (616*a^2*b^6)/5 + 448*a^4*b^4 + (896*a^6*b^2)/5) + tan(c/2 + (d*x)/2)^9*((22*a^8)/5 + (77*b^8)/5 + (616*a^2*b^6)/5 + 448*a^4*b^4 + (896*a^6*b^2)/5) + tan(c/2 + (d*x)/2)^7*((152*a^8)/15 + (412*b^8)/15 + (10976*a^2*b^6)/15 + 896*a^4*b^4 + (3136*a^6*b^2)/15) + (896*a^3*b^5)/15 - (224*a^5*b^3)/15 + tan(c/2 + (d*x)/2)^3*((4*a^8)/3 - (70*b^8)/3 - (560*a^2*b^6)/3 + (224*a^6*b^2)/3) + tan(c/2 + (d*x)/2)^11*((4*a^8)/3 - (70*b^8)/3 - (560*a^2*b^6)/3 + (224*a^6*b^2)/3) + 16*a^7*b*tan(c/2 + (d*x)/2)^12)/(d*(3*tan(c/2 + (d*x)/2)^2 - tan(c/2 + (d*x)/2)^4 - 5*tan(c/2 + (d*x)/2)^6 + 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 3*tan(c/2 + (d*x)/2)^12 + tan(c/2 + (d*x)/2)^14 - 1)) - (7*b^6*atan((7*b^6*tan(c/2 + (d*x)/2)*(8*a^2 + b^2))/(7*b^8 + 56*a^2*b^6))*(8*a^2 + b^2))/d

3.423 $\int \sec^8(c+dx)(a+b\sin(c+dx))^8\,dx$

Optimal. Leaf size=404

$$b^8x+\frac{4ab(24a^6-88a^4b^2+125a^2b^4-96b^6)\cos(c+dx)}{105d}+\frac{b^2(48a^6-152a^4b^2+174a^2b^4-105b^6)\cos(c+dx)\,s}{105d}$$

```
[Out] b^8*x+4/105*a*b*(24*a^6-88*a^4*b^2+125*a^2*b^4-96*b^6)*cos(d*x+c)/d+1/105*b
^2*(48*a^6-152*a^4*b^2+174*a^2*b^4-105*b^6)*cos(d*x+c)*sin(d*x+c)/d+2/105*a
*b*(24*a^4-40*a^2*b^2+9*b^4)*cos(d*x+c)*(a+b*sin(d*x+c))^2/d+2/105*b*(24*a^
4+8*a^2*b^2-35*b^4)*cos(d*x+c)*(a+b*sin(d*x+c))^3/d+1/7*sec(d*x+c)^7*(b+a*s
in(d*x+c))*(a+b*sin(d*x+c))^7/d-2/105*sec(d*x+c)^3*(a+b*sin(d*x+c))^5*(b*(6
*a^2-7*b^2)-a*(12*a^2-11*b^2)*sin(d*x+c))/d-1/35*sec(d*x+c)^5*(a+b*sin(d*x+
c))^6*(a*b-(6*a^2-7*b^2)*sin(d*x+c))/d-2/105*sec(d*x+c)*(a+b*sin(d*x+c))^4*
(3*a*b*(12*a^2-11*b^2)-(24*a^4+8*a^2*b^2-35*b^4)*sin(d*x+c))/d
```

Rubi [A]
time = 0.54, antiderivative size = 404, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2770, 2940, 2832, 2813}

$$-\frac{\sec^5(c+dx)(a+b\sin(c+dx))^6\,(ab-(6a^2-7b^2)\sin(c+dx))}{35d}-\frac{2\sec^3(c+dx)(a+b\sin(c+dx))^5\,(b(6a^2-7b^2)-a(12a^2-11b^2)\sin(c+dx))}{105d}+\frac{2b(24a^4+8a^2b^2-35b^4)\cos(c+dx)(a+b\sin(c+dx))^3}{105d}+\frac{2ab(24a^4-40a^2b^2+9b^4)\cos(c+dx)(a+b\sin(c+dx))^2}{105d}-\frac{2\sec(c+dx)(a+b\sin(c+dx))^4\,(3ab(12a^2-11b^2)-(24a^4+8a^2b^2-35b^4)\sin(c+dx))}{105d}+\frac{4ab(24a^6-88a^4b^2+125a^2b^4-96b^6)\cos(c+dx)}{105d}+\frac{b^2(48a^6-152a^4b^2+174a^2b^4-105b^6)\sin(c+dx)\cos(c+dx)}{105d}+\frac{\sec^7(c+dx)(a\sin(c+dx)+b)(a+b\sin(c+dx))^7}{7d}+b^8x$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^8*(a + b*Sin[c + d*x])^8,x]
```

```
[Out] b^8*x + (4*a*b*(24*a^6 - 88*a^4*b^2 + 125*a^2*b^4 - 96*b^6)*Cos[c + d*x])/(
105*d) + (b^2*(48*a^6 - 152*a^4*b^2 + 174*a^2*b^4 - 105*b^6)*Cos[c + d*x]*S
in[c + d*x])/(105*d) + (2*a*b*(24*a^4 - 40*a^2*b^2 + 9*b^4)*Cos[c + d*x]*(a
 + b*Sin[c + d*x])^2)/(105*d) + (2*b*(24*a^4 + 8*a^2*b^2 - 35*b^4)*Cos[c +
d*x]*(a + b*Sin[c + d*x])^3)/(105*d) + (Sec[c + d*x]^7*(b + a*Sin[c + d*x])
*(a + b*Sin[c + d*x])^7)/(7*d) - (2*Sec[c + d*x]^3*(a + b*Sin[c + d*x])^5*(
b*(6*a^2 - 7*b^2) - a*(12*a^2 - 11*b^2)*Sin[c + d*x]))/(105*d) - (Sec[c + d
*x]^5*(a + b*Sin[c + d*x])^6*(a*b - (6*a^2 - 7*b^2)*Sin[c + d*x]))/(35*d) -
 (2*Sec[c + d*x]*(a + b*Sin[c + d*x])^4*(3*a*b*(12*a^2 - 11*b^2) - (24*a^4
+ 8*a^2*b^2 - 35*b^4)*Sin[c + d*x]))/(105*d)
```

Rule 2770

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x
])^(m - 1)*((b + a*Sin[e + f*x])/(f*g*(p + 1))), x] + Dist[1/(g^2*(p + 1)),
 Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a
^2*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g}
, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p
] || IntegerQ[m])
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rule 2940

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(g*
Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x])^m*((d + c*Sin[e + f*x])/(f*g*(p
 + 1))), x] + Dist[1/(g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin
[e + f*x])^(m - 1)*Simp[a*c*(p + 2) + b*d*m + b*c*(m + p + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && GtQ[
m, 0] && LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] &
& SimplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int \sec^8(c+dx)(a+b\sin(c+dx))^8\,dx = \frac{\sec^7(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^7}{7d} - \frac{1}{7}\int \sec^6(c+$$

$$= \frac{\sec^7(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^7}{7d} - \frac{\sec^5(c+dx)(}{}$$

$$= \frac{\sec^7(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^7}{7d} - \frac{2\sec^3(c+dx)}{}$$

$$= \frac{\sec^7(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^7}{7d} - \frac{2\sec^3(c+dx)}{}$$

$$= \frac{2b(24a^4+8a^2b^2-35b^4)\cos(c+dx)(a+b\sin(c+dx))^3}{105d} + \frac{\sec^7(c+}{}$$

$$= \frac{2ab(24a^4-40a^2b^2+9b^4)\cos(c+dx)(a+b\sin(c+dx))^2}{105d} + \frac{2b(24a^4}{}$$

$$= b^8x + \frac{4ab(24a^6-88a^4b^2+125a^2b^4-96b^6)\cos(c+dx)}{105d} + \frac{b^2(48a^6-}{}$$

Mathematica [A]
time = 1.43, size = 479, normalized size = 1.19

$$\frac{\sec^7(c+dx)\left(7680a^7b+16128a^5b^3+25536a^3b^5-5088ab^7+3675b^8(c+dx)\cos(c+dx)-37632a^5b^3\cos(2(c+dx))-12544a^3b^5\cos(2(c+dx))-14448ab^7\cos(2(c+dx))+2205b^8(c+dx)\cos(3(c+dx))+15680a^3b^5\cos(4(c+dx))-3360ab^7\cos(4(c+dx))+735b^8(c+dx)\cos(5(c+dx))-1680ab^7\cos(6(c+dx))+105b^8(c+dx)\cos(7(c+dx))+1680a^8\sin(c+dx)+23520a^6b^2\sin(c+dx)+44100a^4b^4\sin(c+dx)+14700a^2b^6\sin(c+dx)+1008a^8\sin(3(c+dx))-4704a^6b^2\sin(3(c+dx))-20580a^4b^4\sin(3(c+dx))-8820a^2b^6\sin(3(c+dx))-1176b^8\sin(3(c+dx))+336a^8\sin(5(c+dx))-1568a^6b^2\sin(5(c+dx))+2940a^4b^4\sin(5(c+dx))+2940a^2b^6\sin(5(c+dx))-392b^8\sin(5(c+dx))+48a^8\sin(7(c+dx))-224a^6b^2\sin(7(c+dx))+420a^4b^4\sin(7(c+dx))-420a^2b^6\sin(7(c+dx))-176b^8\sin(7(c+dx))\right)}{6720d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^8*(a + b*Sin[c + d*x])^8,x]
```

```
[Out] (Sec[c + d*x]^7*(7680*a^7*b + 16128*a^5*b^3 + 25536*a^3*b^5 - 5088*a*b^7 +
3675*b^8*(c + d*x)*Cos[c + d*x] - 37632*a^5*b^3*Cos[2*(c + d*x)] - 12544*a^
3*b^5*Cos[2*(c + d*x)] - 14448*a*b^7*Cos[2*(c + d*x)] + 2205*b^8*(c + d*x)*
Cos[3*(c + d*x)] + 15680*a^3*b^5*Cos[4*(c + d*x)] - 3360*a*b^7*Cos[4*(c + d
*x)] + 735*b^8*(c + d*x)*Cos[5*(c + d*x)] - 1680*a*b^7*Cos[6*(c + d*x)] + 1
05*b^8*(c + d*x)*Cos[7*(c + d*x)] + 1680*a^8*Sin[c + d*x] + 23520*a^6*b^2*S
in[c + d*x] + 44100*a^4*b^4*Sin[c + d*x] + 14700*a^2*b^6*Sin[c + d*x] + 100
8*a^8*Sin[3*(c + d*x)] - 4704*a^6*b^2*Sin[3*(c + d*x)] - 20580*a^4*b^4*Sin[
3*(c + d*x)] - 8820*a^2*b^6*Sin[3*(c + d*x)] - 1176*b^8*Sin[3*(c + d*x)] +
336*a^8*Sin[5*(c + d*x)] - 1568*a^6*b^2*Sin[5*(c + d*x)] + 2940*a^4*b^4*Sin
[5*(c + d*x)] + 2940*a^2*b^6*Sin[5*(c + d*x)] - 392*b^8*Sin[5*(c + d*x)] +
48*a^8*Sin[7*(c + d*x)] - 224*a^6*b^2*Sin[7*(c + d*x)] + 420*a^4*b^4*Sin[7*
(c + d*x)] - 420*a^2*b^6*Sin[7*(c + d*x)] - 176*b^8*Sin[7*(c + d*x)]))/(672
0*d)
```

Maple [A]
time = 0.67, size = 567, normalized size = 1.40 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^8*(a+b*sin(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^8*(-16/35-1/7*sec(d*x+c)^6-6/35*sec(d*x+c)^4-8/35*sec(d*x+c)^2)*tan
(d*x+c)+8/7*a^7*b/cos(d*x+c)^7+28*a^6*b^2*(1/7*sin(d*x+c)^3/cos(d*x+c)^7+4/
35*sin(d*x+c)^3/cos(d*x+c)^5+8/105*sin(d*x+c)^3/cos(d*x+c)^3)+56*a^5*b^3*(1
/7*sin(d*x+c)^4/cos(d*x+c)^7+3/35*sin(d*x+c)^4/cos(d*x+c)^5+1/35*sin(d*x+c)
^4/cos(d*x+c)^3-1/35*sin(d*x+c)^4/cos(d*x+c)-1/35*(2+sin(d*x+c)^2)*cos(d*x+
c))+70*a^4*b^4*(1/7*sin(d*x+c)^5/cos(d*x+c)^7+2/35*sin(d*x+c)^5/cos(d*x+c)^
5)+56*a^3*b^5*(1/7*sin(d*x+c)^6/cos(d*x+c)^7+1/35*sin(d*x+c)^6/cos(d*x+c)^5
-1/105*sin(d*x+c)^6/cos(d*x+c)^3+1/35*sin(d*x+c)^6/cos(d*x+c)+1/35*(8/3+sin
(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c))+4*a^2*b^6*sin(d*x+c)^7/cos(d*x+c)^7
+8*a*b^7*(1/7*sin(d*x+c)^8/cos(d*x+c)^7-1/35*sin(d*x+c)^8/cos(d*x+c)^5+1/35
*sin(d*x+c)^8/cos(d*x+c)^3-1/7*sin(d*x+c)^8/cos(d*x+c)-1/7*(16/5+sin(d*x+c)
^6+6/5*sin(d*x+c)^4+8/5*sin(d*x+c)^2)*cos(d*x+c))+b^8*(1/7*tan(d*x+c)^7-1/5
*tan(d*x+c)^5+1/3*tan(d*x+c)^3-tan(d*x+c)+d*x+c))
```

Maxima [A]
time = 0.49, size = 310, normalized size = 0.77

$$\frac{420a^2b^6\tan(dx+c)^7+3\left(5\tan(dx+c)^7+21\tan(dx+c)^5+35\tan(dx+c)^3+35\tan(dx+c)\right)a^8+28\left(15\tan(dx+c)^7+42\tan(dx+c)^5+35\tan(dx+c)^3\right)a^6b^2+210\left(5\tan(dx+c)^7+7\tan(dx+c)^5\right)a^4b^4+\left(15\tan(dx+c)^7-21\tan(dx+c)^5+35\tan(dx+c)^3+105dx+105c-105\tan(dx+c)\right)b^8-\frac{168\left(7\cos(dx+c)^2-5\right)a^5b^3}{\cos(dx+c)^7}+\frac{56\left(35\cos(dx+c)^4-42\cos(dx+c)^2+15\right)a^3b^5}{\cos(dx+c)^7}-\frac{24\left(35\cos(dx+c)^6-35\cos(dx+c)^4+21\cos(dx+c)^2-5\right)ab^7}{\cos(dx+c)^7}+\frac{120a^7b}{\cos(dx+c)^7}}{105d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+b*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 1/105*(420*a^2*b^6*tan(d*x + c)^7 + 3*(5*tan(d*x + c)^7 + 21*tan(d*x + c)^5
 + 35*tan(d*x + c)^3 + 35*tan(d*x + c))*a^8 + 28*(15*tan(d*x + c)^7 + 42*ta
n(d*x + c)^5 + 35*tan(d*x + c)^3)*a^6*b^2 + 210*(5*tan(d*x + c)^7 + 7*tan(d
*x + c)^5)*a^4*b^4 + (15*tan(d*x + c)^7 - 21*tan(d*x + c)^5 + 35*tan(d*x +
c)^3 + 105*d*x + 105*c - 105*tan(d*x + c))*b^8 - 168*(7*cos(d*x + c)^2 - 5)
*a^5*b^3/cos(d*x + c)^7 + 56*(35*cos(d*x + c)^4 - 42*cos(d*x + c)^2 + 15)*a
^3*b^5/cos(d*x + c)^7 - 24*(35*cos(d*x + c)^6 - 35*cos(d*x + c)^4 + 21*cos(
d*x + c)^2 - 5)*a*b^7/cos(d*x + c)^7 + 120*a^7*b/cos(d*x + c)^7)/d
```

Fricas [A]
time = 0.37, size = 306, normalized size = 0.76

$$\frac{105\,b^8dx\cos\left(dx+c\right)^7-840\,ab^7\cos\left(dx+c\right)^6+120\,a^7b+840\,a^5b^3+840\,a^3b^5+120\,ab^7+280\left(7\,a^3b^5+3\,ab^7\right)\cos\left(dx+c\right)^4-168\left(7\,a^5b^3+14\,a^3b^5+3\,ab^7\right)\cos\left(dx+c\right)^2+\left(15\,a^8+420\,a^6b^2+1050\,a^4b^4+420\,a^2b^6+15\,b^8+4\left(12\,a^8-56\,a^6b^2+105\,a^4b^4-105\,a^2b^6-44\,b^8\right)\cos\left(dx+c\right)^6+2\left(12\,a^8-56\,a^6b^2+105\,a^4b^4+630\,a^2b^6+61\,b^8\right)\cos\left(dx+c\right)^4+6\left(3\,a^8-14\,a^6b^2-280\,a^4b^4-210\,a^2b^6-11\,b^8\right)\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)}{105\,d\cos\left(dx+c\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+b*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/105*(105*b^8*d*x*cos(d*x + c)^7 - 840*a*b^7*cos(d*x + c)^6 + 120*a^7*b +
840*a^5*b^3 + 840*a^3*b^5 + 120*a*b^7 + 280*(7*a^3*b^5 + 3*a*b^7)*cos(d*x +
 c)^4 - 168*(7*a^5*b^3 + 14*a^3*b^5 + 3*a*b^7)*cos(d*x + c)^2 + (15*a^8 + 4
20*a^6*b^2 + 1050*a^4*b^4 + 420*a^2*b^6 + 15*b^8 + 4*(12*a^8 - 56*a^6*b^2 +
 105*a^4*b^4 - 105*a^2*b^6 - 44*b^8)*cos(d*x + c)^6 + 2*(12*a^8 - 56*a^6*b^
2 + 105*a^4*b^4 + 630*a^2*b^6 + 61*b^8)*cos(d*x + c)^4 + 6*(3*a^8 - 14*a^6*
b^2 - 280*a^4*b^4 - 210*a^2*b^6 - 11*b^8)*cos(d*x + c)^2)*sin(d*x + c))/(d*
cos(d*x + c)^7)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**8*(a+b*sin(d*x+c))**8,x)
```

```
[Out] Timed out
```

Giac [A]
time = 7.94, size = 726, normalized size = 1.80

$105\,(dx+c)b^8 -$ [illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+b*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/105*(105*(d*x + c)*b^8 - 2*(105*a^8*tan(1/2*d*x + 1/2*c)^13 - 105*b^8*tan
(1/2*d*x + 1/2*c)^13 + 840*a^7*b*tan(1/2*d*x + 1/2*c)^12 - 210*a^8*tan(1/2*
d*x + 1/2*c)^11 + 3920*a^6*b^2*tan(1/2*d*x + 1/2*c)^11 + 770*b^8*tan(1/2*d*
x + 1/2*c)^11 + 11760*a^5*b^3*tan(1/2*d*x + 1/2*c)^10 + 903*a^8*tan(1/2*d*x
 + 1/2*c)^9 + 3136*a^6*b^2*tan(1/2*d*x + 1/2*c)^9 + 23520*a^4*b^4*tan(1/2*d
*x + 1/2*c)^9 - 2471*b^8*tan(1/2*d*x + 1/2*c)^9 + 4200*a^7*b*tan(1/2*d*x +
1/2*c)^8 + 11760*a^5*b^3*tan(1/2*d*x + 1/2*c)^8 + 31360*a^3*b^5*tan(1/2*d*x
 + 1/2*c)^8 - 636*a^8*tan(1/2*d*x + 1/2*c)^7 + 12768*a^6*b^2*tan(1/2*d*x +
1/2*c)^7 + 20160*a^4*b^4*tan(1/2*d*x + 1/2*c)^7 + 26880*a^2*b^6*tan(1/2*d*x
 + 1/2*c)^7 + 4572*b^8*tan(1/2*d*x + 1/2*c)^7 + 23520*a^5*b^3*tan(1/2*d*x +
 1/2*c)^6 + 15680*a^3*b^5*tan(1/2*d*x + 1/2*c)^6 + 13440*a*b^7*tan(1/2*d*x
+ 1/2*c)^6 + 903*a^8*tan(1/2*d*x + 1/2*c)^5 + 3136*a^6*b^2*tan(1/2*d*x + 1/
2*c)^5 + 23520*a^4*b^4*tan(1/2*d*x + 1/2*c)^5 - 2471*b^8*tan(1/2*d*x + 1/2*
c)^5 + 2520*a^7*b*tan(1/2*d*x + 1/2*c)^4 + 4704*a^5*b^3*tan(1/2*d*x + 1/2*c
)^4 + 9408*a^3*b^5*tan(1/2*d*x + 1/2*c)^4 - 8064*a*b^7*tan(1/2*d*x + 1/2*c)
^4 - 210*a^8*tan(1/2*d*x + 1/2*c)^3 + 3920*a^6*b^2*tan(1/2*d*x + 1/2*c)^3 +
 770*b^8*tan(1/2*d*x + 1/2*c)^3 + 2352*a^5*b^3*tan(1/2*d*x + 1/2*c)^2 - 313
6*a^3*b^5*tan(1/2*d*x + 1/2*c)^2 + 2688*a*b^7*tan(1/2*d*x + 1/2*c)^2 + 105*
a^8*tan(1/2*d*x + 1/2*c) - 105*b^8*tan(1/2*d*x + 1/2*c) + 120*a^7*b - 336*a
^5*b^3 + 448*a^3*b^5 - 384*a*b^7)/(tan(1/2*d*x + 1/2*c)^2 - 1)^7)/d
```

Mupad [B]
time = 8.85, size = 546, normalized size = 1.35

$$b^8\,x-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(-4\,a^8+\frac{224\,a^6\,b^2}{3}+\frac{44\,b^8}{3}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}\,\left(-4\,a^8+\frac{224\,a^6\,b^2}{3}+\frac{44\,b^8}{3}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}\,\left(2\,a^8-2\,b^8\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(\frac{224\,a^5\,b^3}{5}-\frac{896\,a^3\,b^5}{15}+\frac{256\,a\,b^7}{5}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,\left(448\,a^5\,b^3+\frac{896\,a^3\,b^5}{3}+256\,a\,b^7\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\,\left(80\,a^7\,b+224\,a^5\,b^3+\frac{1792\,a^3\,b^5}{3}\right)-\frac{256\,a\,b^7}{35}+\frac{16\,a^7\,b}{7}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(48\,a^7\,b+\frac{448\,a^5\,b^3}{5}+\frac{896\,a^3\,b^5}{5}-\frac{768\,a\,b^7}{5}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,a^8-2\,b^8\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\,\left(-\frac{424\,a^8}{35}+\frac{1216\,a^6\,b^2}{5}+384\,a^4\,b^4+512\,a^2\,b^6+\frac{3048\,b^8}{35}\right)+\frac{128\,a^3\,b^5}{15}-\frac{32\,a^5\,b^3}{5}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\left(\frac{86\,a^8}{5}+\frac{896\,a^6\,b^2}{15}+448\,a^4\,b^4-\frac{706\,b^8}{15}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9\,\left(\frac{86\,a^8}{5}+\frac{896\,a^6\,b^2}{15}+448\,a^4\,b^4-\frac{706\,b^8}{15}\right)+224\,a^5\,b^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+16\,a^7\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{14}-7\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{12}+21\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}-35\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8+35\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6-21\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+7\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^8/cos(c + d*x)^8,x)
```

```
[Out] b^8*x - (tan(c/2 + (d*x)/2)^3*((44*b^8)/3 - 4*a^8 + (224*a^6*b^2)/3) + tan(
c/2 + (d*x)/2)^11*((44*b^8)/3 - 4*a^8 + (224*a^6*b^2)/3) + tan(c/2 + (d*x)/
2)^13*(2*a^8 - 2*b^8) + tan(c/2 + (d*x)/2)^2*((256*a*b^7)/5 - (896*a^3*b^5)
/15 + (224*a^5*b^3)/5) + tan(c/2 + (d*x)/2)^6*(256*a*b^7 + (896*a^3*b^5)/3
+ 448*a^5*b^3) + tan(c/2 + (d*x)/2)^8*(80*a^7*b + (1792*a^3*b^5)/3 + 224*a^
5*b^3) - (256*a*b^7)/35 + (16*a^7*b)/7 + tan(c/2 + (d*x)/2)^4*(48*a^7*b - (
768*a*b^7)/5 + (896*a^3*b^5)/5 + (448*a^5*b^3)/5) + tan(c/2 + (d*x)/2)*(2*a
^8 - 2*b^8) + tan(c/2 + (d*x)/2)^7*((3048*b^8)/35 - (424*a^8)/35 + 512*a^2*
b^6 + 384*a^4*b^4 + (1216*a^6*b^2)/5) + (128*a^3*b^5)/15 - (32*a^5*b^3)/5 +
 tan(c/2 + (d*x)/2)^5*((86*a^8)/5 - (706*b^8)/15 + 448*a^4*b^4 + (896*a^6*b
^2)/15) + tan(c/2 + (d*x)/2)^9*((86*a^8)/5 - (706*b^8)/15 + 448*a^4*b^4 + (
896*a^6*b^2)/15) + 224*a^5*b^3*tan(c/2 + (d*x)/2)^10 + 16*a^7*b*tan(c/2 + (
d*x)/2)^12)/(d*(7*tan(c/2 + (d*x)/2)^2 - 21*tan(c/2 + (d*x)/2)^4 + 35*tan(c
/2 + (d*x)/2)^6 - 35*tan(c/2 + (d*x)/2)^8 + 21*tan(c/2 + (d*x)/2)^10 - 7*ta
n(c/2 + (d*x)/2)^12 + tan(c/2 + (d*x)/2)^14 - 1))
```

3.424 $\int \sec^{10}(c+dx)(a+b\sin(c+dx))^8\,dx$

Optimal. Leaf size=236

$$\frac{128ab(a^2-b^2)^3\sec(c+dx)}{315d}+\frac{64a(a^2-b^2)^2\sec^3(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^2}{315d}+\frac{16a(a^2-b^2)}{}$$

```
[Out] 128/315*a*b*(a^2-b^2)^3*sec(d*x+c)/d+64/315*a*(a^2-b^2)^2*sec(d*x+c)^3*(b+a
*sin(d*x+c))*(a+b*sin(d*x+c))^2/d+16/105*a*(a^2-b^2)*sec(d*x+c)^5*(b+a*sin(
d*x+c))*(a+b*sin(d*x+c))^4/d+1/9*sec(d*x+c)^9*(b+a*sin(d*x+c))*(a+b*sin(d*x
+c))^7/d+1/63*sec(d*x+c)^7*(a+b*sin(d*x+c))^6*(a*b+(8*a^2-7*b^2)*sin(d*x+c)
)/d+128/315*a^2*(a^2-b^2)^3*tan(d*x+c)/d
```

Rubi [A]
time = 0.25, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$,
Rules used = {2770, 2940, 12, 2748, 3852, 8}

$$\frac{128a^2(a^2-b^2)^3\tan(c+dx)}{315d}+\frac{128ab(a^2-b^2)^3\sec(c+dx)}{315d}+\frac{\sec^7(c+dx)(a+b\sin(c+dx))^6\left((8a^2-7b^2)\sin(c+dx)+ab\right)}{63d}+\frac{16a(a^2-b^2)\sec^5(c+dx)(a\sin(c+dx)+b)(a+b\sin(c+dx))^4}{105d}+\frac{64a(a^2-b^2)^2\sec^3(c+dx)(a\sin(c+dx)+b)(a+b\sin(c+dx))^2}{315d}+\frac{\sec^9(c+dx)(a\sin(c+dx)+b)(a+b\sin(c+dx))^7}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^10*(a + b*Sin[c + d*x])^8,x]
```

```
[Out] (128*a*b*(a^2 - b^2)^3*Sec[c + d*x])/(315*d) + (64*a*(a^2 - b^2)^2*Sec[c +
d*x]^3*(b + a*Sin[c + d*x])*(a + b*Sin[c + d*x])^2)/(315*d) + (16*a*(a^2 -
b^2)*Sec[c + d*x]^5*(b + a*Sin[c + d*x])*(a + b*Sin[c + d*x])^4)/(105*d) +
(Sec[c + d*x]^9*(b + a*Sin[c + d*x])*(a + b*Sin[c + d*x])^7)/(9*d) + (Sec[c
 + d*x]^7*(a + b*Sin[c + d*x])^6*(a*b + (8*a^2 - 7*b^2)*Sin[c + d*x]))/(63*
d) + (128*a^2*(a^2 - b^2)^3*Tan[c + d*x])/(315*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2770

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x
])^(m - 1)*((b + a*Sin[e + f*x])/(f*g*(p + 1))), x] + Dist[1/(g^2*(p + 1)),
 Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a
^2*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g}
, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p
] || IntegerQ[m])
```

Rule 2940

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(g*
Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x])^m*((d + c*Sin[e + f*x])/(f*g*(p
 + 1))), x] + Dist[1/(g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin
[e + f*x])^(m - 1)*Simp[a*c*(p + 2) + b*d*m + b*c*(m + p + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && GtQ[
m, 0] && LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] &
& SimplerQ[c + d*x, a + b*x])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \sec^{10}(c+dx)(a+b\sin(c+dx))^8\,dx = \frac{\sec^9(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^7}{9d} - \frac{1}{9}\int \sec^8(c+$$

$$= \frac{\sec^9(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^7}{9d} + \frac{\sec^7(c+dx)(}{}$$

$$= \frac{\sec^9(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^7}{9d} + \frac{\sec^7(c+dx)(}{}$$

$$= \frac{16a(a^2-b^2)\sec^5(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^4}{105d} +$$

$$= \frac{16a(a^2-b^2)\sec^5(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^4}{105d} +$$

$$= \frac{64a(a^2-b^2)^2\sec^3(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^2}{315d} +$$

$$= \frac{64a(a^2-b^2)^2\sec^3(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^2}{315d} +$$

$$= \frac{128ab(a^2-b^2)^3\sec(c+dx)}{315d} + \frac{64a(a^2-b^2)^2\sec^3(c+dx)(b+a\sin(}{315d}$$

$$= \frac{128ab(a^2-b^2)^3\sec(c+dx)}{315d} + \frac{64a(a^2-b^2)^2\sec^3(c+dx)(b+a\sin(}{315d}$$

$$= \frac{128ab(a^2-b^2)^3\sec(c+dx)}{315d} + \frac{64a(a^2-b^2)^2\sec^3(c+dx)(b+a\sin(}{315d}$$

Mathematica [A]
time = 4.57, size = 313, normalized size = 1.33

$$\frac{\cos(c+dx)\left(-\sec^{10}(c+dx)(a+b\sin(c+dx))^9+\frac{a\left(35(a+b\sin(c+dx))^8+8(a-b)(1-\sin(c+dx))\left(5(a+b\sin(c+dx))^7+(a-b)(1-\sin(c+dx))\left(7(a+b\sin(c+dx))^6+2(a-b)(1-\sin(c+dx))\left(7(a+b\sin(c+dx))^5+(a-b)(1-\sin(c+dx))\left(35(a+b\sin(c+dx))^4-4(a-b)(1-\sin(c+dx))\left(5(a+b\sin(c+dx))^3+(a+b)(1+\sin(c+dx))\left(7a^2+6ab+2b^2+6(a^2+3ab+b^2)\sin(c+dx)+(2a^2+6ab+7b^2)\sin^2(c+dx)\right)\right)\right)\right)\right)\right)\right)}{35(1-\sin(c+dx))^5(1+\sin(c+dx))^4}\right)}{9(a-b)d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^10*(a + b*Sin[c + d*x])^8,x]
```

```
[Out] (Cos[c + d*x]*(-(Sec[c + d*x]^10*(a + b*Sin[c + d*x])^9) + (a*(35*(a + b*Si
n[c + d*x])^8 + 8*(a - b)*(1 - Sin[c + d*x])*(5*(a + b*Sin[c + d*x])^7 + (a
 - b)*(1 - Sin[c + d*x])*(7*(a + b*Sin[c + d*x])^6 + 2*(a - b)*(1 - Sin[c +
 d*x])*(7*(a + b*Sin[c + d*x])^5 + (a - b)*(1 - Sin[c + d*x])*(35*(a + b*Si
n[c + d*x])^4 - 4*(a - b)*(1 - Sin[c + d*x])*(5*(a + b*Sin[c + d*x])^3 + (a
 + b)*(1 + Sin[c + d*x])*(7*a^2 + 6*a*b + 2*b^2 + 6*(a^2 + 3*a*b + b^2)*Sin
[c + d*x] + (2*a^2 + 6*a*b + 7*b^2)*Sin[c + d*x]^2))))))))/(35*(1 - Sin[c +
 d*x])^5*(1 + Sin[c + d*x])^4)))/(9*(a - b)*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 661 vs. $2(224) = 448$.
time = 0.83, size = 662, normalized size = 2.81 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^10*(a+b*sin(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^8*(-128/315-1/9*sec(d*x+c)^8-8/63*sec(d*x+c)^6-16/105*sec(d*x+c)^4-
64/315*sec(d*x+c)^2)*tan(d*x+c)+8/9*a^7*b/cos(d*x+c)^9+28*a^6*b^2*(1/9*sin(
d*x+c)^3/cos(d*x+c)^9+2/21*sin(d*x+c)^3/cos(d*x+c)^7+8/105*sin(d*x+c)^3/cos
(d*x+c)^5+16/315*sin(d*x+c)^3/cos(d*x+c)^3)+56*a^5*b^3*(1/9*sin(d*x+c)^4/co
s(d*x+c)^9+5/63*sin(d*x+c)^4/cos(d*x+c)^7+1/21*sin(d*x+c)^4/cos(d*x+c)^5+1/
63*sin(d*x+c)^4/cos(d*x+c)^3-1/63*sin(d*x+c)^4/cos(d*x+c)-1/63*(2+sin(d*x+c
)^2)*cos(d*x+c))+70*a^4*b^4*(1/9*sin(d*x+c)^5/cos(d*x+c)^9+4/63*sin(d*x+c)^
5/cos(d*x+c)^7+8/315*sin(d*x+c)^5/cos(d*x+c)^5)+56*a^3*b^5*(1/9*sin(d*x+c)^
6/cos(d*x+c)^9+1/21*sin(d*x+c)^6/cos(d*x+c)^7+1/105*sin(d*x+c)^6/cos(d*x+c)
^5-1/315*sin(d*x+c)^6/cos(d*x+c)^3+1/105*sin(d*x+c)^6/cos(d*x+c)+1/105*(8/3
+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c))+28*a^2*b^6*(1/9*sin(d*x+c)^7/co
s(d*x+c)^9+2/63*sin(d*x+c)^7/cos(d*x+c)^7)+8*a*b^7*(1/9*sin(d*x+c)^8/cos(d*
x+c)^9+1/63*sin(d*x+c)^8/cos(d*x+c)^7-1/315*sin(d*x+c)^8/cos(d*x+c)^5+1/315
*sin(d*x+c)^8/cos(d*x+c)^3-1/63*sin(d*x+c)^8/cos(d*x+c)-1/63*(16/5+sin(d*x+
c)^6+6/5*sin(d*x+c)^4+8/5*sin(d*x+c)^2)*cos(d*x+c))+1/9*b^8*sin(d*x+c)^9/co
s(d*x+c)^9)
```

Maxima [A]
time = 0.30, size = 315, normalized size = 1.33

$$\frac{35\,b^8\tan\left(dx+c\right)^9+\left(35\,\tan\left(dx+c\right)^9+180\,\tan\left(dx+c\right)^7+378\,\tan\left(dx+c\right)^5+420\,\tan\left(dx+c\right)^3+315\,\tan\left(dx+c\right)\right)a^8+28\,\left(35\,\tan\left(dx+c\right)^9+135\,\tan\left(dx+c\right)^7+189\,\tan\left(dx+c\right)^5+105\,\tan\left(dx+c\right)^3\right)a^6b^2+70\,\left(35\,\tan\left(dx+c\right)^9+90\,\tan\left(dx+c\right)^7+63\,\tan\left(dx+c\right)^5\right)a^4b^4+140\,\left(7\,\tan\left(dx+c\right)^9+9\,\tan\left(dx+c\right)^7\right)a^2b^6-\frac{280\,\left(9\,\cos\left(dx+c\right)^2-7\right)a^5b^3}{\cos\left(dx+c\right)^9}+\frac{56\,\left(63\,\cos\left(dx+c\right)^4-90\,\cos\left(dx+c\right)^2+35\right)a^3b^5}{\cos\left(dx+c\right)^9}-\frac{8\,\left(105\,\cos\left(dx+c\right)^6-189\,\cos\left(dx+c\right)^4+135\,\cos\left(dx+c\right)^2-35\right)ab^7}{\cos\left(dx+c\right)^9}+\frac{280\,a^7b}{\cos\left(dx+c\right)^9}}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10*(a+b*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 1/315*(35*b^8*tan(d*x + c)^9 + (35*tan(d*x + c)^9 + 180*tan(d*x + c)^7 + 37
8*tan(d*x + c)^5 + 420*tan(d*x + c)^3 + 315*tan(d*x + c))*a^8 + 28*(35*tan(
d*x + c)^9 + 135*tan(d*x + c)^7 + 189*tan(d*x + c)^5 + 105*tan(d*x + c)^3)*
a^6*b^2 + 70*(35*tan(d*x + c)^9 + 90*tan(d*x + c)^7 + 63*tan(d*x + c)^5)*a^
4*b^4 + 140*(7*tan(d*x + c)^9 + 9*tan(d*x + c)^7)*a^2*b^6 - 280*(9*cos(d*x
+ c)^2 - 7)*a^5*b^3/cos(d*x + c)^9 + 56*(63*cos(d*x + c)^4 - 90*cos(d*x + c
)^2 + 35)*a^3*b^5/cos(d*x + c)^9 - 8*(105*cos(d*x + c)^6 - 189*cos(d*x + c)
^4 + 135*cos(d*x + c)^2 - 35)*a*b^7/cos(d*x + c)^9 + 280*a^7*b/cos(d*x + c)
^9)/d
```

Fricas [A]
time = 0.50, size = 336, normalized size = 1.42

$$-\frac{840\,ab^7\cos\left(dx+c\right)^6-280\,a^7b-1960\,a^5b^3-1960\,a^3b^5-280\,ab^7-504\,\left(7\,a^3b^5+3\,ab^7\right)\cos\left(dx+c\right)^4+360\,\left(7\,a^5b^3+14\,a^3b^5+3\,ab^7\right)\cos\left(dx+c\right)^2-\left(\left(128\,a^8-448\,a^6b^2+560\,a^4b^4-280\,a^2b^6+35\,b^8\right)\cos\left(dx+c\right)^8+35\,a^8+980\,a^6b^2+2450\,a^4b^4+980\,a^2b^6+35\,b^8+4\,\left(16\,a^8-56\,a^6b^2+70\,a^4b^4-35\,a^2b^6-35\,b^8\right)\cos\left(dx+c\right)^6+6\,\left(8\,a^8-28\,a^6b^2+35\,a^4b^4+350\,a^2b^6+35\,b^8\right)\cos\left(dx+c\right)^4+20\,\left(2\,a^8-7\,a^6b^2-175\,a^4b^4-133\,a^2b^6-7\,b^8\right)\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)}{315\,d\cos\left(dx+c\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10*(a+b*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] -1/315*(840*a*b^7*cos(d*x + c)^6 - 280*a^7*b - 1960*a^5*b^3 - 1960*a^3*b^5
- 280*a*b^7 - 504*(7*a^3*b^5 + 3*a*b^7)*cos(d*x + c)^4 + 360*(7*a^5*b^3 + 1
4*a^3*b^5 + 3*a*b^7)*cos(d*x + c)^2 - ((128*a^8 - 448*a^6*b^2 + 560*a^4*b^4
 - 280*a^2*b^6 + 35*b^8)*cos(d*x + c)^8 + 35*a^8 + 980*a^6*b^2 + 2450*a^4*b
^4 + 980*a^2*b^6 + 35*b^8 + 4*(16*a^8 - 56*a^6*b^2 + 70*a^4*b^4 - 35*a^2*b^
6 - 35*b^8)*cos(d*x + c)^6 + 6*(8*a^8 - 28*a^6*b^2 + 35*a^4*b^4 + 350*a^2*b
^6 + 35*b^8)*cos(d*x + c)^4 + 20*(2*a^8 - 7*a^6*b^2 - 175*a^4*b^4 - 133*a^2
*b^6 - 7*b^8)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^9)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**10*(a+b*sin(d*x+c))**8,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 892 vs. 2(224) = 448.
time = 7.01, size = 892, normalized size = 3.78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10*(a+b*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -2/315*(315*a^8*tan(1/2*d*x + 1/2*c)^17 + 2520*a^7*b*tan(1/2*d*x + 1/2*c)^1
6 - 840*a^8*tan(1/2*d*x + 1/2*c)^15 + 11760*a^6*b^2*tan(1/2*d*x + 1/2*c)^15
 + 35280*a^5*b^3*tan(1/2*d*x + 1/2*c)^14 + 4788*a^8*tan(1/2*d*x + 1/2*c)^13
 + 14112*a^6*b^2*tan(1/2*d*x + 1/2*c)^13 + 70560*a^4*b^4*tan(1/2*d*x + 1/2*
c)^13 + 23520*a^7*b*tan(1/2*d*x + 1/2*c)^12 + 58800*a^5*b^3*tan(1/2*d*x + 1
/2*c)^12 + 94080*a^3*b^5*tan(1/2*d*x + 1/2*c)^12 - 5112*a^8*tan(1/2*d*x + 1
/2*c)^11 + 79632*a^6*b^2*tan(1/2*d*x + 1/2*c)^11 + 120960*a^4*b^4*tan(1/2*d
*x + 1/2*c)^11 + 80640*a^2*b^6*tan(1/2*d*x + 1/2*c)^11 + 176400*a^5*b^3*tan
(1/2*d*x + 1/2*c)^10 + 141120*a^3*b^5*tan(1/2*d*x + 1/2*c)^10 + 40320*a*b^7
*tan(1/2*d*x + 1/2*c)^10 + 10658*a^8*tan(1/2*d*x + 1/2*c)^9 + 39872*a^6*b^2
*tan(1/2*d*x + 1/2*c)^9 + 244160*a^4*b^4*tan(1/2*d*x + 1/2*c)^9 + 89600*a^2
*b^6*tan(1/2*d*x + 1/2*c)^9 + 8960*b^8*tan(1/2*d*x + 1/2*c)^9 + 35280*a^7*b
*tan(1/2*d*x + 1/2*c)^8 + 105840*a^5*b^3*tan(1/2*d*x + 1/2*c)^8 + 197568*a^
3*b^5*tan(1/2*d*x + 1/2*c)^8 + 24192*a*b^7*tan(1/2*d*x + 1/2*c)^8 - 5112*a^
8*tan(1/2*d*x + 1/2*c)^7 + 79632*a^6*b^2*tan(1/2*d*x + 1/2*c)^7 + 120960*a^
4*b^4*tan(1/2*d*x + 1/2*c)^7 + 80640*a^2*b^6*tan(1/2*d*x + 1/2*c)^7 + 10584
0*a^5*b^3*tan(1/2*d*x + 1/2*c)^6 + 56448*a^3*b^5*tan(1/2*d*x + 1/2*c)^6 + 1
0752*a*b^7*tan(1/2*d*x + 1/2*c)^6 + 4788*a^8*tan(1/2*d*x + 1/2*c)^5 + 14112
```

```
*a^6*b^2*tan(1/2*d*x + 1/2*c)^5 + 70560*a^4*b^4*tan(1/2*d*x + 1/2*c)^5 + 10
080*a^7*b*tan(1/2*d*x + 1/2*c)^4 + 15120*a^5*b^3*tan(1/2*d*x + 1/2*c)^4 + 1
6128*a^3*b^5*tan(1/2*d*x + 1/2*c)^4 - 4608*a*b^7*tan(1/2*d*x + 1/2*c)^4 - 8
40*a^8*tan(1/2*d*x + 1/2*c)^3 + 11760*a^6*b^2*tan(1/2*d*x + 1/2*c)^3 + 5040
*a^5*b^3*tan(1/2*d*x + 1/2*c)^2 - 4032*a^3*b^5*tan(1/2*d*x + 1/2*c)^2 + 115
2*a*b^7*tan(1/2*d*x + 1/2*c)^2 + 315*a^8*tan(1/2*d*x + 1/2*c) + 280*a^7*b -
 560*a^5*b^3 + 448*a^3*b^5 - 128*a*b^7)/((tan(1/2*d*x + 1/2*c)^2 - 1)^9*d)
```

Mupad [B]
time = 6.68, size = 659, normalized size = 2.79

$$\frac{(a-b)^8}{2\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)^8}-\frac{(a+b)^8}{9\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)^9}-\frac{(a+b)^8}{2\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)^8}-\frac{(a-b)^8}{9\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)^9}-\frac{(a+b)^7\,(37\,a+21\,b)}{28\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)^7}-\frac{(a+b)^7\,(55\,a+7\,b)}{24\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)^6}+\frac{(a-b)^5\,(187\,a^3+191\,a^2\,b+65\,a\,b^2+5\,b^3)}{128\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)^2}+\frac{(a-b)^5\,(-463\,a^3-67\,a^2\,b+67\,a\,b^2+15\,b^3)}{192\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)^3}+\frac{(a-b)^6\,(95\,a^2+18\,a\,b-b^2)}{32\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)^4}+\frac{(a-b)^6\,(-241\,a^2+114\,a\,b+15\,b^2)}{80\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)^5}-\frac{(a-b)^7\,(37\,a-21\,b)}{28\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)^7}+\frac{(a-b)^7\,(55\,a-7\,b)}{24\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)^6}+\frac{(a+b)^6\,(-95\,a^2+18\,a\,b+b^2)}{32\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)^4}-\frac{(a+b)^5\,(187\,a^3-191\,a^2\,b+65\,a\,b^2-5\,b^3)}{128\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)^2}+\frac{(a+b)^5\,(-463\,a^3+67\,a^2\,b+67\,a\,b^2-15\,b^3)}{192\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)^3}-\frac{(a+b)^6\,(241\,a^2+114\,a\,b-15\,b^2)}{80\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)^5}-\frac{a\,(a+b)^4\,(16\,a^3-29\,a^2\,b+20\,a\,b^2-5\,b^3)}{16\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)}-\frac{a\,(a-b)^4\,(16\,a^3+29\,a^2\,b+20\,a\,b^2+5\,b^3)}{16\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^8/cos(c + d*x)^10,x)
```

```
[Out] (a - b)^8/(2*d*(tan(c/2 + (d*x)/2) + 1)^8) - (a + b)^8/(9*d*(tan(c/2 + (d*x
)/2) - 1)^9) - (a + b)^8/(2*d*(tan(c/2 + (d*x)/2) - 1)^8) - (a - b)^8/(9*d*
(tan(c/2 + (d*x)/2) + 1)^9) - ((a + b)^7*(37*a + 21*b))/(28*d*(tan(c/2 + (d
*x)/2) - 1)^7) - ((a + b)^7*(55*a + 7*b))/(24*d*(tan(c/2 + (d*x)/2) - 1)^6)
 + ((a - b)^5*(65*a*b^2 + 191*a^2*b + 187*a^3 + 5*b^3))/(128*d*(tan(c/2 + (
d*x)/2) + 1)^2) + ((a - b)^5*(67*a*b^2 - 67*a^2*b - 463*a^3 + 15*b^3))/(192
*d*(tan(c/2 + (d*x)/2) + 1)^3) + ((a - b)^6*(18*a*b + 95*a^2 - b^2))/(32*d*
(tan(c/2 + (d*x)/2) + 1)^4) + ((a - b)^6*(114*a*b - 241*a^2 + 15*b^2))/(80*
d*(tan(c/2 + (d*x)/2) + 1)^5) - ((a - b)^7*(37*a - 21*b))/(28*d*(tan(c/2 +
(d*x)/2) + 1)^7) + ((a - b)^7*(55*a - 7*b))/(24*d*(tan(c/2 + (d*x)/2) + 1)^
6) + ((a + b)^6*(18*a*b - 95*a^2 + b^2))/(32*d*(tan(c/2 + (d*x)/2) - 1)^4)
- ((a + b)^5*(65*a*b^2 - 191*a^2*b + 187*a^3 - 5*b^3))/(128*d*(tan(c/2 + (d
*x)/2) - 1)^2) + ((a + b)^5*(67*a*b^2 + 67*a^2*b - 463*a^3 - 15*b^3))/(192*
d*(tan(c/2 + (d*x)/2) - 1)^3) - ((a + b)^6*(114*a*b + 241*a^2 - 15*b^2))/(8
0*d*(tan(c/2 + (d*x)/2) - 1)^5) - (a*(a + b)^4*(20*a*b^2 - 29*a^2*b + 16*a^
3 - 5*b^3))/(16*d*(tan(c/2 + (d*x)/2) - 1)) - (a*(a - b)^4*(20*a*b^2 + 29*a
^2*b + 16*a^3 + 5*b^3))/(16*d*(tan(c/2 + (d*x)/2) + 1))
```

3.425 $\int \frac{\cos^5(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=118

$$\frac{(a^2-b^2)^2\log(a+b\sin(c+dx))}{b^5d}-\frac{a(a^2-2b^2)\sin(c+dx)}{b^4d}+\frac{(a^2-2b^2)\sin^2(c+dx)}{2b^3d}-\frac{a\sin^3(c+dx)}{3b^2d}+\frac{\sin^4(c+dx)}{4bd}$$

```
[Out] (a^2-b^2)^2*ln(a+b*sin(d*x+c))/b^5/d-a*(a^2-2*b^2)*sin(d*x+c)/b^4/d+1/2*(a^
2-2*b^2)*sin(d*x+c)^2/b^3/d-1/3*a*sin(d*x+c)^3/b^2/d+1/4*sin(d*x+c)^4/b/d
```

Rubi [A]
time = 0.07, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2747, 711}

$$\frac{(a^2-b^2)^2\log(a+b\sin(c+dx))}{b^5d}-\frac{a(a^2-2b^2)\sin(c+dx)}{b^4d}+\frac{(a^2-2b^2)\sin^2(c+dx)}{2b^3d}-\frac{a\sin^3(c+dx)}{3b^2d}+\frac{\sin^4(c+dx)}{4bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((a^2 - b^2)^2*Log[a + b*Sin[c + d*x]])/(b^5*d) - (a*(a^2 - 2*b^2)*Sin[c +
d*x])/(b^4*d) + ((a^2 - 2*b^2)*Sin[c + d*x]^2)/(2*b^3*d) - (a*Sin[c + d*x]^
3)/(3*b^2*d) + Sin[c + d*x]^4/(4*b*d)
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{(b^2-x^2)^2}{a+x}\,dx, x, b\sin(c+dx)\right)}{b^5d}$$
$$= \frac{\text{Subst}\left(\int \left(-a^3\left(1-\frac{2b^2}{a^2}\right)+(a^2-2b^2)\,x-ax^2+x^3+\frac{(a^2-b^2)^2}{a+x}\right)\,dx, x, b\sin(c+dx\right.}{b^5d}$$
$$= \frac{(a^2-b^2)^2\log(a+b\sin(c+dx))}{b^5d} - \frac{a(a^2-2b^2)\sin(c+dx)}{b^4d} + \frac{(a^2-2b^2)\sin^2(c+}{2b^3d}$$

Mathematica [A]
time = 0.27, size = 106, normalized size = 0.90

$$\frac{(-24a^2b^2+36b^4)\cos(2(c+dx))+3b^4\cos(4(c+dx))+96(a^2-b^2)^2\log(a+b\sin(c+dx))-24ab(4a^2-7b^2)\sin(c+dx)+8ab^3\sin(3(c+dx))}{96b^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((-24*a^2*b^2 + 36*b^4)*Cos[2*(c + d*x)] + 3*b^4*Cos[4*(c + d*x)] + 96*(a^2
 - b^2)^2*Log[a + b*Sin[c + d*x]] - 24*a*b*(4*a^2 - 7*b^2)*Sin[c + d*x] + 8
*a*b^3*Sin[3*(c + d*x)])/(96*b^5*d)
```

Maple [A]
time = 0.35, size = 106, normalized size = 0.90

method	result
derivativedivides	$$\frac{-\frac{-\frac{\left(\sin^4(dx+c)\right)b^3}{4}+\frac{a\left(\sin^3(dx+c)\right)b^2}{3}-\frac{\left(a^2-2b^2\right)\left(\sin^2(dx+c)\right)b}{2}+a\left(a^2-2b^2\right)\sin(dx+c)}{b^4}+\frac{\left(a^4-2a^2b^2+b^4\right)\ln(a+b\sin(dx+c))}{b^5}}{d}$$
default	$$\frac{-\frac{-\frac{\left(\sin^4(dx+c)\right)b^3}{4}+\frac{a\left(\sin^3(dx+c)\right)b^2}{3}-\frac{\left(a^2-2b^2\right)\left(\sin^2(dx+c)\right)b}{2}+a\left(a^2-2b^2\right)\sin(dx+c)}{b^4}+\frac{\left(a^4-2a^2b^2+b^4\right)\ln(a+b\sin(dx+c))}{b^5}}{d}$$
risch	$$-\frac{ix}{b}-\frac{2ic}{bd}+\frac{ia^3\mathrm{e}^{i(dx+c)}}{2b^4d}-\frac{\mathrm{e}^{2i(dx+c)}a^2}{8b^3d}+\frac{3\,\mathrm{e}^{2i(dx+c)}}{16bd}-\frac{7ia\,\mathrm{e}^{i(dx+c)}}{8b^2d}+\frac{4ia^2c}{b^3d}-\frac{2ia^4c}{b^5d}+\frac{2ix\,a^2}{b^3}-\frac{\mathrm{e}^{-2i(d}}{8b}$$
norman	$$\frac{\frac{\left(2a^2-4b^2\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^3d}+\frac{\left(2a^2-4b^2\right)\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^3d}+\frac{2\left(3a^2-4b^2\right)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^3d}+\frac{2\left(3a^2-4b^2\right)\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^3d}-\frac{2a\left(a^2-\right.}{}}{\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/b^4*(-1/4*sin(d*x+c)^4*b^3+1/3*a*sin(d*x+c)^3*b^2-1/2*(a^2-2*b^2)*s
in(d*x+c)^2*b+a*(a^2-2*b^2)*sin(d*x+c))+(a^4-2*a^2*b^2+b^4)/b^5*ln(a+b*sin(
d*x+c)))
```

Maxima [A]
time = 0.29, size = 108, normalized size = 0.92

$$\frac{\frac{3 b^3 \sin(dx+c)^4-4 a b^2 \sin(dx+c)^3+6 (a^2 b-2 b^3) \sin(dx+c)^2-12 (a^3-2 a b^2) \sin(dx+c)}{b^4}+\frac{12 (a^4-2 a^2 b^2+b^4) \log(b \sin(dx+c)+a)}{b^5}}{12 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*((3*b^3*sin(d*x + c)^4 - 4*a*b^2*sin(d*x + c)^3 + 6*(a^2*b - 2*b^3)*si
n(d*x + c)^2 - 12*(a^3 - 2*a*b^2)*sin(d*x + c))/b^4 + 12*(a^4 - 2*a^2*b^2 +
 b^4)*log(b*sin(d*x + c) + a)/b^5)/d
```

Fricas [A]
time = 0.36, size = 107, normalized size = 0.91

$$\frac{3 b^4 \cos(dx+c)^4-6 (a^2 b^2-b^4) \cos(dx+c)^2+12 (a^4-2 a^2 b^2+b^4) \log(b \sin(dx+c)+a)+4 (a b^3 \cos(dx+c)^2-3 a^3 b+5 a b^3) \sin(dx+c)}{12 b^5 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^4*cos(d*x + c)^4 - 6*(a^2*b^2 - b^4)*cos(d*x + c)^2 + 12*(a^4 - 2
*a^2*b^2 + b^4)*log(b*sin(d*x + c) + a) + 4*(a*b^3*cos(d*x + c)^2 - 3*a^3*b
 + 5*a*b^3)*sin(d*x + c))/(b^5*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 5.79, size = 120, normalized size = 1.02

$$\frac{\frac{3 b^3 \sin(dx+c)^4-4 a b^2 \sin(dx+c)^3+6 a^2 b \sin(dx+c)^2-12 b^3 \sin(dx+c)^2-12 a^3 \sin(dx+c)+24 a b^2 \sin(dx+c)}{b^4}+\frac{12 (a^4-2 a^2 b^2+b^4) \log(|b \sin(dx+c)+a|)}{b^5}}{12 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/12*((3*b^3*sin(d*x + c)^4 - 4*a*b^2*sin(d*x + c)^3 + 6*a^2*b*sin(d*x + c)
^2 - 12*b^3*sin(d*x + c)^2 - 12*a^3*sin(d*x + c) + 24*a*b^2*sin(d*x + c))/b
^4 + 12*(a^4 - 2*a^2*b^2 + b^4)*log(abs(b*sin(d*x + c) + a))/b^5)/d
```

Mupad [B]
time = 5.07, size = 109, normalized size = 0.92

$$\frac{\frac{\sin(c+d\,x)^4}{4\,b}-\sin\left(c+d\,x\right)^2\,\left(\frac{1}{b}-\frac{a^2}{2\,b^3}\right)+\frac{\ln(a+b\,\sin(c+d\,x))\,\left(a^4-2\,a^2\,b^2+b^4\right)}{b^5}-\frac{a\sin(c+d\,x)^3}{3\,b^2}+\frac{a\,\sin(c+d\,x)\,\left(\frac{2}{b}-\frac{a^2}{b^3}\right)}{b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(a + b*sin(c + d*x)),x)
```

```
[Out] (sin(c + d*x)^4/(4*b) - sin(c + d*x)^2*(1/b - a^2/(2*b^3)) + (log(a + b*sin
(c + d*x))*(a^4 + b^4 - 2*a^2*b^2))/b^5 - (a*sin(c + d*x)^3)/(3*b^2) + (a*s
in(c + d*x)*(2/b - a^2/b^3))/b)/d
```

3.426 $\int \frac{\cos^3(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=61

$$-\frac{(a^2-b^2)\log(a+b\sin(c+dx))}{b^3d}+\frac{a\sin(c+dx)}{b^2d}-\frac{\sin^2(c+dx)}{2bd}$$

```
[Out] -(a^2-b^2)*ln(a+b*sin(d*x+c))/b^3/d+a*sin(d*x+c)/b^2/d-1/2*sin(d*x+c)^2/b/d
```

Rubi [A]
time = 0.05, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2747, 711}

$$-\frac{(a^2-b^2)\log(a+b\sin(c+dx))}{b^3d}+\frac{a\sin(c+dx)}{b^2d}-\frac{\sin^2(c+dx)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + b*Sin[c + d*x]),x]
```

```
[Out] -(((a^2 - b^2)*Log[a + b*Sin[c + d*x]])/(b^3*d)) + (a*Sin[c + d*x])/(b^2*d)
 - Sin[c + d*x]^2/(2*b*d)
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^3(c+dx)}{a+b\sin(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{b^2-x^2}{a+x}\,dx,x,b\sin(c+dx)\right)}{b^3d}\\ &= \frac{\text{Subst}\left(\int \left(a-x+\frac{-a^2+b^2}{a+x}\right)\,dx,x,b\sin(c+dx)\right)}{b^3d}\\ &= -\frac{(a^2-b^2)\log(a+b\sin(c+dx))}{b^3d}+\frac{a\sin(c+dx)}{b^2d}-\frac{\sin^2(c+dx)}{2bd}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 55, normalized size = 0.90

$$\frac{b^2\cos(2(c+dx))+4(-a^2+b^2)\log(a+b\sin(c+dx))+4ab\sin(c+dx)}{4b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + b*Sin[c + d*x]),x]
```

```
[Out] (b^2*Cos[2*(c + d*x)] + 4*(-a^2 + b^2)*Log[a + b*Sin[c + d*x]] + 4*a*b*Sin[
c + d*x])/(4*b^3*d)
```

Maple [A]
time = 0.18, size = 54, normalized size = 0.89

method	result
derivativedivides	$\frac{\frac{-\frac{\left(\sin^2(dx+c)\right)b}{2}+a\sin(dx+c)}{b^2}+\frac{\left(-a^2+b^2\right)\ln(a+b\sin(dx+c))}{b^3}}{d}$
default	$\frac{\frac{-\frac{\left(\sin^2(dx+c)\right)b}{2}+a\sin(dx+c)}{b^2}+\frac{\left(-a^2+b^2\right)\ln(a+b\sin(dx+c))}{b^3}}{d}$
risch	$\frac{ix\,a^2}{b^3}-\frac{ix}{b}+\frac{e^{2i(dx+c)}}{8bd}-\frac{ia\,e^{i(dx+c)}}{2b^2d}+\frac{ia\,e^{-i(dx+c)}}{2b^2d}+\frac{e^{-2i(dx+c)}}{8bd}+\frac{2ia^2c}{b^3d}-\frac{2ic}{bd}-\frac{\ln\left(e^{2i(dx+c)}+\frac{2ia\,e^{i(dx}}{b}\right.}{b^3d}$
norman	$\frac{-\frac{2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{bd}-\frac{2\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{bd}+\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{b^2d}+\frac{4a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^2d}+\frac{2a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^2d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{(a^2-b^2)\ln\left(1+\tan^2\left(\frac{c}{}\right.\right.}{b^3d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/b^2*(-1/2*sin(d*x+c)^2*b+a*sin(d*x+c))+(-a^2+b^2)/b^3*ln(a+b*sin(d*x
+c)))
```

Maxima [A]
time = 0.28, size = 55, normalized size = 0.90

$$-\frac{\frac{b\sin(dx+c)^2-2\,a\sin(dx+c)}{b^2}+\frac{2\,(a^2-b^2)\log(b\sin(dx+c)+a)}{b^3}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*((b*sin(d*x + c)^2 - 2*a*sin(d*x + c))/b^2 + 2*(a^2 - b^2)*log(b*sin(d
*x + c) + a)/b^3)/d
```

Fricas [A]
time = 0.36, size = 53, normalized size = 0.87

$$\frac{b^2\cos{(dx+c)}^2+2\,ab\sin{(dx+c)}-2\,(a^2-b^2)\log{(b\sin{(dx+c)}+a)}}{2\,b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*cos(d*x + c)^2 + 2*a*b*sin(d*x + c) - 2*(a^2 - b^2)*log(b*sin(d*x
+ c) + a))/(b^3*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 4.09, size = 56, normalized size = 0.92

$$-\frac{\frac{b\sin(dx+c)^2-2\,a\sin(dx+c)}{b^2}+\frac{2\,(a^2-b^2)\log(|b\sin(dx+c)+a|)}{b^3}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*((b*sin(d*x + c)^2 - 2*a*sin(d*x + c))/b^2 + 2*(a^2 - b^2)*log(abs(b*s
in(d*x + c) + a))/b^3)/d
```

Mupad [B]
time = 0.08, size = 55, normalized size = 0.90

$$-\frac{\frac{\sin(c+d\,x)^2}{2\,b}+\frac{\ln(a+b\,\sin(c+d\,x))\,(a^2-b^2)}{b^3}-\frac{a\,\sin(c+d\,x)}{b^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a + b*sin(c + d*x)),x)
```

```
[Out] -(sin(c + d*x)^2/(2*b) + (log(a + b*sin(c + d*x))*(a^2 - b^2))/b^3 - (a*sin
(c + d*x))/b^2)/d
```

3.427 $\int \frac{\cos(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=18

$$\frac{\log(a+b\sin(c+dx))}{bd}$$

```
[Out] ln(a+b*sin(d*x+c))/b/d
```

Rubi [A]
time = 0.02, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2747, 31}

$$\frac{\log(a+b\sin(c+dx))}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + b*Sin[c + d*x]),x]
```

```
[Out] Log[a + b*Sin[c + d*x]]/(b*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)}{a+b\sin(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx,x,b\sin(c+dx)\right)}{bd}\\ &= \frac{\log(a+b\sin(c+dx))}{bd}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 18, normalized size = 1.00

$$\frac{\log(a+b\sin(c+dx))}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + b*Sin[c + d*x]),x]
```

```
[Out] Log[a + b*Sin[c + d*x]]/(b*d)
```

Maple [A]
time = 0.09, size = 19, normalized size = 1.06

method	result	size
derivativedivides	$\frac{\ln(a+b\sin(dx+c))}{bd}$	19
default	$\frac{\ln(a+b\sin(dx+c))}{bd}$	19
risch	$-\frac{ix}{b}-\frac{2ic}{bd}+\frac{\ln\left(e^{2i(dx+c)}+\frac{2ia\,e^{i(dx+c)}}{b}-1\right)}{bd}$	54
norman	$\frac{\ln\left(a\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a\right)}{bd}-\frac{\ln\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{bd}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] ln(a+b*sin(d*x+c))/b/d
```

Maxima [A]
time = 0.27, size = 18, normalized size = 1.00

$$\frac{\log\left(b\sin\left(dx+c\right)+a\right)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] log(b*sin(d*x + c) + a)/(b*d)
```

Fricas [A]
time = 0.35, size = 18, normalized size = 1.00

$$\frac{\log\left(b\sin\left(dx+c\right)+a\right)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] log(b*sin(d*x + c) + a)/(b*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(14) = 28$.

time = 0.29, size = 41, normalized size = 2.28

$$\begin{cases} \frac{x\cos(c)}{a} & \text{for } b = 0 \wedge d = 0 \\ \frac{\sin(c+dx)}{ad} & \text{for } b = 0 \\ \frac{x\cos(c)}{a+b\sin(c)} & \text{for } d = 0 \\ \frac{\log\left(\frac{a}{b}+\sin(c+dx)\right)}{bd} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Piecewise((x*cos(c)/a, Eq(b, 0) & Eq(d, 0)), (sin(c + d*x)/(a*d), Eq(b, 0))
, (x*cos(c)/(a + b*sin(c)), Eq(d, 0)), (log(a/b + sin(c + d*x))/(b*d), True
))
```

Giac [A]
time = 2.10, size = 19, normalized size = 1.06

$$\frac{\log(|b\sin(dx+c)+a|)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] log(abs(b*sin(d*x + c) + a))/(b*d)
```

Mupad [B]
time = 5.09, size = 18, normalized size = 1.00

$$\frac{\ln(a+b\sin(c+d\,x))}{b\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + b*sin(c + d*x)),x)
```

```
[Out] log(a + b*sin(c + d*x))/(b*d)
```

3.428 $\int \frac{\sec(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=75

$$-\frac{\log(1-\sin(c+dx))}{2(a+b)d}+\frac{\log(1+\sin(c+dx))}{2(a-b)d}-\frac{b\log(a+b\sin(c+dx))}{(a^2-b^2)\,d}$$

```
[Out] -1/2*ln(1-sin(d*x+c))/(a+b)/d+1/2*ln(1+sin(d*x+c))/(a-b)/d-b*ln(a+b*sin(d*x
+c))/(a^2-b^2)/d
```

Rubi [A]
time = 0.06, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2747, 720, 31, 647}

$$-\frac{b\log(a+b\sin(c+dx))}{d\,(a^2-b^2)}-\frac{\log(1-\sin(c+dx))}{2d(a+b)}+\frac{\log(\sin(c+dx)+1)}{2d(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/2*Log[1 - Sin[c + d*x]]/((a + b)*d) + Log[1 + Sin[c + d*x]]/(2*(a - b)*d
) - (b*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
(-a)*c]
```

Rule 720

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
```

```
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(c+dx)}{a+b\sin(c+dx)}\,dx &= \frac{b\text{Subst}\left(\int \frac{1}{(a+x)(b^2-x^2)}\,dx, x, b\sin(c+dx)\right)}{d}\\ &= -\frac{b\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\sin(c+dx)\right)}{(a^2-b^2)\,d} - \frac{b\text{Subst}\left(\int \frac{-a+x}{b^2-x^2}\,dx, x, b\sin(c+dx)\right)}{(a^2-b^2)\,d}\\ &= -\frac{b\log(a+b\sin(c+dx))}{(a^2-b^2)\,d} - \frac{\text{Subst}\left(\int \frac{1}{-b-x}\,dx, x, b\sin(c+dx)\right)}{2(a-b)d} + \frac{\text{Subst}\left(\int \frac{1}{b-x}\,dx\right.}{2(a}\\ &= -\frac{\log(1-\sin(c+dx))}{2(a+b)d} + \frac{\log(1+\sin(c+dx))}{2(a-b)d} - \frac{b\log(a+b\sin(c+dx))}{(a^2-b^2)\,d}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 64, normalized size = 0.85

$$\frac{(-a+b)\log(1-\sin(c+dx)) + (a+b)\log(1+\sin(c+dx)) - 2b\log(a+b\sin(c+dx))}{2(a-b)(a+b)d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + b*Sin[c + d*x]),x]
[Out] ((-a + b)*Log[1 - Sin[c + d*x]] + (a + b)*Log[1 + Sin[c + d*x]] - 2*b*Log[a
 + b*Sin[c + d*x]])/(2*(a - b)*(a + b)*d)
```

Maple [A]
time = 0.19, size = 71, normalized size = 0.95

method	result
derivativedivides	$\frac{\frac{\ln(1+\sin(dx+c))}{2a-2b} - \frac{\ln(\sin(dx+c)-1)}{2a+2b} - \frac{b\ln(a+b\sin(dx+c))}{(a-b)(a+b)}}{d}$
default	$\frac{\frac{\ln(1+\sin(dx+c))}{2a-2b} - \frac{\ln(\sin(dx+c)-1)}{2a+2b} - \frac{b\ln(a+b\sin(dx+c))}{(a-b)(a+b)}}{d}$
norman	$\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d(a-b)} - \frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d(a+b)} - \frac{b\ln\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a\right)}{d(a^2-b^2)}$
risch	$\frac{ix}{a+b} + \frac{ic}{d(a+b)} - \frac{ix}{a-b} - \frac{ic}{d(a-b)} + \frac{2ibx}{a^2-b^2} + \frac{2ibc}{d(a^2-b^2)} - \frac{\ln(e^{i(dx+c)}-i)}{d(a+b)} + \frac{\ln(e^{i(dx+c)}+i)}{d(a-b)} - \frac{b\ln\left(e^{2i(dx+c}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/(2*a-2*b)*ln(1+sin(d*x+c))-1/(2*a+2*b)*ln(sin(d*x+c)-1)-b/(a-b)/(a+b
)*ln(a+b*sin(d*x+c)))
```

Maxima [A]
time = 0.28, size = 64, normalized size = 0.85

$$-\frac{\frac{2\,b\log\left(b\sin\left(dx+c\right)+a\right)}{a^2-b^2}-\frac{\log\left(\sin\left(dx+c\right)+1\right)}{a-b}+\frac{\log\left(\sin\left(dx+c\right)-1\right)}{a+b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(2*b*log(b*sin(d*x + c) + a)/(a^2 - b^2) - log(sin(d*x + c) + 1)/(a -
b) + log(sin(d*x + c) - 1)/(a + b))/d
```

Fricas [A]
time = 0.36, size = 62, normalized size = 0.83

$$-\frac{2\,b\log\left(b\sin\left(dx+c\right)+a\right)-(a+b)\log\left(\sin\left(dx+c\right)+1\right)+(a-b)\log\left(-\sin\left(dx+c\right)+1\right)}{2\,(a^2-b^2)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*log(b*sin(d*x + c) + a) - (a + b)*log(sin(d*x + c) + 1) + (a - b)
*log(-sin(d*x + c) + 1))/((a^2 - b^2)*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec\left(c+dx\right)}{a+b\sin\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)/(a + b*sin(c + d*x)), x)
```

Giac [A]
time = 5.58, size = 71, normalized size = 0.95

$$-\frac{\frac{2\,b^2\log\left(\left|b\sin\left(dx+c\right)+a\right|\right)}{a^2b-b^3}-\frac{\log\left(\left|\sin\left(dx+c\right)+1\right|\right)}{a-b}+\frac{\log\left(\left|\sin\left(dx+c\right)-1\right|\right)}{a+b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*b^2*log(abs(b*sin(d*x + c) + a))/(a^2*b - b^3) - log(abs(sin(d*x +
c) + 1))/(a - b) + log(abs(sin(d*x + c) - 1))/(a + b))/d
```

Mupad [B]
time = 5.13, size = 69, normalized size = 0.92

$$\frac{\ln\left(\sin\left(c+d\,x\right)+1\right)}{2\,d\,\left(a-b\right)}-\frac{\ln\left(\sin\left(c+d\,x\right)-1\right)}{2\,d\,\left(a+b\right)}-\frac{b\,\ln\left(a+b\,\sin\left(c+d\,x\right)\right)}{d\,\left(a^2-b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + b*sin(c + d*x))),x)
```

```
[Out] log(sin(c + d*x) + 1)/(2*d*(a - b)) - log(sin(c + d*x) - 1)/(2*d*(a + b)) -
 (b*log(a + b*sin(c + d*x)))/(d*(a^2 - b^2))
```

3.429 $\int \frac{\sec^3(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=123

$$-\frac{(a+2b)\log(1-\sin(c+dx))}{4(a+b)^2d}+\frac{(a-2b)\log(1+\sin(c+dx))}{4(a-b)^2d}+\frac{b^3\log(a+b\sin(c+dx))}{(a^2-b^2)^2\,d}-\frac{\sec^2(c+dx)(b-a$$

```
[Out] -1/4*(a+2*b)*ln(1-sin(d*x+c))/(a+b)^2/d+1/4*(a-2*b)*ln(1+sin(d*x+c))/(a-b)^
2/d+b^3*ln(a+b*sin(d*x+c))/(a^2-b^2)^2/d-1/2*sec(d*x+c)^2*(b-a*sin(d*x+c))/
(a^2-b^2)/d
```

Rubi [A]
time = 0.11, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2747, 755, 815}

$$-\frac{\sec^2(c+dx)(b-a\sin(c+dx))}{2d\,(a^2-b^2)}+\frac{b^3\log(a+b\sin(c+dx))}{d\,(a^2-b^2)^2}-\frac{(a+2b)\log(1-\sin(c+dx))}{4d(a+b)^2}+\frac{(a-2b)\log(\sin(c+dx)+1)}{4d(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/4*((a + 2*b)*Log[1 - Sin[c + d*x]])/((a + b)^2*d) + ((a - 2*b)*Log[1 + S
in[c + d*x]])/(4*(a - b)^2*d) + (b^3*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)^
2*d) - (Sec[c + d*x]^2*(b - a*Sin[c + d*x]))/(2*(a^2 - b^2)*d)
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
```

```
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{b^3\text{Subst}\left(\int \frac{1}{(a+x)(b^2-x^2)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= -\frac{\sec^2(c+dx)(b-a\sin(c+dx))}{2(a^2-b^2)d} + \frac{b\text{Subst}\left(\int \frac{a^2-2b^2+ax}{(a+x)(b^2-x^2)}\,dx, x, b\sin(c+dx)\right)}{2(a^2-b^2)d}$$
$$= -\frac{\sec^2(c+dx)(b-a\sin(c+dx))}{2(a^2-b^2)d} + \frac{b\text{Subst}\left(\int \left(\frac{(a-b)(a+2b)}{2b(a+b)(b-x)} + \frac{2b^2}{(a-b)(a+b)(a+x)} + \frac{(a-}{2(a-}\right.\right.}{2(a^2-b^2)d}$$
$$= -\frac{(a+2b)\log(1-\sin(c+dx))}{4(a+b)^2d} + \frac{(a-2b)\log(1+\sin(c+dx))}{4(a-b)^2d} + \frac{b^3\log(a+b\sin}{(a^2-b^2)}$$

Mathematica [A]
time = 0.60, size = 170, normalized size = 1.38

$$\frac{-\frac{2(a+2b)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{(a+b)^2}+\frac{2(a-2b)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{(a-b)^2}+\frac{4b^3\log(a+b\sin(c+dx))}{(a^2-b^2)^2}+\frac{1}{(a+b)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}-\frac{1}{(a-b)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((-2*(a + 2*b)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/(a + b)^2 + (2*(a
- 2*b)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/(a - b)^2 + (4*b^3*Log[a +
 b*Sin[c + d*x]])/(a^2 - b^2)^2 + 1/((a + b)*(Cos[(c + d*x)/2] - Sin[(c + d
*x)/2])^2) - 1/((a - b)*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2))/(4*d)
```

Maple [A]
time = 0.43, size = 121, normalized size = 0.98

method	result
derivativedivides	$\frac{-\frac{1}{(4a-4b)(1+\sin(dx+c))}+\frac{(a-2b)\ln(1+\sin(dx+c))}{4(a-b)^2}-\frac{1}{(4a+4b)(\sin(dx+c)-1)}+\frac{(-a-2b)\ln(\sin(dx+c)-1)}{4(a+b)^2}+\frac{b^3\ln(a+b\sin(dx+c))}{(a+b)^2(a-b)^2}}{d}$
default	$\frac{-\frac{1}{(4a-4b)(1+\sin(dx+c))}+\frac{(a-2b)\ln(1+\sin(dx+c))}{4(a-b)^2}-\frac{1}{(4a+4b)(\sin(dx+c)-1)}+\frac{(-a-2b)\ln(\sin(dx+c)-1)}{4(a+b)^2}+\frac{b^3\ln(a+b\sin(dx+c))}{(a+b)^2(a-b)^2}}{d}$
norman	$\frac{\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{(a^2-b^2)d}+\frac{a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{(a^2-b^2)d}-\frac{2b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d(a^2-b^2)}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{b^3\ln\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a\right)}{d(a^4-2a^2b^2+b^4)}+\frac{(a-2b)\ln\left(\tan\right.}{2d(a^2-}$

risch	$\frac{iax}{2a^2+4ab+2b^2}+\frac{iac}{2(a^2+2ab+b^2)d}-\frac{2ib^3x}{a^4-2a^2b^2+b^4}-\frac{2ib^3c}{d(a^4-2a^2b^2+b^4)}+\frac{ibx}{a^2-2ab+b^2}+\frac{ibc}{d(a^2-2ab+b^2)}-\frac{}{2(a^2-}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/(4*a-4*b)/(1+sin(d*x+c))+1/4*(a-2*b)/(a-b)^2*ln(1+sin(d*x+c))-1/(4*
a+4*b)/(sin(d*x+c)-1)+1/4/(a+b)^2*(-a-2*b)*ln(sin(d*x+c)-1)+b^3/(a+b)^2/(a-
b)^2*ln(a+b*sin(d*x+c)))
```

Maxima [A]
time = 0.27, size = 139, normalized size = 1.13

$$\frac{\frac{4\,b^3\log\left(b\sin\left(dx+c\right)+a\right)}{a^4-2\,a^2b^2+b^4}+\frac{\left(a-2\,b\right)\log\left(\sin\left(dx+c\right)+1\right)}{a^2-2\,ab+b^2}-\frac{\left(a+2\,b\right)\log\left(\sin\left(dx+c\right)-1\right)}{a^2+2\,ab+b^2}-\frac{2\,\left(a\sin\left(dx+c\right)-b\right)}{\left(a^2-b^2\right)\sin\left(dx+c\right)^2-a^2+b^2}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*(4*b^3*log(b*sin(d*x + c) + a)/(a^4 - 2*a^2*b^2 + b^4) + (a - 2*b)*log(
sin(d*x + c) + 1)/(a^2 - 2*a*b + b^2) - (a + 2*b)*log(sin(d*x + c) - 1)/(a^
2 + 2*a*b + b^2) - 2*(a*sin(d*x + c) - b)/((a^2 - b^2)*sin(d*x + c)^2 - a^2
 + b^2))/d
```

Fricas [A]
time = 0.39, size = 153, normalized size = 1.24

$$\frac{4\,b^3\cos\left(dx+c\right)^2\log\left(b\sin\left(dx+c\right)+a\right)+\left(a^3-3\,ab^2-2\,b^3\right)\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-\left(a^3-3\,ab^2+2\,b^3\right)\cos\left(dx+c\right)^2\log\left(-\sin\left(dx+c\right)+1\right)-2\,a^2b+2\,b^3+2\,\left(a^3-ab^2\right)\sin\left(dx+c\right)}{4\,\left(a^4-2\,a^2b^2+b^4\right)d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(4*b^3*cos(d*x + c)^2*log(b*sin(d*x + c) + a) + (a^3 - 3*a*b^2 - 2*b^3)
*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (a^3 - 3*a*b^2 + 2*b^3)*cos(d*x + c
)^2*log(-sin(d*x + c) + 1) - 2*a^2*b + 2*b^3 + 2*(a^3 - a*b^2)*sin(d*x + c)
)/((a^4 - 2*a^2*b^2 + b^4)*d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec^3\left(c+dx\right)}{a+b\sin\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**3/(a + b*sin(c + d*x)), x)
```

Giac [A]

time = 6.24, size = 177, normalized size = 1.44

$$\frac{\frac{4\,b^4\log\left(|b\sin(dx+c)+a|\right)}{a^4b-2\,a^2b^3+b^5}+\frac{(a-2\,b)\log\left(|\sin(dx+c)+1|\right)}{a^2-2\,ab+b^2}-\frac{(a+2\,b)\log\left(|\sin(dx+c)-1|\right)}{a^2+2\,ab+b^2}+\frac{2\left(b^3\sin(dx+c)^2-a^3\sin(dx+c)+ab^2\sin(dx+c)+a^2b-2\,b^3\right)}{(a^4-2\,a^2b^2+b^4)\left(\sin(dx+c)^2-1\right)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/4*(4*b^4*log(abs(b*sin(d*x + c) + a))/(a^4*b - 2*a^2*b^3 + b^5) + (a - 2*
b)*log(abs(sin(d*x + c) + 1))/(a^2 - 2*a*b + b^2) - (a + 2*b)*log(abs(sin(d
*x + c) - 1))/(a^2 + 2*a*b + b^2) + 2*(b^3*sin(d*x + c)^2 - a^3*sin(d*x + c
) + a*b^2*sin(d*x + c) + a^2*b - 2*b^3)/((a^4 - 2*a^2*b^2 + b^4)*(sin(d*x +
 c)^2 - 1)))/d
```

Mupad [B]

time = 5.39, size = 148, normalized size = 1.20

$$\frac{\frac{b}{2\,(a^2-b^2)}-\frac{a\,\sin(c+d\,x)}{2\,(a^2-b^2)}}{d\,\left(\sin\left(c+d\,x\right)^2-1\right)}-\frac{\ln\left(\sin\left(c+d\,x\right)-1\right)\,\left(\frac{b}{4\,(a+b)^2}+\frac{1}{4\,(a+b)}\right)}{d}+\frac{b^3\,\ln\left(a+b\,\sin\left(c+d\,x\right)\right)}{d\,\left(a^4-2\,a^2\,b^2+b^4\right)}+\frac{\ln\left(\sin\left(c+d\,x\right)+1\right)\,\left(a-2\,b\right)}{4\,d\,\left(a-b\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + b*sin(c + d*x))),x)
```

```
[Out] (b/(2*(a^2 - b^2)) - (a*sin(c + d*x))/(2*(a^2 - b^2)))/(d*(sin(c + d*x)^2 -
 1)) - (log(sin(c + d*x) - 1)*(b/(4*(a + b)^2) + 1/(4*(a + b))))/d + (b^3*l
og(a + b*sin(c + d*x)))/(d*(a^4 + b^4 - 2*a^2*b^2)) + (log(sin(c + d*x) + 1
)*(a - 2*b))/(4*d*(a - b)^2)
```

3.430 $\int \frac{\sec^5(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=195

$$-\frac{(3a^2+9ab+8b^2)\log(1-\sin(c+dx))}{16(a+b)^3d}+\frac{(3a^2-9ab+8b^2)\log(1+\sin(c+dx))}{16(a-b)^3d}-\frac{b^5\log(a+b\sin(c+dx))}{(a^2-b^2)^3\,d}-$$

```
[Out] -1/16*(3*a^2+9*a*b+8*b^2)*ln(1-sin(d*x+c))/(a+b)^3/d+1/16*(3*a^2-9*a*b+8*b^
2)*ln(1+sin(d*x+c))/(a-b)^3/d-b^5*ln(a+b*sin(d*x+c))/(a^2-b^2)^3/d-1/4*sec(
d*x+c)^4*(b-a*sin(d*x+c))/(a^2-b^2)/d+1/8*sec(d*x+c)^2*(4*b^3+a*(3*a^2-7*b^
2)*sin(d*x+c))/(a^2-b^2)^2/d
```

Rubi [A]
time = 0.18, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2747, 755, 837, 815}

$$-\frac{(3a^2+9ab+8b^2)\log(1-\sin(c+dx))}{16d(a+b)^3}+\frac{(3a^2-9ab+8b^2)\log(\sin(c+dx)+1)}{16d(a-b)^3}-\frac{\sec^4(c+dx)(b-a\sin(c+dx))}{4d\,(a^2-b^2)}-\frac{b^5\log(a+b\sin(c+dx))}{d\,(a^2-b^2)^3}+\frac{\sec^2(c+dx)\,(a(3a^2-7b^2)\sin(c+dx)+4b^3)}{8d\,(a^2-b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/16*((3*a^2 + 9*a*b + 8*b^2)*Log[1 - Sin[c + d*x]])/((a + b)^3*d) + ((3*a
^2 - 9*a*b + 8*b^2)*Log[1 + Sin[c + d*x]])/(16*(a - b)^3*d) - (b^5*Log[a +
b*Sin[c + d*x]])/((a^2 - b^2)^3*d) - (Sec[c + d*x]^4*(b - a*Sin[c + d*x]))/
(4*(a^2 - b^2)*d) + (Sec[c + d*x]^2*(4*b^3 + a*(3*a^2 - 7*b^2)*Sin[c + d*x]
))/(8*(a^2 - b^2)^2*d)
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
```

```
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^5(c+dx)}{a+b\sin(c+dx)}\,dx &= \frac{b^5\text{Subst}\left(\int \frac{1}{(a+x)(b^2-x^2)^3}\,dx, x, b\sin(c+dx)\right)}{d}\\
&= -\frac{\sec^4(c+dx)(b-a\sin(c+dx))}{4(a^2-b^2)\,d}+\frac{b^3\text{Subst}\left(\int \frac{3a^2-4b^2+3ax}{(a+x)(b^2-x^2)^2}\,dx, x, b\sin(c+dx)\right)}{4(a^2-b^2)\,d}\\
&= -\frac{\sec^4(c+dx)(b-a\sin(c+dx))}{4(a^2-b^2)\,d}+\frac{\sec^2(c+dx)\left(4b^3+a(3a^2-7b^2)\sin(c+dx)\right)}{8(a^2-b^2)^2\,d}\\
&= -\frac{\sec^4(c+dx)(b-a\sin(c+dx))}{4(a^2-b^2)\,d}+\frac{\sec^2(c+dx)\left(4b^3+a(3a^2-7b^2)\sin(c+dx)\right)}{8(a^2-b^2)^2\,d}\\
&= -\frac{(3a^2+9ab+8b^2)\log(1-\sin(c+dx))}{16(a+b)^3d}+\frac{(3a^2-9ab+8b^2)\log(1+\sin(c+dx))}{16(a-b)^3d}
\end{aligned}$$

Mathematica [A]
time = 0.99, size = 266, normalized size = 1.36

$$\frac{-\frac{2(3a^2+9ab+8b^2)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{(a+b)^3}+\frac{2(3a^2-9ab+8b^2)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{(a-b)^3}+\frac{16b^5\log(a+b\sin(c+dx))}{(-a^2+b^2)^3}+\frac{1}{(a+b)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^4}+\frac{3a+5b}{(a+b)^2\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}-\frac{1}{(a-b)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^4}+\frac{-3a+5b}{(a-b)^2\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((-2*(3*a^2 + 9*a*b + 8*b^2)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/(a +
 b)^3 + (2*(3*a^2 - 9*a*b + 8*b^2)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]
)/(a - b)^3 + (16*b^5*Log[a + b*Sin[c + d*x]])/(-a^2 + b^2)^3 + 1/((a + b)*
```

```
(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^4) + (3*a + 5*b)/((a + b)^2*(Cos[(c +
 d*x)/2] - Sin[(c + d*x)/2])^2) - 1/((a - b)*(Cos[(c + d*x)/2] + Sin[(c + d
*x)/2])^4) + (-3*a + 5*b)/((a - b)^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^
2))/(16*d)
```

Maple [A]
time = 0.65, size = 190, normalized size = 0.97 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2/(8*a-8*b)/(1+sin(d*x+c))^2-1/16*(3*a-5*b)/(a-b)^2/(1+sin(d*x+c))+
1/16*(3*a^2-9*a*b+8*b^2)/(a-b)^3*ln(1+sin(d*x+c))+1/2/(8*a+8*b)/(sin(d*x+c)
-1)^2-1/16*(3*a+5*b)/(a+b)^2/(sin(d*x+c)-1)+1/16/(a+b)^3*(-3*a^2-9*a*b-8*b^
2)*ln(sin(d*x+c)-1)-b^5/(a+b)^3/(a-b)^3*ln(a+b*sin(d*x+c)))
```

Maxima [A]
time = 0.30, size = 278, normalized size = 1.43

$$-\frac{\frac{16\,b^5\log\left(b\sin\left(dx+c\right)+a\right)}{a^6-3\,a^4b^2+3\,a^2b^4-b^6}-\frac{\left(3\,a^2-9\,ab+8\,b^2\right)\log\left(\sin\left(dx+c\right)+1\right)}{a^3-3\,a^2b+3\,ab^2-b^3}+\frac{\left(3\,a^2+9\,ab+8\,b^2\right)\log\left(\sin\left(dx+c\right)-1\right)}{a^3+3\,a^2b+3\,ab^2+b^3}+\frac{2\left(4\,b^3\sin\left(dx+c\right)^2+\left(3\,a^3-7\,ab^2\right)\sin\left(dx+c\right)^3+2\,a^2b-6\,b^3-\left(5\,a^3-9\,ab^2\right)\sin\left(dx+c\right)\right)}{\left(a^4-2\,a^2b^2+b^4\right)\sin\left(dx+c\right)^4+a^4-2\,a^2b^2+b^4-2\left(a^4-2\,a^2b^2+b^4\right)\sin\left(dx+c\right)^2}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/16*(16*b^5*log(b*sin(d*x + c) + a)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6) -
 (3*a^2 - 9*a*b + 8*b^2)*log(sin(d*x + c) + 1)/(a^3 - 3*a^2*b + 3*a*b^2 - b
^3) + (3*a^2 + 9*a*b + 8*b^2)*log(sin(d*x + c) - 1)/(a^3 + 3*a^2*b + 3*a*b^
2 + b^3) + 2*(4*b^3*sin(d*x + c)^2 + (3*a^3 - 7*a*b^2)*sin(d*x + c)^3 + 2*a
^2*b - 6*b^3 - (5*a^3 - 9*a*b^2)*sin(d*x + c))/((a^4 - 2*a^2*b^2 + b^4)*sin
(d*x + c)^4 + a^4 - 2*a^2*b^2 + b^4 - 2*(a^4 - 2*a^2*b^2 + b^4)*sin(d*x + c
)^2))/d
```

Fricas [A]
time = 0.44, size = 253, normalized size = 1.30

$$-\frac{16\,b^5\cos\left(dx+c\right)^4\log\left(b\sin\left(dx+c\right)+a\right)-\left(3\,a^5-10\,a^3b^2+15\,ab^4+8\,b^5\right)\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)+\left(3\,a^5-10\,a^3b^2+15\,ab^4-8\,b^5\right)\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+1\right)+4\,a^4b-8\,a^2b^3+4\,b^5-8\left(a^2b^3-b^5\right)\cos\left(dx+c\right)^2-2\left(2\,a^5-4\,a^3b^2+2\,ab^4+\left(3\,a^5-10\,a^3b^2+7\,ab^4\right)\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)}{16\left(a^6-3\,a^4b^2+3\,a^2b^4-b^6\right)d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/16*(16*b^5*cos(d*x + c)^4*log(b*sin(d*x + c) + a) - (3*a^5 - 10*a^3*b^2
+ 15*a*b^4 + 8*b^5)*cos(d*x + c)^4*log(sin(d*x + c) + 1) + (3*a^5 - 10*a^3*
b^2 + 15*a*b^4 - 8*b^5)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 4*a^4*b - 8
*a^2*b^3 + 4*b^5 - 8*(a^2*b^3 - b^5)*cos(d*x + c)^2 - 2*(2*a^5 - 4*a^3*b^2
+ 2*a*b^4 + (3*a^5 - 10*a^3*b^2 + 7*a*b^4)*cos(d*x + c)^2)*sin(d*x + c))/((
a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d*cos(d*x + c)^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^5(c + dx)}{a + b\sin(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**5/(a + b*sin(c + d*x)), x)
```

Giac [A]
time = 4.55, size = 332, normalized size = 1.70

$$-\frac{\frac{16 b^6 \log(|b \sin(dx+c)+a|)}{a^6 b-3 a^4 b^3+3 a^2 b^5-b^7} - \frac{(3 a^2-9 ab+8 b^2) \log(|\sin(dx+c)+1|)}{a^3-3 a^2 b+3 ab^2-b^3} + \frac{(3 a^2+9 ab+8 b^2) \log(|\sin(dx+c)-1|)}{a^3+3 a^2 b+3 ab^2+b^3} + \frac{2 \left(6 b^5 \sin(dx+c)^4+3 a^5 \sin(dx+c)^3-10 a^3 b^2 \sin(dx+c)^3+7 ab^4 \sin(dx+c)^3+4 a^2 b^3 \sin(dx+c)^2-16 b^5 \sin(dx+c)^2-5 a^5 \sin(dx+c)+14 a^3 b^2 \sin(dx+c)-9 ab^4 \sin(dx+c)+2 a^4 b-8 a^2 b^3+12 b^5\right)}{(a^6-3 a^4 b^2+3 a^2 b^4-b^6) \left(\sin(dx+c)^2-1\right)^2}}{16 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/16*(16*b^6*log(abs(b*sin(d*x + c) + a))/(a^6*b - 3*a^4*b^3 + 3*a^2*b^5 -
 b^7) - (3*a^2 - 9*a*b + 8*b^2)*log(abs(sin(d*x + c) + 1))/(a^3 - 3*a^2*b +
 3*a*b^2 - b^3) + (3*a^2 + 9*a*b + 8*b^2)*log(abs(sin(d*x + c) - 1))/(a^3 +
 3*a^2*b + 3*a*b^2 + b^3) + 2*(6*b^5*sin(d*x + c)^4 + 3*a^5*sin(d*x + c)^3
- 10*a^3*b^2*sin(d*x + c)^3 + 7*a*b^4*sin(d*x + c)^3 + 4*a^2*b^3*sin(d*x +
c)^2 - 16*b^5*sin(d*x + c)^2 - 5*a^5*sin(d*x + c) + 14*a^3*b^2*sin(d*x + c)
 - 9*a*b^4*sin(d*x + c) + 2*a^4*b - 8*a^2*b^3 + 12*b^5)/((a^6 - 3*a^4*b^2 +
 3*a^2*b^4 - b^6)*(sin(d*x + c)^2 - 1)^2))/d
```

Mupad [B]
time = 0.59, size = 322, normalized size = 1.65

$$\frac{\ln(\sin(c+dx)+1) \left(\frac{b^2}{8 (a-b)^3} - \frac{3 b}{16 (a-b)^2} + \frac{3}{16 (a-b)}\right)}{d} - \frac{\ln(\sin(c+dx)-1) \left(\frac{3 b}{16 (a+b)^2} + \frac{3}{16 (a+b)} + \frac{b^2}{8 (a+b)^3}\right)}{d} - \frac{\frac{a^2 b-3 b^3}{4 (a^4-2 a^2 b^2+b^4)} + \frac{b^3 \sin(c+dx)^2}{2 (a^4-2 a^2 b^2+b^4)} - \frac{\sin(c+dx)^3 (7 a b^2-3 a^3)}{8 (a^4-2 a^2 b^2+b^4)} + \frac{\sin(c+dx) (9 a b^2-5 a^3)}{8 (a^4-2 a^2 b^2+b^4)}}{d \left(\cos(c+dx)^2+\sin(c+dx)^4-\sin(c+dx)^2\right)} - \frac{b^5 \ln(a+b \sin(c+dx))}{d (a^6-3 a^4 b^2+3 a^2 b^4-b^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^5*(a + b*sin(c + d*x))),x)
```

```
[Out] (log(sin(c + d*x) + 1)*(b^2/(8*(a - b)^3) - (3*b)/(16*(a - b)^2) + 3/(16*(a
 - b))))/d - (log(sin(c + d*x) - 1)*((3*b)/(16*(a + b)^2) + 3/(16*(a + b))
+ b^2/(8*(a + b)^3)))/d - ((a^2*b - 3*b^3)/(4*(a^4 + b^4 - 2*a^2*b^2)) + (b
^3*sin(c + d*x)^2)/(2*(a^4 + b^4 - 2*a^2*b^2)) - (sin(c + d*x)^3*(7*a*b^2 -
 3*a^3))/(8*(a^4 + b^4 - 2*a^2*b^2)) + (sin(c + d*x)*(9*a*b^2 - 5*a^3))/(8*
(a^4 + b^4 - 2*a^2*b^2)))/(d*(cos(c + d*x)^2 - sin(c + d*x)^2 + sin(c + d*x
)^4)) - (b^5*log(a + b*sin(c + d*x)))/(d*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2
))
```

3.431 $\int \frac{\cos^6(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=188

$$\frac{a(8a^4-20a^2b^2+15b^4)\,x}{8b^6}-\frac{2(a^2-b^2)^{5/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^6d}+\frac{\cos^5(c+dx)}{5bd}-\frac{\cos^3(c+dx)\,(4(a^2-b^2)-3}{12b^3d}$$

```
[Out] 1/8*a*(8*a^4-20*a^2*b^2+15*b^4)*x/b^6-2*(a^2-b^2)^(5/2)*arctan((b+a*tan(1/2
*d*x+1/2*c))/(a^2-b^2)^(1/2))/b^6/d+1/5*cos(d*x+c)^5/b/d-1/12*cos(d*x+c)^3*
(4*a^2-4*b^2-3*a*b*sin(d*x+c))/b^3/d+1/8*cos(d*x+c)*(8*(a^2-b^2)^2-a*b*(4*a
^2-7*b^2)*sin(d*x+c))/b^5/d
```

Rubi [A]

time = 0.29, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2774, 2944, 2814, 2739, 632, 210}

$$-\frac{2(a^2-b^2)^{5/2}\,\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^6d}+\frac{\cos(c+dx)\left(8(a^2-b^2)^2-ab(4a^2-7b^2)\sin(c+dx)\right)}{8b^5d}-\frac{\cos^3(c+dx)\,(4(a^2-b^2)-3ab\sin(c+dx))}{12b^3d}+\frac{ax(8a^4-20a^2b^2+15b^4)}{8b^6}+\frac{\cos^5(c+dx)}{5bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6/(a + b*Sin[c + d*x]),x]
```

```
[Out] (a*(8*a^4 - 20*a^2*b^2 + 15*b^4)*x)/(8*b^6) - (2*(a^2 - b^2)^(5/2)*ArcTan[(
b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^6*d) + Cos[c + d*x]^5/(5*b*d)
- (Cos[c + d*x]^3*(4*(a^2 - b^2) - 3*a*b*Sin[c + d*x]))/(12*b^3*d) + (Cos[c
 + d*x]*(8*(a^2 - b^2)^2 - a*b*(4*a^2 - 7*b^2)*Sin[c + d*x]))/(8*b^5*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
```

```
a^2 - b^2, 0]
```

Rule 2774

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(b*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2944

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Dist[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\cos^6(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\cos^5(c+dx)}{5bd} + \frac{\int \frac{\cos^4(c+dx)(b+a\sin(c+dx))}{a+b\sin(c+dx)}\,dx}{b}$$

$$= \frac{\cos^5(c+dx)}{5bd} - \frac{\cos^3(c+dx)\left(4(a^2-b^2)-3ab\sin(c+dx)\right)}{12b^3d} + \frac{\int \frac{\cos^2(c+dx)(-b(a^2-4b^2)}{a+b\sin}}{4}$$

$$= \frac{\cos^5(c+dx)}{5bd} - \frac{\cos^3(c+dx)\left(4(a^2-b^2)-3ab\sin(c+dx)\right)}{12b^3d} + \frac{\cos(c+dx)\left(8(a^2-\right.}{}$$

$$= \frac{a(8a^4-20a^2b^2+15b^4)\,x}{8b^6} + \frac{\cos^5(c+dx)}{5bd} - \frac{\cos^3(c+dx)\left(4(a^2-b^2)-3ab\sin(c+\right.}{12b^3d}$$

$$= \frac{a(8a^4-20a^2b^2+15b^4)\,x}{8b^6} + \frac{\cos^5(c+dx)}{5bd} - \frac{\cos^3(c+dx)\left(4(a^2-b^2)-3ab\sin(c+\right.}{12b^3d}$$

$$= \frac{a(8a^4-20a^2b^2+15b^4)\,x}{8b^6} + \frac{\cos^5(c+dx)}{5bd} - \frac{\cos^3(c+dx)\left(4(a^2-b^2)-3ab\sin(c+\right.}{12b^3d}$$

$$= \frac{a(8a^4-20a^2b^2+15b^4)\,x}{8b^6} - \frac{2(a^2-b^2)^{5/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^6d} + \frac{\cos^5(c+dx)}{5bd}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 2827 vs. $2(188) = 376$.

time = 6.28, size = 2827, normalized size = 15.04

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6/(a + b*Sin[c + d*x]),x]
```

```
[Out] (Cos[c + d*x]^5*((8*Sqrt[2]*b*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b))^(5/
2)*Sqrt[b/(a + b) - (b*Sin[c + d*x])/(a + b)]*(1 + ((a - b)*(-(b/(a - b)) -
 (b*Sin[c + d*x])/(a - b)))/(2*b))^(7/2)*((5/(16*(1 + ((a - b)*(-(b/(a - b)
) - (b*Sin[c + d*x])/(a - b)))/(2*b))^3) + 5/(8*(1 + ((a - b)*(-(b/(a - b))
 - (b*Sin[c + d*x])/(a - b)))/(2*b))^2) + (1 + ((a - b)*(-(b/(a - b)) - (b*
Sin[c + d*x])/(a - b)))/(2*b))^(-1))/2 - (15*b^3*(((a - b)*(-(b/(a - b)) -
(b*Sin[c + d*x])/(a - b)))/b - ((a - b)^2*(-(b/(a - b)) - (b*Sin[c + d*x])/
(a - b))^2)/(3*b^2) - (Sqrt[2]*Sqrt[a - b]*ArcSinh[(Sqrt[a - b]*Sqrt[-(b/(a
 - b)) - (b*Sin[c + d*x])/(a - b)])/(Sqrt[2]*Sqrt[b])]*Sqrt[-(b/(a - b)) -
(b*Sin[c + d*x])/(a - b)])/(Sqrt[b]*Sqrt[1 + ((a - b)*(-(b/(a - b)) - (b*Si
n[c + d*x])/(a - b)))/(2*b)])))/(64*(a - b)^3*(-(b/(a - b)) - (b*Sin[c + d*
x])/(a - b))^3*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*
b))^3)))/(5*(a + b)^2*Sqrt[((a + b)*(b/(a + b) - (b*Sin[c + d*x])/(a + b)))
```

/b]) - ((-((a*b)/(a - b)) + b^2/(a - b))*((8*Sqrt[2]*b*(-(b/(a - b)) - (b*S
in[c + d*x])/(a - b))^(3/2)*Sqrt[b/(a + b) - (b*Sin[c + d*x])/(a + b)]*(1 +
((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^(7/2)*((3*(5/(8
(1 + ((a - b)(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^3) + 5/(6*
(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^2) + (1 + (
(a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^(-1)))/8 + (15*b^
2*(((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/b - (Sqrt[2]*Sqrt[a
- b]*ArcSinh[(Sqrt[a - b]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)])/(S
qrt[2]*Sqrt[b])]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)])/(Sqrt[b]*Sq
rt[1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b)])))/(64*(a
- b)^2*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b))^2*(1 + ((a - b)*(-(b/(a -
b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^3)))/(3*(a + b)^2*Sqrt[((a + b)*(b
/(a + b) - (b*Sin[c + d*x])/(a + b)))/b]) - ((-((a*b)/(a - b)) + b^2/(a - b
))*((8*Sqrt[2]*b*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)]*Sqrt[b/(a +
b) - (b*Sin[c + d*x])/(a + b)]*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x
])/(a - b)))/(2*b))^(7/2)*((5*Sqrt[b]*ArcSinh[(Sqrt[a - b]*Sqrt[-(b/(a - b)
) - (b*Sin[c + d*x])/(a - b)])/(Sqrt[2]*Sqrt[b])])/(8*Sqrt[2]*Sqrt[a - b]*S
qrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)]*(1 + ((a - b)*(-(b/(a - b)) -
(b*Sin[c + d*x])/(a - b)))/(2*b))^(7/2)) + (15/(8*(1 + ((a - b)*(-(b/(a - b
)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^3) + 5/(4*(1 + ((a - b)*(-(b/(a - b)
) - (b*Sin[c + d*x])/(a - b)))/(2*b))^2) + (1 + ((a - b)*(-(b/(a - b)) - (b
*Sin[c + d*x])/(a - b)))/(2*b))^(-1))/6))/((a + b)^2*Sqrt[((a + b)*(b/(a +
b) - (b*Sin[c + d*x])/(a + b)))/b]) - ((-((a*b)/(a - b)) + b^2/(a - b))*(-(
((-((a*b)/(a + b)) - b^2/(a + b))*(-(((-((a*b)/(a + b)) - b^2/(a + b))*((2*
Sqrt[a - b]*ArcTanh[(Sqrt[a - b]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a -
b)])/(Sqrt[a + b]*Sqrt[b/(a + b) - (b*Sin[c + d*x])/(a + b)])])/(b*Sqrt[a +
b]) - (2*Sqrt[-((a*b)/(a + b)) - b^2/(a + b)]*ArcTanh[(Sqrt[-((a*b)/(a + b
)) - b^2/(a + b)]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)])/(Sqrt[-((a
*b)/(a - b)) + b^2/(a - b)]*Sqrt[b/(a + b) - (b*Sin[c + d*x])/(a + b)])])/(
b*Sqrt[-((a*b)/(a - b)) + b^2/(a - b)])))/b) + (2*Sqrt[2]*(a - b)*Sqrt[-(b/
(a - b)) - (b*Sin[c + d*x])/(a - b)]*Sqrt[b/(a + b) - (b*Sin[c + d*x])/(a +
b)]*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^(3/2)*
((Sqrt[b]*ArcSinh[(Sqrt[a - b]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)
])/(Sqrt[2]*Sqrt[b])])/(Sqrt[2]*Sqrt[a - b]*Sqrt[-(b/(a - b)) - (b*Sin[c +
d*x])/(a - b)]*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*
b))^(3/2)) + 1/(2*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/
(2*b)))))/(b*(a + b)*Sqrt[((a + b)*(b/(a + b) - (b*Sin[c + d*x])/(a + b)))/
b])))/b) + (4*Sqrt[2]*(a - b)*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)]
*Sqrt[b/(a + b) - (b*Sin[c + d*x])/(a + b)]*(1 + ((a - b)*(-(b/(a - b)) - (
b*Sin[c + d*x])/(a - b)))/(2*b))^(5/2)*((3*Sqrt[b]*ArcSinh[(Sqrt[a - b]*Sqr
t[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)])/(Sqrt[2]*Sqrt[b])])/(4*Sqrt[2]*
Sqrt[a - b]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)]*(1 + ((a - b)*(-(
b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^(5/2)) + (3/(2*(1 + ((a - b)
*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^2) + (1 + ((a - b)*(-(b/
(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^(-1))/4))/((a + b)^2*Sqrt[((a

```
+ b)*(b/(a + b) - (b*Sin[c + d*x])/(a + b)))/b]))))/b))/b))/b))/(d*(1 - (a +
 b*Sin[c + d*x])/(a - b))^(5/2)*(1 - (a + b*Sin[c + d*x])/(a + b))^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 381 vs. $2(174) = 348$.

time = 0.48, size = 382, normalized size = 2.03

method	result
derivativedivides	$2\left(\left(\frac{1}{2}a^3b^2-\frac{9}{8}a\,b^4\right)\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(b\,a^4-3a^2b^3+3b^5\right)\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(a^3b^2-\frac{5}{4}a\,b^4\right)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(4b\,a^4-10a^2b^3+6b^5\right)\left(\mathrm{ta}\right.\right.$
default	$2\left(\left(\frac{1}{2}a^3b^2-\frac{9}{8}a\,b^4\right)\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(b\,a^4-3a^2b^3+3b^5\right)\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(a^3b^2-\frac{5}{4}a\,b^4\right)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(4b\,a^4-10a^2b^3+6b^5\right)\left(\mathrm{ta}\right.\right.$
risch	$\frac{a^5x}{b^6}-\frac{5a^3x}{2b^4}+\frac{15ax}{8b^2}+\frac{e^{i(dx+c)}a^4}{2b^5d}-\frac{9\,e^{i(dx+c)}a^2}{8b^3d}+\frac{11\,e^{i(dx+c)}}{16bd}+\frac{e^{-i(dx+c)}a^4}{2b^5d}-\frac{9\,e^{-i(dx+c)}a^2}{8b^3d}+\frac{11\,e^{-i(dx+}}{16bd}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/b^6*((((1/2*a^3*b^2-9/8*a*b^4)*tan(1/2*d*x+1/2*c)^9+(a^4*b-3*a^2*b^3+
3*b^5)*tan(1/2*d*x+1/2*c)^8+(a^3*b^2-5/4*a*b^4)*tan(1/2*d*x+1/2*c)^7+(4*a^4
*b-10*a^2*b^3+6*b^5)*tan(1/2*d*x+1/2*c)^6+(6*b*a^4-40/3*a^2*b^3+28/3*b^5)*t
an(1/2*d*x+1/2*c)^4+(-a^3*b^2+5/4*a*b^4)*tan(1/2*d*x+1/2*c)^3+(4*b*a^4-26/3
*a^2*b^3+14/3*b^5)*tan(1/2*d*x+1/2*c)^2+(-1/2*a^3*b^2+9/8*a*b^4)*tan(1/2*d*
x+1/2*c)+b*a^4-7/3*a^2*b^3+23/15*b^5)/(1+tan(1/2*d*x+1/2*c)^2)^5+1/8*a*(8*a
^4-20*a^2*b^2+15*b^4)*arctan(tan(1/2*d*x+1/2*c)))+2*(-a^6+3*a^4*b^2-3*a^2*b
^4+b^6)/b^6/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^
2)^(1/2)))
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [**A**]
time = 0.39, size = 483, normalized size = 2.57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/120*(24*b^5*cos(d*x + c)^5 - 40*(a^2*b^3 - b^5)*cos(d*x + c)^3 + 15*(8*a
^5 - 20*a^3*b^2 + 15*a*b^4)*d*x + 60*(a^4 - 2*a^2*b^2 + b^4)*sqrt(-a^2 + b^
2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(
a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*
x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) + 120*(a^4*b - 2*a^2*b^3 + b^5)
*cos(d*x + c) + 15*(2*a*b^4*cos(d*x + c)^3 - (4*a^3*b^2 - 7*a*b^4)*cos(d*x
+ c))*sin(d*x + c))/(b^6*d), 1/120*(24*b^5*cos(d*x + c)^5 - 40*(a^2*b^3 - b
^5)*cos(d*x + c)^3 + 15*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*d*x + 120*(a^4 - 2*
a^2*b^2 + b^4)*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2
)*cos(d*x + c))) + 120*(a^4*b - 2*a^2*b^3 + b^5)*cos(d*x + c) + 15*(2*a*b^4
*cos(d*x + c)^3 - (4*a^3*b^2 - 7*a*b^4)*cos(d*x + c))*sin(d*x + c))/(b^6*d)
]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 496 vs. 2(173) = 346.
time = 3.20, size = 496, normalized size = 2.64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/120*(15*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*(d*x + c)/b^6 - 240*(a^6 - 3*a^4*
b^2 + 3*a^2*b^4 - b^6)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a
*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*b^6) + 2*(60*
a^3*b*tan(1/2*d*x + 1/2*c)^9 - 135*a*b^3*tan(1/2*d*x + 1/2*c)^9 + 120*a^4*t
an(1/2*d*x + 1/2*c)^8 - 360*a^2*b^2*tan(1/2*d*x + 1/2*c)^8 + 360*b^4*tan(1/
```

```
2*d*x + 1/2*c)^8 + 120*a^3*b*tan(1/2*d*x + 1/2*c)^7 - 150*a*b^3*tan(1/2*d*x
 + 1/2*c)^7 + 480*a^4*tan(1/2*d*x + 1/2*c)^6 - 1200*a^2*b^2*tan(1/2*d*x + 1
/2*c)^6 + 720*b^4*tan(1/2*d*x + 1/2*c)^6 + 720*a^4*tan(1/2*d*x + 1/2*c)^4 -
 1600*a^2*b^2*tan(1/2*d*x + 1/2*c)^4 + 1120*b^4*tan(1/2*d*x + 1/2*c)^4 - 12
0*a^3*b*tan(1/2*d*x + 1/2*c)^3 + 150*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 480*a^4
*tan(1/2*d*x + 1/2*c)^2 - 1040*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 + 560*b^4*tan
(1/2*d*x + 1/2*c)^2 - 60*a^3*b*tan(1/2*d*x + 1/2*c) + 135*a*b^3*tan(1/2*d*x
 + 1/2*c) + 120*a^4 - 280*a^2*b^2 + 184*b^4)/((tan(1/2*d*x + 1/2*c)^2 + 1)^
5*b^5))/d
```

Mupad [B]
time = 7.66, size = 2500, normalized size = 13.30

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(a + b*sin(c + d*x)),x)
```

```
[Out] ((2*(15*a^4 + 23*b^4 - 35*a^2*b^2))/(15*b^5) + (tan(c/2 + (d*x)/2)^3*(5*a*b
^2 - 4*a^3))/(2*b^4) - (tan(c/2 + (d*x)/2)^7*(5*a*b^2 - 4*a^3))/(2*b^4) - (
tan(c/2 + (d*x)/2)^9*(9*a*b^2 - 4*a^3))/(4*b^4) + (2*tan(c/2 + (d*x)/2)^8*(
a^4 + 3*b^4 - 3*a^2*b^2))/b^5 + (4*tan(c/2 + (d*x)/2)^6*(2*a^4 + 3*b^4 - 5*
a^2*b^2))/b^5 + (4*tan(c/2 + (d*x)/2)^2*(6*a^4 + 7*b^4 - 13*a^2*b^2))/(3*b^
5) + (4*tan(c/2 + (d*x)/2)^4*(9*a^4 + 14*b^4 - 20*a^2*b^2))/(3*b^5) + (tan(
c/2 + (d*x)/2)*(9*a*b^2 - 4*a^3))/(4*b^4))/(d*(5*tan(c/2 + (d*x)/2)^2 + 10*
tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 + 5*tan(c/2 + (d*x)/2)^8 + t
an(c/2 + (d*x)/2)^10 + 1)) + (atan((((-(a + b)^5*(a - b)^5)^(1/2)*(((225*a^
4*b^13)/2 - 300*a^6*b^11 + 320*a^8*b^9 - 160*a^10*b^7 + 32*a^12*b^5)/b^14 -
 (tan(c/2 + (d*x)/2)*(64*a*b^17 - 834*a^3*b^15 + 2385*a^5*b^13 - 3160*a^7*b
^11 + 2240*a^9*b^9 - 832*a^11*b^7 + 128*a^13*b^5))/(2*b^15) + ((-(a + b)^5*
(a - b)^5)^(1/2)*((28*a^2*b^16 - 44*a^4*b^14 + 16*a^6*b^12)/b^14 + ((-(a +
b)^5*(a - b)^5)^(1/2)*(32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(192*a*b^19 - 128*a
^3*b^17))/(2*b^15)))/b^6 - (tan(c/2 + (d*x)/2)*(128*a*b^18 - 384*a^3*b^16 +
 384*a^5*b^14 - 128*a^7*b^12))/(2*b^15)))/b^6)*1i)/b^6 + ((-(a + b)^5*(a -
b)^5)^(1/2)*(((225*a^4*b^13)/2 - 300*a^6*b^11 + 320*a^8*b^9 - 160*a^10*b^7
+ 32*a^12*b^5)/b^14 - (tan(c/2 + (d*x)/2)*(64*a*b^17 - 834*a^3*b^15 + 2385*
a^5*b^13 - 3160*a^7*b^11 + 2240*a^9*b^9 - 832*a^11*b^7 + 128*a^13*b^5))/(2*
b^15) + ((-(a + b)^5*(a - b)^5)^(1/2)*(((-(a + b)^5*(a - b)^5)^(1/2)*(32*a^
2*b^3 + (tan(c/2 + (d*x)/2)*(192*a*b^19 - 128*a^3*b^17))/(2*b^15)))/b^6 - (
28*a^2*b^16 - 44*a^4*b^14 + 16*a^6*b^12)/b^14 + (tan(c/2 + (d*x)/2)*(128*a*
b^18 - 384*a^3*b^16 + 384*a^5*b^14 - 128*a^7*b^12))/(2*b^15)))/b^6)*1i)/b^6
)/((32*a^16 - 120*a^2*b^14 + 655*a^4*b^12 - 1549*a^6*b^10 + 2069*a^8*b^8 -
1695*a^10*b^6 + 856*a^12*b^4 - 248*a^14*b^2)/b^14 + ((-(a + b)^5*(a - b)^5)
^(1/2)*(((225*a^4*b^13)/2 - 300*a^6*b^11 + 320*a^8*b^9 - 160*a^10*b^7 + 32*
a^12*b^5)/b^14 - (tan(c/2 + (d*x)/2)*(64*a*b^17 - 834*a^3*b^15 + 2385*a^5*b
```

^13 - 3160*a^7*b^11 + 2240*a^9*b^9 - 832*a^11*b^7 + 128*a^13*b^5))/(2*b^15)
 + ((-(a + b)^5*(a - b)^5)^(1/2)*((28*a^2*b^16 - 44*a^4*b^14 + 16*a^6*b^12)
/b^14 + ((-(a + b)^5*(a - b)^5)^(1/2)*(32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(19
2*a*b^19 - 128*a^3*b^17))/(2*b^15)))/b^6 - (tan(c/2 + (d*x)/2)*(128*a*b^18
 - 384*a^3*b^16 + 384*a^5*b^14 - 128*a^7*b^12))/(2*b^15)))/b^6))/b^6 - ((-(a
 + b)^5*(a - b)^5)^(1/2)*(((225*a^4*b^13)/2 - 300*a^6*b^11 + 320*a^8*b^9 -
160*a^10*b^7 + 32*a^12*b^5)/b^14 - (tan(c/2 + (d*x)/2)*(64*a*b^17 - 834*a^3
*b^15 + 2385*a^5*b^13 - 3160*a^7*b^11 + 2240*a^9*b^9 - 832*a^11*b^7 + 128*a
^13*b^5))/(2*b^15) + ((-(a + b)^5*(a - b)^5)^(1/2)*(((-(a + b)^5*(a - b)^5)
^(1/2)*(32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(192*a*b^19 - 128*a^3*b^17))/(2*b^
15)))/b^6 - (28*a^2*b^16 - 44*a^4*b^14 + 16*a^6*b^12)/b^14 + (tan(c/2 + (d*
x)/2)*(128*a*b^18 - 384*a^3*b^16 + 384*a^5*b^14 - 128*a^7*b^12))/(2*b^15)))
/b^6))/b^6 + (tan(c/2 + (d*x)/2)*(128*a^17 - 450*a^3*b^14 + 2550*a^5*b^12 -
 6230*a^7*b^10 + 8530*a^9*b^8 - 7088*a^11*b^6 + 3584*a^13*b^4 - 1024*a^15*b
^2))/b^15))*(-(a + b)^5*(a - b)^5)^(1/2)*2i)/(b^6*d) + (a*atan(((a*(((225*a
^4*b^13)/2 - 300*a^6*b^11 + 320*a^8*b^9 - 160*a^10*b^7 + 32*a^12*b^5)/b^14
 - (tan(c/2 + (d*x)/2)*(64*a*b^17 - 834*a^3*b^15 + 2385*a^5*b^13 - 3160*a^7*
b^11 + 2240*a^9*b^9 - 832*a^11*b^7 + 128*a^13*b^5))/(2*b^15) + (a*(8*a^4 +
15*b^4 - 20*a^2*b^2)*((28*a^2*b^16 - 44*a^4*b^14 + 16*a^6*b^12)/b^14 - (tan
(c/2 + (d*x)/2)*(128*a*b^18 - 384*a^3*b^16 + 384*a^5*b^14 - 128*a^7*b^12))/
(2*b^15) + (a*(32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(192*a*b^19 - 128*a^3*b^17)
)/(2*b^15))*(8*a^4 + 15*b^4 - 20*a^2*b^2)*1i)/(8*b^6))*1i)/(8*b^6))*(8*a^4
 + 15*b^4 - 20*a^2*b^2))/(8*b^6) + (a*(((225*a^4*b^13)/2 - 300*a^6*b^11 + 32
0*a^8*b^9 - 160*a^10*b^7 + 32*a^12*b^5)/b^14 - (tan(c/2 + (d*x)/2)*(64*a*b^
17 - 834*a^3*b^15 + 2385*a^5*b^13 - 3160*a^7*b^11 + 2240*a^9*b^9 - 832*a^11
*b^7 + 128*a^13*b^5))/(2*b^15) + (a*(8*a^4 + 15*b^4 - 20*a^2*b^2)*((tan(c/2
 + (d*x)/2)*(128*a*b^18 - 384*a^3*b^16 + 384*a^5*b^14 - 128*a^7*b^12))/(2*b
^15) - (28*a^2*b^16 - 44*a^4*b^14 + 16*a^6*b^12)/b^14 + (a*(32*a^2*b^3 + (t
an(c/2 + (d*x)/2)*(192*a*b^19 - 128*a^3*b^17))/(2*b^15))*(8*a^4 + 15*b^4 -
20*a^2*b^2)*1i)/(8*b^6))*1i)/(8*b^6))*(8*a^4 + 15*b^4 - 20*a^2*b^2))/(8*b^6
))/((32*a^16 - 120*a^2*b^14 + 655*a^4*b^12 - 1549*a^6*b^10 + 2069*a^8*b^8 -
 1695*a^10*b^6 + 856*a^12*b^4 - 248*a^14*b^2)/b^14 + (tan(c/2 + (d*x)/2)*(1
28*a^17 - 450*a^3*b^14 + 2550*a^5*b^12 - 6230*a^7*b^10 + 8530*a^9*b^8 - 708
8*a^11*b^6 + 3584*a^13*b^4 - 1024*a^15*b^2))/b^15 + (a*(((225*a^4*b^13)/2 -
 300*a^6*b^11 + 320*a^8*b^9 - 160*a^10*b^7 + 32*a^12*b^5)/b^14 - (tan(c/2 +
 (d*x)/2)*(64*a*b^17 - 834*a^3*b^15 + 2385*a^5*b^13 - 3160*a^7*b^11 + 2240*
a^9*b^9 - 832*a^11*b^7 + 128*a^13*b^5))/(2*b^15) + (a*(8*a^4 + 15*b^4 - 20*
a^2*b^2)*((28*a^2*b^16 - 44*a^4*b^14 + 16*a^6*b^12)/b^14 - (tan(c/2 + (d*x)
/2)*(128*a*b^18 - 384*a^3*b^16 + 384*a^5*b^14 - 128*a^7*b^12))/(2*b^15) + (
a*(32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(192*a*b^19...

3.432 $\int \frac{\cos^4(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=127

$$-\frac{a(2a^2-3b^2)\,x}{2b^4}+\frac{2(a^2-b^2)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^4d}+\frac{\cos^3(c+dx)}{3bd}-\frac{\cos(c+dx)\left(2(a^2-b^2)-ab\sin(c+\right.}{2b^3d}$$

```
[Out] -1/2*a*(2*a^2-3*b^2)*x/b^4+2*(a^2-b^2)^(3/2)*arctan((b+a*tan(1/2*d*x+1/2*c)
)/(a^2-b^2)^(1/2))/b^4/d+1/3*cos(d*x+c)^3/b/d-1/2*cos(d*x+c)*(2*a^2-2*b^2-a
*b*sin(d*x+c))/b^3/d
```

Rubi [A]
time = 0.17, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2774, 2944, 2814, 2739, 632, 210}

$$\frac{2(a^2-b^2)^{3/2}\,\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^4d}-\frac{ax(2a^2-3b^2)}{2b^4}-\frac{\cos(c+dx)\left(2(a^2-b^2)-ab\sin(c+dx)\right)}{2b^3d}+\frac{\cos^3(c+dx)}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/2*(a*(2*a^2 - 3*b^2)*x)/b^4 + (2*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c
+ d*x)/2])/Sqrt[a^2 - b^2]])/(b^4*d) + Cos[c + d*x]^3/(3*b*d) - (Cos[c + d*
x]*(2*(a^2 - b^2) - a*b*Sin[c + d*x]))/(2*b^3*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2774

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(b*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2944

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Dist[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\cos^3(c+dx)}{3bd} + \frac{\int \frac{\cos^2(c+dx)(b+a\sin(c+dx))}{a+b\sin(c+dx)}\,dx}{b}$$

$$= \frac{\cos^3(c+dx)}{3bd} - \frac{\cos(c+dx)\left(2(a^2-b^2)-ab\sin(c+dx)\right)}{2b^3d} + \frac{\int \frac{-b(a^2-2b^2)-a(2a^2-3b^2)\,\mathrm{s}}{a+b\sin(c+dx)}}{2b^3}$$

$$= -\frac{a(2a^2-3b^2)\,x}{2b^4} + \frac{\cos^3(c+dx)}{3bd} - \frac{\cos(c+dx)\left(2(a^2-b^2)-ab\sin(c+dx)\right)}{2b^3d} + \frac{(a^2}{}$$

$$= -\frac{a(2a^2-3b^2)\,x}{2b^4} + \frac{\cos^3(c+dx)}{3bd} - \frac{\cos(c+dx)\left(2(a^2-b^2)-ab\sin(c+dx)\right)}{2b^3d} + \frac{\left(2\right.}{}$$

$$= -\frac{a(2a^2-3b^2)\,x}{2b^4} + \frac{\cos^3(c+dx)}{3bd} - \frac{\cos(c+dx)\left(2(a^2-b^2)-ab\sin(c+dx)\right)}{2b^3d} - \frac{\left(4\right.}{}$$

$$= -\frac{a(2a^2-3b^2)\,x}{2b^4} + \frac{2(a^2-b^2)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^4d} + \frac{\cos^3(c+dx)}{3bd} - \cos(c+$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 428 vs. $2(127) = 254$.
time = 4.20, size = 428, normalized size = 3.37

$$\frac{\cos^3(c+dx)\left(12(a-b)^2(a+b)\tanh^{-1}\left(\frac{\sqrt{a-b}\sqrt{-\frac{b(1+\sin(c+dx))}{a-b}}}{\sqrt{a+b}\sqrt{-\frac{b(-1+\sin(c+dx))}{a+b}}}\right)\sqrt{1-\sin(c+dx)}+\sqrt{a+b}\left(-12\sqrt{a-b}\,(a^2-b^2)\tanh^{-1}\left(\frac{\sqrt{\frac{b(1+\sin(c+dx))}{-a+b}}}{\sqrt{-\frac{b(-1+\sin(c+dx))}{a+b}}}\right)\sqrt{1-\sin(c+dx)}+\sqrt{-\frac{b(-1+\sin(c+dx))}{a+b}}\left(-6\sqrt{b}\,(-2a^2+ab+2b^2)\sinh^{-1}\left(\frac{\sqrt{a-b}\sqrt{-\frac{b(1+\sin(c+dx))}{a-b}}}{\sqrt{2}\sqrt{b}}\right)+\sqrt{a-b}\sqrt{1-\sin(c+dx)}\sqrt{\frac{b(1+\sin(c+dx))}{-a+b}}\,(6a^2-8b^2-3ab\sin(c+dx)+2b^2\sin^2(c+dx))\right)\right)\right)}{6(a-b)^{3/2}b^2\sqrt{a+b}\,d(1-\sin(c+dx))^{3/2}\sqrt{-\frac{b(-1+\sin(c+dx))}{a+b}}\left(-\frac{b(1+\sin(c+dx))}{a-b}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + b*Sin[c + d*x]),x]
```

```
[Out] (Cos[c + d*x]^3*(12*(a - b)^2*(a + b)*ArcTanh[(Sqrt[a - b]*Sqrt[-((b*(1 + S
in[c + d*x]))/(a - b))])/(Sqrt[a + b]*Sqrt[-((b*(-1 + Sin[c + d*x]))/(a + b
))])]*Sqrt[1 - Sin[c + d*x]] + Sqrt[a + b]*(-12*Sqrt[a - b]*(a^2 - b^2)*Arc
Tanh[Sqrt[(b*(1 + Sin[c + d*x]))/(-a + b)]/Sqrt[-((b*(-1 + Sin[c + d*x]))/(
a + b))]]*Sqrt[1 - Sin[c + d*x]] + Sqrt[-((b*(-1 + Sin[c + d*x]))/(a + b))]
*(-6*Sqrt[b]*(-2*a^2 + a*b + 2*b^2)*ArcSinh[(Sqrt[a - b]*Sqrt[-((b*(1 + Sin
[c + d*x]))/(a - b))])/(Sqrt[2]*Sqrt[b])] + Sqrt[a - b]*Sqrt[1 - Sin[c + d*
x]]*Sqrt[(b*(1 + Sin[c + d*x]))/(-a + b)]*(6*a^2 - 8*b^2 - 3*a*b*Sin[c + d*
x] + 2*b^2*Sin[c + d*x]^2)))))/(6*(a - b)^(3/2)*b^2*Sqrt[a + b]*d*(1 - Sin[
c + d*x])^(3/2)*Sqrt[-((b*(-1 + Sin[c + d*x]))/(a + b))]*(-((b*(1 + Sin[c +
 d*x]))/(a - b)))^(3/2))
```

Maple [A]
time = 0.33, size = 203, normalized size = 1.60

method	result
derivativedivides	$-\frac{2\left(\frac{\frac{a\,b^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\left(a^2b-2b^3\right)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(2a^2b-2b^3\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{a\,b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}+a^2b-\frac{4b^3}{3}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{a\left(2a^2-3b^2\right)}{}\right.}{b^4}}{d}$
default	$-\frac{2\left(\frac{\frac{a\,b^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\left(a^2b-2b^3\right)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(2a^2b-2b^3\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{a\,b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}+a^2b-\frac{4b^3}{3}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{a\left(2a^2-3b^2\right)}{}\right.}{b^4}}{d}$
risch	$-\frac{a^3x}{b^4}+\frac{3ax}{2b^2}-\frac{e^{i(dx+c)}a^2}{2b^3d}+\frac{5\,e^{i(dx+c)}}{8bd}-\frac{e^{-i(dx+c)}a^2}{2b^3d}+\frac{5\,e^{-i(dx+c)}}{8bd}+\frac{\sqrt{-a^2+b^2}\,\ln\left(e^{i(dx+c)}+\frac{ia+\sqrt{}}{}\right.}{d\,b^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2/b^4*((1/2*a*b^2*tan(1/2*d*x+1/2*c)^5+(a^2*b-2*b^3)*tan(1/2*d*x+1/2*
c)^4+(2*a^2*b-2*b^3)*tan(1/2*d*x+1/2*c)^2-1/2*a*b^2*tan(1/2*d*x+1/2*c)+a^2*
b-4/3*b^3)/(1+tan(1/2*d*x+1/2*c)^2)^3+1/2*a*(2*a^2-3*b^2)*arctan(tan(1/2*d*
x+1/2*c)))+2*(a^4-2*a^2*b^2+b^4)/b^4/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/
2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2)))
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]

time = 0.37, size = 332, normalized size = 2.61

$$\left[\frac{2\,b^3\cos\left(dx+c\right)^3+3\,ab^2\cos\left(dx+c\right)\sin\left(dx+c\right)-3\left(2\,a^3-3\,ab^2\right)dx-3\left(a^2-b^2\right)\sqrt{-a^2+b^2}\log\left(\frac{\left(2\,a^2-b^2\right)\cos(dx+c)^2-2\,ab\sin(dx+c)-a^2-b^2+2\left(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c)\right)\sqrt{-a^2+b^2}}{b^2\cos(dx+c)^2-2\,ab\sin(dx+c)-a^2-b^2}\right)-6\left(a^2b-b^3\right)\cos\left(dx+c\right)}{6\,b^4d},\ \frac{2\,b^3\cos\left(dx+c\right)^3+3\,ab^2\cos\left(dx+c\right)\sin\left(dx+c\right)-3\left(2\,a^3-3\,ab^2\right)dx-6\left(a^2-b^2\right)^{\frac{3}{2}}\arctan\left(-\frac{a\sin(dx+c)+b}{\sqrt{a^2-b^2}\cos(dx+c)}\right)-6\left(a^2b-b^3\right)\cos\left(dx+c\right)}{6\,b^4d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/6*(2*b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)*sin(d*x + c) - 3*(2*a^3 -
 3*a*b^2)*d*x - 3*(a^2 - b^2)*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x +
```

```
c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b
*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) -
a^2 - b^2)) - 6*(a^2*b - b^3)*cos(d*x + c))/(b^4*d), 1/6*(2*b^3*cos(d*x +
c)^3 + 3*a*b^2*cos(d*x + c)*sin(d*x + c) - 3*(2*a^3 - 3*a*b^2)*d*x - 6*(a^2
- b^2)^(3/2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c)))
- 6*(a^2*b - b^3)*cos(d*x + c))/(b^4*d)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 4.65, size = 226, normalized size = 1.78

$$-\frac{\frac{3\,(2\,a^3-3\,ab^2)(dx+c)}{b^4}-\frac{12\,(a^4-2\,a^2b^2+b^4)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,b^4}+\frac{2\left(3\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+6\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-12\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+12\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-12\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-3\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+6\,a^2-8\,b^2\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3b^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(3*(2*a^3 - 3*a*b^2)*(d*x + c)/b^4 - 12*(a^4 - 2*a^2*b^2 + b^4)*(pi*fl
oor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sq
rt(a^2 - b^2)))/(sqrt(a^2 - b^2)*b^4) + 2*(3*a*b*tan(1/2*d*x + 1/2*c)^5 + 6
*a^2*tan(1/2*d*x + 1/2*c)^4 - 12*b^2*tan(1/2*d*x + 1/2*c)^4 + 12*a^2*tan(1/
2*d*x + 1/2*c)^2 - 12*b^2*tan(1/2*d*x + 1/2*c)^2 - 3*a*b*tan(1/2*d*x + 1/2*
c) + 6*a^2 - 8*b^2)/((tan(1/2*d*x + 1/2*c)^2 + 1)^3*b^3))/d
```

Mupad [B]
time = 6.12, size = 364, normalized size = 2.87

$$\frac{\frac{5\cos(c+d\,x)}{4}+\frac{\cos(3\,c+3\,d\,x)}{12}}{b\,d}+\frac{3\,a\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)+\frac{a\,\sin(2\,c+2\,d\,x)}{4}}{b^2\,d}-\frac{a^2\,\cos\left(c+d\,x\right)}{b^3\,d}-\frac{2\,a^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{b^4\,d}+\frac{2\,\mathrm{atanh}\left(\frac{2\,b^2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{-a^6+3\,a^4\,b^2-3\,a^2\,b^4+b^6}-a^2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{-a^6+3\,a^4\,b^2-3\,a^2\,b^4+b^6}+a\,b\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{-a^6+3\,a^4\,b^2-3\,a^2\,b^4+b^6}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^5+2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^4\,b-2\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^3\,b^2-4\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2\,b^3+\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a\,b^4+2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,b^5}\right)\,\sqrt{-{\left(a+b\right)}^3\,{\left(a-b\right)}^3}}{b^4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(a + b*sin(c + d*x)),x)
```

```
[Out] ((5*cos(c + d*x))/4 + cos(3*c + 3*d*x)/12)/(b*d) + (3*a*atan(sin(c/2 + (d*x
)/2)/cos(c/2 + (d*x)/2)) + (a*sin(2*c + 2*d*x))/4)/(b^2*d) - (a^2*cos(c + d
*x))/(b^3*d) - (2*a^3*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(b^4*d)
+ (2*atanh((2*b^2*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1
/2) - a^2*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) + a*
b*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2))/(a^5*cos(c/
2 + (d*x)/2) + 2*b^5*sin(c/2 + (d*x)/2) + a*b^4*cos(c/2 + (d*x)/2) + 2*a^4*
b*sin(c/2 + (d*x)/2) - 2*a^3*b^2*cos(c/2 + (d*x)/2) - 4*a^2*b^3*sin(c/2 + (
d*x)/2)))*(-(a + b)^3*(a - b)^3)^(1/2))/(b^4*d)
```

3.433 $\int \frac{\cos^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=70

$$\frac{ax}{b^2}-\frac{2\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^2d}+\frac{\cos(c+dx)}{bd}$$

```
[Out] a*x/b^2+cos(d*x+c)/b/d-2*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))*(
a^2-b^2)^(1/2)/b^2/d
```

Rubi [A]
time = 0.08, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.238$, Rules used = {2774, 2814, 2739, 632, 210}

$$-\frac{2\sqrt{a^2-b^2}\,\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^2d}+\frac{ax}{b^2}+\frac{\cos(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + b*Sin[c + d*x]),x]
```

```
[Out] (a*x)/b^2 - (2*Sqrt[a^2 - b^2]*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b
^2]])/(b^2*d) + Cos[c + d*x]/(b*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2774

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(b*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^2(c+dx)}{a+b\sin(c+dx)}\,dx &= \frac{\cos(c+dx)}{bd}+\frac{\int \frac{b+a\sin(c+dx)}{a+b\sin(c+dx)}\,dx}{b}\\
&= \frac{ax}{b^2}+\frac{\cos(c+dx)}{bd}-\frac{(a^2-b^2)\int \frac{1}{a+b\sin(c+dx)}\,dx}{b^2}\\
&= \frac{ax}{b^2}+\frac{\cos(c+dx)}{bd}-\frac{(2(a^2-b^2))\,\mathrm{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx,x,\tan\left(\frac{1}{2}(c+dx)\right)\right)}{b^2d}\\
&= \frac{ax}{b^2}+\frac{\cos(c+dx)}{bd}+\frac{(4(a^2-b^2))\,\mathrm{Subst}\left(\int \frac{1}{-4(a^2-b^2)-x^2}\,dx,x,2b+2a\tan\left(\frac{1}{2}(c+dx\right.\right.}{b^2d}\\
&= \frac{ax}{b^2}-\frac{2\sqrt{a^2-b^2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^2d}+\frac{\cos(c+dx)}{bd}
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 361 vs. 2(70) = 140.
time = 1.25, size = 361, normalized size = 5.16

$$
\frac{\cos(c+dx)\left(2(a-b)\tanh^{-1}\left(\frac{\sqrt{a-b}\sqrt{-\frac{b(1+\sin(c+dx))}{a-b}}}{\sqrt{a+b}\sqrt{-\frac{b(-1+\sin(c+dx))}{a+b}}}\right)\sqrt{1-\sin(c+dx)}+\sqrt{a+b}\left(-2\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{\frac{b(1+\sin(c+dx))}{-a+b}}}{\sqrt{-\frac{b(-1+\sin(c+dx))}{a+b}}}\right)\sqrt{1-\sin(c+dx)}+\sqrt{-\frac{b(-1+\sin(c+dx))}{a+b}}\left(2\sqrt{b}\sinh^{-1}\left(\frac{\sqrt{a-b}\sqrt{-\frac{b(1+\sin(c+dx))}{a-b}}}{\sqrt{2}\sqrt{b}}\right)+\sqrt{a-b}\sqrt{1-\sin(c+dx)}\sqrt{\frac{b(1+\sin(c+dx))}{-a+b}}\right)\right)\right)}{\sqrt{a-b}\,b\sqrt{a+b}\,d\sqrt{1-\sin(c+dx)}\sqrt{-\frac{b(-1+\sin(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sin(c+dx))}{a-b}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + b*Sin[c + d*x]),x]
```

```
[Out] (Cos[c + d*x]*(2*(a - b)*ArcTanh[(Sqrt[a - b]*Sqrt[-((b*(1 + Sin[c + d*x]))
/(a - b))])/(Sqrt[a + b]*Sqrt[-((b*(-1 + Sin[c + d*x]))/(a + b))])]*Sqrt[1
- Sin[c + d*x]] + Sqrt[a + b]*(-2*Sqrt[a - b]*ArcTanh[Sqrt[(b*(1 + Sin[c +
d*x]))/(-a + b)]/Sqrt[-((b*(-1 + Sin[c + d*x]))/(a + b))]]*Sqrt[1 - Sin[c +
```

```
d*x]] + Sqrt[-((b*(-1 + Sin[c + d*x]))/(a + b))]*(2*Sqrt[b]*ArcSinh[(Sqrt[
a - b]*Sqrt[-((b*(1 + Sin[c + d*x]))/(a - b))])/(Sqrt[2]*Sqrt[b])] + Sqrt[a
 - b]*Sqrt[1 - Sin[c + d*x]]*Sqrt[(b*(1 + Sin[c + d*x]))/(-a + b)])))/(Sqr
t[a - b]*b*Sqrt[a + b]*d*Sqrt[1 - Sin[c + d*x]]*Sqrt[-((b*(-1 + Sin[c + d*x
]))/(a + b))]*Sqrt[-((b*(1 + Sin[c + d*x]))/(a - b))])
```

Maple [A]
time = 0.16, size = 96, normalized size = 1.37

method	result
derivativedivides	$\frac{\frac{\frac{2b}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}+2a\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^2}+\frac{2\left(-a^2+b^2\right)\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{b^2\sqrt{a^2-b^2}}}{d}$
default	$\frac{\frac{\frac{2b}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}+2a\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^2}+\frac{2\left(-a^2+b^2\right)\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{b^2\sqrt{a^2-b^2}}}{d}$
risch	$\frac{ax}{b^2}+\frac{e^{i(dx+c)}}{2bd}+\frac{e^{-i(dx+c)}}{2bd}+\frac{\sqrt{-a^2+b^2}\,\ln\left(e^{i(dx+c)}-\frac{-ia+\sqrt{-a^2+b^2}}{b}\right)}{d\,b^2}-\frac{\sqrt{-a^2+b^2}\,\ln\left(e^{i(d\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/b^2*(b/(1+tan(1/2*d*x+1/2*c)^2)+a*arctan(tan(1/2*d*x+1/2*c)))+2*(-a^
2+b^2)/b^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2
)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.37, size = 214, normalized size = 3.06

$$\left[\frac{2\,adx+2\,b\cos\left(dx+c\right)+\sqrt{-a^2+b^2}\,\log\left(\frac{\left(2\,a^2-b^2\right)\cos\left(dx+c\right)^2-2\,ab\sin\left(dx+c\right)-a^2-b^2+2\left(a\cos\left(dx+c\right)\sin\left(dx+c\right)+b\cos\left(dx+c\right)\right)\sqrt{-a^2+b^2}}{b^2\cos\left(dx+c\right)^2-2\,ab\sin\left(dx+c\right)-a^2-b^2}\right)}{2\,b^2d},\ \frac{adx+b\cos\left(dx+c\right)+\sqrt{a^2-b^2}\,\arctan\left(-\frac{a\sin\left(dx+c\right)+b}{\sqrt{a^2-b^2}\,\cos\left(dx+c\right)}\right)}{b^2d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*d*x + 2*b*cos(d*x + c) + sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(
d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c
) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x +
 c) - a^2 - b^2)))/(b^2*d), (a*d*x + b*cos(d*x + c) + sqrt(a^2 - b^2)*arcta
n(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))))/(b^2*d)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1923 vs. $2(58) = 116$.
time = 166.89, size = 1923, normalized size = 27.47

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Piecewise((zoo*x*cos(c)**2/sin(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), ((log(t
an(c/2 + d*x/2))*tan(c/2 + d*x/2)**2/(d*tan(c/2 + d*x/2)**2 + d) + log(tan(
c/2 + d*x/2))/(d*tan(c/2 + d*x/2)**2 + d) + 2/(d*tan(c/2 + d*x/2)**2 + d))/
b, Eq(a, 0)), (b*d*x*tan(c/2 + d*x/2)**3/(b**2*d*tan(c/2 + d*x/2)**2 + b**2
*d - b*d*sqrt(b**2)*tan(c/2 + d*x/2)**3 - b*d*sqrt(b**2)*tan(c/2 + d*x/2))
+ b*d*x*tan(c/2 + d*x/2)/(b**2*d*tan(c/2 + d*x/2)**2 + b**2*d - b*d*sqrt(b*
*2)*tan(c/2 + d*x/2)**3 - b*d*sqrt(b**2)*tan(c/2 + d*x/2)) + 2*b/(b**2*d*ta
n(c/2 + d*x/2)**2 + b**2*d - b*d*sqrt(b**2)*tan(c/2 + d*x/2)**3 - b*d*sqrt(
b**2)*tan(c/2 + d*x/2)) - d*x*sqrt(b**2)*tan(c/2 + d*x/2)**2/(b**2*d*tan(c/
2 + d*x/2)**2 + b**2*d - b*d*sqrt(b**2)*tan(c/2 + d*x/2)**3 - b*d*sqrt(b**2
)*tan(c/2 + d*x/2)) - d*x*sqrt(b**2)/(b**2*d*tan(c/2 + d*x/2)**2 + b**2*d -
 b*d*sqrt(b**2)*tan(c/2 + d*x/2)**3 - b*d*sqrt(b**2)*tan(c/2 + d*x/2)) - 2*
sqrt(b**2)*tan(c/2 + d*x/2)/(b**2*d*tan(c/2 + d*x/2)**2 + b**2*d - b*d*sqrt
(b**2)*tan(c/2 + d*x/2)**3 - b*d*sqrt(b**2)*tan(c/2 + d*x/2)), Eq(a, -sqrt(
b**2))), (b*d*x*tan(c/2 + d*x/2)**3/(b**2*d*tan(c/2 + d*x/2)**2 + b**2*d +
b*d*sqrt(b**2)*tan(c/2 + d*x/2)**3 + b*d*sqrt(b**2)*tan(c/2 + d*x/2)) + b*d
*x*tan(c/2 + d*x/2)/(b**2*d*tan(c/2 + d*x/2)**2 + b**2*d + b*d*sqrt(b**2)*t
an(c/2 + d*x/2)**3 + b*d*sqrt(b**2)*tan(c/2 + d*x/2)) + 2*b/(b**2*d*tan(c/2
 + d*x/2)**2 + b**2*d + b*d*sqrt(b**2)*tan(c/2 + d*x/2)**3 + b*d*sqrt(b**2)
*tan(c/2 + d*x/2)) + d*x*sqrt(b**2)*tan(c/2 + d*x/2)**2/(b**2*d*tan(c/2 + d
*x/2)**2 + b**2*d + b*d*sqrt(b**2)*tan(c/2 + d*x/2)**3 + b*d*sqrt(b**2)*tan
(c/2 + d*x/2)) + d*x*sqrt(b**2)/(b**2*d*tan(c/2 + d*x/2)**2 + b**2*d + b*d*
sqrt(b**2)*tan(c/2 + d*x/2)**3 + b*d*sqrt(b**2)*tan(c/2 + d*x/2)) + 2*sqrt(
b**2)*tan(c/2 + d*x/2)/(b**2*d*tan(c/2 + d*x/2)**2 + b**2*d + b*d*sqrt(b**2
)*tan(c/2 + d*x/2)**3 + b*d*sqrt(b**2)*tan(c/2 + d*x/2)), Eq(a, sqrt(b**2))
), ((x*sin(c + d*x)**2/2 + x*cos(c + d*x)**2/2 + sin(c + d*x)*cos(c + d*x)/
(2*d))/a, Eq(b, 0)), (x*cos(c)**2/(a + b*sin(c)), Eq(d, 0)), (-a**2*log(tan
```

```
(c/2 + d*x/2) + b/a - sqrt(-a**2 + b**2)/a)*tan(c/2 + d*x/2)**2/(b**2*d*sqr
t(-a**2 + b**2)*tan(c/2 + d*x/2)**2 + b**2*d*sqrt(-a**2 + b**2)) - a**2*log
(tan(c/2 + d*x/2) + b/a - sqrt(-a**2 + b**2)/a)/(b**2*d*sqrt(-a**2 + b**2)*
tan(c/2 + d*x/2)**2 + b**2*d*sqrt(-a**2 + b**2)) + a**2*log(tan(c/2 + d*x/2
) + b/a + sqrt(-a**2 + b**2)/a)*tan(c/2 + d*x/2)**2/(b**2*d*sqrt(-a**2 + b*
*2)*tan(c/2 + d*x/2)**2 + b**2*d*sqrt(-a**2 + b**2)) + a**2*log(tan(c/2 + d
*x/2) + b/a + sqrt(-a**2 + b**2)/a)/(b**2*d*sqrt(-a**2 + b**2)*tan(c/2 + d*
x/2)**2 + b**2*d*sqrt(-a**2 + b**2)) + a*d*x*sqrt(-a**2 + b**2)*tan(c/2 + d
*x/2)**2/(b**2*d*sqrt(-a**2 + b**2)*tan(c/2 + d*x/2)**2 + b**2*d*sqrt(-a**2
 + b**2)) + a*d*x*sqrt(-a**2 + b**2)/(b**2*d*sqrt(-a**2 + b**2)*tan(c/2 + d
*x/2)**2 + b**2*d*sqrt(-a**2 + b**2)) + b**2*log(tan(c/2 + d*x/2) + b/a - s
qrt(-a**2 + b**2)/a)*tan(c/2 + d*x/2)**2/(b**2*d*sqrt(-a**2 + b**2)*tan(c/2
 + d*x/2)**2 + b**2*d*sqrt(-a**2 + b**2)) + b**2*log(tan(c/2 + d*x/2) + b/a
 - sqrt(-a**2 + b**2)/a)/(b**2*d*sqrt(-a**2 + b**2)*tan(c/2 + d*x/2)**2 + b
**2*d*sqrt(-a**2 + b**2)) - b**2*log(tan(c/2 + d*x/2) + b/a + sqrt(-a**2 +
b**2)/a)*tan(c/2 + d*x/2)**2/(b**2*d*sqrt(-a**2 + b**2)*tan(c/2 + d*x/2)**2
 + b**2*d*sqrt(-a**2 + b**2)) - b**2*log(tan(c/2 + d*x/2) + b/a + sqrt(-a**
2 + b**2)/a)/(b**2*d*sqrt(-a**2 + b**2)*tan(c/2 + d*x/2)**2 + b**2*d*sqrt(-
a**2 + b**2)) + 2*b*sqrt(-a**2 + b**2)/(b**2*d*sqrt(-a**2 + b**2)*tan(c/2 +
 d*x/2)**2 + b**2*d*sqrt(-a**2 + b**2)), True))
```

Giac [A]
time = 6.84, size = 95, normalized size = 1.36

$$\frac{\frac{(dx+c)a}{b^2} - \frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)\sqrt{a^2-b^2}}{b^2} + \frac{2}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] ((d*x + c)*a/b^2 - 2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*t
an(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))*sqrt(a^2 - b^2)/b^2 + 2/((tan(1/
2*d*x + 1/2*c)^2 + 1)*b))/d
```

Mupad [B]
time = 5.37, size = 318, normalized size = 4.54

$$\frac{2}{b\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}+\frac{2\,a\,\mathrm{atan}\left(\frac{64\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64\,a^2-\frac{64\,a^4}{b^2}}+\frac{64\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64\,a^4-64\,a^2\,b^2}\right)}{b^2\,d}+\frac{2\,\mathrm{atanh}\left(\frac{64\,a^2\sqrt{b^2-a^2}}{64\,a^2\,b-\frac{64\,a^4}{b}-128\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+128\,a\,b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}+\frac{128\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\sqrt{b^2-a^2}}{64\,a^2-\frac{64\,a^4}{b^2}-\frac{128\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{b}+128\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}+\frac{64\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\sqrt{b^2-a^2}}{64\,a^4+128\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)a^3\,b-64\,a^2\,b^2-128\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)a\,b^3}\right)\sqrt{b^2-a^2}}{b^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a + b*sin(c + d*x)),x)
```

```
[Out] 2/(b*d*(tan(c/2 + (d*x)/2)^2 + 1)) + (2*a*atan((64*a^2*tan(c/2 + (d*x)/2))/
(64*a^2 - (64*a^4)/b^2) + (64*a^4*tan(c/2 + (d*x)/2))/(64*a^4 - 64*a^2*b^2)
```

```
))/(b^2*d) + (2*atanh((64*a^2*(b^2 - a^2)^(1/2))/(64*a^2*b - (64*a^4)/b - 1
28*a^3*tan(c/2 + (d*x)/2) + 128*a*b^2*tan(c/2 + (d*x)/2)) + (128*a*tan(c/2
+ (d*x)/2)*(b^2 - a^2)^(1/2))/(64*a^2 - (64*a^4)/b^2 - (128*a^3*tan(c/2 + (
d*x)/2))/b + 128*a*b*tan(c/2 + (d*x)/2)) + (64*a^3*tan(c/2 + (d*x)/2)*(b^2
- a^2)^(1/2))/(64*a^4 - 64*a^2*b^2 - 128*a*b^3*tan(c/2 + (d*x)/2) + 128*a^3
*b*tan(c/2 + (d*x)/2)))*(b^2 - a^2)^(1/2))/(b^2*d)
```

3.434 $\int \frac{\sec^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=84

$$-\frac{2b^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}\,d}-\frac{\sec(c+dx)(b-a\sin(c+dx))}{(a^2-b^2)\,d}$$

```
[Out] -2*b^2*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/(a^2-b^2)^(3/2)/d-s
ec(d*x+c)*(b-a*sin(d*x+c))/(a^2-b^2)/d
```

Rubi [A]
time = 0.07, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2775, 12, 2739, 632, 210}

$$-\frac{2b^2\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\,(a^2-b^2)^{3/2}}-\frac{\sec(c+dx)(b-a\sin(c+dx))}{d\,(a^2-b^2)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*b^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(3/2
)*d) - (Sec[c + d*x]*(b - a*Sin[c + d*x]))/((a^2 - b^2)*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2775

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^
(m + 1)*((b - a*Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^2(c+dx)}{a+b\sin(c+dx)}\,dx &= -\frac{\sec(c+dx)(b-a\sin(c+dx))}{(a^2-b^2)\,d}+\frac{\int \frac{b^2}{a+b\sin(c+dx)}\,dx}{-a^2+b^2}\\
&= -\frac{\sec(c+dx)(b-a\sin(c+dx))}{(a^2-b^2)\,d}-\frac{b^2\int \frac{1}{a+b\sin(c+dx)}\,dx}{a^2-b^2}\\
&= -\frac{\sec(c+dx)(b-a\sin(c+dx))}{(a^2-b^2)\,d}-\frac{(2b^2)\,\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx,x,\tan\left(\frac{1}{2}(c+dx)\right)\right)}{(a^2-b^2)\,d}\\
&= -\frac{\sec(c+dx)(b-a\sin(c+dx))}{(a^2-b^2)\,d}+\frac{(4b^2)\,\text{Subst}\left(\int \frac{1}{-4(a^2-b^2)-x^2}\,dx,x,2b+2a\tan\left(\frac{1}{2}\right.\right.}{(a^2-b^2)\,d}\\
&= -\frac{2b^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}\,d}-\frac{\sec(c+dx)(b-a\sin(c+dx))}{(a^2-b^2)\,d}
\end{aligned}$$

Mathematica [A]
time = 0.30, size = 152, normalized size = 1.81

$$\frac{2b^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)\cos(c+dx)+\sqrt{a^2-b^2}\,(b-b\cos(c+dx)-a\sin(c+dx))}{(-a+b)(a+b)\sqrt{a^2-b^2}\,d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + b*Sin[c + d*x]),x]
```

```
[Out] (2*b^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]*Cos[c + d*x] + Sqrt
[a^2 - b^2]*(b - b*Cos[c + d*x] - a*Sin[c + d*x]))/((-a + b)*(a + b)*Sqrt[a
```

```
^2 - b^2]*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(
c + d*x)/2]))
```

Maple [A]
time = 0.21, size = 112, normalized size = 1.33

method	result
derivativedivides	$\frac{-\frac{2}{(2a+2b)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{2}{(2a-2b)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{2b^2\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{(a-b)(a+b)\sqrt{a^2-b^2}}}{d}$
default	$\frac{-\frac{2}{(2a+2b)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{2}{(2a-2b)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{2b^2\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{(a-b)(a+b)\sqrt{a^2-b^2}}}{d}$
risch	$\frac{-2ia+2b\,e^{i(dx+c)}}{d(-a^2+b^2)\left(1+e^{2i(dx+c)}\right)}-\frac{b^2\ln\left(e^{i(dx+c)}+\frac{ia\sqrt{-a^2+b^2}+a^2-b^2}{b\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,(a+b)(a-b)d}+\frac{b^2\ln\left(e^{i(dx+c)}+\frac{ia\sqrt{-a^2+b^2}-a^2\cdots}{b\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,(a+b)(a-b)d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2/(2*a+2*b)/(tan(1/2*d*x+1/2*c)-1)-2/(2*a-2*b)/(tan(1/2*d*x+1/2*c)+1)
-2*b^2/(a-b)/(a+b)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/
(a^2-b^2)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.36, size = 305, normalized size = 3.63

$$\left[\frac{\sqrt{-a^2+b^2}\,b^2\cos(dx+c)\log\left(\frac{(2a^2-b^2)\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2+2(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c))\sqrt{-a^2+b^2}}{b^2\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2}\right)-2a^2b+2b^3+2(a^3-ab^2)\sin(dx+c)}{2(a^4-2a^2b^2+b^4)d\cos(dx+c)},\ \frac{\sqrt{a^2-b^2}\,b^2\arctan\left(-\frac{a\sin(dx+c)+b}{\sqrt{a^2-b^2}\cos(dx+c)}\right)\cos(dx+c)-a^2b+b^3+(a^3-ab^2)\sin(dx+c)}{(a^4-2a^2b^2+b^4)d\cos(dx+c)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-a^2 + b^2)*b^2*cos(d*x + c)*log(((2*a^2 - b^2)*cos(d*x + c)^2 -
 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*
x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 -
b^2)) - 2*a^2*b + 2*b^3 + 2*(a^3 - a*b^2)*sin(d*x + c))/((a^4 - 2*a^2*b^2 +
 b^4)*d*cos(d*x + c)), (sqrt(a^2 - b^2)*b^2*arctan(-(a*sin(d*x + c) + b)/(s
qrt(a^2 - b^2)*cos(d*x + c)))*cos(d*x + c) - a^2*b + b^3 + (a^3 - a*b^2)*si
n(d*x + c))/((a^4 - 2*a^2*b^2 + b^4)*d*cos(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**2/(a + b*sin(c + d*x)), x)
```

Giac [A]
time = 6.97, size = 107, normalized size = 1.27

$$-\frac{2\left(\frac{\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)b^2}{(a^2-b^2)^{\frac{3}{2}}}+\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b}{(a^2-b^2)\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -2*((pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*
c) + b)/sqrt(a^2 - b^2)))*b^2/(a^2 - b^2)^(3/2) + (a*tan(1/2*d*x + 1/2*c) -
 b)/((a^2 - b^2)*(tan(1/2*d*x + 1/2*c)^2 - 1)))/d
```

Mupad [B]
time = 5.26, size = 149, normalized size = 1.77

$$\frac{\frac{2\,b}{a^2-b^2}-\frac{2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^2-b^2}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}-\frac{2\,b^2\,\mathrm{atan}\left(\frac{\frac{b^2\,\left(2\,a^2\,b-2\,b^3\right)}{(a+b)^{3/2}\,(a-b)^{3/2}}+\frac{2\,a\,b^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(a^2-b^2\right)}{(a+b)^{3/2}\,(a-b)^{3/2}}}{2\,b^2}\right)}{d\,(a+b)^{3/2}\,(a-b)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + b*sin(c + d*x))),x)
```

```
[Out] ((2*b)/(a^2 - b^2) - (2*a*tan(c/2 + (d*x)/2))/(a^2 - b^2))/(d*(tan(c/2 + (d
*x)/2)^2 - 1)) - (2*b^2*atan(((b^2*(2*a^2*b - 2*b^3))/((a + b)^(3/2)*(a - b
)^(3/2)) + (2*a*b^2*tan(c/2 + (d*x)/2)*(a^2 - b^2))/((a + b)^(3/2)*(a - b)^
(3/2)))/(2*b^2)))/(d*(a + b)^(3/2)*(a - b)^(3/2))
```

3.435 $\int \frac{\sec^4(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=137

$$\frac{2b^4\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}\,d}-\frac{\sec^3(c+dx)(b-a\sin(c+dx))}{3\,(a^2-b^2)\,d}+\frac{\sec(c+dx)\,(3b^3+a(2a^2-5b^2)\sin(c+dx))}{3\,(a^2-b^2)^2\,d}$$

```
[Out] 2*b^4*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/(a^2-b^2)^(5/2)/d-1/
3*sec(d*x+c)^3*(b-a*sin(d*x+c))/(a^2-b^2)/d+1/3*sec(d*x+c)*(3*b^3+a*(2*a^2-
5*b^2)*sin(d*x+c))/(a^2-b^2)^2/d
```

Rubi [A]
time = 0.17, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2775, 2945, 12, 2739, 632, 210}

$$\frac{2b^4\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\,(a^2-b^2)^{5/2}}-\frac{\sec^3(c+dx)(b-a\sin(c+dx))}{3d\,(a^2-b^2)}+\frac{\sec(c+dx)\,(a(2a^2-5b^2)\sin(c+dx)+3b^3)}{3d\,(a^2-b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + b*Sin[c + d*x]),x]
```

```
[Out] (2*b^4*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(5/2)
*d) - (Sec[c + d*x]^3*(b - a*Sin[c + d*x]))/(3*(a^2 - b^2)*d) + (Sec[c + d*
x]*(3*b^3 + a*(2*a^2 - 5*b^2)*Sin[c + d*x]))/(3*(a^2 - b^2)^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2775

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^
(m + 1)*((b - a*Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rule 2945

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{\sec^3(c+dx)(b-a\sin(c+dx))}{3(a^2-b^2)\,d} - \frac{\int \frac{\sec^2(c+dx)(-2a^2+3b^2-2ab\sin(c+dx))}{a+b\sin(c+dx)}\,dx}{3(a^2-b^2)}$$

$$= -\frac{\sec^3(c+dx)(b-a\sin(c+dx))}{3(a^2-b^2)\,d} + \frac{\sec(c+dx)\,(3b^3+a(2a^2-5b^2)\sin(c+dx))}{3(a^2-b^2)^2\,d}$$

$$= -\frac{\sec^3(c+dx)(b-a\sin(c+dx))}{3(a^2-b^2)\,d} + \frac{\sec(c+dx)\,(3b^3+a(2a^2-5b^2)\sin(c+dx))}{3(a^2-b^2)^2\,d}$$

$$= -\frac{\sec^3(c+dx)(b-a\sin(c+dx))}{3(a^2-b^2)\,d} + \frac{\sec(c+dx)\,(3b^3+a(2a^2-5b^2)\sin(c+dx))}{3(a^2-b^2)^2\,d}$$

$$= -\frac{\sec^3(c+dx)(b-a\sin(c+dx))}{3(a^2-b^2)\,d} + \frac{\sec(c+dx)\,(3b^3+a(2a^2-5b^2)\sin(c+dx))}{3(a^2-b^2)^2\,d}$$

$$= \frac{2b^4\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}\,d} - \frac{\sec^3(c+dx)(b-a\sin(c+dx))}{3(a^2-b^2)\,d} + \frac{\sec(c+dx)\,(3b^3}{3}$$

Mathematica [A]

time = 1.31, size = 202, normalized size = 1.47

$$\frac{\frac{24b^4\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}} + \frac{\sec^3(c+dx)\left(-4a^2b+10b^3+\frac{3}{2}b(a^2-7b^2)\cos(c+dx)+6b^3\cos(2(c+dx))+\frac{1}{2}a^2b\cos(3(c+dx))-\frac{7}{2}b^3\cos(3(c+dx))+6a^3\sin(c+dx)-9ab^2\sin(c+dx)+2a^3\sin(3(c+dx))-5ab^2\sin(3(c+dx))\right)}{(a-b)^2(a+b)^2}}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((24*b^4*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2
) + (Sec[c + d*x]^3*(-4*a^2*b + 10*b^3 + (3*b*(a^2 - 7*b^2)*Cos[c + d*x])/2
 + 6*b^3*Cos[2*(c + d*x)] + (a^2*b*Cos[3*(c + d*x)])/2 - (7*b^3*Cos[3*(c +
d*x)])/2 + 6*a^3*Sin[c + d*x] - 9*a*b^2*Sin[c + d*x] + 2*a^3*Sin[3*(c + d*x
)] - 5*a*b^2*Sin[3*(c + d*x)]))/((a - b)^2*(a + b)^2))/(12*d)
```

Maple [A]

time = 0.51, size = 215, normalized size = 1.57

method	result
derivativedivides	$\frac{\frac{2b^4\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{(a-b)^2(a+b)^2\sqrt{a^2-b^2}} - \frac{2}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3(2a-2b)} + \frac{1}{(2a-2b)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2} - \frac{2a-3b}{2(a-b)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)} - \frac{}{3(\tan}}{d}$

default	$\frac{\frac{2b^4 \arctan\left(\frac{2a \tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{(a-b)^2(a+b)^2\sqrt{a^2-b^2}} - \frac{2}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3(2a-2b)} + \frac{1}{(2a-2b)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2} - \frac{2a-3b}{2(a-b)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)} - \frac{\cdots}{3\left(\tan\left(\cdots\right.\right.}}{d}$
risch	$\frac{-2iab^2 e^{4i(dx+c)} + 2b^3 e^{5i(dx+c)} + 4ia^3 e^{2i(dx+c)} - 8iab^2 e^{2i(dx+c)} - \frac{8a^2 b e^{3i(dx+c)}}{3} + \frac{20b^3 e^{3i(dx+c)}}{3} + \frac{4ia^3}{3} - \frac{10iab^2}{3} + 2b^3 e^{i(dx \cdots}}{d(-a^2+b^2)^2\left(1+e^{2i(dx+c)}\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*b^4/(a-b)^2/(a+b)^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*
c)+2*b)/(a^2-b^2)^(1/2))-2/3/(tan(1/2*d*x+1/2*c)+1)^3/(2*a-2*b)+1/(2*a-2*b)
/(tan(1/2*d*x+1/2*c)+1)^2-1/2*(2*a-3*b)/(a-b)^2/(tan(1/2*d*x+1/2*c)+1)-2/3/
(tan(1/2*d*x+1/2*c)-1)^3/(2*a+2*b)-1/(2*a+2*b)/(tan(1/2*d*x+1/2*c)-1)^2-1/2
*(2*a+3*b)/(a+b)^2/(tan(1/2*d*x+1/2*c)-1))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.36, size = 466, normalized size = 3.40

$$\left[-\frac{3\sqrt{-a^2+b^2}\,b^4\cos(dx+c)^3\log\left(\frac{(2a^2-b^2)\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2+2(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c))\sqrt{-a^2+b^2}}{b^2\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2}\right)+2a^4b-4a^2b^3+2b^5-6(a^2b^3-b^5)\cos(dx+c)^2-2(a^5-2a^3b^2+ab^4+(2a^5-7a^3b^2+5ab^4)\cos(dx+c)^2)\sin(dx+c)}{6(a^6-3a^4b^2+3a^2b^4-b^6)d\cos(dx+c)^3},\ -\frac{3\sqrt{a^2-b^2}\,b^4\arctan\left(-\frac{a\sin(dx+c)+b}{\sqrt{a^2-b^2}\cos(dx+c)}\right)\cos(dx+c)^3+a^4b-2a^2b^3+b^5-3(a^2b^3-b^5)\cos(dx+c)^2-(a^5-2a^3b^2+ab^4+(2a^5-7a^3b^2+5ab^4)\cos(dx+c)^2)\sin(dx+c)}{3(a^6-3a^4b^2+3a^2b^4-b^6)d\cos(dx+c)^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*sqrt(-a^2 + b^2)*b^4*cos(d*x + c)^3*log(((2*a^2 - b^2)*cos(d*x + c
)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*c
os(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a
^2 - b^2)) + 2*a^4*b - 4*a^2*b^3 + 2*b^5 - 6*(a^2*b^3 - b^5)*cos(d*x + c)^2
 - 2*(a^5 - 2*a^3*b^2 + a*b^4 + (2*a^5 - 7*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^
2)*sin(d*x + c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d*cos(d*x + c)^3), -1
/3*(3*sqrt(a^2 - b^2)*b^4*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos
```

```
(d*x + c)))*cos(d*x + c)^3 + a^4*b - 2*a^2*b^3 + b^5 - 3*(a^2*b^3 - b^5)*co
s(d*x + c)^2 - (a^5 - 2*a^3*b^2 + a*b^4 + (2*a^5 - 7*a^3*b^2 + 5*a*b^4)*cos
(d*x + c)^2)*sin(d*x + c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d*cos(d*x +
 c)^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^4(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**4/(a + b*sin(c + d*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 273 vs. 2(129) = 258.
time = 6.48, size = 273, normalized size = 1.99

$$\frac{2\left(\frac{3\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)b^4}{(a^4-2a^2b^2+b^4)\sqrt{a^2-b^2}}-\frac{3a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-6ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-3a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+6b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-2a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+8ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-6b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+3a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-6ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-a^2b+4b^3}{(a^4-2a^2b^2+b^4)\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^3}\right)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 2/3*(3*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1
/2*c) + b)/sqrt(a^2 - b^2)))*b^4/((a^4 - 2*a^2*b^2 + b^4)*sqrt(a^2 - b^2))
- (3*a^3*tan(1/2*d*x + 1/2*c)^5 - 6*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 3*a^2*b*
tan(1/2*d*x + 1/2*c)^4 + 6*b^3*tan(1/2*d*x + 1/2*c)^4 - 2*a^3*tan(1/2*d*x +
 1/2*c)^3 + 8*a*b^2*tan(1/2*d*x + 1/2*c)^3 - 6*b^3*tan(1/2*d*x + 1/2*c)^2 +
 3*a^3*tan(1/2*d*x + 1/2*c) - 6*a*b^2*tan(1/2*d*x + 1/2*c) - a^2*b + 4*b^3)
/((a^4 - 2*a^2*b^2 + b^4)*(tan(1/2*d*x + 1/2*c)^2 - 1)^3))/d
```

Mupad [B]
time = 7.95, size = 387, normalized size = 2.82

$$\frac{\frac{2(a^2b-4b^3)}{3(a^4-2a^2b^2+b^4)}+\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(2ab^2-a^3)}{a^4-2a^2b^2+b^4}+\frac{4b^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{a^4-2a^2b^2+b^4}-\frac{4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3(4ab^2-a^3)}{3(a^4-2a^2b^2+b^4)}+\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5(2ab^2-a^3)}{a^4-2a^2b^2+b^4}+\frac{2b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4(a^2-2b^2)}{a^4-2a^2b^2+b^4}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6-3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-1\right)}+\frac{2b^4\,\mathrm{atan}\left(\frac{\frac{b^4(2a^4b-4a^2b^3+2b^5)}{(a+b)^{5/2}(a-b)^{5/2}}+\frac{2ab^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(a^4-2a^2b^2+b^4)}{(a+b)^{5/2}(a-b)^{5/2}}}{2b^4}\right)}{d(a+b)^{5/2}(a-b)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a + b*sin(c + d*x))),x)
```

```
[Out] ((2*(a^2*b - 4*b^3))/(3*(a^4 + b^4 - 2*a^2*b^2)) + (2*tan(c/2 + (d*x)/2)*(2
*a*b^2 - a^3))/(a^4 + b^4 - 2*a^2*b^2) + (4*b^3*tan(c/2 + (d*x)/2)^2)/(a^4
+ b^4 - 2*a^2*b^2) - (4*tan(c/2 + (d*x)/2)^3*(4*a*b^2 - a^3))/(3*(a^4 + b^4
```

```
 - 2*a^2*b^2)) + (2*tan(c/2 + (d*x)/2)^5*(2*a*b^2 - a^3))/(a^4 + b^4 - 2*a^
2*b^2) + (2*b*tan(c/2 + (d*x)/2)^4*(a^2 - 2*b^2))/(a^4 + b^4 - 2*a^2*b^2))/
(d*(3*tan(c/2 + (d*x)/2)^2 - 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6
- 1)) + (2*b^4*atan(((b^4*(2*a^4*b + 2*b^5 - 4*a^2*b^3))/((a + b)^(5/2)*(a
- b)^(5/2)) + (2*a*b^4*tan(c/2 + (d*x)/2)*(a^4 + b^4 - 2*a^2*b^2))/((a + b)
^(5/2)*(a - b)^(5/2)))/(2*b^4)))/(d*(a + b)^(5/2)*(a - b)^(5/2))
```

3.436 $\int \frac{\sec^6(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=197

$$-\frac{2b^6\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{7/2}d}-\frac{\sec^5(c+dx)(b-a\sin(c+dx))}{5\left(a^2-b^2\right)d}+\frac{\sec^3(c+dx)\left(5b^3+a\left(4a^2-9b^2\right)\sin(c+dx\right.}{15\left(a^2-b^2\right)^2d}$$

```
[Out] -2*b^6*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/(a^2-b^2)^(7/2)/d-1
/5*sec(d*x+c)^5*(b-a*sin(d*x+c))/(a^2-b^2)/d+1/15*sec(d*x+c)^3*(5*b^3+a*(4*
a^2-9*b^2)*sin(d*x+c))/(a^2-b^2)^2/d-1/15*sec(d*x+c)*(15*b^5-a*(8*a^4-26*a^
2*b^2+33*b^4)*sin(d*x+c))/(a^2-b^2)^3/d
```

Rubi [A]
time = 0.31, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2775, 2945, 12, 2739, 632, 210}

$$-\frac{2b^6\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{7/2}}-\frac{\sec^5(c+dx)(b-a\sin(c+dx))}{5d\left(a^2-b^2\right)}+\frac{\sec^3(c+dx)\left(a\left(4a^2-9b^2\right)\sin(c+dx)+5b^3\right)}{15d\left(a^2-b^2\right)^2}-\frac{\sec(c+dx)\left(15b^5-a\left(8a^4-26a^2b^2+33b^4\right)\sin(c+dx)\right)}{15d\left(a^2-b^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*b^6*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(7/2
)*d) - (Sec[c + d*x]^5*(b - a*Sin[c + d*x]))/(5*(a^2 - b^2)*d) + (Sec[c + d
*x]^3*(5*b^3 + a*(4*a^2 - 9*b^2)*Sin[c + d*x]))/(15*(a^2 - b^2)^2*d) - (Sec
[c + d*x]*(15*b^5 - a*(8*a^4 - 26*a^2*b^2 + 33*b^4)*Sin[c + d*x]))/(15*(a^2
 - b^2)^3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2775

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^
(m + 1)*((b - a*Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rule 2945

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\sec^6(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{\sec^5(c+dx)(b-a\sin(c+dx))}{5\left(a^2-b^2\right)d} - \frac{\int \frac{\sec^4(c+dx)\left(-4a^2+5b^2-4ab\sin(c+dx)\right)}{a+b\sin(c+dx)}\,dx}{5\left(a^2-b^2\right)}$$

$$= -\frac{\sec^5(c+dx)(b-a\sin(c+dx))}{5\left(a^2-b^2\right)d} + \frac{\sec^3(c+dx)\left(5b^3+a(4a^2-9b^2)\sin(c+dx)\right)}{15\left(a^2-b^2\right)^2 d}$$

$$= -\frac{\sec^5(c+dx)(b-a\sin(c+dx))}{5\left(a^2-b^2\right)d} + \frac{\sec^3(c+dx)\left(5b^3+a(4a^2-9b^2)\sin(c+dx)\right)}{15\left(a^2-b^2\right)^2 d}$$

$$= -\frac{\sec^5(c+dx)(b-a\sin(c+dx))}{5\left(a^2-b^2\right)d} + \frac{\sec^3(c+dx)\left(5b^3+a(4a^2-9b^2)\sin(c+dx)\right)}{15\left(a^2-b^2\right)^2 d}$$

$$= -\frac{\sec^5(c+dx)(b-a\sin(c+dx))}{5\left(a^2-b^2\right)d} + \frac{\sec^3(c+dx)\left(5b^3+a(4a^2-9b^2)\sin(c+dx)\right)}{15\left(a^2-b^2\right)^2 d}$$

$$= -\frac{\sec^5(c+dx)(b-a\sin(c+dx))}{5\left(a^2-b^2\right)d} + \frac{\sec^3(c+dx)\left(5b^3+a(4a^2-9b^2)\sin(c+dx)\right)}{15\left(a^2-b^2\right)^2 d}$$

$$= -\frac{2b^6\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{7/2}d} - \frac{\sec^5(c+dx)(b-a\sin(c+dx))}{5\left(a^2-b^2\right)d} + \frac{\sec^3(c+dx)\,(5}{}$$

Mathematica [A]
time = 2.53, size = 370, normalized size = 1.88

$$-\frac{2b^6\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{7/2}d} + \frac{\sec^5(c+dx)\left(-384a^4b+1088a^2b^3-1424b^5+10b(9a^4-38a^2b^2+149b^4)\cos(c+dx)+320b^3(a^2-4b^2)\cos(2(c+dx))+45a^4b\cos(3(c+dx))-190a^2b^3\cos(3(c+dx))+745b^5\cos(3(c+dx))-240b^5\cos(4(c+dx))+9a^4b\cos(5(c+dx))-38a^2b^3\cos(5(c+dx))+149b^5\cos(5(c+dx))+640a^5\sin(c+dx)-1600a^3b^2\sin(c+dx)+1200ab^4\sin(c+dx)+320a^5\sin(3(c+dx))-1040a^3b^2\sin(3(c+dx))+1080ab^4\sin(3(c+dx))+64a^5\sin(5(c+dx))-208a^3b^2\sin(5(c+dx))+264ab^4\sin(5(c+dx))\right)}{1920(a-b)^3(a+b)^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*b^6*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(7/2
)*d) + (Sec[c + d*x]^5*(-384*a^4*b + 1088*a^2*b^3 - 1424*b^5 + 10*b*(9*a^4
- 38*a^2*b^2 + 149*b^4)*Cos[c + d*x] + 320*b^3*(a^2 - 4*b^2)*Cos[2*(c + d*x
)] + 45*a^4*b*Cos[3*(c + d*x)] - 190*a^2*b^3*Cos[3*(c + d*x)] + 745*b^5*Cos
[3*(c + d*x)] - 240*b^5*Cos[4*(c + d*x)] + 9*a^4*b*Cos[5*(c + d*x)] - 38*a^
2*b^3*Cos[5*(c + d*x)] + 149*b^5*Cos[5*(c + d*x)] + 640*a^5*Sin[c + d*x] -
1600*a^3*b^2*Sin[c + d*x] + 1200*a*b^4*Sin[c + d*x] + 320*a^5*Sin[3*(c + d*
x)] - 1040*a^3*b^2*Sin[3*(c + d*x)] + 1080*a*b^4*Sin[3*(c + d*x)] + 64*a^5*
Sin[5*(c + d*x)] - 208*a^3*b^2*Sin[5*(c + d*x)] + 264*a*b^4*Sin[5*(c + d*x)
]))/(1920*(a - b)^3*(a + b)^3*d)
```

Maple [A]
time = 0.79, size = 338, normalized size = 1.72

method	result
derivativedivides	$-\frac{2b^6\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{(a-b)^3(a+b)^3\sqrt{a^2-b^2}}-\frac{2}{5(2a-2b)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}+\frac{1}{2(a-b)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}-\frac{-7a+9b}{8(a-b)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{\cdots}{12(a\cdots}$
default	$-\frac{2b^6\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{(a-b)^3(a+b)^3\sqrt{a^2-b^2}}-\frac{2}{5(2a-2b)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}+\frac{1}{2(a-b)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}-\frac{-7a+9b}{8(a-b)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{\cdots}{12(a\cdots}$
risch	$\frac{-20iab^4e^{2i(dx+c)}+2b^5e^{9i(dx+c)}-\frac{16ia^5e^{2i(dx+c)}}{3}-\frac{22iab^4}{5}-\frac{8a^2b^3e^{7i(dx+c)}}{3}+\frac{32b^5e^{7i(dx+c)}}{3}-32iab^4e^{4i(dx+c)}-12iab^4e^{6i(\cdots}}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*b^6/(a-b)^3/(a+b)^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2
*c)+2*b)/(a^2-b^2)^(1/2))-2/5/(2*a-2*b)/(tan(1/2*d*x+1/2*c)+1)^5+1/2/(a-b)/
(tan(1/2*d*x+1/2*c)+1)^4-1/8*(-7*a+9*b)/(a-b)^2/(tan(1/2*d*x+1/2*c)+1)^2-1/
12*(11*a-13*b)/(a-b)^2/(tan(1/2*d*x+1/2*c)+1)^3-1/8*(8*a^2-21*a*b+15*b^2)/(
a-b)^3/(tan(1/2*d*x+1/2*c)+1)-1/2/(a+b)/(tan(1/2*d*x+1/2*c)-1)^4-2/5/(2*a+2
*b)/(tan(1/2*d*x+1/2*c)-1)^5-1/8*(7*a+9*b)/(a+b)^2/(tan(1/2*d*x+1/2*c)-1)^2
-1/12*(11*a+13*b)/(a+b)^2/(tan(1/2*d*x+1/2*c)-1)^3-1/8*(8*a^2+21*a*b+15*b^2
)/(a+b)^3/(tan(1/2*d*x+1/2*c)-1))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.39, size = 666, normalized size = 3.38

$$\left[\frac{15\sqrt{-a^2+b^2}\,b^6\cos(dx+c)^5\log\left(\frac{(2a^2-b^2)\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2+2(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c))\sqrt{-a^2+b^2}}{b^2\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2}\right)-6a^6b+18a^4b^3-18a^2b^5+6b^7-30(a^2b^5-b^7)\cos(dx+c)^4+10(a^4b^3-2a^2b^5+b^7)\cos(dx+c)^2+2(3a^7-9a^5b^2+9a^3b^4-3ab^6+(8a^7-34a^5b^2+59a^3b^4-33ab^6)\cos(dx+c)^4+(4a^7-17a^5b^2+22a^3b^4-9ab^6)\cos(dx+c)^2)\sin(dx+c)}{30(a^8-4a^6b^2+6a^4b^4-4a^2b^6+b^8)d\cos(dx+c)^5},\ \frac{15\sqrt{a^2-b^2}\,b^6\arctan\left(-\frac{a\sin(dx+c)+b}{\sqrt{a^2-b^2}\cos(dx+c)}\right)\cos(dx+c)^5-3a^6b+9a^4b^3-9a^2b^5+3b^7-15(a^2b^5-b^7)\cos(dx+c)^4+5(a^4b^3-2a^2b^5+b^7)\cos(dx+c)^2+(3a^7-9a^5b^2+9a^3b^4-3ab^6+(8a^7-34a^5b^2+59a^3b^4-33ab^6)\cos(dx+c)^4+(4a^7-17a^5b^2+22a^3b^4-9ab^6)\cos(dx+c)^2)\sin(dx+c)}{15(a^8-4a^6b^2+6a^4b^4-4a^2b^6+b^8)d\cos(dx+c)^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/30*(15*sqrt(-a^2 + b^2)*b^6*cos(d*x + c)^5*log(((2*a^2 - b^2)*cos(d*x +
c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*
cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) -
a^2 - b^2)) - 6*a^6*b + 18*a^4*b^3 - 18*a^2*b^5 + 6*b^7 - 30*(a^2*b^5 - b^7
)*cos(d*x + c)^4 + 10*(a^4*b^3 - 2*a^2*b^5 + b^7)*cos(d*x + c)^2 + 2*(3*a^7
 - 9*a^5*b^2 + 9*a^3*b^4 - 3*a*b^6 + (8*a^7 - 34*a^5*b^2 + 59*a^3*b^4 - 33*
a*b^6)*cos(d*x + c)^4 + (4*a^7 - 17*a^5*b^2 + 22*a^3*b^4 - 9*a*b^6)*cos(d*x
 + c)^2)*sin(d*x + c))/((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*d*c
os(d*x + c)^5), 1/15*(15*sqrt(a^2 - b^2)*b^6*arctan(-(a*sin(d*x + c) + b)/(
sqrt(a^2 - b^2)*cos(d*x + c)))*cos(d*x + c)^5 - 3*a^6*b + 9*a^4*b^3 - 9*a^2
*b^5 + 3*b^7 - 15*(a^2*b^5 - b^7)*cos(d*x + c)^4 + 5*(a^4*b^3 - 2*a^2*b^5 +
 b^7)*cos(d*x + c)^2 + (3*a^7 - 9*a^5*b^2 + 9*a^3*b^4 - 3*a*b^6 + (8*a^7 -
34*a^5*b^2 + 59*a^3*b^4 - 33*a*b^6)*cos(d*x + c)^4 + (4*a^7 - 17*a^5*b^2 +
22*a^3*b^4 - 9*a*b^6)*cos(d*x + c)^2)*sin(d*x + c))/((a^8 - 4*a^6*b^2 + 6*a
^4*b^4 - 4*a^2*b^6 + b^8)*d*cos(d*x + c)^5)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^6(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**6/(a + b*sin(c + d*x)), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 584 vs. 2(187) = 374.
time = 5.17, size = 584, normalized size = 2.96

$$-\frac{2\left(\frac{15\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)b^6}{(a^6-3a^4b^2+3a^2b^4-b^6)\sqrt{a^2-b^2}}+\frac{15a^5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9-45a^3b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9+45ab^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9-15a^4b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^8+45a^2b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^8-45b^5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^8-20a^5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+80a^3b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-120ab^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-30a^2b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6+90b^5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6+58a^5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-166a^3b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+198ab^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-30a^4b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+80a^2b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-140b^5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-20a^5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+80a^3b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-120ab^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-10a^2b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+70b^5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+15a^5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-45a^3b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+45ab^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-3a^4b+11a^2b^3-23b^5}{(a^6-3a^4b^2+3a^2b^4-b^6)\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^5}\right)}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -2/15*(15*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x
+ 1/2*c) + b)/sqrt(a^2 - b^2)))*b^6/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sq
rt(a^2 - b^2)) + (15*a^5*tan(1/2*d*x + 1/2*c)^9 - 45*a^3*b^2*tan(1/2*d*x +
1/2*c)^9 + 45*a*b^4*tan(1/2*d*x + 1/2*c)^9 - 15*a^4*b*tan(1/2*d*x + 1/2*c)^
8 + 45*a^2*b^3*tan(1/2*d*x + 1/2*c)^8 - 45*b^5*tan(1/2*d*x + 1/2*c)^8 - 20*
a^5*tan(1/2*d*x + 1/2*c)^7 + 80*a^3*b^2*tan(1/2*d*x + 1/2*c)^7 - 120*a*b^4*
tan(1/2*d*x + 1/2*c)^7 - 30*a^2*b^3*tan(1/2*d*x + 1/2*c)^6 + 90*b^5*tan(1/2
*d*x + 1/2*c)^6 + 58*a^5*tan(1/2*d*x + 1/2*c)^5 - 166*a^3*b^2*tan(1/2*d*x +
```

```
1/2*c)^5 + 198*a*b^4*tan(1/2*d*x + 1/2*c)^5 - 30*a^4*b*tan(1/2*d*x + 1/2*c
)^4 + 80*a^2*b^3*tan(1/2*d*x + 1/2*c)^4 - 140*b^5*tan(1/2*d*x + 1/2*c)^4 -
20*a^5*tan(1/2*d*x + 1/2*c)^3 + 80*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 - 120*a*b
^4*tan(1/2*d*x + 1/2*c)^3 - 10*a^2*b^3*tan(1/2*d*x + 1/2*c)^2 + 70*b^5*tan(
1/2*d*x + 1/2*c)^2 + 15*a^5*tan(1/2*d*x + 1/2*c) - 45*a^3*b^2*tan(1/2*d*x +
 1/2*c) + 45*a*b^4*tan(1/2*d*x + 1/2*c) - 3*a^4*b + 11*a^2*b^3 - 23*b^5)/((
a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(tan(1/2*d*x + 1/2*c)^2 - 1)^5))/d
```

Mupad [B]
time = 8.06, size = 774, normalized size = 3.93

$$\frac{\frac{2\,\left(3\,a^4\,b-11\,a^2\,b^3+23\,b^5\right)}{15\,\left(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6\right)}-\frac{2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(a^5-3\,a^3\,b^2+3\,a\,b^4\right)}{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}-\frac{4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(7\,b^5-a^2\,b^3\right)}{3\,\left(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6\right)}-\frac{4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6\,\left(3\,b^5-a^2\,b^3\right)}{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}+\frac{8\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(a^5-4\,a^3\,b^2+6\,a\,b^4\right)}{3\,\left(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6\right)}-\frac{2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^9\,\left(a^5-3\,a^3\,b^2+3\,a\,b^4\right)}{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}+\frac{8\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7\,\left(a^5-4\,a^3\,b^2+6\,a\,b^4\right)}{3\,\left(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6\right)}-\frac{4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5\,\left(29\,a^5-83\,a^3\,b^2+99\,a\,b^4\right)}{15\,\left(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6\right)}+\frac{2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8\,\left(a^4\,b-3\,a^2\,b^3+3\,b^5\right)}{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}+\frac{4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(3\,a^4\,b-8\,a^2\,b^3+14\,b^5\right)}{3\,\left(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6\right)}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}-5\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8+10\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6-10\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+5\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}-\frac{2\,b^6\,\mathrm{atan}\left(\frac{\frac{b^6\,\left(2\,a^6\,b-6\,a^4\,b^3+6\,a^2\,b^5-2\,b^7\right)}{{\left(a+b\right)}^{7/2}\,{\left(a-b\right)}^{7/2}}+\frac{2\,a\,b^6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6\right)}{{\left(a+b\right)}^{7/2}\,{\left(a-b\right)}^{7/2}}}{2\,b^6}\right)}{d\,{\left(a+b\right)}^{7/2}\,{\left(a-b\right)}^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^6*(a + b*sin(c + d*x))),x)
```

```
[Out] ((2*(3*a^4*b + 23*b^5 - 11*a^2*b^3))/(15*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2
)) - (2*tan(c/2 + (d*x)/2)*(3*a*b^4 + a^5 - 3*a^3*b^2))/(a^6 - b^6 + 3*a^2*
b^4 - 3*a^4*b^2) - (4*tan(c/2 + (d*x)/2)^2*(7*b^5 - a^2*b^3))/(3*(a^6 - b^6
 + 3*a^2*b^4 - 3*a^4*b^2)) - (4*tan(c/2 + (d*x)/2)^6*(3*b^5 - a^2*b^3))/(a^
6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2) + (8*tan(c/2 + (d*x)/2)^3*(6*a*b^4 + a^5 -
 4*a^3*b^2))/(3*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) - (2*tan(c/2 + (d*x)/2
)^9*(3*a*b^4 + a^5 - 3*a^3*b^2))/(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2) + (8*t
an(c/2 + (d*x)/2)^7*(6*a*b^4 + a^5 - 4*a^3*b^2))/(3*(a^6 - b^6 + 3*a^2*b^4
- 3*a^4*b^2)) - (4*tan(c/2 + (d*x)/2)^5*(99*a*b^4 + 29*a^5 - 83*a^3*b^2))/(
15*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) + (2*tan(c/2 + (d*x)/2)^8*(a^4*b +
3*b^5 - 3*a^2*b^3))/(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2) + (4*tan(c/2 + (d*x
)/2)^4*(3*a^4*b + 14*b^5 - 8*a^2*b^3))/(3*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^
2)))/(d*(5*tan(c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d
*x)/2)^6 - 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1)) - (2*b^6*at
an(((b^6*(2*a^6*b - 2*b^7 + 6*a^2*b^5 - 6*a^4*b^3))/((a + b)^(7/2)*(a - b)^
(7/2)) + (2*a*b^6*tan(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2))/(
(a + b)^(7/2)*(a - b)^(7/2)))/(2*b^6)))/(d*(a + b)^(7/2)*(a - b)^(7/2))
```

3.437 $\int \frac{\cos^7(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=184

$$\frac{6a(a^2-b^2)^2\log(a+b\sin(c+dx))}{b^7d}-\frac{(5a^4-9a^2b^2+3b^4)\sin(c+dx)}{b^6d}+\frac{a(2a^2-3b^2)\sin^2(c+dx)}{b^5d}-\frac{(a^2-b^2)\ldots}{\ldots}$$

```
[Out] 6*a*(a^2-b^2)^2*ln(a+b*sin(d*x+c))/b^7/d-(5*a^4-9*a^2*b^2+3*b^4)*sin(d*x+c)
/b^6/d+a*(2*a^2-3*b^2)*sin(d*x+c)^2/b^5/d-(a^2-b^2)*sin(d*x+c)^3/b^4/d+1/2*
a*sin(d*x+c)^4/b^3/d-1/5*sin(d*x+c)^5/b^2/d+(a^2-b^2)^3/b^7/d/(a+b*sin(d*x+
c))
```

Rubi [A]

time = 0.12, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2747, 711}

$$\frac{(a^2-b^2)^3}{b^7d(a+b\sin(c+dx))}+\frac{6a(a^2-b^2)^2\log(a+b\sin(c+dx))}{b^7d}+\frac{a(2a^2-3b^2)\sin^2(c+dx)}{b^5d}-\frac{(a^2-b^2)\sin^3(c+dx)}{b^4d}-\frac{(5a^4-9a^2b^2+3b^4)\sin(c+dx)}{b^6d}+\frac{a\sin^4(c+dx)}{2b^3d}-\frac{\sin^5(c+dx)}{5b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (6*a*(a^2 - b^2)^2*Log[a + b*Sin[c + d*x]])/(b^7*d) - ((5*a^4 - 9*a^2*b^2 +
 3*b^4)*Sin[c + d*x])/(b^6*d) + (a*(2*a^2 - 3*b^2)*Sin[c + d*x]^2)/(b^5*d)
- ((a^2 - b^2)*Sin[c + d*x]^3)/(b^4*d) + (a*Sin[c + d*x]^4)/(2*b^3*d) - Sin
[c + d*x]^5/(5*b^2*d) + (a^2 - b^2)^3/(b^7*d*(a + b*Sin[c + d*x]))
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^7(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{(b^2-x^2)^3}{(a+x)^2}\,dx, x, b\sin(c+dx)\right)}{b^7 d}$$
$$= \frac{\text{Subst}\left(\int \left(-5a^4\left(1+\frac{3b^2(-3a^2+b^2)}{5a^4}\right)+2a(2a^2-3b^2)\,x-3(a^2-b^2)\,x^2+2ax^3-x\right.\right.}{b^7d}$$
$$= \frac{6a(a^2-b^2)^2\log(a+b\sin(c+dx))}{b^7d} - \frac{(5a^4-9a^2b^2+3b^4)\sin(c+dx)}{b^6d} + \frac{a(2a^2-}{}$$

Mathematica [A]
time = 1.43, size = 165, normalized size = 0.90

$$-\frac{20ab^2(4a^2-5b^2)\cos(2(c+dx))-5ab^4\cos(4(c+dx))-480a(a^2-b^2)^2\log(a+b\sin(c+dx))+10b(40a^4-66a^2b^2+19b^4)\sin(c+dx)-\frac{80(a-b)^3(a+b)^3}{a+b\sin(c+dx)}+5b^3(-4a^2+3b^2)\sin(3(c+dx))+b^5\sin(5(c+dx))}{80b^7d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -1/80*(20*a*b^2*(4*a^2 - 5*b^2)*Cos[2*(c + d*x)] - 5*a*b^4*Cos[4*(c + d*x)]
 - 480*a*(a^2 - b^2)^2*Log[a + b*Sin[c + d*x]] + 10*b*(40*a^4 - 66*a^2*b^2
+ 19*b^4)*Sin[c + d*x] - (80*(a - b)^3*(a + b)^3)/(a + b*Sin[c + d*x]) + 5*
b^3*(-4*a^2 + 3*b^2)*Sin[3*(c + d*x)] + b^5*Sin[5*(c + d*x)])/(b^7*d)
```

Maple [A]
time = 0.80, size = 205, normalized size = 1.11 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/b^6*(1/5*sin(d*x+c)^5*b^4-1/2*a*b^3*sin(d*x+c)^4+a^2*b^2*sin(d*x+c)
^3-b^4*sin(d*x+c)^3-2*a^3*b*sin(d*x+c)^2+3*a*b^3*sin(d*x+c)^2+5*a^4*sin(d*x
+c)-9*a^2*b^2*sin(d*x+c)+3*b^4*sin(d*x+c))-1/b^7*(-a^6+3*a^4*b^2-3*a^2*b^4+
b^6)/(a+b*sin(d*x+c))+6*a/b^7*(a^4-2*a^2*b^2+b^4)*ln(a+b*sin(d*x+c)))
```

Maxima [A]
time = 0.28, size = 190, normalized size = 1.03

$$\frac{\frac{10\,(a^6-3\,a^4b^2+3\,a^2b^4-b^6)}{b^8\sin(dx+c)+ab^7}-\frac{2\,b^4\sin(dx+c)^5-5\,ab^3\sin(dx+c)^4+10\,(a^2b^2-b^4)\sin(dx+c)^3-10\,(2\,a^3b-3\,ab^3)\sin(dx+c)^2+10\,(5\,a^4-9\,a^2b^2+3\,b^4)\sin(dx+c)}{b^6}+\frac{60\,(a^5-2\,a^3b^2+ab^4)\log(b\sin(dx+c)+a)}{b^7}}{10\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/10*(10*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)/(b^8*sin(d*x + c) + a*b^7) - (
2*b^4*sin(d*x + c)^5 - 5*a*b^3*sin(d*x + c)^4 + 10*(a^2*b^2 - b^4)*sin(d*x
```

```
+ c)^3 - 10*(2*a^3*b - 3*a*b^3)*sin(d*x + c)^2 + 10*(5*a^4 - 9*a^2*b^2 + 3*
b^4)*sin(d*x + c))/b^6 + 60*(a^5 - 2*a^3*b^2 + a*b^4)*log(b*sin(d*x + c) +
a)/b^7)/d
```

Fricas [A]
time = 0.42, size = 243, normalized size = 1.32

$$\frac{16\,b^{6}\cos\left(dx+c\right)^{6}+80\,a^{6}-560\,a^{4}b^{2}+785\,a^{2}b^{4}-256\,b^{6}-8\,\left(5\,a^{2}b^{4}-4\,b^{6}\right)\cos\left(dx+c\right)^{4}+16\,\left(15\,a^{4}b^{2}-25\,a^{2}b^{4}+8\,b^{6}\right)\cos\left(dx+c\right)^{2}+480\,\left(a^{6}-2\,a^{4}b^{2}+a^{2}b^{4}+\left(a^{5}b-2\,a^{3}b^{3}+ab^{5}\right)\sin\left(dx+c\right)\right)\log\left(b\sin\left(dx+c\right)+a\right)+\left(24\,ab^{5}\cos\left(dx+c\right)^{4}-400\,a^{5}b+720\,a^{3}b^{3}-271\,ab^{5}-16\,\left(5\,a^{3}b^{3}-7\,ab^{5}\right)\cos\left(dx+c\right)^{2}\right)\sin\left(dx+c\right)}{80\,\left(b^{8}d\sin\left(dx+c\right)+ab^{7}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/80*(16*b^6*cos(d*x + c)^6 + 80*a^6 - 560*a^4*b^2 + 785*a^2*b^4 - 256*b^6
- 8*(5*a^2*b^4 - 4*b^6)*cos(d*x + c)^4 + 16*(15*a^4*b^2 - 25*a^2*b^4 + 8*b^
6)*cos(d*x + c)^2 + 480*(a^6 - 2*a^4*b^2 + a^2*b^4 + (a^5*b - 2*a^3*b^3 + a
*b^5)*sin(d*x + c))*log(b*sin(d*x + c) + a) + (24*a*b^5*cos(d*x + c)^4 - 40
0*a^5*b + 720*a^3*b^3 - 271*a*b^5 - 16*(5*a^3*b^3 - 7*a*b^5)*cos(d*x + c)^2
)*sin(d*x + c))/(b^8*d*sin(d*x + c) + a*b^7*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 5.60, size = 251, normalized size = 1.36

$$\frac{\frac{60\,\left(a^{5}-2\,a^{3}b^{2}+ab^{4}\right)\log\left(\left|b\sin\left(dx+c\right)+a\right|\right)}{b^{7}}-\frac{10\,\left(6\,a^{5}b\sin\left(dx+c\right)-12\,a^{3}b^{3}\sin\left(dx+c\right)+6\,ab^{5}\sin\left(dx+c\right)+5\,a^{6}-9\,a^{4}b^{2}+3\,a^{2}b^{4}+b^{6}\right)}{\left(b\sin\left(dx+c\right)+a\right)b^{7}}-\frac{2\,b^{8}\sin\left(dx+c\right)^{5}-5\,ab^{7}\sin\left(dx+c\right)^{4}+10\,a^{2}b^{6}\sin\left(dx+c\right)^{3}-10\,b^{8}\sin\left(dx+c\right)^{3}-20\,a^{3}b^{5}\sin\left(dx+c\right)^{2}+30\,ab^{7}\sin\left(dx+c\right)^{2}+50\,a^{4}b^{4}\sin\left(dx+c\right)-90\,a^{2}b^{6}\sin\left(dx+c\right)+30\,b^{8}\sin\left(dx+c\right)}{b^{10}}}{10\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/10*(60*(a^5 - 2*a^3*b^2 + a*b^4)*log(abs(b*sin(d*x + c) + a))/b^7 - 10*(6
*a^5*b*sin(d*x + c) - 12*a^3*b^3*sin(d*x + c) + 6*a*b^5*sin(d*x + c) + 5*a^
6 - 9*a^4*b^2 + 3*a^2*b^4 + b^6)/((b*sin(d*x + c) + a)*b^7) - (2*b^8*sin(d*
x + c)^5 - 5*a*b^7*sin(d*x + c)^4 + 10*a^2*b^6*sin(d*x + c)^3 - 10*b^8*sin(
d*x + c)^3 - 20*a^3*b^5*sin(d*x + c)^2 + 30*a*b^7*sin(d*x + c)^2 + 50*a^4*b
^4*sin(d*x + c) - 90*a^2*b^6*sin(d*x + c) + 30*b^8*sin(d*x + c))/b^10)/d
```

Mupad [B]
time = 0.12, size = 259, normalized size = 1.41

$$\frac{{\sin\left(c+d\,x\right)}^3\,\left(\frac{1}{b^2}-\frac{a^2}{b^4}\right)}{d}-\frac{{\sin\left(c+d\,x\right)}^5}{5\,b^2\,d}-\frac{{\sin\left(c+d\,x\right)}^2\,\left(\frac{a^3}{b^5}+\frac{a\,\left(\frac{3}{b^2}-\frac{3\,a^2}{b^4}\right)}{b}\right)}{d}-\frac{\sin\left(c+d\,x\right)\,\left(\frac{3}{b^2}+\frac{a^2\,\left(\frac{3}{b^2}-\frac{3\,a^2}{b^4}\right)}{b^2}-\frac{2\,a\,\left(\frac{2\,a^3}{b^5}+\frac{2\,a\,\left(\frac{3}{b^2}-\frac{3\,a^2}{b^4}\right)}{b}\right)}{b}\right)}{d}+\frac{a\,{\sin\left(c+d\,x\right)}^4}{2\,b^3\,d}+\frac{\ln\left(a+b\,\sin\left(c+d\,x\right)\right)\,\left(6\,a^5-12\,a^3\,b^2+6\,a\,b^4\right)}{b^7\,d}+\frac{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}{b\,d\,\left(\sin\left(c+d\,x\right)\,b^7+a\,b^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7/(a + b*sin(c + d*x))^2,x)
```

```
[Out] (sin(c + d*x)^3*(1/b^2 - a^2/b^4))/d - sin(c + d*x)^5/(5*b^2*d) - (sin(c +
d*x)^2*(a^3/b^5 + (a*(3/b^2 - (3*a^2)/b^4))/b))/d - (sin(c + d*x)*(3/b^2 +
(a^2*(3/b^2 - (3*a^2)/b^4))/b^2 - (2*a*((2*a^3)/b^5 + (2*a*(3/b^2 - (3*a^2)
/b^4))/b))/b))/d + (a*sin(c + d*x)^4)/(2*b^3*d) + (log(a + b*sin(c + d*x))*
(6*a*b^4 + 6*a^5 - 12*a^3*b^2))/(b^7*d) + (a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^
2)/(b*d*(a*b^6 + b^7*sin(c + d*x)))
```

3.438 $\int \frac{\cos^5(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=120

$$-\frac{4a(a^2-b^2)\log(a+b\sin(c+dx))}{b^5d}+\frac{(3a^2-2b^2)\sin(c+dx)}{b^4d}-\frac{a\sin^2(c+dx)}{b^3d}+\frac{\sin^3(c+dx)}{3b^2d}-\frac{(a^2-b^2)^2}{b^5d(a+b\sin(c+dx))}$$

```
[Out] -4*a*(a^2-b^2)*ln(a+b*sin(d*x+c))/b^5/d+(3*a^2-2*b^2)*sin(d*x+c)/b^4/d-a*si
n(d*x+c)^2/b^3/d+1/3*sin(d*x+c)^3/b^2/d-(a^2-b^2)^2/b^5/d/(a+b*sin(d*x+c))
```

Rubi [A]
time = 0.07, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2747, 711}

$$-\frac{(a^2-b^2)^2}{b^5d(a+b\sin(c+dx))}-\frac{4a(a^2-b^2)\log(a+b\sin(c+dx))}{b^5d}+\frac{(3a^2-2b^2)\sin(c+dx)}{b^4d}-\frac{a\sin^2(c+dx)}{b^3d}+\frac{\sin^3(c+dx)}{3b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-4*a*(a^2 - b^2)*Log[a + b*Sin[c + d*x]])/(b^5*d) + ((3*a^2 - 2*b^2)*Sin[c
 + d*x])/(b^4*d) - (a*Sin[c + d*x]^2)/(b^3*d) + Sin[c + d*x]^3/(3*b^2*d) -
(a^2 - b^2)^2/(b^5*d*(a + b*Sin[c + d*x]))
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{(b^2-x^2)^2}{(a+x)^2}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$

$$= \frac{\text{Subst}\left(\int \left(3a^2\left(1-\frac{2b^2}{3a^2}\right)-2ax+x^2+\frac{(a^2-b^2)^2}{(a+x)^2}-\frac{4(a^3-ab^2)}{a+x}\right)\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$

$$= -\frac{4a(a^2-b^2)\log(a+b\sin(c+dx))}{b^5 d}+\frac{(3a^2-2b^2)\sin(c+dx)}{b^4 d}-\frac{a\sin^2(c+dx)}{b^3 d}$$

Mathematica [A]
time = 0.81, size = 106, normalized size = 0.88

$$-\frac{-6ab^2\cos(2(c+dx))+48a(a^2-b^2)\log(a+b\sin(c+dx))+3b(-12a^2+7b^2)\sin(c+dx)+\frac{12(a-b)^2(a+b)^2}{a+b\sin(c+dx)}+b^3\sin(3(c+dx))}{12b^5 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -1/12*(-6*a*b^2*Cos[2*(c + d*x)] + 48*a*(a^2 - b^2)*Log[a + b*Sin[c + d*x]]
 + 3*b*(-12*a^2 + 7*b^2)*Sin[c + d*x] + (12*(a - b)^2*(a + b)^2)/(a + b*Sin
[c + d*x]) + b^3*Sin[3*(c + d*x)])/(b^5*d)
```

Maple [A]
time = 0.61, size = 116, normalized size = 0.97

method	result
derivativedivides	$\frac{\frac{\frac{\left(\sin^3(dx+c)\right)b^2}{3}-\left(\sin^2(dx+c)\right)ab+3a^2\sin(dx+c)-2b^2\sin(dx+c)}{b^4}-\frac{a^4-2a^2b^2+b^4}{b^5(a+b\sin(dx+c))}-\frac{4a\left(a^2-b^2\right)\ln(a+b\sin(dx+c))}{b^5}}{d}$
default	$\frac{\frac{\frac{\left(\sin^3(dx+c)\right)b^2}{3}-\left(\sin^2(dx+c)\right)ab+3a^2\sin(dx+c)-2b^2\sin(dx+c)}{b^4}-\frac{a^4-2a^2b^2+b^4}{b^5(a+b\sin(dx+c))}-\frac{4a\left(a^2-b^2\right)\ln(a+b\sin(dx+c))}{b^5}}{d}$
risch	$\frac{3i\,e^{-i(dx+c)}a^2}{2b^4d}+\frac{7i\,e^{i(dx+c)}}{8b^2d}+\frac{a\,e^{2i(dx+c)}}{4b^3d}+\frac{8ia^3c}{b^5d}-\frac{8iac}{b^3d}-\frac{4iax}{b^3}+\frac{4ia^3x}{b^5}+\frac{a\,e^{-2i(dx+c)}}{4b^3d}-\frac{3i\,e^{i(dx+c)}a^2}{2b^4d}-$
norman	$\frac{4\left(36a^2-28b^2\right)\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3b^3d}+\frac{\left(96a^2-80b^2\right)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3b^3d}+\frac{\left(96a^2-80b^2\right)\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3b^3d}+\frac{2\left(4a^2-4b^2\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^3d}+\frac{2(}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/b^4*(1/3*sin(d*x+c)^3*b^2-sin(d*x+c)^2*a*b+3*a^2*sin(d*x+c)-2*b^2*si
n(d*x+c))-1/b^5*(a^4-2*a^2*b^2+b^4)/(a+b*sin(d*x+c))-4*a/b^5*(a^2-b^2)*ln(a
+b*sin(d*x+c)))
```

Maxima [A]
time = 0.30, size = 116, normalized size = 0.97

$$-\frac{\frac{3\,(a^4-2\,a^2b^2+b^4)}{b^6\sin(dx+c)+ab^5}-\frac{b^2\sin(dx+c)^3-3\,ab\sin(dx+c)^2+3\,(3\,a^2-2\,b^2)\sin(dx+c)}{b^4}+\frac{12\,(a^3-ab^2)\log(b\sin(dx+c)+a)}{b^5}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/3*(3*(a^4 - 2*a^2*b^2 + b^4)/(b^6*sin(d*x + c) + a*b^5) - (b^2*sin(d*x +
 c)^3 - 3*a*b*sin(d*x + c)^2 + 3*(3*a^2 - 2*b^2)*sin(d*x + c))/b^4 + 12*(a^
3 - a*b^2)*log(b*sin(d*x + c) + a)/b^5)/d
```

Fricas [A]
time = 0.36, size = 156, normalized size = 1.30

$$\frac{2\,b^4\cos(dx+c)^4-6\,a^4+27\,a^2b^2-16\,b^4-4\,(3\,a^2b^2-2\,b^4)\cos(dx+c)^2-24\,(a^4-a^2b^2+(a^3b-ab^3)\sin(dx+c))\log(b\sin(dx+c)+a)+(4\,ab^3\cos(dx+c)^2+18\,a^3b-13\,ab^3)\sin(dx+c)}{6\,(b^6d\sin(dx+c)+ab^5d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^4*cos(d*x + c)^4 - 6*a^4 + 27*a^2*b^2 - 16*b^4 - 4*(3*a^2*b^2 - 2*
b^4)*cos(d*x + c)^2 - 24*(a^4 - a^2*b^2 + (a^3*b - a*b^3)*sin(d*x + c))*log
(b*sin(d*x + c) + a) + (4*a*b^3*cos(d*x + c)^2 + 18*a^3*b - 13*a*b^3)*sin(d
*x + c))/(b^6*d*sin(d*x + c) + a*b^5*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 3.84, size = 150, normalized size = 1.25

$$-\frac{\frac{12\,(a^3-ab^2)\log(|b\sin(dx+c)+a|)}{b^5}-\frac{b^4\sin(dx+c)^3-3\,ab^3\sin(dx+c)^2+9\,a^2b^2\sin(dx+c)-6\,b^4\sin(dx+c)}{b^6}-\frac{3\,(4\,a^3b\sin(dx+c)-4\,ab^3\sin(dx+c)+3\,a^4-2\,a^2b^2-b^4)}{(b\sin(dx+c)+a)b^5}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/3*(12*(a^3 - a*b^2)*log(abs(b*sin(d*x + c) + a))/b^5 - (b^4*sin(d*x + c)
^3 - 3*a*b^3*sin(d*x + c)^2 + 9*a^2*b^2*sin(d*x + c) - 6*b^4*sin(d*x + c))/
```

```
b^6 - 3*(4*a^3*b*sin(d*x + c) - 4*a*b^3*sin(d*x + c) + 3*a^4 - 2*a^2*b^2 -
b^4)/((b*sin(d*x + c) + a)*b^5))/d
```

Mupad [B]
time = 0.08, size = 118, normalized size = 0.98

$$-\frac{\sin(c+dx)\left(\frac{2}{b^2}-\frac{3a^2}{b^4}\right)-\frac{\sin(c+dx)^3}{3b^2}+\frac{a\sin(c+dx)^2}{b^3}-\frac{\ln(a+b\sin(c+dx))\left(4ab^2-4a^3\right)}{b^5}+\frac{a^4-2a^2b^2+b^4}{b\left(\sin(c+dx)\,b^5+a\,b^4\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(a + b*sin(c + d*x))^2,x)
```

```
[Out] -(sin(c + d*x)*(2/b^2 - (3*a^2)/b^4) - sin(c + d*x)^3/(3*b^2) + (a*sin(c +
d*x)^2)/b^3 - (log(a + b*sin(c + d*x))*(4*a*b^2 - 4*a^3))/b^5 + (a^4 + b^4
- 2*a^2*b^2)/(b*(a*b^4 + b^5*sin(c + d*x))))/d
```

3.439 $\int \frac{\cos^3(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=63

$$\frac{2a\log(a+b\sin(c+dx))}{b^3d}-\frac{\sin(c+dx)}{b^2d}+\frac{a^2-b^2}{b^3d(a+b\sin(c+dx))}$$

```
[Out] 2*a*ln(a+b*sin(d*x+c))/b^3/d-sin(d*x+c)/b^2/d+(a^2-b^2)/b^3/d/(a+b*sin(d*x+
c))
```

Rubi [A]

time = 0.05, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2747, 711}

$$\frac{a^2-b^2}{b^3d(a+b\sin(c+dx))}+\frac{2a\log(a+b\sin(c+dx))}{b^3d}-\frac{\sin(c+dx)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (2*a*Log[a + b*Sin[c + d*x]])/(b^3*d) - Sin[c + d*x]/(b^2*d) + (a^2 - b^2)/
(b^3*d*(a + b*Sin[c + d*x]))
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^3(c+dx)}{(a+b\sin(c+dx))^2}\,dx &= \frac{\text{Subst}\left(\int \frac{b^2-x^2}{(a+x)^2}\,dx,x,b\sin(c+dx)\right)}{b^3d}\\ &= \frac{\text{Subst}\left(\int \left(-1+\frac{-a^2+b^2}{(a+x)^2}+\frac{2a}{a+x}\right)\,dx,x,b\sin(c+dx)\right)}{b^3d}\\ &= \frac{2a\log(a+b\sin(c+dx))}{b^3d}-\frac{\sin(c+dx)}{b^2d}+\frac{a^2-b^2}{b^3d(a+b\sin(c+dx))}\end{aligned}$$

Mathematica [A]
time = 0.16, size = 52, normalized size = 0.83

$$\frac{2a\log(a+b\sin(c+dx))-b\sin(c+dx)+\frac{(a-b)(a+b)}{a+b\sin(c+dx)}}{b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (2*a*Log[a + b*Sin[c + d*x]] - b*Sin[c + d*x] + ((a - b)*(a + b))/(a + b*Si
n[c + d*x]))/(b^3*d)
```

Maple [A]
time = 0.48, size = 60, normalized size = 0.95

method	result
derivativedivides	$\frac{-\frac{\sin(dx+c)}{b^2}-\frac{-a^2+b^2}{b^3(a+b\sin(dx+c))}+\frac{2a\ln(a+b\sin(dx+c))}{b^3}}{d}$
default	$\frac{-\frac{\sin(dx+c)}{b^2}-\frac{-a^2+b^2}{b^3(a+b\sin(dx+c))}+\frac{2a\ln(a+b\sin(dx+c))}{b^3}}{d}$
risch	$-\frac{2iax}{b^3}+\frac{i\mathrm{e}^{i(dx+c)}}{2b^2d}-\frac{i\mathrm{e}^{-i(dx+c)}}{2b^2d}-\frac{4iac}{b^3d}+\frac{2(a^2-b^2)\mathrm{e}^{i(dx+c)}}{d\,b^3\left(-ib\,\mathrm{e}^{2i(dx+c)}+ib+2a\,\mathrm{e}^{i(dx+c)}\right)}+\frac{2a\ln\left(\mathrm{e}^{2i(dx+c)}+\frac{2ia\,\mathrm{e}^{i(dx+c)}}{b}-\right.}{b^3d}$
norman	$\frac{-\frac{4\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{bd}-\frac{4\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{bd}-\frac{8\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{bd}-\frac{2\left(2a^2-b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a\,b^2d}-\frac{6\left(2a^2-b^2\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a\,b^2d}-\frac{6\left(2a^2-b^2\right)\left(\right.}{a}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-sin(d*x+c)/b^2-1/b^3*(-a^2+b^2)/(a+b*sin(d*x+c))+2/b^3*a*ln(a+b*sin(d
*x+c)))
```

Maxima [A]
time = 0.27, size = 61, normalized size = 0.97

$$\frac{\frac{a^2-b^2}{b^4\sin(dx+c)+ab^3}+\frac{2\,a\log(b\sin(dx+c)+a)}{b^3}-\frac{\sin(dx+c)}{b^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] ((a^2 - b^2)/(b^4*sin(d*x + c) + a*b^3) + 2*a*log(b*sin(d*x + c) + a)/b^3 -
 sin(d*x + c)/b^2)/d
```

Fricas [A]
time = 0.35, size = 78, normalized size = 1.24

$$\frac{b^2 \cos(dx+c)^2 - ab\sin(dx+c) + a^2 - 2b^2 + 2(ab\sin(dx+c) + a^2)\log(b\sin(dx+c)+a)}{b^4 d\sin(dx+c) + ab^3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] (b^2*cos(d*x + c)^2 - a*b*sin(d*x + c) + a^2 - 2*b^2 + 2*(a*b*sin(d*x + c)
+ a^2)*log(b*sin(d*x + c) + a))/(b^4*d*sin(d*x + c) + a*b^3*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(53) = 106$.
time = 0.62, size = 221, normalized size = 3.51

$$\begin{cases} \frac{x\cos^3(c)}{a^2} & \text{for } b = 0 \wedge d = 0 \\ \frac{\frac{2\sin^3(c+dx)}{3d} + \frac{\sin(c+dx)\cos^2(c+dx)}{d}}{a^2} & \text{for } b = 0 \\ \frac{x\cos^3(c)}{(a+b\sin(c))^2} & \text{for } d = 0 \\ \frac{2a^2\log\left(\frac{a}{b}+\sin(c+dx)\right)}{ab^3d+b^4d\sin(c+dx)} + \frac{2a^2}{ab^3d+b^4d\sin(c+dx)} + \frac{2ab\log\left(\frac{a}{b}+\sin(c+dx)\right)\sin(c+dx)}{ab^3d+b^4d\sin(c+dx)} - \frac{2b^2\sin^2(c+dx)}{ab^3d+b^4d\sin(c+dx)} - \frac{b^2\cos^2(c+dx)}{ab^3d+b^4d\sin(c+dx)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((x*cos(c)**3/a**2, Eq(b, 0) & Eq(d, 0)), ((2*sin(c + d*x)**3/(3*d
) + sin(c + d*x)*cos(c + d*x)**2/d)/a**2, Eq(b, 0)), (x*cos(c)**3/(a + b*si
n(c))**2, Eq(d, 0)), (2*a**2*log(a/b + sin(c + d*x))/(a*b**3*d + b**4*d*sin
(c + d*x)) + 2*a**2/(a*b**3*d + b**4*d*sin(c + d*x)) + 2*a*b*log(a/b + sin(
c + d*x))*sin(c + d*x)/(a*b**3*d + b**4*d*sin(c + d*x)) - 2*b**2*sin(c + d*
x)**2/(a*b**3*d + b**4*d*sin(c + d*x)) - b**2*cos(c + d*x)**2/(a*b**3*d + b
**4*d*sin(c + d*x)), True))
```

Giac [A]
time = 6.40, size = 91, normalized size = 1.44

$$-\frac{\frac{2a\log\left(\frac{|b\sin(dx+c)+a|}{(b\sin(dx+c)+a)^2|b|}\right)}{b^3} + \frac{b\sin(dx+c)+a}{b^3} - \frac{a^2}{(b\sin(dx+c)+a)b^3} + \frac{1}{(b\sin(dx+c)+a)b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -(2*a*log(abs(b*sin(d*x + c) + a)/((b*sin(d*x + c) + a)^2*abs(b)))/b^3 + (b
*sin(d*x + c) + a)/b^3 - a^2/((b*sin(d*x + c) + a)*b^3) + 1/((b*sin(d*x + c
) + a)*b))/d
```

Mupad [B]
time = 0.08, size = 69, normalized size = 1.10

$$\frac{2\,a\,\ln\left(a+b\,\sin\left(c+d\,x\right)\right)}{b^3\,d}-\frac{\sin\left(c+d\,x\right)}{b^2\,d}+\frac{a^2-b^2}{b\,d\,\left(\sin\left(c+d\,x\right)\,b^3+a\,b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a + b*sin(c + d*x))^2,x)
```

```
[Out] (2*a*log(a + b*sin(c + d*x)))/(b^3*d) - sin(c + d*x)/(b^2*d) + (a^2 - b^2)/
(b*d*(a*b^2 + b^3*sin(c + d*x)))
```

3.440 $\int \frac{\cos(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=20

$$-\frac{1}{bd(a+b\sin(c+dx))}$$

```
[Out] -1/b/d/(a+b*sin(d*x+c))
```

Rubi [A]
time = 0.02, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2747, 32}

$$-\frac{1}{bd(a+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -(1/(b*d*(a + b*Sin[c + d*x])))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)}{(a+b\sin(c+dx))^2}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{(a+x)^2}\,dx, x, b\sin(c+dx)\right)}{bd}\\ &= -\frac{1}{bd(a+b\sin(c+dx))}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 20, normalized size = 1.00

$$-\frac{1}{bd(a+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -(1/(b*d*(a + b*Sin[c + d*x])))
```

Maple [A]
time = 0.24, size = 21, normalized size = 1.05

method	result	size
derivativedivides	$-\frac{1}{bd(a+b\sin(dx+c))}$	21
default	$-\frac{1}{bd(a+b\sin(dx+c))}$	21
risch	$-\frac{2ie^{i(dx+c)}}{bd\left(b\,\mathrm{e}^{2i(dx+c)}-b+2ia\,\mathrm{e}^{i(dx+c)}\right)}$	49
norman	$\frac{\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{da}+\frac{2\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(a\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a\right)}$	83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b/d/(a+b*sin(d*x+c))
```

Maxima [A]
time = 0.26, size = 20, normalized size = 1.00

$$-\frac{1}{(b\sin(dx+c)+a)bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/((b*sin(d*x + c) + a)*b*d)
```

Fricas [A]
time = 0.33, size = 20, normalized size = 1.00

$$-\frac{1}{b^2d\sin(dx+c)+abd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/(b^2*d*sin(d*x + c) + a*b*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(15) = 30$.

time = 0.50, size = 51, normalized size = 2.55

$$\begin{cases} \frac{x\cos(c)}{a^2} & \text{for } b = 0 \wedge d = 0 \\ \frac{\sin(c+dx)}{a^2d} & \text{for } b = 0 \\ \frac{x\cos(c)}{(a+b\sin(c))^2} & \text{for } d = 0 \\ -\frac{1}{abd+b^2d\sin(c+dx)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((x*cos(c)/a**2, Eq(b, 0) & Eq(d, 0)), (sin(c + d*x)/(a**2*d), Eq(
b, 0)), (x*cos(c)/(a + b*sin(c))**2, Eq(d, 0)), (-1/(a*b*d + b**2*d*sin(c +
 d*x)), True))
```

Giac [A]

time = 5.82, size = 20, normalized size = 1.00

$$-\frac{1}{(b\sin(dx+c)+a)bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/((b*sin(d*x + c) + a)*b*d)
```

Mupad [B]

time = 5.07, size = 20, normalized size = 1.00

$$-\frac{1}{b\,d\,\left(a+b\,\sin\left(c+d\,x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + b*sin(c + d*x))^2,x)
```

```
[Out] -1/(b*d*(a + b*sin(c + d*x)))
```

3.441 $\int \frac{\sec(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=104

$$-\frac{\log(1-\sin(c+dx))}{2(a+b)^2d}+\frac{\log(1+\sin(c+dx))}{2(a-b)^2d}-\frac{2ab\log(a+b\sin(c+dx))}{(a^2-b^2)^2\,d}+\frac{b}{(a^2-b^2)\,d(a+b\sin(c+dx))}$$

```
[Out] -1/2*ln(1-sin(d*x+c))/(a+b)^2/d+1/2*ln(1+sin(d*x+c))/(a-b)^2/d-2*a*b*ln(a+b
*sin(d*x+c))/(a^2-b^2)^2/d+b/(a^2-b^2)/d/(a+b*sin(d*x+c))
```

Rubi [A]
time = 0.08, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2747, 724, 815}

$$\frac{b}{d(a^2-b^2)(a+b\sin(c+dx))}-\frac{2ab\log(a+b\sin(c+dx))}{d(a^2-b^2)^2}-\frac{\log(1-\sin(c+dx))}{2d(a+b)^2}+\frac{\log(\sin(c+dx)+1)}{2d(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -1/2*Log[1 - Sin[c + d*x]]/((a + b)^2*d) + Log[1 + Sin[c + d*x]]/(2*(a - b)
^2*d) - (2*a*b*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)^2*d) + b/((a^2 - b^2)*
d*(a + b*Sin[c + d*x]))
```

Rule 724

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Simp[e*((d
 + e*x)^(m + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + Dist[c/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*((d - e*x)/(a + c*x^2)), x], x] /; FreeQ[{a, c, d, e, m
}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{b\text{Subst}\left(\int \frac{1}{(a+x)^2(b^2-x^2)}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{b}{(a^2-b^2)\,d(a+b\sin(c+dx))} + \frac{b\text{Subst}\left(\int \frac{a-x}{(a+x)(b^2-x^2)}\,dx, x, b\sin(c+dx)\right)}{(a^2-b^2)\,d}$$
$$= \frac{b}{(a^2-b^2)\,d(a+b\sin(c+dx))} + \frac{b\text{Subst}\left(\int \left(\frac{a-b}{2b(a+b)(b-x)} - \frac{2a}{(a-b)(a+b)(a+x)} + \frac{}{2(a-}\right.\right.}{(a^2-b^2)\,d}$$
$$= -\frac{\log(1-\sin(c+dx))}{2(a+b)^2d} + \frac{\log(1+\sin(c+dx))}{2(a-b)^2d} - \frac{2ab\log(a+b\sin(c+dx))}{(a^2-b^2)^2\,d} +$$

Mathematica [A]
time = 0.24, size = 102, normalized size = 0.98

$$\frac{b\left(-\frac{\log(1-\sin(c+dx))}{2b(a+b)^2} + \frac{\log(1+\sin(c+dx))}{2(a-b)^2b} - \frac{2a\log(a+b\sin(c+dx))}{(a-b)^2(a+b)^2} + \frac{1}{(a^2-b^2)(a+b\sin(c+dx))}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (b*(-1/2*Log[1 - Sin[c + d*x]]/(b*(a + b)^2) + Log[1 + Sin[c + d*x]]/(2*(a
- b)^2*b) - (2*a*Log[a + b*Sin[c + d*x]])/((a - b)^2*(a + b)^2) + 1/((a^2 -
 b^2)*(a + b*Sin[c + d*x]))))/d
```

Maple [A]
time = 0.51, size = 93, normalized size = 0.89

method	result
derivativedivides	$\frac{\frac{\ln(1+\sin(dx+c))}{2(a-b)^2} - \frac{\ln(\sin(dx+c)-1)}{2(a+b)^2} + \frac{b}{(a-b)(a+b)(a+b\sin(dx+c))} - \frac{2ab\ln(a+b\sin(dx+c))}{(a+b)^2(a-b)^2}}{d}$
default	$\frac{\frac{\ln(1+\sin(dx+c))}{2(a-b)^2} - \frac{\ln(\sin(dx+c)-1)}{2(a+b)^2} + \frac{b}{(a-b)(a+b)(a+b\sin(dx+c))} - \frac{2ab\ln(a+b\sin(dx+c))}{(a+b)^2(a-b)^2}}{d}$
norman	$-\frac{2b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{da(a^2-b^2)\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a\right)} + \frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d(a^2-2ab+b^2)} - \frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{(a^2+2ab+b^2)d} - \frac{2ab\ln\left(a\left(\right.\right.}{}$
risch	$\frac{ix}{a^2+2ab+b^2} + \frac{ic}{(a^2+2ab+b^2)d} - \frac{ix}{a^2-2ab+b^2} - \frac{ic}{d(a^2-2ab+b^2)} + \frac{4iabx}{a^4-2a^2b^2+b^4} + \frac{4iabc}{d(a^4-2a^2b^2+b^4)} - \frac{}{d(-a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2/(a-b)^2*ln(1+sin(d*x+c))-1/2/(a+b)^2*ln(sin(d*x+c)-1)+b/(a-b)/(a+b
)/(a+b*sin(d*x+c))-2*a*b/(a+b)^2/(a-b)^2*ln(a+b*sin(d*x+c)))
```

Maxima [A]
time = 0.28, size = 118, normalized size = 1.13

$$-\frac{\frac{4\,ab\log\left(b\sin\left(dx+c\right)+a\right)}{a^4-2\,a^2b^2+b^4}-\frac{2\,b}{a^3-ab^2+\left(a^2b-b^3\right)\sin\left(dx+c\right)}-\frac{\log\left(\sin\left(dx+c\right)+1\right)}{a^2-2\,ab+b^2}+\frac{\log\left(\sin\left(dx+c\right)-1\right)}{a^2+2\,ab+b^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/2*(4*a*b*log(b*sin(d*x + c) + a)/(a^4 - 2*a^2*b^2 + b^4) - 2*b/(a^3 - a*
b^2 + (a^2*b - b^3)*sin(d*x + c)) - log(sin(d*x + c) + 1)/(a^2 - 2*a*b + b^
2) + log(sin(d*x + c) - 1)/(a^2 + 2*a*b + b^2))/d
```

Fricas [A]
time = 0.38, size = 188, normalized size = 1.81

$$\frac{2\,a^2b-2\,b^3-4\,\left(ab^2\sin\left(dx+c\right)+a^2b\right)\log\left(b\sin\left(dx+c\right)+a\right)+\left(a^3+2\,a^2b+ab^2+\left(a^2b+2\,ab^2+b^3\right)\sin\left(dx+c\right)\right)\log\left(\sin\left(dx+c\right)+1\right)-\left(a^3-2\,a^2b+ab^2+\left(a^2b-2\,ab^2+b^3\right)\sin\left(dx+c\right)\right)\log\left(-\sin\left(dx+c\right)+1\right)}{2\,\left(\left(a^4b-2\,a^2b^3+b^5\right)d\sin\left(dx+c\right)+\left(a^5-2\,a^3b^2+ab^4\right)d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a^2*b - 2*b^3 - 4*(a*b^2*sin(d*x + c) + a^2*b)*log(b*sin(d*x + c) +
a) + (a^3 + 2*a^2*b + a*b^2 + (a^2*b + 2*a*b^2 + b^3)*sin(d*x + c))*log(sin
(d*x + c) + 1) - (a^3 - 2*a^2*b + a*b^2 + (a^2*b - 2*a*b^2 + b^3)*sin(d*x +
 c))*log(-sin(d*x + c) + 1))/((a^4*b - 2*a^2*b^3 + b^5)*d*sin(d*x + c) + (a
^5 - 2*a^3*b^2 + a*b^4)*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(c+dx\right)}{\left(a+b\sin\left(c+dx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)/(a + b*sin(c + d*x))**2, x)
```

Giac [A]
time = 5.73, size = 147, normalized size = 1.41

$$-\frac{\frac{4\,ab^2\log\left(\left|b\sin\left(dx+c\right)+a\right|\right)}{a^4b-2\,a^2b^3+b^5}-\frac{\log\left(\left|\sin\left(dx+c\right)+1\right|\right)}{a^2-2\,ab+b^2}+\frac{\log\left(\left|\sin\left(dx+c\right)-1\right|\right)}{a^2+2\,ab+b^2}-\frac{2\,\left(2\,ab^2\sin\left(dx+c\right)+3\,a^2b-b^3\right)}{\left(a^4-2\,a^2b^2+b^4\right)\left(b\sin\left(dx+c\right)+a\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*(4*a*b^2*log(abs(b*sin(d*x + c) + a))/(a^4*b - 2*a^2*b^3 + b^5) - log(
abs(sin(d*x + c) + 1))/(a^2 - 2*a*b + b^2) + log(abs(sin(d*x + c) - 1))/(a^
2 + 2*a*b + b^2) - 2*(2*a*b^2*sin(d*x + c) + 3*a^2*b - b^3)/((a^4 - 2*a^2*b
^2 + b^4)*(b*sin(d*x + c) + a)))/d
```

Mupad [B]
time = 0.21, size = 98, normalized size = 0.94

$$\frac{\ln\left(\sin\left(c+d\,x\right)+1\right)}{2\,d\,{\left(a-b\right)}^2}-\frac{\ln\left(\sin\left(c+d\,x\right)-1\right)}{2\,d\,{\left(a+b\right)}^2}+\frac{b}{d\,\left(a^2-b^2\right)\,\left(a+b\,\sin\left(c+d\,x\right)\right)}-\frac{2\,a\,b\,\ln\left(a+b\,\sin\left(c+d\,x\right)\right)}{d\,{\left(a^2-b^2\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + b*sin(c + d*x))^2),x)
```

```
[Out] log(sin(c + d*x) + 1)/(2*d*(a - b)^2) - log(sin(c + d*x) - 1)/(2*d*(a + b)^
2) + b/(d*(a^2 - b^2)*(a + b*sin(c + d*x))) - (2*a*b*log(a + b*sin(c + d*x)
))/(d*(a^2 - b^2)^2)
```

3.442 $\int \frac{\sec^3(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=177

$$-\frac{(a+3b)\log(1-\sin(c+dx))}{4(a+b)^3d}+\frac{(a-3b)\log(1+\sin(c+dx))}{4(a-b)^3d}+\frac{4ab^3\log(a+b\sin(c+dx))}{(a^2-b^2)^3\,d}-\frac{b(a^2 -}{2\,(a^2-b^2)^2\,d(a}$$

```
[Out] -1/4*(a+3*b)*ln(1-sin(d*x+c))/(a+b)^3/d+1/4*(a-3*b)*ln(1+sin(d*x+c))/(a-b)^
3/d+4*a*b^3*ln(a+b*sin(d*x+c))/(a^2-b^2)^3/d-1/2*b*(a^2+3*b^2)/(a^2-b^2)^2/
d/(a+b*sin(d*x+c))-1/2*sec(d*x+c)^2*(b-a*sin(d*x+c))/(a^2-b^2)/d/(a+b*sin(d
*x+c))
```

Rubi [A]
time = 0.15, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2747, 755, 815}

$$-\frac{b(a^2+3b^2)}{2d\,(a^2-b^2)^2\,(a+b\sin(c+dx))}-\frac{\sec^2(c+dx)(b-a\sin(c+dx))}{2d\,(a^2-b^2)\,(a+b\sin(c+dx))}+\frac{4ab^3\log(a+b\sin(c+dx))}{d\,(a^2-b^2)^3}-\frac{(a+3b)\log(1-\sin(c+dx))}{4d(a+b)^3}+\frac{(a-3b)\log(\sin(c+dx)+1)}{4d(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -1/4*((a + 3*b)*Log[1 - Sin[c + d*x]])/((a + b)^3*d) + ((a - 3*b)*Log[1 + S
in[c + d*x]])/(4*(a - b)^3*d) + (4*a*b^3*Log[a + b*Sin[c + d*x]])/((a^2 - b
^2)^3*d) - (b*(a^2 + 3*b^2))/(2*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x])) - (Se
c[c + d*x]^2*(b - a*Sin[c + d*x]))/(2*(a^2 - b^2)*d*(a + b*Sin[c + d*x]))
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
```

```
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{b^3\text{Subst}\left(\int \frac{1}{(a+x)^2(b^2-x^2)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= -\frac{\sec^2(c+dx)(b-a\sin(c+dx))}{2(a^2-b^2)\,d(a+b\sin(c+dx))} + \frac{b\text{Subst}\left(\int \frac{a^2-3b^2+2ax}{(a+x)^2(b^2-x^2)}\,dx, x, b\sin(c+dx)\right)}{2(a^2-b^2)\,d}$$
$$= -\frac{\sec^2(c+dx)(b-a\sin(c+dx))}{2(a^2-b^2)\,d(a+b\sin(c+dx))} + \frac{b\text{Subst}\left(\int \left(\frac{(a-b)(a+3b)}{2b(a+b)^2(b-x)} + \frac{a^2+3b^2}{(a-b)(a+b)(a+x)^2} - \right.\right.}{2}$$
$$= -\frac{(a+3b)\log(1-\sin(c+dx))}{4(a+b)^3d} + \frac{(a-3b)\log(1+\sin(c+dx))}{4(a-b)^3d} + \frac{4ab^3\log(a+}{(a^2-}$$

Mathematica [A]

time = 1.87, size = 222, normalized size = 1.25

$$\frac{\frac{a((a-b)\log(1-\sin(c+dx))-(a+b)\log(1+\sin(c+dx))+2b\log(a+b\sin(c+dx)))}{(a-b)(a+b)} + \frac{\sec^2(c+dx)(b-a\sin(c+dx))}{a+b\sin(c+dx)} - b(-a^2-3b^2)\left(-\frac{\log(1-\sin(c+dx))}{2b(a+b)^2} + \frac{\log(1+\sin(c+dx))}{2(a-b)^2b} - \frac{2a\log(a+b\sin(c+dx))}{(a-b)^2(a+b)^2} + \frac{1}{(a^2-b^2)(a+b\sin(c+dx))}\right)}{2(-a^2+b^2)\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((a*((a - b)*Log[1 - Sin[c + d*x]] - (a + b)*Log[1 + Sin[c + d*x]] + 2*b*Lo
g[a + b*Sin[c + d*x]]))/((a - b)*(a + b)) + (Sec[c + d*x]^2*(b - a*Sin[c +
d*x]))/(a + b*Sin[c + d*x]) - b*(-a^2 - 3*b^2)*(-1/2*Log[1 - Sin[c + d*x]]/
(b*(a + b)^2) + Log[1 + Sin[c + d*x]]/(2*(a - b)^2*b) - (2*a*Log[a + b*Sin[
c + d*x]])/((a - b)^2*(a + b)^2) + 1/((a^2 - b^2)*(a + b*Sin[c + d*x]))))/(
2*(-a^2 + b^2)*d)
```

Maple [A]

time = 0.82, size = 146, normalized size = 0.82

method	result
derivativedivides	$\frac{-\frac{1}{4(a-b)^2(1+\sin(dx+c))}+\frac{(a-3b)\ln(1+\sin(dx+c))}{4(a-b)^3}-\frac{1}{4(a+b)^2(\sin(dx+c)-1)}+\frac{(-a-3b)\ln(\sin(dx+c)-1)}{4(a+b)^3}-\frac{b^3}{(a+b)^2(a-b)^2(a+b\sin(d}}{d}$
default	$\frac{-\frac{1}{4(a-b)^2(1+\sin(dx+c))}+\frac{(a-3b)\ln(1+\sin(dx+c))}{4(a-b)^3}-\frac{1}{4(a+b)^2(\sin(dx+c)-1)}+\frac{(-a-3b)\ln(\sin(dx+c)-1)}{4(a+b)^3}-\frac{b^3}{(a+b)^2(a-b)^2(a+b\sin(d}}{d}$

norman	$\frac{\frac{4a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(a^2-b^2\right)d}+\frac{-a^4-a^2b^2-2b^4}{2bd\left(a^4-2a^2b^2+b^4\right)}-\frac{\left(-a^4+3a^2b^2-6b^4\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2db\left(a^4-2a^2b^2+b^4\right)}-\frac{\left(-a^4+3a^2b^2-6b^4\right)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2db\left(a^4-2a^2b^2+b^4\right)}+\frac{\left(-a^4-a^2b^2\right.}{2bd\left(\right.}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a\right)}$
risch	$-\frac{8iab^3x}{a^6-3a^4b^2+3a^2b^4-b^6}+\frac{iac}{2(a^3+3a^2b+3ab^2+b^3)d}+\frac{3ibx}{2(a^3+3a^2b+3ab^2+b^3)}+\frac{3ibc}{2d(a^3-3a^2b+3ab^2-b^3)}-\frac{}{d(a^6-3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/4/(a-b)^2/(1+sin(d*x+c))+1/4*(a-3*b)/(a-b)^3*ln(1+sin(d*x+c))-1/4/(
a+b)^2/(sin(d*x+c)-1)+1/4/(a+b)^3*(-a-3*b)*ln(sin(d*x+c)-1)-b^3/(a+b)^2/(a-
b)^2/(a+b*sin(d*x+c))+4*b^3*a/(a+b)^3/(a-b)^3*ln(a+b*sin(d*x+c)))
```

Maxima [A]
time = 0.29, size = 275, normalized size = 1.55

$$\frac{\frac{16\,ab^3\log\left(b\sin\left(dx+c\right)+a\right)}{a^6-3\,a^4b^2+3\,a^2b^4-b^6}+\frac{(a-3\,b)\log\left(\sin\left(dx+c\right)+1\right)}{a^3-3\,a^2b+3\,ab^2-b^3}-\frac{(a+3\,b)\log\left(\sin\left(dx+c\right)-1\right)}{a^3+3\,a^2b+3\,ab^2+b^3}-\frac{2\left(2\,a^2b+2\,b^3-\left(a^2b+3\,b^3\right)\sin\left(dx+c\right)^2-\left(a^3-ab^2\right)\sin\left(dx+c\right)\right)}{a^5-2\,a^3b^2+ab^4-\left(a^4b-2\,a^2b^3+b^5\right)\sin\left(dx+c\right)^3-\left(a^5-2\,a^3b^2+ab^4\right)\sin\left(dx+c\right)^2+\left(a^4b-2\,a^2b^3+b^5\right)\sin\left(dx+c\right)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/4*(16*a*b^3*log(b*sin(d*x + c) + a)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6) +
 (a - 3*b)*log(sin(d*x + c) + 1)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) - (a + 3*b
)*log(sin(d*x + c) - 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - 2*(2*a^2*b + 2*b^
3 - (a^2*b + 3*b^3)*sin(d*x + c)^2 - (a^3 - a*b^2)*sin(d*x + c))/(a^5 - 2*a
^3*b^2 + a*b^4 - (a^4*b - 2*a^2*b^3 + b^5)*sin(d*x + c)^3 - (a^5 - 2*a^3*b^
2 + a*b^4)*sin(d*x + c)^2 + (a^4*b - 2*a^2*b^3 + b^5)*sin(d*x + c)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 381 vs. 2(170) = 340.
time = 0.42, size = 381, normalized size = 2.15

$$-\frac{2\,a^4b-4\,a^2b^3+2\,b^5+2\left(a^4b+2\,a^2b^3-3\,b^5\right)\cos\left(dx+c\right)^2-16\left(ab^4\cos\left(dx+c\right)^2\sin\left(dx+c\right)+a^2b^3\cos\left(dx+c\right)^2\right)\log\left(b\sin\left(dx+c\right)+a\right)-\left(\left(a^4b-6\,a^2b^3-8\,ab^4-3\,b^5\right)\cos\left(dx+c\right)^2\sin\left(dx+c\right)+\left(a^5-6\,a^3b^2-8\,a^2b^3-3\,ab^4\right)\cos\left(dx+c\right)^2\right)\log\left(\sin\left(dx+c\right)+1\right)+\left(\left(a^4b-6\,a^2b^3+8\,ab^4-3\,b^5\right)\cos\left(dx+c\right)^2\sin\left(dx+c\right)+\left(a^5-6\,a^3b^2+8\,a^2b^3-3\,ab^4\right)\cos\left(dx+c\right)^2\right)\log\left(-\sin\left(dx+c\right)+1\right)-2\left(a^5-2\,a^3b^2+ab^4\right)\sin\left(dx+c\right)}{4\left(\left(a^6b-3\,a^4b^3+3\,a^2b^5-b^7\right)d\cos\left(dx+c\right)^2\sin\left(dx+c\right)+\left(a^7-3\,a^5b^2+3\,a^3b^4-ab^6\right)d\cos\left(dx+c\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/4*(2*a^4*b - 4*a^2*b^3 + 2*b^5 + 2*(a^4*b + 2*a^2*b^3 - 3*b^5)*cos(d*x +
 c)^2 - 16*(a*b^4*cos(d*x + c)^2*sin(d*x + c) + a^2*b^3*cos(d*x + c)^2)*log
(b*sin(d*x + c) + a) - ((a^4*b - 6*a^2*b^3 - 8*a*b^4 - 3*b^5)*cos(d*x + c)^
2*sin(d*x + c) + (a^5 - 6*a^3*b^2 - 8*a^2*b^3 - 3*a*b^4)*cos(d*x + c)^2)*lo
g(sin(d*x + c) + 1) + ((a^4*b - 6*a^2*b^3 + 8*a*b^4 - 3*b^5)*cos(d*x + c)^2
*sin(d*x + c) + (a^5 - 6*a^3*b^2 + 8*a^2*b^3 - 3*a*b^4)*cos(d*x + c)^2)*log
(-sin(d*x + c) + 1) - 2*(a^5 - 2*a^3*b^2 + a*b^4)*sin(d*x + c))/((a^6*b - 3
```

```
*a^4*b^3 + 3*a^2*b^5 - b^7)*d*cos(d*x + c)^2*sin(d*x + c) + (a^7 - 3*a^5*b^
2 + 3*a^3*b^4 - a*b^6)*d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**3/(a + b*sin(c + d*x))**2, x)
```

Giac [A]
time = 5.55, size = 244, normalized size = 1.38

$$\frac{\frac{16\,ab^4\log(|b\sin(dx+c)+a|)}{a^6b-3\,a^4b^3+3\,a^2b^5-b^7}+\frac{(a-3\,b)\log(|\sin(dx+c)+1|)}{a^3-3\,a^2b+3\,ab^2-b^3}-\frac{(a+3\,b)\log(|\sin(dx+c)-1|)}{a^3+3\,a^2b+3\,ab^2+b^3}-\frac{2\left(a^2b\sin(dx+c)^2+3\,b^3\sin(dx+c)^2+a^3\sin(dx+c)-ab^2\sin(dx+c)-2\,a^2b-2\,b^3\right)}{(a^4-2\,a^2b^2+b^4)\left(b\sin(dx+c)^3+a\sin(dx+c)^2-b\sin(dx+c)-a\right)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/4*(16*a*b^4*log(abs(b*sin(d*x + c) + a))/(a^6*b - 3*a^4*b^3 + 3*a^2*b^5 -
 b^7) + (a - 3*b)*log(abs(sin(d*x + c) + 1))/(a^3 - 3*a^2*b + 3*a*b^2 - b^3
) - (a + 3*b)*log(abs(sin(d*x + c) - 1))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) -
2*(a^2*b*sin(d*x + c)^2 + 3*b^3*sin(d*x + c)^2 + a^3*sin(d*x + c) - a*b^2*s
in(d*x + c) - 2*a^2*b - 2*b^3)/((a^4 - 2*a^2*b^2 + b^4)*(b*sin(d*x + c)^3 +
 a*sin(d*x + c)^2 - b*sin(d*x + c) - a)))/d
```

Mupad [B]
time = 5.47, size = 227, normalized size = 1.28

$$\frac{\frac{\sin(c+d\,x)^2\,(a^2\,b+3\,b^3)}{2\,(a^4-2\,a^2\,b^2+b^4)}-\frac{a^2\,b+b^3}{(a^2-b^2)^2}+\frac{a\,\sin(c+d\,x)}{2\,(a^2-b^2)}}{d\,\left(-b\sin\left(c+d\,x\right)^3-a\sin\left(c+d\,x\right)^2+b\,\sin\left(c+d\,x\right)+a\right)}-\frac{\ln\left(\sin\left(c+d\,x\right)-1\right)\,\left(\frac{b}{2\,(a+b)^3}+\frac{1}{4\,(a+b)^2}\right)}{d}+\frac{\ln\left(\sin\left(c+d\,x\right)+1\right)\,\left(a-3\,b\right)}{4\,d\,{\left(a-b\right)}^3}+\frac{4\,a\,b^3\,\ln\left(a+b\,\sin\left(c+d\,x\right)\right)}{d\,\left(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + b*sin(c + d*x))^2),x)
```

```
[Out] ((sin(c + d*x)^2*(a^2*b + 3*b^3))/(2*(a^4 + b^4 - 2*a^2*b^2)) - (a^2*b + b^
3)/(a^2 - b^2)^2 + (a*sin(c + d*x))/(2*(a^2 - b^2)))/(d*(a + b*sin(c + d*x)
 - a*sin(c + d*x)^2 - b*sin(c + d*x)^3)) - (log(sin(c + d*x) - 1)*(b/(2*(a
+ b)^3) + 1/(4*(a + b)^2)))/d + (log(sin(c + d*x) + 1)*(a - 3*b))/(4*d*(a -
 b)^3) + (4*a*b^3*log(a + b*sin(c + d*x)))/(d*(a^6 - b^6 + 3*a^2*b^4 - 3*a^
4*b^2))
```

3.443 $\int \frac{\sec^5(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=269

$$-\frac{3(a^2+4ab+5b^2)\log(1-\sin(c+dx))}{16(a+b)^4d}+\frac{3(a^2-4ab+5b^2)\log(1+\sin(c+dx))}{16(a-b)^4d}-\frac{6ab^5\log(a+b\sin(c+dx)}{(a^2-b^2)^4\,d}$$

```
[Out] -3/16*(a^2+4*a*b+5*b^2)*ln(1-sin(d*x+c))/(a+b)^4/d+3/16*(a^2-4*a*b+5*b^2)*l
n(1+sin(d*x+c))/(a-b)^4/d-6*a*b^5*ln(a+b*sin(d*x+c))/(a^2-b^2)^4/d-3/8*b*(a
^4-4*a^2*b^2-5*b^4)/(a^2-b^2)^3/d/(a+b*sin(d*x+c))-1/4*sec(d*x+c)^4*(b-a*si
n(d*x+c))/(a^2-b^2)/d/(a+b*sin(d*x+c))+1/8*sec(d*x+c)^2*(b*(a^2+5*b^2)+3*a*
(a^2-3*b^2)*sin(d*x+c))/(a^2-b^2)^2/d/(a+b*sin(d*x+c))
```

Rubi [A]
time = 0.22, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2747, 755, 837, 815}

$$-\frac{3(a^2+4ab+5b^2)\log(1-\sin(c+dx))}{16d(a+b)^4}+\frac{3(a^2-4ab+5b^2)\log(\sin(c+dx)+1)}{16d(a-b)^4}-\frac{\sec^4(c+dx)(b-a\sin(c+dx))}{4d(a^2-b^2)(a+b\sin(c+dx))}+\frac{\sec^2(c+dx)\left(3a(a^2-3b^2)\sin(c+dx)+b(a^2+5b^2)\right)}{8d(a^2-b^2)^2(a+b\sin(c+dx))}-\frac{6ab^5\log(a+b\sin(c+dx))}{d(a^2-b^2)^4}-\frac{3b(a^4-4a^2b^2-5b^4)}{8d(a^2-b^2)^3(a+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-3*(a^2 + 4*a*b + 5*b^2)*Log[1 - Sin[c + d*x]])/(16*(a + b)^4*d) + (3*(a^2
 - 4*a*b + 5*b^2)*Log[1 + Sin[c + d*x]])/(16*(a - b)^4*d) - (6*a*b^5*Log[a
+ b*Sin[c + d*x]])/((a^2 - b^2)^4*d) - (3*b*(a^4 - 4*a^2*b^2 - 5*b^4))/(8*(
a^2 - b^2)^3*d*(a + b*Sin[c + d*x])) - (Sec[c + d*x]^4*(b - a*Sin[c + d*x])
)/(4*(a^2 - b^2)*d*(a + b*Sin[c + d*x])) + (Sec[c + d*x]^2*(b*(a^2 + 5*b^2)
 + 3*a*(a^2 - 3*b^2)*Sin[c + d*x]))/(8*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x])
)
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{b^5\text{Subst}\left(\int \frac{1}{(a+x)^2(b^2-x^2)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= -\frac{\sec^4(c+dx)(b-a\sin(c+dx))}{4\left(a^2-b^2\right)d(a+b\sin(c+dx))} + \frac{b^3\text{Subst}\left(\int \frac{3a^2-5b^2+4ax}{(a+x)^2(b^2-x^2)^2}\,dx, x, b\sin(c+dx\right.}{4\left(a^2-b^2\right)d}$$

$$= -\frac{\sec^4(c+dx)(b-a\sin(c+dx))}{4\left(a^2-b^2\right)d(a+b\sin(c+dx))} + \frac{\sec^2(c+dx)\left(b(a^2+5b^2)+3a(a^2-3b^2)\text{si}\right.}{8\left(a^2-b^2\right)^2d(a+b\sin(c+dx))}$$

$$= -\frac{\sec^4(c+dx)(b-a\sin(c+dx))}{4\left(a^2-b^2\right)d(a+b\sin(c+dx))} + \frac{\sec^2(c+dx)\left(b(a^2+5b^2)+3a(a^2-3b^2)\text{si}\right.}{8\left(a^2-b^2\right)^2d(a+b\sin(c+dx))}$$

$$= -\frac{3(a^2+4ab+5b^2)\log(1-\sin(c+dx))}{16(a+b)^4d} + \frac{3(a^2-4ab+5b^2)\log(1+\sin(c+a}{16(a-b)^4d}$$

Mathematica [A]
time = 6.12, size = 406, normalized size = 1.51

$$\frac{b^5\left(\frac{\sec^4(c+dx)\left(b^2-ab\sin(c+dx)\right)}{4b^6(-a^2+b^2)(a+b\sin(c+dx))}-\frac{-\frac{\sec^2(c+dx)\left(4a^2b^2-b^2\left(3a^2-5b^2\right)-b\left(4ab^2-a\left(3a^2-5b^2\right)\right)\sin(c+dx)\right)}{2b^4\left(-a^2+b^2\right)(a+b\sin(c+dx))}+\frac{-6a\left(a^2-3b^2\right)\left(-\frac{\log(1-\sin(c+dx))}{2b(a+b)}+\frac{\log(1+\sin(c+dx))}{2(a-b)b}-\frac{\log(a+b\sin(c+dx))}{a^2-b^2}\right)+\left(6a^2\left(a^2-3b^2\right)-3\left(a^4-2a^2b^2+5b^4\right)\right)\left(-\frac{\log(1-\sin(c+dx))}{2b(a+b)^2}+\frac{\log(1+\sin(c+dx))}{2(a-b)^2b}-\frac{2a\log(a+b\sin(c+dx))}{(a-b)^2(a+b)^2}+\frac{1}{\left(a^2-b^2\right)(a+b\sin(c+dx))}\right)}{2b^2\left(-a^2+b^2\right)}}{4b^2(-a^2+b^2)}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (b^5*((Sec[c + d*x]^4*(b^2 - a*b*Sin[c + d*x]))/(4*b^6*(-a^2 + b^2)*(a + b*
Sin[c + d*x])) - (-1/2*(Sec[c + d*x]^2*(4*a^2*b^2 - b^2*(3*a^2 - 5*b^2) - b
*(4*a*b^2 - a*(3*a^2 - 5*b^2))*Sin[c + d*x]))/(b^4*(-a^2 + b^2)*(a + b*Sin[
c + d*x])) + (-6*a*(a^2 - 3*b^2)*(-1/2*Log[1 - Sin[c + d*x]]/(b*(a + b)) +
Log[1 + Sin[c + d*x]]/(2*(a - b)*b) - Log[a + b*Sin[c + d*x]]/(a^2 - b^2))
+ (6*a^2*(a^2 - 3*b^2) - 3*(a^4 - 2*a^2*b^2 + 5*b^4))*(-1/2*Log[1 - Sin[c +
 d*x]]/(b*(a + b)^2) + Log[1 + Sin[c + d*x]]/(2*(a - b)^2*b) - (2*a*Log[a +
 b*Sin[c + d*x]])/((a - b)^2*(a + b)^2) + 1/((a^2 - b^2)*(a + b*Sin[c + d*x
]))))/(2*b^2*(-a^2 + b^2)))/(4*b^2*(-a^2 + b^2))))/d
```

Maple [A]
time = 1.16, size = 213, normalized size = 0.79

method	result
derivativedivides	$\frac{-\frac{1}{16(a-b)^2(1+\sin(dx+c))^2}-\frac{-7b+3a}{16(a-b)^3(1+\sin(dx+c))}+\frac{\left(3a^2-12ab+15b^2\right)\ln(1+\sin(dx+c))}{16(a-b)^4}+\frac{1}{16(a+b)^2(\sin(dx+c)-1)^2}-\frac{7b\cdots}{16(a+b)^3(\mathrm{si}\cdots}}{d}$
default	$\frac{-\frac{1}{16(a-b)^2(1+\sin(dx+c))^2}-\frac{-7b+3a}{16(a-b)^3(1+\sin(dx+c))}+\frac{\left(3a^2-12ab+15b^2\right)\ln(1+\sin(dx+c))}{16(a-b)^4}+\frac{1}{16(a+b)^2(\sin(dx+c)-1)^2}-\frac{7b\cdots}{16(a+b)^3(\mathrm{si}\cdots}}{d}$
norman	$-\frac{3\left(a^2b-3b^3\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d\left(a^4-2a^2b^2+b^4\right)}-\frac{3\left(a^2b-3b^3\right)\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d\left(a^4-2a^2b^2+b^4\right)}+\frac{\left(-5a^2b-b^3\right)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d\left(a^4-2a^2b^2+b^4\right)}+\frac{\left(-5a^2b-b^3\right)\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d\left(a^4-2a^2b^2+b^4\right)}+\frac{(5a\cdots}{\cdots}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/16/(a-b)^2/(1+sin(d*x+c))^2-1/16*(-7*b+3*a)/(a-b)^3/(1+sin(d*x+c))+
1/16/(a-b)^4*(3*a^2-12*a*b+15*b^2)*ln(1+sin(d*x+c))+1/16/(a+b)^2/(sin(d*x+c
)-1)^2-1/16*(7*b+3*a)/(a+b)^3/(sin(d*x+c)-1)+1/16/(a+b)^4*(-3*a^2-12*a*b-15
*b^2)*ln(sin(d*x+c)-1)+b^5/(a+b)^3/(a-b)^3/(a+b*sin(d*x+c))-6*b^5*a/(a+b)^4
/(a-b)^4*ln(a+b*sin(d*x+c)))
```

Maxima [A]
time = 0.30, size = 505, normalized size = 1.88

$$-\frac{\frac{96\,ab^5\log(b\sin(dx+c)+a)}{a^8-4\,a^6b^2+6\,a^4b^4-4\,a^2b^6+b^8}-\frac{3\left(a^2-4\,ab+5\,b^2\right)\log(\sin(dx+c)+1)}{a^4-4\,a^3b+6\,a^2b^2-4\,ab^3+b^4}+\frac{3\left(a^2+4\,ab+5\,b^2\right)\log(\sin(dx+c)-1)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4}+\frac{2\left(4\,a^4b-20\,a^2b^3-8\,b^5+3\left(a^4b-4\,a^2b^3-5\,b^5\right)\sin(dx+c)^4+3\left(a^5-4\,a^3b^2+3\,ab^4\right)\sin(dx+c)^3-\left(5\,a^4b-28\,a^2b^3-25\,b^5\right)\sin(dx+c)^2-\left(5\,a^5-16\,a^3b^2+11\,ab^4\right)\sin(dx+c)\right)}{a^7-3\,a^5b^2+3\,a^3b^4-ab^6+\left(a^6b-3\,a^4b^3+3\,a^2b^5-b^7\right)\sin(dx+c)^5+\left(a^7-3\,a^5b^2+3\,a^3b^4-ab^6\right)\sin(dx+c)^4-2\left(a^6b-3\,a^4b^3+3\,a^2b^5-b^7\right)\sin(dx+c)^3-2\left(a^7-3\,a^5b^2+3\,a^3b^4-ab^6\right)\sin(dx+c)^2+\left(a^6b-3\,a^4b^3+3\,a^2b^5-b^7\right)\sin(dx+c)}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/16*(96*a*b^5*log(b*sin(d*x + c) + a)/(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^
2*b^6 + b^8) - 3*(a^2 - 4*a*b + 5*b^2)*log(sin(d*x + c) + 1)/(a^4 - 4*a^3*b
 + 6*a^2*b^2 - 4*a*b^3 + b^4) + 3*(a^2 + 4*a*b + 5*b^2)*log(sin(d*x + c) -
1)/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) + 2*(4*a^4*b - 20*a^2*b^3 -
8*b^5 + 3*(a^4*b - 4*a^2*b^3 - 5*b^5)*sin(d*x + c)^4 + 3*(a^5 - 4*a^3*b^2 +
```

```
3*a*b^4)*sin(d*x + c)^3 - (5*a^4*b - 28*a^2*b^3 - 25*b^5)*sin(d*x + c)^2 -
(5*a^5 - 16*a^3*b^2 + 11*a*b^4)*sin(d*x + c))/(a^7 - 3*a^5*b^2 + 3*a^3*b^4
- a*b^6 + (a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*sin(d*x + c)^5 + (a^7 - 3*
a^5*b^2 + 3*a^3*b^4 - a*b^6)*sin(d*x + c)^4 - 2*(a^6*b - 3*a^4*b^3 + 3*a^2*
b^5 - b^7)*sin(d*x + c)^3 - 2*(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*sin(d*x
+ c)^2 + (a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*sin(d*x + c)))/d
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 527 vs. $2(260) = 520$.
time = 0.52, size = 527, normalized size = 1.96

$$-\frac{4\,a^{6}b-12\,a^{4}b^{3}+12\,a^{2}b^{5}-4\,b^{7}+6\,\left(a^{6}b-5\,a^{4}b^{3}-a^{2}b^{5}+5\,b^{7}\right)\cos\left(dx+c\right)^{4}-2\,\left(a^{6}b+3\,a^{4}b^{3}-9\,a^{2}b^{5}+5\,b^{7}\right)\cos\left(dx+c\right)^{2}+96\,\left(ab^{6}\cos\left(dx+c\right)^{4}\sin\left(dx+c\right)+a^{2}b^{5}\cos\left(dx+c\right)^{4}\right)\log\left(b\sin\left(dx+c\right)+a\right)-3\,\left(\left(a^{6}b-5\,a^{4}b^{3}+15\,a^{2}b^{5}+16\,ab^{6}+5\,b^{7}\right)\cos\left(dx+c\right)^{4}\sin\left(dx+c\right)+\left(a^{7}-5\,a^{5}b^{2}+15\,a^{3}b^{4}+16\,a^{2}b^{5}+5\,ab^{6}\right)\cos\left(dx+c\right)^{4}\right)\log\left(\sin\left(dx+c\right)+1\right)+3\,\left(\left(a^{6}b-5\,a^{4}b^{3}+15\,a^{2}b^{5}-16\,ab^{6}+5\,b^{7}\right)\cos\left(dx+c\right)^{4}\sin\left(dx+c\right)+\left(a^{7}-5\,a^{5}b^{2}+15\,a^{3}b^{4}-16\,a^{2}b^{5}+5\,ab^{6}\right)\cos\left(dx+c\right)^{4}\right)\log\left(-\sin\left(dx+c\right)+1\right)-2\,\left(2\,a^{7}-6\,a^{5}b^{2}+6\,a^{3}b^{4}-2\,ab^{6}+3\,\left(a^{7}-5\,a^{5}b^{2}+7\,a^{3}b^{4}-3\,ab^{6}\right)\cos\left(dx+c\right)^{2}\right)\sin\left(dx+c\right)}{16\,\left(\left(a^{8}b-4\,a^{6}b^{3}+6\,a^{4}b^{5}-4\,a^{2}b^{7}+b^{9}\right)d\cos\left(dx+c\right)^{4}\sin\left(dx+c\right)+\left(a^{9}-4\,a^{7}b^{2}+6\,a^{5}b^{4}-4\,a^{3}b^{6}+ab^{8}\right)d\cos\left(dx+c\right)^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/16*(4*a^6*b - 12*a^4*b^3 + 12*a^2*b^5 - 4*b^7 + 6*(a^6*b - 5*a^4*b^3 - a
^2*b^5 + 5*b^7)*cos(d*x + c)^4 - 2*(a^6*b + 3*a^4*b^3 - 9*a^2*b^5 + 5*b^7)*
cos(d*x + c)^2 + 96*(a*b^6*cos(d*x + c)^4*sin(d*x + c) + a^2*b^5*cos(d*x +
c)^4)*log(b*sin(d*x + c) + a) - 3*((a^6*b - 5*a^4*b^3 + 15*a^2*b^5 + 16*a*b
^6 + 5*b^7)*cos(d*x + c)^4*sin(d*x + c) + (a^7 - 5*a^5*b^2 + 15*a^3*b^4 + 1
6*a^2*b^5 + 5*a*b^6)*cos(d*x + c)^4)*log(sin(d*x + c) + 1) + 3*((a^6*b - 5*
a^4*b^3 + 15*a^2*b^5 - 16*a*b^6 + 5*b^7)*cos(d*x + c)^4*sin(d*x + c) + (a^7
 - 5*a^5*b^2 + 15*a^3*b^4 - 16*a^2*b^5 + 5*a*b^6)*cos(d*x + c)^4)*log(-sin(
d*x + c) + 1) - 2*(2*a^7 - 6*a^5*b^2 + 6*a^3*b^4 - 2*a*b^6 + 3*(a^7 - 5*a^5
*b^2 + 7*a^3*b^4 - 3*a*b^6)*cos(d*x + c)^2)*sin(d*x + c))/((a^8*b - 4*a^6*b
^3 + 6*a^4*b^5 - 4*a^2*b^7 + b^9)*d*cos(d*x + c)^4*sin(d*x + c) + (a^9 - 4*
a^7*b^2 + 6*a^5*b^4 - 4*a^3*b^6 + a*b^8)*d*cos(d*x + c)^4)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{5}\left(c+dx\right)}{\left(a+b\sin\left(c+dx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**5/(a + b*sin(c + d*x))**2, x)
```

Giac [**A**]
time = 5.02, size = 460, normalized size = 1.71

$$-\frac{\frac{96\,ab^{6}\log\left(|b\sin(dx+c)+a|\right)}{a^{8}b-4\,a^{6}b^{3}+6\,a^{4}b^{5}-4\,a^{2}b^{7}+b^{9}}-\frac{3\,\left(a^{2}-4\,ab+5\,b^{2}\right)\log\left(|\sin(dx+c)+1|\right)}{a^{4}-4\,a^{3}b+6\,a^{2}b^{2}-4\,ab^{3}+b^{4}}+\frac{3\,\left(a^{2}+4\,ab+5\,b^{2}\right)\log\left(|\sin(dx+c)-1|\right)}{a^{4}+4\,a^{3}b+6\,a^{2}b^{2}+4\,ab^{3}+b^{4}}-\frac{16\,\left(6\,ab^{6}\sin(dx+c)+7\,a^{2}b^{5}-b^{7}\right)}{\left(a^{8}-4\,a^{6}b^{2}+6\,a^{4}b^{4}-4\,a^{2}b^{6}+b^{8}\right)\left(b\sin(dx+c)+a\right)}+\frac{2\,\left(36\,ab^{5}\sin(dx+c)^{4}+3\,a^{6}\sin(dx+c)^{3}-15\,a^{4}b^{2}\sin(dx+c)^{3}+5\,a^{2}b^{4}\sin(dx+c)^{3}+7\,b^{6}\sin(dx+c)^{3}+16\,a^{3}b^{3}\sin(dx+c)^{2}-88\,ab^{5}\sin(dx+c)^{2}-5\,a^{6}\sin(dx+c)+17\,a^{4}b^{2}\sin(dx+c)-3\,a^{2}b^{4}\sin(dx+c)-9\,b^{6}\sin(dx+c)+4\,a^{5}b-24\,a^{3}b^{3}+56\,ab^{5}\right)}{\left(a^{8}-4\,a^{6}b^{2}+6\,a^{4}b^{4}-4\,a^{2}b^{6}+b^{8}\right)\left(\sin(dx+c)^{2}-1\right)^{2}}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/16*(96*a*b^6*log(abs(b*sin(d*x + c) + a))/(a^8*b - 4*a^6*b^3 + 6*a^4*b^5
 - 4*a^2*b^7 + b^9) - 3*(a^2 - 4*a*b + 5*b^2)*log(abs(sin(d*x + c) + 1))/(a
^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4) + 3*(a^2 + 4*a*b + 5*b^2)*log(abs
(sin(d*x + c) - 1))/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) - 16*(6*a*b
^6*sin(d*x + c) + 7*a^2*b^5 - b^7)/((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^
6 + b^8)*(b*sin(d*x + c) + a)) + 2*(36*a*b^5*sin(d*x + c)^4 + 3*a^6*sin(d*x
 + c)^3 - 15*a^4*b^2*sin(d*x + c)^3 + 5*a^2*b^4*sin(d*x + c)^3 + 7*b^6*sin(
d*x + c)^3 + 16*a^3*b^3*sin(d*x + c)^2 - 88*a*b^5*sin(d*x + c)^2 - 5*a^6*si
n(d*x + c) + 17*a^4*b^2*sin(d*x + c) - 3*a^2*b^4*sin(d*x + c) - 9*b^6*sin(d
*x + c) + 4*a^5*b - 24*a^3*b^3 + 56*a*b^5)/((a^8 - 4*a^6*b^2 + 6*a^4*b^4 -
4*a^2*b^6 + b^8)*(sin(d*x + c)^2 - 1)^2))/d
```

Mupad [B]
time = 5.94, size = 449, normalized size = 1.67

$$\frac{\ln\left(\sin\left(c+d\,x\right)+1\right)\,\left(\frac{3\,b^2}{8\,{\left(a-b\right)}^4}-\frac{3\,b}{8\,{\left(a-b\right)}^3}+\frac{3}{16\,{\left(a-b\right)}^2}\right)}{d}-\frac{\ln\left(\sin\left(c+d\,x\right)-1\right)\,\left(\frac{3\,b}{8\,{\left(a+b\right)}^3}+\frac{3}{16\,{\left(a+b\right)}^2}+\frac{3\,b^2}{8\,{\left(a+b\right)}^4}\right)}{d}+\frac{\frac{-a^4\,b+5\,a^2\,b^3+2\,b^5}{2\,\left(a^2-b^2\right)\,\left(a^4-2\,a^2\,b^2+b^4\right)}+\frac{3\,{\sin\left(c+d\,x\right)}^3\,\left(3\,a\,b^2-a^3\right)}{8\,\left(a^4-2\,a^2\,b^2+b^4\right)}+\frac{3\,{\sin\left(c+d\,x\right)}^4\,\left(-a^4\,b+4\,a^2\,b^3+5\,b^5\right)}{8\,\left(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6\right)}-\frac{\sin\left(c+d\,x\right)\,\left(11\,a\,b^2-5\,a^3\right)}{8\,\left(a^4-2\,a^2\,b^2+b^4\right)}-\frac{{\sin\left(c+d\,x\right)}^2\,\left(-5\,a^4\,b+28\,a^2\,b^3+25\,b^5\right)}{8\,\left(a^2-b^2\right)\,\left(a^4-2\,a^2\,b^2+b^4\right)}}{d\,\left(b\,{\sin\left(c+d\,x\right)}^5+a\,{\sin\left(c+d\,x\right)}^4-2\,b\,{\sin\left(c+d\,x\right)}^3-2\,a\,{\sin\left(c+d\,x\right)}^2+b\,\sin\left(c+d\,x\right)+a\right)}-\frac{6\,a\,b^5\,\ln\left(a+b\,\sin\left(c+d\,x\right)\right)}{d\,\left(a^8-4\,a^6\,b^2+6\,a^4\,b^4-4\,a^2\,b^6+b^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^5*(a + b*sin(c + d*x))^2),x)
```

```
[Out] (log(sin(c + d*x) + 1)*((3*b^2)/(8*(a - b)^4) - (3*b)/(8*(a - b)^3) + 3/(16
*(a - b)^2)))/d - (log(sin(c + d*x) - 1)*((3*b)/(8*(a + b)^3) + 3/(16*(a +
b)^2) + (3*b^2)/(8*(a + b)^4)))/d + ((2*b^5 - a^4*b + 5*a^2*b^3)/(2*(a^2 -
b^2)*(a^4 + b^4 - 2*a^2*b^2)) + (3*sin(c + d*x)^3*(3*a*b^2 - a^3))/(8*(a^4
+ b^4 - 2*a^2*b^2)) + (3*sin(c + d*x)^4*(5*b^5 - a^4*b + 4*a^2*b^3))/(8*(a^
6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) - (sin(c + d*x)*(11*a*b^2 - 5*a^3))/(8*(a
^4 + b^4 - 2*a^2*b^2)) - (sin(c + d*x)^2*(25*b^5 - 5*a^4*b + 28*a^2*b^3))/(
8*(a^2 - b^2)*(a^4 + b^4 - 2*a^2*b^2)))/(d*(a + b*sin(c + d*x) - 2*a*sin(c
+ d*x)^2 + a*sin(c + d*x)^4 - 2*b*sin(c + d*x)^3 + b*sin(c + d*x)^5)) - (6*
a*b^5*log(a + b*sin(c + d*x)))/(d*(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^
6*b^2))
```

3.444 $\int \frac{\cos^6(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=187

$$-\frac{5(8a^4-12a^2b^2+3b^4)\,x}{8b^6}+\frac{10a(a^2-b^2)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^6d}+\frac{5\cos^3(c+dx)(4a-3b\sin(c+dx))}{12b^3d}-$$

```
[Out] -5/8*(8*a^4-12*a^2*b^2+3*b^4)*x/b^6+10*a*(a^2-b^2)^(3/2)*arctan((b+a*tan(1/
2*d*x+1/2*c))/(a^2-b^2)^(1/2))/b^6/d+5/12*cos(d*x+c)^3*(4*a-3*b*sin(d*x+c))
/b^3/d-cos(d*x+c)^5/b/d/(a+b*sin(d*x+c))-5/8*cos(d*x+c)*(8*a*(a^2-b^2)-b*(4
*a^2-3*b^2)*sin(d*x+c))/b^5/d
```

Rubi [A]
time = 0.25, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2772, 2944, 2814, 2739, 632, 210}

$$\frac{10a(a^2-b^2)^{3/2}\,\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^6d}-\frac{5\cos(c+dx)\left(8a(a^2-b^2)-b(4a^2-3b^2)\sin(c+dx)\right)}{8b^5d}-\frac{5x(8a^4-12a^2b^2+3b^4)}{8b^6}+\frac{5\cos^3(c+dx)(4a-3b\sin(c+dx))}{12b^3d}-\frac{\cos^5(c+dx)}{bd(a+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-5*(8*a^4 - 12*a^2*b^2 + 3*b^4)*x)/(8*b^6) + (10*a*(a^2 - b^2)^(3/2)*ArcTa
n[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^6*d) + (5*Cos[c + d*x]^3*(4
*a - 3*b*Sin[c + d*x]))/(12*b^3*d) - Cos[c + d*x]^5/(b*d*(a + b*Sin[c + d*x
])) - (5*Cos[c + d*x]*(8*a*(a^2 - b^2) - b*(4*a^2 - 3*b^2)*Sin[c + d*x]))/(
8*b^5*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2772

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Dist[g^2*((p - 1)/(b*(m + 1))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2944

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Dist[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\cos^6(c+dx)}{(a+b\sin(c+dx))^2}\,dx = -\frac{\cos^5(c+dx)}{bd(a+b\sin(c+dx))} - \frac{5\int \frac{\cos^4(c+dx)\sin(c+dx)}{a+b\sin(c+dx)}\,dx}{b}$$

$$= \frac{5\cos^3(c+dx)(4a-3b\sin(c+dx))}{12b^3d} - \frac{\cos^5(c+dx)}{bd(a+b\sin(c+dx))} - \frac{5\int \frac{\cos^2(c+dx)(-a}{a+}$$

$$= \frac{5\cos^3(c+dx)(4a-3b\sin(c+dx))}{12b^3d} - \frac{\cos^5(c+dx)}{bd(a+b\sin(c+dx))} - \frac{5\cos(c+dx)\,(8}{}$$

$$= -\frac{5(8a^4-12a^2b^2+3b^4)\,x}{8b^6} + \frac{5\cos^3(c+dx)(4a-3b\sin(c+dx))}{12b^3d} - \frac{\cos^5(c}{bd(a+b\sin}$$

$$= -\frac{5(8a^4-12a^2b^2+3b^4)\,x}{8b^6} + \frac{5\cos^3(c+dx)(4a-3b\sin(c+dx))}{12b^3d} - \frac{\cos^5(c}{bd(a+b\sin}$$

$$= -\frac{5(8a^4-12a^2b^2+3b^4)\,x}{8b^6} + \frac{5\cos^3(c+dx)(4a-3b\sin(c+dx))}{12b^3d} - \frac{\cos^5(c}{bd(a+b\sin}$$

$$= -\frac{5(8a^4-12a^2b^2+3b^4)\,x}{8b^6} + \frac{10a(a^2-b^2)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^6d} + \frac{5\cos^3(}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 3679 vs. $2(187) = 374$.

time = 6.50, size = 3679, normalized size = 19.67

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (Cos[c + d*x]^5*(-((b*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b))^(7/2)*(b/(a
 + b) - (b*Sin[c + d*x])/(a + b))^(7/2))/(((a*b)/(a - b) - b^2/(a - b))*((a
*b)/(a + b) + b^2/(a + b))*(a + b*Sin[c + d*x]))) - ((48*Sqrt[2]*(a - b)*b^
3*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b))^(7/2)*Sqrt[b/(a + b) - (b*Sin[c
 + d*x])/(a + b)]*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/
(2*b))^(7/2)*((7*(3/(16*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a -
 b)))/(2*b))^3) + 1/(2*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a -
b)))/(2*b))^2) + (1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(
2*b))^(-1)))/12 + (35*b^4*(((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b
)))/b - ((a - b)^2*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b))^2)/(3*b^2) + (
2*(a - b)^3*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b))^3)/(15*b^3) - (Sqrt[2
]*Sqrt[a - b]*ArcSinh[(Sqrt[a - b]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a
```

- b)])/(Sqrt[2]*Sqrt[b])]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)])/(S
qrt[b]*Sqrt[1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b)])
))/(128*(a - b)^4*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b))^4*(1 + ((a - b)
*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^3)))/(7*(a + b)^2*(a^2 -
b^2)*Sqrt[((a + b)*(b/(a + b) - (b*Sin[c + d*x])/(a + b)))/b]) + (5*a*b^2*
((8*Sqrt[2]*b*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b))^(5/2)*Sqrt[b/(a + b
) - (b*Sin[c + d*x])/(a + b)]*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x]
)/(a - b)))/(2*b))^(7/2)*((5/(16*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d
*x])/(a - b)))/(2*b))^3) + 5/(8*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*
x])/(a - b)))/(2*b))^2) + (1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a
- b)))/(2*b))^(-1))/2 - (15*b^3*(((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])
/(a - b)))/b - ((a - b)^2*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b))^2)/(3*b
^2) - (Sqrt[2]*Sqrt[a - b]*ArcSinh[(Sqrt[a - b]*Sqrt[-(b/(a - b)) - (b*Sin[
c + d*x])/(a - b)])/(Sqrt[2]*Sqrt[b])]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])
/(a - b)])/(Sqrt[b]*Sqrt[1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a -
b)))/(2*b)])))/(64*(a - b)^3*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b))^3*(
1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^3)))/(5*(a +
b)^2*Sqrt[((a + b)*(b/(a + b) - (b*Sin[c + d*x])/(a + b)))/b]) - ((-((a*b)
/(a - b)) + b^2/(a - b))*((8*Sqrt[2]*b*(-(b/(a - b)) - (b*Sin[c + d*x])/(a
- b))^(3/2)*Sqrt[b/(a + b) - (b*Sin[c + d*x])/(a + b)]*(1 + ((a - b)*(-(b/(
a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^(7/2)*((3*(5/(8*(1 + ((a - b)*(
-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^3) + 5/(6*(1 + ((a - b)*(-
(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^2) + (1 + ((a - b)*(-(b/(a
- b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^(-1)))/8 + (15*b^2*(((a - b)*(-(b
/(a - b)) - (b*Sin[c + d*x])/(a - b)))/b - (Sqrt[2]*Sqrt[a - b]*ArcSinh[(Sq
rt[a - b]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)])/(Sqrt[2]*Sqrt[b])]
*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)])/(Sqrt[b]*Sqrt[1 + ((a - b)*
(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b)])))/(64*(a - b)^2*(-(b/(a
- b)) - (b*Sin[c + d*x])/(a - b))^2*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c
+ d*x])/(a - b)))/(2*b))^3)))/(3*(a + b)^2*Sqrt[((a + b)*(b/(a + b) - (b*Si
n[c + d*x])/(a + b)))/b]) - ((-((a*b)/(a - b)) + b^2/(a - b))*((8*Sqrt[2]*b
*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)]*Sqrt[b/(a + b) - (b*Sin[c +
d*x])/(a + b)]*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*
b))^(7/2)*((5*Sqrt[b]*ArcSinh[(Sqrt[a - b]*Sqrt[-(b/(a - b)) - (b*Sin[c + d
*x])/(a - b)])/(Sqrt[2]*Sqrt[b])])/(8*Sqrt[2]*Sqrt[a - b]*Sqrt[-(b/(a - b))
- (b*Sin[c + d*x])/(a - b)]*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])
/(a - b)))/(2*b))^(7/2)) + (15/(8*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c +
d*x])/(a - b)))/(2*b))^3) + 5/(4*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d
*x])/(a - b)))/(2*b))^2) + (1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(
a - b)))/(2*b))^(-1))/6)))/((a + b)^2*Sqrt[((a + b)*(b/(a + b) - (b*Sin[c +
d*x])/(a + b)))/b]) - ((-((a*b)/(a - b)) + b^2/(a - b))*(-(((-((a*b)/(a + b
)) - b^2/(a + b))*(-(((-((a*b)/(a + b)) - b^2/(a + b))*((2*Sqrt[a - b]*ArcT
anh[(Sqrt[a - b]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)])/(Sqrt[a + b
]*Sqrt[b/(a + b) - (b*Sin[c + d*x])/(a + b)])])/(b*Sqrt[a + b]) - (2*Sqrt[-
((a*b)/(a + b)) - b^2/(a + b)]*ArcTanh[(Sqrt[-((a*b)/(a + b)) - b^2/(a + b)

```
]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)])/(Sqrt[-((a*b)/(a - b)) + b
^2/(a - b)]*Sqrt[b/(a + b) - (b*Sin[c + d*x])/(a + b)])])/(b*Sqrt[-((a*b)/(
a - b)) + b^2/(a - b)]))))/b) + (2*Sqrt[2]*(a - b)*Sqrt[-(b/(a - b)) - (b*Si
n[c + d*x])/(a - b)]*Sqrt[b/(a + b) - (b*Sin[c + d*x])/(a + b)]*(1 + ((a -
b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^(3/2)*((Sqrt[b]*ArcSin
h[(Sqrt[a - b]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)])/(Sqrt[2]*Sqrt
[b])])/(Sqrt[2]*Sqrt[a - b]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)]*(
1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^(3/2)) + 1/(
2*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b)))))/(b*(a
+ b)*Sqrt[((a + b)*(b/(a + b) - (b*Sin[c + d*x]...
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 394 vs. $2(176) = 352$.

time = 0.82, size = 395, normalized size = 2.11

method	result
derivativedivides	$-\frac{2\left(\frac{\left(\frac{3}{2}a^2b^2-\frac{9}{8}b^4\right)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(4b\,a^3-6a\,b^3\right)\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{3}{2}a^2b^2-\frac{1}{8}b^4\right)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(12b\,a^3-14a\,b^3\right)\left(\tan^4\left(\frac{dx}{2}\right.\right.}{\left(1+\tan^2\left(\frac{d}{}\right.\right.}\right.}{}$
default	$-\frac{2\left(\frac{\left(\frac{3}{2}a^2b^2-\frac{9}{8}b^4\right)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(4b\,a^3-6a\,b^3\right)\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{3}{2}a^2b^2-\frac{1}{8}b^4\right)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(12b\,a^3-14a\,b^3\right)\left(\tan^4\left(\frac{dx}{2}\right.\right.}{\left(1+\tan^2\left(\frac{d}{}\right.\right.}\right.}{}$
risch	$-\frac{5x\,a^4}{b^6}+\frac{15x\,a^2}{2b^4}-\frac{15x}{8b^2}-\frac{i\mathrm{e}^{-2i(dx+c)}}{4b^2d}+\frac{3i\mathrm{e}^{-2i(dx+c)}a^2}{8b^4d}-\frac{2a^3\mathrm{e}^{i(dx+c)}}{b^5d}+\frac{9a\,\mathrm{e}^{i(dx+c)}}{4b^3d}-\frac{2a^3\mathrm{e}^{-i(dx+c)}}{b^5d}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2/b^6*(((3/2*a^2*b^2-9/8*b^4)*tan(1/2*d*x+1/2*c)^7+(4*a^3*b-6*a*b^3)*
tan(1/2*d*x+1/2*c)^6+(3/2*a^2*b^2-1/8*b^4)*tan(1/2*d*x+1/2*c)^5+(12*a^3*b-1
4*a*b^3)*tan(1/2*d*x+1/2*c)^4+(-3/2*a^2*b^2+1/8*b^4)*tan(1/2*d*x+1/2*c)^3+(
12*b*a^3-38/3*a*b^3)*tan(1/2*d*x+1/2*c)^2+(-3/2*a^2*b^2+9/8*b^4)*tan(1/2*d*
x+1/2*c)+4*b*a^3-14/3*a*b^3)/(1+tan(1/2*d*x+1/2*c)^2)^4+5/8*(8*a^4-12*a^2*b
^2+3*b^4)*arctan(tan(1/2*d*x+1/2*c)))+2/b^6*((-(a^4-2*a^2*b^2+b^4)*b^2/a*ta
n(1/2*d*x+1/2*c)-b*(a^4-2*a^2*b^2+b^4))/(a*tan(1/2*d*x+1/2*c)^2+2*b*tan(1/2
*d*x+1/2*c)+a)+5*a*(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(
1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))))
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.42, size = 599, normalized size = 3.20

$$\left[\frac{6\,b^{5}\cos\left(dx+c\right)^{5}-5\,\left(4\,a^{2}b^{3}-3\,b^{5}\right)\cos\left(dx+c\right)^{3}-15\,\left(8\,a^{5}-12\,a^{3}b^{2}+3\,ab^{4}\right)dx-60\,\left(a^{4}-a^{2}b^{2}+\left(a^{3}b-ab^{3}\right)\sin\left(dx+c\right)\right)\sqrt{-a^{2}+b^{2}}\log\left(\frac{\left(2\,a^{2}-b^{2}\right)\cos\left(dx+c\right)^{2}-2\,ab\sin\left(dx+c\right)-a^{2}-b^{2}+2\,\left(a\cos\left(dx+c\right)\sin\left(dx+c\right)+b\cos\left(dx+c\right)\right)\sqrt{-a^{2}+b^{2}}}{b^{2}\cos\left(dx+c\right)^{2}-2\,ab\sin\left(dx+c\right)-a^{2}-b^{2}}\right)-15\,\left(8\,a^{4}b-12\,a^{2}b^{3}+3\,b^{5}\right)\cos\left(dx+c\right)+5\,\left(2\,ab^{4}\cos\left(dx+c\right)^{3}-3\,\left(8\,a^{4}b-12\,a^{2}b^{3}+3\,b^{5}\right)dx-3\,\left(4\,a^{3}b^{2}-5\,ab^{4}\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{24\,\left(b^{7}d\sin\left(dx+c\right)+ab^{6}d\right)}, \frac{6\,b^{5}\cos\left(dx+c\right)^{5}-5\,\left(4\,a^{2}b^{3}-3\,b^{5}\right)\cos\left(dx+c\right)^{3}-15\,\left(8\,a^{5}-12\,a^{3}b^{2}+3\,ab^{4}\right)dx-120\,\left(a^{4}-a^{2}b^{2}+\left(a^{3}b-ab^{3}\right)\sin\left(dx+c\right)\right)\sqrt{a^{2}-b^{2}}\arctan\left(-\frac{a\sin\left(dx+c\right)+b}{\sqrt{a^{2}-b^{2}}\cos\left(dx+c\right)}\right)-15\,\left(8\,a^{4}b-12\,a^{2}b^{3}+3\,b^{5}\right)\cos\left(dx+c\right)+5\,\left(2\,ab^{4}\cos\left(dx+c\right)^{3}-3\,\left(8\,a^{4}b-12\,a^{2}b^{3}+3\,b^{5}\right)dx-3\,\left(4\,a^{3}b^{2}-5\,ab^{4}\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{24\,\left(b^{7}d\sin\left(dx+c\right)+ab^{6}d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/24*(6*b^5*cos(d*x + c)^5 - 5*(4*a^2*b^3 - 3*b^5)*cos(d*x + c)^3 - 15*(8*
a^5 - 12*a^3*b^2 + 3*a*b^4)*d*x - 60*(a^4 - a^2*b^2 + (a^3*b - a*b^3)*sin(d
*x + c))*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x
 + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-
a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 15*(8*
a^4*b - 12*a^2*b^3 + 3*b^5)*cos(d*x + c) + 5*(2*a*b^4*cos(d*x + c)^3 - 3*(8
*a^4*b - 12*a^2*b^3 + 3*b^5)*d*x - 3*(4*a^3*b^2 - 5*a*b^4)*cos(d*x + c))*si
n(d*x + c))/(b^7*d*sin(d*x + c) + a*b^6*d), 1/24*(6*b^5*cos(d*x + c)^5 - 5*
(4*a^2*b^3 - 3*b^5)*cos(d*x + c)^3 - 15*(8*a^5 - 12*a^3*b^2 + 3*a*b^4)*d*x
- 120*(a^4 - a^2*b^2 + (a^3*b - a*b^3)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan
(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - 15*(8*a^4*b - 12*a
^2*b^3 + 3*b^5)*cos(d*x + c) + 5*(2*a*b^4*cos(d*x + c)^3 - 3*(8*a^4*b - 12*
a^2*b^3 + 3*b^5)*d*x - 3*(4*a^3*b^2 - 5*a*b^4)*cos(d*x + c))*sin(d*x + c))/
(b^7*d*sin(d*x + c) + a*b^6*d)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 469 vs. 2(175) = 350.
time = 4.61, size = 469, normalized size = 2.51

$$-\frac{\frac{15\,\left(8\,a^{4}-12\,a^{2}b^{2}+3\,b^{4}\right)\left(dx+c\right)}{b^{6}}-\frac{240\,\left(a^{5}-2\,a^{3}b^{2}+ab^{4}\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(a\right)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^{2}-b^{2}}}\right)\right)}{\sqrt{a^{2}-b^{2}}\,b^{6}}+\frac{48\,\left(a^{4}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,a^{2}b^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b^{5}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^{5}-2\,a^{3}b^{2}+ab^{4}\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a\right)ab^{5}}+\frac{2\,\left(36\,a^{2}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-27\,b^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+96\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-144\,ab^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+36\,a^{2}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-3\,b^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+288\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-336\,ab^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-36\,a^{2}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+3\,b^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+288\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-304\,ab^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-36\,a^{2}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+27\,b^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+96\,a^{3}-112\,ab^{2}\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{4}b^{5}}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^6/(a+b*sin(d*x+c))^2,x, algorithm="giac")

[Out] -1/24*(15*(8*a^4 - 12*a^2*b^2 + 3*b^4)*(d*x + c)/b^6 - 240*(a^5 - 2*a^3*b^2 + a*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*b^6) + 48*(a^4*b*tan(1/2*d*x + 1/2*c) - 2*a^2*b^3*tan(1/2*d*x + 1/2*c) + b^5*tan(1/2*d*x + 1/2*c) + a^5 - 2*a^3*b^2 + a*b^4)/((a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)*a*b^5) + 2*(36*a^2*b*tan(1/2*d*x + 1/2*c)^7 - 27*b^3*tan(1/2*d*x + 1/2*c)^7 + 96*a^3*tan(1/2*d*x + 1/2*c)^6 - 144*a*b^2*tan(1/2*d*x + 1/2*c)^6 + 36*a^2*b*tan(1/2*d*x + 1/2*c)^5 - 3*b^3*tan(1/2*d*x + 1/2*c)^5 + 288*a^3*tan(1/2*d*x + 1/2*c)^4 - 336*a*b^2*tan(1/2*d*x + 1/2*c)^4 - 36*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 3*b^3*tan(1/2*d*x + 1/2*c)^3 + 288*a^3*tan(1/2*d*x + 1/2*c)^2 - 304*a*b^2*tan(1/2*d*x + 1/2*c)^2 - 36*a^2*b*tan(1/2*d*x + 1/2*c) + 27*b^3*tan(1/2*d*x + 1/2*c) + 96*a^3 - 112*a*b^2)/((tan(1/2*d*x + 1/2*c)^2 + 1)^4*b^5))/d

Mupad [B]
time = 7.58, size = 2530, normalized size = 13.53

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^6/(a + b*sin(c + d*x))^2,x)

[Out] - ((2*(15*a^4 + 3*b^4 - 20*a^2*b^2))/(3*b^5) - (5*tan(c/2 + (d*x)/2)^8*(b^4 - 4*a^4 + 4*a^2*b^2))/(2*b^5) + (5*tan(c/2 + (d*x)/2)^6*(16*a^4 + 3*b^4 - 20*a^2*b^2))/(2*b^5) + (5*tan(c/2 + (d*x)/2)^2*(48*a^4 + 15*b^4 - 68*a^2*b^2))/(6*b^5) + (5*tan(c/2 + (d*x)/2)^4*(72*a^4 + 15*b^4 - 100*a^2*b^2))/(6*b^5) + (tan(c/2 + (d*x)/2)*(180*a^4 + 24*b^4 - 245*a^2*b^2))/(12*a*b^4) + (4*tan(c/2 + (d*x)/2)^5*(15*a^4 + 3*b^4 - 20*a^2*b^2))/(a*b^4) + (tan(c/2 + (d*x)/2)^9*(20*a^4 + 8*b^4 - 25*a^2*b^2))/(4*a*b^4) + (tan(c/2 + (d*x)/2)^7*(60*a^4 + 16*b^4 - 85*a^2*b^2))/(2*a*b^4) + (tan(c/2 + (d*x)/2)^3*(300*a^4 + 48*b^4 - 385*a^2*b^2))/(6*a*b^4))/(d*(a + 2*b*tan(c/2 + (d*x)/2) + 5*a*tan(c/2 + (d*x)/2)^2 + 10*a*tan(c/2 + (d*x)/2)^4 + 10*a*tan(c/2 + (d*x)/2)^6 + 5*a*tan(c/2 + (d*x)/2)^8 + a*tan(c/2 + (d*x)/2)^10 + 8*b*tan(c/2 + (d*x)/2)^3 + 12*b*tan(c/2 + (d*x)/2)^5 + 8*b*tan(c/2 + (d*x)/2)^7 + 2*b*tan(c/2 + (d*x)/2)^9)) - (atan((((a^4*8i + b^4*3i - a^2*b^2*12i)*(((225*a^2*b^13)/2 - 900*a^4*b^11 + 2400*a^6*b^9 - 2400*a^8*b^7 + 800*a^10*b^5)/b^14 + (tan(c/2 + (d*x)/2)*(450*a*b^15 - 5425*a^3*b^13 + 17800*a^5*b^11 - 24000*a^7*b^9 + 14400*a^9*b^7 - 3200*a^11*b^5))/(2*b^15) - (5*(a^4*8i + b^4*3i - a^2*b^2*12i)*((60*a*b^16 - 140*a^3*b^14 + 80*a^5*b^12)/b^14 - (5*(32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(192*a*b^19 - 128*a^3*b^17))/(2*b^15))*(a^4*8i + b^4*3i - a^2*b^2*12i))/(8*b^6) + (tan(c/2 + (d*x)/2)*(640*a^2*b^16 - 1280*a^4*b^14 + 640*a^6*b^12))/(2*b^15)))/(8*b^6))*5i)/(8*b^6) + ((a^4*8i + b^4*3i - a^2*b^2

12i)(((225*a^2*b^13)/2 - 900*a^4*b^11 + 2400*a^6*b^9 - 2400*a^8*b^7 + 800
*a^10*b^5)/b^14 + (tan(c/2 + (d*x)/2)*(450*a*b^15 - 5425*a^3*b^13 + 17800*a
^5*b^11 - 24000*a^7*b^9 + 14400*a^9*b^7 - 3200*a^11*b^5))/(2*b^15) + (5*(a^
4*8i + b^4*3i - a^2*b^2*12i)*((60*a*b^16 - 140*a^3*b^14 + 80*a^5*b^12)/b^14
 + (5*(32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(192*a*b^19 - 128*a^3*b^17))/(2*b^1
5))*(a^4*8i + b^4*3i - a^2*b^2*12i))/(8*b^6) + (tan(c/2 + (d*x)/2)*(640*a^2
*b^16 - 1280*a^4*b^14 + 640*a^6*b^12))/(2*b^15)))/(8*b^6))*5i)/(8*b^6))/((4
000*a^13 - 1875*a^3*b^10 + 12750*a^5*b^8 - 30875*a^7*b^6 + 35000*a^9*b^4 -
19000*a^11*b^2)/b^14 - (5*(a^4*8i + b^4*3i - a^2*b^2*12i)*(((225*a^2*b^13)/
2 - 900*a^4*b^11 + 2400*a^6*b^9 - 2400*a^8*b^7 + 800*a^10*b^5)/b^14 + (tan(
c/2 + (d*x)/2)*(450*a*b^15 - 5425*a^3*b^13 + 17800*a^5*b^11 - 24000*a^7*b^9
 + 14400*a^9*b^7 - 3200*a^11*b^5))/(2*b^15) - (5*(a^4*8i + b^4*3i - a^2*b^2
12i)((60*a*b^16 - 140*a^3*b^14 + 80*a^5*b^12)/b^14 - (5*(32*a^2*b^3 + (ta
n(c/2 + (d*x)/2)*(192*a*b^19 - 128*a^3*b^17))/(2*b^15))*(a^4*8i + b^4*3i -
a^2*b^2*12i))/(8*b^6) + (tan(c/2 + (d*x)/2)*(640*a^2*b^16 - 1280*a^4*b^14 +
 640*a^6*b^12))/(2*b^15)))/(8*b^6)))/(8*b^6) + (5*(a^4*8i + b^4*3i - a^2*b^
2*12i)*(((225*a^2*b^13)/2 - 900*a^4*b^11 + 2400*a^6*b^9 - 2400*a^8*b^7 + 80
0*a^10*b^5)/b^14 + (tan(c/2 + (d*x)/2)*(450*a*b^15 - 5425*a^3*b^13 + 17800*
a^5*b^11 - 24000*a^7*b^9 + 14400*a^9*b^7 - 3200*a^11*b^5))/(2*b^15) + (5*(a
^4*8i + b^4*3i - a^2*b^2*12i)*((60*a*b^16 - 140*a^3*b^14 + 80*a^5*b^12)/b^1
4 + (5*(32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(192*a*b^19 - 128*a^3*b^17))/(2*b^
15))*(a^4*8i + b^4*3i - a^2*b^2*12i))/(8*b^6) + (tan(c/2 + (d*x)/2)*(640*a^
2*b^16 - 1280*a^4*b^14 + 640*a^6*b^12))/(2*b^15)))/(8*b^6)))/(8*b^6) + (tan
(c/2 + (d*x)/2)*(16000*a^14 + 2250*a^2*b^12 - 22500*a^4*b^10 + 86250*a^6*b^
8 - 162000*a^8*b^6 + 160000*a^10*b^4 - 80000*a^12*b^2))/b^15))*(a^4*8i + b^
4*3i - a^2*b^2*12i)*5i)/(4*b^6*d) - (10*a*atanh((1125*a^3*(b^6 - a^6 - 3*a^
2*b^4 + 3*a^4*b^2)^(1/2))/(3250*a^5*b - 1125*a^3*b^3 - (3125*a^7)/b + (1000
*a^9)/b^3 - 6250*a^6*tan(c/2 + (d*x)/2) - 2250*a^2*b^4*tan(c/2 + (d*x)/2) +
 6500*a^4*b^2*tan(c/2 + (d*x)/2) + (2000*a^8*tan(c/2 + (d*x)/2))/b^2) + (10
00*a^5*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2))/(3125*a^7*b + 1125*a^3*b^
5 - 3250*a^5*b^3 - (1000*a^9)/b - 2000*a^8*tan(c/2 + (d*x)/2) + 2250*a^2*b^
6*tan(c/2 + (d*x)/2) - 6500*a^4*b^4*tan(c/2 + (d*x)/2) + 6250*a^6*b^2*tan(c
/2 + (d*x)/2)) + (2250*a^2*tan(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^
4*b^2)^(1/2))/(3250*a^5 - 1125*a^3*b^2 - (3125*a^7)/b^2 + (1000*a^9)/b^4 +
6500*a^4*b*tan(c/2 + (d*x)/2) - 2250*a^2*b^3*tan(c/2 + (d*x)/2) - (6250*a^6
*tan(c/2 + (d*x)/2))/b + (2000*a^8*tan(c/2 + (d*x)/2))/b^3) + (3125*a^4*tan
(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2))/(3125*a^7 + 1125
*a^3*b^4 - 3250*a^5*b^2 - (1000*a^9)/b^2 + 6250*a^6*b*tan(c/2 + (d*x)/2) +
2250*a^2*b^5*tan(c/2 + (d*x)/2) - 6500*a^4*b^3*tan(c/2 + (d*x)/2) - (2000*a
^8*tan(c/2 + (d*x)/2))/b) + (1000*a^6*tan(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2
*b^4 + 3*a^4*b^2)^(1/2))/(1000*a^9 - 1125*a^3*b^6 + 3250*a^5*b^4 - 3125*a^7
*b^2 + 2000*a^8*b*tan(c/2 + (d*x)/2) - 2250*a^2*b^7*tan(c/2 + (d*x)/2) + 65
00*a^4*b^5*tan(c/2 + (d*x)/2) - 6250*a^6*b^3*tan(c/2 + (d*x)/2)))*(-(a + b)
^3*(a - b)^3)^(1/2))/(b^6*d)

3.445 $\int \frac{\cos^4(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=128

$$\frac{3(2a^2-b^2)\,x}{2b^4}-\frac{6a\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^4d}+\frac{3\cos(c+dx)(2a-b\sin(c+dx))}{2b^3d}-\frac{\cos^3(c+dx)}{bd(a+b\sin(c+d$$

```
[Out] 3/2*(2*a^2-b^2)*x/b^4+3/2*cos(d*x+c)*(2*a-b*sin(d*x+c))/b^3/d-cos(d*x+c)^3/
b/d/(a+b*sin(d*x+c))-6*a*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))*(
a^2-b^2)^(1/2)/b^4/d
```

Rubi [A]

time = 0.14, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2772, 2944, 2814, 2739, 632, 210}

$$-\frac{6a\sqrt{a^2-b^2}\,\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^4d}+\frac{3x(2a^2-b^2)}{2b^4}+\frac{3\cos(c+dx)(2a-b\sin(c+dx))}{2b^3d}-\frac{\cos^3(c+dx)}{bd(a+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (3*(2*a^2 - b^2)*x)/(2*b^4) - (6*a*Sqrt[a^2 - b^2]*ArcTan[(b + a*Tan[(c + d
*x)/2])/Sqrt[a^2 - b^2]])/(b^4*d) + (3*Cos[c + d*x]*(2*a - b*Sin[c + d*x]))
/(2*b^3*d) - Cos[c + d*x]^3/(b*d*(a + b*Sin[c + d*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2772

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Dist[g^2*((p - 1)/(b*(m + 1))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2944

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Dist[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^4(c+dx)}{(a+b\sin(c+dx))^2}\,dx &= -\frac{\cos^3(c+dx)}{bd(a+b\sin(c+dx))} - \frac{3\int \frac{\cos^2(c+dx)\sin(c+dx)}{a+b\sin(c+dx)}\,dx}{b}\\
&= \frac{3\cos(c+dx)(2a-b\sin(c+dx))}{2b^3d} - \frac{\cos^3(c+dx)}{bd(a+b\sin(c+dx))} - \frac{3\int \frac{-ab-(2a^2-b^2)\sin}{a+b\sin(c+dx}}{2b^3}\\
&= \frac{3(2a^2-b^2)\,x}{2b^4} + \frac{3\cos(c+dx)(2a-b\sin(c+dx))}{2b^3d} - \frac{\cos^3(c+dx)}{bd(a+b\sin(c+dx))} - \frac{(}{}\\
&= \frac{3(2a^2-b^2)\,x}{2b^4} + \frac{3\cos(c+dx)(2a-b\sin(c+dx))}{2b^3d} - \frac{\cos^3(c+dx)}{bd(a+b\sin(c+dx))} - \frac{(}{}\\
&= \frac{3(2a^2-b^2)\,x}{2b^4} + \frac{3\cos(c+dx)(2a-b\sin(c+dx))}{2b^3d} - \frac{\cos^3(c+dx)}{bd(a+b\sin(c+dx))} + \frac{(}{}\\
&= \frac{3(2a^2-b^2)\,x}{2b^4} - \frac{6a\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^4d} + \frac{3\cos(c+dx)(2a-b\,\mathrm{s}}{2b^3d}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 448 vs. 2(128) = 256.
time = 5.02, size = 448, normalized size = 3.50

$$\frac{\cos^3(c+dx)\left(-12a(a-b)\tanh^{-1}\left(\frac{\sqrt{a-b}\sqrt{-\frac{b(1+\sin(c+dx))}{a-b}}}{\sqrt{a+b}\sqrt{-\frac{b(-1+\sin(c+dx))}{a+b}}}\right)\sqrt{1-\sin(c+dx)}\,(a+b\sin(c+dx))+\sqrt{a+b}\left(12a\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{\frac{b(1+\sin(c+dx))}{-a+b}}}{\sqrt{-\frac{b(-1+\sin(c+dx))}{a+b}}}\right)\sqrt{1-\sin(c+dx)}\,(a+b\sin(c+dx))+\sqrt{-\frac{b(-1+\sin(c+dx))}{a+b}}\left(6\sqrt{b}\,(-2a+b)\sinh^{-1}\left(\frac{\sqrt{a-b}\sqrt{-\frac{b(1+\sin(c+dx))}{a-b}}}{\sqrt{2}\sqrt{b}}\right)(a+b\sin(c+dx))+\sqrt{a-b}\sqrt{1-\sin(c+dx)}\sqrt{\frac{b(1+\sin(c+dx))}{-a+b}}\left(-6a^2+2b^2-3ab\sin(c+dx)+b^2\sin^2(c+dx)\right)\right)\right)\right)}{2(a-b)^{3/2}b^2\sqrt{a+b}\,d(1-\sin(c+dx))^{3/2}\sqrt{-\frac{b(-1+\sin(c+dx))}{a+b}}\left(-\frac{b(1+\sin(c+dx))}{a-b}\right)^{3/2}(a+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (Cos[c + d*x]^3*(-12*a*(a - b)*ArcTanh[(Sqrt[a - b]*Sqrt[-((b*(1 + Sin[c +
d*x]))/(a - b))])/(Sqrt[a + b]*Sqrt[-((b*(-1 + Sin[c + d*x]))/(a + b))])]*S
qrt[1 - Sin[c + d*x]]*(a + b*Sin[c + d*x]) + Sqrt[a + b]*(12*a*Sqrt[a - b]*
ArcTanh[Sqrt[(b*(1 + Sin[c + d*x]))/(-a + b)]/Sqrt[-((b*(-1 + Sin[c + d*x])
)/(a + b))]]*Sqrt[1 - Sin[c + d*x]]*(a + b*Sin[c + d*x]) + Sqrt[-((b*(-1 +
Sin[c + d*x]))/(a + b))]*(6*Sqrt[b]*(-2*a + b)*ArcSinh[(Sqrt[a - b]*Sqrt[-(
(b*(1 + Sin[c + d*x]))/(a - b))])/(Sqrt[2]*Sqrt[b])]*(a + b*Sin[c + d*x]) +
 Sqrt[a - b]*Sqrt[1 - Sin[c + d*x]]*Sqrt[(b*(1 + Sin[c + d*x]))/(-a + b)]*(
-6*a^2 + 2*b^2 - 3*a*b*Sin[c + d*x] + b^2*Sin[c + d*x]^2)))))/(2*(a - b)^(3
/2)*b^2*Sqrt[a + b]*d*(1 - Sin[c + d*x])^(3/2)*Sqrt[-((b*(-1 + Sin[c + d*x]
))/(a + b))]*(-((b*(1 + Sin[c + d*x]))/(a - b)))^(3/2)*(a + b*Sin[c + d*x])
)
```

Maple [A]
time = 0.66, size = 220, normalized size = 1.72

method	result
derivativedivides	$\frac{\frac{2\left(\frac{\frac{b^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+2ab\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}+2ab}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+3\left(2a^2-b^2\right)\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{b^4}-\frac{2\left(\frac{-\frac{\left(a^2-b^2\right)b^2\tan\left(\frac{dx}{2}+\right.}{a}}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\,\mathrm{t}}\right.}{}}{d}$
default	$\frac{\frac{2\left(\frac{\frac{b^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+2ab\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}+2ab}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+3\left(2a^2-b^2\right)\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{b^4}-\frac{2\left(\frac{-\frac{\left(a^2-b^2\right)b^2\tan\left(\frac{dx}{2}+\right.}{a}}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\,\mathrm{t}}\right.}{}}{d}$
risch	$\frac{3x\,a^2}{b^4}-\frac{3x}{2b^2}+\frac{i\mathrm{e}^{2i(dx+c)}}{8b^2d}+\frac{a\,\mathrm{e}^{i(dx+c)}}{b^3d}+\frac{a\,\mathrm{e}^{-i(dx+c)}}{b^3d}-\frac{i\mathrm{e}^{-2i(dx+c)}}{8b^2d}-\frac{2i\left(a^2-b^2\right)\left(ib+a\,\mathrm{e}^{i(dx+c)}\right)}{b^4d\left(-ib\,\mathrm{e}^{2i(dx+c)}+ib+2a\,\mathrm{e}^{i(dx+c)}\right)}+\ldots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/b^4*((1/2*b^2*tan(1/2*d*x+1/2*c)^3+2*a*b*tan(1/2*d*x+1/2*c)^2-1/2*b^
2*tan(1/2*d*x+1/2*c)+2*a*b)/(1+tan(1/2*d*x+1/2*c)^2)^2+3/2*(2*a^2-b^2)*arct
an(tan(1/2*d*x+1/2*c)))-2/b^4*((-(a^2-b^2)*b^2/a*tan(1/2*d*x+1/2*c)-b*(a^2-
b^2))/(a*tan(1/2*d*x+1/2*c)^2+2*b*tan(1/2*d*x+1/2*c)+a)+3*a*(a^2-b^2)^(1/2)
*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.40, size = 411, normalized size = 3.21

$$\left[\frac{b^3\cos(dx+c)^3+3(2a^3-ab^2)dx+3(ab\sin(dx+c)+a^2)\sqrt{-a^2+b^2}\log\left(\frac{(2a^2-b^2)\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2+2(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c))\sqrt{-a^2+b^2}}{b^2\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2}\right)+3(2a^2b-b^3)\cos(dx+c)+3(ab^2\cos(dx+c)+(2a^2b-b^3)dx)\sin(dx+c)}{2(b^5d\sin(dx+c)+ab^4d)}, \frac{b^3\cos(dx+c)^3+3(2a^3-ab^2)dx+6(ab\sin(dx+c)+a^2)\sqrt{a^2-b^2}\arctan\left(-\frac{a\sin(dx+c)+b}{\sqrt{a^2-b^2}\cos(dx+c)}\right)+3(2a^2b-b^3)\cos(dx+c)+3(ab^2\cos(dx+c)+(2a^2b-b^3)dx)\sin(dx+c)}{2(b^5d\sin(dx+c)+ab^4d)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/2*(b^3*cos(d*x + c)^3 + 3*(2*a^3 - a*b^2)*d*x + 3*(a*b*sin(d*x + c) + a^
2)*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c)
```

```
- a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 +
b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) + 3*(2*a^2*b -
 b^3)*cos(d*x + c) + 3*(a*b^2*cos(d*x + c) + (2*a^2*b - b^3)*d*x)*sin(d*x +
 c))/(b^5*d*sin(d*x + c) + a*b^4*d), 1/2*(b^3*cos(d*x + c)^3 + 3*(2*a^3 - a
*b^2)*d*x + 6*(a*b*sin(d*x + c) + a^2)*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x +
 c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) + 3*(2*a^2*b - b^3)*cos(d*x + c) +
 3*(a*b^2*cos(d*x + c) + (2*a^2*b - b^3)*d*x)*sin(d*x + c))/(b^5*d*sin(d*x
+ c) + a*b^4*d)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 5.58, size = 235, normalized size = 1.84

$$\frac{\frac{3\,(2\,a^2-b^2)(dx+c)}{b^4}-\frac{12\,(a^3-ab^2)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,b^4}+\frac{2\left(b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+4\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+4\,a\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2b^3}+\frac{4\,(a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^3-ab^2)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a\right)ab^3}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(3*(2*a^2 - b^2)*(d*x + c)/b^4 - 12*(a^3 - a*b^2)*(pi*floor(1/2*(d*x +
c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))
/(sqrt(a^2 - b^2)*b^4) + 2*(b*tan(1/2*d*x + 1/2*c)^3 + 4*a*tan(1/2*d*x + 1/
2*c)^2 - b*tan(1/2*d*x + 1/2*c) + 4*a)/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*b^3)
 + 4*(a^2*b*tan(1/2*d*x + 1/2*c) - b^3*tan(1/2*d*x + 1/2*c) + a^3 - a*b^2)/
((a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)*a*b^3))/d
```

Mupad [**B**]
time = 6.40, size = 601, normalized size = 4.70

$$\frac{\frac{2\,(3\,a^2-b^2)}{b^3}+\frac{6\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{b^3}+\frac{6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2(2\,a^2-b^2)}{b^3}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)(9\,a^2-2\,b^2)}{a\,b^2}+\frac{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3(3\,a^2-b^2)}{a\,b^2}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5(3\,a^2-2\,b^2)}{a\,b^2}}{d\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+4\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)}+\frac{6\,a\,\mathrm{atanh}\left(\frac{432\,a^3\sqrt{b^2-a^2}}{432\,a^3\,b-\frac{432\,a^5}{b}-864\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+864\,a^2\,b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}+\frac{864\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\sqrt{b^2-a^2}}{432\,a^3-\frac{432\,a^5}{b^2}+864\,a^2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\frac{864\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{b}}+\frac{432\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\sqrt{b^2-a^2}}{432\,a^5+864\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)a^4\,b-432\,a^3\,b^2-864\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)a^2\,b^3}\right)\sqrt{b^2-a^2}}{b^4\,d}-\frac{\mathrm{atan}\left(\frac{432\,a^5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{432\,a^5-648\,a^3\,b^2+216\,a\,b^4}-\frac{648\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{216\,a\,b^2-648\,a^3+\frac{432\,a^5}{b^2}}+\frac{216\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{216\,a-\frac{648\,a^3}{b^2}+\frac{432\,a^5}{b^4}}\right)(a^2\,2\mathrm{i}-b^2\,1\mathrm{i})\,3\mathrm{i}}{b^4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(a + b*sin(c + d*x))^2,x)
```

```
[Out] ((2*(3*a^2 - b^2))/b^3 + (6*a^2*tan(c/2 + (d*x)/2)^4)/b^3 + (6*tan(c/2 + (d
*x)/2)^2*(2*a^2 - b^2))/b^3 + (tan(c/2 + (d*x)/2)*(9*a^2 - 2*b^2))/(a*b^2)
+ (4*tan(c/2 + (d*x)/2)^3*(3*a^2 - b^2))/(a*b^2) + (tan(c/2 + (d*x)/2)^5*(3
```

```
*a^2 - 2*b^2))/(a*b^2))/(d*(a + 2*b*tan(c/2 + (d*x)/2) + 3*a*tan(c/2 + (d*x
)/2)^2 + 3*a*tan(c/2 + (d*x)/2)^4 + a*tan(c/2 + (d*x)/2)^6 + 4*b*tan(c/2 +
(d*x)/2)^3 + 2*b*tan(c/2 + (d*x)/2)^5)) - (atan((432*a^5*tan(c/2 + (d*x)/2)
)/(216*a*b^4 + 432*a^5 - 648*a^3*b^2) - (648*a^3*tan(c/2 + (d*x)/2))/(216*a
*b^2 - 648*a^3 + (432*a^5)/b^2) + (216*a*tan(c/2 + (d*x)/2))/(216*a - (648*
a^3)/b^2 + (432*a^5)/b^4))*(a^2*2i - b^2*1i)*3i)/(b^4*d) + (6*a*atanh((432*
a^3*(b^2 - a^2)^(1/2))/(432*a^3*b - (432*a^5)/b - 864*a^4*tan(c/2 + (d*x)/2
) + 864*a^2*b^2*tan(c/2 + (d*x)/2)) + (864*a^2*tan(c/2 + (d*x)/2)*(b^2 - a^
2)^(1/2))/(432*a^3 - (432*a^5)/b^2 + 864*a^2*b*tan(c/2 + (d*x)/2) - (864*a^
4*tan(c/2 + (d*x)/2))/b) + (432*a^4*tan(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2))/(
432*a^5 - 432*a^3*b^2 + 864*a^4*b*tan(c/2 + (d*x)/2) - 864*a^2*b^3*tan(c/2
+ (d*x)/2)))*(b^2 - a^2)^(1/2))/(b^4*d)
```

3.446 $\int \frac{\cos^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=84

$$-\frac{x}{b^2}+\frac{2a\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^2\sqrt{a^2-b^2}\,d}-\frac{\cos(c+dx)}{bd(a+b\sin(c+dx))}$$

```
[Out] -x/b^2-cos(d*x+c)/b/d/(a+b*sin(d*x+c))+2*a*arctan((b+a*tan(1/2*d*x+1/2*c))/
(a^2-b^2)^(1/2))/b^2/d/(a^2-b^2)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2772, 2814, 2739, 632, 210}

$$\frac{2a\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^2d\sqrt{a^2-b^2}}-\frac{\cos(c+dx)}{bd(a+b\sin(c+dx))}-\frac{x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -(x/b^2) + (2*a*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^2*Sqrt
[a^2 - b^2]*d) - Cos[c + d*x]/(b*d*(a + b*Sin[c + d*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2772

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Dist[g^2*((p - 1)/(b*(m + 1))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx &= -\frac{\cos(c+dx)}{bd(a+b\sin(c+dx))}-\frac{\int \frac{\sin(c+dx)}{a+b\sin(c+dx)}\,dx}{b}\\
&= -\frac{x}{b^2}-\frac{\cos(c+dx)}{bd(a+b\sin(c+dx))}+\frac{a\int \frac{1}{a+b\sin(c+dx)}\,dx}{b^2}\\
&= -\frac{x}{b^2}-\frac{\cos(c+dx)}{bd(a+b\sin(c+dx))}+\frac{(2a)\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx,x,\tan\left(\frac{1}{2}(c+dx)\right)\right)}{b^2d}\\
&= -\frac{x}{b^2}-\frac{\cos(c+dx)}{bd(a+b\sin(c+dx))}-\frac{(4a)\text{Subst}\left(\int \frac{1}{-4(a^2-b^2)-x^2}\,dx,x,2b+2a\tan\left(\frac{1}{2}(c\right.\right.}{b^2d}\\
&= -\frac{x}{b^2}+\frac{2a\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^2\sqrt{a^2-b^2}\,d}-\frac{\cos(c+dx)}{bd(a+b\sin(c+dx))}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 414 vs. 2(84) = 168.

time = 2.54, size = 414, normalized size = 4.93

$$\frac{\cos(c+dx)\left(-2a(a-b)\tanh^{-1}\left(\frac{\sqrt{a-b}\sqrt{-\frac{b(1+\sin(c+dx))}{a-b}}}{\sqrt{a+b}\sqrt{-\frac{b(-1+\sin(c+dx))}{a+b}}}\right)\sqrt{1-\sin(c+dx)}\,(a+b\sin(c+dx))+\sqrt{a+b}\left(2a\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{\frac{b(1+\sin(c+dx))}{-a+b}}}{\sqrt{-\frac{b(-1+\sin(c+dx))}{a+b}}}\right)\sqrt{1-\sin(c+dx)}\,(a+b\sin(c+dx))+(-a+b)\sqrt{-\frac{b(-1+\sin(c+dx))}{a+b}}\left(\sqrt{a-b}\,(a+b)\sqrt{1-\sin(c+dx)}\sqrt{-\frac{b(1+\sin(c+dx))}{a-b}}+2\sqrt{b}\sinh^{-1}\left(\frac{\sqrt{a-b}\sqrt{-\frac{b(1+\sin(c+dx))}{a-b}}}{\sqrt{2}\sqrt{b}}\right)(a+b\sin(c+dx))\right)\right)\right)}{(a-b)^{3/2}b(a+b)^{3/2}d\sqrt{1-\sin(c+dx)}\sqrt{-\frac{b(-1+\sin(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sin(c+dx))}{a-b}}\,(a+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (Cos[c + d*x]*(-2*a*(a - b)*ArcTanh[(Sqrt[a - b]*Sqrt[-((b*(1 + Sin[c + d*x
]))/(a - b))])/(Sqrt[a + b]*Sqrt[-((b*(-1 + Sin[c + d*x]))/(a + b))])]*Sqrt
[1 - Sin[c + d*x]]*(a + b*Sin[c + d*x]) + Sqrt[a + b]*(2*a*Sqrt[a - b]*ArcT
```

```
anh[Sqrt[(b*(1 + Sin[c + d*x]))/(-a + b)]/Sqrt[-((b*(-1 + Sin[c + d*x]))/(a
 + b))]]*Sqrt[1 - Sin[c + d*x]]*(a + b*Sin[c + d*x]) + (-a + b)*Sqrt[-((b*(
-1 + Sin[c + d*x]))/(a + b))]*(Sqrt[a - b]*(a + b)*Sqrt[1 - Sin[c + d*x]]*S
qrt[-((b*(1 + Sin[c + d*x]))/(a - b))] + 2*Sqrt[b]*ArcSinh[(Sqrt[a - b]*Sqr
t[-((b*(1 + Sin[c + d*x]))/(a - b))])/(Sqrt[2]*Sqrt[b])]*(a + b*Sin[c + d*x
])))))/((a - b)^(3/2)*b*(a + b)^(3/2)*d*Sqrt[1 - Sin[c + d*x]]*Sqrt[-((b*(-
1 + Sin[c + d*x]))/(a + b))]*Sqrt[-((b*(1 + Sin[c + d*x]))/(a - b))]*(a + b
*Sin[c + d*x]))
```

Maple [A]
time = 0.47, size = 121, normalized size = 1.44

method	result
derivativedivides	$\frac{-\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^2}+\frac{\frac{2\left(-\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a}-b\right)}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a}+\frac{2a\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}}{b^2}}{d}$
default	$\frac{-\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^2}+\frac{\frac{2\left(-\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a}-b\right)}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a}+\frac{2a\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}}{b^2}}{d}$
risch	$-\frac{x}{b^2}-\frac{2\left(ib+a\,e^{i(dx+c)}\right)}{b^2 d\left(b\,e^{2i(dx+c)}-b+2ia\,e^{i(dx+c)}\right)}-\frac{a\ln\left(e^{i(dx+c)}+\frac{ia\sqrt{-a^2+b^2}-a^2+b^2}{b\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,d\,b^2}+\frac{a\ln\left(e^{i(dx+c)}+\frac{ia\sqrt{-\cdots}}{b\sqrt{\cdots}}\right)}{\sqrt{-a^2+\cdots}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2/b^2*arctan(tan(1/2*d*x+1/2*c))+2/b^2*((-b^2/a*tan(1/2*d*x+1/2*c)-b)
/(a*tan(1/2*d*x+1/2*c)^2+2*b*tan(1/2*d*x+1/2*c)+a)+a/(a^2-b^2)^(1/2)*arctan
(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [**A**]
time = 0.37, size = 388, normalized size = 4.62

$$\left[-\frac{2\,(a^2b-b^3)dx\sin\left(dx+c\right)+2\,(a^3-ab^2)dx+(ab\sin\left(dx+c\right)+a^2)\sqrt{-a^2+b^2}\,\log\left(\frac{(2\,a^2-b^2)\cos(dx+c)^2-2\,ab\sin(dx+c)-a^2-b^2+2\,(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c))\sqrt{-a^2+b^2}}{b^2\cos(dx+c)^2-2\,ab\sin(dx+c)-a^2-b^2}\right)+2\,(a^2b-b^3)\cos\left(dx+c\right)}{2\,((a^2b^3-b^5)d\sin\left(dx+c\right)+(a^3b^2-ab^4)d)},\,-\frac{(a^2b-b^3)dx\sin\left(dx+c\right)+(a^3-ab^2)dx+(ab\sin\left(dx+c\right)+a^2)\sqrt{a^2-b^2}\,\arctan\left(-\frac{a\sin(dx+c)+b}{\sqrt{a^2-b^2}\cos(dx+c)}\right)+(a^2b-b^3)\cos\left(dx+c\right)}{(a^2b^3-b^5)d\sin\left(dx+c\right)+(a^3b^2-ab^4)d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(a^2*b - b^3)*d*x*sin(d*x + c) + 2*(a^3 - a*b^2)*d*x + (a*b*sin(d*
x + c) + a^2)*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*si
n(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*s
qrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) + 2
*(a^2*b - b^3)*cos(d*x + c))/((a^2*b^3 - b^5)*d*sin(d*x + c) + (a^3*b^2 - a
*b^4)*d), -((a^2*b - b^3)*d*x*sin(d*x + c) + (a^3 - a*b^2)*d*x + (a*b*sin(d
*x + c) + a^2)*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2
)*cos(d*x + c))) + (a^2*b - b^3)*cos(d*x + c))/((a^2*b^3 - b^5)*d*sin(d*x +
 c) + (a^3*b^2 - a*b^4)*d)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 6.07, size = 126, normalized size = 1.50

$$\frac{\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)a}{\sqrt{a^2-b^2}\,b^2}-\frac{dx+c}{b^2}-\frac{2\,\left(b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a\right)ab}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] (2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c
) + b)/sqrt(a^2 - b^2)))*a/(sqrt(a^2 - b^2)*b^2) - (d*x + c)/b^2 - 2*(b*tan
(1/2*d*x + 1/2*c) + a)/((a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c
) + a)*a*b))/d
```

Mupad [**B**]
time = 5.56, size = 329, normalized size = 3.92

$$-\frac{b^2\,\sin\left(c+d\,x\right)+\frac{\left(2\,a^3\,\mathrm{atan}\left(\frac{\left(-\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2+\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a\,b+2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,b^2\right)\,1{}\mathrm{i}}{\sqrt{b^2-a^2}\,\left(a\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+2\,b\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}\right)-a^2\,\sqrt{b^2-a^2}\,\left(c+d\,x\right)\,1{}\mathrm{i}\right)\,1{}\mathrm{i}}{\sqrt{b^2-a^2}}-\frac{b\,\left(a\,\sqrt{b^2-a^2}\,1{}\mathrm{i}+a\,\cos\left(c+d\,x\right)\,\sqrt{b^2-a^2}\,1{}\mathrm{i}-2\,a^2\,\mathrm{atan}\left(\frac{\left(-\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2+\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a\,b+2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,b^2\right)\,1{}\mathrm{i}}{\sqrt{b^2-a^2}\,\left(a\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+2\,b\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}\right)\,\sin\left(c+d\,x\right)+a\,\sin\left(c+d\,x\right)\,\sqrt{b^2-a^2}\,\left(c+d\,x\right)\,1{}\mathrm{i}\right)\,1{}\mathrm{i}}{\sqrt{b^2-a^2}}}{a\,b^2\,d\,\left(a+b\,\sin\left(c+d\,x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a + b*sin(c + d*x))^2,x)
```

```
[Out] -(b^2*sin(c + d*x) + ((2*a^3*atan(((2*b^2*sin(c/2 + (d*x)/2) - a^2*sin(c/2
+ (d*x)/2) + a*b*cos(c/2 + (d*x)/2))*1i)/((b^2 - a^2)^(1/2)*(a*cos(c/2 + (d
*x)/2) + 2*b*sin(c/2 + (d*x)/2)))) - a^2*(b^2 - a^2)^(1/2)*(c + d*x)*1i)*1i
)/(b^2 - a^2)^(1/2) - (b*(a*(b^2 - a^2)^(1/2)*1i + a*cos(c + d*x)*(b^2 - a^
2)^(1/2)*1i - 2*a^2*atan(((2*b^2*sin(c/2 + (d*x)/2) - a^2*sin(c/2 + (d*x)/2
) + a*b*cos(c/2 + (d*x)/2))*1i)/((b^2 - a^2)^(1/2)*(a*cos(c/2 + (d*x)/2) +
2*b*sin(c/2 + (d*x)/2))))*sin(c + d*x) + a*sin(c + d*x)*(b^2 - a^2)^(1/2)*(
c + d*x)*1i)*1i)/(b^2 - a^2)^(1/2))/(a*b^2*d*(a + b*sin(c + d*x)))
```

3.447 $\int \frac{\sec^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=130

$$-\frac{6ab^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}\,d}+\frac{b\sec(c+dx)}{(a^2-b^2)\,d(a+b\sin(c+dx))}-\frac{\sec(c+dx)\,(3ab-(a^2+2b^2)\sin(c+dx))}{(a^2-b^2)^2\,d}$$

```
[Out] -6*a*b^2*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/(a^2-b^2)^(5/2)/d
+b*sec(d*x+c)/(a^2-b^2)/d/(a+b*sin(d*x+c))-sec(d*x+c)*(3*a*b-(a^2+2*b^2)*si
n(d*x+c))/(a^2-b^2)^2/d
```

Rubi [A]
time = 0.13, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2773, 2945, 12, 2739, 632, 210}

$$-\frac{6ab^2\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\,(a^2-b^2)^{5/2}}+\frac{b\sec(c+dx)}{d\,(a^2-b^2)\,(a+b\sin(c+dx))}-\frac{\sec(c+dx)\,(3ab-(a^2+2b^2)\sin(c+dx))}{d\,(a^2-b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-6*a*b^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(5
/2)*d) + (b*Sec[c + d*x])/((a^2 - b^2)*d*(a + b*Sin[c + d*x])) - (Sec[c + d
*x]*(3*a*b - (a^2 + 2*b^2)*Sin[c + d*x]))/((a^2 - b^2)^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2773

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m + 1)/(f*g*(a^2 - b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1))
, Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p
 + 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^
2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2945

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx &= \frac{b\sec(c+dx)}{(a^2-b^2)\,d(a+b\sin(c+dx))} + \frac{\int \frac{\sec^2(c+dx)(-a+2b\sin(c+dx))}{a+b\sin(c+dx)}\,dx}{-a^2+b^2}\\
&= \frac{b\sec(c+dx)}{(a^2-b^2)\,d(a+b\sin(c+dx))} - \frac{\sec(c+dx)\left(3ab-(a^2+2b^2)\sin(c+dx)\right)}{(a^2-b^2)^2\,d} + \\
&= \frac{b\sec(c+dx)}{(a^2-b^2)\,d(a+b\sin(c+dx))} - \frac{\sec(c+dx)\left(3ab-(a^2+2b^2)\sin(c+dx)\right)}{(a^2-b^2)^2\,d} - \\
&= \frac{b\sec(c+dx)}{(a^2-b^2)\,d(a+b\sin(c+dx))} - \frac{\sec(c+dx)\left(3ab-(a^2+2b^2)\sin(c+dx)\right)}{(a^2-b^2)^2\,d} - \\
&= \frac{b\sec(c+dx)}{(a^2-b^2)\,d(a+b\sin(c+dx))} - \frac{\sec(c+dx)\left(3ab-(a^2+2b^2)\sin(c+dx)\right)}{(a^2-b^2)^2\,d} + \\
&= -\frac{6ab^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}\,d} + \frac{b\sec(c+dx)}{(a^2-b^2)\,d(a+b\sin(c+dx))} - \frac{\sec(c+dx)}{}
\end{aligned}$$

Mathematica [A]

time = 1.13, size = 162, normalized size = 1.25

$$\frac{-\frac{6ab^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}}+\sin\left(\frac{1}{2}(c+dx)\right)\left(\frac{1}{(a+b)^2\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}+\frac{1}{(a-b)^2\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}\right)-\frac{b^3\cos(c+dx)}{(a-b)^2(a+b)^2(a+b\sin(c+dx))}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((-6*a*b^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5
/2) + Sin[(c + d*x)/2]*(1/((a + b)^2*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]))
 + 1/((a - b)^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))) - (b^3*Cos[c + d*x]
)/((a - b)^2*(a + b)^2*(a + b*Sin[c + d*x])))/d
```

Maple [A]

time = 0.49, size = 158, normalized size = 1.22

method	result
derivativedivides	$\frac{-\frac{2b^2\left(\frac{\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a}+b}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a}+\frac{3a\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}\right)}{(a-b)^2(a+b)^2}-\frac{1}{(a-b)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{1}{(a+b)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\cdots\right)}}{d}$
default	$\frac{-\frac{2b^2\left(\frac{\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a}+b}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a}+\frac{3a\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}\right)}{(a-b)^2(a+b)^2}-\frac{1}{(a-b)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{1}{(a+b)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\cdots\right)}}{d}$
risch	$\frac{2i\left(2a^3e^{i(dx+c)}+3ia^2b\,e^{2i(dx+c)}+ia^2b+2ib^3+3ab^2e^{3i(dx+c)}+b^2a\,e^{i(dx+c)}\right)}{\left(1+e^{2i(dx+c)}\right)\left(-ib\,e^{2i(dx+c)}+ib+2a\,e^{i(dx+c)}\right)(a^2-b^2)^2d}-\frac{3b^2a\ln\left(e^{i(dx+c)}+\frac{ia\sqrt{-a^2+b^2}+a^2\cdots}{b\sqrt{-a^2+b^2}}\cdots\right)}{\sqrt{-a^2+b^2}\,(a+b)^2(a-b)^2d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*b^2/(a-b)^2/(a+b)^2*((b^2/a*tan(1/2*d*x+1/2*c)+b)/(a*tan(1/2*d*x+1/
2*c)^2+2*b*tan(1/2*d*x+1/2*c)+a)+3*a/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/
2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2)))-1/(a-b)^2/(tan(1/2*d*x+1/2*c)+1)-1/(a+b
)^2/(tan(1/2*d*x+1/2*c)-1))
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.38, size = 538, normalized size = 4.14

$$\left[-\frac{2\,a^4b-4\,a^2b^3+2\,b^5+2\,(a^4b+a^2b^3-2\,b^5)\cos(dx+c)^2+3\,(ab^3\cos(dx+c)\sin(dx+c)+a^2b^2\cos(dx+c))\sqrt{-a^2+b^2}\log\left(-\frac{(2\,a^2-b^2)\cos(dx+c)^2-2\,ab\sin(dx+c)-a^2-b^2-2\,(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c))\sqrt{-a^2+b^2}}{b^2\cos(dx+c)^2-2\,ab\sin(dx+c)-a^2-b^2}\right)-2\,(a^5-2\,a^3b^2+ab^4)\sin(dx+c)}{2\,((a^6b-3\,a^4b^3+3\,a^2b^5-b^7)d\cos(dx+c)\sin(dx+c)+(a^7-3\,a^5b^2+3\,a^3b^4-ab^6)d\cos(dx+c))},\ -\frac{a^4b-2\,a^2b^3+b^5+(a^4b+a^2b^3-2\,b^5)\cos(dx+c)^2-3\,(ab^3\cos(dx+c)\sin(dx+c)+a^2b^2\cos(dx+c))\sqrt{a^2-b^2}\arctan\left(-\frac{a\sin(dx+c)+b}{\sqrt{a^2-b^2}\cos(dx+c)}\right)-(a^5-2\,a^3b^2+ab^4)\sin(dx+c)}{(a^6b-3\,a^4b^3+3\,a^2b^5-b^7)d\cos(dx+c)\sin(dx+c)+(a^7-3\,a^5b^2+3\,a^3b^4-ab^6)d\cos(dx+c)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*a^4*b - 4*a^2*b^3 + 2*b^5 + 2*(a^4*b + a^2*b^3 - 2*b^5)*cos(d*x +
c)^2 + 3*(a*b^3*cos(d*x + c)*sin(d*x + c) + a^2*b^2*cos(d*x + c))*sqrt(-a^2
 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2
 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*
cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 2*(a^5 - 2*a^3*b^2 + a*
b^4)*sin(d*x + c))/((a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*d*cos(d*x + c)*si
n(d*x + c) + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d*cos(d*x + c)), -(a^4*b
 - 2*a^2*b^3 + b^5 + (a^4*b + a^2*b^3 - 2*b^5)*cos(d*x + c)^2 - 3*(a*b^3*co
s(d*x + c)*sin(d*x + c) + a^2*b^2*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*
sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - (a^5 - 2*a^3*b^2 + a*b^
4)*sin(d*x + c))/((a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*d*cos(d*x + c)*sin(
d*x + c) + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d*cos(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**2/(a + b*sin(c + d*x))**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 271 vs. 2(125) = 250.
time = 6.84, size = 271, normalized size = 2.08

$$-\frac{2\left(\frac{3\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)ab^2}{(a^4-2\,a^2b^2+b^4)\sqrt{a^2-b^2}}+\frac{a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+a^2b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+b^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+3\,ab^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-3\,a^2b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2\,a^3b-ab^3}{(a^5-2\,a^3b^2+ab^4)\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+2\,b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-2\,b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-a\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -2*(3*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/
2*c) + b)/sqrt(a^2 - b^2)))*a*b^2/((a^4 - 2*a^2*b^2 + b^4)*sqrt(a^2 - b^2))
 + (a^4*tan(1/2*d*x + 1/2*c)^3 + a^2*b^2*tan(1/2*d*x + 1/2*c)^3 + b^4*tan(1
/2*d*x + 1/2*c)^3 + 3*a*b^3*tan(1/2*d*x + 1/2*c)^2 + a^4*tan(1/2*d*x + 1/2*
c) - 3*a^2*b^2*tan(1/2*d*x + 1/2*c) - b^4*tan(1/2*d*x + 1/2*c) - 2*a^3*b -
a*b^3)/((a^5 - 2*a^3*b^2 + a*b^4)*(a*tan(1/2*d*x + 1/2*c)^4 + 2*b*tan(1/2*d
*x + 1/2*c)^3 - 2*b*tan(1/2*d*x + 1/2*c) - a)))/d
```

Mupad [B]
time = 7.40, size = 303, normalized size = 2.33

$$-\frac{\frac{2\,\left(2\,a^2\,b+b^3\right)}{{\left(a^2-b^2\right)}^2}-\frac{6\,b^3\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}{a^4-2\,a^2\,b^2+b^4}+\frac{2\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(-a^4+3\,a^2\,b^2+b^4\right)}{a\,{\left(a^2-b^2\right)}^2}-\frac{2\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(a^4+a^2\,b^2+b^4\right)}{a\,{\left(a^2-b^2\right)}^2}}{d\,\left(-a\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4-2\,b\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3+2\,b\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)}-\frac{6\,a\,b^2\,\mathrm{atan}\left(\frac{\frac{3\,a\,b^2\,\left(2\,a^4\,b-4\,a^2\,b^3+2\,b^5\right)}{{\left(a+b\right)}^{5/2}\,{\left(a-b\right)}^{5/2}}+\frac{6\,a^2\,b^2\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(a^4-2\,a^2\,b^2+b^4\right)}{{\left(a+b\right)}^{5/2}\,{\left(a-b\right)}^{5/2}}}{6\,a\,b^2}\right)}{d\,{\left(a+b\right)}^{5/2}\,{\left(a-b\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + b*sin(c + d*x))^2),x)
```

```
[Out] - ((2*(2*a^2*b + b^3))/(a^2 - b^2)^2 - (6*b^3*tan(c/2 + (d*x)/2)^2)/(a^4 +
b^4 - 2*a^2*b^2) + (2*tan(c/2 + (d*x)/2)*(b^4 - a^4 + 3*a^2*b^2))/(a*(a^2 -
 b^2)^2) - (2*tan(c/2 + (d*x)/2)^3*(a^4 + b^4 + a^2*b^2))/(a*(a^2 - b^2)^2)
)/(d*(a + 2*b*tan(c/2 + (d*x)/2) - a*tan(c/2 + (d*x)/2)^4 - 2*b*tan(c/2 + (
d*x)/2)^3)) - (6*a*b^2*atan(((3*a*b^2*(2*a^4*b + 2*b^5 - 4*a^2*b^3))/((a +
b)^(5/2)*(a - b)^(5/2)) + (6*a^2*b^2*tan(c/2 + (d*x)/2)*(a^4 + b^4 - 2*a^2*
b^2))/((a + b)^(5/2)*(a - b)^(5/2)))/(6*a*b^2)))/(d*(a + b)^(5/2)*(a - b)^(
5/2))
```

3.448 $\int \frac{\sec^4(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=193

$$\frac{10ab^4\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2}\,d}+\frac{b\sec^3(c+dx)}{(a^2-b^2)\,d(a+b\sin(c+dx))}-\frac{\sec^3(c+dx)\,(5ab-(a^2+4b^2)\sin(c+dx))}{3\,(a^2-b^2)^2\,d}$$

```
[Out] 10*a*b^4*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/(a^2-b^2)^(7/2)/d
+b*sec(d*x+c)^3/(a^2-b^2)/d/(a+b*sin(d*x+c))-1/3*sec(d*x+c)^3*(5*a*b-(a^2+4
*b^2)*sin(d*x+c))/(a^2-b^2)^2/d+1/3*sec(d*x+c)*(15*a*b^3+(2*a^4-9*a^2*b^2-8
*b^4)*sin(d*x+c))/(a^2-b^2)^3/d
```

Rubi [A]

time = 0.24, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2773, 2945, 12, 2739, 632, 210}

$$\frac{10ab^4\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\,(a^2-b^2)^{7/2}}+\frac{b\sec^3(c+dx)}{d\,(a^2-b^2)\,(a+b\sin(c+dx))}-\frac{\sec^3(c+dx)\,(5ab-(a^2+4b^2)\sin(c+dx))}{3d\,(a^2-b^2)^2}+\frac{\sec(c+dx)\,((2a^4-9a^2b^2-8b^4)\sin(c+dx)+15ab^3)}{3d\,(a^2-b^2)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (10*a*b^4*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(7
/2)*d) + (b*Sec[c + d*x]^3)/((a^2 - b^2)*d*(a + b*Sin[c + d*x])) - (Sec[c +
 d*x]^3*(5*a*b - (a^2 + 4*b^2)*Sin[c + d*x]))/(3*(a^2 - b^2)^2*d) + (Sec[c
+ d*x]*(15*a*b^3 + (2*a^4 - 9*a^2*b^2 - 8*b^4)*Sin[c + d*x]))/(3*(a^2 - b^2
)^3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2773

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m + 1)/(f*g*(a^2 - b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1))
, Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p
 + 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^
2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2945

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{b\sec^3(c+dx)}{(a^2-b^2)\,d(a+b\sin(c+dx))} + \frac{\int \frac{\sec^4(c+dx)(-a+4b\sin(c+dx))}{a+b\sin(c+dx)}\,dx}{-a^2+b^2}$$

$$= \frac{b\sec^3(c+dx)}{(a^2-b^2)\,d(a+b\sin(c+dx))} - \frac{\sec^3(c+dx)\,(5ab-(a^2+4b^2)\sin(c+dx))}{3\,(a^2-b^2)^2\,d} +$$

$$= \frac{b\sec^3(c+dx)}{(a^2-b^2)\,d(a+b\sin(c+dx))} - \frac{\sec^3(c+dx)\,(5ab-(a^2+4b^2)\sin(c+dx))}{3\,(a^2-b^2)^2\,d} +$$

$$= \frac{b\sec^3(c+dx)}{(a^2-b^2)\,d(a+b\sin(c+dx))} - \frac{\sec^3(c+dx)\,(5ab-(a^2+4b^2)\sin(c+dx))}{3\,(a^2-b^2)^2\,d} +$$

$$= \frac{b\sec^3(c+dx)}{(a^2-b^2)\,d(a+b\sin(c+dx))} - \frac{\sec^3(c+dx)\,(5ab-(a^2+4b^2)\sin(c+dx))}{3\,(a^2-b^2)^2\,d} +$$

$$= \frac{b\sec^3(c+dx)}{(a^2-b^2)\,d(a+b\sin(c+dx))} - \frac{\sec^3(c+dx)\,(5ab-(a^2+4b^2)\sin(c+dx))}{3\,(a^2-b^2)^2\,d} +$$

$$= \frac{10ab^4\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2}\,d} + \frac{b\sec^3(c+dx)}{(a^2-b^2)\,d(a+b\sin(c+dx))} - \frac{\sec^3(c+dx)}{}$$

Mathematica [A]
time = 1.86, size = 336, normalized size = 1.74

$$\frac{\frac{120ab^4\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2}} + \frac{1}{(a+b)^2\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2} + \frac{2\sin\left(\frac{1}{2}(c+dx)\right)}{(a+b)^2\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3} + \frac{4(2a+5b)\sin\left(\frac{1}{2}(c+dx)\right)}{(a+b)^3\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)} + \frac{2\sin\left(\frac{1}{2}(c+dx)\right)}{(a-b)^2\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3} - \frac{1}{(a-b)^2\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2} + \frac{4(2a-5b)\sin\left(\frac{1}{2}(c+dx)\right)}{(a-b)^3\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)} + \frac{12b^5\cos(c+dx)}{(a-b)^3(a+b)^3(a+b\sin(c+dx))}}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((120*a*b^4*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(
7/2) + 1/((a + b)^2*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2) + (2*Sin[(c +
d*x)/2])/((a + b)^2*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3) + (4*(2*a + 5*
b)*Sin[(c + d*x)/2])/((a + b)^3*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) + (2
*Sin[(c + d*x)/2])/((a - b)^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3) - 1/
((a - b)^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + (4*(2*a - 5*b)*Sin[(c
 + d*x)/2])/((a - b)^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])) + (12*b^5*Cos
[c + d*x])/((a - b)^3*(a + b)^3*(a + b*Sin[c + d*x])))/(12*d)
```

Maple [A]
time = 0.93, size = 252, normalized size = 1.31

method	result

derivativedivides	$\frac{\frac{2b^4\left(\frac{\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a}+b}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a}+\frac{5a\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}\right)}{(a-b)^3(a+b)^3}-\frac{1}{3(a-b)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}+\frac{1}{2(a-b)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}\ldots}{d}$
default	$\frac{\frac{2b^4\left(\frac{\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a}+b}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a}+\frac{5a\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}\right)}{(a-b)^3(a+b)^3}-\frac{1}{3(a-b)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}+\frac{1}{2(a-b)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}\ldots}{d}$
risch	$\frac{\frac{110ia^2b^3e^{4i(dx+c)}}{3}+\frac{32ib^5e^{2i(dx+c)}}{3}+10ia^2b^3e^{6i(dx+c)}+22ia^2b^3e^{2i(dx+c)}+6ia^2b^3-\frac{20ia^4b\,e^{4i(dx+c)}}{3}+\frac{2b^4a\,e^{i(dx+c)}}{3}+12b^2a\ldots}{\left(1+e^{2i(dx+c)}\right)^3(\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*b^4/(a-b)^3/(a+b)^3*((b^2/a*tan(1/2*d*x+1/2*c)+b)/(a*tan(1/2*d*x+1/2
*c)^2+2*b*tan(1/2*d*x+1/2*c)+a)+5*a/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2
*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2)))-1/3/(a-b)^2/(tan(1/2*d*x+1/2*c)+1)^3+1/2
/(a-b)^2/(tan(1/2*d*x+1/2*c)+1)^2-1/(a-b)^3*(a-2*b)/(tan(1/2*d*x+1/2*c)+1)-
1/3/(a+b)^2/(tan(1/2*d*x+1/2*c)-1)^3-1/2/(a+b)^2/(tan(1/2*d*x+1/2*c)-1)^2-1
/(a+b)^3*(a+2*b)/(tan(1/2*d*x+1/2*c)-1))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.39, size = 782, normalized size = 4.05

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/6*(2*a^6*b - 6*a^4*b^3 + 6*a^2*b^5 - 2*b^7 + 2*(2*a^6*b - 11*a^4*b^3 +
a^2*b^5 + 8*b^7)*cos(d*x + c)^4 - 2*(a^6*b + 2*a^4*b^3 - 7*a^2*b^5 + 4*b^7)
*cos(d*x + c)^2 - 15*(a*b^5*cos(d*x + c)^3*sin(d*x + c) + a^2*b^4*cos(d*x +
 c)^3)*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x
+ c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a
^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 2*(a^7
- 3*a^5*b^2 + 3*a^3*b^4 - a*b^6 + (2*a^7 - 11*a^5*b^2 + 16*a^3*b^4 - 7*a*b^
6)*cos(d*x + c)^2)*sin(d*x + c))/((a^8*b - 4*a^6*b^3 + 6*a^4*b^5 - 4*a^2*b^
7 + b^9)*d*cos(d*x + c)^3*sin(d*x + c) + (a^9 - 4*a^7*b^2 + 6*a^5*b^4 - 4*a
^3*b^6 + a*b^8)*d*cos(d*x + c)^3), -1/3*(a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^
7 + (2*a^6*b - 11*a^4*b^3 + a^2*b^5 + 8*b^7)*cos(d*x + c)^4 - (a^6*b + 2*a^
4*b^3 - 7*a^2*b^5 + 4*b^7)*cos(d*x + c)^2 + 15*(a*b^5*cos(d*x + c)^3*sin(d*
x + c) + a^2*b^4*cos(d*x + c)^3)*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) +
b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6 +
 (2*a^7 - 11*a^5*b^2 + 16*a^3*b^4 - 7*a*b^6)*cos(d*x + c)^2)*sin(d*x + c))/
((a^8*b - 4*a^6*b^3 + 6*a^4*b^5 - 4*a^2*b^7 + b^9)*d*cos(d*x + c)^3*sin(d*x
 + c) + (a^9 - 4*a^7*b^2 + 6*a^5*b^4 - 4*a^3*b^6 + a*b^8)*d*cos(d*x + c)^3)
]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^4(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**4/(a + b*sin(c + d*x))**2, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 427 vs. 2(184) = 368.
time = 3.92, size = 427, normalized size = 2.21

$$\frac{2\left(\frac{15\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)ab^4}{(a^6-3a^4b^2+3a^2b^4-b^6)\sqrt{a^2-b^2}}+\frac{3\left(b^6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+ab^5\right)}{(a^7-3a^5b^2+3a^3b^4-ab^6)\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+a\right)}-\frac{3a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-9a^2b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-6b^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-6a^3b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+18ab^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-2a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+18a^2b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+8b^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-24ab^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+3a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-9a^2b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-6b^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2a^3b+14ab^3}{(a^6-3a^4b^2+3a^2b^4-b^6)\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^3}\right)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 2/3*(15*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x +
1/2*c) + b)/sqrt(a^2 - b^2)))*a*b^4/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sq
rt(a^2 - b^2)) + 3*(b^6*tan(1/2*d*x + 1/2*c) + a*b^5)/((a^7 - 3*a^5*b^2 + 3
*a^3*b^4 - a*b^6)*(a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)
) - (3*a^4*tan(1/2*d*x + 1/2*c)^5 - 9*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 - 6*b^
```

```
4*tan(1/2*d*x + 1/2*c)^5 - 6*a^3*b*tan(1/2*d*x + 1/2*c)^4 + 18*a*b^3*tan(1/
2*d*x + 1/2*c)^4 - 2*a^4*tan(1/2*d*x + 1/2*c)^3 + 18*a^2*b^2*tan(1/2*d*x +
1/2*c)^3 + 8*b^4*tan(1/2*d*x + 1/2*c)^3 - 24*a*b^3*tan(1/2*d*x + 1/2*c)^2 +
 3*a^4*tan(1/2*d*x + 1/2*c) - 9*a^2*b^2*tan(1/2*d*x + 1/2*c) - 6*b^4*tan(1/
2*d*x + 1/2*c) - 2*a^3*b + 14*a*b^3)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(
tan(1/2*d*x + 1/2*c)^2 - 1)^3))/d
```

Mupad [B]
time = 8.51, size = 727, normalized size = 3.77

$$\frac{\frac{2\left(-2a^4b+14a^2b^3+3b^5\right)}{3\left(a^6-3a^4b^2+3a^2b^4-b^6\right)}-\frac{10b^5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6}{a^6-3a^4b^2+3a^2b^4-b^6}-\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(-4a^4b+28a^2b^3+21b^5\right)}{3\left(a^6-3a^4b^2+3a^2b^4-b^6\right)}+\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(3a^6-13a^4b^2+22a^2b^4+3b^6\right)}{3a\left(a^6-3a^4b^2+3a^2b^4-b^6\right)}-\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7\left(-a^6+3a^4b^2+2a^2b^4+b^6\right)}{a\left(a^6-3a^4b^2+3a^2b^4-b^6\right)}+\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5\left(a^6-3a^4b^2+38a^2b^4+9b^6\right)}{3a\left(a^6-3a^4b^2+3a^2b^4-b^6\right)}+\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(a^6+9a^4b^2-46a^2b^4-9b^6\right)}{3a\left(a^6-3a^4b^2+3a^2b^4-b^6\right)}+\frac{10b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(-2a^4+6a^2b^2+5b^4\right)}{3\left(a^6-3a^4b^2+3a^2b^4-b^6\right)}}{d\left(-a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8-2b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+2a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+6b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5-6b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3-2a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+2b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a\right)}+\frac{10ab^4\,\mathrm{atan}\left(\frac{\frac{5ab^4\left(2a^6b-6a^4b^3+6a^2b^5-2b^7\right)}{(a+b)^{7/2}(a-b)^{7/2}}+\frac{10a^2b^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(a^6-3a^4b^2+3a^2b^4-b^6\right)}{(a+b)^{7/2}(a-b)^{7/2}}}{10ab^4}\right)}{d\,(a+b)^{7/2}\,(a-b)^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a + b*sin(c + d*x))^2),x)
```

```
[Out] ((2*(3*b^5 - 2*a^4*b + 14*a^2*b^3))/(3*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2))
 - (10*b^5*tan(c/2 + (d*x)/2)^6)/(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2) - (2*t
an(c/2 + (d*x)/2)^2*(21*b^5 - 4*a^4*b + 28*a^2*b^3))/(3*(a^6 - b^6 + 3*a^2*
b^4 - 3*a^4*b^2)) + (2*tan(c/2 + (d*x)/2)*(3*a^6 + 3*b^6 + 22*a^2*b^4 - 13*
a^4*b^2))/(3*a*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) - (2*tan(c/2 + (d*x)/2)
^7*(b^6 - a^6 + 2*a^2*b^4 + 3*a^4*b^2))/(a*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b
^2)) + (2*tan(c/2 + (d*x)/2)^5*(a^6 + 9*b^6 + 38*a^2*b^4 - 3*a^4*b^2))/(3*a
*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) + (2*tan(c/2 + (d*x)/2)^3*(a^6 - 9*b^
6 - 46*a^2*b^4 + 9*a^4*b^2))/(3*a*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) + (1
0*b*tan(c/2 + (d*x)/2)^4*(5*b^4 - 2*a^4 + 6*a^2*b^2))/(3*(a^6 - b^6 + 3*a^2
*b^4 - 3*a^4*b^2)))/(d*(a + 2*b*tan(c/2 + (d*x)/2) - 2*a*tan(c/2 + (d*x)/2)
^2 + 2*a*tan(c/2 + (d*x)/2)^6 - a*tan(c/2 + (d*x)/2)^8 - 6*b*tan(c/2 + (d*x
)/2)^3 + 6*b*tan(c/2 + (d*x)/2)^5 - 2*b*tan(c/2 + (d*x)/2)^7)) + (10*a*b^4*
atan(((5*a*b^4*(2*a^6*b - 2*b^7 + 6*a^2*b^5 - 6*a^4*b^3))/((a + b)^(7/2)*(a
 - b)^(7/2)) + (10*a^2*b^4*tan(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^
4*b^2))/((a + b)^(7/2)*(a - b)^(7/2)))/(10*a*b^4)))/(d*(a + b)^(7/2)*(a - b
)^(7/2))
```

3.449 $\int \frac{\cos^7(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=190

$$-\frac{3(5a^4-6a^2b^2+b^4)\log(a+b\sin(c+dx))}{b^7d}+\frac{a(10a^2-9b^2)\sin(c+dx)}{b^6d}-\frac{3(2a^2-b^2)\sin^2(c+dx)}{2b^5d}+\frac{a\sin^3(c}{b^4}$$

```
[Out] -3*(5*a^4-6*a^2*b^2+b^4)*ln(a+b*sin(d*x+c))/b^7/d+a*(10*a^2-9*b^2)*sin(d*x+
c)/b^6/d-3/2*(2*a^2-b^2)*sin(d*x+c)^2/b^5/d+a*sin(d*x+c)^3/b^4/d-1/4*sin(d*
x+c)^4/b^3/d+1/2*(a^2-b^2)^3/b^7/d/(a+b*sin(d*x+c))^2-6*a*(a^2-b^2)^2/b^7/d
/(a+b*sin(d*x+c))
```

Rubi [A]
time = 0.11, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2747, 711}

$$-\frac{6a(a^2-b^2)^2}{b^7d(a+b\sin(c+dx))}+\frac{(a^2-b^2)^3}{2b^7d(a+b\sin(c+dx))^2}+\frac{a(10a^2-9b^2)\sin(c+dx)}{b^6d}-\frac{3(2a^2-b^2)\sin^2(c+dx)}{2b^5d}-\frac{3(5a^4-6a^2b^2+b^4)\log(a+b\sin(c+dx))}{b^7d}+\frac{a\sin^3(c+dx)}{b^4d}-\frac{\sin^4(c+dx)}{4b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-3*(5*a^4 - 6*a^2*b^2 + b^4)*Log[a + b*Sin[c + d*x]])/(b^7*d) + (a*(10*a^2
 - 9*b^2)*Sin[c + d*x])/(b^6*d) - (3*(2*a^2 - b^2)*Sin[c + d*x]^2)/(2*b^5*d
) + (a*Sin[c + d*x]^3)/(b^4*d) - Sin[c + d*x]^4/(4*b^3*d) + (a^2 - b^2)^3/(
2*b^7*d*(a + b*Sin[c + d*x])^2) - (6*a*(a^2 - b^2)^2)/(b^7*d*(a + b*Sin[c +
 d*x]))
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^7(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{(b^2-x^2)^3}{(a+x)^3}\,dx, x, b\sin(c+dx)\right)}{b^7 d}$$

$$= \frac{\text{Subst}\left(\int \left(10a^3\left(1-\frac{9b^2}{10a^2}\right) - 3(2a^2-b^2)\,x + 3ax^2 - x^3 - \frac{(a^2-b^2)^3}{(a+x)^3} + \frac{6a(a^2-b^2)^2}{(a+x)^2} - \right.\right.}{b^7 d}$$

$$= -\frac{3(5a^4-6a^2b^2+b^4)\log(a+b\sin(c+dx))}{b^7 d} + \frac{a(10a^2-9b^2)\sin(c+dx)}{b^6 d} - \frac{3(2a}{}$$

Mathematica [A]

time = 3.18, size = 165, normalized size = 0.87

$$-\frac{4b^2(-12a^2+5b^2)\cos(2(c+dx)) + b^4\cos(4(c+dx)) + 96(5a^4-6a^2b^2+b^4)\log(a+b\sin(c+dx)) - 8ab(40a^2-33b^2)\sin(c+dx) - \frac{16(a-b)^3(a+b)^3}{(a+b\sin(c+dx))^2} + \frac{192a(a-b)^2(a+b)^2}{a+b\sin(c+dx)} + 8ab^3\sin(3(c+dx))}{32b^7 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -1/32*(4*b^2*(-12*a^2 + 5*b^2)*Cos[2*(c + d*x)] + b^4*Cos[4*(c + d*x)] + 96
*(5*a^4 - 6*a^2*b^2 + b^4)*Log[a + b*Sin[c + d*x]] - 8*a*b*(40*a^2 - 33*b^2
)*Sin[c + d*x] - (16*(a - b)^3*(a + b)^3)/(a + b*Sin[c + d*x])^2 + (192*a*(
a - b)^2*(a + b)^2)/(a + b*Sin[c + d*x]) + 8*a*b^3*Sin[3*(c + d*x)])/(b^7*d
)
```

Maple [A]

time = 0.63, size = 196, normalized size = 1.03

method	result
derivativedivides	$\frac{\frac{-\frac{\left(\sin^4(dx+c)\right)b^3}{4}+a\left(\sin^3(dx+c)\right)b^2-3\left(\sin^2(dx+c)\right)a^2b+\frac{3\left(\sin^2(dx+c)\right)b^3}{2}+10a^3\sin(dx+c)-9\sin(dx+c)a\,b^2}{b^6}-\frac{-a^6+3a^4b^2-3a^2b^4-}{2b^7(a+b\sin(dx+c))}}{d}$
default	$\frac{\frac{-\frac{\left(\sin^4(dx+c)\right)b^3}{4}+a\left(\sin^3(dx+c)\right)b^2-3\left(\sin^2(dx+c)\right)a^2b+\frac{3\left(\sin^2(dx+c)\right)b^3}{2}+10a^3\sin(dx+c)-9\sin(dx+c)a\,b^2}{b^6}-\frac{-a^6+3a^4b^2-3a^2b^4-}{2b^7(a+b\sin(dx+c))}}{d}$
risch	$\frac{5ia^3e^{-i(dx+c)}}{b^6d}+\frac{30ia^4c}{d\,b^7}+\frac{6ic}{d\,b^3}-\frac{5ia^3e^{i(dx+c)}}{b^6d}+\frac{3\,e^{2i(dx+c)}a^2}{4b^5d}-\frac{5\,e^{2i(dx+c)}}{16b^3d}+\frac{ia\,e^{3i(dx+c)}}{8b^4d}-\frac{18ix\,a^2}{b^5}-33i$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7/(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/b^6*(-1/4*sin(d*x+c)^4*b^3+a*sin(d*x+c)^3*b^2-3*sin(d*x+c)^2*a^2*b+3
/2*sin(d*x+c)^2*b^3+10*a^3*sin(d*x+c)-9*sin(d*x+c)*a*b^2)-1/2/b^7*(-a^6+3*a
^4*b^2-3*a^2*b^4+b^6)/(a+b*sin(d*x+c))^2-6*a/b^7*(a^4-2*a^2*b^2+b^4)/(a+b*s
in(d*x+c))+(-15*a^4+18*a^2*b^2-3*b^4)/b^7*ln(a+b*sin(d*x+c)))
```

Maxima [A]
time = 0.27, size = 200, normalized size = 1.05

$$-\frac{\frac{2\,\left(11\,a^{6}-21\,a^{4}b^{2}+9\,a^{2}b^{4}+b^{6}+12\,\left(a^{5}b-2\,a^{3}b^{3}+ab^{5}\right)\sin\left(dx+c\right)\right)}{b^{9}\sin\left(dx+c\right)^{2}+2\,ab^{8}\sin\left(dx+c\right)+a^{2}b^{7}}+\frac{b^{3}\sin\left(dx+c\right)^{4}-4\,ab^{2}\sin\left(dx+c\right)^{3}+6\,\left(2\,a^{2}b-b^{3}\right)\sin\left(dx+c\right)^{2}-4\,\left(10\,a^{3}-9\,ab^{2}\right)\sin\left(dx+c\right)}{b^{6}}+\frac{12\,\left(5\,a^{4}-6\,a^{2}b^{2}+b^{4}\right)\log\left(b\sin\left(dx+c\right)+a\right)}{b^{7}}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/4*(2*(11*a^6 - 21*a^4*b^2 + 9*a^2*b^4 + b^6 + 12*(a^5*b - 2*a^3*b^3 + a*
b^5)*sin(d*x + c))/(b^9*sin(d*x + c)^2 + 2*a*b^8*sin(d*x + c) + a^2*b^7) +
(b^3*sin(d*x + c)^4 - 4*a*b^2*sin(d*x + c)^3 + 6*(2*a^2*b - b^3)*sin(d*x +
c)^2 - 4*(10*a^3 - 9*a*b^2)*sin(d*x + c))/b^6 + 12*(5*a^4 - 6*a^2*b^2 + b^4
)*log(b*sin(d*x + c) + a)/b^7)/d
```

Fricas [A]
time = 0.39, size = 304, normalized size = 1.60

$$-\frac{8\,b^{6}\cos\left(dx+c\right)^{6}-176\,a^{6}+928\,a^{4}b^{2}-685\,a^{2}b^{4}+3\,b^{6}-8\,\left(5\,a^{2}b^{4}-3\,b^{6}\right)\cos\left(dx+c\right)^{4}-\left(544\,a^{4}b^{2}-560\,a^{2}b^{4}+51\,b^{6}\right)\cos\left(dx+c\right)^{2}-96\,\left(5\,a^{6}-a^{4}b^{2}-5\,a^{2}b^{4}+b^{6}-\left(5\,a^{4}b^{2}-6\,a^{2}b^{4}+b^{6}\right)\cos\left(dx+c\right)^{2}+2\,\left(5\,a^{5}b-6\,a^{3}b^{3}+ab^{5}\right)\sin\left(dx+c\right)\right)\log\left(b\sin\left(dx+c\right)+a\right)+2\,\left(8\,ab^{5}\cos\left(dx+c\right)^{4}+64\,a^{5}b+176\,a^{3}b^{3}-205\,ab^{5}-80\,\left(a^{3}b^{3}-ab^{5}\right)\cos\left(dx+c\right)^{2}\right)\sin\left(dx+c\right)}{32\,\left(b^{9}d\cos\left(dx+c\right)^{2}-2\,ab^{8}d\sin\left(dx+c\right)-\left(a^{2}b^{7}+b^{9}\right)d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/32*(8*b^6*cos(d*x + c)^6 - 176*a^6 + 928*a^4*b^2 - 685*a^2*b^4 + 3*b^6 -
 8*(5*a^2*b^4 - 3*b^6)*cos(d*x + c)^4 - (544*a^4*b^2 - 560*a^2*b^4 + 51*b^6
)*cos(d*x + c)^2 - 96*(5*a^6 - a^4*b^2 - 5*a^2*b^4 + b^6 - (5*a^4*b^2 - 6*a
^2*b^4 + b^6)*cos(d*x + c)^2 + 2*(5*a^5*b - 6*a^3*b^3 + a*b^5)*sin(d*x + c)
)*log(b*sin(d*x + c) + a) + 2*(8*a*b^5*cos(d*x + c)^4 + 64*a^5*b + 176*a^3*
b^3 - 205*a*b^5 - 80*(a^3*b^3 - a*b^5)*cos(d*x + c)^2)*sin(d*x + c))/(b^9*d
*cos(d*x + c)^2 - 2*a*b^8*d*sin(d*x + c) - (a^2*b^7 + b^9)*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 3.21, size = 245, normalized size = 1.29

$$-\frac{\frac{12\,\left(5\,a^{4}-6\,a^{2}b^{2}+b^{4}\right)\log\left(\left|b\sin\left(dx+c\right)+a\right|\right)}{b^{7}}-\frac{2\,\left(45\,a^{4}b^{2}\sin\left(dx+c\right)^{2}-54\,a^{2}b^{4}\sin\left(dx+c\right)^{2}+9\,b^{6}\sin\left(dx+c\right)^{2}+78\,a^{5}b\sin\left(dx+c\right)-84\,a^{3}b^{3}\sin\left(dx+c\right)+6\,ab^{5}\sin\left(dx+c\right)+34\,a^{6}-33\,a^{4}b^{2}-b^{6}\right)}{\left(b\sin\left(dx+c\right)+a\right)^{2}b^{7}}+\frac{b^{9}\sin\left(dx+c\right)^{4}-4\,ab^{8}\sin\left(dx+c\right)^{3}+12\,a^{2}b^{7}\sin\left(dx+c\right)^{2}-6\,b^{9}\sin\left(dx+c\right)^{2}-40\,a^{3}b^{6}\sin\left(dx+c\right)+36\,ab^{8}\sin\left(dx+c\right)}{b^{12}}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/4*(12*(5*a^4 - 6*a^2*b^2 + b^4)*log(abs(b*sin(d*x + c) + a))/b^7 - 2*(45
*a^4*b^2*sin(d*x + c)^2 - 54*a^2*b^4*sin(d*x + c)^2 + 9*b^6*sin(d*x + c)^2
+ 78*a^5*b*sin(d*x + c) - 84*a^3*b^3*sin(d*x + c) + 6*a*b^5*sin(d*x + c) +
34*a^6 - 33*a^4*b^2 - b^6)/((b*sin(d*x + c) + a)^2*b^7) + (b^9*sin(d*x + c)
^4 - 4*a*b^8*sin(d*x + c)^3 + 12*a^2*b^7*sin(d*x + c)^2 - 6*b^9*sin(d*x + c
)^2 - 40*a^3*b^6*sin(d*x + c) + 36*a*b^8*sin(d*x + c))/b^12)/d
```

Mupad [B]
time = 0.12, size = 234, normalized size = 1.23

$$\frac{\sin(c+d\,x)^2\,\left(\frac{3}{2\,b^3}-\frac{3\,a^2}{b^5}\right)}{d}-\frac{\sin(c+d\,x)^4}{4\,b^3\,d}-\frac{\sin(c+d\,x)\,\left(\frac{8\,a^3}{b^6}+\frac{3\,a\,\left(\frac{3}{b^3}-\frac{6\,a^2}{b^5}\right)}{b}\right)}{d}-\frac{\frac{11\,a^6-21\,a^4\,b^2+9\,a^2\,b^4+b^6}{2\,b}+\sin(c+d\,x)\,\left(6\,a^5-12\,a^3\,b^2+6\,a\,b^4\right)}{d\,\left(a^2\,b^6+2\,a\,b^7\,\sin(c+d\,x)+b^8\,\sin(c+d\,x)^2\right)}+\frac{a\sin(c+d\,x)^3}{b^4\,d}-\frac{\ln(a+b\,\sin(c+d\,x))\,\left(15\,a^4-18\,a^2\,b^2+3\,b^4\right)}{b^7\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7/(a + b*sin(c + d*x))^3,x)
```

```
[Out] (sin(c + d*x)^2*(3/(2*b^3) - (3*a^2)/b^5))/d - sin(c + d*x)^4/(4*b^3*d) - (
sin(c + d*x)*((8*a^3)/b^6 + (3*a*(3/b^3 - (6*a^2)/b^5))/b))/d - ((11*a^6 +
b^6 + 9*a^2*b^4 - 21*a^4*b^2)/(2*b) + sin(c + d*x)*(6*a*b^4 + 6*a^5 - 12*a^
3*b^2))/(d*(a^2*b^6 + b^8*sin(c + d*x)^2 + 2*a*b^7*sin(c + d*x))) + (a*sin(
c + d*x)^3)/(b^4*d) - (log(a + b*sin(c + d*x))*(15*a^4 + 3*b^4 - 18*a^2*b^2
))/(b^7*d)
```

3.450 $\int \frac{\cos^5(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=127

$$\frac{2(3a^2-b^2)\log(a+b\sin(c+dx))}{b^5d}-\frac{3a\sin(c+dx)}{b^4d}+\frac{\sin^2(c+dx)}{2b^3d}-\frac{(a^2-b^2)^2}{2b^5d(a+b\sin(c+dx))^2}+\frac{4a(a^2-b^2)}{b^5d(a+b\sin(c+dx))}$$

```
[Out] 2*(3*a^2-b^2)*ln(a+b*sin(d*x+c))/b^5/d-3*a*sin(d*x+c)/b^4/d+1/2*sin(d*x+c)^
2/b^3/d-1/2*(a^2-b^2)^2/b^5/d/(a+b*sin(d*x+c))^2+4*a*(a^2-b^2)/b^5/d/(a+b*s
in(d*x+c))
```

Rubi [A]
time = 0.07, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2747, 711}

$$-\frac{(a^2-b^2)^2}{2b^5d(a+b\sin(c+dx))^2}+\frac{4a(a^2-b^2)}{b^5d(a+b\sin(c+dx))}+\frac{2(3a^2-b^2)\log(a+b\sin(c+dx))}{b^5d}-\frac{3a\sin(c+dx)}{b^4d}+\frac{\sin^2(c+dx)}{2b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (2*(3*a^2 - b^2)*Log[a + b*Sin[c + d*x]])/(b^5*d) - (3*a*Sin[c + d*x])/(b^4
*d) + Sin[c + d*x]^2/(2*b^3*d) - (a^2 - b^2)^2/(2*b^5*d*(a + b*Sin[c + d*x]
)^2) + (4*a*(a^2 - b^2))/(b^5*d*(a + b*Sin[c + d*x]))
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \frac{\text{Subst}\Big(\int \frac{(b^2-x^2)^2}{(a+x)^3}\,dx, x, b\sin(c+dx)\Big)}{b^5 d}$$

$$= \frac{\text{Subst}\Big(\int \Big(-3a+x+\frac{(a^2-b^2)^2}{(a+x)^3}-\frac{4(a^3-ab^2)}{(a+x)^2}+\frac{2(3a^2-b^2)}{a+x}\Big)\,dx, x, b\sin(c+dx)\Big)}{b^5 d}$$

$$= \frac{2(3a^2-b^2)\log(a+b\sin(c+dx))}{b^5 d} - \frac{3a\sin(c+dx)}{b^4 d} + \frac{\sin^2(c+dx)}{2b^3 d} - \frac{(a^2}{2b^5 d(a+b}$$

Mathematica [A]
time = 1.14, size = 106, normalized size = 0.83

$$-\frac{b^2\cos(2(c+dx)) - 8(3a^2-b^2)\log(a+b\sin(c+dx)) + 12ab\sin(c+dx) + \frac{2(a-b)^2(a+b)^2}{(a+b\sin(c+dx))^2} - \frac{16a(a-b)(a+b)}{a+b\sin(c+dx)}}{4b^5 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -1/4*(b^2*Cos[2*(c + d*x)] - 8*(3*a^2 - b^2)*Log[a + b*Sin[c + d*x]] + 12*a
*b*Sin[c + d*x] + (2*(a - b)^2*(a + b)^2)/(a + b*Sin[c + d*x])^2 - (16*a*(a
 - b)*(a + b))/(a + b*Sin[c + d*x]))/(b^5*d)
```

Maple [A]
time = 0.90, size = 117, normalized size = 0.92

method	result
derivativedivides	$\frac{-\frac{-\frac{(\sin^2(dx+c))b}{2}+3a\sin(dx+c)}{b^4}-\frac{a^4-2a^2b^2+b^4}{2b^5(a+b\sin(dx+c))^2}+\frac{4a(a^2-b^2)}{b^5(a+b\sin(dx+c))}+\frac{(6a^2-2b^2)\ln(a+b\sin(dx+c))}{b^5}}{d}$
default	$\frac{-\frac{-\frac{(\sin^2(dx+c))b}{2}+3a\sin(dx+c)}{b^4}-\frac{a^4-2a^2b^2+b^4}{2b^5(a+b\sin(dx+c))^2}+\frac{4a(a^2-b^2)}{b^5(a+b\sin(dx+c))}+\frac{(6a^2-2b^2)\ln(a+b\sin(dx+c))}{b^5}}{d}$
risch	$-\frac{6ix a^2}{b^5}+\frac{2ix}{b^3}-\frac{e^{2i(dx+c)}}{8b^3d}+\frac{3ia\,e^{i(dx+c)}}{2b^4d}-\frac{3ia\,e^{-i(dx+c)}}{2b^4d}-\frac{e^{-2i(dx+c)}}{8b^3d}-\frac{12ia^2c}{d\,b^5}+\frac{4ic}{d\,b^3}+\frac{-8ib\,a^3e^{3i(dx+c}}{}$
norman	$\frac{-\frac{(180a^4-68a^2b^2-10b^4)(\tan^4(\frac{dx}{2}+\frac{c}{2}))}{a^2b^3d}-\frac{(180a^4-68a^2b^2-10b^4)(\tan^{10}(\frac{dx}{2}+\frac{c}{2}))}{a^2b^3d}-\frac{(36a^4-12a^2b^2-2b^4)(\tan^2(\frac{dx}{2}+\frac{c}{2}))}{a^2b^3d}-\frac{(36a}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/b^4*(-1/2*sin(d*x+c)^2*b+3*a*sin(d*x+c))-1/2/b^5*(a^4-2*a^2*b^2+b^4
)/(a+b*sin(d*x+c))^2+4*a/b^5*(a^2-b^2)/(a+b*sin(d*x+c))+(6*a^2-2*b^2)/b^5*l
n(a+b*sin(d*x+c)))
```

Maxima [A]
time = 0.28, size = 131, normalized size = 1.03

$$\frac{\frac{7\,a^4-6\,a^2b^2-b^4+8\,\left(a^3b-ab^3\right)\sin\left(dx+c\right)}{b^7\sin\left(dx+c\right)^2+2\,ab^6\sin\left(dx+c\right)+a^2b^5}+\frac{b\sin\left(dx+c\right)^2-6\,a\sin\left(dx+c\right)}{b^4}+\frac{4\,\left(3\,a^2-b^2\right)\log\left(b\sin\left(dx+c\right)+a\right)}{b^5}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*((7*a^4 - 6*a^2*b^2 - b^4 + 8*(a^3*b - a*b^3)*sin(d*x + c))/(b^7*sin(d*
x + c)^2 + 2*a*b^6*sin(d*x + c) + a^2*b^5) + (b*sin(d*x + c)^2 - 6*a*sin(d*
x + c))/b^4 + 4*(3*a^2 - b^2)*log(b*sin(d*x + c) + a)/b^5)/d
```

Fricas [A]
time = 0.37, size = 212, normalized size = 1.67

$$-\frac{2\,b^4\cos\left(dx+c\right)^4+14\,a^4-35\,a^2b^2-b^4+\left(22\,a^2b^2-3\,b^4\right)\cos\left(dx+c\right)^2+8\,\left(3\,a^4+2\,a^2b^2-b^4-\left(3\,a^2b^2-b^4\right)\cos\left(dx+c\right)^2+2\,\left(3\,a^3b-ab^3\right)\sin\left(dx+c\right)\right)\log\left(b\sin\left(dx+c\right)+a\right)+2\,\left(4\,ab^3\cos\left(dx+c\right)^2+2\,a^3b-13\,ab^3\right)\sin\left(dx+c\right)}{4\,\left(b^7d\cos\left(dx+c\right)^2-2\,ab^6d\sin\left(dx+c\right)-\left(a^2b^5+b^7\right)d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*b^4*cos(d*x + c)^4 + 14*a^4 - 35*a^2*b^2 - b^4 + (22*a^2*b^2 - 3*b^
4)*cos(d*x + c)^2 + 8*(3*a^4 + 2*a^2*b^2 - b^4 - (3*a^2*b^2 - b^4)*cos(d*x
+ c)^2 + 2*(3*a^3*b - a*b^3)*sin(d*x + c))*log(b*sin(d*x + c) + a) + 2*(4*a
*b^3*cos(d*x + c)^2 + 2*a^3*b - 13*a*b^3)*sin(d*x + c))/(b^7*d*cos(d*x + c)
^2 - 2*a*b^6*d*sin(d*x + c) - (a^2*b^5 + b^7)*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 2.84, size = 142, normalized size = 1.12

$$\frac{\frac{4\,\left(3\,a^2-b^2\right)\log\left(\left|b\sin\left(dx+c\right)+a\right|\right)}{b^5}+\frac{b^3\sin\left(dx+c\right)^2-6\,ab^2\sin\left(dx+c\right)}{b^6}-\frac{18\,a^2b^2\sin\left(dx+c\right)^2-6\,b^4\sin\left(dx+c\right)^2+28\,a^3b\sin\left(dx+c\right)-4\,ab^3\sin\left(dx+c\right)+11\,a^4+b^4}{\left(b\sin\left(dx+c\right)+a\right)^2b^5}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(4*(3*a^2 - b^2)*log(abs(b*sin(d*x + c) + a))/b^5 + (b^3*sin(d*x + c)^2
 - 6*a*b^2*sin(d*x + c))/b^6 - (18*a^2*b^2*sin(d*x + c)^2 - 6*b^4*sin(d*x +
 c)^2 + 28*a^3*b*sin(d*x + c) - 4*a*b^3*sin(d*x + c) + 11*a^4 + b^4)/((b*si
n(d*x + c) + a)^2*b^5))/d
```

Mupad [B]
time = 5.14, size = 142, normalized size = 1.12

$$\frac{\sin(c+d\,x)^2}{2\,b^3\,d}-\frac{\frac{-7\,a^4+6\,a^2\,b^2+b^4}{2\,b}+\sin(c+d\,x)\,\left(4\,a\,b^2-4\,a^3\right)}{d\,\left(a^2\,b^4+2\,a\,b^5\,\sin(c+d\,x)+b^6\,\sin(c+d\,x)^2\right)}-\frac{3\,a\,\sin(c+d\,x)}{b^4\,d}+\frac{\ln\left(a+b\,\sin(c+d\,x)\right)\,\left(6\,a^2-2\,b^2\right)}{b^5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(a + b*sin(c + d*x))^3,x)
```

```
[Out] sin(c + d*x)^2/(2*b^3*d) - ((b^4 - 7*a^4 + 6*a^2*b^2)/(2*b) + sin(c + d*x)*
(4*a*b^2 - 4*a^3))/(d*(a^2*b^4 + b^6*sin(c + d*x)^2 + 2*a*b^5*sin(c + d*x))
) - (3*a*sin(c + d*x))/(b^4*d) + (log(a + b*sin(c + d*x))*(6*a^2 - 2*b^2))/
(b^5*d)
```

3.451 $\int \frac{\cos^3(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=72

$$-\frac{\log(a+b\sin(c+dx))}{b^3d}+\frac{a^2-b^2}{2b^3d(a+b\sin(c+dx))^2}-\frac{2a}{b^3d(a+b\sin(c+dx))}$$

```
[Out] -ln(a+b*sin(d*x+c))/b^3/d+1/2*(a^2-b^2)/b^3/d/(a+b*sin(d*x+c))^2-2*a/b^3/d/
(a+b*sin(d*x+c))
```

Rubi [A]
time = 0.05, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2747, 711}

$$\frac{a^2-b^2}{2b^3d(a+b\sin(c+dx))^2}-\frac{2a}{b^3d(a+b\sin(c+dx))}-\frac{\log(a+b\sin(c+dx))}{b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -(Log[a + b*Sin[c + d*x]]/(b^3*d)) + (a^2 - b^2)/(2*b^3*d*(a + b*Sin[c + d*
x])^2) - (2*a)/(b^3*d*(a + b*Sin[c + d*x]))
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^3(c+dx)}{(a+b\sin(c+dx))^3}\,dx &= \frac{\text{Subst}\left(\int \frac{b^2-x^2}{(a+x)^3}\,dx,x,b\sin(c+dx)\right)}{b^3d}\\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{-a-x}+\frac{-a^2+b^2}{(a+x)^3}+\frac{2a}{(a+x)^2}\right)dx,x,b\sin(c+dx)\right)}{b^3d}\\
&= -\frac{\log(a+b\sin(c+dx))}{b^3d}+\frac{a^2-b^2}{2b^3d(a+b\sin(c+dx))^2}-\frac{2a}{b^3d(a+b\sin(c+dx))}
\end{aligned}$$

Mathematica [A]
time = 0.34, size = 56, normalized size = 0.78

$$-\frac{2\log(a+b\sin(c+dx))+\frac{3a^2+b^2+4ab\sin(c+dx)}{(a+b\sin(c+dx))^2}}{2b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -1/2*(2*Log[a + b*Sin[c + d*x]] + (3*a^2 + b^2 + 4*a*b*Sin[c + d*x])/(a + b
*Sin[c + d*x])^2)/(b^3*d)
```

Maple [A]
time = 0.68, size = 66, normalized size = 0.92

method	result
derivativedivides	$\frac{-\frac{-a^2+b^2}{2b^3(a+b\sin(dx+c))^2}-\frac{2a}{b^3(a+b\sin(dx+c))}-\frac{\ln(a+b\sin(dx+c))}{b^3}}{d}$
default	$\frac{-\frac{-a^2+b^2}{2b^3(a+b\sin(dx+c))^2}-\frac{2a}{b^3(a+b\sin(dx+c))}-\frac{\ln(a+b\sin(dx+c))}{b^3}}{d}$
risch	$\frac{ix}{b^3}+\frac{2ic}{d\,b^3}-\frac{2i\left(3ia^2e^{2i(dx+c)}+ib^2e^{2i(dx+c)}+2ab\,e^{3i(dx+c)}-2a\,e^{i(dx+c)}b\right)}{\left(b\,e^{2i(dx+c)}-b+2ia\,e^{i(dx+c)}\right)^2d\,b^3}-\frac{\ln\left(e^{2i(dx+c)}+\frac{2ia\,e^{i(dx+c)}}{b}-1\right)}{d\,b^3}$
norman	$\frac{\frac{\left(6a^2+2b^2\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2bd}+\frac{\left(6a^2+2b^2\right)\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2bd}+\frac{2\left(9a^2+3b^2\right)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{bd\,a^2}+\frac{2\left(9a^2+3b^2\right)\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{bd\,a^2}+\frac{2\left(a^2+b^2\right.}{}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\left(a\left(\tan^2\left(\right.\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2*(-a^2+b^2)/b^3/(a+b*sin(d*x+c))^2-2*a/b^3/(a+b*sin(d*x+c))-1/b^3*
ln(a+b*sin(d*x+c)))
```

Maxima [A]
time = 0.29, size = 76, normalized size = 1.06

$$-\frac{\frac{4\,ab\sin(dx+c)+3\,a^2+b^2}{b^5\sin(dx+c)^2+2\,ab^4\sin(dx+c)+a^2b^3}+\frac{2\,\log(b\sin(dx+c)+a)}{b^3}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*((4*a*b*sin(d*x + c) + 3*a^2 + b^2)/(b^5*sin(d*x + c)^2 + 2*a*b^4*sin(
d*x + c) + a^2*b^3) + 2*log(b*sin(d*x + c) + a)/b^3)/d
```

Fricas [A]
time = 0.36, size = 110, normalized size = 1.53

$$\frac{4\,ab\sin\left(dx+c\right)+3\,a^{2}+b^{2}-2\,\left(b^{2}\cos\left(dx+c\right)^{2}-2\,ab\sin\left(dx+c\right)-a^{2}-b^{2}\right)\log\left(b\sin\left(dx+c\right)+a\right)}{2\,\left(b^{5}d\cos\left(dx+c\right)^{2}-2\,ab^{4}d\sin\left(dx+c\right)-\left(a^{2}b^{3}+b^{5}\right)d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(4*a*b*sin(d*x + c) + 3*a^2 + b^2 - 2*(b^2*cos(d*x + c)^2 - 2*a*b*sin(d
*x + c) - a^2 - b^2)*log(b*sin(d*x + c) + a))/(b^5*d*cos(d*x + c)^2 - 2*a*b
^4*d*sin(d*x + c) - (a^2*b^3 + b^5)*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 398 vs. $2(61) = 122$.
time = 0.78, size = 398, normalized size = 5.53

$$\begin{cases} \frac{x\cos^{3}\left(c\right)}{a^{3}} & \text{for}\: b=0 \wedge d=0 \\ \frac{\frac{2\sin^{3}\left(c+dx\right)}{3d}+\frac{\sin\left(c+dx\right)\cos^{2}\left(c+dx\right)}{d}}{a^{3}} & \text{for}\: b=0 \\ \frac{x\cos^{3}\left(c\right)}{\left(a+b\sin\left(c\right)\right)^{3}} & \text{for}\: d=0 \\ -\frac{2a^{2}\log\left(\frac{a}{b}+\sin\left(c+dx\right)\right)}{2a^{2}b^{3}d+4ab^{4}d\sin\left(c+dx\right)+2b^{5}d\sin^{2}\left(c+dx\right)}-\frac{2a^{2}}{2a^{2}b^{3}d+4ab^{4}d\sin\left(c+dx\right)+2b^{5}d\sin^{2}\left(c+dx\right)}-\frac{4ab\log\left(\frac{a}{b}+\sin\left(c+dx\right)\right)\sin\left(c+dx\right)}{2a^{2}b^{3}d+4ab^{4}d\sin\left(c+dx\right)+2b^{5}d\sin^{2}\left(c+dx\right)}-\frac{2ab\sin\left(c+dx\right)}{2a^{2}b^{3}d+4ab^{4}d\sin\left(c+dx\right)+2b^{5}d\sin^{2}\left(c+dx\right)}-\frac{2b^{2}\log\left(\frac{a}{b}+\sin\left(c+dx\right)\right)\sin^{2}\left(c+dx\right)}{2a^{2}b^{3}d+4ab^{4}d\sin\left(c+dx\right)+2b^{5}d\sin^{2}\left(c+dx\right)}-\frac{b^{2}\cos^{2}\left(c+dx\right)}{2a^{2}b^{3}d+4ab^{4}d\sin\left(c+dx\right)+2b^{5}d\sin^{2}\left(c+dx\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((x*cos(c)**3/a**3, Eq(b, 0) & Eq(d, 0)), ((2*sin(c + d*x)**3/(3*d
) + sin(c + d*x)*cos(c + d*x)**2/d)/a**3, Eq(b, 0)), (x*cos(c)**3/(a + b*si
n(c))**3, Eq(d, 0)), (-2*a**2*log(a/b + sin(c + d*x))/(2*a**2*b**3*d + 4*a*
b**4*d*sin(c + d*x) + 2*b**5*d*sin(c + d*x)**2) - 2*a**2/(2*a**2*b**3*d + 4
*a*b**4*d*sin(c + d*x) + 2*b**5*d*sin(c + d*x)**2) - 4*a*b*log(a/b + sin(c
+ d*x))*sin(c + d*x)/(2*a**2*b**3*d + 4*a*b**4*d*sin(c + d*x) + 2*b**5*d*si
n(c + d*x)**2) - 2*a*b*sin(c + d*x)/(2*a**2*b**3*d + 4*a*b**4*d*sin(c + d*x
) + 2*b**5*d*sin(c + d*x)**2) - 2*b**2*log(a/b + sin(c + d*x))*sin(c + d*x)
**2/(2*a**2*b**3*d + 4*a*b**4*d*sin(c + d*x) + 2*b**5*d*sin(c + d*x)**2) -
b**2*cos(c + d*x)**2/(2*a**2*b**3*d + 4*a*b**4*d*sin(c + d*x) + 2*b**5*d*si
n(c + d*x)**2), True))
```

Giac [A]
time = 5.45, size = 62, normalized size = 0.86

$$-\frac{\frac{2\,\log\left(\left|b\sin\left(dx+c\right)+a\right|\right)}{b^{3}}+\frac{4\,a\sin\left(dx+c\right)+\frac{3\,a^{2}+b^{2}}{b}}{\left(b\sin\left(dx+c\right)+a\right)^{2}b^{2}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*log(abs(b*sin(d*x + c) + a))/b^3 + (4*a*sin(d*x + c) + (3*a^2 + b^2
)/b)/((b*sin(d*x + c) + a)^2*b^2))/d
```

Mupad [B]
time = 0.09, size = 80, normalized size = 1.11

$$-\frac{\ln\left(a+b\,\sin\left(c+d\,x\right)\right)}{b^3\,d}-\frac{\frac{3\,a^2+b^2}{2\,b^3}+\frac{2\,a\,\sin(c+d\,x)}{b^2}}{d\,\left(a^2+2\,a\,b\,\sin\left(c+d\,x\right)+b^2\,{\sin\left(c+d\,x\right)}^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a + b*sin(c + d*x))^3,x)
```

```
[Out] - log(a + b*sin(c + d*x))/(b^3*d) - ((3*a^2 + b^2)/(2*b^3) + (2*a*sin(c + d
*x))/b^2)/(d*(a^2 + b^2*sin(c + d*x)^2 + 2*a*b*sin(c + d*x)))
```

3.452 $\int \frac{\cos(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=22

$$-\frac{1}{2bd(a+b\sin(c+dx))^2}$$

```
[Out] -1/2/b/d/(a+b*sin(d*x+c))^2
```

Rubi [A]
time = 0.02, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2747, 32}

$$-\frac{1}{2bd(a+b\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -1/2*1/(b*d*(a + b*Sin[c + d*x])^2)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)}{(a+b\sin(c+dx))^3}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{(a+x)^3}\,dx, x, b\sin(c+dx)\right)}{bd}\\ &= -\frac{1}{2bd(a+b\sin(c+dx))^2}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 22, normalized size = 1.00

$$-\frac{1}{2bd(a+b\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + b*Sin[c + d*x])^3,x]
[Out] -1/2*1/(b*d*(a + b*Sin[c + d*x])^2)
```

Maple [A]
time = 0.35, size = 21, normalized size = 0.95

method	result	size
derivativedivides	$-\frac{1}{2bd(a+b\sin(dx+c))^2}$	21
default	$-\frac{1}{2bd(a+b\sin(dx+c))^2}$	21
risch	$\frac{2\,\mathrm{e}^{2i(dx+c)}}{\left(b\,\mathrm{e}^{2i(dx+c)}-b+2ia\,\mathrm{e}^{i(dx+c)}\right)^2 db}$	48
norman	$\frac{\frac{2b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2d}+\frac{2b\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2d}+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{da}+\frac{4\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}+\frac{2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a\right)^2}$	142

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
[Out] -1/2/b/d/(a+b*sin(d*x+c))^2
```

Maxima [A]
time = 0.26, size = 20, normalized size = 0.91

$$-\frac{1}{2\,(b\sin\left(dx+c\right)+a)^2bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
[Out] -1/2/((b*sin(d*x + c) + a)^2*b*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 43 vs. $2(20) = 40$.
time = 0.35, size = 43, normalized size = 1.95

$$\frac{1}{2\left(b^3d\cos\left(dx+c\right)^2-2\,ab^2d\sin\left(dx+c\right)-(a^2b+b^3)d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
[Out] 1/2/(b^3*d*cos(d*x + c)^2 - 2*a*b^2*d*sin(d*x + c) - (a^2*b + b^3)*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(19) = 38$.
time = 0.73, size = 73, normalized size = 3.32

$$\begin{cases} \frac{x\cos(c)}{a^3} & \text{for } b = 0 \wedge d = 0 \\ \frac{\sin(c+dx)}{a^3d} & \text{for } b = 0 \\ \frac{x\cos(c)}{(a+b\sin(c))^3} & \text{for } d = 0 \\ -\frac{1}{2a^2bd+4ab^2d\sin(c+dx)+2b^3d\sin^2(c+dx)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((x*cos(c)/a**3, Eq(b, 0) & Eq(d, 0)), (sin(c + d*x)/(a**3*d), Eq(
b, 0)), (x*cos(c)/(a + b*sin(c))**3, Eq(d, 0)), (-1/(2*a**2*b*d + 4*a*b**2*
d*sin(c + d*x) + 2*b**3*d*sin(c + d*x)**2), True))
```

Giac [A]
time = 5.43, size = 20, normalized size = 0.91

$$-\frac{1}{2(b\sin(dx+c)+a)^2bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2/((b*sin(d*x + c) + a)^2*b*d)
```

Mupad [B]
time = 0.06, size = 39, normalized size = 1.77

$$-\frac{1}{d\left(2a^2b+4ab^2\sin(c+dx)+2b^3\sin(c+dx)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + b*sin(c + d*x))^3,x)
```

```
[Out] -1/(d*(2*a^2*b + 2*b^3*sin(c + d*x)^2 + 4*a*b^2*sin(c + d*x)))
```

3.453 $\int \frac{\sec(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=145

$$-\frac{\log(1-\sin(c+dx))}{2(a+b)^3d}+\frac{\log(1+\sin(c+dx))}{2(a-b)^3d}-\frac{b(3a^2+b^2)\log(a+b\sin(c+dx))}{(a^2-b^2)^3\,d}+\frac{b}{2\,(a^2-b^2)\,d(a+b\sin(c+$$

```
[Out] -1/2*ln(1-sin(d*x+c))/(a+b)^3/d+1/2*ln(1+sin(d*x+c))/(a-b)^3/d-b*(3*a^2+b^2
)*ln(a+b*sin(d*x+c))/(a^2-b^2)^3/d+1/2*b/(a^2-b^2)/d/(a+b*sin(d*x+c))^2+2*a
*b/(a^2-b^2)^2/d/(a+b*sin(d*x+c))
```

Rubi [A]
time = 0.11, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2747, 724, 815}

$$\frac{2ab}{d\,(a^2-b^2)^2\,(a+b\sin(c+dx))}+\frac{b}{2d\,(a^2-b^2)\,(a+b\sin(c+dx))^2}-\frac{b(3a^2+b^2)\log(a+b\sin(c+dx))}{d\,(a^2-b^2)^3}-\frac{\log(1-\sin(c+dx))}{2d(a+b)^3}+\frac{\log(\sin(c+dx)+1)}{2d(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -1/2*Log[1 - Sin[c + d*x]]/((a + b)^3*d) + Log[1 + Sin[c + d*x]]/(2*(a - b)
^3*d) - (b*(3*a^2 + b^2)*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)^3*d) + b/(2*
(a^2 - b^2)*d*(a + b*Sin[c + d*x])^2) + (2*a*b)/((a^2 - b^2)^2*d*(a + b*Sin
[c + d*x]))
```

Rule 724

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Simp[e*((d
 + e*x)^(m + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + Dist[c/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*((d - e*x)/(a + c*x^2)), x], x] /; FreeQ[{a, c, d, e, m
}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \frac{b\text{Subst}\left(\int \frac{1}{(a+x)^3(b^2-x^2)}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{b}{2(a^2-b^2)\,d(a+b\sin(c+dx))^2} + \frac{b\text{Subst}\left(\int \frac{a-x}{(a+x)^2(b^2-x^2)}\,dx, x, b\sin(c+dx)\right)}{(a^2-b^2)\,d}$$
$$= \frac{b}{2(a^2-b^2)\,d(a+b\sin(c+dx))^2} + \frac{b\text{Subst}\left(\int \left(\frac{a-b}{2b(a+b)^2(b-x)} - \frac{2a}{(a-b)(a+b)(a+x)^2} + \right.\right.}{(}$$
$$= -\frac{\log(1-\sin(c+dx))}{2(a+b)^3d} + \frac{\log(1+\sin(c+dx))}{2(a-b)^3d} - \frac{b(3a^2+b^2)\log(a+b\sin(c+}{(a^2-b^2)^3\,d}$$

Mathematica [A]

time = 0.63, size = 135, normalized size = 0.93

$$\frac{b\left(-\frac{\log(1-\sin(c+dx))}{b(a+b)^3} + \frac{\log(1+\sin(c+dx))}{(a-b)^3b} - \frac{2(3a^2+b^2)\log(a+b\sin(c+dx))}{(a-b)^3(a+b)^3} + \frac{1}{(a^2-b^2)(a+b\sin(c+dx))^2} + \frac{4a}{(a-b)^2(a+b)^2(a+b\sin(c+dx))}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (b*(-(Log[1 - Sin[c + d*x]]/(b*(a + b)^3)) + Log[1 + Sin[c + d*x]]/((a - b)
^3*b) - (2*(3*a^2 + b^2)*Log[a + b*Sin[c + d*x]])/((a - b)^3*(a + b)^3) + 1
/((a^2 - b^2)*(a + b*Sin[c + d*x])^2) + (4*a)/((a - b)^2*(a + b)^2*(a + b*S
in[c + d*x]))))/(2*d)
```

Maple [A]

time = 0.74, size = 130, normalized size = 0.90

method	result
derivativedivides	$\frac{\frac{\ln(1+\sin(dx+c))}{2(a-b)^3} - \frac{\ln(\sin(dx+c)-1)}{2(a+b)^3} + \frac{b}{2(a-b)(a+b)(a+b\sin(dx+c))^2} + \frac{2ab}{(a+b)^2(a-b)^2(a+b\sin(dx+c))} - \frac{b\left(3a^2+b^2\right)\ln(a+b\sin(dx+c}{(a-b)^3(a+b)^3}}{d}$
default	$\frac{\frac{\ln(1+\sin(dx+c))}{2(a-b)^3} - \frac{\ln(\sin(dx+c)-1)}{2(a+b)^3} + \frac{b}{2(a-b)(a+b)(a+b\sin(dx+c))^2} + \frac{2ab}{(a+b)^2(a-b)^2(a+b\sin(dx+c))} - \frac{b\left(3a^2+b^2\right)\ln(a+b\sin(dx+c}{(a-b)^3(a+b)^3}}{d}$
norman	$\frac{\frac{6a^2b^2-2b^4}{4db\left(a^4-2a^2b^2+b^4\right)} + \frac{\left(6a^2b^2-2b^4\right)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4db\left(a^4-2a^2b^2+b^4\right)} + \frac{\left(6a^3b^2-10a\,b^4\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2adb\left(a^4-2a^2b^2+b^4\right)}}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a\right)^2} + \frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d(a^3-3a^2b+3a\,b^2-b^3)} - \frac{\ln\left(\text{ta}\right.}{(a^3+3}$
risch	$\frac{ix}{a^3+3a^2b+3a\,b^2+b^3} + \frac{ic}{(a^3+3a^2b+3a\,b^2+b^3)d} - \frac{ix}{a^3-3a^2b+3a\,b^2-b^3} - \frac{ic}{d(a^3-3a^2b+3a\,b^2-b^3)} + \frac{6ib\,a^2x}{a^6-3a^4b^2+3a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2/(a-b)^3*ln(1+sin(d*x+c))-1/2/(a+b)^3*ln(sin(d*x+c)-1)+1/2*b/(a-b)/
(a+b)/(a+b*sin(d*x+c))^2+2*a*b/(a+b)^2/(a-b)^2/(a+b*sin(d*x+c))-b*(3*a^2+b^
2)/(a-b)^3/(a+b)^3*ln(a+b*sin(d*x+c)))
```

Maxima [A]
time = 0.27, size = 223, normalized size = 1.54

$$-\frac{\frac{2\,(3\,a^2b+b^3)\log(b\sin(dx+c)+a)}{a^6-3\,a^4b^2+3\,a^2b^4-b^6}-\frac{4\,ab^2\sin(dx+c)+5\,a^2b-b^3}{a^6-2\,a^4b^2+a^2b^4+(a^4b^2-2\,a^2b^4+b^6)\sin(dx+c)^2+2\,(a^5b-2\,a^3b^3+ab^5)\sin(dx+c)}-\frac{\log(\sin(dx+c)+1)}{a^3-3\,a^2b+3\,ab^2-b^3}+\frac{\log(\sin(dx+c)-1)}{a^3+3\,a^2b+3\,ab^2+b^3}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*(2*(3*a^2*b + b^3)*log(b*sin(d*x + c) + a)/(a^6 - 3*a^4*b^2 + 3*a^2*b^
4 - b^6) - (4*a*b^2*sin(d*x + c) + 5*a^2*b - b^3)/(a^6 - 2*a^4*b^2 + a^2*b^
4 + (a^4*b^2 - 2*a^2*b^4 + b^6)*sin(d*x + c)^2 + 2*(a^5*b - 2*a^3*b^3 + a*b
^5)*sin(d*x + c)) - log(sin(d*x + c) + 1)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) +
 log(sin(d*x + c) - 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 462 vs. $2(139) = 278$.
time = 0.43, size = 462, normalized size = 3.19

$$-\frac{5\,a^4b-6\,a^2b^3+b^5-2\,(3\,a^4b+4\,a^2b^3+b^5-(3\,a^2b^3+b^5)\cos(dx+c)^2+2\,(3\,a^3b^2+ab^4)\sin(dx+c))\log(b\sin(dx+c)+a)+(a^5+3\,a^4b+4\,a^3b^2+4\,a^2b^3+3\,ab^4+b^5-(a^3b^2+3\,a^2b^3+3\,ab^4+b^5)\cos(dx+c)^2+2\,(a^4b+3\,a^3b^2+3\,a^2b^3+ab^4)\sin(dx+c))\log(\sin(dx+c)+1)-(a^5-3\,a^4b+4\,a^3b^2-4\,a^2b^3+3\,ab^4-b^5-(a^3b^2-3\,a^2b^3+3\,ab^4-b^5)\cos(dx+c)^2+2\,(a^4b-3\,a^3b^2+3\,a^2b^3-ab^4)\sin(dx+c))\log(-\sin(dx+c)+1)+4\,(a^3b^2-ab^4)\sin(dx+c)}{2\,((a^6b^2-3\,a^4b^4+3\,a^2b^6-b^8)d\cos(dx+c)^2-2\,(a^7b-3\,a^5b^3+3\,a^3b^5-ab^7)d\sin(dx+c)-(a^8-2\,a^6b^2+2\,a^2b^6-b^8)d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(5*a^4*b - 6*a^2*b^3 + b^5 - 2*(3*a^4*b + 4*a^2*b^3 + b^5 - (3*a^2*b^3
 + b^5)*cos(d*x + c)^2 + 2*(3*a^3*b^2 + a*b^4)*sin(d*x + c))*log(b*sin(d*x
+ c) + a) + (a^5 + 3*a^4*b + 4*a^3*b^2 + 4*a^2*b^3 + 3*a*b^4 + b^5 - (a^3*b
^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*cos(d*x + c)^2 + 2*(a^4*b + 3*a^3*b^2 + 3*a
^2*b^3 + a*b^4)*sin(d*x + c))*log(sin(d*x + c) + 1) - (a^5 - 3*a^4*b + 4*a^
3*b^2 - 4*a^2*b^3 + 3*a*b^4 - b^5 - (a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*c
os(d*x + c)^2 + 2*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*sin(d*x + c))*log
(-sin(d*x + c) + 1) + 4*(a^3*b^2 - a*b^4)*sin(d*x + c))/((a^6*b^2 - 3*a^4*b
^4 + 3*a^2*b^6 - b^8)*d*cos(d*x + c)^2 - 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 -
 a*b^7)*d*sin(d*x + c) - (a^8 - 2*a^6*b^2 + 2*a^2*b^6 - b^8)*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(c+dx)}{(a+b\sin(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)/(a + b*sin(c + d*x))**3, x)
```

Giac [A]
time = 4.94, size = 242, normalized size = 1.67

$$-\frac{\frac{2\,\left(3\,a^2b^2+b^4\right)\log(|b\sin(dx+c)+a|)}{a^6b-3\,a^4b^3+3\,a^2b^5-b^7}-\frac{\log(|\sin(dx+c)+1|)}{a^3-3\,a^2b+3\,ab^2-b^3}+\frac{\log(|\sin(dx+c)-1|)}{a^3+3\,a^2b+3\,ab^2+b^3}-\frac{9\,a^2b^3\sin(dx+c)^2+3\,b^5\sin(dx+c)^2+22\,a^3b^2\sin(dx+c)+2\,ab^4\sin(dx+c)+14\,a^4b-3\,a^2b^3+b^5}{(a^6-3\,a^4b^2+3\,a^2b^4-b^6)(b\sin(dx+c)+a)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*(3*a^2*b^2 + b^4)*log(abs(b*sin(d*x + c) + a))/(a^6*b - 3*a^4*b^3 +
 3*a^2*b^5 - b^7) - log(abs(sin(d*x + c) + 1))/(a^3 - 3*a^2*b + 3*a*b^2 - b
^3) + log(abs(sin(d*x + c) - 1))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - (9*a^2*b
^3*sin(d*x + c)^2 + 3*b^5*sin(d*x + c)^2 + 22*a^3*b^2*sin(d*x + c) + 2*a*b^
4*sin(d*x + c) + 14*a^4*b - 3*a^2*b^3 + b^5)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4
- b^6)*(b*sin(d*x + c) + a)^2))/d
```

Mupad [B]
time = 5.40, size = 169, normalized size = 1.17

$$\frac{\ln\left(a+b\,\sin\left(c+d\,x\right)\right)\,\left(\frac{1}{2\,{\left(a+b\right)}^3}-\frac{1}{2\,{\left(a-b\right)}^3}\right)}{d}+\frac{\frac{5\,a^2\,b-b^3}{2\,\left(a^4-2\,a^2\,b^2+b^4\right)}+\frac{2\,a\,b^2\,\sin\left(c+d\,x\right)}{a^4-2\,a^2\,b^2+b^4}}{d\,\left(a^2+2\,a\,b\,\sin\left(c+d\,x\right)+b^2\,{\sin\left(c+d\,x\right)}^2\right)}+\frac{\ln\left(\sin\left(c+d\,x\right)+1\right)}{2\,d\,{\left(a-b\right)}^3}-\frac{\ln\left(\sin\left(c+d\,x\right)-1\right)}{2\,d\,{\left(a+b\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + b*sin(c + d*x))^3),x)
```

```
[Out] (log(a + b*sin(c + d*x))*(1/(2*(a + b)^3) - 1/(2*(a - b)^3)))/d + ((5*a^2*b
 - b^3)/(2*(a^4 + b^4 - 2*a^2*b^2)) + (2*a*b^2*sin(c + d*x))/(a^4 + b^4 - 2
*a^2*b^2))/(d*(a^2 + b^2*sin(c + d*x)^2 + 2*a*b*sin(c + d*x))) + log(sin(c
+ d*x) + 1)/(2*d*(a - b)^3) - log(sin(c + d*x) - 1)/(2*d*(a + b)^3)
```

3.454 $\int \frac{\sec^3(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=226

$$-\frac{(a+4b)\log(1-\sin(c+dx))}{4(a+b)^4d}+\frac{(a-4b)\log(1+\sin(c+dx))}{4(a-b)^4d}+\frac{2b^3(5a^2+b^2)\log(a+b\sin(c+dx))}{(a^2-b^2)^4\,d}-\frac{}{2\,(a^2 -}$$

```
[Out] -1/4*(a+4*b)*ln(1-sin(d*x+c))/(a+b)^4/d+1/4*(a-4*b)*ln(1+sin(d*x+c))/(a-b)^
4/d+2*b^3*(5*a^2+b^2)*ln(a+b*sin(d*x+c))/(a^2-b^2)^4/d-1/2*b*(a^2+2*b^2)/(a
^2-b^2)^2/d/(a+b*sin(d*x+c))^2-1/2*sec(d*x+c)^2*(b-a*sin(d*x+c))/(a^2-b^2)/
d/(a+b*sin(d*x+c))^2-1/2*a*b*(a^2+11*b^2)/(a^2-b^2)^3/d/(a+b*sin(d*x+c))
```

Rubi [A]

time = 0.19, antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2747, 755, 815}

$$-\frac{ab(a^2+11b^2)}{2d\,(a^2-b^2)^3\,(a+b\sin(c+dx))}-\frac{b(a^2+2b^2)}{2d\,(a^2-b^2)^2\,(a+b\sin(c+dx))^2}-\frac{\sec^2(c+dx)(b-a\sin(c+dx))}{2d\,(a^2-b^2)\,(a+b\sin(c+dx))^2}+\frac{2b^3(5a^2+b^2)\log(a+b\sin(c+dx))}{d\,(a^2-b^2)^4}-\frac{(a+4b)\log(1-\sin(c+dx))}{4d(a+b)^4}+\frac{(a-4b)\log(\sin(c+dx)+1)}{4d(a-b)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -1/4*((a + 4*b)*Log[1 - Sin[c + d*x]])/((a + b)^4*d) + ((a - 4*b)*Log[1 + S
in[c + d*x]])/(4*(a - b)^4*d) + (2*b^3*(5*a^2 + b^2)*Log[a + b*Sin[c + d*x]
])/((a^2 - b^2)^4*d) - (b*(a^2 + 2*b^2))/(2*(a^2 - b^2)^2*d*(a + b*Sin[c +
d*x])^2) - (Sec[c + d*x]^2*(b - a*Sin[c + d*x]))/(2*(a^2 - b^2)*d*(a + b*Si
n[c + d*x])^2) - (a*b*(a^2 + 11*b^2))/(2*(a^2 - b^2)^3*d*(a + b*Sin[c + d*x
]))
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \frac{b^3\text{Subst}\left(\int \frac{1}{(a+x)^3(b^2-x^2)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= -\frac{\sec^2(c+dx)(b-a\sin(c+dx))}{2(a^2-b^2)\,d(a+b\sin(c+dx))^2} + \frac{b\text{Subst}\left(\int \frac{a^2-4b^2+3ax}{(a+x)^3(b^2-x^2)}\,dx, x, b\sin(c+dx)\right)}{2(a^2-b^2)\,d}$$

$$= -\frac{\sec^2(c+dx)(b-a\sin(c+dx))}{2(a^2-b^2)\,d(a+b\sin(c+dx))^2} + \frac{b\text{Subst}\left(\int \left(\frac{(a-b)(a+4b)}{2b(a+b)^3(b-x)} + \frac{2(a^2+2b^2)}{(a-b)(a+b)(a+x)^3}\right.\right.}{}$$

$$= -\frac{(a+4b)\log(1-\sin(c+dx))}{4(a+b)^4d} + \frac{(a-4b)\log(1+\sin(c+dx))}{4(a-b)^4d} + \frac{2b^3(5a^2+b^2}{}$$

Mathematica [A]
time = 4.08, size = 283, normalized size = 1.25

$$\frac{\frac{\sec^2(c+dx)(b-a\sin(c+dx))}{(a+b\sin(c+dx))^2} + b(a^2+2b^2)\left(-\frac{\log(1-\sin(c+dx))}{b(a+b)^3} + \frac{\log(1+\sin(c+dx))}{(a-b)^3b} - \frac{2(3a^2+b^2)\log(a+b\sin(c+dx))}{(a-b)^3(a+b)^3} + \frac{1}{(a^2-b^2)(a+b\sin(c+dx))^2} + \frac{4a}{(a-b)^2(a+b)^2(a+b\sin(c+dx))}\right) + \frac{3}{2}a\left(\frac{\log(1-\sin(c+dx))}{(a+b)^2} - \frac{\log(1+\sin(c+dx))}{(a-b)^2} + \frac{4ab\log(a+b\sin(c+dx))}{(a-b)^2(a+b)^2} + \frac{2b}{(-a^2+b^2)(a+b\sin(c+dx))}\right)}{2(-a^2+b^2)\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((Sec[c + d*x]^2*(b - a*Sin[c + d*x]))/(a + b*Sin[c + d*x])^2 + b*(a^2 + 2*
b^2)*(-(Log[1 - Sin[c + d*x]]/(b*(a + b)^3)) + Log[1 + Sin[c + d*x]]/((a -
b)^3*b) - (2*(3*a^2 + b^2)*Log[a + b*Sin[c + d*x]])/((a - b)^3*(a + b)^3) +
 1/((a^2 - b^2)*(a + b*Sin[c + d*x])^2) + (4*a)/((a - b)^2*(a + b)^2*(a + b
*Sin[c + d*x]))) + (3*a*(Log[1 - Sin[c + d*x]]/(a + b)^2 - Log[1 + Sin[c +
d*x]]/(a - b)^2 + (4*a*b*Log[a + b*Sin[c + d*x]])/((a - b)^2*(a + b)^2) + (
2*b)/((-a^2 + b^2)*(a + b*Sin[c + d*x]))))/2)/(2*(-a^2 + b^2)*d)
```

Maple [A]
time = 1.18, size = 184, normalized size = 0.81 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/4/(a-b)^3/(1+sin(d*x+c))+1/4*(a-4*b)/(a-b)^4*ln(1+sin(d*x+c))-1/4/(
a+b)^3/(sin(d*x+c)-1)+1/4/(a+b)^4*(-a-4*b)*ln(sin(d*x+c)-1)-1/2*b^3/(a+b)^2
```

```
/(a-b)^2/(a+b*sin(d*x+c))^2-4*a*b^3/(a-b)^3/(a+b)^3/(a+b*sin(d*x+c))+2*b^3*
(5*a^2+b^2)/(a+b)^4/(a-b)^4*ln(a+b*sin(d*x+c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 438 vs. $2(217) = 434$.
time = 0.29, size = 438, normalized size = 1.94

$$\frac{\frac{8\,(5\,a^2b^3+b^5)\log(b\sin(dx+c)+a)}{a^8-4\,a^6b^2+6\,a^4b^4-4\,a^2b^6+b^8}+\frac{(a-4\,b)\log(\sin(dx+c)+1)}{a^4-4\,a^3b+6\,a^2b^2-4\,ab^3+b^4}-\frac{(a+4\,b)\log(\sin(dx+c)-1)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4}-\frac{2\left(3\,a^4b+10\,a^2b^3-b^5-(a^3b^2+11\,ab^4)\sin(dx+c)^3-2\,(a^4b+6\,a^2b^3-b^5)\sin(dx+c)^2-(a^5-3\,a^3b^2-10\,ab^4)\sin(dx+c)\right)}{a^8-3\,a^6b^2+3\,a^4b^4-a^2b^6-(a^6b^2-3\,a^4b^4+3\,a^2b^6-b^8)\sin(dx+c)^4-2\,(a^7b-3\,a^5b^3+3\,a^3b^5-ab^7)\sin(dx+c)^3-(a^8-4\,a^6b^2+6\,a^4b^4-4\,a^2b^6+b^8)\sin(dx+c)^2+2\,(a^7b-3\,a^5b^3+3\,a^3b^5-ab^7)\sin(dx+c)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/4*(8*(5*a^2*b^3 + b^5)*log(b*sin(d*x + c) + a)/(a^8 - 4*a^6*b^2 + 6*a^4*b
^4 - 4*a^2*b^6 + b^8) + (a - 4*b)*log(sin(d*x + c) + 1)/(a^4 - 4*a^3*b + 6*
a^2*b^2 - 4*a*b^3 + b^4) - (a + 4*b)*log(sin(d*x + c) - 1)/(a^4 + 4*a^3*b +
 6*a^2*b^2 + 4*a*b^3 + b^4) - 2*(3*a^4*b + 10*a^2*b^3 - b^5 - (a^3*b^2 + 11
*a*b^4)*sin(d*x + c)^3 - 2*(a^4*b + 6*a^2*b^3 - b^5)*sin(d*x + c)^2 - (a^5
- 3*a^3*b^2 - 10*a*b^4)*sin(d*x + c))/(a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^
6 - (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*sin(d*x + c)^4 - 2*(a^7*b - 3*a
^5*b^3 + 3*a^3*b^5 - a*b^7)*sin(d*x + c)^3 - (a^8 - 4*a^6*b^2 + 6*a^4*b^4 -
 4*a^2*b^6 + b^8)*sin(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7
)*sin(d*x + c)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 707 vs. $2(217) = 434$.
time = 0.52, size = 707, normalized size = 3.13

$$\frac{2\,a^6b-6\,a^4b^3+6\,a^2b^5-2\,b^7+4\,(a^6b+5\,a^4b^3-7\,a^2b^5+b^7)\cos(dx+c)^2+8\left((5\,a^2b^5+b^7)\cos(dx+c)^4-2\,(5\,a^3b^4+ab^6)\cos(dx+c)^2\sin(dx+c)-(5\,a^4b^3+6\,a^2b^5+b^7)\cos(dx+c)^2\right)\log(b\sin(dx+c)+a)+\left((a^5b^2-10\,a^3b^4-20\,a^2b^5-15\,ab^6-4\,b^7)\cos(dx+c)^4-2\,(a^6b-10\,a^4b^3-20\,a^3b^4-15\,a^2b^5-4\,ab^6)\cos(dx+c)^2\sin(dx+c)-(a^7-9\,a^5b^2-20\,a^4b^3-25\,a^3b^4-24\,a^2b^5-15\,ab^6-4\,b^7)\cos(dx+c)^2\right)\log(\sin(dx+c)+1)-\left((a^5b^2-10\,a^3b^4+20\,a^2b^5-15\,ab^6+4\,b^7)\cos(dx+c)^4-2\,(a^6b-10\,a^4b^3+20\,a^3b^4-15\,a^2b^5+4\,ab^6)\cos(dx+c)^2\sin(dx+c)-(a^7-9\,a^5b^2+20\,a^4b^3-25\,a^3b^4+24\,a^2b^5-15\,ab^6+4\,b^7)\cos(dx+c)^2\right)\log(-\sin(dx+c)+1)-2\left(a^7-3\,a^5b^2+3\,a^3b^4-ab^6-(a^5b^2+10\,a^3b^4-11\,ab^6)\cos(dx+c)^2\right)\sin(dx+c)}{4\left((a^8b^2-4\,a^6b^4+6\,a^4b^6-4\,a^2b^8+b^{10})d\cos(dx+c)^4-2\,(a^9b-4\,a^7b^3+6\,a^5b^5-4\,a^3b^7+ab^9)d\cos(dx+c)^2\sin(dx+c)-(a^{10}-3\,a^8b^2+2\,a^6b^4+2\,a^4b^6-3\,a^2b^8+b^{10})d\cos(dx+c)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*a^6*b - 6*a^4*b^3 + 6*a^2*b^5 - 2*b^7 + 4*(a^6*b + 5*a^4*b^3 - 7*a^2
*b^5 + b^7)*cos(d*x + c)^2 + 8*((5*a^2*b^5 + b^7)*cos(d*x + c)^4 - 2*(5*a^3
*b^4 + a*b^6)*cos(d*x + c)^2*sin(d*x + c) - (5*a^4*b^3 + 6*a^2*b^5 + b^7)*c
os(d*x + c)^2)*log(b*sin(d*x + c) + a) + ((a^5*b^2 - 10*a^3*b^4 - 20*a^2*b^
5 - 15*a*b^6 - 4*b^7)*cos(d*x + c)^4 - 2*(a^6*b - 10*a^4*b^3 - 20*a^3*b^4 -
 15*a^2*b^5 - 4*a*b^6)*cos(d*x + c)^2*sin(d*x + c) - (a^7 - 9*a^5*b^2 - 20*
a^4*b^3 - 25*a^3*b^4 - 24*a^2*b^5 - 15*a*b^6 - 4*b^7)*cos(d*x + c)^2)*log(s
in(d*x + c) + 1) - ((a^5*b^2 - 10*a^3*b^4 + 20*a^2*b^5 - 15*a*b^6 + 4*b^7)*
cos(d*x + c)^4 - 2*(a^6*b - 10*a^4*b^3 + 20*a^3*b^4 - 15*a^2*b^5 + 4*a*b^6)
*cos(d*x + c)^2*sin(d*x + c) - (a^7 - 9*a^5*b^2 + 20*a^4*b^3 - 25*a^3*b^4 +
 24*a^2*b^5 - 15*a*b^6 + 4*b^7)*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) - 2*
(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6 - (a^5*b^2 + 10*a^3*b^4 - 11*a*b^6)*co
s(d*x + c)^2)*sin(d*x + c))/((a^8*b^2 - 4*a^6*b^4 + 6*a^4*b^6 - 4*a^2*b^8 +
```

```
 b^10)*d*cos(d*x + c)^4 - 2*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b^7 + a*
b^9)*d*cos(d*x + c)^2*sin(d*x + c) - (a^10 - 3*a^8*b^2 + 2*a^6*b^4 + 2*a^4*
b^6 - 3*a^2*b^8 + b^10)*d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(c+dx)}{(a+b\sin(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**3/(a + b*sin(c + d*x))**3, x)
```

Giac [A]
time = 5.41, size = 413, normalized size = 1.83

$$\frac{\frac{8\,(5\,a^2b^4+b^6)\log(|b\sin(dx+c)+a|)}{a^8b-4\,a^6b^3+6\,a^4b^5-4\,a^2b^7+b^9}+\frac{(a-4\,b)\log(|\sin(dx+c)+1|)}{a^4-4\,a^3b+6\,a^2b^2-4\,ab^3+b^4}-\frac{(a+4\,b)\log(|\sin(dx+c)-1|)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4}+\frac{2\left(10\,a^2b^3\sin(dx+c)^2+2\,b^5\sin(dx+c)^2-a^5\sin(dx+c)-2\,a^3b^2\sin(dx+c)+3\,ab^4\sin(dx+c)+3\,a^4b-12\,a^2b^3-3\,b^5\right)}{(a^8-4\,a^6b^2+6\,a^4b^4-4\,a^2b^6+b^8)\left(\sin(dx+c)^2-1\right)}-\frac{2\left(30\,a^2b^5\sin(dx+c)^2+6\,b^7\sin(dx+c)^2+68\,a^3b^4\sin(dx+c)+4\,ab^6\sin(dx+c)+39\,a^4b^3-4\,a^2b^5+b^7\right)}{(a^8-4\,a^6b^2+6\,a^4b^4-4\,a^2b^6+b^8)(b\sin(dx+c)+a)^2}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/4*(8*(5*a^2*b^4 + b^6)*log(abs(b*sin(d*x + c) + a))/(a^8*b - 4*a^6*b^3 +
6*a^4*b^5 - 4*a^2*b^7 + b^9) + (a - 4*b)*log(abs(sin(d*x + c) + 1))/(a^4 -
4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4) - (a + 4*b)*log(abs(sin(d*x + c) - 1))
/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) + 2*(10*a^2*b^3*sin(d*x + c)^2
 + 2*b^5*sin(d*x + c)^2 - a^5*sin(d*x + c) - 2*a^3*b^2*sin(d*x + c) + 3*a*b
^4*sin(d*x + c) + 3*a^4*b - 12*a^2*b^3 - 3*b^5)/((a^8 - 4*a^6*b^2 + 6*a^4*b
^4 - 4*a^2*b^6 + b^8)*(sin(d*x + c)^2 - 1)) - 2*(30*a^2*b^5*sin(d*x + c)^2
+ 6*b^7*sin(d*x + c)^2 + 68*a^3*b^4*sin(d*x + c) + 4*a*b^6*sin(d*x + c) + 3
9*a^4*b^3 - 4*a^2*b^5 + b^7)/((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^
8)*(b*sin(d*x + c) + a)^2))/d
```

Mupad [B]
time = 5.78, size = 388, normalized size = 1.72

$$\frac{\ln(a+b\sin(c+d\,x))\left(\frac{3\,b}{4\,(a+b)^4}+\frac{1}{4\,(a+b)^3}+\frac{3\,b}{4\,(a-b)^4}-\frac{1}{4\,(a-b)^3}\right)}{d}-\frac{\ln(\sin(c+d\,x)-1)\left(\frac{3\,b}{4\,(a+b)^4}+\frac{1}{4\,(a+b)^3}\right)}{d}+\frac{\frac{3\,a^4\,b+10\,a^2\,b^3-b^5}{2\,(a^2-b^2)\,(a^4-2\,a^2\,b^2+b^4)}-\frac{\sin(c+d\,x)^3\,(a^3\,b^2+11\,a\,b^4)}{2\,(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6)}-\frac{\sin(c+d\,x)^2\,(a^4\,b+6\,a^2\,b^3-b^5)}{(a^2-b^2)\,(a^4-2\,a^2\,b^2+b^4)}+\frac{a\,\sin(c+d\,x)\,(-a^4+3\,a^2\,b^2+10\,b^4)}{2\,(a^2-b^2)\,(a^4-2\,a^2\,b^2+b^4)}}{d\,\left(\sin(c+d\,x)^2\,(a^2-b^2)-a^2+b^2\sin(c+d\,x)^4-2\,a\,b\,\sin(c+d\,x)+2\,a\,b\sin(c+d\,x)^3\right)}+\frac{\ln(\sin(c+d\,x)+1)\,(a-4\,b)}{4\,d\,(a-b)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + b*sin(c + d*x))^3),x)
```

```
[Out] (log(a + b*sin(c + d*x))*((3*b)/(4*(a + b)^4) + 1/(4*(a + b)^3) + (3*b)/(4*
(a - b)^4) - 1/(4*(a - b)^3)))/d - (log(sin(c + d*x) - 1)*((3*b)/(4*(a + b)
^4) + 1/(4*(a + b)^3)))/d + ((3*a^4*b - b^5 + 10*a^2*b^3)/(2*(a^2 - b^2)*(a
^4 + b^4 - 2*a^2*b^2)) - (sin(c + d*x)^3*(11*a*b^4 + a^3*b^2))/(2*(a^6 - b^
```

```
6 + 3*a^2*b^4 - 3*a^4*b^2)) - (sin(c + d*x)^2*(a^4*b - b^5 + 6*a^2*b^3))/((
a^2 - b^2)*(a^4 + b^4 - 2*a^2*b^2)) + (a*sin(c + d*x)*(10*b^4 - a^4 + 3*a^2
*b^2))/(2*(a^2 - b^2)*(a^4 + b^4 - 2*a^2*b^2)))/(d*(sin(c + d*x)^2*(a^2 - b
^2) - a^2 + b^2*sin(c + d*x)^4 - 2*a*b*sin(c + d*x) + 2*a*b*sin(c + d*x)^3)
) + (log(sin(c + d*x) + 1)*(a - 4*b))/(4*d*(a - b)^4)
```

3.455 $\int \frac{\sec^5(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=328

$$-\frac{3(a^2+5ab+8b^2)\log(1-\sin(c+dx))}{16(a+b)^5d}+\frac{3(a^2-5ab+8b^2)\log(1+\sin(c+dx))}{16(a-b)^5d}-\frac{3b^5(7a^2+b^2)\log(a+b\sin(c+dx))}{(a^2-b^2)^5\,d}$$

```
[Out] -3/16*(a^2+5*a*b+8*b^2)*ln(1-sin(d*x+c))/(a+b)^5/d+3/16*(a^2-5*a*b+8*b^2)*l
n(1+sin(d*x+c))/(a-b)^5/d-3*b^5*(7*a^2+b^2)*ln(a+b*sin(d*x+c))/(a^2-b^2)^5/
d-3/8*b*(a^4-5*a^2*b^2-4*b^4)/(a^2-b^2)^3/d/(a+b*sin(d*x+c))^2-1/4*sec(d*x+
c)^4*(b-a*sin(d*x+c))/(a^2-b^2)/d/(a+b*sin(d*x+c))^2-3/8*a*b*(a^4-6*a^2*b^2
-27*b^4)/(a^2-b^2)^4/d/(a+b*sin(d*x+c))+1/8*sec(d*x+c)^2*(2*b*(a^2+3*b^2)+a
*(3*a^2-11*b^2)*sin(d*x+c))/(a^2-b^2)^2/d/(a+b*sin(d*x+c))^2
```

Rubi [A]
time = 0.29, antiderivative size = 328, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2747, 755, 837, 815}

$$-\frac{3(a^2+5ab+8b^2)\log(1-\sin(c+dx))}{16d(a+b)^5}+\frac{3(a^2-5ab+8b^2)\log(\sin(c+dx)+1)}{16d(a-b)^5}-\frac{\sec^4(c+dx)(b-a\sin(c+dx))}{4d(a^2-b^2)(a+b\sin(c+dx))^2}+\frac{\sec^2(c+dx)\left(a(3a^2-11b^2)\sin(c+dx)+2b(a^2+3b^2)\right)}{8d(a^2-b^2)^2(a+b\sin(c+dx))^2}-\frac{3b^5(7a^2+b^2)\log(a+b\sin(c+dx))}{d(a^2-b^2)^5}-\frac{3ab(a^4-6a^2b^2-27b^4)}{8d(a^2-b^2)^4(a+b\sin(c+dx))}-\frac{3b(a^4-5a^2b^2-4b^4)}{8d(a^2-b^2)^3(a+b\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-3*(a^2 + 5*a*b + 8*b^2)*Log[1 - Sin[c + d*x]])/(16*(a + b)^5*d) + (3*(a^2
 - 5*a*b + 8*b^2)*Log[1 + Sin[c + d*x]])/(16*(a - b)^5*d) - (3*b^5*(7*a^2 +
 b^2)*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)^5*d) - (3*b*(a^4 - 5*a^2*b^2 -
4*b^4))/(8*(a^2 - b^2)^3*d*(a + b*Sin[c + d*x])^2) - (Sec[c + d*x]^4*(b - a
*Sin[c + d*x]))/(4*(a^2 - b^2)*d*(a + b*Sin[c + d*x])^2) - (3*a*b*(a^4 - 6*
a^2*b^2 - 27*b^4))/(8*(a^2 - b^2)^4*d*(a + b*Sin[c + d*x])) + (Sec[c + d*x]
^2*(2*b*(a^2 + 3*b^2) + a*(3*a^2 - 11*b^2)*Sin[c + d*x]))/(8*(a^2 - b^2)^2*
d*(a + b*Sin[c + d*x])^2)
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
```

```
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \frac{b^5\text{Subst}\left(\int \frac{1}{(a+x)^3(b^2-x^2)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= -\frac{\sec^4(c+dx)(b-a\sin(c+dx))}{4\left(a^2-b^2\right)d(a+b\sin(c+dx))^2} + \frac{b^3\text{Subst}\left(\int \frac{3(a^2-2b^2)+5ax}{(a+x)^3(b^2-x^2)^2}\,dx, x, b\sin(c+dx)\right)}{4\left(a^2-b^2\right)d}$$

$$= -\frac{\sec^4(c+dx)(b-a\sin(c+dx))}{4\left(a^2-b^2\right)d(a+b\sin(c+dx))^2} + \frac{\sec^2(c+dx)\left(2b(a^2+3b^2)+a(3a^2-11b^2)\right.}{8\left(a^2-b^2\right)^2d(a+b\sin(c+dx))}$$

$$= -\frac{\sec^4(c+dx)(b-a\sin(c+dx))}{4\left(a^2-b^2\right)d(a+b\sin(c+dx))^2} + \frac{\sec^2(c+dx)\left(2b(a^2+3b^2)+a(3a^2-11b^2)\right.}{8\left(a^2-b^2\right)^2d(a+b\sin(c+dx))}$$

$$= -\frac{3(a^2+5ab+8b^2)\log(1-\sin(c+dx))}{16(a+b)^5d} + \frac{3(a^2-5ab+8b^2)\log(1+\sin(c+dx}{16(a-b)^5d}$$

Mathematica [A]
time = 2.84, size = 388, normalized size = 1.18

$$\frac{\frac{2\sec^4(c+dx)(b-a\sin(c+dx))}{(a+b\sin(c+dx))^2}+\frac{\sec^2(c+dx)\left(2b(a^2+3b^2)+a\left(3a^2-11b^2\right)\sin(c+dx)\right)}{(-a^2+b^2)(a+b\sin(c+dx))^2}-\frac{b\left(3\left(a^4-5a^2b^2-4b^4\right)\left(-\frac{\log(1-\sin(c+dx))}{b(a+b)^3}+\frac{\log(1+\sin(c+dx))}{(a-b)^3b}-\frac{2\left(3a^2+b^2\right)\log(a+b\sin(c+dx))}{(a-b)^3(a+b)^3}+\frac{1}{\left(a^2-b^2\right)(a+b\sin(c+dx))^2}+\frac{4a}{(a-b)^2(a+b)^2(a+b\sin(c+dx))}\right)-3a\left(3a^2-11b^2\right)\left(-\frac{\log(1-\sin(c+dx))}{2b(a+b)^2}+\frac{\log(1+\sin(c+dx))}{2(a-b)^2b}-\frac{2a\log(a+b\sin(c+dx))}{(a-b)^2(a+b)^2}+\frac{1}{\left(a^2-b^2\right)(a+b\sin(c+dx))}\right)\right)}{-a^2+b^2}}{8\left(-a^2+b^2\right)d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((2*Sec[c + d*x]^4*(b - a*Sin[c + d*x]))/(a + b*Sin[c + d*x])^2 + (Sec[c +
d*x]^2*(2*b*(a^2 + 3*b^2) + a*(3*a^2 - 11*b^2)*Sin[c + d*x]))/((-a^2 + b^2)
*(a + b*Sin[c + d*x])^2) - (b*(3*(a^4 - 5*a^2*b^2 - 4*b^4)*(-(Log[1 - Sin[c
 + d*x]]/(b*(a + b)^3)) + Log[1 + Sin[c + d*x]]/((a - b)^3*b) - (2*(3*a^2 +
 b^2)*Log[a + b*Sin[c + d*x]])/((a - b)^3*(a + b)^3) + 1/((a^2 - b^2)*(a +
b*Sin[c + d*x])^2) + (4*a)/((a - b)^2*(a + b)^2*(a + b*Sin[c + d*x]))) - 3*
a*(3*a^2 - 11*b^2)*(-1/2*Log[1 - Sin[c + d*x]]/(b*(a + b)^2) + Log[1 + Sin[
c + d*x]]/(2*(a - b)^2*b) - (2*a*Log[a + b*Sin[c + d*x]])/((a - b)^2*(a + b
)^2) + 1/((a^2 - b^2)*(a + b*Sin[c + d*x])))))/(-a^2 + b^2))/(8*(-a^2 + b^2
)*d)
```

Maple [A]
time = 2.19, size = 252, normalized size = 0.77 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/16/(a-b)^3/(1+sin(d*x+c))^2-1/16*(3*a-9*b)/(a-b)^4/(1+sin(d*x+c))+1
/16/(a-b)^5*(3*a^2-15*a*b+24*b^2)*ln(1+sin(d*x+c))+1/16/(a+b)^3/(sin(d*x+c)
-1)^2-1/16*(3*a+9*b)/(a+b)^4/(sin(d*x+c)-1)+1/16/(a+b)^5*(-3*a^2-15*a*b-24*
b^2)*ln(sin(d*x+c)-1)+1/2*b^5/(a+b)^3/(a-b)^3/(a+b*sin(d*x+c))^2+6*a*b^5/(a
+b)^4/(a-b)^4/(a+b*sin(d*x+c))-3*b^5*(7*a^2+b^2)/(a-b)^5/(a+b)^5*ln(a+b*sin
(d*x+c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 725 vs. $2(317) = 634$.
time = 0.30, size = 725, normalized size = 2.21

$$-\frac{\frac{48\,(7\,a^2b^5+b^7)\log(b\sin(dx+c)+a)}{a^{10}-5\,a^8b^2+10\,a^6b^4-10\,a^4b^6+5\,a^2b^8-b^{10}}-\frac{3\,(a^2-5\,ab+8\,b^2)\log(\sin(dx+c)+1)}{a^5-5\,a^4b+10\,a^3b^2-10\,a^2b^3+5\,ab^4-b^5}+\frac{3\,(a^2+5\,ab+8\,b^2)\log(\sin(dx+c)-1)}{a^5+5\,a^4b+10\,a^3b^2+10\,a^2b^3+5\,ab^4+b^5}+\frac{2\left(6\,a^6b-44\,a^4b^3-62\,a^2b^5+4\,b^7+3\,(a^5b^2-6\,a^3b^4-27\,ab^6)\sin(dx+c)^5+6\,(a^6b-6\,a^4b^3-13\,a^2b^5+2\,b^7)\sin(dx+c)^4+(3\,a^7-23\,a^5b^2+61\,a^3b^4+151\,ab^6)\sin(dx+c)^3-2\,(5\,a^6b-37\,a^4b^3-73\,a^2b^5+9\,b^7)\sin(dx+c)^2-(5\,a^7-26\,a^5b^2+49\,a^3b^4+68\,ab^6)\sin(dx+c)\right)}{a^{10}-4\,a^8b^2+6\,a^6b^4-4\,a^4b^6+a^2b^8+(a^8b^2-4\,a^6b^4+6\,a^4b^6-4\,a^2b^8+b^{10})\sin(dx+c)^6+2\,(a^9b-4\,a^7b^3+6\,a^5b^5-4\,a^3b^7+ab^9)\sin(dx+c)^5+(a^{10}-6\,a^8b^2+14\,a^6b^4-16\,a^4b^6+9\,a^2b^8-2\,b^{10})\sin(dx+c)^4-4\,(a^9b-4\,a^7b^3+6\,a^5b^5-4\,a^3b^7+ab^9)\sin(dx+c)^3-(2\,a^{10}-9\,a^8b^2+16\,a^6b^4-14\,a^4b^6+6\,a^2b^8-b^{10})\sin(dx+c)^2+2\,(a^9b-4\,a^7b^3+6\,a^5b^5-4\,a^3b^7+ab^9)\sin(dx+c)}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/16*(48*(7*a^2*b^5 + b^7)*log(b*sin(d*x + c) + a)/(a^10 - 5*a^8*b^2 + 10*
a^6*b^4 - 10*a^4*b^6 + 5*a^2*b^8 - b^10) - 3*(a^2 - 5*a*b + 8*b^2)*log(sin(
d*x + c) + 1)/(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5) + 3
*(a^2 + 5*a*b + 8*b^2)*log(sin(d*x + c) - 1)/(a^5 + 5*a^4*b + 10*a^3*b^2 +
10*a^2*b^3 + 5*a*b^4 + b^5) + 2*(6*a^6*b - 44*a^4*b^3 - 62*a^2*b^5 + 4*b^7
+ 3*(a^5*b^2 - 6*a^3*b^4 - 27*a*b^6)*sin(d*x + c)^5 + 6*(a^6*b - 6*a^4*b^3
- 13*a^2*b^5 + 2*b^7)*sin(d*x + c)^4 + (3*a^7 - 23*a^5*b^2 + 61*a^3*b^4 + 1
51*a*b^6)*sin(d*x + c)^3 - 2*(5*a^6*b - 37*a^4*b^3 - 73*a^2*b^5 + 9*b^7)*si
n(d*x + c)^2 - (5*a^7 - 26*a^5*b^2 + 49*a^3*b^4 + 68*a*b^6)*sin(d*x + c))/(
a^10 - 4*a^8*b^2 + 6*a^6*b^4 - 4*a^4*b^6 + a^2*b^8 + (a^8*b^2 - 4*a^6*b^4 +
 6*a^4*b^6 - 4*a^2*b^8 + b^10)*sin(d*x + c)^6 + 2*(a^9*b - 4*a^7*b^3 + 6*a^
5*b^5 - 4*a^3*b^7 + a*b^9)*sin(d*x + c)^5 + (a^10 - 6*a^8*b^2 + 14*a^6*b^4
```

- 16*a^4*b^6 + 9*a^2*b^8 - 2*b^10)*sin(d*x + c)^4 - 4*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b^7 + a*b^9)*sin(d*x + c)^3 - (2*a^10 - 9*a^8*b^2 + 16*a^6*b^4 - 14*a^4*b^6 + 6*a^2*b^8 - b^10)*sin(d*x + c)^2 + 2*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b^7 + a*b^9)*sin(d*x + c)))/d

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 895 vs. $2(317) = 634$.
time = 0.71, size = 895, normalized size = 2.73

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/16*(4*a^8*b - 16*a^6*b^3 + 24*a^4*b^5 - 16*a^2*b^7 + 4*b^9 + 12*(a^8*b -
7*a^6*b^3 - 7*a^4*b^5 + 15*a^2*b^7 - 2*b^9)*cos(d*x + c)^4 - 4*(a^8*b - 6*a
^4*b^5 + 8*a^2*b^7 - 3*b^9)*cos(d*x + c)^2 - 48*((7*a^2*b^7 + b^9)*cos(d*x
+ c)^6 - 2*(7*a^3*b^6 + a*b^8)*cos(d*x + c)^4*sin(d*x + c) - (7*a^4*b^5 + 8
*a^2*b^7 + b^9)*cos(d*x + c)^4)*log(b*sin(d*x + c) + a) + 3*((a^7*b^2 - 7*a
^5*b^4 + 35*a^3*b^6 + 56*a^2*b^7 + 35*a*b^8 + 8*b^9)*cos(d*x + c)^6 - 2*(a^
8*b - 7*a^6*b^3 + 35*a^4*b^5 + 56*a^3*b^6 + 35*a^2*b^7 + 8*a*b^8)*cos(d*x +
 c)^4*sin(d*x + c) - (a^9 - 6*a^7*b^2 + 28*a^5*b^4 + 56*a^4*b^5 + 70*a^3*b^
6 + 64*a^2*b^7 + 35*a*b^8 + 8*b^9)*cos(d*x + c)^4)*log(sin(d*x + c) + 1) -
3*((a^7*b^2 - 7*a^5*b^4 + 35*a^3*b^6 - 56*a^2*b^7 + 35*a*b^8 - 8*b^9)*cos(d
*x + c)^6 - 2*(a^8*b - 7*a^6*b^3 + 35*a^4*b^5 - 56*a^3*b^6 + 35*a^2*b^7 - 8
*a*b^8)*cos(d*x + c)^4*sin(d*x + c) - (a^9 - 6*a^7*b^2 + 28*a^5*b^4 - 56*a^
4*b^5 + 70*a^3*b^6 - 64*a^2*b^7 + 35*a*b^8 - 8*b^9)*cos(d*x + c)^4)*log(-si
n(d*x + c) + 1) - 2*(2*a^9 - 8*a^7*b^2 + 12*a^5*b^4 - 8*a^3*b^6 + 2*a*b^8 -
 3*(a^7*b^2 - 7*a^5*b^4 - 21*a^3*b^6 + 27*a*b^8)*cos(d*x + c)^4 + (3*a^9 -
20*a^7*b^2 + 42*a^5*b^4 - 36*a^3*b^6 + 11*a*b^8)*cos(d*x + c)^2)*sin(d*x +
c))/((a^10*b^2 - 5*a^8*b^4 + 10*a^6*b^6 - 10*a^4*b^8 + 5*a^2*b^10 - b^12)*d
*cos(d*x + c)^6 - 2*(a^11*b - 5*a^9*b^3 + 10*a^7*b^5 - 10*a^5*b^7 + 5*a^3*b
^9 - a*b^11)*d*cos(d*x + c)^4*sin(d*x + c) - (a^12 - 4*a^10*b^2 + 5*a^8*b^4
 - 5*a^4*b^8 + 4*a^2*b^10 - b^12)*d*cos(d*x + c)^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^5(c+dx)}{(a+b\sin(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**5/(a + b*sin(c + d*x))**3, x)
```

Giac [A]
time = 4.47, size = 575, normalized size = 1.75

$$-\frac{\frac{48\,(7\,a^2b^6+b^8)\log(|b\sin(dx+c)+a|)}{a^{10}b-5\,a^8b^3+10\,a^6b^5-10\,a^4b^7+5\,a^2b^9-b^{11}}-\frac{3\,(a^2-5\,ab+8\,b^2)\log(|\sin(dx+c)+1|)}{a^5-5\,a^4b+10\,a^3b^2-10\,a^2b^3+5\,ab^4-b^5}+\frac{3\,(a^2+5\,ab+8\,b^2)\log(|\sin(dx+c)-1|)}{a^5+5\,a^4b+10\,a^3b^2+10\,a^2b^3+5\,ab^4+b^5}+\frac{2\left(3\,a^5b^2\sin(dx+c)^5-18\,a^3b^4\sin(dx+c)^5-81\,ab^6\sin(dx+c)^5+6\,a^6b\sin(dx+c)^4-36\,a^4b^3\sin(dx+c)^4-78\,a^2b^5\sin(dx+c)^4+12\,b^7\sin(dx+c)^4+3\,a^7\sin(dx+c)^3-23\,a^5b^2\sin(dx+c)^3+61\,a^3b^4\sin(dx+c)^3+151\,ab^6\sin(dx+c)^3-10\,a^6b\sin(dx+c)^2+74\,a^4b^3\sin(dx+c)^2+146\,a^2b^5\sin(dx+c)^2-18\,b^7\sin(dx+c)^2-5\,a^7\sin(dx+c)+26\,a^5b^2\sin(dx+c)-49\,a^3b^4\sin(dx+c)-68\,ab^6\sin(dx+c)+6\,a^6b-44\,a^4b^3-62\,a^2b^5+4\,b^7\right)}{(a^8-4\,a^6b^2+6\,a^4b^4-4\,a^2b^6+b^8)\left(b\sin(dx+c)^3+a\sin(dx+c)^2-b\sin(dx+c)-a\right)^2}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/16*(48*(7*a^2*b^6 + b^8)*log(abs(b*sin(d*x + c) + a))/(a^10*b - 5*a^8*b^
3 + 10*a^6*b^5 - 10*a^4*b^7 + 5*a^2*b^9 - b^11) - 3*(a^2 - 5*a*b + 8*b^2)*l
og(abs(sin(d*x + c) + 1))/(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^
4 - b^5) + 3*(a^2 + 5*a*b + 8*b^2)*log(abs(sin(d*x + c) - 1))/(a^5 + 5*a^4*
b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5) + 2*(3*a^5*b^2*sin(d*x + c)^5
- 18*a^3*b^4*sin(d*x + c)^5 - 81*a*b^6*sin(d*x + c)^5 + 6*a^6*b*sin(d*x + c
)^4 - 36*a^4*b^3*sin(d*x + c)^4 - 78*a^2*b^5*sin(d*x + c)^4 + 12*b^7*sin(d*
x + c)^4 + 3*a^7*sin(d*x + c)^3 - 23*a^5*b^2*sin(d*x + c)^3 + 61*a^3*b^4*si
n(d*x + c)^3 + 151*a*b^6*sin(d*x + c)^3 - 10*a^6*b*sin(d*x + c)^2 + 74*a^4*
b^3*sin(d*x + c)^2 + 146*a^2*b^5*sin(d*x + c)^2 - 18*b^7*sin(d*x + c)^2 - 5
*a^7*sin(d*x + c) + 26*a^5*b^2*sin(d*x + c) - 49*a^3*b^4*sin(d*x + c) - 68*
a*b^6*sin(d*x + c) + 6*a^6*b - 44*a^4*b^3 - 62*a^2*b^5 + 4*b^7)/((a^8 - 4*a
^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*(b*sin(d*x + c)^3 + a*sin(d*x + c)^2
- b*sin(d*x + c) - a)^2))/d
```

Mupad [B]
time = 6.56, size = 688, normalized size = 2.10

$$\frac{\ln(\sin(c+d\,x)+1)\left(\frac{3\,b^2}{4\,(a-b)^5}-\frac{9\,b}{16\,(a-b)^4}+\frac{3}{16\,(a-b)^3}\right)}{d}-\frac{\ln(\sin(c+d\,x)-1)\left(\frac{9\,b}{16\,(a+b)^4}+\frac{3}{16\,(a+b)^3}+\frac{3\,b^2}{4\,(a+b)^5}\right)}{d}-\frac{\frac{3\,a^6\,b-22\,a^4\,b^3-31\,a^2\,b^5+2\,b^7}{4\,(a^8-4\,a^6\,b^2+6\,a^4\,b^4-4\,a^2\,b^6+b^8)}-\frac{\sin(c+d\,x)\,(5\,a^7-26\,a^5\,b^2+49\,a^3\,b^4+68\,a\,b^6)}{8\,(a^8-4\,a^6\,b^2+6\,a^4\,b^4-4\,a^2\,b^6+b^8)}-\frac{3\sin(c+d\,x)^5\,(-a^5\,b^2+6\,a^3\,b^4+27\,a\,b^6)}{8\,(a^8-4\,a^6\,b^2+6\,a^4\,b^4-4\,a^2\,b^6+b^8)}+\frac{\sin(c+d\,x)^3\,(3\,a^7-23\,a^5\,b^2+61\,a^3\,b^4+151\,a\,b^6)}{8\,(a^8-4\,a^6\,b^2+6\,a^4\,b^4-4\,a^2\,b^6+b^8)}+\frac{3\sin(c+d\,x)^4\,(a^6\,b-6\,a^4\,b^3-13\,a^2\,b^5+2\,b^7)}{4\,(a^8-4\,a^6\,b^2+6\,a^4\,b^4-4\,a^2\,b^6+b^8)}-\frac{\sin(c+d\,x)^2\,(5\,a^6\,b-37\,a^4\,b^3-73\,a^2\,b^5+9\,b^7)}{4\,(a^8-4\,a^6\,b^2+6\,a^4\,b^4-4\,a^2\,b^6+b^8)}}{d\left(\sin(c+d\,x)^4\,(a^2-2\,b^2)+a^2-\sin(c+d\,x)^2\,(2\,a^2-b^2)+b^2\sin(c+d\,x)^6+2\,a\,b\sin(c+d\,x)-4\,a\,b\sin(c+d\,x)^3+2\,a\,b\sin(c+d\,x)^5\right)}-\frac{\ln(a+b\sin(c+d\,x))\,(21\,a^2\,b^5+3\,b^7)}{d\,(a^{10}-5\,a^8\,b^2+10\,a^6\,b^4-10\,a^4\,b^6+5\,a^2\,b^8-b^{10})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^5*(a + b*sin(c + d*x))^3),x)
```

```
[Out] (log(sin(c + d*x) + 1)*((3*b^2)/(4*(a - b)^5) - (9*b)/(16*(a - b)^4) + 3/(1
6*(a - b)^3)))/d - (log(sin(c + d*x) - 1)*((9*b)/(16*(a + b)^4) + 3/(16*(a
+ b)^3) + (3*b^2)/(4*(a + b)^5)))/d - ((3*a^6*b + 2*b^7 - 31*a^2*b^5 - 22*a
^4*b^3)/(4*(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6*b^2)) - (sin(c + d*x)
*(68*a*b^6 + 5*a^7 + 49*a^3*b^4 - 26*a^5*b^2))/(8*(a^8 + b^8 - 4*a^2*b^6 +
6*a^4*b^4 - 4*a^6*b^2)) - (3*sin(c + d*x)^5*(27*a*b^6 + 6*a^3*b^4 - a^5*b^2
))/(8*(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6*b^2)) + (sin(c + d*x)^3*(1
51*a*b^6 + 3*a^7 + 61*a^3*b^4 - 23*a^5*b^2))/(8*(a^8 + b^8 - 4*a^2*b^6 + 6*
a^4*b^4 - 4*a^6*b^2)) + (3*sin(c + d*x)^4*(a^6*b + 2*b^7 - 13*a^2*b^5 - 6*a
^4*b^3))/(4*(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6*b^2)) - (sin(c + d*x
)^2*(5*a^6*b + 9*b^7 - 73*a^2*b^5 - 37*a^4*b^3))/(4*(a^8 + b^8 - 4*a^2*b^6
+ 6*a^4*b^4 - 4*a^6*b^2)))/(d*(sin(c + d*x)^4*(a^2 - 2*b^2) + a^2 - sin(c +
 d*x)^2*(2*a^2 - b^2) + b^2*sin(c + d*x)^6 + 2*a*b*sin(c + d*x) - 4*a*b*sin
(c + d*x)^3 + 2*a*b*sin(c + d*x)^5)) - (log(a + b*sin(c + d*x))*(3*b^7 + 21
*a^2*b^5))/(d*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^
2))
```

3.456 $\int \frac{\cos^6(c+dx)}{(a+b\sin(c+dx))^3} \, dx$

Optimal. Leaf size=197

$$\frac{5a(4a^2-3b^2)\,x}{2b^6}-\frac{5(4a^4-5a^2b^2+b^4)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^6\sqrt{a^2-b^2}\,d}-\frac{\cos^5(c+dx)}{2bd(a+b\sin(c+dx))^2}-\frac{5\cos^3(c+dx)(4a+}{6b^3d(a+b\sin}$$

```
[Out] 5/2*a*(4*a^2-3*b^2)*x/b^6-1/2*cos(d*x+c)^5/b/d/(a+b*sin(d*x+c))^2-5/6*cos(d
*x+c)^3*(4*a+b*sin(d*x+c))/b^3/d/(a+b*sin(d*x+c))+5/2*cos(d*x+c)*(4*a^2-b^2
-2*a*b*sin(d*x+c))/b^5/d-5*(4*a^4-5*a^2*b^2+b^4)*arctan((b+a*tan(1/2*d*x+1/
2*c))/(a^2-b^2)^(1/2))/b^6/d/(a^2-b^2)^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2772, 2942, 2944, 2814, 2739, 632, 210}

$$\frac{5ax(4a^2-3b^2)}{2b^6}+\frac{5\cos(c+dx)\left(4a^2-2ab\sin(c+dx)-b^2\right)}{2b^5d}-\frac{5(4a^4-5a^2b^2+b^4)\,\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^6d\sqrt{a^2-b^2}}-\frac{5\cos^3(c+dx)(4a+b\sin(c+dx))}{6b^3d(a+b\sin(c+dx))}-\frac{\cos^5(c+dx)}{2bd(a+b\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (5*a*(4*a^2 - 3*b^2)*x)/(2*b^6) - (5*(4*a^4 - 5*a^2*b^2 + b^4)*ArcTan[(b +
a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^6*Sqrt[a^2 - b^2]*d) - Cos[c + d*x
]^5/(2*b*d*(a + b*Sin[c + d*x])^2) - (5*Cos[c + d*x]^3*(4*a + b*Sin[c + d*x
]))/(6*b^3*d*(a + b*Sin[c + d*x])) + (5*Cos[c + d*x]*(4*a^2 - b^2 - 2*a*b*S
in[c + d*x]))/(2*b^5*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2772

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Dist[g^2*((p - 1)/(b*(m + 1))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2942

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*C
os[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x])/(b^2*f*(m + 1)*(m + p + 1))), x] + Dist[g^2*((
p - 1)/(b^2*(m + 1)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rule 2944

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Dist[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\cos^6(c+dx)}{(a+b\sin(c+dx))^3}\,dx = -\frac{\cos^5(c+dx)}{2bd(a+b\sin(c+dx))^2} - \frac{5\int \frac{\cos^4(c+dx)\sin(c+dx)}{(a+b\sin(c+dx))^2}\,dx}{2b}$$

$$= -\frac{\cos^5(c+dx)}{2bd(a+b\sin(c+dx))^2} - \frac{5\cos^3(c+dx)(4a+b\sin(c+dx))}{6b^3d(a+b\sin(c+dx))} + \frac{5\int \frac{\cos^2(c+dx)(-}{a+b\sin}}{2}$$

$$= -\frac{\cos^5(c+dx)}{2bd(a+b\sin(c+dx))^2} - \frac{5\cos^3(c+dx)(4a+b\sin(c+dx))}{6b^3d(a+b\sin(c+dx))} + \frac{5\cos(c+dx)\,(}{}$$

$$= \frac{5a(4a^2-3b^2)\,x}{2b^6} - \frac{\cos^5(c+dx)}{2bd(a+b\sin(c+dx))^2} - \frac{5\cos^3(c+dx)(4a+b\sin(c+dx))}{6b^3d(a+b\sin(c+dx))}$$

$$= \frac{5a(4a^2-3b^2)\,x}{2b^6} - \frac{\cos^5(c+dx)}{2bd(a+b\sin(c+dx))^2} - \frac{5\cos^3(c+dx)(4a+b\sin(c+dx))}{6b^3d(a+b\sin(c+dx))}$$

$$= \frac{5a(4a^2-3b^2)\,x}{2b^6} - \frac{\cos^5(c+dx)}{2bd(a+b\sin(c+dx))^2} - \frac{5\cos^3(c+dx)(4a+b\sin(c+dx))}{6b^3d(a+b\sin(c+dx))}$$

$$= \frac{5a(4a^2-3b^2)\,x}{2b^6} - \frac{5(4a^4-5a^2b^2+b^4)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^6\sqrt{a^2-b^2}\,d} - \frac{\cos^5(c+}{2bd(a+b\sin(}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 3889 vs. $2(197) = 394$.
time = 6.58, size = 3889, normalized size = 19.74

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (Cos[c + d*x]^5*(-1/2*(b*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b))^(7/2)*(b
/(a + b) - (b*Sin[c + d*x])/(a + b))^(7/2))/(((a*b)/(a - b) - b^2/(a - b))*
((a*b)/(a + b) + b^2/(a + b))*(a + b*Sin[c + d*x])^2) - ((-3*a*b^3*(-(b/(a
- b)) - (b*Sin[c + d*x])/(a - b))^(7/2)*(b/(a + b) - (b*Sin[c + d*x])/(a +
b))^(7/2))/((a^2 - b^2)*((a*b)/(a - b) - b^2/(a - b))*((a*b)/(a + b) + b^2/
(a + b))*(a + b*Sin[c + d*x])) - ((144*Sqrt[2]*a*b^5*(-(b/(a - b)) - (b*Sin
[c + d*x])/(a - b))^(7/2)*Sqrt[b/(a + b) - (b*Sin[c + d*x])/(a + b)]*(1 + (
(a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^(7/2)*((7*(3/(16*
(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^3) + 1/(2*(
1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^2) + (1 + ((
a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^(-1)))/12 + (35*b^
4*(((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/b - ((a - b)^2*(-(b/
(a - b)) - (b*Sin[c + d*x])/(a - b))^2)/(3*b^2) + (2*(a - b)^3*(-(b/(a - b)
```

```
) - (b*Sin[c + d*x])/(a - b))^3)/(15*b^3) - (Sqrt[2]*Sqrt[a - b]*ArcSinh[(S
qrt[a - b]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)])/(Sqrt[2]*Sqrt[b])
]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)])/(Sqrt[b]*Sqrt[1 + ((a - b)
*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b)])))/(128*(a - b)^4*(-(b/(
a - b)) - (b*Sin[c + d*x])/(a - b))^4*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[
c + d*x])/(a - b)))/(2*b))^3)))/(7*(a - b)*(a + b)^4*Sqrt[((a + b)*(b/(a +
b) - (b*Sin[c + d*x])/(a + b)))/b]) + (((18*a^2*b^5)/((a - b)^2*(a + b)^2)
+ (b^5*(2*a^2 - 5*b^2))/((a - b)^2*(a + b)^2))*((8*Sqrt[2]*b*(-(b/(a - b))
- (b*Sin[c + d*x])/(a - b))^(5/2)*Sqrt[b/(a + b) - (b*Sin[c + d*x])/(a + b)
]*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^(7/2)*((5
/(16*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^3) + 5
/(8*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^2) + (1
 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^(-1))/2 - (15
*b^3*(((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/b - ((a - b)^2*(-
(b/(a - b)) - (b*Sin[c + d*x])/(a - b))^2)/(3*b^2) - (Sqrt[2]*Sqrt[a - b]*A
rcSinh[(Sqrt[a - b]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)])/(Sqrt[2]
*Sqrt[b])]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)])/(Sqrt[b]*Sqrt[1 +
 ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b)])))/(64*(a - b)^
3*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b))^3*(1 + ((a - b)*(-(b/(a - b)) -
 (b*Sin[c + d*x])/(a - b)))/(2*b))^3)))/(5*(a + b)^2*Sqrt[((a + b)*(b/(a +
b) - (b*Sin[c + d*x])/(a + b)))/b]) - ((-((a*b)/(a - b)) + b^2/(a - b))*((8
*Sqrt[2]*b*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b))^(3/2)*Sqrt[b/(a + b) -
 (b*Sin[c + d*x])/(a + b)]*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(
a - b)))/(2*b))^(7/2)*((3*(5/(8*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*
x])/(a - b)))/(2*b))^3) + 5/(6*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x
])/(a - b)))/(2*b))^2) + (1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a
- b)))/(2*b))^(-1)))/8 + (15*b^2*(((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])
/(a - b)))/b - (Sqrt[2]*Sqrt[a - b]*ArcSinh[(Sqrt[a - b]*Sqrt[-(b/(a - b))
- (b*Sin[c + d*x])/(a - b)])/(Sqrt[2]*Sqrt[b])]*Sqrt[-(b/(a - b)) - (b*Sin[
c + d*x])/(a - b)])/(Sqrt[b]*Sqrt[1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d
*x])/(a - b)))/(2*b)])))/(64*(a - b)^2*(-(b/(a - b)) - (b*Sin[c + d*x])/(a
- b))^2*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^3))
)/(3*(a + b)^2*Sqrt[((a + b)*(b/(a + b) - (b*Sin[c + d*x])/(a + b)))/b]) -
((-((a*b)/(a - b)) + b^2/(a - b))*((8*Sqrt[2]*b*Sqrt[-(b/(a - b)) - (b*Sin[
c + d*x])/(a - b)]*Sqrt[b/(a + b) - (b*Sin[c + d*x])/(a + b)]*(1 + ((a - b)
*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^(7/2)*((5*Sqrt[b]*ArcSin
h[(Sqrt[a - b]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)])/(Sqrt[2]*Sqrt
[b])])/(8*Sqrt[2]*Sqrt[a - b]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)]
*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^(7/2)) + (
15/(8*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^3) +
5/(4*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^2) + (
1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^(-1))/6))/((
a + b)^2*Sqrt[((a + b)*(b/(a + b) - (b*Sin[c + d*x])/(a + b)))/b]) - ((-((a
*b)/(a - b)) + b^2/(a - b))*(-(((-((a*b)/(a + b)) - b^2/(a + b))*(-(((-((a*
b)/(a + b)) - b^2/(a + b))*((2*Sqrt[a - b]*ArcTanh[(Sqrt[a - b]*Sqrt[-(b/(a
```

```
- b)) - (b*Sin[c + d*x])/(a - b)])/(Sqrt[a + b]*Sqrt[b/(a + b) - (b*Sin[c
+ d*x])/(a + b)])])/(b*Sqrt[a + b]) - (2*Sqrt[-((a*b)/(a + b)) - b^2/(a + b
)]*ArcTanh[(Sqrt[-((a*b)/(a + b)) - b^2/(a + b)]*Sqrt[-(b/(a - b)) - (b*Sin
[c + d*x])/(a - b)])/(Sqrt[-((a*b)/(a - b)) + b^2/(a - b)]*Sqrt[b/(a + b) -
 (b*Sin[c + d*x])/(a + b)])])/(b*Sqrt[-((a*b)/(a - b)) + b^2/(a - b)])))/b)
 + (2*Sqrt[2]*(a - b)*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)]*Sqrt[b/
(a + b) - (b*Sin[c + d*x])/(a + b)]*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c
+ d*x])/(a - b)))/(2*b))^(3/2)*((Sqrt[b]*ArcSinh[(Sqrt[a - b]*Sqrt[-(b/(a -
 b)) - (b*Sin[c + d*x])/(a - b)])/(Sqrt[2]*Sqrt...
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 377 vs. $2(184) = 368$.
time = 1.17, size = 378, normalized size = 1.92

method	result
derivativedivides	$\frac{\frac{2\left(\frac{3ab^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\left(6a^2b-3b^3\right)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(12a^2b-4b^3\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{3ab^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}+6a^2b-\frac{7b^3}{3}\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+5a\left(4a^2-\right.}{b^6}$
default	$\frac{\frac{2\left(\frac{3ab^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\left(6a^2b-3b^3\right)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(12a^2b-4b^3\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{3ab^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}+6a^2b-\frac{7b^3}{3}\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+5a\left(4a^2-\right.}{b^6}$
risch	$\frac{10a^3x}{b^6}-\frac{15ax}{2b^4}-\frac{e^{3i(dx+c)}}{24b^3d}+\frac{3ia\,e^{2i(dx+c)}}{8b^4d}+\frac{3\,e^{i(dx+c)}a^2}{b^5d}-\frac{9\,e^{i(dx+c)}}{8b^3d}+\frac{3\,e^{-i(dx+c)}a^2}{b^5d}-\frac{9\,e^{-i(dx+c)}}{8b^3d}-\frac{3ia}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6/(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/b^6*((3/2*a*b^2*tan(1/2*d*x+1/2*c)^5+(6*a^2*b-3*b^3)*tan(1/2*d*x+1/2
*c)^4+(12*a^2*b-4*b^3)*tan(1/2*d*x+1/2*c)^2-3/2*a*b^2*tan(1/2*d*x+1/2*c)+6*
a^2*b-7/3*b^3)/(1+tan(1/2*d*x+1/2*c)^2)^3+5/2*a*(4*a^2-3*b^2)*arctan(tan(1/
2*d*x+1/2*c)))-2/b^6*((-1/2*b^2*(7*a^4-5*a^2*b^2-2*b^4)/a*tan(1/2*d*x+1/2*c
)^3-1/2*b*(8*a^6+9*a^4*b^2-15*a^2*b^4-2*b^6)/a^2*tan(1/2*d*x+1/2*c)^2-1/2*b
^2*(25*a^4-23*a^2*b^2-2*b^4)/a*tan(1/2*d*x+1/2*c)-4*b*a^4+7/2*a^2*b^3+1/2*b
^5)/(a*tan(1/2*d*x+1/2*c)^2+2*b*tan(1/2*d*x+1/2*c)+a)^2+5/2*(4*a^4-5*a^2*b^
2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1
/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [**A**]
time = 0.42, size = 752, normalized size = 3.82

$$\left[-\frac{4b^5\cos(dx+c)^5 - 30(4a^3b^2-3ab^4)dx\cos(dx+c)^2 - 20(2a^2b^3-b^5)\cos(dx+c)^3 + 30(4a^5+a^3b^2-3ab^4)dx - 15(4a^4+3a^2b^2-b^4-(4a^2b^2-b^4)\cos(dx+c)^2+2(4a^3b-ab^3)\sin(dx+c))\sqrt{-a^2+b^2}\log\left(-\frac{(2a^2-b^2)\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2-2(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c))\sqrt{-a^2+b^2}}{b^2\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2}\right)+30(4a^4b-a^2b^3-b^5)\cos(dx+c)+10(ab^4\cos(dx+c)^3+6(4a^4b-3a^2b^3)dx+6(3a^3b^2-2ab^4)\cos(dx+c))\sin(dx+c)}{12(b^8d\cos(dx+c)^2-2ab^7d\sin(dx+c)-(a^2b^6+b^8)d)}, -\frac{2b^5\cos(dx+c)^5 - 15(4a^3b^2-3ab^4)dx\cos(dx+c)^2 - 10(2a^2b^3-b^5)\cos(dx+c)^3 + 15(4a^5+a^3b^2-3ab^4)dx + 15(4a^4+3a^2b^2-b^4-(4a^2b^2-b^4)\cos(dx+c)^2+2(4a^3b-ab^3)\sin(dx+c))\sqrt{a^2-b^2}\arctan\left(-\frac{a\sin(dx+c)+b}{\sqrt{a^2-b^2}\cos(dx+c)}\right)+15(4a^4b-a^2b^3-b^5)\cos(dx+c)+5(ab^4\cos(dx+c)^3+6(4a^4b-3a^2b^3)dx+6(3a^3b^2-2ab^4)\cos(dx+c))\sin(dx+c)}{6(b^8d\cos(dx+c)^2-2ab^7d\sin(dx+c)-(a^2b^6+b^8)d)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [-1/12*(4*b^5*cos(d*x + c)^5 - 30*(4*a^3*b^2 - 3*a*b^4)*d*x*cos(d*x + c)^2
- 20*(2*a^2*b^3 - b^5)*cos(d*x + c)^3 + 30*(4*a^5 + a^3*b^2 - 3*a*b^4)*d*x
- 15*(4*a^4 + 3*a^2*b^2 - b^4 - (4*a^2*b^2 - b^4)*cos(d*x + c)^2 + 2*(4*a^3
*b - a*b^3)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)
^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*co
s(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^
2 - b^2)) + 30*(4*a^4*b - a^2*b^3 - b^5)*cos(d*x + c) + 10*(a*b^4*cos(d*x +
 c)^3 + 6*(4*a^4*b - 3*a^2*b^3)*d*x + 6*(3*a^3*b^2 - 2*a*b^4)*cos(d*x + c))
*sin(d*x + c))/(b^8*d*cos(d*x + c)^2 - 2*a*b^7*d*sin(d*x + c) - (a^2*b^6 +
b^8)*d), -1/6*(2*b^5*cos(d*x + c)^5 - 15*(4*a^3*b^2 - 3*a*b^4)*d*x*cos(d*x
+ c)^2 - 10*(2*a^2*b^3 - b^5)*cos(d*x + c)^3 + 15*(4*a^5 + a^3*b^2 - 3*a*b^
4)*d*x + 15*(4*a^4 + 3*a^2*b^2 - b^4 - (4*a^2*b^2 - b^4)*cos(d*x + c)^2 + 2
*(4*a^3*b - a*b^3)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) +
b)/(sqrt(a^2 - b^2)*cos(d*x + c))) + 15*(4*a^4*b - a^2*b^3 - b^5)*cos(d*x +
 c) + 5*(a*b^4*cos(d*x + c)^3 + 6*(4*a^4*b - 3*a^2*b^3)*d*x + 6*(3*a^3*b^2
- 2*a*b^4)*cos(d*x + c))*sin(d*x + c))/(b^8*d*cos(d*x + c)^2 - 2*a*b^7*d*si
n(d*x + c) - (a^2*b^6 + b^8)*d)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 457 vs. 2(182) = 364.

time = 5.79, size = 457, normalized size = 2.32

$$\frac{\frac{15\,(4\,a^3-3\,ab^2)(dx+c)}{b^6}-\frac{30\,(4\,a^4-5\,a^2b^2+b^4)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,b^6}+\frac{2\left(9\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+36\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-18\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+72\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-24\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-9\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+36\,a^2-14\,b^2\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3b^5}+\frac{6\left(7\,a^5b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-5\,a^3b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-2\,ab^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+8\,a^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+9\,a^4b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-15\,a^2b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-2\,b^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+25\,a^5b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-23\,a^3b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,ab^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+8\,a^6-7\,a^4b^2-a^2b^4\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a\right)^2a^2b^5}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/6*(15*(4*a^3 - 3*a*b^2)*(d*x + c)/b^6 - 30*(4*a^4 - 5*a^2*b^2 + b^4)*(pi*
floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/
sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*b^6) + 2*(9*a*b*tan(1/2*d*x + 1/2*c)^5 +
 36*a^2*tan(1/2*d*x + 1/2*c)^4 - 18*b^2*tan(1/2*d*x + 1/2*c)^4 + 72*a^2*tan
(1/2*d*x + 1/2*c)^2 - 24*b^2*tan(1/2*d*x + 1/2*c)^2 - 9*a*b*tan(1/2*d*x + 1
/2*c) + 36*a^2 - 14*b^2)/((tan(1/2*d*x + 1/2*c)^2 + 1)^3*b^5) + 6*(7*a^5*b*
tan(1/2*d*x + 1/2*c)^3 - 5*a^3*b^3*tan(1/2*d*x + 1/2*c)^3 - 2*a*b^5*tan(1/2
*d*x + 1/2*c)^3 + 8*a^6*tan(1/2*d*x + 1/2*c)^2 + 9*a^4*b^2*tan(1/2*d*x + 1/
2*c)^2 - 15*a^2*b^4*tan(1/2*d*x + 1/2*c)^2 - 2*b^6*tan(1/2*d*x + 1/2*c)^2 +
 25*a^5*b*tan(1/2*d*x + 1/2*c) - 23*a^3*b^3*tan(1/2*d*x + 1/2*c) - 2*a*b^5*
tan(1/2*d*x + 1/2*c) + 8*a^6 - 7*a^4*b^2 - a^2*b^4)/((a*tan(1/2*d*x + 1/2*c
)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)^2*a^2*b^5))/d
```

Mupad [B]
time = 8.58, size = 1226, normalized size = 6.22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(a + b*sin(c + d*x))^3,x)
```

```
[Out] (atanh((1000*a^2*(b^2 - a^2)^(1/2))/(1000*a^2*b - (5000*a^4)/b + (4000*a^6)
/b^3 - 10000*a^3*tan(c/2 + (d*x)/2) + 2000*a*b^2*tan(c/2 + (d*x)/2) + (8000
*a^5*tan(c/2 + (d*x)/2))/b^2) - (4000*a^4*(b^2 - a^2)^(1/2))/(1000*a^2*b^3
- 5000*a^4*b + (4000*a^6)/b + 8000*a^5*tan(c/2 + (d*x)/2) + 2000*a*b^4*tan(
c/2 + (d*x)/2) - 10000*a^3*b^2*tan(c/2 + (d*x)/2)) + (2000*a*tan(c/2 + (d*x
)/2)*(b^2 - a^2)^(1/2))/(1000*a^2 - (5000*a^4)/b^2 + (4000*a^6)/b^4 - (1000
0*a^3*tan(c/2 + (d*x)/2))/b + (8000*a^5*tan(c/2 + (d*x)/2))/b^3 + 2000*a*b*
tan(c/2 + (d*x)/2)) - (9000*a^3*tan(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2))/(1000
*a^2*b^2 - 5000*a^4 + (4000*a^6)/b^2 + 2000*a*b^3*tan(c/2 + (d*x)/2) - 1000
0*a^3*b*tan(c/2 + (d*x)/2) + (8000*a^5*tan(c/2 + (d*x)/2))/b) + (4000*a^5*t
an(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2))/(4000*a^6 + 1000*a^2*b^4 - 5000*a^4*b^
2 + 2000*a*b^5*tan(c/2 + (d*x)/2) + 8000*a^5*b*tan(c/2 + (d*x)/2) - 10000*a
^3*b^3*tan(c/2 + (d*x)/2)))*(20*a^2*(b^2 - a^2)^(1/2) - 5*b^2*(b^2 - a^2)^(
1/2)))/(b^6*d) - ((3*b^4 - 60*a^4 + 35*a^2*b^2)/(3*b^5) + (tan(c/2 + (d*x)/
2)*(6*b^4 - 210*a^4 + 125*a^2*b^2))/(3*a*b^4) - (tan(c/2 + (d*x)/2)^8*(20*a
^6 - 2*b^6 - 15*a^2*b^4 + 15*a^4*b^2))/(a^2*b^5) - (2*tan(c/2 + (d*x)/2)^6*
(40*a^6 - 3*b^6 - 35*a^2*b^4 + 30*a^4*b^2))/(a^2*b^5) - (2*tan(c/2 + (d*x)/
```

```
2)^2*(120*a^6 - 3*b^6 - 55*a^2*b^4 + 10*a^4*b^2))/(3*a^2*b^5) - (2*tan(c/2
+ (d*x)/2)^4*(180*a^6 - 9*b^6 - 120*a^2*b^4 + 95*a^4*b^2))/(3*a^2*b^5) + (t
an(c/2 + (d*x)/2)^9*(2*b^4 - 10*a^4 + 5*a^2*b^2))/(a*b^4) + (2*tan(c/2 + (d
*x)/2)^7*(4*b^4 - 50*a^4 + 25*a^2*b^2))/(a*b^4) + (4*tan(c/2 + (d*x)/2)^5*(
3*b^4 - 60*a^4 + 35*a^2*b^2))/(a*b^4) + (2*tan(c/2 + (d*x)/2)^3*(12*b^4 - 3
30*a^4 + 205*a^2*b^2))/(3*a*b^4))/(d*(tan(c/2 + (d*x)/2)^2*(5*a^2 + 4*b^2)
+ tan(c/2 + (d*x)/2)^8*(5*a^2 + 4*b^2) + tan(c/2 + (d*x)/2)^4*(10*a^2 + 12*
b^2) + tan(c/2 + (d*x)/2)^6*(10*a^2 + 12*b^2) + a^2*tan(c/2 + (d*x)/2)^10 +
 a^2 + 16*a*b*tan(c/2 + (d*x)/2)^3 + 24*a*b*tan(c/2 + (d*x)/2)^5 + 16*a*b*t
an(c/2 + (d*x)/2)^7 + 4*a*b*tan(c/2 + (d*x)/2)^9 + 4*a*b*tan(c/2 + (d*x)/2)
)) + (5*a*atan((3000*a^2*tan(c/2 + (d*x)/2))/(3000*a^2 - (7000*a^4)/b^2 + (
4000*a^6)/b^4) - (7000*a^4*tan(c/2 + (d*x)/2))/(3000*a^2*b^2 - 7000*a^4 + (
4000*a^6)/b^2) + (4000*a^6*tan(c/2 + (d*x)/2))/(4000*a^6 + 3000*a^2*b^4 - 7
000*a^4*b^2))*(4*a^2 - 3*b^2))/(b^6*d)
```

3.457 $\int \frac{\cos^4(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=139

$$-\frac{3ax}{b^4}+\frac{3(2a^2-b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^4\sqrt{a^2-b^2}\,d}-\frac{\cos^3(c+dx)}{2bd(a+b\sin(c+dx))^2}-\frac{3\cos(c+dx)(2a+b\sin(c+dx))}{2b^3d(a+b\sin(c+dx))}$$

```
[Out] -3*a*x/b^4-1/2*cos(d*x+c)^3/b/d/(a+b*sin(d*x+c))^2-3/2*cos(d*x+c)*(2*a+b*si
n(d*x+c))/b^3/d/(a+b*sin(d*x+c))+3*(2*a^2-b^2)*arctan((b+a*tan(1/2*d*x+1/2*
c))/(a^2-b^2)^(1/2))/b^4/d/(a^2-b^2)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2772, 2942, 2814, 2739, 632, 210}

$$\frac{3(2a^2-b^2)\,\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^4d\sqrt{a^2-b^2}}-\frac{3ax}{b^4}-\frac{3\cos(c+dx)(2a+b\sin(c+dx))}{2b^3d(a+b\sin(c+dx))}-\frac{\cos^3(c+dx)}{2bd(a+b\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-3*a*x)/b^4 + (3*(2*a^2 - b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 -
b^2]])/(b^4*Sqrt[a^2 - b^2]*d) - Cos[c + d*x]^3/(2*b*d*(a + b*Sin[c + d*x])
^2) - (3*Cos[c + d*x]*(2*a + b*Sin[c + d*x]))/(2*b^3*d*(a + b*Sin[c + d*x])
)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
```

```
a^2 - b^2, 0]
```

Rule 2772

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Dist[g^2*((p - 1)/(b*(m + 1))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2942

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*C
os[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x])/(b^2*f*(m + 1)*(m + p + 1))), x] + Dist[g^2*((
p - 1)/(b^2*(m + 1)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{(a+b\sin(c+dx))^3}\,dx = -\frac{\cos^3(c+dx)}{2bd(a+b\sin(c+dx))^2} - \frac{3\int \frac{\cos^2(c+dx)\sin(c+dx)}{(a+b\sin(c+dx))^2}dx}{2b}$$

$$= -\frac{\cos^3(c+dx)}{2bd(a+b\sin(c+dx))^2} - \frac{3\cos(c+dx)(2a+b\sin(c+dx))}{2b^3d(a+b\sin(c+dx))} + \frac{3\int \frac{-b-2a\sin(c+d}{a+b\sin(c+dx)}}{2b^3}$$

$$= -\frac{3ax}{b^4} - \frac{\cos^3(c+dx)}{2bd(a+b\sin(c+dx))^2} - \frac{3\cos(c+dx)(2a+b\sin(c+dx))}{2b^3d(a+b\sin(c+dx))} + \frac{(3(2a^2 -}{}$$

$$= -\frac{3ax}{b^4} - \frac{\cos^3(c+dx)}{2bd(a+b\sin(c+dx))^2} - \frac{3\cos(c+dx)(2a+b\sin(c+dx))}{2b^3d(a+b\sin(c+dx))} + \frac{(3(2a^2 -}{}$$

$$= -\frac{3ax}{b^4} - \frac{\cos^3(c+dx)}{2bd(a+b\sin(c+dx))^2} - \frac{3\cos(c+dx)(2a+b\sin(c+dx))}{2b^3d(a+b\sin(c+dx))} - \frac{(6(2a^2 -}{}$$

$$= -\frac{3ax}{b^4} + \frac{3(2a^2-b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^4\sqrt{a^2-b^2}\,d} - \frac{\cos^3(c+dx)}{2bd(a+b\sin(c+dx))^2} - \frac{3\cos(}{2}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 2641 vs. $2(139) = 278$.
time = 6.29, size = 2641, normalized size = 19.00

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (Cos[c + d*x]^3*(-1/2*(b*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b))^(5/2)*(b
/(a + b) - (b*Sin[c + d*x])/(a + b))^(5/2))/(((a*b)/(a - b) - b^2/(a - b))*
((a*b)/(a + b) + b^2/(a + b))*(a + b*Sin[c + d*x])^2) - (-((a*b^3*(-(b/(a -
 b)) - (b*Sin[c + d*x])/(a - b))^(5/2)*(b/(a + b) - (b*Sin[c + d*x])/(a + b
))^(5/2))/((a^2 - b^2)*((a*b)/(a - b) - b^2/(a - b))*((a*b)/(a + b) + b^2/(
a + b))*(a + b*Sin[c + d*x]))) - ((16*Sqrt[2]*a*b^4*(-(b/(a - b)) - (b*Sin[
c + d*x])/(a - b))^(5/2)*Sqrt[b/(a + b) - (b*Sin[c + d*x])/(a + b)]*(1 + ((
a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^(5/2)*((5*(1/(2*(1
 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^2) + (1 + ((a
 - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^(-1)))/8 - (15*b^3*
(((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/b - ((a - b)^2*(-(b/(a
 - b)) - (b*Sin[c + d*x])/(a - b))^2)/(3*b^2) - (Sqrt[2]*Sqrt[a - b]*ArcSin
h[(Sqrt[a - b]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)])/(Sqrt[2]*Sqrt
[b])]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)])/(Sqrt[b]*Sqrt[1 + ((a
- b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b)])))/(32*(a - b)^3*(-(
b/(a - b)) - (b*Sin[c + d*x])/(a - b))^3*(1 + ((a - b)*(-(b/(a - b)) - (b*S
```

```
in[c + d*x])/(a - b)))/(2*b))^2)))/(5*(a - b)*(a + b)^3*Sqrt[((a + b)*(b/(a
 + b) - (b*Sin[c + d*x])/(a + b)))/b]) + (((4*a^2*b^5)/((a - b)^2*(a + b)^2
) + (b^5*(2*a^2 - 3*b^2))/((a - b)^2*(a + b)^2))*((4*Sqrt[2]*(-(b/(a - b))
- (b*Sin[c + d*x])/(a - b))^(3/2)*Sqrt[b/(a + b) - (b*Sin[c + d*x])/(a + b)
]*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^(5/2)*((3
/(4*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^2) + (1
 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^(-1))/2 + (3*
b^2*(((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/b - (Sqrt[2]*Sqrt[
a - b]*ArcSinh[(Sqrt[a - b]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)])/
(Sqrt[2]*Sqrt[b])]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)])/(Sqrt[b]*
Sqrt[1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b)])))/(8*(
a - b)^2*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b))^2*(1 + ((a - b)*(-(b/(a
- b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^2)))/(3*(a + b)*Sqrt[((a + b)*(b/
(a + b) - (b*Sin[c + d*x])/(a + b)))/b]) - ((-((a*b)/(a - b)) + b^2/(a - b)
)*(-(((-((a*b)/(a - b)) + b^2/(a - b))*(-(((-((a*b)/(a + b)) - b^2/(a + b))
*((2*Sqrt[a - b]*ArcTanh[(Sqrt[a - b]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/
(a - b)])/(Sqrt[a + b]*Sqrt[b/(a + b) - (b*Sin[c + d*x])/(a + b)])])/(b*Sqr
t[a + b]) - (2*Sqrt[-((a*b)/(a + b)) - b^2/(a + b)]*ArcTanh[(Sqrt[-((a*b)/(
a + b)) - b^2/(a + b)]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)])/(Sqrt
[-((a*b)/(a - b)) + b^2/(a - b)]*Sqrt[b/(a + b) - (b*Sin[c + d*x])/(a + b)]
)])/(b*Sqrt[-((a*b)/(a - b)) + b^2/(a - b)])))/b) + (2*Sqrt[2]*(a - b)*Sqrt
[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)]*Sqrt[b/(a + b) - (b*Sin[c + d*x])
/(a + b)]*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^(
3/2)*((Sqrt[b]*ArcSinh[(Sqrt[a - b]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a
 - b)])/(Sqrt[2]*Sqrt[b])])/(Sqrt[2]*Sqrt[a - b]*Sqrt[-(b/(a - b)) - (b*Sin
[c + d*x])/(a - b)]*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a - b))
)/(2*b))^(3/2)) + 1/(2*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[c + d*x])/(a -
b)))/(2*b)))))/(b*(a + b)*Sqrt[((a + b)*(b/(a + b) - (b*Sin[c + d*x])/(a +
b)))/b])))/b) + (4*Sqrt[2]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)]*Sq
rt[b/(a + b) - (b*Sin[c + d*x])/(a + b)]*(1 + ((a - b)*(-(b/(a - b)) - (b*S
in[c + d*x])/(a - b)))/(2*b))^(5/2)*((3*Sqrt[b]*ArcSinh[(Sqrt[a - b]*Sqrt[-
(b/(a - b)) - (b*Sin[c + d*x])/(a - b)])/(Sqrt[2]*Sqrt[b])])/(4*Sqrt[2]*Sqr
t[a - b]*Sqrt[-(b/(a - b)) - (b*Sin[c + d*x])/(a - b)]*(1 + ((a - b)*(-(b/(
a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^(5/2)) + (3/(2*(1 + ((a - b)*(-
(b/(a - b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^2) + (1 + ((a - b)*(-(b/(a
- b)) - (b*Sin[c + d*x])/(a - b)))/(2*b))^(-1))/4))/((a + b)*Sqrt[((a + b)*
(b/(a + b) - (b*Sin[c + d*x])/(a + b)))/b])))/b))/b)/(((a*b)/(a - b) - b^2/
(a - b))*((a*b)/(a + b) + b^2/(a + b))))/(2*((a*b)/(a - b) - b^2/(a - b))*(
(a*b)/(a + b) + b^2/(a + b)))))/(d*(1 - (a + b*Sin[c + d*x])/(a - b))^(3/2)
*(1 - (a + b*Sin[c + d*x])/(a + b))^(3/2))
```

Maple [A]
time = 0.90, size = 238, normalized size = 1.71

method	result

derivativedivides	$\frac{-\frac{2\left(\frac{b}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}+3a\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{b^4}+\frac{2\left(-\frac{b^2(3a^2+2b^2)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}-\frac{b(4a^4+9a^2b^2+2b^4)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a^2}-\frac{b^2(}{}\right.}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a\right)^2}}{d}$
default	$\frac{-\frac{2\left(\frac{b}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}+3a\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{b^4}+\frac{2\left(-\frac{b^2(3a^2+2b^2)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}-\frac{b(4a^4+9a^2b^2+2b^4)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a^2}-\frac{b^2(}{}\right.}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a\right)^2}}{d}$
risch	$-\frac{3ax}{b^4}-\frac{e^{i(dx+c)}}{2b^3d}-\frac{e^{-i(dx+c)}}{2b^3d}+\frac{i\left(-6ia^2b\,e^{3i(dx+c)}+ib^3e^{3i(dx+c)}+14ia^2b\,e^{i(dx+c)}+ib^3e^{i(dx+c)}+10a^3e^{2i(dx+c)}+5a\right.}{\left(-ib\,e^{2i(dx+c)}+ib+2a\,e^{i(dx+c)}\right)^2d\,b^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2/b^4*(b/(1+tan(1/2*d*x+1/2*c)^2)+3*a*arctan(tan(1/2*d*x+1/2*c)))+2/b
^4*((-1/2*b^2*(3*a^2+2*b^2)/a*tan(1/2*d*x+1/2*c)^3-1/2*b*(4*a^4+9*a^2*b^2+2
*b^4)/a^2*tan(1/2*d*x+1/2*c)^2-1/2*b^2*(13*a^2+2*b^2)/a*tan(1/2*d*x+1/2*c)-
2*a^2*b-1/2*b^3)/(a*tan(1/2*d*x+1/2*c)^2+2*b*tan(1/2*d*x+1/2*c)+a)^2+3/2*(2
*a^2-b^2)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)
^(1/2))))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 316 vs. $2(130) = 260$.
time = 0.42, size = 716, normalized size = 5.15

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(12*(a^3*b^2 - a*b^4)*d*x*cos(d*x + c)^2 + 4*(a^2*b^3 - b^5)*cos(d*x
+ c)^3 - 12*(a^5 - a*b^4)*d*x + 3*(2*a^4 + a^2*b^2 - b^4 - (2*a^2*b^2 - b^4
)*cos(d*x + c)^2 + 2*(2*a^3*b - a*b^3)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(-
((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d
*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^
2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 6*(2*a^4*b - a^2*b^3 - b^5)*cos(d*x
+ c) - 6*(4*(a^4*b - a^2*b^3)*d*x + 3*(a^3*b^2 - a*b^4)*cos(d*x + c))*sin(d
*x + c))/((a^2*b^6 - b^8)*d*cos(d*x + c)^2 - 2*(a^3*b^5 - a*b^7)*d*sin(d*x
+ c) - (a^4*b^4 - b^8)*d), -1/2*(6*(a^3*b^2 - a*b^4)*d*x*cos(d*x + c)^2 + 2
*(a^2*b^3 - b^5)*cos(d*x + c)^3 - 6*(a^5 - a*b^4)*d*x - 3*(2*a^4 + a^2*b^2
- b^4 - (2*a^2*b^2 - b^4)*cos(d*x + c)^2 + 2*(2*a^3*b - a*b^3)*sin(d*x + c)
)*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c
))) - 3*(2*a^4*b - a^2*b^3 - b^5)*cos(d*x + c) - 3*(4*(a^4*b - a^2*b^3)*d*x
 + 3*(a^3*b^2 - a*b^4)*cos(d*x + c))*sin(d*x + c))/((a^2*b^6 - b^8)*d*cos(d
*x + c)^2 - 2*(a^3*b^5 - a*b^7)*d*sin(d*x + c) - (a^4*b^4 - b^8)*d)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 272 vs. 2(130) = 260.
time = 7.11, size = 272, normalized size = 1.96

$$-\frac{\frac{3\,(dx+c)a}{b^4}-\frac{3\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)(2\,a^2-b^2)}{\sqrt{a^2-b^2}\,b^4}+\frac{2}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)b^3}+\frac{3\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2\,ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+4\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+9\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+13\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\,ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+4\,a^4+a^2b^2}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a\right)^2a^2b^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -(3*(d*x + c)*a/b^4 - 3*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((
a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))*(2*a^2 - b^2)/(sqrt(a^2 - b^2
)*b^4) + 2/((tan(1/2*d*x + 1/2*c)^2 + 1)*b^3) + (3*a^3*b*tan(1/2*d*x + 1/2*
c)^3 + 2*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 4*a^4*tan(1/2*d*x + 1/2*c)^2 + 9*a^
2*b^2*tan(1/2*d*x + 1/2*c)^2 + 2*b^4*tan(1/2*d*x + 1/2*c)^2 + 13*a^3*b*tan(
1/2*d*x + 1/2*c) + 2*a*b^3*tan(1/2*d*x + 1/2*c) + 4*a^4 + a^2*b^2)/((a*tan(
1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)^2*a^2*b^3))/d
```

Mupad [**B**]
time = 7.55, size = 1360, normalized size = 9.78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(a + b*sin(c + d*x))^3,x)
```

```
[Out] - ((6*a^2 + b^2)/b^3 + (2*tan(c/2 + (d*x)/2)^2*(6*a^4 + b^4 + 9*a^2*b^2))/(
a^2*b^3) + (tan(c/2 + (d*x)/2)*(21*a^2 + 2*b^2))/(a*b^2) + (4*tan(c/2 + (d*
x)/2)^3*(6*a^2 + b^2))/(a*b^2) + (tan(c/2 + (d*x)/2)^4*(6*a^4 + 2*b^4 + 9*a
^2*b^2))/(a^2*b^3) + (tan(c/2 + (d*x)/2)^5*(3*a^2 + 2*b^2))/(a*b^2))/(d*(ta
n(c/2 + (d*x)/2)^2*(3*a^2 + 4*b^2) + tan(c/2 + (d*x)/2)^4*(3*a^2 + 4*b^2) +
 a^2*tan(c/2 + (d*x)/2)^6 + a^2 + 8*a*b*tan(c/2 + (d*x)/2)^3 + 4*a*b*tan(c/
2 + (d*x)/2)^5 + 4*a*b*tan(c/2 + (d*x)/2))) - (3*a*x)/b^4 - (atan((((-(a +
b)*(a - b))^(1/2)*(2*a^2 - b^2)*((288*a^4)/b^5 - (8*tan(c/2 + (d*x)/2)*(9*a
*b^7 - 108*a^3*b^5 + 72*a^5*b^3))/b^9 + (3*(-(a + b)*(a - b))^(1/2)*(2*a^2
- b^2)*((8*tan(c/2 + (d*x)/2)*(12*a*b^10 - 24*a^3*b^8))/b^9 - 48*a^2 + (3*(
-(a + b)*(a - b))^(1/2)*(2*a^2 - b^2)*(32*a^2*b^3 + (8*tan(c/2 + (d*x)/2)*(
12*a*b^13 - 8*a^3*b^11))/b^9))/(2*(b^6 - a^2*b^4))))/(2*(b^6 - a^2*b^4)))*3
i)/(2*(b^6 - a^2*b^4)) + ((-(a + b)*(a - b))^(1/2)*(2*a^2 - b^2)*((288*a^4)
/b^5 - (8*tan(c/2 + (d*x)/2)*(9*a*b^7 - 108*a^3*b^5 + 72*a^5*b^3))/b^9 + (3
*(-(a + b)*(a - b))^(1/2)*(2*a^2 - b^2)*(48*a^2 - (8*tan(c/2 + (d*x)/2)*(12
*a*b^10 - 24*a^3*b^8))/b^9 + (3*(-(a + b)*(a - b))^(1/2)*(2*a^2 - b^2)*(32*
a^2*b^3 + (8*tan(c/2 + (d*x)/2)*(12*a*b^13 - 8*a^3*b^11))/b^9))/(2*(b^6 - a
^2*b^4))))/(2*(b^6 - a^2*b^4)))*3i)/(2*(b^6 - a^2*b^4)))/((16*(54*a^4 - 27*
a^2*b^2))/b^8 + (16*tan(c/2 + (d*x)/2)*(216*a^5 - 108*a^3*b^2))/b^9 - (3*(-
(a + b)*(a - b))^(1/2)*(2*a^2 - b^2)*((288*a^4)/b^5 - (8*tan(c/2 + (d*x)/2)
*(9*a*b^7 - 108*a^3*b^5 + 72*a^5*b^3))/b^9 + (3*(-(a + b)*(a - b))^(1/2)*(2
*a^2 - b^2)*((8*tan(c/2 + (d*x)/2)*(12*a*b^10 - 24*a^3*b^8))/b^9 - 48*a^2 +
 (3*(-(a + b)*(a - b))^(1/2)*(2*a^2 - b^2)*(32*a^2*b^3 + (8*tan(c/2 + (d*x)
/2)*(12*a*b^13 - 8*a^3*b^11))/b^9))/(2*(b^6 - a^2*b^4))))/(2*(b^6 - a^2*b^4
))))/(2*(b^6 - a^2*b^4)) + (3*(-(a + b)*(a - b))^(1/2)*(2*a^2 - b^2)*((288*
a^4)/b^5 - (8*tan(c/2 + (d*x)/2)*(9*a*b^7 - 108*a^3*b^5 + 72*a^5*b^3))/b^9
+ (3*(-(a + b)*(a - b))^(1/2)*(2*a^2 - b^2)*(48*a^2 - (8*tan(c/2 + (d*x)/2)
*(12*a*b^10 - 24*a^3*b^8))/b^9 + (3*(-(a + b)*(a - b))^(1/2)*(2*a^2 - b^2)*
(32*a^2*b^3 + (8*tan(c/2 + (d*x)/2)*(12*a*b^13 - 8*a^3*b^11))/b^9))/(2*(b^6
 - a^2*b^4))))/(2*(b^6 - a^2*b^4))))/(2*(b^6 - a^2*b^4))))*(-(a + b)*(a - b
))^(1/2)*(2*a^2 - b^2)*3i)/(d*(b^6 - a^2*b^4))
```

3.458 $\int \frac{\cos^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=115

$$\frac{\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}\,d}-\frac{\cos(c+dx)}{2bd(a+b\sin(c+dx))^2}+\frac{a\cos(c+dx)}{2b\,(a^2-b^2)\,d(a+b\sin(c+dx))}$$

```
[Out] arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/(a^2-b^2)^(3/2)/d-1/2*cos(
d*x+c)/b/d/(a+b*sin(d*x+c))^2+1/2*a*cos(d*x+c)/b/(a^2-b^2)/d/(a+b*sin(d*x+c
))
```

Rubi [A]
time = 0.09, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2772, 2833, 12, 2739, 632, 210}

$$\frac{\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\,(a^2-b^2)^{3/2}}+\frac{a\cos(c+dx)}{2bd\,(a^2-b^2)\,(a+b\sin(c+dx))}-\frac{\cos(c+dx)}{2bd(a+b\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]/((a^2 - b^2)^(3/2)*d) - Co
s[c + d*x]/(2*b*d*(a + b*Sin[c + d*x])^2) + (a*Cos[c + d*x])/(2*b*(a^2 - b^
2)*d*(a + b*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2772

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Dist[g^2*((p - 1)/(b*(m + 1))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx = -\frac{\cos(c+dx)}{2bd(a+b\sin(c+dx))^2}-\frac{\int \frac{\sin(c+dx)}{(a+b\sin(c+dx))^2}\,dx}{2b}$$

$$= -\frac{\cos(c+dx)}{2bd(a+b\sin(c+dx))^2}+\frac{a\cos(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sin(c+dx))}+\frac{\int \frac{b}{a+b\sin(c+dx)}\,dx}{2b\left(a^2-b^2\right)}$$

$$= -\frac{\cos(c+dx)}{2bd(a+b\sin(c+dx))^2}+\frac{a\cos(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sin(c+dx))}+\frac{\int \frac{1}{a+b\sin(c+dx)}\,dx}{2\left(a^2-b^2\right)}$$

$$= -\frac{\cos(c+dx)}{2bd(a+b\sin(c+dx))^2}+\frac{a\cos(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sin(c+dx))}+\frac{\text{Subst}\left(\int \frac{1}{a+2bx+a\cdots}\right.}{(\cdots}$$

$$= -\frac{\cos(c+dx)}{2bd(a+b\sin(c+dx))^2}+\frac{a\cos(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sin(c+dx))}-\frac{2\text{Subst}\left(\int \frac{\cdots}{-4(a^2-\cdots}\right.}{\cdots}$$

$$= \frac{\tan^{-1}\left(\frac{\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{}}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}d}-\frac{\cos(c+dx)}{2bd(a+b\sin(c+dx))^2}+\frac{a\cos(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sin(c+\cdots}$$

Mathematica [A]
time = 0.27, size = 93, normalized size = 0.81

$$\frac{\frac{2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+\frac{\cos(c+dx)(b+a\sin(c+dx))}{(a+b\sin(c+dx))^2}}{2(a-b)(a+b)d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] + (Co
s[c + d*x]*(b + a*Sin[c + d*x]))/(a + b*Sin[c + d*x])^2)/(2*(a - b)*(a + b)
*d)
```

Maple [A]
time = 0.64, size = 201, normalized size = 1.75

method	result
derivativedivides	$\frac{\frac{-\frac{(a^2-2b^2)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a(a^2-b^2)}+\frac{(a^2+2b^2)b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{(a^2-b^2)a^2}+\frac{(a^2+2b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a(a^2-b^2)}+\frac{2b}{2a^2-2b^2}}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a\right)^2}+\frac{\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{\frac{3}{2}}}}{d}$
default	$\frac{\frac{-\frac{(a^2-2b^2)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a(a^2-b^2)}+\frac{(a^2+2b^2)b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{(a^2-b^2)a^2}+\frac{(a^2+2b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a(a^2-b^2)}+\frac{2b}{2a^2-2b^2}}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a\right)^2}+\frac{\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{\frac{3}{2}}}}{d}$
risch	$-\frac{i\left(-2ia^2b\,e^{3i(dx+c)}+ib^3e^{3i(dx+c)}+2ia^2b\,e^{i(dx+c)}+ib^3e^{i(dx+c)}+2a^3e^{2i(dx+c)}+a\,b^2e^{2i(dx+c)}-a\,b^2\right)}{\left(-ib\,e^{2i(dx+c)}+ib+2a\,e^{i(dx+c)}\right)^2(a^2-b^2)d\,b^2}-\frac{\ln\left(e^{i(dx+c)}+\right.}{2\sqrt{-a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*(-1/2*(a^2-2*b^2)/a/(a^2-b^2)*tan(1/2*d*x+1/2*c)^3+1/2*(a^2+2*b^2)*b
/(a^2-b^2)/a^2*tan(1/2*d*x+1/2*c)^2+1/2*(a^2+2*b^2)/a/(a^2-b^2)*tan(1/2*d*x
+1/2*c)+1/2*b/(a^2-b^2))/(a*tan(1/2*d*x+1/2*c)^2+2*b*tan(1/2*d*x+1/2*c)+a)^
2+1/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2)
))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.36, size = 501, normalized size = 4.36

$$\left[-\frac{2\,(a^3-ab^2)\cos(dx+c)\sin(dx+c)-(b^2\cos(dx+c)^2-2\,ab\sin(dx+c)-a^2-b^2)\sqrt{-a^2+b^2}\,\log\left(-\frac{(2\,a^2-b^2)\cos(dx+c)^2-2\,ab\sin(dx+c)-a^2-b^2-2\,(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c))\sqrt{-a^2+b^2}}{b^2\cos(dx+c)^2-2\,ab\sin(dx+c)-a^2-b^2}\right)+2\,(a^2b-b^3)\cos(dx+c)}{4\,((a^4b^2-2\,a^2b^4+b^6)d\cos(dx+c)^2-2\,(a^5b-2\,a^3b^3+ab^5)d\sin(dx+c)-(a^6-a^4b^2-a^2b^4+b^6)d)},\ -\frac{(a^3-ab^2)\cos(dx+c)\sin(dx+c)+(b^2\cos(dx+c)^2-2\,ab\sin(dx+c)-a^2-b^2)\sqrt{a^2-b^2}\,\arctan\left(-\frac{a\sin(dx+c)+b}{\sqrt{a^2-b^2}\cos(dx+c)}\right)+(a^2b-b^3)\cos(dx+c)}{2\,((a^4b^2-2\,a^2b^4+b^6)d\cos(dx+c)^2-2\,(a^5b-2\,a^3b^3+ab^5)d\sin(dx+c)-(a^6-a^4b^2-a^2b^4+b^6)d)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*(a^3 - a*b^2)*cos(d*x + c)*sin(d*x + c) - (b^2*cos(d*x + c)^2 - 2*
a*b*sin(d*x + c) - a^2 - b^2)*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x
+ c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) +
b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c)
- a^2 - b^2)) + 2*(a^2*b - b^3)*cos(d*x + c))/((a^4*b^2 - 2*a^2*b^4 + b^6)*
d*cos(d*x + c)^2 - 2*(a^5*b - 2*a^3*b^3 + a*b^5)*d*sin(d*x + c) - (a^6 - a^
4*b^2 - a^2*b^4 + b^6)*d), -1/2*((a^3 - a*b^2)*cos(d*x + c)*sin(d*x + c) +
(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)*sqrt(a^2 - b^2)*arcta
n(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) + (a^2*b - b^3)*cos
(d*x + c))/((a^4*b^2 - 2*a^2*b^4 + b^6)*d*cos(d*x + c)^2 - 2*(a^5*b - 2*a^3
*b^3 + a*b^5)*d*sin(d*x + c) - (a^6 - a^4*b^2 - a^2*b^4 + b^6)*d)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 4.84, size = 207, normalized size = 1.80

$$\frac{\frac{\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{\frac{3}{2}}}-\frac{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-2\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-2\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a^2b}{(a^4-a^2b^2)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a\right)^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] ((pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c)
+ b)/sqrt(a^2 - b^2)))/(a^2 - b^2)^(3/2) - (a^3*tan(1/2*d*x + 1/2*c)^3 - 2*
a*b^2*tan(1/2*d*x + 1/2*c)^3 - a^2*b*tan(1/2*d*x + 1/2*c)^2 - 2*b^3*tan(1/2
*d*x + 1/2*c)^2 - a^3*tan(1/2*d*x + 1/2*c) - 2*a*b^2*tan(1/2*d*x + 1/2*c) -
 a^2*b)/((a^4 - a^2*b^2)*(a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*
c) + a)^2))/d
```

Mupad [B]
time = 7.37, size = 282, normalized size = 2.45

$$\frac{\frac{b}{a^2-b^2}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(a^2-2\,b^2\right)}{a\,\left(a^2-b^2\right)}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(a^2+2\,b^2\right)}{a\,\left(a^2-b^2\right)}+\frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(a^2+2\,b^2\right)}{a^2\,\left(a^2-b^2\right)}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(2\,a^2+4\,b^2\right)+a^2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+a^2+4\,a\,b\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3+4\,a\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}+\frac{\mathrm{atan}\left(\left(a^2-b^2\right)\,\left(\frac{a^2\,b-b^3}{{\left(a+b\right)}^{3/2}\,\left(a^2-b^2\right)\,{\left(a-b\right)}^{3/2}}+\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{{\left(a+b\right)}^{3/2}\,{\left(a-b\right)}^{3/2}}\right)\right)}{d\,{\left(a+b\right)}^{3/2}\,{\left(a-b\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a + b*sin(c + d*x))^3,x)
```

```
[Out] (b/(a^2 - b^2) - (tan(c/2 + (d*x)/2)^3*(a^2 - 2*b^2))/(a*(a^2 - b^2)) + (ta
n(c/2 + (d*x)/2)*(a^2 + 2*b^2))/(a*(a^2 - b^2)) + (b*tan(c/2 + (d*x)/2)^2*(
a^2 + 2*b^2))/(a^2*(a^2 - b^2)))/(d*(tan(c/2 + (d*x)/2)^2*(2*a^2 + 4*b^2) +
 a^2*tan(c/2 + (d*x)/2)^4 + a^2 + 4*a*b*tan(c/2 + (d*x)/2)^3 + 4*a*b*tan(c/
2 + (d*x)/2))) + atan((a^2 - b^2)*((a^2*b - b^3)/((a + b)^(3/2)*(a^2 - b^2)
*(a - b)^(3/2)) + (a*tan(c/2 + (d*x)/2))/((a + b)^(3/2)*(a - b)^(3/2))))/(d
*(a + b)^(3/2)*(a - b)^(3/2))
```

3.459 $\int \frac{\sec^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=192

$$-\frac{3b^2(4a^2+b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2}\,d}+\frac{b\sec(c+dx)}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))^2}+\frac{5ab\sec(c+dx)}{2\left(a^2-b^2\right)^2d(a+b\sin(c+dx))}-$$

```
[Out] -3*b^2*(4*a^2+b^2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/(a^2-b^
2)^(7/2)/d+1/2*b*sec(d*x+c)/(a^2-b^2)/d/(a+b*sin(d*x+c))^2+5/2*a*b*sec(d*x+
c)/(a^2-b^2)^2/d/(a+b*sin(d*x+c))-1/2*sec(d*x+c)*(3*b*(4*a^2+b^2)-a*(2*a^2+
13*b^2)*sin(d*x+c))/(a^2-b^2)^3/d
```

Rubi [A]

time = 0.25, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2773, 2943, 2945, 12, 2739, 632, 210}

$$-\frac{3b^2(4a^2+b^2)\,\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{7/2}}+\frac{5ab\sec(c+dx)}{2d\left(a^2-b^2\right)^2(a+b\sin(c+dx))}+\frac{b\sec(c+dx)}{2d\left(a^2-b^2\right)(a+b\sin(c+dx))^2}-\frac{\sec(c+dx)\left(3b(4a^2+b^2)-a(2a^2+13b^2)\sin(c+dx)\right)}{2d\left(a^2-b^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-3*b^2*(4*a^2 + b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a
^2 - b^2)^(7/2)*d) + (b*Sec[c + d*x])/(2*(a^2 - b^2)*d*(a + b*Sin[c + d*x])
^2) + (5*a*b*Sec[c + d*x])/(2*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x])) - (Sec[
c + d*x]*(3*b*(4*a^2 + b^2) - a*(2*a^2 + 13*b^2)*Sin[c + d*x]))/(2*(a^2 - b
^2)^3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2773

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m + 1)/(f*g*(a^2 - b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1))
, Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p
 + 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^
2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2943

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c
 - a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^(m + 1)/(f*g*(a^2 -
 b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*
(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p
+ 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[
a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2945

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \frac{b\sec(c+dx)}{2(a^2-b^2)\,d(a+b\sin(c+dx))^2} - \frac{\int \frac{\sec^2(c+dx)(-2a+3b\sin(c+dx))}{(a+b\sin(c+dx))^2}\,dx}{2(a^2-b^2)}$$

$$= \frac{b\sec(c+dx)}{2(a^2-b^2)\,d(a+b\sin(c+dx))^2} + \frac{5ab\sec(c+dx)}{2(a^2-b^2)^2\,d(a+b\sin(c+dx))} + \frac{\int \frac{\sec^2(c+dx}{}}{}$$

$$= \frac{b\sec(c+dx)}{2(a^2-b^2)\,d(a+b\sin(c+dx))^2} + \frac{5ab\sec(c+dx)}{2(a^2-b^2)^2\,d(a+b\sin(c+dx))} - \frac{\sec(c+d}{}$$

$$= \frac{b\sec(c+dx)}{2(a^2-b^2)\,d(a+b\sin(c+dx))^2} + \frac{5ab\sec(c+dx)}{2(a^2-b^2)^2\,d(a+b\sin(c+dx))} - \frac{\sec(c+d}{}$$

$$= \frac{b\sec(c+dx)}{2(a^2-b^2)\,d(a+b\sin(c+dx))^2} + \frac{5ab\sec(c+dx)}{2(a^2-b^2)^2\,d(a+b\sin(c+dx))} - \frac{\sec(c+d}{}$$

$$= \frac{b\sec(c+dx)}{2(a^2-b^2)\,d(a+b\sin(c+dx))^2} + \frac{5ab\sec(c+dx)}{2(a^2-b^2)^2\,d(a+b\sin(c+dx))} - \frac{\sec(c+d}{}$$

$$= -\frac{3b^2(4a^2+b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2}\,d} + \frac{b\sec(c+dx)}{2(a^2-b^2)\,d(a+b\sin(c+dx))^2} + \frac{}{2}$$

Mathematica [A]
time = 3.17, size = 193, normalized size = 1.01

$$\frac{-\frac{6b^2(4a^2+b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2}} + \sin\left(\frac{1}{2}(c+dx)\right)\left(\frac{2}{(a+b)^3\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)} + \frac{2}{(a-b)^3\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}\right) + \frac{b^3\cos(c+dx)(-8a^2+b^2-7ab\sin(c+dx))}{(a-b)^3(a+b)^3(a+b\sin(c+dx))^2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((-6*b^2*(4*a^2 + b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a
^2 - b^2)^(7/2) + Sin[(c + d*x)/2]*(2/((a + b)^3*(Cos[(c + d*x)/2] - Sin[(c
 + d*x)/2])) + 2/((a - b)^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))) + (b^3*
Cos[c + d*x]*(-8*a^2 + b^2 - 7*a*b*Sin[c + d*x]))/((a - b)^3*(a + b)^3*(a +
 b*Sin[c + d*x])^2))/(2*d)
```

Maple [A]
time = 0.81, size = 254, normalized size = 1.32

method	result

derivativedivides	$\frac{-\frac{1}{(a+b)^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{2b^2\left(\frac{\frac{b^2(9a^2-2b^2)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}+\frac{b(8a^4+15a^2b^2-2b^4)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a^2}+\frac{b^2(23a^2-2b^2)\tan\left(\frac{dx}{2}\right.}{2a}}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a\right)^2}\right.}{(a-b)^3(a+b)^3}}{d}$
default	$\frac{-\frac{1}{(a+b)^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{2b^2\left(\frac{\frac{b^2(9a^2-2b^2)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}+\frac{b(8a^4+15a^2b^2-2b^4)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a^2}+\frac{b^2(23a^2-2b^2)\tan\left(\frac{dx}{2}\right.}{2a}}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a\right)^2}\right.}{(a-b)^3(a+b)^3}}{d}$
risch	$\frac{i\left(-12ia^2b^3e^{5i(dx+c)}-3ib^5e^{5i(dx+c)}+16ia^4b\,e^{3i(dx+c)}+12ia^2b^3e^{3i(dx+c)}+2ib^5e^{3i(dx+c)}+36a^3b^2e^{4i(dx+c)}+9a\,b^4e^{4i(dx+c)}\right.}{\left(1+e^{2i(dx+c)}\right)\left(-ib\,e^{2i(dx+c)}+\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/(a+b)^3/(tan(1/2*d*x+1/2*c)-1)-2*b^2/(a-b)^3/(a+b)^3*((1/2*b^2*(9*a
^2-2*b^2)/a*tan(1/2*d*x+1/2*c)^3+1/2*b*(8*a^4+15*a^2*b^2-2*b^4)/a^2*tan(1/2
*d*x+1/2*c)^2+1/2*b^2*(23*a^2-2*b^2)/a*tan(1/2*d*x+1/2*c)+4*a^2*b-1/2*b^3)/
(a*tan(1/2*d*x+1/2*c)^2+2*b*tan(1/2*d*x+1/2*c)+a)^2+3/2*(4*a^2+b^2)/(a^2-b^
2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2)))-1/(a-b)^
3/(tan(1/2*d*x+1/2*c)+1))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 405 vs. $2(180) = 360$.
time = 0.40, size = 894, normalized size = 4.66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [1/4*(4*a^6*b - 12*a^4*b^3 + 12*a^2*b^5 - 4*b^7 + 2*(4*a^6*b + 10*a^4*b^3 -
 17*a^2*b^5 + 3*b^7)*cos(d*x + c)^2 + 3*((4*a^2*b^4 + b^6)*cos(d*x + c)^3 -
 2*(4*a^3*b^3 + a*b^5)*cos(d*x + c)*sin(d*x + c) - (4*a^4*b^2 + 5*a^2*b^4 +
 b^6)*cos(d*x + c))*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*
a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x +
 c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2
)) - 2*(2*a^7 - 6*a^5*b^2 + 6*a^3*b^4 - 2*a*b^6 - (2*a^5*b^2 + 11*a^3*b^4 -
 13*a*b^6)*cos(d*x + c)^2)*sin(d*x + c))/((a^8*b^2 - 4*a^6*b^4 + 6*a^4*b^6
- 4*a^2*b^8 + b^10)*d*cos(d*x + c)^3 - 2*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4
*a^3*b^7 + a*b^9)*d*cos(d*x + c)*sin(d*x + c) - (a^10 - 3*a^8*b^2 + 2*a^6*b
^4 + 2*a^4*b^6 - 3*a^2*b^8 + b^10)*d*cos(d*x + c)), 1/2*(2*a^6*b - 6*a^4*b^
3 + 6*a^2*b^5 - 2*b^7 + (4*a^6*b + 10*a^4*b^3 - 17*a^2*b^5 + 3*b^7)*cos(d*x
 + c)^2 + 3*((4*a^2*b^4 + b^6)*cos(d*x + c)^3 - 2*(4*a^3*b^3 + a*b^5)*cos(d
*x + c)*sin(d*x + c) - (4*a^4*b^2 + 5*a^2*b^4 + b^6)*cos(d*x + c))*sqrt(a^2
 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - (2*a
^7 - 6*a^5*b^2 + 6*a^3*b^4 - 2*a*b^6 - (2*a^5*b^2 + 11*a^3*b^4 - 13*a*b^6)*
cos(d*x + c)^2)*sin(d*x + c))/((a^8*b^2 - 4*a^6*b^4 + 6*a^4*b^6 - 4*a^2*b^8
 + b^10)*d*cos(d*x + c)^3 - 2*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b^7 +
a*b^9)*d*cos(d*x + c)*sin(d*x + c) - (a^10 - 3*a^8*b^2 + 2*a^6*b^4 + 2*a^4*
b^6 - 3*a^2*b^8 + b^10)*d*cos(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**2/(a + b*sin(c + d*x))**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 385 vs. 2(180) = 360.
time = 4.49, size = 385, normalized size = 2.01

$$-\frac{\frac{3\,(4\,a^2b^2+b^4)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{(a^6-3\,a^4b^2+3\,a^2b^4-b^6)\sqrt{a^2-b^2}}+\frac{2\,(a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3\,a^2b-b^3)}{(a^6-3\,a^4b^2+3\,a^2b^4-b^6)\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)}+\frac{9\,a^3b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-2\,ab^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+8\,a^4b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+15\,a^2b^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-2\,b^7\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+23\,a^3b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,ab^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+8\,a^4b^3-a^2b^5}{(a^8-3\,a^6b^2+3\,a^4b^4-a^2b^6)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a\right)^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -(3*(4*a^2*b^2 + b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*
tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 -
 b^6)*sqrt(a^2 - b^2)) + 2*(a^3*tan(1/2*d*x + 1/2*c) + 3*a*b^2*tan(1/2*d*x
```

```
+ 1/2*c) - 3*a^2*b - b^3)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(tan(1/2*d*x
 + 1/2*c)^2 - 1)) + (9*a^3*b^4*tan(1/2*d*x + 1/2*c)^3 - 2*a*b^6*tan(1/2*d*x
 + 1/2*c)^3 + 8*a^4*b^3*tan(1/2*d*x + 1/2*c)^2 + 15*a^2*b^5*tan(1/2*d*x + 1
/2*c)^2 - 2*b^7*tan(1/2*d*x + 1/2*c)^2 + 23*a^3*b^4*tan(1/2*d*x + 1/2*c) -
2*a*b^6*tan(1/2*d*x + 1/2*c) + 8*a^4*b^3 - a^2*b^5)/((a^8 - 3*a^6*b^2 + 3*a
^4*b^4 - a^2*b^6)*(a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)
^2))/d
```

Mupad [B]
time = 8.82, size = 650, normalized size = 3.39

$$-\frac{\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(2\,a^5-2\,a^3\,b^2+15\,a\,b^4\right)}{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}-\frac{6\,a^4\,b+10\,a^2\,b^3-b^5}{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\left(2\,a^6+6\,a^4\,b^2+9\,a^2\,b^4-2\,b^6\right)}{a\,\left(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6\right)}-\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(2\,a^6\,b+2\,a^4\,b^3+12\,a^2\,b^5-b^7\right)}{a^2\,\left(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6\right)}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(2\,a^6\,b+30\,a^4\,b^3+15\,a^2\,b^5-2\,b^7\right)}{a^2\,\left(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6\right)}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,a^6-18\,a^4\,b^2-31\,a^2\,b^4+2\,b^6\right)}{a\,\left(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6\right)}}{d\,\left(a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-a^2-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(a^2+4\,b^2\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(a^2+4\,b^2\right)+4\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5-4\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}-\frac{3\,b^2\,\mathrm{atan}\left(\frac{\frac{3\,b^2\,\left(4\,a^2+b^2\right)\,\left(2\,a^6\,b-6\,a^4\,b^3+6\,a^2\,b^5-2\,b^7\right)}{2\,{\left(a+b\right)}^{7/2}\,{\left(a-b\right)}^{7/2}}+\frac{3\,a\,b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(4\,a^2+b^2\right)\,\left(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6\right)}{{\left(a+b\right)}^{7/2}\,{\left(a-b\right)}^{7/2}}}{12\,a^2\,b^2+3\,b^4}\right)\,\left(4\,a^2+b^2\right)}{d\,{\left(a+b\right)}^{7/2}\,{\left(a-b\right)}^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + b*sin(c + d*x))^3),x)
```

```
[Out] - ((2*tan(c/2 + (d*x)/2)^3*(15*a*b^4 + 2*a^5 - 2*a^3*b^2))/(a^6 - b^6 + 3*a
^2*b^4 - 3*a^4*b^2) - (6*a^4*b - b^5 + 10*a^2*b^3)/(a^6 - b^6 + 3*a^2*b^4 -
 3*a^4*b^2) + (tan(c/2 + (d*x)/2)^5*(2*a^6 - 2*b^6 + 9*a^2*b^4 + 6*a^4*b^2)
)/(a*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) - (2*tan(c/2 + (d*x)/2)^2*(2*a^6*
b - b^7 + 12*a^2*b^5 + 2*a^4*b^3))/(a^2*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)
) + (tan(c/2 + (d*x)/2)^4*(2*a^6*b - 2*b^7 + 15*a^2*b^5 + 30*a^4*b^3))/(a^2
*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) + (tan(c/2 + (d*x)/2)*(2*a^6 + 2*b^6
- 31*a^2*b^4 - 18*a^4*b^2))/(a*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)))/(d*(a^
2*tan(c/2 + (d*x)/2)^6 - a^2 - tan(c/2 + (d*x)/2)^2*(a^2 + 4*b^2) + tan(c/2
 + (d*x)/2)^4*(a^2 + 4*b^2) + 4*a*b*tan(c/2 + (d*x)/2)^5 - 4*a*b*tan(c/2 +
(d*x)/2))) - (3*b^2*atan(((3*b^2*(4*a^2 + b^2)*(2*a^6*b - 2*b^7 + 6*a^2*b^5
 - 6*a^4*b^3))/(2*(a + b)^(7/2)*(a - b)^(7/2)) + (3*a*b^2*tan(c/2 + (d*x)/2
)*(4*a^2 + b^2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2))/((a + b)^(7/2)*(a - b)
^(7/2)))/(3*b^4 + 12*a^2*b^2))*(4*a^2 + b^2))/(d*(a + b)^(7/2)*(a - b)^(7/2
))
```

3.460 $\int \frac{\sec^4(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=264

$$\frac{5b^4(6a^2+b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{9/2}\,d}+\frac{b\sec^3(c+dx)}{2\,(a^2-b^2)\,d(a+b\sin(c+dx))^2}+\frac{7ab\sec^3(c+dx)}{2\,(a^2-b^2)^2\,d(a+b\sin(c+dx))}-$$

```
[Out] 5*b^4*(6*a^2+b^2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/(a^2-b^2
)^(9/2)/d+1/2*b*sec(d*x+c)^3/(a^2-b^2)/d/(a+b*sin(d*x+c))^2+7/2*a*b*sec(d*x
+c)^3/(a^2-b^2)^2/d/(a+b*sin(d*x+c))-1/6*sec(d*x+c)^3*(5*b*(6*a^2+b^2)-a*(2
*a^2+33*b^2)*sin(d*x+c))/(a^2-b^2)^3/d+1/6*sec(d*x+c)*(15*b^3*(6*a^2+b^2)+a
*(4*a^4-28*a^2*b^2-81*b^4)*sin(d*x+c))/(a^2-b^2)^4/d
```

Rubi [A]
time = 0.42, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2773, 2943, 2945, 12, 2739, 632, 210}

$$\frac{5b^4(6a^2+b^2)\,\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\,(a^2-b^2)^{9/2}}+\frac{7ab\sec^3(c+dx)}{2d\,(a^2-b^2)^2\,(a+b\sin(c+dx))}+\frac{b\sec^3(c+dx)}{2d\,(a^2-b^2)\,(a+b\sin(c+dx))^2}-\frac{\sec^3(c+dx)\,(5b(6a^2+b^2)-a(2a^2+33b^2)\sin(c+dx))}{6d\,(a^2-b^2)^3}+\frac{\sec(c+dx)\,(15b^3(6a^2+b^2)+a(4a^4-28a^2b^2-81b^4)\sin(c+dx))}{6d\,(a^2-b^2)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (5*b^4*(6*a^2 + b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^
2 - b^2)^(9/2)*d) + (b*Sec[c + d*x]^3)/(2*(a^2 - b^2)*d*(a + b*Sin[c + d*x]
)^2) + (7*a*b*Sec[c + d*x]^3)/(2*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x])) - (S
ec[c + d*x]^3*(5*b*(6*a^2 + b^2) - a*(2*a^2 + 33*b^2)*Sin[c + d*x]))/(6*(a^
2 - b^2)^3*d) + (Sec[c + d*x]*(15*b^3*(6*a^2 + b^2) + a*(4*a^4 - 28*a^2*b^2
 - 81*b^4)*Sin[c + d*x]))/(6*(a^2 - b^2)^4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2773

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m + 1)/(f*g*(a^2 - b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1))
, Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p
 + 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^
2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2943

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c
 - a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^(m + 1)/(f*g*(a^2 -
 b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*
(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p
+ 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[
a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2945

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \frac{b\sec^3(c+dx)}{2(a^2-b^2)\,d(a+b\sin(c+dx))^2} - \frac{\int \frac{\sec^4(c+dx)(-2a+5b\sin(c+dx))}{(a+b\sin(c+dx))^2}\,dx}{2(a^2-b^2)}$$

$$= \frac{b\sec^3(c+dx)}{2(a^2-b^2)\,d(a+b\sin(c+dx))^2} + \frac{7ab\sec^3(c+dx)}{2(a^2-b^2)^2\,d(a+b\sin(c+dx))} + \frac{\int \frac{\sec^4(c+dx}{\cdots}}{\cdots}$$

$$= \frac{b\sec^3(c+dx)}{2(a^2-b^2)\,d(a+b\sin(c+dx))^2} + \frac{7ab\sec^3(c+dx)}{2(a^2-b^2)^2\,d(a+b\sin(c+dx))} - \frac{\sec^3(c+}{\cdots}$$

$$= \frac{b\sec^3(c+dx)}{2(a^2-b^2)\,d(a+b\sin(c+dx))^2} + \frac{7ab\sec^3(c+dx)}{2(a^2-b^2)^2\,d(a+b\sin(c+dx))} - \frac{\sec^3(c+}{\cdots}$$

$$= \frac{b\sec^3(c+dx)}{2(a^2-b^2)\,d(a+b\sin(c+dx))^2} + \frac{7ab\sec^3(c+dx)}{2(a^2-b^2)^2\,d(a+b\sin(c+dx))} - \frac{\sec^3(c+}{\cdots}$$

$$= \frac{b\sec^3(c+dx)}{2(a^2-b^2)\,d(a+b\sin(c+dx))^2} + \frac{7ab\sec^3(c+dx)}{2(a^2-b^2)^2\,d(a+b\sin(c+dx))} - \frac{\sec^3(c+}{\cdots}$$

$$= \frac{b\sec^3(c+dx)}{2(a^2-b^2)\,d(a+b\sin(c+dx))^2} + \frac{7ab\sec^3(c+dx)}{2(a^2-b^2)^2\,d(a+b\sin(c+dx))} - \frac{\sec^3(c+}{\cdots}$$

$$= \frac{5b^4(6a^2+b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{9/2}\,d} + \frac{b\sec^3(c+dx)}{2(a^2-b^2)\,d(a+b\sin(c+dx))^2} + \frac{\cdots}{2(a\cdots}$$

Mathematica [A]
time = 2.86, size = 380, normalized size = 1.44

$$\frac{\frac{60b^4(6a^2+b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{9/2}} + \frac{1}{(a+b)^3\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2} + \frac{2\sin\left(\frac{1}{2}(c+dx)\right)}{(a+b)^3\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3} + \frac{2(4a+13b)\sin\left(\frac{1}{2}(c+dx)\right)}{(a+b)^4\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)} + \frac{2\sin\left(\frac{1}{2}(c+dx)\right)}{(a-b)^3\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3} - \frac{1}{(a-b)^3\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2} + \frac{2(4a-13b)\sin\left(\frac{1}{2}(c+dx)\right)}{(a-b)^4\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)} + \frac{6b^5\cos(c+dx)}{(a-b)^3(a+b)^3(a+b\sin(c+dx))^2} + \frac{66ab^5\cos(c+dx)}{(a-b)^4(a+b)^4(a+b\sin(c+dx))}}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((60*b^4*(6*a^2 + b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a
^2 - b^2)^(9/2) + 1/((a + b)^3*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2) + (
2*Sin[(c + d*x)/2])/((a + b)^3*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3) + (
2*(4*a + 13*b)*Sin[(c + d*x)/2])/((a + b)^4*(Cos[(c + d*x)/2] - Sin[(c + d*
x)/2])) + (2*Sin[(c + d*x)/2])/((a - b)^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)
/2])^3) - 1/((a - b)^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + (2*(4*a -
 13*b)*Sin[(c + d*x)/2])/((a - b)^4*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
+ (6*b^5*Cos[c + d*x])/((a - b)^3*(a + b)^3*(a + b*Sin[c + d*x])^2) + (66*a
*b^5*Cos[c + d*x])/((a - b)^4*(a + b)^4*(a + b*Sin[c + d*x])))/(12*d)
```

Maple [A]
time = 1.53, size = 352, normalized size = 1.33 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/3/(a+b)^3/(tan(1/2*d*x+1/2*c)-1)^3-1/2/(a+b)^3/(tan(1/2*d*x+1/2*c)-
1)^2-1/2*(2*a+5*b)/(a+b)^4/(tan(1/2*d*x+1/2*c)-1)+2*b^4/(a-b)^4/(a+b)^4*((1
/2*b^2*(13*a^2-2*b^2)/a*tan(1/2*d*x+1/2*c)^3+1/2*b*(12*a^4+23*a^2*b^2-2*b^4
)/a^2*tan(1/2*d*x+1/2*c)^2+1/2*b^2*(35*a^2-2*b^2)/a*tan(1/2*d*x+1/2*c)+6*a^
2*b-1/2*b^3)/(a*tan(1/2*d*x+1/2*c)^2+2*b*tan(1/2*d*x+1/2*c)+a)^2+5/2*(6*a^2
+b^2)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/
2)))-1/3/(a-b)^3/(tan(1/2*d*x+1/2*c)+1)^3+1/2/(a-b)^3/(tan(1/2*d*x+1/2*c)+1
)^2-1/2*(2*a-5*b)/(a-b)^4/(tan(1/2*d*x+1/2*c)+1))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 558 vs. $2(250) = 500$.
time = 0.43, size = 1200, normalized size = 4.55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [1/12*(4*a^8*b - 16*a^6*b^3 + 24*a^4*b^5 - 16*a^2*b^7 + 4*b^9 + 2*(8*a^8*b
- 64*a^6*b^3 - 16*a^4*b^5 + 87*a^2*b^7 - 15*b^9)*cos(d*x + c)^4 - 4*(2*a^8*
b - a^6*b^3 - 9*a^4*b^5 + 13*a^2*b^7 - 5*b^9)*cos(d*x + c)^2 - 15*((6*a^2*b
^6 + b^8)*cos(d*x + c)^5 - 2*(6*a^3*b^5 + a*b^7)*cos(d*x + c)^3*sin(d*x + c
) - (6*a^4*b^4 + 7*a^2*b^6 + b^8)*cos(d*x + c)^3)*sqrt(-a^2 + b^2)*log(((2*
a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x +
 c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 -
2*a*b*sin(d*x + c) - a^2 - b^2)) - 2*(2*a^9 - 8*a^7*b^2 + 12*a^5*b^4 - 8*a^
3*b^6 + 2*a*b^8 - (4*a^7*b^2 - 32*a^5*b^4 - 53*a^3*b^6 + 81*a*b^8)*cos(d*x
+ c)^4 + 2*(2*a^9 - 15*a^7*b^2 + 33*a^5*b^4 - 29*a^3*b^6 + 9*a*b^8)*cos(d*x
 + c)^2)*sin(d*x + c))/((a^10*b^2 - 5*a^8*b^4 + 10*a^6*b^6 - 10*a^4*b^8 + 5
```

```
*a^2*b^10 - b^12)*d*cos(d*x + c)^5 - 2*(a^11*b - 5*a^9*b^3 + 10*a^7*b^5 - 1
0*a^5*b^7 + 5*a^3*b^9 - a*b^11)*d*cos(d*x + c)^3*sin(d*x + c) - (a^12 - 4*a
^10*b^2 + 5*a^8*b^4 - 5*a^4*b^8 + 4*a^2*b^10 - b^12)*d*cos(d*x + c)^3), 1/6
*(2*a^8*b - 8*a^6*b^3 + 12*a^4*b^5 - 8*a^2*b^7 + 2*b^9 + (8*a^8*b - 64*a^6*
b^3 - 16*a^4*b^5 + 87*a^2*b^7 - 15*b^9)*cos(d*x + c)^4 - 2*(2*a^8*b - a^6*b
^3 - 9*a^4*b^5 + 13*a^2*b^7 - 5*b^9)*cos(d*x + c)^2 - 15*((6*a^2*b^6 + b^8)
*cos(d*x + c)^5 - 2*(6*a^3*b^5 + a*b^7)*cos(d*x + c)^3*sin(d*x + c) - (6*a^
4*b^4 + 7*a^2*b^6 + b^8)*cos(d*x + c)^3)*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x
 + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - (2*a^9 - 8*a^7*b^2 + 12*a^5*b^
4 - 8*a^3*b^6 + 2*a*b^8 - (4*a^7*b^2 - 32*a^5*b^4 - 53*a^3*b^6 + 81*a*b^8)*
cos(d*x + c)^4 + 2*(2*a^9 - 15*a^7*b^2 + 33*a^5*b^4 - 29*a^3*b^6 + 9*a*b^8)
*cos(d*x + c)^2)*sin(d*x + c))/((a^10*b^2 - 5*a^8*b^4 + 10*a^6*b^6 - 10*a^4
*b^8 + 5*a^2*b^10 - b^12)*d*cos(d*x + c)^5 - 2*(a^11*b - 5*a^9*b^3 + 10*a^7
*b^5 - 10*a^5*b^7 + 5*a^3*b^9 - a*b^11)*d*cos(d*x + c)^3*sin(d*x + c) - (a^
12 - 4*a^10*b^2 + 5*a^8*b^4 - 5*a^4*b^8 + 4*a^2*b^10 - b^12)*d*cos(d*x + c)
^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^4(c+dx)}{(a+b\sin(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**4/(a + b*sin(c + d*x))**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 622 vs. 2(250) = 500.
time = 5.15, size = 622, normalized size = 2.36

$$\frac{\frac{15\,(6\,a^2b^4+b^6)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{(a^8-4\,a^6b^2+6\,a^4b^4-4\,a^2b^6+b^8)\sqrt{a^2-b^2}}+\frac{3\left(13\,a^3b^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-2\,ab^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+12\,a^4b^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+23\,a^2b^7\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-2\,b^9\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+35\,a^3b^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,ab^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+12\,a^4b^5-a^2b^7\right)}{(a^{10}-4\,a^8b^2+6\,a^6b^4-4\,a^4b^6+a^2b^8)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a\right)^2}-\frac{2\left(3\,a^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-12\,a^3b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-27\,ab^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-9\,a^4b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+36\,a^2b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+9\,b^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-2\,a^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+32\,a^3b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+42\,ab^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-60\,a^2b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-12\,b^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+3\,a^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-12\,a^3b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-27\,ab^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3\,a^4b+32\,a^2b^3+7\,b^5\right)}{(a^8-4\,a^6b^2+6\,a^4b^4-4\,a^2b^6+b^8)\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/3*(15*(6*a^2*b^4 + b^6)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan
((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/((a^8 - 4*a^6*b^2 + 6*a^4*b
^4 - 4*a^2*b^6 + b^8)*sqrt(a^2 - b^2)) + 3*(13*a^3*b^6*tan(1/2*d*x + 1/2*c)
^3 - 2*a*b^8*tan(1/2*d*x + 1/2*c)^3 + 12*a^4*b^5*tan(1/2*d*x + 1/2*c)^2 + 2
3*a^2*b^7*tan(1/2*d*x + 1/2*c)^2 - 2*b^9*tan(1/2*d*x + 1/2*c)^2 + 35*a^3*b^
6*tan(1/2*d*x + 1/2*c) - 2*a*b^8*tan(1/2*d*x + 1/2*c) + 12*a^4*b^5 - a^2*b^
7)/((a^10 - 4*a^8*b^2 + 6*a^6*b^4 - 4*a^4*b^6 + a^2*b^8)*(a*tan(1/2*d*x + 1
/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)^2) - 2*(3*a^5*tan(1/2*d*x + 1/2*c)^
5 - 12*a^3*b^2*tan(1/2*d*x + 1/2*c)^5 - 27*a*b^4*tan(1/2*d*x + 1/2*c)^5 - 9
```

```
*a^4*b*tan(1/2*d*x + 1/2*c)^4 + 36*a^2*b^3*tan(1/2*d*x + 1/2*c)^4 + 9*b^5*t
an(1/2*d*x + 1/2*c)^4 - 2*a^5*tan(1/2*d*x + 1/2*c)^3 + 32*a^3*b^2*tan(1/2*d
*x + 1/2*c)^3 + 42*a*b^4*tan(1/2*d*x + 1/2*c)^3 - 60*a^2*b^3*tan(1/2*d*x +
1/2*c)^2 - 12*b^5*tan(1/2*d*x + 1/2*c)^2 + 3*a^5*tan(1/2*d*x + 1/2*c) - 12*
a^3*b^2*tan(1/2*d*x + 1/2*c) - 27*a*b^4*tan(1/2*d*x + 1/2*c) - 3*a^4*b + 32
*a^2*b^3 + 7*b^5)/((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*(tan(1/2
*d*x + 1/2*c)^2 - 1)^3))/d
```

Mupad [B]

time = 9.37, size = 1167, normalized size = 4.42

$$\frac{5\,b^4\,\mathrm{atan}\left(\frac{\frac{5\,b^4\,\left(6\,a^2+b^2\right)\,\left(2\,a^8\,b-8\,a^6\,b^3+12\,a^4\,b^5-8\,a^2\,b^7+2\,b^9\right)}{2\,{\left(a+b\right)}^{9/2}\,{\left(a-b\right)}^{9/2}}+\frac{5\,a\,b^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(6\,a^2+b^2\right)\,\left(a^8-4\,a^6\,b^2+6\,a^4\,b^4-4\,a^2\,b^6+b^8\right)}{{\left(a+b\right)}^{9/2}\,{\left(a-b\right)}^{9/2}}}{30\,a^2\,b^4+5\,b^6}\right)\,\left(6\,a^2+b^2\right)}{d\,{\left(a+b\right)}^{9/2}\,{\left(a-b\right)}^{9/2}}-\frac{\frac{2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5\,\left(2\,a^7-4\,a^5\,b^2+62\,a^3\,b^4+255\,a\,b^6\right)}{3\,\left(a^8-4\,a^6\,b^2+6\,a^4\,b^4-4\,a^2\,b^6+b^8\right)}-\frac{6\,a^6\,b-64\,a^4\,b^3-50\,a^2\,b^5+3\,b^7}{3\,\left(a^2-b^2\right)\,\left(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6\right)}+\frac{4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7\,\left(2\,a^6-6\,a^4\,b^2+36\,a^2\,b^4+3\,b^6\right)}{3\,a\,\left(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6\right)}+\frac{2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(6\,a^6\,b-50\,a^4\,b^3-64\,a^2\,b^5+3\,b^7\right)}{3\,a^2\,\left(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6\right)}-\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^9\,\left(-2\,a^8+8\,a^6\,b^2+18\,a^4\,b^4+13\,a^2\,b^6-2\,b^8\right)}{a\,\left(a^8-4\,a^6\,b^2+6\,a^4\,b^4-4\,a^2\,b^6+b^8\right)}-\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8\,\left(-2\,a^8\,b+8\,a^6\,b^3+78\,a^4\,b^5+23\,a^2\,b^7-2\,b^9\right)}{a^2\,\left(a^8-4\,a^6\,b^2+6\,a^4\,b^4-4\,a^2\,b^6+b^8\right)}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(6\,a^8-48\,a^6\,b^2+202\,a^4\,b^4+161\,a^2\,b^6-6\,b^8\right)}{3\,a\,\left(a^2-b^2\right)\,\left(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6\right)}+\frac{4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(2\,a^8+4\,a^6\,b^2-86\,a^4\,b^4-133\,a^2\,b^6+3\,b^8\right)}{3\,a\,\left(a^2-b^2\right)\,\left(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6\right)}-\frac{2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(8\,a^8\,b-28\,a^6\,b^3+188\,a^4\,b^5+156\,a^2\,b^7-9\,b^9\right)}{3\,a^2\,\left(a^2-b^2\right)\,\left(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6\right)}+\frac{2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6\,\left(-14\,a^8\,b+56\,a^6\,b^3+246\,a^4\,b^5+141\,a^2\,b^7-9\,b^9\right)}{3\,a^2\,\left(a^2-b^2\right)\,\left(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6\right)}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(2\,a^2+12\,b^2\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6\,\left(2\,a^2+12\,b^2\right)+a^2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}-a^2+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(a^2-4\,b^2\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8\,\left(a^2-4\,b^2\right)+8\,a\,b\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3-8\,a\,b\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7+4\,a\,b\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^9-4\,a\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a + b*sin(c + d*x))^3),x)
```

```
[Out] (5*b^4*atan(((5*b^4*(6*a^2 + b^2)*(2*a^8*b + 2*b^9 - 8*a^2*b^7 + 12*a^4*b^5
 - 8*a^6*b^3))/(2*(a + b)^(9/2)*(a - b)^(9/2)) + (5*a*b^4*tan(c/2 + (d*x)/2
)*(6*a^2 + b^2)*(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6*b^2))/((a + b)^(
9/2)*(a - b)^(9/2)))/(5*b^6 + 30*a^2*b^4))*(6*a^2 + b^2))/(d*(a + b)^(9/2)*
(a - b)^(9/2)) - ((2*tan(c/2 + (d*x)/2)^5*(255*a*b^6 + 2*a^7 + 62*a^3*b^4 -
 4*a^5*b^2))/(3*(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6*b^2)) - (6*a^6*b
 + 3*b^7 - 50*a^2*b^5 - 64*a^4*b^3)/(3*(a^2 - b^2)*(a^6 - b^6 + 3*a^2*b^4 -
 3*a^4*b^2)) + (4*tan(c/2 + (d*x)/2)^7*(2*a^6 + 3*b^6 + 36*a^2*b^4 - 6*a^4*
b^2))/(3*a*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) + (2*tan(c/2 + (d*x)/2)^2*(
6*a^6*b + 3*b^7 - 64*a^2*b^5 - 50*a^4*b^3))/(3*a^2*(a^6 - b^6 + 3*a^2*b^4 -
 3*a^4*b^2)) - (tan(c/2 + (d*x)/2)^9*(13*a^2*b^6 - 2*b^8 - 2*a^8 + 18*a^4*b
^4 + 8*a^6*b^2))/(a*(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6*b^2)) - (tan
(c/2 + (d*x)/2)^8*(23*a^2*b^7 - 2*b^9 - 2*a^8*b + 78*a^4*b^5 + 8*a^6*b^3))/
(a^2*(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6*b^2)) + (tan(c/2 + (d*x)/2)
*(6*a^8 - 6*b^8 + 161*a^2*b^6 + 202*a^4*b^4 - 48*a^6*b^2))/(3*a*(a^2 - b^2)
*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) + (4*tan(c/2 + (d*x)/2)^3*(2*a^8 + 3*
b^8 - 133*a^2*b^6 - 86*a^4*b^4 + 4*a^6*b^2))/(3*a*(a^2 - b^2)*(a^6 - b^6 +
3*a^2*b^4 - 3*a^4*b^2)) - (2*tan(c/2 + (d*x)/2)^4*(8*a^8*b - 9*b^9 + 156*a^
2*b^7 + 188*a^4*b^5 - 28*a^6*b^3))/(3*a^2*(a^2 - b^2)*(a^6 - b^6 + 3*a^2*b^
4 - 3*a^4*b^2)) + (2*tan(c/2 + (d*x)/2)^6*(141*a^2*b^7 - 9*b^9 - 14*a^8*b +
 246*a^4*b^5 + 56*a^6*b^3))/(3*a^2*(a^2 - b^2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a
^4*b^2)))/(d*(tan(c/2 + (d*x)/2)^4*(2*a^2 + 12*b^2) - tan(c/2 + (d*x)/2)^6*
(2*a^2 + 12*b^2) + a^2*tan(c/2 + (d*x)/2)^10 - a^2 + tan(c/2 + (d*x)/2)^2*(
a^2 - 4*b^2) - tan(c/2 + (d*x)/2)^8*(a^2 - 4*b^2) + 8*a*b*tan(c/2 + (d*x)/2
)^3 - 8*a*b*tan(c/2 + (d*x)/2)^7 + 4*a*b*tan(c/2 + (d*x)/2)^9 - 4*a*b*tan(c
/2 + (d*x)/2)))
```

3.461 $\int \frac{\cos^7(c+dx)}{(a+b\sin(c+dx))^8}\,dx$

Optimal. Leaf size=207

$$\frac{(a^2-b^2)^3}{7b^7d(a+b\sin(c+dx))^7}-\frac{a(a^2-b^2)^2}{b^7d(a+b\sin(c+dx))^6}+\frac{3(5a^4-6a^2b^2+b^4)}{5b^7d(a+b\sin(c+dx))^5}-\frac{a(5a^2-3b^2)}{b^7d(a+b\sin(c+dx))^4}+\frac{}{b^7d(a}$$

```
[Out] 1/7*(a^2-b^2)^3/b^7/d/(a+b*sin(d*x+c))^7-a*(a^2-b^2)^2/b^7/d/(a+b*sin(d*x+c
))^6+3/5*(5*a^4-6*a^2*b^2+b^4)/b^7/d/(a+b*sin(d*x+c))^5-a*(5*a^2-3*b^2)/b^7
/d/(a+b*sin(d*x+c))^4+(5*a^2-b^2)/b^7/d/(a+b*sin(d*x+c))^3-3*a/b^7/d/(a+b*s
in(d*x+c))^2+1/b^7/d/(a+b*sin(d*x+c))
```

Rubi [A]
time = 0.12, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2747, 711}

$$\frac{(a^2-b^2)^3}{7b^7d(a+b\sin(c+dx))^7}-\frac{a(a^2-b^2)^2}{b^7d(a+b\sin(c+dx))^6}+\frac{5a^2-b^2}{b^7d(a+b\sin(c+dx))^3}-\frac{a(5a^2-3b^2)}{b^7d(a+b\sin(c+dx))^4}+\frac{3(5a^4-6a^2b^2+b^4)}{5b^7d(a+b\sin(c+dx))^5}+\frac{1}{b^7d(a+b\sin(c+dx))}-\frac{3a}{b^7d(a+b\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7/(a + b*Sin[c + d*x])^8,x]
```

```
[Out] (a^2 - b^2)^3/(7*b^7*d*(a + b*Sin[c + d*x])^7) - (a*(a^2 - b^2)^2)/(b^7*d*(
a + b*Sin[c + d*x])^6) + (3*(5*a^4 - 6*a^2*b^2 + b^4))/(5*b^7*d*(a + b*Sin[
c + d*x])^5) - (a*(5*a^2 - 3*b^2))/(b^7*d*(a + b*Sin[c + d*x])^4) + (5*a^2
- b^2)/(b^7*d*(a + b*Sin[c + d*x])^3) - (3*a)/(b^7*d*(a + b*Sin[c + d*x])^2
) + 1/(b^7*d*(a + b*Sin[c + d*x]))
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^7(c+dx)}{(a+b\sin(c+dx))^8}\,dx = \frac{\text{Subst}\left(\int \frac{(b^2-x^2)^3}{(a+x)^8}\,dx, x, b\sin(c+dx)\right)}{b^7 d}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{(a^2-b^2)^3}{(a+x)^8} + \frac{6a(a^2-b^2)^2}{(a+x)^7} - \frac{3(5a^4-6a^2b^2+b^4)}{(a+x)^6} + \frac{4(5a^3-3ab^2)}{(a+x)^5} - \frac{3(5a^2-b^2)}{(a+x)^4} + \frac{}{(a}\right.\right.}{b^7 d}$$
$$= \frac{(a^2-b^2)^3}{7b^7 d(a+b\sin(c+dx))^7} - \frac{a(a^2-b^2)^2}{b^7 d(a+b\sin(c+dx))^6} + \frac{3(5a^4-6a^2b^2+b^4)}{5b^7 d(a+b\sin(c+dx))^5}$$

Mathematica [A]
time = 0.45, size = 179, normalized size = 0.86

$$\frac{5a^6-a^4b^2+a^2b^4-5b^6+7ab(5a^4-a^2b^2+b^4)\sin(c+dx)+21b^2(5a^4-a^2b^2+b^4)\sin^2(c+dx)+35ab^3(5a^2-b^2)\sin^3(c+dx)-35b^4(-5a^2+b^2)\sin^4(c+dx)+105ab^5\sin^5(c+dx)+35b^6\sin^6(c+dx)}{35b^7d(a+b\sin(c+dx))^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7/(a + b*Sin[c + d*x])^8,x]
```

```
[Out] (5*a^6 - a^4*b^2 + a^2*b^4 - 5*b^6 + 7*a*b*(5*a^4 - a^2*b^2 + b^4)*Sin[c +
d*x] + 21*b^2*(5*a^4 - a^2*b^2 + b^4)*Sin[c + d*x]^2 + 35*a*b^3*(5*a^2 - b^
2)*Sin[c + d*x]^3 - 35*b^4*(-5*a^2 + b^2)*Sin[c + d*x]^4 + 105*a*b^5*Sin[c
+ d*x]^5 + 35*b^6*Sin[c + d*x]^6)/(35*b^7*d*(a + b*Sin[c + d*x])^7)
```

Maple [A]
time = 3.35, size = 208, normalized size = 1.00

method	result
derivativedivides	$\frac{-\frac{-a^6+3a^4b^2-3a^2b^4+b^6}{7b^7(a+b\sin(dx+c))^7}-\frac{a\left(5a^2-3b^2\right)}{b^7(a+b\sin(dx+c))^4}+\frac{1}{b^7(a+b\sin(dx+c))}-\frac{3a}{b^7(a+b\sin(dx+c))^2}-\frac{-15a^4+18a^2b^2-3b^4}{5b^7(a+b\sin(dx+c))^5}-\frac{a\left(a^4-2a^2b^2\right.}{b^7(a+b\sin(dx}}{d}$
default	$\frac{-\frac{-a^6+3a^4b^2-3a^2b^4+b^6}{7b^7(a+b\sin(dx+c))^7}-\frac{a\left(5a^2-3b^2\right)}{b^7(a+b\sin(dx+c))^4}+\frac{1}{b^7(a+b\sin(dx+c))}-\frac{3a}{b^7(a+b\sin(dx+c))^2}-\frac{-15a^4+18a^2b^2-3b^4}{5b^7(a+b\sin(dx+c))^5}-\frac{a\left(a^4-2a^2b^2\right.}{b^7(a+b\sin(dx}}{d}$
risch	$\frac{2i\left(210iab^5e^{12i(dx+c)}-770iab^5e^{10i(dx+c)}+1484iab^5e^{8i(dx+c)}-1400ia^3b^3e^{10i(dx+c)}-1484iab^5e^{6i(dx+c)}+3976ia^3b^3e^{8i}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7/(a+b*sin(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/7*(-a^6+3*a^4*b^2-3*a^2*b^4+b^6)/b^7/(a+b*sin(d*x+c))^7-a*(5*a^2-3*
b^2)/b^7/(a+b*sin(d*x+c))^4+1/b^7/(a+b*sin(d*x+c))-3*a/b^7/(a+b*sin(d*x+c))
^2-1/5*(-15*a^4+18*a^2*b^2-3*b^4)/b^7/(a+b*sin(d*x+c))^5-a*(a^4-2*a^2*b^2+b
^4)/b^7/(a+b*sin(d*x+c))^6-1/3*(-15*a^2+3*b^2)/b^7/(a+b*sin(d*x+c))^3)
```

Maxima [A]
time = 0.28, size = 279, normalized size = 1.35

$$\frac{35\,b^6\sin\left(dx+c\right)^6+105\,ab^5\sin\left(dx+c\right)^5+5\,a^6-a^4b^2+a^2b^4-5\,b^6+35\,\left(5\,a^2b^4-b^6\right)\sin\left(dx+c\right)^4+35\,\left(5\,a^3b^3-ab^5\right)\sin\left(dx+c\right)^3+21\,\left(5\,a^4b^2-a^2b^4+b^6\right)\sin\left(dx+c\right)^2+7\,\left(5\,a^5b-a^3b^3+ab^5\right)\sin\left(dx+c\right)}{35\,\left(b^{14}\sin\left(dx+c\right)^7+7\,ab^{13}\sin\left(dx+c\right)^6+21\,a^2b^{12}\sin\left(dx+c\right)^5+35\,a^3b^{11}\sin\left(dx+c\right)^4+35\,a^4b^{10}\sin\left(dx+c\right)^3+21\,a^5b^9\sin\left(dx+c\right)^2+7\,a^6b^8\sin\left(dx+c\right)+a^7b^7\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+b*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 1/35*(35*b^6*sin(d*x + c)^6 + 105*a*b^5*sin(d*x + c)^5 + 5*a^6 - a^4*b^2 +
a^2*b^4 - 5*b^6 + 35*(5*a^2*b^4 - b^6)*sin(d*x + c)^4 + 35*(5*a^3*b^3 - a*b
^5)*sin(d*x + c)^3 + 21*(5*a^4*b^2 - a^2*b^4 + b^6)*sin(d*x + c)^2 + 7*(5*a
^5*b - a^3*b^3 + a*b^5)*sin(d*x + c))/((b^14*sin(d*x + c)^7 + 7*a*b^13*sin(
d*x + c)^6 + 21*a^2*b^12*sin(d*x + c)^5 + 35*a^3*b^11*sin(d*x + c)^4 + 35*a
^4*b^10*sin(d*x + c)^3 + 21*a^5*b^9*sin(d*x + c)^2 + 7*a^6*b^8*sin(d*x + c)
 + a^7*b^7)*d)
```

Fricas [A]
time = 0.40, size = 382, normalized size = 1.85

$$\frac{35\,b^6\cos\left(dx+c\right)^6-5\,a^6-104\,a^4b^2-155\,a^2b^4-16\,b^6-35\,\left(5\,a^2b^4+2\,b^6\right)\cos\left(dx+c\right)^4+7\,\left(15\,a^4b^2+47\,a^2b^4+8\,b^6\right)\cos\left(dx+c\right)^2-7\,\left(15\,ab^5\cos\left(dx+c\right)^4+5\,a^5b+24\,a^3b^3+11\,ab^5-25\,\left(a^3b^3+ab^5\right)\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)}{35\,\left(7\,ab^{13}d\cos\left(dx+c\right)^6-7\,\left(5\,a^3b^{11}+3\,ab^{13}\right)d\cos\left(dx+c\right)^4+7\,\left(3\,a^5b^9+10\,a^3b^{11}+3\,ab^{13}\right)d\cos\left(dx+c\right)^2-\left(a^7b^7+21\,a^5b^9+35\,a^3b^{11}+7\,ab^{13}\right)d+\left(b^{14}d\cos\left(dx+c\right)^6-3\,\left(7\,a^2b^{12}+b^{14}\right)d\cos\left(dx+c\right)^4+\left(35\,a^4b^{10}+42\,a^2b^{12}+3\,b^{14}\right)d\cos\left(dx+c\right)^2-\left(7\,a^6b^8+35\,a^4b^{10}+21\,a^2b^{12}+b^{14}\right)d\right)\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+b*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/35*(35*b^6*cos(d*x + c)^6 - 5*a^6 - 104*a^4*b^2 - 155*a^2*b^4 - 16*b^6 -
35*(5*a^2*b^4 + 2*b^6)*cos(d*x + c)^4 + 7*(15*a^4*b^2 + 47*a^2*b^4 + 8*b^6)
*cos(d*x + c)^2 - 7*(15*a*b^5*cos(d*x + c)^4 + 5*a^5*b + 24*a^3*b^3 + 11*a*
b^5 - 25*(a^3*b^3 + a*b^5)*cos(d*x + c)^2)*sin(d*x + c))/(7*a*b^13*d*cos(d*
x + c)^6 - 7*(5*a^3*b^11 + 3*a*b^13)*d*cos(d*x + c)^4 + 7*(3*a^5*b^9 + 10*a
^3*b^11 + 3*a*b^13)*d*cos(d*x + c)^2 - (a^7*b^7 + 21*a^5*b^9 + 35*a^3*b^11
+ 7*a*b^13)*d + (b^14*d*cos(d*x + c)^6 - 3*(7*a^2*b^12 + b^14)*d*cos(d*x +
c)^4 + (35*a^4*b^10 + 42*a^2*b^12 + 3*b^14)*d*cos(d*x + c)^2 - (7*a^6*b^8 +
 35*a^4*b^10 + 21*a^2*b^12 + b^14)*d)*sin(d*x + c))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 2530 vs. $2(184) = 368$.
time = 15.57, size = 2530, normalized size = 12.22

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7/(a+b*sin(d*x+c))**8,x)
```

```
[Out] Piecewise((x*cos(c)**7/a**8, Eq(b, 0) & Eq(d, 0)), ((16*sin(c + d*x)**7/(35
*d) + 8*sin(c + d*x)**5*cos(c + d*x)**2/(5*d) + 2*sin(c + d*x)**3*cos(c + d
```

*x)**4/d + sin(c + d*x)*cos(c + d*x)**6/d)/a**8, Eq(b, 0)), (x*cos(c)**7/(a
 + b*sin(c))**8, Eq(d, 0)), (5*a**6/(35*a**7*b**7*d + 245*a**6*b**8*d*sin(c
 + d*x) + 735*a**5*b**9*d*sin(c + d*x)**2 + 1225*a**4*b**10*d*sin(c + d*x)*
*3 + 1225*a**3*b**11*d*sin(c + d*x)**4 + 735*a**2*b**12*d*sin(c + d*x)**5 +
 245*a*b**13*d*sin(c + d*x)**6 + 35*b**14*d*sin(c + d*x)**7) + 35*a**5*b*si
n(c + d*x)/(35*a**7*b**7*d + 245*a**6*b**8*d*sin(c + d*x) + 735*a**5*b**9*d
*sin(c + d*x)**2 + 1225*a**4*b**10*d*sin(c + d*x)**3 + 1225*a**3*b**11*d*si
n(c + d*x)**4 + 735*a**2*b**12*d*sin(c + d*x)**5 + 245*a*b**13*d*sin(c + d*
x)**6 + 35*b**14*d*sin(c + d*x)**7) + 104*a**4*b**2*sin(c + d*x)**2/(35*a**
7*b**7*d + 245*a**6*b**8*d*sin(c + d*x) + 735*a**5*b**9*d*sin(c + d*x)**2 +
 1225*a**4*b**10*d*sin(c + d*x)**3 + 1225*a**3*b**11*d*sin(c + d*x)**4 + 73
5*a**2*b**12*d*sin(c + d*x)**5 + 245*a*b**13*d*sin(c + d*x)**6 + 35*b**14*d
*sin(c + d*x)**7) - a**4*b**2*cos(c + d*x)**2/(35*a**7*b**7*d + 245*a**6*b*
*8*d*sin(c + d*x) + 735*a**5*b**9*d*sin(c + d*x)**2 + 1225*a**4*b**10*d*sin
(c + d*x)**3 + 1225*a**3*b**11*d*sin(c + d*x)**4 + 735*a**2*b**12*d*sin(c +
 d*x)**5 + 245*a*b**13*d*sin(c + d*x)**6 + 35*b**14*d*sin(c + d*x)**7) + 16
8*a**3*b**3*sin(c + d*x)**3/(35*a**7*b**7*d + 245*a**6*b**8*d*sin(c + d*x)
 + 735*a**5*b**9*d*sin(c + d*x)**2 + 1225*a**4*b**10*d*sin(c + d*x)**3 + 122
5*a**3*b**11*d*sin(c + d*x)**4 + 735*a**2*b**12*d*sin(c + d*x)**5 + 245*a*b
13*d*sin(c + d*x)6 + 35*b**14*d*sin(c + d*x)**7) - 7*a**3*b**3*sin(c +
d*x)*cos(c + d*x)**2/(35*a**7*b**7*d + 245*a**6*b**8*d*sin(c + d*x) + 735*a
5*b9*d*sin(c + d*x)**2 + 1225*a**4*b**10*d*sin(c + d*x)**3 + 1225*a**3*
b**11*d*sin(c + d*x)**4 + 735*a**2*b**12*d*sin(c + d*x)**5 + 245*a*b**13*d*
sin(c + d*x)**6 + 35*b**14*d*sin(c + d*x)**7) + 155*a**2*b**4*sin(c + d*x)*
*4/(35*a**7*b**7*d + 245*a**6*b**8*d*sin(c + d*x) + 735*a**5*b**9*d*sin(c +
 d*x)**2 + 1225*a**4*b**10*d*sin(c + d*x)**3 + 1225*a**3*b**11*d*sin(c + d*
x)**4 + 735*a**2*b**12*d*sin(c + d*x)**5 + 245*a*b**13*d*sin(c + d*x)**6 +
35*b**14*d*sin(c + d*x)**7) - 19*a**2*b**4*sin(c + d*x)**2*cos(c + d*x)**2/
(35*a**7*b**7*d + 245*a**6*b**8*d*sin(c + d*x) + 735*a**5*b**9*d*sin(c + d*
x)**2 + 1225*a**4*b**10*d*sin(c + d*x)**3 + 1225*a**3*b**11*d*sin(c + d*x)*
*4 + 735*a**2*b**12*d*sin(c + d*x)**5 + 245*a*b**13*d*sin(c + d*x)**6 + 35*
b**14*d*sin(c + d*x)**7) + a**2*b**4*cos(c + d*x)**4/(35*a**7*b**7*d + 245*
a**6*b**8*d*sin(c + d*x) + 735*a**5*b**9*d*sin(c + d*x)**2 + 1225*a**4*b**1
0*d*sin(c + d*x)**3 + 1225*a**3*b**11*d*sin(c + d*x)**4 + 735*a**2*b**12*d*
sin(c + d*x)**5 + 245*a*b**13*d*sin(c + d*x)**6 + 35*b**14*d*sin(c + d*x)**
7) + 77*a*b**5*sin(c + d*x)**5/(35*a**7*b**7*d + 245*a**6*b**8*d*sin(c + d*
x) + 735*a**5*b**9*d*sin(c + d*x)**2 + 1225*a**4*b**10*d*sin(c + d*x)**3 +
1225*a**3*b**11*d*sin(c + d*x)**4 + 735*a**2*b**12*d*sin(c + d*x)**5 + 245*
a*b**13*d*sin(c + d*x)**6 + 35*b**14*d*sin(c + d*x)**7) - 21*a*b**5*sin(c +
 d*x)**3*cos(c + d*x)**2/(35*a**7*b**7*d + 245*a**6*b**8*d*sin(c + d*x) + 7
35*a**5*b**9*d*sin(c + d*x)**2 + 1225*a**4*b**10*d*sin(c + d*x)**3 + 1225*a
3*b11*d*sin(c + d*x)**4 + 735*a**2*b**12*d*sin(c + d*x)**5 + 245*a*b**1
3*d*sin(c + d*x)**6 + 35*b**14*d*sin(c + d*x)**7) + 7*a*b**5*sin(c + d*x)*c
os(c + d*x)**4/(35*a**7*b**7*d + 245*a**6*b**8*d*sin(c + d*x) + 735*a**5*b*
*9*d*sin(c + d*x)**2 + 1225*a**4*b**10*d*sin(c + d*x)**3 + 1225*a**3*b**11*

```
d*sin(c + d*x)**4 + 735*a**2*b**12*d*sin(c + d*x)**5 + 245*a*b**13*d*sin(c
+ d*x)**6 + 35*b**14*d*sin(c + d*x)**7) + 16*b**6*sin(c + d*x)**6/(35*a**7*
b**7*d + 245*a**6*b**8*d*sin(c + d*x) + 735*a**5*b**9*d*sin(c + d*x)**2 + 1
225*a**4*b**10*d*sin(c + d*x)**3 + 1225*a**3*b**11*d*sin(c + d*x)**4 + 735*
a**2*b**12*d*sin(c + d*x)**5 + 245*a*b**13*d*sin(c + d*x)**6 + 35*b**14*d*s
in(c + d*x)**7) - 8*b**6*sin(c + d*x)**4*cos(c + d*x)**2/(35*a**7*b**7*d +
245*a**6*b**8*d*sin(c + d*x) + 735*a**5*b**9*d*sin(c + d*x)**2 + 1225*a**4*
b**10*d*sin(c + d*x)**3 + 1225*a**3*b**11*d*sin(c + d*x)**4 + 735*a**2*b**1
2*d*sin(c + d*x)**5 + 245*a*b**13*d*sin(c + d*x)**6 + 35*b**14*d*sin(c + d*
x)**7) + 6*b**6*sin(c + d*x)**2*cos(c + d*x)**4/(35*a**7*b**7*d + 245*a**6*
b**8*d*sin(c + d*x) + 735*a**5*b**9*d*sin(c + d*x)**2 + 1225*a**4*b**10*d*s
in(c + d*x)**3 + 1225*a**3*b**11*d*sin(c + d*x)**4 + 735*a**2*b**12*d*sin(c
 + d*x)**5 + 245*a*b**13*d*sin(c + d*x)**6 + 35*b**14*d*sin(c + d*x)**7) -
5*b**6*cos(c + d*x)**6/(35*a**7*b**7*d + 245*a**6*b**8*d*sin(c + d*x) + 735
*a**5*b**9*d*sin(c + d*x)**2 + 1225*a**4*b**10*d*sin(c + d*x)**3 + 1225*a**
3*b**11*d*sin(c + d*x)**4 + 735*a**2*b**12*d*sin(c + d*x)**5 + 245*a*b**13*
d*sin(c + d*x)**6 + 35*b**14*d*sin(c + d*x)**7), True))
```

Giac [A]
time = 6.99, size = 215, normalized size = 1.04

$$\frac{35\,b^6\sin\left(dx+c\right)^6+105\,ab^5\sin\left(dx+c\right)^5+175\,a^2b^4\sin\left(dx+c\right)^4-35\,b^6\sin\left(dx+c\right)^4+175\,a^3b^3\sin\left(dx+c\right)^3-35\,ab^5\sin\left(dx+c\right)^3+105\,a^4b^2\sin\left(dx+c\right)^2-21\,a^2b^4\sin\left(dx+c\right)^2+21\,b^6\sin\left(dx+c\right)^2+35\,a^5b\sin\left(dx+c\right)-7\,a^3b^3\sin\left(dx+c\right)+7\,ab^5\sin\left(dx+c\right)+5\,a^6-a^4b^2+a^2b^4-5\,b^6}{35\,\left(b\sin\left(dx+c\right)+a\right)^7b^7d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+b*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/35*(35*b^6*sin(d*x + c)^6 + 105*a*b^5*sin(d*x + c)^5 + 175*a^2*b^4*sin(d*
x + c)^4 - 35*b^6*sin(d*x + c)^4 + 175*a^3*b^3*sin(d*x + c)^3 - 35*a*b^5*si
n(d*x + c)^3 + 105*a^4*b^2*sin(d*x + c)^2 - 21*a^2*b^4*sin(d*x + c)^2 + 21*
b^6*sin(d*x + c)^2 + 35*a^5*b*sin(d*x + c) - 7*a^3*b^3*sin(d*x + c) + 7*a*b
^5*sin(d*x + c) + 5*a^6 - a^4*b^2 + a^2*b^4 - 5*b^6)/((b*sin(d*x + c) + a)^
7*b^7*d)
```

Mupad [B]
time = 0.24, size = 276, normalized size = 1.33

$$\frac{\frac{5\,a^6-a^4\,b^2+a^2\,b^4-5\,b^6}{35\,b^7}+\frac{\sin(c+d\,x)^6}{b}+\frac{3\sin(c+d\,x)^2\,\left(5\,a^4-a^2\,b^2+b^4\right)}{5\,b^5}+\frac{3\,a\sin(c+d\,x)^5}{b^2}+\frac{\sin(c+d\,x)^4\,\left(5\,a^2-b^2\right)}{b^3}+\frac{a\,\sin(c+d\,x)\,\left(5\,a^4-a^2\,b^2+b^4\right)}{5\,b^6}+\frac{a\sin(c+d\,x)^3\,\left(5\,a^2-b^2\right)}{b^4}}{d\,\left(a^7+7\,a^6\,b\,\sin\left(c+d\,x\right)+21\,a^5\,b^2\sin\left(c+d\,x\right)^2+35\,a^4\,b^3\sin\left(c+d\,x\right)^3+35\,a^3\,b^4\sin\left(c+d\,x\right)^4+21\,a^2\,b^5\sin\left(c+d\,x\right)^5+7\,a\,b^6\sin\left(c+d\,x\right)^6+b^7\sin\left(c+d\,x\right)^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7/(a + b*sin(c + d*x))^8,x)
```

```
[Out] ((5*a^6 - 5*b^6 + a^2*b^4 - a^4*b^2)/(35*b^7) + sin(c + d*x)^6/b + (3*sin(c
 + d*x)^2*(5*a^4 + b^4 - a^2*b^2))/(5*b^5) + (3*a*sin(c + d*x)^5)/b^2 + (si
n(c + d*x)^4*(5*a^2 - b^2))/b^3 + (a*sin(c + d*x)*(5*a^4 + b^4 - a^2*b^2))/
(5*b^6) + (a*sin(c + d*x)^3*(5*a^2 - b^2))/b^4)/(d*(a^7 + b^7*sin(c + d*x)^
7 + 7*a*b^6*sin(c + d*x)^6 + 21*a^5*b^2*sin(c + d*x)^2 + 35*a^4*b^3*sin(c +
 d*x)^3 + 35*a^3*b^4*sin(c + d*x)^4 + 21*a^2*b^5*sin(c + d*x)^5 + 7*a^6*b*s
in(c + d*x)))
```

3.462 $\int \frac{\cos^5(c+dx)}{(a+b\sin(c+dx))^8}\,dx$

Optimal. Leaf size=141

$$-\frac{(a^2-b^2)^2}{7b^5d(a+b\sin(c+dx))^7}+\frac{2a(a^2-b^2)}{3b^5d(a+b\sin(c+dx))^6}-\frac{2(3a^2-b^2)}{5b^5d(a+b\sin(c+dx))^5}+\frac{a}{b^5d(a+b\sin(c+dx))^4}-3$$

```
[Out] -1/7*(a^2-b^2)^2/b^5/d/(a+b*sin(d*x+c))^7+2/3*a*(a^2-b^2)/b^5/d/(a+b*sin(d*
x+c))^6-2/5*(3*a^2-b^2)/b^5/d/(a+b*sin(d*x+c))^5+a/b^5/d/(a+b*sin(d*x+c))^4
-1/3/b^5/d/(a+b*sin(d*x+c))^3
```

Rubi [A]
time = 0.08, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2747, 711}

$$-\frac{(a^2-b^2)^2}{7b^5d(a+b\sin(c+dx))^7}+\frac{2a(a^2-b^2)}{3b^5d(a+b\sin(c+dx))^6}-\frac{2(3a^2-b^2)}{5b^5d(a+b\sin(c+dx))^5}-\frac{1}{3b^5d(a+b\sin(c+dx))^3}+\frac{a}{b^5d(a+b\sin(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/(a + b*Sin[c + d*x])^8,x]
```

```
[Out] -1/7*(a^2 - b^2)^2/(b^5*d*(a + b*Sin[c + d*x])^7) + (2*a*(a^2 - b^2))/(3*b^
5*d*(a + b*Sin[c + d*x])^6) - (2*(3*a^2 - b^2))/(5*b^5*d*(a + b*Sin[c + d*x
])^5) + a/(b^5*d*(a + b*Sin[c + d*x])^4) - 1/(3*b^5*d*(a + b*Sin[c + d*x])^
3)
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)}{(a+b\sin(c+dx))^8}\,dx = \frac{\text{Subst}\left(\int \frac{(b^2-x^2)^2}{(a+x)^8}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{(a^2-b^2)^2}{(a+x)^8} - \frac{4(a^3-ab^2)}{(a+x)^7} + \frac{2(3a^2-b^2)}{(a+x)^6} - \frac{4a}{(a+x)^5} + \frac{1}{(a+x)^4}\right) dx, x, b\sin(c+dx)\right)}{b^5 d}$$
$$= -\frac{(a^2-b^2)^2}{7b^5d(a+b\sin(c+dx))^7} + \frac{2a(a^2-b^2)}{3b^5d(a+b\sin(c+dx))^6} - \frac{2(3a^2-b^2)}{5b^5d(a+b\sin(c+dx))}$$

Mathematica [A]
time = 0.33, size = 107, normalized size = 0.76

$$-\frac{a^4-2a^2b^2+15b^4+7ab(a^2-2b^2)\sin(c+dx)+21b^2(a^2-2b^2)\sin^2(c+dx)+35ab^3\sin^3(c+dx)+35b^4\sin^4(c+dx)}{105b^5d(a+b\sin(c+dx))^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/(a + b*Sin[c + d*x])^8,x]
```

```
[Out] -1/105*(a^4 - 2*a^2*b^2 + 15*b^4 + 7*a*b*(a^2 - 2*b^2)*Sin[c + d*x] + 21*b^
2*(a^2 - 2*b^2)*Sin[c + d*x]^2 + 35*a*b^3*Sin[c + d*x]^3 + 35*b^4*Sin[c + d
*x]^4)/(b^5*d*(a + b*Sin[c + d*x])^7)
```

Maple [A]
time = 3.21, size = 127, normalized size = 0.90

method	result
derivativedivides	$\frac{\frac{2a(a^2-b^2)}{3b^5(a+b\sin(dx+c))^6} - \frac{a^4-2a^2b^2+b^4}{7b^5(a+b\sin(dx+c))^7} - \frac{1}{3b^5(a+b\sin(dx+c))^3} - \frac{6a^2-2b^2}{5b^5(a+b\sin(dx+c))^5} + \frac{a}{b^5(a+b\sin(dx+c))^4}}{d}$
default	$\frac{\frac{2a(a^2-b^2)}{3b^5(a+b\sin(dx+c))^6} - \frac{a^4-2a^2b^2+b^4}{7b^5(a+b\sin(dx+c))^7} - \frac{1}{3b^5(a+b\sin(dx+c))^3} - \frac{6a^2-2b^2}{5b^5(a+b\sin(dx+c))^5} + \frac{a}{b^5(a+b\sin(dx+c))^4}}{d}$
risch	$\frac{8i(70iab^3e^{10i(dx+c)}+35b^4e^{11i(dx+c)}-56iba^3e^{8i(dx+c)}-98iab^3e^{8i(dx+c)}-84a^2b^2e^{9i(dx+c)}+28b^4e^{9i(dx+c)}+56iba^3e^{6i(dx}}{105(be^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a+b*sin(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/3*a*(a^2-b^2)/b^5/(a+b*sin(d*x+c))^6-1/7*(a^4-2*a^2*b^2+b^4)/b^5/(a+
b*sin(d*x+c))^7-1/3/b^5/(a+b*sin(d*x+c))^3-1/5*(6*a^2-2*b^2)/b^5/(a+b*sin(d
*x+c))^5+a/b^5/(a+b*sin(d*x+c))^4)
```

Maxima [A]
time = 0.29, size = 206, normalized size = 1.46

$$-\frac{35\,b^4\sin(dx+c)^4+35\,ab^3\sin(dx+c)^3+a^4-2\,a^2b^2+15\,b^4+21\,(a^2b^2-2\,b^4)\sin(dx+c)^2+7\,(a^3b-2\,ab^3)\sin(dx+c)}{105\,(b^{12}\sin(dx+c)^7+7\,ab^{11}\sin(dx+c)^6+21\,a^2b^{10}\sin(dx+c)^5+35\,a^3b^9\sin(dx+c)^4+35\,a^4b^8\sin(dx+c)^3+21\,a^5b^7\sin(dx+c)^2+7\,a^6b^6\sin(dx+c)+a^7b^5)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/105*(35*b^4*sin(d*x + c)^4 + 35*a*b^3*sin(d*x + c)^3 + a^4 - 2*a^2*b^2 +
 15*b^4 + 21*(a^2*b^2 - 2*b^4)*sin(d*x + c)^2 + 7*(a^3*b - 2*a*b^3)*sin(d*x
 + c))/((b^12*sin(d*x + c)^7 + 7*a*b^11*sin(d*x + c)^6 + 21*a^2*b^10*sin(d*
x + c)^5 + 35*a^3*b^9*sin(d*x + c)^4 + 35*a^4*b^8*sin(d*x + c)^3 + 21*a^5*b
^7*sin(d*x + c)^2 + 7*a^6*b^6*sin(d*x + c) + a^7*b^5)*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 309 vs. $2(133) = 266$.
time = 0.39, size = 309, normalized size = 2.19

$$\frac{35\, b^{4} \cos\left(dx + c\right)^{4} + a^{4} + 19\, a^{2} b^{2} + 8\, b^{4} - 7\, {\left(3\, a^{2} b^{2} + 4\, b^{4}\right)} \cos\left(dx + c\right)^{2} - 7\, {\left(5\, a b^{3} \cos\left(dx + c\right)^{2} - a^{3} b - 3\, a b^{3}\right)} \sin\left(dx + c\right)}{105\, {\left(7\, a b^{11} d \cos\left(dx + c\right)^{6} - 7\, {\left(5\, a^{3} b^{9} + 3\, a b^{11}\right)} d \cos\left(dx + c\right)^{4} + 7\, {\left(3\, a^{5} b^{7} + 10\, a^{3} b^{9} + 3\, a b^{11}\right)} d \cos\left(dx + c\right)^{2} - {\left(a^{7} b^{5} + 21\, a^{5} b^{7} + 35\, a^{3} b^{9} + 7\, a b^{11}\right)} d + {\left(b^{12} d \cos\left(dx + c\right)^{6} - 3\, {\left(7\, a^{2} b^{10} + b^{12}\right)} d \cos\left(dx + c\right)^{4} + {\left(35\, a^{4} b^{8} + 42\, a^{2} b^{10} + 3\, b^{12}\right)} d \cos\left(dx + c\right)^{2} - {\left(7\, a^{6} b^{6} + 35\, a^{4} b^{8} + 21\, a^{2} b^{10} + b^{12}\right)} d\right)} \sin\left(dx + c\right)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/105*(35*b^4*cos(d*x + c)^4 + a^4 + 19*a^2*b^2 + 8*b^4 - 7*(3*a^2*b^2 + 4*
b^4)*cos(d*x + c)^2 - 7*(5*a*b^3*cos(d*x + c)^2 - a^3*b - 3*a*b^3)*sin(d*x
+ c))/(7*a*b^11*d*cos(d*x + c)^6 - 7*(5*a^3*b^9 + 3*a*b^11)*d*cos(d*x + c)^
4 + 7*(3*a^5*b^7 + 10*a^3*b^9 + 3*a*b^11)*d*cos(d*x + c)^2 - (a^7*b^5 + 21*
a^5*b^7 + 35*a^3*b^9 + 7*a*b^11)*d + (b^12*d*cos(d*x + c)^6 - 3*(7*a^2*b^10
 + b^12)*d*cos(d*x + c)^4 + (35*a^4*b^8 + 42*a^2*b^10 + 3*b^12)*d*cos(d*x +
 c)^2 - (7*a^6*b^6 + 35*a^4*b^8 + 21*a^2*b^10 + b^12)*d)*sin(d*x + c))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1425 vs. $2(124) = 248$.
time = 14.99, size = 1425, normalized size = 10.11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a+b*sin(d*x+c))**8,x)
```

```
[Out] Piecewise((x*cos(c)**5/a**8, Eq(b, 0) & Eq(d, 0)), ((8*sin(c + d*x)**5/(15*
d) + 4*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + sin(c + d*x)*cos(c + d*x)**4
/d)/a**8, Eq(b, 0)), (x*cos(c)**5/(a + b*sin(c))**8, Eq(d, 0)), (-a**4/(105
*a**7*b**5*d + 735*a**6*b**6*d*sin(c + d*x) + 2205*a**5*b**7*d*sin(c + d*x)
**2 + 3675*a**4*b**8*d*sin(c + d*x)**3 + 3675*a**3*b**9*d*sin(c + d*x)**4 +
 2205*a**2*b**10*d*sin(c + d*x)**5 + 735*a*b**11*d*sin(c + d*x)**6 + 105*b*
*12*d*sin(c + d*x)**7) - 7*a**3*b*sin(c + d*x)/(105*a**7*b**5*d + 735*a**6*
b**6*d*sin(c + d*x) + 2205*a**5*b**7*d*sin(c + d*x)**2 + 3675*a**4*b**8*d*s
in(c + d*x)**3 + 3675*a**3*b**9*d*sin(c + d*x)**4 + 2205*a**2*b**10*d*sin(c
 + d*x)**5 + 735*a*b**11*d*sin(c + d*x)**6 + 105*b**12*d*sin(c + d*x)**7) -
```

```
19*a**2*b**2*sin(c + d*x)**2/(105*a**7*b**5*d + 735*a**6*b**6*d*sin(c + d*
x) + 2205*a**5*b**7*d*sin(c + d*x)**2 + 3675*a**4*b**8*d*sin(c + d*x)**3 +
3675*a**3*b**9*d*sin(c + d*x)**4 + 2205*a**2*b**10*d*sin(c + d*x)**5 + 735*
a*b**11*d*sin(c + d*x)**6 + 105*b**12*d*sin(c + d*x)**7) + 2*a**2*b**2*cos(
c + d*x)**2/(105*a**7*b**5*d + 735*a**6*b**6*d*sin(c + d*x) + 2205*a**5*b**
7*d*sin(c + d*x)**2 + 3675*a**4*b**8*d*sin(c + d*x)**3 + 3675*a**3*b**9*d*s
in(c + d*x)**4 + 2205*a**2*b**10*d*sin(c + d*x)**5 + 735*a*b**11*d*sin(c +
d*x)**6 + 105*b**12*d*sin(c + d*x)**7) - 21*a*b**3*sin(c + d*x)**3/(105*a**
7*b**5*d + 735*a**6*b**6*d*sin(c + d*x) + 2205*a**5*b**7*d*sin(c + d*x)**2
+ 3675*a**4*b**8*d*sin(c + d*x)**3 + 3675*a**3*b**9*d*sin(c + d*x)**4 + 220
5*a**2*b**10*d*sin(c + d*x)**5 + 735*a*b**11*d*sin(c + d*x)**6 + 105*b**12*
d*sin(c + d*x)**7) + 14*a*b**3*sin(c + d*x)*cos(c + d*x)**2/(105*a**7*b**5*
d + 735*a**6*b**6*d*sin(c + d*x) + 2205*a**5*b**7*d*sin(c + d*x)**2 + 3675*
a**4*b**8*d*sin(c + d*x)**3 + 3675*a**3*b**9*d*sin(c + d*x)**4 + 2205*a**2*
b**10*d*sin(c + d*x)**5 + 735*a*b**11*d*sin(c + d*x)**6 + 105*b**12*d*sin(c
 + d*x)**7) - 8*b**4*sin(c + d*x)**4/(105*a**7*b**5*d + 735*a**6*b**6*d*sin
(c + d*x) + 2205*a**5*b**7*d*sin(c + d*x)**2 + 3675*a**4*b**8*d*sin(c + d*x
)**3 + 3675*a**3*b**9*d*sin(c + d*x)**4 + 2205*a**2*b**10*d*sin(c + d*x)**5
 + 735*a*b**11*d*sin(c + d*x)**6 + 105*b**12*d*sin(c + d*x)**7) + 12*b**4*s
in(c + d*x)**2*cos(c + d*x)**2/(105*a**7*b**5*d + 735*a**6*b**6*d*sin(c + d
*x) + 2205*a**5*b**7*d*sin(c + d*x)**2 + 3675*a**4*b**8*d*sin(c + d*x)**3 +
 3675*a**3*b**9*d*sin(c + d*x)**4 + 2205*a**2*b**10*d*sin(c + d*x)**5 + 735
*a*b**11*d*sin(c + d*x)**6 + 105*b**12*d*sin(c + d*x)**7) - 15*b**4*cos(c +
 d*x)**4/(105*a**7*b**5*d + 735*a**6*b**6*d*sin(c + d*x) + 2205*a**5*b**7*d
*sin(c + d*x)**2 + 3675*a**4*b**8*d*sin(c + d*x)**3 + 3675*a**3*b**9*d*sin(
c + d*x)**4 + 2205*a**2*b**10*d*sin(c + d*x)**5 + 735*a*b**11*d*sin(c + d*x
)**6 + 105*b**12*d*sin(c + d*x)**7), True))
```

Giac [A]
time = 4.95, size = 117, normalized size = 0.83

$$-\frac{35\,b^4\sin\left(dx+c\right)^4+35\,ab^3\sin\left(dx+c\right)^3+21\,a^2b^2\sin\left(dx+c\right)^2-42\,b^4\sin\left(dx+c\right)^2+7\,a^3b\sin\left(dx+c\right)-14\,ab^3\sin\left(dx+c\right)+a^4-2\,a^2b^2+15\,b^4}{105\,\left(b\sin\left(dx+c\right)+a\right)^7b^5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -1/105*(35*b^4*sin(d*x + c)^4 + 35*a*b^3*sin(d*x + c)^3 + 21*a^2*b^2*sin(d*
x + c)^2 - 42*b^4*sin(d*x + c)^2 + 7*a^3*b*sin(d*x + c) - 14*a*b^3*sin(d*x
+ c) + a^4 - 2*a^2*b^2 + 15*b^4)/((b*sin(d*x + c) + a)^7*b^5*d)
```

Mupad [B]
time = 0.14, size = 206, normalized size = 1.46

$$-\frac{\frac{a^4-2\,a^2\,b^2+15\,b^4}{105\,b^5}+\frac{{\sin\left(c+d\,x\right)}^4}{3\,b}+\frac{{\sin\left(c+d\,x\right)}^2\,\left(a^2-2\,b^2\right)}{5\,b^3}+\frac{a\,{\sin\left(c+d\,x\right)}^3}{3\,b^2}+\frac{a\,\sin\left(c+d\,x\right)\,\left(a^2-2\,b^2\right)}{15\,b^4}}{d\,\left(a^7+7\,a^6\,b\,\sin\left(c+d\,x\right)+21\,a^5\,b^2\,{\sin\left(c+d\,x\right)}^2+35\,a^4\,b^3\,{\sin\left(c+d\,x\right)}^3+35\,a^3\,b^4\,{\sin\left(c+d\,x\right)}^4+21\,a^2\,b^5\,{\sin\left(c+d\,x\right)}^5+7\,a\,b^6\,{\sin\left(c+d\,x\right)}^6+b^7\,{\sin\left(c+d\,x\right)}^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(a + b*sin(c + d*x))^8,x)
```

```
[Out] -((a^4 + 15*b^4 - 2*a^2*b^2)/(105*b^5) + sin(c + d*x)^4/(3*b) + (sin(c + d*
x)^2*(a^2 - 2*b^2))/(5*b^3) + (a*sin(c + d*x)^3)/(3*b^2) + (a*sin(c + d*x)*
(a^2 - 2*b^2))/(15*b^4))/(d*(a^7 + b^7*sin(c + d*x)^7 + 7*a*b^6*sin(c + d*x
)^6 + 21*a^5*b^2*sin(c + d*x)^2 + 35*a^4*b^3*sin(c + d*x)^3 + 35*a^3*b^4*si
n(c + d*x)^4 + 21*a^2*b^5*sin(c + d*x)^5 + 7*a^6*b*sin(c + d*x)))
```

3.463 $\int \frac{\cos^3(c+dx)}{(a+b\sin(c+dx))^8}\,dx$

Optimal. Leaf size=77

$$\frac{a^2-b^2}{7b^3d(a+b\sin(c+dx))^7}-\frac{a}{3b^3d(a+b\sin(c+dx))^6}+\frac{1}{5b^3d(a+b\sin(c+dx))^5}$$

```
[Out] 1/7*(a^2-b^2)/b^3/d/(a+b*sin(d*x+c))^7-1/3*a/b^3/d/(a+b*sin(d*x+c))^6+1/5/b
^3/d/(a+b*sin(d*x+c))^5
```

Rubi [A]
time = 0.05, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2747, 711}

$$\frac{a^2-b^2}{7b^3d(a+b\sin(c+dx))^7}-\frac{a}{3b^3d(a+b\sin(c+dx))^6}+\frac{1}{5b^3d(a+b\sin(c+dx))^5}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + b*Sin[c + d*x])^8,x]
```

```
[Out] (a^2 - b^2)/(7*b^3*d*(a + b*Sin[c + d*x])^7) - a/(3*b^3*d*(a + b*Sin[c + d*
x])^6) + 1/(5*b^3*d*(a + b*Sin[c + d*x])^5)
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^3(c+dx)}{(a+b\sin(c+dx))^8}\,dx &= \frac{\text{Subst}\left(\int \frac{b^2-x^2}{(a+x)^8}\,dx,x,b\sin(c+dx)\right)}{b^3d}\\ &= \frac{\text{Subst}\left(\int \left(\frac{-a^2+b^2}{(a+x)^8}+\frac{2a}{(a+x)^7}-\frac{1}{(a+x)^6}\right)dx,x,b\sin(c+dx)\right)}{b^3d}\\ &= \frac{a^2-b^2}{7b^3d(a+b\sin(c+dx))^7}-\frac{a}{3b^3d(a+b\sin(c+dx))^6}+\frac{1}{5b^3d(a+b\sin(c+dx))^5}\end{aligned}$$

Mathematica [A]
time = 0.24, size = 54, normalized size = 0.70

$$\frac{a^2 - 15b^2 + 7ab\sin(c+dx) + 21b^2\sin^2(c+dx)}{105b^3d(a+b\sin(c+dx))^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + b*Sin[c + d*x])^8,x]
```

```
[Out] (a^2 - 15*b^2 + 7*a*b*Sin[c + d*x] + 21*b^2*Sin[c + d*x]^2)/(105*b^3*d*(a +
 b*Sin[c + d*x])^7)
```

Maple [A]
time = 3.05, size = 67, normalized size = 0.87

method	result	size
derivativedivides	$\frac{-\frac{-a^2+b^2}{7b^3(a+b\sin(dx+c))^7}+\frac{1}{5b^3(a+b\sin(dx+c))^5}-\frac{a}{3b^3(a+b\sin(dx+c))^6}}{d}$	67
default	$\frac{-\frac{-a^2+b^2}{7b^3(a+b\sin(dx+c))^7}+\frac{1}{5b^3(a+b\sin(dx+c))^5}-\frac{a}{3b^3(a+b\sin(dx+c))^6}}{d}$	67
risch	$\frac{32i\left(14iab\,\mathrm{e}^{8i(dx+c)}+21b^2\mathrm{e}^{9i(dx+c)}-14iab\,\mathrm{e}^{6i(dx+c)}-4a^2\mathrm{e}^{7i(dx+c)}+18b^2\mathrm{e}^{7i(dx+c)}+21b^2\mathrm{e}^{5i(dx+c)}\right)}{105\left(b\,\mathrm{e}^{2i(dx+c)}-b+2ia\,\mathrm{e}^{i(dx+c)}\right)^7 d\,b^3}$	125

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+b*sin(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/7*(-a^2+b^2)/b^3/(a+b*sin(d*x+c))^7+1/5/b^3/(a+b*sin(d*x+c))^5-1/3*
a/b^3/(a+b*sin(d*x+c))^6)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 151 vs. 2(71) = 142.
time = 0.27, size = 151, normalized size = 1.96

$$\frac{21\,b^2\sin(dx+c)^2+7\,ab\sin(dx+c)+a^2-15\,b^2}{105\,(b^{10}\sin(dx+c)^7+7\,ab^9\sin(dx+c)^6+21\,a^2b^8\sin(dx+c)^5+35\,a^3b^7\sin(dx+c)^4+35\,a^4b^6\sin(dx+c)^3+21\,a^5b^5\sin(dx+c)^2+7\,a^6b^4\sin(dx+c)+a^7b^3)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 1/105*(21*b^2*sin(d*x + c)^2 + 7*a*b*sin(d*x + c) + a^2 - 15*b^2)/((b^10*si
n(d*x + c)^7 + 7*a*b^9*sin(d*x + c)^6 + 21*a^2*b^8*sin(d*x + c)^5 + 35*a^3*
b^7*sin(d*x + c)^4 + 35*a^4*b^6*sin(d*x + c)^3 + 21*a^5*b^5*sin(d*x + c)^2
+ 7*a^6*b^4*sin(d*x + c) + a^7*b^3)*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 254 vs. 2(71) = 142.
time = 0.39, size = 254, normalized size = 3.30

$$\frac{21\,b^2\cos(dx+c)^2-7\,ab\sin(dx+c)-a^2-6\,b^2}{105\left(7\,ab^9d\cos(dx+c)^6-7\,(5\,a^3b^7+3\,ab^9)d\cos(dx+c)^4+7\,(3\,a^5b^5+10\,a^3b^7+3\,ab^9)d\cos(dx+c)^2-(a^7b^3+21\,a^5b^5+35\,a^3b^7+7\,ab^9)d+\left(b^{10}d\cos(dx+c)^6-3\,(7\,a^2b^8+b^{10})d\cos(dx+c)^4+(35\,a^4b^6+42\,a^2b^8+3\,b^{10})d\cos(dx+c)^2-(7\,a^6b^4+35\,a^4b^6+21\,a^2b^8+b^{10})d\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/105*(21*b^2*cos(d*x + c)^2 - 7*a*b*sin(d*x + c) - a^2 - 6*b^2)/(7*a*b^9*d
*cos(d*x + c)^6 - 7*(5*a^3*b^7 + 3*a*b^9)*d*cos(d*x + c)^4 + 7*(3*a^5*b^5 +
 10*a^3*b^7 + 3*a*b^9)*d*cos(d*x + c)^2 - (a^7*b^3 + 21*a^5*b^5 + 35*a^3*b^
7 + 7*a*b^9)*d + (b^10*d*cos(d*x + c)^6 - 3*(7*a^2*b^8 + b^10)*d*cos(d*x +
c)^4 + (35*a^4*b^6 + 42*a^2*b^8 + 3*b^10)*d*cos(d*x + c)^2 - (7*a^6*b^4 + 3
5*a^4*b^6 + 21*a^2*b^8 + b^10)*d)*sin(d*x + c))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 636 vs. $2(66) = 132$.
time = 14.73, size = 636, normalized size = 8.26

$$\begin{cases} \frac{x\cos^3(c)}{a^8} & \text{for } b = 0 \wedge d = 0 \\ \frac{\frac{2\sin^3(c+dx)}{3d} + \frac{\sin(c+dx)\cos^2(c+dx)}{d}}{a^8} & \text{for } b = 0 \\ \frac{x\cos^3(c)}{(a+b\sin(c))^8} & \text{for } d = 0 \\ \frac{a^2}{D} + \frac{7ab\sin(c+dx)}{D} + \frac{6b^2\sin^2(c+dx)}{D} - \frac{15b^2\cos^2(c+dx)}{D} & \text{otherwise} \end{cases}$$

where $D = 105a^7b^3d + 735a^6b^4d\sin(c+dx) + 2205a^5b^5d\sin^2(c+dx) + 3675a^4b^6d\sin^3(c+dx) + 3675a^3b^7d\sin^4(c+dx) + 2205a^2b^8d\sin^5(c+dx) + 735ab^9d\sin^6(c+dx) + 105b^{10}d\sin^7(c+dx)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+b*sin(d*x+c))**8,x)
```

```
[Out] Piecewise((x*cos(c)**3/a**8, Eq(b, 0) & Eq(d, 0)), ((2*sin(c + d*x)**3/(3*d
) + sin(c + d*x)*cos(c + d*x)**2/d)/a**8, Eq(b, 0)), (x*cos(c)**3/(a + b*si
n(c))**8, Eq(d, 0)), (a**2/(105*a**7*b**3*d + 735*a**6*b**4*d*sin(c + d*x)
+ 2205*a**5*b**5*d*sin(c + d*x)**2 + 3675*a**4*b**6*d*sin(c + d*x)**3 + 367
5*a**3*b**7*d*sin(c + d*x)**4 + 2205*a**2*b**8*d*sin(c + d*x)**5 + 735*a*b*
*9*d*sin(c + d*x)**6 + 105*b**10*d*sin(c + d*x)**7) + 7*a*b*sin(c + d*x)/(1
05*a**7*b**3*d + 735*a**6*b**4*d*sin(c + d*x) + 2205*a**5*b**5*d*sin(c + d*
x)**2 + 3675*a**4*b**6*d*sin(c + d*x)**3 + 3675*a**3*b**7*d*sin(c + d*x)**4
 + 2205*a**2*b**8*d*sin(c + d*x)**5 + 735*a*b**9*d*sin(c + d*x)**6 + 105*b*
*10*d*sin(c + d*x)**7) + 6*b**2*sin(c + d*x)**2/(105*a**7*b**3*d + 735*a**6
*b**4*d*sin(c + d*x) + 2205*a**5*b**5*d*sin(c + d*x)**2 + 3675*a**4*b**6*d*
sin(c + d*x)**3 + 3675*a**3*b**7*d*sin(c + d*x)**4 + 2205*a**2*b**8*d*sin(c
 + d*x)**5 + 735*a*b**9*d*sin(c + d*x)**6 + 105*b**10*d*sin(c + d*x)**7) -
15*b**2*cos(c + d*x)**2/(105*a**7*b**3*d + 735*a**6*b**4*d*sin(c + d*x) + 2
205*a**5*b**5*d*sin(c + d*x)**2 + 3675*a**4*b**6*d*sin(c + d*x)**3 + 3675*a
**3*b**7*d*sin(c + d*x)**4 + 2205*a**2*b**8*d*sin(c + d*x)**5 + 735*a*b**9*
d*sin(c + d*x)**6 + 105*b**10*d*sin(c + d*x)**7), True))
```

Giac [A]
time = 4.85, size = 52, normalized size = 0.68

$$\frac{21\,b^2\sin(dx+c)^2 + 7\,ab\sin(dx+c) + a^2 - 15\,b^2}{105\,(b\sin(dx+c)+a)^7 b^3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sin(d*x+c))^8,x, algorithm="giac")
```

[Out] 1/105*(21*b^2*sin(d*x + c)^2 + 7*a*b*sin(d*x + c) + a^2 - 15*b^2)/((b*sin(d*x + c) + a)^7*b^3*d)

Mupad [B]
time = 5.22, size = 152, normalized size = 1.97

$$\frac{\frac{a^2-15\,b^2}{105\,b^3}+\frac{\sin(c+d\,x)^2}{5\,b}+\frac{a\,\sin(c+d\,x)}{15\,b^2}}{d\,\left(a^7+7\,a^6\,b\,\sin\left(c+d\,x\right)+21\,a^5\,b^2\sin\left(c+d\,x\right)^2+35\,a^4\,b^3\sin\left(c+d\,x\right)^3+35\,a^3\,b^4\sin\left(c+d\,x\right)^4+21\,a^2\,b^5\sin\left(c+d\,x\right)^5+7\,a\,b^6\sin\left(c+d\,x\right)^6+b^7\sin\left(c+d\,x\right)^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^3/(a + b*sin(c + d*x))^8,x)

[Out] ((a^2 - 15*b^2)/(105*b^3) + sin(c + d*x)^2/(5*b) + (a*sin(c + d*x))/(15*b^2))/(d*(a^7 + b^7*sin(c + d*x)^7 + 7*a*b^6*sin(c + d*x)^6 + 21*a^5*b^2*sin(c + d*x)^2 + 35*a^4*b^3*sin(c + d*x)^3 + 35*a^3*b^4*sin(c + d*x)^4 + 21*a^2*b^5*sin(c + d*x)^5 + 7*a^6*b*sin(c + d*x)))

3.464 $\int \frac{\cos(c+dx)}{(a+b\sin(c+dx))^8}\,dx$

Optimal. Leaf size=22

$$-\frac{1}{7bd(a+b\sin(c+dx))^7}$$

```
[Out] -1/7/b/d/(a+b*sin(d*x+c))^7
```

Rubi [A]
time = 0.02, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2747, 32}

$$-\frac{1}{7bd(a+b\sin(c+dx))^7}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + b*Sin[c + d*x])^8,x]
```

```
[Out] -1/7*1/(b*d*(a + b*Sin[c + d*x])^7)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)}{(a+b\sin(c+dx))^8}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{(a+x)^8}\,dx, x, b\sin(c+dx)\right)}{bd}\\ &= -\frac{1}{7bd(a+b\sin(c+dx))^7}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 22, normalized size = 1.00

$$-\frac{1}{7bd(a+b\sin(c+dx))^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + b*Sin[c + d*x])^8,x]
```

```
[Out] -1/7*1/(b*d*(a + b*Sin[c + d*x])^7)
```

Maple [A]
time = 3.23, size = 21, normalized size = 0.95

method	result
derivativedivides	$-\frac{1}{7 b d \left(a+b \sin \left(d x+c\right)\right)^{7}}$
default	$-\frac{1}{7 b d \left(a+b \sin \left(d x+c\right)\right)^{7}}$
risch	$\frac{128 i \,{\mathrm e}^{7 i \left(d x+c\right)}}{7 \left(b\,{\mathrm e}^{2 i \left(d x+c\right)}-b+2 i a\,{\mathrm e}^{i \left(d x+c\right)}\right)^{7} d b}$
norman	$\frac{\frac{2 \left(245 a^{6}+1400 a^{4} b^{2}+1008 a^{2} b^{4}+64 b^{6}\right) \left(\tan^{7}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)}{7 d\,a^{7}}+\frac{2 \left(245 a^{6}+1400 a^{4} b^{2}+1008 a^{2} b^{4}+64 b^{6}\right) \left(\tan^{9}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)}{7 d\,a^{7}}+\frac{4 \left(45 b\,a^{4}+80 a^{2} b^{3}\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+b*sin(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] -1/7/b/d/(a+b*sin(d*x+c))^7
```

Maxima [A]
time = 0.28, size = 20, normalized size = 0.91

$$-\frac{1}{7 \, {\left(b \sin \left(d x + c\right) + a\right)}^{7} b d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/7/((b*sin(d*x + c) + a)^7*b*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 218 vs. 2(20) = 40.
time = 0.38, size = 218, normalized size = 9.91

$$\frac{1}{7 \, {\left(7 \, a b^{7} d \cos \left(d x + c\right)^{6} - 7 \, {\left(5 \, a^{3} b^{5} + 3 \, a b^{7}\right)} d \cos \left(d x + c\right)^{4} + 7 \, {\left(3 \, a^{5} b^{3} + 10 \, a^{3} b^{5} + 3 \, a b^{7}\right)} d \cos \left(d x + c\right)^{2} - {\left(a^{7} b + 21 \, a^{5} b^{3} + 35 \, a^{3} b^{5} + 7 \, a b^{7}\right)} d + {\left(b^{8} d \cos \left(d x + c\right)^{6} - 3 \, {\left(7 \, a^{2} b^{6} + b^{8}\right)} d \cos \left(d x + c\right)^{4} + {\left(35 \, a^{4} b^{4} + 42 \, a^{2} b^{6} + 3 \, b^{8}\right)} d \cos \left(d x + c\right)^{2} - {\left(7 \, a^{6} b^{2} + 35 \, a^{4} b^{4} + 21 \, a^{2} b^{6} + b^{8}\right)} d\right)} \sin \left(d x + c\right)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/7/(7*a*b^7*d*cos(d*x + c)^6 - 7*(5*a^3*b^5 + 3*a*b^7)*d*cos(d*x + c)^4 +
7*(3*a^5*b^3 + 10*a^3*b^5 + 3*a*b^7)*d*cos(d*x + c)^2 - (a^7*b + 21*a^5*b^3
 + 35*a^3*b^5 + 7*a*b^7)*d + (b^8*d*cos(d*x + c)^6 - 3*(7*a^2*b^6 + b^8)*d*
```

cos(d*x + c)^4 + (35*a^4*b^4 + 42*a^2*b^6 + 3*b^8)*d*cos(d*x + c)^2 - (7*a^6*b^2 + 35*a^4*b^4 + 21*a^2*b^6 + b^8)*d)*sin(d*x + c))

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(19) = 38$.
time = 14.71, size = 167, normalized size = 7.59

$$\begin{cases} \frac{x\cos(c)}{a^8} & \text{for } b = 0 \wedge d = 0 \\ \frac{\sin(c+dx)}{a^8 d} & \text{for } b = 0 \\ \frac{x\cos(c)}{(a+b\sin(c))^8} & \text{for } d = 0 \\ -\frac{1}{7a^7bd+49a^6b^2d\sin(c+dx)+147a^5b^3d\sin^2(c+dx)+245a^4b^4d\sin^3(c+dx)+245a^3b^5d\sin^4(c+dx)+147a^2b^6d\sin^5(c+dx)+49ab^7d\sin^6(c+dx)+7b^8d\sin^7(c+dx)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c))**8,x)
```

```
[Out] Piecewise((x*cos(c)/a**8, Eq(b, 0) & Eq(d, 0)), (sin(c + d*x)/(a**8*d), Eq(
b, 0)), (x*cos(c)/(a + b*sin(c))**8, Eq(d, 0)), (-1/(7*a**7*b*d + 49*a**6*b
**2*d*sin(c + d*x) + 147*a**5*b**3*d*sin(c + d*x)**2 + 245*a**4*b**4*d*sin(
c + d*x)**3 + 245*a**3*b**5*d*sin(c + d*x)**4 + 147*a**2*b**6*d*sin(c + d*x
)**5 + 49*a*b**7*d*sin(c + d*x)**6 + 7*b**8*d*sin(c + d*x)**7), True))
```

Giac [A]
time = 6.28, size = 20, normalized size = 0.91

$$-\frac{1}{7\,(b\sin(dx+c)+a)^7bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -1/7/((b*sin(d*x + c) + a)^7*b*d)
```

Mupad [B]
time = 5.20, size = 119, normalized size = 5.41

$$-\frac{1}{d\,\left(7\,a^7\,b+49\,a^6\,b^2\,\sin(c+d\,x)+147\,a^5\,b^3\,\sin(c+d\,x)^2+245\,a^4\,b^4\,\sin(c+d\,x)^3+245\,a^3\,b^5\,\sin(c+d\,x)^4+147\,a^2\,b^6\,\sin(c+d\,x)^5+49\,a\,b^7\,\sin(c+d\,x)^6+7\,b^8\,\sin(c+d\,x)^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + b*sin(c + d*x))^8,x)
```

```
[Out] -1/(d*(7*a^7*b + 7*b^8*sin(c + d*x)^7 + 49*a^6*b^2*sin(c + d*x) + 49*a*b^7*
sin(c + d*x)^6 + 147*a^5*b^3*sin(c + d*x)^2 + 245*a^4*b^4*sin(c + d*x)^3 +
245*a^3*b^5*sin(c + d*x)^4 + 147*a^2*b^6*sin(c + d*x)^5))
```

3.465 $\int \frac{\sec(c+dx)}{(a+b\sin(c+dx))^8}\,dx$

Optimal. Leaf size=385

$$-\frac{\log(1-\sin(c+dx))}{2(a+b)^8d}+\frac{\log(1+\sin(c+dx))}{2(a-b)^8d}-\frac{8ab(a^2+b^2)\left(a^4+6a^2b^2+b^4\right)\log(a+b\sin(c+dx))}{\left(a^2-b^2\right)^8d}+\frac{}{7\left(a^2-\right.}$$

```
[Out] -1/2*ln(1-sin(d*x+c))/(a+b)^8/d+1/2*ln(1+sin(d*x+c))/(a-b)^8/d-8*a*b*(a^2+b
^2)*(a^4+6*a^2*b^2+b^4)*ln(a+b*sin(d*x+c))/(a^2-b^2)^8/d+1/7*b/(a^2-b^2)/d/
(a+b*sin(d*x+c))^7+1/3*a*b/(a^2-b^2)^2/d/(a+b*sin(d*x+c))^6+1/5*b*(3*a^2+b^
2)/(a^2-b^2)^3/d/(a+b*sin(d*x+c))^5+a*b*(a^2+b^2)/(a^2-b^2)^4/d/(a+b*sin(d*
x+c))^4+1/3*b*(5*a^4+10*a^2*b^2+b^4)/(a^2-b^2)^5/d/(a+b*sin(d*x+c))^3+a*b*(
3*a^4+10*a^2*b^2+3*b^4)/(a^2-b^2)^6/d/(a+b*sin(d*x+c))^2+b*(7*a^6+35*a^4*b^
2+21*a^2*b^4+b^6)/(a^2-b^2)^7/d/(a+b*sin(d*x+c))
```

Rubi [A]
time = 0.37, antiderivative size = 385, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2747, 724, 815}

$$\frac{ab(3a^2+b^2)(a^2+3b^2)}{d(a^2-b^2)^6(a+b\sin(c+dx))^2}+\frac{ab(a^2+b^2)}{d(a^2-b^2)^4(a+b\sin(c+dx))^4}+\frac{b(3a^2+b^2)}{5d(a^2-b^2)^3(a+b\sin(c+dx))^5}+\frac{ab}{3d(a^2-b^2)^2(a+b\sin(c+dx))^6}+\frac{b}{7d(a^2-b^2)(a+b\sin(c+dx))^7}+\frac{b(5a^4+10a^2b^2+b^4)}{3d(a^2-b^2)^5(a+b\sin(c+dx))^3}-\frac{8ab(a^2+b^2)(a^4+6a^2b^2+b^4)\log(a+b\sin(c+dx))}{d(a^2-b^2)^8}+\frac{b(7a^6+35a^4b^2+21a^2b^4+b^6)}{d(a^2-b^2)^7(a+b\sin(c+dx))}-\frac{\log(1-\sin(c+dx))}{2d(a+b)^8}+\frac{\log(\sin(c+dx)+1)}{2d(a-b)^8}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + b*Sin[c + d*x])^8,x]
```

```
[Out] -1/2*Log[1 - Sin[c + d*x]]/((a + b)^8*d) + Log[1 + Sin[c + d*x]]/(2*(a - b)
^8*d) - (8*a*b*(a^2 + b^2)*(a^4 + 6*a^2*b^2 + b^4)*Log[a + b*Sin[c + d*x]])
/((a^2 - b^2)^8*d) + b/(7*(a^2 - b^2)*d*(a + b*Sin[c + d*x])^7) + (a*b)/(3*
(a^2 - b^2)^2*d*(a + b*Sin[c + d*x])^6) + (b*(3*a^2 + b^2))/(5*(a^2 - b^2)^
3*d*(a + b*Sin[c + d*x])^5) + (a*b*(a^2 + b^2))/((a^2 - b^2)^4*d*(a + b*Sin
[c + d*x])^4) + (b*(5*a^4 + 10*a^2*b^2 + b^4))/(3*(a^2 - b^2)^5*d*(a + b*Si
n[c + d*x])^3) + (a*b*(3*a^2 + b^2)*(a^2 + 3*b^2))/((a^2 - b^2)^6*d*(a + b*
Sin[c + d*x])^2) + (b*(7*a^6 + 35*a^4*b^2 + 21*a^2*b^4 + b^6))/((a^2 - b^2)
^7*d*(a + b*Sin[c + d*x]))
```

Rule 724

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Simp[e*((d
 + e*x)^(m + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + Dist[c/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*((d - e*x)/(a + c*x^2)), x], x] /; FreeQ[{a, c, d, e, m
}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
```

```
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(c+dx)}{(a+b\sin(c+dx))^8}\,dx &= \frac{b\text{Subst}\left(\int \frac{1}{(a+x)^8(b^2-x^2)}\,dx, x, b\sin(c+dx)\right)}{d}\\ &= \frac{b}{7\,(a^2-b^2)\,d(a+b\sin(c+dx))^7} + \frac{b\text{Subst}\left(\int \frac{a-x}{(a+x)^7(b^2-x^2)}\,dx, x, b\sin(c+dx)\right)}{(a^2-b^2)\,d}\\ &= \frac{b}{7\,(a^2-b^2)\,d(a+b\sin(c+dx))^7} + \frac{b\text{Subst}\left(\int \left(\frac{a-b}{2b(a+b)^7(b-x)} - \frac{2a}{(a-b)(a+b)(a+x)^7} + \cdots\right.\right.}{\cdots}\\ &= -\frac{\log(1-\sin(c+dx))}{2(a+b)^8d} + \frac{\log(1+\sin(c+dx))}{2(a-b)^8d} - \frac{8ab(a^2+b^2)\,(a^4+6a^2b^2+b^4)\cdots}{(a^2-b^2)^8\cdots}\end{aligned}$$

Mathematica [A]
time = 2.68, size = 365, normalized size = 0.95

$$\frac{b\left(-\frac{\log(1-\sin(c+dx))}{2b(a+b)^8} + \frac{\log(1+\sin(c+dx))}{2(a-b)^8b} - \frac{8a(a^2+b^2)\,(a^4+6a^2b^2+b^4)\log(a+b\sin(c+dx))}{(a-b)^8(a+b)^8} + \frac{1}{7(a^2-b^2)(a+b\sin(c+dx))^7} + \frac{a}{3(a-b)^2(a+b)^2(a+b\sin(c+dx))^6} + \frac{3a^2+b^2}{5(a-b)^3(a+b)^3(a+b\sin(c+dx))^5} + \frac{a(a^2+b^2)}{(a-b)^4(a+b)^4(a+b\sin(c+dx))^4} + \frac{5a^4+10a^2b^2+b^4}{3(a-b)^5(a+b)^5(a+b\sin(c+dx))^3} + \frac{a(3a^2+b^2)\,(a^2+3b^2)}{(a-b)^6(a+b)^6(a+b\sin(c+dx))^2} + \frac{7a^6+35a^4b^2+21a^2b^4+b^6}{(a-b)^7(a+b)^7(a+b\sin(c+dx))}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + b*Sin[c + d*x])^8,x]
```

```
[Out] (b*(-1/2*Log[1 - Sin[c + d*x]]/(b*(a + b)^8) + Log[1 + Sin[c + d*x]]/(2*(a
- b)^8*b) - (8*a*(a^2 + b^2)*(a^4 + 6*a^2*b^2 + b^4)*Log[a + b*Sin[c + d*x]
])/((a - b)^8*(a + b)^8) + 1/(7*(a^2 - b^2)*(a + b*Sin[c + d*x])^7) + a/(3*
(a - b)^2*(a + b)^2*(a + b*Sin[c + d*x])^6) + (3*a^2 + b^2)/(5*(a - b)^3*(a
 + b)^3*(a + b*Sin[c + d*x])^5) + (a*(a^2 + b^2))/((a - b)^4*(a + b)^4*(a +
 b*Sin[c + d*x])^4) + (5*a^4 + 10*a^2*b^2 + b^4)/(3*(a - b)^5*(a + b)^5*(a
+ b*Sin[c + d*x])^3) + (a*(3*a^2 + b^2)*(a^2 + 3*b^2))/((a - b)^6*(a + b)^6
*(a + b*Sin[c + d*x])^2) + (7*a^6 + 35*a^4*b^2 + 21*a^2*b^4 + b^6)/((a - b)
^7*(a + b)^7*(a + b*Sin[c + d*x]))))/d
```

Maple [A]
time = 5.88, size = 356, normalized size = 0.92

method	result
derivativedivides	$\frac{\frac{\ln(1+\sin(dx+c))}{2(a-b)^8}-\frac{\ln(\sin(dx+c)-1)}{2(a+b)^8}+\frac{b}{7(a-b)(a+b)(a+b\sin(dx+c))^7}+\frac{ab}{3(a+b)^2(a-b)^2(a+b\sin(dx+c))^6}+\frac{b(3a^2+b^2)}{5(a-b)^3(a+b)^3(a+b\sin(d}}{}$
default	$\frac{\frac{\ln(1+\sin(dx+c))}{2(a-b)^8}-\frac{\ln(\sin(dx+c)-1)}{2(a+b)^8}+\frac{b}{7(a-b)(a+b)(a+b\sin(dx+c))^7}+\frac{ab}{3(a+b)^2(a-b)^2(a+b\sin(dx+c))^6}+\frac{b(3a^2+b^2)}{5(a-b)^3(a+b)^3(a+b\sin(d}}{}$
norman	Expression too large to display
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+b*sin(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2/(a-b)^8*ln(1+sin(d*x+c))-1/2/(a+b)^8*ln(sin(d*x+c)-1)+1/7*b/(a-b)/
(a+b)/(a+b*sin(d*x+c))^7+1/3*a*b/(a+b)^2/(a-b)^2/(a+b*sin(d*x+c))^6+1/5*b*(
3*a^2+b^2)/(a-b)^3/(a+b)^3/(a+b*sin(d*x+c))^5+1/3*b*(5*a^4+10*a^2*b^2+b^4)/
(a+b)^5/(a-b)^5/(a+b*sin(d*x+c))^3+b*(7*a^6+35*a^4*b^2+21*a^2*b^4+b^6)/(a-b
)^7/(a+b)^7/(a+b*sin(d*x+c))+b*a*(a^2+b^2)/(a+b)^4/(a-b)^4/(a+b*sin(d*x+c))
^4+b*a*(3*a^4+10*a^2*b^2+3*b^4)/(a+b)^6/(a-b)^6/(a+b*sin(d*x+c))^2-8*b*(a^6
+7*a^4*b^2+7*a^2*b^4+b^6)*a/(a+b)^8/(a-b)^8*ln(a+b*sin(d*x+c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1160 vs. $2(374) = 748$.
time = 0.35, size = 1160, normalized size = 3.01

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/210*(1680*(a^7*b + 7*a^5*b^3 + 7*a^3*b^5 + a*b^7)*log(b*sin(d*x + c) + a
)/(a^16 - 8*a^14*b^2 + 28*a^12*b^4 - 56*a^10*b^6 + 70*a^8*b^8 - 56*a^6*b^10
 + 28*a^4*b^12 - 8*a^2*b^14 + b^16) - 2*(1443*a^12*b + 3704*a^10*b^3 + 1849
*a^8*b^5 - 496*a^6*b^7 + 309*a^4*b^9 - 104*a^2*b^11 + 15*b^13 + 105*(7*a^6*
b^7 + 35*a^4*b^9 + 21*a^2*b^11 + b^13)*sin(d*x + c)^6 + 105*(45*a^7*b^6 + 2
17*a^5*b^8 + 119*a^3*b^10 + 3*a*b^12)*sin(d*x + c)^5 + 35*(365*a^8*b^5 + 16
80*a^6*b^7 + 826*a^4*b^9 + 8*a^2*b^11 + b^13)*sin(d*x + c)^4 + 35*(533*a^9*
b^4 + 2304*a^7*b^6 + 994*a^5*b^8 + 8*a^3*b^10 + a*b^12)*sin(d*x + c)^3 + 21
*(743*a^10*b^3 + 2934*a^8*b^5 + 1099*a^6*b^7 + 29*a^4*b^9 - 6*a^2*b^11 + b^
13)*sin(d*x + c)^2 + 7*(1023*a^11*b^2 + 3494*a^9*b^4 + 1219*a^7*b^6 + 29*a^
5*b^8 - 6*a^3*b^10 + a*b^12)*sin(d*x + c))/(a^21 - 7*a^19*b^2 + 21*a^17*b^4
 - 35*a^15*b^6 + 35*a^13*b^8 - 21*a^11*b^10 + 7*a^9*b^12 - a^7*b^14 + (a^14
*b^7 - 7*a^12*b^9 + 21*a^10*b^11 - 35*a^8*b^13 + 35*a^6*b^15 - 21*a^4*b^17
+ 7*a^2*b^19 - b^21)*sin(d*x + c)^7 + 7*(a^15*b^6 - 7*a^13*b^8 + 21*a^11*b^
10 - 35*a^9*b^12 + 35*a^7*b^14 - 21*a^5*b^16 + 7*a^3*b^18 - a*b^20)*sin(d*x
```

```
 + c)^6 + 21*(a^16*b^5 - 7*a^14*b^7 + 21*a^12*b^9 - 35*a^10*b^11 + 35*a^8*b
^13 - 21*a^6*b^15 + 7*a^4*b^17 - a^2*b^19)*sin(d*x + c)^5 + 35*(a^17*b^4 -
7*a^15*b^6 + 21*a^13*b^8 - 35*a^11*b^10 + 35*a^9*b^12 - 21*a^7*b^14 + 7*a^5
*b^16 - a^3*b^18)*sin(d*x + c)^4 + 35*(a^18*b^3 - 7*a^16*b^5 + 21*a^14*b^7
- 35*a^12*b^9 + 35*a^10*b^11 - 21*a^8*b^13 + 7*a^6*b^15 - a^4*b^17)*sin(d*x
 + c)^3 + 21*(a^19*b^2 - 7*a^17*b^4 + 21*a^15*b^6 - 35*a^13*b^8 + 35*a^11*b
^10 - 21*a^9*b^12 + 7*a^7*b^14 - a^5*b^16)*sin(d*x + c)^2 + 7*(a^20*b - 7*a
^18*b^3 + 21*a^16*b^5 - 35*a^14*b^7 + 35*a^12*b^9 - 21*a^10*b^11 + 7*a^8*b^
13 - a^6*b^15)*sin(d*x + c)) - 105*log(sin(d*x + c) + 1)/(a^8 - 8*a^7*b + 2
8*a^6*b^2 - 56*a^5*b^3 + 70*a^4*b^4 - 56*a^3*b^5 + 28*a^2*b^6 - 8*a*b^7 + b
^8) + 105*log(sin(d*x + c) - 1)/(a^8 + 8*a^7*b + 28*a^6*b^2 + 56*a^5*b^3 +
70*a^4*b^4 + 56*a^3*b^5 + 28*a^2*b^6 + 8*a*b^7 + b^8))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3165 vs. $2(374) = 748$.
time = 1.57, size = 3165, normalized size = 8.22

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] -1/210*(2886*a^14*b + 35728*a^12*b^3 + 113862*a^10*b^5 + 11760*a^8*b^7 - 97
230*a^6*b^9 - 62496*a^4*b^11 - 4158*a^2*b^13 - 352*b^15 - 210*(7*a^8*b^7 +
28*a^6*b^9 - 14*a^4*b^11 - 20*a^2*b^13 - b^15)*cos(d*x + c)^6 + 70*(365*a^1
0*b^5 + 1378*a^8*b^7 - 602*a^6*b^9 - 944*a^4*b^11 - 187*a^2*b^13 - 10*b^15)
*cos(d*x + c)^4 - 14*(2229*a^12*b^3 + 10223*a^10*b^5 + 7960*a^8*b^7 - 10490
*a^6*b^9 - 8915*a^4*b^11 - 949*a^2*b^13 - 58*b^15)*cos(d*x + c)^2 - 1680*(a
^14*b + 28*a^12*b^3 + 189*a^10*b^5 + 400*a^8*b^7 + 315*a^6*b^9 + 84*a^4*b^1
1 + 7*a^2*b^13 - 7*(a^8*b^7 + 7*a^6*b^9 + 7*a^4*b^11 + a^2*b^13)*cos(d*x +
c)^6 + 7*(5*a^10*b^5 + 38*a^8*b^7 + 56*a^6*b^9 + 26*a^4*b^11 + 3*a^2*b^13)*
cos(d*x + c)^4 - 7*(3*a^12*b^3 + 31*a^10*b^5 + 94*a^8*b^7 + 94*a^6*b^9 + 31
*a^4*b^11 + 3*a^2*b^13)*cos(d*x + c)^2 + (7*a^13*b^2 + 84*a^11*b^4 + 315*a^
9*b^6 + 400*a^7*b^8 + 189*a^5*b^10 + 28*a^3*b^12 + a*b^14 - (a^7*b^8 + 7*a^
5*b^10 + 7*a^3*b^12 + a*b^14)*cos(d*x + c)^6 + 3*(7*a^9*b^6 + 50*a^7*b^8 +
56*a^5*b^10 + 14*a^3*b^12 + a*b^14)*cos(d*x + c)^4 - (35*a^11*b^4 + 287*a^9
*b^6 + 542*a^7*b^8 + 350*a^5*b^10 + 63*a^3*b^12 + 3*a*b^14)*cos(d*x + c)^2)
*sin(d*x + c))*log(b*sin(d*x + c) + a) + 105*(a^15 + 8*a^14*b + 49*a^13*b^2
 + 224*a^12*b^3 + 693*a^11*b^4 + 1512*a^10*b^5 + 2485*a^9*b^6 + 3200*a^8*b^
7 + 3235*a^7*b^8 + 2520*a^6*b^9 + 1491*a^5*b^10 + 672*a^4*b^11 + 231*a^3*b^
12 + 56*a^2*b^13 + 7*a*b^14 - 7*(a^9*b^6 + 8*a^8*b^7 + 28*a^7*b^8 + 56*a^6*
b^9 + 70*a^5*b^10 + 56*a^4*b^11 + 28*a^3*b^12 + 8*a^2*b^13 + a*b^14)*cos(d*
x + c)^6 + 7*(5*a^11*b^4 + 40*a^10*b^5 + 143*a^9*b^6 + 304*a^8*b^7 + 434*a^
7*b^8 + 448*a^6*b^9 + 350*a^5*b^10 + 208*a^4*b^11 + 89*a^3*b^12 + 24*a^2*b^
13 + 3*a*b^14)*cos(d*x + c)^4 - 7*(3*a^13*b^2 + 24*a^12*b^3 + 94*a^11*b^4 +
```

248*a^10*b^5 + 493*a^9*b^6 + 752*a^8*b^7 + 868*a^7*b^8 + 752*a^6*b^9 + 493
*a^5*b^10 + 248*a^4*b^11 + 94*a^3*b^12 + 24*a^2*b^13 + 3*a*b^14)*cos(d*x +
c)^2 + (7*a^14*b + 56*a^13*b^2 + 231*a^12*b^3 + 672*a^11*b^4 + 1491*a^10*b^
5 + 2520*a^9*b^6 + 3235*a^8*b^7 + 3200*a^7*b^8 + 2485*a^6*b^9 + 1512*a^5*b^
10 + 693*a^4*b^11 + 224*a^3*b^12 + 49*a^2*b^13 + 8*a*b^14 + b^15 - (a^8*b^7
+ 8*a^7*b^8 + 28*a^6*b^9 + 56*a^5*b^10 + 70*a^4*b^11 + 56*a^3*b^12 + 28*a^
2*b^13 + 8*a*b^14 + b^15)*cos(d*x + c)^6 + 3*(7*a^10*b^5 + 56*a^9*b^6 + 197
*a^8*b^7 + 400*a^7*b^8 + 518*a^6*b^9 + 448*a^5*b^10 + 266*a^4*b^11 + 112*a^
3*b^12 + 35*a^2*b^13 + 8*a*b^14 + b^15)*cos(d*x + c)^4 - (35*a^12*b^3 + 280
*a^11*b^4 + 1022*a^10*b^5 + 2296*a^9*b^6 + 3629*a^8*b^7 + 4336*a^7*b^8 + 40
04*a^6*b^9 + 2800*a^5*b^10 + 1421*a^4*b^11 + 504*a^3*b^12 + 126*a^2*b^13 +
24*a*b^14 + 3*b^15)*cos(d*x + c)^2)*sin(d*x + c))*log(sin(d*x + c) + 1) - 1
05*(a^15 - 8*a^14*b + 49*a^13*b^2 - 224*a^12*b^3 + 693*a^11*b^4 - 1512*a^10
*b^5 + 2485*a^9*b^6 - 3200*a^8*b^7 + 3235*a^7*b^8 - 2520*a^6*b^9 + 1491*a^5
*b^10 - 672*a^4*b^11 + 231*a^3*b^12 - 56*a^2*b^13 + 7*a*b^14 - 7*(a^9*b^6 -
8*a^8*b^7 + 28*a^7*b^8 - 56*a^6*b^9 + 70*a^5*b^10 - 56*a^4*b^11 + 28*a^3*b
^12 - 8*a^2*b^13 + a*b^14)*cos(d*x + c)^6 + 7*(5*a^11*b^4 - 40*a^10*b^5 + 1
43*a^9*b^6 - 304*a^8*b^7 + 434*a^7*b^8 - 448*a^6*b^9 + 350*a^5*b^10 - 208*a
^4*b^11 + 89*a^3*b^12 - 24*a^2*b^13 + 3*a*b^14)*cos(d*x + c)^4 - 7*(3*a^13*
b^2 - 24*a^12*b^3 + 94*a^11*b^4 - 248*a^10*b^5 + 493*a^9*b^6 - 752*a^8*b^7
+ 868*a^7*b^8 - 752*a^6*b^9 + 493*a^5*b^10 - 248*a^4*b^11 + 94*a^3*b^12 - 2
4*a^2*b^13 + 3*a*b^14)*cos(d*x + c)^2 + (7*a^14*b - 56*a^13*b^2 + 231*a^12*
b^3 - 672*a^11*b^4 + 1491*a^10*b^5 - 2520*a^9*b^6 + 3235*a^8*b^7 - 3200*a^7
*b^8 + 2485*a^6*b^9 - 1512*a^5*b^10 + 693*a^4*b^11 - 224*a^3*b^12 + 49*a^2*
b^13 - 8*a*b^14 + b^15 - (a^8*b^7 - 8*a^7*b^8 + 28*a^6*b^9 - 56*a^5*b^10 +
70*a^4*b^11 - 56*a^3*b^12 + 28*a^2*b^13 - 8*a*b^14 + b^15)*cos(d*x + c)^6 +
3*(7*a^10*b^5 - 56*a^9*b^6 + 197*a^8*b^7 - 400*a^7*b^8 + 518*a^6*b^9 - 448
*a^5*b^10 + 266*a^4*b^11 - 112*a^3*b^12 + 35*a^2*b^13 - 8*a*b^14 + b^15)*co
s(d*x + c)^4 - (35*a^12*b^3 - 280*a^11*b^4 + 1022*a^10*b^5 - 2296*a^9*b^6 +
3629*a^8*b^7 - 4336*a^7*b^8 + 4004*a^6*b^9 - 2800*a^5*b^10 + 1421*a^4*b^11
- 504*a^3*b^12 + 126*a^2*b^13 - 24*a*b^14 + 3*b^15)*cos(d*x + c)^2)*sin(d*
x + c))*log(-sin(d*x + c) + 1) + 14*(1023*a^13*b^2 + 5136*a^11*b^4 + 7255*a
^9*b^6 - 5160*a^7*b^8 - 6435*a^5*b^10 - 1768*a^3*b^12 - 51*a*b^14 + 15*(45*
a^9*b^6 + 172*a^7*b^8 - 98*a^5*b^10 - 116*a^3*b^12 - 3*a*b^14)*cos(d*x + c)
^4 - 5*(533*a^11*b^4 + 2041*a^9*b^6 - 278*a^7*b^8 - 1574*a^5*b^10 - 703*a^3
*b^12 - 19*a*b^14)*cos(d*x + c)^2)*sin(d*x + c))/(7*(a^17*b^6 - 8*a^15*b^8
+ 28*a^13*b^10 - 56*a^11*b^12 + 70*a^9*b^14 - 56*a^7*b^16 + 28*a^5*b^18 - 8
*a^3*b^20 + a*b^22)*d*cos(d*x + c)^6 - 7*(5*a^19*b^4 - 37*a^17*b^6 + 116*a^
15*b^8 - 196*a^13*b^10 + 182*a^11*b^12 - 70*a^9*b^14 - 28*a^7*b^16 + 44*a^5
*b^18 - 19*a^3*b^20 + 3*a*b^22)*d*cos(d*x + c)^4 + 7*(3*a^21*b^2 - 14*a^19*
b^4 + 7*a^17*b^6 + 88*a^15*b^8 - 266*a^13*b^10 + 364*a^11*b^12 - 266*a^9*b^
14 + 88*a^7*b^16 + 7*a^5*b^18 - 14*a^3*b^20 + 3*a*b^22)*d*cos(d*x + c)^2 -
(a^23 + 13*a^21*b^2 - 105*a^19*b^4 + 259*a^17*b^6 - 182*a^15*b^8 - 350*a^13
*b^10 + 910*a^11*b^12 - 890*a^9*b^14 + 421*a^7*...

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c))**8,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1010 vs. 2(374) = 748.
time = 6.26, size = 1010, normalized size = 2.62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -1/210*(1680*(a^7*b^2 + 7*a^5*b^4 + 7*a^3*b^6 + a*b^8)*log(abs(b*sin(d*x +
c) + a))/(a^16*b - 8*a^14*b^3 + 28*a^12*b^5 - 56*a^10*b^7 + 70*a^8*b^9 - 56
*a^6*b^11 + 28*a^4*b^13 - 8*a^2*b^15 + b^17) - 105*log(abs(sin(d*x + c) + 1
))/(a^8 - 8*a^7*b + 28*a^6*b^2 - 56*a^5*b^3 + 70*a^4*b^4 - 56*a^3*b^5 + 28*
a^2*b^6 - 8*a*b^7 + b^8) + 105*log(abs(sin(d*x + c) - 1))/(a^8 + 8*a^7*b +
28*a^6*b^2 + 56*a^5*b^3 + 70*a^4*b^4 + 56*a^3*b^5 + 28*a^2*b^6 + 8*a*b^7 +
b^8) - 2*(2178*a^7*b^8*sin(d*x + c)^7 + 15246*a^5*b^10*sin(d*x + c)^7 + 152
46*a^3*b^12*sin(d*x + c)^7 + 2178*a*b^14*sin(d*x + c)^7 + 15981*a^8*b^7*sin
(d*x + c)^6 + 109662*a^6*b^9*sin(d*x + c)^6 + 105252*a^4*b^11*sin(d*x + c)^
6 + 13146*a^2*b^13*sin(d*x + c)^6 - 105*b^15*sin(d*x + c)^6 + 50463*a^9*b^6
*sin(d*x + c)^5 + 338226*a^7*b^8*sin(d*x + c)^5 + 309876*a^5*b^10*sin(d*x +
 c)^5 + 33558*a^3*b^12*sin(d*x + c)^5 - 315*a*b^14*sin(d*x + c)^5 + 89005*a
^10*b^5*sin(d*x + c)^4 + 579635*a^8*b^7*sin(d*x + c)^4 + 503720*a^6*b^9*sin
(d*x + c)^4 + 47600*a^4*b^11*sin(d*x + c)^4 - 245*a^2*b^13*sin(d*x + c)^4 -
 35*b^15*sin(d*x + c)^4 + 94885*a^11*b^4*sin(d*x + c)^3 + 595595*a^9*b^6*si
n(d*x + c)^3 + 487760*a^7*b^8*sin(d*x + c)^3 + 41720*a^5*b^10*sin(d*x + c)^
3 - 245*a^3*b^12*sin(d*x + c)^3 - 35*a*b^14*sin(d*x + c)^3 + 61341*a^12*b^3
*sin(d*x + c)^2 + 366177*a^10*b^5*sin(d*x + c)^2 + 281631*a^8*b^7*sin(d*x +
 c)^2 + 23268*a^6*b^9*sin(d*x + c)^2 - 735*a^4*b^11*sin(d*x + c)^2 + 147*a^
2*b^13*sin(d*x + c)^2 - 21*b^15*sin(d*x + c)^2 + 22407*a^13*b^2*sin(d*x + c
) + 124019*a^11*b^4*sin(d*x + c) + 90797*a^9*b^6*sin(d*x + c) + 6916*a^7*b^
8*sin(d*x + c) - 245*a^5*b^10*sin(d*x + c) + 49*a^3*b^12*sin(d*x + c) - 7*a
*b^14*sin(d*x + c) + 3621*a^14*b + 17507*a^12*b^3 + 13391*a^10*b^5 - 167*a^
8*b^7 + 805*a^6*b^9 - 413*a^4*b^11 + 119*a^2*b^13 - 15*b^15)/((a^16 - 8*a^1
4*b^2 + 28*a^12*b^4 - 56*a^10*b^6 + 70*a^8*b^8 - 56*a^6*b^10 + 28*a^4*b^12
- 8*a^2*b^14 + b^16)*(b*sin(d*x + c) + a)^7))/d
```

Mupad [B]
time = 7.64, size = 937, normalized size = 2.43

$$\frac{\ln\left(a+b\,\sin\left(c+d\,x\right)\right)\,\left(\frac{1}{2\,{\left(a+b\right)}^8}-\frac{1}{2\,{\left(a-b\right)}^8}\right)}{d}+\frac{\frac{1443\,a^{12}\,b+3704\,a^{10}\,b^3+1849\,a^8\,b^5-496\,a^6\,b^7+309\,a^4\,b^9-104\,a^2\,b^{11}+15\,b^{13}}{105\,\left(a^{14}-7\,a^{12}\,b^2+21\,a^{10}\,b^4-35\,a^8\,b^6+35\,a^6\,b^8-21\,a^4\,b^{10}+7\,a^2\,b^{12}-b^{14}\right)}+\frac{\sin(c+d\,x)\,\left(1023\,a^{11}\,b^2+3494\,a^9\,b^4+1219\,a^7\,b^6+29\,a^5\,b^8-6\,a^3\,b^{10}+a\,b^{12}\right)}{15\,\left(a^{14}-7\,a^{12}\,b^2+21\,a^{10}\,b^4-35\,a^8\,b^6+35\,a^6\,b^8-21\,a^4\,b^{10}+7\,a^2\,b^{12}-b^{14}\right)}+\frac{\sin(c+d\,x)^3\,\left(533\,a^9\,b^4+2304\,a^7\,b^6+994\,a^5\,b^8+8\,a^3\,b^{10}+a\,b^{12}\right)}{3\,\left(a^{14}-7\,a^{12}\,b^2+21\,a^{10}\,b^4-35\,a^8\,b^6+35\,a^6\,b^8-21\,a^4\,b^{10}+7\,a^2\,b^{12}-b^{14}\right)}+\frac{\sin(c+d\,x)^5\,\left(45\,a^7\,b^6+217\,a^5\,b^8+119\,a^3\,b^{10}+3\,a\,b^{12}\right)}{a^{14}-7\,a^{12}\,b^2+21\,a^{10}\,b^4-35\,a^8\,b^6+35\,a^6\,b^8-21\,a^4\,b^{10}+7\,a^2\,b^{12}-b^{14}}+\frac{\sin(c+d\,x)^2\,\left(743\,a^{10}\,b^3+2934\,a^8\,b^5+1099\,a^6\,b^7+29\,a^4\,b^9-6\,a^2\,b^{11}+b^{13}\right)}{5\,\left(a^{14}-7\,a^{12}\,b^2+21\,a^{10}\,b^4-35\,a^8\,b^6+35\,a^6\,b^8-21\,a^4\,b^{10}+7\,a^2\,b^{12}-b^{14}\right)}+\frac{\sin(c+d\,x)^4\,\left(365\,a^8\,b^5+1680\,a^6\,b^7+826\,a^4\,b^9+8\,a^2\,b^{11}+b^{13}\right)}{3\,\left(a^{14}-7\,a^{12}\,b^2+21\,a^{10}\,b^4-35\,a^8\,b^6+35\,a^6\,b^8-21\,a^4\,b^{10}+7\,a^2\,b^{12}-b^{14}\right)}+\frac{\sin(c+d\,x)^6\,\left(7\,a^6\,b^7+35\,a^4\,b^9+21\,a^2\,b^{11}+b^{13}\right)}{a^{14}-7\,a^{12}\,b^2+21\,a^{10}\,b^4-35\,a^8\,b^6+35\,a^6\,b^8-21\,a^4\,b^{10}+7\,a^2\,b^{12}-b^{14}}}{d\,\left(a^7+7\,a^6\,b\,\sin\left(c+d\,x\right)+21\,a^5\,b^2\,\sin\left(c+d\,x\right)^2+35\,a^4\,b^3\,\sin\left(c+d\,x\right)^3+35\,a^3\,b^4\,\sin\left(c+d\,x\right)^4+21\,a^2\,b^5\,\sin\left(c+d\,x\right)^5+7\,a\,b^6\,\sin\left(c+d\,x\right)^6+b^7\,\sin\left(c+d\,x\right)^7\right)}+\frac{\ln\left(\sin\left(c+d\,x\right)+1\right)}{2\,d\,{\left(a-b\right)}^8}-\frac{\ln\left(\sin\left(c+d\,x\right)-1\right)}{2\,d\,{\left(a+b\right)}^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + b*sin(c + d*x))^8),x)
```

```
[Out] (log(a + b*sin(c + d*x))*(1/(2*(a + b)^8) - 1/(2*(a - b)^8)))/d + ((1443*a^
12*b + 15*b^13 - 104*a^2*b^11 + 309*a^4*b^9 - 496*a^6*b^7 + 1849*a^8*b^5 +
3704*a^10*b^3)/(105*(a^14 - b^14 + 7*a^2*b^12 - 21*a^4*b^10 + 35*a^6*b^8 -
35*a^8*b^6 + 21*a^10*b^4 - 7*a^12*b^2)) + (sin(c + d*x)*(a*b^12 - 6*a^3*b^1
0 + 29*a^5*b^8 + 1219*a^7*b^6 + 3494*a^9*b^4 + 1023*a^11*b^2))/(15*(a^14 -
b^14 + 7*a^2*b^12 - 21*a^4*b^10 + 35*a^6*b^8 - 35*a^8*b^6 + 21*a^10*b^4 - 7
*a^12*b^2)) + (sin(c + d*x)^3*(a*b^12 + 8*a^3*b^10 + 994*a^5*b^8 + 2304*a^7
*b^6 + 533*a^9*b^4))/(3*(a^14 - b^14 + 7*a^2*b^12 - 21*a^4*b^10 + 35*a^6*b^
8 - 35*a^8*b^6 + 21*a^10*b^4 - 7*a^12*b^2)) + (sin(c + d*x)^5*(3*a*b^12 + 1
19*a^3*b^10 + 217*a^5*b^8 + 45*a^7*b^6))/(a^14 - b^14 + 7*a^2*b^12 - 21*a^4
*b^10 + 35*a^6*b^8 - 35*a^8*b^6 + 21*a^10*b^4 - 7*a^12*b^2) + (sin(c + d*x)
^2*(b^13 - 6*a^2*b^11 + 29*a^4*b^9 + 1099*a^6*b^7 + 2934*a^8*b^5 + 743*a^10
*b^3))/(5*(a^14 - b^14 + 7*a^2*b^12 - 21*a^4*b^10 + 35*a^6*b^8 - 35*a^8*b^6
 + 21*a^10*b^4 - 7*a^12*b^2)) + (sin(c + d*x)^4*(b^13 + 8*a^2*b^11 + 826*a^
4*b^9 + 1680*a^6*b^7 + 365*a^8*b^5))/(3*(a^14 - b^14 + 7*a^2*b^12 - 21*a^4*
b^10 + 35*a^6*b^8 - 35*a^8*b^6 + 21*a^10*b^4 - 7*a^12*b^2)) + (sin(c + d*x)
^6*(b^13 + 21*a^2*b^11 + 35*a^4*b^9 + 7*a^6*b^7))/(a^14 - b^14 + 7*a^2*b^12
 - 21*a^4*b^10 + 35*a^6*b^8 - 35*a^8*b^6 + 21*a^10*b^4 - 7*a^12*b^2))/(d*(a
^7 + b^7*sin(c + d*x)^7 + 7*a*b^6*sin(c + d*x)^6 + 21*a^5*b^2*sin(c + d*x)^
2 + 35*a^4*b^3*sin(c + d*x)^3 + 35*a^3*b^4*sin(c + d*x)^4 + 21*a^2*b^5*sin(
c + d*x)^5 + 7*a^6*b*sin(c + d*x))) + log(sin(c + d*x) + 1)/(2*d*(a - b)^8)
 - log(sin(c + d*x) - 1)/(2*d*(a + b)^8)
```

3.466 $\int \frac{\sec^3(c+dx)}{(a+b\sin(c+dx))^8}\,dx$

Optimal. Leaf size=527

$$-\frac{(a+9b)\log(1-\sin(c+dx))}{4(a+b)^9d}+\frac{(a-9b)\log(1+\sin(c+dx))}{4(a-b)^9d}+\frac{8ab^3(15a^6+63a^4b^2+45a^2b^4+5b^6)\log(a+b}{(a^2-b^2)^9\,d}$$

```
[Out] -1/4*(a+9*b)*ln(1-sin(d*x+c))/(a+b)^9/d+1/4*(a-9*b)*ln(1+sin(d*x+c))/(a-b)^
9/d+8*a*b^3*(15*a^6+63*a^4*b^2+45*a^2*b^4+5*b^6)*ln(a+b*sin(d*x+c))/(a^2-b^
2)^9/d-1/14*b*(7*a^2+9*b^2)/(a^2-b^2)^2/d/(a+b*sin(d*x+c))^7-1/2*sec(d*x+c)
^2*(b-a*sin(d*x+c))/(a^2-b^2)/d/(a+b*sin(d*x+c))^7-1/6*a*b*(3*a^2+13*b^2)/(
a^2-b^2)^3/d/(a+b*sin(d*x+c))^6-1/10*b*(5*a^4+50*a^2*b^2+9*b^4)/(a^2-b^2)^4
/d/(a+b*sin(d*x+c))^5-1/2*a*b*(a^4+20*a^2*b^2+11*b^4)/(a^2-b^2)^5/d/(a+b*si
n(d*x+c))^4-1/6*b*(3*a^6+115*a^4*b^2+129*a^2*b^4+9*b^6)/(a^2-b^2)^6/d/(a+b*
sin(d*x+c))^3-1/2*a*b*(a^6+77*a^4*b^2+147*a^2*b^4+31*b^6)/(a^2-b^2)^7/d/(a+
b*sin(d*x+c))^2-1/2*b*(a^8+196*a^6*b^2+574*a^4*b^4+244*a^2*b^6+9*b^8)/(a^2-
b^2)^8/d/(a+b*sin(d*x+c))
```

Rubi [A]

time = 0.52, antiderivative size = 527, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2747, 755, 815}

$$-\frac{ab(3a^2+13b^2)}{6d(a^2-b^2)^3(a+b\sin(c+dx))^6}-\frac{b(7a^2+9b^2)}{14d(a^2-b^2)^2(a+b\sin(c+dx))^7}-\frac{\sec^2(c+dx)(b-a\sin(c+dx))}{2d(a^2-b^2)(a+b\sin(c+dx))^7}-\frac{ab(a^4+20a^2b^2+11b^4)}{2d(a^2-b^2)^5(a+b\sin(c+dx))^4}-\frac{b(5a^4+50a^2b^2+9b^4)}{10d(a^2-b^2)^4(a+b\sin(c+dx))^5}-\frac{ab(a^6+77a^4b^2+147a^2b^4+31b^6)}{2d(a^2-b^2)^7(a+b\sin(c+dx))^2}-\frac{b(3a^6+115a^4b^2+129a^2b^4+9b^6)}{6d(a^2-b^2)^6(a+b\sin(c+dx))^3}+\frac{8ab^3(15a^6+63a^4b^2+45a^2b^4+5b^6)\log(a+b\sin(c+dx))}{d(a^2-b^2)^9}-\frac{b(a^8+196a^6b^2+574a^4b^4+244a^2b^6+9b^8)}{2d(a^2-b^2)^8(a+b\sin(c+dx))}-\frac{(a+9b)\log(1-\sin(c+dx))}{4d(a+b)^9}+\frac{(a-9b)\log(\sin(c+dx)+1)}{4d(a-b)^9}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + b*Sin[c + d*x])^8,x]
```

```
[Out] -1/4*((a + 9*b)*Log[1 - Sin[c + d*x]])/((a + b)^9*d) + ((a - 9*b)*Log[1 + S
in[c + d*x]])/(4*(a - b)^9*d) + (8*a*b^3*(15*a^6 + 63*a^4*b^2 + 45*a^2*b^4
+ 5*b^6)*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)^9*d) - (b*(7*a^2 + 9*b^2))/(
14*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x])^7) - (Sec[c + d*x]^2*(b - a*Sin[c +
 d*x]))/(2*(a^2 - b^2)*d*(a + b*Sin[c + d*x])^7) - (a*b*(3*a^2 + 13*b^2))/(
6*(a^2 - b^2)^3*d*(a + b*Sin[c + d*x])^6) - (b*(5*a^4 + 50*a^2*b^2 + 9*b^4)
)/(10*(a^2 - b^2)^4*d*(a + b*Sin[c + d*x])^5) - (a*b*(a^4 + 20*a^2*b^2 + 11
*b^4))/(2*(a^2 - b^2)^5*d*(a + b*Sin[c + d*x])^4) - (b*(3*a^6 + 115*a^4*b^2
 + 129*a^2*b^4 + 9*b^6))/(6*(a^2 - b^2)^6*d*(a + b*Sin[c + d*x])^3) - (a*b*
(a^6 + 77*a^4*b^2 + 147*a^2*b^4 + 31*b^6))/(2*(a^2 - b^2)^7*d*(a + b*Sin[c
+ d*x])^2) - (b*(a^8 + 196*a^6*b^2 + 574*a^4*b^4 + 244*a^2*b^6 + 9*b^8))/(2
*(a^2 - b^2)^8*d*(a + b*Sin[c + d*x]))
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
```

```
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a+b\sin(c+dx))^8}\,dx = \frac{b^3\text{Subst}\left(\int \frac{1}{(a+x)^8(b^2-x^2)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= -\frac{\sec^2(c+dx)(b-a\sin(c+dx))}{2(a^2-b^2)\,d(a+b\sin(c+dx))^7} + \frac{b\text{Subst}\left(\int \frac{a^2-9b^2+8ax}{(a+x)^8(b^2-x^2)}\,dx, x, b\sin(c+dx)\right)}{2(a^2-b^2)\,d}$$

$$= -\frac{\sec^2(c+dx)(b-a\sin(c+dx))}{2(a^2-b^2)\,d(a+b\sin(c+dx))^7} + \frac{b\text{Subst}\left(\int \left(\frac{(a-b)(a+9b)}{2b(a+b)^8(b-x)} + \frac{7a^2+9b^2}{(a-b)(a+b)(a+x)^8}\right.\right.}{}$$

$$= -\frac{(a+9b)\log(1-\sin(c+dx))}{4(a+b)^9d} + \frac{(a-9b)\log(1+\sin(c+dx))}{4(a-b)^9d} + \frac{8ab^3(15a^6+}{}$$

Mathematica [A]
time = 6.72, size = 770, normalized size = 1.46

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + b*Sin[c + d*x])^8,x]
```

```
[Out] (b^3*((Sec[c + d*x]^2*(b^2 - a*b*Sin[c + d*x]))/(2*b^4*(-a^2 + b^2)*(a + b*
Sin[c + d*x])^7) - (8*a*(-1/2*Log[1 - Sin[c + d*x]]/(b*(a + b)^7) + Log[1 +
 Sin[c + d*x]]/(2*(a - b)^7*b) - ((7*a^6 + 35*a^4*b^2 + 21*a^2*b^4 + b^6)*L
```

```
og[a + b*Sin[c + d*x]])/((a - b)^7*(a + b)^7) + 1/(6*(a^2 - b^2)*(a + b*Sin
[c + d*x])^6) + (2*a)/(5*(a - b)^2*(a + b)^2*(a + b*Sin[c + d*x])^5) + (3*a
^2 + b^2)/(4*(a - b)^3*(a + b)^3*(a + b*Sin[c + d*x])^4) + (4*a*(a^2 + b^2)
)/(3*(a - b)^4*(a + b)^4*(a + b*Sin[c + d*x])^3) + (5*a^4 + 10*a^2*b^2 + b^
4)/(2*(a - b)^5*(a + b)^5*(a + b*Sin[c + d*x])^2) + (2*a*(3*a^2 + b^2)*(a^2
 + 3*b^2))/((a - b)^6*(a + b)^6*(a + b*Sin[c + d*x]))) + (-7*a^2 - 9*b^2)*(
-1/2*Log[1 - Sin[c + d*x]]/(b*(a + b)^8) + Log[1 + Sin[c + d*x]]/(2*(a - b)
^8*b) - (8*a*(a^2 + b^2)*(a^4 + 6*a^2*b^2 + b^4)*Log[a + b*Sin[c + d*x]])/(
(a - b)^8*(a + b)^8) + 1/(7*(a^2 - b^2)*(a + b*Sin[c + d*x])^7) + a/(3*(a -
 b)^2*(a + b)^2*(a + b*Sin[c + d*x])^6) + (3*a^2 + b^2)/(5*(a - b)^3*(a + b
)^3*(a + b*Sin[c + d*x])^5) + (a*(a^2 + b^2))/((a - b)^4*(a + b)^4*(a + b*S
in[c + d*x])^4) + (5*a^4 + 10*a^2*b^2 + b^4)/(3*(a - b)^5*(a + b)^5*(a + b*
Sin[c + d*x])^3) + (a*(3*a^2 + b^2)*(a^2 + 3*b^2))/((a - b)^6*(a + b)^6*(a
+ b*Sin[c + d*x])^2) + (7*a^6 + 35*a^4*b^2 + 21*a^2*b^4 + b^6)/((a - b)^7*(
a + b)^7*(a + b*Sin[c + d*x]))))/(2*b^2*(-a^2 + b^2))))/d
```

Maple [A]
time = 5.32, size = 433, normalized size = 0.82

method	result
derivativedivides	$-\frac{1}{4(a-b)^8(1+\sin(dx+c))}+\frac{(a-9b)\ln(1+\sin(dx+c))}{4(a-b)^9}-\frac{1}{4(a+b)^8(\sin(dx+c)-1)}+\frac{(-a-9b)\ln(\sin(dx+c)-1)}{4(a+b)^9}-\frac{b^3}{7(a+b)^2(a-b)^2(a+b\sin(dx}$
default	$-\frac{1}{4(a-b)^8(1+\sin(dx+c))}+\frac{(a-9b)\ln(1+\sin(dx+c))}{4(a-b)^9}-\frac{1}{4(a+b)^8(\sin(dx+c)-1)}+\frac{(-a-9b)\ln(\sin(dx+c)-1)}{4(a+b)^9}-\frac{b^3}{7(a+b)^2(a-b)^2(a+b\sin(dx}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+b*sin(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/4/(a-b)^8/(1+sin(d*x+c))+1/4*(a-9*b)/(a-b)^9*ln(1+sin(d*x+c))-1/4/(
a+b)^8/(sin(d*x+c)-1)+1/4/(a+b)^9*(-a-9*b)*ln(sin(d*x+c)-1)-1/7*b^3/(a+b)^2
/(a-b)^2/(a+b*sin(d*x+c))^7-1/3*b^3*(35*a^4+42*a^2*b^2+3*b^4)/(a+b)^6/(a-b)
^6/(a+b*sin(d*x+c))^3-2/3*a*b^3/(a-b)^3/(a+b)^3/(a+b*sin(d*x+c))^6-2/5*b^3*
(5*a^2+b^2)/(a+b)^4/(a-b)^4/(a+b*sin(d*x+c))^5-4*b^3*(21*a^6+63*a^4*b^2+27*
a^2*b^4+b^6)/(a+b)^8/(a-b)^8/(a+b*sin(d*x+c))-b^3*a*(5*a^2+3*b^2)/(a+b)^5/(
a-b)^5/(a+b*sin(d*x+c))^4-4*b^3*a*(7*a^4+14*a^2*b^2+3*b^4)/(a-b)^7/(a+b)^7/
(a+b*sin(d*x+c))^2+8*b^3*a*(15*a^6+63*a^4*b^2+45*a^2*b^4+5*b^6)/(a-b)^9/(a+
b)^9*ln(a+b*sin(d*x+c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1670 vs. 2(508) = 1016.
time = 0.39, size = 1670, normalized size = 3.17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 1/420*(3360*(15*a^7*b^3 + 63*a^5*b^5 + 45*a^3*b^7 + 5*a*b^9)*log(b*sin(d*x
+ c) + a)/(a^18 - 9*a^16*b^2 + 36*a^14*b^4 - 84*a^12*b^6 + 126*a^10*b^8 - 1
26*a^8*b^10 + 84*a^6*b^12 - 36*a^4*b^14 + 9*a^2*b^16 - b^18) + 105*(a - 9*b
)*log(sin(d*x + c) + 1)/(a^9 - 9*a^8*b + 36*a^7*b^2 - 84*a^6*b^3 + 126*a^5*
b^4 - 126*a^4*b^5 + 84*a^3*b^6 - 36*a^2*b^7 + 9*a*b^8 - b^9) - 105*(a + 9*b
)*log(sin(d*x + c) - 1)/(a^9 + 9*a^8*b + 36*a^7*b^2 + 84*a^6*b^3 + 126*a^5*
b^4 + 126*a^4*b^5 + 84*a^3*b^6 + 36*a^2*b^7 + 9*a*b^8 + b^9) - 2*(840*a^14*
b + 33490*a^12*b^3 + 57724*a^10*b^5 + 16354*a^8*b^7 - 1496*a^6*b^9 + 814*a^
4*b^11 - 236*a^2*b^13 + 30*b^15 - 105*(a^8*b^7 + 196*a^6*b^9 + 574*a^4*b^11
 + 244*a^2*b^13 + 9*b^15)*sin(d*x + c)^8 - 105*(7*a^9*b^6 + 1252*a^7*b^8 +
3514*a^5*b^10 + 1348*a^3*b^12 + 23*a*b^14)*sin(d*x + c)^7 - 35*(63*a^10*b^5
 + 10066*a^8*b^7 + 26194*a^6*b^9 + 7384*a^4*b^11 - 681*a^2*b^13 - 18*b^15)*
sin(d*x + c)^6 - 35*(105*a^11*b^4 + 14506*a^9*b^6 + 32254*a^7*b^8 + 160*a^5
*b^10 - 3951*a^3*b^12 - 66*a*b^14)*sin(d*x + c)^5 - 7*(525*a^12*b^3 + 59310
*a^10*b^5 + 83812*a^8*b^7 - 98528*a^6*b^9 - 44663*a^4*b^11 - 438*a^2*b^13 -
 18*b^15)*sin(d*x + c)^4 - 7*(315*a^13*b^2 + 25930*a^11*b^4 - 20896*a^9*b^6
 - 166336*a^7*b^8 - 53641*a^5*b^10 - 386*a^3*b^12 - 26*a*b^14)*sin(d*x + c)
^3 - (735*a^14*b + 30550*a^12*b^3 - 361856*a^10*b^5 - 919070*a^8*b^7 - 2528
45*a^6*b^9 - 3050*a^4*b^11 + 310*a^2*b^13 - 54*b^15)*sin(d*x + c)^2 - 7*(15
*a^15 - 420*a^13*b^2 - 26140*a^11*b^4 - 52264*a^9*b^6 - 13189*a^7*b^8 - 184
*a^5*b^10 + 26*a^3*b^12 - 4*a*b^14)*sin(d*x + c))/(a^23 - 8*a^21*b^2 + 28*a
^19*b^4 - 56*a^17*b^6 + 70*a^15*b^8 - 56*a^13*b^10 + 28*a^11*b^12 - 8*a^9*b
^14 + a^7*b^16 - (a^16*b^7 - 8*a^14*b^9 + 28*a^12*b^11 - 56*a^10*b^13 + 70*
a^8*b^15 - 56*a^6*b^17 + 28*a^4*b^19 - 8*a^2*b^21 + b^23)*sin(d*x + c)^9 -
7*(a^17*b^6 - 8*a^15*b^8 + 28*a^13*b^10 - 56*a^11*b^12 + 70*a^9*b^14 - 56*a
^7*b^16 + 28*a^5*b^18 - 8*a^3*b^20 + a*b^22)*sin(d*x + c)^8 - (21*a^18*b^5
- 169*a^16*b^7 + 596*a^14*b^9 - 1204*a^12*b^11 + 1526*a^10*b^13 - 1246*a^8*
b^15 + 644*a^6*b^17 - 196*a^4*b^19 + 29*a^2*b^21 - b^23)*sin(d*x + c)^7 - 7
*(5*a^19*b^4 - 41*a^17*b^6 + 148*a^15*b^8 - 308*a^13*b^10 + 406*a^11*b^12 -
 350*a^9*b^14 + 196*a^7*b^16 - 68*a^5*b^18 + 13*a^3*b^20 - a*b^22)*sin(d*x
+ c)^6 - 7*(5*a^20*b^3 - 43*a^18*b^5 + 164*a^16*b^7 - 364*a^14*b^9 + 518*a^
12*b^11 - 490*a^10*b^13 + 308*a^8*b^15 - 124*a^6*b^17 + 29*a^4*b^19 - 3*a^2
*b^21)*sin(d*x + c)^5 - 7*(3*a^21*b^2 - 29*a^19*b^4 + 124*a^17*b^6 - 308*a^
15*b^8 + 490*a^13*b^10 - 518*a^11*b^12 + 364*a^9*b^14 - 164*a^7*b^16 + 43*a
^5*b^18 - 5*a^3*b^20)*sin(d*x + c)^4 - 7*(a^22*b - 13*a^20*b^3 + 68*a^18*b^
5 - 196*a^16*b^7 + 350*a^14*b^9 - 406*a^12*b^11 + 308*a^10*b^13 - 148*a^8*b
^15 + 41*a^6*b^17 - 5*a^4*b^19)*sin(d*x + c)^3 - (a^23 - 29*a^21*b^2 + 196*
a^19*b^4 - 644*a^17*b^6 + 1246*a^15*b^8 - 1526*a^13*b^10 + 1204*a^11*b^12 -
 596*a^9*b^14 + 169*a^7*b^16 - 21*a^5*b^18)*sin(d*x + c)^2 + 7*(a^22*b - 8*
a^20*b^3 + 28*a^18*b^5 - 56*a^16*b^7 + 70*a^14*b^9 - 56*a^12*b^11 + 28*a^10
*b^13 - 8*a^8*b^15 + a^6*b^17)*sin(d*x + c)))/d
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3678 vs. $2(508) = 1016$.

time = 2.58, size = 3678, normalized size = 6.98

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/420*(210*a^16*b - 1680*a^14*b^3 + 5880*a^12*b^5 - 11760*a^10*b^7 + 14700*
a^8*b^9 - 11760*a^6*b^11 + 5880*a^4*b^13 - 1680*a^2*b^15 + 210*b^17 - 210*(
a^10*b^7 + 195*a^8*b^9 + 378*a^6*b^11 - 330*a^4*b^13 - 235*a^2*b^15 - 9*b^1
7)*cos(d*x + c)^8 + 70*(63*a^12*b^5 + 10015*a^10*b^7 + 18468*a^8*b^9 - 1427
4*a^6*b^11 - 12025*a^4*b^13 - 2157*a^2*b^15 - 90*b^17)*cos(d*x + c)^6 - 14*
(525*a^14*b^3 + 59730*a^12*b^5 + 174637*a^10*b^7 + 77130*a^8*b^9 - 194265*a
^6*b^11 - 106450*a^4*b^13 - 10785*a^2*b^15 - 522*b^17)*cos(d*x + c)^4 + 2*(
735*a^16*b + 37165*a^14*b^3 + 437199*a^12*b^5 + 836549*a^10*b^7 - 111195*a^
8*b^9 - 812385*a^6*b^11 - 362915*a^4*b^13 - 23569*a^2*b^15 - 1584*b^17)*cos
(d*x + c)^2 + 3360*(7*(15*a^8*b^9 + 63*a^6*b^11 + 45*a^4*b^13 + 5*a^2*b^15)
*cos(d*x + c)^8 - 7*(75*a^10*b^7 + 360*a^8*b^9 + 414*a^6*b^11 + 160*a^4*b^1
3 + 15*a^2*b^15)*cos(d*x + c)^6 + 7*(45*a^12*b^5 + 339*a^10*b^7 + 810*a^8*b
^9 + 654*a^6*b^11 + 185*a^4*b^13 + 15*a^2*b^15)*cos(d*x + c)^4 - (15*a^14*b
^3 + 378*a^12*b^5 + 1893*a^10*b^7 + 3260*a^8*b^9 + 2121*a^6*b^11 + 490*a^4*
b^13 + 35*a^2*b^15)*cos(d*x + c)^2 + ((15*a^7*b^10 + 63*a^5*b^12 + 45*a^3*b
^14 + 5*a*b^16)*cos(d*x + c)^8 - 3*(105*a^9*b^8 + 456*a^7*b^10 + 378*a^5*b^
12 + 80*a^3*b^14 + 5*a*b^16)*cos(d*x + c)^6 + (525*a^11*b^6 + 2835*a^9*b^8
+ 4266*a^7*b^10 + 2254*a^5*b^12 + 345*a^3*b^14 + 15*a*b^16)*cos(d*x + c)^4
- (105*a^13*b^4 + 966*a^11*b^6 + 2835*a^9*b^8 + 2948*a^7*b^10 + 1183*a^5*b^
12 + 150*a^3*b^14 + 5*a*b^16)*cos(d*x + c)^2)*sin(d*x + c))*log(b*sin(d*x +
 c) + a) + 105*(7*(a^11*b^6 - 45*a^9*b^8 - 240*a^8*b^9 - 630*a^7*b^10 - 100
8*a^6*b^11 - 1050*a^5*b^12 - 720*a^4*b^13 - 315*a^3*b^14 - 80*a^2*b^15 - 9*
a*b^16)*cos(d*x + c)^8 - 7*(5*a^13*b^4 - 222*a^11*b^6 - 1200*a^10*b^7 - 328
5*a^9*b^8 - 5760*a^8*b^9 - 7140*a^7*b^10 - 6624*a^6*b^11 - 4725*a^5*b^12 -
2560*a^4*b^13 - 990*a^3*b^14 - 240*a^2*b^15 - 27*a*b^16)*cos(d*x + c)^6 + 7
*(3*a^15*b^2 - 125*a^13*b^4 - 720*a^12*b^5 - 2337*a^11*b^6 - 5424*a^10*b^7
- 9585*a^9*b^8 - 12960*a^8*b^9 - 13335*a^7*b^10 - 10464*a^6*b^11 - 6327*a^5
*b^12 - 2960*a^4*b^13 - 1035*a^3*b^14 - 240*a^2*b^15 - 27*a*b^16)*cos(d*x +
 c)^4 - (a^17 - 24*a^15*b^2 - 240*a^14*b^3 - 1540*a^13*b^4 - 6048*a^12*b^5
- 15848*a^11*b^6 - 30288*a^10*b^7 - 44730*a^9*b^8 - 52160*a^8*b^9 - 47784*a
^7*b^10 - 33936*a^6*b^11 - 18564*a^5*b^12 - 7840*a^4*b^13 - 2520*a^3*b^14 -
 560*a^2*b^15 - 63*a*b^16)*cos(d*x + c)^2 + ((a^10*b^7 - 45*a^8*b^9 - 240*a
^7*b^10 - 630*a^6*b^11 - 1008*a^5*b^12 - 1050*a^4*b^13 - 720*a^3*b^14 - 315
*a^2*b^15 - 80*a*b^16 - 9*b^17)*cos(d*x + c)^8 - 3*(7*a^12*b^5 - 314*a^10*b
^7 - 1680*a^9*b^8 - 4455*a^8*b^9 - 7296*a^7*b^10 - 7980*a^6*b^11 - 6048*a^5
*b^12 - 3255*a^4*b^13 - 1280*a^3*b^14 - 378*a^2*b^15 - 80*a*b^16 - 9*b^17)*
cos(d*x + c)^6 + (35*a^14*b^3 - 1533*a^12*b^5 - 8400*a^11*b^6 - 23937*a^10*
b^7 - 45360*a^9*b^8 - 63345*a^8*b^9 - 68256*a^7*b^10 - 57015*a^6*b^11 - 360
```

```
64*a^5*b^12 - 16695*a^4*b^13 - 5520*a^3*b^14 - 1323*a^2*b^15 - 240*a*b^16 -
 27*b^17)*cos(d*x + c)^4 - (7*a^16*b - 280*a^14*b^3 - 1680*a^13*b^4 - 5964*
a^12*b^5 - 15456*a^11*b^6 - 30344*a^10*b^7 - 45360*a^9*b^8 - 52230*a^8*b^9
- 47168*a^7*b^10 - 33768*a^6*b^11 - 18928*a^5*b^12 - 7980*a^4*b^13 - 2400*a
^3*b^14 - 504*a^2*b^15 - 80*a*b^16 - 9*b^17)*cos(d*x + c)^2)*sin(d*x + c))*
log(sin(d*x + c) + 1) - 105*(7*(a^11*b^6 - 45*a^9*b^8 + 240*a^8*b^9 - 630*a
^7*b^10 + 1008*a^6*b^11 - 1050*a^5*b^12 + 720*a^4*b^13 - 315*a^3*b^14 + 80*
a^2*b^15 - 9*a*b^16)*cos(d*x + c)^8 - 7*(5*a^13*b^4 - 222*a^11*b^6 + 1200*a
^10*b^7 - 3285*a^9*b^8 + 5760*a^8*b^9 - 7140*a^7*b^10 + 6624*a^6*b^11 - 472
5*a^5*b^12 + 2560*a^4*b^13 - 990*a^3*b^14 + 240*a^2*b^15 - 27*a*b^16)*cos(d
*x + c)^6 + 7*(3*a^15*b^2 - 125*a^13*b^4 + 720*a^12*b^5 - 2337*a^11*b^6 + 5
424*a^10*b^7 - 9585*a^9*b^8 + 12960*a^8*b^9 - 13335*a^7*b^10 + 10464*a^6*b^
11 - 6327*a^5*b^12 + 2960*a^4*b^13 - 1035*a^3*b^14 + 240*a^2*b^15 - 27*a*b^
16)*cos(d*x + c)^4 - (a^17 - 24*a^15*b^2 + 240*a^14*b^3 - 1540*a^13*b^4 + 6
048*a^12*b^5 - 15848*a^11*b^6 + 30288*a^10*b^7 - 44730*a^9*b^8 + 52160*a^8*
b^9 - 47784*a^7*b^10 + 33936*a^6*b^11 - 18564*a^5*b^12 + 7840*a^4*b^13 - 25
20*a^3*b^14 + 560*a^2*b^15 - 63*a*b^16)*cos(d*x + c)^2 + ((a^10*b^7 - 45*a^
8*b^9 + 240*a^7*b^10 - 630*a^6*b^11 + 1008*a^5*b^12 - 1050*a^4*b^13 + 720*a
^3*b^14 - 315*a^2*b^15 + 80*a*b^16 - 9*b^17)*cos(d*x + c)^8 - 3*(7*a^12*b^5
 - 314*a^10*b^7 + 1680*a^9*b^8 - 4455*a^8*b^9 + 7296*a^7*b^10 - 7980*a^6*b^
11 + 6048*a^5*b^12 - 3255*a^4*b^13 + 1280*a^3*b^14 - 378*a^2*b^15 + 80*a*b^
16 - 9*b^17)*cos(d*x + c)^6 + (35*a^14*b^3 - 1533*a^12*b^5 + 8400*a^11*b^6
- 23937*a^10*b^7 + 45360*a^9*b^8 - 63345*a^8*b^9 + 68256*a^7*b^10 - 57015*a
^6*b^11 + 36064*a^5*b^12 - 16695*a^4*b^13 + 5520*a^3*b^14 - 1323*a^2*b^15 +
 240*a*b^16 - 27*b^17)*cos(d*x + c)^4 - (7*a^16*b - 280*a^14*b^3 + 1680*a^1
3*b^4 - 5964*a^12*b^5 + 15456*a^11*b^6 - 30344*a^10*b^7 + 45360*a^9*b^8 - 5
2230*a^8*b^9 + 47168*a^7*b^10 - 33768*a^6*b^11 + 18928*a^5*b^12 - 7980*a^4*
b^13 + 2400*a^3*b^14 - 504*a^2*b^15 + 80*a*b^16 - 9*b^17)*cos(d*x + c)^2)*s
in(d*x + c))*log(-sin(d*x + c) + 1) - 14*(15*a^...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+b*sin(d*x+c))**8,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1327 vs. 2(508) = 1016.
time = 7.44, size = 1327, normalized size = 2.52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/420*(3360*(15*a^7*b^4 + 63*a^5*b^6 + 45*a^3*b^8 + 5*a*b^10)*log(abs(b*sin
(d*x + c) + a))/(a^18*b - 9*a^16*b^3 + 36*a^14*b^5 - 84*a^12*b^7 + 126*a^10
*b^9 - 126*a^8*b^11 + 84*a^6*b^13 - 36*a^4*b^15 + 9*a^2*b^17 - b^19) + 105*
(a - 9*b)*log(abs(sin(d*x + c) + 1))/(a^9 - 9*a^8*b + 36*a^7*b^2 - 84*a^6*b
^3 + 126*a^5*b^4 - 126*a^4*b^5 + 84*a^3*b^6 - 36*a^2*b^7 + 9*a*b^8 - b^9) -
 105*(a + 9*b)*log(abs(sin(d*x + c) - 1))/(a^9 + 9*a^8*b + 36*a^7*b^2 + 84*
a^6*b^3 + 126*a^5*b^4 + 126*a^4*b^5 + 84*a^3*b^6 + 36*a^2*b^7 + 9*a*b^8 + b
^9) + 210*(120*a^7*b^3*sin(d*x + c)^2 + 504*a^5*b^5*sin(d*x + c)^2 + 360*a^
3*b^7*sin(d*x + c)^2 + 40*a*b^9*sin(d*x + c)^2 - a^10*sin(d*x + c) - 27*a^8
*b^2*sin(d*x + c) - 42*a^6*b^4*sin(d*x + c) + 42*a^4*b^6*sin(d*x + c) + 27*
a^2*b^8*sin(d*x + c) + b^10*sin(d*x + c) + 8*a^9*b - 72*a^7*b^3 - 504*a^5*b
^5 - 408*a^3*b^7 - 48*a*b^9)/((a^18 - 9*a^16*b^2 + 36*a^14*b^4 - 84*a^12*b^
6 + 126*a^10*b^8 - 126*a^8*b^10 + 84*a^6*b^12 - 36*a^4*b^14 + 9*a^2*b^16 -
b^18)*(sin(d*x + c)^2 - 1)) - 4*(32670*a^7*b^10*sin(d*x + c)^7 + 137214*a^5
*b^12*sin(d*x + c)^7 + 98010*a^3*b^14*sin(d*x + c)^7 + 10890*a*b^16*sin(d*x
 + c)^7 + 237510*a^8*b^9*sin(d*x + c)^6 + 978138*a^6*b^11*sin(d*x + c)^6 +
670950*a^4*b^13*sin(d*x + c)^6 + 65310*a^2*b^15*sin(d*x + c)^6 - 420*b^17*s
in(d*x + c)^6 + 741930*a^9*b^8*sin(d*x + c)^5 + 2987334*a^7*b^10*sin(d*x +
c)^5 + 1959930*a^5*b^12*sin(d*x + c)^5 + 166530*a^3*b^14*sin(d*x + c)^5 - 1
260*a*b^16*sin(d*x + c)^5 + 1291675*a^10*b^7*sin(d*x + c)^4 + 5064885*a^8*b
^9*sin(d*x + c)^4 + 3165120*a^6*b^11*sin(d*x + c)^4 + 237020*a^4*b^13*sin(d
*x + c)^4 - 1155*a^2*b^15*sin(d*x + c)^4 - 105*b^17*sin(d*x + c)^4 + 135467
5*a^11*b^6*sin(d*x + c)^3 + 5144685*a^9*b^8*sin(d*x + c)^3 + 3051720*a^7*b^
10*sin(d*x + c)^3 + 207620*a^5*b^12*sin(d*x + c)^3 - 1155*a^3*b^14*sin(d*x
+ c)^3 - 105*a*b^16*sin(d*x + c)^3 + 856905*a^12*b^5*sin(d*x + c)^2 + 31265
01*a^10*b^7*sin(d*x + c)^2 + 1759590*a^8*b^9*sin(d*x + c)^2 + 113400*a^6*b^
11*sin(d*x + c)^2 - 2205*a^4*b^13*sin(d*x + c)^2 + 315*a^2*b^15*sin(d*x + c
)^2 - 42*b^17*sin(d*x + c)^2 + 303275*a^13*b^4*sin(d*x + c) + 1049727*a^11*
b^6*sin(d*x + c) + 565530*a^9*b^8*sin(d*x + c) + 33600*a^7*b^10*sin(d*x + c
) - 735*a^5*b^12*sin(d*x + c) + 105*a^3*b^14*sin(d*x + c) - 14*a*b^16*sin(d
*x + c) + 46475*a^14*b^3 + 149331*a^12*b^5 + 79845*a^10*b^7 + 2385*a^8*b^9
+ 1155*a^6*b^11 - 525*a^4*b^13 + 133*a^2*b^15 - 15*b^17)/((a^18 - 9*a^16*b^
2 + 36*a^14*b^4 - 84*a^12*b^6 + 126*a^10*b^8 - 126*a^8*b^10 + 84*a^6*b^12 -
 36*a^4*b^14 + 9*a^2*b^16 - b^18)*(b*sin(d*x + c) + a)^7))/d
```

Mupad [B]
time = 9.89, size = 1443, normalized size = 2.74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + b*sin(c + d*x))^8),x)
```

```
[Out] ((sin(c + d*x)^7*(23*a*b^14 + 1348*a^3*b^12 + 3514*a^5*b^10 + 1252*a^7*b^8
+ 7*a^9*b^6))/(2*(a^16 + b^16 - 8*a^2*b^14 + 28*a^4*b^12 - 56*a^6*b^10 + 70
```

*a^8*b^8 - 56*a^10*b^6 + 28*a^12*b^4 - 8*a^14*b^2)) - (420*a^14*b + 15*b^15
 - 118*a^2*b^13 + 407*a^4*b^11 - 748*a^6*b^9 + 8177*a^8*b^7 + 28862*a^10*b^
5 + 16745*a^12*b^3)/(105*(a^2 - b^2)*(a^14 - b^14 + 7*a^2*b^12 - 21*a^4*b^1
0 + 35*a^6*b^8 - 35*a^8*b^6 + 21*a^10*b^4 - 7*a^12*b^2)) + (sin(c + d*x)^6*
(7384*a^4*b^11 - 681*a^2*b^13 - 18*b^15 + 26194*a^6*b^9 + 10066*a^8*b^7 + 6
3*a^10*b^5))/(6*(a^16 + b^16 - 8*a^2*b^14 + 28*a^4*b^12 - 56*a^6*b^10 + 70*
a^8*b^8 - 56*a^10*b^6 + 28*a^12*b^4 - 8*a^14*b^2)) + (sin(c + d*x)^8*(9*b^1
5 + 244*a^2*b^13 + 574*a^4*b^11 + 196*a^6*b^9 + a^8*b^7))/(2*(a^16 + b^16 -
 8*a^2*b^14 + 28*a^4*b^12 - 56*a^6*b^10 + 70*a^8*b^8 - 56*a^10*b^6 + 28*a^1
2*b^4 - 8*a^14*b^2)) + (sin(c + d*x)^5*(160*a^5*b^10 - 3951*a^3*b^12 - 66*a
*b^14 + 32254*a^7*b^8 + 14506*a^9*b^6 + 105*a^11*b^4))/(6*(a^16 + b^16 - 8*
a^2*b^14 + 28*a^4*b^12 - 56*a^6*b^10 + 70*a^8*b^8 - 56*a^10*b^6 + 28*a^12*b
^4 - 8*a^14*b^2)) + (sin(c + d*x)^4*(18*b^13 + 456*a^2*b^11 + 45119*a^4*b^9
 + 143647*a^6*b^7 + 59835*a^8*b^5 + 525*a^10*b^3))/(30*(a^14 - b^14 + 7*a^2
*b^12 - 21*a^4*b^10 + 35*a^6*b^8 - 35*a^8*b^6 + 21*a^10*b^4 - 7*a^12*b^2))
- (sin(c + d*x)^2*(54*b^15 - 735*a^14*b - 310*a^2*b^13 + 3050*a^4*b^11 + 25
2845*a^6*b^9 + 919070*a^8*b^7 + 361856*a^10*b^5 - 30550*a^12*b^3))/(210*(a^
2 - b^2)*(a^14 - b^14 + 7*a^2*b^12 - 21*a^4*b^10 + 35*a^6*b^8 - 35*a^8*b^6
+ 21*a^10*b^4 - 7*a^12*b^2)) - (sin(c + d*x)^3*(26*a*b^14 + 386*a^3*b^12 +
53641*a^5*b^10 + 166336*a^7*b^8 + 20896*a^9*b^6 - 25930*a^11*b^4 - 315*a^13
b^2))/(30(a^2 - b^2)*(a^14 - b^14 + 7*a^2*b^12 - 21*a^4*b^10 + 35*a^6*b^8
 - 35*a^8*b^6 + 21*a^10*b^4 - 7*a^12*b^2)) - (a*sin(c + d*x)*(4*b^14 - 15*a
^14 - 26*a^2*b^12 + 184*a^4*b^10 + 13189*a^6*b^8 + 52264*a^8*b^6 + 26140*a^
10*b^4 + 420*a^12*b^2))/(30*(a^2 - b^2)*(a^14 - b^14 + 7*a^2*b^12 - 21*a^4*
b^10 + 35*a^6*b^8 - 35*a^8*b^6 + 21*a^10*b^4 - 7*a^12*b^2)))/(d*(sin(c + d*
x)^7*(b^7 - 21*a^2*b^5) - sin(c + d*x)^2*(a^7 - 21*a^5*b^2) + sin(c + d*x)^
4*(35*a^3*b^4 - 21*a^5*b^2) + sin(c + d*x)^5*(21*a^2*b^5 - 35*a^4*b^3) + a^
7 - b^7*sin(c + d*x)^9 - sin(c + d*x)^3*(7*a^6*b - 35*a^4*b^3) + sin(c + d*
x)^6*(7*a*b^6 - 35*a^3*b^4) - 7*a*b^6*sin(c + d*x)^8 + 7*a^6*b*sin(c + d*x)
)) - (log(sin(c + d*x) - 1)*((2*b)/(a + b)^9 + 1/(4*(a + b)^8)))/d + (log(a
 + b*sin(c + d*x))*((2*b)/(a + b)^9 + 1/(4*(a + b)^8) + (2*b)/(a - b)^9 - 1
/(4*(a - b)^8)))/d + (log(sin(c + d*x) + 1)*(a - 9*b))/(4*d*(a - b)^9)

3.467 $\int \frac{\cos^8(c+dx)}{(a+b\sin(c+dx))^8}\,dx$

Optimal. Leaf size=491

$$\frac{x}{b^8}-\frac{a(16a^6-56a^4b^2+70a^2b^4-35b^6)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{8b^8\left(a^2-b^2\right)^{7/2}d}-\frac{\cos^7(c+dx)}{7bd(a+b\sin(c+dx))^7}+\frac{a\cos^7(c+}{6b\left(a^2-b^2\right)d(a+b}$$

```
[Out] x/b^8-1/8*a*(16*a^6-56*a^4*b^2+70*a^2*b^4-35*b^6)*arctan((b+a*tan(1/2*d*x+1
/2*c))/(a^2-b^2)^(1/2))/b^8/(a^2-b^2)^(7/2)/d-1/7*cos(d*x+c)^7/b/d/(a+b*sin
(d*x+c))^7+1/6*a*cos(d*x+c)^7/b/(a^2-b^2)/d/(a+b*sin(d*x+c))^6-1/24*a*(6*a^
2-11*b^2)*cos(d*x+c)^5/b^3/(a^2-b^2)^2/d/(a+b*sin(d*x+c))^4+1/16*a*(8*a^4-2
2*a^2*b^2+19*b^4)*cos(d*x+c)^3/b^5/(a^2-b^2)^3/d/(a+b*sin(d*x+c))^2+1/30*co
s(d*x+c)^5*(6*a^2-6*b^2+5*a*b*sin(d*x+c))/b^3/(a^2-b^2)/d/(a+b*sin(d*x+c))^
5-1/24*cos(d*x+c)^3*(8*(a^2-b^2)^2+a*b*(6*a^2-11*b^2)*sin(d*x+c))/b^5/(a^2-
b^2)^2/d/(a+b*sin(d*x+c))^3+1/16*cos(d*x+c)*(16*(a^2-b^2)^3+a*b*(8*a^4-22*a
^2*b^2+19*b^4)*sin(d*x+c))/b^7/(a^2-b^2)^3/d/(a+b*sin(d*x+c))
```

Rubi [A]

time = 0.84, antiderivative size = 491, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2772, 2943, 2942, 2814, 2739, 632, 210}

$$\frac{a\cos^7(c+dx)}{6bd\left(a^2-b^2\right)(a+b\sin(c+dx))^6}-\frac{\cos^3(c+dx)\left(ab(6a^2-11b^2)\sin(c+dx)+8(a^2-b^2)^2\right)}{24b^5d\left(a^2-b^2\right)^2(a+b\sin(c+dx))^3}+\frac{\cos^5(c+dx)\left(6(a^2-b^2)+5ab\sin(c+dx)\right)}{30b^3d\left(a^2-b^2\right)(a+b\sin(c+dx))^5}-\frac{a(6a^2-11b^2)\cos^5(c+dx)}{24b^3d\left(a^2-b^2\right)^2(a+b\sin(c+dx))^4}+\frac{\cos(c+dx)\left(16(a^2-b^2)^3+ab(8a^4-22a^2b^2+19b^4)\sin(c+dx)\right)}{16b^7d\left(a^2-b^2\right)^3(a+b\sin(c+dx))}+\frac{a(8a^4-22a^2b^2+19b^4)\cos^3(c+dx)}{16b^5d\left(a^2-b^2\right)^3(a+b\sin(c+dx))^2}-\frac{a(16a^6-56a^4b^2+70a^2b^4-35b^6)\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{8b^8d\left(a^2-b^2\right)^{7/2}}-\frac{\cos^7(c+dx)}{7bd(a+b\sin(c+dx))^7}+\frac{x}{b^8}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^8/(a + b*Sin[c + d*x])^8,x]
```

```
[Out] x/b^8 - (a*(16*a^6 - 56*a^4*b^2 + 70*a^2*b^4 - 35*b^6)*ArcTan[(b + a*Tan[(c
 + d*x)/2])/Sqrt[a^2 - b^2]])/(8*b^8*(a^2 - b^2)^(7/2)*d) - Cos[c + d*x]^7/
(7*b*d*(a + b*Sin[c + d*x])^7) + (a*Cos[c + d*x]^7)/(6*b*(a^2 - b^2)*d*(a +
 b*Sin[c + d*x])^6) - (a*(6*a^2 - 11*b^2)*Cos[c + d*x]^5)/(24*b^3*(a^2 - b^
2)^2*d*(a + b*Sin[c + d*x])^4) + (a*(8*a^4 - 22*a^2*b^2 + 19*b^4)*Cos[c + d
*x]^3)/(16*b^5*(a^2 - b^2)^3*d*(a + b*Sin[c + d*x])^2) + (Cos[c + d*x]^5*(6
*(a^2 - b^2) + 5*a*b*Sin[c + d*x]))/(30*b^3*(a^2 - b^2)*d*(a + b*Sin[c + d*
x])^5) - (Cos[c + d*x]^3*(8*(a^2 - b^2)^2 + a*b*(6*a^2 - 11*b^2)*Sin[c + d*
x]))/(24*b^5*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x])^3) + (Cos[c + d*x]*(16*(a
^2 - b^2)^3 + a*b*(8*a^4 - 22*a^2*b^2 + 19*b^4)*Sin[c + d*x]))/(16*b^7*(a^2
 - b^2)^3*d*(a + b*Sin[c + d*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2772

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Dist[g^2*((p - 1)/(b*(m + 1))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2942

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*C
os[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x])/(b^2*f*(m + 1)*(m + p + 1))), x] + Dist[g^2*((
p - 1)/(b^2*(m + 1)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rule 2943

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c
 - a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^(m + 1)/(f*g*(a^2 -
 b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*
(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p
```

```
+ 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[
a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\cos^8(c+dx)}{(a+b\sin(c+dx))^8}\,dx = -\frac{\cos^7(c+dx)}{7bd(a+b\sin(c+dx))^7} - \frac{\int \frac{\cos^6(c+dx)\sin(c+dx)}{(a+b\sin(c+dx))^7}\,dx}{b}$$

$$= -\frac{\cos^7(c+dx)}{7bd(a+b\sin(c+dx))^7} + \frac{a\cos^7(c+dx)}{6b(a^2-b^2)d(a+b\sin(c+dx))^6} + \frac{\int \frac{\cos^6(c+dx)(6b+a}{(a+b\sin(c+}}{6b(a^2-}$$

$$= -\frac{\cos^7(c+dx)}{7bd(a+b\sin(c+dx))^7} + \frac{a\cos^7(c+dx)}{6b(a^2-b^2)d(a+b\sin(c+dx))^6} + \frac{\cos^5(c+dx)(6}{30b^3(a^2-b}$$

$$= -\frac{\cos^7(c+dx)}{7bd(a+b\sin(c+dx))^7} + \frac{a\cos^7(c+dx)}{6b(a^2-b^2)d(a+b\sin(c+dx))^6} - \frac{a(6a^2-11}{24b^3(a^2-b^2)^2}$$

$$= -\frac{\cos^7(c+dx)}{7bd(a+b\sin(c+dx))^7} + \frac{a\cos^7(c+dx)}{6b(a^2-b^2)d(a+b\sin(c+dx))^6} - \frac{a(6a^2-11}{24b^3(a^2-b^2)^2}$$

$$= -\frac{\cos^7(c+dx)}{7bd(a+b\sin(c+dx))^7} + \frac{a\cos^7(c+dx)}{6b(a^2-b^2)d(a+b\sin(c+dx))^6} - \frac{a(6a^2-11}{24b^3(a^2-b^2)^2}$$

$$= -\frac{\cos^7(c+dx)}{7bd(a+b\sin(c+dx))^7} + \frac{a\cos^7(c+dx)}{6b(a^2-b^2)d(a+b\sin(c+dx))^6} - \frac{a(6a^2-11}{24b^3(a^2-b^2)^2}$$

$$= \frac{x}{b^8} - \frac{\cos^7(c+dx)}{7bd(a+b\sin(c+dx))^7} + \frac{a\cos^7(c+dx)}{6b(a^2-b^2)d(a+b\sin(c+dx))^6} - \frac{a(6a^2-}{24b^3(a^2-b}$$

$$= \frac{x}{b^8} - \frac{\cos^7(c+dx)}{7bd(a+b\sin(c+dx))^7} + \frac{a\cos^7(c+dx)}{6b(a^2-b^2)d(a+b\sin(c+dx))^6} - \frac{a(6a^2-}{24b^3(a^2-b}$$

$$= \frac{x}{b^8} - \frac{\cos^7(c+dx)}{7bd(a+b\sin(c+dx))^7} + \frac{a\cos^7(c+dx)}{6b(a^2-b^2)d(a+b\sin(c+dx))^6} - \frac{a(6a^2-}{24b^3(a^2-b}$$

$$= \frac{x}{b^8} - \frac{a(16a^6-56a^4b^2+70a^2b^4-35b^6)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{8b^8(a^2-b^2)^{7/2}d} - \frac{\cos^7(c+}{7bd(a+b\sin}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 6570 vs. 2(491) = 982.
time = 8.36, size = 6570, normalized size = 13.38

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^8/(a + b*Sin[c + d*x])^8,x]
```

```
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1651 vs. $2(469) = 938$.
time = 3.68, size = 1652, normalized size = 3.36

method	result	size
derivativedivides	Expression too large to display	1652
default	Expression too large to display	1652
risch	Expression too large to display	1998

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8/(a+b*sin(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2/b^8*((-1/16*b^2*(8*a^12-22*a^10*b^2+19*a^8*b^4-16*a^6*b^6+48*a^4*b^
8-48*a^2*b^10+16*b^12)/a/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)*tan(1/2*d*x+1/2*c)^1
3-1/16*b*(16*a^14+56*a^12*b^2-238*a^10*b^4+231*a^8*b^6-96*a^6*b^8+288*a^4*b
^10-288*a^2*b^12+96*b^14)/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/a^2*tan(1/2*d*x+1/2
*c)^12-1/24/a^3*b^2*(384*a^14-276*a^12*b^2-1252*a^10*b^4+1697*a^8*b^6-384*a
^6*b^8+1344*a^4*b^10-1408*a^2*b^12+480*b^14)/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)*
tan(1/2*d*x+1/2*c)^11-1/24/a^4*b*(144*a^16+2064*a^14*b^2-4284*a^12*b^4-224*
a^10*b^6+3949*a^8*b^8+528*a^6*b^10+1392*a^4*b^12-2384*a^2*b^14+960*b^16)/(a
^6-3*a^4*b^2+3*a^2*b^4-b^6)*tan(1/2*d*x+1/2*c)^10-1/240/a^5*b^2*(17400*a^16
+34110*a^14*b^2-152515*a^12*b^4+93770*a^10*b^6+49308*a^8*b^8+30576*a^6*b^10
-12496*a^4*b^12-18048*a^2*b^14+11520*b^16)/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)*ta
n(1/2*d*x+1/2*c)^9-1/240/a^6*b*(3600*a^18+73480*a^16*b^2-70050*a^14*b^4-198
555*a^12*b^6+252090*a^10*b^8+28792*a^8*b^10+43584*a^6*b^12-48064*a^4*b^14+3
968*a^2*b^16+7680*b^18)/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)*tan(1/2*d*x+1/2*c)^8-
1/420/a^7*b^2*(58800*a^18+186200*a^16*b^2-565950*a^14*b^4+95655*a^12*b^6+44
4108*a^10*b^8+51212*a^8*b^10-11904*a^6*b^12-64640*a^4*b^14+27904*a^2*b^16+3
840*b^18)/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)*tan(1/2*d*x+1/2*c)^7-1/60/a^6*b*(12
00*a^18+26080*a^16*b^2-19080*a^14*b^4-80730*a^12*b^6+87285*a^10*b^8+21208*a
^8*b^10+5316*a^6*b^12-12016*a^4*b^14+992*a^2*b^16+1920*b^18)/(a^6-3*a^4*b^2
+3*a^2*b^4-b^6)*tan(1/2*d*x+1/2*c)^6-1/240/a^5*b^2*(33000*a^16+91890*a^14*b
^2-318025*a^12*b^4+107140*a^10*b^6+211848*a^8*b^8-10304*a^6*b^10-12496*a^4*
b^12-18048*a^2*b^14+11520*b^16)/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)*tan(1/2*d*x+1
/2*c)^5-1/240/a^4*b*(3600*a^16+65880*a^14*b^2-95382*a^12*b^4-122429*a^10*b^
6+240884*a^8*b^8-41568*a^6*b^10+16880*a^4*b^12-23840*a^2*b^14+9600*b^16)/(a
^6-3*a^4*b^2+3*a^2*b^4-b^6)*tan(1/2*d*x+1/2*c)^4-1/120/a^3*b^2*(8160*a^14+6
420*a^12*b^2-62984*a^10*b^4+71177*a^8*b^6-15192*a^6*b^8+7784*a^4*b^10-7040*
a^2*b^12+2400*b^14)/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)*tan(1/2*d*x+1/2*c)^3-1/12
```

```
0*b*(720*a^14+7280*a^12*b^2-24612*a^10*b^4+23652*a^8*b^6-5659*a^6*b^8+3200*
a^4*b^10-2376*a^2*b^12+720*b^14)/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/a^2*tan(1/2*
d*x+1/2*c)^2-1/240*b^2*(3240*a^12-9190*a^10*b^2+8367*a^8*b^4-2046*a^6*b^6+1
196*a^4*b^8-832*a^2*b^10+240*b^12)/a/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)*tan(1/2*
d*x+1/2*c)-1/1680*(1680*a^12-4760*a^10*b^2+4326*a^8*b^4-1143*a^6*b^6+958*a^
4*b^8-776*a^2*b^10+240*b^12)*b/(a^6-3*a^4*b^2+3*a^2*b^4-b^6))/(a*tan(1/2*d*
x+1/2*c)^2+2*b*tan(1/2*d*x+1/2*c)+a)^7+1/16*a*(16*a^6-56*a^4*b^2+70*a^2*b^4
-35*b^6)/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(
1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2)))+2/b^8*arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8/(a+b*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1818 vs. $2(469) = 938$.
time = 0.73, size = 3721, normalized size = 7.58

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8/(a+b*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] [1/3360*(23520*(a^9*b^6 - 4*a^7*b^8 + 6*a^5*b^10 - 4*a^3*b^12 + a*b^14)*d*x
*cos(d*x + c)^6 + 2*(4356*a^8*b^7 - 16864*a^6*b^9 + 24001*a^4*b^11 - 14309*
a^2*b^13 + 2816*b^15)*cos(d*x + c)^7 - 23520*(5*a^11*b^4 - 17*a^9*b^6 + 18*
a^7*b^8 - 2*a^5*b^10 - 7*a^3*b^12 + 3*a*b^14)*d*x*cos(d*x + c)^4 - 28*(2754
*a^10*b^5 - 9717*a^8*b^7 + 11528*a^6*b^9 - 3782*a^4*b^11 - 1247*a^2*b^13 +
464*b^15)*cos(d*x + c)^5 + 23520*(3*a^13*b^2 - 2*a^11*b^4 - 19*a^9*b^6 + 36
*a^7*b^8 - 19*a^5*b^10 - 2*a^3*b^12 + 3*a*b^14)*d*x*cos(d*x + c)^2 + 70*(85
6*a^12*b^3 - 1090*a^10*b^5 - 3477*a^8*b^7 + 7907*a^6*b^9 - 4423*a^4*b^11 +
67*a^2*b^13 + 160*b^15)*cos(d*x + c)^3 - 3360*(a^15 + 17*a^13*b^2 - 43*a^11
*b^4 - 11*a^9*b^6 + 99*a^7*b^8 - 77*a^5*b^10 + 7*a^3*b^12 + 7*a*b^14)*d*x +
 105*(16*a^14 + 280*a^12*b^2 - 546*a^10*b^4 - 413*a^8*b^6 + 1323*a^6*b^8 -
735*a^4*b^10 - 245*a^2*b^12 - 7*(16*a^8*b^6 - 56*a^6*b^8 + 70*a^4*b^10 - 35
*a^2*b^12)*cos(d*x + c)^6 + 7*(80*a^10*b^4 - 232*a^8*b^6 + 182*a^6*b^8 + 35
*a^4*b^10 - 105*a^2*b^12)*cos(d*x + c)^4 - 7*(48*a^12*b^2 - 8*a^10*b^4 - 30
```

2*a^8*b^6 + 427*a^6*b^8 - 140*a^4*b^10 - 105*a^2*b^12)*cos(d*x + c)^2 + (11
2*a^13*b + 168*a^11*b^3 - 1134*a^9*b^5 + 1045*a^7*b^7 + 189*a^5*b^9 - 665*a
^3*b^11 - 35*a*b^13 - (16*a^7*b^7 - 56*a^5*b^9 + 70*a^3*b^11 - 35*a*b^13)*c
os(d*x + c)^6 + 3*(112*a^9*b^5 - 376*a^7*b^7 + 434*a^5*b^9 - 175*a^3*b^11 -
 35*a*b^13)*cos(d*x + c)^4 - (560*a^11*b^3 - 1288*a^9*b^5 + 146*a^7*b^7 + 1
547*a^5*b^9 - 1260*a^3*b^11 - 105*a*b^13)*cos(d*x + c)^2)*sin(d*x + c))*sqr
t(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2
 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))
/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 420*(8*a^14*b + 1
12*a^12*b^3 - 322*a^10*b^5 + 63*a^8*b^7 + 479*a^6*b^9 - 379*a^4*b^11 + 31*a
^2*b^13 + 8*b^15)*cos(d*x + c) + 14*(240*(a^8*b^7 - 4*a^6*b^9 + 6*a^4*b^11
 - 4*a^2*b^13 + b^15)*d*x*cos(d*x + c)^6 - 720*(7*a^10*b^5 - 27*a^8*b^7 + 38
*a^6*b^9 - 22*a^4*b^11 + 3*a^2*b^13 + b^15)*d*x*cos(d*x + c)^4 - (2676*a^9*
b^6 - 10264*a^7*b^8 + 14371*a^5*b^10 - 8204*a^3*b^12 + 1421*a*b^14)*cos(d*x
 + c)^5 + 240*(35*a^12*b^3 - 98*a^10*b^5 + 45*a^8*b^7 + 100*a^6*b^9 - 115*a
^4*b^11 + 30*a^2*b^13 + 3*b^15)*d*x*cos(d*x + c)^2 + 10*(638*a^11*b^4 - 192
5*a^9*b^6 + 1427*a^7*b^8 + 861*a^5*b^10 - 1253*a^3*b^12 + 252*a*b^14)*cos(d
x + c)^3 - 240(7*a^14*b + 7*a^12*b^3 - 77*a^10*b^5 + 99*a^8*b^7 - 11*a^6*
b^9 - 43*a^4*b^11 + 17*a^2*b^13 + b^15)*d*x - 15*(104*a^13*b^2 + 26*a^11*b^
4 - 897*a^9*b^6 + 1306*a^7*b^8 - 308*a^5*b^10 - 308*a^3*b^12 + 77*a*b^14)*c
os(d*x + c))*sin(d*x + c))/(7*(a^9*b^14 - 4*a^7*b^16 + 6*a^5*b^18 - 4*a^3*b
^20 + a*b^22)*d*cos(d*x + c)^6 - 7*(5*a^11*b^12 - 17*a^9*b^14 + 18*a^7*b^16
 - 2*a^5*b^18 - 7*a^3*b^20 + 3*a*b^22)*d*cos(d*x + c)^4 + 7*(3*a^13*b^10 -
2*a^11*b^12 - 19*a^9*b^14 + 36*a^7*b^16 - 19*a^5*b^18 - 2*a^3*b^20 + 3*a*b^
22)*d*cos(d*x + c)^2 - (a^15*b^8 + 17*a^13*b^10 - 43*a^11*b^12 - 11*a^9*b^1
4 + 99*a^7*b^16 - 77*a^5*b^18 + 7*a^3*b^20 + 7*a*b^22)*d + ((a^8*b^15 - 4*a
^6*b^17 + 6*a^4*b^19 - 4*a^2*b^21 + b^23)*d*cos(d*x + c)^6 - 3*(7*a^10*b^13
 - 27*a^8*b^15 + 38*a^6*b^17 - 22*a^4*b^19 + 3*a^2*b^21 + b^23)*d*cos(d*x +
 c)^4 + (35*a^12*b^11 - 98*a^10*b^13 + 45*a^8*b^15 + 100*a^6*b^17 - 115*a^4
*b^19 + 30*a^2*b^21 + 3*b^23)*d*cos(d*x + c)^2 - (7*a^14*b^9 + 7*a^12*b^11
 - 77*a^10*b^13 + 99*a^8*b^15 - 11*a^6*b^17 - 43*a^4*b^19 + 17*a^2*b^21 + b^
23)*d)*sin(d*x + c)), 1/1680*(11760*(a^9*b^6 - 4*a^7*b^8 + 6*a^5*b^10 - 4*a
^3*b^12 + a*b^14)*d*x*cos(d*x + c)^6 + (4356*a^8*b^7 - 16864*a^6*b^9 + 2400
1*a^4*b^11 - 14309*a^2*b^13 + 2816*b^15)*cos(d*x + c)^7 - 11760*(5*a^11*b^4
 - 17*a^9*b^6 + 18*a^7*b^8 - 2*a^5*b^10 - 7*a^3*b^12 + 3*a*b^14)*d*x*cos(d*
x + c)^4 - 14*(2754*a^10*b^5 - 9717*a^8*b^7 + 11528*a^6*b^9 - 3782*a^4*b^11
 - 1247*a^2*b^13 + 464*b^15)*cos(d*x + c)^5 + 11760*(3*a^13*b^2 - 2*a^11*b^
4 - 19*a^9*b^6 + 36*a^7*b^8 - 19*a^5*b^10 - 2*a^3*b^12 + 3*a*b^14)*d*x*cos(
d*x + c)^2 + 35*(856*a^12*b^3 - 1090*a^10*b^5 - 3477*a^8*b^7 + 7907*a^6*b^9
 - 4423*a^4*b^11 + 67*a^2*b^13 + 160*b^15)*cos(d*x + c)^3 - 1680*(a^15 + 17
*a^13*b^2 - 43*a^11*b^4 - 11*a^9*b^6 + 99*a^7*b^8 - 77*a^5*b^10 + 7*a^3*b^1
2 + 7*a*b^14)*d*x - 105*(16*a^14 + 280*a^12*b^2 - 546*a^10*b^4 - 413*a^8*b^
6 + 1323*a^6*b^8 - 735*a^4*b^10 - 245*a^2*b^12 - 7*(16*a^8*b^6 - 56*a^6*b^8
 + 70*a^4*b^10 - 35*a^2*b^12)*cos(d*x + c)^6 + 7*(80*a^10*b^4 - 232*a^8*b^6
 + 182*a^6*b^8 + 35*a^4*b^10 - 105*a^2*b^12)*cos(d*x + c)^4 - 7*(48*a^12*b^

```
2 - 8*a^10*b^4 - 302*a^8*b^6 + 427*a^6*b^8 - 140*a^4*b^10 - 105*a^2*b^12)*c
os(d*x + c)^2 + (112*a^13*b + 168*a^11*b^3 - 1134*a^9*b^5 + 1045*a^7*b^7 +
189*a^5*b^9 - 665*a^3*b^11 - 35*a*b^13 - (16*a^7*b^7 - 56*a^5*b^9 + 70*a^3*
b^11 - 35*a*b^13)*cos(d*x + c)^6 + 3*(112*a^9*b^5 - 376*a^7*b^7 + 434*a^5*b
^9 - 175*a^3*b^11 - 35*a*b^13)*cos(d*x + c)^4 - (560*a^11*b^3 - 1288*a^9*b^
5 + 146*a^7*b^7 + 1547*a^5*b^9 - 1260*a^3*b^11 - 105*a*b^13)*cos(d*x + c)^2
)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^
2)*cos(d*x + c))) - 210*(8*a^14*b + 112*a^12*b^...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8/(a+b*sin(d*x+c))**8,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2326 vs. 2(469) = 938.
time = 4.26, size = 2326, normalized size = 4.74

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8/(a+b*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -1/840*(105*(16*a^7 - 56*a^5*b^2 + 70*a^3*b^4 - 35*a*b^6)*(pi*floor(1/2*(d*
x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^
2)))/((a^6*b^8 - 3*a^4*b^10 + 3*a^2*b^12 - b^14)*sqrt(a^2 - b^2)) - (840*a^
18*b*tan(1/2*d*x + 1/2*c)^13 - 2310*a^16*b^3*tan(1/2*d*x + 1/2*c)^13 + 1995
*a^14*b^5*tan(1/2*d*x + 1/2*c)^13 - 1680*a^12*b^7*tan(1/2*d*x + 1/2*c)^13 +
 5040*a^10*b^9*tan(1/2*d*x + 1/2*c)^13 - 5040*a^8*b^11*tan(1/2*d*x + 1/2*c)
^13 + 1680*a^6*b^13*tan(1/2*d*x + 1/2*c)^13 + 1680*a^19*tan(1/2*d*x + 1/2*c
)^12 + 5880*a^17*b^2*tan(1/2*d*x + 1/2*c)^12 - 24990*a^15*b^4*tan(1/2*d*x +
 1/2*c)^12 + 24255*a^13*b^6*tan(1/2*d*x + 1/2*c)^12 - 10080*a^11*b^8*tan(1/
2*d*x + 1/2*c)^12 + 30240*a^9*b^10*tan(1/2*d*x + 1/2*c)^12 - 30240*a^7*b^12
*tan(1/2*d*x + 1/2*c)^12 + 10080*a^5*b^14*tan(1/2*d*x + 1/2*c)^12 + 26880*a
^18*b*tan(1/2*d*x + 1/2*c)^11 - 19320*a^16*b^3*tan(1/2*d*x + 1/2*c)^11 - 87
640*a^14*b^5*tan(1/2*d*x + 1/2*c)^11 + 118790*a^12*b^7*tan(1/2*d*x + 1/2*c)
^11 - 26880*a^10*b^9*tan(1/2*d*x + 1/2*c)^11 + 94080*a^8*b^11*tan(1/2*d*x +
 1/2*c)^11 - 98560*a^6*b^13*tan(1/2*d*x + 1/2*c)^11 + 33600*a^4*b^15*tan(1/
2*d*x + 1/2*c)^11 + 10080*a^19*tan(1/2*d*x + 1/2*c)^10 + 144480*a^17*b^2*ta
n(1/2*d*x + 1/2*c)^10 - 299880*a^15*b^4*tan(1/2*d*x + 1/2*c)^10 - 15680*a^1
3*b^6*tan(1/2*d*x + 1/2*c)^10 + 276430*a^11*b^8*tan(1/2*d*x + 1/2*c)^10 + 3
```

6960*a^9*b^10*tan(1/2*d*x + 1/2*c)^10 + 97440*a^7*b^12*tan(1/2*d*x + 1/2*c)
^10 - 166880*a^5*b^14*tan(1/2*d*x + 1/2*c)^10 + 67200*a^3*b^16*tan(1/2*d*x
+ 1/2*c)^10 + 121800*a^18*b*tan(1/2*d*x + 1/2*c)^9 + 238770*a^16*b^3*tan(1/
2*d*x + 1/2*c)^9 - 1067605*a^14*b^5*tan(1/2*d*x + 1/2*c)^9 + 656390*a^12*b^
7*tan(1/2*d*x + 1/2*c)^9 + 345156*a^10*b^9*tan(1/2*d*x + 1/2*c)^9 + 214032*
a^8*b^11*tan(1/2*d*x + 1/2*c)^9 - 87472*a^6*b^13*tan(1/2*d*x + 1/2*c)^9 - 1
26336*a^4*b^15*tan(1/2*d*x + 1/2*c)^9 + 80640*a^2*b^17*tan(1/2*d*x + 1/2*c)
^9 + 25200*a^19*tan(1/2*d*x + 1/2*c)^8 + 514360*a^17*b^2*tan(1/2*d*x + 1/2*
c)^8 - 490350*a^15*b^4*tan(1/2*d*x + 1/2*c)^8 - 1389885*a^13*b^6*tan(1/2*d*
x + 1/2*c)^8 + 1764630*a^11*b^8*tan(1/2*d*x + 1/2*c)^8 + 201544*a^9*b^10*ta
n(1/2*d*x + 1/2*c)^8 + 305088*a^7*b^12*tan(1/2*d*x + 1/2*c)^8 - 336448*a^5*
b^14*tan(1/2*d*x + 1/2*c)^8 + 27776*a^3*b^16*tan(1/2*d*x + 1/2*c)^8 + 53760
*a*b^18*tan(1/2*d*x + 1/2*c)^8 + 235200*a^18*b*tan(1/2*d*x + 1/2*c)^7 + 744
800*a^16*b^3*tan(1/2*d*x + 1/2*c)^7 - 2263800*a^14*b^5*tan(1/2*d*x + 1/2*c)
^7 + 382620*a^12*b^7*tan(1/2*d*x + 1/2*c)^7 + 1776432*a^10*b^9*tan(1/2*d*x
+ 1/2*c)^7 + 204848*a^8*b^11*tan(1/2*d*x + 1/2*c)^7 - 47616*a^6*b^13*tan(1/
2*d*x + 1/2*c)^7 - 258560*a^4*b^15*tan(1/2*d*x + 1/2*c)^7 + 111616*a^2*b^17
*tan(1/2*d*x + 1/2*c)^7 + 15360*b^19*tan(1/2*d*x + 1/2*c)^7 + 33600*a^19*ta
n(1/2*d*x + 1/2*c)^6 + 730240*a^17*b^2*tan(1/2*d*x + 1/2*c)^6 - 534240*a^15
*b^4*tan(1/2*d*x + 1/2*c)^6 - 2260440*a^13*b^6*tan(1/2*d*x + 1/2*c)^6 + 244
3980*a^11*b^8*tan(1/2*d*x + 1/2*c)^6 + 593824*a^9*b^10*tan(1/2*d*x + 1/2*c)
^6 + 148848*a^7*b^12*tan(1/2*d*x + 1/2*c)^6 - 336448*a^5*b^14*tan(1/2*d*x +
1/2*c)^6 + 27776*a^3*b^16*tan(1/2*d*x + 1/2*c)^6 + 53760*a*b^18*tan(1/2*d*
x + 1/2*c)^6 + 231000*a^18*b*tan(1/2*d*x + 1/2*c)^5 + 643230*a^16*b^3*tan(1
/2*d*x + 1/2*c)^5 - 2226175*a^14*b^5*tan(1/2*d*x + 1/2*c)^5 + 749980*a^12*b
^7*tan(1/2*d*x + 1/2*c)^5 + 1482936*a^10*b^9*tan(1/2*d*x + 1/2*c)^5 - 72128
*a^8*b^11*tan(1/2*d*x + 1/2*c)^5 - 87472*a^6*b^13*tan(1/2*d*x + 1/2*c)^5 -
126336*a^4*b^15*tan(1/2*d*x + 1/2*c)^5 + 80640*a^2*b^17*tan(1/2*d*x + 1/2*c
)^5 + 25200*a^19*tan(1/2*d*x + 1/2*c)^4 + 461160*a^17*b^2*tan(1/2*d*x + 1/2
*c)^4 - 667674*a^15*b^4*tan(1/2*d*x + 1/2*c)^4 - 857003*a^13*b^6*tan(1/2*d*
x + 1/2*c)^4 + 1686188*a^11*b^8*tan(1/2*d*x + 1/2*c)^4 - 290976*a^9*b^10*ta
n(1/2*d*x + 1/2*c)^4 + 118160*a^7*b^12*tan(1/2*d*x + 1/2*c)^4 - 166880*a^5*
b^14*tan(1/2*d*x + 1/2*c)^4 + 67200*a^3*b^16*tan(1/2*d*x + 1/2*c)^4 + 11424
0*a^18*b*tan(1/2*d*x + 1/2*c)^3 + 89880*a^16*b^3*tan(1/2*d*x + 1/2*c)^3 - 8
81776*a^14*b^5*tan(1/2*d*x + 1/2*c)^3 + 996478*a^12*b^7*tan(1/2*d*x + 1/2*c
)^3 - 212688*a^10*b^9*tan(1/2*d*x + 1/2*c)^3 + 108976*a^8*b^11*tan(1/2*d*x
+ 1/2*c)^3 - 98560*a^6*b^13*tan(1/2*d*x + 1/2*c)^3 + 33600*a^4*b^15*tan(1/2
*d*x + 1/2*c)^3 + 10080*a^19*tan(1/2*d*x + 1/2*c)^2 + 101920*a^17*b^2*tan(1
/2*d*x + 1/2*c)^2 - 344568*a^15*b^4*tan(1/2*d*x + 1/2*c)^2 + 331128*a^13*b^
6*tan(1/2*d*x + 1/2*c)^2 - 79226*a^11*b^8*tan(1/2*d*x + 1/2*c)^2 + 44800*a^
9*b^10*tan(1/2*d*x + 1/2*c)^2 - 33264*a^7*b^12*tan(1/2*d*x + 1/2*c)^2 + 100
80*a^5*b^14*tan(1/2*d*x + 1/2*c)^2 + 22680*a^18*b*tan(1/2*d*x + 1/2*c) - 64
330*a^16*b^3*tan(1/2*d*x + 1/2*c) + 58569*a^14*b^5*tan(1/2*d*x + 1/2*c) - 1
4322*a^12*b^7*tan(1/2*d*x + 1/2*c) + 8372*a^10*b^9*tan(1/2*d*x + 1/2*c) - 5
824*a^8*b^11*tan(1/2*d*x + 1/2*c) + 1680*a^6*b^13*tan(1/2*d*x + 1/2*c) + 16

```
80*a^19 - 4760*a^17*b^2 + 4326*a^15*b^4 - 1143*a^13*b^6 + 958*a^11*b^8 - 77
6*a^9*b^10 + 240*a^7*b^12)/((a^13*b^7 - 3*a^11*b^9 + 3*a^9*b^11 - a^7*b^13)
*(a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)^7) - 840*(d*x +
c)/b^8)/d
```

Mupad [B]
time = 32.22, size = 2500, normalized size = 5.09

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^8/(a + b*sin(c + d*x))^8,x)
```

```
[Out] (2*atan((((((((32*a^2*b^35 - 192*a^4*b^33 + 480*a^6*b^31 - 640*a^8*b^29 + 4
80*a^10*b^27 - 192*a^12*b^25 + 32*a^14*b^23)*1i)/(b^32 - 6*a^2*b^30 + 15*a^
4*b^28 - 20*a^6*b^26 + 15*a^8*b^24 - 6*a^10*b^22 + a^12*b^20) + (tan(c/2 +
(d*x)/2)*(768*a*b^37 - 5120*a^3*b^35 + 14592*a^5*b^33 - 23040*a^7*b^31 + 21
760*a^9*b^29 - 12288*a^11*b^27 + 3840*a^13*b^25 - 512*a^15*b^23)*1i)/(8*(b^
33 - 6*a^2*b^31 + 15*a^4*b^29 - 20*a^6*b^27 + 15*a^8*b^25 - 6*a^10*b^23 + a
^12*b^21)))*1i)/b^8 + ((32*a*b^28 - 154*a^3*b^26 + 322*a^5*b^24 - 378*a^7*b
^22 + 262*a^9*b^20 - 100*a^11*b^18 + 16*a^13*b^16)*1i)/(b^32 - 6*a^2*b^30 +
 15*a^4*b^28 - 20*a^6*b^26 + 15*a^8*b^24 - 6*a^10*b^22 + a^12*b^20) + (tan(
c/2 + (d*x)/2)*(1120*a^2*b^28 - 5600*a^4*b^26 + 11872*a^6*b^24 - 13728*a^8*
b^22 + 9152*a^10*b^20 - 3328*a^12*b^18 + 512*a^14*b^16)*1i)/(8*(b^33 - 6*a^
2*b^31 + 15*a^4*b^29 - 20*a^6*b^27 + 15*a^8*b^25 - 6*a^10*b^23 + a^12*b^21)
))/b^8 + (32*a^2*b^19 - 192*a^4*b^17 + 480*a^6*b^15 - 640*a^8*b^13 + 480*a^
10*b^11 - 192*a^12*b^9 + 32*a^14*b^7)/(b^32 - 6*a^2*b^30 + 15*a^4*b^28 - 20
*a^6*b^26 + 15*a^8*b^24 - 6*a^10*b^22 + a^12*b^20) + (tan(c/2 + (d*x)/2)*(5
12*a*b^21 - 4553*a^3*b^19 + 14116*a^5*b^17 - 22900*a^7*b^15 + 21760*a^9*b^1
3 - 12288*a^11*b^11 + 3840*a^13*b^9 - 512*a^15*b^7))/(8*(b^33 - 6*a^2*b^31
+ 15*a^4*b^29 - 20*a^6*b^27 + 15*a^8*b^25 - 6*a^10*b^23 + a^12*b^21)))/b^8
+ ((32*a^2*b^19 - 192*a^4*b^17 + 480*a^6*b^15 - 640*a^8*b^13 + 480*a^10*b^1
1 - 192*a^12*b^9 + 32*a^14*b^7)/(b^32 - 6*a^2*b^30 + 15*a^4*b^28 - 20*a^6*b
^26 + 15*a^8*b^24 - 6*a^10*b^22 + a^12*b^20) - (((32*a*b^28 - 154*a^3*b^26
+ 322*a^5*b^24 - 378*a^7*b^22 + 262*a^9*b^20 - 100*a^11*b^18 + 16*a^13*b^16
)*1i)/(b^32 - 6*a^2*b^30 + 15*a^4*b^28 - 20*a^6*b^26 + 15*a^8*b^24 - 6*a^10
*b^22 + a^12*b^20) - ((((32*a^2*b^35 - 192*a^4*b^33 + 480*a^6*b^31 - 640*a^
8*b^29 + 480*a^10*b^27 - 192*a^12*b^25 + 32*a^14*b^23)*1i)/(b^32 - 6*a^2*b^
30 + 15*a^4*b^28 - 20*a^6*b^26 + 15*a^8*b^24 - 6*a^10*b^22 + a^12*b^20) + (
tan(c/2 + (d*x)/2)*(768*a*b^37 - 5120*a^3*b^35 + 14592*a^5*b^33 - 23040*a^7
*b^31 + 21760*a^9*b^29 - 12288*a^11*b^27 + 3840*a^13*b^25 - 512*a^15*b^23)*
1i)/(8*(b^33 - 6*a^2*b^31 + 15*a^4*b^29 - 20*a^6*b^27 + 15*a^8*b^25 - 6*a^1
0*b^23 + a^12*b^21)))*1i)/b^8 + (tan(c/2 + (d*x)/2)*(1120*a^2*b^28 - 5600*a
^4*b^26 + 11872*a^6*b^24 - 13728*a^8*b^22 + 9152*a^10*b^20 - 3328*a^12*b^18
 + 512*a^14*b^16)*1i)/(8*(b^33 - 6*a^2*b^31 + 15*a^4*b^29 - 20*a^6*b^27 + 1
```

5*a^8*b^25 - 6*a^10*b^23 + a^12*b^21)))/b^8 + (tan(c/2 + (d*x)/2)*(512*a*b^
21 - 4553*a^3*b^19 + 14116*a^5*b^17 - 22900*a^7*b^15 + 21760*a^9*b^13 - 122
88*a^11*b^11 + 3840*a^13*b^9 - 512*a^15*b^7))/(8*(b^33 - 6*a^2*b^31 + 15*a^
4*b^29 - 20*a^6*b^27 + 15*a^8*b^25 - 6*a^10*b^23 + a^12*b^21)))/b^8)/((32*a
^13 - (665*a^3*b^10)/4 + 525*a^5*b^8 - 721*a^7*b^6 + 524*a^9*b^4 - 200*a^11
*b^2)/(b^32 - 6*a^2*b^30 + 15*a^4*b^28 - 20*a^6*b^26 + 15*a^8*b^24 - 6*a^10
*b^22 + a^12*b^20) - (((32*a^2*b^19 - 192*a^4*b^17 + 480*a^6*b^15 - 640*a^8
*b^13 + 480*a^10*b^11 - 192*a^12*b^9 + 32*a^14*b^7)*1i)/(b^32 - 6*a^2*b^30
+ 15*a^4*b^28 - 20*a^6*b^26 + 15*a^8*b^24 - 6*a^10*b^22 + a^12*b^20) - ((((
32*a*b^28 - 154*a^3*b^26 + 322*a^5*b^24 - 378*a^7*b^22 + 262*a^9*b^20 - 100
*a^11*b^18 + 16*a^13*b^16)*1i)/(b^32 - 6*a^2*b^30 + 15*a^4*b^28 - 20*a^6*b^
26 + 15*a^8*b^24 - 6*a^10*b^22 + a^12*b^20) - ((((32*a^2*b^35 - 192*a^4*b^3
3 + 480*a^6*b^31 - 640*a^8*b^29 + 480*a^10*b^27 - 192*a^12*b^25 + 32*a^14*b
^23)*1i)/(b^32 - 6*a^2*b^30 + 15*a^4*b^28 - 20*a^6*b^26 + 15*a^8*b^24 - 6*a
^10*b^22 + a^12*b^20) + (tan(c/2 + (d*x)/2)*(768*a*b^37 - 5120*a^3*b^35 + 1
4592*a^5*b^33 - 23040*a^7*b^31 + 21760*a^9*b^29 - 12288*a^11*b^27 + 3840*a^
13*b^25 - 512*a^15*b^23)*1i)/(8*(b^33 - 6*a^2*b^31 + 15*a^4*b^29 - 20*a^6*b
^27 + 15*a^8*b^25 - 6*a^10*b^23 + a^12*b^21)))*1i)/b^8 + (tan(c/2 + (d*x)/2
)*(1120*a^2*b^28 - 5600*a^4*b^26 + 11872*a^6*b^24 - 13728*a^8*b^22 + 9152*a
^10*b^20 - 3328*a^12*b^18 + 512*a^14*b^16)*1i)/(8*(b^33 - 6*a^2*b^31 + 15*a
^4*b^29 - 20*a^6*b^27 + 15*a^8*b^25 - 6*a^10*b^23 + a^12*b^21)))*1i)/b^8 +
(tan(c/2 + (d*x)/2)*(512*a*b^21 - 4553*a^3*b^19 + 14116*a^5*b^17 - 22900*a^
7*b^15 + 21760*a^9*b^13 - 12288*a^11*b^11 + 3840*a^13*b^9 - 512*a^15*b^7)*1
i)/(8*(b^33 - 6*a^2*b^31 + 15*a^4*b^29 - 20*a^6*b^27 + 15*a^8*b^25 - 6*a^10
*b^23 + a^12*b^21)))/b^8 + (((((((32*a^2*b^35 - 192*a^4*b^33 + 480*a^6*b^31
 - 640*a^8*b^29 + 480*a^10*b^27 - 192*a^12*b^25 + 32*a^14*b^23)*1i)/(b^32 -
 6*a^2*b^30 + 15*a^4*b^28 - 20*a^6*b^26 + 15*a^8*b^24 - 6*a^10*b^22 + a^12*
b^20) + (tan(c/2 + (d*x)/2)*(768*a*b^37 - 5120*a^3*b^35 + 14592*a^5*b^33 -
23040*a^7*b^31 + 21760*a^9*b^29 - 12288*a^11*b^27 + 3840*a^13*b^25 - 512*a^
15*b^23)*1i)/(8*(b^33 - 6*a^2*b^31 + 15*a^4*b^29 - 20*a^6*b^27 + 15*a^8*b^2
5 - 6*a^10*b^23 + a^12*b^21)))*1i)/b^8 + ((32*a*b^28 - 154*a^3*b^26 + 322*a
^5*b^24 - 378*a^7*b^22 + 262*a^9*b^20 - 100*a^11*b^18 + 16*a^13*b^16)*1i)/(
b^32 - 6*a^2*b^30 + 15*a^4*b^28 - 20*a^6*b^26 + 15*a^8*b^24 - 6*a^10*b^22 +
 a^12*b^20) + (tan(c/2 + (d*x)/2)*(1120*a^2*b^28 - 5600*a^4*b^26 + 11872*a^
6*b^24 - 13728*a^8*b^22 + 9152*a^10*b^20 - 3328...

3.468 $\int \frac{\cos^6(c+dx)}{(a+b\sin(c+dx))^8}\,dx$

Optimal. Leaf size=407

$$\frac{5a\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{8\left(a^2-b^2\right)^{9/2}d}-\frac{\cos^5(c+dx)}{7bd(a+b\sin(c+dx))^7}+\frac{a(4a^2-b^2)\cos(c+dx)}{168b^5\left(a^2-b^2\right)d(a+b\sin(c+dx))^4}+\frac{(4a^4-9a^2b^2}{168b^5\left(a^2-b^2\right)^2}$$

```
[Out] 5/8*a*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/(a^2-b^2)^(9/2)/d-1/
7*cos(d*x+c)^5/b/d/(a+b*sin(d*x+c))^7+1/168*a*(4*a^2-b^2)*cos(d*x+c)/b^5/(a
^2-b^2)/d/(a+b*sin(d*x+c))^4+1/168*(4*a^4-9*a^2*b^2+12*b^4)*cos(d*x+c)/b^5/
(a^2-b^2)^2/d/(a+b*sin(d*x+c))^3+1/336*a*(8*a^4-30*a^2*b^2+57*b^4)*cos(d*x+
c)/b^5/(a^2-b^2)^3/d/(a+b*sin(d*x+c))^2+1/336*(8*a^6-38*a^4*b^2+87*a^2*b^4+
48*b^6)*cos(d*x+c)/b^5/(a^2-b^2)^4/d/(a+b*sin(d*x+c))+5/42*cos(d*x+c)^3*(2*
a+3*b*sin(d*x+c))/b^3/d/(a+b*sin(d*x+c))^6-1/42*cos(d*x+c)*(4*a^2+9*b^2+10*
a*b*sin(d*x+c))/b^5/d/(a+b*sin(d*x+c))^5
```

Rubi [A]
time = 0.51, antiderivative size = 407, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2772, 2942, 2833, 12, 2739, 632, 210}

$$\frac{5a\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{8d\left(a^2-b^2\right)^{9/2}}-\frac{\cos(c+dx)\left(4a^2+10ab\sin(c+dx)+9b^2\right)}{42b^5d(a+b\sin(c+dx))^5}+\frac{a(4a^2-b^2)\cos(c+dx)}{168b^5d\left(a^2-b^2\right)(a+b\sin(c+dx))^4}+\frac{a(8a^4-30a^2b^2+57b^4)\cos(c+dx)}{336b^5d\left(a^2-b^2\right)^3(a+b\sin(c+dx))^2}+\frac{(4a^4-9a^2b^2+12b^4)\cos(c+dx)}{168b^5d\left(a^2-b^2\right)^2(a+b\sin(c+dx))^3}+\frac{(8a^6-38a^4b^2+87a^2b^4+48b^6)\cos(c+dx)}{336b^5d\left(a^2-b^2\right)^4(a+b\sin(c+dx))}+\frac{5\cos^3(c+dx)(2a+3b\sin(c+dx))}{42b^3d(a+b\sin(c+dx))^6}-\frac{\cos^5(c+dx)}{7bd(a+b\sin(c+dx))^7}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6/(a + b*Sin[c + d*x])^8,x]
```

```
[Out] (5*a*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(8*(a^2 - b^2)^(9/2)
*d) - Cos[c + d*x]^5/(7*b*d*(a + b*Sin[c + d*x])^7) + (a*(4*a^2 - b^2)*Cos[
c + d*x])/(168*b^5*(a^2 - b^2)*d*(a + b*Sin[c + d*x])^4) + ((4*a^4 - 9*a^2*
b^2 + 12*b^4)*Cos[c + d*x])/(168*b^5*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x])^3
) + (a*(8*a^4 - 30*a^2*b^2 + 57*b^4)*Cos[c + d*x])/(336*b^5*(a^2 - b^2)^3*d
*(a + b*Sin[c + d*x])^2) + ((8*a^6 - 38*a^4*b^2 + 87*a^2*b^4 + 48*b^6)*Cos[
c + d*x])/(336*b^5*(a^2 - b^2)^4*d*(a + b*Sin[c + d*x])) + (5*Cos[c + d*x]^
3*(2*a + 3*b*Sin[c + d*x]))/(42*b^3*d*(a + b*Sin[c + d*x])^6) - (Cos[c + d*
x]*(4*a^2 + 9*b^2 + 10*a*b*Sin[c + d*x]))/(42*b^5*d*(a + b*Sin[c + d*x])^5)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2772

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Dist[g^2*((p - 1)/(b*(m + 1))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2942

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*C
os[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x])/(b^2*f*(m + 1)*(m + p + 1))), x] + Dist[g^2*((
p - 1)/(b^2*(m + 1)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\cos^6(c+dx)}{(a+b\sin(c+dx))^8}\,dx = -\frac{\cos^5(c+dx)}{7bd(a+b\sin(c+dx))^7} - \frac{5\int \frac{\cos^4(c+dx)\sin(c+dx)}{(a+b\sin(c+dx))^7}\,dx}{7b}$$

$$= -\frac{\cos^5(c+dx)}{7bd(a+b\sin(c+dx))^7} + \frac{5\cos^3(c+dx)(2a+3b\sin(c+dx))}{42b^3d(a+b\sin(c+dx))^6} - \frac{5\int \frac{\cos^2(c+dx)(\cdots}{(a+b\cdots}}{\cdots}$$

$$= -\frac{\cos^5(c+dx)}{7bd(a+b\sin(c+dx))^7} + \frac{5\cos^3(c+dx)(2a+3b\sin(c+dx))}{42b^3d(a+b\sin(c+dx))^6} - \frac{\cos(c+dx)\,(\cdots}{42b^5\cdots}$$

$$= -\frac{\cos^5(c+dx)}{7bd(a+b\sin(c+dx))^7} + \frac{a(4a^2-b^2)\cos(c+dx)}{168b^5(a^2-b^2)\,d(a+b\sin(c+dx))^4} + \frac{5\cos^3(c+d\cdots}{42b^3d(\cdots}$$

$$= -\frac{\cos^5(c+dx)}{7bd(a+b\sin(c+dx))^7} + \frac{a(4a^2-b^2)\cos(c+dx)}{168b^5(a^2-b^2)\,d(a+b\sin(c+dx))^4} + \frac{(4a^4-9a\cdots}{168b^5(a^2-\cdots}$$

$$= -\frac{\cos^5(c+dx)}{7bd(a+b\sin(c+dx))^7} + \frac{a(4a^2-b^2)\cos(c+dx)}{168b^5(a^2-b^2)\,d(a+b\sin(c+dx))^4} + \frac{(4a^4-9a\cdots}{168b^5(a^2-\cdots}$$

$$= -\frac{\cos^5(c+dx)}{7bd(a+b\sin(c+dx))^7} + \frac{a(4a^2-b^2)\cos(c+dx)}{168b^5(a^2-b^2)\,d(a+b\sin(c+dx))^4} + \frac{(4a^4-9a\cdots}{168b^5(a^2-\cdots}$$

$$= -\frac{\cos^5(c+dx)}{7bd(a+b\sin(c+dx))^7} + \frac{a(4a^2-b^2)\cos(c+dx)}{168b^5(a^2-b^2)\,d(a+b\sin(c+dx))^4} + \frac{(4a^4-9a\cdots}{168b^5(a^2-\cdots}$$

$$= -\frac{\cos^5(c+dx)}{7bd(a+b\sin(c+dx))^7} + \frac{a(4a^2-b^2)\cos(c+dx)}{168b^5(a^2-b^2)\,d(a+b\sin(c+dx))^4} + \frac{(4a^4-9a\cdots}{168b^5(a^2-\cdots}$$

$$= -\frac{\cos^5(c+dx)}{7bd(a+b\sin(c+dx))^7} + \frac{a(4a^2-b^2)\cos(c+dx)}{168b^5(a^2-b^2)\,d(a+b\sin(c+dx))^4} + \frac{(4a^4-9a\cdots}{168b^5(a^2-\cdots}$$

$$= \frac{5a\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{8\left(a^2-b^2\right)^{9/2}d} - \frac{\cos^5(c+dx)}{7bd(a+b\sin(c+dx))^7} + \frac{a(4a^2-b^2)\cos(c\cdots}{168b^5(a^2-b^2)\,d(a+b\cdots}$$

Mathematica [A]

time = 5.79, size = 386, normalized size = 0.95

$$\frac{\cos(c+dx)\left(\frac{6\cos^6(c+dx)}{(a+b\sin(c+dx))^7} + \frac{6a(-1+\sin(c+dx))^2(1+\sin(c+dx))^4}{(a-b)(a+b\sin(c+dx))^7} - \frac{5a(-1+\sin(c+dx))(1+\sin(c+dx))^4}{(a-b)^2(a+b\sin(c+dx))^6} + \frac{3a(1+\sin(c+dx))^4}{(a-b)^3(a+b\sin(c+dx))^5} + \frac{3a(1+\sin(c+dx))^3}{4(-a+b)^3(a+b)(a+b\sin(c+dx))^4} + \frac{7a(1+\sin(c+dx))^2}{4(-a+b)^3(a+b)^2(a+b\sin(c+dx))^3} + \frac{35a(1+\sin(c+dx))}{8(-a+b)^3(a+b)^3(a+b\sin(c+dx))^2} + \frac{105}{8}a\left(\frac{2\tanh^{-1}\left(\frac{\sqrt{a-b}\sqrt{1-\sin(c+dx)}}{\sqrt{-a-b}\sqrt{1+\sin(c+dx)}}\right)}{(-a-b)^{9/2}(a-b)^{7/2}\sqrt{\cos^2(c+dx)}} - \frac{1}{(a-b)^3(a+b)^4(a+b\sin(c+dx))}\right)\right)}{42(a-b)d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6/(a + b*Sin[c + d*x])^8,x]
```

```
[Out] (Cos[c + d*x]*((6*Cos[c + d*x]^6)/(a + b*Sin[c + d*x])^7 + (6*a*(-1 + Sin[c
 + d*x])^2*(1 + Sin[c + d*x])^4)/((a - b)*(a + b*Sin[c + d*x])^7) - (5*a*(-
```

```
1 + Sin[c + d*x])*(1 + Sin[c + d*x])^4)/((a - b)^2*(a + b*Sin[c + d*x])^6)
+ (3*a*(1 + Sin[c + d*x])^4)/((a - b)^3*(a + b*Sin[c + d*x])^5) + (3*a*(1 +
 Sin[c + d*x])^3)/(4*(-a + b)^3*(a + b)*(a + b*Sin[c + d*x])^4) + (7*a*(1 +
 Sin[c + d*x])^2)/(4*(-a + b)^3*(a + b)^2*(a + b*Sin[c + d*x])^3) + (35*a*(
1 + Sin[c + d*x]))/(8*(-a + b)^3*(a + b)^3*(a + b*Sin[c + d*x])^2) + (105*a
*((2*ArcTanh[(Sqrt[a - b]*Sqrt[1 - Sin[c + d*x]])/(Sqrt[-a - b]*Sqrt[1 + Si
n[c + d*x]])])/((-a - b)^(9/2)*(a - b)^(7/2)*Sqrt[Cos[c + d*x]^2]) - 1/((a
- b)^3*(a + b)^4*(a + b*Sin[c + d*x]))))/8))/(42*(a - b)*d)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1388 vs. $2(386) = 772$.

time = 2.41, size = 1389, normalized size = 3.41

method	result	size
risch	Expression too large to display	1388
derivativedivides	Expression too large to display	1389
default	Expression too large to display	1389

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6/(a+b*sin(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*(-1/16*(11*a^8-64*a^6*b^2+96*a^4*b^4-64*a^2*b^6+16*b^8)/a/(a^8-4*a^6
*b^2+6*a^4*b^4-4*a^2*b^6+b^8)*tan(1/2*d*x+1/2*c)^13-1/16*b*(31*a^8-384*a^6*
b^2+576*a^4*b^4-384*a^2*b^6+96*b^8)/a^2/(a^8-4*a^6*b^2+6*a^4*b^4-4*a^2*b^6+
b^8)*tan(1/2*d*x+1/2*c)^12-1/24/a^3*(14*a^10-311*a^8*b^2-1536*a^6*b^4+2624*
a^4*b^6-1856*a^2*b^8+480*b^10)/(a^8-4*a^6*b^2+6*a^4*b^4-4*a^2*b^6+b^8)*tan(
1/2*d*x+1/2*c)^11-5/24/a^4*b*(2*a^10-575*a^8*b^2-96*a^6*b^4+704*a^4*b^6-656
*a^2*b^8+192*b^10)/(a^8-4*a^6*b^2+6*a^4*b^4-4*a^2*b^6+b^8)*tan(1/2*d*x+1/2*
c)^10-1/48/a^5*(85*a^12-2950*a^10*b^2-14820*a^8*b^4+10048*a^6*b^6+368*a^4*b
^8-5760*a^2*b^10+2304*b^12)/(a^8-4*a^6*b^2+6*a^4*b^4-4*a^2*b^6+b^8)*tan(1/2
*d*x+1/2*c)^9+1/48/a^6*b*(435*a^12+14350*a^10*b^2+13160*a^8*b^4-18432*a^6*b
^6+10688*a^4*b^8+640*a^2*b^10-1536*b^12)/(a^8-4*a^6*b^2+6*a^4*b^4-4*a^2*b^6
+b^8)*tan(1/2*d*x+1/2*c)^8+1/84/a^7*b^2*(9765*a^12+47180*a^10*b^2-14588*a^8
*b^4-9984*a^6*b^6+18304*a^4*b^8-4864*a^2*b^10-768*b^12)/(a^8-4*a^6*b^2+6*a^
4*b^4-4*a^2*b^6+b^8)*tan(1/2*d*x+1/2*c)^7+1/12/a^6*b*(240*a^12+4375*a^10*b^
2+3920*a^8*b^4-4548*a^6*b^6+2672*a^4*b^8+160*a^2*b^10-384*b^12)/(a^8-4*a^6*
b^2+6*a^4*b^4-4*a^2*b^6+b^8)*tan(1/2*d*x+1/2*c)^6+1/48/a^5*(85*a^12+5420*a^
10*b^2+20040*a^8*b^4-9328*a^6*b^6-368*a^4*b^8+5760*a^2*b^10-2304*b^12)/(a^8
-4*a^6*b^2+6*a^4*b^4-4*a^2*b^6+b^8)*tan(1/2*d*x+1/2*c)^5+1/48/a^4*b*(857*a^
10+10012*a^8*b^2+2400*a^6*b^4-7184*a^4*b^6+6560*a^2*b^8-1920*b^10)/(a^8-4*a
^6*b^2+6*a^4*b^4-4*a^2*b^6+b^8)*tan(1/2*d*x+1/2*c)^4+1/24/a^3*(14*a^10+1363
*a^8*b^2+2088*a^6*b^4-2696*a^4*b^6+1856*a^2*b^8-480*b^10)/(a^8-4*a^6*b^2+6*
a^4*b^4-4*a^2*b^6+b^8)*tan(1/2*d*x+1/2*c)^3+1/24*b*(186*a^8+935*a^6*b^2-992
*a^4*b^4+600*a^2*b^6-144*b^8)/a^2/(a^8-4*a^6*b^2+6*a^4*b^4-4*a^2*b^6+b^8)*t
an(1/2*d*x+1/2*c)^2+1/48*(33*a^8+366*a^6*b^2-364*a^4*b^4+208*a^2*b^6-48*b^8
```

```
)/a/(a^8-4*a^6*b^2+6*a^4*b^4-4*a^2*b^6+b^8)*tan(1/2*d*x+1/2*c)+1/336*(279*a
^6-326*a^4*b^2+200*a^2*b^4-48*b^6)*b/(a^8-4*a^6*b^2+6*a^4*b^4-4*a^2*b^6+b^8
))/(a*tan(1/2*d*x+1/2*c)^2+2*b*tan(1/2*d*x+1/2*c)+a)^7+5/8*a/(a^8-4*a^6*b^2
+6*a^4*b^4-4*a^2*b^6+b^8)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c
)+2*b)/(a^2-b^2)^(1/2)))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+b*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1083 vs. 2(386) = 772.
time = 0.49, size = 2250, normalized size = 5.53

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+b*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] [1/672*(2*(8*a^8*b - 46*a^6*b^3 + 125*a^4*b^5 - 39*a^2*b^7 - 48*b^9)*cos(d*
x + c)^7 + 28*(7*a^8*b - 56*a^6*b^3 - 44*a^4*b^5 + 93*a^2*b^7)*cos(d*x + c)
^5 + 70*(7*a^8*b + 83*a^6*b^3 - 43*a^4*b^5 - 47*a^2*b^7)*cos(d*x + c)^3 - 1
05*(7*a^2*b^6*cos(d*x + c)^6 - a^8 - 21*a^6*b^2 - 35*a^4*b^4 - 7*a^2*b^6 -
7*(5*a^4*b^4 + 3*a^2*b^6)*cos(d*x + c)^4 + 7*(3*a^6*b^2 + 10*a^4*b^4 + 3*a^
2*b^6)*cos(d*x + c)^2 + (a*b^7*cos(d*x + c)^6 - 7*a^7*b - 35*a^5*b^3 - 21*a
^3*b^5 - a*b^7 - 3*(7*a^3*b^5 + a*b^7)*cos(d*x + c)^4 + (35*a^5*b^3 + 42*a^
3*b^5 + 3*a*b^7)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(((2*a^2
 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)
*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a
*b*sin(d*x + c) - a^2 - b^2)) - 420*(3*a^8*b + 7*a^6*b^3 - 7*a^4*b^5 - 3*a^
2*b^7)*cos(d*x + c) - 14*((8*a^9 - 46*a^7*b^2 + 125*a^5*b^4 - 54*a^3*b^6 -
33*a*b^8)*cos(d*x + c)^5 + 10*(a^9 - 11*a^7*b^2 - 25*a^5*b^4 + 31*a^3*b^6 +
 4*a*b^8)*cos(d*x + c)^3 + 15*(a^9 + 14*a^7*b^2 - 14*a^3*b^6 - a*b^8)*cos(d
*x + c))*sin(d*x + c))/(7*(a^11*b^6 - 5*a^9*b^8 + 10*a^7*b^10 - 10*a^5*b^12
 + 5*a^3*b^14 - a*b^16)*d*cos(d*x + c)^6 - 7*(5*a^13*b^4 - 22*a^11*b^6 + 35
*a^9*b^8 - 20*a^7*b^10 - 5*a^5*b^12 + 10*a^3*b^14 - 3*a*b^16)*d*cos(d*x + c
)^4 + 7*(3*a^15*b^2 - 5*a^13*b^4 - 17*a^11*b^6 + 55*a^9*b^8 - 55*a^7*b^10 +
```

```
 17*a^5*b^12 + 5*a^3*b^14 - 3*a*b^16)*d*cos(d*x + c)^2 - (a^17 + 16*a^15*b^
2 - 60*a^13*b^4 + 32*a^11*b^6 + 110*a^9*b^8 - 176*a^7*b^10 + 84*a^5*b^12 -
7*a*b^16)*d + ((a^10*b^7 - 5*a^8*b^9 + 10*a^6*b^11 - 10*a^4*b^13 + 5*a^2*b^
15 - b^17)*d*cos(d*x + c)^6 - 3*(7*a^12*b^5 - 34*a^10*b^7 + 65*a^8*b^9 - 60
*a^6*b^11 + 25*a^4*b^13 - 2*a^2*b^15 - b^17)*d*cos(d*x + c)^4 + (35*a^14*b^
3 - 133*a^12*b^5 + 143*a^10*b^7 + 55*a^8*b^9 - 215*a^6*b^11 + 145*a^4*b^13
- 27*a^2*b^15 - 3*b^17)*d*cos(d*x + c)^2 - (7*a^16*b - 84*a^12*b^5 + 176*a^
10*b^7 - 110*a^8*b^9 - 32*a^6*b^11 + 60*a^4*b^13 - 16*a^2*b^15 - b^17)*d)*s
in(d*x + c)), 1/336*((8*a^8*b - 46*a^6*b^3 + 125*a^4*b^5 - 39*a^2*b^7 - 48*
b^9)*cos(d*x + c)^7 + 14*(7*a^8*b - 56*a^6*b^3 - 44*a^4*b^5 + 93*a^2*b^7)*c
os(d*x + c)^5 + 35*(7*a^8*b + 83*a^6*b^3 - 43*a^4*b^5 - 47*a^2*b^7)*cos(d*x
 + c)^3 - 105*(7*a^2*b^6*cos(d*x + c)^6 - a^8 - 21*a^6*b^2 - 35*a^4*b^4 - 7
*a^2*b^6 - 7*(5*a^4*b^4 + 3*a^2*b^6)*cos(d*x + c)^4 + 7*(3*a^6*b^2 + 10*a^4
*b^4 + 3*a^2*b^6)*cos(d*x + c)^2 + (a*b^7*cos(d*x + c)^6 - 7*a^7*b - 35*a^5
*b^3 - 21*a^3*b^5 - a*b^7 - 3*(7*a^3*b^5 + a*b^7)*cos(d*x + c)^4 + (35*a^5*
b^3 + 42*a^3*b^5 + 3*a*b^7)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(a^2 - b^2)*a
rctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - 210*(3*a^8*b
+ 7*a^6*b^3 - 7*a^4*b^5 - 3*a^2*b^7)*cos(d*x + c) - 7*((8*a^9 - 46*a^7*b^2
+ 125*a^5*b^4 - 54*a^3*b^6 - 33*a*b^8)*cos(d*x + c)^5 + 10*(a^9 - 11*a^7*b^
2 - 25*a^5*b^4 + 31*a^3*b^6 + 4*a*b^8)*cos(d*x + c)^3 + 15*(a^9 + 14*a^7*b^
2 - 14*a^3*b^6 - a*b^8)*cos(d*x + c))*sin(d*x + c))/(7*(a^11*b^6 - 5*a^9*b^
8 + 10*a^7*b^10 - 10*a^5*b^12 + 5*a^3*b^14 - a*b^16)*d*cos(d*x + c)^6 - 7*(
5*a^13*b^4 - 22*a^11*b^6 + 35*a^9*b^8 - 20*a^7*b^10 - 5*a^5*b^12 + 10*a^3*b
^14 - 3*a*b^16)*d*cos(d*x + c)^4 + 7*(3*a^15*b^2 - 5*a^13*b^4 - 17*a^11*b^6
 + 55*a^9*b^8 - 55*a^7*b^10 + 17*a^5*b^12 + 5*a^3*b^14 - 3*a*b^16)*d*cos(d*
x + c)^2 - (a^17 + 16*a^15*b^2 - 60*a^13*b^4 + 32*a^11*b^6 + 110*a^9*b^8 -
176*a^7*b^10 + 84*a^5*b^12 - 7*a*b^16)*d + ((a^10*b^7 - 5*a^8*b^9 + 10*a^6*
b^11 - 10*a^4*b^13 + 5*a^2*b^15 - b^17)*d*cos(d*x + c)^6 - 3*(7*a^12*b^5 -
34*a^10*b^7 + 65*a^8*b^9 - 60*a^6*b^11 + 25*a^4*b^13 - 2*a^2*b^15 - b^17)*d
*cos(d*x + c)^4 + (35*a^14*b^3 - 133*a^12*b^5 + 143*a^10*b^7 + 55*a^8*b^9 -
 215*a^6*b^11 + 145*a^4*b^13 - 27*a^2*b^15 - 3*b^17)*d*cos(d*x + c)^2 - (7*
a^16*b - 84*a^12*b^5 + 176*a^10*b^7 - 110*a^8*b^9 - 32*a^6*b^11 + 60*a^4*b^
13 - 16*a^2*b^15 - b^17)*d)*sin(d*x + c))]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6/(a+b*sin(d*x+c))**8,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1650 vs. $2(386) = 772$.

time = 3.44, size = 1650, normalized size = 4.05

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+b*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/168*(105*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x
 + 1/2*c) + b)/sqrt(a^2 - b^2)))*a/((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^
6 + b^8)*sqrt(a^2 - b^2)) - (231*a^14*tan(1/2*d*x + 1/2*c)^13 - 1344*a^12*b
^2*tan(1/2*d*x + 1/2*c)^13 + 2016*a^10*b^4*tan(1/2*d*x + 1/2*c)^13 - 1344*a
^8*b^6*tan(1/2*d*x + 1/2*c)^13 + 336*a^6*b^8*tan(1/2*d*x + 1/2*c)^13 + 651*
a^13*b*tan(1/2*d*x + 1/2*c)^12 - 8064*a^11*b^3*tan(1/2*d*x + 1/2*c)^12 + 12
096*a^9*b^5*tan(1/2*d*x + 1/2*c)^12 - 8064*a^7*b^7*tan(1/2*d*x + 1/2*c)^12
+ 2016*a^5*b^9*tan(1/2*d*x + 1/2*c)^12 + 196*a^14*tan(1/2*d*x + 1/2*c)^11 -
 4354*a^12*b^2*tan(1/2*d*x + 1/2*c)^11 - 21504*a^10*b^4*tan(1/2*d*x + 1/2*c
)^11 + 36736*a^8*b^6*tan(1/2*d*x + 1/2*c)^11 - 25984*a^6*b^8*tan(1/2*d*x +
1/2*c)^11 + 6720*a^4*b^10*tan(1/2*d*x + 1/2*c)^11 + 140*a^13*b*tan(1/2*d*x
+ 1/2*c)^10 - 40250*a^11*b^3*tan(1/2*d*x + 1/2*c)^10 - 6720*a^9*b^5*tan(1/2
*d*x + 1/2*c)^10 + 49280*a^7*b^7*tan(1/2*d*x + 1/2*c)^10 - 45920*a^5*b^9*ta
n(1/2*d*x + 1/2*c)^10 + 13440*a^3*b^11*tan(1/2*d*x + 1/2*c)^10 + 595*a^14*t
an(1/2*d*x + 1/2*c)^9 - 20650*a^12*b^2*tan(1/2*d*x + 1/2*c)^9 - 103740*a^10
*b^4*tan(1/2*d*x + 1/2*c)^9 + 70336*a^8*b^6*tan(1/2*d*x + 1/2*c)^9 + 2576*a
^6*b^8*tan(1/2*d*x + 1/2*c)^9 - 40320*a^4*b^10*tan(1/2*d*x + 1/2*c)^9 + 161
28*a^2*b^12*tan(1/2*d*x + 1/2*c)^9 - 3045*a^13*b*tan(1/2*d*x + 1/2*c)^8 - 1
00450*a^11*b^3*tan(1/2*d*x + 1/2*c)^8 - 92120*a^9*b^5*tan(1/2*d*x + 1/2*c)^
8 + 129024*a^7*b^7*tan(1/2*d*x + 1/2*c)^8 - 74816*a^5*b^9*tan(1/2*d*x + 1/2
*c)^8 - 4480*a^3*b^11*tan(1/2*d*x + 1/2*c)^8 + 10752*a*b^13*tan(1/2*d*x + 1
/2*c)^8 - 39060*a^12*b^2*tan(1/2*d*x + 1/2*c)^7 - 188720*a^10*b^4*tan(1/2*d
*x + 1/2*c)^7 + 58352*a^8*b^6*tan(1/2*d*x + 1/2*c)^7 + 39936*a^6*b^8*tan(1/
2*d*x + 1/2*c)^7 - 73216*a^4*b^10*tan(1/2*d*x + 1/2*c)^7 + 19456*a^2*b^12*t
an(1/2*d*x + 1/2*c)^7 + 3072*b^14*tan(1/2*d*x + 1/2*c)^7 - 6720*a^13*b*tan(
1/2*d*x + 1/2*c)^6 - 122500*a^11*b^3*tan(1/2*d*x + 1/2*c)^6 - 109760*a^9*b^
5*tan(1/2*d*x + 1/2*c)^6 + 127344*a^7*b^7*tan(1/2*d*x + 1/2*c)^6 - 74816*a^
5*b^9*tan(1/2*d*x + 1/2*c)^6 - 4480*a^3*b^11*tan(1/2*d*x + 1/2*c)^6 + 10752
*a*b^13*tan(1/2*d*x + 1/2*c)^6 - 595*a^14*tan(1/2*d*x + 1/2*c)^5 - 37940*a^
12*b^2*tan(1/2*d*x + 1/2*c)^5 - 140280*a^10*b^4*tan(1/2*d*x + 1/2*c)^5 + 65
296*a^8*b^6*tan(1/2*d*x + 1/2*c)^5 + 2576*a^6*b^8*tan(1/2*d*x + 1/2*c)^5 -
40320*a^4*b^10*tan(1/2*d*x + 1/2*c)^5 + 16128*a^2*b^12*tan(1/2*d*x + 1/2*c)
^5 - 5999*a^13*b*tan(1/2*d*x + 1/2*c)^4 - 70084*a^11*b^3*tan(1/2*d*x + 1/2*
c)^4 - 16800*a^9*b^5*tan(1/2*d*x + 1/2*c)^4 + 50288*a^7*b^7*tan(1/2*d*x + 1
/2*c)^4 - 45920*a^5*b^9*tan(1/2*d*x + 1/2*c)^4 + 13440*a^3*b^11*tan(1/2*d*x
 + 1/2*c)^4 - 196*a^14*tan(1/2*d*x + 1/2*c)^3 - 19082*a^12*b^2*tan(1/2*d*x
+ 1/2*c)^3 - 29232*a^10*b^4*tan(1/2*d*x + 1/2*c)^3 + 37744*a^8*b^6*tan(1/2*
d*x + 1/2*c)^3 - 25984*a^6*b^8*tan(1/2*d*x + 1/2*c)^3 + 6720*a^4*b^10*tan(1
```

```
/2*d*x + 1/2*c)^3 - 2604*a^13*b*tan(1/2*d*x + 1/2*c)^2 - 13090*a^11*b^3*tan
(1/2*d*x + 1/2*c)^2 + 13888*a^9*b^5*tan(1/2*d*x + 1/2*c)^2 - 8400*a^7*b^7*t
an(1/2*d*x + 1/2*c)^2 + 2016*a^5*b^9*tan(1/2*d*x + 1/2*c)^2 - 231*a^14*tan(
1/2*d*x + 1/2*c) - 2562*a^12*b^2*tan(1/2*d*x + 1/2*c) + 2548*a^10*b^4*tan(1
/2*d*x + 1/2*c) - 1456*a^8*b^6*tan(1/2*d*x + 1/2*c) + 336*a^6*b^8*tan(1/2*d
*x + 1/2*c) - 279*a^13*b + 326*a^11*b^3 - 200*a^9*b^5 + 48*a^7*b^7)/((a^15
- 4*a^13*b^2 + 6*a^11*b^4 - 4*a^9*b^6 + a^7*b^8)*(a*tan(1/2*d*x + 1/2*c)^2
+ 2*b*tan(1/2*d*x + 1/2*c) + a)^7))/d
```

Mupad [B]
time = 12.47, size = 1868, normalized size = 4.59

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(a + b*sin(c + d*x))^8,x)
```

```
[Out] ((279*a^6*b - 48*b^7 + 200*a^2*b^5 - 326*a^4*b^3)/(168*(a^8 + b^8 - 4*a^2*b
^6 + 6*a^4*b^4 - 4*a^6*b^2)) + (tan(c/2 + (d*x)/2)*(33*a^8 - 48*b^8 + 208*a
^2*b^6 - 364*a^4*b^4 + 366*a^6*b^2))/(24*a*(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b
^4 - 4*a^6*b^2)) - (tan(c/2 + (d*x)/2)^9*(85*a^12 + 2304*b^12 - 5760*a^2*b^
10 + 368*a^4*b^8 + 10048*a^6*b^6 - 14820*a^8*b^4 - 2950*a^10*b^2))/(24*a^5*
(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6*b^2)) + (tan(c/2 + (d*x)/2)^5*(8
5*a^12 - 2304*b^12 + 5760*a^2*b^10 - 368*a^4*b^8 - 9328*a^6*b^6 + 20040*a^8
*b^4 + 5420*a^10*b^2))/(24*a^5*(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6*b
^2)) - (tan(c/2 + (d*x)/2)^11*(14*a^10 + 480*b^10 - 1856*a^2*b^8 + 2624*a^4
*b^6 - 1536*a^6*b^4 - 311*a^8*b^2))/(12*a^3*(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*
b^4 - 4*a^6*b^2)) + (tan(c/2 + (d*x)/2)^3*(14*a^10 - 480*b^10 + 1856*a^2*b^
8 - 2696*a^4*b^6 + 2088*a^6*b^4 + 1363*a^8*b^2))/(12*a^3*(a^8 + b^8 - 4*a^2
*b^6 + 6*a^4*b^4 - 4*a^6*b^2)) - (tan(c/2 + (d*x)/2)^13*(11*a^8 + 16*b^8 -
64*a^2*b^6 + 96*a^4*b^4 - 64*a^6*b^2))/(8*a*(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*
b^4 - 4*a^6*b^2)) + (tan(c/2 + (d*x)/2)^6*(240*a^12*b - 384*b^13 + 160*a^2*
b^11 + 2672*a^4*b^9 - 4548*a^6*b^7 + 3920*a^8*b^5 + 4375*a^10*b^3))/(6*a^6*
(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6*b^2)) + (tan(c/2 + (d*x)/2)^8*(4
35*a^12*b - 1536*b^13 + 640*a^2*b^11 + 10688*a^4*b^9 - 18432*a^6*b^7 + 1316
0*a^8*b^5 + 14350*a^10*b^3))/(24*a^6*(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4
*a^6*b^2)) - (5*tan(c/2 + (d*x)/2)^10*(2*a^10*b + 192*b^11 - 656*a^2*b^9 +
704*a^4*b^7 - 96*a^6*b^5 - 575*a^8*b^3))/(12*a^4*(a^8 + b^8 - 4*a^2*b^6 + 6
*a^4*b^4 - 4*a^6*b^2)) + (tan(c/2 + (d*x)/2)^4*(857*a^10*b - 1920*b^11 + 65
60*a^2*b^9 - 7184*a^4*b^7 + 2400*a^6*b^5 + 10012*a^8*b^3))/(24*a^4*(a^8 + b
^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6*b^2)) - (tan(c/2 + (d*x)/2)^12*(31*a^8*b
 + 96*b^9 - 384*a^2*b^7 + 576*a^4*b^5 - 384*a^6*b^3))/(8*a^2*(a^8 + b^8 - 4
*a^2*b^6 + 6*a^4*b^4 - 4*a^6*b^2)) + (tan(c/2 + (d*x)/2)^2*(186*a^8*b - 144
*b^9 + 600*a^2*b^7 - 992*a^4*b^5 + 935*a^6*b^3))/(12*a^2*(a^8 + b^8 - 4*a^2
*b^6 + 6*a^4*b^4 - 4*a^6*b^2)) + (b*tan(c/2 + (d*x)/2)^7*(35*a^6 + 16*b^6 +
```

```
 168*a^2*b^4 + 210*a^4*b^2)*(279*a^6*b - 48*b^7 + 200*a^2*b^5 - 326*a^4*b^3
))/(42*a^7*(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6*b^2)))/(d*(tan(c/2 +
(d*x)/2)^5*(210*a^6*b + 672*a^2*b^5 + 1120*a^4*b^3) + tan(c/2 + (d*x)/2)^9*
(210*a^6*b + 672*a^2*b^5 + 1120*a^4*b^3) + a^7*tan(c/2 + (d*x)/2)^14 + tan(
c/2 + (d*x)/2)^3*(84*a^6*b + 280*a^4*b^3) + tan(c/2 + (d*x)/2)^11*(84*a^6*b
 + 280*a^4*b^3) + tan(c/2 + (d*x)/2)^6*(448*a*b^6 + 35*a^7 + 1680*a^3*b^4 +
 840*a^5*b^2) + tan(c/2 + (d*x)/2)^8*(448*a*b^6 + 35*a^7 + 1680*a^3*b^4 + 8
40*a^5*b^2) + tan(c/2 + (d*x)/2)^7*(280*a^6*b + 128*b^7 + 1344*a^2*b^5 + 16
80*a^4*b^3) + a^7 + tan(c/2 + (d*x)/2)^4*(21*a^7 + 560*a^3*b^4 + 420*a^5*b^
2) + tan(c/2 + (d*x)/2)^10*(21*a^7 + 560*a^3*b^4 + 420*a^5*b^2) + tan(c/2 +
 (d*x)/2)^2*(7*a^7 + 84*a^5*b^2) + tan(c/2 + (d*x)/2)^12*(7*a^7 + 84*a^5*b^
2) + 14*a^6*b*tan(c/2 + (d*x)/2) + 14*a^6*b*tan(c/2 + (d*x)/2)^13)) + (5*a*
atan((8*((5*a^2*tan(c/2 + (d*x)/2))/(8*(a + b)^(9/2)*(a - b)^(9/2)) + (5*a*
(16*a^8*b + 16*b^9 - 64*a^2*b^7 + 96*a^4*b^5 - 64*a^6*b^3))/(128*(a + b)^(9
/2)*(a - b)^(9/2)*(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6*b^2)))*(a^8 +
b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6*b^2))/(5*a)))/(8*d*(a + b)^(9/2)*(a - b
)^(9/2))
```

3.469 $\int \frac{\cos^4(c+dx)}{(a+b\sin(c+dx))^8}\, dx$

Optimal. Leaf size=411

$$\frac{3a(2a^2+b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{8\left(a^2-b^2\right)^{11/2}d}-\frac{\cos^3(c+dx)}{7bd(a+b\sin(c+dx))^7}-\frac{(a^2-3b^2)\cos(c+dx)}{140b^3\left(a^2-b^2\right)d(a+b\sin(c+dx))^5}-\frac{}{280b}$$

```
[Out] 3/8*a*(2*a^2+b^2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/(a^2-b^2
)^(11/2)/d-1/7*cos(d*x+c)^3/b/d/(a+b*sin(d*x+c))^7-1/140*(a^2-3*b^2)*cos(d*
x+c)/b^3/(a^2-b^2)/d/(a+b*sin(d*x+c))^5-1/280*a*(2*a^2-11*b^2)*cos(d*x+c)/b
^3/(a^2-b^2)^2/d/(a+b*sin(d*x+c))^4-1/280*(2*a^4-15*a^2*b^2-8*b^4)*cos(d*x+
c)/b^3/(a^2-b^2)^3/d/(a+b*sin(d*x+c))^3-1/560*a*(4*a^4-36*a^2*b^2-73*b^4)*c
os(d*x+c)/b^3/(a^2-b^2)^4/d/(a+b*sin(d*x+c))^2-1/560*(4*a^6-40*a^4*b^2-247*
a^2*b^4-32*b^6)*cos(d*x+c)/b^3/(a^2-b^2)^5/d/(a+b*sin(d*x+c))+1/28*cos(d*x+
c)*(a+3*b*sin(d*x+c))/b^3/d/(a+b*sin(d*x+c))^6
```

Rubi [A]

time = 0.52, antiderivative size = 411, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2772, 2942, 2833, 12, 2739, 632, 210}

$$\frac{3a(2a^2+b^2)\,\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{8d\left(a^2-b^2\right)^{11/2}}-\frac{a(2a^2-11b^2)\cos(c+dx)}{280b^3d\left(a^2-b^2\right)^2(a+b\sin(c+dx))^4}-\frac{(a^2-3b^2)\cos(c+dx)}{140b^3d\left(a^2-b^2\right)(a+b\sin(c+dx))^5}-\frac{a(4a^4-36a^2b^2-73b^4)\cos(c+dx)}{560b^3d\left(a^2-b^2\right)^4(a+b\sin(c+dx))^2}-\frac{(2a^4-15a^2b^2-8b^4)\cos(c+dx)}{280b^3d\left(a^2-b^2\right)^3(a+b\sin(c+dx))^3}-\frac{(4a^6-40a^4b^2-247a^2b^4-32b^6)\cos(c+dx)}{560b^3d\left(a^2-b^2\right)^5(a+b\sin(c+dx))}+\frac{\cos(c+dx)(a+3b\sin(c+dx))}{28b^3d(a+b\sin(c+dx))^6}-\frac{\cos^3(c+dx)}{7bd(a+b\sin(c+dx))^7}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + b*Sin[c + d*x])^8,x]
```

```
[Out] (3*a*(2*a^2 + b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(8*(a^
2 - b^2)^(11/2)*d) - Cos[c + d*x]^3/(7*b*d*(a + b*Sin[c + d*x])^7) - ((a^2
- 3*b^2)*Cos[c + d*x])/(140*b^3*(a^2 - b^2)*d*(a + b*Sin[c + d*x])^5) - (a*
(2*a^2 - 11*b^2)*Cos[c + d*x])/(280*b^3*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x]
)^4) - ((2*a^4 - 15*a^2*b^2 - 8*b^4)*Cos[c + d*x])/(280*b^3*(a^2 - b^2)^3*d
*(a + b*Sin[c + d*x])^3) - (a*(4*a^4 - 36*a^2*b^2 - 73*b^4)*Cos[c + d*x])/(
560*b^3*(a^2 - b^2)^4*d*(a + b*Sin[c + d*x])^2) - ((4*a^6 - 40*a^4*b^2 - 24
7*a^2*b^4 - 32*b^6)*Cos[c + d*x])/(560*b^3*(a^2 - b^2)^5*d*(a + b*Sin[c + d
*x])) + (Cos[c + d*x]*(a + 3*b*Sin[c + d*x]))/(28*b^3*d*(a + b*Sin[c + d*x]
)^6)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2772

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Dist[g^2*((p - 1)/(b*(m + 1))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2942

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*C
os[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x])/(b^2*f*(m + 1)*(m + p + 1))), x] + Dist[g^2*((
p - 1)/(b^2*(m + 1)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{(a+b\sin(c+dx))^8}\,dx = -\frac{\cos^3(c+dx)}{7bd(a+b\sin(c+dx))^7} - \frac{3\int \frac{\cos^2(c+dx)\sin(c+dx)}{(a+b\sin(c+dx))^7}\,dx}{7b}$$

$$= -\frac{\cos^3(c+dx)}{7bd(a+b\sin(c+dx))^7} + \frac{\cos(c+dx)(a+3b\sin(c+dx))}{28b^3d(a+b\sin(c+dx))^6} - \frac{\int \frac{-6b-2a\sin(c+dx}{(a+b\sin(c+dx))^6}}{56b^3}$$

$$= -\frac{\cos^3(c+dx)}{7bd(a+b\sin(c+dx))^7} - \frac{(a^2-3b^2)\cos(c+dx)}{140b^3(a^2-b^2)\,d(a+b\sin(c+dx))^5} + \frac{\cos(c+dx}{28b^3d(a}$$

$$= -\frac{\cos^3(c+dx)}{7bd(a+b\sin(c+dx))^7} - \frac{(a^2-3b^2)\cos(c+dx)}{140b^3(a^2-b^2)\,d(a+b\sin(c+dx))^5} - \frac{a(2a}{280b^3(a^2-}$$

$$= -\frac{\cos^3(c+dx)}{7bd(a+b\sin(c+dx))^7} - \frac{(a^2-3b^2)\cos(c+dx)}{140b^3(a^2-b^2)\,d(a+b\sin(c+dx))^5} - \frac{a(2a}{280b^3(a^2-}$$

$$= -\frac{\cos^3(c+dx)}{7bd(a+b\sin(c+dx))^7} - \frac{(a^2-3b^2)\cos(c+dx)}{140b^3(a^2-b^2)\,d(a+b\sin(c+dx))^5} - \frac{a(2a}{280b^3(a^2-}$$

$$= -\frac{\cos^3(c+dx)}{7bd(a+b\sin(c+dx))^7} - \frac{(a^2-3b^2)\cos(c+dx)}{140b^3(a^2-b^2)\,d(a+b\sin(c+dx))^5} - \frac{a(2a}{280b^3(a^2-}$$

$$= -\frac{\cos^3(c+dx)}{7bd(a+b\sin(c+dx))^7} - \frac{(a^2-3b^2)\cos(c+dx)}{140b^3(a^2-b^2)\,d(a+b\sin(c+dx))^5} - \frac{a(2a}{280b^3(a^2-}$$

$$= -\frac{\cos^3(c+dx)}{7bd(a+b\sin(c+dx))^7} - \frac{(a^2-3b^2)\cos(c+dx)}{140b^3(a^2-b^2)\,d(a+b\sin(c+dx))^5} - \frac{a(2a}{280b^3(a^2-}$$

$$= -\frac{\cos^3(c+dx)}{7bd(a+b\sin(c+dx))^7} - \frac{(a^2-3b^2)\cos(c+dx)}{140b^3(a^2-b^2)\,d(a+b\sin(c+dx))^5} - \frac{a(2a}{280b^3(a^2-}$$

$$= \frac{3a(2a^2+b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{8\left(a^2-b^2\right)^{11/2}d} - \frac{\cos^3(c+dx)}{7bd(a+b\sin(c+dx))^7} - \frac{(a^2-}{140b^3(a^2-}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1167 vs. 2(411) = 822.

time = 6.09, size = 1167, normalized size = 2.84

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + b*Sin[c + d*x])^8,x]
```

```
[Out] Cos[c + d*x]^5/(5*(a - b)*d*(a + b*Sin[c + d*x])^7) + (a*Cos[c + d*x]*(-1/7
*(b*(1 - Sin[c + d*x])^(5/2)*(1 + Sin[c + d*x])^(7/2))/((-a + b)*(a + b)*(a
 + b*Sin[c + d*x])^7) - (-1/6*((a*b + (7*a - b)*b)*(1 - Sin[c + d*x])^(5/2)
*(1 + Sin[c + d*x])^(7/2))/((-a + b)*(a + b)*(a + b*Sin[c + d*x])^6) - (7*(
6*a^2 - 2*a*b + b^2)*(-1/5*((1 - Sin[c + d*x])^(3/2)*(1 + Sin[c + d*x])^(7/
2))/((-a + b)*(a + b*Sin[c + d*x])^5) - (3*(-1/4*(Sqrt[1 - Sin[c + d*x]]*(1
 + Sin[c + d*x])^(7/2))/((-a + b)*(a + b*Sin[c + d*x])^4) - (-1/3*(Sqrt[1 -
 Sin[c + d*x]]*(1 + Sin[c + d*x])^(5/2))/((a + b)*(a + b*Sin[c + d*x])^3) +
 (5*(-1/2*(Sqrt[1 - Sin[c + d*x]]*(1 + Sin[c + d*x])^(3/2))/((a + b)*(a + b
*Sin[c + d*x])^2) + (3*((-2*ArcTanh[(Sqrt[a - b]*Sqrt[1 - Sin[c + d*x]])/(S
qrt[-a - b]*Sqrt[1 + Sin[c + d*x]])])/((-a - b)^(3/2)*Sqrt[a - b]) + (Sqrt[
1 - Sin[c + d*x]]*Sqrt[1 + Sin[c + d*x]])/((-a - b)*(a + b*Sin[c + d*x]))))
/(2*(a + b))))/(3*(a + b)))/(4*(-a + b))))/(5*(-a + b))))/(6*(-a + b)*(a +
b)))/(7*(-a + b)*(a + b))))/((a - b)*d*Sqrt[1 - Sin[c + d*x]]*Sqrt[1 + Sin[
c + d*x]]) + (2*b*(Cos[c + d*x]^7/(7*(a - b)*d*(a + b*Sin[c + d*x])^7) + (a
*Cos[c + d*x]*(-1/7*((1 - Sin[c + d*x])^(5/2)*(1 + Sin[c + d*x])^(9/2))/((-
a + b)*(a + b*Sin[c + d*x])^7) - (5*(-1/6*((1 - Sin[c + d*x])^(3/2)*(1 + Si
n[c + d*x])^(9/2))/((-a + b)*(a + b*Sin[c + d*x])^6) - (-1/5*(Sqrt[1 - Sin[
c + d*x]]*(1 + Sin[c + d*x])^(9/2))/((-a + b)*(a + b*Sin[c + d*x])^5) - (-1
```

```
/4*(Sqrt[1 - Sin[c + d*x]]*(1 + Sin[c + d*x])^(7/2))/((a + b)*(a + b*Sin[c
+ d*x])^4) + (7*(-1/3*(Sqrt[1 - Sin[c + d*x]]*(1 + Sin[c + d*x])^(5/2))/((a
 + b)*(a + b*Sin[c + d*x])^3) + (5*(-1/2*(Sqrt[1 - Sin[c + d*x]]*(1 + Sin[c
 + d*x])^(3/2))/((a + b)*(a + b*Sin[c + d*x])^2) + (3*((-2*ArcTanh[(Sqrt[a
- b]*Sqrt[1 - Sin[c + d*x]])/(Sqrt[-a - b]*Sqrt[1 + Sin[c + d*x]])])/((-a -
 b)^(3/2)*Sqrt[a - b]) + (Sqrt[1 - Sin[c + d*x]]*Sqrt[1 + Sin[c + d*x]])/((
-a - b)*(a + b*Sin[c + d*x]))))/(2*(a + b))))/(3*(a + b))))/(4*(a + b)))/(5
*(-a + b)))/(2*(-a + b))))/(7*(-a + b))))/((a - b)*d*Sqrt[1 - Sin[c + d*x]]
*Sqrt[1 + Sin[c + d*x]])))/(5*(a - b))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1659 vs. $2(390) = 780$.
time = 2.10, size = 1660, normalized size = 4.04

method	result	size
risch	Expression too large to display	1533
derivativedivides	Expression too large to display	1660
default	Expression too large to display	1660

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+b*sin(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*(-1/16*(10*a^10-83*a^8*b^2+160*a^6*b^4-160*a^4*b^6+80*a^2*b^8-16*b^1
0)/a/(a^10-5*a^8*b^2+10*a^6*b^4-10*a^4*b^6+5*a^2*b^8-b^10)*tan(1/2*d*x+1/2*
c)^13-3/16*b*(6*a^10-173*a^8*b^2+320*a^6*b^4-320*a^4*b^6+160*a^2*b^8-32*b^1
0)/a^2/(a^10-5*a^8*b^2+10*a^6*b^4-10*a^4*b^6+5*a^2*b^8-b^10)*tan(1/2*d*x+1/
2*c)^12-1/8/a^3*(12*a^12-224*a^10*b^2-587*a^8*b^4+1280*a^6*b^6-1440*a^4*b^8
+768*a^2*b^10-160*b^12)/(a^10-5*a^8*b^2+10*a^6*b^4-10*a^4*b^6+5*a^2*b^8-b^1
0)*tan(1/2*d*x+1/2*c)^11+1/8/a^4*b*(12*a^12+1468*a^10*b^2-161*a^8*b^4-1120*
a^6*b^6+2160*a^4*b^8-1392*a^2*b^10+320*b^12)/(a^10-5*a^8*b^2+10*a^6*b^4-10*
a^4*b^6+5*a^2*b^8-b^10)*tan(1/2*d*x+1/2*c)^10-1/80/a^5*(90*a^14-7415*a^12*b
^2-35630*a^10*b^4+28892*a^8*b^6-18480*a^6*b^8-8976*a^4*b^10+13184*a^2*b^12-
3840*b^14)/(a^10-5*a^8*b^2+10*a^6*b^4-10*a^4*b^6+5*a^2*b^8-b^10)*tan(1/2*d*
x+1/2*c)^9+1/80/a^6*b*(1190*a^14+35535*a^12*b^2+27230*a^10*b^4-36248*a^8*b^
6+48320*a^6*b^8-17216*a^4*b^10-3456*a^2*b^12+2560*b^14)/(a^10-5*a^8*b^2+10*
a^6*b^4-10*a^4*b^6+5*a^2*b^8-b^10)*tan(1/2*d*x+1/2*c)^8+1/140/a^7*b^2*(2401
0*a^14+113085*a^12*b^2-41132*a^10*b^4+24836*a^8*b^6+44416*a^6*b^8-38272*a^4
*b^10+6912*a^2*b^12+1280*b^14)/(a^10-5*a^8*b^2+10*a^6*b^4-10*a^4*b^6+5*a^2*
b^8-b^10)*tan(1/2*d*x+1/2*c)^7+1/20/a^6*b*(560*a^14+10590*a^12*b^2+8855*a^1
0*b^4-8312*a^8*b^6+12140*a^6*b^8-4304*a^4*b^10-864*a^2*b^12+640*b^14)/(a^10
-5*a^8*b^2+10*a^6*b^4-10*a^4*b^6+5*a^2*b^8-b^10)*tan(1/2*d*x+1/2*c)^6+1/80/
a^5*(90*a^14+13165*a^12*b^2+47580*a^10*b^4-21592*a^8*b^6+19040*a^6*b^8+8976
*a^4*b^10-13184*a^2*b^12+3840*b^14)/(a^10-5*a^8*b^2+10*a^6*b^4-10*a^4*b^6+5
*a^2*b^8-b^10)*tan(1/2*d*x+1/2*c)^5+1/80/a^4*b*(1938*a^12+23825*a^10*b^2+59
16*a^8*b^4-10304*a^6*b^6+21520*a^4*b^8-13920*a^2*b^10+3200*b^12)/(a^10-5*a^
```

```
8*b^2+10*a^6*b^4-10*a^4*b^6+5*a^2*b^8-b^10)*tan(1/2*d*x+1/2*c)^4+1/40/a^3*(
60*a^12+2996*a^10*b^2+5475*a^8*b^4-6248*a^6*b^6+7192*a^4*b^8-3840*a^2*b^10+
800*b^12)/(a^10-5*a^8*b^2+10*a^6*b^4-10*a^4*b^6+5*a^2*b^8-b^10)*tan(1/2*d*x
+1/2*c)^3+1/40*b*(388*a^10+2376*a^8*b^2-2489*a^6*b^4+2448*a^4*b^6-1208*a^2*
b^8+240*b^10)/a^2/(a^10-5*a^8*b^2+10*a^6*b^4-10*a^4*b^6+5*a^2*b^8-b^10)*tan
(1/2*d*x+1/2*c)^2+1/80*(50*a^10+957*a^8*b^2-970*a^6*b^4+884*a^4*b^6-416*a^2
*b^8+80*b^10)/a/(a^10-5*a^8*b^2+10*a^6*b^4-10*a^4*b^6+5*a^2*b^8-b^10)*tan(1
/2*d*x+1/2*c)+1/560*(686*a^8-885*a^6*b^2+842*a^4*b^4-408*a^2*b^6+80*b^8)*b/
(a^10-5*a^8*b^2+10*a^6*b^4-10*a^4*b^6+5*a^2*b^8-b^10))/(a*tan(1/2*d*x+1/2*c
)^2+2*b*tan(1/2*d*x+1/2*c)+a)^7+3/8*a*(2*a^2+b^2)/(a^10-5*a^8*b^2+10*a^6*b^
4-10*a^4*b^6+5*a^2*b^8-b^10)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/
2*c)+2*b)/(a^2-b^2)^(1/2)))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1286 vs. $2(390) = 780$.
time = 0.52, size = 2657, normalized size = 6.46

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] [-1/1120*(2*(4*a^8*b^3 - 44*a^6*b^5 - 207*a^4*b^7 + 215*a^2*b^9 + 32*b^11)*
cos(d*x + c)^7 - 28*(6*a^10*b - 65*a^8*b^3 - 224*a^6*b^5 + 222*a^4*b^7 + 53
*a^2*b^9 + 8*b^11)*cos(d*x + c)^5 - 70*(14*a^10*b + 173*a^8*b^3 - 3*a^6*b^5
 - 137*a^4*b^7 - 47*a^2*b^9)*cos(d*x + c)^3 + 105*(2*a^10 + 43*a^8*b^2 + 91
*a^6*b^4 + 49*a^4*b^6 + 7*a^2*b^8 - 7*(2*a^4*b^6 + a^2*b^8)*cos(d*x + c)^6
+ 7*(10*a^6*b^4 + 11*a^4*b^6 + 3*a^2*b^8)*cos(d*x + c)^4 - 7*(6*a^8*b^2 + 2
3*a^6*b^4 + 16*a^4*b^6 + 3*a^2*b^8)*cos(d*x + c)^2 + (14*a^9*b + 77*a^7*b^3
 + 77*a^5*b^5 + 23*a^3*b^7 + a*b^9 - (2*a^3*b^7 + a*b^9)*cos(d*x + c)^6 + 3
*(14*a^5*b^5 + 9*a^3*b^7 + a*b^9)*cos(d*x + c)^4 - (70*a^7*b^3 + 119*a^5*b^
5 + 48*a^3*b^7 + 3*a*b^9)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(-a^2 + b^2)*lo
g(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*co
```

s(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) + 420*(6*a^10*b + 17*a^8*b^3 - 7*a^6*b^5 - 13*a^4*b^7 - 3*a^2*b^9)*cos(d*x + c) - 14*((4*a^9*b^2 - 44*a^7*b^4 - 177*a^5*b^6 + 200*a^3*b^8 + 17*a*b^10)*cos(d*x + c)^5 - 10*(2*a^11 - 21*a^9*b^2 - 61*a^7*b^4 + 37*a^5*b^6 + 39*a^3*b^8 + 4*a*b^10)*cos(d*x + c)^3 - 15*(2*a^11 + 29*a^9*b^2 + 14*a^7*b^4 - 28*a^5*b^6 - 16*a^3*b^8 - a*b^10)*cos(d*x + c))*sin(d*x + c))/(7*(a^13*b^6 - 6*a^11*b^8 + 15*a^9*b^10 - 20*a^7*b^12 + 15*a^5*b^14 - 6*a^3*b^16 + a*b^18)*d*cos(d*x + c)^6 - 7*(5*a^15*b^4 - 27*a^13*b^6 + 57*a^11*b^8 - 55*a^9*b^10 + 15*a^7*b^12 + 15*a^5*b^14 - 13*a^3*b^16 + 3*a*b^18)*d*cos(d*x + c)^4 + 7*(3*a^17*b^2 - 8*a^15*b^4 - 12*a^13*b^6 + 72*a^11*b^8 - 110*a^9*b^10 + 72*a^7*b^12 - 12*a^5*b^14 - 8*a^3*b^16 + 3*a*b^18)*d*cos(d*x + c)^2 - (a^19 + 15*a^17*b^2 - 76*a^15*b^4 + 92*a^13*b^6 + 78*a^11*b^8 - 286*a^9*b^10 + 260*a^7*b^12 - 84*a^5*b^14 - 7*a^3*b^16 + 7*a*b^18)*d + ((a^12*b^7 - 6*a^10*b^9 + 15*a^8*b^11 - 20*a^6*b^13 + 15*a^4*b^15 - 6*a^2*b^17 + b^19)*d*cos(d*x + c)^6 - 3*(7*a^14*b^5 - 41*a^12*b^7 + 99*a^10*b^9 - 125*a^8*b^11 + 85*a^6*b^13 - 27*a^4*b^15 + a^2*b^17 + b^19)*d*cos(d*x + c)^4 + (35*a^16*b^3 - 168*a^14*b^5 + 276*a^12*b^7 - 88*a^10*b^9 - 270*a^8*b^11 + 360*a^6*b^13 - 172*a^4*b^15 + 24*a^2*b^17 + 3*b^19)*d*cos(d*x + c)^2 - (7*a^18*b - 7*a^16*b^3 - 84*a^14*b^5 + 260*a^12*b^7 - 286*a^10*b^9 + 78*a^8*b^11 + 92*a^6*b^13 - 76*a^4*b^15 + 15*a^2*b^17 + b^19)*d)*sin(d*x + c)), -1/560*((4*a^8*b^3 - 44*a^6*b^5 - 207*a^4*b^7 + 215*a^2*b^9 + 32*b^11)*cos(d*x + c)^7 - 14*(6*a^10*b - 65*a^8*b^3 - 224*a^6*b^5 + 222*a^4*b^7 + 53*a^2*b^9 + 8*b^11)*cos(d*x + c)^5 - 35*(14*a^10*b + 173*a^8*b^3 - 3*a^6*b^5 - 137*a^4*b^7 - 47*a^2*b^9)*cos(d*x + c)^3 - 105*(2*a^10 + 43*a^8*b^2 + 91*a^6*b^4 + 49*a^4*b^6 + 7*a^2*b^8 - 7*(2*a^4*b^6 + a^2*b^8)*cos(d*x + c)^6 + 7*(10*a^6*b^4 + 11*a^4*b^6 + 3*a^2*b^8)*cos(d*x + c)^4 - 7*(6*a^8*b^2 + 23*a^6*b^4 + 16*a^4*b^6 + 3*a^2*b^8)*cos(d*x + c)^2 + (14*a^9*b + 77*a^7*b^3 + 77*a^5*b^5 + 23*a^3*b^7 + a*b^9 - (2*a^3*b^7 + a*b^9)*cos(d*x + c)^6 + 3*(14*a^5*b^5 + 9*a^3*b^7 + a*b^9)*cos(d*x + c)^4 - (70*a^7*b^3 + 119*a^5*b^5 + 48*a^3*b^7 + 3*a*b^9)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) + 210*(6*a^10*b + 17*a^8*b^3 - 7*a^6*b^5 - 13*a^4*b^7 - 3*a^2*b^9)*cos(d*x + c) - 7*((4*a^9*b^2 - 44*a^7*b^4 - 177*a^5*b^6 + 200*a^3*b^8 + 17*a*b^10)*cos(d*x + c)^5 - 10*(2*a^11 - 21*a^9*b^2 - 61*a^7*b^4 + 37*a^5*b^6 + 39*a^3*b^8 + 4*a*b^10)*cos(d*x + c)^3 - 15*(2*a^11 + 29*a^9*b^2 + 14*a^7*b^4 - 28*a^5*b^6 - 16*a^3*b^8 - a*b^10)*cos(d*x + c))*sin(d*x + c))/(7*(a^13*b^6 - 6*a^11*b^8 + 15*a^9*b^10 - 20*a^7*b^12 + 15*a^5*b^14 - 6*a^3*b^16 + a*b^18)*d*cos(d*x + c)^6 - 7*(5*a^15*b^4 - 27*a^13*b^6 + 57*a^11*b^8 - 55*a^9*b^10 + 15*a^7*b^12 + 15*a^5*b^14 - 13*a^3*b^16 + 3*a*b^18)*d*cos(d*x + c)^4 + 7*(3*a^17*b^2 - 8*a^15*b^4 - 12*a^13*b^6 + 72*a^11*b^8 - 110*a^9*b^10 + 72*a^7*b^12 - 12*a^5*b^14 - 8*a^3*b^16 + 3*a*b^18)*d*cos(d*x + c)^2 - (a^19 + 15*a^17*b^2 - 76*a^15*b^4 + 92*a^13*b^6 + 78*a^11*b^8 - 286*a^9*b^10 + 260*a^7*b^12 - 84*a^5*b^14 - 7*a^3*b^16 + 7*a*b^18)*d + ((a^12*b^7 - 6*a^10*b^9 + 15*a^8*b^11 - 20*a^6*b^13 + 15*a^4*b^15 - 6*a^2*b^17 + b^19)*d*cos(d*x + c)^6 - 3*(7*a^14*b^5 - 41*a^12*b^7 + 99*a^10*b^9 - 125*a^8*b^11 + 85*a^6*b^13 - 2

```
7*a^4*b^15 + a^2*b^17 + b^19)*d*cos(d*x + c)^4 + (35*a^16*b^3 - 168*a^14*b^
5 + 276*a^12*b^7 - 88*a^10*b^9 - 270*a^8*b^11 + 360*a^6*b^13 - 172*a^4*b^15
 + 24*a^2*b^17 + 3*b^19)*d*cos(d*x + c)^2 - (7*a^18*b - 7*a^16*b^3 - 84*a^1
4*b^5 + 260*a^12*b^7 - 286*a^10*b^9 + 78*a^8*b^11 + 92*a^6*b^13 - 76*a^4*b^
15 + 15*a^2*b^17 + b^19)*d)*sin(d*x + c))]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+b*sin(d*x+c))**8,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1932 vs. $2(390) = 780$.
time = 5.29, size = 1932, normalized size = 4.70

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/280*(105*(2*a^3 + a*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arcta
n((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/((a^10 - 5*a^8*b^2 + 10*a^
6*b^4 - 10*a^4*b^6 + 5*a^2*b^8 - b^10)*sqrt(a^2 - b^2)) - (350*a^16*tan(1/2
*d*x + 1/2*c)^13 - 2905*a^14*b^2*tan(1/2*d*x + 1/2*c)^13 + 5600*a^12*b^4*ta
n(1/2*d*x + 1/2*c)^13 - 5600*a^10*b^6*tan(1/2*d*x + 1/2*c)^13 + 2800*a^8*b^
8*tan(1/2*d*x + 1/2*c)^13 - 560*a^6*b^10*tan(1/2*d*x + 1/2*c)^13 + 630*a^15
*b*tan(1/2*d*x + 1/2*c)^12 - 18165*a^13*b^3*tan(1/2*d*x + 1/2*c)^12 + 33600
*a^11*b^5*tan(1/2*d*x + 1/2*c)^12 - 33600*a^9*b^7*tan(1/2*d*x + 1/2*c)^12 +
 16800*a^7*b^9*tan(1/2*d*x + 1/2*c)^12 - 3360*a^5*b^11*tan(1/2*d*x + 1/2*c)
^12 + 840*a^16*tan(1/2*d*x + 1/2*c)^11 - 15680*a^14*b^2*tan(1/2*d*x + 1/2*c
)^11 - 41090*a^12*b^4*tan(1/2*d*x + 1/2*c)^11 + 89600*a^10*b^6*tan(1/2*d*x
+ 1/2*c)^11 - 100800*a^8*b^8*tan(1/2*d*x + 1/2*c)^11 + 53760*a^6*b^10*tan(1
/2*d*x + 1/2*c)^11 - 11200*a^4*b^12*tan(1/2*d*x + 1/2*c)^11 - 840*a^15*b*ta
n(1/2*d*x + 1/2*c)^10 - 102760*a^13*b^3*tan(1/2*d*x + 1/2*c)^10 + 11270*a^1
1*b^5*tan(1/2*d*x + 1/2*c)^10 + 78400*a^9*b^7*tan(1/2*d*x + 1/2*c)^10 - 151
200*a^7*b^9*tan(1/2*d*x + 1/2*c)^10 + 97440*a^5*b^11*tan(1/2*d*x + 1/2*c)^1
0 - 22400*a^3*b^13*tan(1/2*d*x + 1/2*c)^10 + 630*a^16*tan(1/2*d*x + 1/2*c)^
9 - 51905*a^14*b^2*tan(1/2*d*x + 1/2*c)^9 - 249410*a^12*b^4*tan(1/2*d*x + 1
/2*c)^9 + 202244*a^10*b^6*tan(1/2*d*x + 1/2*c)^9 - 129360*a^8*b^8*tan(1/2*d
*x + 1/2*c)^9 - 62832*a^6*b^10*tan(1/2*d*x + 1/2*c)^9 + 92288*a^4*b^12*tan(
1/2*d*x + 1/2*c)^9 - 26880*a^2*b^14*tan(1/2*d*x + 1/2*c)^9 - 8330*a^15*b*ta
```

```
n(1/2*d*x + 1/2*c)^8 - 248745*a^13*b^3*tan(1/2*d*x + 1/2*c)^8 - 190610*a^11
*b^5*tan(1/2*d*x + 1/2*c)^8 + 253736*a^9*b^7*tan(1/2*d*x + 1/2*c)^8 - 33824
0*a^7*b^9*tan(1/2*d*x + 1/2*c)^8 + 120512*a^5*b^11*tan(1/2*d*x + 1/2*c)^8 +
 24192*a^3*b^13*tan(1/2*d*x + 1/2*c)^8 - 17920*a*b^15*tan(1/2*d*x + 1/2*c)^
8 - 96040*a^14*b^2*tan(1/2*d*x + 1/2*c)^7 - 452340*a^12*b^4*tan(1/2*d*x + 1
/2*c)^7 + 164528*a^10*b^6*tan(1/2*d*x + 1/2*c)^7 - 99344*a^8*b^8*tan(1/2*d*
x + 1/2*c)^7 - 177664*a^6*b^10*tan(1/2*d*x + 1/2*c)^7 + 153088*a^4*b^12*tan
(1/2*d*x + 1/2*c)^7 - 27648*a^2*b^14*tan(1/2*d*x + 1/2*c)^7 - 5120*b^16*tan
(1/2*d*x + 1/2*c)^7 - 15680*a^15*b*tan(1/2*d*x + 1/2*c)^6 - 296520*a^13*b^3
*tan(1/2*d*x + 1/2*c)^6 - 247940*a^11*b^5*tan(1/2*d*x + 1/2*c)^6 + 232736*a
^9*b^7*tan(1/2*d*x + 1/2*c)^6 - 339920*a^7*b^9*tan(1/2*d*x + 1/2*c)^6 + 120
512*a^5*b^11*tan(1/2*d*x + 1/2*c)^6 + 24192*a^3*b^13*tan(1/2*d*x + 1/2*c)^6
 - 17920*a*b^15*tan(1/2*d*x + 1/2*c)^6 - 630*a^16*tan(1/2*d*x + 1/2*c)^5 -
92155*a^14*b^2*tan(1/2*d*x + 1/2*c)^5 - 333060*a^12*b^4*tan(1/2*d*x + 1/2*c
)^5 + 151144*a^10*b^6*tan(1/2*d*x + 1/2*c)^5 - 133280*a^8*b^8*tan(1/2*d*x +
 1/2*c)^5 - 62832*a^6*b^10*tan(1/2*d*x + 1/2*c)^5 + 92288*a^4*b^12*tan(1/2*
d*x + 1/2*c)^5 - 26880*a^2*b^14*tan(1/2*d*x + 1/2*c)^5 - 13566*a^15*b*tan(1
/2*d*x + 1/2*c)^4 - 166775*a^13*b^3*tan(1/2*d*x + 1/2*c)^4 - 41412*a^11*b^5
*tan(1/2*d*x + 1/2*c)^4 + 72128*a^9*b^7*tan(1/2*d*x + 1/2*c)^4 - 150640*a^7
*b^9*tan(1/2*d*x + 1/2*c)^4 + 97440*a^5*b^11*tan(1/2*d*x + 1/2*c)^4 - 22400
*a^3*b^13*tan(1/2*d*x + 1/2*c)^4 - 840*a^16*tan(1/2*d*x + 1/2*c)^3 - 41944*
a^14*b^2*tan(1/2*d*x + 1/2*c)^3 - 76650*a^12*b^4*tan(1/2*d*x + 1/2*c)^3 + 8
7472*a^10*b^6*tan(1/2*d*x + 1/2*c)^3 - 100688*a^8*b^8*tan(1/2*d*x + 1/2*c)^
3 + 53760*a^6*b^10*tan(1/2*d*x + 1/2*c)^3 - 11200*a^4*b^12*tan(1/2*d*x + 1/
2*c)^3 - 5432*a^15*b*tan(1/2*d*x + 1/2*c)^2 - 33264*a^13*b^3*tan(1/2*d*x +
1/2*c)^2 + 34846*a^11*b^5*tan(1/2*d*x + 1/2*c)^2 - 34272*a^9*b^7*tan(1/2*d*
x + 1/2*c)^2 + 16912*a^7*b^9*tan(1/2*d*x + 1/2*c)^2 - 3360*a^5*b^11*tan(1/2
*d*x + 1/2*c)^2 - 350*a^16*tan(1/2*d*x + 1/2*c) - 6699*a^14*b^2*tan(1/2*d*x
 + 1/2*c) + 6790*a^12*b^4*tan(1/2*d*x + 1/2*c) - 6188*a^10*b^6*tan(1/2*d*x
+ 1/2*c) + 2912*a^8*b^8*tan(1/2*d*x + 1/2*c) - 560*a^6*b^10*tan(1/2*d*x + 1
/2*c) - 686*a^15*b + 885*a^13*b^3 - 842*a^11*b^5 + 408*a^9*b^7 - 80*a^7*b^9
)/((a^17 - 5*a^15*b^2 + 10*a^13*b^4 - 10*a^11*b^6 + 5*a^9*b^8 - a^7*b^10)*(
a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)^7))/d
```

Mupad [B]
time = 10.85, size = 2184, normalized size = 5.31

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(a + b*sin(c + d*x))^8,x)
```

```
[Out] ((686*a^8*b + 80*b^9 - 408*a^2*b^7 + 842*a^4*b^5 - 885*a^6*b^3)/(280*(a^10
- b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)) + (tan(c/2 + (d*
x)/2)*(50*a^10 + 80*b^10 - 416*a^2*b^8 + 884*a^4*b^6 - 970*a^6*b^4 + 957*a^
```

8*b^2))/(40*a*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^
2)) + (tan(c/2 + (d*x)/2)^9*(3840*b^14 - 90*a^14 - 13184*a^2*b^12 + 8976*a^
4*b^10 + 18480*a^6*b^8 - 28892*a^8*b^6 + 35630*a^10*b^4 + 7415*a^12*b^2))/(
40*a^5*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)) + (
tan(c/2 + (d*x)/2)^5*(90*a^14 + 3840*b^14 - 13184*a^2*b^12 + 8976*a^4*b^10
+ 19040*a^6*b^8 - 21592*a^8*b^6 + 47580*a^10*b^4 + 13165*a^12*b^2))/(40*a^5
*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)) + (tan(c/
2 + (d*x)/2)^11*(160*b^12 - 12*a^12 - 768*a^2*b^10 + 1440*a^4*b^8 - 1280*a^
6*b^6 + 587*a^8*b^4 + 224*a^10*b^2))/(4*a^3*(a^10 - b^10 + 5*a^2*b^8 - 10*a
^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)) + (tan(c/2 + (d*x)/2)^3*(60*a^12 + 800*b^
12 - 3840*a^2*b^10 + 7192*a^4*b^8 - 6248*a^6*b^6 + 5475*a^8*b^4 + 2996*a^10
*b^2))/(20*a^3*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b
^2)) - (tan(c/2 + (d*x)/2)^13*(10*a^10 - 16*b^10 + 80*a^2*b^8 - 160*a^4*b^6
 + 160*a^6*b^4 - 83*a^8*b^2))/(8*a*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 +
10*a^6*b^4 - 5*a^8*b^2)) + (tan(c/2 + (d*x)/2)^6*(560*a^14*b + 640*b^15 - 8
64*a^2*b^13 - 4304*a^4*b^11 + 12140*a^6*b^9 - 8312*a^8*b^7 + 8855*a^10*b^5
+ 10590*a^12*b^3))/(10*a^6*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b
^4 - 5*a^8*b^2)) + (tan(c/2 + (d*x)/2)^8*(1190*a^14*b + 2560*b^15 - 3456*a^
2*b^13 - 17216*a^4*b^11 + 48320*a^6*b^9 - 36248*a^8*b^7 + 27230*a^10*b^5 +
35535*a^12*b^3))/(40*a^6*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4
 - 5*a^8*b^2)) + (tan(c/2 + (d*x)/2)^10*(12*a^12*b + 320*b^13 - 1392*a^2*b^
11 + 2160*a^4*b^9 - 1120*a^6*b^7 - 161*a^8*b^5 + 1468*a^10*b^3))/(4*a^4*(a^
10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)) + (tan(c/2 +
(d*x)/2)^4*(1938*a^12*b + 3200*b^13 - 13920*a^2*b^11 + 21520*a^4*b^9 - 1030
4*a^6*b^7 + 5916*a^8*b^5 + 23825*a^10*b^3))/(40*a^4*(a^10 - b^10 + 5*a^2*b^
8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)) - (3*tan(c/2 + (d*x)/2)^12*(6*a^1
0*b - 32*b^11 + 160*a^2*b^9 - 320*a^4*b^7 + 320*a^6*b^5 - 173*a^8*b^3))/(8*
a^2*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)) + (tan
(c/2 + (d*x)/2)^2*(388*a^10*b + 240*b^11 - 1208*a^2*b^9 + 2448*a^4*b^7 - 24
89*a^6*b^5 + 2376*a^8*b^3))/(20*a^2*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 +
 10*a^6*b^4 - 5*a^8*b^2)) + (b*tan(c/2 + (d*x)/2)^7*(35*a^6 + 16*b^6 + 168*
a^2*b^4 + 210*a^4*b^2)*(686*a^8*b + 80*b^9 - 408*a^2*b^7 + 842*a^4*b^5 - 88
5*a^6*b^3))/(70*a^7*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*
a^8*b^2)))/(d*(tan(c/2 + (d*x)/2)^5*(210*a^6*b + 672*a^2*b^5 + 1120*a^4*b^3
) + tan(c/2 + (d*x)/2)^9*(210*a^6*b + 672*a^2*b^5 + 1120*a^4*b^3) + a^7*tan
(c/2 + (d*x)/2)^14 + tan(c/2 + (d*x)/2)^3*(84*a^6*b + 280*a^4*b^3) + tan(c/
2 + (d*x)/2)^11*(84*a^6*b + 280*a^4*b^3) + tan(c/2 + (d*x)/2)^6*(448*a*b^6
+ 35*a^7 + 1680*a^3*b^4 + 840*a^5*b^2) + tan(c/2 + (d*x)/2)^8*(448*a*b^6 +
35*a^7 + 1680*a^3*b^4 + 840*a^5*b^2) + tan(c/2 + (d*x)/2)^7*(280*a^6*b + 12
8*b^7 + 1344*a^2*b^5 + 1680*a^4*b^3) + a^7 + tan(c/2 + (d*x)/2)^4*(21*a^7 +
 560*a^3*b^4 + 420*a^5*b^2) + tan(c/2 + (d*x)/2)^10*(21*a^7 + 560*a^3*b^4 +
 420*a^5*b^2) + tan(c/2 + (d*x)/2)^2*(7*a^7 + 84*a^5*b^2) + tan(c/2 + (d*x)
/2)^12*(7*a^7 + 84*a^5*b^2) + 14*a^6*b*tan(c/2 + (d*x)/2) + 14*a^6*b*tan(c/
2 + (d*x)/2)^13)) + (3*a*atan((8*((3*a^2*tan(c/2 + (d*x)/2)*(2*a^2 + b^2))/
(8*(a + b)^(11/2)*(a - b)^(11/2)) + (3*a*(2*a^2 + b^2)*(16*a^10*b - 16*b^11

```
 + 80*a^2*b^9 - 160*a^4*b^7 + 160*a^6*b^5 - 80*a^8*b^3))/(128*(a + b)^(11/2
)*(a - b)^(11/2)*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8
*b^2)))*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2))/(3
*a*b^2 + 6*a^3))*(2*a^2 + b^2))/(8*d*(a + b)^(11/2)*(a - b)^(11/2))
```

3.470 $\int \frac{\cos^2(c+dx)}{(a+b\sin(c+dx))^8}\,dx$

Optimal. Leaf size=422

$$\frac{a\left(8a^4+20a^2b^2+5b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{8\left(a^2-b^2\right)^{13/2}d}-\frac{\cos(c+dx)}{7bd(a+b\sin(c+dx))^7}+\frac{a\cos(c+dx)}{42b\left(a^2-b^2\right)d(a+b\sin(c+dx))^6}$$

```
[Out] 1/8*a*(8*a^4+20*a^2*b^2+5*b^4)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1
/2))/(a^2-b^2)^(13/2)/d-1/7*cos(d*x+c)/b/d/(a+b*sin(d*x+c))^7+1/42*a*cos(d*
x+c)/b/(a^2-b^2)/d/(a+b*sin(d*x+c))^6+1/210*(5*a^2+6*b^2)*cos(d*x+c)/b/(a^2
-b^2)^2/d/(a+b*sin(d*x+c))^5+1/840*a*(20*a^2+79*b^2)*cos(d*x+c)/b/(a^2-b^2)
^3/d/(a+b*sin(d*x+c))^4+1/840*(20*a^4+179*a^2*b^2+32*b^4)*cos(d*x+c)/b/(a^2
-b^2)^4/d/(a+b*sin(d*x+c))^3+1/1680*a*(40*a^4+718*a^2*b^2+397*b^4)*cos(d*x+
c)/b/(a^2-b^2)^5/d/(a+b*sin(d*x+c))^2+1/1680*(40*a^6+1518*a^4*b^2+1779*a^2*
b^4+128*b^6)*cos(d*x+c)/b/(a^2-b^2)^6/d/(a+b*sin(d*x+c))
```

Rubi [A]
time = 0.50, antiderivative size = 422, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2772, 2833, 12, 2739, 632, 210}

$$\frac{a\left(20a^2+79b^2\right)\cos(c+dx)}{840bd\left(a^2-b^2\right)^3(a+b\sin(c+dx))^4}+\frac{\left(5a^2+6b^2\right)\cos(c+dx)}{210bd\left(a^2-b^2\right)^2(a+b\sin(c+dx))^5}+\frac{a\cos(c+dx)}{42bd\left(a^2-b^2\right)(a+b\sin(c+dx))^6}+\frac{a\left(8a^4+20a^2b^2+5b^4\right)\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{8d\left(a^2-b^2\right)^{13/2}}+\frac{a\left(40a^4+718a^2b^2+397b^4\right)\cos(c+dx)}{1680bd\left(a^2-b^2\right)^5(a+b\sin(c+dx))^2}+\frac{\left(20a^4+179a^2b^2+32b^4\right)\cos(c+dx)}{840bd\left(a^2-b^2\right)^4(a+b\sin(c+dx))^3}+\frac{\left(40a^6+1518a^4b^2+1779a^2b^4+128b^6\right)\cos(c+dx)}{1680bd\left(a^2-b^2\right)^6(a+b\sin(c+dx))}-\frac{\cos(c+dx)}{7bd(a+b\sin(c+dx))^7}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + b*Sin[c + d*x])^8,x]
```

```
[Out] (a*(8*a^4 + 20*a^2*b^2 + 5*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 -
b^2]])/(8*(a^2 - b^2)^(13/2)*d) - Cos[c + d*x]/(7*b*d*(a + b*Sin[c + d*x])^
7) + (a*Cos[c + d*x])/(42*b*(a^2 - b^2)*d*(a + b*Sin[c + d*x])^6) + ((5*a^2
 + 6*b^2)*Cos[c + d*x])/(210*b*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x])^5) + (a
*(20*a^2 + 79*b^2)*Cos[c + d*x])/(840*b*(a^2 - b^2)^3*d*(a + b*Sin[c + d*x]
)^4) + ((20*a^4 + 179*a^2*b^2 + 32*b^4)*Cos[c + d*x])/(840*b*(a^2 - b^2)^4*
d*(a + b*Sin[c + d*x])^3) + (a*(40*a^4 + 718*a^2*b^2 + 397*b^4)*Cos[c + d*x
])/(1680*b*(a^2 - b^2)^5*d*(a + b*Sin[c + d*x])^2) + ((40*a^6 + 1518*a^4*b^
2 + 1779*a^2*b^4 + 128*b^6)*Cos[c + d*x])/(1680*b*(a^2 - b^2)^6*d*(a + b*Si
n[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2772

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Dist[g^2*((p - 1)/(b*(m + 1))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a+b\sin(c+dx))^8}\,dx = -\frac{\cos(c+dx)}{7bd(a+b\sin(c+dx))^7} - \frac{\int \frac{\sin(c+dx)}{(a+b\sin(c+dx))^7}\,dx}{7b}$$

$$= -\frac{\cos(c+dx)}{7bd(a+b\sin(c+dx))^7} + \frac{a\cos(c+dx)}{42b\,(a^2-b^2)\,d(a+b\sin(c+dx))^6} + \frac{\int \frac{6b-5a\sin(c+dx}{(a+b\sin(c+dx))}}{42b\,(a^2-b}$$

$$= -\frac{\cos(c+dx)}{7bd(a+b\sin(c+dx))^7} + \frac{a\cos(c+dx)}{42b\,(a^2-b^2)\,d(a+b\sin(c+dx))^6} + \frac{(5a^2+6}{210b\,(a^2-b^2)}$$

$$= -\frac{\cos(c+dx)}{7bd(a+b\sin(c+dx))^7} + \frac{a\cos(c+dx)}{42b\,(a^2-b^2)\,d(a+b\sin(c+dx))^6} + \frac{(5a^2+6}{210b\,(a^2-b^2)}$$

$$= -\frac{\cos(c+dx)}{7bd(a+b\sin(c+dx))^7} + \frac{a\cos(c+dx)}{42b\,(a^2-b^2)\,d(a+b\sin(c+dx))^6} + \frac{(5a^2+6}{210b\,(a^2-b^2)}$$

$$= -\frac{\cos(c+dx)}{7bd(a+b\sin(c+dx))^7} + \frac{a\cos(c+dx)}{42b\,(a^2-b^2)\,d(a+b\sin(c+dx))^6} + \frac{(5a^2+6}{210b\,(a^2-b^2)}$$

$$= -\frac{\cos(c+dx)}{7bd(a+b\sin(c+dx))^7} + \frac{a\cos(c+dx)}{42b\,(a^2-b^2)\,d(a+b\sin(c+dx))^6} + \frac{(5a^2+6}{210b\,(a^2-b^2)}$$

$$= -\frac{\cos(c+dx)}{7bd(a+b\sin(c+dx))^7} + \frac{a\cos(c+dx)}{42b\,(a^2-b^2)\,d(a+b\sin(c+dx))^6} + \frac{(5a^2+6}{210b\,(a^2-b^2)}$$

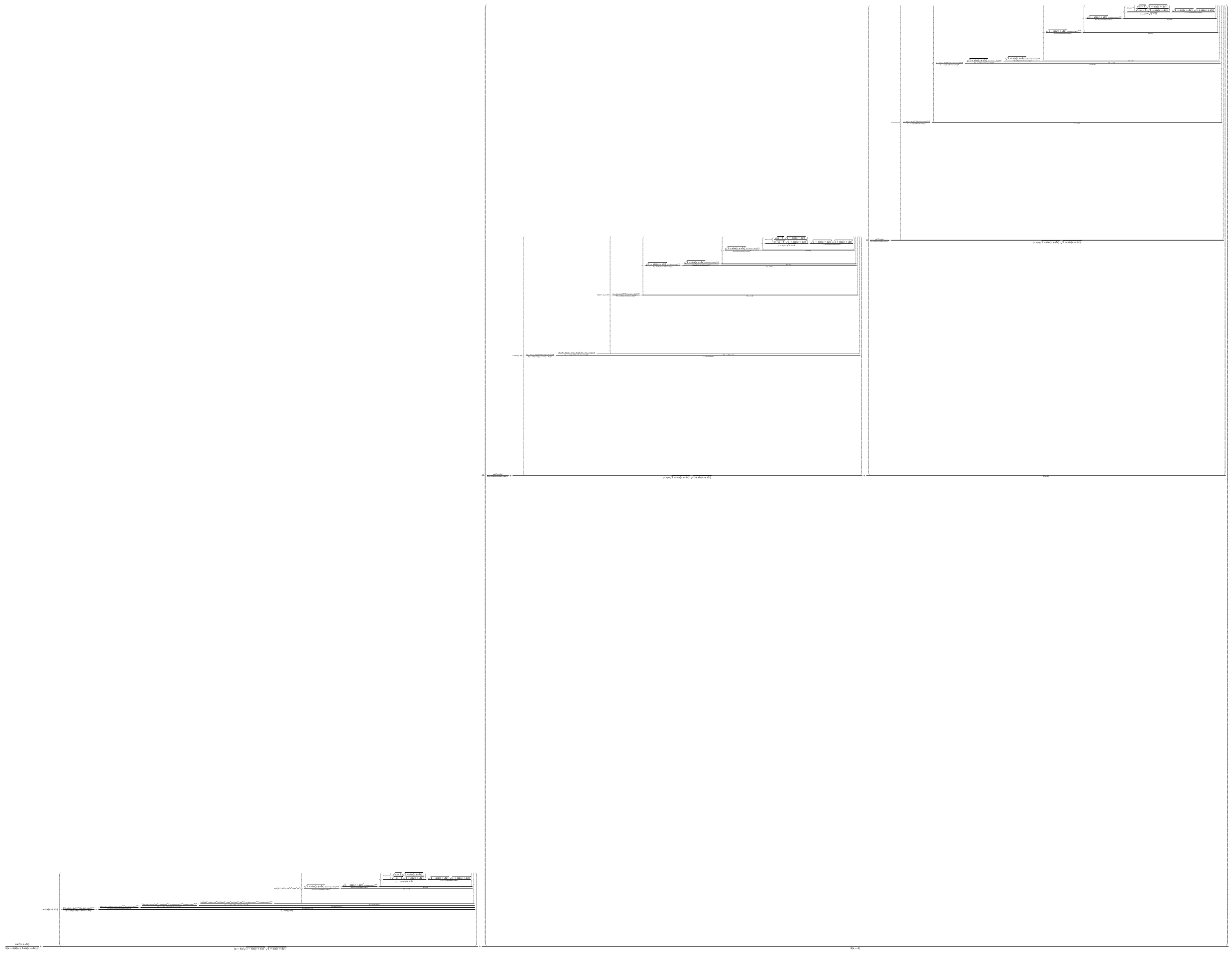
$$= -\frac{\cos(c+dx)}{7bd(a+b\sin(c+dx))^7} + \frac{a\cos(c+dx)}{42b\,(a^2-b^2)\,d(a+b\sin(c+dx))^6} + \frac{(5a^2+6}{210b\,(a^2-b^2)}$$

$$= -\frac{\cos(c+dx)}{7bd(a+b\sin(c+dx))^7} + \frac{a\cos(c+dx)}{42b\,(a^2-b^2)\,d(a+b\sin(c+dx))^6} + \frac{(5a^2+6}{210b\,(a^2-b^2)}$$

$$= \frac{a(8a^4+20a^2b^2+5b^4)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{8\,(a^2-b^2)^{13/2}\,d} - \frac{\cos(c+dx)}{7bd(a+b\sin(c+dx))^7} + \frac{}{42b}$$

<u>**Mathematica**</u> [**B**] Leaf count is larger than twice the leaf count of optimal. 1896 vs. $2(422) = 844$.

time = 6.20, size = 1896, normalized size = 4.49



Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + b*Sin[c + d*x])^8,x]
```

```
[Out] Cos[c + d*x]^3/(3*(a - b)*d*(a + b*Sin[c + d*x])^7) + (a*Cos[c + d*x]*(-1/7
*(b*(1 - Sin[c + d*x])^(3/2)*(1 + Sin[c + d*x])^(5/2))/((-a + b)*(a + b)*(a
 + b*Sin[c + d*x])^7) - (-1/6*((3*a*b + (7*a - b)*b)*(1 - Sin[c + d*x])^(3/
2)*(1 + Sin[c + d*x])^(5/2))/((-a + b)*(a + b)*(a + b*Sin[c + d*x])^6) - (-
1/5*((2*a*(10*a - b)*b + b*(42*a^2 - 16*a*b + 19*b^2))*(1 - Sin[c + d*x])^(
3/2)*(1 + Sin[c + d*x])^(5/2))/((-a + b)*(a + b)*(a + b*Sin[c + d*x])^5) -
(-1/4*((a*b*(62*a^2 - 18*a*b + 19*b^2) + b*(210*a^3 - 142*a^2*b + 213*a*b^2
 - 29*b^3))*(1 - Sin[c + d*x])^(3/2)*(1 + Sin[c + d*x])^(5/2))/((-a + b)*(a
 + b)*(a + b*Sin[c + d*x])^4) - (105*(8*a^4 - 8*a^3*b + 12*a^2*b^2 - 4*a*b^
3 + b^4)*(-1/3*(Sqrt[1 - Sin[c + d*x]]*(1 + Sin[c + d*x])^(5/2))/((-a + b)*
(a + b*Sin[c + d*x])^3) - (-1/2*(Sqrt[1 - Sin[c + d*x]]*(1 + Sin[c + d*x])^
(3/2))/((a + b)*(a + b*Sin[c + d*x])^2) + (3*((-2*ArcTanh[(Sqrt[a - b]*Sqrt
[1 - Sin[c + d*x]])/(Sqrt[-a - b]*Sqrt[1 + Sin[c + d*x]])])/((-a - b)^(3/2)
*Sqrt[a - b]) + (Sqrt[1 - Sin[c + d*x]]*Sqrt[1 + Sin[c + d*x]])/((-a - b)*(
```

```
a + b*Sin[c + d*x]))))/(2*(a + b)))/(3*(-a + b))))/(4*(-a + b)*(a + b)))/(5
*(-a + b)*(a + b)))/(6*(-a + b)*(a + b)))/(7*(-a + b)*(a + b))))/((a - b)*d
*Sqrt[1 - Sin[c + d*x]]*Sqrt[1 + Sin[c + d*x]]) + (4*b*(Cos[c + d*x]^5/(5*(
a - b)*d*(a + b*Sin[c + d*x])^7) + (a*Cos[c + d*x]*(-1/7*(b*(1 - Sin[c + d*
x])^(5/2)*(1 + Sin[c + d*x])^(7/2))/((-a + b)*(a + b)*(a + b*Sin[c + d*x])^
7) - (-1/6*((a*b + (7*a - b)*b)*(1 - Sin[c + d*x])^(5/2)*(1 + Sin[c + d*x])
^(7/2))/((-a + b)*(a + b)*(a + b*Sin[c + d*x])^6) - (7*(6*a^2 - 2*a*b + b^2
)*(-1/5*((1 - Sin[c + d*x])^(3/2)*(1 + Sin[c + d*x])^(7/2))/((-a + b)*(a +
b*Sin[c + d*x])^5) - (3*(-1/4*(Sqrt[1 - Sin[c + d*x]]*(1 + Sin[c + d*x])^(7
/2))/((-a + b)*(a + b*Sin[c + d*x])^4) - (-1/3*(Sqrt[1 - Sin[c + d*x]]*(1 +
 Sin[c + d*x])^(5/2))/((a + b)*(a + b*Sin[c + d*x])^3) + (5*(-1/2*(Sqrt[1 -
 Sin[c + d*x]]*(1 + Sin[c + d*x])^(3/2))/((a + b)*(a + b*Sin[c + d*x])^2) +
 (3*((-2*ArcTanh[(Sqrt[a - b]*Sqrt[1 - Sin[c + d*x]])/(Sqrt[-a - b]*Sqrt[1
+ Sin[c + d*x]])])/((-a - b)^(3/2)*Sqrt[a - b]) + (Sqrt[1 - Sin[c + d*x]]*S
qrt[1 + Sin[c + d*x]])/((-a - b)*(a + b*Sin[c + d*x]))))/(2*(a + b))))/(3*(
a + b)))/(4*(-a + b))))/(5*(-a + b))))/(6*(-a + b)*(a + b)))/(7*(-a + b)*(a
 + b))))/((a - b)*d*Sqrt[1 - Sin[c + d*x]]*Sqrt[1 + Sin[c + d*x]]) + (2*b*(
Cos[c + d*x]^7/(7*(a - b)*d*(a + b*Sin[c + d*x])^7) + (a*Cos[c + d*x]*(-1/7
*((1 - Sin[c + d*x])^(5/2)*(1 + Sin[c + d*x])^(9/2))/((-a + b)*(a + b*Sin[c
 + d*x])^7) - (5*(-1/6*((1 - Sin[c + d*x])^(3/2)*(1 + Sin[c + d*x])^(9/2))/
((-a + b)*(a + b*Sin[c + d*x])^6) - (-1/5*(Sqrt[1 - Sin[c + d*x]]*(1 + Sin[
c + d*x])^(9/2))/((-a + b)*(a + b*Sin[c + d*x])^5) - (-1/4*(Sqrt[1 - Sin[c
+ d*x]]*(1 + Sin[c + d*x])^(7/2))/((a + b)*(a + b*Sin[c + d*x])^4) + (7*(-1
/3*(Sqrt[1 - Sin[c + d*x]]*(1 + Sin[c + d*x])^(5/2))/((a + b)*(a + b*Sin[c
+ d*x])^3) + (5*(-1/2*(Sqrt[1 - Sin[c + d*x]]*(1 + Sin[c + d*x])^(3/2))/((a
 + b)*(a + b*Sin[c + d*x])^2) + (3*((-2*ArcTanh[(Sqrt[a - b]*Sqrt[1 - Sin[c
 + d*x]])/(Sqrt[-a - b]*Sqrt[1 + Sin[c + d*x]])])/((-a - b)^(3/2)*Sqrt[a -
b]) + (Sqrt[1 - Sin[c + d*x]]*Sqrt[1 + Sin[c + d*x]])/((-a - b)*(a + b*Sin[
c + d*x]))))/(2*(a + b))))/(3*(a + b))))/(4*(a + b)))/(5*(-a + b)))/(2*(-a
+ b))))/(7*(-a + b))))/((a - b)*d*Sqrt[1 - Sin[c + d*x]]*Sqrt[1 + Sin[c + d
*x]])))/(5*(a - b))))/(3*(a - b))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1871 vs. $2(401) = 802$.
time = 2.06, size = 1872, normalized size = 4.44

method	result	size
risch	Expression too large to display	1703
derivativedivides	Expression too large to display	1872
default	Expression too large to display	1872

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+b*sin(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*(-1/16*(8*a^12-116*a^10*b^2+235*a^8*b^4-320*a^6*b^6+240*a^4*b^8-96*a
^2*b^10+16*b^12)/a/(a^12-6*a^10*b^2+15*a^8*b^4-20*a^6*b^6+15*a^4*b^8-6*a^2*
b^10+b^12)*tan(1/2*d*x+1/2*c)^13+1/16*b*(8*a^12+836*a^10*b^2-1375*a^8*b^4+1
920*a^6*b^6-1440*a^4*b^8+576*a^2*b^10-96*b^12)/a^2/(a^12-6*a^10*b^2+15*a^8*
b^4-20*a^6*b^6+15*a^4*b^8-6*a^2*b^10+b^12)*tan(1/2*d*x+1/2*c)^12-1/24/a^3*(
48*a^14-1344*a^12*b^2-3150*a^10*b^4+4105*a^8*b^6-7680*a^6*b^8+6432*a^4*b^10
-2752*a^2*b^12+480*b^14)/(a^12-6*a^10*b^2+15*a^8*b^4-20*a^6*b^6+15*a^4*b^8-
6*a^2*b^10+b^12)*tan(1/2*d*x+1/2*c)^11+1/24/a^4*b*(192*a^14+7920*a^12*b^2+2
370*a^10*b^4+955*a^8*b^6+8880*a^6*b^8-10272*a^4*b^10+5072*a^2*b^12-960*b^14
)/(a^12-6*a^10*b^2+15*a^8*b^4-20*a^6*b^6+15*a^4*b^8-6*a^2*b^10+b^12)*tan(1/
2*d*x+1/2*c)^10-1/240/a^5*(600*a^16-43500*a^14*b^2-202575*a^12*b^4+21010*a^
10*b^6-188100*a^8*b^8+34656*a^6*b^10+62768*a^4*b^12-50304*a^2*b^14+11520*b^
16)/(a^12-6*a^10*b^2+15*a^8*b^4-20*a^6*b^6+15*a^4*b^8-6*a^2*b^10+b^12)*tan(
1/2*d*x+1/2*c)^9+1/240/a^6*b*(7000*a^16+193900*a^14*b^2+246615*a^12*b^4+495
10*a^10*b^6+281800*a^8*b^8-194304*a^6*b^10+42688*a^4*b^12+17536*a^2*b^14-76
80*b^16)/(a^12-6*a^10*b^2+15*a^8*b^4-20*a^6*b^6+15*a^4*b^8-6*a^2*b^10+b^12)
*tan(1/2*d*x+1/2*c)^8+1/420/a^7*b^2*(127400*a^16+671300*a^14*b^2+225225*a^1
2*b^4+517580*a^10*b^6+69524*a^8*b^8-238080*a^6*b^10+134528*a^4*b^12-17152*a
^2*b^14-3840*b^16)/(a^12-6*a^10*b^2+15*a^8*b^4-20*a^6*b^6+15*a^4*b^8-6*a^2*
b^10+b^12)*tan(1/2*d*x+1/2*c)^7+1/60/a^6*b*(2800*a^16+57400*a^14*b^2+83100*
a^12*b^4+29395*a^10*b^6+74800*a^8*b^8-48276*a^6*b^10+10672*a^4*b^12+4384*a^
2*b^14-1920*b^16)/(a^12-6*a^10*b^2+15*a^8*b^4-20*a^6*b^6+15*a^4*b^8-6*a^2*b
^10+b^12)*tan(1/2*d*x+1/2*c)^6+1/240/a^5*(600*a^16+65700*a^14*b^2+300025*a^
12*b^4+92540*a^10*b^6+234840*a^8*b^8-32656*a^6*b^10-62768*a^4*b^12+50304*a^
2*b^14-11520*b^16)/(a^12-6*a^10*b^2+15*a^8*b^4-20*a^6*b^6+15*a^4*b^8-6*a^2*
b^10+b^12)*tan(1/2*d*x+1/2*c)^5+1/240/a^4*b*(9000*a^14+131220*a^12*b^2+1226
69*a^10*b^4+62092*a^8*b^6+90624*a^6*b^8-102800*a^4*b^10+50720*a^2*b^12-9600
*b^14)/(a^12-6*a^10*b^2+15*a^8*b^4-20*a^6*b^6+15*a^4*b^8-6*a^2*b^10+b^12)*t
an(1/2*d*x+1/2*c)^4+1/120/a^3*(240*a^14+15120*a^12*b^2+41090*a^10*b^4-3137*
a^8*b^6+38184*a^6*b^8-32072*a^4*b^10+13760*a^2*b^12-2400*b^14)/(a^12-6*a^10
*b^2+15*a^8*b^4-20*a^6*b^6+15*a^4*b^8-6*a^2*b^10+b^12)*tan(1/2*d*x+1/2*c)^3
+1/120*b*(1760*a^12+14240*a^10*b^2-3186*a^8*b^4+13315*a^6*b^6-10352*a^4*b^8
+4248*a^2*b^10-720*b^12)/a^2/(a^12-6*a^10*b^2+15*a^8*b^4-20*a^6*b^6+15*a^4*
b^8-6*a^2*b^10+b^12)*tan(1/2*d*x+1/2*c)^2+1/240*(120*a^12+5540*a^10*b^2-179
5*a^8*b^4+5046*a^6*b^6-3692*a^4*b^8+1456*a^2*b^10-240*b^12)/a/(a^12-6*a^10*
b^2+15*a^8*b^4-20*a^6*b^6+15*a^4*b^8-6*a^2*b^10+b^12)*tan(1/2*d*x+1/2*c)+1/
1680*(3640*a^10-2660*a^8*b^2+4923*a^6*b^4-3646*a^4*b^6+1448*a^2*b^8-240*b^1
0)*b/(a^12-6*a^10*b^2+15*a^8*b^4-20*a^6*b^6+15*a^4*b^8-6*a^2*b^10+b^12))/(a
*tan(1/2*d*x+1/2*c)^2+2*b*tan(1/2*d*x+1/2*c)+a)^7+1/8*a*(8*a^4+20*a^2*b^2+5
*b^4)/(a^12-6*a^10*b^2+15*a^8*b^4-20*a^6*b^6+15*a^4*b^8-6*a^2*b^10+b^12)/(a
^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2)))
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1444 vs. $2(401) = 802$.
time = 0.57, size = 2972, normalized size = 7.04

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] [1/3360*(2*(40*a^8*b^5 + 1478*a^6*b^7 + 261*a^4*b^9 - 1651*a^2*b^11 - 128*b
^13)*cos(d*x + c)^7 - 28*(60*a^10*b^3 + 1837*a^8*b^5 + 176*a^6*b^7 - 1680*a
^4*b^9 - 361*a^2*b^11 - 32*b^13)*cos(d*x + c)^5 + 70*(40*a^12*b + 900*a^10*
b^3 + 1111*a^8*b^5 - 501*a^6*b^7 - 1395*a^4*b^9 - 139*a^2*b^11 - 16*b^13)*c
os(d*x + c)^3 + 105*(8*a^12 + 188*a^10*b^2 + 705*a^8*b^4 + 861*a^6*b^6 + 31
5*a^4*b^8 + 35*a^2*b^10 - 7*(8*a^6*b^6 + 20*a^4*b^8 + 5*a^2*b^10)*cos(d*x +
 c)^6 + 7*(40*a^8*b^4 + 124*a^6*b^6 + 85*a^4*b^8 + 15*a^2*b^10)*cos(d*x + c
)^4 - 7*(24*a^10*b^2 + 140*a^8*b^4 + 239*a^6*b^6 + 110*a^4*b^8 + 15*a^2*b^1
0)*cos(d*x + c)^2 + (56*a^11*b + 420*a^9*b^3 + 903*a^7*b^5 + 603*a^5*b^7 +
125*a^3*b^9 + 5*a*b^11 - (8*a^5*b^7 + 20*a^3*b^9 + 5*a*b^11)*cos(d*x + c)^6
 + 3*(56*a^7*b^5 + 148*a^5*b^7 + 55*a^3*b^9 + 5*a*b^11)*cos(d*x + c)^4 - (2
80*a^9*b^3 + 1036*a^7*b^5 + 1039*a^5*b^7 + 270*a^3*b^9 + 15*a*b^11)*cos(d*x
 + c)^2)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 -
 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*
x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 -
b^2)) - 420*(24*a^12*b + 116*a^10*b^3 + 99*a^8*b^5 - 129*a^6*b^7 - 95*a^4*b
^9 - 15*a^2*b^11)*cos(d*x + c) - 14*((40*a^9*b^4 + 1358*a^7*b^6 + 81*a^5*b^
8 - 1426*a^3*b^10 - 53*a*b^12)*cos(d*x + c)^5 - 10*(20*a^11*b^2 + 535*a^9*b
^4 + 147*a^7*b^6 - 407*a^5*b^8 - 283*a^3*b^10 - 12*a*b^12)*cos(d*x + c)^3 +
 15*(8*a^13 + 132*a^11*b^2 + 285*a^9*b^4 - 42*a^7*b^6 - 288*a^5*b^8 - 90*a^
3*b^10 - 5*a*b^12)*cos(d*x + c))*sin(d*x + c))/(7*(a^15*b^6 - 7*a^13*b^8 +
21*a^11*b^10 - 35*a^9*b^12 + 35*a^7*b^14 - 21*a^5*b^16 + 7*a^3*b^18 - a*b^2
0)*d*cos(d*x + c)^6 - 7*(5*a^17*b^4 - 32*a^15*b^6 + 84*a^13*b^8 - 112*a^11*
b^10 + 70*a^9*b^12 - 28*a^5*b^16 + 16*a^3*b^18 - 3*a*b^20)*d*cos(d*x + c)^4
 + 7*(3*a^19*b^2 - 11*a^17*b^4 - 4*a^15*b^6 + 84*a^13*b^8 - 182*a^11*b^10 +
 182*a^9*b^12 - 84*a^7*b^14 + 4*a^5*b^16 + 11*a^3*b^18 - 3*a*b^20)*d*cos(d*
x + c)^2 - (a^21 + 14*a^19*b^2 - 91*a^17*b^4 + 168*a^15*b^6 - 14*a^13*b^8 -
 364*a^11*b^10 + 546*a^9*b^12 - 344*a^7*b^14 + 77*a^5*b^16 + 14*a^3*b^18 -
```

```
7*a*b^20)*d + ((a^14*b^7 - 7*a^12*b^9 + 21*a^10*b^11 - 35*a^8*b^13 + 35*a^6
*b^15 - 21*a^4*b^17 + 7*a^2*b^19 - b^21)*d*cos(d*x + c)^6 - 3*(7*a^16*b^5 -
 48*a^14*b^7 + 140*a^12*b^9 - 224*a^10*b^11 + 210*a^8*b^13 - 112*a^6*b^15 +
 28*a^4*b^17 - b^21)*d*cos(d*x + c)^4 + (35*a^18*b^3 - 203*a^16*b^5 + 444*a
^14*b^7 - 364*a^12*b^9 - 182*a^10*b^11 + 630*a^8*b^13 - 532*a^6*b^15 + 196*
a^4*b^17 - 21*a^2*b^19 - 3*b^21)*d*cos(d*x + c)^2 - (7*a^20*b - 14*a^18*b^3
 - 77*a^16*b^5 + 344*a^14*b^7 - 546*a^12*b^9 + 364*a^10*b^11 + 14*a^8*b^13
- 168*a^6*b^15 + 91*a^4*b^17 - 14*a^2*b^19 - b^21)*d)*sin(d*x + c)), 1/1680
*((40*a^8*b^5 + 1478*a^6*b^7 + 261*a^4*b^9 - 1651*a^2*b^11 - 128*b^13)*cos(
d*x + c)^7 - 14*(60*a^10*b^3 + 1837*a^8*b^5 + 176*a^6*b^7 - 1680*a^4*b^9 -
361*a^2*b^11 - 32*b^13)*cos(d*x + c)^5 + 35*(40*a^12*b + 900*a^10*b^3 + 111
1*a^8*b^5 - 501*a^6*b^7 - 1395*a^4*b^9 - 139*a^2*b^11 - 16*b^13)*cos(d*x +
c)^3 + 105*(8*a^12 + 188*a^10*b^2 + 705*a^8*b^4 + 861*a^6*b^6 + 315*a^4*b^8
 + 35*a^2*b^10 - 7*(8*a^6*b^6 + 20*a^4*b^8 + 5*a^2*b^10)*cos(d*x + c)^6 + 7
*(40*a^8*b^4 + 124*a^6*b^6 + 85*a^4*b^8 + 15*a^2*b^10)*cos(d*x + c)^4 - 7*(
24*a^10*b^2 + 140*a^8*b^4 + 239*a^6*b^6 + 110*a^4*b^8 + 15*a^2*b^10)*cos(d*
x + c)^2 + (56*a^11*b + 420*a^9*b^3 + 903*a^7*b^5 + 603*a^5*b^7 + 125*a^3*b
^9 + 5*a*b^11 - (8*a^5*b^7 + 20*a^3*b^9 + 5*a*b^11)*cos(d*x + c)^6 + 3*(56*
a^7*b^5 + 148*a^5*b^7 + 55*a^3*b^9 + 5*a*b^11)*cos(d*x + c)^4 - (280*a^9*b^
3 + 1036*a^7*b^5 + 1039*a^5*b^7 + 270*a^3*b^9 + 15*a*b^11)*cos(d*x + c)^2)*
sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)
*cos(d*x + c))) - 210*(24*a^12*b + 116*a^10*b^3 + 99*a^8*b^5 - 129*a^6*b^7
 - 95*a^4*b^9 - 15*a^2*b^11)*cos(d*x + c) - 7*((40*a^9*b^4 + 1358*a^7*b^6 +
81*a^5*b^8 - 1426*a^3*b^10 - 53*a*b^12)*cos(d*x + c)^5 - 10*(20*a^11*b^2 +
535*a^9*b^4 + 147*a^7*b^6 - 407*a^5*b^8 - 283*a^3*b^10 - 12*a*b^12)*cos(d*x
 + c)^3 + 15*(8*a^13 + 132*a^11*b^2 + 285*a^9*b^4 - 42*a^7*b^6 - 288*a^5*b^
8 - 90*a^3*b^10 - 5*a*b^12)*cos(d*x + c))*sin(d*x + c))/(7*(a^15*b^6 - 7*a^
13*b^8 + 21*a^11*b^10 - 35*a^9*b^12 + 35*a^7*b^14 - 21*a^5*b^16 + 7*a^3*b^1
8 - a*b^20)*d*cos(d*x + c)^6 - 7*(5*a^17*b^4 - 32*a^15*b^6 + 84*a^13*b^8 -
112*a^11*b^10 + 70*a^9*b^12 - 28*a^5*b^16 + 16*a^3*b^18 - 3*a*b^20)*d*cos(d
*x + c)^4 + 7*(3*a^19*b^2 - 11*a^17*b^4 - 4*a^15*b^6 + 84*a^13*b^8 - 182*a^
11*b^10 + 182*a^9*b^12 - 84*a^7*b^14 + 4*a^5*b^16 + 11*a^3*b^18 - 3*a*b^20)
*d*cos(d*x + c)^2 - (a^21 + 14*a^19*b^2 - 91*a^17*b^4 + 168*a^15*b^6 - 14*a
^13*b^8 - 364*a^11*b^10 + 546*a^9*b^12 - 344*a^7*b^14 + 77*a^5*b^16 + 14*a^
3*b^18 - 7*a*b^20)*d + ((a^14*b^7 - 7*a^12*b^9 + 21*a^10*b^11 - 35*a^8*b^13
 + 35*a^6*b^15 - 21*a^4*b^17 + 7*a^2*b^19 - b^21)*d*cos(d*x + c)^6 - 3*(7*a
^16*b^5 - 48*a^14*b^7 + 140*a^12*b^9 - 224*a^10*b^11 + 210*a^8*b^13 - 112*a
^6*b^15 + 28*a^4*b^17 - b^21)*d*cos(d*x + c)^4 + (35*a^18*b^3 - 203*a^16*b^
5 + 444*a^14*b^7 - 364*a^12*b^9 - 182*a^10*b^11...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+b*sin(d*x+c))**8,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 2207 vs. $2(401) = 802$.
time = 8.75, size = 2207, normalized size = 5.23

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/840*(105*(8*a^5 + 20*a^3*b^2 + 5*a*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)
*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/((a^12 - 6*
a^10*b^2 + 15*a^8*b^4 - 20*a^6*b^6 + 15*a^4*b^8 - 6*a^2*b^10 + b^12)*sqrt(a
^2 - b^2)) - (840*a^18*tan(1/2*d*x + 1/2*c)^13 - 12180*a^16*b^2*tan(1/2*d*x
 + 1/2*c)^13 + 24675*a^14*b^4*tan(1/2*d*x + 1/2*c)^13 - 33600*a^12*b^6*tan(
1/2*d*x + 1/2*c)^13 + 25200*a^10*b^8*tan(1/2*d*x + 1/2*c)^13 - 10080*a^8*b^
10*tan(1/2*d*x + 1/2*c)^13 + 1680*a^6*b^12*tan(1/2*d*x + 1/2*c)^13 - 840*a^
17*b*tan(1/2*d*x + 1/2*c)^12 - 87780*a^15*b^3*tan(1/2*d*x + 1/2*c)^12 + 144
375*a^13*b^5*tan(1/2*d*x + 1/2*c)^12 - 201600*a^11*b^7*tan(1/2*d*x + 1/2*c)
^12 + 151200*a^9*b^9*tan(1/2*d*x + 1/2*c)^12 - 60480*a^7*b^11*tan(1/2*d*x +
 1/2*c)^12 + 10080*a^5*b^13*tan(1/2*d*x + 1/2*c)^12 + 3360*a^18*tan(1/2*d*x
 + 1/2*c)^11 - 94080*a^16*b^2*tan(1/2*d*x + 1/2*c)^11 - 220500*a^14*b^4*tan
(1/2*d*x + 1/2*c)^11 + 287350*a^12*b^6*tan(1/2*d*x + 1/2*c)^11 - 537600*a^1
0*b^8*tan(1/2*d*x + 1/2*c)^11 + 450240*a^8*b^10*tan(1/2*d*x + 1/2*c)^11 - 1
92640*a^6*b^12*tan(1/2*d*x + 1/2*c)^11 + 33600*a^4*b^14*tan(1/2*d*x + 1/2*c
)^11 - 13440*a^17*b*tan(1/2*d*x + 1/2*c)^10 - 554400*a^15*b^3*tan(1/2*d*x +
 1/2*c)^10 - 165900*a^13*b^5*tan(1/2*d*x + 1/2*c)^10 - 66850*a^11*b^7*tan(1
/2*d*x + 1/2*c)^10 - 621600*a^9*b^9*tan(1/2*d*x + 1/2*c)^10 + 719040*a^7*b^
11*tan(1/2*d*x + 1/2*c)^10 - 355040*a^5*b^13*tan(1/2*d*x + 1/2*c)^10 + 6720
0*a^3*b^15*tan(1/2*d*x + 1/2*c)^10 + 4200*a^18*tan(1/2*d*x + 1/2*c)^9 - 304
500*a^16*b^2*tan(1/2*d*x + 1/2*c)^9 - 1418025*a^14*b^4*tan(1/2*d*x + 1/2*c)
^9 + 147070*a^12*b^6*tan(1/2*d*x + 1/2*c)^9 - 1316700*a^10*b^8*tan(1/2*d*x
+ 1/2*c)^9 + 242592*a^8*b^10*tan(1/2*d*x + 1/2*c)^9 + 439376*a^6*b^12*tan(1
/2*d*x + 1/2*c)^9 - 352128*a^4*b^14*tan(1/2*d*x + 1/2*c)^9 + 80640*a^2*b^16
*tan(1/2*d*x + 1/2*c)^9 - 49000*a^17*b*tan(1/2*d*x + 1/2*c)^8 - 1357300*a^1
5*b^3*tan(1/2*d*x + 1/2*c)^8 - 1726305*a^13*b^5*tan(1/2*d*x + 1/2*c)^8 - 34
6570*a^11*b^7*tan(1/2*d*x + 1/2*c)^8 - 1972600*a^9*b^9*tan(1/2*d*x + 1/2*c)
^8 + 1360128*a^7*b^11*tan(1/2*d*x + 1/2*c)^8 - 298816*a^5*b^13*tan(1/2*d*x
+ 1/2*c)^8 - 122752*a^3*b^15*tan(1/2*d*x + 1/2*c)^8 + 53760*a*b^17*tan(1/2*
d*x + 1/2*c)^8 - 509600*a^16*b^2*tan(1/2*d*x + 1/2*c)^7 - 2685200*a^14*b^4*
tan(1/2*d*x + 1/2*c)^7 - 900900*a^12*b^6*tan(1/2*d*x + 1/2*c)^7 - 2070320*a
^10*b^8*tan(1/2*d*x + 1/2*c)^7 - 278096*a^8*b^10*tan(1/2*d*x + 1/2*c)^7 + 9
52320*a^6*b^12*tan(1/2*d*x + 1/2*c)^7 - 538112*a^4*b^14*tan(1/2*d*x + 1/2*c
```

```
)^7 + 68608*a^2*b^16*tan(1/2*d*x + 1/2*c)^7 + 15360*b^18*tan(1/2*d*x + 1/2*
c)^7 - 78400*a^17*b*tan(1/2*d*x + 1/2*c)^6 - 1607200*a^15*b^3*tan(1/2*d*x +
 1/2*c)^6 - 2326800*a^13*b^5*tan(1/2*d*x + 1/2*c)^6 - 823060*a^11*b^7*tan(1
/2*d*x + 1/2*c)^6 - 2094400*a^9*b^9*tan(1/2*d*x + 1/2*c)^6 + 1351728*a^7*b^
11*tan(1/2*d*x + 1/2*c)^6 - 298816*a^5*b^13*tan(1/2*d*x + 1/2*c)^6 - 122752
*a^3*b^15*tan(1/2*d*x + 1/2*c)^6 + 53760*a*b^17*tan(1/2*d*x + 1/2*c)^6 - 42
00*a^18*tan(1/2*d*x + 1/2*c)^5 - 459900*a^16*b^2*tan(1/2*d*x + 1/2*c)^5 - 2
100175*a^14*b^4*tan(1/2*d*x + 1/2*c)^5 - 647780*a^12*b^6*tan(1/2*d*x + 1/2*
c)^5 - 1643880*a^10*b^8*tan(1/2*d*x + 1/2*c)^5 + 228592*a^8*b^10*tan(1/2*d*
x + 1/2*c)^5 + 439376*a^6*b^12*tan(1/2*d*x + 1/2*c)^5 - 352128*a^4*b^14*tan
(1/2*d*x + 1/2*c)^5 + 80640*a^2*b^16*tan(1/2*d*x + 1/2*c)^5 - 63000*a^17*b*
tan(1/2*d*x + 1/2*c)^4 - 918540*a^15*b^3*tan(1/2*d*x + 1/2*c)^4 - 858683*a^
13*b^5*tan(1/2*d*x + 1/2*c)^4 - 434644*a^11*b^7*tan(1/2*d*x + 1/2*c)^4 - 63
4368*a^9*b^9*tan(1/2*d*x + 1/2*c)^4 + 719600*a^7*b^11*tan(1/2*d*x + 1/2*c)^
4 - 355040*a^5*b^13*tan(1/2*d*x + 1/2*c)^4 + 67200*a^3*b^15*tan(1/2*d*x + 1
/2*c)^4 - 3360*a^18*tan(1/2*d*x + 1/2*c)^3 - 211680*a^16*b^2*tan(1/2*d*x +
1/2*c)^3 - 575260*a^14*b^4*tan(1/2*d*x + 1/2*c)^3 + 43918*a^12*b^6*tan(1/2*
d*x + 1/2*c)^3 - 534576*a^10*b^8*tan(1/2*d*x + 1/2*c)^3 + 449008*a^8*b^10*t
an(1/2*d*x + 1/2*c)^3 - 192640*a^6*b^12*tan(1/2*d*x + 1/2*c)^3 + 33600*a^4*
b^14*tan(1/2*d*x + 1/2*c)^3 - 24640*a^17*b*tan(1/2*d*x + 1/2*c)^2 - 199360*
a^15*b^3*tan(1/2*d*x + 1/2*c)^2 + 44604*a^13*b^5*tan(1/2*d*x + 1/2*c)^2 - 1
86410*a^11*b^7*tan(1/2*d*x + 1/2*c)^2 + 144928*a^9*b^9*tan(1/2*d*x + 1/2*c)
^2 - 59472*a^7*b^11*tan(1/2*d*x + 1/2*c)^2 + 10080*a^5*b^13*tan(1/2*d*x + 1
/2*c)^2 - 840*a^18*tan(1/2*d*x + 1/2*c) - 38780*a^16*b^2*tan(1/2*d*x + 1/2*
c) + 12565*a^14*b^4*tan(1/2*d*x + 1/2*c) - 35322*a^12*b^6*tan(1/2*d*x + 1/2
*c) + 25844*a^10*b^8*tan(1/2*d*x + 1/2*c) - 10192*a^8*b^10*tan(1/2*d*x + 1/
2*c) + 1680*a^6*b^12*tan(1/2*d*x + 1/2*c) - 3640*a^17*b + 2660*a^15*b^3 - 4
923*a^13*b^5 + 3646*a^11*b^7 - 1448*a^9*b^9 + 240*a^7*b^11)/((a^19 - 6*a^17
*b^2 + 15*a^15*b^4 - 20*a^13*b^6 + 15*a^11*b^8 - 6*a^9*b^10 + a^7*b^12)*(a*
tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)^7))/d
```

Mupad [B]
time = 10.34, size = 2440, normalized size = 5.78

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a + b*sin(c + d*x))^8,x)
```

```
[Out] ((3640*a^10*b - 240*b^11 + 1448*a^2*b^9 - 3646*a^4*b^7 + 4923*a^6*b^5 - 266
0*a^8*b^3)/(840*(a^12 + b^12 - 6*a^2*b^10 + 15*a^4*b^8 - 20*a^6*b^6 + 15*a^
8*b^4 - 6*a^10*b^2)) + (tan(c/2 + (d*x)/2)^6*(2800*a^16*b - 1920*b^17 + 438
4*a^2*b^15 + 10672*a^4*b^13 - 48276*a^6*b^11 + 74800*a^8*b^9 + 29395*a^10*b
^7 + 83100*a^12*b^5 + 57400*a^14*b^3))/(30*a^6*(a^12 + b^12 - 6*a^2*b^10 +
15*a^4*b^8 - 20*a^6*b^6 + 15*a^8*b^4 - 6*a^10*b^2)) + (tan(c/2 + (d*x)/2)^8
```

*(7000*a^16*b - 7680*b^17 + 17536*a^2*b^15 + 42688*a^4*b^13 - 194304*a^6*b^
11 + 281800*a^8*b^9 + 49510*a^10*b^7 + 246615*a^12*b^5 + 193900*a^14*b^3))/
(120*a^6*(a^12 + b^12 - 6*a^2*b^10 + 15*a^4*b^8 - 20*a^6*b^6 + 15*a^8*b^4 -
6*a^10*b^2)) + (tan(c/2 + (d*x)/2)^10*(192*a^14*b - 960*b^15 + 5072*a^2*b^
13 - 10272*a^4*b^11 + 8880*a^6*b^9 + 955*a^8*b^7 + 2370*a^10*b^5 + 7920*a^1
2*b^3))/(12*a^4*(a^12 + b^12 - 6*a^2*b^10 + 15*a^4*b^8 - 20*a^6*b^6 + 15*a^
8*b^4 - 6*a^10*b^2)) + (tan(c/2 + (d*x)/2)^4*(9000*a^14*b - 9600*b^15 + 507
20*a^2*b^13 - 102800*a^4*b^11 + 90624*a^6*b^9 + 62092*a^8*b^7 + 122669*a^10
*b^5 + 131220*a^12*b^3))/(120*a^4*(a^12 + b^12 - 6*a^2*b^10 + 15*a^4*b^8 -
20*a^6*b^6 + 15*a^8*b^4 - 6*a^10*b^2)) + (tan(c/2 + (d*x)/2)^12*(8*a^12*b -
96*b^13 + 576*a^2*b^11 - 1440*a^4*b^9 + 1920*a^6*b^7 - 1375*a^8*b^5 + 836*
a^10*b^3))/(8*a^2*(a^12 + b^12 - 6*a^2*b^10 + 15*a^4*b^8 - 20*a^6*b^6 + 15*
a^8*b^4 - 6*a^10*b^2)) + (tan(c/2 + (d*x)/2)^2*(1760*a^12*b - 720*b^13 + 42
48*a^2*b^11 - 10352*a^4*b^9 + 13315*a^6*b^7 - 3186*a^8*b^5 + 14240*a^10*b^3
))/(60*a^2*(a^12 + b^12 - 6*a^2*b^10 + 15*a^4*b^8 - 20*a^6*b^6 + 15*a^8*b^4
- 6*a^10*b^2)) + (tan(c/2 + (d*x)/2)*(120*a^12 - 240*b^12 + 1456*a^2*b^10
- 3692*a^4*b^8 + 5046*a^6*b^6 - 1795*a^8*b^4 + 5540*a^10*b^2))/(120*a*(a^12
+ b^12 - 6*a^2*b^10 + 15*a^4*b^8 - 20*a^6*b^6 + 15*a^8*b^4 - 6*a^10*b^2))
- (tan(c/2 + (d*x)/2)^9*(600*a^16 + 11520*b^16 - 50304*a^2*b^14 + 62768*a^4
*b^12 + 34656*a^6*b^10 - 188100*a^8*b^8 + 21010*a^10*b^6 - 202575*a^12*b^4
- 43500*a^14*b^2))/(120*a^5*(a^12 + b^12 - 6*a^2*b^10 + 15*a^4*b^8 - 20*a^6
*b^6 + 15*a^8*b^4 - 6*a^10*b^2)) + (tan(c/2 + (d*x)/2)^5*(600*a^16 - 11520*
b^16 + 50304*a^2*b^14 - 62768*a^4*b^12 - 32656*a^6*b^10 + 234840*a^8*b^8 +
92540*a^10*b^6 + 300025*a^12*b^4 + 65700*a^14*b^2))/(120*a^5*(a^12 + b^12 -
6*a^2*b^10 + 15*a^4*b^8 - 20*a^6*b^6 + 15*a^8*b^4 - 6*a^10*b^2)) - (tan(c/
2 + (d*x)/2)^11*(48*a^14 + 480*b^14 - 2752*a^2*b^12 + 6432*a^4*b^10 - 7680*
a^6*b^8 + 4105*a^8*b^6 - 3150*a^10*b^4 - 1344*a^12*b^2))/(12*a^3*(a^12 + b^
12 - 6*a^2*b^10 + 15*a^4*b^8 - 20*a^6*b^6 + 15*a^8*b^4 - 6*a^10*b^2)) + (ta
n(c/2 + (d*x)/2)^3*(240*a^14 - 2400*b^14 + 13760*a^2*b^12 - 32072*a^4*b^10
+ 38184*a^6*b^8 - 3137*a^8*b^6 + 41090*a^10*b^4 + 15120*a^12*b^2))/(60*a^3*
(a^12 + b^12 - 6*a^2*b^10 + 15*a^4*b^8 - 20*a^6*b^6 + 15*a^8*b^4 - 6*a^10*b
^2)) - (tan(c/2 + (d*x)/2)^13*(8*a^12 + 16*b^12 - 96*a^2*b^10 + 240*a^4*b^8
- 320*a^6*b^6 + 235*a^8*b^4 - 116*a^10*b^2))/(8*a*(a^12 + b^12 - 6*a^2*b^1
0 + 15*a^4*b^8 - 20*a^6*b^6 + 15*a^8*b^4 - 6*a^10*b^2)) + (b*tan(c/2 + (d*x
)/2)^7*(35*a^6 + 16*b^6 + 168*a^2*b^4 + 210*a^4*b^2)*(3640*a^10*b - 240*b^1
1 + 1448*a^2*b^9 - 3646*a^4*b^7 + 4923*a^6*b^5 - 2660*a^8*b^3))/(210*a^7*(a
^12 + b^12 - 6*a^2*b^10 + 15*a^4*b^8 - 20*a^6*b^6 + 15*a^8*b^4 - 6*a^10*b^2
)))/(d*(tan(c/2 + (d*x)/2)^5*(210*a^6*b + 672*a^2*b^5 + 1120*a^4*b^3) + tan
(c/2 + (d*x)/2)^9*(210*a^6*b + 672*a^2*b^5 + 1120*a^4*b^3) + a^7*tan(c/2 +
(d*x)/2)^14 + tan(c/2 + (d*x)/2)^3*(84*a^6*b + 280*a^4*b^3) + tan(c/2 + (d*
x)/2)^11*(84*a^6*b + 280*a^4*b^3) + tan(c/2 + (d*x)/2)^6*(448*a*b^6 + 35*a^
7 + 1680*a^3*b^4 + 840*a^5*b^2) + tan(c/2 + (d*x)/2)^8*(448*a*b^6 + 35*a^7
+ 1680*a^3*b^4 + 840*a^5*b^2) + tan(c/2 + (d*x)/2)^7*(280*a^6*b + 128*b^7 +
1344*a^2*b^5 + 1680*a^4*b^3) + a^7 + tan(c/2 + (d*x)/2)^4*(21*a^7 + 560*a^
3*b^4 + 420*a^5*b^2) + tan(c/2 + (d*x)/2)^10*(21*a^7 + 560*a^3*b^4 + 420*a^

```
5*b^2) + tan(c/2 + (d*x)/2)^2*(7*a^7 + 84*a^5*b^2) + tan(c/2 + (d*x)/2)^12*
(7*a^7 + 84*a^5*b^2) + 14*a^6*b*tan(c/2 + (d*x)/2) + 14*a^6*b*tan(c/2 + (d*
x)/2)^13)) + (a*atan((8*((a^2*tan(c/2 + (d*x)/2)*(8*a^4 + 5*b^4 + 20*a^2*b^
2))/(8*(a + b)^(13/2)*(a - b)^(13/2)) + (a*(8*a^4 + 5*b^4 + 20*a^2*b^2)*(16
*a^12*b + 16*b^13 - 96*a^2*b^11 + 240*a^4*b^9 - 320*a^6*b^7 + 240*a^8*b^5 -
 96*a^10*b^3))/(128*(a + b)^(13/2)*(a - b)^(13/2)*(a^12 + b^12 - 6*a^2*b^10
 + 15*a^4*b^8 - 20*a^6*b^6 + 15*a^8*b^4 - 6*a^10*b^2)))*(a^12 + b^12 - 6*a^
2*b^10 + 15*a^4*b^8 - 20*a^6*b^6 + 15*a^8*b^4 - 6*a^10*b^2))/(5*a*b^4 + 8*a
^5 + 20*a^3*b^2))*(8*a^4 + 5*b^4 + 20*a^2*b^2))/(8*d*(a + b)^(13/2)*(a - b)
^(13/2))
```

3.471 $\int \frac{\sec^2(c+dx)}{(a+b\sin(c+dx))^8}\,dx$

Optimal. Leaf size=529

$$-\frac{9ab^2(64a^6+336a^4b^2+280a^2b^4+35b^6)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{8\left(a^2-b^2\right)^{17/2}d}+\frac{b\sec(c+dx)}{7\left(a^2-b^2\right)d(a+b\sin(c+dx))^7}+\frac{}{14\left(a^2\right.}$$

```
[Out] -9/8*a*b^2*(64*a^6+336*a^4*b^2+280*a^2*b^4+35*b^6)*arctan((b+a*tan(1/2*d*x+
1/2*c))/(a^2-b^2)^(1/2))/(a^2-b^2)^(17/2)/d+1/7*b*sec(d*x+c)/(a^2-b^2)/d/(a
+b*sin(d*x+c))^7+5/14*a*b*sec(d*x+c)/(a^2-b^2)^2/d/(a+b*sin(d*x+c))^6+1/70*
b*(49*a^2+16*b^2)*sec(d*x+c)/(a^2-b^2)^3/d/(a+b*sin(d*x+c))^5+13/280*a*b*(2
8*a^2+27*b^2)*sec(d*x+c)/(a^2-b^2)^4/d/(a+b*sin(d*x+c))^4+1/280*b*(700*a^4+
1317*a^2*b^2+128*b^4)*sec(d*x+c)/(a^2-b^2)^5/d/(a+b*sin(d*x+c))^3+11/560*a*
b*(280*a^4+844*a^2*b^2+241*b^4)*sec(d*x+c)/(a^2-b^2)^6/d/(a+b*sin(d*x+c))^2
+1/560*b*(9800*a^6+41484*a^4*b^2+22767*a^2*b^4+1024*b^6)*sec(d*x+c)/(a^2-b^
2)^7/d/(a+b*sin(d*x+c))-1/560*sec(d*x+c)*(315*a*b*(64*a^6+336*a^4*b^2+280*a
^2*b^4+35*b^6)-(560*a^8+42472*a^6*b^2+125634*a^4*b^4+54511*a^2*b^6+2048*b^8
)*sin(d*x+c))/(a^2-b^2)^8/d
```

Rubi [A]
time = 1.15, antiderivative size = 529, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2773, 2943, 2945, 12, 2739, 632, 210}

$$\frac{13ab(28a^2+27b^2)\sec(c+dx)}{280d\left(a^2-b^2\right)^4(a+b\sin(c+dx))^4}+\frac{b(49a^2+16b^2)\sec(c+dx)}{70d\left(a^2-b^2\right)^3(a+b\sin(c+dx))^5}+\frac{5ab\sec(c+dx)}{14d\left(a^2-b^2\right)^2(a+b\sin(c+dx))^6}+\frac{b\sec(c+dx)}{7d\left(a^2-b^2\right)(a+b\sin(c+dx))^7}+\frac{11ab(280a^4+844a^2b^2+241b^4)\sec(c+dx)}{560d\left(a^2-b^2\right)^6(a+b\sin(c+dx))^2}+\frac{b(700a^4+1317a^2b^2+128b^4)\sec(c+dx)}{280d\left(a^2-b^2\right)^5(a+b\sin(c+dx))^3}-\frac{9ab^2(64a^6+336a^4b^2+280a^2b^4+35b^6)\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{8d\left(a^2-b^2\right)^{17/2}}+\frac{b(9800a^6+41484a^4b^2+22767a^2b^4+1024b^6)\sec(c+dx)}{560d\left(a^2-b^2\right)^7(a+b\sin(c+dx))}-\frac{\sec(c+dx)\left(315ab(64a^6+336a^4b^2+280a^2b^4+35b^6)-(560a^8+42472a^6b^2+125634a^4b^4+54511a^2b^6+2048b^8)\sin(c+dx)\right)}{560d\left(a^2-b^2\right)^8}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + b*Sin[c + d*x])^8,x]
```

```
[Out] (-9*a*b^2*(64*a^6 + 336*a^4*b^2 + 280*a^2*b^4 + 35*b^6)*ArcTan[(b + a*Tan[(
c + d*x)/2])/Sqrt[a^2 - b^2]])/(8*(a^2 - b^2)^(17/2)*d) + (b*Sec[c + d*x])/
(7*(a^2 - b^2)*d*(a + b*Sin[c + d*x])^7) + (5*a*b*Sec[c + d*x])/(14*(a^2 -
b^2)^2*d*(a + b*Sin[c + d*x])^6) + (b*(49*a^2 + 16*b^2)*Sec[c + d*x])/(70*(
a^2 - b^2)^3*d*(a + b*Sin[c + d*x])^5) + (13*a*b*(28*a^2 + 27*b^2)*Sec[c +
d*x])/(280*(a^2 - b^2)^4*d*(a + b*Sin[c + d*x])^4) + (b*(700*a^4 + 1317*a^2
*b^2 + 128*b^4)*Sec[c + d*x])/(280*(a^2 - b^2)^5*d*(a + b*Sin[c + d*x])^3)
+ (11*a*b*(280*a^4 + 844*a^2*b^2 + 241*b^4)*Sec[c + d*x])/(560*(a^2 - b^2)^
6*d*(a + b*Sin[c + d*x])^2) + (b*(9800*a^6 + 41484*a^4*b^2 + 22767*a^2*b^4
+ 1024*b^6)*Sec[c + d*x])/(560*(a^2 - b^2)^7*d*(a + b*Sin[c + d*x])) - (Sec
[c + d*x]*(315*a*b*(64*a^6 + 336*a^4*b^2 + 280*a^2*b^4 + 35*b^6) - (560*a^8
 + 42472*a^6*b^2 + 125634*a^4*b^4 + 54511*a^2*b^6 + 2048*b^8)*Sin[c + d*x])
)/(560*(a^2 - b^2)^8*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2773

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m + 1)/(f*g*(a^2 - b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1))
, Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p
 + 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^
2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2943

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c
 - a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^(m + 1)/(f*g*(a^2 -
 b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*
(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p
+ 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[
a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2945

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
```

```
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(a+b\sin(c+dx))^8}\,dx = \frac{b\sec(c+dx)}{7(a^2-b^2)d(a+b\sin(c+dx))^7} - \frac{\int \frac{\sec^2(c+dx)(-7a+8b\sin(c+dx))}{(a+b\sin(c+dx))^7}\,dx}{7(a^2-b^2)}$$

$$= \frac{b\sec(c+dx)}{7(a^2-b^2)d(a+b\sin(c+dx))^7} + \frac{5ab\sec(c+dx)}{14(a^2-b^2)^2d(a+b\sin(c+dx))^6} + \frac{\int \frac{\sec^2(c-}{}}{}$$

$$= \frac{b\sec(c+dx)}{7(a^2-b^2)d(a+b\sin(c+dx))^7} + \frac{5ab\sec(c+dx)}{14(a^2-b^2)^2d(a+b\sin(c+dx))^6} + \frac{b(49}{70(a^2-}$$

$$= \frac{b\sec(c+dx)}{7(a^2-b^2)d(a+b\sin(c+dx))^7} + \frac{5ab\sec(c+dx)}{14(a^2-b^2)^2d(a+b\sin(c+dx))^6} + \frac{b(49}{70(a^2-}$$

$$= \frac{b\sec(c+dx)}{7(a^2-b^2)d(a+b\sin(c+dx))^7} + \frac{5ab\sec(c+dx)}{14(a^2-b^2)^2d(a+b\sin(c+dx))^6} + \frac{b(49}{70(a^2-}$$

$$= \frac{b\sec(c+dx)}{7(a^2-b^2)d(a+b\sin(c+dx))^7} + \frac{5ab\sec(c+dx)}{14(a^2-b^2)^2d(a+b\sin(c+dx))^6} + \frac{b(49}{70(a^2-}$$

$$= \frac{b\sec(c+dx)}{7(a^2-b^2)d(a+b\sin(c+dx))^7} + \frac{5ab\sec(c+dx)}{14(a^2-b^2)^2d(a+b\sin(c+dx))^6} + \frac{b(49}{70(a^2-}$$

$$= \frac{b\sec(c+dx)}{7(a^2-b^2)d(a+b\sin(c+dx))^7} + \frac{5ab\sec(c+dx)}{14(a^2-b^2)^2d(a+b\sin(c+dx))^6} + \frac{b(49}{70(a^2-}$$

$$= \frac{b\sec(c+dx)}{7(a^2-b^2)d(a+b\sin(c+dx))^7} + \frac{5ab\sec(c+dx)}{14(a^2-b^2)^2d(a+b\sin(c+dx))^6} + \frac{b(49}{70(a^2-}$$

$$= \frac{b\sec(c+dx)}{7(a^2-b^2)d(a+b\sin(c+dx))^7} + \frac{5ab\sec(c+dx)}{14(a^2-b^2)^2d(a+b\sin(c+dx))^6} + \frac{b(49}{70(a^2-}$$

$$= \frac{b\sec(c+dx)}{7(a^2-b^2)d(a+b\sin(c+dx))^7} + \frac{5ab\sec(c+dx)}{14(a^2-b^2)^2d(a+b\sin(c+dx))^6} + \frac{b(49}{70(a^2-}$$

$$= -\frac{9ab^2(64a^6+336a^4b^2+280a^2b^4+35b^6)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{8(a^2-b^2)^{17/2}d} + \frac{b}{7(a^2-b^2)}$$

Mathematica [A]
time = 4.34, size = 494, normalized size = 0.93

$$-\frac{\frac{630ab^2(64a^6+336a^4b^2+280a^2b^4+35b^6)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{17/2}}+\frac{80b^3\cos(c+dx)}{(a^2-b^2)^2(a+b\sin(c+dx))^7}+\frac{360ab^3\cos(c+dx)}{(a^2-b^2)^3(a+b\sin(c+dx))^6}+\frac{8b^3(129a^2+26b^2)\cos(c+dx)}{(a^2-b^2)^4(a+b\sin(c+dx))^5}+\frac{2ab^3(1216a^2+739b^2)\cos(c+dx)}{(a^2-b^2)^5(a+b\sin(c+dx))^4}+\frac{2b^3(2616a^4+3207a^2b^2+232b^4)\cos(c+dx)}{(a^2-b^2)^6(a+b\sin(c+dx))^3}+\frac{ab^3(11112a^4+23066a^2b^2+5057b^4)\cos(c+dx)}{(a^2-b^2)^7(a+b\sin(c+dx))^2}+\frac{b^3(26792a^6+86434a^4b^2+38831a^2b^4+1488b^6)\cos(c+dx)}{(a^2-b^2)^8(a+b\sin(c+dx))}-\frac{560\sec(c+dx)\left(-8ab(a^6+7a^4b^2+7a^2b^4+b^6)+(a^8+28a^6b^2+70a^4b^4+28a^2b^6+b^8)\sin(c+dx)\right)}{(a^2-b^2)^8}}{560d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + b*Sin[c + d*x])^8,x]
```

```
[Out] -1/560*((630*a*b^2*(64*a^6 + 336*a^4*b^2 + 280*a^2*b^4 + 35*b^6)*ArcTan[(b
+ a*Tan[(c + d*x)/2]])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(17/2) + (80*b^3*Cos[c
+ d*x])/((a^2 - b^2)^2*(a + b*Sin[c + d*x])^7) + (360*a*b^3*Cos[c + d*x])/(
(a^2 - b^2)^3*(a + b*Sin[c + d*x])^6) + (8*b^3*(129*a^2 + 26*b^2)*Cos[c + d
*x])/((a^2 - b^2)^4*(a + b*Sin[c + d*x])^5) + (2*a*b^3*(1216*a^2 + 739*b^2)
*Cos[c + d*x])/((a^2 - b^2)^5*(a + b*Sin[c + d*x])^4) + (2*b^3*(2616*a^4 +
3207*a^2*b^2 + 232*b^4)*Cos[c + d*x])/((a^2 - b^2)^6*(a + b*Sin[c + d*x])^3
) + (a*b^3*(11112*a^4 + 23066*a^2*b^2 + 5057*b^4)*Cos[c + d*x])/((a^2 - b^2
)^7*(a + b*Sin[c + d*x])^2) + (b^3*(26792*a^6 + 86434*a^4*b^2 + 38831*a^2*b
^4 + 1488*b^6)*Cos[c + d*x])/((a^2 - b^2)^8*(a + b*Sin[c + d*x])) - (560*Se
c[c + d*x]*(-8*a*b*(a^6 + 7*a^4*b^2 + 7*a^2*b^4 + b^6) + (a^8 + 28*a^6*b^2
+ 70*a^4*b^4 + 28*a^2*b^6 + b^8)*Sin[c + d*x]))/(a^2 - b^2)^8)/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1285 vs. $2(506) = 1012$.
time = 2.86, size = 1286, normalized size = 2.43

method	result	size
derivativedivides	Expression too large to display	1286
default	Expression too large to display	1286
risch	Expression too large to display	2333

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+b*sin(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/(a+b)^8/(tan(1/2*d*x+1/2*c)-1)-1/(a-b)^8/(tan(1/2*d*x+1/2*c)+1)-2*b
^2/(a-b)^8/(a+b)^8*((1/16*b^2*(2352*a^12+1176*a^10*b^2+1419*a^8*b^4-896*a^6
*b^6+448*a^4*b^8-128*a^2*b^10+16*b^12)/a*tan(1/2*d*x+1/2*c)^13+1/16*b*(1344
*a^14+27216*a^12*b^2+21240*a^10*b^4+10591*a^8*b^6-5376*a^6*b^8+2688*a^4*b^1
0-768*a^2*b^12+96*b^14)/a^2*tan(1/2*d*x+1/2*c)^12+1/8/a^3*b^2*(14112*a^14+7
4592*a^12*b^2+68466*a^10*b^4+18179*a^8*b^6-7168*a^6*b^8+3968*a^4*b^10-1216*
a^2*b^12+160*b^14)*tan(1/2*d*x+1/2*c)^11+1/8/a^4*b*(4032*a^16+100800*a^14*b
^2+267072*a^12*b^4+222502*a^10*b^6+38423*a^8*b^8-10304*a^6*b^10+6784*a^4*b^
12-2288*a^2*b^14+320*b^16)*tan(1/2*d*x+1/2*c)^10+1/80/a^5*b^2*(529200*a^16+
3849720*a^14*b^2+6666195*a^12*b^4+4159190*a^10*b^6+487396*a^8*b^8-43648*a^6
*b^10+55568*a^4*b^12-23936*a^2*b^14+3840*b^16)*tan(1/2*d*x+1/2*c)^9+1/80/a^
```

```
6*b*(100800*a^18+2682960*a^16*b^2+9329400*a^14*b^4+11096155*a^12*b^6+464051
0*a^10*b^8+305704*a^8*b^10+83968*a^6*b^12-1088*a^4*b^14-10624*a^2*b^16+2560
*b^18)*tan(1/2*d*x+1/2*c)^8+1/140/a^7*b^2*(1646400*a^18+12759600*a^16*b^2+2
6124840*a^14*b^4+20046285*a^12*b^6+5007436*a^10*b^8+165284*a^8*b^10+170752*
a^6*b^12-54400*a^4*b^14+3328*a^2*b^16+1280*b^18)*tan(1/2*d*x+1/2*c)^7+1/20/
a^6*b*(33600*a^18+843360*a^16*b^2+2993040*a^14*b^4+3713960*a^12*b^6+1637615
*a^10*b^8+166096*a^8*b^10+24732*a^6*b^12-272*a^4*b^14-2656*a^2*b^16+640*b^1
8)*tan(1/2*d*x+1/2*c)^6+1/80/a^5*b^2*(882000*a^16+6146280*a^14*b^2+11822205
*a^12*b^4+7892620*a^10*b^6+1476776*a^8*b^8-25008*a^6*b^10+55568*a^4*b^12-23
936*a^2*b^14+3840*b^16)*tan(1/2*d*x+1/2*c)^5+1/80/a^4*b*(100800*a^16+206136
0*a^14*b^2+6111816*a^12*b^4+5950817*a^10*b^6+1509628*a^8*b^8-100576*a^6*b^1
0+69360*a^4*b^12-22880*a^2*b^14+3200*b^16)*tan(1/2*d*x+1/2*c)^4+1/40/a^3*b^
2*(211680*a^14+1061760*a^12*b^2+1464342*a^10*b^4+418379*a^8*b^6-34264*a^6*b
^8+20088*a^4*b^10-6080*a^2*b^12+800*b^14)*tan(1/2*d*x+1/2*c)^3+1/40*b*(2016
0*a^14+255360*a^12*b^2+454176*a^10*b^4+131714*a^8*b^6-10433*a^6*b^8+6080*a^
4*b^10-1832*a^2*b^12+240*b^14)/a^2*tan(1/2*d*x+1/2*c)^2+1/80*b^2*(82320*a^1
2+158760*a^10*b^2+46329*a^8*b^4-3842*a^6*b^6+2132*a^4*b^8-624*a^2*b^10+80*b
^12)/a*tan(1/2*d*x+1/2*c)+1/560*(47040*a^12+82320*a^10*b^2+26712*a^8*b^4-41
61*a^6*b^6+2186*a^4*b^8-632*a^2*b^10+80*b^12)*b)/(a*tan(1/2*d*x+1/2*c)^2+2*
b*tan(1/2*d*x+1/2*c)+a)^7+9/16*a*(64*a^6+336*a^4*b^2+280*a^2*b^4+35*b^6)/(a
^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1899 vs. $2(506) = 1012$.
time = 0.70, size = 3882, normalized size = 7.34

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sin(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] [1/1120*(1120*a^16*b - 8960*a^14*b^3 + 31360*a^12*b^5 - 62720*a^10*b^7 + 78
400*a^8*b^9 - 62720*a^6*b^11 + 31360*a^4*b^13 - 8960*a^2*b^15 + 1120*b^17 -
```

2*(560*a^10*b^7 + 41912*a^8*b^9 + 83162*a^6*b^11 - 71123*a^4*b^13 - 52463*a^2*b^15 - 2048*b^17)*cos(d*x + c)^8 + 28*(840*a^12*b^5 + 53648*a^10*b^7 + 95441*a^8*b^9 - 77704*a^6*b^11 - 60644*a^4*b^13 - 11069*a^2*b^15 - 512*b^17)*cos(d*x + c)^6 - 70*(560*a^14*b^3 + 27440*a^12*b^5 + 71064*a^10*b^7 + 29927*a^8*b^9 - 81421*a^6*b^11 - 43131*a^4*b^13 - 4183*a^2*b^15 - 256*b^17)*cos(d*x + c)^4 + 140*(56*a^16*b + 1400*a^14*b^3 + 13832*a^12*b^5 + 24080*a^10*b^7 - 4591*a^8*b^9 - 23443*a^6*b^11 - 10717*a^4*b^13 - 553*a^2*b^15 - 64*b^17)*cos(d*x + c)^2 - 315*(7*(64*a^8*b^8 + 336*a^6*b^10 + 280*a^4*b^12 + 35*a^2*b^14)*cos(d*x + c)^7 - 7*(320*a^10*b^6 + 1872*a^8*b^8 + 2408*a^6*b^10 + 1015*a^4*b^12 + 105*a^2*b^14)*cos(d*x + c)^5 + 7*(192*a^12*b^4 + 1648*a^10*b^6 + 4392*a^8*b^8 + 3913*a^6*b^10 + 1190*a^4*b^12 + 105*a^2*b^14)*cos(d*x + c)^3 - (64*a^14*b^2 + 1680*a^12*b^4 + 9576*a^10*b^6 + 18123*a^8*b^8 + 12887*a^6*b^10 + 3185*a^4*b^12 + 245*a^2*b^14)*cos(d*x + c) + ((64*a^7*b^9 + 336*a^5*b^11 + 280*a^3*b^13 + 35*a*b^15)*cos(d*x + c)^7 - 3*(448*a^9*b^7 + 2416*a^7*b^9 + 2296*a^5*b^11 + 525*a^3*b^13 + 35*a*b^15)*cos(d*x + c)^5 + (2240*a^11*b^5 + 14448*a^9*b^7 + 24104*a^7*b^9 + 13993*a^5*b^11 + 2310*a^3*b^13 + 105*a*b^15)*cos(d*x + c)^3 - (448*a^13*b^3 + 4592*a^11*b^5 + 15064*a^9*b^7 + 17165*a^7*b^9 + 7441*a^5*b^11 + 1015*a^3*b^13 + 35*a*b^15)*cos(d*x + c))*sin(d*x + c))*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 14*(80*a^17 - 640*a^15*b^2 + 2240*a^13*b^4 - 4480*a^11*b^6 + 5600*a^9*b^8 - 4480*a^7*b^10 + 2240*a^5*b^12 - 640*a^3*b^14 + 80*a*b^16 - (560*a^11*b^6 + 39032*a^9*b^8 + 70922*a^7*b^10 - 68603*a^5*b^12 - 41438*a^3*b^14 - 473*a*b^16)*cos(d*x + c)^6 + 10*(280*a^13*b^4 + 15960*a^11*b^6 + 29463*a^9*b^8 - 13541*a^7*b^10 - 23679*a^5*b^12 - 8391*a^3*b^14 - 92*a*b^16)*cos(d*x + c)^4 - 15*(112*a^15*b^2 + 4256*a^13*b^4 + 13272*a^11*b^6 + 11977*a^9*b^8 - 15634*a^7*b^10 - 11088*a^5*b^12 - 2870*a^3*b^14 - 25*a*b^16)*cos(d*x + c)^2)*sin(d*x + c))/(7*(a^19*b^6 - 9*a^17*b^8 + 36*a^15*b^10 - 84*a^13*b^12 + 126*a^11*b^14 - 126*a^9*b^16 + 84*a^7*b^18 - 36*a^5*b^20 + 9*a^3*b^22 - a*b^24)*d*cos(d*x + c)^7 - 7*(5*a^21*b^4 - 42*a^19*b^6 + 153*a^17*b^8 - 312*a^15*b^10 + 378*a^13*b^12 - 252*a^11*b^14 + 42*a^9*b^16 + 72*a^7*b^18 - 63*a^5*b^20 + 22*a^3*b^22 - 3*a*b^24)*d*cos(d*x + c)^5 + 7*(3*a^23*b^2 - 17*a^21*b^4 + 21*a^19*b^6 + 81*a^17*b^8 - 354*a^15*b^10 + 630*a^13*b^12 - 630*a^11*b^14 + 354*a^9*b^16 - 81*a^7*b^18 - 21*a^5*b^20 + 17*a^3*b^22 - 3*a*b^24)*d*cos(d*x + c)^3 - (a^25 + 12*a^23*b^2 - 118*a^21*b^4 + 364*a^19*b^6 - 441*a^17*b^8 - 168*a^15*b^10 + 1260*a^13*b^12 - 1800*a^11*b^14 + 1311*a^9*b^16 - 484*a^7*b^18 + 42*a^5*b^20 + 28*a^3*b^22 - 7*a*b^24)*d*cos(d*x + c) + ((a^18*b^7 - 9*a^16*b^9 + 36*a^14*b^11 - 84*a^12*b^13 + 126*a^10*b^15 - 126*a^8*b^17 + 84*a^6*b^19 - 36*a^4*b^21 + 9*a^2*b^23 - b^25)*d*cos(d*x + c)^7 - 3*(7*a^20*b^5 - 62*a^18*b^7 + 243*a^16*b^9 - 552*a^14*b^11 + 798*a^12*b^13 - 756*a^10*b^15 + 462*a^8*b^17 - 168*a^6*b^19 + 27*a^4*b^21 + 2*a^2*b^23 - b^25)*d*cos(d*x + c)^5 + (35*a^22*b^3 - 273*a^20*b^5 + 885*a^18*b^7 - 1455*a^16*b^9 + 990*a^14*b^11 + 630*a^12*b^13 - 1974*a^10*b^15 + 1890*a^8*b^17 - 945*a^6*b^19 + 235*a^4*b^21 - 15*a^2*b^23 - 3*b^25)*d*cos(d*x + c)^3 - (7

```
*a^24*b - 28*a^22*b^3 - 42*a^20*b^5 + 484*a^18*b^7 - 1311*a^16*b^9 + 1800*a
^14*b^11 - 1260*a^12*b^13 + 168*a^10*b^15 + 441*a^8*b^17 - 364*a^6*b^19 + 1
18*a^4*b^21 - 12*a^2*b^23 - b^25)*d*cos(d*x + c))*sin(d*x + c)), 1/560*(560
*a^16*b - 4480*a^14*b^3 + 15680*a^12*b^5 - 31360*a^10*b^7 + 39200*a^8*b^9 -
 31360*a^6*b^11 + 15680*a^4*b^13 - 4480*a^2*b^15 + 560*b^17 - (560*a^10*b^7
 + 41912*a^8*b^9 + 83162*a^6*b^11 - 71123*a^4*b^13 - 52463*a^2*b^15 - 2048*
b^17)*cos(d*x + c)^8 + 14*(840*a^12*b^5 + 53648*a^10*b^7 + 95441*a^8*b^9 -
77704*a^6*b^11 - 60644*a^4*b^13 - 11069*a^2*b^15 - 512*b^17)*cos(d*x + c)^6
 - 35*(560*a^14*b^3 + 27440*a^12*b^5 + 71064*a^10*b^7 + 29927*a^8*b^9 - 814
21*a^6*b^11 - 43131*a^4*b^13 - 4183*a^2*b^15 - 256*b^17)*cos(d*x + c)^4 + 7
0*(56*a^16*b + 1400*a^14*b^3 + 13832*a^12*b^5 + 24080*a^10*b^7 - 4591*a^8*b
^9 - 23443*a^6*b^11 - 10717*a^4*b^13 - 553*a^2*b^15 - 64*b^17)*cos(d*x + c)
^2 + 315*(7*(64*a^8*b^8 + 336*a^6*b^10 + 280*a^4*b^12 + 35*a^2*b^14)*cos(d*
x + c)^7 - 7*(320*a^10*b^6 + 1872*a^8*b^8 + 2408*a^6*b^10 + 1015*a^4*b^12 +
 105*a^2*b^14)*cos(d*x + c)^5 + 7*(192*a^12*b^4 + 1648*a^10*b^6 + 4392*a^8*
b^8 + 3913*a^6*b^10 + 1190*a^4*b^12 + 105*a^2*b^14)*cos(d*x + c)^3 - (64*a^
14*b^2 + 1680*a^12*b^4 + 9576*a^10*b^6 + 18123*a^8*b^8 + 12887*a^6*b^10 + 3
185*a^4*b^12 + 245*a^2*b^14)*cos(d*x + c) + ((64*a^7*b^9 + 336*a^5*b^11 + 2
80*a^3*b^13 + 35*a*b^15)*cos(d*x + c)^7 - 3*(448*a^9*b^7 + 2416*a^7*b^9 + 2
296*a^5*b^11 + 525*a^3*b^13 + 35*a*b^15)*cos(d*...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+b*sin(d*x+c))**8,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 2610 vs. 2(506) = 1012.
time = 6.57, size = 2610, normalized size = 4.93

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -1/280*(315*(64*a^7*b^2 + 336*a^5*b^4 + 280*a^3*b^6 + 35*a*b^8)*(pi*floor(1
/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^
2 - b^2)))/((a^16 - 8*a^14*b^2 + 28*a^12*b^4 - 56*a^10*b^6 + 70*a^8*b^8 - 5
6*a^6*b^10 + 28*a^4*b^12 - 8*a^2*b^14 + b^16)*sqrt(a^2 - b^2)) + 560*(a^8*t
an(1/2*d*x + 1/2*c) + 28*a^6*b^2*tan(1/2*d*x + 1/2*c) + 70*a^4*b^4*tan(1/2*
d*x + 1/2*c) + 28*a^2*b^6*tan(1/2*d*x + 1/2*c) + b^8*tan(1/2*d*x + 1/2*c) -
```

8*a^7*b - 56*a^5*b^3 - 56*a^3*b^5 - 8*a*b^7)/((a^16 - 8*a^14*b^2 + 28*a^12
*b^4 - 56*a^10*b^6 + 70*a^8*b^8 - 56*a^6*b^10 + 28*a^4*b^12 - 8*a^2*b^14 +
b^16)*(tan(1/2*d*x + 1/2*c)^2 - 1)) + (82320*a^18*b^4*tan(1/2*d*x + 1/2*c)^
13 + 41160*a^16*b^6*tan(1/2*d*x + 1/2*c)^13 + 49665*a^14*b^8*tan(1/2*d*x +
1/2*c)^13 - 31360*a^12*b^10*tan(1/2*d*x + 1/2*c)^13 + 15680*a^10*b^12*tan(1
/2*d*x + 1/2*c)^13 - 4480*a^8*b^14*tan(1/2*d*x + 1/2*c)^13 + 560*a^6*b^16*t
an(1/2*d*x + 1/2*c)^13 + 47040*a^19*b^3*tan(1/2*d*x + 1/2*c)^12 + 952560*a^
17*b^5*tan(1/2*d*x + 1/2*c)^12 + 743400*a^15*b^7*tan(1/2*d*x + 1/2*c)^12 +
370685*a^13*b^9*tan(1/2*d*x + 1/2*c)^12 - 188160*a^11*b^11*tan(1/2*d*x + 1/
2*c)^12 + 94080*a^9*b^13*tan(1/2*d*x + 1/2*c)^12 - 26880*a^7*b^15*tan(1/2*d
*x + 1/2*c)^12 + 3360*a^5*b^17*tan(1/2*d*x + 1/2*c)^12 + 987840*a^18*b^4*ta
n(1/2*d*x + 1/2*c)^11 + 5221440*a^16*b^6*tan(1/2*d*x + 1/2*c)^11 + 4792620*
a^14*b^8*tan(1/2*d*x + 1/2*c)^11 + 1272530*a^12*b^10*tan(1/2*d*x + 1/2*c)^1
1 - 501760*a^10*b^12*tan(1/2*d*x + 1/2*c)^11 + 277760*a^8*b^14*tan(1/2*d*x
+ 1/2*c)^11 - 85120*a^6*b^16*tan(1/2*d*x + 1/2*c)^11 + 11200*a^4*b^18*tan(1
/2*d*x + 1/2*c)^11 + 282240*a^19*b^3*tan(1/2*d*x + 1/2*c)^10 + 7056000*a^17
*b^5*tan(1/2*d*x + 1/2*c)^10 + 18695040*a^15*b^7*tan(1/2*d*x + 1/2*c)^10 +
15575140*a^13*b^9*tan(1/2*d*x + 1/2*c)^10 + 2689610*a^11*b^11*tan(1/2*d*x +
1/2*c)^10 - 721280*a^9*b^13*tan(1/2*d*x + 1/2*c)^10 + 474880*a^7*b^15*tan(
1/2*d*x + 1/2*c)^10 - 160160*a^5*b^17*tan(1/2*d*x + 1/2*c)^10 + 22400*a^3*b
^19*tan(1/2*d*x + 1/2*c)^10 + 3704400*a^18*b^4*tan(1/2*d*x + 1/2*c)^9 + 269
48040*a^16*b^6*tan(1/2*d*x + 1/2*c)^9 + 46663365*a^14*b^8*tan(1/2*d*x + 1/2
*c)^9 + 29114330*a^12*b^10*tan(1/2*d*x + 1/2*c)^9 + 3411772*a^10*b^12*tan(1
/2*d*x + 1/2*c)^9 - 305536*a^8*b^14*tan(1/2*d*x + 1/2*c)^9 + 388976*a^6*b^1
6*tan(1/2*d*x + 1/2*c)^9 - 167552*a^4*b^18*tan(1/2*d*x + 1/2*c)^9 + 26880*a
^2*b^20*tan(1/2*d*x + 1/2*c)^9 + 705600*a^19*b^3*tan(1/2*d*x + 1/2*c)^8 + 1
8780720*a^17*b^5*tan(1/2*d*x + 1/2*c)^8 + 65305800*a^15*b^7*tan(1/2*d*x + 1
/2*c)^8 + 77673085*a^13*b^9*tan(1/2*d*x + 1/2*c)^8 + 32483570*a^11*b^11*tan
(1/2*d*x + 1/2*c)^8 + 2139928*a^9*b^13*tan(1/2*d*x + 1/2*c)^8 + 587776*a^7*
b^15*tan(1/2*d*x + 1/2*c)^8 - 7616*a^5*b^17*tan(1/2*d*x + 1/2*c)^8 - 74368*
a^3*b^19*tan(1/2*d*x + 1/2*c)^8 + 17920*a*b^21*tan(1/2*d*x + 1/2*c)^8 + 658
5600*a^18*b^4*tan(1/2*d*x + 1/2*c)^7 + 51038400*a^16*b^6*tan(1/2*d*x + 1/2*
c)^7 + 104499360*a^14*b^8*tan(1/2*d*x + 1/2*c)^7 + 80185140*a^12*b^10*tan(1
/2*d*x + 1/2*c)^7 + 20029744*a^10*b^12*tan(1/2*d*x + 1/2*c)^7 + 661136*a^8*
b^14*tan(1/2*d*x + 1/2*c)^7 + 683008*a^6*b^16*tan(1/2*d*x + 1/2*c)^7 - 2176
00*a^4*b^18*tan(1/2*d*x + 1/2*c)^7 + 13312*a^2*b^20*tan(1/2*d*x + 1/2*c)^7
+ 5120*b^22*tan(1/2*d*x + 1/2*c)^7 + 940800*a^19*b^3*tan(1/2*d*x + 1/2*c)^6
+ 23614080*a^17*b^5*tan(1/2*d*x + 1/2*c)^6 + 83805120*a^15*b^7*tan(1/2*d*x
+ 1/2*c)^6 + 103990880*a^13*b^9*tan(1/2*d*x + 1/2*c)^6 + 45853220*a^11*b^1
1*tan(1/2*d*x + 1/2*c)^6 + 4650688*a^9*b^13*tan(1/2*d*x + 1/2*c)^6 + 692496
*a^7*b^15*tan(1/2*d*x + 1/2*c)^6 - 7616*a^5*b^17*tan(1/2*d*x + 1/2*c)^6 - 7
4368*a^3*b^19*tan(1/2*d*x + 1/2*c)^6 + 17920*a*b^21*tan(1/2*d*x + 1/2*c)^6
+ 6174000*a^18*b^4*tan(1/2*d*x + 1/2*c)^5 + 43023960*a^16*b^6*tan(1/2*d*x +
1/2*c)^5 + 82755435*a^14*b^8*tan(1/2*d*x + 1/2*c)^5 + 55248340*a^12*b^10*t
an(1/2*d*x + 1/2*c)^5 + 10337432*a^10*b^12*tan(1/2*d*x + 1/2*c)^5 - 175056*

```
a^8*b^14*tan(1/2*d*x + 1/2*c)^5 + 388976*a^6*b^16*tan(1/2*d*x + 1/2*c)^5 -
167552*a^4*b^18*tan(1/2*d*x + 1/2*c)^5 + 26880*a^2*b^20*tan(1/2*d*x + 1/2*c
)^5 + 705600*a^19*b^3*tan(1/2*d*x + 1/2*c)^4 + 14429520*a^17*b^5*tan(1/2*d*
x + 1/2*c)^4 + 42782712*a^15*b^7*tan(1/2*d*x + 1/2*c)^4 + 41655719*a^13*b^9
*tan(1/2*d*x + 1/2*c)^4 + 10567396*a^11*b^11*tan(1/2*d*x + 1/2*c)^4 - 70403
2*a^9*b^13*tan(1/2*d*x + 1/2*c)^4 + 485520*a^7*b^15*tan(1/2*d*x + 1/2*c)^4
- 160160*a^5*b^17*tan(1/2*d*x + 1/2*c)^4 + 22400*a^3*b^19*tan(1/2*d*x + 1/2
*c)^4 + 2963520*a^18*b^4*tan(1/2*d*x + 1/2*c)^3 + 14864640*a^16*b^6*tan(1/2
*d*x + 1/2*c)^3 + 20500788*a^14*b^8*tan(1/2*d*x + 1/2*c)^3 + 5857306*a^12*b
^10*tan(1/2*d*x + 1/2*c)^3 - 479696*a^10*b^12*tan(1/2*d*x + 1/2*c)^3 + 2812
32*a^8*b^14*tan(1/2*d*x + 1/2*c)^3 - 85120*a^6*b^16*tan(1/2*d*x + 1/2*c)^3
+ 11200*a^4*b^18*tan(1/2*d*x + 1/2*c)^3 + 282240*a^19*b^3*tan(1/2*d*x + 1/2
*c)^2 + 3575040*a^17*b^5*tan(1/2*d*x + 1/2*c)^2 + 6358464*a^15*b^7*tan(1/2*
d*x + 1/2*c)^2 + 1843996*a^13*b^9*tan(1/2*d*x + 1/2*c)^2 - 146062*a^11*b^11
*tan(1/2*d*x + 1/2*c)^2 + 85120*a^9*b^13*tan(1/2*d*x + 1/2*c)^2 - 25648*a^7
*b^15*tan(1/2*d*x + 1/2*c)^2 + 3360*a^5*b^17*ta...
```

Mupad [B]
time = 53.32, size = 2500, normalized size = 4.73

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + b*sin(c + d*x))^8),x)
```

```
[Out] - ((4480*a^14*b + 80*b^15 - 632*a^2*b^13 + 2186*a^4*b^11 - 4161*a^6*b^9 + 3
1192*a^8*b^7 + 113680*a^10*b^5 + 78400*a^12*b^3)/(280*(a^16 + b^16 - 8*a^2*
b^14 + 28*a^4*b^12 - 56*a^6*b^10 + 70*a^8*b^8 - 56*a^10*b^6 + 28*a^12*b^4 -
 8*a^14*b^2)) - (9*tan(c/2 + (d*x)/2)^8*(560*b^15 + 10360*a^2*b^13 + 59766*
a^4*b^11 + 117497*a^6*b^9 + 91112*a^8*b^7 + 25200*a^10*b^5 + 2240*a^12*b^3)
)/(8*(a^16 + b^16 - 8*a^2*b^14 + 28*a^4*b^12 - 56*a^6*b^10 + 70*a^8*b^8 - 5
6*a^10*b^6 + 28*a^12*b^4 - 8*a^14*b^2)) + (tan(c/2 + (d*x)/2)*(80*b^16 - 80
*a^16 - 624*a^2*b^14 + 2132*a^4*b^12 - 3842*a^6*b^10 + 55209*a^8*b^8 + 2192
40*a^10*b^6 + 139440*a^12*b^4 + 6720*a^14*b^2))/(40*a*(a^16 + b^16 - 8*a^2*
b^14 + 28*a^4*b^12 - 56*a^6*b^10 + 70*a^8*b^8 - 56*a^10*b^6 + 28*a^12*b^4 -
 8*a^14*b^2)) + (tan(c/2 + (d*x)/2)^7*(5120*b^22 - 19600*a^22 - 13568*a^2*b
^20 - 50048*a^4*b^18 + 294032*a^6*b^16 + 1158752*a^8*b^14 + 11762072*a^10*b
^12 + 34250720*a^12*b^10 + 32332965*a^14*b^8 + 15431080*a^16*b^6 + 1234800*
a^18*b^4 + 235200*a^20*b^2))/(280*a^7*(a^16 + b^16 - 8*a^2*b^14 + 28*a^4*b^
12 - 56*a^6*b^10 + 70*a^8*b^8 - 56*a^10*b^6 + 28*a^12*b^4 - 8*a^14*b^2)) -
(tan(c/2 + (d*x)/2)^9*(19600*a^22 + 5120*b^22 - 13568*a^2*b^20 - 50048*a^4*
b^18 + 294032*a^6*b^16 + 1217552*a^8*b^14 + 21572852*a^10*b^12 + 69353690*a
^12*b^10 + 86769515*a^14*b^8 + 39441080*a^16*b^6 + 6762000*a^18*b^4 + 78400
*a^20*b^2))/(280*a^7*(a^16 + b^16 - 8*a^2*b^14 + 28*a^4*b^12 - 56*a^6*b^10
+ 70*a^8*b^8 - 56*a^10*b^6 + 28*a^12*b^4 - 8*a^14*b^2)) - (3*tan(c/2 + (d*x
```

)/2)^11*(560*a^20 + 1280*b^20 - 8512*a^2*b^18 + 22576*a^4*b^16 - 27776*a^6*
b^14 + 201292*a^8*b^12 + 1695400*a^10*b^10 + 2917285*a^12*b^8 + 1708840*a^1
4*b^6 + 311920*a^16*b^4 + 8960*a^18*b^2))/(40*a^5*(a^16 + b^16 - 8*a^2*b^14
 + 28*a^4*b^12 - 56*a^6*b^10 + 70*a^8*b^8 - 56*a^10*b^6 + 28*a^12*b^4 - 8*a
^14*b^2)) + (3*tan(c/2 + (d*x)/2)^5*(1280*b^20 - 560*a^20 - 8512*a^2*b^18 +
 22576*a^4*b^16 - 21728*a^6*b^14 + 643528*a^8*b^12 + 3165074*a^10*b^10 + 43
25867*a^12*b^8 + 2252600*a^14*b^6 + 337680*a^16*b^4 + 17920*a^18*b^2))/(40*
a^5*(a^16 + b^16 - 8*a^2*b^14 + 28*a^4*b^12 - 56*a^6*b^10 + 70*a^8*b^8 - 56
*a^10*b^6 + 28*a^12*b^4 - 8*a^14*b^2)) - (tan(c/2 + (d*x)/2)^13*(112*a^18 +
 320*b^18 - 2448*a^2*b^16 + 8064*a^4*b^14 - 14784*a^6*b^12 + 38598*a^8*b^10
 + 171465*a^10*b^8 + 232680*a^12*b^6 + 58800*a^14*b^4 + 2688*a^16*b^2))/(8*
a^3*(a^16 + b^16 - 8*a^2*b^14 + 28*a^4*b^12 - 56*a^6*b^10 + 70*a^8*b^8 - 56
*a^10*b^6 + 28*a^12*b^4 - 8*a^14*b^2)) + (tan(c/2 + (d*x)/2)^3*(320*b^18 -
112*a^18 - 2448*a^2*b^16 + 8160*a^4*b^14 - 14132*a^6*b^12 + 202616*a^8*b^10
 + 800359*a^10*b^8 + 621880*a^12*b^6 + 133840*a^14*b^4 + 6272*a^16*b^2))/(8
a^3(a^16 + b^16 - 8*a^2*b^14 + 28*a^4*b^12 - 56*a^6*b^10 + 70*a^8*b^8 - 5
6*a^10*b^6 + 28*a^12*b^4 - 8*a^14*b^2)) - (tan(c/2 + (d*x)/2)^15*(16*a^16 +
 16*b^16 - 128*a^2*b^14 + 448*a^4*b^12 - 896*a^6*b^10 + 1435*a^8*b^8 + 1624
*a^10*b^6 + 3472*a^12*b^4 + 448*a^14*b^2))/(8*a*(a^16 + b^16 - 8*a^2*b^14 +
 28*a^4*b^12 - 56*a^6*b^10 + 70*a^8*b^8 - 56*a^10*b^6 + 28*a^12*b^4 - 8*a^1
4*b^2)) + (tan(c/2 + (d*x)/2)^6*(5600*a^20*b + 2560*b^21 - 13824*a^2*b^19 +
 21792*a^4*b^17 + 29568*a^6*b^15 + 997920*a^8*b^13 + 6528192*a^10*b^11 + 12
687263*a^12*b^9 + 9211384*a^14*b^7 + 2568720*a^16*b^5 + 168000*a^18*b^3))/(
40*a^6*(a^16 + b^16 - 8*a^2*b^14 + 28*a^4*b^12 - 56*a^6*b^10 + 70*a^8*b^8 -
 56*a^10*b^6 + 28*a^12*b^4 - 8*a^14*b^2)) - (tan(c/2 + (d*x)/2)^10*(3360*a^
20*b + 2560*b^21 - 13824*a^2*b^19 + 21792*a^4*b^17 + 16128*a^6*b^15 + 46250
4*a^8*b^13 + 5492760*a^10*b^11 + 12382335*a^12*b^9 + 10502520*a^14*b^7 + 30
79440*a^16*b^5 + 257600*a^18*b^3))/(40*a^6*(a^16 + b^16 - 8*a^2*b^14 + 28*a
^4*b^12 - 56*a^6*b^10 + 70*a^8*b^8 - 56*a^10*b^6 + 28*a^12*b^4 - 8*a^14*b^2
)) - (tan(c/2 + (d*x)/2)^12*(448*a^18*b + 640*b^19 - 4672*a^2*b^17 + 14336*
a^4*b^15 - 23296*a^6*b^13 + 86702*a^8*b^11 + 550445*a^10*b^9 + 787976*a^12*
b^7 + 312368*a^14*b^5 + 31808*a^16*b^3))/(8*a^4*(a^16 + b^16 - 8*a^2*b^14 +
 28*a^4*b^12 - 56*a^6*b^10 + 70*a^8*b^8 - 56*a^10*b^6 + 28*a^12*b^4 - 8*a^1
4*b^2)) + (tan(c/2 + (d*x)/2)^4*(6720*a^18*b + 3200*b^19 - 23360*a^2*b^17 +
 73024*a^4*b^15 - 112736*a^6*b^13 + 1866494*a^8*b^11 + 7831069*a^10*b^9 + 7
851144*a^12*b^7 + 2787120*a^14*b^5 + 212800*a^16*b^3))/(40*a^4*(a^16 + b^16
 - 8*a^2*b^14 + 28*a^4*b^12 - 56*a^6*b^10 + 70*a^8*b^8 - 56*a^10*b^6 + 28*a
^12*b^4 - 8*a^14*b^2)) - (3*tan(c/2 + (d*x)/2)^14*(32*a^16*b + 32*b^17 - 25
6*a^2*b^15 + 896*a^4*b^13 - 1792*a^6*b^11 + 3605*a^8*b^9 + 9128*a^10*b^7 +
14000*a^12*b^5 + 2240*a^14*b^3))/(8*a^2*(a^16 + b^16 - 8*a^2*b^14 + 28*a^4*
b^12 - 56*a^6*b^10 + 70*a^8*b^8 - 56*a^10*b^6 + 28*a^12*b^4 - 8*a^14*b^2))
 + (3*tan(c/2 + (d*x)/2)^2*(7840*a^16*b + 1120*b^17 - 8576*a^2*b^15 + 28584*
a^4*b^13 - 49416*a^6*b^11 + 738879*a^8*b^9 + 2925944*a^10*b^7 + 1932560*a^1
2*b^5 + 203840*a^14*b^3))/(280*a^2*(a^16 + b^16 - 8*a^2*b^14 + 28*a^4*b^12
 - 56*a^6*b^10 + 70*a^8*b^8 - 56*a^10*b^6 + 28*a^12*b^4 - 8*a^14*b^2)))/(d*(

```
tan(c/2 + (d*x)/2)^5*(126*a^6*b + 672*a^2*b^5 + 840*a^4*b^3) - tan(c/2 + (d
*x)/2)^11*(126*a^6*b + 672*a^2*b^5 + 840*a^4*b^...
```

3.472 $\int \frac{\sec^4(c+dx)}{(a+b\sin(c+dx))^8}\,dx$

Optimal. Leaf size=653

$$\frac{165ab^4(32a^6+112a^4b^2+70a^2b^4+7b^6)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{8\left(a^2-b^2\right)^{19/2}d}+\frac{b\sec^3(c+dx)}{7\left(a^2-b^2\right)d(a+b\sin(c+dx))^7}+\frac{}{42\left(a^2-\right.}$$

```
[Out] 165/8*a*b^4*(32*a^6+112*a^4*b^2+70*a^2*b^4+7*b^6)*arctan((b+a*tan(1/2*d*x+1
/2*c))/(a^2-b^2)^(1/2))/(a^2-b^2)^(19/2)/d+1/7*b*sec(d*x+c)^3/(a^2-b^2)/d/(
a+b*sin(d*x+c))^7+17/42*a*b*sec(d*x+c)^3/(a^2-b^2)^2/d/(a+b*sin(d*x+c))^6+1
/14*b*(13*a^2+4*b^2)*sec(d*x+c)^3/(a^2-b^2)^3/d/(a+b*sin(d*x+c))^5+1/56*a*b
*(118*a^2+103*b^2)*sec(d*x+c)^3/(a^2-b^2)^4/d/(a+b*sin(d*x+c))^4+1/168*b*(8
82*a^4+1421*a^2*b^2+128*b^4)*sec(d*x+c)^3/(a^2-b^2)^5/d/(a+b*sin(d*x+c))^3+
13/112*a*b*(140*a^4+336*a^2*b^2+85*b^4)*sec(d*x+c)^3/(a^2-b^2)^6/d/(a+b*sin
(d*x+c))^2+1/112*b*(9212*a^6+28420*a^4*b^2+12907*a^2*b^4+512*b^6)*sec(d*x+c
)^3/(a^2-b^2)^7/d/(a+b*sin(d*x+c))-1/336*sec(d*x+c)^3*(1155*a*b*(32*a^6+112
*a^4*b^2+70*a^2*b^4+7*b^6)-(112*a^8+52528*a^6*b^2+142902*a^4*b^4+57665*a^2*
b^6+2048*b^8)*sin(d*x+c))/(a^2-b^2)^8/d+1/336*sec(d*x+c)*(3465*a*b^3*(32*a^
6+112*a^4*b^2+70*a^2*b^4+7*b^6)+(224*a^10-6048*a^8*b^2-207332*a^6*b^4-41302
4*a^4*b^6-135489*a^2*b^8-4096*b^10)*sin(d*x+c))/(a^2-b^2)^9/d
```

Rubi [A]

time = 1.38, antiderivative size = 653, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2773, 2943, 2945, 12, 2739, 632, 210}

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + b*Sin[c + d*x])^8,x]
```

```
[Out] (165*a*b^4*(32*a^6 + 112*a^4*b^2 + 70*a^2*b^4 + 7*b^6)*ArcTan[(b + a*Tan[(c
 + d*x)/2])/Sqrt[a^2 - b^2]])/(8*(a^2 - b^2)^(19/2)*d) + (b*Sec[c + d*x]^3)
/(7*(a^2 - b^2)*d*(a + b*Sin[c + d*x])^7) + (17*a*b*Sec[c + d*x]^3)/(42*(a^
2 - b^2)^2*d*(a + b*Sin[c + d*x])^6) + (b*(13*a^2 + 4*b^2)*Sec[c + d*x]^3)/
(14*(a^2 - b^2)^3*d*(a + b*Sin[c + d*x])^5) + (a*b*(118*a^2 + 103*b^2)*Sec[
c + d*x]^3)/(56*(a^2 - b^2)^4*d*(a + b*Sin[c + d*x])^4) + (b*(882*a^4 + 142
1*a^2*b^2 + 128*b^4)*Sec[c + d*x]^3)/(168*(a^2 - b^2)^5*d*(a + b*Sin[c + d*
x])^3) + (13*a*b*(140*a^4 + 336*a^2*b^2 + 85*b^4)*Sec[c + d*x]^3)/(112*(a^2
 - b^2)^6*d*(a + b*Sin[c + d*x])^2) + (b*(9212*a^6 + 28420*a^4*b^2 + 12907*
a^2*b^4 + 512*b^6)*Sec[c + d*x]^3)/(112*(a^2 - b^2)^7*d*(a + b*Sin[c + d*x]
)) - (Sec[c + d*x]^3*(1155*a*b*(32*a^6 + 112*a^4*b^2 + 70*a^2*b^4 + 7*b^6)
- (112*a^8 + 52528*a^6*b^2 + 142902*a^4*b^4 + 57665*a^2*b^6 + 2048*b^8)*Sin
```

```
[c + d*x]))/(336*(a^2 - b^2)^8*d) + (Sec[c + d*x]*(3465*a*b^3*(32*a^6 + 112
*a^4*b^2 + 70*a^2*b^4 + 7*b^6) + (224*a^10 - 6048*a^8*b^2 - 207332*a^6*b^4
- 413024*a^4*b^6 - 135489*a^2*b^8 - 4096*b^10)*Sin[c + d*x]))/(336*(a^2 - b
^2)^9*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2773

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m + 1)/(f*g*(a^2 - b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1))
, Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p
 + 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^
2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2943

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c
 - a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^(m + 1)/(f*g*(a^2 -
 b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*
(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p
+ 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[
a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2945

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{(a+b\sin(c+dx))^8}\,dx = \frac{b\sec^3(c+dx)}{7(a^2-b^2)\,d(a+b\sin(c+dx))^7} - \frac{\int \frac{\sec^4(c+dx)(-7a+10b\sin(c+dx))}{(a+b\sin(c+dx))^7}\,dx}{7(a^2-b^2)}$$

$$= \frac{b\sec^3(c+dx)}{7(a^2-b^2)\,d(a+b\sin(c+dx))^7} + \frac{17ab\sec^3(c+dx)}{42(a^2-b^2)^2\,d(a+b\sin(c+dx))^6} + \frac{\int \frac{\sec^4(c-}{}}{}$$

$$= \frac{b\sec^3(c+dx)}{7(a^2-b^2)\,d(a+b\sin(c+dx))^7} + \frac{17ab\sec^3(c+dx)}{42(a^2-b^2)^2\,d(a+b\sin(c+dx))^6} + \frac{b(13}{14(a^2-}$$

$$= \frac{b\sec^3(c+dx)}{7(a^2-b^2)\,d(a+b\sin(c+dx))^7} + \frac{17ab\sec^3(c+dx)}{42(a^2-b^2)^2\,d(a+b\sin(c+dx))^6} + \frac{b(13}{14(a^2-}$$

$$= \frac{b\sec^3(c+dx)}{7(a^2-b^2)\,d(a+b\sin(c+dx))^7} + \frac{17ab\sec^3(c+dx)}{42(a^2-b^2)^2\,d(a+b\sin(c+dx))^6} + \frac{b(13}{14(a^2-}$$

$$= \frac{b\sec^3(c+dx)}{7(a^2-b^2)\,d(a+b\sin(c+dx))^7} + \frac{17ab\sec^3(c+dx)}{42(a^2-b^2)^2\,d(a+b\sin(c+dx))^6} + \frac{b(13}{14(a^2-}$$

$$= \frac{b\sec^3(c+dx)}{7(a^2-b^2)\,d(a+b\sin(c+dx))^7} + \frac{17ab\sec^3(c+dx)}{42(a^2-b^2)^2\,d(a+b\sin(c+dx))^6} + \frac{b(13}{14(a^2-}$$

$$= \frac{b\sec^3(c+dx)}{7(a^2-b^2)\,d(a+b\sin(c+dx))^7} + \frac{17ab\sec^3(c+dx)}{42(a^2-b^2)^2\,d(a+b\sin(c+dx))^6} + \frac{b(13}{14(a^2-}$$

$$= \frac{b\sec^3(c+dx)}{7(a^2-b^2)\,d(a+b\sin(c+dx))^7} + \frac{17ab\sec^3(c+dx)}{42(a^2-b^2)^2\,d(a+b\sin(c+dx))^6} + \frac{b(13}{14(a^2-}$$

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$$= \frac{165ab^4(32a^6+112a^4b^2+70a^2b^4+7b^6)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{8(a^2-b^2)^{19/2}\,d} + \frac{b\sec}{7(a^2-b^2)\,d(}$$

Mathematica [A]
time = 5.25, size = 597, normalized size = 0.91

$$\frac{\frac{6930ab^4(32a^6+112a^4b^2+70a^2b^4+7b^6)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{19/2}} + \frac{48b^5\cos(c+dx)}{(a^2-b^2)^3(a+b\sin(c+dx))^7} + \frac{328ab^5\cos(c+dx)}{(a^2-b^2)^4(a+b\sin(c+dx))^6} + \frac{8b^5(167a^2+24b^2)\cos(c+dx)}{(a^2-b^2)^5(a+b\sin(c+dx))^5} + \frac{2ab^5(2138a^2+925b^2)\cos(c+dx)}{(a^2-b^2)^6(a+b\sin(c+dx))^4} + \frac{2b^5(6058a^4+5273a^2b^2+296b^4)\cos(c+dx)}{(a^2-b^2)^7(a+b\sin(c+dx))^3} + \frac{ab^5(33284a^4+48820a^2b^2+8287b^4)\cos(c+dx)}{(a^2-b^2)^8(a+b\sin(c+dx))^2} + \frac{b^5(103844a^6+234272a^4b^2+81057a^2b^4+2528b^6)\cos(c+dx)}{(a^2-b^2)^9(a+b\sin(c+dx))} + \frac{112\sec^3(c+dx)(-8ab(a^6+7a^4b^2+7a^2b^4+b^6)+(a^8+28a^6b^2+70a^4b^4+28a^2b^6+b^8)\sin(c+dx))}{(a^2-b^2)^8} + \frac{224\sec(c+dx)(12(15a^7b^3+63a^5b^5+45a^3b^7+5ab^9)+(a^{10}-27a^8b^2-462a^6b^4-798a^4b^6-243a^2b^8-7b^{10})\sin(c+dx))}{(a^2-b^2)^9}}{336d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + b*Sin[c + d*x])^8,x]
```

```
[Out] ((6930*a*b^4*(32*a^6 + 112*a^4*b^2 + 70*a^2*b^4 + 7*b^6)*ArcTan[(b + a*Tan[
(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(19/2) + (48*b^5*Cos[c + d*x])/
((a^2 - b^2)^3*(a + b*Sin[c + d*x])^7) + (328*a*b^5*Cos[c + d*x])/((a^2 - b
^2)^4*(a + b*Sin[c + d*x])^6) + (8*b^5*(167*a^2 + 24*b^2)*Cos[c + d*x])/((a
^2 - b^2)^5*(a + b*Sin[c + d*x])^5) + (2*a*b^5*(2138*a^2 + 925*b^2)*Cos[c +
 d*x])/((a^2 - b^2)^6*(a + b*Sin[c + d*x])^4) + (2*b^5*(6058*a^4 + 5273*a^2
*b^2 + 296*b^4)*Cos[c + d*x])/((a^2 - b^2)^7*(a + b*Sin[c + d*x])^3) + (a*b
^5*(33284*a^4 + 48820*a^2*b^2 + 8287*b^4)*Cos[c + d*x])/((a^2 - b^2)^8*(a +
 b*Sin[c + d*x])^2) + (b^5*(103844*a^6 + 234272*a^4*b^2 + 81057*a^2*b^4 + 2
528*b^6)*Cos[c + d*x])/((a^2 - b^2)^9*(a + b*Sin[c + d*x])) + (112*Sec[c +
d*x]^3*(-8*a*b*(a^6 + 7*a^4*b^2 + 7*a^2*b^4 + b^6) + (a^8 + 28*a^6*b^2 + 70
*a^4*b^4 + 28*a^2*b^6 + b^8)*Sin[c + d*x]))/(a^2 - b^2)^8 + (224*Sec[c + d*
x]*(12*(15*a^7*b^3 + 63*a^5*b^5 + 45*a^3*b^7 + 5*a*b^9) + (a^10 - 27*a^8*b^
2 - 462*a^6*b^4 - 798*a^4*b^6 - 243*a^2*b^8 - 7*b^10)*Sin[c + d*x]))/(a^2 -
 b^2)^9)/(336*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1379 vs. $2(628) = 1256$.
time = 4.30, size = 1380, normalized size = 2.11

method	result	size
derivativedivides	Expression too large to display	1380
default	Expression too large to display	1380
risch	Expression too large to display	3064

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+b*sin(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/3/(a+b)^8/(tan(1/2*d*x+1/2*c)-1)^3-1/2/(a+b)^8/(tan(1/2*d*x+1/2*c)-
1)^2-(a+5*b)/(a+b)^9/(tan(1/2*d*x+1/2*c)-1)-1/3/(a-b)^8/(tan(1/2*d*x+1/2*c)
+1)^3+1/2/(a-b)^8/(tan(1/2*d*x+1/2*c)+1)^2-(a-5*b)/(a-b)^9/(tan(1/2*d*x+1/2
*c)+1)+2*b^4/(a-b)^9/(a+b)^9*((1/16*b^2*(11088*a^12+6798*a^10*b^2+3091*a^8*
b^4-1344*a^6*b^6+576*a^4*b^8-144*a^2*b^10+16*b^12)/a*tan(1/2*d*x+1/2*c)^13+
1/16*b*(7392*a^14+132528*a^12*b^2+100518*a^10*b^4+25991*a^8*b^6-8064*a^6*b^
8+3456*a^4*b^10-864*a^2*b^12+96*b^14)/a^2*tan(1/2*d*x+1/2*c)^12+1/24/a^3*b^
2*(221760*a^14+1107612*a^12*b^2+885544*a^10*b^4+155169*a^8*b^6-32256*a^6*b^
8+15264*a^4*b^10-4096*a^2*b^12+480*b^14)*tan(1/2*d*x+1/2*c)^11+1/24/a^4*b*(
66528*a^16+1574496*a^14*b^2+3884100*a^12*b^4+2736860*a^10*b^6+381885*a^8*b^
8-48960*a^6*b^10+26640*a^4*b^12-7760*a^2*b^14+960*b^16)*tan(1/2*d*x+1/2*c)^
10+1/48/a^5*b^2*(1718640*a^16+11886930*a^14*b^2+18491825*a^12*b^4+9856770*a
^10*b^6+1146588*a^8*b^8-59760*a^6*b^10+46960*a^4*b^12-16512*a^2*b^14+2304*b
```

```
^16)*tan(1/2*d*x+1/2*c)^9+1/48/a^6*b*(332640*a^18+8651280*a^16*b^2+27807890
*a^14*b^4+29152473*a^12*b^6+10622738*a^10*b^8+917592*a^8*b^10+48960*a^6*b^1
2+5440*a^4*b^14-7808*a^2*b^16+1536*b^18)*tan(1/2*d*x+1/2*c)^8+1/84/a^7*b^2*
(5433120*a^18+40230960*a^16*b^2+73645726*a^14*b^4+49633899*a^12*b^6+1131281
2*a^10*b^8+549276*a^8*b^10+136320*a^6*b^12-34432*a^4*b^14+1280*a^2*b^16+768
*b^18)*tan(1/2*d*x+1/2*c)^7+1/12/a^6*b*(110880*a^18+2766720*a^16*b^2+896720
0*a^14*b^4+9794970*a^12*b^6+3768737*a^10*b^8+417528*a^8*b^10+18420*a^6*b^12
+1360*a^4*b^14-1952*a^2*b^16+384*b^18)*tan(1/2*d*x+1/2*c)^6+1/48/a^5*b^2*(2
938320*a^16+19492110*a^14*b^2+32820667*a^12*b^4+19077284*a^10*b^6+3198648*a
^8*b^8-27040*a^6*b^10+46960*a^4*b^12-16512*a^2*b^14+2304*b^16)*tan(1/2*d*x+
1/2*c)^5+1/48/a^4*b*(332640*a^16+6819120*a^14*b^2+17886198*a^12*b^4+1500069
5*a^10*b^6+3081220*a^8*b^8-87552*a^6*b^10+55024*a^4*b^12-15520*a^2*b^14+192
0*b^16)*tan(1/2*d*x+1/2*c)^4+1/24/a^3*b^2*(709632*a^14+3305412*a^12*b^2+377
9732*a^10*b^4+836821*a^8*b^6-28824*a^6*b^8+15592*a^4*b^10-4096*a^2*b^12+480
*b^14)*tan(1/2*d*x+1/2*c)^3+1/24*b*(66528*a^14+841632*a^12*b^2+1182940*a^10
*b^4+263232*a^8*b^6-8399*a^6*b^8+4624*a^4*b^10-1224*a^2*b^12+144*b^14)/a^2*
tan(1/2*d*x+1/2*c)^2+1/48*b^2*(277200*a^12+415734*a^10*b^2+92059*a^8*b^4-30
78*a^6*b^6+1612*a^4*b^8-416*a^2*b^10+48*b^12)/a*tan(1/2*d*x+1/2*c)+1/336*(1
55232*a^12+218064*a^10*b^2+50666*a^8*b^4-3555*a^6*b^6+1670*a^4*b^8-424*a^2*
b^10+48*b^12)*b)/(a*tan(1/2*d*x+1/2*c)^2+2*b*tan(1/2*d*x+1/2*c)+a)^7+165/16
*a*(32*a^6+112*a^4*b^2+70*a^2*b^4+7*b^6)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*ta
n(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sin(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2208 vs. $2(628) = 1256$.
time = 0.90, size = 4500, normalized size = 6.89

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sin(d*x+c))^8,x, algorithm="fricas")
```

[Out] [1/672*(224*a^18*b - 2016*a^16*b^3 + 8064*a^14*b^5 - 18816*a^12*b^7 + 28224
*a^10*b^9 - 28224*a^8*b^11 + 18816*a^6*b^13 - 8064*a^4*b^15 + 2016*a^2*b^17
 - 224*b^19 - 2*(224*a^12*b^7 - 6272*a^10*b^9 - 201284*a^8*b^11 - 205692*a^
6*b^13 + 277535*a^4*b^15 + 131393*a^2*b^17 + 4096*b^19)*cos(d*x + c)^10 + 2
8*(336*a^14*b^5 - 9352*a^12*b^7 - 252014*a^10*b^9 - 230159*a^8*b^11 + 29731
2*a^6*b^13 + 165122*a^4*b^15 + 27731*a^2*b^17 + 1024*b^19)*cos(d*x + c)^8 -
 70*(224*a^16*b^3 - 5936*a^14*b^5 - 126448*a^12*b^7 - 243082*a^10*b^9 - 297
47*a^8*b^11 + 284285*a^6*b^13 + 109607*a^4*b^15 + 10585*a^2*b^17 + 512*b^19
)*cos(d*x + c)^6 + 28*(112*a^18*b - 2296*a^16*b^3 - 35224*a^14*b^5 - 308392
*a^12*b^7 - 337750*a^10*b^9 + 149783*a^8*b^11 + 394751*a^6*b^13 + 130949*a^
4*b^15 + 7427*a^2*b^17 + 640*b^19)*cos(d*x + c)^4 - 224*(7*a^18*b - 46*a^16
*b^3 + 116*a^14*b^5 - 112*a^12*b^7 - 70*a^10*b^9 + 308*a^8*b^11 - 364*a^6*b
^13 + 224*a^4*b^15 - 73*a^2*b^17 + 10*b^19)*cos(d*x + c)^2 + 3465*(7*(32*a^
8*b^10 + 112*a^6*b^12 + 70*a^4*b^14 + 7*a^2*b^16)*cos(d*x + c)^9 - 7*(160*a
^10*b^8 + 656*a^8*b^10 + 686*a^6*b^12 + 245*a^4*b^14 + 21*a^2*b^16)*cos(d*x
 + c)^7 + 7*(96*a^12*b^6 + 656*a^10*b^8 + 1426*a^8*b^10 + 1057*a^6*b^12 + 2
80*a^4*b^14 + 21*a^2*b^16)*cos(d*x + c)^5 - (32*a^14*b^4 + 784*a^12*b^6 + 3
542*a^10*b^8 + 5621*a^8*b^10 + 3381*a^6*b^12 + 735*a^4*b^14 + 49*a^2*b^16)*
cos(d*x + c)^3 + ((32*a^7*b^11 + 112*a^5*b^13 + 70*a^3*b^15 + 7*a*b^17)*cos
(d*x + c)^9 - 3*(224*a^9*b^9 + 816*a^7*b^11 + 602*a^5*b^13 + 119*a^3*b^15 +
 7*a*b^17)*cos(d*x + c)^7 + (1120*a^11*b^7 + 5264*a^9*b^9 + 7250*a^7*b^11 +
 3521*a^5*b^13 + 504*a^3*b^15 + 21*a*b^17)*cos(d*x + c)^5 - (224*a^13*b^5 +
 1904*a^11*b^7 + 5082*a^9*b^9 + 4883*a^7*b^11 + 1827*a^5*b^13 + 217*a^3*b^1
5 + 7*a*b^17)*cos(d*x + c)^3)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(-((2*a^2 -
 b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*s
in(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b
*sin(d*x + c) - a^2 - b^2)) - 14*(16*a^19 - 144*a^17*b^2 + 576*a^15*b^4 - 1
344*a^13*b^6 + 2016*a^11*b^8 - 2016*a^9*b^10 + 1344*a^7*b^12 - 576*a^5*b^14
 + 144*a^3*b^16 - 16*a*b^18 - (224*a^13*b^6 - 6272*a^11*b^8 - 185444*a^9*b^
10 - 166092*a^7*b^12 + 256745*a^5*b^14 + 100208*a^3*b^16 + 631*a*b^18)*cos(
d*x + c)^8 + 10*(112*a^15*b^4 - 3080*a^13*b^6 - 73962*a^11*b^8 - 78323*a^9*
b^10 + 60829*a^7*b^12 + 73923*a^5*b^14 + 20401*a^3*b^16 + 100*a*b^18)*cos(d
x + c)^6 - 3(224*a^17*b^2 - 5712*a^15*b^4 - 95648*a^13*b^6 - 254254*a^11*
b^8 - 120855*a^9*b^10 + 282886*a^7*b^12 + 157892*a^5*b^14 + 35448*a^3*b^16
+ 19*a*b^18)*cos(d*x + c)^4 + 16*(2*a^19 - 35*a^17*b^2 + 208*a^15*b^4 - 644
*a^13*b^6 + 1204*a^11*b^8 - 1442*a^9*b^10 + 1120*a^7*b^12 - 548*a^5*b^14 +
154*a^3*b^16 - 19*a*b^18)*cos(d*x + c)^2)*sin(d*x + c))/(7*(a^21*b^6 - 10*a
^19*b^8 + 45*a^17*b^10 - 120*a^15*b^12 + 210*a^13*b^14 - 252*a^11*b^16 + 21
0*a^9*b^18 - 120*a^7*b^20 + 45*a^5*b^22 - 10*a^3*b^24 + a*b^26)*d*cos(d*x +
 c)^9 - 7*(5*a^23*b^4 - 47*a^21*b^6 + 195*a^19*b^8 - 465*a^17*b^10 + 690*a^
15*b^12 - 630*a^13*b^14 + 294*a^11*b^16 + 30*a^9*b^18 - 135*a^7*b^20 + 85*a
^5*b^22 - 25*a^3*b^24 + 3*a*b^26)*d*cos(d*x + c)^7 + 7*(3*a^25*b^2 - 20*a^2
3*b^4 + 38*a^21*b^6 + 60*a^19*b^8 - 435*a^17*b^10 + 984*a^15*b^12 - 1260*a^
13*b^14 + 984*a^11*b^16 - 435*a^9*b^18 + 60*a^7*b^20 + 38*a^5*b^22 - 20*a^3
*b^24 + 3*a*b^26)*d*cos(d*x + c)^5 - (a^27 + 11*a^25*b^2 - 130*a^23*b^4 + 4

```
82*a^21*b^6 - 805*a^19*b^8 + 273*a^17*b^10 + 1428*a^15*b^12 - 3060*a^13*b^1
4 + 3111*a^11*b^16 - 1795*a^9*b^18 + 526*a^7*b^20 - 14*a^5*b^22 - 35*a^3*b^
24 + 7*a*b^26)*d*cos(d*x + c)^3 + ((a^20*b^7 - 10*a^18*b^9 + 45*a^16*b^11 -
 120*a^14*b^13 + 210*a^12*b^15 - 252*a^10*b^17 + 210*a^8*b^19 - 120*a^6*b^2
1 + 45*a^4*b^23 - 10*a^2*b^25 + b^27)*d*cos(d*x + c)^9 - 3*(7*a^22*b^5 - 69
*a^20*b^7 + 305*a^18*b^9 - 795*a^16*b^11 + 1350*a^14*b^13 - 1554*a^12*b^15
+ 1218*a^10*b^17 - 630*a^8*b^19 + 195*a^6*b^21 - 25*a^4*b^23 - 3*a^2*b^25 +
 b^27)*d*cos(d*x + c)^7 + (35*a^24*b^3 - 308*a^22*b^5 + 1158*a^20*b^7 - 234
0*a^18*b^9 + 2445*a^16*b^11 - 360*a^14*b^13 - 2604*a^12*b^15 + 3864*a^10*b^
17 - 2835*a^8*b^19 + 1180*a^6*b^21 - 250*a^4*b^23 + 12*a^2*b^25 + 3*b^27)*d
*cos(d*x + c)^5 - (7*a^26*b - 35*a^24*b^3 - 14*a^22*b^5 + 526*a^20*b^7 - 17
95*a^18*b^9 + 3111*a^16*b^11 - 3060*a^14*b^13 + 1428*a^12*b^15 + 273*a^10*b
^17 - 805*a^8*b^19 + 482*a^6*b^21 - 130*a^4*b^23 + 11*a^2*b^25 + b^27)*d*co
s(d*x + c)^3)*sin(d*x + c)), 1/336*(112*a^18*b - 1008*a^16*b^3 + 4032*a^14*
b^5 - 9408*a^12*b^7 + 14112*a^10*b^9 - 14112*a^8*b^11 + 9408*a^6*b^13 - 403
2*a^4*b^15 + 1008*a^2*b^17 - 112*b^19 - (224*a^12*b^7 - 6272*a^10*b^9 - 201
284*a^8*b^11 - 205692*a^6*b^13 + 277535*a^4*b^15 + 131393*a^2*b^17 + 4096*b
^19)*cos(d*x + c)^10 + 14*(336*a^14*b^5 - 9352*a^12*b^7 - 252014*a^10*b^9 -
 230159*a^8*b^11 + 297312*a^6*b^13 + 165122*a^4*b^15 + 27731*a^2*b^17 + 102
4*b^19)*cos(d*x + c)^8 - 35*(224*a^16*b^3 - 5936*a^14*b^5 - 126448*a^12*b^7
 - 243082*a^10*b^9 - 29747*a^8*b^11 + 284285*a^6*b^13 + 109607*a^4*b^15 + 1
0585*a^2*b^17 + 512*b^19)*cos(d*x + c)^6 + 14*(...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+b*sin(d*x+c))**8,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3047 vs. 2(628) = 1256.
time = 6.09, size = 3047, normalized size = 4.67

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/168*(3465*(32*a^7*b^4 + 112*a^5*b^6 + 70*a^3*b^8 + 7*a*b^10)*(pi*floor(1/
2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2
 - b^2)))/((a^18 - 9*a^16*b^2 + 36*a^14*b^4 - 84*a^12*b^6 + 126*a^10*b^8 -
126*a^8*b^10 + 84*a^6*b^12 - 36*a^4*b^14 + 9*a^2*b^16 - b^18)*sqrt(a^2 - b^
```

2)) - 112*(3*a^10*tan(1/2*d*x + 1/2*c)^5 - 27*a^8*b^2*tan(1/2*d*x + 1/2*c)^
5 - 882*a^6*b^4*tan(1/2*d*x + 1/2*c)^5 - 1638*a^4*b^6*tan(1/2*d*x + 1/2*c)^
5 - 513*a^2*b^8*tan(1/2*d*x + 1/2*c)^5 - 15*b^10*tan(1/2*d*x + 1/2*c)^5 - 2
4*a^9*b*tan(1/2*d*x + 1/2*c)^4 + 216*a^7*b^3*tan(1/2*d*x + 1/2*c)^4 + 1512*
a^5*b^5*tan(1/2*d*x + 1/2*c)^4 + 1224*a^3*b^7*tan(1/2*d*x + 1/2*c)^4 + 144*
a*b^9*tan(1/2*d*x + 1/2*c)^4 - 2*a^10*tan(1/2*d*x + 1/2*c)^3 + 162*a^8*b^2*
tan(1/2*d*x + 1/2*c)^3 + 1932*a^6*b^4*tan(1/2*d*x + 1/2*c)^3 + 3108*a^4*b^6
*tan(1/2*d*x + 1/2*c)^3 + 918*a^2*b^8*tan(1/2*d*x + 1/2*c)^3 + 26*b^10*tan(
1/2*d*x + 1/2*c)^3 - 720*a^7*b^3*tan(1/2*d*x + 1/2*c)^2 - 3024*a^5*b^5*tan(
1/2*d*x + 1/2*c)^2 - 2160*a^3*b^7*tan(1/2*d*x + 1/2*c)^2 - 240*a*b^9*tan(1/
2*d*x + 1/2*c)^2 + 3*a^10*tan(1/2*d*x + 1/2*c) - 27*a^8*b^2*tan(1/2*d*x + 1
/2*c) - 882*a^6*b^4*tan(1/2*d*x + 1/2*c) - 1638*a^4*b^6*tan(1/2*d*x + 1/2*c
) - 513*a^2*b^8*tan(1/2*d*x + 1/2*c) - 15*b^10*tan(1/2*d*x + 1/2*c) - 8*a^9
*b + 312*a^7*b^3 + 1512*a^5*b^5 + 1128*a^3*b^7 + 128*a*b^9)/((a^18 - 9*a^16
*b^2 + 36*a^14*b^4 - 84*a^12*b^6 + 126*a^10*b^8 - 126*a^8*b^10 + 84*a^6*b^1
2 - 36*a^4*b^14 + 9*a^2*b^16 - b^18)*(tan(1/2*d*x + 1/2*c)^2 - 1)^3) + (232
848*a^18*b^6*tan(1/2*d*x + 1/2*c)^13 + 142758*a^16*b^8*tan(1/2*d*x + 1/2*c)
^13 + 64911*a^14*b^10*tan(1/2*d*x + 1/2*c)^13 - 28224*a^12*b^12*tan(1/2*d*x
 + 1/2*c)^13 + 12096*a^10*b^14*tan(1/2*d*x + 1/2*c)^13 - 3024*a^8*b^16*tan(
1/2*d*x + 1/2*c)^13 + 336*a^6*b^18*tan(1/2*d*x + 1/2*c)^13 + 155232*a^19*b^
5*tan(1/2*d*x + 1/2*c)^12 + 2783088*a^17*b^7*tan(1/2*d*x + 1/2*c)^12 + 2110
878*a^15*b^9*tan(1/2*d*x + 1/2*c)^12 + 545811*a^13*b^11*tan(1/2*d*x + 1/2*c
)^12 - 169344*a^11*b^13*tan(1/2*d*x + 1/2*c)^12 + 72576*a^9*b^15*tan(1/2*d*
x + 1/2*c)^12 - 18144*a^7*b^17*tan(1/2*d*x + 1/2*c)^12 + 2016*a^5*b^19*tan(
1/2*d*x + 1/2*c)^12 + 3104640*a^18*b^6*tan(1/2*d*x + 1/2*c)^11 + 15506568*a
^16*b^8*tan(1/2*d*x + 1/2*c)^11 + 12397616*a^14*b^10*tan(1/2*d*x + 1/2*c)^1
1 + 2172366*a^12*b^12*tan(1/2*d*x + 1/2*c)^11 - 451584*a^10*b^14*tan(1/2*d*
x + 1/2*c)^11 + 213696*a^8*b^16*tan(1/2*d*x + 1/2*c)^11 - 57344*a^6*b^18*ta
n(1/2*d*x + 1/2*c)^11 + 6720*a^4*b^20*tan(1/2*d*x + 1/2*c)^11 + 931392*a^19
*b^5*tan(1/2*d*x + 1/2*c)^10 + 22042944*a^17*b^7*tan(1/2*d*x + 1/2*c)^10 +
54377400*a^15*b^9*tan(1/2*d*x + 1/2*c)^10 + 38316040*a^13*b^11*tan(1/2*d*x
+ 1/2*c)^10 + 5346390*a^11*b^13*tan(1/2*d*x + 1/2*c)^10 - 685440*a^9*b^15*t
an(1/2*d*x + 1/2*c)^10 + 372960*a^7*b^17*tan(1/2*d*x + 1/2*c)^10 - 108640*a
^5*b^19*tan(1/2*d*x + 1/2*c)^10 + 13440*a^3*b^21*tan(1/2*d*x + 1/2*c)^10 +
12030480*a^18*b^6*tan(1/2*d*x + 1/2*c)^9 + 83208510*a^16*b^8*tan(1/2*d*x +
1/2*c)^9 + 129442775*a^14*b^10*tan(1/2*d*x + 1/2*c)^9 + 68997390*a^12*b^12*
tan(1/2*d*x + 1/2*c)^9 + 8026116*a^10*b^14*tan(1/2*d*x + 1/2*c)^9 - 418320*
a^8*b^16*tan(1/2*d*x + 1/2*c)^9 + 328720*a^6*b^18*tan(1/2*d*x + 1/2*c)^9 -
115584*a^4*b^20*tan(1/2*d*x + 1/2*c)^9 + 16128*a^2*b^22*tan(1/2*d*x + 1/2*c
)^9 + 2328480*a^19*b^5*tan(1/2*d*x + 1/2*c)^8 + 60558960*a^17*b^7*tan(1/2*d
*x + 1/2*c)^8 + 194655230*a^15*b^9*tan(1/2*d*x + 1/2*c)^8 + 204067311*a^13*
b^11*tan(1/2*d*x + 1/2*c)^8 + 74359166*a^11*b^13*tan(1/2*d*x + 1/2*c)^8 + 6
423144*a^9*b^15*tan(1/2*d*x + 1/2*c)^8 + 342720*a^7*b^17*tan(1/2*d*x + 1/2*
c)^8 + 38080*a^5*b^19*tan(1/2*d*x + 1/2*c)^8 - 54656*a^3*b^21*tan(1/2*d*x +
 1/2*c)^8 + 10752*a*b^23*tan(1/2*d*x + 1/2*c)^8 + 21732480*a^18*b^6*tan(1/2

```
*d*x + 1/2*c)^7 + 160923840*a^16*b^8*tan(1/2*d*x + 1/2*c)^7 + 294582904*a^1
4*b^10*tan(1/2*d*x + 1/2*c)^7 + 198535596*a^12*b^12*tan(1/2*d*x + 1/2*c)^7
+ 45251248*a^10*b^14*tan(1/2*d*x + 1/2*c)^7 + 2197104*a^8*b^16*tan(1/2*d*x
+ 1/2*c)^7 + 545280*a^6*b^18*tan(1/2*d*x + 1/2*c)^7 - 137728*a^4*b^20*tan(1
/2*d*x + 1/2*c)^7 + 5120*a^2*b^22*tan(1/2*d*x + 1/2*c)^7 + 3072*b^24*tan(1/
2*d*x + 1/2*c)^7 + 3104640*a^19*b^5*tan(1/2*d*x + 1/2*c)^6 + 77468160*a^17*
b^7*tan(1/2*d*x + 1/2*c)^6 + 251081600*a^15*b^9*tan(1/2*d*x + 1/2*c)^6 + 27
4259160*a^13*b^11*tan(1/2*d*x + 1/2*c)^6 + 105524636*a^11*b^13*tan(1/2*d*x
+ 1/2*c)^6 + 11690784*a^9*b^15*tan(1/2*d*x + 1/2*c)^6 + 515760*a^7*b^17*tan
(1/2*d*x + 1/2*c)^6 + 38080*a^5*b^19*tan(1/2*d*x + 1/2*c)^6 - 54656*a^3*b^2
1*tan(1/2*d*x + 1/2*c)^6 + 10752*a*b^23*tan(1/2*d*x + 1/2*c)^6 + 20568240*a
^18*b^6*tan(1/2*d*x + 1/2*c)^5 + 136444770*a^16*b^8*tan(1/2*d*x + 1/2*c)^5
+ 229744669*a^14*b^10*tan(1/2*d*x + 1/2*c)^5 + 133540988*a^12*b^12*tan(1/2*
d*x + 1/2*c)^5 + 22390536*a^10*b^14*tan(1/2*d*x + 1/2*c)^5 - 189280*a^8*b^1
6*tan(1/2*d*x + 1/2*c)^5 + 328720*a^6*b^18*tan(1/2*d*x + 1/2*c)^5 - 115584*
a^4*b^20*tan(1/2*d*x + 1/2*c)^5 + 16128*a^2*b^22*tan(1/2*d*x + 1/2*c)^5 + 2
328480*a^19*b^5*tan(1/2*d*x + 1/2*c)^4 + 47733840*a^17*b^7*tan(1/2*d*x + 1/
2*c)^4 + 125203386*a^15*b^9*tan(1/2*d*x + 1/2*c...
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a + b*sin(c + d*x))^8),x)
[Out] \text{Hanged}
```

3.473 $\int \cos^5(c+dx)\sqrt{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=154

$$\frac{2(a^2-b^2)^2(a+b\sin(c+dx))^{3/2}}{3b^5d}-\frac{8a(a^2-b^2)(a+b\sin(c+dx))^{5/2}}{5b^5d}+\frac{4(3a^2-b^2)(a+b\sin(c+dx))^{7/2}}{7b^5d}-8$$

```
[Out] 2/3*(a^2-b^2)^2*(a+b*sin(d*x+c))^(3/2)/b^5/d-8/5*a*(a^2-b^2)*(a+b*sin(d*x+c
))^(5/2)/b^5/d+4/7*(3*a^2-b^2)*(a+b*sin(d*x+c))^(7/2)/b^5/d-8/9*a*(a+b*sin(
d*x+c))^(9/2)/b^5/d+2/11*(a+b*sin(d*x+c))^(11/2)/b^5/d
```

Rubi [A]
time = 0.08, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2747, 711}

$$\frac{4(3a^2-b^2)(a+b\sin(c+dx))^{7/2}}{7b^5d}-\frac{8a(a^2-b^2)(a+b\sin(c+dx))^{5/2}}{5b^5d}+\frac{2(a^2-b^2)^2(a+b\sin(c+dx))^{3/2}}{3b^5d}+\frac{2(a+b\sin(c+dx))^{11/2}}{11b^5d}-\frac{8a(a+b\sin(c+dx))^{9/2}}{9b^5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (2*(a^2 - b^2)^2*(a + b*Sin[c + d*x])^(3/2))/(3*b^5*d) - (8*a*(a^2 - b^2)*(
a + b*Sin[c + d*x])^(5/2))/(5*b^5*d) + (4*(3*a^2 - b^2)*(a + b*Sin[c + d*x]
)^(7/2))/(7*b^5*d) - (8*a*(a + b*Sin[c + d*x])^(9/2))/(9*b^5*d) + (2*(a + b
*Sin[c + d*x])^(11/2))/(11*b^5*d)
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^5(c+dx)\sqrt{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \sqrt{a+x}\left(b^2-x^2\right)^2\,dx, x, b\sin(c+dx)\right)}{b^5d}$$

$$= \frac{\text{Subst}\left(\int \left(\left(a^2-b^2\right)^2\sqrt{a+x} - 4\left(a^3-ab^2\right)(a+x)^{3/2} + 2\left(3a^2-b^2\right.\right.\right.}{b^5d}$$

$$= \frac{2\left(a^2-b^2\right)^2(a+b\sin(c+dx))^{3/2}}{3b^5d} - \frac{8a\left(a^2-b^2\right)(a+b\sin(c+dx))^{5}}{5b^5d}$$

Mathematica [A]
time = 0.33, size = 117, normalized size = 0.76

$$\frac{2(a+b\sin(c+dx))^{3/2}\left(315b^4\cos^4(c+dx)+8(16a^4-66a^2b^2+105b^4+(-24a^3b+99ab^3)\sin(c+dx)+15b^2(2a^2-3b^2)\sin^2(c+dx)-35ab^3\sin^3(c+dx))\right)}{3465b^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (2*(a + b*Sin[c + d*x])^(3/2)*(315*b^4*Cos[c + d*x]^4 + 8*(16*a^4 - 66*a^2*
b^2 + 105*b^4 + (-24*a^3*b + 99*a*b^3)*Sin[c + d*x] + 15*b^2*(2*a^2 - 3*b^2
)*Sin[c + d*x]^2 - 35*a*b^3*Sin[c + d*x]^3)))/(3465*b^5*d)
```

Maple [A]
time = 1.83, size = 126, normalized size = 0.82

method	result
default	$\frac{2(a+b\sin(dx+c))^{\frac{3}{2}}\left(315b^4\left(\cos^4(dx+c)\right)+280a\,b^3\left(\cos^2(dx+c)\right)\sin(dx+c)-240a^2b^2\left(\cos^2(dx+c)\right)+360b^4\left(\cos^2(dx+c)\right)-192a^3b\sin(dx\right.}{3465b^5d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+b*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3465/b^5*(a+b*sin(d*x+c))^(3/2)*(315*b^4*cos(d*x+c)^4+280*a*b^3*cos(d*x+c
)^2*sin(d*x+c)-240*a^2*b^2*cos(d*x+c)^2+360*b^4*cos(d*x+c)^2-192*a^3*b*sin(
d*x+c)+512*a*b^3*sin(d*x+c)+128*a^4-288*a^2*b^2+480*b^4)/d
```

Maxima [A]
time = 0.28, size = 116, normalized size = 0.75

$$\frac{2\left(315\,(b\sin(dx+c)+a)^{\frac{11}{2}}-1540\,(b\sin(dx+c)+a)^{\frac{9}{2}}a+990\,(3\,a^2-b^2)(b\sin(dx+c)+a)^{\frac{7}{2}}-2772\,(a^3-ab^2)(b\sin(dx+c)+a)^{\frac{5}{2}}+1155\,(a^4-2\,a^2b^2+b^4)(b\sin(dx+c)+a)^{\frac{3}{2}}\right)}{3465\,b^5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3465*(315*(b*sin(d*x + c) + a)^(11/2) - 1540*(b*sin(d*x + c) + a)^(9/2)*a
 + 990*(3*a^2 - b^2)*(b*sin(d*x + c) + a)^(7/2) - 2772*(a^3 - a*b^2)*(b*sin
(d*x + c) + a)^(5/2) + 1155*(a^4 - 2*a^2*b^2 + b^4)*(b*sin(d*x + c) + a)^(3
/2))/(b^5*d)
```

Fricas [A]
time = 0.36, size = 142, normalized size = 0.92

$$\frac{2\,\left(35\,ab^{4}\cos\left(dx+c\right)^{4}+128\,a^{5}-480\,a^{3}b^{2}+992\,ab^{4}-16\,\left(3\,a^{3}b^{2}-8\,ab^{4}\right)\cos\left(dx+c\right)^{2}+\left(315\,b^{5}\cos\left(dx+c\right)^{4}-64\,a^{4}b+224\,a^{2}b^{3}+480\,b^{5}+40\,\left(a^{2}b^{3}+9\,b^{5}\right)\cos\left(dx+c\right)^{2}\right)\sin\left(dx+c\right)\right)\sqrt{b\sin\left(dx+c\right)+a}}{3465\,b^{5}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
[Out] 2/3465*(35*a*b^4*cos(d*x + c)^4 + 128*a^5 - 480*a^3*b^2 + 992*a*b^4 - 16*(3
*a^3*b^2 - 8*a*b^4)*cos(d*x + c)^2 + (315*b^5*cos(d*x + c)^4 - 64*a^4*b + 2
24*a^2*b^3 + 480*b^5 + 40*(a^2*b^3 + 9*b^5)*cos(d*x + c)^2)*sin(d*x + c))*s
qrt(b*sin(d*x + c) + a)/(b^5*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+b*sin(d*x+c))**(1/2),x)
[Out] Timed out
```

Giac [A]
time = 5.08, size = 161, normalized size = 1.05

$$\frac{2\,\left(315\,\left(b\sin\left(dx+c\right)+a\right)^{\frac{11}{2}}-1540\,\left(b\sin\left(dx+c\right)+a\right)^{\frac{9}{2}}a+2970\,\left(b\sin\left(dx+c\right)+a\right)^{\frac{7}{2}}a^{2}-2772\,\left(b\sin\left(dx+c\right)+a\right)^{\frac{5}{2}}a^{3}+1155\,\left(b\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}a^{4}-990\,\left(b\sin\left(dx+c\right)+a\right)^{\frac{7}{2}}b^{2}+2772\,\left(b\sin\left(dx+c\right)+a\right)^{\frac{5}{2}}ab^{2}-2310\,\left(b\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}a^{2}b^{2}+1155\,\left(b\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}b^{4}\right)}{3465\,b^{5}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sin(d*x+c))^(1/2),x, algorithm="giac")
[Out] 2/3465*(315*(b*sin(d*x + c) + a)^(11/2) - 1540*(b*sin(d*x + c) + a)^(9/2)*a
 + 2970*(b*sin(d*x + c) + a)^(7/2)*a^2 - 2772*(b*sin(d*x + c) + a)^(5/2)*a^
3 + 1155*(b*sin(d*x + c) + a)^(3/2)*a^4 - 990*(b*sin(d*x + c) + a)^(7/2)*b^
2 + 2772*(b*sin(d*x + c) + a)^(5/2)*a*b^2 - 2310*(b*sin(d*x + c) + a)^(3/2)
*a^2*b^2 + 1155*(b*sin(d*x + c) + a)^(3/2)*b^4)/(b^5*d)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^5\,\sqrt{a+b\,\sin\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(a + b*sin(c + d*x))^(1/2),x)
[Out] int(cos(c + d*x)^5*(a + b*sin(c + d*x))^(1/2), x)
```

3.474 $\int \cos^3(c+dx)\sqrt{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=83

$$-\frac{2(a^2-b^2)\,(a+b\sin(c+dx))^{3/2}}{3b^3d}+\frac{4a(a+b\sin(c+dx))^{5/2}}{5b^3d}-\frac{2(a+b\sin(c+dx))^{7/2}}{7b^3d}$$

```
[Out] -2/3*(a^2-b^2)*(a+b*sin(d*x+c))^(3/2)/b^3/d+4/5*a*(a+b*sin(d*x+c))^(5/2)/b^
3/d-2/7*(a+b*sin(d*x+c))^(7/2)/b^3/d
```

Rubi [A]
time = 0.05, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2747, 711}

$$-\frac{2(a^2-b^2)\,(a+b\sin(c+dx))^{3/2}}{3b^3d}-\frac{2(a+b\sin(c+dx))^{7/2}}{7b^3d}+\frac{4a(a+b\sin(c+dx))^{5/2}}{5b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (-2*(a^2 - b^2)*(a + b*Sin[c + d*x])^(3/2))/(3*b^3*d) + (4*a*(a + b*Sin[c +
 d*x])^(5/2))/(5*b^3*d) - (2*(a + b*Sin[c + d*x])^(7/2))/(7*b^3*d)
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^3(c+dx)\sqrt{a+b\sin(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \sqrt{a+x}\,(b^2-x^2)\,dx,x,b\sin(c+dx)\right)}{b^3d}\\ &= \frac{\text{Subst}\left(\int \left((-a^2+b^2)\sqrt{a+x}+2a(a+x)^{3/2}-(a+x)^{5/2}\right)\,dx,x,b\right.}{b^3d}\\ &= -\frac{2(a^2-b^2)\,(a+b\sin(c+dx))^{3/2}}{3b^3d}+\frac{4a(a+b\sin(c+dx))^{5/2}}{5b^3d}-\frac{2(a}{}\end{aligned}$$

Mathematica [A]
time = 0.13, size = 58, normalized size = 0.70

$$\frac{(a+b\sin(c+dx))^{3/2}(-16a^2+55b^2+15b^2\cos(2(c+dx))+24ab\sin(c+dx))}{105b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] ((a + b*Sin[c + d*x])^(3/2)*(-16*a^2 + 55*b^2 + 15*b^2*Cos[2*(c + d*x)] + 2
4*a*b*Sin[c + d*x]))/(105*b^3*d)
```

Maple [A]
time = 1.09, size = 55, normalized size = 0.66

method	result	size
default	$-\frac{2(a+b\sin(dx+c))^{\frac{3}{2}}(-15b^2(\cos^2(dx+c))-12ab\sin(dx+c)+8a^2-20b^2)}{105b^3d}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/105/b^3*(a+b*sin(d*x+c))^(3/2)*(-15*b^2*cos(d*x+c)^2-12*a*b*sin(d*x+c)+8
*a^2-20*b^2)/d
```

Maxima [A]
time = 0.27, size = 61, normalized size = 0.73

$$-\frac{2\left(15(b\sin(dx+c)+a)^{\frac{7}{2}}-42(b\sin(dx+c)+a)^{\frac{5}{2}}a+35(a^2-b^2)(b\sin(dx+c)+a)^{\frac{3}{2}}\right)}{105\,b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -2/105*(15*(b*sin(d*x + c) + a)^(7/2) - 42*(b*sin(d*x + c) + a)^(5/2)*a + 3
5*(a^2 - b^2)*(b*sin(d*x + c) + a)^(3/2))/(b^3*d)
```

Fricas [A]
time = 0.35, size = 78, normalized size = 0.94

$$\frac{2\left(3\,ab^2\cos(dx+c)^2-8\,a^3+32\,ab^2+\left(15\,b^3\cos(dx+c)^2+4\,a^2b+20\,b^3\right)\sin(dx+c)\right)\sqrt{b\sin(dx+c)+a}}{105\,b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/105*(3*a*b^2*cos(d*x + c)^2 - 8*a^3 + 32*a*b^2 + (15*b^3*cos(d*x + c)^2 +
 4*a^2*b + 20*b^3)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)/(b^3*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 4.98, size = 72, normalized size = 0.87

$$-\frac{2\left(15\left(b\sin\left(dx+c\right)+a\right)^{\frac{7}{2}}-42\left(b\sin\left(dx+c\right)+a\right)^{\frac{5}{2}}a+35\left(b\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}a^{2}-35\left(b\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}b^{2}\right)}{105\,b^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -2/105*(15*(b*sin(d*x + c) + a)^(7/2) - 42*(b*sin(d*x + c) + a)^(5/2)*a + 3
5*(b*sin(d*x + c) + a)^(3/2)*a^2 - 35*(b*sin(d*x + c) + a)^(3/2)*b^2)/(b^3*
d)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^3\,\sqrt{a+b\,\sin\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a + b*sin(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^3*(a + b*sin(c + d*x))^(1/2), x)
```

3.475 $\int \cos(c+dx)\sqrt{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=24

$$\frac{2(a+b\sin(c+dx))^{3/2}}{3bd}$$

```
[Out] 2/3*(a+b*sin(d*x+c))^(3/2)/b/d
```

Rubi [A]
time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2747, 32}

$$\frac{2(a+b\sin(c+dx))^{3/2}}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (2*(a + b*Sin[c + d*x])^(3/2))/(3*b*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)\sqrt{a+b\sin(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \sqrt{a+x}\,dx, x, b\sin(c+dx)\right)}{bd}\\ &= \frac{2(a+b\sin(c+dx))^{3/2}}{3bd}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 24, normalized size = 1.00

$$\frac{2(a+b\sin(c+dx))^{3/2}}{3bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (2*(a + b*Sin[c + d*x])^(3/2))/(3*b*d)
```

Maple [A]
time = 0.05, size = 21, normalized size = 0.88

method	result	size
derivativedivides	$\frac{2(a+b\sin(dx+c))^{\frac{3}{2}}}{3bd}$	21
default	$\frac{2(a+b\sin(dx+c))^{\frac{3}{2}}}{3bd}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(a+b*sin(d*x+c))^(3/2)/b/d
```

Maxima [A]
time = 0.27, size = 20, normalized size = 0.83

$$\frac{2\left(b\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}}{3\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(b*sin(d*x + c) + a)^(3/2)/(b*d)
```

Fricas [A]
time = 0.34, size = 20, normalized size = 0.83

$$\frac{2\left(b\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}}{3\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(b*sin(d*x + c) + a)^(3/2)/(b*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(19) = 38$.

time = 0.15, size = 83, normalized size = 3.46

$$\begin{cases} \sqrt{a}\, x \cos{(c)} & \text{for } b = 0 \wedge d = 0 \\ \frac{\sqrt{a} \sin{(c+dx)}}{d} & \text{for } b = 0 \\ x\sqrt{a + b\sin{(c)}} \cos{(c)} & \text{for } d = 0 \\ \frac{2a\sqrt{a + b\sin{(c+dx)}}}{3bd} + \frac{2\sqrt{a + b\sin{(c+dx)}} \sin{(c+dx)}}{3d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Piecewise((sqrt(a)*x*cos(c), Eq(b, 0) & Eq(d, 0)), (sqrt(a)*sin(c + d*x)/d,
 Eq(b, 0)), (x*sqrt(a + b*sin(c))*cos(c), Eq(d, 0)), (2*a*sqrt(a + b*sin(c
+ d*x))/(3*b*d) + 2*sqrt(a + b*sin(c + d*x))*sin(c + d*x)/(3*d), True))
```

Giac [A]
time = 6.58, size = 20, normalized size = 0.83

$$\frac{2\,(b\sin{(dx+c)} + a)^{\frac{3}{2}}}{3\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*(b*sin(d*x + c) + a)^(3/2)/(b*d)
```

Mupad [B]
time = 5.20, size = 20, normalized size = 0.83

$$\frac{2\,{\left(a+b\,\sin\left(c+d\,x\right)\right)}^{3/2}}{3\,b\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + b*sin(c + d*x))^(1/2),x)
```

```
[Out] (2*(a + b*sin(c + d*x))^(3/2))/(3*b*d)
```

3.476 $\int \sec(c+dx)\sqrt{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=74

$$-\frac{\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{d}+\frac{\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{d}$$

```
[Out] -arctanh((a+b*sin(d*x+c))^(1/2)/(a-b)^(1/2))*(a-b)^(1/2)/d+arctanh((a+b*sin
(d*x+c))^(1/2)/(a+b)^(1/2))*(a+b)^(1/2)/d
```

Rubi [A]
time = 0.07, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2747, 714, 1144, 212}

$$\frac{\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{d}-\frac{\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] -((Sqrt[a - b]*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a - b]])/d) + (Sqrt[a
+ b]*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a + b]])/d
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 714

```
Int[Sqrt[(d_) + (e_.)*(x_)]/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[2*e, S
ubst[Int[x^2/(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x]
 /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1144

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2/2)*(b/q + 1), Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2/2)*(b/q - 1), Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec(c+dx)\sqrt{a+b\sin(c+dx)}\,dx = \frac{b\text{Subst}\left(\int \frac{\sqrt{a+x}}{b^2-x^2}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{(2b)\text{Subst}\left(\int \frac{x^2}{-a^2+b^2+2ax^2-x^4}\,dx, x, \sqrt{a+b\sin(c+dx)}\right)}{d}$$
$$= -\frac{(a-b)\text{Subst}\left(\int \frac{1}{a-b-x^2}\,dx, x, \sqrt{a+b\sin(c+dx)}\right)}{d} + \frac{(a+b)\text{Sub}}{}$$
$$= -\frac{\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{d} + \frac{\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a}}{}\right.}{d}$$

Mathematica [A]
time = 0.06, size = 74, normalized size = 1.00

$$-\frac{\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{d}+\frac{\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] -((Sqrt[a - b]*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a - b]])/d) + (Sqrt[a
+ b]*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a + b]])/d
```

Maple [A]
time = 1.12, size = 61, normalized size = 0.82

method	result	size
default	$\frac{\sqrt{a+b}\,\text{arctanh}\left(\frac{\sqrt{a+b\sin(dx+c)}}{\sqrt{a+b}}\right)-\sqrt{-a+b}\,\arctan\left(\frac{\sqrt{a+b\sin(dx+c)}}{\sqrt{-a+b}}\right)}{d}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((a+b)^(1/2)*arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))-(-a+b)^(1/2)*arcta
n((a+b*sin(d*x+c))^(1/2)/(-a+b)^(1/2)))/d
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 294 vs. 2(62) = 124.
time = 0.60, size = 1729, normalized size = 23.36

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(a + b)*log((b^4*cos(d*x + c)^4 + 128*a^4 + 256*a^3*b + 320*a^2*b
^2 + 256*a*b^3 + 72*b^4 - 8*(20*a^2*b^2 + 28*a*b^3 + 9*b^4)*cos(d*x + c)^2
+ 8*(16*a^3 + 24*a^2*b + 20*a*b^2 + 8*b^3 - (10*a*b^2 + 7*b^3)*cos(d*x + c)
^2 - (b^3*cos(d*x + c)^2 - 24*a^2*b - 28*a*b^2 - 8*b^3)*sin(d*x + c))*sqrt(
b*sin(d*x + c) + a)*sqrt(a + b) + 4*(64*a^3*b + 112*a^2*b^2 + 64*a*b^3 + 14
*b^4 - (8*a*b^3 + 7*b^4)*cos(d*x + c)^2)*sin(d*x + c))/(cos(d*x + c)^4 - 8*
cos(d*x + c)^2 + 4*(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8)) + sqrt(a - b)*lo
g((b^4*cos(d*x + c)^4 + 128*a^4 - 256*a^3*b + 320*a^2*b^2 - 256*a*b^3 + 72*
b^4 - 8*(20*a^2*b^2 - 28*a*b^3 + 9*b^4)*cos(d*x + c)^2 - 8*(16*a^3 - 24*a^2
*b + 20*a*b^2 - 8*b^3 - (10*a*b^2 - 7*b^3)*cos(d*x + c)^2 - (b^3*cos(d*x +
c)^2 - 24*a^2*b + 28*a*b^2 - 8*b^3)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)*
sqrt(a - b) + 4*(64*a^3*b - 112*a^2*b^2 + 64*a*b^3 - 14*b^4 - (8*a*b^3 - 7*
b^4)*cos(d*x + c)^2)*sin(d*x + c))/(cos(d*x + c)^4 - 8*cos(d*x + c)^2 - 4*(
cos(d*x + c)^2 - 2)*sin(d*x + c) + 8)))/d, -1/8*(2*sqrt(-a - b)*arctan(-1/4
*(b^2*cos(d*x + c)^2 - 8*a^2 - 8*a*b - 2*b^2 - 2*(4*a*b + 3*b^2)*sin(d*x +
```

```
c))*sqrt(b*sin(d*x + c) + a)*sqrt(-a - b)/(2*a^3 + 3*a^2*b + 2*a*b^2 + b^3
- (a*b^2 + b^3)*cos(d*x + c)^2 + (3*a^2*b + 4*a*b^2 + b^3)*sin(d*x + c))) -
 sqrt(a - b)*log((b^4*cos(d*x + c)^4 + 128*a^4 - 256*a^3*b + 320*a^2*b^2 -
256*a*b^3 + 72*b^4 - 8*(20*a^2*b^2 - 28*a*b^3 + 9*b^4)*cos(d*x + c)^2 - 8*(
16*a^3 - 24*a^2*b + 20*a*b^2 - 8*b^3 - (10*a*b^2 - 7*b^3)*cos(d*x + c)^2 -
(b^3*cos(d*x + c)^2 - 24*a^2*b + 28*a*b^2 - 8*b^3)*sin(d*x + c))*sqrt(b*sin
(d*x + c) + a)*sqrt(a - b) + 4*(64*a^3*b - 112*a^2*b^2 + 64*a*b^3 - 14*b^4
- (8*a*b^3 - 7*b^4)*cos(d*x + c)^2)*sin(d*x + c))/(cos(d*x + c)^4 - 8*cos(d
*x + c)^2 - 4*(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8)))/d, -1/8*(2*sqrt(-a +
 b)*arctan(1/4*(b^2*cos(d*x + c)^2 - 8*a^2 + 8*a*b - 2*b^2 - 2*(4*a*b - 3*b
^2)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)*sqrt(-a + b)/(2*a^3 - 3*a^2*b +
2*a*b^2 - b^3 - (a*b^2 - b^3)*cos(d*x + c)^2 + (3*a^2*b - 4*a*b^2 + b^3)*si
n(d*x + c))) - sqrt(a + b)*log((b^4*cos(d*x + c)^4 + 128*a^4 + 256*a^3*b +
320*a^2*b^2 + 256*a*b^3 + 72*b^4 - 8*(20*a^2*b^2 + 28*a*b^3 + 9*b^4)*cos(d*
x + c)^2 + 8*(16*a^3 + 24*a^2*b + 20*a*b^2 + 8*b^3 - (10*a*b^2 + 7*b^3)*cos
(d*x + c)^2 - (b^3*cos(d*x + c)^2 - 24*a^2*b - 28*a*b^2 - 8*b^3)*sin(d*x +
c))*sqrt(b*sin(d*x + c) + a)*sqrt(a + b) + 4*(64*a^3*b + 112*a^2*b^2 + 64*a
*b^3 + 14*b^4 - (8*a*b^3 + 7*b^4)*cos(d*x + c)^2)*sin(d*x + c))/(cos(d*x +
c)^4 - 8*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8)))/d, -1/
4*(sqrt(-a + b)*arctan(1/4*(b^2*cos(d*x + c)^2 - 8*a^2 + 8*a*b - 2*b^2 - 2*
(4*a*b - 3*b^2)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)*sqrt(-a + b)/(2*a^3
- 3*a^2*b + 2*a*b^2 - b^3 - (a*b^2 - b^3)*cos(d*x + c)^2 + (3*a^2*b - 4*a*b
^2 + b^3)*sin(d*x + c))) + sqrt(-a - b)*arctan(-1/4*(b^2*cos(d*x + c)^2 - 8
*a^2 - 8*a*b - 2*b^2 - 2*(4*a*b + 3*b^2)*sin(d*x + c))*sqrt(b*sin(d*x + c)
+ a)*sqrt(-a - b)/(2*a^3 + 3*a^2*b + 2*a*b^2 + b^3 - (a*b^2 + b^3)*cos(d*x
+ c)^2 + (3*a^2*b + 4*a*b^2 + b^3)*sin(d*x + c))))/d]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sin(c+dx)}\,\sec(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(c + d*x))*sec(c + d*x), x)
```

Giac [A]
time = 5.07, size = 83, normalized size = 1.12

$$\frac{b\left(\frac{(a-b)\arctan\left(\frac{\sqrt{b\sin(dx+c)+a}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}\,b}-\frac{(a+b)\arctan\left(\frac{\sqrt{b\sin(dx+c)+a}}{\sqrt{-a-b}}\right)}{\sqrt{-a-b}\,b}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] b*((a - b)*arctan(sqrt(b*sin(d*x + c) + a)/sqrt(-a + b))/(sqrt(-a + b)*b) -
 (a + b)*arctan(sqrt(b*sin(d*x + c) + a)/sqrt(-a - b))/(sqrt(-a - b)*b))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+b\,\sin\left(c+d\,x\right)}}{\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^(1/2)/cos(c + d*x),x)
```

```
[Out] int((a + b*sin(c + d*x))^(1/2)/cos(c + d*x), x)
```

3.477 $\int \sec^3(c+dx)\sqrt{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=124

$$-\frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{4\sqrt{a-b}\,d}+\frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{4\sqrt{a+b}\,d}+\frac{\sec(c+dx)\sqrt{a+b\sin}}{}$$

```
[Out] -1/4*(2*a-b)*arctanh((a+b*sin(d*x+c))^(1/2)/(a-b)^(1/2))/d/(a-b)^(1/2)+1/4*
(2*a+b)*arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))/d/(a+b)^(1/2)+1/2*sec(d
*x+c)*(a+b*sin(d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A]
time = 0.11, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2747, 751, 841, 1180, 212}

$$-\frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{4d\sqrt{a-b}}+\frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{4d\sqrt{a+b}}+\frac{\tan(c+dx)\sec(c+dx)\sqrt{a+b\sin(c+dx)}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] -1/4*((2*a - b)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a - b]])/(Sqrt[a - b]
*d) + ((2*a + b)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a + b]])/(4*Sqrt[a +
 b]*d) + (Sec[c + d*x]*Sqrt[a + b*Sin[c + d*x]]*Tan[c + d*x])/(2*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 751

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-x)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*a*(p + 1))), x] + Dist[1/(2*a*(p +
 1)), Int[(d + e*x)^(m - 1)*(d*(2*p + 3) + e*(m + 2*p + 3)*x)*(a + c*x^2)^(
p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p,
 -1] && GtQ[m, 0] && (LtQ[m, 1] || (ILtQ[m + 2*p + 3, 0] && NeQ[m, 2])) &&
IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 841

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
```

```
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^3(c+dx)\sqrt{a+b\sin(c+dx)}\,dx = \frac{b^3\text{Subst}\left(\int \frac{\sqrt{a+x}}{(b^2-x^2)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec(c+dx)\sqrt{a+b\sin(c+dx)}\tan(c+dx)}{2d} - \frac{b\text{Subst}\left(\int \frac{-a-\frac{x}{2}}{\sqrt{a+x}\,(\ldots}\right.}{\ldots}$$

$$= \frac{\sec(c+dx)\sqrt{a+b\sin(c+dx)}\tan(c+dx)}{2d} - \frac{b\text{Subst}\left(\int \frac{-\frac{a}{2}-\frac{x^2}{2}\ldots}{-a^2+b^2+2a\ldots}\right.}{\ldots}$$

$$= \frac{\sec(c+dx)\sqrt{a+b\sin(c+dx)}\tan(c+dx)}{2d} - \frac{(2a-b)\text{Subst}\left(\int \frac{\ldots}{a-\ldots}\right.}{\ldots}$$

$$= -\frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{4\sqrt{a-b}\,d} + \frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a\ldots}}{\ldots}\right)}{4\sqrt{a+\ldots}}$$

Mathematica [A]
time = 0.75, size = 143, normalized size = 1.15

$$\frac{-\sqrt{a-b}\,(2a^2+ab-b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)+(a-b)\left(\sqrt{a+b}\,(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)+2(a+b)\sec(c+dx)\sqrt{a+b\sin(c+dx)}\tan(c+dx)\right)}{4(a^2-b^2)\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (-(Sqrt[a - b]*(2*a^2 + a*b - b^2)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a
- b]]) + (a - b)*(Sqrt[a + b]*(2*a + b)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sq
rt[a + b]] + 2*(a + b)*Sec[c + d*x]*Sqrt[a + b*Sin[c + d*x]]*Tan[c + d*x]))
/(4*(a^2 - b^2)*d)
```

Maple [A]
time = 1.80, size = 185, normalized size = 1.49

method	result
default	$\frac{2\sqrt{a+b\sin(dx+c)}\,\sqrt{a+b}\,\sqrt{-a+b}\,\sin(dx+c)-\left(-2\arctan\left(\frac{\sqrt{a+b\sin(dx+c)}}{\sqrt{-a+b}}\right)a\sqrt{a+b}+\mathrm{arcta}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+b*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(2*(a+b*sin(d*x+c))^(1/2)*(a+b)^(1/2)*(-a+b)^(1/2)*sin(d*x+c)-(-2*arcta
n((a+b*sin(d*x+c))^(1/2)/(-a+b)^(1/2))*a*(a+b)^(1/2)+arctan((a+b*sin(d*x+c)
)^(1/2)/(-a+b)^(1/2))*b*(a+b)^(1/2)-2*arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^
(1/2))*a*(-a+b)^(1/2)-arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))*b*(-a+b)^
(1/2))*cos(d*x+c)^2)/(a+b)^(1/2)/(-a+b)^(1/2)/cos(d*x+c)^2/d
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 387 vs. $2(104) = 208$.
time = 0.62, size = 2101, normalized size = 16.94

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/32*((2*a^2 - a*b - b^2)*sqrt(a + b)*cos(d*x + c)^2*log((b^4*cos(d*x + c)
^4 + 128*a^4 + 256*a^3*b + 320*a^2*b^2 + 256*a*b^3 + 72*b^4 - 8*(20*a^2*b^2
 + 28*a*b^3 + 9*b^4)*cos(d*x + c)^2 + 8*(16*a^3 + 24*a^2*b + 20*a*b^2 + 8*b
^3 - (10*a*b^2 + 7*b^3)*cos(d*x + c)^2 - (b^3*cos(d*x + c)^2 - 24*a^2*b - 2
8*a*b^2 - 8*b^3)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)*sqrt(a + b) + 4*(64
*a^3*b + 112*a^2*b^2 + 64*a*b^3 + 14*b^4 - (8*a*b^3 + 7*b^4)*cos(d*x + c)^2
)*sin(d*x + c))/(cos(d*x + c)^4 - 8*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 - 2)
*sin(d*x + c) + 8)) - (2*a^2 + a*b - b^2)*sqrt(a - b)*cos(d*x + c)^2*log((b
^4*cos(d*x + c)^4 + 128*a^4 - 256*a^3*b + 320*a^2*b^2 - 256*a*b^3 + 72*b^4
- 8*(20*a^2*b^2 - 28*a*b^3 + 9*b^4)*cos(d*x + c)^2 + 8*(16*a^3 - 24*a^2*b +
 20*a*b^2 - 8*b^3 - (10*a*b^2 - 7*b^3)*cos(d*x + c)^2 - (b^3*cos(d*x + c)^2
 - 24*a^2*b + 28*a*b^2 - 8*b^3)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)*sqrt
(a - b) + 4*(64*a^3*b - 112*a^2*b^2 + 64*a*b^3 - 14*b^4 - (8*a*b^3 - 7*b^4)
*cos(d*x + c)^2)*sin(d*x + c))/(cos(d*x + c)^4 - 8*cos(d*x + c)^2 - 4*(cos(
d*x + c)^2 - 2)*sin(d*x + c) + 8)) + 16*(a^2 - b^2)*sqrt(b*sin(d*x + c) + a
)*sin(d*x + c))/((a^2 - b^2)*d*cos(d*x + c)^2), -1/32*(2*(2*a^2 - a*b - b^2
)*sqrt(-a - b)*arctan(-1/4*(b^2*cos(d*x + c)^2 - 8*a^2 - 8*a*b - 2*b^2 - 2*
(4*a*b + 3*b^2)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)*sqrt(-a - b)/(2*a^3
+ 3*a^2*b + 2*a*b^2 + b^3 - (a*b^2 + b^3)*cos(d*x + c)^2 + (3*a^2*b + 4*a*b
^2 + b^3)*sin(d*x + c)))*cos(d*x + c)^2 + (2*a^2 + a*b - b^2)*sqrt(a - b)*c
os(d*x + c)^2*log((b^4*cos(d*x + c)^4 + 128*a^4 - 256*a^3*b + 320*a^2*b^2 -
 256*a*b^3 + 72*b^4 - 8*(20*a^2*b^2 - 28*a*b^3 + 9*b^4)*cos(d*x + c)^2 + 8*
(16*a^3 - 24*a^2*b + 20*a*b^2 - 8*b^3 - (10*a*b^2 - 7*b^3)*cos(d*x + c)^2 -
 (b^3*cos(d*x + c)^2 - 24*a^2*b + 28*a*b^2 - 8*b^3)*sin(d*x + c))*sqrt(b*si
n(d*x + c) + a)*sqrt(a - b) + 4*(64*a^3*b - 112*a^2*b^2 + 64*a*b^3 - 14*b^4
 - (8*a*b^3 - 7*b^4)*cos(d*x + c)^2)*sin(d*x + c))/(cos(d*x + c)^4 - 8*cos(
d*x + c)^2 - 4*(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8)) - 16*(a^2 - b^2)*sqr
t(b*sin(d*x + c) + a)*sin(d*x + c))/((a^2 - b^2)*d*cos(d*x + c)^2), -1/32*(
2*(2*a^2 + a*b - b^2)*sqrt(-a + b)*arctan(1/4*(b^2*cos(d*x + c)^2 - 8*a^2 +
 8*a*b - 2*b^2 - 2*(4*a*b - 3*b^2)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)*s
qrt(-a + b)/(2*a^3 - 3*a^2*b + 2*a*b^2 - b^3 - (a*b^2 - b^3)*cos(d*x + c)^2
 + (3*a^2*b - 4*a*b^2 + b^3)*sin(d*x + c)))*cos(d*x + c)^2 - (2*a^2 - a*b -
 b^2)*sqrt(a + b)*cos(d*x + c)^2*log((b^4*cos(d*x + c)^4 + 128*a^4 + 256*a^
3*b + 320*a^2*b^2 + 256*a*b^3 + 72*b^4 - 8*(20*a^2*b^2 + 28*a*b^3 + 9*b^4)*
cos(d*x + c)^2 + 8*(16*a^3 + 24*a^2*b + 20*a*b^2 + 8*b^3 - (10*a*b^2 + 7*b^
3)*cos(d*x + c)^2 - (b^3*cos(d*x + c)^2 - 24*a^2*b - 28*a*b^2 - 8*b^3)*sin(
d*x + c))*sqrt(b*sin(d*x + c) + a)*sqrt(a + b) + 4*(64*a^3*b + 112*a^2*b^2
+ 64*a*b^3 + 14*b^4 - (8*a*b^3 + 7*b^4)*cos(d*x + c)^2)*sin(d*x + c))/(cos(
d*x + c)^4 - 8*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8)) -
 16*(a^2 - b^2)*sqrt(b*sin(d*x + c) + a)*sin(d*x + c))/((a^2 - b^2)*d*cos(d
*x + c)^2), -1/16*((2*a^2 + a*b - b^2)*sqrt(-a + b)*arctan(1/4*(b^2*cos(d*x
 + c)^2 - 8*a^2 + 8*a*b - 2*b^2 - 2*(4*a*b - 3*b^2)*sin(d*x + c))*sqrt(b*si
n(d*x + c) + a)*sqrt(-a + b)/(2*a^3 - 3*a^2*b + 2*a*b^2 - b^3 - (a*b^2 - b^
3)*cos(d*x + c)^2 + (3*a^2*b - 4*a*b^2 + b^3)*sin(d*x + c)))*cos(d*x + c)^2
 + (2*a^2 - a*b - b^2)*sqrt(-a - b)*arctan(-1/4*(b^2*cos(d*x + c)^2 - 8*a^2
```

```
 - 8*a*b - 2*b^2 - 2*(4*a*b + 3*b^2)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)
*sqrt(-a - b)/(2*a^3 + 3*a^2*b + 2*a*b^2 + b^3 - (a*b^2 + b^3)*cos(d*x + c)
^2 + (3*a^2*b + 4*a*b^2 + b^3)*sin(d*x + c)))*cos(d*x + c)^2 - 8*(a^2 - b^2
)*sqrt(b*sin(d*x + c) + a)*sin(d*x + c))/((a^2 - b^2)*d*cos(d*x + c)^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\sin(c + dx)}\,\sec^3(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+b*sin(d*x+c))**(1/2),x)
[Out] Integral(sqrt(a + b*sin(c + d*x))*sec(c + d*x)**3, x)
```

Giac [A]
time = 5.95, size = 159, normalized size = 1.28

$$\frac{b^3\left(\frac{(2a-b)\arctan\left(\frac{\sqrt{b\sin(dx+c)+a}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}\,b^3} - \frac{(2a+b)\arctan\left(\frac{\sqrt{b\sin(dx+c)+a}}{\sqrt{-a-b}}\right)}{\sqrt{-a-b}\,b^3} - \frac{2\left((b\sin(dx+c)+a)^{\frac{3}{2}} - \sqrt{b\sin(dx+c)+a}\,a\right)}{\left((b\sin(dx+c)+a)^2 - 2(b\sin(dx+c)+a)a + a^2 - b^2\right)b^2}\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sin(d*x+c))^(1/2),x, algorithm="giac")
[Out] 1/4*b^3*((2*a - b)*arctan(sqrt(b*sin(d*x + c) + a)/sqrt(-a + b))/(sqrt(-a +
 b)*b^3) - (2*a + b)*arctan(sqrt(b*sin(d*x + c) + a)/sqrt(-a - b))/(sqrt(-a
 - b)*b^3) - 2*((b*sin(d*x + c) + a)^(3/2) - sqrt(b*sin(d*x + c) + a)*a)/((
(b*sin(d*x + c) + a)^2 - 2*(b*sin(d*x + c) + a)*a + a^2 - b^2)*b^2))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a + b\,\sin(c + d\,x)}}{\cos(c + d\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^(1/2)/cos(c + d*x)^3,x)
[Out] int((a + b*sin(c + d*x))^(1/2)/cos(c + d*x)^3, x)
```

3.478 $\int \sec^5(c+dx)\sqrt{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=207

$$-\frac{(12a^2-18ab+5b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{32(a-b)^{3/2}d}+\frac{(12a^2+18ab+5b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{32(a+b)^{3/2}d}$$

```
[Out] -1/32*(12*a^2-18*a*b+5*b^2)*arctanh((a+b*sin(d*x+c))^(1/2)/(a-b)^(1/2))/(a-
b)^(3/2)/d+1/32*(12*a^2+18*a*b+5*b^2)*arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^
(1/2))/(a+b)^(3/2)/d-1/16*sec(d*x+c)^2*(a*b-(6*a^2-5*b^2)*sin(d*x+c))*(a+b*
sin(d*x+c))^(1/2)/(a^2-b^2)/d+1/4*sec(d*x+c)^3*(a+b*sin(d*x+c))^(1/2)*tan(d
*x+c)/d
```

Rubi [A]
time = 0.21, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2747, 751, 837, 841, 1180, 212}

$$-\frac{(12a^2-18ab+5b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{32d(a-b)^{3/2}}+\frac{(12a^2+18ab+5b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{32d(a+b)^{3/2}}-\frac{\sec^2(c+dx)\sqrt{a+b\sin(c+dx)}\,(ab-(6a^2-5b^2)\sin(c+dx))}{16d\,(a^2-b^2)}+\frac{\tan(c+dx)\sec^3(c+dx)\sqrt{a+b\sin(c+dx)}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] -1/32*((12*a^2 - 18*a*b + 5*b^2)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a -
b]])/((a - b)^(3/2)*d) + ((12*a^2 + 18*a*b + 5*b^2)*ArcTanh[Sqrt[a + b*Sin[
c + d*x]]/Sqrt[a + b]])/(32*(a + b)^(3/2)*d) - (Sec[c + d*x]^2*Sqrt[a + b*S
in[c + d*x]]*(a*b - (6*a^2 - 5*b^2)*Sin[c + d*x]))/(16*(a^2 - b^2)*d) + (Se
c[c + d*x]^3*Sqrt[a + b*Sin[c + d*x]]*Tan[c + d*x])/(4*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 751

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-x)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*a*(p + 1))), x] + Dist[1/(2*a*(p +
 1)), Int[(d + e*x)^(m - 1)*(d*(2*p + 3) + e*(m + 2*p + 3)*x)*(a + c*x^2)^(
p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p,
 -1] && GtQ[m, 0] && (LtQ[m, 1] || (ILtQ[m + 2*p + 3, 0] && NeQ[m, 2])) &&
IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 841

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^5(c+dx)\sqrt{a+b\sin(c+dx)}\,dx = \frac{b^5\text{Subst}\left(\int \frac{\sqrt{a+x}}{(b^2-x^2)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^3(c+dx)\sqrt{a+b\sin(c+dx)}\,\tan(c+dx)}{4d} - \frac{b^3\text{Subst}\left(\int \frac{-3a}{\sqrt{a+x}}\right.}{}$$

$$= -\frac{\sec^2(c+dx)\sqrt{a+b\sin(c+dx)}\,(ab-(6a^2-5b^2)\sin(c+dx))}{16\,(a^2-b^2)\,d} +$$

$$= -\frac{\sec^2(c+dx)\sqrt{a+b\sin(c+dx)}\,(ab-(6a^2-5b^2)\sin(c+dx))}{16\,(a^2-b^2)\,d} +$$

$$= -\frac{\sec^2(c+dx)\sqrt{a+b\sin(c+dx)}\,(ab-(6a^2-5b^2)\sin(c+dx))}{16\,(a^2-b^2)\,d} +$$

$$= -\frac{(12a^2-18ab+5b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{32(a-b)^{3/2}d} + \frac{(12a^2+18}{}$$

Mathematica [A]
time = 1.52, size = 224, normalized size = 1.08

$$\frac{-\sqrt{a-b}\,(a+b)^2\,(12a^2-18ab+5b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)+(a-b)^2\sqrt{a+b}\,(12a^2+18ab+5b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)+\frac{1}{2}(a^2-b^2)\sec^4(c+dx)\sqrt{a+b\sin(c+dx)}\,(-2ab-2ab\cos(2(c+dx))+(22a^2-21b^2)\sin(c+dx)+6a^2\sin(3(c+dx))-5b^2\sin(3(c+dx)))}{32\,(a^2-b^2)^2\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (-(Sqrt[a - b]*(a + b)^2*(12*a^2 - 18*a*b + 5*b^2)*ArcTanh[Sqrt[a + b*Sin[c
 + d*x]]/Sqrt[a - b]]) + (a - b)^2*Sqrt[a + b]*(12*a^2 + 18*a*b + 5*b^2)*Ar
cTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a + b]] + ((a^2 - b^2)*Sec[c + d*x]^4*S
qrt[a + b*Sin[c + d*x]]*(-2*a*b - 2*a*b*Cos[2*(c + d*x)] + (22*a^2 - 21*b^2
)*Sin[c + d*x] + 6*a^2*Sin[3*(c + d*x)] - 5*b^2*Sin[3*(c + d*x)]))/2)/(32*(
a^2 - b^2)^2*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 470 vs. $2(183) = 366$.
time = 2.44, size = 471, normalized size = 2.28

method	result

default	$$-\frac{3b(a+b\sin(dx+c))^{\frac{3}{2}}a}{16(b\sin(dx+c)-b)^2(a+b)}-\frac{5b^2(a+b\sin(dx+c))^{\frac{3}{2}}}{32(b\sin(dx+c)-b)^2(a+b)}+\frac{3\sqrt{a+b\sin(dx+c)}\,ab}{16(b\sin(dx+c)-b)^2}+\frac{7\sqrt{a+b\sin(dx+c)}\,b^2}{32(b\sin(dx+c)-b)^2}+\frac{3\operatorname{arctanh}\left(\frac{\sqrt{}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^5*(a+b*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)

[Out] (-3/16/(b*sin(d*x+c)-b)^2*b/(a+b)*(a+b*sin(d*x+c))^(3/2)*a-5/32/(b*sin(d*x+c)-b)^2*b^2/(a+b)*(a+b*sin(d*x+c))^(3/2)+3/16/(b*sin(d*x+c)-b)^2*(a+b*sin(d*x+c))^(1/2)*a*b+7/32/(b*sin(d*x+c)-b)^2*(a+b*sin(d*x+c))^(1/2)*b^2+3/8/(a+b)^(3/2)*arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))*a^2+9/16/(a+b)^(3/2)*arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))*a*b+5/32/(a+b)^(3/2)*arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))*b^2-3/16/(b*sin(d*x+c)+b)^2*b/(a-b)*(a+b*sin(d*x+c))^(3/2)*a+5/32/(b*sin(d*x+c)+b)^2*b^2/(a-b)*(a+b*sin(d*x+c))^(3/2)+3/16/(b*sin(d*x+c)+b)^2*(a+b*sin(d*x+c))^(1/2)*a*b-7/32/(b*sin(d*x+c)+b)^2*(a+b*sin(d*x+c))^(1/2)*b^2+3/8/(a-b)/(-a+b)^(1/2)*arctan((a+b*sin(d*x+c))^(1/2)/(-a+b)^(1/2))*a^2-9/16/(a-b)/(-a+b)^(1/2)*arctan((a+b*sin(d*x+c))^(1/2)/(-a+b)^(1/2))*a*b+5/32/(a-b)/(-a+b)^(1/2)*arctan((a+b*sin(d*x+c))^(1/2)/(-a+b)^(1/2))*b^2)/d

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^5*(a+b*sin(d*x+c))^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for more detail

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^5*(a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(b*sin(d*x + c) + a)*sec(d*x + c)^5, x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sin\left(c+dx\right)}\,\sec^5\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(c + d*x))*sec(c + d*x)**5, x)
```

Giac [A]
time = 5.93, size = 358, normalized size = 1.73

$$\frac{b^5\left(\frac{\left(12\,a^2-18\,ab+5\,b^2\right)\arctan\left(\frac{\sqrt{b\sin\left(dx+c\right)+a}}{\sqrt{-a+b}}\right)}{\left(ab^5-b^6\right)\sqrt{-a+b}}-\frac{\left(12\,a^2+18\,ab+5\,b^2\right)\arctan\left(\frac{\sqrt{b\sin\left(dx+c\right)+a}}{\sqrt{-a-b}}\right)}{\left(ab^5+b^6\right)\sqrt{-a-b}}-\frac{2\left(6\left(b\sin(dx+c)+a\right)^{\frac{7}{2}}a^2-18\left(b\sin(dx+c)+a\right)^{\frac{5}{2}}a^3+18\left(b\sin(dx+c)+a\right)^{\frac{3}{2}}a^4-6\sqrt{b\sin\left(dx+c\right)+a}\,a^5-5\left(b\sin(dx+c)+a\right)^{\frac{7}{2}}b^2+14\left(b\sin(dx+c)+a\right)^{\frac{5}{2}}ab^2-23\left(b\sin(dx+c)+a\right)^{\frac{3}{2}}a^2b^2+14\sqrt{b\sin\left(dx+c\right)+a}\,a^3b^2+9\left(b\sin(dx+c)+a\right)^{\frac{3}{2}}b^4-8\sqrt{b\sin\left(dx+c\right)+a}\,ab^4\right)}{\left(a^2b^4-b^6\right)\left(\left(b\sin(dx+c)+a\right)^2-2\left(b\sin(dx+c)+a\right)a+a^2-b^2\right)^2}\right)}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/32*b^5*((12*a^2 - 18*a*b + 5*b^2)*arctan(sqrt(b*sin(d*x + c) + a)/sqrt(-a
 + b))/((a*b^5 - b^6)*sqrt(-a + b)) - (12*a^2 + 18*a*b + 5*b^2)*arctan(sqrt
(b*sin(d*x + c) + a)/sqrt(-a - b))/((a*b^5 + b^6)*sqrt(-a - b)) - 2*(6*(b*s
in(d*x + c) + a)^(7/2)*a^2 - 18*(b*sin(d*x + c) + a)^(5/2)*a^3 + 18*(b*sin(
d*x + c) + a)^(3/2)*a^4 - 6*sqrt(b*sin(d*x + c) + a)*a^5 - 5*(b*sin(d*x + c
) + a)^(7/2)*b^2 + 14*(b*sin(d*x + c) + a)^(5/2)*a*b^2 - 23*(b*sin(d*x + c)
 + a)^(3/2)*a^2*b^2 + 14*sqrt(b*sin(d*x + c) + a)*a^3*b^2 + 9*(b*sin(d*x +
c) + a)^(3/2)*b^4 - 8*sqrt(b*sin(d*x + c) + a)*a*b^4)/((a^2*b^4 - b^6)*((b*
sin(d*x + c) + a)^2 - 2*(b*sin(d*x + c) + a)*a + a^2 - b^2)^2))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a+b\,\sin\left(c+d\,x\right)}}{{\cos\left(c+d\,x\right)}^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^(1/2)/cos(c + d*x)^5,x)
```

```
[Out] int((a + b*sin(c + d*x))^(1/2)/cos(c + d*x)^5, x)
```

3.479 $\int \cos^4(c+dx)\sqrt{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=298

$$-\frac{4a\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}}{21bd}+\frac{2\cos^3(c+dx)(a+b\sin(c+dx))^{3/2}}{9bd}-\frac{8(4a^4-15a^2b^2-21b^4)\,E\left(\frac{1}{2}(c\right.}{315b^4d\sqrt{\cdots}}$$

```
[Out] 2/9*cos(d*x+c)^3*(a+b*sin(d*x+c))^(3/2)/b/d-4/21*a*cos(d*x+c)^3*(a+b*sin(d*
x+c))^(1/2)/b/d-4/315*cos(d*x+c)*(4*a*(a^2-3*b^2)-3*b*(a^2+7*b^2)*sin(d*x+c
))*(a+b*sin(d*x+c))^(1/2)/b^3/d+8/315*(4*a^4-15*a^2*b^2-21*b^4)*(sin(1/2*c+
1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*
Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/b^4/d/((a+b*sin
(d*x+c))/(a+b))^(1/2)-32/315*a*(a^4-4*a^2*b^2+3*b^4)*(sin(1/2*c+1/4*Pi+1/2*
d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x)
,2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/b^4/d/(a+b*sin(d*x
+c))^(1/2)
```

Rubi [A]
time = 0.35, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {2774, 2941, 2944, 2831, 2742, 2740, 2734, 2732}

$$-\frac{4\cos(c+dx)\sqrt{a+b\sin(c+dx)}\,(4a(a^2-3b^2)-3b(a^2+7b^2)\sin(c+dx))}{315b^3d}+\frac{32a(a^4-4a^2b^2+3b^4)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{315b^4d\sqrt{a+b\sin(c+dx)}}-\frac{8(4a^4-15a^2b^2-21b^4)\sqrt{a+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{315b^4d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}+\frac{2\cos^3(c+dx)(a+b\sin(c+dx))^{3/2}}{9bd}-\frac{4a\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}}{21bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (-4*a*Cos[c + d*x]^3*Sqrt[a + b*Sin[c + d*x]])/(21*b*d) + (2*Cos[c + d*x]^3
*(a + b*Sin[c + d*x])^(3/2))/(9*b*d) - (8*(4*a^4 - 15*a^2*b^2 - 21*b^4)*Ell
ipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(315*b^
4*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + (32*a*(a^4 - 4*a^2*b^2 + 3*b^4)*E
llipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a +
b)])/(315*b^4*d*Sqrt[a + b*Sin[c + d*x]]) - (4*Cos[c + d*x]*Sqrt[a + b*Sin[
c + d*x]]*(4*a*(a^2 - 3*b^2) - 3*b*(a^2 + 7*b^2)*Sin[c + d*x]))/(315*b^3*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
```

```
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2774

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(b*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2941

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp
[a*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x],
x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
&&  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Si
mplerQ[c + d*x, a + b*x])
```

Rule 2944

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
```

```
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Dist[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \cos^4(c+dx)\sqrt{a+b\sin(c+dx)}\,dx = \frac{2\cos^3(c+dx)(a+b\sin(c+dx))^{3/2}}{9bd} + \frac{2\int \cos^2(c+dx)(b+a\sin(c$$

$$= -\frac{4a\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}}{21bd} + \frac{2\cos^3(c+dx)(a+b\sin(c}{9bd}$$

$$= -\frac{4a\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}}{21bd} + \frac{2\cos^3(c+dx)(a+b\sin(c}{9bd}$$

$$= -\frac{4a\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}}{21bd} + \frac{2\cos^3(c+dx)(a+b\sin(c}{9bd}$$

$$= -\frac{4a\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}}{21bd} + \frac{2\cos^3(c+dx)(a+b\sin(c}{9bd}$$

$$= -\frac{4a\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}}{21bd} + \frac{2\cos^3(c+dx)(a+b\sin(c}{9bd}$$

Mathematica [A]
time = 0.88, size = 233, normalized size = 0.78

$$\frac{32\left(ab^2(a^2-33b^2)\,F\left(\frac{1}{4}(-2c+\pi-2dx)|\frac{2b}{a+b}\right)+(4a^4-15a^2b^2-21b^4)\left((a+b)E\left(\frac{1}{4}(-2c+\pi-2dx)|\frac{2b}{a+b}\right)-aF\left(\frac{1}{4}(-2c+\pi-2dx)|\frac{2b}{a+b}\right)\right)\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}+2b\cos(c+dx)(a+b\sin(c+dx))\left(-32a^3+106ab^2+10ab^2\cos(2(c+dx))+b(24a^2+203b^2)\sin(c+dx)+35b^3\sin(3(c+dx))\right)}{1260b^4d\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (32*(a*b^2*(a^2 - 33*b^2)*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)] +
 (4*a^4 - 15*a^2*b^2 - 21*b^4)*((a + b)*EllipticE[(-2*c + Pi - 2*d*x)/4, (2
```

```
*b)/(a + b)] - a*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]))*Sqrt[(a
+ b*Sin[c + d*x])/(a + b)] + 2*b*Cos[c + d*x]*(a + b*Sin[c + d*x])*(-32*a^3
 + 106*a*b^2 + 10*a*b^2*Cos[2*(c + d*x)] + b*(24*a^2 + 203*b^2)*Sin[c + d*x
] + 35*b^3*Sin[3*(c + d*x)]))/(1260*b^4*d*Sqrt[a + b*Sin[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1188 vs. $2(340) = 680$.
time = 2.14, size = 1189, normalized size = 3.99

method	result	size
default	Expression too large to display	1189

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+b*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/315*(-35*b^6*sin(d*x+c)^6-40*a*b^5*sin(d*x+c)^5+16*((a+b*sin(d*x+c))/(a-
b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*E
llipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*b-12*((a+b
*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*
b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2)
)*a^4*b^2-64*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)
*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),(
(a-b)/(a+b))^(1/2))*a^3*b^3-72*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)
-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+
c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^4+48*((a+b*sin(d*x+c))/(a-b))^(
1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Ellipt
icF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^5+84*((a+b*sin(
d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-
b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b^6
-16*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin
(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+
b))^(1/2))*a^6+76*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^
(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1
/2),((a-b)/(a+b))^(1/2))*a^4*b^2+24*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d
*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin
(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^4-84*((a+b*sin(d*x+c))/(a-
b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*E
llipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b^6+a^2*b^4*si
n(d*x+c)^4+112*b^6*sin(d*x+c)^4-2*a^3*b^3*sin(d*x+c)^3+146*a*b^5*sin(d*x+c)
^3-8*a^4*b^2*sin(d*x+c)^2+28*a^2*b^4*sin(d*x+c)^2-77*b^6*sin(d*x+c)^2+2*a^3
*b^3*sin(d*x+c)-106*a*b^5*sin(d*x+c)+8*a^4*b^2-29*a^2*b^4)/b^5/cos(d*x+c)/(
a+b*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(d*x + c) + a)*cos(d*x + c)^4, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.15, size = 534, normalized size = 1.79

$$\frac{2\left(2\sqrt{2}\left(8a^5-33a^3b^2+57ab^4\right)\sqrt{ib}\,\mathrm{weierstrassPInverse}\left(-\frac{4\left(4a^2-3b^2\right)}{3b^2},-\frac{8\left(8ia^3-9iab^2\right)}{27b^3},\frac{3b\cos(dx+c)-3ib\sin(dx+c)-2ia}{3b}\right)+2\sqrt{2}\left(8a^5-33a^3b^2+57ab^4\right)\sqrt{-ib}\,\mathrm{weierstrassPInverse}\left(-\frac{4\left(4a^2-3b^2\right)}{3b^2},-\frac{8\left(-8ia^3+9iab^2\right)}{27b^3},\frac{3b\cos(dx+c)+3ib\sin(dx+c)+2ia}{3b}\right)-6\sqrt{2}\left(-4ia^4b+15ia^2b^3+21ib^5\right)\sqrt{ib}\,\mathrm{weierstrassZeta}\left(-\frac{4\left(4a^2-3b^2\right)}{3b^2},-\frac{8\left(8ia^3-9iab^2\right)}{27b^3},\mathrm{weierstrassPInverse}\left(-\frac{4\left(4a^2-3b^2\right)}{3b^2},-\frac{8\left(8ia^3-9iab^2\right)}{27b^3},\frac{3b\cos(dx+c)-3ib\sin(dx+c)-2ia}{3b}\right)\right)-6\sqrt{2}\left(4ia^4b-15ia^2b^3-21ib^5\right)\sqrt{-ib}\,\mathrm{weierstrassZeta}\left(-\frac{4\left(4a^2-3b^2\right)}{3b^2},-\frac{8\left(-8ia^3+9iab^2\right)}{27b^3},\mathrm{weierstrassPInverse}\left(-\frac{4\left(4a^2-3b^2\right)}{3b^2},-\frac{8\left(-8ia^3+9iab^2\right)}{27b^3},\frac{3b\cos(dx+c)+3ib\sin(dx+c)+2ia}{3b}\right)\right)+3\left(5ab^4\cos(dx+c)^3-8\left(a^3b^2-3ab^4\right)\cos(dx+c)+\left(35b^5\cos(dx+c)^3+6\left(a^2b^3+7b^5\right)\cos(dx+c)\right)\sin(dx+c)\right)\sqrt{b\sin(dx+c)+a}\right)}{945\,b^5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/945*(2*sqrt(2)*(8*a^5 - 33*a^3*b^2 + 57*a*b^4)*sqrt(I*b)*weierstrassPInve
rse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, 1/3*(3*b*cos
(d*x + c) - 3*I*b*sin(d*x + c) - 2*I*a)/b) + 2*sqrt(2)*(8*a^5 - 33*a^3*b^2
+ 57*a*b^4)*sqrt(-I*b)*weierstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*
(-8*I*a^3 + 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*
I*a)/b) - 6*sqrt(2)*(-4*I*a^4*b + 15*I*a^2*b^3 + 21*I*b^5)*sqrt(I*b)*weiers
trassZeta(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, weiers
trassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, 1/
3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) - 2*I*a)/b)) - 6*sqrt(2)*(4*I*a^4*
b - 15*I*a^2*b^3 - 21*I*b^5)*sqrt(-I*b)*weierstrassZeta(-4/3*(4*a^2 - 3*b^2
)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, weierstrassPInverse(-4/3*(4*a^2 -
3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b
*sin(d*x + c) + 2*I*a)/b)) + 3*(5*a*b^4*cos(d*x + c)^3 - 8*(a^3*b^2 - 3*a*b
^4)*cos(d*x + c) + (35*b^5*cos(d*x + c)^3 + 6*(a^2*b^3 + 7*b^5)*cos(d*x + c
))*sin(d*x + c))*sqrt(b*sin(d*x + c) + a))/(b^5*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sin(c+dx)}\,\cos^4(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(c + d*x))*cos(c + d*x)**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sin(d*x + c) + a)*cos(d*x + c)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(c+d\,x\right)^4\,\sqrt{a+b\,\sin\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(a + b*sin(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^4*(a + b*sin(c + d*x))^(1/2), x)
```

3.480 $\int \cos^2(c+dx)\sqrt{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=215

$$-\frac{4a\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{15bd}+\frac{2\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{5bd}+\frac{4(a^2+3b^2)\,E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)|\frac{2}{a-}\right.}{15b^2d\sqrt{\frac{a+b\sin(}{a+}}}$$

```
[Out] 2/5*cos(d*x+c)*(a+b*sin(d*x+c))^(3/2)/b/d-4/15*a*cos(d*x+c)*(a+b*sin(d*x+c)
)^(1/2)/b/d-4/15*(a^2+3*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+
1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2)
)*(a+b*sin(d*x+c))^(1/2)/b^2/d/((a+b*sin(d*x+c))/(a+b))^(1/2)+4/15*a*(a^2-b
^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF
(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b)
)^(1/2)/b^2/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {2774, 2832, 2831, 2742, 2740, 2734, 2732}

$$-\frac{4a(a^2-b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{15b^2d\sqrt{a+b\sin(c+dx)}}+\frac{4(a^2+3b^2)\sqrt{a+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{15b^2d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}+\frac{2\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{5bd}-\frac{4a\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{15bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (-4*a*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(15*b*d) + (2*Cos[c + d*x]*(a
+ b*Sin[c + d*x])^(3/2))/(5*b*d) + (4*(a^2 + 3*b^2)*EllipticE[(c - Pi/2 + d
*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(15*b^2*d*Sqrt[(a + b*Sin[c
 + d*x])/(a + b)]) - (4*a*(a^2 - b^2)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(
a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(15*b^2*d*Sqrt[a + b*Sin[c + d*
x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2774

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(b*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rubi steps

$$\int \cos^2(c+dx)\sqrt{a+b\sin(c+dx)}\,dx = \frac{2\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{5bd} + \frac{2\int(b+a\sin(c+dx))\sqrt{a+}}{5b}$$

$$= -\frac{4a\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{15bd} + \frac{2\cos(c+dx)(a+b\sin(c+}{5bd}$$

$$= -\frac{4a\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{15bd} + \frac{2\cos(c+dx)(a+b\sin(c+}{5bd}$$

$$= -\frac{4a\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{15bd} + \frac{2\cos(c+dx)(a+b\sin(c+}{5bd}$$

$$= -\frac{4a\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{15bd} + \frac{2\cos(c+dx)(a+b\sin(c+}{5bd}$$

Mathematica [A]
time = 0.82, size = 185, normalized size = 0.86

$$\frac{-4(a^3+a^2b+3ab^2+3b^3)\,E\left(\frac{1}{4}(-2c+\pi-2dx)\middle|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}+4a(a^2-b^2)\,F\left(\frac{1}{4}(-2c+\pi-2dx)\middle|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}+b\cos(c+dx)\left(2a^2+3b^2-3b^2\cos(2(c+dx))+8ab\sin(c+dx)\right)}{15b^2d\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (-4*(a^3 + a^2*b + 3*a*b^2 + 3*b^3)*EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b)/
(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] + 4*a*(a^2 - b^2)*EllipticF[(-2
*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] + b*C
os[c + d*x]*(2*a^2 + 3*b^2 - 3*b^2*Cos[2*(c + d*x)] + 8*a*b*Sin[c + d*x]))/
(15*b^2*d*Sqrt[a + b*Sin[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 791 vs. $2(261) = 522$.
time = 1.81, size = 792, normalized size = 3.68

method	result

default	$\frac{\frac{4\sqrt{\frac{a+b\sin(dx+c)}{a-b}}\,\sqrt{-\frac{(\sin(dx+c)-1)b}{a+b}}\,\sqrt{-\frac{(1+\sin(dx+c))b}{a-b}}\,\text{EllipticF}\left(\sqrt{\frac{a+b\sin(dx+c)}{a-b}},\sqrt{\frac{a-b}{a+b}}\right)a^3b}{15}+\frac{4a^2\sqrt{\frac{a+b\sin(dx}{a-b}}}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15*(2*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1
+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)
/(a+b))^(1/2))*a^3*b+6*a^2*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*
b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/
(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b^2-2*a*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-
(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a
+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b^3-6*((a+b*sin(d*x+c))/(a
-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*
EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b^4-2*((a+b*s
in(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/
(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*
a^4-4*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+s
in(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(
a+b))^(1/2))*a^2*b^2+6*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a
+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b
))^(1/2),((a-b)/(a+b))^(1/2))*b^4-3*b^4*sin(d*x+c)^4-4*a*b^3*sin(d*x+c)^3-a
^2*b^2*sin(d*x+c)^2+3*b^4*sin(d*x+c)^2+4*a*b^3*sin(d*x+c)+a^2*b^2)/b^3/cos(
d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(d*x + c) + a)*cos(d*x + c)^2, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 448, normalized size = 2.08

$$-\frac{2\left(2\sqrt{2}\,(a^3-3ab^2)\sqrt{ib}\,\text{weierstrassPInverse}\left(-\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(8ia^3-9iab^2)}{27b^3},\frac{3b\cos(dx+c)-3ib\sin(dx+c)-2ia}{3b}\right)+2\sqrt{2}\,(a^3-3ab^2)\sqrt{-ib}\,\text{weierstrassPInverse}\left(-\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(-8ia^3+9iab^2)}{27b^3},\frac{3b\cos(dx+c)+3ib\sin(dx+c)+2ia}{3b}\right)+3\sqrt{2}\,(ia^2b+3ib^3)\sqrt{ib}\,\text{weierstrassZeta}\left(-\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(8ia^3-9iab^2)}{27b^3},\text{weierstrassPInverse}\left(-\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(8ia^3-9iab^2)}{27b^3},\frac{3b\cos(dx+c)-3ib\sin(dx+c)-2ia}{3b}\right)\right)+3\sqrt{2}\,(-ia^2b-3ib^3)\sqrt{-ib}\,\text{weierstrassZeta}\left(-\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(-8ia^3+9iab^2)}{27b^3},\text{weierstrassPInverse}\left(-\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(-8ia^3+9iab^2)}{27b^3},\frac{3b\cos(dx+c)+3ib\sin(dx+c)+2ia}{3b}\right)\right)-3\left(3b^3\cos(dx+c)\sin(dx+c)+ab^2\cos(dx+c)\right)\sqrt{b\sin(dx+c)+a}\right)}{45\,b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/45*(2*sqrt(2)*(a^3 - 3*a*b^2)*sqrt(I*b)*weierstrassPInverse(-4/3*(4*a^2
- 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*
b*sin(d*x + c) - 2*I*a)/b) + 2*sqrt(2)*(a^3 - 3*a*b^2)*sqrt(-I*b)*weierstra
ssPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, 1/3*
(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*I*a)/b) + 3*sqrt(2)*(I*a^2*b + 3
*I*b^3)*sqrt(I*b)*weierstrassZeta(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3
- 9*I*a*b^2)/b^3, weierstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I*
a^3 - 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) - 2*I*a)/b
)) + 3*sqrt(2)*(-I*a^2*b - 3*I*b^3)*sqrt(-I*b)*weierstrassZeta(-4/3*(4*a^2
- 3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, weierstrassPInverse(-4/3*(4
*a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c)
+ 3*I*b*sin(d*x + c) + 2*I*a)/b)) - 3*(3*b^3*cos(d*x + c)*sin(d*x + c) + a*
b^2*cos(d*x + c))*sqrt(b*sin(d*x + c) + a))/(b^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\sin{(c + dx)}}\cos^2{(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(c + d*x))*cos(c + d*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sin(d*x + c) + a)*cos(d*x + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(c+d\,x\right)^2\,\sqrt{a+b\,\sin\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a + b*sin(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^2*(a + b*sin(c + d*x))^(1/2), x)
```

3.481 $\int \sec^2(c+dx)\sqrt{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=149

$$-\frac{E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)|\frac{2b}{a+b}\right)\sqrt{a+b\sin(c+dx)}}{d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}+\frac{aF\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}{d\sqrt{a+b\sin(c+dx)}}+\frac{\sqrt{a+b\sin(c}}{}$$

```
[Out] (sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos
(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/d/((
a+b*sin(d*x+c))/(a+b))^(1/2)-a*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*
c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/
2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/d/(a+b*sin(d*x+c))^(1/2)+(a+b*sin(d*x+c)
)^(1/2)*tan(d*x+c)/d
```

Rubi [A]
time = 0.11, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {2769, 12, 2831, 2742, 2740, 2734, 2732}

$$\frac{\tan(c+dx)\sqrt{a+b\sin(c+dx)}}{d}+\frac{a\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\sin(c+dx)}}-\frac{\sqrt{a+b\sin(c+dx)}E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] -((EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(
d*Sqrt[(a + b*Sin[c + d*x])/(a + b)])) + (a*EllipticF[(c - Pi/2 + d*x)/2, (
2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(d*Sqrt[a + b*Sin[c + d*x
]]) + (Sqrt[a + b*Sin[c + d*x]]*Tan[c + d*x])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2769

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x
])^m*(Sin[e + f*x]/(f*g*(p + 1))), x] + Dist[1/(g^2*(p + 1)), Int[(g*Cos[e
+ f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1)*(a*(p + 2) + b*(m + p + 2)*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[0, m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^2(c+dx)\sqrt{a+b\sin(c+dx)}\,dx = \frac{\sqrt{a+b\sin(c+dx)}\,\tan(c+dx)}{d} - \int \frac{b\sin(c+dx)}{2\sqrt{a+b\sin(c+dx)}}\,dx$$

$$= \frac{\sqrt{a+b\sin(c+dx)}\,\tan(c+dx)}{d} - \frac{1}{2}b\int \frac{\sin(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx$$

$$= \frac{\sqrt{a+b\sin(c+dx)}\,\tan(c+dx)}{d} - \frac{1}{2}\int \sqrt{a+b\sin(c+dx)}\,dx + \frac{1}{2}$$

$$= \frac{\sqrt{a+b\sin(c+dx)}\,\tan(c+dx)}{d} - \frac{\sqrt{a+b\sin(c+dx)}\int\sqrt{\frac{a}{a+b}}}{2\sqrt{\frac{a+b\sin(c+}{a+b}}}$$

$$= -\frac{E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)|\frac{2b}{a+b}\right)\sqrt{a+b\sin(c+dx)}}{d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}} + \frac{aF\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\right.}{d\sqrt{a-}}$$

Mathematica [A]
time = 2.82, size = 127, normalized size = 0.85

$$\frac{(a+b)E\left(\frac{1}{4}(-2c+\pi-2dx)|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}} - aF\left(\frac{1}{4}(-2c+\pi-2dx)|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}} + (a+b\sin(c+dx))\tan(c+dx)}{d\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] ((a + b)*EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c
+ d*x])/(a + b)] - a*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(
a + b*Sin[c + d*x])/(a + b)] + (a + b*Sin[c + d*x])*Tan[c + d*x])/(d*Sqrt[a
 + b*Sin[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 613 vs. 2(205) = 410.
time = 2.33, size = 614, normalized size = 4.12

method	result
default	$\frac{\sqrt{b(\cos^2(dx+c))\sin(dx+c)+a(\cos^2(dx+c))}\left(\text{EllipticE}\left(\sqrt{\frac{b\sin(dx+c)}{a-b}+\frac{a}{a-b}},\sqrt{\frac{a-b}{a+b}}\right)\sqrt{-\frac{b\sin(dx}{a-}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+b*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(b*cos(d*x+c)^2*sin(d*x+c)+a*cos(d*x+c)^2)^(1/2)*(EllipticE((b/(a-b)*si
n(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a-b)*sin(d*x+c)-b/(a-b)
)^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^
(1/2)*a^2-EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2
))*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)
*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*b^2-EllipticF((b/(a-b)*sin(d*x+c)+1/(a
-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(b/(a
-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*a*b+Ell
ipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a-b)*
sin(d*x+c)-b/(a-b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a+b)*si
n(d*x+c)+b/(a+b))^(1/2)*b^2-b^2*cos(d*x+c)^2+a*b*sin(d*x+c)+b^2)/(-(a+b*sin
(d*x+c))*(sin(d*x+c)-1)*(1+sin(d*x+c)))^(1/2)/cos(d*x+c)/(a+b*sin(d*x+c))^(
1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(d*x + c) + a)*sec(d*x + c)^2, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 418, normalized size = 2.81

$$\frac{2\sqrt{2}\,a\sqrt{i\,b}\cos\left(dx+c\right)\mathrm{weierstrassPInverse}\left(-\frac{4\left(4a^2-3b^2\right)}{3b^2},-\frac{8\left(8i\,a^3-9i\,ab^2\right)}{27b^3},\frac{3b\cos(dx+c)-3i\,b\sin(dx+c)-2i\,a}{3b}\right)+2\sqrt{2}\,a\sqrt{-i\,b}\cos\left(dx+c\right)\mathrm{weierstrassPInverse}\left(-\frac{4\left(4a^2-3b^2\right)}{3b^2},-\frac{8\left(-8i\,a^3+9i\,ab^2\right)}{27b^3},\frac{3b\cos(dx+c)+3i\,b\sin(dx+c)+2i\,a}{3b}\right)+3i\sqrt{2}\sqrt{i\,b}\,b\cos\left(dx+c\right)\mathrm{weierstrassZeta}\left(-\frac{4\left(4a^2-3b^2\right)}{3b^2},-\frac{8\left(8i\,a^3-9i\,ab^2\right)}{27b^3},\mathrm{weierstrassPInverse}\left(-\frac{4\left(4a^2-3b^2\right)}{3b^2},-\frac{8\left(8i\,a^3-9i\,ab^2\right)}{27b^3},\frac{3b\cos(dx+c)-3i\,b\sin(dx+c)-2i\,a}{3b}\right)\right)-3i\sqrt{2}\sqrt{-i\,b}\,b\cos\left(dx+c\right)\mathrm{weierstrassZeta}\left(-\frac{4\left(4a^2-3b^2\right)}{3b^2},-\frac{8\left(-8i\,a^3+9i\,ab^2\right)}{27b^3},\mathrm{weierstrassPInverse}\left(-\frac{4\left(4a^2-3b^2\right)}{3b^2},-\frac{8\left(-8i\,a^3+9i\,ab^2\right)}{27b^3},\frac{3b\cos(dx+c)+3i\,b\sin(dx+c)+2i\,a}{3b}\right)\right)+6\sqrt{b\sin\left(dx+c\right)+a}\,b\sin\left(dx+c\right)}{6\,bd\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(2*sqrt(2)*a*sqrt(I*b)*cos(d*x + c)*weierstrassPInverse(-4/3*(4*a^2 - 3
*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*s
in(d*x + c) - 2*I*a)/b) + 2*sqrt(2)*a*sqrt(-I*b)*cos(d*x + c)*weierstrassPI
nverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, 1/3*(3*b
*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*I*a)/b) + 3*I*sqrt(2)*sqrt(I*b)*b*co
s(d*x + c)*weierstrassZeta(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a
*b^2)/b^3, weierstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9
*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) - 2*I*a)/b)) - 3*
```

```
I*sqrt(2)*sqrt(-I*b)*b*cos(d*x + c)*weierstrassZeta(-4/3*(4*a^2 - 3*b^2)/b^
2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, weierstrassPInverse(-4/3*(4*a^2 - 3*b^
2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin
(d*x + c) + 2*I*a)/b)) + 6*sqrt(b*sin(d*x + c) + a)*b*sin(d*x + c))/(b*d*co
s(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\sin\left(c + dx\right)}\,\sec^2\left(c + dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*sin(d*x+c))**(1/2),x)
[Out] Integral(sqrt(a + b*sin(c + d*x))*sec(c + d*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sin(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sin(d*x + c) + a)*sec(d*x + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a + b\,\sin\left(c + d\,x\right)}}{\cos\left(c + d\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^(1/2)/cos(c + d*x)^2,x)
[Out] int((a + b*sin(c + d*x))^(1/2)/cos(c + d*x)^2, x)
```

3.482 $\int \sec^4(c+dx)\sqrt{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=248

$$-\frac{(4a^2-3b^2)\,E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)|\frac{2b}{a+b}\right)\sqrt{a+b\sin(c+dx)}}{6\,(a^2-b^2)\,d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}+\frac{2aF\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}{3d\sqrt{a+b\sin(c+dx)}}-$$

```
[Out] -1/6*sec(d*x+c)*(a*b-(4*a^2-3*b^2)*sin(d*x+c))*(a+b*sin(d*x+c))^(1/2)/(a^2-
b^2)/d+1/6*(4*a^2-3*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*
Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*(a
+b*sin(d*x+c))^(1/2)/(a^2-b^2)/d/((a+b*sin(d*x+c))/(a+b))^(1/2)-2/3*a*(sin(
1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*
c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/d
/(a+b*sin(d*x+c))^(1/2)+1/3*sec(d*x+c)^2*(a+b*sin(d*x+c))^(1/2)*tan(d*x+c)/
d
```

Rubi [A]
time = 0.23, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {2769, 2945, 2831, 2742, 2740, 2734, 2732}

$$-\frac{\sec(c+dx)\sqrt{a+b\sin(c+dx)}\,(ab-(4a^2-3b^2)\sin(c+dx))}{6d\,(a^2-b^2)}-\frac{(4a^2-3b^2)\sqrt{a+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{6d\,(a^2-b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}+\frac{2a\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{3d\sqrt{a+b\sin(c+dx)}}+\frac{\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\sin(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] -1/6*((4*a^2 - 3*b^2)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a +
 b*Sin[c + d*x]])/((a^2 - b^2)*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + (2*a
*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a
+ b)])/(3*d*Sqrt[a + b*Sin[c + d*x]]) - (Sec[c + d*x]*Sqrt[a + b*Sin[c + d*
x]]*(a*b - (4*a^2 - 3*b^2)*Sin[c + d*x]))/(6*(a^2 - b^2)*d) + (Sec[c + d*x]
^2*Sqrt[a + b*Sin[c + d*x]]*Tan[c + d*x])/(3*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
```

```
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2769

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x
])^m*(Sin[e + f*x]/(f*g*(p + 1))), x] + Dist[1/(g^2*(p + 1)), Int[(g*Cos[e
+ f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1)*(a*(p + 2) + b*(m + p + 2)*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[0, m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2945

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \sec^4(c+dx)\sqrt{a+b\sin(c+dx)}\,dx = \frac{\sec^2(c+dx)\sqrt{a+b\sin(c+dx)}\,\tan(c+dx)}{3d} - \frac{1}{3}\int \frac{\sec^2(c+dx)}{\sqrt{a-}}$$

$$= -\frac{\sec(c+dx)\sqrt{a+b\sin(c+dx)}\,(ab-(4a^2-3b^2)\sin(c+dx))}{6(a^2-b^2)d} +$$

$$= -\frac{\sec(c+dx)\sqrt{a+b\sin(c+dx)}\,(ab-(4a^2-3b^2)\sin(c+dx))}{6(a^2-b^2)d} +$$

$$= -\frac{\sec(c+dx)\sqrt{a+b\sin(c+dx)}\,(ab-(4a^2-3b^2)\sin(c+dx))}{6(a^2-b^2)d} +$$

$$= -\frac{(4a^2-3b^2)\,E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)|\frac{2b}{a+b}\right)\sqrt{a+b\sin(c+dx)}}{6(a^2-b^2)\,d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}} + \frac{2aF(}{}$$

Mathematica [A]
time = 3.25, size = 270, normalized size = 1.09

$$\frac{(4a^3+4a^2b-3ab^2-3b^3)\,E\left(\frac{1}{4}(-2c+\pi-2dx)|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}} - 4a(a^2-b^2)\,F\left(\frac{1}{4}(-2c+\pi-2dx)|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}} + \frac{1}{8}\sec^3(c+dx)\,(8a^2b-11b^3+(-12a^2b+8b^3)\cos(2(c+dx))+(-4a^2b+3b^3)\cos(4(c+dx))+24a^3\sin(c+dx)-24ab^2\sin(c+dx)+8a^3\sin(3(c+dx))-8ab^2\sin(3(c+dx)))}{6(a-b)(a+b)d\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] ((4*a^3 + 4*a^2*b - 3*a*b^2 - 3*b^3)*EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b)
/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] - 4*a*(a^2 - b^2)*EllipticF[(-
2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] + (S
ec[c + d*x]^3*(8*a^2*b - 11*b^3 + (-12*a^2*b + 8*b^3)*Cos[2*(c + d*x)] + (-
4*a^2*b + 3*b^3)*Cos[4*(c + d*x)] + 24*a^3*Sin[c + d*x] - 24*a*b^2*Sin[c +
d*x] + 8*a^3*Sin[3*(c + d*x)] - 8*a*b^2*Sin[3*(c + d*x)]))/8)/(6*(a - b)*(a
 + b)*d*Sqrt[a + b*Sin[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1260 vs. $2(294) = 588$.
time = 2.27, size = 1261, normalized size = 5.08

method	result	size
default	Expression too large to display	1261

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+b*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(-4*(b*cos(d*x+c)^2*sin(d*x+c)+a*cos(d*x+c)^2)^(1/2)*a*b*(a^2-b^2)*sin(
d*x+c)*cos(d*x+c)^2-2*(b*cos(d*x+c)^2*sin(d*x+c)+a*cos(d*x+c)^2)^(1/2)*a*b*
(a^2-b^2)*sin(d*x+c)+(b*cos(d*x+c)^2*sin(d*x+c)+a*cos(d*x+c)^2)^(1/2)*b^2*(
4*a^2-3*b^2)*cos(d*x+c)^4-(b*cos(d*x+c)^2*sin(d*x+c)+a*cos(d*x+c)^2)^(1/2)*
(4*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b
)*a)^(1/2),((a-b)/(a+b))^(1/2))*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a
-b)*sin(d*x+c)-b/(a-b))^(1/2)*a^4-7*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*Ell
ipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(b/(a-b)*s
in(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*a^2*b^2+3*(-
b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^
(1/2),((a-b)/(a+b))^(1/2))*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a-b)*s
in(d*x+c)-b/(a-b))^(1/2)*b^4-4*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(b/(a-b)
*sin(d*x+c)+1/(a-b)*a)^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)
,((a-b)/(a+b))^(1/2))*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*a^3*b+3*(-b/(a+b)
*sin(d*x+c)+b/(a+b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*EllipticF((
b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a-b)*sin(d*x+
c)-b/(a-b))^(1/2)*a^2*b^2+4*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(b/(a-b)*si
n(d*x+c)+1/(a-b)*a)^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((
a-b)/(a+b))^(1/2))*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*a*b^3-3*(-b/(a+b)*si
n(d*x+c)+b/(a+b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*EllipticF((b/(
a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a-b)*sin(d*x+c)-
b/(a-b))^(1/2)*b^4+a^2*b^2-b^4)*cos(d*x+c)^2-2*(b*cos(d*x+c)^2*sin(d*x+c)+a
*cos(d*x+c)^2)^(1/2)*a^2*b^2+2*(b*cos(d*x+c)^2*sin(d*x+c)+a*cos(d*x+c)^2)^(
1/2)*b^4)/(1+sin(d*x+c))/(a-b)/(-(a+b*sin(d*x+c))*(sin(d*x+c)-1)*(1+sin(d*x
+c)))^(1/2)/(a+b)/(sin(d*x+c)-1)/b/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(d*x + c) + a)*sec(d*x + c)^4, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 526, normalized size = 2.12

$$\frac{\sqrt{2}\,(8a^3-9ab^2)\sqrt{ib}\,\cos(dx+c)^3\,\mathrm{weierstrassPInverse}\left(-\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(8ia^3-9iab^2)}{27b^3},\frac{3b\cos(dx+c)-3ib\sin(dx+c)-2ia}{3b}\right)+\sqrt{2}\,(8a^3-9ab^2)\sqrt{-ib}\,\cos(dx+c)^3\,\mathrm{weierstrassPInverse}\left(-\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(-8ia^3+9iab^2)}{27b^3},\frac{3b\cos(dx+c)+3ib\sin(dx+c)+2ia}{3b}\right)+3\sqrt{2}\,(4ia^2b-3ib^3)\sqrt{ib}\,\cos(dx+c)^3\,\mathrm{weierstrassZeta}\left(-\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(8ia^3-9iab^2)}{27b^3},\mathrm{weierstrassPInverse}\left(-\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(8ia^3-9iab^2)}{27b^3},\frac{3b\cos(dx+c)-3ib\sin(dx+c)-2ia}{3b}\right)\right)+3\sqrt{2}\,(-4ia^2b+3ib^3)\sqrt{-ib}\,\cos(dx+c)^3\,\mathrm{weierstrassZeta}\left(-\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(-8ia^3+9iab^2)}{27b^3},\mathrm{weierstrassPInverse}\left(-\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(-8ia^3+9iab^2)}{27b^3},\frac{3b\cos(dx+c)+3ib\sin(dx+c)+2ia}{3b}\right)\right)-6\left(ab^2\cos(dx+c)^2-(2a^2b-2b^3+(4a^2b-3b^3)\cos(dx+c)^2)\sin(dx+c)\right)\sqrt{b\sin(dx+c)+a}}{36(a^2b-b^3)d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/36*(sqrt(2)*(8*a^3 - 9*a*b^2)*sqrt(I*b)*cos(d*x + c)^3*weierstrassPInvers
e(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d
*x + c) - 3*I*b*sin(d*x + c) - 2*I*a)/b) + sqrt(2)*(8*a^3 - 9*a*b^2)*sqrt(-
I*b)*cos(d*x + c)^3*weierstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-8
*I*a^3 + 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*I*a
)/b) + 3*sqrt(2)*(4*I*a^2*b - 3*I*b^3)*sqrt(I*b)*cos(d*x + c)^3*weierstrass
Zeta(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, weierstrass
PInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, 1/3*(3*
b*cos(d*x + c) - 3*I*b*sin(d*x + c) - 2*I*a)/b)) + 3*sqrt(2)*(-4*I*a^2*b +
3*I*b^3)*sqrt(-I*b)*cos(d*x + c)^3*weierstrassZeta(-4/3*(4*a^2 - 3*b^2)/b^2
, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, weierstrassPInverse(-4/3*(4*a^2 - 3*b^2
)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(
d*x + c) + 2*I*a)/b)) - 6*(a*b^2*cos(d*x + c)^2 - (2*a^2*b - 2*b^3 + (4*a^2
*b - 3*b^3)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a))/((a^2*b
 - b^3)*d*cos(d*x + c)^3)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sin(c+dx)}\,\sec^4(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(c + d*x))*sec(c + d*x)**4, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sin(d*x + c) + a)*sec(d*x + c)^4, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a+b\,\sin\left(c+d\,x\right)}}{{\cos\left(c+d\,x\right)}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^(1/2)/cos(c + d*x)^4,x)
```

```
[Out] int((a + b*sin(c + d*x))^(1/2)/cos(c + d*x)^4, x)
```

3.483 $\int \cos^5(c+dx)(a+b\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=154

$$\frac{2(a^2-b^2)^2\,(a+b\sin(c+dx))^{5/2}}{5b^5d}-\frac{8a(a^2-b^2)\,(a+b\sin(c+dx))^{7/2}}{7b^5d}+\frac{4(3a^2-b^2)\,(a+b\sin(c+dx))^{9/2}}{9b^5d}-8a($$

```
[Out] 2/5*(a^2-b^2)^2*(a+b*sin(d*x+c))^(5/2)/b^5/d-8/7*a*(a^2-b^2)*(a+b*sin(d*x+c
))^(7/2)/b^5/d+4/9*(3*a^2-b^2)*(a+b*sin(d*x+c))^(9/2)/b^5/d-8/11*a*(a+b*sin
(d*x+c))^(11/2)/b^5/d+2/13*(a+b*sin(d*x+c))^(13/2)/b^5/d
```

Rubi [A]
time = 0.07, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.087$, Rules used = {2747, 711}

$$\frac{4(3a^2-b^2)\,(a+b\sin(c+dx))^{9/2}}{9b^5d}-\frac{8a(a^2-b^2)\,(a+b\sin(c+dx))^{7/2}}{7b^5d}+\frac{2(a^2-b^2)^2\,(a+b\sin(c+dx))^{5/2}}{5b^5d}+\frac{2(a+b\sin(c+dx))^{13/2}}{13b^5d}-\frac{8a(a+b\sin(c+dx))^{11/2}}{11b^5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (2*(a^2 - b^2)^2*(a + b*Sin[c + d*x])^(5/2))/(5*b^5*d) - (8*a*(a^2 - b^2)*(
a + b*Sin[c + d*x])^(7/2))/(7*b^5*d) + (4*(3*a^2 - b^2)*(a + b*Sin[c + d*x]
)^(9/2))/(9*b^5*d) - (8*a*(a + b*Sin[c + d*x])^(11/2))/(11*b^5*d) + (2*(a +
 b*Sin[c + d*x])^(13/2))/(13*b^5*d)
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^5(c+dx)(a+b\sin(c+dx))^{3/2}\,dx = \frac{\text{Subst}\left(\int (a+x)^{3/2}\left(b^2-x^2\right)^2\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$

$$= \frac{\text{Subst}\left(\int \left(\left(a^2-b^2\right)^2(a+x)^{3/2} - 4\left(a^3-ab^2\right)(a+x)^{5/2} + 2(3a^2\right.\right.}{}$$

$$= \frac{2\left(a^2-b^2\right)^2(a+b\sin(c+dx))^{5/2}}{5b^5 d} - \frac{8a\left(a^2-b^2\right)(a+b\sin(c+dx}{7b^5 d}$$

Mathematica [A]
time = 0.99, size = 238, normalized size = 1.55

$$\frac{2a(6144a^6-35456a^4b^2+137910a^2b^4+29337b^6)\sqrt{1+\frac{b\sin(c+dx)}{a}}\left(-1+\sqrt{1+\frac{b\sin(c+dx)}{a}}\right)-b(a+b\sin(c+dx))\left(b(2304a^4-12048a^2b^2+35959b^4)\cos(2(c+dx))+70b^3(-6a^2+275b^2)\cos(4(c+dx))+3465b^5\cos(6(c+dx))+8a(768a^4-4216a^2b^2-40197b^4)\sin(c+dx)-20ab^2(48a^2+3515b^2)\sin(3(c+dx))-8820ab^4\sin(5(c+dx))\right)}{720720b^5d\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (2*a*(6144*a^6 - 35456*a^4*b^2 + 137910*a^2*b^4 + 29337*b^6)*Sqrt[1 + (b*Si
n[c + d*x])/a]*(-1 + Sqrt[1 + (b*Sin[c + d*x])/a]) - b*(a + b*Sin[c + d*x])
*(b*(2304*a^4 - 12048*a^2*b^2 + 35959*b^4)*Cos[2*(c + d*x)] + 70*b^3*(-6*a^
2 + 275*b^2)*Cos[4*(c + d*x)] + 3465*b^5*Cos[6*(c + d*x)] + 8*a*(768*a^4 -
4216*a^2*b^2 - 40197*b^4)*Sin[c + d*x] - 20*a*b^2*(48*a^2 + 3515*b^2)*Sin[3
*(c + d*x)] - 8820*a*b^4*Sin[5*(c + d*x)]))/(720720*b^5*d*Sqrt[a + b*Sin[c
+ d*x]])
```

Maple [A]
time = 1.58, size = 126, normalized size = 0.82

method	result
default	$\frac{2(a+b\sin(dx+c))^{\frac{5}{2}}\left(3465b^4\left(\cos^4(dx+c)\right)+2520a\,b^3\left(\cos^2(dx+c)\right)\sin(dx+c)-1680a^2b^2\left(\cos^2(dx+c)\right)+3080b^4\left(\cos^2(dx+c)\right)-960a^3b\right.}{45045b^5d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+b*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/45045/b^5*(a+b*sin(d*x+c))^(5/2)*(3465*b^4*cos(d*x+c)^4+2520*a*b^3*cos(d*
x+c)^2*sin(d*x+c)-1680*a^2*b^2*cos(d*x+c)^2+3080*b^4*cos(d*x+c)^2-960*a^3*b
*sin(d*x+c)+3200*a*b^3*sin(d*x+c)+384*a^4-608*a^2*b^2+2464*b^4)/d
```

Maxima [A]
time = 0.27, size = 116, normalized size = 0.75

$$\frac{2\left(3465\,(b\sin(dx+c)+a)^{\frac{13}{2}} - 16380\,(b\sin(dx+c)+a)^{\frac{11}{2}}a + 10010\,(3\,a^2-b^2)(b\sin(dx+c)+a)^{\frac{9}{2}} - 25740\,(a^3-ab^2)(b\sin(dx+c)+a)^{\frac{7}{2}} + 9009\,(a^4-2\,a^2b^2+b^4)(b\sin(dx+c)+a)^{\frac{5}{2}}\right)}{45045\,b^5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/45045*(3465*(b*sin(d*x + c) + a)^(13/2) - 16380*(b*sin(d*x + c) + a)^(11/
2)*a + 10010*(3*a^2 - b^2)*(b*sin(d*x + c) + a)^(9/2) - 25740*(a^3 - a*b^2)
*(b*sin(d*x + c) + a)^(7/2) + 9009*(a^4 - 2*a^2*b^2 + b^4)*(b*sin(d*x + c)
+ a)^(5/2))/(b^5*d)
```

Fricas [A]
time = 0.39, size = 184, normalized size = 1.19

$$-\frac{2\,\left(3465\,b^{6}\cos\left(dx+c\right)^{6}-384\,a^{6}+2144\,a^{4}b^{2}-8256\,a^{2}b^{4}-2464\,b^{6}-35\,\left(3\,a^{2}b^{4}+11\,b^{6}\right)\cos\left(dx+c\right)^{4}+8\,\left(18\,a^{4}b^{2}-81\,a^{2}b^{4}-77\,b^{6}\right)\cos\left(dx+c\right)^{2}-2\,\left(2205\,ab^{5}\cos\left(dx+c\right)^{4}-96\,a^{5}b+512\,a^{3}b^{3}+4064\,ab^{5}+20\,\left(3\,a^{3}b^{3}+137\,ab^{5}\right)\cos\left(dx+c\right)^{2}\right)\sin\left(dx+c\right)\right)\sqrt{b\sin\left(dx+c\right)+a}}{45045\,b^{5}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/45045*(3465*b^6*cos(d*x + c)^6 - 384*a^6 + 2144*a^4*b^2 - 8256*a^2*b^4 -
 2464*b^6 - 35*(3*a^2*b^4 + 11*b^6)*cos(d*x + c)^4 + 8*(18*a^4*b^2 - 81*a^2
*b^4 - 77*b^6)*cos(d*x + c)^2 - 2*(2205*a*b^5*cos(d*x + c)^4 - 96*a^5*b + 5
12*a^3*b^3 + 4064*a*b^5 + 20*(3*a^3*b^3 + 137*a*b^5)*cos(d*x + c)^2)*sin(d*
x + c))*sqrt(b*sin(d*x + c) + a)/(b^5*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^5\,{\left(a+b\,\sin\left(c+d\,x\right)\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(a + b*sin(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^5*(a + b*sin(c + d*x))^(3/2), x)
```

3.484 $\int \cos^3(c+dx)(a+b\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=83

$$-\frac{2(a^2-b^2)(a+b\sin(c+dx))^{5/2}}{5b^3d}+\frac{4a(a+b\sin(c+dx))^{7/2}}{7b^3d}-\frac{2(a+b\sin(c+dx))^{9/2}}{9b^3d}$$

```
[Out] -2/5*(a^2-b^2)*(a+b*sin(d*x+c))^(5/2)/b^3/d+4/7*a*(a+b*sin(d*x+c))^(7/2)/b^
3/d-2/9*(a+b*sin(d*x+c))^(9/2)/b^3/d
```

Rubi [A]
time = 0.06, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2747, 711}

$$-\frac{2(a^2-b^2)(a+b\sin(c+dx))^{5/2}}{5b^3d}-\frac{2(a+b\sin(c+dx))^{9/2}}{9b^3d}+\frac{4a(a+b\sin(c+dx))^{7/2}}{7b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*(a^2 - b^2)*(a + b*Sin[c + d*x])^(5/2))/(5*b^3*d) + (4*a*(a + b*Sin[c +
 d*x])^(7/2))/(7*b^3*d) - (2*(a + b*Sin[c + d*x])^(9/2))/(9*b^3*d)
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^3(c+dx)(a+b\sin(c+dx))^{3/2}\,dx &= \frac{\text{Subst}\left(\int (a+x)^{3/2}\left(b^2-x^2\right)\,dx,x,b\sin(c+dx)\right)}{b^3d}\\ &= \frac{\text{Subst}\left(\int \left(\left(-a^2+b^2\right)(a+x)^{3/2}+2a(a+x)^{5/2}-(a+x)^{7/2}\right)\,dx,\right.}{b^3d}\\ &= -\frac{2(a^2-b^2)(a+b\sin(c+dx))^{5/2}}{5b^3d}+\frac{4a(a+b\sin(c+dx))^{7/2}}{7b^3d}-\end{aligned}$$

Mathematica [A]
time = 0.18, size = 58, normalized size = 0.70

$$\frac{(a+b\sin(c+dx))^{5/2}\left(-16a^2+91b^2+35b^2\cos(2(c+dx))+40ab\sin(c+dx)\right)}{315b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((a + b*Sin[c + d*x])^(5/2)*(-16*a^2 + 91*b^2 + 35*b^2*Cos[2*(c + d*x)] + 4
0*a*b*Sin[c + d*x]))/(315*b^3*d)
```

Maple [A]
time = 0.97, size = 55, normalized size = 0.66

method	result	size
default	$-\frac{2(a+b\sin(dx+c))^{\frac{5}{2}}\left(-35b^2\left(\cos^2(dx+c)\right)-20ab\sin(dx+c)+8a^2-28b^2\right)}{315b^3d}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/315/b^3*(a+b*sin(d*x+c))^(5/2)*(-35*b^2*cos(d*x+c)^2-20*a*b*sin(d*x+c)+8
*a^2-28*b^2)/d
```

Maxima [A]
time = 0.26, size = 61, normalized size = 0.73

$$-\frac{2\left(35\left(b\sin\left(dx+c\right)+a\right)^{\frac{9}{2}}-90\left(b\sin\left(dx+c\right)+a\right)^{\frac{7}{2}}a+63\left(a^2-b^2\right)\left(b\sin\left(dx+c\right)+a\right)^{\frac{5}{2}}\right)}{315\,b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -2/315*(35*(b*sin(d*x + c) + a)^(9/2) - 90*(b*sin(d*x + c) + a)^(7/2)*a + 6
3*(a^2 - b^2)*(b*sin(d*x + c) + a)^(5/2))/(b^3*d)
```

Fricas [A]
time = 0.34, size = 111, normalized size = 1.34

$$-\frac{2\left(35\,b^4\cos\left(dx+c\right)^4+8\,a^4-60\,a^2b^2-28\,b^4-\left(3\,a^2b^2+7\,b^4\right)\cos\left(dx+c\right)^2-2\left(25\,ab^3\cos\left(dx+c\right)^2+2\,a^3b+38\,ab^3\right)\sin\left(dx+c\right)\right)\sqrt{b\sin\left(dx+c\right)+a}}{315\,b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/315*(35*b^4*cos(d*x + c)^4 + 8*a^4 - 60*a^2*b^2 - 28*b^4 - (3*a^2*b^2 +
7*b^4)*cos(d*x + c)^2 - 2*(25*a*b^3*cos(d*x + c)^2 + 2*a^3*b + 38*a*b^3)*si
n(d*x + c))*sqrt(b*sin(d*x + c) + a)/(b^3*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 314 vs. $2(76) = 152$.
time = 5.62, size = 314, normalized size = 3.78

$$\begin{cases} a^{\frac{3}{2}} x \cos^{3}{\left(c \right)} & \text{for}\: b = 0 \wedge d = 0 \\ a^{\frac{3}{2}} \cdot \left(\frac{2 \sin^{3}{\left(c + d x \right)}}{3 d} + \frac{\sin{\left(c + d x \right)} \cos^{2}{\left(c + d x \right)}}{d}\right) & \text{for}\: b = 0 \\ x \left(a + b \sin{\left(c \right)}\right)^{\frac{3}{2}} \cos^{3}{\left(c \right)} & \text{for}\: d = 0 \\ - \frac{16 a^{4} \sqrt{a + b \sin{\left(c + d x \right)}}}{315 b^{3} d} + \frac{8 a^{3} \sqrt{a + b \sin{\left(c + d x \right)}} \sin{\left(c + d x \right)}}{315 b^{2} d} + \frac{8 a^{2} \sqrt{a + b \sin{\left(c + d x \right)}} \sin^{2}{\left(c + d x \right)}}{21 b d} + \frac{2 a^{2} \sqrt{a + b \sin{\left(c + d x \right)}} \cos^{2}{\left(c + d x \right)}}{5 b d} + \frac{152 a \sqrt{a + b \sin{\left(c + d x \right)}} \sin^{3}{\left(c + d x \right)}}{315 d} + \frac{4 a \sqrt{a + b \sin{\left(c + d x \right)}} \sin{\left(c + d x \right)} \cos^{2}{\left(c + d x \right)}}{5 d} + \frac{8 b \sqrt{a + b \sin{\left(c + d x \right)}} \sin^{4}{\left(c + d x \right)}}{45 d} + \frac{2 b \sqrt{a + b \sin{\left(c + d x \right)}} \sin^{2}{\left(c + d x \right)} \cos^{2}{\left(c + d x \right)}}{5 d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Piecewise((a**(3/2)*x*cos(c)**3, Eq(b, 0) & Eq(d, 0)), (a**(3/2)*(2*sin(c +
 d*x)**3/(3*d) + sin(c + d*x)*cos(c + d*x)**2/d), Eq(b, 0)), (x*(a + b*sin(
c))**(3/2)*cos(c)**3, Eq(d, 0)), (-16*a**4*sqrt(a + b*sin(c + d*x))/(315*b*
*3*d) + 8*a**3*sqrt(a + b*sin(c + d*x))*sin(c + d*x)/(315*b**2*d) + 8*a**2*
sqrt(a + b*sin(c + d*x))*sin(c + d*x)**2/(21*b*d) + 2*a**2*sqrt(a + b*sin(c
 + d*x))*cos(c + d*x)**2/(5*b*d) + 152*a*sqrt(a + b*sin(c + d*x))*sin(c + d
*x)**3/(315*d) + 4*a*sqrt(a + b*sin(c + d*x))*sin(c + d*x)*cos(c + d*x)**2/
(5*d) + 8*b*sqrt(a + b*sin(c + d*x))*sin(c + d*x)**4/(45*d) + 2*b*sqrt(a +
b*sin(c + d*x))*sin(c + d*x)**2*cos(c + d*x)**2/(5*d), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^3\,{\left(a+b\,\sin\left(c+d\,x\right)\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a + b*sin(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^3*(a + b*sin(c + d*x))^(3/2), x)
```

3.485 $\int \cos(c+dx)(a+b\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=24

$$\frac{2(a+b\sin(c+dx))^{5/2}}{5bd}$$

```
[Out] 2/5*(a+b*sin(d*x+c))^(5/2)/b/d
```

Rubi [A]
time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2747, 32}

$$\frac{2(a+b\sin(c+dx))^{5/2}}{5bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (2*(a + b*Sin[c + d*x])^(5/2))/(5*b*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a+b\sin(c+dx))^{3/2}\,dx &= \frac{\text{Subst}\left(\int (a+x)^{3/2}\,dx, x, b\sin(c+dx)\right)}{bd}\\ &= \frac{2(a+b\sin(c+dx))^{5/2}}{5bd}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 24, normalized size = 1.00

$$\frac{2(a+b\sin(c+dx))^{5/2}}{5bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (2*(a + b*Sin[c + d*x])^(5/2))/(5*b*d)
```

Maple [A]
time = 0.05, size = 21, normalized size = 0.88

method	result	size
derivativedivides	$\frac{2(a+b\sin(dx+c))^{\frac{5}{2}}}{5bd}$	21
default	$\frac{2(a+b\sin(dx+c))^{\frac{5}{2}}}{5bd}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*(a+b*sin(d*x+c))^(5/2)/b/d
```

Maxima [A]
time = 0.27, size = 20, normalized size = 0.83

$$\frac{2\,(b\sin{(dx+c)}+a)^{\frac{5}{2}}}{5\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/5*(b*sin(d*x + c) + a)^(5/2)/(b*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(20) = 40$.
time = 0.34, size = 53, normalized size = 2.21

$$-\frac{2\,\left(b^2\cos{(dx+c)}^2-2\,ab\sin{(dx+c)}-a^2-b^2\right)\sqrt{b\sin{(dx+c)}+a}}{5\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/5*(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)*sqrt(b*sin(d*x +
 c) + a)/(b*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 116 vs. $2(19) = 38$.

time = 1.65, size = 116, normalized size = 4.83

$$\begin{cases} a^{\frac{3}{2}} x \cos(c) & \text{for } b = 0 \wedge d = 0 \\ \frac{a^{\frac{3}{2}} \sin(c+dx)}{d} & \text{for } b = 0 \\ x(a + b \sin(c))^{\frac{3}{2}} \cos(c) & \text{for } d = 0 \\ \frac{2a^2 \sqrt{a + b \sin(c + dx)}}{5bd} + \frac{4a \sqrt{a + b \sin(c + dx)} \sin(c+dx)}{5d} + \frac{2b \sqrt{a + b \sin(c + dx)} \sin^2(c+dx)}{5d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Piecewise((a**(3/2)*x*cos(c), Eq(b, 0) & Eq(d, 0)), (a**(3/2)*sin(c + d*x)/
d, Eq(b, 0)), (x*(a + b*sin(c))**(3/2)*cos(c), Eq(d, 0)), (2*a**2*sqrt(a +
b*sin(c + d*x))/(5*b*d) + 4*a*sqrt(a + b*sin(c + d*x))*sin(c + d*x)/(5*d) +
 2*b*sqrt(a + b*sin(c + d*x))*sin(c + d*x)**2/(5*d), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(3/2)*cos(d*x + c), x)
```

Mupad [B]
time = 5.41, size = 20, normalized size = 0.83

$$\frac{2\,{\left(a+b\,\sin\left(c+d\,x\right)\right)}^{5/2}}{5\,b\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + b*sin(c + d*x))^(3/2),x)
```

```
[Out] (2*(a + b*sin(c + d*x))^(5/2))/(5*b*d)
```

3.486 $\int \sec(c+dx)(a+b\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=94

$$-\frac{(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{d}+\frac{(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{d}-\frac{2b\sqrt{a+b\sin(c+}}{d}$$

```
[Out] -(a-b)^(3/2)*arctanh((a+b*sin(d*x+c))^(1/2)/(a-b)^(1/2))/d+(a+b)^(3/2)*arct
anh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))/d-2*b*(a+b*sin(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.11, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2747, 718, 841, 1180, 212}

$$-\frac{2b\sqrt{a+b\sin(c+dx)}}{d}-\frac{(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{d}+\frac{(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] -(((a - b)^(3/2)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a - b]])/d) + ((a +
b)^(3/2)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a + b]])/d - (2*b*Sqrt[a + b
*Sin[c + d*x]])/d
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 718

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Simp[e*((d
 + e*x)^(m - 1)/(c*(m - 1))), x] + Dist[1/c, Int[(d + e*x)^(m - 2)*(Simp[c*
d^2 - a*e^2 + 2*c*d*e*x, x]/(a + c*x^2)), x], x] /; FreeQ[{a, c, d, e}, x]
&& NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 1]
```

Rule 841

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec(c+dx)(a+b\sin(c+dx))^{3/2}\,dx = \frac{b\text{Subst}\left(\int \frac{(a+x)^{3/2}}{b^2-x^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= -\frac{2b\sqrt{a+b\sin(c+dx)}}{d} - \frac{b\text{Subst}\left(\int \frac{-a^2-b^2-2ax}{\sqrt{a+x}\,(b^2-x^2)}\,dx, x, b\sin(c-$$

$$= -\frac{2b\sqrt{a+b\sin(c+dx)}}{d} - \frac{(2b)\text{Subst}\left(\int \frac{a^2-b^2-2ax^2}{-a^2+b^2+2ax^2-x^4}\,dx, x, \sqrt{a+}$$

$$= -\frac{2b\sqrt{a+b\sin(c+dx)}}{d} - \frac{(a-b)^2\text{Subst}\left(\int \frac{1}{a-b-x^2}\,dx, x, \sqrt{a+b\,\text{s}}$$

$$= -\frac{(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{d} + \frac{(a+b)^{3/2}\tanh^{-1}\left(\right.}{}$$

Mathematica [A]
time = 0.10, size = 89, normalized size = 0.95

$$\frac{-(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)+(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)-2b\sqrt{a+b\sin(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sin[c + d*x])^(3/2),x]
[Out] (-((a - b)^(3/2)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a - b]]) + (a + b)^(
3/2)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a + b]] - 2*b*Sqrt[a + b*Sin[c +
 d*x]])/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 200 vs. $2(80) = 160$.

time = 1.27, size = 201, normalized size = 2.14

method	result
default	$\frac{-2b\sqrt{a+b\sin(dx+c)}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sin(dx+c)}}{\sqrt{a+b}}\right)a^2}{\sqrt{a+b}}+\frac{2b\operatorname{arctanh}\left(\frac{\sqrt{a+b\sin(dx+c)}}{\sqrt{a+b}}\right)a}{\sqrt{a+b}}+\frac{b^2\operatorname{arctanh}}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-2*b*(a+b*sin(d*x+c))^(1/2)+1/(a+b)^(1/2)*arctanh((a+b*sin(d*x+c))^(1/2)/(
a+b)^(1/2))*a^2+2*b/(a+b)^(1/2)*arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))
*a+b^2/(a+b)^(1/2)*arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))+1/(-a+b)^(1/
2)*arctan((a+b*sin(d*x+c))^(1/2)/(-a+b)^(1/2))*a^2-2*b/(-a+b)^(1/2)*arctan(
(a+b*sin(d*x+c))^(1/2)/(-a+b)^(1/2))*a+b^2/(-a+b)^(1/2)*arctan((a+b*sin(d*x
+c))^(1/2)/(-a+b)^(1/2)))/d
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for more
 detail
```

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c)*sin(d*x + c) + a*sec(d*x + c))*sqrt(b*sin(d*x + c)
 + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sin (c + dx))^{\frac{3}{2}} \sec (c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))**(3/2),x)
[Out] Integral((a + b*sin(c + d*x))**(3/2)*sec(c + d*x), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))^(3/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\, \sin (c + d\, x))^{3/2}}{\cos (c + d\, x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^(3/2)/cos(c + d*x),x)
[Out] int((a + b*sin(c + d*x))^(3/2)/cos(c + d*x), x)
```

3.487 $\int \sec^3(c+dx)(a+b\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=130

$$-\frac{\sqrt{a-b}\,(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{4d}+\frac{(2a-b)\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{4d}+\frac{\sec^2}{}$$

```
[Out] -1/4*(2*a+b)*arctanh((a+b*sin(d*x+c))^(1/2)/(a-b)^(1/2))*(a-b)^(1/2)/d+1/4*
(2*a-b)*arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))*(a+b)^(1/2)/d+1/2*sec(d
*x+c)^2*(b+a*sin(d*x+c))*(a+b*sin(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.17, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2747, 753, 841, 1180, 212}

$$-\frac{\sqrt{a-b}\,(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{4d}+\frac{(2a-b)\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{4d}+\frac{\sec^2(c+dx)(a\sin(c+dx)+b)\sqrt{a+b\sin(c+dx)}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] -1/4*(Sqrt[a - b]*(2*a + b)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a - b]])/
d + ((2*a - b)*Sqrt[a + b]*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a + b]])/(
4*d) + (Sec[c + d*x]^2*(b + a*Sin[c + d*x])*Sqrt[a + b*Sin[c + d*x]])/(2*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 753

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m - 1)*(a*e - c*d*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 841

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
```

```
eQ[c*d^2 + a*e^2, 0]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^3(c+dx)(a+b\sin(c+dx))^{3/2}\,dx = \frac{b^3\text{Subst}\left(\int \frac{(a+x)^{3/2}}{(b^2-x^2)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^2(c+dx)(b+a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{2d} - \frac{b\text{Subst}\left(\int \cdots\right.}{}$$

$$= \frac{\sec^2(c+dx)(b+a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{2d} - \frac{b\text{Subst}\left(\int \cdots\right.}{}$$

$$= \frac{\sec^2(c+dx)(b+a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{2d} + \frac{((2a-b)(a \cdots}{}$$

$$= -\frac{\sqrt{a-b}\,(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{4d} + \frac{(2a-b)\sqrt{a \cdots}}{}$$

Mathematica [A]
time = 0.71, size = 121, normalized size = 0.93

$$\frac{-\sqrt{a-b}\,(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)+(2a-b)\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)+2\sec^2(c+dx)(b+a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-(Sqrt[a - b]*(2*a + b)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a - b]]) + (
2*a - b)*Sqrt[a + b]*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a + b]] + 2*Sec[
c + d*x]^2*(b + a*Sin[c + d*x])*Sqrt[a + b*Sin[c + d*x]])/(4*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 278 vs. $2(110) = 220$.
time = 1.94, size = 279, normalized size = 2.15

method	result
default	$\frac{2a\sqrt{a+b\sin(dx+c)}\,\sqrt{a+b}\,\sqrt{-a+b}\,\sin(dx+c)-\left(-2\arctan\left(\frac{\sqrt{a+b\sin(dx+c)}}{\sqrt{-a+b}}\right)a^2\sqrt{a+b}+ba\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+b*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(2*a*(a+b*sin(d*x+c))^(1/2)*(a+b)^(1/2)*(-a+b)^(1/2)*sin(d*x+c)-(-2*arc
tan((a+b*sin(d*x+c))^(1/2)/(-a+b)^(1/2))*a^2*(a+b)^(1/2)+b*arctan((a+b*sin(
d*x+c))^(1/2)/(-a+b)^(1/2))*a*(a+b)^(1/2)+b^2*arctan((a+b*sin(d*x+c))^(1/2)
/(-a+b)^(1/2))*(a+b)^(1/2)-2*arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))*a^
2*(-a+b)^(1/2)-b*arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))*a*(-a+b)^(1/2)
+b^2*arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))*(-a+b)^(1/2))*cos(d*x+c)^2
+2*b*(a+b*sin(d*x+c))^(1/2)*(a+b)^(1/2)*(-a+b)^(1/2))/(a+b)^(1/2)/(-a+b)^(1
/2)/cos(d*x+c)^2/d
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 354 vs. $2(110) = 220$.
time = 0.62, size = 1969, normalized size = 15.15

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")

[Out] [-1/32*((2*a - b)*sqrt(a + b)*cos(d*x + c)^2*log((b^4*cos(d*x + c)^4 + 128*
a^4 + 256*a^3*b + 320*a^2*b^2 + 256*a*b^3 + 72*b^4 - 8*(20*a^2*b^2 + 28*a*b
^3 + 9*b^4)*cos(d*x + c)^2 - 8*(16*a^3 + 24*a^2*b + 20*a*b^2 + 8*b^3 - (10*
a*b^2 + 7*b^3)*cos(d*x + c)^2 - (b^3*cos(d*x + c)^2 - 24*a^2*b - 28*a*b^2 -
 8*b^3)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)*sqrt(a + b) + 4*(64*a^3*b +
112*a^2*b^2 + 64*a*b^3 + 14*b^4 - (8*a*b^3 + 7*b^4)*cos(d*x + c)^2)*sin(d*x
 + c))/(cos(d*x + c)^4 - 8*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 - 2)*sin(d*x
+ c) + 8)) - (2*a + b)*sqrt(a - b)*cos(d*x + c)^2*log((b^4*cos(d*x + c)^4 +
 128*a^4 - 256*a^3*b + 320*a^2*b^2 - 256*a*b^3 + 72*b^4 - 8*(20*a^2*b^2 - 2
8*a*b^3 + 9*b^4)*cos(d*x + c)^2 - 8*(16*a^3 - 24*a^2*b + 20*a*b^2 - 8*b^3 -
 (10*a*b^2 - 7*b^3)*cos(d*x + c)^2 - (b^3*cos(d*x + c)^2 - 24*a^2*b + 28*a*
b^2 - 8*b^3)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)*sqrt(a - b) + 4*(64*a^3
*b - 112*a^2*b^2 + 64*a*b^3 - 14*b^4 - (8*a*b^3 - 7*b^4)*cos(d*x + c)^2)*si
n(d*x + c))/(cos(d*x + c)^4 - 8*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2)*sin
(d*x + c) + 8)) - 16*(a*sin(d*x + c) + b)*sqrt(b*sin(d*x + c) + a))/(d*cos(
d*x + c)^2), -1/32*(2*(2*a - b)*sqrt(-a - b)*arctan(-1/4*(b^2*cos(d*x + c)^
2 - 8*a^2 - 8*a*b - 2*b^2 - 2*(4*a*b + 3*b^2)*sin(d*x + c))*sqrt(b*sin(d*x
+ c) + a)*sqrt(-a - b)/(2*a^3 + 3*a^2*b + 2*a*b^2 + b^3 - (a*b^2 + b^3)*cos
(d*x + c)^2 + (3*a^2*b + 4*a*b^2 + b^3)*sin(d*x + c)))*cos(d*x + c)^2 - (2*
a + b)*sqrt(a - b)*cos(d*x + c)^2*log((b^4*cos(d*x + c)^4 + 128*a^4 - 256*a
^3*b + 320*a^2*b^2 - 256*a*b^3 + 72*b^4 - 8*(20*a^2*b^2 - 28*a*b^3 + 9*b^4)
*cos(d*x + c)^2 - 8*(16*a^3 - 24*a^2*b + 20*a*b^2 - 8*b^3 - (10*a*b^2 - 7*b
^3)*cos(d*x + c)^2 - (b^3*cos(d*x + c)^2 - 24*a^2*b + 28*a*b^2 - 8*b^3)*sin
(d*x + c))*sqrt(b*sin(d*x + c) + a)*sqrt(a - b) + 4*(64*a^3*b - 112*a^2*b^2
 + 64*a*b^3 - 14*b^4 - (8*a*b^3 - 7*b^4)*cos(d*x + c)^2)*sin(d*x + c))/(cos
(d*x + c)^4 - 8*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8))
- 16*(a*sin(d*x + c) + b)*sqrt(b*sin(d*x + c) + a))/(d*cos(d*x + c)^2), -1/
32*(2*(2*a + b)*sqrt(-a + b)*arctan(1/4*(b^2*cos(d*x + c)^2 - 8*a^2 + 8*a*b
 - 2*b^2 - 2*(4*a*b - 3*b^2)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)*sqrt(-a
 + b)/(2*a^3 - 3*a^2*b + 2*a*b^2 - b^3 - (a*b^2 - b^3)*cos(d*x + c)^2 + (3*
a^2*b - 4*a*b^2 + b^3)*sin(d*x + c)))*cos(d*x + c)^2 + (2*a - b)*sqrt(a + b
)*cos(d*x + c)^2*log((b^4*cos(d*x + c)^4 + 128*a^4 + 256*a^3*b + 320*a^2*b^
2 + 256*a*b^3 + 72*b^4 - 8*(20*a^2*b^2 + 28*a*b^3 + 9*b^4)*cos(d*x + c)^2 -
 8*(16*a^3 + 24*a^2*b + 20*a*b^2 + 8*b^3 - (10*a*b^2 + 7*b^3)*cos(d*x + c)^
2 - (b^3*cos(d*x + c)^2 - 24*a^2*b - 28*a*b^2 - 8*b^3)*sin(d*x + c))*sqrt(b
*sin(d*x + c) + a)*sqrt(a + b) + 4*(64*a^3*b + 112*a^2*b^2 + 64*a*b^3 + 14*
b^4 - (8*a*b^3 + 7*b^4)*cos(d*x + c)^2)*sin(d*x + c))/(cos(d*x + c)^4 - 8*c
os(d*x + c)^2 + 4*(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8)) - 16*(a*sin(d*x +
 c) + b)*sqrt(b*sin(d*x + c) + a))/(d*cos(d*x + c)^2), -1/16*((2*a + b)*sqr
t(-a + b)*arctan(1/4*(b^2*cos(d*x + c)^2 - 8*a^2 + 8*a*b - 2*b^2 - 2*(4*a*b
 - 3*b^2)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)*sqrt(-a + b)/(2*a^3 - 3*a^
2*b + 2*a*b^2 - b^3 - (a*b^2 - b^3)*cos(d*x + c)^2 + (3*a^2*b - 4*a*b^2 + b
^3)*sin(d*x + c)))*cos(d*x + c)^2 + (2*a - b)*sqrt(-a - b)*arctan(-1/4*(b^2
```

```
*cos(d*x + c)^2 - 8*a^2 - 8*a*b - 2*b^2 - 2*(4*a*b + 3*b^2)*sin(d*x + c))*s
qrt(b*sin(d*x + c) + a)*sqrt(-a - b)/(2*a^3 + 3*a^2*b + 2*a*b^2 + b^3 - (a*
b^2 + b^3)*cos(d*x + c)^2 + (3*a^2*b + 4*a*b^2 + b^3)*sin(d*x + c)))*cos(d*
x + c)^2 - 8*(a*sin(d*x + c) + b)*sqrt(b*sin(d*x + c) + a))/(d*cos(d*x + c)
^2)]
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+b*sin(d*x+c))**(3/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 5006 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sin(d*x+c))^(3/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,\sin\left(c+d\,x\right)\right)^{3/2}}{{\cos\left(c+d\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^(3/2)/cos(c + d*x)^3,x)
[Out] int((a + b*sin(c + d*x))^(3/2)/cos(c + d*x)^3, x)
```

3.488 $\int \sec^5(c+dx)(a+b\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=188

$$-\frac{3(4a^2-2ab-b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{32\sqrt{a-b}\,d}+\frac{3(4a^2+2ab-b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{32\sqrt{a+b}\,d}-\frac{\sec$$

```
[Out] -3/32*(4*a^2-2*a*b-b^2)*arctanh((a+b*sin(d*x+c))^(1/2)/(a-b)^(1/2))/d/(a-b)
^(1/2)+3/32*(4*a^2+2*a*b-b^2)*arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))/d
/(a+b)^(1/2)-1/16*sec(d*x+c)^2*(b-6*a*sin(d*x+c))*(a+b*sin(d*x+c))^(1/2)/d+
1/4*sec(d*x+c)^4*(b+a*sin(d*x+c))*(a+b*sin(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.20, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2747, 753, 837, 841, 1180, 212}

$$-\frac{3(4a^2-2ab-b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{32d\sqrt{a-b}}+\frac{3(4a^2+2ab-b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{32d\sqrt{a+b}}+\frac{\sec^4(c+dx)(a\sin(c+dx)+b)\sqrt{a+b\sin(c+dx)}}{4d}-\frac{\sec^2(c+dx)(b-6a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-3*(4*a^2 - 2*a*b - b^2)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a - b]])/(3
2*Sqrt[a - b]*d) + (3*(4*a^2 + 2*a*b - b^2)*ArcTanh[Sqrt[a + b*Sin[c + d*x]
]/Sqrt[a + b]])/(32*Sqrt[a + b]*d) - (Sec[c + d*x]^2*(b - 6*a*Sin[c + d*x])
*Sqrt[a + b*Sin[c + d*x]])/(16*d) + (Sec[c + d*x]^4*(b + a*Sin[c + d*x])*Sq
rt[a + b*Sin[c + d*x]])/(4*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 753

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m - 1)*(a*e - c*d*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 841

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^5(c+dx)(a+b\sin(c+dx))^{3/2}\,dx = \frac{b^5\text{Subst}\left(\int \frac{(a+x)^{3/2}}{(b^2-x^2)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^4(c+dx)(b+a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{4d} - \frac{b^3\text{Subst}\Big(\int$$

$$= -\frac{\sec^2(c+dx)(b-6a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{16d} + \frac{\sec^4(c+$$

$$= -\frac{\sec^2(c+dx)(b-6a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{16d} + \frac{\sec^4(c+$$

$$= -\frac{\sec^2(c+dx)(b-6a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{16d} + \frac{\sec^4(c+$$

$$= -\frac{3(4a^2-2ab-b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{32\sqrt{a-b}\,d} + \frac{3(4a^2+2al$$

Mathematica [A]
time = 2.49, size = 297, normalized size = 1.58

$$-\frac{3\sqrt{a-b}\,(a+b)^2(4a^3-6a^2b+ab^2+b^3)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)-3(a-b)^2\sqrt{a+b}\,(4a^3+6a^2b+ab^2-b^3)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)+8(-a^2+b^2)\sec^4(c+dx)(-b+a\sin(c+dx))(a+b\sin(c+dx))^{5/2}+2\sec^2(c+dx)(a+b\sin(c+dx))^{5/2}(5a^2b-3b^3+(-6a^3+4ab^2)\sin(c+dx))-2b\sqrt{a+b\sin(c+dx)}\,(12a^4-13a^2b^2+3b^4+(6a^3b-4ab^3)\sin(c+dx))}{32(a^2-b^2)^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] -1/32*(3*Sqrt[a - b]*(a + b)^2*(4*a^3 - 6*a^2*b + a*b^2 + b^3)*ArcTanh[Sqrt
[a + b*Sin[c + d*x]]/Sqrt[a - b]] - 3*(a - b)^2*Sqrt[a + b]*(4*a^3 + 6*a^2*
b + a*b^2 - b^3)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a + b]] + 8*(-a^2 +
b^2)*Sec[c + d*x]^4*(-b + a*Sin[c + d*x])*(a + b*Sin[c + d*x])^(5/2) + 2*Se
c[c + d*x]^2*(a + b*Sin[c + d*x])^(5/2)*(5*a^2*b - 3*b^3 + (-6*a^3 + 4*a*b^
2)*Sin[c + d*x]) - 2*b*Sqrt[a + b*Sin[c + d*x]]*(12*a^4 - 13*a^2*b^2 + 3*b^
4 + (6*a^3*b - 4*a*b^3)*Sin[c + d*x]))/((a^2 - b^2)^2*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 408 vs. $2(164) = 328$.
time = 2.05, size = 409, normalized size = 2.18

method	result

default	$\frac{4\sqrt{a+b}\,\sqrt{-a+b}\,\sqrt{a+b\sin(dx+c)}\,b(b(\cos^2(dx+c))+8a\sin(dx+c)-b)+3b\left(4\arctan\left(\frac{\sqrt{a+b\sin(dx+}}{\sqrt{-a+b}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(a+b*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/32*(4*(a+b)^(1/2)*(-a+b)^(1/2)*(a+b*sin(d*x+c))^(1/2)*b*(b*cos(d*x+c)^2+8
*a*sin(d*x+c)-b)+3*b*(4*arctan((a+b*sin(d*x+c))^(1/2)/(-a+b)^(1/2))*a^2*(a+
b)^(1/2)-2*b*arctan((a+b*sin(d*x+c))^(1/2)/(-a+b)^(1/2))*a*(a+b)^(1/2)-b^2*
arctan((a+b*sin(d*x+c))^(1/2)/(-a+b)^(1/2))*(a+b)^(1/2)+4*arctanh((a+b*sin(
d*x+c))^(1/2)/(a+b)^(1/2))*a^2*(-a+b)^(1/2)+2*b*arctanh((a+b*sin(d*x+c))^(1
/2)/(a+b)^(1/2))*a*(-a+b)^(1/2)-b^2*arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1
/2))*(-a+b)^(1/2))*cos(d*x+c)^4+6*(a+b)^(1/2)*(-a+b)^(1/2)*(a+b*sin(d*x+c))
^(1/2)*b*(2*a*sin(d*x+c)-b)*cos(d*x+c)^2-24*(a+b*sin(d*x+c))^(3/2)*a*(a+b)^
(1/2)*(-a+b)^(1/2)+24*(a+b*sin(d*x+c))^(1/2)*a^2*(a+b)^(1/2)*(-a+b)^(1/2)+1
2*(a+b*sin(d*x+c))^(1/2)*b^2*(a+b)^(1/2)*(-a+b)^(1/2))/(a+b)^(1/2)/(-a+b)^(
1/2)/b/cos(d*x+c)^4/d
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for more
 detail
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c)^5*sin(d*x + c) + a*sec(d*x + c)^5)*sqrt(b*sin(d*x
+ c) + a), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\sin(c+d\,x))^{3/2}}{\cos(c+d\,x)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^(3/2)/cos(c + d*x)^5,x)
```

```
[Out] int((a + b*sin(c + d*x))^(3/2)/cos(c + d*x)^5, x)
```

3.489 $\int \cos^4(c+dx)(a+b\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=329

$$-\frac{2b\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{11d}-\frac{32a(a^4-6a^2b^2-27b^4)\,E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)|\frac{2b}{a+b}\right)\sqrt{a+b\sin(c+dx)}}{1155b^4d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}$$

```
[Out] -2/11*b*cos(d*x+c)^5*(a+b*sin(d*x+c))^(1/2)/d+2/231*cos(d*x+c)^3*(a^2+3*b^2
+28*a*b*sin(d*x+c))*(a+b*sin(d*x+c))^(1/2)/b/d-4/1155*cos(d*x+c)*(4*a^4-21*
a^2*b^2-15*b^4-3*a*b*(a^2+31*b^2)*sin(d*x+c))*(a+b*sin(d*x+c))^(1/2)/b^3/d+
32/1155*a*(a^4-6*a^2*b^2-27*b^4)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/
2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(
1/2))*(a+b*sin(d*x+c))^(1/2)/b^4/d/((a+b*sin(d*x+c))/(a+b))^(1/2)-8/1155*(4
*a^6-25*a^4*b^2+6*a^2*b^4+15*b^6)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1
/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^
(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/b^4/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A]
time = 0.44, antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {2771, 2944, 2831, 2742, 2740, 2734, 2732}

$$\frac{2\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}\,(a^2+28ab\sin(c+dx)+3b^2)}{231bd}-\frac{32a(a^4-6a^2b^2-27b^4)\sqrt{a+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{1155b^4d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}-\frac{4\cos(c+dx)\sqrt{a+b\sin(c+dx)}\,(4a^4-3ab(a^2+31b^2)\sin(c+dx)-21a^2b^2-15b^4)}{1155b^3d}+\frac{8(4a^6-25a^4b^2+6a^2b^4+15b^6)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{1155b^4d\sqrt{a+b\sin(c+dx)}}-\frac{2b\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{11d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*b*Cos[c + d*x]^5*Sqrt[a + b*Sin[c + d*x]])/(11*d) - (32*a*(a^4 - 6*a^2*
b^2 - 27*b^4)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c
 + d*x]])/(1155*b^4*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + (8*(4*a^6 - 25*
a^4*b^2 + 6*a^2*b^4 + 15*b^6)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*
Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(1155*b^4*d*Sqrt[a + b*Sin[c + d*x]]) +
 (2*Cos[c + d*x]^3*Sqrt[a + b*Sin[c + d*x]]*(a^2 + 3*b^2 + 28*a*b*Sin[c + d
*x]))/(231*b*d) - (4*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]]*(4*a^4 - 21*a^2*
b^2 - 15*b^4 - 3*a*b*(a^2 + 31*b^2)*Sin[c + d*x]))/(1155*b^3*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2771

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(
a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2944

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Dist[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \cos^4(c+dx)(a+b\sin(c+dx))^{3/2}\,dx = -\frac{2b\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{11d} + \frac{2}{11}\int \frac{\cos^4(c+dx)\left(\frac{11a^2}{2}\right.}{\sqrt{a+}}$$

$$= -\frac{2b\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{11d} + \frac{2\cos^3(c+dx)\sqrt{a+b\sin}}{}$$

$$= -\frac{2b\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{11d} + \frac{2\cos^3(c+dx)\sqrt{a+b\sin}}{}$$

$$= -\frac{2b\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{11d} + \frac{2\cos^3(c+dx)\sqrt{a+b\sin}}{}$$

$$= -\frac{2b\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{11d} + \frac{2\cos^3(c+dx)\sqrt{a+b\sin}}{}$$

$$= -\frac{2b\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{11d} - \frac{32a(a^4-6a^2b^2-27b^4)}{1155}$$

Mathematica [A]
time = 1.09, size = 278, normalized size = 0.84

$$\frac{64\left(b^2(a^4-114a^2b^2-15b^4)F\left(\frac{1}{4}(-2c+\pi-2dx)\middle|\frac{2b}{a+b}\right)+4(a^5-6a^3b^2-27ab^4)\left((a+b)E\left(\frac{1}{4}(-2c+\pi-2dx)\middle|\frac{2b}{a+b}\right)-aF\left(\frac{1}{4}(-2c+\pi-2dx)\middle|\frac{2b}{a+b}\right)\right)\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}-b(a+b\sin(c+dx))\left(2(64a^4-366a^2b^2+195b^4)\cos(c+dx)+5b^2(-4a^2+93b^2)\cos(3(c+dx))+105b^4\cos(5(c+dx))-16ab(3a^2+128b^2)\sin(2(c+dx))-280ab^3\sin(4(c+dx))\right)}{9240b^4d\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (64*(b^2*(a^4 - 114*a^2*b^2 - 15*b^4)*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b
)/(a + b)] + 4*(a^5 - 6*a^3*b^2 - 27*a*b^4)*((a + b)*EllipticE[(-2*c + Pi -
 2*d*x)/4, (2*b)/(a + b)] - a*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b
)]))*Sqrt[(a + b*Sin[c + d*x])/(a + b)] - b*(a + b*Sin[c + d*x])*(2*(64*a^4
 - 366*a^2*b^2 + 195*b^4)*Cos[c + d*x] + 5*b^2*(-4*a^2 + 93*b^2)*Cos[3*(c +
 d*x)] + 105*b^4*Cos[5*(c + d*x)] - 16*a*b*(3*a^2 + 128*b^2)*Sin[2*(c + d*x
)] - 280*a*b^3*Sin[4*(c + d*x)]))/(9240*b^4*d*Sqrt[a + b*Sin[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1354 vs. $2(371) = 742$.
time = 2.59, size = 1355, normalized size = 4.12

method	result	size
default	Expression too large to display	1355

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+b*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/1155*(-100*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2
)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),
((a-b)/(a+b))^(1/2))*a^4*b^3-360*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+
c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*
x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^4+24*((a+b*sin(d*x+c))/(a-b))
^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Elli
pticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^5+60*a*b^6-
8*a^5*b^2*sin(d*x+c)^2+46*a^3*b^4*sin(d*x+c)^2-581*a*b^6*sin(d*x+c)^2+2*a^4
*b^3*sin(d*x+c)-373*a^2*b^5*sin(d*x+c)-245*a*b^6*sin(d*x+c)^6-145*a^2*b^5*s
in(d*x+c)^5+a^3*b^4*sin(d*x+c)^4+766*a*b^6*sin(d*x+c)^4-2*a^4*b^3*sin(d*x+c
)^3+518*a^2*b^5*sin(d*x+c)^3-12*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c
)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x
+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*b^2-105*b^7*sin(d*x+c)^7+300*b^7
*sin(d*x+c)^5-255*b^7*sin(d*x+c)^3+60*b^7*sin(d*x+c)+60*((a+b*sin(d*x+c))/(
a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)
*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b^7-16*((a+b
*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*
b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2)
)*a^7+8*a^5*b^2+372*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b)
)^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^
(1/2),((a-b)/(a+b))^(1/2))*a*b^6+112*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(
d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*si
n(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*b^2+16*((a+b*sin(d*x+c))/(a
-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*
EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^6*b+336*((a
+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c)
)*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/
2))*a^3*b^4-47*a^3*b^4-432*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*
b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/
(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^6)/b^5/cos(d*x+c)/(a+b*sin(d*x+c))^(1
/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^4, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.15, size = 583, normalized size = 1.77

$$\frac{2\left(2\sqrt{2}\left(8a^6-51a^4b^2+126a^2b^4+45b^6\right)\sqrt{ib}\,\mathrm{weierstrassPInverse}\left(-\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(8ia^3-9iab^2)}{27b^3},\frac{3b\cos(dx+c)-3ib\sin(dx+c)-2ia}{3b}\right)+2\sqrt{2}\left(8a^6-51a^4b^2+126a^2b^4+45b^6\right)\sqrt{-ib}\,\mathrm{weierstrassPInverse}\left(-\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(-8ia^3+9iab^2)}{27b^3},\frac{3b\cos(dx+c)+3ib\sin(dx+c)+2ia}{3b}\right)-24\sqrt{2}\left(-ia^5b+6ia^3b^3+27iab^5\right)\sqrt{ib}\,\mathrm{weierstrassZeta}\left(-\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(8ia^3-9iab^2)}{27b^3},\mathrm{weierstrassPInverse}\left(-\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(8ia^3-9iab^2)}{27b^3},\frac{3b\cos(dx+c)-3ib\sin(dx+c)-2ia}{3b}\right)\right)-24\sqrt{2}\left(ia^5b-6ia^3b^3-27iab^5\right)\sqrt{-ib}\,\mathrm{weierstrassZeta}\left(-\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(-8ia^3+9iab^2)}{27b^3},\mathrm{weierstrassPInverse}\left(-\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(-8ia^3+9iab^2)}{27b^3},\frac{3b\cos(dx+c)+3ib\sin(dx+c)+2ia}{3b}\right)\right)-3\left(105b^6\cos(dx+c)^5-5(a^2b^4+3b^6)\cos(dx+c)^3+2(4a^4b^2-21a^2b^4-15b^6)\cos(dx+c)-2\left(70ab^5\cos(dx+c)^3+3(a^3b^3+31ab^5)\cos(dx+c)\right)\sin(dx+c)\right)\sqrt{b\sin(dx+c)+a}\right)}{3465\,b^5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3465*(2*sqrt(2)*(8*a^6 - 51*a^4*b^2 + 126*a^2*b^4 + 45*b^6)*sqrt(I*b)*wei
erstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3,
 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) - 2*I*a)/b) + 2*sqrt(2)*(8*a^6
- 51*a^4*b^2 + 126*a^2*b^4 + 45*b^6)*sqrt(-I*b)*weierstrassPInverse(-4/3*(4
*a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c)
+ 3*I*b*sin(d*x + c) + 2*I*a)/b) - 24*sqrt(2)*(-I*a^5*b + 6*I*a^3*b^3 + 27*
I*a*b^5)*sqrt(I*b)*weierstrassZeta(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3
 - 9*I*a*b^2)/b^3, weierstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I
*a^3 - 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) - 2*I*a)/
b)) - 24*sqrt(2)*(I*a^5*b - 6*I*a^3*b^3 - 27*I*a*b^5)*sqrt(-I*b)*weierstras
sZeta(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, weierstra
ssPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, 1/3*
(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*I*a)/b)) - 3*(105*b^6*cos(d*x +
c)^5 - 5*(a^2*b^4 + 3*b^6)*cos(d*x + c)^3 + 2*(4*a^4*b^2 - 21*a^2*b^4 - 15*
b^6)*cos(d*x + c) - 2*(70*a*b^5*cos(d*x + c)^3 + 3*(a^3*b^3 + 31*a*b^5)*cos
(d*x + c))*sin(d*x + c))*sqrt(b*sin(d*x + c) + a))/(b^5*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sin(c+dx))^{\frac{3}{2}}\cos^4(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral((a + b*sin(c + d*x))**(3/2)*cos(c + d*x)**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c+dx)^4 (a+b\sin(c+dx))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(a + b*sin(c + d*x))^(3/2),x)
[Out] int(cos(c + d*x)^4*(a + b*sin(c + d*x))^(3/2), x)
```

3.490 $\int \cos^2(c+dx)(a+b\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=247

$$-\frac{2b\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}}{7d}+\frac{4a(3a^2+29b^2)\,E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)|\frac{2b}{a+b}\right)\sqrt{a+b\sin(c+dx)}}{105b^2d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}-\frac{4(3a^4}{}$$

```
[Out] -2/7*b*cos(d*x+c)^3*(a+b*sin(d*x+c))^(1/2)/d+2/105*cos(d*x+c)*(3*a^2+5*b^2+
24*a*b*sin(d*x+c))*(a+b*sin(d*x+c))^(1/2)/b/d-4/105*a*(3*a^2+29*b^2)*(sin(1
/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c
+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/b^2/d/((a+
b*sin(d*x+c))/(a+b))^(1/2)+4/105*(3*a^4+2*a^2*b^2-5*b^4)*(sin(1/2*c+1/4*Pi+
1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*
d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/b^2/d/(a+b*sin
(d*x+c))^(1/2)
```

Rubi [A]
time = 0.28, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {2771, 2944, 2831, 2742, 2740, 2734, 2732}

$$\frac{2\cos(c+dx)\sqrt{a+b\sin(c+dx)}\,(3a^2+24ab\sin(c+dx)+5b^2)}{105bd}+\frac{4a(3a^2+29b^2)\sqrt{a+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{105b^2d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}-\frac{4(3a^4+2a^2b^2-5b^4)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{105b^2d\sqrt{a+b\sin(c+dx)}}-\frac{2b\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*b*Cos[c + d*x]^3*Sqrt[a + b*Sin[c + d*x]])/(7*d) + (4*a*(3*a^2 + 29*b^2
)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(1
05*b^2*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) - (4*(3*a^4 + 2*a^2*b^2 - 5*b^
4)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(
a + b)])/(105*b^2*d*Sqrt[a + b*Sin[c + d*x]]) + (2*Cos[c + d*x]*Sqrt[a + b*
Sin[c + d*x]]*(3*a^2 + 5*b^2 + 24*a*b*Sin[c + d*x]))/(105*b*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
```

```
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2771

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(
a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2944

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Dist[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \cos^2(c+dx)(a+b\sin(c+dx))^{3/2}\,dx = -\frac{2b\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}}{7d} + \frac{2}{7}\int \frac{\cos^2(c+dx)\left(\frac{7a^2}{2} + \right.}{\sqrt{a+b\sin}}$$

$$= -\frac{2b\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}}{7d} + \frac{2\cos(c+dx)\sqrt{a+b\sin}}{}$$

$$= -\frac{2b\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}}{7d} + \frac{2\cos(c+dx)\sqrt{a+b\sin}}{}$$

$$= -\frac{2b\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}}{7d} + \frac{2\cos(c+dx)\sqrt{a+b\sin}}{}$$

$$= -\frac{2b\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}}{7d} + \frac{4a\left(29+\frac{3a^2}{b^2}\right)E\left(\frac{1}{2}\left(c-\right.\right.}{105d\sqrt{}}$$

Mathematica [A]
time = 1.03, size = 222, normalized size = 0.90

$$\frac{-8a(3a^3+3a^2b+29ab^2+29b^3)\,E\left(\frac{1}{4}(-2c+\pi-2dx)\middle|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}+8(3a^4+2a^2b^2-5b^4)\,F\left(\frac{1}{4}(-2c+\pi-2dx)\middle|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}+b\cos(c+dx)\,(12a^3+38ab^2-78ab^2\cos(2(c+dx))+b(108a^2+5b^2)\sin(c+dx)-15b^3\sin(3(c+dx)))}{210b^2d\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-8*a*(3*a^3 + 3*a^2*b + 29*a*b^2 + 29*b^3)*EllipticE[(-2*c + Pi - 2*d*x)/4
, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] + 8*(3*a^4 + 2*a^2*b^2
- 5*b^4)*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c
+ d*x])/(a + b)] + b*Cos[c + d*x]*(12*a^3 + 38*a*b^2 - 78*a*b^2*Cos[2*(c +
d*x)] + b*(108*a^2 + 5*b^2)*Sin[c + d*x] - 15*b^3*Sin[3*(c + d*x)]))/(210*b
^2*d*Sqrt[a + b*Sin[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 942 vs. $2(293) = 586$.
time = 2.21, size = 943, normalized size = 3.82

method	result

default	$-\frac{\frac{2b^5\left(\sin^5(dx+c)\right)}{7}+\frac{4\sqrt{\frac{a+b\sin(dx+c)}{a-b}}\,\sqrt{-\frac{(\sin(dx+c)-1)b}{a+b}}\,\sqrt{-\frac{(1+\sin(dx+c))b}{a-b}}\,\mathrm{EllipticF}\left(\sqrt{\frac{a+b\sin(dx+c)}{a-b}},\sqrt{\frac{a-b}{a+b}}\right)a^4b}{35}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/105*(-15*b^5*sin(d*x+c)^5+6*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-
1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c
))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b+48*((a+b*sin(d*x+c))/(a-b))^(1/2
)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF
(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^2+4*((a+b*sin(d*
x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b)
)^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b
^3-48*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+s
in(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(
a+b))^(1/2))*a*b^4-10*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+
b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b)
)^(1/2),((a-b)/(a+b))^(1/2))*b^5-6*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*
x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(
d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5-52*((a+b*sin(d*x+c))/(a-b))^(
1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Ellipt
icE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^2+58*((a+b*si
n(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(
a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a
*b^4-39*a*b^4*sin(d*x+c)^4-27*a^2*b^3*sin(d*x+c)^3+25*b^5*sin(d*x+c)^3-3*a^
3*b^2*sin(d*x+c)^2+49*a*b^4*sin(d*x+c)^2+27*a^2*b^3*sin(d*x+c)-10*b^5*sin(d
*x+c)+3*a^3*b^2-10*a*b^4)/b^3/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^2, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 491, normalized size = 1.99

$$-\frac{2\left(\sqrt{2}\left(6a^4-23a^2b^2-15b^4\right)\sqrt{ib}\,\mathrm{weierstrassPInverse}\left(-\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(8ia^3-9iab^2)}{27b^3},\frac{3b\cos(dx+c)-3ib\sin(dx+c)-2ia}{3b}\right)+\sqrt{2}\left(6a^4-23a^2b^2-15b^4\right)\sqrt{-ib}\,\mathrm{weierstrassPInverse}\left(-\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(-8ia^3+9iab^2)}{27b^3},\frac{3b\cos(dx+c)+3ib\sin(dx+c)+2ia}{3b}\right)+3\sqrt{2}\left(3ia^3b+29iab^3\right)\sqrt{ib}\,\mathrm{weierstrassZeta}\left(-\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(8ia^3-9iab^2)}{27b^3},\mathrm{weierstrassPInverse}\left(-\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(8ia^3-9iab^2)}{27b^3},\frac{3b\cos(dx+c)-3ib\sin(dx+c)-2ia}{3b}\right)\right)+3\sqrt{2}\left(-3ia^3b-29iab^3\right)\sqrt{-ib}\,\mathrm{weierstrassZeta}\left(-\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(-8ia^3+9iab^2)}{27b^3},\mathrm{weierstrassPInverse}\left(-\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(-8ia^3+9iab^2)}{27b^3},\frac{3b\cos(dx+c)+3ib\sin(dx+c)+2ia}{3b}\right)\right)+3\left(15b^4\cos(dx+c)^3-24ab^3\cos(dx+c)\sin(dx+c)-(3a^2b^2+5b^4)\cos(dx+c)\right)\sqrt{b\sin(dx+c)+a}\right)}{315b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/315*(sqrt(2)*(6*a^4 - 23*a^2*b^2 - 15*b^4)*sqrt(I*b)*weierstrassPInverse
(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*
x + c) - 3*I*b*sin(d*x + c) - 2*I*a)/b) + sqrt(2)*(6*a^4 - 23*a^2*b^2 - 15*
b^4)*sqrt(-I*b)*weierstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-8*I*a
^3 + 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*I*a)/b)
 + 3*sqrt(2)*(3*I*a^3*b + 29*I*a*b^3)*sqrt(I*b)*weierstrassZeta(-4/3*(4*a^2
 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, weierstrassPInverse(-4/3*(4
*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) -
 3*I*b*sin(d*x + c) - 2*I*a)/b)) + 3*sqrt(2)*(-3*I*a^3*b - 29*I*a*b^3)*sqrt
(-I*b)*weierstrassZeta(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^
2)/b^3, weierstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I
*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*I*a)/b)) + 3*(1
5*b^4*cos(d*x + c)^3 - 24*a*b^3*cos(d*x + c)*sin(d*x + c) - (3*a^2*b^2 + 5*
b^4)*cos(d*x + c))*sqrt(b*sin(d*x + c) + a))/(b^3*d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sin(c+dx))^{\frac{3}{2}}\cos^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral((a + b*sin(c + d*x))**(3/2)*cos(c + d*x)**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c+d\,x)^2\,(a+b\,\sin(c+d\,x))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a + b*sin(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^2*(a + b*sin(c + d*x))^(3/2), x)
```

3.491 $\int \sec^2(c+dx)(a+b\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=168

$$\frac{\sec(c+dx)(b+a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{d}-\frac{aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)|\frac{2b}{a+b}\right)\sqrt{a+b\sin(c+dx)}}{d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}+\frac{(a^2-b^2)}{}$$

```
[Out] sec(d*x+c)*(b+a*sin(d*x+c))*(a+b*sin(d*x+c))^(1/2)/d+a*(sin(1/2*c+1/4*Pi+1/
2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*
x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/d/((a+b*sin(d*x+c))/(a+b
))^(1/2)-(a^2-b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2
*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*si
n(d*x+c))/(a+b))^(1/2)/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A]

time = 0.13, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2770, 2831, 2742, 2740, 2734, 2732}

$$\frac{(a^2-b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\sin(c+dx)}}+\frac{\sec(c+dx)(a\sin(c+dx)+b)\sqrt{a+b\sin(c+dx)}}{d}-\frac{a\sqrt{a+b\sin(c+dx)}E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (Sec[c + d*x]*(b + a*Sin[c + d*x])*Sqrt[a + b*Sin[c + d*x]])/d - (a*Ellipti
cE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(d*Sqrt[(a
+ b*Sin[c + d*x])/(a + b)]) + ((a^2 - b^2)*EllipticF[(c - Pi/2 + d*x)/2, (2
*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(d*Sqrt[a + b*Sin[c + d*x]
])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2770

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x
])^(m - 1)*((b + a*Sin[e + f*x])/(f*g*(p + 1))), x] + Dist[1/(g^2*(p + 1)),
 Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a
^2*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g}
, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p
] || IntegerQ[m])
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^2(c+dx)(a+b\sin(c+dx))^{3/2}\,dx = \frac{\sec(c+dx)(b+a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{d} - \int \frac{\frac{b^2}{2}+\frac{1}{2}ab\,\mathrm{s}}{\sqrt{a+b\,\mathrm{si}}}$$

$$= \frac{\sec(c+dx)(b+a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{d} - \frac{1}{2}a\int \sqrt{a+}$$

$$= \frac{\sec(c+dx)(b+a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{d} - \frac{\left(a\sqrt{a+b\,\mathrm{si}}\right.}{}$$

$$= \frac{\sec(c+dx)(b+a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{d} - \frac{aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}\right.\right.}{d}$$

Mathematica [A]
time = 0.66, size = 163, normalized size = 0.97

$$\frac{ab\sec(c+dx)+a(a+b)E\left(\frac{1}{4}(-2c+\pi-2dx)|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}-(a^2-b^2)F\left(\frac{1}{4}(-2c+\pi-2dx)|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}+a^2\tan(c+dx)+b^2\tan(c+dx)+ab\sin(c+dx)\tan(c+dx)}{d\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (a*b*Sec[c + d*x] + a*(a + b)*EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b
)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] - (a^2 - b^2)*EllipticF[(-2*c + Pi -
2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] + a^2*Tan[c + d
*x] + b^2*Tan[c + d*x] + a*b*Sin[c + d*x]*Tan[c + d*x])/(d*Sqrt[a + b*Sin[c
 + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 634 vs. $2(224) = 448$.
time = 2.29, size = 635, normalized size = 3.78

method	result
default	$-\frac{\sqrt{b\left(\cos^2(dx+c)\right)\sin(dx+c)+a\left(\cos^2(dx+c)\right)}\left(\sqrt{\frac{b\sin(dx+c)}{a-b}+\frac{a}{a-b}}\sqrt{-\frac{b\sin(dx+c)}{a+b}+\frac{b}{a+b}}\sqrt{-}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+b*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b*(b*cos(d*x+c)^2*sin(d*x+c)+a*cos(d*x+c)^2)^(1/2)*((b/(a-b)*sin(d*x+c)+
1/(a-b)*a)^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b
/(a-b))^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^
(1/2))*a^2*b-(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a
+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticF((b/(a-b)*sin(d*x+c
)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*b^3-(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^
(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/
2)*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^3+
(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-
b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^
(1/2),((a-b)/(a+b))^(1/2))*a*b^2+a*b^2*cos(d*x+c)^2-a^2*b*sin(d*x+c)-b^3*si
n(d*x+c)-2*a*b^2)/(-(a+b*sin(d*x+c))*(sin(d*x+c)-1)*(1+sin(d*x+c)))^(1/2)/c
os(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(3/2)*sec(d*x + c)^2, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 444, normalized size = 2.64

$$\frac{3i\sqrt{2}\,a\sqrt{i\,b}\,b\cos(dx+c)\,\mathrm{weierstrassZeta}\left(-\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(8i\,a^3-9i\,ab^2)}{27b^3},\mathrm{weierstrassPInverse}\left(-\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(8i\,a^3-9i\,ab^2)}{27b^3},\frac{3b\cos(dx+c)-3i\,b\sin(dx+c)-2i\,a}{3b}\right)\right)-3i\sqrt{2}\,a\sqrt{-i\,b}\,b\cos(dx+c)\,\mathrm{weierstrassZeta}\left(-\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(-8i\,a^3+9i\,ab^2)}{27b^3},\mathrm{weierstrassPInverse}\left(-\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(-8i\,a^3+9i\,ab^2)}{27b^3},\frac{3b\cos(dx+c)+3i\,b\sin(dx+c)+2i\,a}{3b}\right)\right)+\sqrt{2}(2a^2-3b^2)\sqrt{i\,b}\cos(dx+c)\,\mathrm{weierstrassPInverse}\left(-\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(8i\,a^3-9i\,ab^2)}{27b^3},\frac{3b\cos(dx+c)-3i\,b\sin(dx+c)-2i\,a}{3b}\right)+\sqrt{2}(2a^2-3b^2)\sqrt{-i\,b}\cos(dx+c)\,\mathrm{weierstrassPInverse}\left(-\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(-8i\,a^3+9i\,ab^2)}{27b^3},\frac{3b\cos(dx+c)+3i\,b\sin(dx+c)+2i\,a}{3b}\right)+6(ab\sin(dx+c)+b^2)\sqrt{b\sin(dx+c)+a}}{6\,bd\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/6*(3*I*sqrt(2)*a*sqrt(I*b)*b*cos(d*x + c)*weierstrassZeta(-4/3*(4*a^2 - 3
*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, weierstrassPInverse(-4/3*(4*a^2
 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I
*b*sin(d*x + c) - 2*I*a)/b)) - 3*I*sqrt(2)*a*sqrt(-I*b)*b*cos(d*x + c)*weie
rstrassZeta(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, wei
erstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3
, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*I*a)/b)) + sqrt(2)*(2*a^2
- 3*b^2)*sqrt(I*b)*cos(d*x + c)*weierstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^
2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x +
 c) - 2*I*a)/b) + sqrt(2)*(2*a^2 - 3*b^2)*sqrt(-I*b)*cos(d*x + c)*weierstra
ssPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, 1/3*
(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*I*a)/b) + 6*(a*b*sin(d*x + c) +
b^2)*sqrt(b*sin(d*x + c) + a))/(b*d*cos(d*x + c))
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*sin(d*x+c))**(3/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sin(d*x+c))^(3/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b \sin(c + d\,x))^{3/2}}{\cos(c + d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^(3/2)/cos(c + d*x)^2,x)
[Out] int((a + b*sin(c + d*x))^(3/2)/cos(c + d*x)^2, x)
```

3.492 $\int \sec^4(c+dx)(a+b\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=218

$$-\frac{\sec(c+dx)(b-4a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{6d}+\frac{\sec^3(c+dx)(b+a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{3d}$$

```
[Out] -1/6*sec(d*x+c)*(b-4*a*sin(d*x+c))*(a+b*sin(d*x+c))^(1/2)/d+1/3*sec(d*x+c)^
3*(b+a*sin(d*x+c))*(a+b*sin(d*x+c))^(1/2)/d+2/3*a*(sin(1/2*c+1/4*Pi+1/2*d*x
)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^
(1/2)*(b/(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/d/((a+b*sin(d*x+c))/(a+b))^(1
/2)-1/6*(4*a^2-b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/
2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*s
in(d*x+c))/(a+b))^(1/2)/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A]
time = 0.28, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {2770, 2945, 2831, 2742, 2740, 2734, 2732}

$$\frac{(4a^2-b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{6d\sqrt{a+b\sin(c+dx)}}+\frac{\sec^3(c+dx)(a\sin(c+dx)+b)\sqrt{a+b\sin(c+dx)}}{3d}-\frac{\sec(c+dx)(b-4a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{6d}-\frac{2a\sqrt{a+b\sin(c+dx)}E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{3d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] -1/6*(Sec[c + d*x]*(b - 4*a*Sin[c + d*x])*Sqrt[a + b*Sin[c + d*x]])/d + (Se
c[c + d*x]^3*(b + a*Sin[c + d*x])*Sqrt[a + b*Sin[c + d*x]])/(3*d) - (2*a*El
lipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(3*d*S
qrt[(a + b*Sin[c + d*x])/(a + b)]) + ((4*a^2 - b^2)*EllipticF[(c - Pi/2 + d
*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(6*d*Sqrt[a + b*S
in[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
0] && !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && !GtQ[a + b, 0]
```

Rule 2770

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x
])^(m - 1)*((b + a*Sin[e + f*x])/(f*g*(p + 1))), x] + Dist[1/(g^2*(p + 1)),
 Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a
^2*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g}
, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p
] || IntegerQ[m])
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2945

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \sec^4(c+dx)(a+b\sin(c+dx))^{3/2}\,dx = \frac{\sec^3(c+dx)(b+a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{3d} - \frac{1}{3}\int \frac{\sec^2(}{}$$

$$= -\frac{\sec(c+dx)(b-4a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{6d} + \frac{\sec^3(c+}{}$$

$$= -\frac{\sec(c+dx)(b-4a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{6d} + \frac{\sec^3(c+}{}$$

$$= -\frac{\sec(c+dx)(b-4a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{6d} + \frac{\sec^3(c+}{}$$

$$= -\frac{\sec(c+dx)(b-4a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{6d} + \frac{\sec^3(c+}{}$$

Mathematica [A]
time = 2.35, size = 211, normalized size = 0.97

$$\frac{16a(a+b)E\left(\frac{1}{4}(-2c+\pi-2dx)|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}} - 4(4a^2-b^2)\,F\left(\frac{1}{4}(-2c+\pi-2dx)|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}} + \sec^3(c+dx)\,(12ab-6ab\cos(2(c+dx))-2ab\cos(4(c+dx))+12a^2\sin(c+dx)+7b^2\sin(c+dx)+4a^2\sin(3(c+dx))-b^2\sin(3(c+dx)))}{24d\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (16*a*(a + b)*EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*S
in[c + d*x])/(a + b)] - 4*(4*a^2 - b^2)*EllipticF[(-2*c + Pi - 2*d*x)/4, (2
*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] + Sec[c + d*x]^3*(12*a*b -
6*a*b*Cos[2*(c + d*x)] - 2*a*b*Cos[4*(c + d*x)] + 12*a^2*Sin[c + d*x] + 7*b
^2*Sin[c + d*x] + 4*a^2*Sin[3*(c + d*x)] - b^2*Sin[3*(c + d*x)]))/(24*d*Sqr
t[a + b*Sin[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 937 vs. $2(264) = 528$.
time = 2.54, size = 938, normalized size = 4.30

method	result

default	$\frac{-\sqrt{b\left(\cos^{2}\left(dx+c\right)\right)\sin\left(dx+c\right)+a\left(\cos^{2}\left(dx+c\right)\right)}\, b\left(4a^{2}-b^{2}\right)\sin\left(dx+c\right)\left(\cos^{2}\left(dx+c\right)\right)-2\sqrt{b\left(\cos^{2}\left(dx+c\right)\right.}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+b*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(-(b*cos(d*x+c)^2*sin(d*x+c)+a*cos(d*x+c)^2)^(1/2)*b*(4*a^2-b^2)*sin(d*
x+c)*cos(d*x+c)^2-2*(b*cos(d*x+c)^2*sin(d*x+c)+a*cos(d*x+c)^2)^(1/2)*b*(a^2
+b^2)*sin(d*x+c)+4*(b*cos(d*x+c)^2*sin(d*x+c)+a*cos(d*x+c)^2)^(1/2)*a*b^2*c
os(d*x+c)^4+(b*cos(d*x+c)^2*sin(d*x+c)+a*cos(d*x+c)^2)^(1/2)*(4*(b/(a-b)*si
n(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin
(d*x+c)-b/(a-b))^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b
)/(a+b))^(1/2))*a^2*b-3*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-
b)/(a+b))^(1/2))*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b
/(a-b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*a*b^2-(b/(a-b)*sin(d*x+c
)+1/(a-b)*a)^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)
-b/(a-b))^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b)
)^(1/2))*b^3-4*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a+b)*sin(d*x+c)+b/
(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticE((b/(a-b)*sin(d*x
+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^3+4*(b/(a-b)*sin(d*x+c)+1/(a-b)
*a)^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))
^(1/2)*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*
a*b^2-a*b^2)*cos(d*x+c)^2-4*(b*cos(d*x+c)^2*sin(d*x+c)+a*cos(d*x+c)^2)^(1/2
)*a*b^2)/(1+sin(d*x+c))/(-(a+b*sin(d*x+c))*(sin(d*x+c)-1)*(1+sin(d*x+c)))^(
1/2)/(sin(d*x+c)-1)/b/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(3/2)*sec(d*x + c)^4, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 481, normalized size = 2.21

$$\frac{12i\sqrt{2}\,a\sqrt{ib}\,b\cos\left(dx+c\right)^{3}\mathrm{weierstrassZeta}\left(-\frac{4\left(4a^{2}-3b^{2}\right)}{3b^{2}},-\frac{8\left(8ia^{3}-9iab^{2}\right)}{27b^{3}},\mathrm{weierstrassPInverse}\left(-\frac{4\left(4a^{2}-3b^{2}\right)}{3b^{2}},-\frac{8\left(8ia^{3}-9iab^{2}\right)}{27b^{3}},\frac{3b\cos(dx+c)-3ib\sin(dx+c)-2ia}{3b}\right)\right)-12i\sqrt{2}\,a\sqrt{-ib}\,b\cos\left(dx+c\right)^{3}\mathrm{weierstrassZeta}\left(-\frac{4\left(4a^{2}-3b^{2}\right)}{3b^{2}},-\frac{8\left(-8ia^{3}+9iab^{2}\right)}{27b^{3}},\mathrm{weierstrassPInverse}\left(-\frac{4\left(4a^{2}-3b^{2}\right)}{3b^{2}},-\frac{8\left(-8ia^{3}+9iab^{2}\right)}{27b^{3}},\frac{3b\cos(dx+c)+3ib\sin(dx+c)+2ia}{3b}\right)\right)+\sqrt{2}\left(8a^{2}-3b^{2}\right)\sqrt{ib}\cos\left(dx+c\right)^{3}\mathrm{weierstrassPInverse}\left(-\frac{4\left(4a^{2}-3b^{2}\right)}{3b^{2}},-\frac{8\left(8ia^{3}-9iab^{2}\right)}{27b^{3}},\frac{3b\cos(dx+c)-3ib\sin(dx+c)-2ia}{3b}\right)+\sqrt{2}\left(8a^{2}-3b^{2}\right)\sqrt{-ib}\cos\left(dx+c\right)^{3}\mathrm{weierstrassPInverse}\left(-\frac{4\left(4a^{2}-3b^{2}\right)}{3b^{2}},-\frac{8\left(-8ia^{3}+9iab^{2}\right)}{27b^{3}},\frac{3b\cos(dx+c)+3ib\sin(dx+c)+2ia}{3b}\right)-6\left(b^{2}\cos\left(dx+c\right)^{2}-2b^{2}-2\left(2ab\cos\left(dx+c\right)^{2}+ab\right)\sin\left(dx+c\right)\right)\sqrt{b\sin\left(dx+c\right)+a}}{36\,bd\cos\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/36*(12*I*sqrt(2)*a*sqrt(I*b)*b*cos(d*x + c)^3*weierstrassZeta(-4/3*(4*a^2
 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, weierstrassPInverse(-4/3*(4
*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) -
 3*I*b*sin(d*x + c) - 2*I*a)/b)) - 12*I*sqrt(2)*a*sqrt(-I*b)*b*cos(d*x + c)
^3*weierstrassZeta(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b
^3, weierstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b
^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*I*a)/b)) + sqrt(2)*
(8*a^2 - 3*b^2)*sqrt(I*b)*cos(d*x + c)^3*weierstrassPInverse(-4/3*(4*a^2 -
3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*
sin(d*x + c) - 2*I*a)/b) + sqrt(2)*(8*a^2 - 3*b^2)*sqrt(-I*b)*cos(d*x + c)^
3*weierstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2
)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*I*a)/b) - 6*(b^2*cos(
d*x + c)^2 - 2*b^2 - 2*(2*a*b*cos(d*x + c)^2 + a*b)*sin(d*x + c))*sqrt(b*si
n(d*x + c) + a))/(b*d*cos(d*x + c)^3)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8009 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\sin(c+d\,x))^{3/2}}{\cos(c+d\,x)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^(3/2)/cos(c + d*x)^4,x)
```

```
[Out] int((a + b*sin(c + d*x))^(3/2)/cos(c + d*x)^4, x)
```

3.493 $\int \sec^6(c+dx)(a+b\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=330

$$-\frac{\sec^3(c+dx)(b-8a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{30d}+\frac{\sec^5(c+dx)(b+a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{5d}$$

```
[Out] -1/30*sec(d*x+c)^3*(b-8*a*sin(d*x+c))*(a+b*sin(d*x+c))^(1/2)/d+1/5*sec(d*x+
c)^5*(b+a*sin(d*x+c))*(a+b*sin(d*x+c))^(1/2)/d-1/60*sec(d*x+c)*(b*(8*a^4-13
*a^2*b^2+5*b^4)-a*(32*a^4-61*a^2*b^2+29*b^4)*sin(d*x+c))*(a+b*sin(d*x+c))^(
1/2)/(a^2-b^2)^2/d+1/60*a*(32*a^2-29*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/
2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b
/(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/(a^2-b^2)/d/((a+b*sin(d*x+c))/(a+b))^
(1/2)-1/60*(32*a^2-5*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4
*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*(
(a+b*sin(d*x+c))/(a+b))^(1/2)/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A]
time = 0.45, antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {2770, 2945, 2831, 2742, 2740, 2734, 2732}

$$\frac{(32a^2-5b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{60d\sqrt{a+b\sin(c+dx)}}-\frac{a(32a^2-29b^2)\sqrt{a+b\sin(c+dx)}E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{60d(a^2-b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}-\frac{\sec(c+dx)\sqrt{a+b\sin(c+dx)}\left(b(8a^4-13a^2b^2+5b^4)-a(32a^4-61a^2b^2+29b^4)\sin(c+dx)\right)}{60d(a^2-b^2)^2}+\frac{\sec^5(c+dx)(a\sin(c+dx)+b)\sqrt{a+b\sin(c+dx)}}{5d}-\frac{\sec^3(c+dx)(b-8a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{30d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] -1/30*(Sec[c + d*x]^3*(b - 8*a*Sin[c + d*x])*Sqrt[a + b*Sin[c + d*x]])/d +
(Sec[c + d*x]^5*(b + a*Sin[c + d*x])*Sqrt[a + b*Sin[c + d*x]])/(5*d) - (a*(
32*a^2 - 29*b^2)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Si
n[c + d*x]])/(60*(a^2 - b^2)*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + ((32*a
^2 - 5*b^2)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c
+ d*x])/(a + b)])/(60*d*Sqrt[a + b*Sin[c + d*x]]) - (Sec[c + d*x]*Sqrt[a +
b*Sin[c + d*x]]*(b*(8*a^4 - 13*a^2*b^2 + 5*b^4) - a*(32*a^4 - 61*a^2*b^2 +
29*b^4)*Sin[c + d*x]))/(60*(a^2 - b^2)^2*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2770

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x
])^(m - 1)*((b + a*Sin[e + f*x])/(f*g*(p + 1))), x] + Dist[1/(g^2*(p + 1)),
 Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a
^2*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g}
, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p
] || IntegerQ[m])
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2945

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \sec^6(c+dx)(a+b\sin(c+dx))^{3/2}\,dx = \frac{\sec^5(c+dx)(b+a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{5d} - \frac{1}{5}\int \frac{\sec^4(c\cdots}{\cdots}$$

$$= -\frac{\sec^3(c+dx)(b-8a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{30d} + \frac{\sec^5(c+\cdots}{\cdots}$$

$$= -\frac{\sec^3(c+dx)(b-8a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{30d} + \frac{\sec^5(c+\cdots}{\cdots}$$

$$= -\frac{\sec^3(c+dx)(b-8a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{30d} + \frac{\sec^5(c+\cdots}{\cdots}$$

$$= -\frac{\sec^3(c+dx)(b-8a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{30d} + \frac{\sec^5(c+\cdots}{\cdots}$$

$$= -\frac{\sec^3(c+dx)(b-8a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{30d} + \frac{\sec^5(c+\cdots}{\cdots}$$

Mathematica [A]
time = 6.28, size = 364, normalized size = 1.10

$$\frac{\sqrt{a+b\sin(c+dx)}\left(\frac{1}{5}\sec^5(c+dx)(b+a\sin(c+dx))+\frac{1}{30}\sec^3(c+dx)(-b+8a\sin(c+dx))+\frac{\sec(c+dx)\left(-8a^2b+5b^3+32a^3\sin(c+dx)-29ab^2\sin(c+dx)\right)}{60(a^2-b^2)}\right)}{d} - \frac{b\left(-\frac{2(8a^2b-5b^3)F\left(\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}{\sqrt{a+b\sin(c+dx)}} - \frac{(32a^3-29ab^2)\left(\frac{2(a+b)E\left(\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}{\sqrt{a+b\sin(c+dx)}} - \frac{2aF\left(\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}{\sqrt{a+b\sin(c+dx)}}\right)}{b}\right)}{120(a-b)(a+b)d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (Sqrt[a + b*Sin[c + d*x]]*((Sec[c + d*x]^5*(b + a*Sin[c + d*x]))/5 + (Sec[c
 + d*x]^3*(-b + 8*a*Sin[c + d*x]))/30 + (Sec[c + d*x]*(-8*a^2*b + 5*b^3 + 3
2*a^3*Sin[c + d*x] - 29*a*b^2*Sin[c + d*x]))/(60*(a^2 - b^2))))/d - (b*((-2
*(8*a^2*b - 5*b^3)*EllipticF[(-c + Pi/2 - d*x)/2, (2*b)/(a + b)]*Sqrt[(a +
b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]] - ((32*a^3 - 29*a*b^2)*(
(2*(a + b)*EllipticE[(-c + Pi/2 - d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c
+ d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]] - (2*a*EllipticF[(-c + Pi/2 - d*
```

```
x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c +
 d*x]]))/b))/(120*(a - b)*(a + b)*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1518 vs. $2(372) = 744$.

time = 3.00, size = 1519, normalized size = 4.60

method	result	size
default	Expression too large to display	1519

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(a+b*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/120*(2*(b*cos(d*x+c)^2*sin(d*x+c)+a*cos(d*x+c)^2)^(1/2)*b*(32*a^4-37*a^2*
b^2+5*b^4)*sin(d*x+c)*cos(d*x+c)^4+4*(b*cos(d*x+c)^2*sin(d*x+c)+a*cos(d*x+c
)^2)^(1/2)*b*(8*a^4-9*a^2*b^2+b^4)*cos(d*x+c)^2*sin(d*x+c)+24*(b*cos(d*x+c)
^2*sin(d*x+c)+a*cos(d*x+c)^2)^(1/2)*b*(a^4-b^4)*sin(d*x+c)-2*(b*cos(d*x+c)^
2*sin(d*x+c)+a*cos(d*x+c)^2)^(1/2)*a*b^2*(32*a^2-29*b^2)*cos(d*x+c)^6+2*(b*
cos(d*x+c)^2*sin(d*x+c)+a*cos(d*x+c)^2)^(1/2)*(32*EllipticE((b/(a-b)*sin(d*
x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1
/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2
)*a^5-61*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2)
)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(
b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*a^3*b^2+29*EllipticE((b/(a-b)*sin(d*x+c
)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)
*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*a
*b^4-32*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))
*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(b
/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*a^4*b+24*EllipticF((b/(a-b)*sin(d*x+c)+1
/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(-
b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*a^3*
b^2+37*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*
(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(b/
(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*a^2*b^3-24*EllipticF((b/(a-b)*sin(d*x+c)+
1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(
-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*a*b
^4-5*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-
b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(b/(a
-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*b^5+8*a^3*b^2-8*a*b^4)*cos(d*x+c)^4+4*(b*co
s(d*x+c)^2*sin(d*x+c)+a*cos(d*x+c)^2)^(1/2)*a*b^2*(a^2-b^2)*cos(d*x+c)^2+48
*(b*cos(d*x+c)^2*sin(d*x+c)+a*cos(d*x+c)^2)^(1/2)*a^3*b^2-48*(b*cos(d*x+c)^
2*sin(d*x+c)+a*cos(d*x+c)^2)^(1/2)*a*b^4)/(1+sin(d*x+c))^2/(a-b)/(-(a+b*sin
(d*x+c))*(sin(d*x+c)-1)*(1+sin(d*x+c)))^(1/2)/(a+b)/(sin(d*x+c)-1)^2/b/cos(
d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+b*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(3/2)*sec(d*x + c)^6, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.15, size = 612, normalized size = 1.85

$$\frac{\sqrt{2}\left(64a^4-82a^2b^2+15b^4\right)\sqrt{ib}\cos(dx+c)^5\operatorname{weierstrassPInverse}\left(-\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(8ia^3-9iab^2)}{27b^3},\frac{3b\cos(dx+c)-3ib\sin(dx+c)-2ia}{3b}\right)+\sqrt{2}\left(64a^4-82a^2b^2+15b^4\right)\sqrt{-ib}\cos(dx+c)^5\operatorname{weierstrassPInverse}\left(-\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(-8ia^3+9iab^2)}{27b^3},\frac{3b\cos(dx+c)+3ib\sin(dx+c)+2ia}{3b}\right)+3\sqrt{2}\left(32ia^3b-29iab^3\right)\sqrt{ib}\cos(dx+c)^5\operatorname{weierstrassZeta}\left(-\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(8ia^3-9iab^2)}{27b^3},\operatorname{weierstrassPInverse}\left(-\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(8ia^3-9iab^2)}{27b^3},\frac{3b\cos(dx+c)-3ib\sin(dx+c)-2ia}{3b}\right)\right)+3\sqrt{2}\left(-32ia^3b+29iab^3\right)\sqrt{-ib}\cos(dx+c)^5\operatorname{weierstrassZeta}\left(-\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(-8ia^3+9iab^2)}{27b^3},\operatorname{weierstrassPInverse}\left(-\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(-8ia^3+9iab^2)}{27b^3},\frac{3b\cos(dx+c)+3ib\sin(dx+c)+2ia}{3b}\right)\right)-6\left((8a^2b^2-5b^4)\cos(dx+c)^4-12a^2b^2+12b^4+2(a^2b^2-b^4)\cos(dx+c)^2-\left((32a^3b-29ab^3)\cos(dx+c)^4+12a^3b-12ab^3+16(a^3b-ab^3)\cos(dx+c)^2\right)\sin(dx+c)\right)\sqrt{b\sin(dx+c)+a}}{360\left(a^2b-b^3\right)d\cos(dx+c)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/360*(sqrt(2)*(64*a^4 - 82*a^2*b^2 + 15*b^4)*sqrt(I*b)*cos(d*x + c)^5*weie
rstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3,
1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) - 2*I*a)/b) + sqrt(2)*(64*a^4 -
82*a^2*b^2 + 15*b^4)*sqrt(-I*b)*cos(d*x + c)^5*weierstrassPInverse(-4/3*(4*
a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) +
 3*I*b*sin(d*x + c) + 2*I*a)/b) + 3*sqrt(2)*(32*I*a^3*b - 29*I*a*b^3)*sqrt(
I*b)*cos(d*x + c)^5*weierstrassZeta(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^
3 - 9*I*a*b^2)/b^3, weierstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*
I*a^3 - 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) - 2*I*a)
/b)) + 3*sqrt(2)*(-32*I*a^3*b + 29*I*a*b^3)*sqrt(-I*b)*cos(d*x + c)^5*weier
strassZeta(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, weie
rstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3,
 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*I*a)/b)) - 6*((8*a^2*b^2 -
5*b^4)*cos(d*x + c)^4 - 12*a^2*b^2 + 12*b^4 + 2*(a^2*b^2 - b^4)*cos(d*x + c
)^2 - ((32*a^3*b - 29*a*b^3)*cos(d*x + c)^4 + 12*a^3*b - 12*a*b^3 + 16*(a^3
*b - a*b^3)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a))/((a^2*b
 - b^3)*d*cos(d*x + c)^5)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+b*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b \sin(c + d\,x))^{3/2}}{\cos(c + d\,x)^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^(3/2)/cos(c + d*x)^6,x)
```

```
[Out] int((a + b*sin(c + d*x))^(3/2)/cos(c + d*x)^6, x)
```

3.494 $\int \cos^5(c+dx)(a+b\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=154

$$\frac{2(a^2-b^2)^2(a+b\sin(c+dx))^{7/2}}{7b^5d}-\frac{8a(a^2-b^2)(a+b\sin(c+dx))^{9/2}}{9b^5d}+\frac{4(3a^2-b^2)(a+b\sin(c+dx))^{11/2}}{11b^5d}-8a$$

```
[Out] 2/7*(a^2-b^2)^2*(a+b*sin(d*x+c))^(7/2)/b^5/d-8/9*a*(a^2-b^2)*(a+b*sin(d*x+c
))^(9/2)/b^5/d+4/11*(3*a^2-b^2)*(a+b*sin(d*x+c))^(11/2)/b^5/d-8/13*a*(a+b*s
in(d*x+c))^(13/2)/b^5/d+2/15*(a+b*sin(d*x+c))^(15/2)/b^5/d
```

Rubi [A]
time = 0.07, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = $\{2747, 711\}$

$$\frac{4(3a^2-b^2)(a+b\sin(c+dx))^{11/2}}{11b^5d}-\frac{8a(a^2-b^2)(a+b\sin(c+dx))^{9/2}}{9b^5d}+\frac{2(a^2-b^2)^2(a+b\sin(c+dx))^{7/2}}{7b^5d}+\frac{2(a+b\sin(c+dx))^{15/2}}{15b^5d}-\frac{8a(a+b\sin(c+dx))^{13/2}}{13b^5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (2*(a^2 - b^2)^2*(a + b*Sin[c + d*x])^(7/2))/(7*b^5*d) - (8*a*(a^2 - b^2)*(
a + b*Sin[c + d*x])^(9/2))/(9*b^5*d) + (4*(3*a^2 - b^2)*(a + b*Sin[c + d*x]
)^(11/2))/(11*b^5*d) - (8*a*(a + b*Sin[c + d*x])^(13/2))/(13*b^5*d) + (2*(a
 + b*Sin[c + d*x])^(15/2))/(15*b^5*d)
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^5(c+dx)(a+b\sin(c+dx))^{5/2}\,dx = \frac{\text{Subst}\left(\int (a+x)^{5/2}\left(b^2-x^2\right)^2\,dx, x, b\sin(c+dx)\right)}{b^5d}$$
$$= \frac{\text{Subst}\left(\int \left(\left(a^2-b^2\right)^2(a+x)^{5/2}-4\left(a^3-ab^2\right)(a+x)^{7/2}+2\left(3a^2\right.\right.\right.}{}$$
$$= \frac{2\left(a^2-b^2\right)^2(a+b\sin(c+dx))^{7/2}}{7b^5d} - \frac{8a\left(a^2-b^2\right)(a+b\sin(c+dx\right.}{9b^5d}$$

Mathematica [A]
time = 1.39, size = 279, normalized size = 1.81

$$\frac{4a^2\left(2048a^6-16256a^4b^2+100050a^2b^4+64665b^6\right)\sqrt{1+\frac{b\sin(c+dx)}{a}}\left(-1+\sqrt{1+\frac{b\sin(c+dx)}{a}}\right)-b(a+b\sin(c+dx))\left(2ab\left(768a^4-5680a^2b^2+81415b^4\right)\cos(2(c+dx))-28ab^3\left(10a^2-2911b^2\right)\cos(4(c+dx))+14322ab^5\cos(6(c+dx))+\left(4096a^6-31360a^4b^2-709320a^2b^4-86385b^6\right)\sin(c+dx)+b^2\left(-640a^4-145580a^2b^2+2223b^4\right)\sin(3(c+dx))+21b^4\left(-852a^2+559b^2\right)\sin(5(c+dx))+3003b^6\sin(7(c+dx))\right)}{1441440b^5d\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (4*a^2*(2048*a^6 - 16256*a^4*b^2 + 100050*a^2*b^4 + 64665*b^6)*Sqrt[1 + (b*
Sin[c + d*x])/a]*(-1 + Sqrt[1 + (b*Sin[c + d*x])/a]) - b*(a + b*Sin[c + d*x
])*(2*a*b*(768*a^4 - 5680*a^2*b^2 + 81415*b^4)*Cos[2*(c + d*x)] - 28*a*b^3*
(10*a^2 - 2911*b^2)*Cos[4*(c + d*x)] + 14322*a*b^5*Cos[6*(c + d*x)] + (4096
*a^6 - 31360*a^4*b^2 - 709320*a^2*b^4 - 86385*b^6)*Sin[c + d*x] + b^2*(-640
*a^4 - 145580*a^2*b^2 + 2223*b^4)*Sin[3*(c + d*x)] + 21*b^4*(-852*a^2 + 559
*b^2)*Sin[5*(c + d*x)] + 3003*b^6*Sin[7*(c + d*x)]))/(1441440*b^5*d*Sqrt[a
+ b*Sin[c + d*x]])
```

Maple [A]
time = 1.93, size = 126, normalized size = 0.82

method	result
default	$\frac{2(a+b\sin(dx+c))^{\frac{7}{2}}\left(3003b^4\left(\cos^4(dx+c)\right)+1848a\,b^3\left(\cos^2(dx+c)\right)\sin(dx+c)-1008a^2b^2\left(\cos^2(dx+c)\right)+2184b^4\left(\cos^2(dx+c)\right)-448a^3b\right.}{45045b^5d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+b*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/45045/b^5*(a+b*sin(d*x+c))^(7/2)*(3003*b^4*cos(d*x+c)^4+1848*a*b^3*cos(d*
x+c)^2*sin(d*x+c)-1008*a^2*b^2*cos(d*x+c)^2+2184*b^4*cos(d*x+c)^2-448*a^3*b
*sin(d*x+c)+1792*a*b^3*sin(d*x+c)+128*a^4-32*a^2*b^2+1248*b^4)/d
```

Maxima [A]
time = 0.29, size = 116, normalized size = 0.75

$$\frac{2\left(3003\,(b\sin(dx+c)+a)^{\frac{15}{2}}-13860\,(b\sin(dx+c)+a)^{\frac{13}{2}}a+8190\,(3\,a^2-b^2)(b\sin(dx+c)+a)^{\frac{11}{2}}-20020\,(a^3-ab^2)(b\sin(dx+c)+a)^{\frac{9}{2}}+6435\,(a^4-2\,a^2b^2+b^4)(b\sin(dx+c)+a)^{\frac{7}{2}}\right)}{45045\,b^5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 2/45045*(3003*(b*sin(d*x + c) + a)^(15/2) - 13860*(b*sin(d*x + c) + a)^(13/
2)*a + 8190*(3*a^2 - b^2)*(b*sin(d*x + c) + a)^(11/2) - 20020*(a^3 - a*b^2)
*(b*sin(d*x + c) + a)^(9/2) + 6435*(a^4 - 2*a^2*b^2 + b^4)*(b*sin(d*x + c)
+ a)^(7/2))/(b^5*d)
```

Fricas [A]
time = 0.41, size = 224, normalized size = 1.45

$$-\frac{2\left(7161\,ab^{6}\cos\left(dx+c\right)^{6}-128\,a^{7}+992\,a^{5}b^{2}-6080\,a^{3}b^{4}-5536\,ab^{6}-7\left(5\,a^{3}b^{4}+79\,ab^{6}\right)\cos\left(dx+c\right)^{4}+16\left(3\,a^{5}b^{2}-20\,a^{3}b^{4}-67\,ab^{6}\right)\cos\left(dx+c\right)^{2}+\left(3003\,b^{7}\cos\left(dx+c\right)^{6}+64\,a^{6}b-480\,a^{4}b^{3}-9088\,a^{2}b^{5}-1248\,b^{7}-63\left(71\,a^{2}b^{5}+13\,b^{7}\right)\cos\left(dx+c\right)^{4}-8\left(5\,a^{4}b^{3}+718\,a^{2}b^{5}+117\,b^{7}\right)\cos\left(dx+c\right)^{2}\right)\sin\left(dx+c\right)\right)\sqrt{b\sin\left(dx+c\right)+a}}{45045\,b^{5}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/45045*(7161*a*b^6*cos(d*x + c)^6 - 128*a^7 + 992*a^5*b^2 - 6080*a^3*b^4
- 5536*a*b^6 - 7*(5*a^3*b^4 + 79*a*b^6)*cos(d*x + c)^4 + 16*(3*a^5*b^2 - 20
*a^3*b^4 - 67*a*b^6)*cos(d*x + c)^2 + (3003*b^7*cos(d*x + c)^6 + 64*a^6*b -
 480*a^4*b^3 - 9088*a^2*b^5 - 1248*b^7 - 63*(71*a^2*b^5 + 13*b^7)*cos(d*x +
 c)^4 - 8*(5*a^4*b^3 + 718*a^2*b^5 + 117*b^7)*cos(d*x + c)^2)*sin(d*x + c))
*sqrt(b*sin(d*x + c) + a)/(b^5*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3876 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(5/2)*cos(d*x + c)^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^5\,{\left(a+b\,\sin\left(c+d\,x\right)\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(a + b*sin(c + d*x))^(5/2),x)
[Out] int(cos(c + d*x)^5*(a + b*sin(c + d*x))^(5/2), x)
```

3.495 $\int \cos^3(c+dx)(a+b\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=83

$$-\frac{2(a^2-b^2)\,(a+b\sin(c+dx))^{7/2}}{7b^3d}+\frac{4a(a+b\sin(c+dx))^{9/2}}{9b^3d}-\frac{2(a+b\sin(c+dx))^{11/2}}{11b^3d}$$

```
[Out] -2/7*(a^2-b^2)*(a+b*sin(d*x+c))^(7/2)/b^3/d+4/9*a*(a+b*sin(d*x+c))^(9/2)/b^
3/d-2/11*(a+b*sin(d*x+c))^(11/2)/b^3/d
```

Rubi [A]
time = 0.06, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2747, 711}

$$-\frac{2(a^2-b^2)\,(a+b\sin(c+dx))^{7/2}}{7b^3d}-\frac{2(a+b\sin(c+dx))^{11/2}}{11b^3d}+\frac{4a(a+b\sin(c+dx))^{9/2}}{9b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*(a^2 - b^2)*(a + b*Sin[c + d*x])^(7/2))/(7*b^3*d) + (4*a*(a + b*Sin[c +
 d*x])^(9/2))/(9*b^3*d) - (2*(a + b*Sin[c + d*x])^(11/2))/(11*b^3*d)
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^3(c+dx)(a+b\sin(c+dx))^{5/2}\,dx &= \frac{\text{Subst}\left(\int (a+x)^{5/2}\left(b^2-x^2\right)\,dx,x,b\sin(c+dx)\right)}{b^3d}\\ &= \frac{\text{Subst}\left(\int \left(\left(-a^2+b^2\right)(a+x)^{5/2}+2a(a+x)^{7/2}-(a+x)^{9/2}\right)\,dx,x\right.}{b^3d}\\ &= -\frac{2(a^2-b^2)\,(a+b\sin(c+dx))^{7/2}}{7b^3d}+\frac{4a(a+b\sin(c+dx))^{9/2}}{9b^3d}-\frac{2(}{}\end{aligned}$$

Mathematica [A]
time = 0.12, size = 58, normalized size = 0.70

$$-\frac{2(a+b\sin(c+dx))^{7/2}\left(8a^2-99b^2-28ab\sin(c+dx)+63b^2\sin^2(c+dx)\right)}{693b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*(a + b*Sin[c + d*x])^(7/2)*(8*a^2 - 99*b^2 - 28*a*b*Sin[c + d*x] + 63*b
^2*Sin[c + d*x]^2))/(693*b^3*d)
```

Maple [A]
time = 1.20, size = 55, normalized size = 0.66

method	result	size
default	$-\frac{2(a+b\sin(dx+c))^{\frac{7}{2}}\left(-63b^2\left(\cos^2(dx+c)\right)-28ab\sin(dx+c)+8a^2-36b^2\right)}{693b^3d}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/693/b^3*(a+b*sin(d*x+c))^(7/2)*(-63*b^2*cos(d*x+c)^2-28*a*b*sin(d*x+c)+8
*a^2-36*b^2)/d
```

Maxima [A]
time = 0.28, size = 61, normalized size = 0.73

$$-\frac{2\left(63\left(b\sin\left(dx+c\right)+a\right)^{\frac{11}{2}}-154\left(b\sin\left(dx+c\right)+a\right)^{\frac{9}{2}}a+99\left(a^2-b^2\right)\left(b\sin\left(dx+c\right)+a\right)^{\frac{7}{2}}\right)}{693\,b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -2/693*(63*(b*sin(d*x + c) + a)^(11/2) - 154*(b*sin(d*x + c) + a)^(9/2)*a +
 99*(a^2 - b^2)*(b*sin(d*x + c) + a)^(7/2))/(b^3*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(71) = 142$.
time = 0.36, size = 143, normalized size = 1.72

$$-\frac{2\left(161\,ab^4\cos\left(dx+c\right)^4+8\,a^5-96\,a^3b^2-136\,ab^4-\left(3\,a^3b^2+25\,ab^4\right)\cos\left(dx+c\right)^2+\left(63\,b^5\cos\left(dx+c\right)^4-4\,a^4b-184\,a^2b^3-36\,b^5-\left(113\,a^2b^3+27\,b^5\right)\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)\right)\sqrt{b\sin\left(dx+c\right)+a}}{693\,b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/693*(161*a*b^4*cos(d*x + c)^4 + 8*a^5 - 96*a^3*b^2 - 136*a*b^4 - (3*a^3*
b^2 + 25*a*b^4)*cos(d*x + c)^2 + (63*b^5*cos(d*x + c)^4 - 4*a^4*b - 184*a^2
*b^3 - 36*b^5 - (113*a^2*b^3 + 27*b^5)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(b
*sin(d*x + c) + a)/(b^3*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 391 vs. $2(76) = 152$.
time = 48.74, size = 391, normalized size = 4.71

$$\begin{cases} a^{\frac{5}{2}} x \cos^{3}{\left(c \right)} & \text{for}\: b = 0 \wedge d = 0 \\ a^{\frac{5}{2}} \cdot \left(\frac{2 \sin^{3}{\left(c + d x \right)}}{3 d} + \frac{\sin{\left(c + d x \right)} \cos^{2}{\left(c + d x \right)}}{d}\right) & \text{for}\: b = 0 \\ x \left(a + b \sin{\left(c \right)}\right)^{\frac{5}{2}} \cos^{3}{\left(c \right)} & \text{for}\: d = 0 \\ - \frac{16 a^{5} \sqrt{a + b \sin{\left(c + d x \right)}}}{693 b^{3} d} + \frac{8 a^{4} \sqrt{a + b \sin{\left(c + d x \right)}} \sin{\left(c + d x \right)}}{693 b^{2} d} + \frac{64 a^{3} \sqrt{a + b \sin{\left(c + d x \right)}} \sin^{2}{\left(c + d x \right)}}{231 b d} + \frac{2 a^{3} \sqrt{a + b \sin{\left(c + d x \right)}} \cos^{2}{\left(c + d x \right)}}{7 b d} + \frac{368 a^{2} \sqrt{a + b \sin{\left(c + d x \right)}} \sin^{3}{\left(c + d x \right)}}{693 d} + \frac{6 a^{2} \sqrt{a + b \sin{\left(c + d x \right)}} \sin{\left(c + d x \right)} \cos^{2}{\left(c + d x \right)}}{7 d} + \frac{272 a b \sqrt{a + b \sin{\left(c + d x \right)}} \sin^{4}{\left(c + d x \right)}}{693 d} + \frac{6 a b \sqrt{a + b \sin{\left(c + d x \right)}} \sin^{2}{\left(c + d x \right)} \cos^{2}{\left(c + d x \right)}}{7 d} + \frac{8 b^{2} \sqrt{a + b \sin{\left(c + d x \right)}} \sin^{5}{\left(c + d x \right)}}{77 d} + \frac{2 b^{2} \sqrt{a + b \sin{\left(c + d x \right)}} \sin^{3}{\left(c + d x \right)} \cos^{2}{\left(c + d x \right)}}{7 d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Piecewise((a**(5/2)*x*cos(c)**3, Eq(b, 0) & Eq(d, 0)), (a**(5/2)*(2*sin(c +
 d*x)**3/(3*d) + sin(c + d*x)*cos(c + d*x)**2/d), Eq(b, 0)), (x*(a + b*sin(
c))**(5/2)*cos(c)**3, Eq(d, 0)), (-16*a**5*sqrt(a + b*sin(c + d*x))/(693*b*
*3*d) + 8*a**4*sqrt(a + b*sin(c + d*x))*sin(c + d*x)/(693*b**2*d) + 64*a**3
*sqrt(a + b*sin(c + d*x))*sin(c + d*x)**2/(231*b*d) + 2*a**3*sqrt(a + b*sin
(c + d*x))*cos(c + d*x)**2/(7*b*d) + 368*a**2*sqrt(a + b*sin(c + d*x))*sin(
c + d*x)**3/(693*d) + 6*a**2*sqrt(a + b*sin(c + d*x))*sin(c + d*x)*cos(c +
d*x)**2/(7*d) + 272*a*b*sqrt(a + b*sin(c + d*x))*sin(c + d*x)**4/(693*d) +
6*a*b*sqrt(a + b*sin(c + d*x))*sin(c + d*x)**2*cos(c + d*x)**2/(7*d) + 8*b*
*2*sqrt(a + b*sin(c + d*x))*sin(c + d*x)**5/(77*d) + 2*b**2*sqrt(a + b*sin(
c + d*x))*sin(c + d*x)**3*cos(c + d*x)**2/(7*d), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(5/2)*cos(d*x + c)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^3\,{\left(a+b\,\sin\left(c+d\,x\right)\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a + b*sin(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^3*(a + b*sin(c + d*x))^(5/2), x)
```

3.496 $\int \cos(c+dx)(a+b\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=24

$$\frac{2(a+b\sin(c+dx))^{7/2}}{7bd}$$

```
[Out] 2/7*(a+b*sin(d*x+c))^(7/2)/b/d
```

Rubi [A]
time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2747, 32}

$$\frac{2(a+b\sin(c+dx))^{7/2}}{7bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (2*(a + b*Sin[c + d*x])^(7/2))/(7*b*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a+b\sin(c+dx))^{5/2}\,dx &= \frac{\text{Subst}\left(\int (a+x)^{5/2}\,dx,x,b\sin(c+dx)\right)}{bd}\\ &= \frac{2(a+b\sin(c+dx))^{7/2}}{7bd}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 24, normalized size = 1.00

$$\frac{2(a+b\sin(c+dx))^{7/2}}{7bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (2*(a + b*Sin[c + d*x])^(7/2))/(7*b*d)
```

Maple [A]
time = 0.07, size = 21, normalized size = 0.88

method	result	size
derivativedivides	$\frac{2(a+b\sin(dx+c))^{\frac{7}{2}}}{7bd}$	21
default	$\frac{2(a+b\sin(dx+c))^{\frac{7}{2}}}{7bd}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/7*(a+b*sin(d*x+c))^(7/2)/b/d
```

Maxima [A]
time = 0.27, size = 20, normalized size = 0.83

$$\frac{2\,(b\sin{(dx+c)}+a)^{\frac{7}{2}}}{7\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 2/7*(b*sin(d*x + c) + a)^(7/2)/(b*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 77 vs. $2(20) = 40$.
time = 0.35, size = 77, normalized size = 3.21

$$-\frac{2\,\left(3\,ab^2\cos{(dx+c)}^2-a^3-3\,ab^2+\left(b^3\cos{(dx+c)}^2-3\,a^2b-b^3\right)\sin{(dx+c)}\right)\sqrt{b\sin{(dx+c)}+a}}{7\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/7*(3*a*b^2*cos(d*x + c)^2 - a^3 - 3*a*b^2 + (b^3*cos(d*x + c)^2 - 3*a^2*
b - b^3)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)/(b*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 150 vs. $2(19) = 38$.

time = 17.32, size = 150, normalized size = 6.25

$$\begin{cases} a^{\frac{5}{2}}x\cos{(c)} & \text{for } b=0 \wedge d=0 \\ \frac{a^{\frac{5}{2}}\sin{(c+dx)}}{d} & \text{for } b=0 \\ x(a+b\sin{(c)})^{\frac{5}{2}}\cos{(c)} & \text{for } d=0 \\ \frac{2a^3\sqrt{a+b\sin{(c+dx)}}}{7bd} + \frac{6a^2\sqrt{a+b\sin{(c+dx)}}\sin{(c+dx)}}{7d} + \frac{6ab\sqrt{a+b\sin{(c+dx)}}\sin^2{(c+dx)}}{7d} + \frac{2b^2\sqrt{a+b\sin{(c+dx)}}\sin^3{(c+dx)}}{7d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))**(5/2),x)

[Out] Piecewise((a**(5/2)*x*cos(c), Eq(b, 0) & Eq(d, 0)), (a**(5/2)*sin(c + d*x)/d, Eq(b, 0)), (x*(a + b*sin(c))**(5/2)*cos(c), Eq(d, 0)), (2*a**3*sqrt(a + b*sin(c + d*x))/(7*b*d) + 6*a**2*sqrt(a + b*sin(c + d*x))*sin(c + d*x)/(7*d) + 6*a*b*sqrt(a + b*sin(c + d*x))*sin(c + d*x)**2/(7*d) + 2*b**2*sqrt(a + b*sin(c + d*x))*sin(c + d*x)**3/(7*d), True))

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate((b*sin(d*x + c) + a)^(5/2)*cos(d*x + c), x)

Mupad [B]
time = 5.57, size = 20, normalized size = 0.83

$$\frac{2\,{\left(a+b\,\sin\left(c+d\,x\right)\right)}^{7/2}}{7\,b\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)*(a + b*sin(c + d*x))^(5/2),x)

[Out] (2*(a + b*sin(c + d*x))^(7/2))/(7*b*d)

3.497 $\int \sec(c+dx)(a+b\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=117

$$-\frac{(a-b)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{d}+\frac{(a+b)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{d}-\frac{4ab\sqrt{a+b\sin(c+}}{d}$$

```
[Out] -(a-b)^(5/2)*arctanh((a+b*sin(d*x+c))^(1/2)/(a-b)^(1/2))/d+(a+b)^(5/2)*arct
anh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))/d-2/3*b*(a+b*sin(d*x+c))^(3/2)/d-4*
a*b*(a+b*sin(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.15, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2747, 718, 839, 841, 1180, 212}

$$-\frac{2b(a+b\sin(c+dx))^{3/2}}{3d}-\frac{4ab\sqrt{a+b\sin(c+dx)}}{d}-\frac{(a-b)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{d}+\frac{(a+b)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] -(((a - b)^(5/2)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a - b]])/d) + ((a +
b)^(5/2)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a + b]])/d - (4*a*b*Sqrt[a +
 b*Sin[c + d*x]])/d - (2*b*(a + b*Sin[c + d*x])^(3/2))/(3*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 718

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Simp[e*((d
 + e*x)^(m - 1)/(c*(m - 1))), x] + Dist[1/c, Int[(d + e*x)^(m - 2)*(Simp[c*
d^2 - a*e^2 + 2*c*d*e*x, x]/(a + c*x^2)), x], x] /; FreeQ[{a, c, d, e}, x]
&& NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 1]
```

Rule 839

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Simp[g*((d + e*x)^m/(c*m)), x] + Dist[1/c, Int[(d + e*x)^(m -
 1)*(Simp[c*d*f - a*e*g + (g*c*d + c*e*f)*x, x]/(a + c*x^2)), x], x] /; Fre
eQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && FractionQ[m] && GtQ[m
```

```
, 0]
```

Rule 841

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec(c+dx)(a+b\sin(c+dx))^{5/2}\,dx = \frac{b\text{Subst}\left(\int \frac{(a+x)^{5/2}}{b^2-x^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= -\frac{2b(a+b\sin(c+dx))^{3/2}}{3d} - \frac{b\text{Subst}\left(\int \frac{\sqrt{a+x}\,(-a^2-b^2-2ax)}{b^2-x^2}\,dx, x,\right.}{d}$$

$$= -\frac{4ab\sqrt{a+b\sin(c+dx)}}{d} - \frac{2b(a+b\sin(c+dx))^{3/2}}{3d} + \frac{b\text{Subst}\left(\int\right.}{}$$

$$= -\frac{4ab\sqrt{a+b\sin(c+dx)}}{d} - \frac{2b(a+b\sin(c+dx))^{3/2}}{3d} + \frac{(2b)\text{Subst}}{}$$

$$= -\frac{4ab\sqrt{a+b\sin(c+dx)}}{d} - \frac{2b(a+b\sin(c+dx))^{3/2}}{3d} - \frac{(a-b)^3\text{Su}}{}$$

$$= -\frac{(a-b)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{d} + \frac{(a+b)^{5/2}\tanh^{-1}\left(\right.}{}$$

Mathematica [A]
time = 0.16, size = 105, normalized size = 0.90

$$\frac{-3(a-b)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)+3(a+b)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)-2b\sqrt{a+b\sin(c+dx)}\,(7a+b\sin(c+dx))}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-3*(a - b)^(5/2)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a - b]] + 3*(a + b)
^(5/2)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a + b]] - 2*b*Sqrt[a + b*Sin[c
 + d*x]]*(7*a + b*Sin[c + d*x]))/(3*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 285 vs. $2(99) = 198$.
time = 1.58, size = 286, normalized size = 2.44

method	result
default	$-\frac{2b(a+b\sin(dx+c))^{\frac{3}{2}}}{3}-4ba\sqrt{a+b\sin(dx+c)}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sin(dx+c)}}{\sqrt{a+b}}\right)a^3}{\sqrt{a+b}}+\frac{3b\operatorname{arctanh}\left(\frac{\sqrt{a+b\sin(dx+}}{\sqrt{a+b}}\right.}{\sqrt{a+b}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-2/3*b*(a+b*sin(d*x+c))^(3/2)-4*b*a*(a+b*sin(d*x+c))^(1/2)+1/(a+b)^(1/2)*a
rctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))*a^3+3*b/(a+b)^(1/2)*arctanh((a+b
*sin(d*x+c))^(1/2)/(a+b)^(1/2))*a^2+3*b^2/(a+b)^(1/2)*arctanh((a+b*sin(d*x+
c))^(1/2)/(a+b)^(1/2))*a+b^3/(a+b)^(1/2)*arctanh((a+b*sin(d*x+c))^(1/2)/(a+
b)^(1/2))+1/(-a+b)^(1/2)*arctan((a+b*sin(d*x+c))^(1/2)/(-a+b)^(1/2))*a^3-3*
b/(-a+b)^(1/2)*arctan((a+b*sin(d*x+c))^(1/2)/(-a+b)^(1/2))*a^2+3*b^2/(-a+b)
^(1/2)*arctan((a+b*sin(d*x+c))^(1/2)/(-a+b)^(1/2))*a-b^3/(-a+b)^(1/2)*arcta
n((a+b*sin(d*x+c))^(1/2)/(-a+b)^(1/2)))/d
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for more
 detail
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
[Out] integral((2*a*b*sec(d*x + c)*sin(d*x + c) - (b^2*cos(d*x + c)^2 - a^2 - b^2
)*sec(d*x + c))*sqrt(b*sin(d*x + c) + a), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))**(5/2),x)
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))^(5/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\sin(c+d\,x))^{5/2}}{\cos(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^(5/2)/cos(c + d*x),x)
[Out] int((a + b*sin(c + d*x))^(5/2)/cos(c + d*x), x)
```

3.498 $\int \sec^3(c+dx)(a+b\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=155

$$-\frac{(a-b)^{3/2}(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{4d}+\frac{(2a-3b)(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{4d}+$$

```
[Out] -1/4*(a-b)^(3/2)*(2*a+3*b)*arctanh((a+b*sin(d*x+c))^(1/2)/(a-b)^(1/2))/d+1/
4*(2*a-3*b)*(a+b)^(3/2)*arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))/d+1/2*s
ec(d*x+c)^2*(b+a*sin(d*x+c))*(a+b*sin(d*x+c))^(3/2)/d+1/2*a*b*(a+b*sin(d*x+
c))^(1/2)/d
```

Rubi [A]
time = 0.18, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2747, 753, 839, 841, 1180, 212}

$$\frac{ab\sqrt{a+b\sin(c+dx)}}{2d}-\frac{(a-b)^{3/2}(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{4d}+\frac{(2a-3b)(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{4d}+\frac{\sec^2(c+dx)(a\sin(c+dx)+b)(a+b\sin(c+dx))^{3/2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] -1/4*((a - b)^(3/2)*(2*a + 3*b)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a - b
]])/d + ((2*a - 3*b)*(a + b)^(3/2)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a
+ b]])/(4*d) + (a*b*Sqrt[a + b*Sin[c + d*x]])/(2*d) + (Sec[c + d*x]^2*(b +
a*Sin[c + d*x])*(a + b*Sin[c + d*x])^(3/2))/(2*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 753

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m - 1)*(a*e - c*d*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 839

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Simp[g*((d + e*x)^m/(c*m)), x] + Dist[1/c, Int[(d + e*x)^(m -
```

```
 1)*(Simp[c*d*f - a*e*g + (g*c*d + c*e*f)*x, x]/(a + c*x^2)), x], x] /; Fre
eQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && FractionQ[m] && GtQ[m
, 0]
```

Rule 841

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^3(c+dx)(a+b\sin(c+dx))^{5/2}\,dx = \frac{b^3\text{Subst}\left(\int \frac{(a+x)^{5/2}}{(b^2-x^2)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^2(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^{3/2}}{2d} - \frac{b\text{Subst}\left(\int\right.}{}$$

$$= \frac{ab\sqrt{a+b\sin(c+dx)}}{2d} + \frac{\sec^2(c+dx)(b+a\sin(c+dx))(a+b\sin}{2d}$$

$$= \frac{ab\sqrt{a+b\sin(c+dx)}}{2d} + \frac{\sec^2(c+dx)(b+a\sin(c+dx))(a+b\sin}{2d}$$

$$= \frac{ab\sqrt{a+b\sin(c+dx)}}{2d} + \frac{\sec^2(c+dx)(b+a\sin(c+dx))(a+b\sin}{2d}$$

$$= -\frac{(a-b)^{3/2}(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{4d} + \frac{(2a-3b)(}{}$$

Mathematica [A]
time = 0.89, size = 147, normalized size = 0.95

$$\frac{-\sqrt{a-b}\,(2a^2+ab-3b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)+\sqrt{a+b}\,(2a^2-ab-3b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)+2\sec^2(c+dx)\sqrt{a+b\sin(c+dx)}\,(2ab+(a^2+b^2)\sin(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-(Sqrt[a - b]*(2*a^2 + a*b - 3*b^2)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[
a - b]]) + Sqrt[a + b]*(2*a^2 - a*b - 3*b^2)*ArcTanh[Sqrt[a + b*Sin[c + d*x
]]/Sqrt[a + b]] + 2*Sec[c + d*x]^2*Sqrt[a + b*Sin[c + d*x]]*(2*a*b + (a^2 +
 b^2)*Sin[c + d*x]))/(4*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 355 vs. $2(131) = 262$.
time = 2.11, size = 356, normalized size = 2.30

method	result
default	$\frac{2\sin(dx+c)\sqrt{a+b}\,\sqrt{-a+b}\,\sqrt{a+b\sin(dx+c)}\,(a^2+b^2)-\left(-2\arctan\left(\frac{\sqrt{a+b\sin(dx+c)}}{\sqrt{-a+b}}\right)a^3\sqrt{a+b}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+b*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(2*sin(d*x+c)*(a+b)^(1/2)*(-a+b)^(1/2)*(a+b*sin(d*x+c))^(1/2)*(a^2+b^2)
-(-2*arctan((a+b*sin(d*x+c))^(1/2)/(-a+b)^(1/2))*a^3*(a+b)^(1/2)+b*arctan((
a+b*sin(d*x+c))^(1/2)/(-a+b)^(1/2))*a^2*(a+b)^(1/2)+4*b^2*arctan((a+b*sin(d
*x+c))^(1/2)/(-a+b)^(1/2))*a*(a+b)^(1/2)-3*b^3*arctan((a+b*sin(d*x+c))^(1/2
)/(-a+b)^(1/2))*(a+b)^(1/2)-2*arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))*a
^3*(-a+b)^(1/2)-b*arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))*a^2*(-a+b)^(1
/2)+4*b^2*arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))*a*(-a+b)^(1/2)+3*b^3*
arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))*(-a+b)^(1/2))*cos(d*x+c)^2+4*b*
(a+b*sin(d*x+c))^(1/2)*a*(a+b)^(1/2)*(-a+b)^(1/2))/(a+b)^(1/2)/(-a+b)^(1/2)
/cos(d*x+c)^2/d
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 380 vs. 2(131) = 262.
time = 0.65, size = 2071, normalized size = 13.36

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/32*((2*a^2 - a*b - 3*b^2)*sqrt(a + b)*cos(d*x + c)^2*log((b^4*cos(d*x +
 c)^4 + 128*a^4 + 256*a^3*b + 320*a^2*b^2 + 256*a*b^3 + 72*b^4 - 8*(20*a^2*
b^2 + 28*a*b^3 + 9*b^4)*cos(d*x + c)^2 - 8*(16*a^3 + 24*a^2*b + 20*a*b^2 +
8*b^3 - (10*a*b^2 + 7*b^3)*cos(d*x + c)^2 - (b^3*cos(d*x + c)^2 - 24*a^2*b
- 28*a*b^2 - 8*b^3)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)*sqrt(a + b) + 4*
(64*a^3*b + 112*a^2*b^2 + 64*a*b^3 + 14*b^4 - (8*a*b^3 + 7*b^4)*cos(d*x + c
)^2)*sin(d*x + c))/(cos(d*x + c)^4 - 8*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 -
 2)*sin(d*x + c) + 8)) + (2*a^2 + a*b - 3*b^2)*sqrt(a - b)*cos(d*x + c)^2*l
og((b^4*cos(d*x + c)^4 + 128*a^4 - 256*a^3*b + 320*a^2*b^2 - 256*a*b^3 + 72
*b^4 - 8*(20*a^2*b^2 - 28*a*b^3 + 9*b^4)*cos(d*x + c)^2 + 8*(16*a^3 - 24*a^
2*b + 20*a*b^2 - 8*b^3 - (10*a*b^2 - 7*b^3)*cos(d*x + c)^2 - (b^3*cos(d*x +
```

```
c)^2 - 24*a^2*b + 28*a*b^2 - 8*b^3)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)
*sqrt(a - b) + 4*(64*a^3*b - 112*a^2*b^2 + 64*a*b^3 - 14*b^4 - (8*a*b^3 - 7
*b^4)*cos(d*x + c)^2)*sin(d*x + c))/(cos(d*x + c)^4 - 8*cos(d*x + c)^2 - 4*
(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8)) - 16*(2*a*b + (a^2 + b^2)*sin(d*x +
 c))*sqrt(b*sin(d*x + c) + a))/(d*cos(d*x + c)^2), -1/32*(2*(2*a^2 - a*b -
3*b^2)*sqrt(-a - b)*arctan(-1/4*(b^2*cos(d*x + c)^2 - 8*a^2 - 8*a*b - 2*b^2
 - 2*(4*a*b + 3*b^2)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)*sqrt(-a - b)/(2
*a^3 + 3*a^2*b + 2*a*b^2 + b^3 - (a*b^2 + b^3)*cos(d*x + c)^2 + (3*a^2*b +
4*a*b^2 + b^3)*sin(d*x + c)))*cos(d*x + c)^2 + (2*a^2 + a*b - 3*b^2)*sqrt(a
 - b)*cos(d*x + c)^2*log((b^4*cos(d*x + c)^4 + 128*a^4 - 256*a^3*b + 320*a^
2*b^2 - 256*a*b^3 + 72*b^4 - 8*(20*a^2*b^2 - 28*a*b^3 + 9*b^4)*cos(d*x + c)
^2 + 8*(16*a^3 - 24*a^2*b + 20*a*b^2 - 8*b^3 - (10*a*b^2 - 7*b^3)*cos(d*x +
 c)^2 - (b^3*cos(d*x + c)^2 - 24*a^2*b + 28*a*b^2 - 8*b^3)*sin(d*x + c))*sq
rt(b*sin(d*x + c) + a)*sqrt(a - b) + 4*(64*a^3*b - 112*a^2*b^2 + 64*a*b^3 -
 14*b^4 - (8*a*b^3 - 7*b^4)*cos(d*x + c)^2)*sin(d*x + c))/(cos(d*x + c)^4 -
 8*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8)) - 16*(2*a*b +
 (a^2 + b^2)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a))/(d*cos(d*x + c)^2), -1
/32*(2*(2*a^2 + a*b - 3*b^2)*sqrt(-a + b)*arctan(1/4*(b^2*cos(d*x + c)^2 -
8*a^2 + 8*a*b - 2*b^2 - 2*(4*a*b - 3*b^2)*sin(d*x + c))*sqrt(b*sin(d*x + c)
 + a)*sqrt(-a + b)/(2*a^3 - 3*a^2*b + 2*a*b^2 - b^3 - (a*b^2 - b^3)*cos(d*x
 + c)^2 + (3*a^2*b - 4*a*b^2 + b^3)*sin(d*x + c)))*cos(d*x + c)^2 + (2*a^2
- a*b - 3*b^2)*sqrt(a + b)*cos(d*x + c)^2*log((b^4*cos(d*x + c)^4 + 128*a^4
 + 256*a^3*b + 320*a^2*b^2 + 256*a*b^3 + 72*b^4 - 8*(20*a^2*b^2 + 28*a*b^3
+ 9*b^4)*cos(d*x + c)^2 - 8*(16*a^3 + 24*a^2*b + 20*a*b^2 + 8*b^3 - (10*a*b
^2 + 7*b^3)*cos(d*x + c)^2 - (b^3*cos(d*x + c)^2 - 24*a^2*b - 28*a*b^2 - 8*
b^3)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)*sqrt(a + b) + 4*(64*a^3*b + 112
*a^2*b^2 + 64*a*b^3 + 14*b^4 - (8*a*b^3 + 7*b^4)*cos(d*x + c)^2)*sin(d*x +
c))/(cos(d*x + c)^4 - 8*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 - 2)*sin(d*x + c
) + 8)) - 16*(2*a*b + (a^2 + b^2)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a))/(
d*cos(d*x + c)^2), -1/16*((2*a^2 + a*b - 3*b^2)*sqrt(-a + b)*arctan(1/4*(b^
2*cos(d*x + c)^2 - 8*a^2 + 8*a*b - 2*b^2 - 2*(4*a*b - 3*b^2)*sin(d*x + c))*
sqrt(b*sin(d*x + c) + a)*sqrt(-a + b)/(2*a^3 - 3*a^2*b + 2*a*b^2 - b^3 - (a
*b^2 - b^3)*cos(d*x + c)^2 + (3*a^2*b - 4*a*b^2 + b^3)*sin(d*x + c)))*cos(d
*x + c)^2 + (2*a^2 - a*b - 3*b^2)*sqrt(-a - b)*arctan(-1/4*(b^2*cos(d*x + c
)^2 - 8*a^2 - 8*a*b - 2*b^2 - 2*(4*a*b + 3*b^2)*sin(d*x + c))*sqrt(b*sin(d*
x + c) + a)*sqrt(-a - b)/(2*a^3 + 3*a^2*b + 2*a*b^2 + b^3 - (a*b^2 + b^3)*c
os(d*x + c)^2 + (3*a^2*b + 4*a*b^2 + b^3)*sin(d*x + c)))*cos(d*x + c)^2 - 8
*(2*a*b + (a^2 + b^2)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a))/(d*cos(d*x +
c)^2)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,\sin\left(c+d\,x\right)\right)}^{5/2}}{{\cos\left(c+d\,x\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^(5/2)/cos(c + d*x)^3,x)
```

```
[Out] int((a + b*sin(c + d*x))^(5/2)/cos(c + d*x)^3, x)
```

3.499 $\int \sec^5(c+dx)(a+b\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=199

$$-\frac{3\sqrt{a-b}\,(4a^2+2ab-b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{32d}+\frac{3\sqrt{a+b}\,(4a^2-2ab-b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sin}}{\sqrt{a+}}\right.}{32d}$$

[Out] 1/4*sec(d*x+c)^4*(b+a*sin(d*x+c))*(a+b*sin(d*x+c))^(3/2)/d-3/32*(4*a^2+2*a*b-b^2)*arctanh((a+b*sin(d*x+c))^(1/2)/(a-b)^(1/2))*(a-b)^(1/2)/d+3/32*(4*a^2-2*a*b-b^2)*arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))*(a+b)^(1/2)/d+3/16*sec(d*x+c)^2*(a*b+(2*a^2-b^2)*sin(d*x+c))*(a+b*sin(d*x+c))^(1/2)/d

Rubi [A]
time = 0.18, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2747, 753, 835, 841, 1180, 212}

$$-\frac{3\sqrt{a-b}\,(4a^2+2ab-b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{32d}+\frac{3\sqrt{a+b}\,(4a^2-2ab-b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{32d}+\frac{3\sec^2(c+dx)\sqrt{a+b\sin(c+dx)}\,((2a^2-b^2)\sin(c+dx)+ab)}{16d}+\frac{\sec^4(c+dx)(a\sin(c+dx)+b)(a+b\sin(c+dx))^{3/2}}{4d}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d*x]^5*(a + b*Sin[c + d*x])^(5/2),x]

[Out] (-3*Sqrt[a - b]*(4*a^2 + 2*a*b - b^2)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a - b]])/(32*d) + (3*Sqrt[a + b]*(4*a^2 - 2*a*b - b^2)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a + b]])/(32*d) + (Sec[c + d*x]^4*(b + a*Sin[c + d*x])*(a + b*Sin[c + d*x])^(3/2))/(4*d) + (3*Sec[c + d*x]^2*Sqrt[a + b*Sin[c + d*x]]*(a*b + (2*a^2 - b^2)*Sin[c + d*x]))/(16*d)

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 753

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m - 1)*(a*e - c*d*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c
*(p + 1))), x] - Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^
(p + 1)*Simp[a*e*g*m - c*d*f*(2*p + 3) - c*e*f*(m + 2*p + 3)*x, x], x], x]
/; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && G
tQ[m, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 841

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^5(c+dx)(a+b\sin(c+dx))^{5/2}\,dx = \frac{b^5\text{Subst}\left(\int \frac{(a+x)^{5/2}}{(b^2-x^2)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^4(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^{3/2}}{4d} - \frac{b^3\text{Subst}\Big(\int$$

$$= \frac{\sec^4(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^{3/2}}{4d} + \frac{3\sec^2(c+$$

$$= \frac{\sec^4(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^{3/2}}{4d} + \frac{3\sec^2(c+$$

$$= \frac{\sec^4(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^{3/2}}{4d} + \frac{3\sec^2(c+$$

$$= -\frac{3\sqrt{a-b}\,(4a^2+2ab-b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{32d} + \frac{3\sqrt{}}{}$$

Mathematica [A]
time = 3.06, size = 307, normalized size = 1.54

$$-\frac{3\sqrt{a-b}\,(a^2-b^2)^2(4a^2+2ab-b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)-3\sqrt{a+b}\,(a^2-b^2)^2(4a^2-2ab-b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)+8(-a^2+b^2)\sec^4(c+dx)(-b+a\sin(c+dx))(a+b\sin(c+dx))^{7/2}-2\sec^2(c+dx)\left(-7a^2b+b^3+6a^3\sin(c+dx)\right)(a+b\sin(c+dx))^{7/2}-2b\sqrt{a+b\sin(c+dx)}\,(18a^5-16a^3b^2+7ab^4-3a^3b^2\cos(2(c+dx))+b(18a^4-7a^2b^2+b^4)\sin(c+dx))}{32(a^2-b^2)^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] -1/32*(3*Sqrt[a - b]*(a^2 - b^2)^2*(4*a^2 + 2*a*b - b^2)*ArcTanh[Sqrt[a + b
*Sin[c + d*x]]/Sqrt[a - b]] - 3*Sqrt[a + b]*(a^2 - b^2)^2*(4*a^2 - 2*a*b -
b^2)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a + b]] + 8*(-a^2 + b^2)*Sec[c +
 d*x]^4*(-b + a*Sin[c + d*x])*(a + b*Sin[c + d*x])^(7/2) - 2*Sec[c + d*x]^2
*(-7*a^2*b + b^3 + 6*a^3*Sin[c + d*x])*(a + b*Sin[c + d*x])^(7/2) - 2*b*Sqr
t[a + b*Sin[c + d*x]]*(18*a^5 - 16*a^3*b^2 + 7*a*b^4 - 3*a^3*b^2*Cos[2*(c +
 d*x)] + b*(18*a^4 - 7*a^2*b^2 + b^4)*Sin[c + d*x]))/((a^2 - b^2)^2*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 537 vs. 2(175) = 350.
time = 2.39, size = 538, normalized size = 2.70

method	result

default	$\frac{4\sqrt{a+b\sin(dx+c)}\,\sqrt{a+b}\,\sqrt{-a+b}\,b(3ab(\cos^2(dx+c))+8a^2\sin(dx+c)-b^2\sin(dx+c)-3ab)+3b\left(4\operatorname{arctanh}\left(\frac{\sqrt{}}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(a+b*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/32*(4*(a+b*sin(d*x+c))^(1/2)*(a+b)^(1/2)*(-a+b)^(1/2)*b*(3*a*b*cos(d*x+c)
^2+8*a^2*sin(d*x+c)-b^2*sin(d*x+c)-3*a*b)+3*b*(4*arctanh((a+b*sin(d*x+c))^(
1/2)/(a+b)^(1/2))*a^3*(-a+b)^(1/2)+2*b*arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)
^(1/2))*a^2*(-a+b)^(1/2)-3*b^2*arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))*
a*(-a+b)^(1/2)-b^3*arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))*(-a+b)^(1/2)
+4*arctan((a+b*sin(d*x+c))^(1/2)/(-a+b)^(1/2))*a^3*(a+b)^(1/2)-2*b*arctan((
a+b*sin(d*x+c))^(1/2)/(-a+b)^(1/2))*a^2*(a+b)^(1/2)-3*b^2*arctan((a+b*sin(d
*x+c))^(1/2)/(-a+b)^(1/2))*a*(a+b)^(1/2)+b^3*arctan((a+b*sin(d*x+c))^(1/2)/
(-a+b)^(1/2))*(a+b)^(1/2))*cos(d*x+c)^4+2*(a+b*sin(d*x+c))^(1/2)*(a+b)^(1/2
)*(-a+b)^(1/2)*b*(6*a^2*sin(d*x+c)-3*b^2*sin(d*x+c)-7*a*b)*cos(d*x+c)^2-24*
(a+b*sin(d*x+c))^(3/2)*a^2*(a+b)^(1/2)*(-a+b)^(1/2)+12*(a+b*sin(d*x+c))^(3/
2)*b^2*(a+b)^(1/2)*(-a+b)^(1/2)+24*(a+b*sin(d*x+c))^(1/2)*a^3*(a+b)^(1/2)*(
-a+b)^(1/2)+16*a*(a+b*sin(d*x+c))^(1/2)*b^2*(a+b)^(1/2)*(-a+b)^(1/2))/(a+b)
^(1/2)/(-a+b)^(1/2)/b/cos(d*x+c)^4/d
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 420 vs. $2(175) = 350$.
time = 0.66, size = 2229, normalized size = 11.20

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/256*(3*(4*a^2 - 2*a*b - b^2)*sqrt(a + b)*cos(d*x + c)^4*log((b^4*cos(d*
x + c)^4 + 128*a^4 + 256*a^3*b + 320*a^2*b^2 + 256*a*b^3 + 72*b^4 - 8*(20*a
^2*b^2 + 28*a*b^3 + 9*b^4)*cos(d*x + c)^2 - 8*(16*a^3 + 24*a^2*b + 20*a*b^2
 + 8*b^3 - (10*a*b^2 + 7*b^3)*cos(d*x + c)^2 - (b^3*cos(d*x + c)^2 - 24*a^2
*b - 28*a*b^2 - 8*b^3)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)*sqrt(a + b) +
 4*(64*a^3*b + 112*a^2*b^2 + 64*a*b^3 + 14*b^4 - (8*a*b^3 + 7*b^4)*cos(d*x
+ c)^2)*sin(d*x + c))/(cos(d*x + c)^4 - 8*cos(d*x + c)^2 + 4*(cos(d*x + c)^
2 - 2)*sin(d*x + c) + 8)) + 3*(4*a^2 + 2*a*b - b^2)*sqrt(a - b)*cos(d*x + c
)^4*log((b^4*cos(d*x + c)^4 + 128*a^4 - 256*a^3*b + 320*a^2*b^2 - 256*a*b^3
 + 72*b^4 - 8*(20*a^2*b^2 - 28*a*b^3 + 9*b^4)*cos(d*x + c)^2 + 8*(16*a^3 -
24*a^2*b + 20*a*b^2 - 8*b^3 - (10*a*b^2 - 7*b^3)*cos(d*x + c)^2 - (b^3*cos(
d*x + c)^2 - 24*a^2*b + 28*a*b^2 - 8*b^3)*sin(d*x + c))*sqrt(b*sin(d*x + c)
 + a)*sqrt(a - b) + 4*(64*a^3*b - 112*a^2*b^2 + 64*a*b^3 - 14*b^4 - (8*a*b^
3 - 7*b^4)*cos(d*x + c)^2)*sin(d*x + c))/(cos(d*x + c)^4 - 8*cos(d*x + c)^2
 - 4*(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8)) + 16*(a*b*cos(d*x + c)^2 - 8*a
*b - (3*(2*a^2 - b^2)*cos(d*x + c)^2 + 4*a^2 + 4*b^2)*sin(d*x + c))*sqrt(b*
sin(d*x + c) + a))/(d*cos(d*x + c)^4), -1/256*(6*(4*a^2 - 2*a*b - b^2)*sqrt
(-a - b)*arctan(-1/4*(b^2*cos(d*x + c)^2 - 8*a^2 - 8*a*b - 2*b^2 - 2*(4*a*b
 + 3*b^2)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)*sqrt(-a - b)/(2*a^3 + 3*a^
2*b + 2*a*b^2 + b^3 - (a*b^2 + b^3)*cos(d*x + c)^2 + (3*a^2*b + 4*a*b^2 + b
^3)*sin(d*x + c)))*cos(d*x + c)^4 + 3*(4*a^2 + 2*a*b - b^2)*sqrt(a - b)*cos
(d*x + c)^4*log((b^4*cos(d*x + c)^4 + 128*a^4 - 256*a^3*b + 320*a^2*b^2 - 2
56*a*b^3 + 72*b^4 - 8*(20*a^2*b^2 - 28*a*b^3 + 9*b^4)*cos(d*x + c)^2 + 8*(1
6*a^3 - 24*a^2*b + 20*a*b^2 - 8*b^3 - (10*a*b^2 - 7*b^3)*cos(d*x + c)^2 - (
b^3*cos(d*x + c)^2 - 24*a^2*b + 28*a*b^2 - 8*b^3)*sin(d*x + c))*sqrt(b*sin(
d*x + c) + a)*sqrt(a - b) + 4*(64*a^3*b - 112*a^2*b^2 + 64*a*b^3 - 14*b^4 -
 (8*a*b^3 - 7*b^4)*cos(d*x + c)^2)*sin(d*x + c))/(cos(d*x + c)^4 - 8*cos(d*
x + c)^2 - 4*(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8)) + 16*(a*b*cos(d*x + c)
^2 - 8*a*b - (3*(2*a^2 - b^2)*cos(d*x + c)^2 + 4*a^2 + 4*b^2)*sin(d*x + c))
*sqrt(b*sin(d*x + c) + a))/(d*cos(d*x + c)^4), -1/256*(6*(4*a^2 + 2*a*b - b
^2)*sqrt(-a + b)*arctan(1/4*(b^2*cos(d*x + c)^2 - 8*a^2 + 8*a*b - 2*b^2 - 2
*(4*a*b - 3*b^2)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)*sqrt(-a + b)/(2*a^3
 - 3*a^2*b + 2*a*b^2 - b^3 - (a*b^2 - b^3)*cos(d*x + c)^2 + (3*a^2*b - 4*a*
b^2 + b^3)*sin(d*x + c)))*cos(d*x + c)^4 + 3*(4*a^2 - 2*a*b - b^2)*sqrt(a +
 b)*cos(d*x + c)^4*log((b^4*cos(d*x + c)^4 + 128*a^4 + 256*a^3*b + 320*a^2*
b^2 + 256*a*b^3 + 72*b^4 - 8*(20*a^2*b^2 + 28*a*b^3 + 9*b^4)*cos(d*x + c)^2
 - 8*(16*a^3 + 24*a^2*b + 20*a*b^2 + 8*b^3 - (10*a*b^2 + 7*b^3)*cos(d*x + c
)^2 - (b^3*cos(d*x + c)^2 - 24*a^2*b - 28*a*b^2 - 8*b^3)*sin(d*x + c))*sqrt
(b*sin(d*x + c) + a)*sqrt(a + b) + 4*(64*a^3*b + 112*a^2*b^2 + 64*a*b^3 + 1
4*b^4 - (8*a*b^3 + 7*b^4)*cos(d*x + c)^2)*sin(d*x + c))/(cos(d*x + c)^4 - 8
*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8)) + 16*(a*b*cos(d
*x + c)^2 - 8*a*b - (3*(2*a^2 - b^2)*cos(d*x + c)^2 + 4*a^2 + 4*b^2)*sin(d*
x + c))*sqrt(b*sin(d*x + c) + a))/(d*cos(d*x + c)^4), -1/128*(3*(4*a^2 + 2*
a*b - b^2)*sqrt(-a + b)*arctan(1/4*(b^2*cos(d*x + c)^2 - 8*a^2 + 8*a*b - 2*
b^2 - 2*(4*a*b - 3*b^2)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)*sqrt(-a + b)
```

```
/(2*a^3 - 3*a^2*b + 2*a*b^2 - b^3 - (a*b^2 - b^3)*cos(d*x + c)^2 + (3*a^2*b
 - 4*a*b^2 + b^3)*sin(d*x + c)))*cos(d*x + c)^4 + 3*(4*a^2 - 2*a*b - b^2)*s
qrt(-a - b)*arctan(-1/4*(b^2*cos(d*x + c)^2 - 8*a^2 - 8*a*b - 2*b^2 - 2*(4*
a*b + 3*b^2)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)*sqrt(-a - b)/(2*a^3 + 3
*a^2*b + 2*a*b^2 + b^3 - (a*b^2 + b^3)*cos(d*x + c)^2 + (3*a^2*b + 4*a*b^2
+ b^3)*sin(d*x + c)))*cos(d*x + c)^4 + 8*(a*b*cos(d*x + c)^2 - 8*a*b - (3*(
2*a^2 - b^2)*cos(d*x + c)^2 + 4*a^2 + 4*b^2)*sin(d*x + c))*sqrt(b*sin(d*x +
 c) + a))/(d*cos(d*x + c)^4)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,\sin\left(c+d\,x\right)\right)^{5/2}}{{\cos\left(c+d\,x\right)}^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^(5/2)/cos(c + d*x)^5,x)
```

```
[Out] int((a + b*sin(c + d*x))^(5/2)/cos(c + d*x)^5, x)
```

3.500 $\int \cos^4(c+dx)(a+b\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=398

$$-\frac{32ab\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{143d}-\frac{2b\cos^5(c+dx)(a+b\sin(c+dx))^{3/2}}{13d}-\frac{8(20a^6-175a^4b^2-1662a^2b \cdots}{15 \cdots}$$

```
[Out] -2/13*b*cos(d*x+c)^5*(a+b*sin(d*x+c))^(3/2)/d-32/143*a*b*cos(d*x+c)^5*(a+b*
sin(d*x+c))^(1/2)/d+2/3003*cos(d*x+c)^3*(a*(5*a^2+59*b^2)+7*b*(53*a^2+11*b^
2)*sin(d*x+c))*(a+b*sin(d*x+c))^(1/2)/b/d-4/15015*cos(d*x+c)*(4*a*(5*a^4-40
*a^2*b^2-93*b^4)-3*b*(5*a^4+430*a^2*b^2+77*b^4)*sin(d*x+c))*(a+b*sin(d*x+c)
)^(1/2)/b^3/d+8/15015*(20*a^6-175*a^4*b^2-1662*a^2*b^4-231*b^6)*(sin(1/2*c+
1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*
Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/b^4/d/((a+b*sin
(d*x+c))/(a+b))^(1/2)-32/15015*a*(5*a^6-45*a^4*b^2-53*a^2*b^4+93*b^6)*(sin(
1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*
c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/b
^4/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A]
time = 0.59, antiderivative size = 398, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {2771, 2941, 2944, 2831, 2742, 2740, 2734, 2732}

$$\frac{2\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}\,(7b(53a^2+11b^2)\sin(c+dx)+a(5a^2+59b^2))}{3003bd}-\frac{4\cos(c+dx)\sqrt{a+b\sin(c+dx)}\,(4a(5a^4-40a^2b^2-93b^4)-3b(5a^4+430a^2b^2+77b^4)\sin(c+dx))}{15015b^3d}+\frac{32a(5a^6-45a^4b^2-53a^2b^4+93b^6)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{15015b^4d\sqrt{a+b\sin(c+dx)}}-\frac{8(20a^6-175a^4b^2-1662a^2b^4-231b^6)\sqrt{a+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{15015b^4d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}-\frac{2b\cos^5(c+dx)(a+b\sin(c+dx))^{3/2}}{13d}-\frac{32ab\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{143d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-32*a*b*Cos[c + d*x]^5*Sqrt[a + b*Sin[c + d*x]])/(143*d) - (2*b*Cos[c + d*
x]^5*(a + b*Sin[c + d*x])^(3/2))/(13*d) - (8*(20*a^6 - 175*a^4*b^2 - 1662*a
^2*b^4 - 231*b^6)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*S
in[c + d*x]])/(15015*b^4*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + (32*a*(5*a
^6 - 45*a^4*b^2 - 53*a^2*b^4 + 93*b^6)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/
(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(15015*b^4*d*Sqrt[a + b*Sin[c
+ d*x]]) + (2*Cos[c + d*x]^3*Sqrt[a + b*Sin[c + d*x]]*(a*(5*a^2 + 59*b^2) +
 7*b*(53*a^2 + 11*b^2)*Sin[c + d*x]))/(3003*b*d) - (4*Cos[c + d*x]*Sqrt[a +
 b*Sin[c + d*x]]*(4*a*(5*a^4 - 40*a^2*b^2 - 93*b^4) - 3*b*(5*a^4 + 430*a^2*
b^2 + 77*b^4)*Sin[c + d*x]))/(15015*b^3*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2771

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(
a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2941

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp
[a*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x],
x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

```
&& !LtQ[p, -1] && IntegerQ[2*m] && !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Si
mplerQ[c + d*x, a + b*x])
```

Rule 2944

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Dist[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \cos^4(c+dx)(a+b\sin(c+dx))^{5/2}\,dx = -\frac{2b\cos^5(c+dx)(a+b\sin(c+dx))^{3/2}}{13d} + \frac{2}{13}\int \cos^4(c+dx)\sqrt{a+$$

$$= -\frac{32ab\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{143d} - \frac{2b\cos^5(c+dx)(a+b\,\mathrm{si}}{13d}$$

$$= -\frac{32ab\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{143d} - \frac{2b\cos^5(c+dx)(a+b\,\mathrm{si}}{13d}$$

$$= -\frac{32ab\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{143d} - \frac{2b\cos^5(c+dx)(a+b\,\mathrm{si}}{13d}$$

$$= -\frac{32ab\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{143d} - \frac{2b\cos^5(c+dx)(a+b\,\mathrm{si}}{13d}$$

$$= -\frac{32ab\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{143d} - \frac{2b\cos^5(c+dx)(a+b\,\mathrm{si}}{13d}$$

$$= -\frac{32ab\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{143d} - \frac{2b\cos^5(c+dx)(a+b\,\mathrm{si}}{13d}$$

Mathematica [A]

time = 1.27, size = 321, normalized size = 0.81

$$\frac{128\left(b(5a^5b-1450a^3b^3-603ab^5)\,F\left(\frac{1}{4}(-2c+\pi-2dx)\middle|\frac{2b}{a+b}\right)+(20a^6-175a^4b^2-1662a^2b^4-231b^6)\left((a+b)E\left(\frac{1}{4}(-2c+\pi-2dx)\middle|\frac{2b}{a+b}\right)-aF\left(\frac{1}{4}(-2c+\pi-2dx)\middle|\frac{2b}{a+b}\right)\right)\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}-b(a+b\sin(c+dx))\left(4a(320a^4-2710a^2b^2+6453b^4)\cos(c+dx)-10ab^2(20a^2-2599b^2)\cos(3(c+dx))+5670ab^4\cos(5(c+dx))-b(480a^4+56120a^2b^2+4697b^4)\sin(2(c+dx))+140b^3(-53a^2+22b^2)\sin(4(c+dx))+1155b^5\sin(6(c+dx))\right)}{240240b^4d\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (128*(b*(5*a^5*b - 1450*a^3*b^3 - 603*a*b^5)*EllipticF[(-2*c + Pi - 2*d*x)/
4, (2*b)/(a + b)] + (20*a^6 - 175*a^4*b^2 - 1662*a^2*b^4 - 231*b^6)*((a + b
)*EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)] - a*EllipticF[(-2*c + Pi
- 2*d*x)/4, (2*b)/(a + b)]))*Sqrt[(a + b*Sin[c + d*x])/(a + b)] - b*(a + b*
Sin[c + d*x])*(4*a*(320*a^4 - 2710*a^2*b^2 + 6453*b^4)*Cos[c + d*x] - 10*a*
b^2*(20*a^2 - 2599*b^2)*Cos[3*(c + d*x)] + 5670*a*b^4*Cos[5*(c + d*x)] - b*
(480*a^4 + 56120*a^2*b^2 + 4697*b^4)*Sin[2*(c + d*x)] + 140*b^3*(-53*a^2 +
22*b^2)*Sin[4*(c + d*x)] + 1155*b^5*Sin[6*(c + d*x)]))/(240240*b^4*d*Sqrt[a
 + b*Sin[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1618 vs. $2(436) = 872$.
time = 2.60, size = 1619, normalized size = 4.07

method	result	size
default	Expression too large to display	1619

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+b*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/15015*(10*a^5*b^3*sin(d*x+c)-4780*a^3*b^5*sin(d*x+c)+2104*a*b^7*sin(d*x+
c)-3990*a*b^7*sin(d*x+c)^7-4690*a^2*b^6*sin(d*x+c)^6-1880*a^3*b^5*sin(d*x+c
)^5+11290*a*b^7*sin(d*x+c)^5+5*a^4*b^4*sin(d*x+c)^4+14500*a^2*b^6*sin(d*x+c
)^4-10*a^5*b^3*sin(d*x+c)^3+6660*a^3*b^5*sin(d*x+c)^3-9404*a*b^7*sin(d*x+c)
^3-40*a^6*b^2*sin(d*x+c)^2+340*a^4*b^4*sin(d*x+c)^2-11606*a^2*b^6*sin(d*x+c
)^2+40*a^6*b^2+1796*a^2*b^6-345*a^4*b^4-1155*b^8*sin(d*x+c)^8+3080*b^8*sin(
d*x+c)^6-2233*b^8*sin(d*x+c)^4+308*b^8*sin(d*x+c)^2+924*((a+b*sin(d*x+c))/(
a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)
*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b^8-80*((a+b
*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*
b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2)
)*a^8-924*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-
(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-
b)/(a+b))^(1/2))*b^8+5948*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b
/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(
a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b^4-5724*((a+b*sin(d*x+c))/(a-b))^(1/2
)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE
(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^6-60*((a+b*sin(d
```

```
*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b
))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^6*
b^2-720*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1
+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)
/(a+b))^(1/2))*a^5*b^3-5100*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)
*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))
/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b^4-848*((a+b*sin(d*x+c))/(a-b))^(1/
2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Elliptic
F(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^5+80*((a+b*sin(
d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-
b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^7
*b+1488*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1
+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)
/(a+b))^(1/2))*a*b^7+780*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/
(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a
-b))^(1/2),((a-b)/(a+b))^(1/2))*a^6*b^2+4236*((a+b*sin(d*x+c))/(a-b))^(1/2)
*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(
((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^6)/b^5/cos(d*x+c)
/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(5/2)*cos(d*x + c)^4, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.17, size = 633, normalized size = 1.59

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/45045*(2*sqrt(2)*(40*a^7 - 365*a^5*b^2 + 1026*a^3*b^4 + 1347*a*b^6)*sqrt(
I*b)*weierstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b
^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) - 2*I*a)/b) + 2*sqrt(2)
*(40*a^7 - 365*a^5*b^2 + 1026*a^3*b^4 + 1347*a*b^6)*sqrt(-I*b)*weierstrassP
Inverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, 1/3*(3*
b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*I*a)/b) - 6*sqrt(2)*(-20*I*a^6*b +
175*I*a^4*b^3 + 1662*I*a^2*b^5 + 231*I*b^7)*sqrt(I*b)*weierstrassZeta(-4/3*
(4*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, weierstrassPInverse(-
```

```
4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x
+ c) - 3*I*b*sin(d*x + c) - 2*I*a)/b)) - 6*sqrt(2)*(20*I*a^6*b - 175*I*a^4*
b^3 - 1662*I*a^2*b^5 - 231*I*b^7)*sqrt(-I*b)*weierstrassZeta(-4/3*(4*a^2 -
3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, weierstrassPInverse(-4/3*(4*a
^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) +
3*I*b*sin(d*x + c) + 2*I*a)/b)) - 3*(2835*a*b^6*cos(d*x + c)^5 - 5*(5*a^3*b
^4 + 59*a*b^6)*cos(d*x + c)^3 + 8*(5*a^5*b^2 - 40*a^3*b^4 - 93*a*b^6)*cos(d
*x + c) + (1155*b^7*cos(d*x + c)^5 - 35*(53*a^2*b^5 + 11*b^7)*cos(d*x + c)^
3 - 6*(5*a^4*b^3 + 430*a^2*b^5 + 77*b^7)*cos(d*x + c))*sin(d*x + c))*sqrt(b
*sin(d*x + c) + a))/(b^5*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+b*sin(d*x+c))**(5/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3060 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sin(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((b*sin(d*x + c) + a)^(5/2)*cos(d*x + c)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(c+d\,x\right)^4\,{\left(a+b\,\sin\left(c+d\,x\right)\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(a + b*sin(c + d*x))^(5/2),x)
[Out] int(cos(c + d*x)^4*(a + b*sin(c + d*x))^(5/2), x)
```

3.501 $\int \cos^2(c+dx)(a+b\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=299

$$-\frac{8ab\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}}{21d}-\frac{2b\cos^3(c+dx)(a+b\sin(c+dx))^{3/2}}{9d}+\frac{4(5a^4+102a^2b^2+21b^4)\,E\big(\tfrac{1}{2}}{315b^2d\sqrt{}}$$

```
[Out] -2/9*b*cos(d*x+c)^3*(a+b*sin(d*x+c))^(3/2)/d-8/21*a*b*cos(d*x+c)^3*(a+b*sin
(d*x+c))^(1/2)/d+2/315*cos(d*x+c)*(a*(5*a^2+27*b^2)+3*b*(25*a^2+7*b^2)*sin(
d*x+c))*(a+b*sin(d*x+c))^(1/2)/b/d-4/315*(5*a^4+102*a^2*b^2+21*b^4)*(sin(1/
2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+
1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/b^2/d/((a+b
*sin(d*x+c))/(a+b))^(1/2)+4/315*a*(5*a^4+22*a^2*b^2-27*b^4)*(sin(1/2*c+1/4*
Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1
/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/b^2/d/(a+b*
sin(d*x+c))^(1/2)
```

Rubi [A]
time = 0.42, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {2771, 2941, 2944, 2831, 2742, 2740, 2734, 2732}

$$\frac{2\cos(c+dx)\sqrt{a+b\sin(c+dx)}\,(3b(25a^2+7b^2)\sin(c+dx)+a(5a^2+27b^2))}{315bd}-\frac{4a(5a^4+22a^2b^2-27b^4)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\big(\tfrac{1}{2}(c+dx-\tfrac{\pi}{2})\,|\,\tfrac{2b}{a+b}\big)}{315b^2d\sqrt{a+b\sin(c+dx)}}+\frac{4(5a^4+102a^2b^2+21b^4)\sqrt{a+b\sin(c+dx)}\,E\big(\tfrac{1}{2}(c+dx-\tfrac{\pi}{2})\,|\,\tfrac{2b}{a+b}\big)}{315b^2d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}-\frac{2b\cos^3(c+dx)(a+b\sin(c+dx))^{3/2}}{9d}-\frac{8ab\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-8*a*b*Cos[c + d*x]^3*Sqrt[a + b*Sin[c + d*x]])/(21*d) - (2*b*Cos[c + d*x]
^3*(a + b*Sin[c + d*x])^(3/2))/(9*d) + (4*(5*a^4 + 102*a^2*b^2 + 21*b^4)*El
lipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(315*b
^2*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) - (4*a*(5*a^4 + 22*a^2*b^2 - 27*b^
4)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(
a + b)])/(315*b^2*d*Sqrt[a + b*Sin[c + d*x]]) + (2*Cos[c + d*x]*Sqrt[a + b*
Sin[c + d*x]]*(a*(5*a^2 + 27*b^2) + 3*b*(25*a^2 + 7*b^2)*Sin[c + d*x]))/(31
5*b*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2771

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(
a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2941

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp
[a*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x],
x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
&&  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Si
mplerQ[c + d*x, a + b*x])
```

Rule 2944

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Dist[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \cos^2(c+dx)(a+b\sin(c+dx))^{5/2}\,dx = -\frac{2b\cos^3(c+dx)(a+b\sin(c+dx))^{3/2}}{9d} + \frac{2}{9}\int \cos^2(c+dx)\sqrt{a+b$$

$$= -\frac{8ab\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}}{21d} - \frac{2b\cos^3(c+dx)(a+b\sin$$

$$= -\frac{8ab\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}}{21d} - \frac{2b\cos^3(c+dx)(a+b\sin$$

$$= -\frac{8ab\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}}{21d} - \frac{2b\cos^3(c+dx)(a+b\sin$$

$$= -\frac{8ab\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}}{21d} - \frac{2b\cos^3(c+dx)(a+b\sin$$

$$= -\frac{8ab\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}}{21d} - \frac{2b\cos^3(c+dx)(a+b\sin$$

Mathematica [A]
time = 1.01, size = 239, normalized size = 0.80

$$\frac{-16\left(16b(5a^3b+3ab^3)F\left(\frac{1}{4}(-2c+\pi-2dx)\middle|\frac{2b}{a+b}\right)+(5a^4+102a^2b^2+21b^4)\left((a+b)E\left(\frac{1}{4}(-2c+\pi-2dx)\middle|\frac{2b}{a+b}\right)-aF\left(\frac{1}{4}(-2c+\pi-2dx)\middle|\frac{2b}{a+b}\right)\right)\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}+b(a+b\sin(c+dx))\left((40a^3-354ab^2)\cos(c+dx)+2b(-95ab\cos(3(c+dx))+(150a^2+7b^2-35b^2\cos(2(c+dx)))\sin(2(c+dx)))\right)}{1260b^2d\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-16*(16*b*(5*a^3*b + 3*a*b^3)*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a +
b)] + (5*a^4 + 102*a^2*b^2 + 21*b^4)*((a + b)*EllipticE[(-2*c + Pi - 2*d*x)
/4, (2*b)/(a + b)] - a*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]))*Sq
rt[(a + b*Sin[c + d*x])/(a + b)] + b*(a + b*Sin[c + d*x])*((40*a^3 - 354*a*
b^2)*Cos[c + d*x] + 2*b*(-95*a*b*Cos[3*(c + d*x)] + (150*a^2 + 7*b^2 - 35*b
^2*Cos[2*(c + d*x)])*Sin[2*(c + d*x)])))/(1260*b^2*d*Sqrt[a + b*Sin[c + d*x
]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1189 vs. $2(341) = 682$.
time = 2.31, size = 1190, normalized size = 3.98

method	result	size
default	Expression too large to display	1190

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/315*(-35*b^6*sin(d*x+c)^6-130*a*b^5*sin(d*x+c)^5+10*((a+b*sin(d*x+c))/(a-
b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*E
llipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*b+150*((a+
b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))
*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2
))*a^4*b^2+44*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2
)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),
((a-b)/(a+b))^(1/2))*a^3*b^3-108*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+
c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*
x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^4-54*((a+b*sin(d*x+c))/(a-b))
^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Elli
pticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^5-42*((a+b*si
n(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(
a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b
^6-10*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+s
in(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(
a+b))^(1/2))*a^6-194*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b
))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))
^(1/2),((a-b)/(a+b))^(1/2))*a^4*b^2+162*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(s
in(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b
*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^4+42*((a+b*sin(d*x+c))
/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/
2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b^6-170*a^
2*b^4*sin(d*x+c)^4+49*b^6*sin(d*x+c)^4-80*a^3*b^3*sin(d*x+c)^3+212*a*b^5*si
n(d*x+c)^3-5*a^4*b^2*sin(d*x+c)^2+238*a^2*b^4*sin(d*x+c)^2-14*b^6*sin(d*x+c
)^2+80*a^3*b^3*sin(d*x+c)-82*a*b^5*sin(d*x+c)+5*a^4*b^2-68*a^2*b^4)/b^3/cos
(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(5/2)*cos(d*x + c)^2, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 536, normalized size = 1.79

$$-\frac{2\left(2\sqrt{2}\left(5a^{5}-18a^{3}b^{2}-51ab^{4}\right)\sqrt{ib}\,\mathrm{weierstrassPInverse}\left(-\frac{4(4a^{2}-3b^{2})}{3b^{2}},-\frac{8(8ia^{3}-9iab^{2})}{27b^{3}},\frac{3b\cos(dx+c)-3ib\sin(dx+c)-2ia}{3b}\right)+2\sqrt{2}\left(5a^{5}-18a^{3}b^{2}-51ab^{4}\right)\sqrt{-ib}\,\mathrm{weierstrassPInverse}\left(-\frac{4(4a^{2}-3b^{2})}{3b^{2}},-\frac{8(-8ia^{3}+9iab^{2})}{27b^{3}},\frac{3b\cos(dx+c)+3ib\sin(dx+c)+2ia}{3b}\right)+3\sqrt{2}\left(5ia^{4}b+102ia^{2}b^{3}+21ib^{5}\right)\sqrt{ib}\,\mathrm{weierstrassZeta}\left(-\frac{4(4a^{2}-3b^{2})}{3b^{2}},-\frac{8(8ia^{3}-9iab^{2})}{27b^{3}},\mathrm{weierstrassPInverse}\left(-\frac{4(4a^{2}-3b^{2})}{3b^{2}},-\frac{8(8ia^{3}-9iab^{2})}{27b^{3}},\frac{3b\cos(dx+c)-3ib\sin(dx+c)-2ia}{3b}\right)\right)+3\sqrt{2}\left(-5ia^{4}b-102ia^{2}b^{3}-21ib^{5}\right)\sqrt{-ib}\,\mathrm{weierstrassZeta}\left(-\frac{4(4a^{2}-3b^{2})}{3b^{2}},-\frac{8(-8ia^{3}+9iab^{2})}{27b^{3}},\mathrm{weierstrassPInverse}\left(-\frac{4(4a^{2}-3b^{2})}{3b^{2}},-\frac{8(-8ia^{3}+9iab^{2})}{27b^{3}},\frac{3b\cos(dx+c)+3ib\sin(dx+c)+2ia}{3b}\right)\right)+3\left(95ab^{4}\cos(dx+c)^{3}-(5a^{3}b^{2}+27ab^{4})\cos(dx+c)+(35b^{5}\cos(dx+c)^{3}-3(25a^{2}b^{3}+7b^{5})\cos(dx+c))\sin(dx+c)\right)\sqrt{b\sin(dx+c)+a}\right)}{945b^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/945*(2*sqrt(2)*(5*a^5 - 18*a^3*b^2 - 51*a*b^4)*sqrt(I*b)*weierstrassPInv
erse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, 1/3*(3*b*co
s(d*x + c) - 3*I*b*sin(d*x + c) - 2*I*a)/b) + 2*sqrt(2)*(5*a^5 - 18*a^3*b^2
 - 51*a*b^4)*sqrt(-I*b)*weierstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27
*(-8*I*a^3 + 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2
*I*a)/b) + 3*sqrt(2)*(5*I*a^4*b + 102*I*a^2*b^3 + 21*I*b^5)*sqrt(I*b)*weier
strassZeta(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, weier
strassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, 1
/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) - 2*I*a)/b)) + 3*sqrt(2)*(-5*I*a^
4*b - 102*I*a^2*b^3 - 21*I*b^5)*sqrt(-I*b)*weierstrassZeta(-4/3*(4*a^2 - 3*
b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, weierstrassPInverse(-4/3*(4*a^2
 - 3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*
I*b*sin(d*x + c) + 2*I*a)/b)) + 3*(95*a*b^4*cos(d*x + c)^3 - (5*a^3*b^2 + 2
7*a*b^4)*cos(d*x + c) + (35*b^5*cos(d*x + c)^3 - 3*(25*a^2*b^3 + 7*b^5)*cos
(d*x + c))*sin(d*x + c))*sqrt(b*sin(d*x + c) + a))/(b^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\sin\left(c + dx\right)\right)^{\frac{5}{2}}\cos^{2}\left(c + dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Integral((a + b*sin(c + d*x))**(5/2)*cos(c + d*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sin(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((b*sin(d*x + c) + a)^(5/2)*cos(d*x + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(c+d\,x\right)^2\,{\left(a+b\,\sin\left(c+d\,x\right)\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a + b*sin(c + d*x))^(5/2),x)
[Out] int(cos(c + d*x)^2*(a + b*sin(c + d*x))^(5/2), x)
```

3.502 $\int \sec^2(c+dx)(a+b\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=203

$$\frac{ab\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{d}+\frac{\sec(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^{3/2}}{d}-\frac{(a^2+3b^2)\,E\left(\frac{1}{2}(c-\right.}{d\sqrt{}}$$

```
[Out] sec(d*x+c)*(b+a*sin(d*x+c))*(a+b*sin(d*x+c))^(3/2)/d+a*b*cos(d*x+c)*(a+b*si
n(d*x+c))^(1/2)/d+(a^2+3*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c
+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2
))*(a+b*sin(d*x+c))^(1/2)/d/((a+b*sin(d*x+c))/(a+b))^(1/2)-a*(a^2-b^2)*(sin
(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2
*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/
d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {2770, 2832, 2831, 2742, 2740, 2734, 2732}

$$\frac{a(a^2-b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\sin(c+dx)}}-\frac{(a^2+3b^2)\sqrt{a+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}+\frac{ab\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{d}+\frac{\sec(c+dx)(a\sin(c+dx)+b)(a+b\sin(c+dx))^{3/2}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (a*b*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/d + (Sec[c + d*x]*(b + a*Sin[c
+ d*x])*(a + b*Sin[c + d*x])^(3/2))/d - ((a^2 + 3*b^2)*EllipticE[(c - Pi/2
+ d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(d*Sqrt[(a + b*Sin[c + d
*x])/(a + b)]) + (a*(a^2 - b^2)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)
]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(d*Sqrt[a + b*Sin[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] && !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2770

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x
])^(m - 1)*((b + a*Sin[e + f*x])/(f*g*(p + 1))), x] + Dist[1/(g^2*(p + 1)),
 Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a
^2*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g}
, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p
] || IntegerQ[m])
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rubi steps

$$\int \sec^2(c+dx)(a+b\sin(c+dx))^{5/2}\,dx = \frac{\sec(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^{3/2}}{d} - \int \sqrt{a+b\,\mathrm{s}}$$

$$= \frac{ab\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{d} + \frac{\sec(c+dx)(b+a\sin(c+dx}{d}$$

$$= \frac{ab\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{d} + \frac{\sec(c+dx)(b+a\sin(c+dx}{d}$$

$$= \frac{ab\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{d} + \frac{\sec(c+dx)(b+a\sin(c+dx}{d}$$

$$= \frac{ab\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{d} + \frac{\sec(c+dx)(b+a\sin(c+dx}{d}$$

Mathematica [A]
time = 0.87, size = 203, normalized size = 1.00

$$\frac{2a^2b\sec(c+dx)+(a^3+a^2b+3ab^2+3b^3)\,E\left(\frac{1}{4}(-2c+\pi-2dx)\middle|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}-a(a^2-b^2)\,F\left(\frac{1}{4}(-2c+\pi-2dx)\middle|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}+a^3\tan(c+dx)+3ab^2\tan(c+dx)+a^2b\sin(c+dx)\tan(c+dx)+b^3\sin(c+dx)\tan(c+dx)}{d\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (2*a^2*b*Sec[c + d*x] + (a^3 + a^2*b + 3*a*b^2 + 3*b^3)*EllipticE[(-2*c + P
i - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] - a*(a^2 -
b^2)*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*
x])/(a + b)] + a^3*Tan[c + d*x] + 3*a*b^2*Tan[c + d*x] + a^2*b*Sin[c + d*x]
*Tan[c + d*x] + b^3*Sin[c + d*x]*Tan[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1041 vs. $2(257) = 514$.
time = 2.40, size = 1042, normalized size = 5.13

method	result
default	$-\frac{\sqrt{b\left(\cos^2(dx+c)\right)\sin(dx+c)+a\left(\cos^2(dx+c)\right)}\left(\sqrt{-\frac{b\sin(dx+c)}{a+b}+\frac{b}{a+b}}\sqrt{\frac{b\sin(dx+c)}{a-b}+\frac{a}{a-b}}\ \mathrm{Ellip}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+b*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b*(b*cos(d*x+c)^2*sin(d*x+c)+a*cos(d*x+c)^2)^(1/2)*((-b/(a+b)*sin(d*x+c)
+b/(a+b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*EllipticF((b/(a-b)*sin
(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a-b)*sin(d*x+c)-b/(a-b))
^(1/2)*a^3*b+3*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a
-b)*a)^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(
1/2))*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*a^2*b^2-(-b/(a+b)*sin(d*x+c)+b/(a
+b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*EllipticF((b/(a-b)*sin(d*x+
c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2
)*a*b^3-3*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a
)^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))
*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*b^4-(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2
)*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(b/(a
-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*a^4-2*(
-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)
^(1/2),((a-b)/(a+b))^(1/2))*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a-b)*
sin(d*x+c)-b/(a-b))^(1/2)*a^2*b^2+3*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*Ell
ipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(b/(a-b)*s
in(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*b^4+a^2*b^2*
cos(d*x+c)^2+b^4*cos(d*x+c)^2-a^3*b*sin(d*x+c)-3*a*b^3*sin(d*x+c)-3*a^2*b^2
-b^4)/(-(a+b*sin(d*x+c))*(sin(d*x+c)-1)*(1+sin(d*x+c)))^(1/2)/cos(d*x+c)/(a
+b*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(5/2)*sec(d*x + c)^2, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 474, normalized size = 2.33

$$\frac{2\sqrt{2}\,(a^3-3ab^2)\sqrt{ib}\cos(dx+c)\,\mathrm{weierstrassPInverse}\left(-\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(8ia^3-9iab^2)}{27b^3},\frac{3b\cos(dx+c)-3ib\sin(dx+c)-2ia}{3b}\right)+2\sqrt{2}\,(a^3-3ab^2)\sqrt{-ib}\cos(dx+c)\,\mathrm{weierstrassPInverse}\left(-\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(-8ia^3+9iab^2)}{27b^3},\frac{3b\cos(dx+c)+3ib\sin(dx+c)+2ia}{3b}\right)-3\sqrt{2}\,(-ia^2b-3ib^3)\sqrt{ib}\cos(dx+c)\,\mathrm{weierstrassZeta}\left(-\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(8ia^3-9iab^2)}{27b^3},\mathrm{weierstrassPInverse}\left(-\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(8ia^3-9iab^2)}{27b^3},\frac{3b\cos(dx+c)-3ib\sin(dx+c)-2ia}{3b}\right)\right)-3\sqrt{2}\,(ia^2b+3ib^3)\sqrt{-ib}\cos(dx+c)\,\mathrm{weierstrassZeta}\left(-\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(-8ia^3+9iab^2)}{27b^3},\mathrm{weierstrassPInverse}\left(-\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(-8ia^3+9iab^2)}{27b^3},\frac{3b\cos(dx+c)+3ib\sin(dx+c)+2ia}{3b}\right)\right)+6\,(2ab^2+(a^2b+b^3)\sin(dx+c))\sqrt{b\sin(dx+c)+a}}{6\,bd\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/6*(2*sqrt(2)*(a^3 - 3*a*b^2)*sqrt(I*b)*cos(d*x + c)*weierstrassPInverse(-
4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x
+ c) - 3*I*b*sin(d*x + c) - 2*I*a)/b) + 2*sqrt(2)*(a^3 - 3*a*b^2)*sqrt(-I*b
)*cos(d*x + c)*weierstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^
3 + 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*I*a)/b)
- 3*sqrt(2)*(-I*a^2*b - 3*I*b^3)*sqrt(I*b)*cos(d*x + c)*weierstrassZeta(-4/
3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, weierstrassPInverse
(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*
x + c) - 3*I*b*sin(d*x + c) - 2*I*a)/b)) - 3*sqrt(2)*(I*a^2*b + 3*I*b^3)*sq
rt(-I*b)*cos(d*x + c)*weierstrassZeta(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-8*I
*a^3 + 9*I*a*b^2)/b^3, weierstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*
(-8*I*a^3 + 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*
I*a)/b)) + 6*(2*a*b^2 + (a^2*b + b^3)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a
))/(b*d*cos(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\sin\left(c+d\,x\right)\right)}^{5/2}}{{\cos\left(c+d\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^(5/2)/cos(c + d*x)^2,x)
```

```
[Out] int((a + b*sin(c + d*x))^(5/2)/cos(c + d*x)^2, x)
```

3.503 $\int \sec^4(c+dx)(a+b\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=238

$$\frac{\sec^3(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^{3/2}}{3d}-\frac{(4a^2-3b^2)\,E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)|\frac{2b}{a+b}\right)\sqrt{a+b\sin(c+d}}{6d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}$$

```
[Out] 1/3*sec(d*x+c)^3*(b+a*sin(d*x+c))*(a+b*sin(d*x+c))^(3/2)/d+1/6*sec(d*x+c)*(
a*b+(4*a^2-3*b^2)*sin(d*x+c))*(a+b*sin(d*x+c))^(1/2)/d+1/6*(4*a^2-3*b^2)*(s
in(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1
/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/d/((a+
b*sin(d*x+c))/(a+b))^(1/2)-2/3*a*(a^2-b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1
/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(
b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {2770, 2940, 2831, 2742, 2740, 2734, 2732}

$$\frac{\sec(c+dx)\sqrt{a+b\sin(c+dx)}\left((4a^2-3b^2)\sin(c+dx)+ab\right)}{6d}+\frac{2a(a^2-b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{3d\sqrt{a+b\sin(c+dx)}}-\frac{(4a^2-3b^2)\sqrt{a+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{6d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}+\frac{\sec^3(c+dx)(a\sin(c+dx)+b)(a+b\sin(c+dx))^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (Sec[c + d*x]^3*(b + a*Sin[c + d*x])*(a + b*Sin[c + d*x])^(3/2))/(3*d) - ((
4*a^2 - 3*b^2)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[
c + d*x]])/(6*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + (2*a*(a^2 - b^2)*Elli
pticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)]
)/(3*d*Sqrt[a + b*Sin[c + d*x]]) + (Sec[c + d*x]*Sqrt[a + b*Sin[c + d*x]]*(
a*b + (4*a^2 - 3*b^2)*Sin[c + d*x]))/(6*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
0] && !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && !GtQ[a + b, 0]
```

Rule 2770

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x
])^(m - 1)*((b + a*Sin[e + f*x])/(f*g*(p + 1))), x] + Dist[1/(g^2*(p + 1)),
 Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a
^2*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g}
, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p
] || IntegerQ[m])
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2940

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(g*
Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x])^m*((d + c*Sin[e + f*x])/(f*g*(p
 + 1))), x] + Dist[1/(g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin
[e + f*x])^(m - 1)*Simp[a*c*(p + 2) + b*d*m + b*c*(m + p + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && GtQ[
m, 0] && LtQ[p, -1] && IntegerQ[2*m] && !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] &
& SimplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int \sec^4(c+dx)(a+b\sin(c+dx))^{5/2}\,dx = \frac{\sec^3(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^{3/2}}{3d} - \frac{1}{3}\int \sec^2($$

$$= \frac{\sec^3(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^{3/2}}{3d} + \frac{\sec(c+d}{}$$

$$= \frac{\sec^3(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^{3/2}}{3d} + \frac{\sec(c+d}{}$$

$$= \frac{\sec^3(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^{3/2}}{3d} + \frac{\sec(c+d}{}$$

$$= \frac{\sec^3(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^{3/2}}{3d} - \frac{(4a^2-3b}{}$$

Mathematica [A]
time = 3.30, size = 259, normalized size = 1.09

$$\frac{(4a^3+4a^2b-3ab^2-3b^3)\,E\left(\frac{1}{4}(-2c+\pi-2dx)\middle|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}} - 4a(a^2-b^2)\,F\left(\frac{1}{4}(-2c+\pi-2dx)\middle|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}} + \frac{1}{8}\sec^3(c+dx)\,(40a^2b+5b^3-4(3a^2b+2b^3)\cos(2(c+dx))+(-4a^2b+3b^3)\cos(4(c+dx))+24a^3\sin(c+dx)+40ab^2\sin(c+dx)+8a^3\sin(3(c+dx))-8ab^2\sin(3(c+dx)))}{6d\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] ((4*a^3 + 4*a^2*b - 3*a*b^2 - 3*b^3)*EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b)
/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] - 4*a*(a^2 - b^2)*EllipticF[(-
2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] + (S
ec[c + d*x]^3*(40*a^2*b + 5*b^3 - 4*(3*a^2*b + 2*b^3)*Cos[2*(c + d*x)] + (-
4*a^2*b + 3*b^3)*Cos[4*(c + d*x)] + 24*a^3*Sin[c + d*x] + 40*a*b^2*Sin[c +
d*x] + 8*a^3*Sin[3*(c + d*x)] - 8*a*b^2*Sin[3*(c + d*x)]))/8)/(6*d*Sqrt[a +
 b*Sin[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1248 vs. $2(284) = 568$.
time = 3.03, size = 1249, normalized size = 5.25

method	result	size
default	Expression too large to display	1249

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+b*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(-4*(b*cos(d*x+c)^2*sin(d*x+c)+a*cos(d*x+c)^2)^(1/2)*a*b*(a^2-b^2)*sin(
d*x+c)*cos(d*x+c)^2-2*(b*cos(d*x+c)^2*sin(d*x+c)+a*cos(d*x+c)^2)^(1/2)*a*b*
(a^2+3*b^2)*sin(d*x+c)+(b*cos(d*x+c)^2*sin(d*x+c)+a*cos(d*x+c)^2)^(1/2)*b^2
*(4*a^2-3*b^2)*cos(d*x+c)^4+(b*cos(d*x+c)^2*sin(d*x+c)+a*cos(d*x+c)^2)^(1/2
)*(4*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/
2)*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/
(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*a^3*b-3*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)
*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)
*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*a^2*b^2-
4*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*
EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a-
b)*sin(d*x+c)-b/(a-b))^(1/2)*a*b^3+3*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(b
/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)
^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*b^4-4*(-b/(
a+b)*sin(d*x+c)+b/(a+b))^(1/2)*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/
2),((a-b)/(a+b))^(1/2))*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a-b)*sin(
d*x+c)-b/(a-b))^(1/2)*a^4+7*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*EllipticE((
b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(b/(a-b)*sin(d*x+c
)+1/(a-b)*a)^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*a^2*b^2-3*(-b/(a+b)*
sin(d*x+c)+b/(a+b))^(1/2)*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((
a-b)/(a+b))^(1/2))*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a-b)*sin(d*x+c
)-b/(a-b))^(1/2)*b^4-a^2*b^2+5*b^4)*cos(d*x+c)^2-6*(b*cos(d*x+c)^2*sin(d*x+
c)+a*cos(d*x+c)^2)^(1/2)*a^2*b^2-2*(b*cos(d*x+c)^2*sin(d*x+c)+a*cos(d*x+c)^
2)^(1/2)*b^4)/(1+sin(d*x+c))/(-(a+b*sin(d*x+c))*(sin(d*x+c)-1)*(1+sin(d*x+c
)))^(1/2)/(sin(d*x+c)-1)/b/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(5/2)*sec(d*x + c)^4, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 522, normalized size = 2.19

$$\frac{\sqrt{2}\,(8a^3-9ab^2)\sqrt{ib}\,\cos(dx+c)^3\,\mathrm{weierstrassPInverse}\left(-\frac{4(4a^2-3b^2)}{3b^2}, -\frac{8(8ia^3-9iab^2)}{27b^3}, \frac{3b\cos(dx+c)-3ib\sin(dx+c)-2ia}{3b}\right) + \sqrt{2}\,(8a^3-9ab^2)\sqrt{-ib}\,\cos(dx+c)^3\,\mathrm{weierstrassPInverse}\left(-\frac{4(4a^2-3b^2)}{3b^2}, -\frac{8(-8ia^3+9iab^2)}{27b^3}, \frac{3b\cos(dx+c)+3ib\sin(dx+c)+2ia}{3b}\right) - 3\sqrt{2}\,(-4ia^2b+3ib^3)\sqrt{ib}\,\cos(dx+c)^3\,\mathrm{weierstrassZeta}\left(-\frac{4(4a^2-3b^2)}{3b^2}, -\frac{8(8ia^3-9iab^2)}{27b^3}, \mathrm{weierstrassPInverse}\left(-\frac{4(4a^2-3b^2)}{3b^2}, -\frac{8(8ia^3-9iab^2)}{27b^3}, \frac{3b\cos(dx+c)-3ib\sin(dx+c)-2ia}{3b}\right)\right) - 3\sqrt{2}\,(4ia^2b-3ib^3)\sqrt{-ib}\,\cos(dx+c)^3\,\mathrm{weierstrassZeta}\left(-\frac{4(4a^2-3b^2)}{3b^2}, -\frac{8(-8ia^3+9iab^2)}{27b^3}, \mathrm{weierstrassPInverse}\left(-\frac{4(4a^2-3b^2)}{3b^2}, -\frac{8(-8ia^3+9iab^2)}{27b^3}, \frac{3b\cos(dx+c)+3ib\sin(dx+c)+2ia}{3b}\right)\right) - 6\left(ab^2\cos(dx+c)^2 - 4ab^2 - \left(2a^2b+2b^3+(4a^2b-3b^3)\cos(dx+c)^2\right)\sin(dx+c)\right)\sqrt{b\sin(dx+c)+a}}{36\,bd\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/36*(sqrt(2)*(8*a^3 - 9*a*b^2)*sqrt(I*b)*cos(d*x + c)^3*weierstrassPInvers
e(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d
*x + c) - 3*I*b*sin(d*x + c) - 2*I*a)/b) + sqrt(2)*(8*a^3 - 9*a*b^2)*sqrt(-
I*b)*cos(d*x + c)^3*weierstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-8
*I*a^3 + 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*I*a
)/b) - 3*sqrt(2)*(-4*I*a^2*b + 3*I*b^3)*sqrt(I*b)*cos(d*x + c)^3*weierstras
sZeta(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, weierstras
sPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, 1/3*(3
*b*cos(d*x + c) - 3*I*b*sin(d*x + c) - 2*I*a)/b)) - 3*sqrt(2)*(4*I*a^2*b -
3*I*b^3)*sqrt(-I*b)*cos(d*x + c)^3*weierstrassZeta(-4/3*(4*a^2 - 3*b^2)/b^2
, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, weierstrassPInverse(-4/3*(4*a^2 - 3*b^2
)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(
d*x + c) + 2*I*a)/b)) - 6*(a*b^2*cos(d*x + c)^2 - 4*a*b^2 - (2*a^2*b + 2*b^
3 + (4*a^2*b - 3*b^3)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a
))/(b*d*cos(d*x + c)^3)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\sin(c+d\,x))^{5/2}}{\cos(c+d\,x)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^(5/2)/cos(c + d*x)^4,x)
```

```
[Out] int((a + b*sin(c + d*x))^(5/2)/cos(c + d*x)^4, x)
```

3.504 $\int \sec^6(c+dx)(a+b\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=322

$$\frac{\sec^5(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^{3/2}}{5d}-\frac{(32a^2-9b^2)\,E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\frac{2b}{a+b}\right)\sqrt{a+b\sin(c+dx)}}{60d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}$$

```
[Out] 1/5*sec(d*x+c)^5*(b+a*sin(d*x+c))*(a+b*sin(d*x+c))^(3/2)/d+1/30*sec(d*x+c)^
3*(5*a*b+(8*a^2-3*b^2)*sin(d*x+c))*(a+b*sin(d*x+c))^(1/2)/d-1/60*sec(d*x+c)
*(8*a*b*(a^2-b^2)-(32*a^4-41*a^2*b^2+9*b^4)*sin(d*x+c))*(a+b*sin(d*x+c))^(1
/2)/(a^2-b^2)/d+1/60*(32*a^2-9*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin
(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b)
)^(1/2))*(a+b*sin(d*x+c))^(1/2)/d/((a+b*sin(d*x+c))/(a+b))^(1/2)-1/60*a*(32
*a^2-17*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*
EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+
c))/(a+b))^(1/2)/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A]

time = 0.42, antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {2770, 2940, 2945, 2831, 2742, 2740, 2734, 2732}

$$\frac{\sec^3(c+dx)\sqrt{a+b\sin(c+dx)}\left((8a^2-3b^2)\sin(c+dx)+5ab\right)}{30d}+\frac{a(32a^2-17b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|\frac{2b}{a+b}\right)}{60d\sqrt{a+b\sin(c+dx)}}-\frac{(32a^2-9b^2)\sqrt{a+b\sin(c+dx)}E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|\frac{2b}{a+b}\right)}{60d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}-\frac{\sec(c+dx)\sqrt{a+b\sin(c+dx)}\left(8ab(a^2-b^2)-(32a^4-41a^2b^2+9b^4)\sin(c+dx)\right)}{60d(a^2-b^2)}+\frac{\sec^5(c+dx)(a\sin(c+dx)+b)(a+b\sin(c+dx))^{3/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (Sec[c + d*x]^5*(b + a*Sin[c + d*x])*(a + b*Sin[c + d*x])^(3/2))/(5*d) - ((
32*a^2 - 9*b^2)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin
[c + d*x]])/(60*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + (a*(32*a^2 - 17*b^2
)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a
 + b)])/(60*d*Sqrt[a + b*Sin[c + d*x]]) + (Sec[c + d*x]^3*Sqrt[a + b*Sin[c
+ d*x]]*(5*a*b + (8*a^2 - 3*b^2)*Sin[c + d*x]))/(30*d) - (Sec[c + d*x]*Sqrt
[a + b*Sin[c + d*x]]*(8*a*b*(a^2 - b^2) - (32*a^4 - 41*a^2*b^2 + 9*b^4)*Sin
[c + d*x]))/(60*(a^2 - b^2)*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2770

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x
])^(m - 1)*((b + a*Sin[e + f*x])/(f*g*(p + 1))), x] + Dist[1/(g^2*(p + 1)),
 Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a
^2*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g}
, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p
] || IntegerQ[m])
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2940

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(g*
Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x])^m*((d + c*Sin[e + f*x])/(f*g*(p
 + 1))), x] + Dist[1/(g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin
[e + f*x])^(m - 1)*Simp[a*c*(p + 2) + b*d*m + b*c*(m + p + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && GtQ[
m, 0] && LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] &
& SimplerQ[c + d*x, a + b*x])
```

Rule 2945

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \sec^6(c+dx)(a+b\sin(c+dx))^{5/2}\,dx = \frac{\sec^5(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^{3/2}}{5d} - \frac{1}{5}\int \sec^4(c$$

$$= \frac{\sec^5(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^{3/2}}{5d} + \frac{\sec^3(c+dx$$

$$= \frac{\sec^5(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^{3/2}}{5d} + \frac{\sec^3(c+dx$$

$$= \frac{\sec^5(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^{3/2}}{5d} + \frac{\sec^3(c+dx$$

$$= \frac{\sec^5(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^{3/2}}{5d} + \frac{\sec^3(c+dx$$

$$= \frac{\sec^5(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^{3/2}}{5d} - \frac{(32a^2-9b$$

Mathematica [A]
time = 6.26, size = 351, normalized size = 1.09

$$\frac{\sqrt{a+b\sin(c+dx)}\left(\frac{1}{60}\sec(c+dx)\left(-8ab+32a^2\sin(c+dx)-9b^2\sin(c+dx)\right)+\frac{1}{30}\sec^3(c+dx)\left(-ab+8a^2\sin(c+dx)-3b^2\sin(c+dx)\right)+\frac{1}{5}\sec^5(c+dx)\left(2ab+a^2\sin(c+dx)+b^2\sin(c+dx)\right)\right)}{d}-\frac{b\left(-\frac{16abF\left(\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}{\sqrt{a+b\sin(c+dx)}}-\frac{(32a^2-9b^2)\left(\frac{2(a+b)E\left(\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}{\sqrt{a+b\sin(c+dx)}}-\frac{2aF\left(\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}{\sqrt{a+b\sin(c+dx)}}\right)}{b}\right)}{120d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (Sqrt[a + b*Sin[c + d*x]]*((Sec[c + d*x]*(-8*a*b + 32*a^2*Sin[c + d*x] - 9*
b^2*Sin[c + d*x]))/60 + (Sec[c + d*x]^3*(-(a*b) + 8*a^2*Sin[c + d*x] - 3*b^
2*Sin[c + d*x]))/30 + (Sec[c + d*x]^5*(2*a*b + a^2*Sin[c + d*x] + b^2*Sin[c
 + d*x]))/5))/d - (b*((-16*a*b*EllipticF[(-c + Pi/2 - d*x)/2, (2*b)/(a + b)
]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]] - ((32*a^2 -
 9*b^2)*((2*(a + b)*EllipticE[(-c + Pi/2 - d*x)/2, (2*b)/(a + b)]*Sqrt[(a +
 b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]] - (2*a*EllipticF[(-c +
Pi/2 - d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/Sqrt[a +
b*Sin[c + d*x]]))/b))/(120*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1359 vs. $2(364) = 728$.
time = 2.75, size = 1360, normalized size = 4.22

method	result	size
default	Expression too large to display	1360

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(a+b*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/60*((b*cos(d*x+c)^2*sin(d*x+c)+a*cos(d*x+c)^2)^(1/2)*a*b*(32*a^2-17*b^2)*
sin(d*x+c)*cos(d*x+c)^4+8*(b*cos(d*x+c)^2*sin(d*x+c)+a*cos(d*x+c)^2)^(1/2)*
a*b*(2*a^2-b^2)*cos(d*x+c)^2*sin(d*x+c)+12*(b*cos(d*x+c)^2*sin(d*x+c)+a*cos
(d*x+c)^2)^(1/2)*a*b*(a^2+3*b^2)*sin(d*x+c)-(b*cos(d*x+c)^2*sin(d*x+c)+a*co
s(d*x+c)^2)^(1/2)*b^2*(32*a^2-9*b^2)*cos(d*x+c)^6+(b*cos(d*x+c)^2*sin(d*x+c
)+a*cos(d*x+c)^2)^(1/2)*(32*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*EllipticE((
b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(b/(a-b)*sin(d*x+c
)+1/(a-b)*a)^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*a^4-41*(-b/(a+b)*sin
(d*x+c)+b/(a+b))^(1/2)*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b
)/(a+b))^(1/2))*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a-b)*sin(d*x+c)-b
/(a-b))^(1/2)*a^2*b^2+9*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*EllipticE((b/(a
-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(b/(a-b)*sin(d*x+c)+1/
(a-b)*a)^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*b^4-32*(-b/(a+b)*sin(d*x
+c)+b/(a+b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*EllipticF((b/(a-b)*
sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a-b)*sin(d*x+c)-b/(a-
b))^(1/2)*a^3*b+24*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(b/(a-b)*sin(d*x+c)+
1/(a-b)*a)^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b
))^(1/2))*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*a^2*b^2+17*(-b/(a+b)*sin(d*x+
c)+b/(a+b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*EllipticF((b/(a-b)*s
in(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a-b)*sin(d*x+c)-b/(a-b
))^(1/2)*a*b^3-9*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/
(a-b)*a)^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))
^(1/2))*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*b^4+8*a^2*b^2-3*b^4)*cos(d*x+c)
^4+2*(b*cos(d*x+c)^2*sin(d*x+c)+a*cos(d*x+c)^2)^(1/2)*b^2*(a^2-9*b^2)*cos(d
```

```
*x+c)^2+36*(b*cos(d*x+c)^2*sin(d*x+c)+a*cos(d*x+c)^2)^(1/2)*a^2*b^2+12*(b*c
os(d*x+c)^2*sin(d*x+c)+a*cos(d*x+c)^2)^(1/2)*b^4)/(1+sin(d*x+c))^2/(-(a+b*s
in(d*x+c))*(sin(d*x+c)-1)*(1+sin(d*x+c)))^(1/2)/(sin(d*x+c)-1)^2/b/cos(d*x+
c)/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+b*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(5/2)*sec(d*x + c)^6, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 561, normalized size = 1.74

$$\frac{2\sqrt{2}\,(32a^3-21ab^2)\sqrt{ib}\cos(dx+c)^5\,\mathrm{weierstrassPInverse}\left(-\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(8ia^3-9iab^2)}{27b^3},\frac{3b\cos(dx+c)-3ib\sin(dx+c)-2ia}{3b}\right)+2\sqrt{2}\,(32a^3-21ab^2)\sqrt{-ib}\cos(dx+c)^5\,\mathrm{weierstrassPInverse}\left(-\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(-8ia^3+9iab^2)}{27b^3},\frac{3b\cos(dx+c)+3ib\sin(dx+c)+2ia}{3b}\right)-3\sqrt{2}\,(-32ia^2b+9ib^3)\sqrt{ib}\cos(dx+c)^5\,\mathrm{weierstrassZeta}\left(-\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(8ia^3-9iab^2)}{27b^3},\mathrm{weierstrassPInverse}\left(-\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(8ia^3-9iab^2)}{27b^3},\frac{3b\cos(dx+c)-3ib\sin(dx+c)-2ia}{3b}\right)\right)-3\sqrt{2}\,(32ia^2b-9ib^3)\sqrt{-ib}\cos(dx+c)^5\,\mathrm{weierstrassZeta}\left(-\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(-8ia^3+9iab^2)}{27b^3},\mathrm{weierstrassPInverse}\left(-\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(-8ia^3+9iab^2)}{27b^3},\frac{3b\cos(dx+c)+3ib\sin(dx+c)+2ia}{3b}\right)\right)-6\left(8ab^2\cos(dx+c)^4+2ab^2\cos(dx+c)^2-24ab^2-\left((32a^2b-9b^3)\cos(dx+c)^4+12a^2b+12b^3+2(8a^2b-3b^3)\cos(dx+c)^2\right)\sin(dx+c)\right)\sqrt{b\sin(dx+c)+a}}{360\,bd\cos(dx+c)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/360*(2*sqrt(2)*(32*a^3 - 21*a*b^2)*sqrt(I*b)*cos(d*x + c)^5*weierstrassPI
nverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, 1/3*(3*b*
cos(d*x + c) - 3*I*b*sin(d*x + c) - 2*I*a)/b) + 2*sqrt(2)*(32*a^3 - 21*a*b^
2)*sqrt(-I*b)*cos(d*x + c)^5*weierstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2,
-8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c
) + 2*I*a)/b) - 3*sqrt(2)*(-32*I*a^2*b + 9*I*b^3)*sqrt(I*b)*cos(d*x + c)^5*
weierstrassZeta(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3,
weierstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b
^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) - 2*I*a)/b)) - 3*sqrt(2)*(32
*I*a^2*b - 9*I*b^3)*sqrt(-I*b)*cos(d*x + c)^5*weierstrassZeta(-4/3*(4*a^2 -
 3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, weierstrassPInverse(-4/3*(4*
a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) +
 3*I*b*sin(d*x + c) + 2*I*a)/b)) - 6*(8*a*b^2*cos(d*x + c)^4 + 2*a*b^2*cos(
d*x + c)^2 - 24*a*b^2 - ((32*a^2*b - 9*b^3)*cos(d*x + c)^4 + 12*a^2*b + 12*
b^3 + 2*(8*a^2*b - 3*b^3)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(b*sin(d*x + c)
 + a))/(b*d*cos(d*x + c)^5)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+b*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^(5/2)/cos(c + d*x)^6,x)
```

```
[Out] \text{Hanged}
```

3.505 $\int \sec^8(c+dx)(a+b\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=439

$$\frac{\sec^7(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^{3/2}}{7d}-\frac{(128a^4-144a^2b^2+21b^4)\,E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\frac{2b}{a+b}\right)\sqrt{a\cdots}}{280\,(a^2-b^2)\,d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}$$

```
[Out] 1/7*sec(d*x+c)^7*(b+a*sin(d*x+c))*(a+b*sin(d*x+c))^(3/2)/d+3/70*sec(d*x+c)^
5*(3*a*b+(4*a^2-b^2)*sin(d*x+c))*(a+b*sin(d*x+c))^(1/2)/d-1/140*sec(d*x+c)^
3*(4*a*b*(a^2-b^2)-(32*a^4-39*a^2*b^2+7*b^4)*sin(d*x+c))*(a+b*sin(d*x+c))^(
1/2)/(a^2-b^2)/d-1/280*sec(d*x+c)*(a*b*(32*a^4-59*a^2*b^2+27*b^4)-(128*a^6-
272*a^4*b^2+165*a^2*b^4-21*b^6)*sin(d*x+c))*(a+b*sin(d*x+c))^(1/2)/(a^2-b^2
)^2/d+1/280*(128*a^4-144*a^2*b^2+21*b^4)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2
)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/
(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/(a^2-b^2)/d/((a+b*sin(d*x+c))/(a+b))^(
1/2)-2/35*a*(8*a^2-3*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4
*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*(
(a+b*sin(d*x+c))/(a+b))^(1/2)/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A]
time = 0.59, antiderivative size = 439, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {2770, 2940, 2945, 2831, 2742, 2740, 2734, 2732}

$$\frac{3\sec^5(c+dx)\sqrt{a+b\sin(c+dx)}\left((4a^2-b^2)\sin(c+dx)+3ab\right)}{70d}+\frac{2a(8a^2-3b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|\frac{2b}{a+b}\right)}{35d\sqrt{a+b\sin(c+dx)}}-\frac{\sec^3(c+dx)\sqrt{a+b\sin(c+dx)}\left(4ab(a^2-b^2)-(32a^4-39a^2b^2+7b^4)\sin(c+dx)\right)}{140d\,(a^2-b^2)}-\frac{(128a^4-144a^2b^2+21b^4)\sqrt{a+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|\frac{2b}{a+b}\right)}{280d\,(a^2-b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}-\frac{\sec(c+dx)\sqrt{a+b\sin(c+dx)}\left(ab(32a^4-59a^2b^2+27b^4)-(128a^6-272a^4b^2+165a^2b^4-21b^6)\sin(c+dx)\right)}{280d\,(a^2-b^2)^2}+\frac{\sec^7(c+dx)(a\sin(c+dx)+b)(a+b\sin(c+dx))^{3/2}}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^8*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (Sec[c + d*x]^7*(b + a*Sin[c + d*x])*(a + b*Sin[c + d*x])^(3/2))/(7*d) - ((
128*a^4 - 144*a^2*b^2 + 21*b^4)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)
]*Sqrt[a + b*Sin[c + d*x]])/(280*(a^2 - b^2)*d*Sqrt[(a + b*Sin[c + d*x])/(a
 + b)]) + (2*a*(8*a^2 - 3*b^2)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]
*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(35*d*Sqrt[a + b*Sin[c + d*x]]) + (3*S
ec[c + d*x]^5*Sqrt[a + b*Sin[c + d*x]]*(3*a*b + (4*a^2 - b^2)*Sin[c + d*x])
)/(70*d) - (Sec[c + d*x]^3*Sqrt[a + b*Sin[c + d*x]]*(4*a*b*(a^2 - b^2) - (3
2*a^4 - 39*a^2*b^2 + 7*b^4)*Sin[c + d*x]))/(140*(a^2 - b^2)*d) - (Sec[c + d
*x]*Sqrt[a + b*Sin[c + d*x]]*(a*b*(32*a^4 - 59*a^2*b^2 + 27*b^4) - (128*a^6
 - 272*a^4*b^2 + 165*a^2*b^4 - 21*b^6)*Sin[c + d*x]))/(280*(a^2 - b^2)^2*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
```

b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2770

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x
])^(m - 1)*((b + a*Sin[e + f*x])/(f*g*(p + 1))), x] + Dist[1/(g^2*(p + 1)),
 Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a
^2*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g}
, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p
] || IntegerQ[m])
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2940

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(g*
Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x])^m*((d + c*Sin[e + f*x])/(f*g*(p
 + 1))), x] + Dist[1/(g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin
[e + f*x])^(m - 1)*Simp[a*c*(p + 2) + b*d*m + b*c*(m + p + 2)*Sin[e + f*x],
```

```
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && GtQ[
m, 0] && LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] &
& SimplerQ[c + d*x, a + b*x])
```

Rule 2945

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \sec^8(c+dx)(a+b\sin(c+dx))^{5/2}\,dx = \frac{\sec^7(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^{3/2}}{7d} - \frac{1}{7}\int \sec^6(c$$

$$= \frac{\sec^7(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^{3/2}}{7d} + \frac{3\sec^5(c+}{}$$

$$= \frac{\sec^7(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^{3/2}}{7d} + \frac{3\sec^5(c+}{}$$

$$= \frac{\sec^7(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^{3/2}}{7d} + \frac{3\sec^5(c+}{}$$

$$= \frac{\sec^7(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^{3/2}}{7d} + \frac{3\sec^5(c+}{}$$

$$= \frac{\sec^7(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^{3/2}}{7d} + \frac{3\sec^5(c+}{}$$

$$= \frac{\sec^7(c+dx)(b+a\sin(c+dx))(a+b\sin(c+dx))^{3/2}}{7d} - \frac{(128a^4-1}{}$$

Mathematica [**A**]

time = 4.20, size = 338, normalized size = 0.77

$$\frac{\frac{\left(\left(128a^4-144a^2b^2+21b^4\right)E\left(\frac{1}{4}(-2c+\pi-2dx)|\frac{2b}{a+b}\right)-16a\left(8a^3-8a^2b-3ab^2+3b^3\right)F\left(\frac{1}{4}(-2c+\pi-2dx)|\frac{2b}{a+b}\right)\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}{a-b}+\frac{\sec(c+dx)(a+b\sin(c+dx))\left(-32a^3b+27ab^3+128a^4\sin(c+dx)-144a^2b^2\sin(c+dx)+21b^4\sin(c+dx)+2\left(a^2-b^2\right)\sec^2(c+dx)\left(-4ab+\left(32a^2-7b^2\right)\sin(c+dx)\right)-4\left(a^2-b^2\right)\sec^4(c+dx)\left(ab+3\left(-4a^2+b^2\right)\sin(c+dx)\right)+40\left(a^2-b^2\right)\sec^6(c+dx)\left(2ab+\left(a^2+b^2\right)\sin(c+dx)\right)\right)}{a^2-b^2}}{280d\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^8*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] ((((128*a^4 - 144*a^2*b^2 + 21*b^4)*EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b)/
(a + b)] - 16*a*(8*a^3 - 8*a^2*b - 3*a*b^2 + 3*b^3)*EllipticF[(-2*c + Pi -
2*d*x)/4, (2*b)/(a + b)])*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(a - b) + (Se
c[c + d*x]*(a + b*Sin[c + d*x])*(-32*a^3*b + 27*a*b^3 + 128*a^4*Sin[c + d*x
] - 144*a^2*b^2*Sin[c + d*x] + 21*b^4*Sin[c + d*x] + 2*(a^2 - b^2)*Sec[c +
d*x]^2*(-4*a*b + (32*a^2 - 7*b^2)*Sin[c + d*x]) - 4*(a^2 - b^2)*Sec[c + d*x
]^4*(a*b + 3*(-4*a^2 + b^2)*Sin[c + d*x]) + 40*(a^2 - b^2)*Sec[c + d*x]^6*(
2*a*b + (a^2 + b^2)*Sin[c + d*x])))/(a^2 - b^2))/(280*d*Sqrt[a + b*Sin[c +
d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1886 vs. $2(477) = 954$.
time = 27.70, size = 1887, normalized size = 4.30

method	result	size
default	Expression too large to display	1887

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^8*(a+b*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/280*(-(-a-b*sin(d*x+c))*cos(d*x+c)^2)^(1/2)/cos(d*x+c)^9/(a+b*sin(d*x+c))
^(3/2)/b/(a^2-b^2)*(2*cos(d*x+c)^4*(b*cos(d*x+c)^2*sin(d*x+c)+a*cos(d*x+c)^
2)^(1/2)*b^2*(4*a^4-5*a^2*b^2+b^4)+40*(b*cos(d*x+c)^2*sin(d*x+c)+a*cos(d*x+
c)^2)^(1/2)*b^2*(3*a^4-2*a^2*b^2-b^4)+4*cos(d*x+c)^2*(b*cos(d*x+c)^2*sin(d*
x+c)+a*cos(d*x+c)^2)^(1/2)*b^2*(a^4-14*a^2*b^2+13*b^4)-cos(d*x+c)^8*(b*cos(
d*x+c)^2*sin(d*x+c)+a*cos(d*x+c)^2)^(1/2)*b^2*(128*a^4-144*a^2*b^2+21*b^4)+
16*cos(d*x+c)^2*(b*cos(d*x+c)^2*sin(d*x+c)+a*cos(d*x+c)^2)^(1/2)*a*b*(3*a^4
-4*a^2*b^2+b^4)*sin(d*x+c)+40*(b*cos(d*x+c)^2*sin(d*x+c)+a*cos(d*x+c)^2)^(1
/2)*a*b*(a^4+2*a^2*b^2-3*b^4)*sin(d*x+c)+16*cos(d*x+c)^6*(b*cos(d*x+c)^2*si
n(d*x+c)+a*cos(d*x+c)^2)^(1/2)*a*b*(8*a^4-11*a^2*b^2+3*b^4)*sin(d*x+c)+2*co
s(d*x+c)^4*(b*cos(d*x+c)^2*sin(d*x+c)+a*cos(d*x+c)^2)^(1/2)*a*b*(32*a^4-43*
a^2*b^2+11*b^4)*sin(d*x+c)+cos(d*x+c)^6*(b*cos(d*x+c)^2*sin(d*x+c)+a*cos(d*
x+c)^2)^(1/2)*(128*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a
+b))^(1/2))*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b
))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*a^6-272*EllipticE((b/(a-b)*si
n(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a+b)*sin(d*x+c)+b/(a+b)
)^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^
(1/2)*a^4*b^2+165*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+
```

```
b))^(1/2))*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b)
)^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*a^2*b^4-21*EllipticE((b/(a-b)*
sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a+b)*sin(d*x+c)+b/(a+
b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a
)^(1/2)*b^6-128*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/
(a-b))^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(
1/2))*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*a^5*b+96*(-b/(a+b)*sin(d*x+c)+b/
(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticF((b/(a-b)*sin(d*x
+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1
/2)*a^4*b^2+176*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/
(a-b))^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(
1/2))*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*a^3*b^3-117*(-b/(a+b)*sin(d*x+c)
+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticF((b/(a-b)*sin(
d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)
^(1/2)*a^2*b^4-48*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-
b/(a-b))^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))
^(1/2))*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*a*b^5+21*(-b/(a+b)*sin(d*x+c)+
b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticF((b/(a-b)*sin(d
*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^
(1/2)*b^6+32*a^4*b^2-39*a^2*b^4+7*b^6))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+b*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(5/2)*sec(d*x + c)^8, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.18, size = 697, normalized size = 1.59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/1680*(sqrt(2)*(256*a^5 - 384*a^3*b^2 + 123*a*b^4)*sqrt(I*b)*cos(d*x + c)^
7*weierstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)
/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) - 2*I*a)/b) + sqrt(2)*(256
*a^5 - 384*a^3*b^2 + 123*a*b^4)*sqrt(-I*b)*cos(d*x + c)^7*weierstrassPInver
se(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, 1/3*(3*b*cos
(d*x + c) + 3*I*b*sin(d*x + c) + 2*I*a)/b) + 3*sqrt(2)*(128*I*a^4*b - 144*I
*a^2*b^3 + 21*I*b^5)*sqrt(I*b)*cos(d*x + c)^7*weierstrassZeta(-4/3*(4*a^2 -
```

```
 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, weierstrassPInverse(-4/3*(4*a
^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3
*I*b*sin(d*x + c) - 2*I*a)/b)) + 3*sqrt(2)*(-128*I*a^4*b + 144*I*a^2*b^3 -
21*I*b^5)*sqrt(-I*b)*cos(d*x + c)^7*weierstrassZeta(-4/3*(4*a^2 - 3*b^2)/b^
2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, weierstrassPInverse(-4/3*(4*a^2 - 3*b^
2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin
(d*x + c) + 2*I*a)/b)) - 6*((32*a^3*b^2 - 27*a*b^4)*cos(d*x + c)^6 - 80*a^3
*b^2 + 80*a*b^4 + 8*(a^3*b^2 - a*b^4)*cos(d*x + c)^4 + 4*(a^3*b^2 - a*b^4)*
cos(d*x + c)^2 - ((128*a^4*b - 144*a^2*b^3 + 21*b^5)*cos(d*x + c)^6 + 40*a^
4*b - 40*b^5 + 2*(32*a^4*b - 39*a^2*b^3 + 7*b^5)*cos(d*x + c)^4 + 12*(4*a^4
*b - 5*a^2*b^3 + b^5)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a
))/((a^2*b - b^3)*d*cos(d*x + c)^7)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**8*(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+b*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^(5/2)/cos(c + d*x)^8,x)
```

```
[Out] \text{Hanged}
```

3.506 $\int \frac{\cos^5(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx$

Optimal. Leaf size=152

$$\frac{2(a^2-b^2)^2\sqrt{a+b\sin(c+dx)}}{b^5d}-\frac{8a(a^2-b^2)(a+b\sin(c+dx))^{3/2}}{3b^5d}+\frac{4(3a^2-b^2)(a+b\sin(c+dx))^{5/2}}{5b^5d}-\frac{8a(c$$

```
[Out] -8/3*a*(a^2-b^2)*(a+b*sin(d*x+c))^(3/2)/b^5/d+4/5*(3*a^2-b^2)*(a+b*sin(d*x+
c))^(5/2)/b^5/d-8/7*a*(a+b*sin(d*x+c))^(7/2)/b^5/d+2/9*(a+b*sin(d*x+c))^(9/
2)/b^5/d+2*(a^2-b^2)^2*(a+b*sin(d*x+c))^(1/2)/b^5/d
```

Rubi [A]
time = 0.07, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2747, 711}

$$\frac{4(3a^2-b^2)(a+b\sin(c+dx))^{5/2}}{5b^5d}-\frac{8a(a^2-b^2)(a+b\sin(c+dx))^{3/2}}{3b^5d}+\frac{2(a^2-b^2)^2\sqrt{a+b\sin(c+dx)}}{b^5d}+\frac{2(a+b\sin(c+dx))^{9/2}}{9b^5d}-\frac{8a(a+b\sin(c+dx))^{7/2}}{7b^5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (2*(a^2 - b^2)^2*Sqrt[a + b*Sin[c + d*x]])/(b^5*d) - (8*a*(a^2 - b^2)*(a +
b*Sin[c + d*x])^(3/2))/(3*b^5*d) + (4*(3*a^2 - b^2)*(a + b*Sin[c + d*x])^(5
/2))/(5*b^5*d) - (8*a*(a + b*Sin[c + d*x])^(7/2))/(7*b^5*d) + (2*(a + b*Sin
[c + d*x])^(9/2))/(9*b^5*d)
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{(b^2-x^2)^2}{\sqrt{a+x}}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{(a^2-b^2)^2}{\sqrt{a+x}} - 4(a^3-ab^2)\sqrt{a+x} + 2(3a^2-b^2)(a+x)^{3/2} - 4a(a+\right.\right.}{b^5 d}$$

$$= \frac{2(a^2-b^2)^2\sqrt{a+b\sin(c+dx)}}{b^5 d} - \frac{8a(a^2-b^2)(a+b\sin(c+dx))^{3/2}}{3b^5 d} + \frac{4(3a^2}{}$$

Mathematica [A]

time = 1.37, size = 161, normalized size = 1.06

$$\frac{a(1024a^4-2496a^2b^2+2121b^4)\sqrt{1+\frac{b\sin(c+dx)}{a}}\left(-1+\sqrt{1+\frac{b\sin(c+dx)}{a}}\right) - b(a+b\sin(c+dx))(-35b^3\cos(4(c+dx)) + 32a(16a^2-37b^2)\sin(c+dx) - 4b\cos(2(c+dx))(-48a^2+91b^2+40ab\sin(c+dx)))}{1260b^5 d\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (a*(1024*a^4 - 2496*a^2*b^2 + 2121*b^4)*Sqrt[1 + (b*Sin[c + d*x])/a]*(-1 +
Sqrt[1 + (b*Sin[c + d*x])/a]) - b*(a + b*Sin[c + d*x])*(-35*b^3*Cos[4*(c +
d*x)] + 32*a*(16*a^2 - 37*b^2)*Sin[c + d*x] - 4*b*Cos[2*(c + d*x)]*(-48*a^2
 + 91*b^2 + 40*a*b*Sin[c + d*x])))/(1260*b^5*d*Sqrt[a + b*Sin[c + d*x]])
```

Maple [A]

time = 2.63, size = 126, normalized size = 0.83

method	result
default	$\frac{2\sqrt{a+b\sin(dx+c)}\,(35b^4(\cos^4(dx+c))+40ab^3(\cos^2(dx+c))\sin(dx+c)-48a^2b^2(\cos^2(dx+c))+56b^4(\cos^2(dx+c))-64a^3b}{315b^5 d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a+b*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/315/b^5*(a+b*sin(d*x+c))^(1/2)*(35*b^4*cos(d*x+c)^4+40*a*b^3*cos(d*x+c)^2
*sin(d*x+c)-48*a^2*b^2*cos(d*x+c)^2+56*b^4*cos(d*x+c)^2-64*a^3*b*sin(d*x+c)
+128*a*b^3*sin(d*x+c)+128*a^4-288*a^2*b^2+224*b^4)/d
```

Maxima [A]

time = 0.30, size = 160, normalized size = 1.05

$$\frac{2\left(315\sqrt{b\sin(dx+c)+a} - \frac{42\left(3(b\sin(dx+c)+a)^{\frac{5}{2}} - 10(b\sin(dx+c)+a)^{\frac{3}{2}}a + 15\sqrt{b\sin(dx+c)+a}\,a^2\right)}{b^2} + \frac{35(b\sin(dx+c)+a)^{\frac{9}{2}} - 180(b\sin(dx+c)+a)^{\frac{7}{2}}a + 378(b\sin(dx+c)+a)^{\frac{5}{2}}a^2 - 420(b\sin(dx+c)+a)^{\frac{3}{2}}a^3 + 315\sqrt{b\sin(dx+c)+a}\,a^4}{b^4}\right)}{315\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/315*(315*sqrt(b*sin(d*x + c) + a) - 42*(3*(b*sin(d*x + c) + a)^(5/2) - 10
*(b*sin(d*x + c) + a)^(3/2)*a + 15*sqrt(b*sin(d*x + c) + a)*a^2)/b^2 + (35*
(b*sin(d*x + c) + a)^(9/2) - 180*(b*sin(d*x + c) + a)^(7/2)*a + 378*(b*sin(
d*x + c) + a)^(5/2)*a^2 - 420*(b*sin(d*x + c) + a)^(3/2)*a^3 + 315*sqrt(b*s
in(d*x + c) + a)*a^4)/b^4)/(b*d)
```

Fricas [A]
time = 0.35, size = 111, normalized size = 0.73

$$\frac{2\left(35\,b^{4}\cos\left(dx+c\right)^{4}+128\,a^{4}-288\,a^{2}b^{2}+224\,b^{4}-8\left(6\,a^{2}b^{2}-7\,b^{4}\right)\cos\left(dx+c\right)^{2}+8\left(5\,ab^{3}\cos\left(dx+c\right)^{2}-8\,a^{3}b+16\,ab^{3}\right)\sin\left(dx+c\right)\right)\sqrt{b\sin\left(dx+c\right)+a}}{315\,b^{5}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*b^4*cos(d*x + c)^4 + 128*a^4 - 288*a^2*b^2 + 224*b^4 - 8*(6*a^2*b
^2 - 7*b^4)*cos(d*x + c)^2 + 8*(5*a*b^3*cos(d*x + c)^2 - 8*a^3*b + 16*a*b^3
)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)/(b^5*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 6.25, size = 161, normalized size = 1.06

$$\frac{2\left(35\left(b\sin\left(dx+c\right)+a\right)^{\frac{9}{2}}-180\left(b\sin\left(dx+c\right)+a\right)^{\frac{7}{2}}a+378\left(b\sin\left(dx+c\right)+a\right)^{\frac{5}{2}}a^{2}-420\left(b\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}a^{3}+315\sqrt{b\sin\left(dx+c\right)+a}\,a^{4}-126\left(b\sin\left(dx+c\right)+a\right)^{\frac{5}{2}}b^{2}+420\left(b\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}ab^{2}-630\sqrt{b\sin\left(dx+c\right)+a}\,a^{2}b^{2}+315\sqrt{b\sin\left(dx+c\right)+a}\,b^{4}\right)}{315\,b^{5}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 2/315*(35*(b*sin(d*x + c) + a)^(9/2) - 180*(b*sin(d*x + c) + a)^(7/2)*a + 3
78*(b*sin(d*x + c) + a)^(5/2)*a^2 - 420*(b*sin(d*x + c) + a)^(3/2)*a^3 + 31
5*sqrt(b*sin(d*x + c) + a)*a^4 - 126*(b*sin(d*x + c) + a)^(5/2)*b^2 + 420*(
b*sin(d*x + c) + a)^(3/2)*a*b^2 - 630*sqrt(b*sin(d*x + c) + a)*a^2*b^2 + 31
5*sqrt(b*sin(d*x + c) + a)*b^4)/(b^5*d)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^5}{\sqrt{a+b\,\sin\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(a + b*sin(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^5/(a + b*sin(c + d*x))^(1/2), x)
```

3.507 $\int \frac{\cos^3(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx$

Optimal. Leaf size=81

$$-\frac{2(a^2-b^2)\sqrt{a+b\sin(c+dx)}}{b^3d}+\frac{4a(a+b\sin(c+dx))^{3/2}}{3b^3d}-\frac{2(a+b\sin(c+dx))^{5/2}}{5b^3d}$$

```
[Out] 4/3*a*(a+b*sin(d*x+c))^(3/2)/b^3/d-2/5*(a+b*sin(d*x+c))^(5/2)/b^3/d-2*(a^2-
b^2)*(a+b*sin(d*x+c))^(1/2)/b^3/d
```

Rubi [A]
time = 0.05, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2747, 711}

$$-\frac{2(a^2-b^2)\sqrt{a+b\sin(c+dx)}}{b^3d}-\frac{2(a+b\sin(c+dx))^{5/2}}{5b^3d}+\frac{4a(a+b\sin(c+dx))^{3/2}}{3b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (-2*(a^2 - b^2)*Sqrt[a + b*Sin[c + d*x]])/(b^3*d) + (4*a*(a + b*Sin[c + d*x
])^(3/2))/(3*b^3*d) - (2*(a + b*Sin[c + d*x])^(5/2))/(5*b^3*d)
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{b^2-x^2}{\sqrt{a+x}}\,dx, x, b\sin(c+dx)\right)}{b^3d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{-a^2+b^2}{\sqrt{a+x}} + 2a\sqrt{a+x} - (a+x)^{3/2}\right)\,dx, x, b\sin(c+dx)\right)}{b^3d}$$
$$= -\frac{2(a^2-b^2)\sqrt{a+b\sin(c+dx)}}{b^3d} + \frac{4a(a+b\sin(c+dx))^{3/2}}{3b^3d} - \frac{2(a+b\sin(c+}{5b^3d}$$

Mathematica [A]
time = 0.65, size = 108, normalized size = 1.33

$$\frac{b(3b\cos(2(c+dx)) + 8a\sin(c+dx))(a+b\sin(c+dx)) - a(16a^2-27b^2)\sqrt{1+\frac{b\sin(c+dx)}{a}}\left(-1+\sqrt{1+\frac{b\sin(c+dx)}{a}}\right)}{15b^3d\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (b*(3*b*Cos[2*(c + d*x)] + 8*a*Sin[c + d*x])*(a + b*Sin[c + d*x]) - a*(16*a
^2 - 27*b^2)*Sqrt[1 + (b*Sin[c + d*x])/a]*(-1 + Sqrt[1 + (b*Sin[c + d*x])/a
]))/(15*b^3*d*Sqrt[a + b*Sin[c + d*x]])
```

Maple [A]
time = 1.29, size = 55, normalized size = 0.68

method	result	size
default	$-\frac{2\sqrt{a+b\sin(dx+c)}\,(-3b^2(\cos^2(dx+c))-4ab\sin(dx+c)+8a^2-12b^2)}{15b^3d}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+b*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/15/b^3*(a+b*sin(d*x+c))^(1/2)*(-3*b^2*cos(d*x+c)^2-4*a*b*sin(d*x+c)+8*a^
2-12*b^2)/d
```

Maxima [A]
time = 0.27, size = 75, normalized size = 0.93

$$\frac{2\left(15\sqrt{b\sin(dx+c)+a} - \frac{3(b\sin(dx+c)+a)^{\frac{5}{2}} - 10(b\sin(dx+c)+a)^{\frac{3}{2}}a + 15\sqrt{b\sin(dx+c)+a}\,a^2}{b^2}\right)}{15\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/15*(15*sqrt(b*sin(d*x + c) + a) - (3*(b*sin(d*x + c) + a)^(5/2) - 10*(b*s
in(d*x + c) + a)^(3/2)*a + 15*sqrt(b*sin(d*x + c) + a)*a^2)/b^2)/(b*d)
```

Fricas [A]
time = 0.35, size = 54, normalized size = 0.67

$$\frac{2\left(3\,b^2\cos\left(dx+c\right)^2+4\,ab\sin\left(dx+c\right)-8\,a^2+12\,b^2\right)\sqrt{b\sin\left(dx+c\right)+a}}{15\,b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*b^2*cos(d*x + c)^2 + 4*a*b*sin(d*x + c) - 8*a^2 + 12*b^2)*sqrt(b*si
n(d*x + c) + a)/(b^3*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 5.00, size = 72, normalized size = 0.89

$$-\frac{2\left(3\left(b\sin\left(dx+c\right)+a\right)^{\frac{5}{2}}-10\left(b\sin\left(dx+c\right)+a\right)^{\frac{3}{2}}a+15\,\sqrt{b\sin\left(dx+c\right)+a}\,a^2-15\,\sqrt{b\sin\left(dx+c\right)+a}\,b^2\right)}{15\,b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -2/15*(3*(b*sin(d*x + c) + a)^(5/2) - 10*(b*sin(d*x + c) + a)^(3/2)*a + 15*
sqrt(b*sin(d*x + c) + a)*a^2 - 15*sqrt(b*sin(d*x + c) + a)*b^2)/(b^3*d)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^3}{\sqrt{a+b\,\sin\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a + b*sin(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^3/(a + b*sin(c + d*x))^(1/2), x)
```

3.508 $\int \frac{\cos(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx$

Optimal. Leaf size=22

$$\frac{2\sqrt{a+b\sin(c+dx)}}{bd}$$

```
[Out] 2*(a+b*sin(d*x+c))^(1/2)/b/d
```

Rubi [A]
time = 0.02, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2747, 32}

$$\frac{2\sqrt{a+b\sin(c+dx)}}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (2*Sqrt[a + b*Sin[c + d*x]])/(b*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+x}}\,dx, x, b\sin(c+dx)\right)}{bd}\\ &= \frac{2\sqrt{a+b\sin(c+dx)}}{bd}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 22, normalized size = 1.00

$$\frac{2\sqrt{a+b\sin(c+dx)}}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (2*Sqrt[a + b*Sin[c + d*x]])/(b*d)
```

Maple [A]
time = 0.08, size = 21, normalized size = 0.95

method	result	size
derivativedivides	$\frac{2\sqrt{a+b\sin(dx+c)}}{bd}$	21
default	$\frac{2\sqrt{a+b\sin(dx+c)}}{bd}$	21
risch	$-\frac{i\sqrt{2}\,(2ia+2ib\sin(dx+c))}{\sqrt{2b\sin(dx+c)+2a}\,db}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+b*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(a+b*sin(d*x+c))^(1/2)/b/d
```

Maxima [A]
time = 0.30, size = 20, normalized size = 0.91

$$\frac{2\sqrt{b\sin(dx+c)+a}}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(b*sin(d*x + c) + a)/(b*d)
```

Fricas [A]
time = 0.34, size = 20, normalized size = 0.91

$$\frac{2\sqrt{b\sin(dx+c)+a}}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(b*sin(d*x + c) + a)/(b*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(17) = 34$.
time = 0.55, size = 54, normalized size = 2.45

$$\begin{cases} \frac{x\cos(c)}{\sqrt{a}} & \text{for } b = 0 \wedge d = 0 \\ \frac{\sin(c+dx)}{\sqrt{a}\,d} & \text{for } b = 0 \\ \frac{x\cos(c)}{\sqrt{a+b\sin(c)}} & \text{for } d = 0 \\ \frac{2\sqrt{a+b\sin(c+dx)}}{bd} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Piecewise((x*cos(c)/sqrt(a), Eq(b, 0) & Eq(d, 0)), (sin(c + d*x)/(sqrt(a)*d
), Eq(b, 0)), (x*cos(c)/sqrt(a + b*sin(c)), Eq(d, 0)), (2*sqrt(a + b*sin(c
+ d*x))/(b*d), True))
```

Giac [A]
time = 6.40, size = 20, normalized size = 0.91

$$\frac{2\sqrt{b\sin(dx+c)+a}}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(b*sin(d*x + c) + a)/(b*d)
```

Mupad [B]
time = 6.22, size = 20, normalized size = 0.91

$$\frac{2\sqrt{a+b\sin(c+d\,x)}}{b\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + b*sin(c + d*x))^(1/2),x)
```

```
[Out] (2*(a + b*sin(c + d*x))^(1/2))/(b*d)
```

3.509 $\int \frac{\sec(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx$

Optimal. Leaf size=74

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}\,d}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}\,d}$$

```
[Out] -arctanh((a+b*sin(d*x+c))^(1/2)/(a-b)^(1/2))/d/(a-b)^(1/2)+arctanh((a+b*sin
(d*x+c))^(1/2)/(a+b)^(1/2))/d/(a+b)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2747, 722, 1107, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{d\sqrt{a+b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{d\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a - b]]/(Sqrt[a - b]*d)) + ArcTanh[
Sqrt[a + b*Sin[c + d*x]]/Sqrt[a + b]]/(Sqrt[a + b]*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 722

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[2*
e, Subst[Int[1/(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4), x], x, Sqrt[d + e*x]],
x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1107

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx = \frac{b\text{Subst}\left(\int \frac{1}{\sqrt{a+x}\,(b^2-x^2)}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{(2b)\text{Subst}\left(\int \frac{1}{-a^2+b^2+2ax^2-x^4}\,dx, x, \sqrt{a+b\sin(c+dx)}\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{a-b-x^2}\,dx, x, \sqrt{a+b\sin(c+dx)}\right)}{d} + \frac{\text{Subst}\left(\int \frac{1}{a+b-x^2}\,dx, x, \sqrt{a+b\sin(c+dx)}\right)}{d}$$
$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}\,d} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}\,d}$$

Mathematica [A]
time = 0.06, size = 74, normalized size = 1.00

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}\,d} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a - b]]/(Sqrt[a - b]*d)) + ArcTanh[
Sqrt[a + b*Sin[c + d*x]]/Sqrt[a + b]]/(Sqrt[a + b]*d)
```

Maple [A]
time = 1.27, size = 60, normalized size = 0.81

method	result	size

default	$\frac{\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sin(dx+c)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}+\frac{\arctan\left(\frac{\sqrt{a+b\sin(dx+c)}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}}{d}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+b*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (1/(a+b)^(1/2)*arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))+1/(-a+b)^(1/2)*a
rctan((a+b*sin(d*x+c))^(1/2)/(-a+b)^(1/2)))/d
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for more
 detail
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)/sqrt(b*sin(d*x + c) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)/sqrt(a + b*sin(c + d*x)), x)
```

Giac [A]
time = 4.49, size = 75, normalized size = 1.01

$$\frac{b\left(\frac{\arctan\left(\frac{\sqrt{b\sin\left(dx+c\right)+a}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}\,b}-\frac{\arctan\left(\frac{\sqrt{b\sin\left(dx+c\right)+a}}{\sqrt{-a-b}}\right)}{\sqrt{-a-b}\,b}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] b*(arctan(sqrt(b*sin(d*x + c) + a)/sqrt(-a + b))/(sqrt(-a + b)*b) - arctan(
sqrt(b*sin(d*x + c) + a)/sqrt(-a - b))/(sqrt(-a - b)*b))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)\,\sqrt{a+b\,\sin\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + b*sin(c + d*x))^(1/2)),x)
```

```
[Out] int(1/(cos(c + d*x)*(a + b*sin(c + d*x))^(1/2)), x)
```

3.510 $\int \frac{\sec^3(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx$

Optimal. Leaf size=144

$$-\frac{(2a-3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{4(a-b)^{3/2}d}+\frac{(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{4(a+b)^{3/2}d}-\frac{\sec^2(c+dx)(b-a\sin$$

```
[Out] -1/4*(2*a-3*b)*arctanh((a+b*sin(d*x+c))^(1/2)/(a-b)^(1/2))/(a-b)^(3/2)/d+1/
4*(2*a+3*b)*arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))/(a+b)^(3/2)/d-1/2*s
ec(d*x+c)^2*(b-a*sin(d*x+c))*(a+b*sin(d*x+c))^(1/2)/(a^2-b^2)/d
```

Rubi [A]
time = 0.20, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2747, 755, 841, 1180, 212}

$$-\frac{\sec^2(c+dx)(b-a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{2d\left(a^2-b^2\right)}-\frac{(2a-3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{4d(a-b)^{3/2}}+\frac{(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{4d(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] -1/4*((2*a - 3*b)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a - b]])/((a - b)^(
3/2)*d) + ((2*a + 3*b)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a + b]])/(4*(a
 + b)^(3/2)*d) - (Sec[c + d*x]^2*(b - a*Sin[c + d*x])*Sqrt[a + b*Sin[c + d*
x]])/(2*(a^2 - b^2)*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 841

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx = \frac{b^3\text{Subst}\left(\int \frac{1}{\sqrt{a+x}\,(b^2-x^2)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= -\frac{\sec^2(c+dx)(b-a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{2(a^2-b^2)d} + \frac{b\text{Subst}\left(\int \frac{\frac{1}{2}(2a^2-3b^2)\cdots}{\sqrt{a+x}\,(b^2\cdots}\cdots\right)}{2(a^2\cdots}$$

$$= -\frac{\sec^2(c+dx)(b-a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{2(a^2-b^2)d} + \frac{b\text{Subst}\left(\int \frac{-\frac{a^2}{2}+\frac{1}{2}(2a^2-\cdots}{-a^2+b^2+2\cdots}\cdots\right)}{\cdots}$$

$$= -\frac{\sec^2(c+dx)(b-a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{2(a^2-b^2)d} - \frac{(2a-3b)\text{Subst}\left(\int \frac{\cdots}{a-\cdots}\cdots\right)}{\cdots}$$

$$= -\frac{(2a-3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{4(a-b)^{3/2}d} + \frac{(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(\cdots}}{\sqrt{a+\cdots}}\right)}{4(a+b)^{3/2}d}$$

Mathematica [A]
time = 0.54, size = 176, normalized size = 1.22

$$\frac{\sqrt{a+b}\,(2a^2-ab-3b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)-\sqrt{a-b}\left((2a^2+ab-3b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)+2\sqrt{a+b}\,\sec^2(c+dx)(-b+a\sin(c+dx))\sqrt{a+b\sin(c+dx)}\right)}{4\sqrt{a-b}\,\sqrt{a+b}\,(-a^2+b^2)\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (Sqrt[a + b]*(2*a^2 - a*b - 3*b^2)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a
- b]] - Sqrt[a - b]*((2*a^2 + a*b - 3*b^2)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]
/Sqrt[a + b]] + 2*Sqrt[a + b]*Sec[c + d*x]^2*(-b + a*Sin[c + d*x])*Sqrt[a +
 b*Sin[c + d*x]]))/(4*Sqrt[a - b]*Sqrt[a + b]*(-a^2 + b^2)*d)
```

Maple [A]
time = 2.31, size = 204, normalized size = 1.42

method	result
default	$\frac{-\frac{b\sqrt{a+b\sin(dx+c)}}{4(a+b)(b\sin(dx+c)-b)}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sin(dx+c)}}{\sqrt{a+b}}\right)a}{2(a+b)^{\frac{3}{2}}}+\frac{3\operatorname{arctanh}\left(\frac{\sqrt{a+b\sin(dx+c)}}{\sqrt{a+b}}\right)b}{4(a+b)^{\frac{3}{2}}}-\frac{b\sqrt{a+b\,\mathrm{si}}}{4(a-b)(b\,\mathrm{si}}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+b*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-1/4*b/(a+b)*(a+b*sin(d*x+c))^(1/2)/(b*sin(d*x+c)-b)+1/2/(a+b)^(3/2)*arcta
nh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))*a+3/4/(a+b)^(3/2)*arctanh((a+b*sin(d
*x+c))^(1/2)/(a+b)^(1/2))*b-1/4*b/(a-b)*(a+b*sin(d*x+c))^(1/2)/(b*sin(d*x+c
)+b)+1/2/(a-b)/(-a+b)^(1/2)*arctan((a+b*sin(d*x+c))^(1/2)/(-a+b)^(1/2))*a-3
/4/(a-b)/(-a+b)^(1/2)*arctan((a+b*sin(d*x+c))^(1/2)/(-a+b)^(1/2))*b)/d
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for more
 detail
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (residue poly has multiple non-linear facto
rs)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**3/sqrt(a + b*sin(c + d*x)), x)
```

Giac [A]
time = 2.21, size = 212, normalized size = 1.47

$$\frac{b^3\left(\frac{(2a-3b)\arctan\left(\frac{\sqrt{b\sin(dx+c)+a}}{\sqrt{-a+b}}\right)}{(ab^3-b^4)\sqrt{-a+b}}-\frac{(2a+3b)\arctan\left(\frac{\sqrt{b\sin(dx+c)+a}}{\sqrt{-a-b}}\right)}{(ab^3+b^4)\sqrt{-a-b}}-\frac{2\left((b\sin(dx+c)+a)^{\frac{3}{2}}a-\sqrt{b\sin(dx+c)+a}\,a^2-\sqrt{b\sin(dx+c)+a}\,b^2\right)}{(a^2b^2-b^4)\left((b\sin(dx+c)+a)^2-2(b\sin(dx+c)+a)a+a^2-b^2\right)}\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*b^3*((2*a - 3*b)*arctan(sqrt(b*sin(d*x + c) + a)/sqrt(-a + b))/((a*b^3
- b^4)*sqrt(-a + b)) - (2*a + 3*b)*arctan(sqrt(b*sin(d*x + c) + a)/sqrt(-a
- b))/((a*b^3 + b^4)*sqrt(-a - b)) - 2*((b*sin(d*x + c) + a)^(3/2)*a - sqrt
(b*sin(d*x + c) + a)*a^2 - sqrt(b*sin(d*x + c) + a)*b^2)/((a^2*b^2 - b^4)*(
(b*sin(d*x + c) + a)^2 - 2*(b*sin(d*x + c) + a)*a + a^2 - b^2)))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\cos\left(c+d\,x\right)}^3\,\sqrt{a+b\,\sin\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + b*sin(c + d*x))^(1/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^3*(a + b*sin(c + d*x))^(1/2)), x)
```

3.511 $\int \frac{\sec^5(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx$

Optimal. Leaf size=230

$$-\frac{3(4a^2-10ab+7b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{32(a-b)^{5/2}d}+\frac{3(4a^2+10ab+7b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{32(a+b)^{5/2}d}\cdot$$

```
[Out] -3/32*(4*a^2-10*a*b+7*b^2)*arctanh((a+b*sin(d*x+c))^(1/2)/(a-b)^(1/2))/(a-b
)^(5/2)/d+3/32*(4*a^2+10*a*b+7*b^2)*arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1
/2))/(a+b)^(5/2)/d-1/4*sec(d*x+c)^4*(b-a*sin(d*x+c))*(a+b*sin(d*x+c))^(1/2)
/(a^2-b^2)/d-1/16*sec(d*x+c)^2*(b*(a^2-7*b^2)-6*a*(a^2-2*b^2)*sin(d*x+c))*(
a+b*sin(d*x+c))^(1/2)/(a^2-b^2)^2/d
```

Rubi [A]

time = 0.25, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2747, 755, 837, 841, 1180, 212}

$$-\frac{3(4a^2-10ab+7b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{32d(a-b)^{5/2}}+\frac{3(4a^2+10ab+7b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{32d(a+b)^{5/2}}-\frac{\sec^4(c+dx)(b-a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{4d(a^2-b^2)}-\frac{\sec^2(c+dx)\sqrt{a+b\sin(c+dx)}\,(b(a^2-7b^2)-6a(a^2-2b^2)\sin(c+dx))}{16d(a^2-b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (-3*(4*a^2 - 10*a*b + 7*b^2)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a - b]])
/(32*(a - b)^(5/2)*d) + (3*(4*a^2 + 10*a*b + 7*b^2)*ArcTanh[Sqrt[a + b*Sin[
c + d*x]]/Sqrt[a + b]])/(32*(a + b)^(5/2)*d) - (Sec[c + d*x]^4*(b - a*Sin[c
 + d*x])*Sqrt[a + b*Sin[c + d*x]])/(4*(a^2 - b^2)*d) - (Sec[c + d*x]^2*Sqrt
[a + b*Sin[c + d*x]]*(b*(a^2 - 7*b^2) - 6*a*(a^2 - 2*b^2)*Sin[c + d*x]))/(1
6*(a^2 - b^2)^2*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
```

```
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 841

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx = \frac{b^5\text{Subst}\left(\int \frac{1}{\sqrt{a+x}\,(b^2-x^2)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= -\frac{\sec^4(c+dx)(b-a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{4(a^2-b^2)\,d} + \frac{b^3\text{Subst}\left(\int \frac{\frac{1}{2}(6a^2-7b^2)+}{\sqrt{a+x}\,(b^2}\right.}{4\,(a^2}$$

$$= -\frac{\sec^4(c+dx)(b-a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{4(a^2-b^2)\,d} - \frac{\sec^2(c+dx)\sqrt{a+b\sin}}{}$$

$$= -\frac{\sec^4(c+dx)(b-a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{4(a^2-b^2)\,d} - \frac{\sec^2(c+dx)\sqrt{a+b\sin}}{}$$

$$= -\frac{\sec^4(c+dx)(b-a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{4(a^2-b^2)\,d} - \frac{\sec^2(c+dx)\sqrt{a+b\sin}}{}$$

$$= -\frac{3(4a^2-10ab+7b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{32(a-b)^{5/2}d} + \frac{3(4a^2+10ab+7b^2)\,\text{ta}}{32}$$

Mathematica [A]
time = 1.90, size = 244, normalized size = 1.06

$$\frac{-3(a+b)^{5/2}(4a^2-10ab+7b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)+\sqrt{a-b}\left(3(a-b)^2(4a^2+10ab+7b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)+\sqrt{a+b}\sec^4(c+dx)\sqrt{a+b\sin(c+dx)}\,(-9a^2b+15b^3+(-a^2b+7b^3)\cos(2(c+dx))+a(11a^2-14b^2)\sin(c+dx)+3(a^3-2ab^2)\sin(3(c+dx)))\right)}{32\sqrt{a-b}\sqrt{a+b}\,(a^2-b^2)^2\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (-3*(a + b)^(5/2)*(4*a^2 - 10*a*b + 7*b^2)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]
/Sqrt[a - b]] + Sqrt[a - b]*(3*(a - b)^2*(4*a^2 + 10*a*b + 7*b^2)*ArcTanh[S
qrt[a + b*Sin[c + d*x]]/Sqrt[a + b]] + Sqrt[a + b]*Sec[c + d*x]^4*Sqrt[a +
b*Sin[c + d*x]]*(-9*a^2*b + 15*b^3 + (-(a^2*b) + 7*b^3)*Cos[2*(c + d*x)] +
a*(11*a^2 - 14*b^2)*Sin[c + d*x] + 3*(a^3 - 2*a*b^2)*Sin[3*(c + d*x)])))/(3
2*Sqrt[a - b]*Sqrt[a + b]*(a^2 - b^2)^2*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 579 vs. $2(206) = 412$.
time = 3.43, size = 580, normalized size = 2.52

method	result

default	$\frac{-\frac{3b(a+b\sin(dx+c))^{\frac{3}{2}}a}{16(b\sin(dx+c)-b)^2(a^2+2ab+b^2)}-\frac{9b^2(a+b\sin(dx+c))^{\frac{3}{2}}}{32(b\sin(dx+c)-b)^2(a^2+2ab+b^2)}+\frac{3b\sqrt{a+b\sin(dx+c)}\,a}{16(b\sin(dx+c)-b)^2(a+b)}+\frac{11b^2\sqrt{a+b\sin(dx+c}}{32(b\sin(dx+c)-b)^2(a+b)}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+b*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-3/16/(b*sin(d*x+c)-b)^2*b/(a^2+2*a*b+b^2)*(a+b*sin(d*x+c))^(3/2)*a-9/32/(
b*sin(d*x+c)-b)^2*b^2/(a^2+2*a*b+b^2)*(a+b*sin(d*x+c))^(3/2)+3/16/(b*sin(d*
x+c)-b)^2*b/(a+b)*(a+b*sin(d*x+c))^(1/2)*a+11/32/(b*sin(d*x+c)-b)^2*b^2/(a+
b)*(a+b*sin(d*x+c))^(1/2)+3/8/(a^2+2*a*b+b^2)/(a+b)^(1/2)*arctanh((a+b*sin(
d*x+c))^(1/2)/(a+b)^(1/2))*a^2+15/16/(a^2+2*a*b+b^2)/(a+b)^(1/2)*arctanh((a
+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))*a*b+21/32/(a^2+2*a*b+b^2)/(a+b)^(1/2)*arc
tanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))*b^2-3/16/(b*sin(d*x+c)+b)^2*b/(a^2
-2*a*b+b^2)*(a+b*sin(d*x+c))^(3/2)*a+9/32/(b*sin(d*x+c)+b)^2*b^2/(a^2-2*a*b
+b^2)*(a+b*sin(d*x+c))^(3/2)+3/16/(b*sin(d*x+c)+b)^2*b/(a-b)*(a+b*sin(d*x+c
))^(1/2)*a-11/32/(b*sin(d*x+c)+b)^2*b^2/(a-b)*(a+b*sin(d*x+c))^(1/2)+3/8/(a
^2-2*a*b+b^2)/(-a+b)^(1/2)*arctan((a+b*sin(d*x+c))^(1/2)/(-a+b)^(1/2))*a^2-
15/16/(a^2-2*a*b+b^2)/(-a+b)^(1/2)*arctan((a+b*sin(d*x+c))^(1/2)/(-a+b)^(1/
2))*a*b+21/32/(a^2-2*a*b+b^2)/(-a+b)^(1/2)*arctan((a+b*sin(d*x+c))^(1/2)/(-
a+b)^(1/2))*b^2)/d
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for more
 detail
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)^5/sqrt(b*sin(d*x + c) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^5(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**5/sqrt(a + b*sin(c + d*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 419 vs. 2(208) = 416.
time = 4.70, size = 419, normalized size = 1.82

$$\frac{b^5\left(\frac{3\,(4\,a^2-10\,ab+7\,b^2)\arctan\left(\frac{\sqrt{b\sin(dx+c)+a}}{\sqrt{-a+b}}\right)}{(a^2b^5-2\,ab^6+b^7)\sqrt{-a+b}}-\frac{3\,(4\,a^2+10\,ab+7\,b^2)\arctan\left(\frac{\sqrt{b\sin(dx+c)+a}}{\sqrt{-a-b}}\right)}{(a^2b^5+2\,ab^6+b^7)\sqrt{-a-b}}-\frac{2\left(6\,(b\sin(dx+c)+a)^{\frac{7}{2}}a^3-18\,(b\sin(dx+c)+a)^{\frac{5}{2}}a^4+18\,(b\sin(dx+c)+a)^{\frac{3}{2}}a^5-6\,\sqrt{b\sin(dx+c)+a}\,a^6-12\,(b\sin(dx+c)+a)^{\frac{7}{2}}ab^2+35\,(b\sin(dx+c)+a)^{\frac{5}{2}}a^2b^2-44\,(b\sin(dx+c)+a)^{\frac{3}{2}}a^3b^2+21\,\sqrt{b\sin(dx+c)+a}\,a^4b^2+7\,(b\sin(dx+c)+a)^{\frac{5}{2}}b^4+2\,(b\sin(dx+c)+a)^{\frac{3}{2}}ab^4-4\,\sqrt{b\sin(dx+c)+a}\,a^2b^4-11\,\sqrt{b\sin(dx+c)+a}\,b^6\right)}{(a^4b^4-2\,a^2b^6+b^8)\left((b\sin(dx+c)+a)^2-2\,(b\sin(dx+c)+a)a+a^2-b^2\right)^2}\right)}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/32*b^5*(3*(4*a^2 - 10*a*b + 7*b^2)*arctan(sqrt(b*sin(d*x + c) + a)/sqrt(-
a + b))/((a^2*b^5 - 2*a*b^6 + b^7)*sqrt(-a + b)) - 3*(4*a^2 + 10*a*b + 7*b^
2)*arctan(sqrt(b*sin(d*x + c) + a)/sqrt(-a - b))/((a^2*b^5 + 2*a*b^6 + b^7)
*sqrt(-a - b)) - 2*(6*(b*sin(d*x + c) + a)^(7/2)*a^3 - 18*(b*sin(d*x + c) +
 a)^(5/2)*a^4 + 18*(b*sin(d*x + c) + a)^(3/2)*a^5 - 6*sqrt(b*sin(d*x + c) +
 a)*a^6 - 12*(b*sin(d*x + c) + a)^(7/2)*a*b^2 + 35*(b*sin(d*x + c) + a)^(5/
2)*a^2*b^2 - 44*(b*sin(d*x + c) + a)^(3/2)*a^3*b^2 + 21*sqrt(b*sin(d*x + c)
 + a)*a^4*b^2 + 7*(b*sin(d*x + c) + a)^(5/2)*b^4 + 2*(b*sin(d*x + c) + a)^(
3/2)*a*b^4 - 4*sqrt(b*sin(d*x + c) + a)*a^2*b^4 - 11*sqrt(b*sin(d*x + c) +
a)*b^6)/((a^4*b^4 - 2*a^2*b^6 + b^8)*((b*sin(d*x + c) + a)^2 - 2*(b*sin(d*x
 + c) + a)*a + a^2 - b^2)^2))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos(c+d\,x)^5\,\sqrt{a+b\,\sin(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^5*(a + b*sin(c + d*x))^(1/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^5*(a + b*sin(c + d*x))^(1/2)), x)
```

3.512 $\int \frac{\cos^4(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx$

Optimal. Leaf size=247

$$\frac{2\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}}{7bd}-\frac{32a(a^2-2b^2)\,E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\frac{2b}{a+b}\right)\sqrt{a+b\sin(c+dx)}}{35b^4d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}+\frac{8(4a^4-9a}{}$$

```
[Out] 2/7*cos(d*x+c)^3*(a+b*sin(d*x+c))^(1/2)/b/d-4/35*cos(d*x+c)*(4*a^2-5*b^2-3*
a*b*sin(d*x+c))*(a+b*sin(d*x+c))^(1/2)/b^3/d+32/35*a*(a^2-2*b^2)*(sin(1/2*c
+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4
*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/b^4/d/((a+b*si
n(d*x+c))/(a+b))^(1/2)-8/35*(4*a^4-9*a^2*b^2+5*b^4)*(sin(1/2*c+1/4*Pi+1/2*d
*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),
2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/b^4/d/(a+b*sin(d*x+
c))^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {2774, 2944, 2831, 2742, 2740, 2734, 2732}

$$-\frac{32a(a^2-2b^2)\sqrt{a+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|\frac{2b}{a+b}\right)}{35b^4d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}-\frac{4\cos(c+dx)\sqrt{a+b\sin(c+dx)}\,(4a^2-3ab\sin(c+dx)-5b^2)}{35b^3d}+\frac{8(4a^4-9a^2b^2+5b^4)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|\frac{2b}{a+b}\right)}{35b^4d\sqrt{a+b\sin(c+dx)}}+\frac{2\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}}{7bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (2*Cos[c + d*x]^3*Sqrt[a + b*Sin[c + d*x]])/(7*b*d) - (32*a*(a^2 - 2*b^2)*E
llipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(35*b
^4*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + (8*(4*a^4 - 9*a^2*b^2 + 5*b^4)*E
llipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a +
b)])/(35*b^4*d*Sqrt[a + b*Sin[c + d*x]]) - (4*Cos[c + d*x]*Sqrt[a + b*Sin[c
 + d*x]]*(4*a^2 - 5*b^2 - 3*a*b*Sin[c + d*x]))/(35*b^3*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
```

```
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2774

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(b*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2944

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Dist[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx = \frac{2\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}}{7bd} + \frac{6\int \frac{\cos^2(c+dx)(b+a\sin(c+dx))}{\sqrt{a+b\sin(c+dx)}}\,dx}{7b}$$

$$= \frac{2\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}}{7bd} - \frac{4\cos(c+dx)\sqrt{a+b\sin(c+dx)}\,(4a^2 -}{35b^3d}$$

$$= \frac{2\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}}{7bd} - \frac{4\cos(c+dx)\sqrt{a+b\sin(c+dx)}\,(4a^2 -}{35b^3d}$$

$$= \frac{2\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}}{7bd} - \frac{4\cos(c+dx)\sqrt{a+b\sin(c+dx)}\,(4a^2 -}{35b^3d}$$

$$= \frac{2\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}}{7bd} - \frac{32a(a^2-2b^2)\,E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\frac{2b}{a+b}\right)\sqrt{}}{35b^4d\sqrt{\frac{a+b\sin(c+dx}{a+b}}}$$

Mathematica [**A**]
time = 1.07, size = 219, normalized size = 0.89

$$\frac{64a(a^3+a^2b-2ab^2-2b^3)\,E\left(\frac{1}{4}(-2c+\pi-2dx)\middle|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}} - 16(4a^4-9a^2b^2+5b^4)\,F\left(\frac{1}{4}(-2c+\pi-2dx)\middle|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}} + b\cos(c+dx)\left(-32a^3+62ab^2-2ab^2\cos(2(c+dx))+(-8a^2b+45b^3)\sin(c+dx)+5b^3\sin(3(c+dx))\right)}{70b^4d\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (64*a*(a^3 + a^2*b - 2*a*b^2 - 2*b^3)*EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b
)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] - 16*(4*a^4 - 9*a^2*b^2 + 5*b
^4)*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x
])/(a + b)] + b*Cos[c + d*x]*(-32*a^3 + 62*a*b^2 - 2*a*b^2*Cos[2*(c + d*x)]
 + (-8*a^2*b + 45*b^3)*Sin[c + d*x] + 5*b^3*Sin[3*(c + d*x)]))/(70*b^4*d*Sq
rt[a + b*Sin[c + d*x]])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 941 vs. $2(293) = 586$.
time = 2.11, size = 942, normalized size = 3.81

method	result

default	$-\frac{2\left(-5b^5\left(\sin^5(dx+c)\right)+16\sqrt{\frac{a+b\sin(dx+c)}{a-b}}\,\sqrt{-\frac{(\sin(dx+c)-1)b}{a+b}}\,\sqrt{-\frac{(1+\sin(dx+c))b}{a-b}}\,\text{EllipticF}\left(\sqrt{\frac{a+b\sin(dx+c)}{a-b}},\sqrt{\frac{a}{a}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+b*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/35*(-5*b^5*sin(d*x+c)^5+16*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-
1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c
))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b-12*((a+b*sin(d*x+c))/(a-b))^(1/2
)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF
(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^2-36*((a+b*sin(d
*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b
))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*
b^3+12*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+
sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/
(a+b))^(1/2))*a*b^4+20*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a
+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b
))^(1/2),((a-b)/(a+b))^(1/2))*b^5-16*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(
d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*si
n(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5+48*((a+b*sin(d*x+c))/(a-b))
^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Elli
pticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^2-32*((a+b*
sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b
/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))
*a*b^4+a*b^4*sin(d*x+c)^4-2*a^2*b^3*sin(d*x+c)^3+20*b^5*sin(d*x+c)^3-8*a^3*
b^2*sin(d*x+c)^2+14*a*b^4*sin(d*x+c)^2+2*a^2*b^3*sin(d*x+c)-15*b^5*sin(d*x+
c)+8*a^3*b^2-15*a*b^4)/b^5/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^4/sqrt(b*sin(d*x + c) + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 493, normalized size = 2.00

$$\frac{2\left(2\sqrt{2}\left(8a^4-19a^2b^2+15b^4\right)\sqrt{ib}\,\text{weierstrassPInverse}\left(-\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(8ia^3-9iab^2)}{27b^3},\frac{3b\cos(dx+c)-3ib\sin(dx+c)-2ia}{3b}\right)+2\sqrt{2}\left(8a^4-19a^2b^2+15b^4\right)\sqrt{-ib}\,\text{weierstrassPInverse}\left(-\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(-8ia^3+9iab^2)}{27b^3},\frac{3b\cos(dx+c)+3ib\sin(dx+c)+2ia}{3b}\right)-24\sqrt{2}\left(-ia^3b+2iab^3\right)\sqrt{ib}\,\text{weierstrassZeta}\left(-\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(8ia^3-9iab^2)}{27b^3},\text{weierstrassPInverse}\left(-\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(8ia^3-9iab^2)}{27b^3},\frac{3b\cos(dx+c)-3ib\sin(dx+c)-2ia}{3b}\right)\right)-24\sqrt{2}\left(ia^3b-2iab^3\right)\sqrt{-ib}\,\text{weierstrassZeta}\left(-\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(-8ia^3+9iab^2)}{27b^3},\text{weierstrassPInverse}\left(-\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(-8ia^3+9iab^2)}{27b^3},\frac{3b\cos(dx+c)+3ib\sin(dx+c)+2ia}{3b}\right)\right)+3\left(5b^4\cos(dx+c)^3+6ab^3\cos(dx+c)\sin(dx+c)-2\left(4a^2b^2-5b^4\right)\cos(dx+c)\right)\sqrt{b\sin(dx+c)+a}\right)}{105\,b^5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/105*(2*sqrt(2)*(8*a^4 - 19*a^2*b^2 + 15*b^4)*sqrt(I*b)*weierstrassPInvers
e(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d
*x + c) - 3*I*b*sin(d*x + c) - 2*I*a)/b) + 2*sqrt(2)*(8*a^4 - 19*a^2*b^2 +
15*b^4)*sqrt(-I*b)*weierstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-8*
I*a^3 + 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*I*a)
/b) - 24*sqrt(2)*(-I*a^3*b + 2*I*a*b^3)*sqrt(I*b)*weierstrassZeta(-4/3*(4*a
^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, weierstrassPInverse(-4/3*
(4*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c)
 - 3*I*b*sin(d*x + c) - 2*I*a)/b)) - 24*sqrt(2)*(I*a^3*b - 2*I*a*b^3)*sqrt(
-I*b)*weierstrassZeta(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2
)/b^3, weierstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*
a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*I*a)/b)) + 3*(5*
b^4*cos(d*x + c)^3 + 6*a*b^3*cos(d*x + c)*sin(d*x + c) - 2*(4*a^2*b^2 - 5*b
^4)*cos(d*x + c))*sqrt(b*sin(d*x + c) + a))/(b^5*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^4(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(cos(c + d*x)**4/sqrt(a + b*sin(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^4/sqrt(b*sin(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^4}{\sqrt{a+b\,\sin\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(a + b*sin(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^4/(a + b*sin(c + d*x))^(1/2), x)
```

3.513 $\int \frac{\cos^2(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx$

Optimal. Leaf size=175

$$\frac{2\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{3bd}+\frac{4aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)|\frac{2b}{a+b}\right)\sqrt{a+b\sin(c+dx)}}{3b^2d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}-\frac{4(a^2-b^2)F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+ \ldots\right.\right.}{3b^2d\sqrt{a- \ldots}}$$

[Out] 2/3*cos(d*x+c)*(a+b*sin(d*x+c))^(1/2)/b/d-4/3*a*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/b^2/d/((a+b*sin(d*x+c))/(a+b))^(1/2)+4/3*(a^2-b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/b^2/d/(a+b*sin(d*x+c))^(1/2)

Rubi [A]
time = 0.12, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2774, 2831, 2742, 2740, 2734, 2732}

$$-\frac{4(a^2-b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{3b^2d\sqrt{a+b\sin(c+dx)}}+\frac{4a\sqrt{a+b\sin(c+dx)}E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{3b^2d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}+\frac{2\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{3bd}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^2/Sqrt[a + b*Sin[c + d*x]],x]

[Out] (2*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(3*b*d) + (4*a*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(3*b^2*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) - (4*(a^2 - b^2)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(3*b^2*d*Sqrt[a + b*Sin[c + d*x]])

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2774

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(b*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx = \frac{2\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{3bd} + \frac{2\int \frac{b+a\sin(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx}{3b}$$

$$= \frac{2\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{3bd} + \frac{1}{3}\left(2\left(1-\frac{a^2}{b^2}\right)\right)\int \frac{1}{\sqrt{a+b\sin(c+dx)}}\,dx$$

$$= \frac{2\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{3bd} + \frac{\left(2a\sqrt{a+b\sin(c+dx)}\right)\int \sqrt{\frac{a}{a+b}+\frac{b\sin}{}}}{3b^2\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}$$

$$= \frac{2\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{3bd} + \frac{4aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)|\frac{2b}{a+b}\right)\sqrt{a+b\sin(c+}}{3b^2d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}$$

Mathematica [A]
time = 0.77, size = 145, normalized size = 0.83

$$\frac{2b\cos(c+dx)(a+b\sin(c+dx)) - 4a(a+b)E\left(\frac{1}{4}(-2c+\pi-2dx)|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}} + 4(a^2-b^2)F\left(\frac{1}{4}(-2c+\pi-2dx)|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}{3b^2d\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (2*b*Cos[c + d*x]*(a + b*Sin[c + d*x]) - 4*a*(a + b)*EllipticE[(-2*c + Pi -
 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] + 4*(a^2 - b^2
)*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])
/(a + b)])/(3*b^2*d*Sqrt[a + b*Sin[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 461 vs. $2(225) = 450$.
time = 1.68, size = 462, normalized size = 2.64

method	result
default	$\frac{4\sqrt{\frac{a+b\sin(dx+c)}{a-b}}\sqrt{-\frac{(\sin(dx+c)-1)b}{a+b}}\sqrt{-\frac{(1+\sin(dx+c))b}{a-b}}\,\text{EllipticF}\left(\sqrt{\frac{a+b\sin(dx+c)}{a-b}},\sqrt{\frac{a-b}{a+b}}\right)a^2b - 4\sqrt{\frac{a+b\sin(dx+}{a-b}}}{3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+b*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(2*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+
sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/
(a+b))^(1/2))*a^2*b-2*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+
b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b)
)^(1/2),((a-b)/(a+b))^(1/2))*b^3-2*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*
x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(
d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3+2*((a+b*sin(d*x+c))/(a-b))^(1
/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Ellipti
cE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^2-b^3*sin(d*x+c)
^3-a*b^2*sin(d*x+c)^2+b^3*sin(d*x+c)+a*b^2)/b^3/cos(d*x+c)/(a+b*sin(d*x+c))
^(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^2/sqrt(b*sin(d*x + c) + a), x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 408, normalized size = 2.33

$$-\frac{2\left(3i\sqrt{2}\,a\sqrt{i\,b}\,b\,\mathrm{weierstrassZeta}\left(-\frac{4\,(4\,a^2-3\,b^2)}{3\,b^2},-\frac{8\,(8i\,a^3-9i\,ab^2)}{27\,b^3},\mathrm{weierstrassPInverse}\left(-\frac{4\,(4\,a^2-3\,b^2)}{3\,b^2},-\frac{8\,(8i\,a^3-9i\,ab^2)}{27\,b^3},\frac{3\,b\cos(dx+c)-3i\,b\sin(dx+c)-2i\,a}{3\,b}\right)\right)-3i\sqrt{2}\,a\sqrt{-i\,b}\,b\,\mathrm{weierstrassZeta}\left(-\frac{4\,(4\,a^2-3\,b^2)}{3\,b^2},-\frac{8\,(-8i\,a^3+9i\,ab^2)}{27\,b^3},\mathrm{weierstrassPInverse}\left(-\frac{4\,(4\,a^2-3\,b^2)}{3\,b^2},-\frac{8\,(-8i\,a^3+9i\,ab^2)}{27\,b^3},\frac{3\,b\cos(dx+c)+3i\,b\sin(dx+c)+2i\,a}{3\,b}\right)\right)-3\sqrt{b\sin(dx+c)+a}\,b^2\cos(dx+c)+\sqrt{2}\,(2\,a^2-3\,b^2)\sqrt{i\,b}\,\mathrm{weierstrassPInverse}\left(-\frac{4\,(4\,a^2-3\,b^2)}{3\,b^2},-\frac{8\,(8i\,a^3-9i\,ab^2)}{27\,b^3},\frac{3\,b\cos(dx+c)-3i\,b\sin(dx+c)-2i\,a}{3\,b}\right)+\sqrt{2}\,(2\,a^2-3\,b^2)\sqrt{-i\,b}\,\mathrm{weierstrassPInverse}\left(-\frac{4\,(4\,a^2-3\,b^2)}{3\,b^2},-\frac{8\,(-8i\,a^3+9i\,ab^2)}{27\,b^3},\frac{3\,b\cos(dx+c)+3i\,b\sin(dx+c)+2i\,a}{3\,b}\right)\right)}{9\,b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/9*(3*I*sqrt(2)*a*sqrt(I*b)*b*weierstrassZeta(-4/3*(4*a^2 - 3*b^2)/b^2, -
8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, weierstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^
2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x +
 c) - 2*I*a)/b)) - 3*I*sqrt(2)*a*sqrt(-I*b)*b*weierstrassZeta(-4/3*(4*a^2 -
 3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, weierstrassPInverse(-4/3*(4*
a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) +
 3*I*b*sin(d*x + c) + 2*I*a)/b)) - 3*sqrt(b*sin(d*x + c) + a)*b^2*cos(d*x +
 c) + sqrt(2)*(2*a^2 - 3*b^2)*sqrt(I*b)*weierstrassPInverse(-4/3*(4*a^2 - 3
*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*s
in(d*x + c) - 2*I*a)/b) + sqrt(2)*(2*a^2 - 3*b^2)*sqrt(-I*b)*weierstrassPIn
verse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, 1/3*(3*b*
cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*I*a)/b))/(b^3*d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2{(c+dx)}}{\sqrt{a+b\sin{(c+dx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(cos(c + d*x)**2/sqrt(a + b*sin(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^2/sqrt(b*sin(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^2}{\sqrt{a+b\,\sin\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a + b*sin(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^2/(a + b*sin(c + d*x))^(1/2), x)
```

3.514 $\int \frac{\sec^2(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx$

Optimal. Leaf size=183

$$-\frac{\sec(c+dx)(b-a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{(a^2-b^2)\,d}-\frac{aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)|\frac{2b}{a+b}\right)\sqrt{a+b\sin(c+dx)}}{(a^2-b^2)\,d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}+\frac{F\left(\frac{1}{2}\left(\right.\right.}{}$$

```
[Out] -sec(d*x+c)*(b-a*sin(d*x+c))*(a+b*sin(d*x+c))^(1/2)/(a^2-b^2)/d+a*(sin(1/2*
c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/
4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/(a^2-b^2)/d/(
(a+b*sin(d*x+c))/(a+b))^(1/2)-(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c
+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2
))*((a+b*sin(d*x+c))/(a+b))^(1/2)/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2775, 2831, 2742, 2740, 2734, 2732}

$$-\frac{\sec(c+dx)(b-a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{d\,(a^2-b^2)}-\frac{a\sqrt{a+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{d\,(a^2-b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}+\frac{\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] -((Sec[c + d*x]*(b - a*Sin[c + d*x])*Sqrt[a + b*Sin[c + d*x]])/((a^2 - b^2)
*d)) - (a*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d
*x]])/((a^2 - b^2)*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + (EllipticF[(c -
Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(d*Sqrt[a
 + b*Sin[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2775

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^
(m + 1)*((b - a*Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx = -\frac{\sec(c+dx)(b-a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{(a^2-b^2)\,d} + \frac{\int \frac{\frac{b^2}{2}+\frac{1}{2}ab\sin(c+dx)}{\sqrt{a+b\sin(c+dx)}}}{-a^2+b^2}$$

$$= -\frac{\sec(c+dx)(b-a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{(a^2-b^2)\,d} + \frac{1}{2}\int \frac{1}{\sqrt{a+b\sin(c+d}}$$

$$= -\frac{\sec(c+dx)(b-a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{(a^2-b^2)\,d} - \frac{\left(a\sqrt{a+b\sin(c+dx)}\right.}{2\,(a^2-b^2)}$$

$$= -\frac{\sec(c+dx)(b-a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{(a^2-b^2)\,d} - \frac{aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\frac{2b}{a+}\right.}{(a^2-b^2)\,d\sqrt{\frac{a}{}}}$$

Mathematica [A]
time = 0.65, size = 177, normalized size = 0.97

$$\frac{-ab\sec(c+dx)+a(a+b)E\left(\frac{1}{4}(-2c+\pi-2dx)\middle|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}-(a^2-b^2)\,F\left(\frac{1}{4}(-2c+\pi-2dx)\middle|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}+a^2\tan(c+dx)-b^2\tan(c+dx)+ab\sin(c+dx)\tan(c+dx)}{(a-b)(a+b)d\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (-(a*b*Sec[c + d*x]) + a*(a + b)*EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b)/(a
 + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] - (a^2 - b^2)*EllipticF[(-2*c + Pi
 - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] + a^2*Tan[c
 + d*x] - b^2*Tan[c + d*x] + a*b*Sin[c + d*x]*Tan[c + d*x])/((a - b)*(a + b)
*d*Sqrt[a + b*Sin[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 639 vs. 2(239) = 478.
time = 2.26, size = 640, normalized size = 3.50

method	result
default	$-\frac{\sqrt{b\left(\cos^2\left(dx+c\right)\right)\sin\left(dx+c\right)+a\left(\cos^2\left(dx+c\right)\right)}\left(\sqrt{\frac{b\sin(dx+c)}{a-b}+\frac{a}{a-b}}\sqrt{-\frac{b\sin(dx+c)}{a+b}+\frac{b}{a+b}}\sqrt{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+b*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b*(b*cos(d*x+c)^2*sin(d*x+c)+a*cos(d*x+c)^2)^(1/2)*((b/(a-b)*sin(d*x+c)+
1/(a-b)*a)^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b
/(a-b))^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^
(1/2))*a^2*b-(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a
+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticF((b/(a-b)*sin(d*x+c
)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*b^3-(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^
(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/
2)*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^3+
(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-
b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^
(1/2),((a-b)/(a+b))^(1/2))*a*b^2+a*b^2*cos(d*x+c)^2-a^2*b*sin(d*x+c)+b^3*si
n(d*x+c))/(a+b)/(-(a+b*sin(d*x+c))*(sin(d*x+c)-1)*(1+sin(d*x+c)))^(1/2)/(a-
b)/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^2/sqrt(b*sin(d*x + c) + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 458, normalized size = 2.50

$$-\frac{-3i\sqrt{2}a\sqrt{ib}\,b\cos(dx+c)\,\mathrm{weierstrassZeta}\left(-\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(8ia^3-9iab^2)}{27b^3},\mathrm{weierstrassPInverse}\left(-\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(8ia^3-9iab^2)}{27b^3},\frac{3b\cos(dx+c)-3ib\sin(dx+c)-2ia}{3b}\right)\right)+3i\sqrt{2}a\sqrt{-ib}\,b\cos(dx+c)\,\mathrm{weierstrassZeta}\left(-\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(-8ia^3+9iab^2)}{27b^3},\mathrm{weierstrassPInverse}\left(-\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(-8ia^3+9iab^2)}{27b^3},\frac{3b\cos(dx+c)+3ib\sin(dx+c)+2ia}{3b}\right)\right)-\sqrt{2}(2a^2-3b^2)\sqrt{ib}\cos(dx+c)\,\mathrm{weierstrassPInverse}\left(-\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(8ia^3-9iab^2)}{27b^3},\frac{3b\cos(dx+c)-3ib\sin(dx+c)-2ia}{3b}\right)-\sqrt{2}(2a^2-3b^2)\sqrt{-ib}\cos(dx+c)\,\mathrm{weierstrassPInverse}\left(-\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(-8ia^3+9iab^2)}{27b^3},\frac{3b\cos(dx+c)+3ib\sin(dx+c)+2ia}{3b}\right)-6(ab\sin(dx+c)-b^2)\sqrt{b\sin(dx+c)+a}}{6(a^2b-b^3)d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*(-3*I*sqrt(2)*a*sqrt(I*b)*b*cos(d*x + c)*weierstrassZeta(-4/3*(4*a^2 -
 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, weierstrassPInverse(-4/3*(4*a
^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3
*I*b*sin(d*x + c) - 2*I*a)/b)) + 3*I*sqrt(2)*a*sqrt(-I*b)*b*cos(d*x + c)*we
ierstrassZeta(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, w
eierstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b
^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*I*a)/b)) - sqrt(2)*(2*a^
2 - 3*b^2)*sqrt(I*b)*cos(d*x + c)*weierstrassPInverse(-4/3*(4*a^2 - 3*b^2)/
b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x
 + c) - 2*I*a)/b) - sqrt(2)*(2*a^2 - 3*b^2)*sqrt(-I*b)*cos(d*x + c)*weierst
rassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, 1/
3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*I*a)/b) - 6*(a*b*sin(d*x + c)
- b^2)*sqrt(b*sin(d*x + c) + a))/((a^2*b - b^3)*d*cos(d*x + c))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+b*sin(d*x+c))**(1/2),x)
[Out] Integral(sec(c + d*x)**2/sqrt(a + b*sin(c + d*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sin(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sec(d*x + c)^2/sqrt(b*sin(d*x + c) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(c+d\,x)^2\,\sqrt{a+b\,\sin(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + b*sin(c + d*x))^(1/2)),x)
[Out] int(1/(cos(c + d*x)^2*(a + b*sin(c + d*x))^(1/2)), x)
```

3.515 $\int \frac{\sec^4(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx$

Optimal. Leaf size=291

$$-\frac{\sec^3(c+dx)(b-a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{3\left(a^2-b^2\right)d}-\frac{2a(a^2-2b^2)\,E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\frac{2b}{a+b}\right)\sqrt{a+b\sin(c+d}}{3\left(a^2-b^2\right)^2 d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}$$

```
[Out] -1/3*sec(d*x+c)^3*(b-a*sin(d*x+c))*(a+b*sin(d*x+c))^(1/2)/(a^2-b^2)/d-1/6*s
ec(d*x+c)*(b*(a^2-5*b^2)-4*a*(a^2-2*b^2)*sin(d*x+c))*(a+b*sin(d*x+c))^(1/2)
/(a^2-b^2)^2/d+2/3*a*(a^2-2*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/
2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(
1/2))*(a+b*sin(d*x+c))^(1/2)/(a^2-b^2)^2/d/((a+b*sin(d*x+c))/(a+b))^(1/2)-1
/6*(4*a^2-5*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d
*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(
d*x+c))/(a+b))^(1/2)/(a^2-b^2)/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A]

time = 0.29, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {2775, 2945, 2831, 2742, 2740, 2734, 2732}

$$-\frac{\sec^3(c+dx)(b-a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{3d\left(a^2-b^2\right)}-\frac{\sec(c+dx)\sqrt{a+b\sin(c+dx)}\left(b(a^2-5b^2)-4a(a^2-2b^2)\sin(c+dx)\right)}{6d\left(a^2-b^2\right)^2}+\frac{(4a^2-5b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|\frac{2b}{a+b}\right)}{6d\left(a^2-b^2\right)\sqrt{a+b\sin(c+dx)}}-\frac{2a(a^2-2b^2)\sqrt{a+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|\frac{2b}{a+b}\right)}{3d\left(a^2-b^2\right)^2\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] -1/3*(Sec[c + d*x]^3*(b - a*Sin[c + d*x])*Sqrt[a + b*Sin[c + d*x]])/((a^2 -
 b^2)*d) - (2*a*(a^2 - 2*b^2)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*
Sqrt[a + b*Sin[c + d*x]])/(3*(a^2 - b^2)^2*d*Sqrt[(a + b*Sin[c + d*x])/(a +
 b)]) + ((4*a^2 - 5*b^2)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[
(a + b*Sin[c + d*x])/(a + b)])/(6*(a^2 - b^2)*d*Sqrt[a + b*Sin[c + d*x]]) -
 (Sec[c + d*x]*Sqrt[a + b*Sin[c + d*x]]*(b*(a^2 - 5*b^2) - 4*a*(a^2 - 2*b^2
)*Sin[c + d*x]))/(6*(a^2 - b^2)^2*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2775

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^
(m + 1)*((b - a*Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2945

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx = -\frac{\sec^3(c+dx)(b-a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{3\left(a^2-b^2\right)d} - \frac{\int \frac{\sec^2(c+dx)\left(-2a^2+\frac{5b^2}{2}-\frac{3}{2}c\right.}{\sqrt{a+b\sin(c+}}}{3\left(a^2-b^2\right.}$$

$$= -\frac{\sec^3(c+dx)(b-a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{3\left(a^2-b^2\right)d} - \frac{\sec(c+dx)\sqrt{a+b\sin}}{}$$

$$= -\frac{\sec^3(c+dx)(b-a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{3\left(a^2-b^2\right)d} - \frac{\sec(c+dx)\sqrt{a+b\sin}}{}$$

$$= -\frac{\sec^3(c+dx)(b-a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{3\left(a^2-b^2\right)d} - \frac{\sec(c+dx)\sqrt{a+b\sin}}{}$$

$$= -\frac{\sec^3(c+dx)(b-a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{3\left(a^2-b^2\right)d} - \frac{2a\left(a^2-2b^2\right)E\left(\frac{1}{2}(c-\right.}{3\left(a^2-b^2\right)^2}$$

Mathematica [A]
time = 4.04, size = 306, normalized size = 1.05

$$\frac{16a\left(a^3+a^2b-2ab^2-2b^3\right)E\left(\frac{1}{4}(-2c+\pi-2dx)\middle|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}-4\left(4a^4-9a^2b^2+5b^4\right)F\left(\frac{1}{4}(-2c+\pi-2dx)\middle|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}+\sec^3(c+dx)\left(-4a^3b+10ab^3+\left(-6a^3b+14ab^3\right)\cos(2(c+dx))+\left(-2a^3b+4ab^3\right)\cos(4(c+dx))+12a^4\sin(c+dx)-25a^2b^2\sin(c+dx)+13b^4\sin(c+dx)+4a^4\sin(3(c+dx))-9a^2b^2\sin(3(c+dx))+5b^4\sin(3(c+dx))\right)}{24(a-b)^2(a+b)^2d\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (16*a*(a^3 + a^2*b - 2*a*b^2 - 2*b^3)*EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b
)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] - 4*(4*a^4 - 9*a^2*b^2 + 5*b^
4)*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x]
)/(a + b)] + Sec[c + d*x]^3*(-4*a^3*b + 10*a*b^3 + (-6*a^3*b + 14*a*b^3)*Co
s[2*(c + d*x)] + (-2*a^3*b + 4*a*b^3)*Cos[4*(c + d*x)] + 12*a^4*Sin[c + d*x
] - 25*a^2*b^2*Sin[c + d*x] + 13*b^4*Sin[c + d*x] + 4*a^4*Sin[3*(c + d*x)]
- 9*a^2*b^2*Sin[3*(c + d*x)] + 5*b^4*Sin[3*(c + d*x)]))/(24*(a - b)^2*(a +
b)^2*d*Sqrt[a + b*Sin[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1313 vs. $2(337) = 674$.
time = 8.31, size = 1314, normalized size = 4.52

method	result	size
default	Expression too large to display	1314

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+b*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(-(-a-b*sin(d*x+c))*cos(d*x+c)^2)^(1/2)/cos(d*x+c)^5/(a+b*sin(d*x+c))^(
3/2)/b/(a^4-2*a^2*b^2+b^4)*(-4*cos(d*x+c)^4*(b*cos(d*x+c)^2*sin(d*x+c)+a*co
s(d*x+c)^2)^(1/2)*a*b^2*(a^2-2*b^2)+cos(d*x+c)^2*(b*cos(d*x+c)^2*sin(d*x+c)
+a*cos(d*x+c)^2)^(1/2)*b*(4*a^4-9*a^2*b^2+5*b^4)*sin(d*x+c)+2*(b*cos(d*x+c)
^2*sin(d*x+c)+a*cos(d*x+c)^2)^(1/2)*b*(a^4-2*a^2*b^2+b^4)*sin(d*x+c)-cos(d*
x+c)^2*(b*cos(d*x+c)^2*sin(d*x+c)+a*cos(d*x+c)^2)^(1/2)*(4*EllipticF((b/(a-
b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a-b)*sin(d*x+c)-b/
(a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b
)*a)^(1/2)*a^4*b-3*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a
+b))^(1/2))*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b
))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*a^3*b^2-9*EllipticF((b/(a-b)*
sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a-b)*sin(d*x+c)-b/(a-
b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a
)^(1/2)*a^2*b^3+3*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+
b))^(1/2))*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b)
)^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*a*b^4+5*EllipticF((b/(a-b)*sin
(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a-b)*sin(d*x+c)-b/(a-b))
^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(
1/2)*b^5-4*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/
2))*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)
*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*a^5+12*EllipticE((b/(a-b)*sin(d*x+c)+
1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(
-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*a^3
*b^2-8*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*
(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(b/
(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*a*b^4-a^3*b^2+a*b^4))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^4/sqrt(b*sin(d*x + c) + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 571, normalized size = 1.96

$$\frac{\sqrt{2}\left(8a^{4}-19a^{2}b^{2}+15b^{4}\right)\sqrt{ib}\cos\left(dx+c\right)^{3}\mathrm{weierstrassPInverse}\left(-\frac{4\left(4a^{2}-3b^{2}\right)}{3b^{2}},-\frac{8\left(8ia^{3}-9iab^{2}\right)}{27b^{3}},\frac{3b\cos\left(dx+c\right)-3ib\sin\left(dx+c\right)-2ia}{3b}\right)+\sqrt{2}\left(8a^{4}-19a^{2}b^{2}+15b^{4}\right)\sqrt{-ib}\cos\left(dx+c\right)^{3}\mathrm{weierstrassPInverse}\left(-\frac{4\left(4a^{2}-3b^{2}\right)}{3b^{2}},-\frac{8\left(-8ia^{3}+9iab^{2}\right)}{27b^{3}},\frac{3b\cos\left(dx+c\right)+3ib\sin\left(dx+c\right)+2ia}{3b}\right)-12\sqrt{2}\left(-ia^{3}b+2iab^{3}\right)\sqrt{ib}\cos\left(dx+c\right)^{3}\mathrm{weierstrassZeta}\left(-\frac{4\left(4a^{2}-3b^{2}\right)}{3b^{2}},-\frac{8\left(8ia^{3}-9iab^{2}\right)}{27b^{3}},\mathrm{weierstrassPInverse}\left(-\frac{4\left(4a^{2}-3b^{2}\right)}{3b^{2}},-\frac{8\left(8ia^{3}-9iab^{2}\right)}{27b^{3}},\frac{3b\cos\left(dx+c\right)-3ib\sin\left(dx+c\right)-2ia}{3b}\right)\right)-12\sqrt{2}\left(ia^{3}b-2iab^{3}\right)\sqrt{-ib}\cos\left(dx+c\right)^{3}\mathrm{weierstrassZeta}\left(-\frac{4\left(4a^{2}-3b^{2}\right)}{3b^{2}},-\frac{8\left(-8ia^{3}+9iab^{2}\right)}{27b^{3}},\mathrm{weierstrassPInverse}\left(-\frac{4\left(4a^{2}-3b^{2}\right)}{3b^{2}},-\frac{8\left(-8ia^{3}+9iab^{2}\right)}{27b^{3}},\frac{3b\cos\left(dx+c\right)+3ib\sin\left(dx+c\right)+2ia}{3b}\right)\right)-6\left(2a^{2}b^{2}-2b^{4}+\left(a^{2}b^{2}-5b^{4}\right)\cos\left(dx+c\right)^{2}-2\left(a^{3}b-ab^{3}+2\left(a^{3}b-2ab^{3}\right)\cos\left(dx+c\right)^{2}\right)\sin\left(dx+c\right)\right)\sqrt{b\sin\left(dx+c\right)+a}}{36\left(a^{4}b-2a^{2}b^{3}+b^{5}\right)d\cos\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/36*(sqrt(2)*(8*a^4 - 19*a^2*b^2 + 15*b^4)*sqrt(I*b)*cos(d*x + c)^3*weiers
trassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, 1/
3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) - 2*I*a)/b) + sqrt(2)*(8*a^4 - 19*
a^2*b^2 + 15*b^4)*sqrt(-I*b)*cos(d*x + c)^3*weierstrassPInverse(-4/3*(4*a^2
 - 3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*
I*b*sin(d*x + c) + 2*I*a)/b) - 12*sqrt(2)*(-I*a^3*b + 2*I*a*b^3)*sqrt(I*b)*
cos(d*x + c)^3*weierstrassZeta(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9
*I*a*b^2)/b^3, weierstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3
 - 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) - 2*I*a)/b))
- 12*sqrt(2)*(I*a^3*b - 2*I*a*b^3)*sqrt(-I*b)*cos(d*x + c)^3*weierstrassZet
a(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, weierstrassPI
nverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, 1/3*(3*b
*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*I*a)/b)) - 6*(2*a^2*b^2 - 2*b^4 + (a
^2*b^2 - 5*b^4)*cos(d*x + c)^2 - 2*(a^3*b - a*b^3 + 2*(a^3*b - 2*a*b^3)*cos
(d*x + c)^2)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a))/((a^4*b - 2*a^2*b^3 +
b^5)*d*cos(d*x + c)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{4}\left(c + dx\right)}{\sqrt{a + b\sin\left(c + dx\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**4/sqrt(a + b*sin(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^4/sqrt(b*sin(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos\left(c+d\,x\right)^4\,\sqrt{a+b\,\sin\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a + b*sin(c + d*x))^(1/2)),x)
[Out] int(1/(cos(c + d*x)^4*(a + b*sin(c + d*x))^(1/2)), x)
```

3.516 $\int \frac{\cos^5(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=150

$$-\frac{2(a^2-b^2)^2}{b^5d\sqrt{a+b\sin(c+dx)}}-\frac{8a(a^2-b^2)\sqrt{a+b\sin(c+dx)}}{b^5d}+\frac{4(3a^2-b^2)(a+b\sin(c+dx))^{3/2}}{3b^5d}-\frac{8a(a+b\sin(c+dx))^{5/2}}{5b^5d}$$

```
[Out] 4/3*(3*a^2-b^2)*(a+b*sin(d*x+c))^(3/2)/b^5/d-8/5*a*(a+b*sin(d*x+c))^(5/2)/b
^5/d+2/7*(a+b*sin(d*x+c))^(7/2)/b^5/d-2*(a^2-b^2)^2/b^5/d/(a+b*sin(d*x+c))^
(1/2)-8*a*(a^2-b^2)*(a+b*sin(d*x+c))^(1/2)/b^5/d
```

Rubi [A]
time = 0.08, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2747, 711}

$$\frac{4(3a^2-b^2)(a+b\sin(c+dx))^{3/2}}{3b^5d}-\frac{8a(a^2-b^2)\sqrt{a+b\sin(c+dx)}}{b^5d}-\frac{2(a^2-b^2)^2}{b^5d\sqrt{a+b\sin(c+dx)}}+\frac{2(a+b\sin(c+dx))^{7/2}}{7b^5d}-\frac{8a(a+b\sin(c+dx))^{5/2}}{5b^5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*(a^2 - b^2)^2)/(b^5*d*Sqrt[a + b*Sin[c + d*x]]) - (8*a*(a^2 - b^2)*Sqrt
[a + b*Sin[c + d*x]])/(b^5*d) + (4*(3*a^2 - b^2)*(a + b*Sin[c + d*x])^(3/2)
)/(3*b^5*d) - (8*a*(a + b*Sin[c + d*x])^(5/2))/(5*b^5*d) + (2*(a + b*Sin[c
+ d*x])^(7/2))/(7*b^5*d)
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{(b^2-x^2)^2}{(a+x)^{3/2}}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{(a^2-b^2)^2}{(a+x)^{3/2}} - \frac{4(a^3-ab^2)}{\sqrt{a+x}} + 2(3a^2-b^2)\sqrt{a+x} - 4a(a+x)^{3/2} + (a+\right.\right.}{b^5 d}$$

$$= -\frac{2(a^2-b^2)^2}{b^5 d\sqrt{a+b\sin(c+dx)}} - \frac{8a(a^2-b^2)\sqrt{a+b\sin(c+dx)}}{b^5 d} + \frac{4(3a^2-b^2)\,(}{}$$

Mathematica [A]
time = 0.69, size = 165, normalized size = 1.10

$$\frac{-3072a^4+4672a^2b^2-1075b^4-4(48a^2b^2-55b^4)\cos(2(c+dx))+15b^4\cos(4(c+dx))-1536a^3b\sin(c+dx)+2096ab^3\sin(c+dx)+2232a^4\sqrt{1+\frac{b\sin(c+dx)}{a}}-2644a^2b^2\sqrt{1+\frac{b\sin(c+dx)}{a}}+48ab^3\sin(3(c+dx))}{420b^5d\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-3072*a^4 + 4672*a^2*b^2 - 1075*b^4 - 4*(48*a^2*b^2 - 55*b^4)*Cos[2*(c + d
*x)] + 15*b^4*Cos[4*(c + d*x)] - 1536*a^3*b*Sin[c + d*x] + 2096*a*b^3*Sin[c
 + d*x] + 2232*a^4*Sqrt[1 + (b*Sin[c + d*x])/a] - 2644*a^2*b^2*Sqrt[1 + (b*
Sin[c + d*x])/a] + 48*a*b^3*Sin[3*(c + d*x)])/(420*b^5*d*Sqrt[a + b*Sin[c +
 d*x]])
```

Maple [A]
time = 1.00, size = 116, normalized size = 0.77

method	result
default	$\frac{\frac{16a\,b^3\left(\cos^2(dx+c)\right)\sin(dx+c)}{35}+\frac{2\left(-192b\,a^3+256a\,b^3\right)\sin(dx+c)}{105}+\frac{2b^4\left(\cos^4(dx+c)\right)}{7}+\frac{2\left(-48a^2b^2+40b^4\right)\left(\cos^2(dx+c)\right)}{105}-\frac{256a^4}{35}+\frac{1216a^2b^2}{105}-\frac{64}{2}}{b^5\sqrt{a+b\sin\left(dx+c\right)}\,d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a+b*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/105/b^5*(24*a*b^3*cos(d*x+c)^2*sin(d*x+c)+(-192*a^3*b+256*a*b^3)*sin(d*x+
c)+15*b^4*cos(d*x+c)^4+(-48*a^2*b^2+40*b^4)*cos(d*x+c)^2-384*a^4+608*a^2*b^
2-160*b^4)/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [A]
time = 0.27, size = 124, normalized size = 0.83

$$\frac{2\left(\frac{15\,(b\sin(dx+c)+a)^{\frac{7}{2}}-84\,(b\sin(dx+c)+a)^{\frac{5}{2}}a+70\,(3\,a^2-b^2)(b\sin(dx+c)+a)^{\frac{3}{2}}-420\,(a^3-ab^2)\sqrt{b\sin\left(dx+c\right)+a}}{b^4}-\frac{105\,(a^4-2\,a^2b^2+b^4)}{\sqrt{b\sin\left(dx+c\right)+a}\,b^4}\right)}{105\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/105*((15*(b*sin(d*x + c) + a)^(7/2) - 84*(b*sin(d*x + c) + a)^(5/2)*a + 7
0*(3*a^2 - b^2)*(b*sin(d*x + c) + a)^(3/2) - 420*(a^3 - a*b^2)*sqrt(b*sin(d
*x + c) + a))/b^4 - 105*(a^4 - 2*a^2*b^2 + b^4)/(sqrt(b*sin(d*x + c) + a)*b
^4))/(b*d)
```

Fricas [A]
time = 0.38, size = 125, normalized size = 0.83

$$\frac{2\left(15\,b^{4}\cos\left(dx+c\right)^{4}-384\,a^{4}+608\,a^{2}b^{2}-160\,b^{4}-8\left(6\,a^{2}b^{2}-5\,b^{4}\right)\cos\left(dx+c\right)^{2}+8\left(3\,ab^{3}\cos\left(dx+c\right)^{2}-24\,a^{3}b+32\,ab^{3}\right)\sin\left(dx+c\right)\right)\sqrt{b\sin\left(dx+c\right)+a}}{105\left(b^{6}d\sin\left(dx+c\right)+ab^{5}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/105*(15*b^4*cos(d*x + c)^4 - 384*a^4 + 608*a^2*b^2 - 160*b^4 - 8*(6*a^2*b
^2 - 5*b^4)*cos(d*x + c)^2 + 8*(3*a*b^3*cos(d*x + c)^2 - 24*a^3*b + 32*a*b^
3)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)/(b^6*d*sin(d*x + c) + a*b^5*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^5}{{\left(a+b\,\sin\left(c+d\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(a + b*sin(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^5/(a + b*sin(c + d*x))^(3/2), x)
```

3.517 $\int \frac{\cos^3(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=79

$$\frac{2(a^2-b^2)}{b^3d\sqrt{a+b\sin(c+dx)}}+\frac{4a\sqrt{a+b\sin(c+dx)}}{b^3d}-\frac{2(a+b\sin(c+dx))^{3/2}}{3b^3d}$$

```
[Out] -2/3*(a+b*sin(d*x+c))^(3/2)/b^3/d+2*(a^2-b^2)/b^3/d/(a+b*sin(d*x+c))^(1/2)+
4*a*(a+b*sin(d*x+c))^(1/2)/b^3/d
```

Rubi [A]
time = 0.06, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2747, 711}

$$\frac{2(a^2-b^2)}{b^3d\sqrt{a+b\sin(c+dx)}}-\frac{2(a+b\sin(c+dx))^{3/2}}{3b^3d}+\frac{4a\sqrt{a+b\sin(c+dx)}}{b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (2*(a^2 - b^2))/(b^3*d*Sqrt[a + b*Sin[c + d*x]]) + (4*a*Sqrt[a + b*Sin[c +
d*x]])/(b^3*d) - (2*(a + b*Sin[c + d*x])^(3/2))/(3*b^3*d)
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{b^2-x^2}{(a+x)^{3/2}}\,dx, x, b\sin(c+dx)\right)}{b^3d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{-a^2+b^2}{(a+x)^{3/2}} + \frac{2a}{\sqrt{a+x}} - \sqrt{a+x}\right)\,dx, x, b\sin(c+dx)\right)}{b^3d}$$

$$= \frac{2(a^2-b^2)}{b^3d\sqrt{a+b\sin(c+dx)}} + \frac{4a\sqrt{a+b\sin(c+dx)}}{b^3d} - \frac{2(a+b\sin(c+dx))^{3/2}}{3b^3d}$$

Mathematica [A]
time = 0.07, size = 57, normalized size = 0.72

$$\frac{16a^2-7b^2+b^2\cos(2(c+dx))+8ab\sin(c+dx)}{3b^3d\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (16*a^2 - 7*b^2 + b^2*Cos[2*(c + d*x)] + 8*a*b*Sin[c + d*x])/(3*b^3*d*Sqrt[
a + b*Sin[c + d*x]])
```

Maple [A]
time = 0.82, size = 54, normalized size = 0.68

method	result	size
default	$\frac{\frac{2b^2\left(\cos^2(dx+c)\right)}{3}+\frac{8ab\sin(dx+c)}{3}+\frac{16a^2}{3}-\frac{8b^2}{3}}{b^3\sqrt{a+b\sin(dx+c)}\,d}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+b*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/b^3/(a+b*sin(d*x+c))^(1/2)*(b^2*cos(d*x+c)^2+4*a*b*sin(d*x+c)+8*a^2-4*b
^2)/d
```

Maxima [A]
time = 0.27, size = 67, normalized size = 0.85

$$-\frac{2\left(\frac{(b\sin(dx+c)+a)^{\frac{3}{2}}-6\sqrt{b\sin(dx+c)+a}\,a}{b^2}-\frac{3(a^2-b^2)}{\sqrt{b\sin(dx+c)+a}\,b^2}\right)}{3\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -2/3*(((b*sin(d*x + c) + a)^(3/2) - 6*sqrt(b*sin(d*x + c) + a)*a)/b^2 - 3*(
a^2 - b^2)/(sqrt(b*sin(d*x + c) + a)*b^2))/(b*d)
```

Fricas [A]
time = 0.38, size = 67, normalized size = 0.85

$$\frac{2\left(b^{2}\cos\left(dx+c\right)^{2}+4\,ab\sin\left(dx+c\right)+8\,a^{2}-4\,b^{2}\right)\sqrt{b\sin\left(dx+c\right)+a}}{3\left(b^{4}d\sin\left(dx+c\right)+ab^{3}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(b^2*cos(d*x + c)^2 + 4*a*b*sin(d*x + c) + 8*a^2 - 4*b^2)*sqrt(b*sin(d*
x + c) + a)/(b^4*d*sin(d*x + c) + a*b^3*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^3}{{\left(a+b\,\sin\left(c+d\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a + b*sin(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^3/(a + b*sin(c + d*x))^(3/2), x)
```

3.518 $\int \frac{\cos(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=22

$$-\frac{2}{bd\sqrt{a+b\sin(c+dx)}}$$

```
[Out] -2/b/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2747, 32}

$$-\frac{2}{bd\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] -2/(b*d*Sqrt[a + b*Sin[c + d*x]])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{(a+x)^{3/2}}\,dx, x, b\sin(c+dx)\right)}{bd}\\ &= -\frac{2}{bd\sqrt{a+b\sin(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 22, normalized size = 1.00

$$-\frac{2}{bd\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] -2/(b*d*Sqrt[a + b*Sin[c + d*x]])
```

Maple [A]
time = 0.04, size = 21, normalized size = 0.95

method	result	size
derivativedivides	$-\frac{2}{bd\sqrt{a+b\sin\left(dx+c\right)}}$	21
default	$-\frac{2}{bd\sqrt{a+b\sin\left(dx+c\right)}}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+b*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/b/d/(a+b*sin(d*x+c))^(1/2)
```

Maxima [A]
time = 0.29, size = 20, normalized size = 0.91

$$-\frac{2}{\sqrt{b\sin\left(dx+c\right)+a}\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -2/(sqrt(b*sin(d*x + c) + a)*b*d)
```

Fricas [A]
time = 0.35, size = 32, normalized size = 1.45

$$-\frac{2\sqrt{b\sin\left(dx+c\right)+a}}{b^2d\sin\left(dx+c\right)+abd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(b*sin(d*x + c) + a)/(b^2*d*sin(d*x + c) + a*b*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(19) = 38$.

time = 1.00, size = 56, normalized size = 2.55

$$\begin{cases} \frac{x\cos(c)}{a^{\frac{3}{2}}} & \text{for } b = 0 \wedge d = 0 \\ \frac{\sin(c+dx)}{a^{\frac{3}{2}}d} & \text{for } b = 0 \\ \frac{x\cos(c)}{(a+b\sin(c))^{\frac{3}{2}}} & \text{for } d = 0 \\ -\frac{2}{bd\sqrt{a+b\sin(c+dx)}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Piecewise((x*cos(c)/a**(3/2), Eq(b, 0) & Eq(d, 0)), (sin(c + d*x)/(a**(3/2)
*d), Eq(b, 0)), (x*cos(c)/(a + b*sin(c))**(3/2), Eq(d, 0)), (-2/(b*d*sqrt(a
 + b*sin(c + d*x))), True))
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 6.14, size = 51, normalized size = 2.32

$$-\frac{4\,{\left(a+b\,\sin\left(c+d\,x\right)\right)}^{3/2}}{b\,d\,\left(2\,a^2+4\,a\,b\,\sin\left(c+d\,x\right)+2\,b^2\,{\sin\left(c+d\,x\right)}^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + b*sin(c + d*x))^(3/2),x)
```

```
[Out] -(4*(a + b*sin(c + d*x))^(3/2))/(b*d*(2*a^2 + 2*b^2*sin(c + d*x)^2 + 4*a*b*
sin(c + d*x)))
```

3.519 $\int \frac{\sec(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=105

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}d}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}d}+\frac{2b}{(a^2-b^2)\,d\sqrt{a+b\sin(c+dx)}}$$

```
[Out] -arctanh((a+b*sin(d*x+c))^(1/2)/(a-b)^(1/2))/(a-b)^(3/2)/d+arctanh((a+b*sin
(d*x+c))^(1/2)/(a+b)^(1/2))/(a+b)^(3/2)/d+2*b/(a^2-b^2)/d/(a+b*sin(d*x+c))^
(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2747, 724, 841, 1180, 212}

$$\frac{2b}{d\,(a^2-b^2)\,\sqrt{a+b\sin(c+dx)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{d(a-b)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{d(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a - b]]/((a - b)^(3/2)*d)) + ArcTan
h[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a + b]]/((a + b)^(3/2)*d) + (2*b)/((a^2 - b
^2)*d*Sqrt[a + b*Sin[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Simp[e*((d
 + e*x)^(m + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + Dist[c/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*((d - e*x)/(a + c*x^2)), x], x] /; FreeQ[{a, c, d, e, m
}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 841

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
```

```
eQ[c*d^2 + a*e^2, 0]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx = \frac{b\text{Subst}\left(\int \frac{1}{(a+x)^{3/2}(b^2-x^2)}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{2b}{(a^2-b^2)\,d\sqrt{a+b\sin(c+dx)}} + \frac{b\text{Subst}\left(\int \frac{a-x}{\sqrt{a+x}\,(b^2-x^2)}\,dx, x, b\sin(c+dx$$

$$= \frac{2b}{(a^2-b^2)\,d\sqrt{a+b\sin(c+dx)}} + \frac{(2b)\text{Subst}\left(\int \frac{2a-x^2}{-a^2+b^2+2ax^2-x^4}\,dx, x, \sqrt{a+b\,\text{si}$$

$$= \frac{2b}{(a^2-b^2)\,d\sqrt{a+b\sin(c+dx)}} - \frac{\text{Subst}\left(\int \frac{1}{a-b-x^2}\,dx, x, \sqrt{a+b\sin(c+dx)}\right)}{(a-b)d}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}d} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}d} + \frac{}{(a^2-$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.09, size = 91, normalized size = 0.87

$$\frac{(a+b)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{a+b\sin(c+dx)}{a-b}\right)+(-a+b)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{a+b\sin(c+dx)}{a+b}\right)}{(a-b)(a+b)d\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((a + b)*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Sin[c + d*x])/(a - b)] + (-
a + b)*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Sin[c + d*x])/(a + b)])/((a -
 b)*(a + b)*d*Sqrt[a + b*Sin[c + d*x]])
```

Maple [A]
time = 1.53, size = 94, normalized size = 0.90

method	result	size
default	$\frac{\frac{2b}{(a-b)(a+b)\sqrt{a+b\sin\left(dx+c\right)}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sin\left(dx+c\right)}}{\sqrt{a+b}}\right)}{(a+b)^{\frac{3}{2}}}+\frac{\arctan\left(\frac{\sqrt{a+b\sin\left(dx+c\right)}}{\sqrt{-a+b}}\right)}{(a-b)\sqrt{-a+b}}}{d}$	94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+b*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (2*b/(a-b)/(a+b)/(a+b*sin(d*x+c))^(1/2)+1/(a+b)^(3/2)*arctanh((a+b*sin(d*x+
c))^(1/2)/(a+b)^(1/2))+1/(a-b)/(-a+b)^(1/2)*arctan((a+b*sin(d*x+c))^(1/2)/(
-a+b)^(1/2)))/d
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for more
 detail
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(b*sin(d*x + c) + a)*sec(d*x + c)/(b^2*cos(d*x + c)^2 - 2*a*b
*sin(d*x + c) - a^2 - b^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(c+dx)}{(a+b\sin(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral(sec(c + d*x)/(a + b*sin(c + d*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)/(b*sin(d*x + c) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(c+d\,x)\,\left(a+b\,\sin(c+d\,x)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + b*sin(c + d*x))^(3/2)),x)
```

```
[Out] int(1/(cos(c + d*x)*(a + b*sin(c + d*x))^(3/2)), x)
```

3.520 $\int \frac{\sec^3(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=186

$$-\frac{(2a-5b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{4(a-b)^{5/2}d}+\frac{(2a+5b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{4(a+b)^{5/2}d}-\frac{b(a^2+}{2\left(a^2-b^2\right)^2 d\sqrt{a-}}$$

[Out] -1/4*(2*a-5*b)*arctanh((a+b*sin(d*x+c))^(1/2)/(a-b)^(1/2))/(a-b)^(5/2)/d+1/4*(2*a+5*b)*arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))/(a+b)^(5/2)/d-1/2*b*(a^2+5*b^2)/(a^2-b^2)^2/d/(a+b*sin(d*x+c))^(1/2)-1/2*sec(d*x+c)^2*(b-a*sin(d*x+c))/(a^2-b^2)/d/(a+b*sin(d*x+c))^(1/2)

Rubi [A]
time = 0.24, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2747, 755, 843, 841, 1180, 212}

$$-\frac{b(a^2+5b^2)}{2d\left(a^2-b^2\right)^2\sqrt{a+b\sin(c+dx)}}-\frac{\sec^2(c+dx)(b-a\sin(c+dx))}{2d\left(a^2-b^2\right)\sqrt{a+b\sin(c+dx)}}-\frac{(2a-5b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{4d(a-b)^{5/2}}+\frac{(2a+5b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{4d(a+b)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d*x]^3/(a + b*Sin[c + d*x])^(3/2),x]

[Out] -1/4*((2*a - 5*b)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a - b]])/((a - b)^(5/2)*d) + ((2*a + 5*b)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a + b]])/(4*(a + b)^(5/2)*d) - (b*(a^2 + 5*b^2))/(2*(a^2 - b^2)^2*d*Sqrt[a + b*Sin[c + d*x]]) - (Sec[c + d*x]^2*(b - a*Sin[c + d*x]))/(2*(a^2 - b^2)*d*Sqrt[a + b*Sin[c + d*x]])

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 841

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 843

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Simp[(e*f - d*g)*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 + a*e^2))
), x] + Dist[1/(c*d^2 + a*e^2), Int[(d + e*x)^(m + 1)*(Simp[c*d*f + a*e*g -
 c*(e*f - d*g)*x, x]/(a + c*x^2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x
] && NeQ[c*d^2 + a*e^2, 0] && FractionQ[m] && LtQ[m, -1]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx = \frac{b^3\text{Subst}\left(\int \frac{1}{(a+x)^{3/2}(b^2-x^2)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= -\frac{\sec^2(c+dx)(b-a\sin(c+dx))}{2(a^2-b^2)\,d\sqrt{a+b\sin(c+dx)}} + \frac{b\text{Subst}\left(\int \frac{\frac{1}{2}(2a^2-5b^2)+\frac{3ax}{2}}{(a+x)^{3/2}(b^2-x^2)}\,dx, x, b\sin(c+\right.}{2(a^2-b^2)\,d}$$

$$= -\frac{b(a^2+5b^2)}{2(a^2-b^2)^2\,d\sqrt{a+b\sin(c+dx)}} - \frac{\sec^2(c+dx)(b-a\sin(c+dx))}{2(a^2-b^2)\,d\sqrt{a+b\sin(c+dx)}} - \frac{b\text{S}}{}$$

$$= -\frac{b(a^2+5b^2)}{2(a^2-b^2)^2\,d\sqrt{a+b\sin(c+dx)}} - \frac{\sec^2(c+dx)(b-a\sin(c+dx))}{2(a^2-b^2)\,d\sqrt{a+b\sin(c+dx)}} - \frac{b\text{S}}{}$$

$$= -\frac{b(a^2+5b^2)}{2(a^2-b^2)^2\,d\sqrt{a+b\sin(c+dx)}} - \frac{\sec^2(c+dx)(b-a\sin(c+dx))}{2(a^2-b^2)\,d\sqrt{a+b\sin(c+dx)}} - \frac{(2}{}$$

$$= -\frac{(2a-5b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{4(a-b)^{5/2}d} + \frac{(2a+5b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin}}{\sqrt{a-}}\right.}{4(a+b)^{5/2}d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 1.15, size = 221, normalized size = 1.19

$$\frac{\frac{3a\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}} - \frac{3a\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}} + \frac{(a^2+5b^2)\left((a+b)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{a+b\sin(c+dx)}{a-b}\right)+(-a+b)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{a+b\sin(c+dx)}{a+b}\right)\right)}{(a-b)(a+b)\sqrt{a+b\sin(c+dx)}} + \frac{2\sec^2(c+dx)(b-a\sin(c+dx))}{\sqrt{a+b\sin(c+dx)}}}{4(-a^2+b^2)\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((3*a*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a - b]])/Sqrt[a - b] - (3*a*Arc
Tanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a + b]])/Sqrt[a + b] + ((a^2 + 5*b^2)*((
a + b)*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Sin[c + d*x])/(a - b)] + (-a
+ b)*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Sin[c + d*x])/(a + b)]))/((a -
b)*(a + b)*Sqrt[a + b*Sin[c + d*x]]) + (2*Sec[c + d*x]^2*(b - a*Sin[c + d*x
]))/Sqrt[a + b*Sin[c + d*x]])/(4*(-a^2 + b^2)*d)
```

Maple [A]
time = 2.76, size = 233, normalized size = 1.25

method	result

default	$\frac{-\frac{b\sqrt{a+b\sin(dx+c)}}{4(a+b)^2(b\sin(dx+c)-b)}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sin(dx+c)}}{\sqrt{a+b}}\right)a}{2(a+b)^{\frac{5}{2}}}+\frac{5b\operatorname{arctanh}\left(\frac{\sqrt{a+b\sin(dx+c)}}{\sqrt{a+b}}\right)}{4(a+b)^{\frac{5}{2}}}-\frac{b\sqrt{a+b\sin}}{4(a-b)^2(b\sin}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+b*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-1/4*b/(a+b)^2*(a+b*sin(d*x+c))^(1/2)/(b*sin(d*x+c)-b)+1/2/(a+b)^(5/2)*arc
tanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))*a+5/4*b/(a+b)^(5/2)*arctanh((a+b*s
in(d*x+c))^(1/2)/(a+b)^(1/2))-1/4*b/(a-b)^2*(a+b*sin(d*x+c))^(1/2)/(b*sin(d
*x+c)+b)+1/2/(a-b)^2/(-a+b)^(1/2)*arctan((a+b*sin(d*x+c))^(1/2)/(-a+b)^(1/2
))*a-5/4*b/(a-b)^2/(-a+b)^(1/2)*arctan((a+b*sin(d*x+c))^(1/2)/(-a+b)^(1/2))
-2*b^3/(a-b)^2/(a+b)^2/(a+b*sin(d*x+c))^(1/2))/d
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for more
 detail
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (residue poly has multiple non-linear facto
rs)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(c+dx)}{(a+b\sin(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral(sec(c + d*x)**3/(a + b*sin(c + d*x))**(3/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)^3\,{\left(a+b\,\sin\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + b*sin(c + d*x))^(3/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^3*(a + b*sin(c + d*x))^(3/2)), x)
```

3.521 $\int \frac{\sec^5(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=284

$$-\frac{3(4a^2-14ab+15b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{32(a-b)^{7/2}d}+\frac{3(4a^2+14ab+15b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{32(a+b)^{7/2}d}$$

```
[Out] -3/32*(4*a^2-14*a*b+15*b^2)*arctanh((a+b*sin(d*x+c))^(1/2)/(a-b)^(1/2))/(a-
b)^(7/2)/d+3/32*(4*a^2+14*a*b+15*b^2)*arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^
(1/2))/(a+b)^(7/2)/d-3/16*b*(2*a^4-7*a^2*b^2-15*b^4)/(a^2-b^2)^3/d/(a+b*sin
(d*x+c))^(1/2)-1/4*sec(d*x+c)^4*(b-a*sin(d*x+c))/(a^2-b^2)/d/(a+b*sin(d*x+c
))^(1/2)+1/16*sec(d*x+c)^2*(b*(a^2+9*b^2)+2*a*(3*a^2-8*b^2)*sin(d*x+c))/(a^
2-b^2)^2/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A]
time = 0.35, antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {2747, 755, 837, 843, 841, 1180, 212}

$$-\frac{3(4a^2-14ab+15b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{32d(a-b)^{7/2}}+\frac{3(4a^2+14ab+15b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{32d(a+b)^{7/2}}-\frac{\sec^4(c+dx)(b-a\sin(c+dx))}{4d(a^2-b^2)\sqrt{a+b\sin(c+dx)}}+\frac{\sec^2(c+dx)\left(2a(3a^2-8b^2)\sin(c+dx)+b(a^2+9b^2)\right)}{16d(a^2-b^2)^2\sqrt{a+b\sin(c+dx)}}-\frac{3b(2a^4-7a^2b^2-15b^4)}{16d(a^2-b^2)^3\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-3*(4*a^2 - 14*a*b + 15*b^2)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a - b]]
)/(32*(a - b)^(7/2)*d) + (3*(4*a^2 + 14*a*b + 15*b^2)*ArcTanh[Sqrt[a + b*Si
n[c + d*x]]/Sqrt[a + b]])/(32*(a + b)^(7/2)*d) - (3*b*(2*a^4 - 7*a^2*b^2 -
15*b^4))/(16*(a^2 - b^2)^3*d*Sqrt[a + b*Sin[c + d*x]]) - (Sec[c + d*x]^4*(b
 - a*Sin[c + d*x]))/(4*(a^2 - b^2)*d*Sqrt[a + b*Sin[c + d*x]]) + (Sec[c + d
*x]^2*(b*(a^2 + 9*b^2) + 2*a*(3*a^2 - 8*b^2)*Sin[c + d*x]))/(16*(a^2 - b^2)
^2*d*Sqrt[a + b*Sin[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
```

```
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 841

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 843

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Simp[(e*f - d*g)*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 + a*e^2))
), x] + Dist[1/(c*d^2 + a*e^2), Int[(d + e*x)^(m + 1)*(Simp[c*d*f + a*e*g -
 c*(e*f - d*g)*x, x]/(a + c*x^2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x
] && NeQ[c*d^2 + a*e^2, 0] && FractionQ[m] && LtQ[m, -1]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx = \frac{b^5\text{Subst}\left(\int \frac{1}{(a+x)^{3/2}(b^2-x^2)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= -\frac{\sec^4(c+dx)(b-a\sin(c+dx))}{4\,(a^2-b^2)\,d\sqrt{a+b\sin(c+dx)}} + \frac{b^3\text{Subst}\left(\int \frac{\frac{3}{2}(2a^2-3b^2)+\frac{7ax}{2}}{(a+x)^{3/2}(b^2-x^2)^2}\,dx, x, b\sin(c+\right.}{4\,(a^2-b^2)\,d}$$

$$= -\frac{\sec^4(c+dx)(b-a\sin(c+dx))}{4\,(a^2-b^2)\,d\sqrt{a+b\sin(c+dx)}} + \frac{\sec^2(c+dx)\,(b(a^2+9b^2)+2a(3a^2-8b^2}{16\,(a^2-b^2)^2\,d\sqrt{a+b\sin(c+}}$$

$$= -\frac{3b(2a^4-7a^2b^2-15b^4)}{16\,(a^2-b^2)^3\,d\sqrt{a+b\sin(c+dx)}} - \frac{\sec^4(c+dx)(b-a\sin(c+dx))}{4\,(a^2-b^2)\,d\sqrt{a+b\sin(c+dx)}} + \frac{\text{sec}}{}$$

$$= -\frac{3b(2a^4-7a^2b^2-15b^4)}{16\,(a^2-b^2)^3\,d\sqrt{a+b\sin(c+dx)}} - \frac{\sec^4(c+dx)(b-a\sin(c+dx))}{4\,(a^2-b^2)\,d\sqrt{a+b\sin(c+dx)}} + \frac{\text{sec}}{}$$

$$= -\frac{3b(2a^4-7a^2b^2-15b^4)}{16\,(a^2-b^2)^3\,d\sqrt{a+b\sin(c+dx)}} - \frac{\sec^4(c+dx)(b-a\sin(c+dx))}{4\,(a^2-b^2)\,d\sqrt{a+b\sin(c+dx)}} + \frac{\text{sec}}{}$$

$$= -\frac{3(4a^2-14ab+15b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{32(a-b)^{7/2}d} + \frac{3(4a^2+14ab+15b}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 2.18, size = 324, normalized size = 1.14

$$\frac{\frac{3}{2}(2a^4-7a^2b^2-15b^4)\left((a+b)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{a+b\sin(c+dx)}{a-b}\right)+(-a+b)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{a+b\sin(c+dx)}{a+b}\right)\right)-4(a-b)^2(a+b)^2\sec^4(c+dx)(-b+a\sin(c+dx))+3a\sqrt{a-b}\sqrt{a+b}\,(3a^2-8b^2)\left(\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)-\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)\right)\sqrt{a+b\sin(c+dx)}-(a-b)(a+b)\sec^2(c+dx)\,(b(a^2+9b^2)+2a(3a^2-8b^2)\sin(c+dx))}{16\,(a^2-b^2)^2\,(-a^2+b^2)\,d\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((3*(2*a^4 - 7*a^2*b^2 - 15*b^4)*((a + b)*Hypergeometric2F1[-1/2, 1, 1/2, (
a + b*Sin[c + d*x])/(a - b)] + (-a + b)*Hypergeometric2F1[-1/2, 1, 1/2, (a
+ b*Sin[c + d*x])/(a + b)]))/2 - 4*(a - b)^2*(a + b)^2*Sec[c + d*x]^4*(-b +
 a*Sin[c + d*x]) + 3*a*Sqrt[a - b]*Sqrt[a + b]*(3*a^2 - 8*b^2)*(Sqrt[a + b]
*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a - b]] - Sqrt[a - b]*ArcTanh[Sqrt[a
 + b*Sin[c + d*x]]/Sqrt[a + b]])*Sqrt[a + b*Sin[c + d*x]] - (a - b)*(a + b)
*Sec[c + d*x]^2*(b*(a^2 + 9*b^2) + 2*a*(3*a^2 - 8*b^2)*Sin[c + d*x]))/(16*(
a^2 - b^2)^2*(-a^2 + b^2)*d*Sqrt[a + b*Sin[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 601 vs. $2(256) = 512$.
time = 3.14, size = 602, normalized size = 2.12

method	result
default	$-\frac{3b(a+b\sin(dx+c))^{\frac{3}{2}}a}{16(a+b)^3(b\sin(dx+c)-b)^2}-\frac{13b^2(a+b\sin(dx+c))^{\frac{3}{2}}}{32(a+b)^3(b\sin(dx+c)-b)^2}+\frac{3b\sqrt{a+b\sin(dx+c)}\,a^2}{16(a+b)^3(b\sin(dx+c)-b)^2}+\frac{21b^2\sqrt{a+b\sin(dx+c)}\,a}{32(a+b)^3(b\sin(dx+c)-b)^2}+\frac{15b^3\sqrt{}}{32}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+b*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-3/16*b/(a+b)^3/(b*sin(d*x+c)-b)^2*(a+b*sin(d*x+c))^(3/2)*a-13/32*b^2/(a+b
)^3/(b*sin(d*x+c)-b)^2*(a+b*sin(d*x+c))^(3/2)+3/16*b/(a+b)^3/(b*sin(d*x+c)-
b)^2*(a+b*sin(d*x+c))^(1/2)*a^2+21/32*b^2/(a+b)^3/(b*sin(d*x+c)-b)^2*(a+b*s
in(d*x+c))^(1/2)*a+15/32*b^3/(a+b)^3/(b*sin(d*x+c)-b)^2*(a+b*sin(d*x+c))^(1
/2)+3/8/(a+b)^(7/2)*arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))*a^2+21/16*b
/(a+b)^(7/2)*arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))*a+45/32*b^2/(a+b)^
(7/2)*arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))-3/16*b/(a-b)^3/(b*sin(d*x
+c)+b)^2*(a+b*sin(d*x+c))^(3/2)*a+13/32*b^2/(a-b)^3/(b*sin(d*x+c)+b)^2*(a+b
*sin(d*x+c))^(3/2)+3/16*b/(a-b)^3/(b*sin(d*x+c)+b)^2*(a+b*sin(d*x+c))^(1/2)
*a^2-21/32*b^2/(a-b)^3/(b*sin(d*x+c)+b)^2*(a+b*sin(d*x+c))^(1/2)*a+15/32*b^
3/(a-b)^3/(b*sin(d*x+c)+b)^2*(a+b*sin(d*x+c))^(1/2)+3/8/(a-b)^3/(-a+b)^(1/2
)*arctan((a+b*sin(d*x+c))^(1/2)/(-a+b)^(1/2))*a^2-21/16*b/(a-b)^3/(-a+b)^(1
/2)*arctan((a+b*sin(d*x+c))^(1/2)/(-a+b)^(1/2))*a+45/32*b^2/(a-b)^3/(-a+b)^
(1/2)*arctan((a+b*sin(d*x+c))^(1/2)/(-a+b)^(1/2))+2*b^5/(a-b)^3/(a+b)^3/(a+
b*sin(d*x+c))^(1/2))/d
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for more
 detail
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(b*sin(d*x + c) + a)*sec(d*x + c)^5/(b^2*cos(d*x + c)^2 - 2*a
*b*sin(d*x + c) - a^2 - b^2), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^5{(c+dx)}}{(a+b\sin{(c+dx)})^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral(sec(c + d*x)**5/(a + b*sin(c + d*x))**(3/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos(c+d\,x)^5\,(a+b\,\sin(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^5*(a + b*sin(c + d*x))^(3/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^5*(a + b*sin(c + d*x))^(3/2)), x)
```

3.522 $\int \frac{\cos^6(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=313

$$-\frac{2\cos^5(c+dx)}{bd\sqrt{a+b\sin(c+dx)}}+\frac{20\cos^3(c+dx)(8a-7b\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{63b^3d}-\frac{16(32a^4-57a^2b^2+2\ldots}{\ldots}$$

```
[Out] -2*cos(d*x+c)^5/b/d/(a+b*sin(d*x+c))^(1/2)+20/63*cos(d*x+c)^3*(8*a-7*b*sin(
d*x+c))*(a+b*sin(d*x+c))^(1/2)/b^3/d-8/63*cos(d*x+c)*(a*(32*a^2-33*b^2)-3*b
*(8*a^2-7*b^2)*sin(d*x+c))*(a+b*sin(d*x+c))^(1/2)/b^5/d+16/63*(32*a^4-57*a^
2*b^2+21*b^4)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)
*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*sin(d*x+
c))^(1/2)/b^6/d/((a+b*sin(d*x+c))/(a+b))^(1/2)-16/63*a*(32*a^4-65*a^2*b^2+3
3*b^4)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*Ellipt
icF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a
+b))^(1/2)/b^6/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A]

time = 0.35, antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {2772, 2944, 2831, 2742, 2740, 2734, 2732}

$$-\frac{8\cos(c+dx)\sqrt{a+b\sin(c+dx)}\,(a(32a^2-33b^2)-3b(8a^2-7b^2)\sin(c+dx))}{63b^5d}+\frac{16a(32a^4-65a^2b^2+33b^4)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{63b^6d\sqrt{a+b\sin(c+dx)}}-\frac{16(32a^4-57a^2b^2+21b^4)\sqrt{a+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{63b^6d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}+\frac{20\cos^3(c+dx)(8a-7b\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{63b^3d}-\frac{2\cos^5(c+dx)}{bd\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*Cos[c + d*x]^5)/(b*d*Sqrt[a + b*Sin[c + d*x]]) + (20*Cos[c + d*x]^3*(8*
a - 7*b*Sin[c + d*x])*Sqrt[a + b*Sin[c + d*x]])/(63*b^3*d) - (16*(32*a^4 -
57*a^2*b^2 + 21*b^4)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a +
b*Sin[c + d*x]])/(63*b^6*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + (16*a*(32*
a^4 - 65*a^2*b^2 + 33*b^4)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqr
t[(a + b*Sin[c + d*x])/(a + b)])/(63*b^6*d*Sqrt[a + b*Sin[c + d*x]]) - (8*C
os[c + d*x]*Sqrt[a + b*Sin[c + d*x]]*(a*(32*a^2 - 33*b^2) - 3*b*(8*a^2 - 7*
b^2)*Sin[c + d*x]))/(63*b^5*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2772

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Dist[g^2*((p - 1)/(b*(m + 1))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2944

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Dist[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\cos^6(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx = -\frac{2\cos^5(c+dx)}{bd\sqrt{a+b\sin(c+dx)}} - \frac{10\int \frac{\cos^4(c+dx)\sin(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx}{b}$$

$$= -\frac{2\cos^5(c+dx)}{bd\sqrt{a+b\sin(c+dx)}} + \frac{20\cos^3(c+dx)(8a-7b\sin(c+dx))\sqrt{a+b\sin(c}}{63b^3d}$$

$$= -\frac{2\cos^5(c+dx)}{bd\sqrt{a+b\sin(c+dx)}} + \frac{20\cos^3(c+dx)(8a-7b\sin(c+dx))\sqrt{a+b\sin(c}}{63b^3d}$$

$$= -\frac{2\cos^5(c+dx)}{bd\sqrt{a+b\sin(c+dx)}} + \frac{20\cos^3(c+dx)(8a-7b\sin(c+dx))\sqrt{a+b\sin(c}}{63b^3d}$$

$$= -\frac{2\cos^5(c+dx)}{bd\sqrt{a+b\sin(c+dx)}} + \frac{20\cos^3(c+dx)(8a-7b\sin(c+dx))\sqrt{a+b\sin(c}}{63b^3d}$$

$$= -\frac{2\cos^5(c+dx)}{bd\sqrt{a+b\sin(c+dx)}} + \frac{20\cos^3(c+dx)(8a-7b\sin(c+dx))\sqrt{a+b\sin(c}}{63b^3d}$$

Mathematica [A]
time = 1.50, size = 273, normalized size = 0.87

$$\frac{64(32a^5+32a^4b-57a^3b^2-57a^2b^3+21ab^4+21b^5)\,E\left(\frac{1}{4}(-2c+\pi-2dx)\middle|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}} - 64a(32a^4-65a^2b^2+33b^4)\,F\left(\frac{1}{4}(-2c+\pi-2dx)\middle|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}} + b\cos(c+dx)\left(-1024a^4+1760a^2b^2-595b^4+(-64a^2b^2+84b^4)\cos(2(c+dx))+7b^4\cos(4(c+dx))-256a^3b\sin(c+dx)+404ab^3\sin(c+dx)+20ab^3\sin(3(c+dx))\right)}{252b^6d\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (64*(32*a^5 + 32*a^4*b - 57*a^3*b^2 - 57*a^2*b^3 + 21*a*b^4 + 21*b^5)*Ellip
ticE[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b
)] - 64*a*(32*a^4 - 65*a^2*b^2 + 33*b^4)*EllipticF[(-2*c + Pi - 2*d*x)/4, (
2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] + b*Cos[c + d*x]*(-1024*a^
4 + 1760*a^2*b^2 - 595*b^4 + (-64*a^2*b^2 + 84*b^4)*Cos[2*(c + d*x)] + 7*b^
4*Cos[4*(c + d*x)] - 256*a^3*b*Sin[c + d*x] + 404*a*b^3*Sin[c + d*x] + 20*a
*b^3*Sin[3*(c + d*x)]))/(252*b^6*d*Sqrt[a + b*Sin[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1194 vs. $2(357) = 714$.
time = 2.26, size = 1195, normalized size = 3.82

method	result	size
default	Expression too large to display	1195

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6/(a+b*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/63*(7*b^6*sin(d*x+c)^6-10*a*b^5*sin(d*x+c)^5+256*((a+b*sin(d*x+c))/(a-b)
)^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Ell
ipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*b-192*((a+b*
sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b
/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))
*a^4*b^2-520*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)
*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),(
(a-b)/(a+b))^(1/2))*a^3*b^3+360*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c
)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x
+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^4+264*((a+b*sin(d*x+c))/(a-b))
^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Elli
pticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^5-168*((a+b*s
in(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/
(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*
b^6-256*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1
+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)
/(a+b))^(1/2))*a^6+712*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a
+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b
))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b^2-624*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-
(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a
+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^4+168*((a+b*sin(d*x+
c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^
(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b^6+16*
a^2*b^4*sin(d*x+c)^4-35*b^6*sin(d*x+c)^4-32*a^3*b^3*sin(d*x+c)^3+68*a*b^5*s
in(d*x+c)^3-128*a^4*b^2*sin(d*x+c)^2+196*a^2*b^4*sin(d*x+c)^2-35*b^6*sin(d*
x+c)^2+32*a^3*b^3*sin(d*x+c)-58*a*b^5*sin(d*x+c)+128*a^4*b^2-212*a^2*b^4+63
*b^6)/b^7/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+b*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^6/(b*sin(d*x + c) + a)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.21, size = 723, normalized size = 2.31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/189*(8*(sqrt(2)*(32*a^5*b - 69*a^3*b^3 + 39*a*b^5)*sin(d*x + c) + sqrt(2)
*(32*a^6 - 69*a^4*b^2 + 39*a^2*b^4))*sqrt(I*b)*weierstrassPInverse(-4/3*(4*
a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) -
3*I*b*sin(d*x + c) - 2*I*a)/b) + 8*(sqrt(2)*(32*a^5*b - 69*a^3*b^3 + 39*a*b
^5)*sin(d*x + c) + sqrt(2)*(32*a^6 - 69*a^4*b^2 + 39*a^2*b^4))*sqrt(-I*b)*w
eierstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b
^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*I*a)/b) - 12*(sqrt(2)*(-
32*I*a^4*b^2 + 57*I*a^2*b^4 - 21*I*b^6)*sin(d*x + c) + sqrt(2)*(-32*I*a^5*b
 + 57*I*a^3*b^3 - 21*I*a*b^5))*sqrt(I*b)*weierstrassZeta(-4/3*(4*a^2 - 3*b^
2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, weierstrassPInverse(-4/3*(4*a^2 -
3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*
sin(d*x + c) - 2*I*a)/b)) - 12*(sqrt(2)*(32*I*a^4*b^2 - 57*I*a^2*b^4 + 21*I
*b^6)*sin(d*x + c) + sqrt(2)*(32*I*a^5*b - 57*I*a^3*b^3 + 21*I*a*b^5))*sqrt
(-I*b)*weierstrassZeta(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^
2)/b^3, weierstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I
*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*I*a)/b)) + 3*(7
*b^6*cos(d*x + c)^5 - 2*(8*a^2*b^4 - 7*b^6)*cos(d*x + c)^3 - 4*(32*a^4*b^2
- 57*a^2*b^4 + 21*b^6)*cos(d*x + c) + 2*(5*a*b^5*cos(d*x + c)^3 - 8*(2*a^3*
b^3 - 3*a*b^5)*cos(d*x + c))*sin(d*x + c))*sqrt(b*sin(d*x + c) + a))/(b^8*d
*sin(d*x + c) + a*b^7*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6/(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+b*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^6}{(a+b\,\sin(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(a + b*sin(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^6/(a + b*sin(c + d*x))^(3/2), x)
```

3.523 $\int \frac{\cos^4(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=229

$$-\frac{2\cos^3(c+dx)}{bd\sqrt{a+b\sin(c+dx)}}+\frac{4\cos(c+dx)(4a-3b\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{5b^3d}+\frac{8(4a^2-3b^2)\,E\left(\frac{1}{2}\left(c-\frac{\pi}{2}\right.\right.}{5b^4d\sqrt{\frac{a}{-}}}$$

```
[Out] -2*cos(d*x+c)^3/b/d/(a+b*sin(d*x+c))^(1/2)+4/5*cos(d*x+c)*(4*a-3*b*sin(d*x+
c))*(a+b*sin(d*x+c))^(1/2)/b^3/d-8/5*(4*a^2-3*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*
x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2
^(1/2)*(b/(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/b^4/d/((a+b*sin(d*x+c))/(a+b
))^(1/2)+32/5*a*(a^2-b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4
*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*(
(a+b*sin(d*x+c))/(a+b))^(1/2)/b^4/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A]

time = 0.22, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {2772, 2944, 2831, 2742, 2740, 2734, 2732}

$$-\frac{32a(a^2-b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{5b^4d\sqrt{a+b\sin(c+dx)}}+\frac{8(4a^2-3b^2)\sqrt{a+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{5b^4d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}+\frac{4\cos(c+dx)(4a-3b\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{5b^3d}-\frac{2\cos^3(c+dx)}{bd\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*Cos[c + d*x]^3)/(b*d*Sqrt[a + b*Sin[c + d*x]]) + (4*Cos[c + d*x]*(4*a -
 3*b*Sin[c + d*x])*Sqrt[a + b*Sin[c + d*x]])/(5*b^3*d) + (8*(4*a^2 - 3*b^2)
*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(5*
b^4*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) - (32*a*(a^2 - b^2)*EllipticF[(c
- Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(5*b^4*
d*Sqrt[a + b*Sin[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
```

```
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2772

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Dist[g^2*((p - 1)/(b*(m + 1))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2944

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Dist[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx = -\frac{2\cos^3(c+dx)}{bd\sqrt{a+b\sin(c+dx)}} - \frac{6\int \frac{\cos^2(c+dx)\sin(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx}{b}$$

$$= -\frac{2\cos^3(c+dx)}{bd\sqrt{a+b\sin(c+dx)}} + \frac{4\cos(c+dx)(4a-3b\sin(c+dx))\sqrt{a+b\sin(c+}}{5b^3d}$$

$$= -\frac{2\cos^3(c+dx)}{bd\sqrt{a+b\sin(c+dx)}} + \frac{4\cos(c+dx)(4a-3b\sin(c+dx))\sqrt{a+b\sin(c+}}{5b^3d}$$

$$= -\frac{2\cos^3(c+dx)}{bd\sqrt{a+b\sin(c+dx)}} + \frac{4\cos(c+dx)(4a-3b\sin(c+dx))\sqrt{a+b\sin(c+}}{5b^3d}$$

$$= -\frac{2\cos^3(c+dx)}{bd\sqrt{a+b\sin(c+dx)}} + \frac{4\cos(c+dx)(4a-3b\sin(c+dx))\sqrt{a+b\sin(c+}}{5b^3d}$$

Mathematica [A]
time = 1.07, size = 187, normalized size = 0.82

$$\frac{-8(4a^3+4a^2b-3ab^2-3b^3)\,E\left(\frac{1}{4}(-2c+\pi-2dx)|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}+32a(a^2-b^2)\,F\left(\frac{1}{4}(-2c+\pi-2dx)|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}+b\cos(c+dx)\,(16a^2-11b^2+b^2\cos(2(c+dx))+4ab\sin(c+dx))}{5b^4d\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-8*(4*a^3 + 4*a^2*b - 3*a*b^2 - 3*b^3)*EllipticE[(-2*c + Pi - 2*d*x)/4, (2
*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] + 32*a*(a^2 - b^2)*Elliptic
F[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)]
+ b*Cos[c + d*x]*(16*a^2 - 11*b^2 + b^2*Cos[2*(c + d*x)] + 4*a*b*Sin[c + d*
x]))/(5*b^4*d*Sqrt[a + b*Sin[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 796 vs. $2(277) = 554$.
time = 2.03, size = 797, normalized size = 3.48

method	result

default	$\frac{32\sqrt{\frac{a+b\sin(dx+c)}{a-b}}\sqrt{-\frac{(\sin(dx+c)-1)b}{a+b}}\sqrt{-\frac{(1+\sin(dx+c))b}{a-b}}\,\text{EllipticF}\left(\sqrt{\frac{a+b\sin(dx+c)}{a-b}},\sqrt{\frac{a-b}{a+b}}\right)a^3b}{5} - \frac{24a^2\sqrt{\frac{a+b\sin(}{a-}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+b*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*(16*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1
+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)
/(a+b))^(1/2))*a^3*b-12*a^2*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)
*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))
/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b^2-16*a*((a+b*sin(d*x+c))/(a-b))^(1/2)*
(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF((
(a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b^3+12*((a+b*sin(d*x+c))
/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/
2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b^4-16*((a
+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c)
)*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/
2))*a^4+28*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(
-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a
-b)/(a+b))^(1/2))*a^2*b^2-12*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1
)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c)
)/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b^4+b^4*sin(d*x+c)^4-2*a*b^3*sin(d*x+c)
^3-8*a^2*b^2*sin(d*x+c)^2+4*b^4*sin(d*x+c)^2+2*a*b^3*sin(d*x+c)+8*a^2*b^2-5
*b^4)/b^5/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^4/(b*sin(d*x + c) + a)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.15, size = 598, normalized size = 2.61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/15*(2*(sqrt(2)*(8*a^3*b - 9*a*b^3)*sin(d*x + c) + sqrt(2)*(8*a^4 - 9*a^2
*b^2))*sqrt(I*b)*weierstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I*a
^3 - 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) - 2*I*a)/b)
 + 2*(sqrt(2)*(8*a^3*b - 9*a*b^3)*sin(d*x + c) + sqrt(2)*(8*a^4 - 9*a^2*b^2
))*sqrt(-I*b)*weierstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3
 + 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*I*a)/b) +
 6*(sqrt(2)*(4*I*a^2*b^2 - 3*I*b^4)*sin(d*x + c) + sqrt(2)*(4*I*a^3*b - 3*I
*a*b^3))*sqrt(I*b)*weierstrassZeta(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3
 - 9*I*a*b^2)/b^3, weierstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I
*a^3 - 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) - 2*I*a)/
b)) + 6*(sqrt(2)*(-4*I*a^2*b^2 + 3*I*b^4)*sin(d*x + c) + sqrt(2)*(-4*I*a^3*
b + 3*I*a*b^3))*sqrt(-I*b)*weierstrassZeta(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*
(-8*I*a^3 + 9*I*a*b^2)/b^3, weierstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -
8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c)
 + 2*I*a)/b)) - 3*(b^4*cos(d*x + c)^3 + 2*a*b^3*cos(d*x + c)*sin(d*x + c) +
 2*(4*a^2*b^2 - 3*b^4)*cos(d*x + c))*sqrt(b*sin(d*x + c) + a))/(b^6*d*sin(d
*x + c) + a*b^5*d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^4(c+dx)}{(a+b\sin(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+b*sin(d*x+c))**(3/2),x)
[Out] Integral(cos(c + d*x)**4/(a + b*sin(c + d*x))**(3/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*sin(d*x+c))^(3/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^4}{(a+b\,\sin(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(a + b*sin(c + d*x))^(3/2),x)
[Out] int(cos(c + d*x)^4/(a + b*sin(c + d*x))^(3/2), x)
```

3.524 $\int \frac{\cos^2(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=160

$$-\frac{2\cos(c+dx)}{bd\sqrt{a+b\sin(c+dx)}}-\frac{4E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)|\frac{2b}{a+b}\right)\sqrt{a+b\sin(c+dx)}}{b^2d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}+\frac{4aF\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}{b^2d\sqrt{a+b\sin(c+dx)}}$$

```
[Out] -2*cos(d*x+c)/b/d/(a+b*sin(d*x+c))^(1/2)+4*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1
/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(
b/(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/b^2/d/((a+b*sin(d*x+c))/(a+b))^(1/2)
-4*a*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*Elliptic
F(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b
))^(1/2)/b^2/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2772, 2831, 2742, 2740, 2734, 2732}

$$\frac{4a\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{b^2d\sqrt{a+b\sin(c+dx)}}-\frac{4\sqrt{a+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{b^2d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}-\frac{2\cos(c+dx)}{bd\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*Cos[c + d*x])/(b*d*Sqrt[a + b*Sin[c + d*x]]) - (4*EllipticE[(c - Pi/2 +
 d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(b^2*d*Sqrt[(a + b*Sin[c
+ d*x])/(a + b)]) + (4*a*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[
(a + b*Sin[c + d*x])/(a + b)])/(b^2*d*Sqrt[a + b*Sin[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2772

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Dist[g^2*((p - 1)/(b*(m + 1))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx = -\frac{2\cos(c+dx)}{bd\sqrt{a+b\sin(c+dx)}} - \frac{2\int \frac{\sin(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx}{b}$$

$$= -\frac{2\cos(c+dx)}{bd\sqrt{a+b\sin(c+dx)}} - \frac{2\int \sqrt{a+b\sin(c+dx)}\,dx}{b^2} + \frac{(2a)\int \frac{1}{\sqrt{a+b\sin(c}}}{b^2}$$

$$= -\frac{2\cos(c+dx)}{bd\sqrt{a+b\sin(c+dx)}} - \frac{\left(2\sqrt{a+b\sin(c+dx)}\right)\int \sqrt{\frac{a}{a+b}+\frac{b\sin(c+dx)}{a+b}}}{b^2\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}$$

$$= -\frac{2\cos(c+dx)}{bd\sqrt{a+b\sin(c+dx)}} - \frac{4E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\frac{2b}{a+b}\right)\sqrt{a+b\sin(c+dx)}}{b^2d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}} + \frac{4a}{}$$

Mathematica [A]
time = 2.74, size = 125, normalized size = 0.78

$$\frac{4(a+b)E\left(\frac{1}{4}(-2c+\pi-2dx)\middle|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}-2\left(b\cos(c+dx)+2aF\left(\frac{1}{4}(-2c+\pi-2dx)\middle|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\right)}{b^2d\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (4*(a + b)*EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[
c + d*x])/(a + b)] - 2*(b*Cos[c + d*x] + 2*a*EllipticF[(-2*c + Pi - 2*d*x)/
4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)]))/(b^2*d*Sqrt[a + b*Si
n[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 433 vs. 2(216) = 432.
time = 2.30, size = 434, normalized size = 2.71

method	result
default	$\frac{4\sqrt{\frac{a+b\sin(dx+c)}{a-b}}\sqrt{-\frac{(\sin(dx+c)-1)b}{a+b}}\sqrt{-\frac{(1+\sin(dx+c))b}{a-b}}\,\mathrm{EllipticE}\left(\sqrt{\frac{a+b\sin(dx+c)}{a-b}},\sqrt{\frac{a-b}{a+b}}\right)a^2-4\sqrt{\frac{a+b\sin(dx+}{a-b}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+b*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(2*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+si
n(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a
+b))^(1/2))*a^2-2*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^
(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1
/2),((a-b)/(a+b))^(1/2))*b^2-2*a*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+
c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*
x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b+2*((a+b*sin(d*x+c))/(a-b))^(1/2)*
(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF((
(a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b^2+b^2*sin(d*x+c)^2-b^2
)/b^3/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^2/(b*sin(d*x + c) + a)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 470, normalized size = 2.94

$$-\frac{2\left(3\sqrt{b\sin\left(dx+c\right)+a}\,b^{2}\cos\left(dx+c\right)-2\left(\sqrt{2}\,ab\sin\left(dx+c\right)+\sqrt{2}\,a^{2}\right)\sqrt{i\,b}\,\mathrm{weierstrassPInverse}\left(-\frac{4\left(4\,a^{2}-3\,b^{2}\right)}{3\,b^{2}},-\frac{8\left(8i\,a^{3}-9i\,ab^{2}\right)}{27\,b^{3}},\frac{3\,b\cos\left(dx+c\right)-3i\,b\sin\left(dx+c\right)-2i\,a}{3\,b}\right)-2\left(\sqrt{2}\,ab\sin\left(dx+c\right)+\sqrt{2}\,a^{2}\right)\sqrt{-i\,b}\,\mathrm{weierstrassPInverse}\left(-\frac{4\left(4\,a^{2}-3\,b^{2}\right)}{3\,b^{2}},-\frac{8\left(-8i\,a^{3}+9i\,ab^{2}\right)}{27\,b^{3}},\frac{3\,b\cos\left(dx+c\right)+3i\,b\sin\left(dx+c\right)+2i\,a}{3\,b}\right)+3\left(-i\sqrt{2}\,b^{2}\sin\left(dx+c\right)-i\sqrt{2}\,ab\right)\sqrt{i\,b}\,\mathrm{weierstrassZeta}\left(-\frac{4\left(4\,a^{2}-3\,b^{2}\right)}{3\,b^{2}},-\frac{8\left(8i\,a^{3}-9i\,ab^{2}\right)}{27\,b^{3}},\mathrm{weierstrassPInverse}\left(-\frac{4\left(4\,a^{2}-3\,b^{2}\right)}{3\,b^{2}},-\frac{8\left(8i\,a^{3}-9i\,ab^{2}\right)}{27\,b^{3}},\frac{3\,b\cos\left(dx+c\right)-3i\,b\sin\left(dx+c\right)-2i\,a}{3\,b}\right)\right)+3\left(i\sqrt{2}\,b^{2}\sin\left(dx+c\right)+i\sqrt{2}\,ab\right)\sqrt{-i\,b}\,\mathrm{weierstrassZeta}\left(-\frac{4\left(4\,a^{2}-3\,b^{2}\right)}{3\,b^{2}},-\frac{8\left(-8i\,a^{3}+9i\,ab^{2}\right)}{27\,b^{3}},\mathrm{weierstrassPInverse}\left(-\frac{4\left(4\,a^{2}-3\,b^{2}\right)}{3\,b^{2}},-\frac{8\left(-8i\,a^{3}+9i\,ab^{2}\right)}{27\,b^{3}},\frac{3\,b\cos\left(dx+c\right)+3i\,b\sin\left(dx+c\right)+2i\,a}{3\,b}\right)\right)\right)}{3\left(b^{4}d\sin\left(dx+c\right)+ab^{3}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/3*(3*sqrt(b*sin(d*x + c) + a)*b^2*cos(d*x + c) - 2*(sqrt(2)*a*b*sin(d*x
+ c) + sqrt(2)*a^2)*sqrt(I*b)*weierstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2,
 -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c
) - 2*I*a)/b) - 2*(sqrt(2)*a*b*sin(d*x + c) + sqrt(2)*a^2)*sqrt(-I*b)*weier
strassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3,
1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*I*a)/b) + 3*(-I*sqrt(2)*b^2*
sin(d*x + c) - I*sqrt(2)*a*b)*sqrt(I*b)*weierstrassZeta(-4/3*(4*a^2 - 3*b^2
)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, weierstrassPInverse(-4/3*(4*a^2 - 3
*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*s
in(d*x + c) - 2*I*a)/b)) + 3*(I*sqrt(2)*b^2*sin(d*x + c) + I*sqrt(2)*a*b)*s
qrt(-I*b)*weierstrassZeta(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a
*b^2)/b^3, weierstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3 +
9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*I*a)/b)))/(b
^4*d*sin(d*x + c) + a*b^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(c+dx)}{(a+b\sin(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral(cos(c + d*x)**2/(a + b*sin(c + d*x))**(3/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)^2}{(a+b\,\sin(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a + b*sin(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^2/(a + b*sin(c + d*x))^(3/2), x)
```

3.525 $\int \frac{\sec^2(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=251

$$\frac{2b\sec(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\sin(c+dx)}}-\frac{(a^2+3b^2)\,E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)|\frac{2b}{a+b}\right)\sqrt{a+b\sin(c+dx)}}{(a^2-b^2)^2\,d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}+\frac{aF\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right.\right.}{(a^2-b^2)\,d}$$

```
[Out] 2*b*sec(d*x+c)/(a^2-b^2)/d/(a+b*sin(d*x+c))^(1/2)-sec(d*x+c)*(4*a*b-(a^2+3*
b^2)*sin(d*x+c))*(a+b*sin(d*x+c))^(1/2)/(a^2-b^2)^2/d+(a^2+3*b^2)*(sin(1/2*
c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/
4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/(a^2-b^2)^2/d
/((a+b*sin(d*x+c))/(a+b))^(1/2)-a*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1
/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^
(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/(a^2-b^2)/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {2773, 2945, 2831, 2742, 2740, 2734, 2732}

$$-\frac{\sec(c+dx)\sqrt{a+b\sin(c+dx)}\,(4ab-(a^2+3b^2)\sin(c+dx))}{d\,(a^2-b^2)^2}+\frac{2b\sec(c+dx)}{d\,(a^2-b^2)\sqrt{a+b\sin(c+dx)}}+\frac{a\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{d\,(a^2-b^2)\sqrt{a+b\sin(c+dx)}}-\frac{(a^2+3b^2)\sqrt{a+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{d\,(a^2-b^2)^2\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (2*b*Sec[c + d*x])/((a^2 - b^2)*d*Sqrt[a + b*Sin[c + d*x]]) - ((a^2 + 3*b^2
)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/((
a^2 - b^2)^2*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + (a*EllipticF[(c - Pi/2
 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/((a^2 - b^2)*
d*Sqrt[a + b*Sin[c + d*x]]) - (Sec[c + d*x]*Sqrt[a + b*Sin[c + d*x]]*(4*a*b
 - (a^2 + 3*b^2)*Sin[c + d*x]))/((a^2 - b^2)^2*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
```

```
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2773

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m + 1)/(f*g*(a^2 - b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1))
, Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p
 + 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^
2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2945

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx = \frac{2b\sec(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\sin(c+dx)}} - \frac{2\int \frac{\sec^2(c+dx)\left(-\frac{a}{2}+\frac{3}{2}b\sin(c+dx)\right)}{\sqrt{a+b\sin(c+dx)}}\,dx}{a^2-b^2}$$

$$= \frac{2b\sec(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\sin(c+dx)}} - \frac{\sec(c+dx)\sqrt{a+b\sin(c+dx)}\,(4ab-(a^2}{(a^2-b^2)^2\,d}$$

$$= \frac{2b\sec(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\sin(c+dx)}} - \frac{\sec(c+dx)\sqrt{a+b\sin(c+dx)}\,(4ab-(a^2}{(a^2-b^2)^2\,d}$$

$$= \frac{2b\sec(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\sin(c+dx)}} - \frac{\sec(c+dx)\sqrt{a+b\sin(c+dx)}\,(4ab-(a^2}{(a^2-b^2)^2\,d}$$

$$= \frac{2b\sec(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\sin(c+dx)}} - \frac{(a^2+3b^2)\,E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)|\frac{2b}{a+b}\right)\sqrt{a+b}}{(a^2-b^2)^2\,d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}$$

Mathematica [A]
time = 1.72, size = 205, normalized size = 0.82

$$\frac{(a^3+a^2b+3ab^2+3b^3)\,E\left(\frac{1}{4}(-2c+\pi-2dx)|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}-a(a^2-b^2)\,F\left(\frac{1}{4}(-2c+\pi-2dx)|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}-\frac{1}{2}\sec(c+dx)\left(3a^2b+b^3+b(a^2+3b^2)\cos(2(c+dx))-2a(a^2-b^2)\sin(c+dx)\right)}{(a-b)^2(a+b)^2d\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((a^3 + a^2*b + 3*a*b^2 + 3*b^3)*EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b)/(a
+ b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] - a*(a^2 - b^2)*EllipticF[(-2*c +
Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] - (Sec[c +
 d*x]*(3*a^2*b + b^3 + b*(a^2 + 3*b^2)*Cos[2*(c + d*x)] - 2*a*(a^2 - b^2)*S
in[c + d*x]))/2)/((a - b)^2*(a + b)^2*d*Sqrt[a + b*Sin[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1063 vs. $2(305) = 610$.
time = 3.38, size = 1064, normalized size = 4.24

method	result

default	$-\frac{\sqrt{b\left(\cos^2\left(dx+c\right)\right)\sin\left(dx+c\right)+a\left(\cos^2\left(dx+c\right)\right)}\left(\sqrt{-\frac{b\sin(dx+c)}{a+b}+\frac{b}{a+b}}\sqrt{\frac{b\sin(dx+c)}{a-b}+\frac{a}{a-b}}\ \text{Ellip}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^2/(a+b*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)

[Out] -1/b*(b*cos(d*x+c)^2*sin(d*x+c)+a*cos(d*x+c)^2)^(1/2)*((-b/(a+b)*sin(d*x+c)
+b/(a+b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*EllipticF((b/(a-b)*sin
(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a-b)*sin(d*x+c)-b/(a-b))
^(1/2)*a^3*b+3*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a
-b)*a)^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(
1/2))*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*a^2*b^2-(-b/(a+b)*sin(d*x+c)+b/(a
+b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*EllipticF((b/(a-b)*sin(d*x+
c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2
)*a*b^3-3*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a
)^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))
*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*b^4-(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2
)*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(b/(a
-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*a^4-2*(
-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)
^(1/2),((a-b)/(a+b))^(1/2))*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a-b)*
sin(d*x+c)-b/(a-b))^(1/2)*a^2*b^2+3*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*Ell
ipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(b/(a-b)*s
in(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*b^4+a^2*b^2*
cos(d*x+c)^2+3*b^4*cos(d*x+c)^2-a^3*b*sin(d*x+c)+a*b^3*sin(d*x+c)+a^2*b^2-b
^4)/(a^2-b^2)/(a-b)/(a+b)/(-(a+b*sin(d*x+c))*(sin(d*x+c)-1)*(1+sin(d*x+c)))
^(1/2)/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2/(a+b*sin(d*x+c))^(3/2),x, algorithm="maxima")

[Out] integrate(sec(d*x + c)^2/(b*sin(d*x + c) + a)^(3/2), x)

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 685, normalized size = 2.73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/6*(2*(sqrt(2)*(a^3*b - 3*a*b^3)*cos(d*x + c)*sin(d*x + c) + sqrt(2)*(a^4
- 3*a^2*b^2)*cos(d*x + c))*sqrt(I*b)*weierstrassPInverse(-4/3*(4*a^2 - 3*b^
2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(
d*x + c) - 2*I*a)/b) + 2*(sqrt(2)*(a^3*b - 3*a*b^3)*cos(d*x + c)*sin(d*x +
c) + sqrt(2)*(a^4 - 3*a^2*b^2)*cos(d*x + c))*sqrt(-I*b)*weierstrassPInverse
(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d
*x + c) + 3*I*b*sin(d*x + c) + 2*I*a)/b) - 3*(sqrt(2)*(-I*a^2*b^2 - 3*I*b^4
)*cos(d*x + c)*sin(d*x + c) + sqrt(2)*(-I*a^3*b - 3*I*a*b^3)*cos(d*x + c))*
sqrt(I*b)*weierstrassZeta(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*
b^2)/b^3, weierstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*
I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) - 2*I*a)/b)) - 3*(
sqrt(2)*(I*a^2*b^2 + 3*I*b^4)*cos(d*x + c)*sin(d*x + c) + sqrt(2)*(I*a^3*b
+ 3*I*a*b^3)*cos(d*x + c))*sqrt(-I*b)*weierstrassZeta(-4/3*(4*a^2 - 3*b^2)/
b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, weierstrassPInverse(-4/3*(4*a^2 - 3*
b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*s
in(d*x + c) + 2*I*a)/b)) - 6*(a^2*b^2 - b^4 + (a^2*b^2 + 3*b^4)*cos(d*x + c
)^2 - (a^3*b - a*b^3)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a))/((a^4*b^2 - 2
*a^2*b^4 + b^6)*d*cos(d*x + c)*sin(d*x + c) + (a^5*b - 2*a^3*b^3 + a*b^5)*d
*cos(d*x + c))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(c+dx)}{(a+b\sin(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral(sec(c + d*x)**2/(a + b*sin(c + d*x))**(3/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos(c+d\,x)^2\,(a+b\,\sin(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + b*sin(c + d*x))^(3/2)),x)
[Out] int(1/(cos(c + d*x)^2*(a + b*sin(c + d*x))^(3/2)), x)
```

3.526 $\int \frac{\sec^4(c+dx)}{(a+b\sin(c+dx))^{3/2}} \, dx$

Optimal. Leaf size=359

$$\frac{2b\sec^3(c+dx)}{(a^2-b^2)\, d\sqrt{a+b\sin(c+dx)}}-\frac{(4a^4-15a^2b^2-21b^4)\, E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\frac{2b}{a+b}\right)\sqrt{a+b\sin(c+dx)}}{6\,(a^2-b^2)^3\, d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}+\frac{2a(a^2$$

```
[Out] 2*b*sec(d*x+c)^3/(a^2-b^2)/d/(a+b*sin(d*x+c))^(1/2)-1/3*sec(d*x+c)^3*(8*a*b
-(a^2+7*b^2)*sin(d*x+c))*(a+b*sin(d*x+c))^(1/2)/(a^2-b^2)^2/d-1/6*sec(d*x+c
)*(a*b*(a^2-33*b^2)-(4*a^4-15*a^2*b^2-21*b^4)*sin(d*x+c))*(a+b*sin(d*x+c))^
(1/2)/(a^2-b^2)^3/d+1/6*(4*a^4-15*a^2*b^2-21*b^4)*(sin(1/2*c+1/4*Pi+1/2*d*x
)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^
(1/2)*(b/(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/(a^2-b^2)^3/d/((a+b*sin(d*x+c
))/(a+b))^(1/2)-2/3*a*(a^2-3*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1
/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^
(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/(a^2-b^2)^2/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A]
time = 0.40, antiderivative size = 359, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {2773, 2945, 2831, 2742, 2740, 2734, 2732}

$$-\frac{\sec^3(c+dx)\sqrt{a+b\sin(c+dx)}\,(8ab-(a^2+7b^2)\sin(c+dx))}{3d\,(a^2-b^2)^2}+\frac{2b\sec^3(c+dx)}{d\,(a^2-b^2)\sqrt{a+b\sin(c+dx)}}+\frac{2a(a^2-3b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|\frac{2b}{a+b}\right)}{3d\,(a^2-b^2)^2\sqrt{a+b\sin(c+dx)}}-\frac{\sec(c+dx)\sqrt{a+b\sin(c+dx)}\,(ab(a^2-33b^2)-(4a^4-15a^2b^2-21b^4)\sin(c+dx))}{6d\,(a^2-b^2)^3}-\frac{(4a^4-15a^2b^2-21b^4)\sqrt{a+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|\frac{2b}{a+b}\right)}{6d\,(a^2-b^2)^3\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (2*b*Sec[c + d*x]^3)/((a^2 - b^2)*d*Sqrt[a + b*Sin[c + d*x]]) - ((4*a^4 - 1
5*a^2*b^2 - 21*b^4)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b
*Sin[c + d*x]])/(6*(a^2 - b^2)^3*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + (2
*a*(a^2 - 3*b^2)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*S
in[c + d*x])/(a + b)])/(3*(a^2 - b^2)^2*d*Sqrt[a + b*Sin[c + d*x]]) - (Sec[
c + d*x]^3*Sqrt[a + b*Sin[c + d*x]]*(8*a*b - (a^2 + 7*b^2)*Sin[c + d*x]))/(
3*(a^2 - b^2)^2*d) - (Sec[c + d*x]*Sqrt[a + b*Sin[c + d*x]]*(a*b*(a^2 - 33*
b^2) - (4*a^4 - 15*a^2*b^2 - 21*b^4)*Sin[c + d*x]))/(6*(a^2 - b^2)^3*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2773

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m + 1)/(f*g*(a^2 - b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1))
, Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p
 + 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^
2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2945

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx = \frac{2b\sec^3(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\sin(c+dx)}} - \frac{2\int \frac{\sec^4(c+dx)\left(-\frac{a}{2}+\frac{7}{2}b\sin(c+dx)\right)}{\sqrt{a+b\sin(c+dx)}}\,dx}{a^2-b^2}$$

$$= \frac{2b\sec^3(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\sin(c+dx)}} - \frac{\sec^3(c+dx)\sqrt{a+b\sin(c+dx)}\,(8ab-(a^2}{3\,(a^2-b^2)^2\,d}$$

$$= \frac{2b\sec^3(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\sin(c+dx)}} - \frac{\sec^3(c+dx)\sqrt{a+b\sin(c+dx)}\,(8ab-(a^2}{3\,(a^2-b^2)^2\,d}$$

$$= \frac{2b\sec^3(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\sin(c+dx)}} - \frac{\sec^3(c+dx)\sqrt{a+b\sin(c+dx)}\,(8ab-(a^2}{3\,(a^2-b^2)^2\,d}$$

$$= \frac{2b\sec^3(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\sin(c+dx)}} - \frac{\sec^3(c+dx)\sqrt{a+b\sin(c+dx)}\,(8ab-(a^2}{3\,(a^2-b^2)^2\,d}$$

$$= \frac{2b\sec^3(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\sin(c+dx)}} - \frac{(4a^4-15a^2b^2-21b^4)\,E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)|\frac{2}{a}\right.}{6\,(a^2-b^2)^3\,d\sqrt{\frac{a+b\sin(c}{a+b}}}$$

Mathematica [A]
time = 3.03, size = 348, normalized size = 0.97

$$\frac{(4a^5+4a^4b-15a^3b^2-15a^2b^3-21ab^4-21b^5)\,E\left(\frac{1}{4}(-2c+\pi-2dx)|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}-4a(a^4-4a^2b^2+3b^4)\,F\left(\frac{1}{4}(-2c+\pi-2dx)|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}+\frac{1}{8}\sec^3(c+dx)\left(-24a^4b+101a^2b^3+19b^5+(-12a^4b+84a^2b^3+56b^5)\cos(2(c+dx))+(-4a^4b+15a^2b^3+21b^5)\cos(4(c+dx))+24a^5\sin(c+dx)-64a^3b^2\sin(c+dx)+40ab^4\sin(c+dx)+8a^5\sin(3(c+dx))-32a^3b^2\sin(3(c+dx))+24ab^4\sin(3(c+dx))\right)}{6(a-b)^3(a+b)^3d\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((4*a^5 + 4*a^4*b - 15*a^3*b^2 - 15*a^2*b^3 - 21*a*b^4 - 21*b^5)*EllipticE[
(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] -
4*a*(a^4 - 4*a^2*b^2 + 3*b^4)*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b
)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] + (Sec[c + d*x]^3*(-24*a^4*b + 101*a^
2*b^3 + 19*b^5 + (-12*a^4*b + 84*a^2*b^3 + 56*b^5)*Cos[2*(c + d*x)] + (-4*a
^4*b + 15*a^2*b^3 + 21*b^5)*Cos[4*(c + d*x)] + 24*a^5*Sin[c + d*x] - 64*a^3
*b^2*Sin[c + d*x] + 40*a*b^4*Sin[c + d*x] + 8*a^5*Sin[3*(c + d*x)] - 32*a^3
*b^2*Sin[3*(c + d*x)] + 24*a*b^4*Sin[3*(c + d*x)]))/8)/(6*(a - b)^3*(a + b)
^3*d*Sqrt[a + b*Sin[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1645 vs. $2(403) = 806$.
time = 15.54, size = 1646, normalized size = 4.58

method	result	size
default	Expression too large to display	1646

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+b*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(-(-a-b*sin(d*x+c))*cos(d*x+c)^2)^(1/2)/cos(d*x+c)^5/(a+b*sin(d*x+c))^(
3/2)/b/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)*(-2*(b*cos(d*x+c)^2*sin(d*x+c)+a*cos(d
*x+c)^2)^(1/2)*b^2*(a^4-2*a^2*b^2+b^4)-cos(d*x+c)^4*(b*cos(d*x+c)^2*sin(d*x
+c)+a*cos(d*x+c)^2)^(1/2)*b^2*(4*a^4-15*a^2*b^2-21*b^4)+4*cos(d*x+c)^2*(b*c
os(d*x+c)^2*sin(d*x+c)+a*cos(d*x+c)^2)^(1/2)*a*b*(a^4-4*a^2*b^2+3*b^4)*sin(
d*x+c)+2*(b*cos(d*x+c)^2*sin(d*x+c)+a*cos(d*x+c)^2)^(1/2)*a*b*(a^4-2*a^2*b^
2+b^4)*sin(d*x+c)+cos(d*x+c)^2*(b*cos(d*x+c)^2*sin(d*x+c)+a*cos(d*x+c)^2)^(
1/2)*(4*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))
*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(b
/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*a^6-19*EllipticE((b/(a-b)*sin(d*x+c)+1/(
a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/
(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*a^4*b^
2-6*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b
/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(b/(a-
b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*a^2*b^4+21*EllipticE((b/(a-b)*sin(d*x+c)+1/(
a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/
(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*b^6-4*
(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*Ell
ipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(b/(a-b)*s
in(d*x+c)+1/(a-b)*a)^(1/2)*a^5*b+3*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/
(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1
/2),((a-b)/(a+b))^(1/2))*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*a^4*b^2+16*(-
b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*Ellip
ticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(b/(a-b)*sin
(d*x+c)+1/(a-b)*a)^(1/2)*a^3*b^3+18*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b
/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(
1/2),((a-b)/(a+b))^(1/2))*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*a^2*b^4-12*(
-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*Elli
pticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(b/(a-b)*si
n(d*x+c)+1/(a-b)*a)^(1/2)*a*b^5-21*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/
(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1
/2),((a-b)/(a+b))^(1/2))*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*b^6+a^4*b^2+6
*a^2*b^4-7*b^6))/d
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^4/(b*sin(d*x + c) + a)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.22, size = 878, normalized size = 2.45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/36*((sqrt(2)*(8*a^5*b - 33*a^3*b^3 + 57*a*b^5)*cos(d*x + c)^3*sin(d*x + c
) + sqrt(2)*(8*a^6 - 33*a^4*b^2 + 57*a^2*b^4)*cos(d*x + c)^3)*sqrt(I*b)*wei
erstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3,
 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) - 2*I*a)/b) + (sqrt(2)*(8*a^5*b
 - 33*a^3*b^3 + 57*a*b^5)*cos(d*x + c)^3*sin(d*x + c) + sqrt(2)*(8*a^6 - 33
*a^4*b^2 + 57*a^2*b^4)*cos(d*x + c)^3)*sqrt(-I*b)*weierstrassPInverse(-4/3*
(4*a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c
) + 3*I*b*sin(d*x + c) + 2*I*a)/b) + 3*(sqrt(2)*(4*I*a^4*b^2 - 15*I*a^2*b^4
 - 21*I*b^6)*cos(d*x + c)^3*sin(d*x + c) + sqrt(2)*(4*I*a^5*b - 15*I*a^3*b^
3 - 21*I*a*b^5)*cos(d*x + c)^3)*sqrt(I*b)*weierstrassZeta(-4/3*(4*a^2 - 3*b
^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, weierstrassPInverse(-4/3*(4*a^2 -
 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b
*sin(d*x + c) - 2*I*a)/b)) + 3*(sqrt(2)*(-4*I*a^4*b^2 + 15*I*a^2*b^4 + 21*I
*b^6)*cos(d*x + c)^3*sin(d*x + c) + sqrt(2)*(-4*I*a^5*b + 15*I*a^3*b^3 + 21
*I*a*b^5)*cos(d*x + c)^3)*sqrt(-I*b)*weierstrassZeta(-4/3*(4*a^2 - 3*b^2)/b
^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, weierstrassPInverse(-4/3*(4*a^2 - 3*b
^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*si
n(d*x + c) + 2*I*a)/b)) - 6*(2*a^4*b^2 - 4*a^2*b^4 + 2*b^6 + (4*a^4*b^2 - 1
5*a^2*b^4 - 21*b^6)*cos(d*x + c)^4 - (a^4*b^2 + 6*a^2*b^4 - 7*b^6)*cos(d*x
+ c)^2 - 2*(a^5*b - 2*a^3*b^3 + a*b^5 + 2*(a^5*b - 4*a^3*b^3 + 3*a*b^5)*cos
(d*x + c)^2)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a))/((a^6*b^2 - 3*a^4*b^4
+ 3*a^2*b^6 - b^8)*d*cos(d*x + c)^3*sin(d*x + c) + (a^7*b - 3*a^5*b^3 + 3*a
^3*b^5 - a*b^7)*d*cos(d*x + c)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^4(c+dx)}{(a+b\sin(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+b*sin(d*x+c))**(3/2),x)
[Out] Integral(sec(c + d*x)**4/(a + b*sin(c + d*x))**(3/2), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sin(d*x+c))^(3/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos\left(c+d\,x\right)^4\,{\left(a+b\,\sin\left(c+d\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a + b*sin(c + d*x))^(3/2)),x)
[Out] int(1/(cos(c + d*x)^4*(a + b*sin(c + d*x))^(3/2)), x)
```

3.527 $\int \frac{\cos^5(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=150

$$-\frac{2(a^2-b^2)^2}{3b^5d(a+b\sin(c+dx))^{3/2}}+\frac{8a(a^2-b^2)}{b^5d\sqrt{a+b\sin(c+dx)}}+\frac{4(3a^2-b^2)\sqrt{a+b\sin(c+dx)}}{b^5d}-\frac{8a(a+b\sin(c+d}{3b^5d}$$

```
[Out] -2/3*(a^2-b^2)^2/b^5/d/(a+b*sin(d*x+c))^(3/2)-8/3*a*(a+b*sin(d*x+c))^(3/2)/
b^5/d+2/5*(a+b*sin(d*x+c))^(5/2)/b^5/d+8*a*(a^2-b^2)/b^5/d/(a+b*sin(d*x+c))
^(1/2)+4*(3*a^2-b^2)*(a+b*sin(d*x+c))^(1/2)/b^5/d
```

Rubi [A]
time = 0.08, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2747, 711}

$$\frac{4(3a^2-b^2)\sqrt{a+b\sin(c+dx)}}{b^5d}+\frac{8a(a^2-b^2)}{b^5d\sqrt{a+b\sin(c+dx)}}-\frac{2(a^2-b^2)^2}{3b^5d(a+b\sin(c+dx))^{3/2}}+\frac{2(a+b\sin(c+dx))^{5/2}}{5b^5d}-\frac{8a(a+b\sin(c+dx))^{3/2}}{3b^5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*(a^2 - b^2)^2)/(3*b^5*d*(a + b*Sin[c + d*x])^(3/2)) + (8*a*(a^2 - b^2))
/(b^5*d*Sqrt[a + b*Sin[c + d*x]]) + (4*(3*a^2 - b^2)*Sqrt[a + b*Sin[c + d*x
]])/(b^5*d) - (8*a*(a + b*Sin[c + d*x])^(3/2))/(3*b^5*d) + (2*(a + b*Sin[c
+ d*x])^(5/2))/(5*b^5*d)
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{(b^2-x^2)^2}{(a+x)^{5/2}}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{(a^2-b^2)^2}{(a+x)^{5/2}} - \frac{4(a^3-ab^2)}{(a+x)^{3/2}} + \frac{2(3a^2-b^2)}{\sqrt{a+x}} - 4a\sqrt{a+x} + (a+x)^{3/2}\right) dx, x, b}{b^5 d}$$
$$= -\frac{2(a^2-b^2)^2}{3b^5 d(a+b\sin(c+dx))^{3/2}} + \frac{8a(a^2-b^2)}{b^5 d\sqrt{a+b\sin(c+dx)}} + \frac{4(3a^2-b^2)\sqrt{a+b\text{s}}}{b^5 d}$$

Mathematica [A]
time = 0.30, size = 117, normalized size = 0.78

$$\frac{6b^4\cos^4(c+dx) + 16(16a^4 - 10a^2b^2 - b^4 + 3ab(8a^2-5b^2)\sin(c+dx) + (6a^2b^2-3b^4)\sin^2(c+dx) - ab^3\sin^3(c+dx))}{15b^5 d(a+b\sin(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (6*b^4*Cos[c + d*x]^4 + 16*(16*a^4 - 10*a^2*b^2 - b^4 + 3*a*b*(8*a^2 - 5*b^
2)*Sin[c + d*x] + (6*a^2*b^2 - 3*b^4)*Sin[c + d*x]^2 - a*b^3*Sin[c + d*x]^3
))/(15*b^5*d*(a + b*Sin[c + d*x])^(3/2))
```

Maple [A]
time = 1.09, size = 116, normalized size = 0.77

method	result
default	$\frac{\frac{16a\,b^3\left(\cos^2(dx+c)\right)\sin(dx+c)}{15} + \frac{2\left(192b\,a^3-128a\,b^3\right)\sin(dx+c)}{15} + \frac{2b^4\left(\cos^4(dx+c)\right)}{5} + \frac{2\left(-48a^2b^2+24b^4\right)\left(\cos^2(dx+c)\right)}{15} + \frac{256a^4}{15} - \frac{64a^2b^2}{15} - \frac{64b^4}{15}}{b^5(a+b\sin(dx+c))^{\frac{3}{2}}d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a+b*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15/b^5*(8*a*b^3*cos(d*x+c)^2*sin(d*x+c)+(192*a^3*b-128*a*b^3)*sin(d*x+c)+
3*b^4*cos(d*x+c)^4+(-48*a^2*b^2+24*b^4)*cos(d*x+c)^2+128*a^4-32*a^2*b^2-32*
b^4)/(a+b*sin(d*x+c))^(3/2)/d
```

Maxima [A]
time = 0.27, size = 122, normalized size = 0.81

$$\frac{2\left(\frac{3(b\sin(dx+c)+a)^{\frac{5}{2}} - 20(b\sin(dx+c)+a)^{\frac{3}{2}}a + 30(3a^2-b^2)\sqrt{b\sin(dx+c)+a}}{b^4} - \frac{5(a^4-2a^2b^2+b^4-12(a^3-ab^2)(b\sin(dx+c)+a))}{(b\sin(dx+c)+a)^{\frac{3}{2}}b^4}\right)}{15\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 2/15*((3*(b*sin(d*x + c) + a)^(5/2) - 20*(b*sin(d*x + c) + a)^(3/2)*a + 30*
(3*a^2 - b^2)*sqrt(b*sin(d*x + c) + a))/b^4 - 5*(a^4 - 2*a^2*b^2 + b^4 - 12
*(a^3 - a*b^2)*(b*sin(d*x + c) + a))/((b*sin(d*x + c) + a)^(3/2)*b^4))/(b*d
)
```

Fricas [**A**]
time = 0.38, size = 147, normalized size = 0.98

$$-\frac{2\,\left(3\,b^{4}\cos\left(dx+c\right)^{4}+128\,a^{4}-32\,a^{2}b^{2}-32\,b^{4}-24\,\left(2\,a^{2}b^{2}-b^{4}\right)\cos\left(dx+c\right)^{2}+8\,\left(ab^{3}\cos\left(dx+c\right)^{2}+24\,a^{3}b-16\,ab^{3}\right)\sin\left(dx+c\right)\right)\sqrt{b\sin\left(dx+c\right)+a}}{15\,\left(b^{7}d\cos\left(dx+c\right)^{2}-2\,ab^{6}d\sin\left(dx+c\right)-\left(a^{2}b^{5}+b^{7}\right)d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/15*(3*b^4*cos(d*x + c)^4 + 128*a^4 - 32*a^2*b^2 - 32*b^4 - 24*(2*a^2*b^2
 - b^4)*cos(d*x + c)^2 + 8*(a*b^3*cos(d*x + c)^2 + 24*a^3*b - 16*a*b^3)*sin
(d*x + c))*sqrt(b*sin(d*x + c) + a)/(b^7*d*cos(d*x + c)^2 - 2*a*b^6*d*sin(d
*x + c) - (a^2*b^5 + b^7)*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^5}{{\left(a+b\,\sin\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(a + b*sin(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^5/(a + b*sin(c + d*x))^(5/2), x)
```

3.528 $\int \frac{\cos^3(c+dx)}{(a+b\sin(c+dx))^{5/2}} \, dx$

Optimal. Leaf size=79

$$\frac{2(a^2-b^2)}{3b^3d(a+b\sin(c+dx))^{3/2}}-\frac{4a}{b^3d\sqrt{a+b\sin(c+dx)}}-\frac{2\sqrt{a+b\sin(c+dx)}}{b^3d}$$

```
[Out] 2/3*(a^2-b^2)/b^3/d/(a+b*sin(d*x+c))^(3/2)-4*a/b^3/d/(a+b*sin(d*x+c))^(1/2)
-2*(a+b*sin(d*x+c))^(1/2)/b^3/d
```

Rubi [A]
time = 0.06, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2747, 711}

$$\frac{2(a^2-b^2)}{3b^3d(a+b\sin(c+dx))^{3/2}}-\frac{4a}{b^3d\sqrt{a+b\sin(c+dx)}}-\frac{2\sqrt{a+b\sin(c+dx)}}{b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (2*(a^2 - b^2))/(3*b^3*d*(a + b*Sin[c + d*x])^(3/2)) - (4*a)/(b^3*d*Sqrt[a
+ b*Sin[c + d*x]]) - (2*Sqrt[a + b*Sin[c + d*x]])/(b^3*d)
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{b^2-x^2}{(a+x)^{5/2}}\,dx, x, b\sin(c+dx)\right)}{b^3d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{-a^2+b^2}{(a+x)^{5/2}} + \frac{2a}{(a+x)^{3/2}} - \frac{1}{\sqrt{a+x}}\right) dx, x, b\sin(c+dx)\right)}{b^3d}$$
$$= \frac{2(a^2-b^2)}{3b^3d(a+b\sin(c+dx))^{3/2}} - \frac{4a}{b^3d\sqrt{a+b\sin(c+dx)}} - \frac{2\sqrt{a+b\sin(c+dx)}}{b^3d}$$

Mathematica [A]
time = 0.06, size = 56, normalized size = 0.71

$$-\frac{2(8a^2+b^2+12ab\sin(c+dx)+3b^2\sin^2(c+dx))}{3b^3d(a+b\sin(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*(8*a^2 + b^2 + 12*a*b*Sin[c + d*x] + 3*b^2*Sin[c + d*x]^2))/(3*b^3*d*(a
 + b*Sin[c + d*x])^(3/2))
```

Maple [A]
time = 0.83, size = 55, normalized size = 0.70

method	result	size
default	$-\frac{2\left(-3b^2\left(\cos^2(dx+c)\right)+12ab\sin(dx+c)+8a^2+4b^2\right)}{3b^3(a+b\sin(dx+c))^{\frac{3}{2}}d}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+b*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/b^3*(-3*b^2*cos(d*x+c)^2+12*a*b*sin(d*x+c)+8*a^2+4*b^2)/(a+b*sin(d*x+c
))^(3/2)/d
```

Maxima [A]
time = 0.30, size = 64, normalized size = 0.81

$$-\frac{2\left(\frac{3\sqrt{b\sin(dx+c)+a}}{b^2} + \frac{6(b\sin(dx+c)+a)a-a^2+b^2}{(b\sin(dx+c)+a)^{\frac{3}{2}}b^2}\right)}{3bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3*(3*sqrt(b*sin(d*x + c) + a)/b^2 + (6*(b*sin(d*x + c) + a)*a - a^2 + b^
2)/((b*sin(d*x + c) + a)^(3/2)*b^2))/(b*d)
```

Fricas [A]
time = 0.35, size = 91, normalized size = 1.15

$$-\frac{2\left(3\,b^{2}\cos\left(dx+c\right)^{2}-12\,ab\sin\left(dx+c\right)-8\,a^{2}-4\,b^{2}\right)\sqrt{b\sin\left(dx+c\right)+a}}{3\left(b^{5}d\cos\left(dx+c\right)^{2}-2\,ab^{4}d\sin\left(dx+c\right)-(a^{2}b^{3}+b^{5})d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(3*b^2*cos(d*x + c)^2 - 12*a*b*sin(d*x + c) - 8*a^2 - 4*b^2)*sqrt(b*si
n(d*x + c) + a)/(b^5*d*cos(d*x + c)^2 - 2*a*b^4*d*sin(d*x + c) - (a^2*b^3 +
 b^5)*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 304 vs. $2(73) = 146$.
time = 3.09, size = 304, normalized size = 3.85

$$\begin{cases} \frac{x\cos^{3}\left(c\right)}{a^{\frac{5}{2}}} & \text{for}\: b=0 \wedge d=0 \\ \frac{\frac{2\sin^{3}\left(c+dx\right)}{3d}+\frac{\sin\left(c+dx\right)\cos^{2}\left(c+dx\right)}{d}}{a^{\frac{5}{2}}} & \text{for}\: b=0 \\ \frac{x\cos^{3}\left(c\right)}{\left(a+b\sin\left(c\right)\right)^{\frac{5}{2}}} & \text{for}\: d=0 \\ -\frac{16a^{2}}{3ab^{3}d\sqrt{a+b\sin\left(c+dx\right)}+3b^{4}d\sqrt{a+b\sin\left(c+dx\right)}\sin\left(c+dx\right)}-\frac{24ab\sin\left(c+dx\right)}{3ab^{3}d\sqrt{a+b\sin\left(c+dx\right)}+3b^{4}d\sqrt{a+b\sin\left(c+dx\right)}\sin\left(c+dx\right)}-\frac{8b^{2}\sin^{2}\left(c+dx\right)}{3ab^{3}d\sqrt{a+b\sin\left(c+dx\right)}+3b^{4}d\sqrt{a+b\sin\left(c+dx\right)}\sin\left(c+dx\right)}-\frac{2b^{2}\cos^{2}\left(c+dx\right)}{3ab^{3}d\sqrt{a+b\sin\left(c+dx\right)}+3b^{4}d\sqrt{a+b\sin\left(c+dx\right)}\sin\left(c+dx\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Piecewise((x*cos(c)**3/a**(5/2), Eq(b, 0) & Eq(d, 0)), ((2*sin(c + d*x)**3/
(3*d) + sin(c + d*x)*cos(c + d*x)**2/d)/a**(5/2), Eq(b, 0)), (x*cos(c)**3/(
a + b*sin(c))**(5/2), Eq(d, 0)), (-16*a**2/(3*a*b**3*d*sqrt(a + b*sin(c + d
*x)) + 3*b**4*d*sqrt(a + b*sin(c + d*x))*sin(c + d*x)) - 24*a*b*sin(c + d*x
)/(3*a*b**3*d*sqrt(a + b*sin(c + d*x)) + 3*b**4*d*sqrt(a + b*sin(c + d*x))*
sin(c + d*x)) - 8*b**2*sin(c + d*x)**2/(3*a*b**3*d*sqrt(a + b*sin(c + d*x))
 + 3*b**4*d*sqrt(a + b*sin(c + d*x))*sin(c + d*x)) - 2*b**2*cos(c + d*x)**2
/(3*a*b**3*d*sqrt(a + b*sin(c + d*x)) + 3*b**4*d*sqrt(a + b*sin(c + d*x))*s
in(c + d*x)), True))
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sin(d*x+c))^(5/2),x, algorithm="giac")
```

[Out] Timed out

Mupad [B]
time = 11.89, size = 1402, normalized size = 17.75

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a + b*sin(c + d*x))^(5/2),x)
```

```
[Out] (16*a^2*b^2*(cos(2*d*x) + sin(2*d*x)*1i)*(cos(2*c) + sin(2*c)*1i)*(a + (b*(
cos(d*x) - sin(d*x)*1i)*(cos(c) - sin(c)*1i)*1i)/2 - (b*(cos(d*x) + sin(d*x
)*1i)*(cos(c) + sin(c)*1i)*1i)/2)^(1/2))/(3*(a^2*b^5*d - b^7*d + 2*b^7*d*(c
os(2*d*x) + sin(2*d*x)*1i)*(cos(2*c) + sin(2*c)*1i) - b^7*d*(cos(4*d*x) + s
in(4*d*x)*1i)*(cos(4*c) + sin(4*c)*1i) - a*b^6*d*(cos(3*d*x) + sin(3*d*x)*1
i)*(cos(3*c) + sin(3*c)*1i)*4i + a*b^6*d*(cos(d*x) + sin(d*x)*1i)*(cos(c) +
 sin(c)*1i)*4i + 2*a^2*b^5*d*(cos(2*d*x) + sin(2*d*x)*1i)*(cos(2*c) + sin(2
*c)*1i) - 4*a^4*b^3*d*(cos(2*d*x) + sin(2*d*x)*1i)*(cos(2*c) + sin(2*c)*1i)
 + a^3*b^4*d*(cos(3*d*x) + sin(3*d*x)*1i)*(cos(3*c) + sin(3*c)*1i)*4i + a^2
*b^5*d*(cos(4*d*x) + sin(4*d*x)*1i)*(cos(4*c) + sin(4*c)*1i) - a^3*b^4*d*(c
os(d*x) + sin(d*x)*1i)*(cos(c) + sin(c)*1i)*4i)) - (8*a^4*(cos(2*d*x) + sin
(2*d*x)*1i)*(cos(2*c) + sin(2*c)*1i)*(a + (b*(cos(d*x) - sin(d*x)*1i)*(cos(
c) - sin(c)*1i)*1i)/2 - (b*(cos(d*x) + sin(d*x)*1i)*(cos(c) + sin(c)*1i)*1i
)/2)^(1/2))/(3*(a^2*b^5*d - b^7*d + 2*b^7*d*(cos(2*d*x) + sin(2*d*x)*1i)*(c
os(2*c) + sin(2*c)*1i) - b^7*d*(cos(4*d*x) + sin(4*d*x)*1i)*(cos(4*c) + sin
(4*c)*1i) - a*b^6*d*(cos(3*d*x) + sin(3*d*x)*1i)*(cos(3*c) + sin(3*c)*1i)*4
i + a*b^6*d*(cos(d*x) + sin(d*x)*1i)*(cos(c) + sin(c)*1i)*4i + 2*a^2*b^5*d*
(cos(2*d*x) + sin(2*d*x)*1i)*(cos(2*c) + sin(2*c)*1i) - 4*a^4*b^3*d*(cos(2*
d*x) + sin(2*d*x)*1i)*(cos(2*c) + sin(2*c)*1i) + a^3*b^4*d*(cos(3*d*x) + si
n(3*d*x)*1i)*(cos(3*c) + sin(3*c)*1i)*4i + a^2*b^5*d*(cos(4*d*x) + sin(4*d*
x)*1i)*(cos(4*c) + sin(4*c)*1i) - a^3*b^4*d*(cos(d*x) + sin(d*x)*1i)*(cos(c
) + sin(c)*1i)*4i)) - (8*b^4*(cos(2*d*x) + sin(2*d*x)*1i)*(cos(2*c) + sin(2
*c)*1i)*(a + (b*(cos(d*x) - sin(d*x)*1i)*(cos(c) - sin(c)*1i)*1i)/2 - (b*(c
os(d*x) + sin(d*x)*1i)*(cos(c) + sin(c)*1i)*1i)/2)^(1/2))/(3*(a^2*b^5*d - b
^7*d + 2*b^7*d*(cos(2*d*x) + sin(2*d*x)*1i)*(cos(2*c) + sin(2*c)*1i) - b^7*
d*(cos(4*d*x) + sin(4*d*x)*1i)*(cos(4*c) + sin(4*c)*1i) - a*b^6*d*(cos(3*d*
x) + sin(3*d*x)*1i)*(cos(3*c) + sin(3*c)*1i)*4i + a*b^6*d*(cos(d*x) + sin(d
*x)*1i)*(cos(c) + sin(c)*1i)*4i + 2*a^2*b^5*d*(cos(2*d*x) + sin(2*d*x)*1i)*
(cos(2*c) + sin(2*c)*1i) - 4*a^4*b^3*d*(cos(2*d*x) + sin(2*d*x)*1i)*(cos(2*
c) + sin(2*c)*1i) + a^3*b^4*d*(cos(3*d*x) + sin(3*d*x)*1i)*(cos(3*c) + sin(
3*c)*1i)*4i + a^2*b^5*d*(cos(4*d*x) + sin(4*d*x)*1i)*(cos(4*c) + sin(4*c)*1
i) - a^3*b^4*d*(cos(d*x) + sin(d*x)*1i)*(cos(c) + sin(c)*1i)*4i)) - (a*(cos
(d*x) + sin(d*x)*1i)*(cos(c) + sin(c)*1i)*(a + (b*(cos(d*x) - sin(d*x)*1i)*
(cos(c) - sin(c)*1i)*1i)/2 - (b*(cos(d*x) + sin(d*x)*1i)*(cos(c) + sin(c)*1
i)*1i)/2)^(1/2)*8i)/(b^4*d*(cos(2*d*x) + sin(2*d*x)*1i)*(cos(2*c) + sin(2*c
```

```
)*1i) - b^4*d + a*b^3*d*(cos(d*x) + sin(d*x)*1i)*(cos(c) + sin(c)*1i)*2i) -
 (2*(a + (b*(cos(d*x) - sin(d*x)*1i)*(cos(c) - sin(c)*1i)*1i)/2 - (b*(cos(d
*x) + sin(d*x)*1i)*(cos(c) + sin(c)*1i)*1i)/2)^(1/2))/(b^3*d)
```

3.529 $\int \frac{\cos(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=24

$$-\frac{2}{3bd(a+b\sin(c+dx))^{3/2}}$$

```
[Out] -2/3/b/d/(a+b*sin(d*x+c))^(3/2)
```

Rubi [A]
time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2747, 32}

$$-\frac{2}{3bd(a+b\sin(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] -2/(3*b*d*(a + b*Sin[c + d*x])^(3/2))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{(a+x)^{5/2}}\,dx, x, b\sin(c+dx)\right)}{bd}\\ &= -\frac{2}{3bd(a+b\sin(c+dx))^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 24, normalized size = 1.00

$$-\frac{2}{3bd(a+b\sin(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] -2/(3*b*d*(a + b*Sin[c + d*x])^(3/2))
```

Maple [A]
time = 0.04, size = 21, normalized size = 0.88

method	result	size
derivativedivides	$-\frac{2}{3bd(a+b\sin(dx+c))^{\frac{3}{2}}}$	21
default	$-\frac{2}{3bd(a+b\sin(dx+c))^{\frac{3}{2}}}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+b*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/b/d/(a+b*sin(d*x+c))^(3/2)
```

Maxima [A]
time = 0.41, size = 20, normalized size = 0.83

$$-\frac{2}{3\,(b\sin\left(dx+c\right)+a)^{\frac{3}{2}}bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3/((b*sin(d*x + c) + a)^(3/2)*b*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 55 vs. 2(20) = 40.
time = 0.35, size = 55, normalized size = 2.29

$$\frac{2\,\sqrt{b\sin\left(dx+c\right)+a}}{3\,\left(b^3d\cos\left(dx+c\right)^2-2\,ab^2d\sin\left(dx+c\right)-(a^2b+b^3)d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(b*sin(d*x + c) + a)/(b^3*d*cos(d*x + c)^2 - 2*a*b^2*d*sin(d*x + c)
 - (a^2*b + b^3)*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 87 vs. 2(20) = 40.

time = 3.00, size = 87, normalized size = 3.62

$$\begin{cases} \frac{x\cos(c)}{a^{\frac{5}{2}}} & \text{for } b=0 \wedge d=0 \\ \frac{\sin(c+dx)}{a^{\frac{5}{2}}d} & \text{for } b=0 \\ \frac{x\cos(c)}{(a+b\sin(c))^{\frac{5}{2}}} & \text{for } d=0 \\ -\frac{2}{3abd\sqrt{a+b\sin(c+dx)}+3b^2d\sqrt{a+b\sin(c+dx)}\sin(c+dx)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Piecewise((x*cos(c)/a**(5/2), Eq(b, 0) & Eq(d, 0)), (sin(c + d*x)/(a**(5/2)
*d), Eq(b, 0)), (x*cos(c)/(a + b*sin(c))**(5/2), Eq(d, 0)), (-2/(3*a*b*d*sq
rt(a + b*sin(c + d*x)) + 3*b**2*d*sqrt(a + b*sin(c + d*x))*sin(c + d*x)), T
rue))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)/(b*sin(d*x + c) + a)^(5/2), x)
```

Mupad [B]
time = 7.25, size = 157, normalized size = 6.54

$$-\frac{8\sqrt{a+b\sin(c+dx)}\,(2a^2+b^2-b^2\cos(2c+2dx)+4ab\sin(c+dx))}{3bd\,(8a^4+3b^4+24a^2b^2-4b^4\cos(2c+2dx)+b^4\cos(4c+4dx)-8ab^3\sin(3c+3dx)-24a^2b^2\cos(2c+2dx)+24ab^3\sin(c+dx)+32a^3b\sin(c+dx))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + b*sin(c + d*x))^(5/2),x)
```

```
[Out] -(8*(a + b*sin(c + d*x))^(1/2)*(2*a^2 + b^2 - b^2*cos(2*c + 2*d*x) + 4*a*b*
sin(c + d*x)))/(3*b*d*(8*a^4 + 3*b^4 + 24*a^2*b^2 - 4*b^4*cos(2*c + 2*d*x)
+ b^4*cos(4*c + 4*d*x) - 8*a*b^3*sin(3*c + 3*d*x) - 24*a^2*b^2*cos(2*c + 2*
d*x) + 24*a*b^3*sin(c + d*x) + 32*a^3*b*sin(c + d*x)))
```

3.530 $\int \frac{\sec(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=139

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{(a-b)^{5/2}d}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{(a+b)^{5/2}d}+\frac{2b}{3\left(a^2-b^2\right)d(a+b\sin(c+dx))^{3/2}}+\frac{}{(a^2}$$

[Out] -arctanh((a+b*sin(d*x+c))^(1/2)/(a-b)^(1/2))/(a-b)^(5/2)/d+arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))/(a+b)^(5/2)/d+2/3*b/(a^2-b^2)/d/(a+b*sin(d*x+c))^(3/2)+4*a*b/(a^2-b^2)^2/d/(a+b*sin(d*x+c))^(1/2)

Rubi [A]
time = 0.17, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2747, 724, 843, 841, 1180, 212}

$$\frac{4ab}{d\left(a^2-b^2\right)^2\sqrt{a+b\sin(c+dx)}}+\frac{2b}{3d\left(a^2-b^2\right)(a+b\sin(c+dx))^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{d(a-b)^{5/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{d(a+b)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d*x]/(a + b*Sin[c + d*x])^(5/2),x]

[Out] -(ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a - b]]/((a - b)^(5/2)*d)) + ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a + b]]/((a + b)^(5/2)*d) + (2*b)/(3*(a^2 - b^2)*d*(a + b*Sin[c + d*x])^(3/2)) + (4*a*b)/((a^2 - b^2)^2*d*Sqrt[a + b*Sin[c + d*x]])

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + Dist[c/(c*d^2 + a*e^2), Int[(d + e*x)^(m + 1)*((d - e*x)/(a + c*x^2)), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 841

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
```

```
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 843

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Simp[(e*f - d*g)*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 + a*e^2))
), x] + Dist[1/(c*d^2 + a*e^2), Int[(d + e*x)^(m + 1)*(Simp[c*d*f + a*e*g -
 c*(e*f - d*g)*x, x]/(a + c*x^2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x
] && NeQ[c*d^2 + a*e^2, 0] && FractionQ[m] && LtQ[m, -1]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx = \frac{b\text{Subst}\left(\int \frac{1}{(a+x)^{5/2}(b^2-x^2)}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{2b}{3(a^2-b^2)\,d(a+b\sin(c+dx))^{3/2}} + \frac{b\text{Subst}\left(\int \frac{a-x}{(a+x)^{3/2}(b^2-x^2)}\,dx, x, b\sin(c+dx\right.}{(a^2-b^2)\,d}$$

$$= \frac{2b}{3(a^2-b^2)\,d(a+b\sin(c+dx))^{3/2}} + \frac{4ab}{(a^2-b^2)^2\,d\sqrt{a+b\sin(c+dx)}} - \frac{b\text{Subs}}{}$$

$$= \frac{2b}{3(a^2-b^2)\,d(a+b\sin(c+dx))^{3/2}} + \frac{4ab}{(a^2-b^2)^2\,d\sqrt{a+b\sin(c+dx)}} - \frac{(2b)\text{Su}}{}$$

$$= \frac{2b}{3(a^2-b^2)\,d(a+b\sin(c+dx))^{3/2}} + \frac{4ab}{(a^2-b^2)^2\,d\sqrt{a+b\sin(c+dx)}} - \frac{\text{Subst}}{}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{(a-b)^{5/2}d} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{(a+b)^{5/2}d} + \frac{}{3(a^2 -}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.08, size = 94, normalized size = 0.68

$$\frac{(a+b)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{a+b\sin(c+dx)}{a-b}\right) + (-a+b)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{a+b\sin(c+dx)}{a+b}\right)}{3(a-b)(a+b)d(a+b\sin(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] ((a + b)*Hypergeometric2F1[-3/2, 1, -1/2, (a + b*Sin[c + d*x])/(a - b)] + (
-a + b)*Hypergeometric2F1[-3/2, 1, -1/2, (a + b*Sin[c + d*x])/(a + b)])/(3*
(a - b)*(a + b)*d*(a + b*Sin[c + d*x])^(3/2))
```

Maple [A]
time = 2.07, size = 122, normalized size = 0.88

method	result

default	$\frac{\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sin(dx+c)}}{\sqrt{a+b}}\right)}{(a+b)^{\frac{5}{2}}}+\frac{\arctan\left(\frac{\sqrt{a+b\sin(dx+c)}}{\sqrt{-a+b}}\right)}{(a-b)^2\sqrt{-a+b}}+\frac{2b}{3(a-b)(a+b)(a+b\sin(dx+c))^{\frac{3}{2}}}+\frac{4}{(a-b)^2(a+b)^2\sqrt{a}}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)/(a+b*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)

[Out] (1/(a+b)^(5/2)*arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))+1/(a-b)^2/(-a+b)^(1/2)*arctan((a+b*sin(d*x+c))^(1/2)/(-a+b)^(1/2))+2/3*b/(a-b)/(a+b)/(a+b*sin(d*x+c))^(3/2)+4*b*a/(a-b)^2/(a+b)^2/(a+b*sin(d*x+c))^(1/2))/d

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c))^(5/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for more detail

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 669 vs. $2(121) = 242$.
time = 0.74, size = 3225, normalized size = 23.20

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")

[Out] [-1/24*(3*(a^5 - 3*a^4*b + 4*a^3*b^2 - 4*a^2*b^3 + 3*a*b^4 - b^5 - (a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*cos(d*x + c)^2 + 2*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*sin(d*x + c))*sqrt(a + b)*log((b^4*cos(d*x + c)^4 + 128*a^4 + 256*a^3*b + 320*a^2*b^2 + 256*a*b^3 + 72*b^4 - 8*(20*a^2*b^2 + 28*a*b^3 + 9*b^4)*cos(d*x + c)^2 + 8*(16*a^3 + 24*a^2*b + 20*a*b^2 + 8*b^3 - (10*a*b^2 + 7*b^3)*cos(d*x + c)^2 - (b^3*cos(d*x + c)^2 - 24*a^2*b - 28*a*b^2 - 8*b^3)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)*sqrt(a + b) + 4*(64*a^3*b + 112*a^2*b^2 + 64*a*b^3 + 14*b^4 - (8*a*b^3 + 7*b^4)*cos(d*x + c)^2)*sin(d*x + c))/(cos(d*x + c)^4 - 8*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8)) + 3*(a^5 + 3*a^4*b + 4*a^3*b^2 + 4*a^2*b^3 + 3*a*b^4 + b^5 - (a^3*b^2

+ 3*a^2*b^3 + 3*a*b^4 + b^5)*cos(d*x + c)^2 + 2*(a^4*b + 3*a^3*b^2 + 3*a^2
*b^3 + a*b^4)*sin(d*x + c))*sqrt(a - b)*log((b^4*cos(d*x + c)^4 + 128*a^4 -
256*a^3*b + 320*a^2*b^2 - 256*a*b^3 + 72*b^4 - 8*(20*a^2*b^2 - 28*a*b^3 +
9*b^4)*cos(d*x + c)^2 - 8*(16*a^3 - 24*a^2*b + 20*a*b^2 - 8*b^3 - (10*a*b^2
- 7*b^3)*cos(d*x + c)^2 - (b^3*cos(d*x + c)^2 - 24*a^2*b + 28*a*b^2 - 8*b^
3)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)*sqrt(a - b) + 4*(64*a^3*b - 112*a
^2*b^2 + 64*a*b^3 - 14*b^4 - (8*a*b^3 - 7*b^4)*cos(d*x + c)^2)*sin(d*x + c)
)/(cos(d*x + c)^4 - 8*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2)*sin(d*x + c)
+ 8)) + 16*(7*a^4*b - 8*a^2*b^3 + b^5 + 6*(a^3*b^2 - a*b^4)*sin(d*x + c))*s
qrt(b*sin(d*x + c) + a))/((a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d*cos(d*x
+ c)^2 - 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*sin(d*x + c) - (a^8 -
2*a^6*b^2 + 2*a^2*b^6 - b^8)*d), 1/24*(6*(a^5 - 3*a^4*b + 4*a^3*b^2 - 4*a^
2*b^3 + 3*a*b^4 - b^5 - (a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*cos(d*x + c)^
2 + 2*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*sin(d*x + c))*sqrt(-a - b)*ar
ctan(-1/4*(b^2*cos(d*x + c)^2 - 8*a^2 - 8*a*b - 2*b^2 - 2*(4*a*b + 3*b^2)*s
in(d*x + c))*sqrt(b*sin(d*x + c) + a)*sqrt(-a - b)/(2*a^3 + 3*a^2*b + 2*a*b
^2 + b^3 - (a*b^2 + b^3)*cos(d*x + c)^2 + (3*a^2*b + 4*a*b^2 + b^3)*sin(d*x
+ c))) - 3*(a^5 + 3*a^4*b + 4*a^3*b^2 + 4*a^2*b^3 + 3*a*b^4 + b^5 - (a^3*b
^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*cos(d*x + c)^2 + 2*(a^4*b + 3*a^3*b^2 + 3*a
^2*b^3 + a*b^4)*sin(d*x + c))*sqrt(a - b)*log((b^4*cos(d*x + c)^4 + 128*a^4
- 256*a^3*b + 320*a^2*b^2 - 256*a*b^3 + 72*b^4 - 8*(20*a^2*b^2 - 28*a*b^3
+ 9*b^4)*cos(d*x + c)^2 - 8*(16*a^3 - 24*a^2*b + 20*a*b^2 - 8*b^3 - (10*a*b
^2 - 7*b^3)*cos(d*x + c)^2 - (b^3*cos(d*x + c)^2 - 24*a^2*b + 28*a*b^2 - 8*
b^3)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)*sqrt(a - b) + 4*(64*a^3*b - 112
*a^2*b^2 + 64*a*b^3 - 14*b^4 - (8*a*b^3 - 7*b^4)*cos(d*x + c)^2)*sin(d*x +
c))/(cos(d*x + c)^4 - 8*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2)*sin(d*x + c
) + 8)) - 16*(7*a^4*b - 8*a^2*b^3 + b^5 + 6*(a^3*b^2 - a*b^4)*sin(d*x + c))
*sqrt(b*sin(d*x + c) + a))/((a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d*cos(d
x + c)^2 - 2(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*sin(d*x + c) - (a^8
- 2*a^6*b^2 + 2*a^2*b^6 - b^8)*d), 1/24*(6*(a^5 + 3*a^4*b + 4*a^3*b^2 + 4*
a^2*b^3 + 3*a*b^4 + b^5 - (a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*cos(d*x + c
)^2 + 2*(a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*sin(d*x + c))*sqrt(-a + b)*
arctan(1/4*(b^2*cos(d*x + c)^2 - 8*a^2 + 8*a*b - 2*b^2 - 2*(4*a*b - 3*b^2)*
sin(d*x + c))*sqrt(b*sin(d*x + c) + a)*sqrt(-a + b)/(2*a^3 - 3*a^2*b + 2*a*
b^2 - b^3 - (a*b^2 - b^3)*cos(d*x + c)^2 + (3*a^2*b - 4*a*b^2 + b^3)*sin(d*
x + c))) - 3*(a^5 - 3*a^4*b + 4*a^3*b^2 - 4*a^2*b^3 + 3*a*b^4 - b^5 - (a^3*
b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*cos(d*x + c)^2 + 2*(a^4*b - 3*a^3*b^2 + 3*
a^2*b^3 - a*b^4)*sin(d*x + c))*sqrt(a + b)*log((b^4*cos(d*x + c)^4 + 128*a^
4 + 256*a^3*b + 320*a^2*b^2 + 256*a*b^3 + 72*b^4 - 8*(20*a^2*b^2 + 28*a*b^3
+ 9*b^4)*cos(d*x + c)^2 + 8*(16*a^3 + 24*a^2*b + 20*a*b^2 + 8*b^3 - (10*a*
b^2 + 7*b^3)*cos(d*x + c)^2 - (b^3*cos(d*x + c)^2 - 24*a^2*b - 28*a*b^2 - 8
*b^3)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)*sqrt(a + b) + 4*(64*a^3*b + 11
2*a^2*b^2 + 64*a*b^3 + 14*b^4 - (8*a*b^3 + 7*b^4)*cos(d*x + c)^2)*sin(d*x +
c))/(cos(d*x + c)^4 - 8*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 - 2)*sin(d*x +
c) + 8)) - 16*(7*a^4*b - 8*a^2*b^3 + b^5 + 6*(a^3*b^2 - a*b^4)*sin(d*x + c)

```
)*sqrt(b*sin(d*x + c) + a))/((a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d*cos(
d*x + c)^2 - 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*sin(d*x + c) - (a^
8 - 2*a^6*b^2 + 2*a^2*b^6 - b^8)*d), 1/12*(3*(a^5 + 3*a^4*b + 4*a^3*b^2 + 4
*a^2*b^3 + 3*a*b^4 + b^5 - (a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*cos(d*x +
c)^2 + 2*(a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*sin(d*x + c))*sqrt(-a + b)
*arctan(1/4*(b^2*cos(d*x + c)^2 - 8*a^2 + 8*a*b - 2*b^2 - 2*(4*a*b - 3*b^2)
*sin(d*x + c))*sqrt(b*sin(d*x + c) + a)*sqrt(-a + b)/(2*a^3 - 3*a^2*b + 2*a
*b^2 - b^3 - (a*b^2 - b^3)*cos(d*x + c)^2 + (3*a^2*b - 4*a*b^2 + b^3)*sin(d
*x + c))) + 3*(a^5 - 3*a^4*b + 4*a^3*b^2 - 4*a^2*b^3 + 3*a*b^4 - b^5 - (a^3
*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*cos(d*x + c)^2 + 2*(a^4*b - 3*a^3*b^2 + 3
*a^2*b^3 - a*b^4)*sin(d*x + c))*sqrt(-a - b)*arctan(-1/4*(b^2*cos(d*x + c)^
2 - 8*a^2 - 8*a*b - 2*b^2 - 2*(4*a*b + 3*b^2)*s...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(c+dx)}{(a+b\sin(c+dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c))**(5/2),x)
[Out] Integral(sec(c + d*x)/(a + b*sin(c + d*x))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate(sec(d*x + c)/(b*sin(d*x + c) + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(c+d\,x)\,\left(a+b\,\sin(c+d\,x)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + b*sin(c + d*x))^(5/2)),x)
[Out] int(1/(cos(c + d*x)*(a + b*sin(c + d*x))^(5/2)), x)
```

3.531 $\int \frac{\sec^3(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=231

$$-\frac{(2a-7b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{4(a-b)^{7/2}d}+\frac{(2a+7b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{4(a+b)^{7/2}d}-\frac{b(3a^2+7}{6\left(a^2-b^2\right)^2d(a+b}$$

```
[Out] -1/4*(2*a-7*b)*arctanh((a+b*sin(d*x+c))^(1/2)/(a-b)^(1/2))/(a-b)^(7/2)/d+1/
4*(2*a+7*b)*arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))/(a+b)^(7/2)/d-1/6*b
*(3*a^2+7*b^2)/(a^2-b^2)^2/d/(a+b*sin(d*x+c))^(3/2)-1/2*sec(d*x+c)^2*(b-a*s
in(d*x+c))/(a^2-b^2)/d/(a+b*sin(d*x+c))^(3/2)-1/2*a*b*(a^2+19*b^2)/(a^2-b^2
)^3/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A]
time = 0.28, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2747, 755, 843, 841, 1180, 212}

$$-\frac{ab(a^2+19b^2)}{2d\left(a^2-b^2\right)^3\sqrt{a+b\sin(c+dx)}}-\frac{b(3a^2+7b^2)}{6d\left(a^2-b^2\right)^2(a+b\sin(c+dx))^{3/2}}-\frac{\sec^2(c+dx)(b-a\sin(c+dx))}{2d\left(a^2-b^2\right)(a+b\sin(c+dx))^{3/2}}-\frac{(2a-7b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{4d(a-b)^{7/2}}+\frac{(2a+7b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{4d(a+b)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] -1/4*((2*a - 7*b)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a - b]])/((a - b)^(
7/2)*d) + ((2*a + 7*b)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a + b]])/(4*(a
 + b)^(7/2)*d) - (b*(3*a^2 + 7*b^2))/(6*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x]
)^(3/2)) - (Sec[c + d*x]^2*(b - a*Sin[c + d*x]))/(2*(a^2 - b^2)*d*(a + b*Si
n[c + d*x])^(3/2)) - (a*b*(a^2 + 19*b^2))/(2*(a^2 - b^2)^3*d*Sqrt[a + b*Sin
[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
```

```
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 841

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 843

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Simp[(e*f - d*g)*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 + a*e^2))
), x] + Dist[1/(c*d^2 + a*e^2), Int[(d + e*x)^(m + 1)*(Simp[c*d*f + a*e*g -
 c*(e*f - d*g)*x, x]/(a + c*x^2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x
] && NeQ[c*d^2 + a*e^2, 0] && FractionQ[m] && LtQ[m, -1]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx = \frac{b^3\text{Subst}\left(\int \frac{1}{(a+x)^{5/2}(b^2-x^2)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= -\frac{\sec^2(c+dx)(b-a\sin(c+dx))}{2(a^2-b^2)\,d(a+b\sin(c+dx))^{3/2}} + \frac{b\text{Subst}\left(\int \frac{\frac{1}{2}(2a^2-7b^2)+\frac{5ax}{2}}{(a+x)^{5/2}(b^2-x^2)}\,dx, x, b\sin(c+\right.}{2(a^2-b^2)\,d}$$

$$= -\frac{b(3a^2+7b^2)}{6(a^2-b^2)^2\,d(a+b\sin(c+dx))^{3/2}} - \frac{\sec^2(c+dx)(b-a\sin(c+dx))}{2(a^2-b^2)\,d(a+b\sin(c+dx))^{3/2}} - \frac{b}{}$$

$$= -\frac{b(3a^2+7b^2)}{6(a^2-b^2)^2\,d(a+b\sin(c+dx))^{3/2}} - \frac{\sec^2(c+dx)(b-a\sin(c+dx))}{2(a^2-b^2)\,d(a+b\sin(c+dx))^{3/2}} - \frac{}{2}$$

$$= -\frac{b(3a^2+7b^2)}{6(a^2-b^2)^2\,d(a+b\sin(c+dx))^{3/2}} - \frac{\sec^2(c+dx)(b-a\sin(c+dx))}{2(a^2-b^2)\,d(a+b\sin(c+dx))^{3/2}} - \frac{}{2}$$

$$= -\frac{b(3a^2+7b^2)}{6(a^2-b^2)^2\,d(a+b\sin(c+dx))^{3/2}} - \frac{\sec^2(c+dx)(b-a\sin(c+dx))}{2(a^2-b^2)\,d(a+b\sin(c+dx))^{3/2}} - \frac{}{2}$$

$$= -\frac{(2a-7b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{4(a-b)^{7/2}d} + \frac{(2a+7b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(}}{\sqrt{a+}}\right.}{4(a+b)^{7/2}d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.87, size = 245, normalized size = 1.06

$$\frac{-\left((3a^3+3a^2b+7ab^2+7b^3)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{a+b\sin(c+dx)}{a-b}\right)\right)+(3a^3-3a^2b+7ab^2-7b^3)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{a+b\sin(c+dx)}{a+b}\right)+15a(a+b)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{a+b\sin(c+dx)}{a-b}\right)(a+b\sin(c+dx))-3(a-b)\left(-2(a+b)\sec^2(c+dx)(-b+a\sin(c+dx))+5a\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{a+b\sin(c+dx)}{a+b}\right)(a+b\sin(c+dx))\right)}{12(a-b)^2(a+b)^2d(a+b\sin(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-((3*a^3 + 3*a^2*b + 7*a*b^2 + 7*b^3)*Hypergeometric2F1[-3/2, 1, -1/2, (a
+ b*Sin[c + d*x])/(a - b)]) + (3*a^3 - 3*a^2*b + 7*a*b^2 - 7*b^3)*Hypergeom
etric2F1[-3/2, 1, -1/2, (a + b*Sin[c + d*x])/(a + b)] + 15*a*(a + b)*Hyperg
eometric2F1[-1/2, 1, 1/2, (a + b*Sin[c + d*x])/(a - b)]*(a + b*Sin[c + d*x]
) - 3*(a - b)*(-2*(a + b)*Sec[c + d*x]^2*(-b + a*Sin[c + d*x]) + 5*a*Hyperg
eometric2F1[-1/2, 1, 1/2, (a + b*Sin[c + d*x])/(a + b)]*(a + b*Sin[c + d*x]
)))/(12*(a - b)^2*(a + b)^2*d*(a + b*Sin[c + d*x])^(3/2))
```

Maple [A]
time = 2.73, size = 263, normalized size = 1.14

method	result
default	$\frac{-\frac{b\sqrt{a+b\sin(dx+c)}}{4(a+b)^3(b\sin(dx+c)-b)}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sin(dx+c)}}{\sqrt{a+b}}\right)a}{2(a+b)^{\frac{7}{2}}}+\frac{7b\operatorname{arctanh}\left(\frac{\sqrt{a+b\sin(dx+c)}}{\sqrt{a+b}}\right)}{4(a+b)^{\frac{7}{2}}}-\frac{b\sqrt{a+b}}{4(a-b)^3(}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+b*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-1/4*b/(a+b)^3*(a+b*sin(d*x+c))^(1/2)/(b*sin(d*x+c)-b)+1/2/(a+b)^(7/2)*arc
tanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))*a+7/4*b/(a+b)^(7/2)*arctanh((a+b*s
in(d*x+c))^(1/2)/(a+b)^(1/2))-1/4*b/(a-b)^3*(a+b*sin(d*x+c))^(1/2)/(b*sin(d
*x+c)+b)+1/2/(a-b)^3/(-a+b)^(1/2)*arctan((a+b*sin(d*x+c))^(1/2)/(-a+b)^(1/2
))*a-7/4*b/(a-b)^3/(-a+b)^(1/2)*arctan((a+b*sin(d*x+c))^(1/2)/(-a+b)^(1/2))
-2/3*b^3/(a-b)^2/(a+b)^2/(a+b*sin(d*x+c))^(3/2)-8*b^3*a/(a-b)^3/(a+b)^3/(a+
b*sin(d*x+c))^(1/2))/d
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for more
 detail
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(b*sin(d*x + c) + a)*sec(d*x + c)^3/(3*a*b^2*cos(d*x + c)^2 -
 a^3 - 3*a*b^2 + (b^3*cos(d*x + c)^2 - 3*a^2*b - b^3)*sin(d*x + c)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(c+dx)}{(a+b\sin(c+dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Integral(sec(c + d*x)**3/(a + b*sin(c + d*x))**(5/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos\left(c+d\,x\right)^3\,{\left(a+b\,\sin\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + b*sin(c + d*x))^(5/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^3*(a + b*sin(c + d*x))^(5/2)), x)
```

3.532 $\int \frac{\sec^5(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=339

$$-\frac{(12a^2-54ab+77b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{32(a-b)^{9/2}d}+\frac{(12a^2+54ab+77b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{32(a+b)^{9/2}d}$$

```
[Out] -1/32*(12*a^2-54*a*b+77*b^2)*arctanh((a+b*sin(d*x+c))^(1/2)/(a-b)^(1/2))/(a
-b)^(9/2)/d+1/32*(12*a^2+54*a*b+77*b^2)*arctanh((a+b*sin(d*x+c))^(1/2)/(a+b
)^(1/2))/(a+b)^(9/2)/d-1/48*b*(18*a^4-81*a^2*b^2-77*b^4)/(a^2-b^2)^3/d/(a+b
*sin(d*x+c))^(3/2)-1/4*sec(d*x+c)^4*(b-a*sin(d*x+c))/(a^2-b^2)/d/(a+b*sin(d
*x+c))^(3/2)+1/16*sec(d*x+c)^2*(b*(3*a^2+11*b^2)+2*a*(3*a^2-10*b^2)*sin(d*x
+c))/(a^2-b^2)^2/d/(a+b*sin(d*x+c))^(3/2)-1/8*a*b*(3*a^4-16*a^2*b^2-127*b^4
)/(a^2-b^2)^4/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A]
time = 0.43, antiderivative size = 339, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {2747, 755, 837, 843, 841, 1180, 212}

$$-\frac{(12a^2-54ab+77b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{32d(a-b)^{9/2}}+\frac{(12a^2+54ab+77b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a+b}}\right)}{32d(a+b)^{9/2}}-\frac{\sec^4(c+dx)(b-a\sin(c+dx))}{4d(a^2-b^2)(a+b\sin(c+dx))^{3/2}}+\frac{\sec^2(c+dx)\left(2a(3a^2-10b^2)\sin(c+dx)+b(3a^2+11b^2)\right)}{16d(a^2-b^2)^2(a+b\sin(c+dx))^{3/2}}-\frac{ab(3a^4-16a^2b^2-127b^4)}{8d(a^2-b^2)^4\sqrt{a+b\sin(c+dx)}}-\frac{b(18a^4-81a^2b^2-77b^4)}{48d(a^2-b^2)^3(a+b\sin(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] -1/32*((12*a^2 - 54*a*b + 77*b^2)*ArcTanh[Sqrt[a + b*Sin[c + d*x]]/Sqrt[a -
 b]])/((a - b)^(9/2)*d) + ((12*a^2 + 54*a*b + 77*b^2)*ArcTanh[Sqrt[a + b*Si
n[c + d*x]]/Sqrt[a + b]])/(32*(a + b)^(9/2)*d) - (b*(18*a^4 - 81*a^2*b^2 -
77*b^4))/(48*(a^2 - b^2)^3*d*(a + b*Sin[c + d*x])^(3/2)) - (Sec[c + d*x]^4*
(b - a*Sin[c + d*x]))/(4*(a^2 - b^2)*d*(a + b*Sin[c + d*x])^(3/2)) - (a*b*(
3*a^4 - 16*a^2*b^2 - 127*b^4))/(8*(a^2 - b^2)^4*d*Sqrt[a + b*Sin[c + d*x]])
 + (Sec[c + d*x]^2*(b*(3*a^2 + 11*b^2) + 2*a*(3*a^2 - 10*b^2)*Sin[c + d*x])
)/(16*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
```

```
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 841

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 843

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Simp[(e*f - d*g)*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 + a*e^2))
), x] + Dist[1/(c*d^2 + a*e^2), Int[(d + e*x)^(m + 1)*(Simp[c*d*f + a*e*g -
 c*(e*f - d*g)*x, x]/(a + c*x^2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x
] && NeQ[c*d^2 + a*e^2, 0] && FractionQ[m] && LtQ[m, -1]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx = \frac{b^5\text{Subst}\left(\int \frac{1}{(a+x)^{5/2}(b^2-x^2)^3}\,dx,x,b\sin(c+dx)\right)}{d}$$

$$= -\frac{\sec^4(c+dx)(b-a\sin(c+dx))}{4(a^2-b^2)\,d(a+b\sin(c+dx))^{3/2}} + \frac{b^3\text{Subst}\left(\int \frac{\frac{1}{2}(6a^2-11b^2)+\frac{9ax}{2}}{(a+x)^{5/2}(b^2-x^2)^2}\,dx,x,b\sin(\right.}{4(a^2-b^2)\,d}$$

$$= -\frac{\sec^4(c+dx)(b-a\sin(c+dx))}{4(a^2-b^2)\,d(a+b\sin(c+dx))^{3/2}} + \frac{\sec^2(c+dx)\left(b(3a^2+11b^2)+2a(3a^2\right.}{16(a^2-b^2)^2\,d(a+b\sin(c}$$

$$= -\frac{b(18a^4-81a^2b^2-77b^4)}{48(a^2-b^2)^3\,d(a+b\sin(c+dx))^{3/2}} - \frac{\sec^4(c+dx)(b-a\sin(c+dx))}{4(a^2-b^2)\,d(a+b\sin(c+dx))^{3/2}} +$$

$$= -\frac{b(18a^4-81a^2b^2-77b^4)}{48(a^2-b^2)^3\,d(a+b\sin(c+dx))^{3/2}} - \frac{\sec^4(c+dx)(b-a\sin(c+dx))}{4(a^2-b^2)\,d(a+b\sin(c+dx))^{3/2}} -$$

$$= -\frac{b(18a^4-81a^2b^2-77b^4)}{48(a^2-b^2)^3\,d(a+b\sin(c+dx))^{3/2}} - \frac{\sec^4(c+dx)(b-a\sin(c+dx))}{4(a^2-b^2)\,d(a+b\sin(c+dx))^{3/2}} -$$

$$= -\frac{b(18a^4-81a^2b^2-77b^4)}{48(a^2-b^2)^3\,d(a+b\sin(c+dx))^{3/2}} - \frac{\sec^4(c+dx)(b-a\sin(c+dx))}{4(a^2-b^2)\,d(a+b\sin(c+dx))^{3/2}} -$$

$$= -\frac{(12a^2-54ab+77b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c+dx)}}{\sqrt{a-b}}\right)}{32(a-b)^{9/2}d} + \frac{(12a^2+54ab+77}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 3.21, size = 296, normalized size = 0.87

$$\frac{\frac{1}{2}(18a^4-81a^2b^2-77b^4)\left((a+b)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{a+b\sin(c+dx)}{a-b}\right)+(-a+b)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{a+b\sin(c+dx)}{a+b}\right)\right)-12(a-b)^2(a+b)^2\sec^4(c+dx)(-b+a\sin(c+dx))-15a(3a^2-10b^2)\left((a+b)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{a+b\sin(c+dx)}{a-b}\right)+(-a+b)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{a+b\sin(c+dx)}{a+b}\right)\right)(a+b\sin(c+dx))-3(a-b)(a+b)\sec^2(c+dx)\left(3a^2b+11b^3+(6a^3-20ab^2)\sin(c+dx)\right)}{48(a^2-b^2)^2(-a^2+b^2)\,d(a+b\sin(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (((18*a^4 - 81*a^2*b^2 - 77*b^4)*((a + b)*Hypergeometric2F1[-3/2, 1, -1/2,
(a + b*Sin[c + d*x])/(a - b)] + (-a + b)*Hypergeometric2F1[-3/2, 1, -1/2, (
a + b*Sin[c + d*x])/(a + b)]))/2 - 12*(a - b)^2*(a + b)^2*Sec[c + d*x]^4*(-
b + a*Sin[c + d*x]) - 15*a*(3*a^2 - 10*b^2)*((a + b)*Hypergeometric2F1[-1/2
, 1, 1/2, (a + b*Sin[c + d*x])/(a - b)] + (-a + b)*Hypergeometric2F1[-1/2,
1, 1/2, (a + b*Sin[c + d*x])/(a + b)])*(a + b*Sin[c + d*x]) - 3*(a - b)*(a
```

```
+ b)*Sec[c + d*x]^2*(3*a^2*b + 11*b^3 + (6*a^3 - 20*a*b^2)*Sin[c + d*x]))/(
48*(a^2 - b^2)^2*(-a^2 + b^2)*d*(a + b*Sin[c + d*x])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 631 vs. $2(307) = 614$.

time = 3.16, size = 632, normalized size = 1.86

method	result
default	$-\frac{3b(a+b\sin(dx+c))^{\frac{3}{2}}a}{16(a+b)^4(b\sin(dx+c)-b)^2}-\frac{17b^2(a+b\sin(dx+c))^{\frac{3}{2}}}{32(a+b)^4(b\sin(dx+c)-b)^2}+\frac{3b\sqrt{a+b\sin(dx+c)}\,a^2}{16(a+b)^4(b\sin(dx+c)-b)^2}+\frac{25b^2\sqrt{a+b\sin(dx+c)}\,a}{32(a+b)^4(b\sin(dx+c)-b)^2}+\frac{19b^3\sqrt{}}{32(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+b*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-3/16*b/(a+b)^4/(b*sin(d*x+c)-b)^2*(a+b*sin(d*x+c))^(3/2)*a-17/32*b^2/(a+b
)^4/(b*sin(d*x+c)-b)^2*(a+b*sin(d*x+c))^(3/2)+3/16*b/(a+b)^4/(b*sin(d*x+c)-
b)^2*(a+b*sin(d*x+c))^(1/2)*a^2+25/32*b^2/(a+b)^4/(b*sin(d*x+c)-b)^2*(a+b*s
in(d*x+c))^(1/2)*a+19/32*b^3/(a+b)^4/(b*sin(d*x+c)-b)^2*(a+b*sin(d*x+c))^(1
/2)+3/8/(a+b)^(9/2)*arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))*a^2+27/16*b
/(a+b)^(9/2)*arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))*a+77/32*b^2/(a+b)^
(9/2)*arctanh((a+b*sin(d*x+c))^(1/2)/(a+b)^(1/2))-3/16*b/(a-b)^4/(b*sin(d*x
+c)+b)^2*(a+b*sin(d*x+c))^(3/2)*a+17/32*b^2/(a-b)^4/(b*sin(d*x+c)+b)^2*(a+b
*sin(d*x+c))^(3/2)+3/16*b/(a-b)^4/(b*sin(d*x+c)+b)^2*(a+b*sin(d*x+c))^(1/2)
*a^2-25/32*b^2/(a-b)^4/(b*sin(d*x+c)+b)^2*(a+b*sin(d*x+c))^(1/2)*a+19/32*b^
3/(a-b)^4/(b*sin(d*x+c)+b)^2*(a+b*sin(d*x+c))^(1/2)+3/8/(a-b)^4/(-a+b)^(1/2
)*arctan((a+b*sin(d*x+c))^(1/2)/(-a+b)^(1/2))*a^2-27/16*b/(a-b)^4/(-a+b)^(1
/2)*arctan((a+b*sin(d*x+c))^(1/2)/(-a+b)^(1/2))*a+77/32*b^2/(a-b)^4/(-a+b)^
(1/2)*arctan((a+b*sin(d*x+c))^(1/2)/(-a+b)^(1/2))+2/3*b^5/(a-b)^3/(a+b)^3/(
a+b*sin(d*x+c))^(3/2)+12*b^5*a/(a-b)^4/(a+b)^4/(a+b*sin(d*x+c))^(1/2))/d
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for more
 detail
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(b*sin(d*x + c) + a)*sec(d*x + c)^5/(3*a*b^2*cos(d*x + c)^2 -
 a^3 - 3*a*b^2 + (b^3*cos(d*x + c)^2 - 3*a^2*b - b^3)*sin(d*x + c)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^5(c+dx)}{(a+b\sin(c+dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Integral(sec(c + d*x)**5/(a + b*sin(c + d*x))**(5/2), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^5*(a + b*sin(c + d*x))^(5/2)),x)
```

```
[Out] \text{Hanged}
```

3.533 $\int \frac{\cos^8(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=384

$$-\frac{2\cos^7(c+dx)}{3bd(a+b\sin(c+dx))^{3/2}}-\frac{128a(8a^2-9b^2)\left(4a^2-3b^2\right)E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\frac{2b}{a+b}\right)\sqrt{a+b\sin(c+dx)}}{99b^8d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}+\frac{32(12}{}$$

```
[Out] -2/3*cos(d*x+c)^7/b/d/(a+b*sin(d*x+c))^(3/2)-28/33*cos(d*x+c)^5*(12*a+b*sin
(d*x+c))/b^3/d/(a+b*sin(d*x+c))^(1/2)+40/99*cos(d*x+c)^3*(32*a^2-3*b^2-28*a
*b*sin(d*x+c))*(a+b*sin(d*x+c))^(1/2)/b^5/d-16/99*cos(d*x+c)*(128*a^4-144*a
^2*b^2+15*b^4-3*a*b*(32*a^2-31*b^2)*sin(d*x+c))*(a+b*sin(d*x+c))^(1/2)/b^7/
d+128/99*a*(8*a^2-9*b^2)*(4*a^2-3*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/
sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a
+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/b^8/d/((a+b*sin(d*x+c))/(a+b))^(1/2)-32/
99*(128*a^6-272*a^4*b^2+159*a^2*b^4-15*b^6)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(
1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*
(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/b^8/d/(a+b*sin(d*x+c))^(1/2
)
```

Rubi [A]
time = 0.49, antiderivative size = 384, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {2772, 2942, 2944, 2831, 2742, 2740, 2734, 2732}

$$-\frac{128a(8a^2-9b^2)\left(4a^2-3b^2\right)\sqrt{a+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|\frac{2b}{a+b}\right)}{99b^8d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}+\frac{40\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}\left(32a^2-28ab\sin(c+dx)-3b^2\right)}{99b^5d}-\frac{16\cos(c+dx)\sqrt{a+b\sin(c+dx)}\left(128a^4-3ab(32a^2-31b^2)\sin(c+dx)-144a^2b^2+15b^4\right)}{99b^7d}+\frac{32(128a^6-272a^4b^2+159a^2b^4-15b^6)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|\frac{2b}{a+b}\right)}{99b^8d\sqrt{a+b\sin(c+dx)}}-\frac{28\cos^5(c+dx)(12a+b\sin(c+dx))}{33b^3d\sqrt{a+b\sin(c+dx)}}-\frac{2\cos^7(c+dx)}{3bd(a+b\sin(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^8/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*Cos[c + d*x]^7)/(3*b*d*(a + b*Sin[c + d*x])^(3/2)) - (128*a*(8*a^2 - 9*
b^2)*(4*a^2 - 3*b^2)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a +
b*Sin[c + d*x]])/(99*b^8*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + (32*(128*a
^6 - 272*a^4*b^2 + 159*a^2*b^4 - 15*b^6)*EllipticF[(c - Pi/2 + d*x)/2, (2*b
)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(99*b^8*d*Sqrt[a + b*Sin[c +
 d*x]]) - (28*Cos[c + d*x]^5*(12*a + b*Sin[c + d*x]))/(33*b^3*d*Sqrt[a + b*
Sin[c + d*x]]) + (40*Cos[c + d*x]^3*Sqrt[a + b*Sin[c + d*x]]*(32*a^2 - 3*b^
2 - 28*a*b*Sin[c + d*x]))/(99*b^5*d) - (16*Cos[c + d*x]*Sqrt[a + b*Sin[c +
d*x]]*(128*a^4 - 144*a^2*b^2 + 15*b^4 - 3*a*b*(32*a^2 - 31*b^2)*Sin[c + d*x
]))/(99*b^7*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2772

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Dist[g^2*((p - 1)/(b*(m + 1))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2942

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*C
os[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x])/(b^2*f*(m + 1)*(m + p + 1))), x] + Dist[g^2*((
p - 1)/(b^2*(m + 1)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
```

```
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rule 2944

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Dist[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\cos^8(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx = -\frac{2\cos^7(c+dx)}{3bd(a+b\sin(c+dx))^{3/2}} - \frac{14\int \frac{\cos^6(c+dx)\sin(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx}{3b}$$

$$= -\frac{2\cos^7(c+dx)}{3bd(a+b\sin(c+dx))^{3/2}} - \frac{28\cos^5(c+dx)(12a+b\sin(c+dx))}{33b^3d\sqrt{a+b\sin(c+dx)}} + \frac{280\int \frac{\cos}{}}{}$$

$$= -\frac{2\cos^7(c+dx)}{3bd(a+b\sin(c+dx))^{3/2}} - \frac{28\cos^5(c+dx)(12a+b\sin(c+dx))}{33b^3d\sqrt{a+b\sin(c+dx)}} + \frac{40\cos^3(c}{}$$

$$= -\frac{2\cos^7(c+dx)}{3bd(a+b\sin(c+dx))^{3/2}} - \frac{28\cos^5(c+dx)(12a+b\sin(c+dx))}{33b^3d\sqrt{a+b\sin(c+dx)}} + \frac{40\cos^3(c}{}$$

$$= -\frac{2\cos^7(c+dx)}{3bd(a+b\sin(c+dx))^{3/2}} - \frac{28\cos^5(c+dx)(12a+b\sin(c+dx))}{33b^3d\sqrt{a+b\sin(c+dx)}} + \frac{40\cos^3(c}{}$$

$$= -\frac{2\cos^7(c+dx)}{3bd(a+b\sin(c+dx))^{3/2}} - \frac{28\cos^5(c+dx)(12a+b\sin(c+dx))}{33b^3d\sqrt{a+b\sin(c+dx)}} + \frac{40\cos^3(c}{}$$

$$= -\frac{2\cos^7(c+dx)}{3bd(a+b\sin(c+dx))^{3/2}} - \frac{128a(8a^2-9b^2)(4a^2-3b^2)\,E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\frac{2}{a}\right.}{99b^8d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}$$

Mathematica [**A**]
time = 1.74, size = 356, normalized size = 0.93

$$\frac{256(a+b)\left(b(32a^4b-51a^2b^3+15b^5)F\left(\frac{1}{4}(-2c+\pi-2dx)|\frac{2b}{a+b}\right)+4(32a^5-60a^3b^2+27ab^4)\left((a+b)E\left(\frac{1}{4}(-2c+\pi-2dx)|\frac{2b}{a+b}\right)-aF\left(\frac{1}{4}(-2c+\pi-2dx)|\frac{2b}{a+b}\right)\right)\right)\left(\frac{a+b\sin(c+dx)}{a+b}\right)^{3/2}+\frac{1}{2}b\cos(c+dx)\left(-32768a^6+55296a^4b^2-18144a^2b^4-2574b^6+(2048a^4b^2-3648a^2b^4+1383b^6)\cos(2(c+dx))+(-96a^2b^4+126b^6)\cos(4(c+dx))+9b^6\cos(6(c+dx))-40960a^5b\sin(c+dx)+74112a^3b^3\sin(c+dx)-30920ab^5\sin(c+dx)-384a^3b^3\sin(3(c+dx))+596ab^5\sin(3(c+dx))+28ab^5\sin(5(c+dx))\right)}{792b^8d(a+b\sin(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^8/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (256*(a + b)*(b*(32*a^4*b - 51*a^2*b^3 + 15*b^5)*EllipticF[(-2*c + Pi - 2*d
*x)/4, (2*b)/(a + b)] + 4*(32*a^5 - 60*a^3*b^2 + 27*a*b^4)*((a + b)*Ellipti
cE[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)] - a*EllipticF[(-2*c + Pi - 2*d*x)/
4, (2*b)/(a + b)]))*((a + b*Sin[c + d*x])/(a + b))^(3/2) + (b*Cos[c + d*x]*
(-32768*a^6 + 55296*a^4*b^2 - 18144*a^2*b^4 - 2574*b^6 + (2048*a^4*b^2 - 36
48*a^2*b^4 + 1383*b^6)*Cos[2*(c + d*x)] + (-96*a^2*b^4 + 126*b^6)*Cos[4*(c
+ d*x)] + 9*b^6*Cos[6*(c + d*x)] - 40960*a^5*b*Sin[c + d*x] + 74112*a^3*b^3
*Sin[c + d*x] - 30920*a*b^5*Sin[c + d*x] - 384*a^3*b^3*Sin[3*(c + d*x)] + 5
96*a*b^5*Sin[3*(c + d*x)] + 28*a*b^5*Sin[5*(c + d*x)]))/2)/(792*b^8*d*(a +
b*Sin[c + d*x])^(3/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2252 vs. $2(422) = 844$.
time = 3.03, size = 2253, normalized size = 5.87

method	result	size
default	Expression too large to display	2253

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8/(a+b*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/99*(-14*a*b^7*sin(d*x+c)*cos(d*x+c)^6+(48*a^3*b^5-64*a*b^7)*cos(d*x+c)^4
*sin(d*x+c)+(1280*a^5*b^3-2328*a^3*b^5+984*a*b^7)*cos(d*x+c)^2*sin(d*x+c)+1
6*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*
(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*b*(128*EllipticF((b/(a-b)*sin(d*x+c)+1/
(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^6*b-96*EllipticF((b/(a-b)*sin(d*x+c)+
1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^5*b^2-272*EllipticF((b/(a-b)*sin(d*
x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^4*b^3+189*EllipticF((b/(a-b)*s
in(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^4+159*EllipticF((b/(a
-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^5-93*EllipticF((
b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a*b^6-15*EllipticF
((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*b^7-128*Elliptic
E((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^7+368*Ellipti
cE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^5*b^2-348*El
lipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^4+1
08*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a*b^
6)*sin(d*x+c)-9*b^8*cos(d*x+c)^8+(24*a^2*b^6-18*b^8)*cos(d*x+c)^6+(-128*a^4
```

```
*b^4+204*a^2*b^6-60*b^8)*cos(d*x+c)^4+(1024*a^6*b^2-1664*a^4*b^4+456*a^2*b^
6+120*b^8)*cos(d*x+c)^2+2048*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a+b)
*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticF((b
/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^7*b-1536*(b/(a-b)
*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*
sin(d*x+c)-b/(a-b))^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((
a-b)/(a+b))^(1/2))*a^6*b^2-4352*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a
+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticF
((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^5*b^3+3024*(b/
(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(
a-b)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/
2),((a-b)/(a+b))^(1/2))*a^4*b^4+2544*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(
-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*Elli
pticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^5-148
8*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*
(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a
)^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^6-240*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/
2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*
EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a*b^7-2
048*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2
)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)
*a)^(1/2),((a-b)/(a+b))^(1/2))*a^8+5888*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2
)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*E
llipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^6*b^2-
5568*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/
2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b
)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^4*b^4+1728*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)
^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1
/2)*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^2
*b^6)/(a+b*sin(d*x+c))^(3/2)/b^9/cos(d*x+c)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8/(a+b*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^8/(b*sin(d*x + c) + a)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.41, size = 1043, normalized size = 2.72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8/(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/297*(8*(sqrt(2)*(256*a^6*b^2 - 576*a^4*b^4 + 369*a^2*b^6 - 45*b^8)*cos(d*
x + c)^2 - 2*sqrt(2)*(256*a^7*b - 576*a^5*b^3 + 369*a^3*b^5 - 45*a*b^7)*sin
(d*x + c) - sqrt(2)*(256*a^8 - 320*a^6*b^2 - 207*a^4*b^4 + 324*a^2*b^6 - 45
*b^8))*sqrt(I*b)*weierstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I*a
^3 - 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) - 2*I*a)/b)
 + 8*(sqrt(2)*(256*a^6*b^2 - 576*a^4*b^4 + 369*a^2*b^6 - 45*b^8)*cos(d*x +
c)^2 - 2*sqrt(2)*(256*a^7*b - 576*a^5*b^3 + 369*a^3*b^5 - 45*a*b^7)*sin(d*x
 + c) - sqrt(2)*(256*a^8 - 320*a^6*b^2 - 207*a^4*b^4 + 324*a^2*b^6 - 45*b^8
))*sqrt(-I*b)*weierstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3
 + 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*I*a)/b) +
 96*(sqrt(2)*(32*I*a^5*b^3 - 60*I*a^3*b^5 + 27*I*a*b^7)*cos(d*x + c)^2 + 2*
sqrt(2)*(-32*I*a^6*b^2 + 60*I*a^4*b^4 - 27*I*a^2*b^6)*sin(d*x + c) + sqrt(2
)*(-32*I*a^7*b + 28*I*a^5*b^3 + 33*I*a^3*b^5 - 27*I*a*b^7))*sqrt(I*b)*weier
strassZeta(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, weier
strassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, 1
/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) - 2*I*a)/b)) + 96*(sqrt(2)*(-32*I
*a^5*b^3 + 60*I*a^3*b^5 - 27*I*a*b^7)*cos(d*x + c)^2 + 2*sqrt(2)*(32*I*a^6*
b^2 - 60*I*a^4*b^4 + 27*I*a^2*b^6)*sin(d*x + c) + sqrt(2)*(32*I*a^7*b - 28*
I*a^5*b^3 - 33*I*a^3*b^5 + 27*I*a*b^7))*sqrt(-I*b)*weierstrassZeta(-4/3*(4*
a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, weierstrassPInverse(-4/
3*(4*a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x +
 c) + 3*I*b*sin(d*x + c) + 2*I*a)/b)) - 3*(9*b^8*cos(d*x + c)^7 - 6*(4*a^2*
b^6 - 3*b^8)*cos(d*x + c)^5 + 4*(32*a^4*b^4 - 51*a^2*b^6 + 15*b^8)*cos(d*x
+ c)^3 - 8*(128*a^6*b^2 - 208*a^4*b^4 + 57*a^2*b^6 + 15*b^8)*cos(d*x + c) +
 2*(7*a*b^7*cos(d*x + c)^5 - 8*(3*a^3*b^5 - 4*a*b^7)*cos(d*x + c)^3 - 4*(16
0*a^5*b^3 - 291*a^3*b^5 + 123*a*b^7)*cos(d*x + c))*sin(d*x + c))*sqrt(b*sin
(d*x + c) + a))/(b^11*d*cos(d*x + c)^2 - 2*a*b^10*d*sin(d*x + c) - (a^2*b^9
 + b^11)*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8/(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3278 deep
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8/(a+b*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^8}{{\left(a+b\,\sin\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^8/(a + b*sin(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^8/(a + b*sin(c + d*x))^(5/2), x)
```

3.534 $\int \frac{\cos^6(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=293

$$-\frac{2\cos^5(c+dx)}{3bd(a+b\sin(c+dx))^{3/2}}+\frac{16a(32a^2-29b^2)\,E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\frac{2b}{a+b}\right)\sqrt{a+b\sin(c+dx)}}{21b^6d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}-\frac{16(32a^4-37a^2\ldots}{}$$

```
[Out] -2/3*cos(d*x+c)^5/b/d/(a+b*sin(d*x+c))^(3/2)-20/21*cos(d*x+c)^3*(8*a+b*sin(
d*x+c))/b^3/d/(a+b*sin(d*x+c))^(1/2)+8/21*cos(d*x+c)*(32*a^2-5*b^2-24*a*b*s
in(d*x+c))*(a+b*sin(d*x+c))^(1/2)/b^5/d-16/21*a*(32*a^2-29*b^2)*(sin(1/2*c+
1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*
Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/b^6/d/((a+b*sin
(d*x+c))/(a+b))^(1/2)+16/21*(32*a^4-37*a^2*b^2+5*b^4)*(sin(1/2*c+1/4*Pi+1/2
*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x
),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/b^6/d/(a+b*sin(d*
x+c))^(1/2)
```

Rubi [A]
time = 0.34, antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {2772, 2942, 2944, 2831, 2742, 2740, 2734, 2732}

$$\frac{16a(32a^2-29b^2)\sqrt{a+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|\frac{2b}{a+b}\right)}{21b^6d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}+\frac{8\cos(c+dx)\sqrt{a+b\sin(c+dx)}\,(32a^2-24ab\sin(c+dx)-5b^2)}{21b^5d}-\frac{16(32a^4-37a^2b^2+5b^4)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|\frac{2b}{a+b}\right)}{21b^6d\sqrt{a+b\sin(c+dx)}}-\frac{20\cos^3(c+dx)(8a+b\sin(c+dx))}{21b^3d\sqrt{a+b\sin(c+dx)}}-\frac{2\cos^5(c+dx)}{3bd(a+b\sin(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*Cos[c + d*x]^5)/(3*b*d*(a + b*Sin[c + d*x])^(3/2)) + (16*a*(32*a^2 - 29
*b^2)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]]
)/(21*b^6*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) - (16*(32*a^4 - 37*a^2*b^2
+ 5*b^4)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d
*x])/(a + b)])/(21*b^6*d*Sqrt[a + b*Sin[c + d*x]]) - (20*Cos[c + d*x]^3*(8*
a + b*Sin[c + d*x]))/(21*b^3*d*Sqrt[a + b*Sin[c + d*x]]) + (8*Cos[c + d*x]*
Sqrt[a + b*Sin[c + d*x]]*(32*a^2 - 5*b^2 - 24*a*b*Sin[c + d*x]))/(21*b^5*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2772

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Dist[g^2*((p - 1)/(b*(m + 1))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2942

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*C
os[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x])/(b^2*f*(m + 1)*(m + p + 1))), x] + Dist[g^2*((
p - 1)/(b^2*(m + 1)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rule 2944

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Dist[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\cos^6(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx = -\frac{2\cos^5(c+dx)}{3bd(a+b\sin(c+dx))^{3/2}} - \frac{10\int \frac{\cos^4(c+dx)\sin(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx}{3b}$$

$$= -\frac{2\cos^5(c+dx)}{3bd(a+b\sin(c+dx))^{3/2}} - \frac{20\cos^3(c+dx)(8a+b\sin(c+dx))}{21b^3d\sqrt{a+b\sin(c+dx)}} + \frac{40\int \frac{\cos^2}{\sqrt{}}}{}$$

$$= -\frac{2\cos^5(c+dx)}{3bd(a+b\sin(c+dx))^{3/2}} - \frac{20\cos^3(c+dx)(8a+b\sin(c+dx))}{21b^3d\sqrt{a+b\sin(c+dx)}} + \frac{8\cos(c}{}$$

$$= -\frac{2\cos^5(c+dx)}{3bd(a+b\sin(c+dx))^{3/2}} - \frac{20\cos^3(c+dx)(8a+b\sin(c+dx))}{21b^3d\sqrt{a+b\sin(c+dx)}} + \frac{8\cos(c}{}$$

$$= -\frac{2\cos^5(c+dx)}{3bd(a+b\sin(c+dx))^{3/2}} - \frac{20\cos^3(c+dx)(8a+b\sin(c+dx))}{21b^3d\sqrt{a+b\sin(c+dx)}} + \frac{8\cos(c}{}$$

$$= -\frac{2\cos^5(c+dx)}{3bd(a+b\sin(c+dx))^{3/2}} + \frac{16a(32a^2-29b^2)\,E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\frac{2b}{a+b}\right)\sqrt{a+}}{21b^6d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}$$

Mathematica [A]
time = 1.22, size = 244, normalized size = 0.83

$$\frac{-32(a+b)\left(a(32a^3+32a^2b-29ab^2-29b^3)\,E\left(\frac{1}{4}(-2c+\pi-2dx)\middle|\frac{2b}{a+b}\right)+(-32a^4+37a^2b^2-5b^4)\,F\left(\frac{1}{4}(-2c+\pi-2dx)\middle|\frac{2b}{a+b}\right)\right)\left(\frac{a+b\sin(c+dx)}{a+b}\right)^{3/2}+\frac{1}{2}b\cos(c+dx)\left(1024a^4-736a^2b^2-111b^4+(-64a^2b^2+52b^4)\cos(2(c+dx))+3b^4\cos(4(c+dx))+1280a^3b\sin(c+dx)-1076ab^3\sin(c+dx)+12ab^3\sin(3(c+dx))\right)}{42b^6d(a+b\sin(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-32*(a + b)*(a*(32*a^3 + 32*a^2*b - 29*a*b^2 - 29*b^3)*EllipticE[(-2*c + P
i - 2*d*x)/4, (2*b)/(a + b)] + (-32*a^4 + 37*a^2*b^2 - 5*b^4)*EllipticF[(-2
*c + Pi - 2*d*x)/4, (2*b)/(a + b)])*((a + b*Sin[c + d*x])/(a + b))^(3/2) +
(b*Cos[c + d*x]*(1024*a^4 - 736*a^2*b^2 - 111*b^4 + (-64*a^2*b^2 + 52*b^4)*
Cos[2*(c + d*x)] + 3*b^4*Cos[4*(c + d*x)] + 1280*a^3*b*Sin[c + d*x] - 1076*
a*b^3*Sin[c + d*x] + 12*a*b^3*Sin[3*(c + d*x)]))/2)/(42*b^6*d*(a + b*Sin[c
+ d*x])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1641 vs. $2(335) = 670$.
time = 2.23, size = 1642, normalized size = 5.60

method	result	size
default	Expression too large to display	1642

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6/(a+b*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/21*(6*a*b^5*sin(d*x+c)*cos(d*x+c)^4+(160*a^3*b^3-136*a*b^5)*cos(d*x+c)^2*
sin(d*x+c)+8*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b
)*a)^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*b*(32*EllipticF((b/(a-b)*sin
(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^4*b-24*EllipticF((b/(a-b)*s
in(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^2-37*EllipticF((b/(a-
b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^3+24*EllipticF((b
/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a*b^4+5*EllipticF((
b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*b^5-32*EllipticE((
b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^5+61*EllipticE((
b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^2-29*Ellipti
cE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a*b^4)*sin(d*x
+c)+3*b^6*cos(d*x+c)^6+(-16*a^2*b^4+10*b^6)*cos(d*x+c)^4+(128*a^4*b^2-84*a^
2*b^4-20*b^6)*cos(d*x+c)^2+256*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b
)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),
((a-b)/(a+b))^(1/2))*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*a^5*b-192*(-b/(a+
b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticF(
(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(b/(a-b)*sin(d*x+
c)+1/(a-b)*a)^(1/2)*a^4*b^2-296*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-
b)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)
,((a-b)/(a+b))^(1/2))*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*a^3*b^3+192*(-b/
(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*Ellipti
cF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(b/(a-b)*sin(d
*x+c)+1/(a-b)*a)^(1/2)*a^2*b^4+40*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(
a-b)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/
2),((a-b)/(a+b))^(1/2))*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*a*b^5-256*Elli
pticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a+b)*s
```

```
in(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(b/(a-b)*sin(d
*x+c)+1/(a-b)*a)^(1/2)*a^6+488*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/
2),((a-b)/(a+b))^(1/2))*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d
*x+c)-b/(a-b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*a^4*b^2-232*Ellip
ticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a+b)*si
n(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(b/(a-b)*sin(d*
x+c)+1/(a-b)*a)^(1/2)*a^2*b^4)/(a+b*sin(d*x+c))^(3/2)/b^7/cos(d*x+c)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+b*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^6/(b*sin(d*x + c) + a)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.25, size = 874, normalized size = 2.98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/63*(4*(sqrt(2)*(64*a^4*b^2 - 82*a^2*b^4 + 15*b^6)*cos(d*x + c)^2 - 2*sqr
t(2)*(64*a^5*b - 82*a^3*b^3 + 15*a*b^5)*sin(d*x + c) - sqrt(2)*(64*a^6 - 18
*a^4*b^2 - 67*a^2*b^4 + 15*b^6))*sqrt(I*b)*weierstrassPInverse(-4/3*(4*a^2
- 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*
b*sin(d*x + c) - 2*I*a)/b) + 4*(sqrt(2)*(64*a^4*b^2 - 82*a^2*b^4 + 15*b^6)*
cos(d*x + c)^2 - 2*sqrt(2)*(64*a^5*b - 82*a^3*b^3 + 15*a*b^5)*sin(d*x + c)
- sqrt(2)*(64*a^6 - 18*a^4*b^2 - 67*a^2*b^4 + 15*b^6))*sqrt(-I*b)*weierstra
ssPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, 1/3*
(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*I*a)/b) - 12*(sqrt(2)*(-32*I*a^3
*b^3 + 29*I*a*b^5)*cos(d*x + c)^2 + 2*sqrt(2)*(32*I*a^4*b^2 - 29*I*a^2*b^4)
*sin(d*x + c) + sqrt(2)*(32*I*a^5*b + 3*I*a^3*b^3 - 29*I*a*b^5))*sqrt(I*b)*
weierstrassZeta(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3,
weierstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b
^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) - 2*I*a)/b)) - 12*(sqrt(2)*(
32*I*a^3*b^3 - 29*I*a*b^5)*cos(d*x + c)^2 + 2*sqrt(2)*(-32*I*a^4*b^2 + 29*I
*a^2*b^4)*sin(d*x + c) + sqrt(2)*(-32*I*a^5*b - 3*I*a^3*b^3 + 29*I*a*b^5))*
sqrt(-I*b)*weierstrassZeta(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*
a*b^2)/b^3, weierstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3 +
 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*I*a)/b)) +
3*(3*b^6*cos(d*x + c)^5 - 2*(8*a^2*b^4 - 5*b^6)*cos(d*x + c)^3 + 4*(32*a^4*
```

```
b^2 - 21*a^2*b^4 - 5*b^6)*cos(d*x + c) + 2*(3*a*b^5*cos(d*x + c)^3 + 4*(20*
a^3*b^3 - 17*a*b^5)*cos(d*x + c))*sin(d*x + c))*sqrt(b*sin(d*x + c) + a))/(
b^9*d*cos(d*x + c)^2 - 2*a*b^8*d*sin(d*x + c) - (a^2*b^7 + b^9)*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6/(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+b*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^6}{{\left(a+b\,\sin\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(a + b*sin(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^6/(a + b*sin(c + d*x))^(5/2), x)
```

3.535 $\int \frac{\cos^4(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=221

$$-\frac{2\cos^3(c+dx)}{3bd(a+b\sin(c+dx))^{3/2}}-\frac{32aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)|\frac{2b}{a+b}\right)\sqrt{a+b\sin(c+dx)}}{3b^4d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}+\frac{8(4a^2-b^2)F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\right.}{3b^4d\sqrt{a+b\,\mathrm{s}}}$$

```
[Out] -2/3*cos(d*x+c)^3/b/d/(a+b*sin(d*x+c))^(3/2)-4/3*cos(d*x+c)*(4*a+b*sin(d*x+
c))/b^3/d/(a+b*sin(d*x+c))^(1/2)+32/3*a*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)
/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(
a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/b^4/d/((a+b*sin(d*x+c))/(a+b))^(1/2)-8/
3*(4*a^2-b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)
*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x
+c))/(a+b))^(1/2)/b^4/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {2772, 2942, 2831, 2742, 2740, 2734, 2732}

$$\frac{8(4a^2-b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{3b^4d\sqrt{a+b\sin(c+dx)}}-\frac{32a\sqrt{a+b\sin(c+dx)}E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{3b^4d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}-\frac{4\cos(c+dx)(4a+b\sin(c+dx))}{3b^3d\sqrt{a+b\sin(c+dx)}}-\frac{2\cos^3(c+dx)}{3bd(a+b\sin(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*Cos[c + d*x]^3)/(3*b*d*(a + b*Sin[c + d*x])^(3/2)) - (32*a*EllipticE[(c
 - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(3*b^4*d*Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]) + (8*(4*a^2 - b^2)*EllipticF[(c - Pi/2 + d*x)/
2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(3*b^4*d*Sqrt[a + b*S
in[c + d*x]]) - (4*Cos[c + d*x]*(4*a + b*Sin[c + d*x]))/(3*b^3*d*Sqrt[a + b
*Sin[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
```

```
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2772

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Dist[g^2*((p - 1)/(b*(m + 1))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2942

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*C
os[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x])/(b^2*f*(m + 1)*(m + p + 1))), x] + Dist[g^2*((
p - 1)/(b^2*(m + 1)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx = -\frac{2\cos^3(c+dx)}{3bd(a+b\sin(c+dx))^{3/2}} - \frac{2\int \frac{\cos^2(c+dx)\sin(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx}{b}$$

$$= -\frac{2\cos^3(c+dx)}{3bd(a+b\sin(c+dx))^{3/2}} - \frac{4\cos(c+dx)(4a+b\sin(c+dx))}{3b^3d\sqrt{a+b\sin(c+dx)}} + \frac{8\int \frac{-\frac{b}{2}}{\sqrt{a+}}}{}$$

$$= -\frac{2\cos^3(c+dx)}{3bd(a+b\sin(c+dx))^{3/2}} - \frac{4\cos(c+dx)(4a+b\sin(c+dx))}{3b^3d\sqrt{a+b\sin(c+dx)}} - \frac{(16a)\int \sqrt{}}{}$$

$$= -\frac{2\cos^3(c+dx)}{3bd(a+b\sin(c+dx))^{3/2}} - \frac{4\cos(c+dx)(4a+b\sin(c+dx))}{3b^3d\sqrt{a+b\sin(c+dx)}} - \frac{\left(16a\sqrt{a-}\right.}{}$$

$$= -\frac{2\cos^3(c+dx)}{3bd(a+b\sin(c+dx))^{3/2}} - \frac{32aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\frac{2b}{a+b}\right)\sqrt{a+b\sin(c+dx)}}{3b^4d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}$$

Mathematica [A]
time = 1.03, size = 174, normalized size = 0.79

$$\frac{32a(a+b)^2E\left(\frac{1}{4}(-2c+\pi-2dx)\middle|\frac{2b}{a+b}\right)\left(\frac{a+b\sin(c+dx)}{a+b}\right)^{3/2} - 8(a+b)\left(4a^2-b^2\right)F\left(\frac{1}{4}(-2c+\pi-2dx)\middle|\frac{2b}{a+b}\right)\left(\frac{a+b\sin(c+dx)}{a+b}\right)^{3/2} + b\cos(c+dx)\left(-16a^2-3b^2+b^2\cos(2(c+dx))-20ab\sin(c+dx)\right)}{3b^4d(a+b\sin(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (32*a*(a + b)^2*EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*((a + b*Sin
[c + d*x])/(a + b))^(3/2) - 8*(a + b)*(4*a^2 - b^2)*EllipticF[(-2*c + Pi -
2*d*x)/4, (2*b)/(a + b)]*((a + b*Sin[c + d*x])/(a + b))^(3/2) + b*Cos[c + d
*x]*(-16*a^2 - 3*b^2 + b^2*Cos[2*(c + d*x)] - 20*a*b*Sin[c + d*x]))/(3*b^4*
d*(a + b*Sin[c + d*x])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1045 vs. $2(267) = 534$.
time = 2.24, size = 1046, normalized size = 4.73

method	result

default	$-\frac{2\left(10a\,b^{3}\left(\cos^{2}(dx+c)\right)\sin(dx+c)-4\sqrt{-\frac{b\sin(dx+c)}{a-b}-\frac{b}{a-b}}\,\sqrt{-\frac{b\sin(dx+c)}{a+b}+\frac{b}{a+b}}\,\sqrt{\frac{b\sin(dx+c)}{a-b}+\frac{a}{a-b}}\,b\left(4\,\mathrm{Elliptic}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+b*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(10*a*b^3*cos(d*x+c)^2*sin(d*x+c)-4*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2
)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*
b*(4*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^
3-4*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a*b
^2-4*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^
2*b+3*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a
*b^2+EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*b^
3)*sin(d*x+c)-b^4*cos(d*x+c)^4+(8*a^2*b^2+2*b^4)*cos(d*x+c)^2+16*(-b/(a+b)*
sin(d*x+c)+b/(a+b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*EllipticF((b
/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a-b)*sin(d*x+c
)-b/(a-b))^(1/2)*a^3*b-12*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(b/(a-b)*sin(
d*x+c)+1/(a-b)*a)^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-
b)/(a+b))^(1/2))*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*a^2*b^2-4*(-b/(a+b)*si
n(d*x+c)+b/(a+b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*EllipticF((b/(
a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a-b)*sin(d*x+c)-
b/(a-b))^(1/2)*a*b^3-16*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*EllipticE((b/(a
-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(b/(a-b)*sin(d*x+c)+1/
(a-b)*a)^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*a^4+16*(-b/(a+b)*sin(d*x
+c)+b/(a+b))^(1/2)*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a
+b))^(1/2))*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-
b))^(1/2)*a^2*b^2)/(a+b*sin(d*x+c))^(3/2)/b^5/cos(d*x+c)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^4/(b*sin(d*x + c) + a)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.17, size = 705, normalized size = 3.19

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4/(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")

[Out] 2/9*(2*(sqrt(2)*(8*a^2*b^2 - 3*b^4)*cos(d*x + c)^2 - 2*sqrt(2)*(8*a^3*b - 3*a*b^3)*sin(d*x + c) - sqrt(2)*(8*a^4 + 5*a^2*b^2 - 3*b^4))*sqrt(I*b)*weierstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) - 2*I*a)/b) + 2*(sqrt(2)*(8*a^2*b^2 - 3*b^4)*cos(d*x + c)^2 - 2*sqrt(2)*(8*a^3*b - 3*a*b^3)*sin(d*x + c) - sqrt(2)*(8*a^4 + 5*a^2*b^2 - 3*b^4))*sqrt(-I*b)*weierstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*I*a)/b) + 24*(I*sqrt(2)*a*b^3*cos(d*x + c)^2 - 2*I*sqrt(2)*a^2*b^2*sin(d*x + c) + sqrt(2)*(-I*a^3*b - I*a*b^3))*sqrt(I*b)*weierstrassZeta(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, weierstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) - 2*I*a)/b)) + 24*(-I*sqrt(2)*a*b^3*cos(d*x + c)^2 + 2*I*sqrt(2)*a^2*b^2*sin(d*x + c) + sqrt(2)*(I*a^3*b + I*a*b^3))*sqrt(-I*b)*weierstrassZeta(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, weierstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*I*a)/b)) - 3*(b^4*cos(d*x + c)^3 - 10*a*b^3*cos(d*x + c)*sin(d*x + c) - 2*(4*a^2*b^2 + b^4)*cos(d*x + c))*sqrt(b*sin(d*x + c) + a))/(b^7*d*cos(d*x + c)^2 - 2*a*b^6*d*sin(d*x + c) - (a^2*b^5 + b^7)*d)

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^4(c+dx)}{(a+b\sin(c+dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**4/(a+b*sin(d*x+c))**(5/2),x)

[Out] Integral(cos(c + d*x)**4/(a + b*sin(c + d*x))**(5/2), x)

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4/(a+b*sin(d*x+c))^(5/2),x, algorithm="giac")

[Out] Timed out

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^4}{(a+b\,\sin(c+d\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(a + b*sin(c + d*x))^(5/2),x)
[Out] int(cos(c + d*x)^4/(a + b*sin(c + d*x))^(5/2), x)
```

3.536 $\int \frac{\cos^2(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=219

$$-\frac{2\cos(c+dx)}{3bd(a+b\sin(c+dx))^{3/2}}+\frac{4a\cos(c+dx)}{3b\left(a^2-b^2\right)d\sqrt{a+b\sin(c+dx)}}+\frac{4aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\frac{2b}{a+b}\right)\sqrt{a+b\sin(c+d}}{3b^2\left(a^2-b^2\right)d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}$$

```
[Out] -2/3*cos(d*x+c)/b/d/(a+b*sin(d*x+c))^(3/2)+4/3*a*cos(d*x+c)/b/(a^2-b^2)/d/(
a+b*sin(d*x+c))^(1/2)-4/3*a*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1
/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))
*(a+b*sin(d*x+c))^(1/2)/b^2/(a^2-b^2)/d/((a+b*sin(d*x+c))/(a+b))^(1/2)+4/3*
(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos
(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1
/2)/b^2/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {2772, 2833, 2831, 2742, 2740, 2734, 2732}

$$\frac{4a\cos(c+dx)}{3bd\left(a^2-b^2\right)\sqrt{a+b\sin(c+dx)}}+\frac{4a\sqrt{a+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|\frac{2b}{a+b}\right)}{3b^2d\left(a^2-b^2\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}-\frac{4\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|\frac{2b}{a+b}\right)}{3b^2d\sqrt{a+b\sin(c+dx)}}-\frac{2\cos(c+dx)}{3bd(a+b\sin(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*Cos[c + d*x])/(3*b*d*(a + b*Sin[c + d*x])^(3/2)) + (4*a*Cos[c + d*x])/(
3*b*(a^2 - b^2)*d*Sqrt[a + b*Sin[c + d*x]]) + (4*a*EllipticE[(c - Pi/2 + d*
x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(3*b^2*(a^2 - b^2)*d*Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]) - (4*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a +
b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(3*b^2*d*Sqrt[a + b*Sin[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2772

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Dist[g^2*((p - 1)/(b*(m + 1))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx = -\frac{2\cos(c+dx)}{3bd(a+b\sin(c+dx))^{3/2}} - \frac{2\int \frac{\sin(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx}{3b}$$

$$= -\frac{2\cos(c+dx)}{3bd(a+b\sin(c+dx))^{3/2}} + \frac{4a\cos(c+dx)}{3b(a^2-b^2)\,d\sqrt{a+b\sin(c+dx)}} + \frac{4\int \frac{\frac{b}{2}+}{\sqrt{a+}}}{3b}$$

$$= -\frac{2\cos(c+dx)}{3bd(a+b\sin(c+dx))^{3/2}} + \frac{4a\cos(c+dx)}{3b(a^2-b^2)\,d\sqrt{a+b\sin(c+dx)}} - \frac{2\int \frac{}{\sqrt{a+}}}{}$$

$$= -\frac{2\cos(c+dx)}{3bd(a+b\sin(c+dx))^{3/2}} + \frac{4a\cos(c+dx)}{3b(a^2-b^2)\,d\sqrt{a+b\sin(c+dx)}} + \frac{\left(2a\sqrt{a+}\right.}{}$$

$$= -\frac{2\cos(c+dx)}{3bd(a+b\sin(c+dx))^{3/2}} + \frac{4a\cos(c+dx)}{3b(a^2-b^2)\,d\sqrt{a+b\sin(c+dx)}} + \frac{4aE\left(\frac{1}{2}(c\right.}{3b^2(}$$

Mathematica [A]
time = 1.05, size = 167, normalized size = 0.76

$$\frac{-4a(a+b)^2E\left(\frac{1}{4}(-2c+\pi-2dx)|\frac{2b}{a+b}\right)\left(\frac{a+b\sin(c+dx)}{a+b}\right)^{3/2}+4(a-b)(a+b)^2F\left(\frac{1}{4}(-2c+\pi-2dx)|\frac{2b}{a+b}\right)\left(\frac{a+b\sin(c+dx)}{a+b}\right)^{3/2}+2b\cos(c+dx)\left(a^2+b^2+2ab\sin(c+dx)\right)}{3(a-b)b^2(a+b)d(a+b\sin(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-4*a*(a + b)^2*EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*((a + b*Sin
[c + d*x])/(a + b))^(3/2) + 4*(a - b)*(a + b)^2*EllipticF[(-2*c + Pi - 2*d*
x)/4, (2*b)/(a + b)]*((a + b*Sin[c + d*x])/(a + b))^(3/2) + 2*b*Cos[c + d*x
]*(a^2 + b^2 + 2*a*b*Sin[c + d*x]))/(3*(a - b)*b^2*(a + b)*d*(a + b*Sin[c +
 d*x])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 863 vs. $2(265) = 530$.
time = 2.64, size = 864, normalized size = 3.95

method	result

default	$\frac{4a\,b^3\left(\cos^2(dx+c)\right)\sin(dx+c)}{3} - 4\sqrt{\frac{b\sin(dx+c)}{a-b}+\frac{a}{a-b}}\,\sqrt{-\frac{b\sin(dx+c)}{a+b}+\frac{b}{a+b}}\,\sqrt{-\frac{b\sin(dx+c)}{a-b}-\frac{b}{a-b}}\,b\left(\text{EllipticE}\left(\sqrt{\frac{b\sin(}{a}}\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+b*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(2*a*b^3*cos(d*x+c)^2*sin(d*x+c)-2*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)
*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*b*
(EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^3-El
lipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a*b^2-Ell
ipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*a^2*b+Elli
pticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*b^3)*sin(d*
x+c)+(a^2*b^2+b^4)*cos(d*x+c)^2+2*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(b/(a
-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1
/2),((a-b)/(a+b))^(1/2))*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*a^3*b-2*(-b/(a
+b)*sin(d*x+c)+b/(a+b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*Elliptic
F((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a-b)*sin(d
*x+c)-b/(a-b))^(1/2)*a*b^3-2*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*EllipticE(
(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(b/(a-b)*sin(d*x+
c)+1/(a-b)*a)^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*a^4+2*(-b/(a+b)*sin
(d*x+c)+b/(a+b))^(1/2)*EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b
)/(a+b))^(1/2))*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a-b)*sin(d*x+c)-b
/(a-b))^(1/2)*a^2*b^2)/(a^2-b^2)/(a+b*sin(d*x+c))^(3/2)/b^3/cos(d*x+c)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^2/(b*sin(d*x + c) + a)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 711, normalized size = 3.25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/9*((sqrt(2)*(2*a^2*b^2 - 3*b^4)*cos(d*x + c)^2 - 2*sqrt(2)*(2*a^3*b - 3*
a*b^3)*sin(d*x + c) - sqrt(2)*(2*a^4 - a^2*b^2 - 3*b^4))*sqrt(I*b)*weierstr
assPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, 1/3*
(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) - 2*I*a)/b) + (sqrt(2)*(2*a^2*b^2 -
3*b^4)*cos(d*x + c)^2 - 2*sqrt(2)*(2*a^3*b - 3*a*b^3)*sin(d*x + c) - sqrt(2
)*(2*a^4 - a^2*b^2 - 3*b^4))*sqrt(-I*b)*weierstrassPInverse(-4/3*(4*a^2 - 3
*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*
sin(d*x + c) + 2*I*a)/b) + 3*(I*sqrt(2)*a*b^3*cos(d*x + c)^2 - 2*I*sqrt(2)*
a^2*b^2*sin(d*x + c) + sqrt(2)*(-I*a^3*b - I*a*b^3))*sqrt(I*b)*weierstrassZ
eta(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, weierstrassP
Inverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, 1/3*(3*b
*cos(d*x + c) - 3*I*b*sin(d*x + c) - 2*I*a)/b)) + 3*(-I*sqrt(2)*a*b^3*cos(d
*x + c)^2 + 2*I*sqrt(2)*a^2*b^2*sin(d*x + c) + sqrt(2)*(I*a^3*b + I*a*b^3))
*sqrt(-I*b)*weierstrassZeta(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I
*a*b^2)/b^3, weierstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3
+ 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*I*a)/b)) +
 3*(2*a*b^3*cos(d*x + c)*sin(d*x + c) + (a^2*b^2 + b^4)*cos(d*x + c))*sqrt(
b*sin(d*x + c) + a))/((a^2*b^5 - b^7)*d*cos(d*x + c)^2 - 2*(a^3*b^4 - a*b^6
)*d*sin(d*x + c) - (a^4*b^3 - b^7)*d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2{(c + dx)}}{(a + b\sin{(c + dx)})^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Integral(cos(c + d*x)**2/(a + b*sin(c + d*x))**(5/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\cos\left(c+d\,x\right)}^2}{{\left(a+b\,\sin\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a + b*sin(c + d*x))^(5/2),x)
[Out] int(cos(c + d*x)^2/(a + b*sin(c + d*x))^(5/2), x)
```

3.537 $\int \frac{\sec^2(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=325

$$\frac{2b\sec(c+dx)}{3(a^2-b^2)\,d(a+b\sin(c+dx))^{3/2}}+\frac{16ab\sec(c+dx)}{3(a^2-b^2)^2\,d\sqrt{a+b\sin(c+dx)}}-\frac{a(3a^2+29b^2)\,E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)|\frac{2b}{a+b}\right.}{3(a^2-b^2)^3\,d\sqrt{\frac{a+b\sin}{a}}}$$

```
[Out] 2/3*b*sec(d*x+c)/(a^2-b^2)/d/(a+b*sin(d*x+c))^(3/2)+16/3*a*b*sec(d*x+c)/(a^
2-b^2)^2/d/(a+b*sin(d*x+c))^(1/2)-1/3*sec(d*x+c)*(b*(27*a^2+5*b^2)-a*(3*a^2
+29*b^2)*sin(d*x+c))*(a+b*sin(d*x+c))^(1/2)/(a^2-b^2)^3/d+1/3*a*(3*a^2+29*b
^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE
(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/
(a^2-b^2)^3/d/((a+b*sin(d*x+c))/(a+b))^(1/2)-1/3*(3*a^2+5*b^2)*(sin(1/2*c+1
/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*P
i+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/(a^2-b^2
)^2/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A]
time = 0.39, antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {2773, 2943, 2945, 2831, 2742, 2740, 2734, 2732}

$$-\frac{\sec(c+dx)\sqrt{a+b\sin(c+dx)}\,(b(27a^2+5b^2)-a(3a^2+29b^2)\sin(c+dx))}{3d(a^2-b^2)^3}+\frac{16ab\sec(c+dx)}{3d(a^2-b^2)^2\sqrt{a+b\sin(c+dx)}}+\frac{2b\sec(c+dx)}{3d(a^2-b^2)(a+b\sin(c+dx))^{3/2}}+\frac{(3a^2+5b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{3d(a^2-b^2)^2\sqrt{a+b\sin(c+dx)}}-\frac{a(3a^2+29b^2)\sqrt{a+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{3d(a^2-b^2)^3\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (2*b*Sec[c + d*x])/(3*(a^2 - b^2)*d*(a + b*Sin[c + d*x])^(3/2)) + (16*a*b*S
ec[c + d*x])/(3*(a^2 - b^2)^2*d*Sqrt[a + b*Sin[c + d*x]]) - (a*(3*a^2 + 29*
b^2)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])
/(3*(a^2 - b^2)^3*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + ((3*a^2 + 5*b^2)*
EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a +
 b)])/(3*(a^2 - b^2)^2*d*Sqrt[a + b*Sin[c + d*x]]) - (Sec[c + d*x]*Sqrt[a +
 b*Sin[c + d*x]]*(b*(27*a^2 + 5*b^2) - a*(3*a^2 + 29*b^2)*Sin[c + d*x]))/(3
*(a^2 - b^2)^3*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2773

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m + 1)/(f*g*(a^2 - b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1))
, Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p
 + 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^
2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2943

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c
 - a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^(m + 1)/(f*g*(a^2 -
 b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*
(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p
+ 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[
a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2945

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx = \frac{2b\sec(c+dx)}{3\left(a^2-b^2\right)d(a+b\sin(c+dx))^{3/2}} - \frac{2\int \frac{\sec^2(c+dx)\left(-\frac{3a}{2}+\frac{5}{2}b\sin(c+dx)\right)}{(a+b\sin(c+dx))^{3/2}}\,dx}{3\left(a^2-b^2\right)}$$

$$= \frac{2b\sec(c+dx)}{3\left(a^2-b^2\right)d(a+b\sin(c+dx))^{3/2}} + \frac{16ab\sec(c+dx)}{3\left(a^2-b^2\right)^2 d\sqrt{a+b\sin(c+dx)}} + \frac{4\int}{}$$

$$= \frac{2b\sec(c+dx)}{3\left(a^2-b^2\right)d(a+b\sin(c+dx))^{3/2}} + \frac{16ab\sec(c+dx)}{3\left(a^2-b^2\right)^2 d\sqrt{a+b\sin(c+dx)}} - \frac{\sec}{}$$

$$= \frac{2b\sec(c+dx)}{3\left(a^2-b^2\right)d(a+b\sin(c+dx))^{3/2}} + \frac{16ab\sec(c+dx)}{3\left(a^2-b^2\right)^2 d\sqrt{a+b\sin(c+dx)}} - \frac{\sec}{}$$

$$= \frac{2b\sec(c+dx)}{3\left(a^2-b^2\right)d(a+b\sin(c+dx))^{3/2}} + \frac{16ab\sec(c+dx)}{3\left(a^2-b^2\right)^2 d\sqrt{a+b\sin(c+dx)}} - \frac{\sec}{}$$

$$= \frac{2b\sec(c+dx)}{3\left(a^2-b^2\right)d(a+b\sin(c+dx))^{3/2}} + \frac{16ab\sec(c+dx)}{3\left(a^2-b^2\right)^2 d\sqrt{a+b\sin(c+dx)}} - \frac{a(3}{}$$

Mathematica [A]
time = 1.92, size = 241, normalized size = 0.74

$$\frac{\frac{\left(\left(3a^3+29ab^2\right)E\left(\frac{1}{4}(-2c+\pi-2dx)\middle|\frac{2b}{a+b}\right)+\left(-3a^3+3a^2b-5ab^2+5b^3\right)F\left(\frac{1}{4}(-2c+\pi-2dx)\middle|\frac{2b}{a+b}\right)\right)\left(\frac{a+b\sin(c+dx)}{a+b}\right)^{3/2}}{(a-b)^3(a+b)} - \frac{2b^3\left(a^2-b^2\right)\cos(c+dx)+20ab^3\cos(c+dx)(a+b\sin(c+dx))+3\sec(c+dx)(a+b\sin(c+dx))^2\left(3a^2b+b^3-a\left(a^2+3b^2\right)\sin(c+dx)\right)}{\left(a^2-b^2\right)^3}}{3d(a+b\sin(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] ((((3*a^3 + 29*a*b^2)*EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)] + (-3
*a^3 + 3*a^2*b - 5*a*b^2 + 5*b^3)*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a
 + b)])*((a + b*Sin[c + d*x])/(a + b))^(3/2))/((a - b)^3*(a + b)) - (2*b^3*
(a^2 - b^2)*Cos[c + d*x] + 20*a*b^3*Cos[c + d*x]*(a + b*Sin[c + d*x]) + 3*S
ec[c + d*x]*(a + b*Sin[c + d*x])^2*(3*a^2*b + b^3 - a*(a^2 + 3*b^2)*Sin[c +
 d*x]))/(a^2 - b^2)^3)/(3*d*(a + b*Sin[c + d*x])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1652 vs. $2(367) = 734$.
time = 12.44, size = 1653, normalized size = 5.09

method	result	size
default	Expression too large to display	1653

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+b*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-(-a-b*sin(d*x+c))*cos(d*x+c)^2)^(1/2)*(1/2/(a+b)^3/b/cos(d*x+c)^2/(a+b*si
n(d*x+c))*(b*cos(d*x+c)^2*sin(d*x+c)+a*cos(d*x+c)^2)^(1/2)*(EllipticE((b/(a
-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a-b)*sin(d*x+c)-b
/(a-b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(
a+b))^(1/2)*a^2-EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b)
)^(1/2))*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)
^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*b^2-b^2*cos(d*x+c)^2+a*b*sin(d*x
+c)+b^2*sin(d*x+c)+a*b+b^2)-b^2/(a-b)/(a+b)*(2/3/b/(a^2-b^2)*(-(-a-b*sin(d*
x+c))*cos(d*x+c)^2)^(1/2)/(sin(d*x+c)+a/b)^2+8/3*b*cos(d*x+c)^2/(a^2-b^2)^2
*a/(-(-a-b*sin(d*x+c))*cos(d*x+c)^2)^(1/2)+2*(3*a^2+b^2)/(3*a^4-6*a^2*b^2+3
*b^4)*(a/b-1)*((a+b*sin(d*x+c))/(a-b))^(1/2)*(b*(1-sin(d*x+c))/(a+b))^(1/2)
*((-1-sin(d*x+c))*b/(a-b))^(1/2)/(-(-a-b*sin(d*x+c))*cos(d*x+c)^2)^(1/2)*El
lipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))+8/3*a*b/(a^2-b^
2)^2*(a/b-1)*((a+b*sin(d*x+c))/(a-b))^(1/2)*(b*(1-sin(d*x+c))/(a+b))^(1/2)*
((-1-sin(d*x+c))*b/(a-b))^(1/2)/(-(-a-b*sin(d*x+c))*cos(d*x+c)^2)^(1/2)*((-
a/b-1)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))+Ellipt
icF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))))-2*a*b^2/(a+b)^2/(
a-b)^2*(2*b*cos(d*x+c)^2/(a^2-b^2)/(-(-a-b*sin(d*x+c))*cos(d*x+c)^2)^(1/2)+
2*a/(a^2-b^2)*(a/b-1)*((a+b*sin(d*x+c))/(a-b))^(1/2)*(b*(1-sin(d*x+c))/(a+b
))^(1/2)*((-1-sin(d*x+c))*b/(a-b))^(1/2)/(-(-a-b*sin(d*x+c))*cos(d*x+c)^2)^
(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))+2*b/(a^
2-b^2)*(a/b-1)*((a+b*sin(d*x+c))/(a-b))^(1/2)*(b*(1-sin(d*x+c))/(a+b))^(1/2
)*((-1-sin(d*x+c))*b/(a-b))^(1/2)/(-(-a-b*sin(d*x+c))*cos(d*x+c)^2)^(1/2)*(
(-a/b-1)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))+Elli
pticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))))+1/2/(a-b)^2*(-(
-sin(d*x+c)^2*b-a*sin(d*x+c)+b*sin(d*x+c)+a)/(a-b)/((-a-b*sin(d*x+c))*(sin(
d*x+c)-1)*(1+sin(d*x+c)))^(1/2)-2*b/(2*a-2*b)*(a/b-1)*((a+b*sin(d*x+c))/(a-
b))^(1/2)*(b*(1-sin(d*x+c))/(a+b))^(1/2)*((-1-sin(d*x+c))*b/(a-b))^(1/2)/(-
```

```
(-a-b*sin(d*x+c))*cos(d*x+c)^2)^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1
/2),((a-b)/(a+b))^(1/2))-b/(a-b)*(a/b-1)*((a+b*sin(d*x+c))/(a-b))^(1/2)*(b*
(1-sin(d*x+c))/(a+b))^(1/2)*((-1-sin(d*x+c))*b/(a-b))^(1/2)/(-(-a-b*sin(d*x
+c))*cos(d*x+c)^2)^(1/2)*((-a/b-1)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2)
,((a-b)/(a+b))^(1/2))+EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b)
)^(1/2)))))/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sin(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate(sec(d*x + c)^2/(b*sin(d*x + c) + a)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.18, size = 993, normalized size = 3.06

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
[Out] 1/18*((sqrt(2)*(6*a^4*b^2 - 23*a^2*b^4 - 15*b^6)*cos(d*x + c)^3 - 2*sqrt(2)
*(6*a^5*b - 23*a^3*b^3 - 15*a*b^5)*cos(d*x + c)*sin(d*x + c) - sqrt(2)*(6*a
^6 - 17*a^4*b^2 - 38*a^2*b^4 - 15*b^6)*cos(d*x + c))*sqrt(I*b)*weierstrassP
Inverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, 1/3*(3*b
*cos(d*x + c) - 3*I*b*sin(d*x + c) - 2*I*a)/b) + (sqrt(2)*(6*a^4*b^2 - 23*a
^2*b^4 - 15*b^6)*cos(d*x + c)^3 - 2*sqrt(2)*(6*a^5*b - 23*a^3*b^3 - 15*a*b^
5)*cos(d*x + c)*sin(d*x + c) - sqrt(2)*(6*a^6 - 17*a^4*b^2 - 38*a^2*b^4 - 1
5*b^6)*cos(d*x + c))*sqrt(-I*b)*weierstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^
2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x
+ c) + 2*I*a)/b) - 3*(sqrt(2)*(-3*I*a^3*b^3 - 29*I*a*b^5)*cos(d*x + c)^3 +
2*sqrt(2)*(3*I*a^4*b^2 + 29*I*a^2*b^4)*cos(d*x + c)*sin(d*x + c) + sqrt(2)*
(3*I*a^5*b + 32*I*a^3*b^3 + 29*I*a*b^5)*cos(d*x + c))*sqrt(I*b)*weierstrass
Zeta(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, weierstrass
PInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, 1/3*(3*
b*cos(d*x + c) - 3*I*b*sin(d*x + c) - 2*I*a)/b)) - 3*(sqrt(2)*(3*I*a^3*b^3
+ 29*I*a*b^5)*cos(d*x + c)^3 + 2*sqrt(2)*(-3*I*a^4*b^2 - 29*I*a^2*b^4)*cos(
d*x + c)*sin(d*x + c) + sqrt(2)*(-3*I*a^5*b - 32*I*a^3*b^3 - 29*I*a*b^5)*co
s(d*x + c))*sqrt(-I*b)*weierstrassZeta(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-8*
I*a^3 + 9*I*a*b^2)/b^3, weierstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27
*(-8*I*a^3 + 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2
*I*a)/b)) + 6*(3*a^4*b^2 - 6*a^2*b^4 + 3*b^6 + (6*a^4*b^2 + 31*a^2*b^4 - 5*
```

```
b^6)*cos(d*x + c)^2 - (3*a^5*b - 6*a^3*b^3 + 3*a*b^5 - (3*a^3*b^3 + 29*a*b^
5)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(b*sin(d*x + c) + a))/((a^6*b^3 - 3*a^
4*b^5 + 3*a^2*b^7 - b^9)*d*cos(d*x + c)^3 - 2*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*
b^6 - a*b^8)*d*cos(d*x + c)*sin(d*x + c) - (a^8*b - 2*a^6*b^3 + 2*a^2*b^7 -
 b^9)*d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(c+dx)}{(a+b\sin(c+dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Integral(sec(c + d*x)**2/(a + b*sin(c + d*x))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^2/(b*sin(d*x + c) + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos(c+d\,x)^2\,(a+b\,\sin(c+d\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + b*sin(c + d*x))^(5/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^2*(a + b*sin(c + d*x))^(5/2)), x)
```

3.538 $\int \frac{\sec^4(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=425

$$\frac{2b\sec^3(c+dx)}{3\left(a^2-b^2\right)d(a+b\sin(c+dx))^{3/2}}+\frac{8ab\sec^3(c+dx)}{\left(a^2-b^2\right)^2 d\sqrt{a+b\sin(c+dx)}}-\frac{2a\left(a^4-6a^2b^2-27b^4\right)E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+d\right.\right.}{3\left(a^2-b^2\right)^4 d\sqrt{\frac{a+b}{}}}$$

```
[Out] 2/3*b*sec(d*x+c)^3/(a^2-b^2)/d/(a+b*sin(d*x+c))^(3/2)+8*a*b*sec(d*x+c)^3/(a
^2-b^2)^2/d/(a+b*sin(d*x+c))^(1/2)-1/3*sec(d*x+c)^3*(b*(29*a^2+3*b^2)-a*(a^
2+31*b^2)*sin(d*x+c))*(a+b*sin(d*x+c))^(1/2)/(a^2-b^2)^3/d-1/6*sec(d*x+c)*(
b*(a^4-114*a^2*b^2-15*b^4)-4*a*(a^4-6*a^2*b^2-27*b^4)*sin(d*x+c))*(a+b*sin(
d*x+c))^(1/2)/(a^2-b^2)^4/d+2/3*a*(a^4-6*a^2*b^2-27*b^4)*(sin(1/2*c+1/4*Pi+
1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*
d*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/(a^2-b^2)^4/d/((a+b*si
n(d*x+c))/(a+b))^(1/2)-1/6*(4*a^4-21*a^2*b^2-15*b^4)*(sin(1/2*c+1/4*Pi+1/2*
d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x)
,2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/(a^2-b^2)^3/d/(a+b
*sin(d*x+c))^(1/2)
```

Rubi [A]
time = 0.56, antiderivative size = 425, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {2773, 2943, 2945, 2831, 2742, 2740, 2734, 2732}

$$-\frac{\sec^3(c+dx)\sqrt{a+b\sin(c+dx)}\left(b\left(29a^2+3b^2\right)-a\left(a^2+31b^2\right)\sin(c+dx)\right)}{3d\left(a^2-b^2\right)^3}+\frac{8ab\sec^3(c+dx)}{d\left(a^2-b^2\right)^2\sqrt{a+b\sin(c+dx)}}+\frac{2b\sec^3(c+dx)}{3d\left(a^2-b^2\right)(a+b\sin(c+dx))^{3/2}}-\frac{\sec(c+dx)\sqrt{a+b\sin(c+dx)}\left(b\left(a^4-114a^2b^2-15b^4\right)-4a\left(a^4-6a^2b^2-27b^4\right)\sin(c+dx)\right)}{6d\left(a^2-b^2\right)^4}+\frac{\left(4a^4-21a^2b^2-15b^4\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{6d\left(a^2-b^2\right)^3\sqrt{a+b\sin(c+dx)}}-\frac{2a\left(a^4-6a^2b^2-27b^4\right)\sqrt{a+b\sin(c+dx)}E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{3d\left(a^2-b^2\right)^4\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (2*b*Sec[c + d*x]^3)/(3*(a^2 - b^2)*d*(a + b*Sin[c + d*x])^(3/2)) + (8*a*b*
Sec[c + d*x]^3)/((a^2 - b^2)^2*d*Sqrt[a + b*Sin[c + d*x]]) - (2*a*(a^4 - 6*
a^2*b^2 - 27*b^4)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*S
in[c + d*x]])/(3*(a^2 - b^2)^4*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + ((4*
a^4 - 21*a^2*b^2 - 15*b^4)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqr
t[(a + b*Sin[c + d*x])/(a + b)])/(6*(a^2 - b^2)^3*d*Sqrt[a + b*Sin[c + d*x]
]) - (Sec[c + d*x]^3*Sqrt[a + b*Sin[c + d*x]]*(b*(29*a^2 + 3*b^2) - a*(a^2
+ 31*b^2)*Sin[c + d*x]))/(3*(a^2 - b^2)^3*d) - (Sec[c + d*x]*Sqrt[a + b*Sin
[c + d*x]]*(b*(a^4 - 114*a^2*b^2 - 15*b^4) - 4*a*(a^4 - 6*a^2*b^2 - 27*b^4)
*Sin[c + d*x]))/(6*(a^2 - b^2)^4*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2773

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m + 1)/(f*g*(a^2 - b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1))
, Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p
 + 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^
2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2943

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c
 - a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^(m + 1)/(f*g*(a^2 -
 b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*
(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p
+ 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[
```

```
a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2945

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx = \frac{2b\sec^3(c+dx)}{3(a^2-b^2)\,d(a+b\sin(c+dx))^{3/2}} - \frac{2\int \frac{\sec^4(c+dx)\left(-\frac{3a}{2}+\frac{9}{2}b\sin(c+dx)\right)}{(a+b\sin(c+dx))^{3/2}}\,dx}{3(a^2-b^2)}$$

$$= \frac{2b\sec^3(c+dx)}{3(a^2-b^2)\,d(a+b\sin(c+dx))^{3/2}} + \frac{8ab\sec^3(c+dx)}{(a^2-b^2)^2\,d\sqrt{a+b\sin(c+dx)}} + \frac{4\int \ldots}{\ldots}$$

$$= \frac{2b\sec^3(c+dx)}{3(a^2-b^2)\,d(a+b\sin(c+dx))^{3/2}} + \frac{8ab\sec^3(c+dx)}{(a^2-b^2)^2\,d\sqrt{a+b\sin(c+dx)}} - \frac{\sec^3(\ldots}{\ldots}$$

$$= \frac{2b\sec^3(c+dx)}{3(a^2-b^2)\,d(a+b\sin(c+dx))^{3/2}} + \frac{8ab\sec^3(c+dx)}{(a^2-b^2)^2\,d\sqrt{a+b\sin(c+dx)}} - \frac{\sec^3(\ldots}{\ldots}$$

$$= \frac{2b\sec^3(c+dx)}{3(a^2-b^2)\,d(a+b\sin(c+dx))^{3/2}} + \frac{8ab\sec^3(c+dx)}{(a^2-b^2)^2\,d\sqrt{a+b\sin(c+dx)}} - \frac{\sec^3(\ldots}{\ldots}$$

$$= \frac{2b\sec^3(c+dx)}{3(a^2-b^2)\,d(a+b\sin(c+dx))^{3/2}} + \frac{8ab\sec^3(c+dx)}{(a^2-b^2)^2\,d\sqrt{a+b\sin(c+dx)}} - \frac{\sec^3(\ldots}{\ldots}$$

$$= \frac{2b\sec^3(c+dx)}{3(a^2-b^2)\,d(a+b\sin(c+dx))^{3/2}} + \frac{8ab\sec^3(c+dx)}{(a^2-b^2)^2\,d\sqrt{a+b\sin(c+dx)}} - \frac{2a(a\ldots}{\ldots}$$

Mathematica [A]

time = 2.50, size = 341, normalized size = 0.80

$$\frac{\frac{\left(4\left(a^5-6a^3b^2-27ab^4\right)E\left(\frac{1}{4}(-2c+\pi-2dx)\middle|\frac{2b}{a+b}\right)+\left(-4a^5+4a^4b+21a^3b^2-21a^2b^3+15ab^4-15b^5\right)F\left(\frac{1}{4}(-2c+\pi-2dx)\middle|\frac{2b}{a+b}\right)\right)\left(\frac{a+b\sin(c+dx)}{a+b}\right)^{3/2}}{(a-b)^4(a+b)^2}+\frac{4b^5\left(a^2-b^2\right)\cos(c+dx)+64ab^5\cos(c+dx)(a+b\sin(c+dx))+2\left(a^2-b^2\right)\sec^3(c+dx)(a+b\sin(c+dx))^2\left(-b\left(3a^2+b^2\right)+a\left(a^2+3b^2\right)\sin(c+dx)\right)+\sec(c+dx)(a+b\sin(c+dx))^2\left(-a^4b+54a^2b^3+11b^5+4a\left(a^4-6a^2b^2-11b^4\right)\sin(c+dx)\right)}{\left(a^2-b^2\right)^4}}{6d(a+b\sin(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (((4*(a^5 - 6*a^3*b^2 - 27*a*b^4)*EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b)/(a
 + b)] + (-4*a^5 + 4*a^4*b + 21*a^3*b^2 - 21*a^2*b^3 + 15*a*b^4 - 15*b^5)*E
llipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)])*((a + b*Sin[c + d*x])/(a +
b))^(3/2))/((a - b)^4*(a + b)^2) + (4*b^5*(a^2 - b^2)*Cos[c + d*x] + 64*a*b
^5*Cos[c + d*x]*(a + b*Sin[c + d*x]) + 2*(a^2 - b^2)*Sec[c + d*x]^3*(a + b*
Sin[c + d*x])^2*(-(b*(3*a^2 + b^2)) + a*(a^2 + 3*b^2)*Sin[c + d*x]) + Sec[c
 + d*x]*(a + b*Sin[c + d*x])^2*(-(a^4*b) + 54*a^2*b^3 + 11*b^5 + 4*a*(a^4 -
 6*a^2*b^2 - 11*b^4)*Sin[c + d*x]))/(a^2 - b^2)^4)/(6*d*(a + b*Sin[c + d*x]
)^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2584 vs. $2(465) = 930$.

time = 18.66, size = 2585, normalized size = 6.08

method	result	size
default	Expression too large to display	2585

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+b*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-(-a-b*sin(d*x+c))*cos(d*x+c)^2)^(1/2)*(1/4/(a-b)^2*(-1/3/(a-b)*(-(-a-b*si
n(d*x+c))*cos(d*x+c)^2)^(1/2)/(1+sin(d*x+c))^2-1/3*(-sin(d*x+c)^2*b-a*sin(d
*x+c)+b*sin(d*x+c)+a)/(a-b)^2*(a-3*b)/((-a-b*sin(d*x+c))*(sin(d*x+c)-1)*(1+
sin(d*x+c)))^(1/2)+2*b^2/(3*a^2-6*a*b+3*b^2)*(a/b-1)*((a+b*sin(d*x+c))/(a-b
))^(1/2)*(b*(1-sin(d*x+c))/(a+b))^(1/2)*((-1-sin(d*x+c))*b/(a-b))^(1/2)/(-(
-a-b*sin(d*x+c))*cos(d*x+c)^2)^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/
2),((a-b)/(a+b))^(1/2))-1/3*b*(a-3*b)/(a-b)^2*(a/b-1)*((a+b*sin(d*x+c))/(a-
b))^(1/2)*(b*(1-sin(d*x+c))/(a+b))^(1/2)*((-1-sin(d*x+c))*b/(a-b))^(1/2)/(-
(-a-b*sin(d*x+c))*cos(d*x+c)^2)^(1/2)*((-a/b-1)*EllipticE(((a+b*sin(d*x+c))
/(a-b))^(1/2),((a-b)/(a+b))^(1/2))+EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2)
,((a-b)/(a+b))^(1/2))))-1/4*(-a-3*b)/(a+b)^4/b/cos(d*x+c)^2/(a+b*sin(d*x+c)
)*(b*cos(d*x+c)^2*sin(d*x+c)+a*cos(d*x+c)^2)^(1/2)*(EllipticE((b/(a-b)*sin(
d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))*(-b/(a-b)*sin(d*x+c)-b/(a-b))^
(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1
/2)*a^2-EllipticE((b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2),((a-b)/(a+b))^(1/2))
*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(b/(a-b)*sin(d*x+c)+1/(a-b)*a)^(1/2)*(
-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*b^2-b^2*cos(d*x+c)^2+a*b*sin(d*x+c)+b^2*
sin(d*x+c)+a*b+b^2)+b^4/(a+b)^2/(a-b)^2*(2/3/b/(a^2-b^2)*(-(-a-b*sin(d*x+c)
```

```
)*cos(d*x+c)^2)^(1/2)/(sin(d*x+c)+a/b)^2+8/3*b*cos(d*x+c)^2/(a^2-b^2)^2*a/(
-(-a-b*sin(d*x+c))*cos(d*x+c)^2)^(1/2)+2*(3*a^2+b^2)/(3*a^4-6*a^2*b^2+3*b^4
)*(a/b-1)*((a+b*sin(d*x+c))/(a-b))^(1/2)*(b*(1-sin(d*x+c))/(a+b))^(1/2)*((-
1-sin(d*x+c))*b/(a-b))^(1/2)/(-(-a-b*sin(d*x+c))*cos(d*x+c)^2)^(1/2)*Ellipt
icF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))+8/3*a*b/(a^2-b^2)^2
*(a/b-1)*((a+b*sin(d*x+c))/(a-b))^(1/2)*(b*(1-sin(d*x+c))/(a+b))^(1/2)*((-1
-sin(d*x+c))*b/(a-b))^(1/2)/(-(-a-b*sin(d*x+c))*cos(d*x+c)^2)^(1/2)*((-a/b-
1)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))+EllipticF(
((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))))+4*a*b^4/(a+b)^3/(a-b)
^3*(2*b*cos(d*x+c)^2/(a^2-b^2)/(-(-a-b*sin(d*x+c))*cos(d*x+c)^2)^(1/2)+2*a/
(a^2-b^2)*(a/b-1)*((a+b*sin(d*x+c))/(a-b))^(1/2)*(b*(1-sin(d*x+c))/(a+b))^(
1/2)*((-1-sin(d*x+c))*b/(a-b))^(1/2)/(-(-a-b*sin(d*x+c))*cos(d*x+c)^2)^(1/2
)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))+2*b/(a^2-b^
2)*(a/b-1)*((a+b*sin(d*x+c))/(a-b))^(1/2)*(b*(1-sin(d*x+c))/(a+b))^(1/2)*((
-1-sin(d*x+c))*b/(a-b))^(1/2)/(-(-a-b*sin(d*x+c))*cos(d*x+c)^2)^(1/2)*((-a/
b-1)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))+Elliptic
F(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))))+1/4*(a-3*b)/(a-b)^3
*(-(-sin(d*x+c)^2*b-a*sin(d*x+c)+b*sin(d*x+c)+a)/(a-b)/((-a-b*sin(d*x+c))*(
sin(d*x+c)-1)*(1+sin(d*x+c)))^(1/2)-2*b/(2*a-2*b)*(a/b-1)*((a+b*sin(d*x+c))
/(a-b))^(1/2)*(b*(1-sin(d*x+c))/(a+b))^(1/2)*((-1-sin(d*x+c))*b/(a-b))^(1/2
)/(-(-a-b*sin(d*x+c))*cos(d*x+c)^2)^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b)
)^(1/2),((a-b)/(a+b))^(1/2))-b/(a-b)*(a/b-1)*((a+b*sin(d*x+c))/(a-b))^(1/2)
*(b*(1-sin(d*x+c))/(a+b))^(1/2)*((-1-sin(d*x+c))*b/(a-b))^(1/2)/(-(-a-b*sin
(d*x+c))*cos(d*x+c)^2)^(1/2)*((-a/b-1)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(
1/2),((a-b)/(a+b))^(1/2))+EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(
a+b))^(1/2))))+1/4/(a+b)^2*(1/3/(a+b)*(-(-a-b*sin(d*x+c))*cos(d*x+c)^2)^(1/
2)/(sin(d*x+c)-1)^2-1/3*(-sin(d*x+c)^2*b-a*sin(d*x+c)-b*sin(d*x+c)-a)/(a+b)
^2*(a+3*b)/((-a-b*sin(d*x+c))*(sin(d*x+c)-1)*(1+sin(d*x+c)))^(1/2)+2*b^2/(3
*a^2+6*a*b+3*b^2)*(a/b-1)*((a+b*sin(d*x+c))/(a-b))^(1/2)*(b*(1-sin(d*x+c))/
(a+b))^(1/2)*((-1-sin(d*x+c))*b/(a-b))^(1/2)/(-(-a-b*sin(d*x+c))*cos(d*x+c)
^2)^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))-1/3
*b*(a+3*b)/(a+b)^2*(a/b-1)*((a+b*sin(d*x+c))/(a-b))^(1/2)*(b*(1-sin(d*x+c))
/(a+b))^(1/2)*((-1-sin(d*x+c))*b/(a-b))^(1/2)/(-(-a-b*sin(d*x+c))*cos(d*x+c
)^2)^(1/2)*((-a/b-1)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))
^(1/2))+EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2)))))/co
s(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sin(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate(sec(d*x + c)^4/(b*sin(d*x + c) + a)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.27, size = 1242, normalized size = 2.92

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/36*((sqrt(2)*(8*a^6*b^2 - 51*a^4*b^4 + 126*a^2*b^6 + 45*b^8)*cos(d*x + c)
^5 - 2*sqrt(2)*(8*a^7*b - 51*a^5*b^3 + 126*a^3*b^5 + 45*a*b^7)*cos(d*x + c)
^3*sin(d*x + c) - sqrt(2)*(8*a^8 - 43*a^6*b^2 + 75*a^4*b^4 + 171*a^2*b^6 +
45*b^8)*cos(d*x + c)^3)*sqrt(I*b)*weierstrassPInverse(-4/3*(4*a^2 - 3*b^2)/
b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x
 + c) - 2*I*a)/b) + (sqrt(2)*(8*a^6*b^2 - 51*a^4*b^4 + 126*a^2*b^6 + 45*b^8
)*cos(d*x + c)^5 - 2*sqrt(2)*(8*a^7*b - 51*a^5*b^3 + 126*a^3*b^5 + 45*a*b^7
)*cos(d*x + c)^3*sin(d*x + c) - sqrt(2)*(8*a^8 - 43*a^6*b^2 + 75*a^4*b^4 +
171*a^2*b^6 + 45*b^8)*cos(d*x + c)^3)*sqrt(-I*b)*weierstrassPInverse(-4/3*(
4*a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c)
 + 3*I*b*sin(d*x + c) + 2*I*a)/b) + 12*(sqrt(2)*(I*a^5*b^3 - 6*I*a^3*b^5 -
27*I*a*b^7)*cos(d*x + c)^5 + 2*sqrt(2)*(-I*a^6*b^2 + 6*I*a^4*b^4 + 27*I*a^2
*b^6)*cos(d*x + c)^3*sin(d*x + c) + sqrt(2)*(-I*a^7*b + 5*I*a^5*b^3 + 33*I*
a^3*b^5 + 27*I*a*b^7)*cos(d*x + c)^3)*sqrt(I*b)*weierstrassZeta(-4/3*(4*a^2
 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, weierstrassPInverse(-4/3*(4
*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) -
 3*I*b*sin(d*x + c) - 2*I*a)/b)) + 12*(sqrt(2)*(-I*a^5*b^3 + 6*I*a^3*b^5 +
27*I*a*b^7)*cos(d*x + c)^5 + 2*sqrt(2)*(I*a^6*b^2 - 6*I*a^4*b^4 - 27*I*a^2*
b^6)*cos(d*x + c)^3*sin(d*x + c) + sqrt(2)*(I*a^7*b - 5*I*a^5*b^3 - 33*I*a^
3*b^5 - 27*I*a*b^7)*cos(d*x + c)^3)*sqrt(-I*b)*weierstrassZeta(-4/3*(4*a^2
- 3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, weierstrassPInverse(-4/3*(4
*a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c)
+ 3*I*b*sin(d*x + c) + 2*I*a)/b)) + 6*(2*a^6*b^2 - 6*a^4*b^4 + 6*a^2*b^6 -
2*b^8 + (8*a^6*b^2 - 49*a^4*b^4 - 102*a^2*b^6 + 15*b^8)*cos(d*x + c)^4 - 3*
(a^6*b^2 + a^4*b^4 - 5*a^2*b^6 + 3*b^8)*cos(d*x + c)^2 - 2*(a^7*b - 3*a^5*b
^3 + 3*a^3*b^5 - a*b^7 - 2*(a^5*b^3 - 6*a^3*b^5 - 27*a*b^7)*cos(d*x + c)^4
+ 2*(a^7*b - 6*a^5*b^3 + 9*a^3*b^5 - 4*a*b^7)*cos(d*x + c)^2)*sin(d*x + c))
*sqrt(b*sin(d*x + c) + a))/((a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 +
b^11)*d*cos(d*x + c)^5 - 2*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a
*b^10)*d*cos(d*x + c)^3*sin(d*x + c) - (a^10*b - 3*a^8*b^3 + 2*a^6*b^5 + 2*
a^4*b^7 - 3*a^2*b^9 + b^11)*d*cos(d*x + c)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^4(c+dx)}{(a+b\sin(c+dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Integral(sec(c + d*x)**4/(a + b*sin(c + d*x))**(5/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos\left(c+d\,x\right)^4\,\left(a+b\,\sin\left(c+d\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a + b*sin(c + d*x))^(5/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^4*(a + b*sin(c + d*x))^(5/2)), x)
```

3.539 $\int (e\cos(c+dx))^{7/2}(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=124

$$-\frac{2b(e\cos(c+dx))^{9/2}}{9de}+\frac{10ae^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{e\cos(c+dx)}}+\frac{10ae^3\sqrt{e\cos(c+dx)}\,\sin(c+dx)}{21d}+\frac{2ae(e\cos(}{}$$

```
[Out] -2/9*b*(e*cos(d*x+c))^(9/2)/d/e+2/7*a*e*(e*cos(d*x+c))^(5/2)*sin(d*x+c)/d+1
0/21*a*e^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/
2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(e*cos(d*x+c))^(1/2)+10/21*a*e^3*s
in(d*x+c)*(e*cos(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.06, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2748, 2715, 2721, 2720}

$$\frac{10ae^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{e\cos(c+dx)}}+\frac{10ae^3\sin(c+dx)\sqrt{e\cos(c+dx)}}{21d}+\frac{2ae\sin(c+dx)(e\cos(c+dx))^{5/2}}{7d}-\frac{2b(e\cos(c+dx))^{9/2}}{9de}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(7/2)*(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*b*(e*Cos[c + d*x])^(9/2))/(9*d*e) + (10*a*e^4*Sqrt[Cos[c + d*x]]*Ellipt
icF[(c + d*x)/2, 2])/(21*d*Sqrt[e*Cos[c + d*x]]) + (10*a*e^3*Sqrt[e*Cos[c +
 d*x]]*Sin[c + d*x])/(21*d) + (2*a*e*(e*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(
7*d)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\int (e\cos(c+dx))^{7/2}(a+b\sin(c+dx))\,dx = -\frac{2b(e\cos(c+dx))^{9/2}}{9de} + a\int (e\cos(c+dx))^{7/2}\,dx$$
$$= -\frac{2b(e\cos(c+dx))^{9/2}}{9de} + \frac{2ae(e\cos(c+dx))^{5/2}\sin(c+dx)}{7d} + \frac{1}{7}$$
$$= -\frac{2b(e\cos(c+dx))^{9/2}}{9de} + \frac{10ae^3\sqrt{e\cos(c+dx)}\,\sin(c+dx)}{21d} + $$
$$= -\frac{2b(e\cos(c+dx))^{9/2}}{9de} + \frac{10ae^3\sqrt{e\cos(c+dx)}\,\sin(c+dx)}{21d} + $$
$$= -\frac{2b(e\cos(c+dx))^{9/2}}{9de} + \frac{10ae^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{21d\sqrt{e\cos(c+dx)}}$$

Mathematica [A]
time = 0.85, size = 104, normalized size = 0.84

$$\frac{e^3\sqrt{e\cos(c+dx)}\left(120aF\left(\frac{1}{2}(c+dx)\middle|\,2\right)+\sqrt{\cos(c+dx)}\,(-21b-28b\cos(2(c+dx))-7b\cos(4(c+dx))+138a\sin(c+dx)+18a\sin(3(c+dx)))\right)}{252d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(7/2)*(a + b*Sin[c + d*x]),x]
```

```
[Out] (e^3*Sqrt[e*Cos[c + d*x]]*(120*a*EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d
*x]]*(-21*b - 28*b*Cos[2*(c + d*x)] - 7*b*Cos[4*(c + d*x)] + 138*a*Sin[c +
d*x] + 18*a*Sin[3*(c + d*x)])))/(252*d*Sqrt[Cos[c + d*x]])
```

Maple [A]
time = 4.37, size = 259, normalized size = 2.09

method	result

default	$-\frac{2e^4\left(-224b\left(\sin^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+144a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+560b\left(\sin^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-216a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-560b\left(\mathrm{si}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(7/2)*(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/63/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^4*(-224*b*si
n(1/2*d*x+1/2*c)^11+144*a*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+560*b*sin
(1/2*d*x+1/2*c)^9-216*a*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-560*b*sin(1
/2*d*x+1/2*c)^7+168*a*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+280*b*sin(1/2
*d*x+1/2*c)^5+15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a-48*a*cos(1/2*d*x+1/2*c)*sin(1/2
*d*x+1/2*c)^2-70*b*sin(1/2*d*x+1/2*c)^3+7*b*sin(1/2*d*x+1/2*c))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] e^(7/2)*integrate((b*sin(d*x + c) + a)*cos(d*x + c)^(7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 109, normalized size = 0.88

$$\frac{-15i\sqrt{2}\,ae^{\frac{7}{2}}\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+15i\sqrt{2}\,ae^{\frac{7}{2}}\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-2\left(7\,b\cos(dx+c)^4e^{\frac{7}{2}}-3\left(3\,a\cos(dx+c)^2e^{\frac{7}{2}}+5\,ae^{\frac{7}{2}}\right)\sin(dx+c)\right)\sqrt{\cos(dx+c)}}{63\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/63*(-15*I*sqrt(2)*a*e^(7/2)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*s
in(d*x + c)) + 15*I*sqrt(2)*a*e^(7/2)*weierstrassPInverse(-4, 0, cos(d*x +
c) - I*sin(d*x + c)) - 2*(7*b*cos(d*x + c)^4*e^(7/2) - 3*(3*a*cos(d*x + c)^
2*e^(7/2) + 5*a*e^(7/2))*sin(d*x + c))*sqrt(cos(d*x + c)))/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(7/2)*(a+b*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4846 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)*cos(d*x + c)^(7/2)*e^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (e\,\cos\left(c+d\,x\right))^{7/2}\,\left(a+b\,\sin\left(c+d\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(7/2)*(a + b*sin(c + d*x)),x)
```

```
[Out] int((e*cos(c + d*x))^(7/2)*(a + b*sin(c + d*x)), x)
```

3.540 $\int (e\cos(c+dx))^{5/2}(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=95

$$-\frac{2b(e\cos(c+dx))^{7/2}}{7de}+\frac{6ae^2\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}}+\frac{2ae(e\cos(c+dx))^{3/2}\sin(c+dx)}{5d}$$

```
[Out] -2/7*b*(e*cos(d*x+c))^(7/2)/d/e+2/5*a*e*(e*cos(d*x+c))^(3/2)*sin(d*x+c)/d+6
/5*a*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*
d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2748, 2715, 2721, 2719}

$$\frac{6ae^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2ae\sin(c+dx)(e\cos(c+dx))^{3/2}}{5d}-\frac{2b(e\cos(c+dx))^{7/2}}{7de}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(5/2)*(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*b*(e*Cos[c + d*x])^(7/2))/(7*d*e) + (6*a*e^2*Sqrt[e*Cos[c + d*x]]*Ellip
ticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]) + (2*a*e*(e*Cos[c + d*x])^(3
/2)*Sin[c + d*x])/(5*d)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\int (e\cos(c+dx))^{5/2}(a+b\sin(c+dx))\,dx = -\frac{2b(e\cos(c+dx))^{7/2}}{7de}+a\int (e\cos(c+dx))^{5/2}\,dx$$
$$= -\frac{2b(e\cos(c+dx))^{7/2}}{7de}+\frac{2ae(e\cos(c+dx))^{3/2}\sin(c+dx)}{5d}+\frac{1}{5}$$
$$= -\frac{2b(e\cos(c+dx))^{7/2}}{7de}+\frac{2ae(e\cos(c+dx))^{3/2}\sin(c+dx)}{5d}+$$
$$= -\frac{2b(e\cos(c+dx))^{7/2}}{7de}+\frac{6ae^2\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}}$$

Mathematica [A]
time = 0.50, size = 79, normalized size = 0.83

$$\frac{(e\cos(c+dx))^{5/2}\left(42aE\left(\frac{1}{2}(c+dx)\middle|2\right)+\cos^{\frac{3}{2}}(c+dx)(-5b-5b\cos(2(c+dx))+14a\sin(c+dx))\right)}{35d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(5/2)*(a + b*Sin[c + d*x]),x]
[Out] ((e*Cos[c + d*x])^(5/2)*(42*a*EllipticE[(c + d*x)/2, 2] + Cos[c + d*x]^(3/2
)*(-5*b - 5*b*Cos[2*(c + d*x)] + 14*a*Sin[c + d*x])))/(35*d*Cos[c + d*x]^(5
/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(107) = 214$.
time = 4.20, size = 222, normalized size = 2.34

method	result
default	$\frac{2e^3\left(-80b\left(\sin^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+56a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+160b\left(\sin^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-56a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-120b\left(\sin^5\right.\right.}{35\sin}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(5/2)*(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/35/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^3*(-80*b*sin(
1/2*d*x+1/2*c)^9+56*a*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+160*b*sin(1/2
*d*x+1/2*c)^7-56*a*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-120*b*sin(1/2*d*
x+1/2*c)^5+21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a+14*a*cos(1/2*d*x+1/2*c)*sin(1/2*d*
x+1/2*c)^2+40*b*sin(1/2*d*x+1/2*c)^3-5*b*sin(1/2*d*x+1/2*c))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] e^(5/2)*integrate((b*sin(d*x + c) + a)*cos(d*x + c)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 105, normalized size = 1.11

$$\frac{21i\sqrt{2}\,ae^{\frac{5}{2}}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))) - 21i\sqrt{2}\,ae^{\frac{5}{2}}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c))) - 2\left(5\,b\cos(dx+c)^3e^{\frac{5}{2}} - 7\,a\cos(dx+c)\,e^{\frac{5}{2}}\sin(dx+c)\right)\sqrt{\cos(dx+c)}}{35\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/35*(21*I*sqrt(2)*a*e^(5/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
 0, cos(d*x + c) + I*sin(d*x + c))) - 21*I*sqrt(2)*a*e^(5/2)*weierstrassZet
a(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(5*
b*cos(d*x + c)^3*e^(5/2) - 7*a*cos(d*x + c)*e^(5/2)*sin(d*x + c))*sqrt(cos(
d*x + c)))/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(5/2)*(a+b*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4849 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)*cos(d*x + c)^(5/2)*e^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (e\,\cos(c+d\,x))^{5/2}\,(a+b\,\sin(c+d\,x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(5/2)*(a + b*sin(c + d*x)),x)
```

```
[Out] int((e*cos(c + d*x))^(5/2)*(a + b*sin(c + d*x)), x)
```

3.541 $\int (e\cos(c+dx))^{3/2}(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=95

$$-\frac{2b(e\cos(c+dx))^{5/2}}{5de}+\frac{2ae^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{e\cos(c+dx)}}+\frac{2ae\sqrt{e\cos(c+dx)}\,\sin(c+dx)}{3d}$$

```
[Out] -2/5*b*(e*cos(d*x+c))^(5/2)/d/e+2/3*a*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(
1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(e*
cos(d*x+c))^(1/2)+2/3*a*e*sin(d*x+c)*(e*cos(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.05, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2748, 2715, 2721, 2720}

$$\frac{2ae^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{e\cos(c+dx)}}+\frac{2ae\sin(c+dx)\sqrt{e\cos(c+dx)}}{3d}-\frac{2b(e\cos(c+dx))^{5/2}}{5de}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*b*(e*Cos[c + d*x])^(5/2))/(5*d*e) + (2*a*e^2*Sqrt[Cos[c + d*x]]*Ellipti
cF[(c + d*x)/2, 2])/(3*d*Sqrt[e*Cos[c + d*x]]) + (2*a*e*Sqrt[e*Cos[c + d*x]
]*Sin[c + d*x])/(3*d)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\int (e\cos(c+dx))^{3/2}(a+b\sin(c+dx))\,dx = -\frac{2b(e\cos(c+dx))^{5/2}}{5de} + a\int (e\cos(c+dx))^{3/2}\,dx$$
$$= -\frac{2b(e\cos(c+dx))^{5/2}}{5de} + \frac{2ae\sqrt{e\cos(c+dx)}\,\sin(c+dx)}{3d} + \frac{1}{3}($$
$$= -\frac{2b(e\cos(c+dx))^{5/2}}{5de} + \frac{2ae\sqrt{e\cos(c+dx)}\,\sin(c+dx)}{3d} + \frac{(a}{}$$
$$= -\frac{2b(e\cos(c+dx))^{5/2}}{5de} + \frac{2ae^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{3d\sqrt{e\cos(c+dx)}} +$$

Mathematica [A]
time = 0.49, size = 79, normalized size = 0.83

$$\frac{(e\cos(c+dx))^{3/2}\left(10aF\left(\frac{1}{2}(c+dx)\middle|\,2\right)+\sqrt{\cos(c+dx)}\,(-3b-3b\cos(2(c+dx))+10a\sin(c+dx))\right)}{15d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x]),x]
[Out] ((e*Cos[c + d*x])^(3/2)*(10*a*EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]
]*(-3*b - 3*b*Cos[2*(c + d*x)] + 10*a*Sin[c + d*x])))/(15*d*Cos[c + d*x]^(3
/2))
```

Maple [A]
time = 3.95, size = 185, normalized size = 1.95

method	result
default	$-\frac{2e^2\left(-24b\left(\sin^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+20a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+36b\left(\sin^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+5\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\right.\right.}\right.}{15\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^2\left(\right.\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(3/2)*(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/15/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^2*(-24*b*sin
(1/2*d*x+1/2*c)^7+20*a*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+36*b*sin(1/2
*d*x+1/2*c)^5+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a-10*a*cos(1/2*d*x+1/2*c)*sin(1/2*
d*x+1/2*c)^2-18*b*sin(1/2*d*x+1/2*c)^3+3*b*sin(1/2*d*x+1/2*c))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] e^(3/2)*integrate((b*sin(d*x + c) + a)*cos(d*x + c)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 93, normalized size = 0.98

$$\frac{-5i\sqrt{2}\,ae^{\frac{3}{2}}\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}\,ae^{\frac{3}{2}}\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-2\left(3\,b\cos(dx+c)^{2}e^{\frac{3}{2}}-5\,ae^{\frac{3}{2}}\sin(dx+c)\right)\sqrt{\cos(dx+c)}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/15*(-5*I*sqrt(2)*a*e^(3/2)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*si
n(d*x + c)) + 5*I*sqrt(2)*a*e^(3/2)*weierstrassPInverse(-4, 0, cos(d*x + c)
 - I*sin(d*x + c)) - 2*(3*b*cos(d*x + c)^2*e^(3/2) - 5*a*e^(3/2)*sin(d*x +
c))*sqrt(cos(d*x + c)))/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(3/2)*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)*cos(d*x + c)^(3/2)*e^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (e\,\cos\left(c+d\,x\right))^{3/2}\,\left(a+b\,\sin\left(c+d\,x\right)\right) \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(3/2)*(a + b*sin(c + d*x)),x)
```

```
[Out] int((e*cos(c + d*x))^(3/2)*(a + b*sin(c + d*x)), x)
```

3.542 $\int \sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=63

$$-\frac{2b(e\cos(c+dx))^{3/2}}{3de}+\frac{2a\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}}$$

```
[Out] -2/3*b*(e*cos(d*x+c))^(3/2)/d/e+2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*
x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/d/cos(d
*x+c)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {2748, 2721, 2719}

$$\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{2b(e\cos(c+dx))^{3/2}}{3de}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*b*(e*Cos[c + d*x])^(3/2))/(3*d*e) + (2*a*Sqrt[e*Cos[c + d*x]]*EllipticE
[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\int \sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))\,dx = -\frac{2b(e\cos(c+dx))^{3/2}}{3de} + a\int \sqrt{e\cos(c+dx)}\,dx$$
$$= -\frac{2b(e\cos(c+dx))^{3/2}}{3de} + \frac{\left(a\sqrt{e\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}}{\sqrt{\cos(c+dx)}}$$
$$= -\frac{2b(e\cos(c+dx))^{3/2}}{3de} + \frac{2a\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}}$$

Mathematica [A]
time = 0.11, size = 56, normalized size = 0.89

$$-\frac{2\sqrt{e\cos(c+dx)}\left(b\cos^{\frac{3}{2}}(c+dx)-3aE\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{3d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x]),x]
[Out] (-2*Sqrt[e*Cos[c + d*x]]*(b*Cos[c + d*x]^(3/2) - 3*a*EllipticE[(c + d*x)/2,
 2]))/(3*d*Sqrt[Cos[c + d*x]])
```

Maple [A]
time = 3.09, size = 123, normalized size = 1.95

method	result
default	$\frac{2e\left(-4b\left(\sin^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)a+4b\left(\sin^{3}\left(\frac{dx}{2}+\right.\right.\right.}{3\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)e+e}\,d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))*(e*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/3/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e*(-4*b*sin(1/2*
d*x+1/2*c)^5+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a+4*b*sin(1/2*d*x+1/2*c)^3-b*sin(1/
2*d*x+1/2*c))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))*(e*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] e^(1/2)*integrate((b*sin(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 77, normalized size = 1.22

$$-\frac{2 b \cos(dx+c)^{\frac{3}{2}} e^{\frac{1}{2}} - 3i\sqrt{2}\, a e^{\frac{1}{2}} \mathrm{weierstrassZeta}(-4, 0, \mathrm{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c))) + 3i\sqrt{2}\, a e^{\frac{1}{2}} \mathrm{weierstrassZeta}(-4, 0, \mathrm{weierstrassPInverse}(-4, 0, \cos(dx+c) - i \sin(dx+c)))}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))*(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*b*cos(d*x + c)^(3/2)*e^(1/2) - 3*I*sqrt(2)*a*e^(1/2)*weierstrassZet
a(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*I*s
qrt(2)*a*e^(1/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x
+ c) - I*sin(d*x + c))))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{e \cos(c+dx)}\,(a + b \sin(c+dx))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))*(e*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(e*cos(c + d*x))*(a + b*sin(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))*(e*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)*sqrt(cos(d*x + c))*e^(1/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{e\,\cos(c+d\,x)}\,\left(a + b\,\sin(c+d\,x)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(1/2)*(a + b*sin(c + d*x)),x)
```

```
[Out] int((e*cos(c + d*x))^(1/2)*(a + b*sin(c + d*x)), x)
```

3.543 $\int \frac{a+b\sin(c+dx)}{\sqrt{e\cos(c+dx)}}\,dx$

Optimal. Leaf size=61

$$-\frac{2b\sqrt{e\cos(c+dx)}}{de}+\frac{2a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{d\sqrt{e\cos(c+dx)}}$$

```
[Out] 2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1
/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(e*cos(d*x+c))^(1/2)-2*b*(e*cos(d*x+c))^(
1/2)/d/e
```

Rubi [A]
time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.130$, Rules used = {2748, 2721, 2720}

$$\frac{2a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{d\sqrt{e\cos(c+dx)}}-\frac{2b\sqrt{e\cos(c+dx)}}{de}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])/Sqrt[e*Cos[c + d*x]],x]
```

```
[Out] (-2*b*Sqrt[e*Cos[c + d*x]])/(d*e) + (2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c +
d*x)/2, 2])/(d*Sqrt[e*Cos[c + d*x]])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\int \frac{a+b\sin(c+dx)}{\sqrt{e\cos(c+dx)}}\,dx = -\frac{2b\sqrt{e\cos(c+dx)}}{de}+a\int \frac{1}{\sqrt{e\cos(c+dx)}}\,dx$$
$$= -\frac{2b\sqrt{e\cos(c+dx)}}{de}+\frac{\left(a\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{\sqrt{e\cos(c+dx)}}$$
$$= -\frac{2b\sqrt{e\cos(c+dx)}}{de}+\frac{2a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{d\sqrt{e\cos(c+dx)}}$$

Mathematica [A]
time = 0.20, size = 50, normalized size = 0.82

$$\frac{-2b\cos(c+dx)+2a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{d\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])/Sqrt[e*Cos[c + d*x]],x]
[Out] (-2*b*Cos[c + d*x] + 2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*S
qrt[e*Cos[c + d*x]])
```

Maple [A]
time = 2.27, size = 106, normalized size = 1.74

method	result
default	$-\frac{2\left(\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)a-2b\left(\sin^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)e+e}\,d}$
risch	$-\frac{b\left(1+e^{2i(dx+c)}\right)\sqrt{2}\,e^{-i(dx+c)}}{d\sqrt{e\left(1+e^{2i(dx+c)}\right)e^{-i(dx+c)}}}+\frac{2a\sqrt{-i\left(e^{i(dx+c)}+i\right)}\,\sqrt{i\left(e^{i(dx+c)}-i\right)}\,\sqrt{ie^{i(dx+c)}}\,\text{EllipticF}\left(\sqrt{-i\left(\right.}\right.}{d\sqrt{e^{3i(dx+c)}e+e^{i(dx+c)}e}\,\sqrt{e\left(1-\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))/(e*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*((sin(1/2*d*x+1/2
*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),
2^(1/2))*a-2*b*sin(1/2*d*x+1/2*c)^3+b*sin(1/2*d*x+1/2*c))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))/(e*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] e^(-1/2)*integrate((b*sin(d*x + c) + a)/sqrt(cos(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 66, normalized size = 1.08

$$\frac{\left(-i\sqrt{2}\,a\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}\,a\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-2\,b\sqrt{\cos(dx+c)}\right)e^{\left(-\frac{1}{2}\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))/(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] (-I*sqrt(2)*a*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + I
*sqrt(2)*a*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 2*b*
sqrt(cos(d*x + c)))*e^(-1/2)/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\sin(c+dx)}{\sqrt{e\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))/(e*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((a + b*sin(c + d*x))/sqrt(e*cos(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))/(e*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)*e^(-1/2)/sqrt(cos(d*x + c)), x)
```

Mupad [B]
time = 6.49, size = 47, normalized size = 0.77

$$-\frac{2\sqrt{\cos(c+d\,x)}\,\left(b\sqrt{\cos(c+d\,x)}-a\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)\right)}{d\sqrt{e\,\cos(c+d\,x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))/(e*cos(c + d*x))^(1/2),x)
```

```
[Out] -(2*cos(c + d*x)^(1/2)*(b*cos(c + d*x)^(1/2) - a*ellipticF(c/2 + (d*x)/2, 2
)))/(d*(e*cos(c + d*x))^(1/2))
```

3.544 $\int \frac{a+b\sin(c+dx)}{(e\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=91

$$\frac{2b}{de\sqrt{e\cos(c+dx)}}-\frac{2a\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{de^2\sqrt{\cos(c+dx)}}+\frac{2a\sin(c+dx)}{de\sqrt{e\cos(c+dx)}}$$

```
[Out] 2*b/d/e/(e*cos(d*x+c))^(1/2)+2*a*sin(d*x+c)/d/e/(e*cos(d*x+c))^(1/2)-2*a*(c
os(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),
2^(1/2))*(e*cos(d*x+c))^(1/2)/d/e^2/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.174$, Rules used = {2748, 2716, 2721, 2719}

$$-\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{de^2\sqrt{\cos(c+dx)}}+\frac{2a\sin(c+dx)}{de\sqrt{e\cos(c+dx)}}+\frac{2b}{de\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])/(e*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*b)/(d*e*Sqrt[e*Cos[c + d*x]]) - (2*a*Sqrt[e*Cos[c + d*x]]*EllipticE[(c +
 d*x)/2, 2])/(d*e^2*Sqrt[Cos[c + d*x]]) + (2*a*Sin[c + d*x])/(d*e*Sqrt[e*Co
s[c + d*x]])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\sin(c+dx)}{(e\cos(c+dx))^{3/2}}\,dx &= \frac{2b}{de\sqrt{e\cos(c+dx)}}+a\int \frac{1}{(e\cos(c+dx))^{3/2}}\,dx\\
&= \frac{2b}{de\sqrt{e\cos(c+dx)}}+\frac{2a\sin(c+dx)}{de\sqrt{e\cos(c+dx)}}-\frac{a\int \sqrt{e\cos(c+dx)}\,dx}{e^2}\\
&= \frac{2b}{de\sqrt{e\cos(c+dx)}}+\frac{2a\sin(c+dx)}{de\sqrt{e\cos(c+dx)}}-\frac{\left(a\sqrt{e\cos(c+dx)}\right)\int \sqrt{\cos(c+dx}}{e^2\sqrt{\cos(c+dx)}}\\
&= \frac{2b}{de\sqrt{e\cos(c+dx)}}-\frac{2a\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{de^2\sqrt{\cos(c+dx)}}+\frac{2a\sin(c+dx)}{de\sqrt{e\cos(c+dx)}}
\end{aligned}$$

Mathematica [A]
time = 0.13, size = 54, normalized size = 0.59

$$\frac{2\left(b-a\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+a\sin(c+dx)\right)}{de\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])/(e*Cos[c + d*x])^(3/2),x]
[Out] (2*(b - a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + a*Sin[c + d*x]))/(
d*e*Sqrt[e*Cos[c + d*x]])
```

Maple [A]
time = 5.52, size = 119, normalized size = 1.31

method	result
default	$-\frac{2\left(\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)a-2a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\,\mathrm{si}\right.}{e\sqrt{-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)e+e}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))/(e*cos(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/e/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/sin(1/2*d*x+1/2*c)*((sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))*a-2*a*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-b*sin(1/2*d*x+1/2*
c))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))/(e*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] e^(-3/2)*integrate((b*sin(d*x + c) + a)/cos(d*x + c)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 101, normalized size = 1.11

$$\frac{\left(-i\sqrt{2}\,a\cos\left(dx+c\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)\right)+i\sqrt{2}\,a\cos\left(dx+c\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)\right)+2\left(a\sin\left(dx+c\right)+b\right)\sqrt{\cos\left(dx+c\right)}\right)e^{\left(-\frac{3}{2}\right)}}{d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))/(e*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] (-I*sqrt(2)*a*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0
, cos(d*x + c) + I*sin(d*x + c))) + I*sqrt(2)*a*cos(d*x + c)*weierstrassZet
a(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(a*
sin(d*x + c) + b)*sqrt(cos(d*x + c)))*e^(-3/2)/(d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\sin\left(c + dx\right)}{\left(e\cos\left(c + dx\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))/(e*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral((a + b*sin(c + d*x))/(e*cos(c + d*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))/(e*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)*e^(-3/2)/cos(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\sin\left(c+d\,x\right)}{{\left(e\,\cos\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))/(e*cos(c + d*x))^(3/2),x)
[Out] int((a + b*sin(c + d*x))/(e*cos(c + d*x))^(3/2), x)
```

3.545 $\int \frac{a+b\sin(c+dx)}{(e\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=97

$$\frac{2b}{3de(e\cos(c+dx))^{3/2}}+\frac{2a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3de^2\sqrt{e\cos(c+dx)}}+\frac{2a\sin(c+dx)}{3de(e\cos(c+dx))^{3/2}}$$

```
[Out] 2/3*b/d/e/(e*cos(d*x+c))^(3/2)+2/3*a*sin(d*x+c)/d/e/(e*cos(d*x+c))^(3/2)+2/
3*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1
/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/e^2/(e*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2748, 2716, 2721, 2720}

$$\frac{2a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3de^2\sqrt{e\cos(c+dx)}}+\frac{2a\sin(c+dx)}{3de(e\cos(c+dx))^{3/2}}+\frac{2b}{3de(e\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])/(e*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*b)/(3*d*e*(e*Cos[c + d*x])^(3/2)) + (2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c
 + d*x)/2, 2])/(3*d*e^2*Sqrt[e*Cos[c + d*x]]) + (2*a*Sin[c + d*x])/(3*d*e*(
e*Cos[c + d*x])^(3/2))
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\int \frac{a+b\sin(c+dx)}{(e\cos(c+dx))^{5/2}}\,dx = \frac{2b}{3de(e\cos(c+dx))^{3/2}} + a\int \frac{1}{(e\cos(c+dx))^{5/2}}\,dx$$
$$= \frac{2b}{3de(e\cos(c+dx))^{3/2}} + \frac{2a\sin(c+dx)}{3de(e\cos(c+dx))^{3/2}} + \frac{a\int \frac{1}{\sqrt{e\cos(c+dx)}}\,dx}{3e^2}$$
$$= \frac{2b}{3de(e\cos(c+dx))^{3/2}} + \frac{2a\sin(c+dx)}{3de(e\cos(c+dx))^{3/2}} + \frac{\left(a\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+}}}{3e^2\sqrt{e\cos(c+dx)}}$$
$$= \frac{2b}{3de(e\cos(c+dx))^{3/2}} + \frac{2a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3de^2\sqrt{e\cos(c+dx)}} + \frac{2a\sin(c+dx)}{3de(e\cos(c+dx))^{3}}$$

Mathematica [A]
time = 0.16, size = 55, normalized size = 0.57

$$\frac{2\left(b+a\cos^{\frac{3}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)+a\sin(c+dx)\right)}{3de(e\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])/(e*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*(b + a*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + a*Sin[c + d*x]))/(
3*d*e*(e*Cos[c + d*x])^(3/2))
```

Maple [A]
time = 6.69, size = 193, normalized size = 1.99

method	result
default	$-\frac{2\left(2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ \mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\sqrt{\frac{1}{2}-\frac{\cos(dx+}{2}}}{3\left(2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))/(e*cos(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/(2*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2
*e+e)^(1/2)/e^2*(2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a*sin(1/2*d*x+1/2*c)^2-(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))*a+2*a*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+b*sin(1/2*d*
x+1/2*c))/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))/(e*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] e^(-5/2)*integrate((b*sin(d*x + c) + a)/cos(d*x + c)^(5/2), x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 100, normalized size = 1.03

$$\frac{\left(-i\sqrt{2}\,a\cos\left(dx+c\right)^{2}\operatorname{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+i\sqrt{2}\,a\cos\left(dx+c\right)^{2}\operatorname{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right))+2\left(a\sin\left(dx+c\right)+b\right)\sqrt{\cos\left(dx+c\right)}\right)e^{\left(-\frac{5}{2}\right)}}{3\,d\cos\left(dx+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))/(e*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(-I*sqrt(2)*a*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*x + c) +
I*sin(d*x + c)) + I*sqrt(2)*a*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos
(d*x + c) - I*sin(d*x + c)) + 2*(a*sin(d*x + c) + b)*sqrt(cos(d*x + c)))*e^
(-5/2)/(d*cos(d*x + c)^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))/(e*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))/(e*cos(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((b*sin(d*x + c) + a)*e^(-5/2)/cos(d*x + c)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\sin\left(c+d\,x\right)}{{\left(e\,\cos\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))/(e*cos(c + d*x))^(5/2),x)
[Out] int((a + b*sin(c + d*x))/(e*cos(c + d*x))^(5/2), x)
```

3.546 $\int \frac{a+b\sin(c+dx)}{(e\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=126

$$\frac{2b}{5de(e\cos(c+dx))^{5/2}}-\frac{6a\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5de^4\sqrt{\cos(c+dx)}}+\frac{2a\sin(c+dx)}{5de(e\cos(c+dx))^{5/2}}+\frac{6a\sin(c+dx)}{5de^3\sqrt{e\cos(c+dx)}}$$

```
[Out] 2/5*b/d/e/(e*cos(d*x+c))^(5/2)+2/5*a*sin(d*x+c)/d/e/(e*cos(d*x+c))^(5/2)+6/
5*a*sin(d*x+c)/d/e^3/(e*cos(d*x+c))^(1/2)-6/5*a*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(
1/2)/d/e^4/cos(d*x+c)^(1/2)
```

Rubi [A]

time = 0.06, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2748, 2716, 2721, 2719}

$$-\frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5de^4\sqrt{\cos(c+dx)}}+\frac{6a\sin(c+dx)}{5de^3\sqrt{e\cos(c+dx)}}+\frac{2a\sin(c+dx)}{5de(e\cos(c+dx))^{5/2}}+\frac{2b}{5de(e\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])/(e*Cos[c + d*x])^(7/2),x]
```

```
[Out] (2*b)/(5*d*e*(e*Cos[c + d*x])^(5/2)) - (6*a*Sqrt[e*Cos[c + d*x]]*EllipticE[
(c + d*x)/2, 2])/(5*d*e^4*Sqrt[Cos[c + d*x]]) + (2*a*Sin[c + d*x])/(5*d*e*(
e*Cos[c + d*x])^(5/2)) + (6*a*Sin[c + d*x])/(5*d*e^3*Sqrt[e*Cos[c + d*x]])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\int \frac{a+b\sin(c+dx)}{(e\cos(c+dx))^{7/2}}\,dx = \frac{2b}{5de(e\cos(c+dx))^{5/2}} + a\int \frac{1}{(e\cos(c+dx))^{7/2}}\,dx$$

$$= \frac{2b}{5de(e\cos(c+dx))^{5/2}} + \frac{2a\sin(c+dx)}{5de(e\cos(c+dx))^{5/2}} + \frac{(3a)\int \frac{1}{(e\cos(c+dx))^{3/2}}\,dx}{5e^2}$$

$$= \frac{2b}{5de(e\cos(c+dx))^{5/2}} + \frac{2a\sin(c+dx)}{5de(e\cos(c+dx))^{5/2}} + \frac{6a\sin(c+dx)}{5de^3\sqrt{e\cos(c+dx)}} - \frac{(3a)\int}{}$$

$$= \frac{2b}{5de(e\cos(c+dx))^{5/2}} + \frac{2a\sin(c+dx)}{5de(e\cos(c+dx))^{5/2}} + \frac{6a\sin(c+dx)}{5de^3\sqrt{e\cos(c+dx)}} - \frac{\left(3a\sqrt{e}\right.}{}$$

$$= \frac{2b}{5de(e\cos(c+dx))^{5/2}} - \frac{6a\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5de^4\sqrt{\cos(c+dx)}} + \frac{2a\sin(c+dx)}{5de(e\cos(c+dx)}$$

Mathematica [A]
time = 0.34, size = 70, normalized size = 0.56

$$\frac{4b-12a\cos^{\frac{5}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+7a\sin(c+dx)+3a\sin(3(c+dx))}{10de(e\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])/(e*Cos[c + d*x])^(7/2),x]
[Out] (4*b - 12*a*Cos[c + d*x]^(5/2)*EllipticE[(c + d*x)/2, 2] + 7*a*Sin[c + d*x]
 + 3*a*Sin[3*(c + d*x)])/(10*d*e*(e*Cos[c + d*x])^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 309 vs. $2(134) = 268$.
time = 10.35, size = 310, normalized size = 2.46

method	result

default	$-\frac{2\left(12\,\text{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,a\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-24a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))/(e*cos(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)/(
-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/e^3*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a*sin(1
/2*d*x+1/2*c)^4-24*a*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*a*sin(1/2*d*x+1/2*c)^2+24*a*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/
2*c)^4+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2))*a-8*a*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*
c)^2-b*sin(1/2*d*x+1/2*c))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))/(e*cos(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] e^(-7/2)*integrate((b*sin(d*x + c) + a)/cos(d*x + c)^(7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 118, normalized size = 0.94

$$\frac{\left(-3i\sqrt{2}\,a\cos\left(dx+c\right)^{3}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)))+3i\sqrt{2}\,a\cos\left(dx+c\right)^{3}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)))+2\left(\left(3\,a\cos\left(dx+c\right)^{2}+a\right)\sin\left(dx+c\right)+b\right)\sqrt{\cos\left(dx+c\right)}\right)e^{\left(-\frac{7}{2}\right)}}{5\,d\cos\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))/(e*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/5*(-3*I*sqrt(2)*a*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInver
se(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*I*sqrt(2)*a*cos(d*x + c)^3*we
ierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x +
c))) + 2*((3*a*cos(d*x + c)^2 + a)*sin(d*x + c) + b)*sqrt(cos(d*x + c)))*e^
(-7/2)/(d*cos(d*x + c)^3)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))/(e*cos(d*x+c))**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5990 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))/(e*cos(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)*e^(-7/2)/cos(d*x + c)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\sin\left(c+d\,x\right)}{{\left(e\,\cos\left(c+d\,x\right)\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))/(e*cos(c + d*x))^(7/2),x)
```

```
[Out] int((a + b*sin(c + d*x))/(e*cos(c + d*x))^(7/2), x)
```

3.547 $\int (e\cos(c+dx))^{7/2}(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=188

$$-\frac{26ab(e\cos(c+dx))^{9/2}}{99de}+\frac{10(11a^2+2b^2)\,e^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{231d\sqrt{e\cos(c+dx)}}+\frac{10(11a^2+2b^2)\,e^3\sqrt{e\cos(c+dx}}{231d}$$

```
[Out] -26/99*a*b*(e*cos(d*x+c))^(9/2)/d/e+2/77*(11*a^2+2*b^2)*e*(e*cos(d*x+c))^(5
/2)*sin(d*x+c)/d-2/11*b*(e*cos(d*x+c))^(9/2)*(a+b*sin(d*x+c))/d/e+10/231*(1
1*a^2+2*b^2)*e^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(
sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(e*cos(d*x+c))^(1/2)+10/231*
(11*a^2+2*b^2)*e^3*sin(d*x+c)*(e*cos(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.13, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2771, 2748, 2715, 2721, 2720}

$$\frac{10e^4(11a^2+2b^2)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{231d\sqrt{e\cos(c+dx)}}+\frac{10e^3(11a^2+2b^2)\sin(c+dx)\sqrt{e\cos(c+dx)}}{231d}+\frac{2e(11a^2+2b^2)\sin(c+dx)(e\cos(c+dx))^{5/2}}{77d}-\frac{26ab(e\cos(c+dx))^{9/2}}{99de}-\frac{2b(e\cos(c+dx))^{9/2}(a+b\sin(c+dx))}{11de}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(7/2)*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-26*a*b*(e*Cos[c + d*x])^(9/2))/(99*d*e) + (10*(11*a^2 + 2*b^2)*e^4*Sqrt[C
os[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(231*d*Sqrt[e*Cos[c + d*x]]) + (10*
(11*a^2 + 2*b^2)*e^3*Sqrt[e*Cos[c + d*x]]*Sin[c + d*x])/(231*d) + (2*(11*a^
2 + 2*b^2)*e*(e*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(77*d) - (2*b*(e*Cos[c +
d*x])^(9/2)*(a + b*Sin[c + d*x]))/(11*d*e)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2771

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(
a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rubi steps

$$\int (e\cos(c+dx))^{7/2}(a+b\sin(c+dx))^2\,dx = -\frac{2b(e\cos(c+dx))^{9/2}(a+b\sin(c+dx))}{11de}+\frac{2}{11}\int (e\cos(c+dx$$

$$= -\frac{26ab(e\cos(c+dx))^{9/2}}{99de}-\frac{2b(e\cos(c+dx))^{9/2}(a+b\sin(c+$$

$$11de$$

$$= -\frac{26ab(e\cos(c+dx))^{9/2}}{99de}+\frac{2(11a^2+2b^2)\,e(e\cos(c+dx))^{5/2}\,\mathrm{si}}{77d}$$

$$= -\frac{26ab(e\cos(c+dx))^{9/2}}{99de}+\frac{10(11a^2+2b^2)\,e^3\sqrt{e\cos(c+dx)}\,\,\mathrm{s}}{231d}$$

$$= -\frac{26ab(e\cos(c+dx))^{9/2}}{99de}+\frac{10(11a^2+2b^2)\,e^3\sqrt{e\cos(c+dx)}\,\,\mathrm{s}}{231d}$$

$$= -\frac{26ab(e\cos(c+dx))^{9/2}}{99de}+\frac{10(11a^2+2b^2)\,e^4\sqrt{\cos(c+dx)}\,F}{231d\sqrt{e\cos(c+dx)}}$$

Mathematica [A]
time = 1.92, size = 160, normalized size = 0.85

$$\frac{(e\cos(c+dx))^{7/2}\left(-154ab\sqrt{\cos(c+dx)}+40(11a^2+2b^2)\,F\left(\tfrac{1}{2}(c+dx)\middle|2\right)+\tfrac{1}{6}\sqrt{\cos(c+dx)}\,(6(572a^2+41b^2)\sin(c+dx)-14b\cos(4(c+dx))(22a+9b\sin(c+dx))+8\cos(2(c+dx))\,(-154ab+9(11a^2-5b^2)\sin(c+dx)))\right)}{924d\cos^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(7/2)*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((e*Cos[c + d*x])^(7/2)*(-154*a*b*Sqrt[Cos[c + d*x]] + 40*(11*a^2 + 2*b^2)*
EllipticF[(c + d*x)/2, 2] + (Sqrt[Cos[c + d*x]]*(6*(572*a^2 + 41*b^2)*Sin[c
 + d*x] - 14*b*Cos[4*(c + d*x)]*(22*a + 9*b*Sin[c + d*x]) + 8*Cos[2*(c + d*
x)]*(-154*a*b + 9*(11*a^2 - 5*b^2)*Sin[c + d*x])))/6))/(924*d*Cos[c + d*x]^
(7/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 472 vs. $2(192) = 384$.
time = 5.16, size = 473, normalized size = 2.52

method	result
default	$-\frac{2e^4\left(-4032b^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4928ab\left(\sin^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+10080b^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1584a^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(7/2)*(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/693/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^4*(-4032*b^
2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12-4928*a*b*sin(1/2*d*x+1/2*c)^11+1
0080*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+1584*a^2*cos(1/2*d*x+1/2*
c)*sin(1/2*d*x+1/2*c)^8+12320*a*b*sin(1/2*d*x+1/2*c)^9-9792*b^2*cos(1/2*d*x
+1/2*c)*sin(1/2*d*x+1/2*c)^8-2376*a^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)
^6-12320*a*b*sin(1/2*d*x+1/2*c)^7+4608*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1
/2*c)^6+1848*a^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+6160*a*b*sin(1/2*d
*x+1/2*c)^5-924*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+165*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))*a^2+30*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-528*a^2*cos(1/2*d*x+1/2*c
)*sin(1/2*d*x+1/2*c)^2-1540*a*b*sin(1/2*d*x+1/2*c)^3+30*b^2*cos(1/2*d*x+1/2
*c)*sin(1/2*d*x+1/2*c)^2+154*a*b*sin(1/2*d*x+1/2*c))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] e^(7/2)*integrate((b*sin(d*x + c) + a)^2*cos(d*x + c)^(7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 165, normalized size = 0.88

$$\frac{-15i\sqrt{2}\,(11\,a^2+2\,b^2)e^{\frac{7}{2}}\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+15i\sqrt{2}\,(11\,a^2+2\,b^2)e^{\frac{7}{2}}\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c))-2\left(154\,ab\cos(dx+c)^4e^{\frac{7}{2}}+3\left(21\,b^2\cos(dx+c)^4e^{\frac{7}{2}}-3\,(11\,a^2+2\,b^2)\cos(dx+c)^2e^{\frac{7}{2}}-5\,(11\,a^2+2\,b^2)e^{\frac{7}{2}}\right)\sin(dx+c)\right)\sqrt{\cos(dx+c)}}{693\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*cos(d*x+c))^(7/2)*(a+b*sin(d*x+c))^2,x, algorithm="fricas")

[Out] 1/693*(-15*I*sqrt(2)*(11*a^2 + 2*b^2)*e^(7/2)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 15*I*sqrt(2)*(11*a^2 + 2*b^2)*e^(7/2)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 2*(154*a*b*cos(d*x + c)^4*e^(7/2) + 3*(21*b^2*cos(d*x + c)^4*e^(7/2) - 3*(11*a^2 + 2*b^2)*cos(d*x + c)^2*e^(7/2) - 5*(11*a^2 + 2*b^2)*e^(7/2))*sin(d*x + c))*sqrt(cos(d*x + c)))/d

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*cos(d*x+c))**(7/2)*(a+b*sin(d*x+c))**2,x)

[Out] Exception raised: SystemError >> excessive stack use: stack is 5986 deep

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*cos(d*x+c))^(7/2)*(a+b*sin(d*x+c))^2,x, algorithm="giac")

[Out] integrate((b*sin(d*x + c) + a)^2*cos(d*x + c)^(7/2)*e^(7/2), x)

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (e\,\cos\left(c+d\,x\right))^{7/2}\,{\left(a+b\,\sin\left(c+d\,x\right)\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*cos(c + d*x))^(7/2)*(a + b*sin(c + d*x))^2,x)

[Out] int((e*cos(c + d*x))^(7/2)*(a + b*sin(c + d*x))^2, x)

3.548 $\int (e\cos(c+dx))^{5/2}(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=149

$$-\frac{22ab(e\cos(c+dx))^{7/2}}{63de}+\frac{2(9a^2+2b^2)\,e^2\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{15d\sqrt{\cos(c+dx)}}+\frac{2(9a^2+2b^2)\,e(e\cos(c+dx))^{3/2}\,\mathrm{s}}{45d}$$

```
[Out] -22/63*a*b*(e*cos(d*x+c))^(7/2)/d/e+2/45*(9*a^2+2*b^2)*e*(e*cos(d*x+c))^(3/
2)*sin(d*x+c)/d-2/9*b*(e*cos(d*x+c))^(7/2)*(a+b*sin(d*x+c))/d/e+2/15*(9*a^2
+2*b^2)*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1
/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2771, 2748, 2715, 2721, 2719}

$$\frac{2e^2(9a^2+2b^2)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)\sqrt{e\cos(c+dx)}}{15d\sqrt{\cos(c+dx)}}+\frac{2e(9a^2+2b^2)\sin(c+dx)(e\cos(c+dx))^{3/2}}{45d}-\frac{22ab(e\cos(c+dx))^{7/2}}{63de}-\frac{2b(e\cos(c+dx))^{7/2}(a+b\sin(c+dx))}{9de}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(5/2)*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-22*a*b*(e*Cos[c + d*x])^(7/2))/(63*d*e) + (2*(9*a^2 + 2*b^2)*e^2*Sqrt[e*C
os[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*d*Sqrt[Cos[c + d*x]]) + (2*(9*a
^2 + 2*b^2)*e*(e*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(45*d) - (2*b*(e*Cos[c +
 d*x])^(7/2)*(a + b*Sin[c + d*x]))/(9*d*e)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2771

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(
a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rubi steps

$$\int (e\cos(c+dx))^{5/2}(a+b\sin(c+dx))^2\,dx = -\frac{2b(e\cos(c+dx))^{7/2}(a+b\sin(c+dx))}{9de}+\frac{2}{9}\int (e\cos(c+dx)$$

$$= -\frac{22ab(e\cos(c+dx))^{7/2}}{63de}-\frac{2b(e\cos(c+dx))^{7/2}(a+b\sin(c+}{9de}$$

$$= -\frac{22ab(e\cos(c+dx))^{7/2}}{63de}+\frac{2(9a^2+2b^2)\,e(e\cos(c+dx))^{3/2}\sin}{45d}$$

$$= -\frac{22ab(e\cos(c+dx))^{7/2}}{63de}+\frac{2(9a^2+2b^2)\,e(e\cos(c+dx))^{3/2}\sin}{45d}$$

$$= -\frac{22ab(e\cos(c+dx))^{7/2}}{63de}+\frac{2(9a^2+2b^2)\,e^2\sqrt{e\cos(c+dx)}\,E(}{15d\sqrt{\cos(c+dx)}}$$

Mathematica [A]
time = 0.92, size = 113, normalized size = 0.76

$$\frac{(e\cos(c+dx))^{5/2}\left(84(9a^2+2b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+\cos^{\frac{3}{2}}(c+dx)\left(-180ab\cos(2(c+dx))+21(12a^2+b^2)\sin(c+dx)-5b(36a+7b\sin(3(c+dx)))\right)\right)}{630d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(5/2)*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((e*Cos[c + d*x])^(5/2)*(84*(9*a^2 + 2*b^2)*EllipticE[(c + d*x)/2, 2] + Cos
[c + d*x]^(3/2)*(-180*a*b*Cos[2*(c + d*x)] + 21*(12*a^2 + b^2)*Sin[c + d*x]
 - 5*b*(36*a + 7*b*Sin[3*(c + d*x)]))))/(630*d*Cos[c + d*x]^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 407 vs. $2(157) = 314$.
time = 5.08, size = 408, normalized size = 2.74

method	result
default	$\frac{2e^3\left(-1120b^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1440ab\left(\sin^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2240b^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+504a^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(5/2)*(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/315/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^3*(-1120*b^2
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10-1440*a*b*sin(1/2*d*x+1/2*c)^9+224
0*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+504*a^2*cos(1/2*d*x+1/2*c)*si
n(1/2*d*x+1/2*c)^6+2880*a*b*sin(1/2*d*x+1/2*c)^7-1568*b^2*cos(1/2*d*x+1/2*c
)*sin(1/2*d*x+1/2*c)^6-504*a^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-2160
*a*b*sin(1/2*d*x+1/2*c)^5+448*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+1
89*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))*a^2+42*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^2+126*a^2*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+720*a*b*sin(1/2*d*x+1/2*c)^3-42*b^2*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-90*a*b*sin(1/2*d*x+1/2*c))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] e^(5/2)*integrate((b*sin(d*x + c) + a)^2*cos(d*x + c)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 154, normalized size = 1.03

$$\frac{21i\sqrt{2}\,(9a^2+2b^2)e^{\frac{5}{2}}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-21i\sqrt{2}\,(9a^2+2b^2)e^{\frac{5}{2}}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))-2\left(90ab\cos(dx+c)^3e^{\frac{5}{2}}+7\left(5b^2\cos(dx+c)^3e^{\frac{5}{2}}-(9a^2+2b^2)\cos(dx+c)e^{\frac{5}{2}}\right)\sin(dx+c)\right)\sqrt{\cos(dx+c)}}{315d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/315*(21*I*sqrt(2)*(9*a^2 + 2*b^2)*e^(5/2)*weierstrassZeta(-4, 0, weierstr
assPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 21*I*sqrt(2)*(9*a^2 +
```

```
2*b^2)*e^(5/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x +
c) - I*sin(d*x + c))) - 2*(90*a*b*cos(d*x + c)^3*e^(5/2) + 7*(5*b^2*cos(d*x
 + c)^3*e^(5/2) - (9*a^2 + 2*b^2)*cos(d*x + c)*e^(5/2))*sin(d*x + c))*sqrt(
cos(d*x + c)))/d
```

Sympy [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(5/2)*(a+b*sin(d*x+c))**2,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 7319 deep
```

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+b*sin(d*x+c))^2,x, algorithm="giac")
[Out] integrate((b*sin(d*x + c) + a)^2*cos(d*x + c)^(5/2)*e^(5/2), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int (e\cos(c+dx))^{5/2}\,(a+b\sin(c+dx))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(5/2)*(a + b*sin(c + d*x))^2,x)
[Out] int((e*cos(c + d*x))^(5/2)*(a + b*sin(c + d*x))^2, x)
```

3.549 $\int (e\cos(c+dx))^{3/2}(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=149

$$-\frac{18ab(e\cos(c+dx))^{5/2}}{35de}+\frac{2(7a^2+2b^2)\,e^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{e\cos(c+dx)}}+\frac{2(7a^2+2b^2)\,e\sqrt{e\cos(c+dx)}\,\sin}{21d}$$

```
[Out] -18/35*a*b*(e*cos(d*x+c))^(5/2)/d/e-2/7*b*(e*cos(d*x+c))^(5/2)*(a+b*sin(d*x
+c))/d/e+2/21*(7*a^2+2*b^2)*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/
2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(e*cos(d*x+c)
)^(1/2)+2/21*(7*a^2+2*b^2)*e*sin(d*x+c)*(e*cos(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.11, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2771, 2748, 2715, 2721, 2720}

$$\frac{2e^2(7a^2+2b^2)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{e\cos(c+dx)}}+\frac{2e(7a^2+2b^2)\sin(c+dx)\sqrt{e\cos(c+dx)}}{21d}-\frac{18ab(e\cos(c+dx))^{5/2}}{35de}-\frac{2b(e\cos(c+dx))^{5/2}(a+b\sin(c+dx))}{7de}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-18*a*b*(e*Cos[c + d*x])^(5/2))/(35*d*e) + (2*(7*a^2 + 2*b^2)*e^2*Sqrt[Cos
[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[e*Cos[c + d*x]]) + (2*(7*a
^2 + 2*b^2)*e*Sqrt[e*Cos[c + d*x]]*Sin[c + d*x])/(21*d) - (2*b*(e*Cos[c + d
*x])^(5/2)*(a + b*Sin[c + d*x]))/(7*d*e)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2771

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(
a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rubi steps

$$\int (e\cos(c+dx))^{3/2}(a+b\sin(c+dx))^2\,dx = -\frac{2b(e\cos(c+dx))^{5/2}(a+b\sin(c+dx))}{7de}+\frac{2}{7}\int (e\cos(c+dx)$$

$$= -\frac{18ab(e\cos(c+dx))^{5/2}}{35de}-\frac{2b(e\cos(c+dx))^{5/2}(a+b\sin(c+}{7de}$$

$$= -\frac{18ab(e\cos(c+dx))^{5/2}}{35de}+\frac{2(7a^2+2b^2)\,e\sqrt{e\cos(c+dx)}\,\sin(}{21d}$$

$$= -\frac{18ab(e\cos(c+dx))^{5/2}}{35de}+\frac{2(7a^2+2b^2)\,e\sqrt{e\cos(c+dx)}\,\sin(}{21d}$$

$$= -\frac{18ab(e\cos(c+dx))^{5/2}}{35de}+\frac{2(7a^2+2b^2)\,e^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}\right.}{21d\sqrt{e\cos(c+dx)}}$$

Mathematica [**A**]
time = 1.16, size = 115, normalized size = 0.77

$$\frac{(e\cos(c+dx))^{3/2}\left(20(7a^2+2b^2)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+\sqrt{\cos(c+dx)}\,(-84ab\cos(2(c+dx))+5(28a^2+5b^2)\sin(c+dx)-3b(28a+5b\sin(3(c+dx))))\right)}{210d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((e*Cos[c + d*x])^(3/2)*(20*(7*a^2 + 2*b^2)*EllipticF[(c + d*x)/2, 2] + Sqr
t[Cos[c + d*x]]*(-84*a*b*Cos[2*(c + d*x)] + 5*(28*a^2 + 5*b^2)*Sin[c + d*x]
 - 3*b*(28*a + 5*b*Sin[3*(c + d*x)]))))/(210*d*Cos[c + d*x]^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 342 vs. $2(157) = 314$.
time = 5.21, size = 343, normalized size = 2.30

method	result
default	$-\frac{2e^{2}\left(-240b^{2}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-336ab\left(\sin^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+360b^{2}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+140a^{2}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{4}\left(\frac{dx}{2}\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(3/2)*(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/105/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^2*(-240*b^2
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-336*a*b*sin(1/2*d*x+1/2*c)^7+360*b
^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+140*a^2*cos(1/2*d*x+1/2*c)*sin(1
/2*d*x+1/2*c)^4+504*a*b*sin(1/2*d*x+1/2*c)^5-140*b^2*cos(1/2*d*x+1/2*c)*sin
(1/2*d*x+1/2*c)^4+35*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2+10*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))*b^2-70*a^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-252*a*b*sin(1/2*d*x+1
/2*c)^3+10*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+42*a*b*sin(1/2*d*x+1
/2*c))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] e^(3/2)*integrate((b*sin(d*x + c) + a)^2*cos(d*x + c)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 142, normalized size = 0.95

$$\frac{-5i\sqrt{2}\,(7a^{2}+2b^{2})e^{\frac{3}{2}}\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}\,(7a^{2}+2b^{2})e^{\frac{3}{2}}\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-2\left(42ab\cos(dx+c)^{2}e^{\frac{3}{2}}+5\left(3b^{2}\cos(dx+c)^{2}e^{\frac{3}{2}}-(7a^{2}+2b^{2})e^{\frac{3}{2}}\right)\sin(dx+c)\right)\sqrt{\cos(dx+c)}}{105d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/105*(-5*I*sqrt(2)*(7*a^2 + 2*b^2)*e^(3/2)*weierstrassPInverse(-4, 0, cos(
d*x + c) + I*sin(d*x + c)) + 5*I*sqrt(2)*(7*a^2 + 2*b^2)*e^(3/2)*weierstras
sPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 2*(42*a*b*cos(d*x + c)^2*
```

```
e^(3/2) + 5*(3*b^2*cos(d*x + c)^2*e^(3/2) - (7*a^2 + 2*b^2)*e^(3/2))*sin(d*
x + c))*sqrt(cos(d*x + c)))/d
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(3/2)*(a+b*sin(d*x+c))**2,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3064 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+b*sin(d*x+c))^2,x, algorithm="giac")
[Out] integrate((b*sin(d*x + c) + a)^2*cos(d*x + c)^(3/2)*e^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (e\,\cos\left(c+d\,x\right))^{3/2}\,{\left(a+b\,\sin\left(c+d\,x\right)\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(3/2)*(a + b*sin(c + d*x))^2,x)
[Out] int((e*cos(c + d*x))^(3/2)*(a + b*sin(c + d*x))^2, x)
```

3.550 $\int \sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=109

$$-\frac{14ab(e\cos(c+dx))^{3/2}}{15de}+\frac{2(5a^2+2b^2)\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}}-\frac{2b(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))}{5de}$$

```
[Out] -14/15*a*b*(e*cos(d*x+c))^(3/2)/d/e-2/5*b*(e*cos(d*x+c))^(3/2)*(a+b*sin(d*x
+c))/d/e+2/5*(5*a^2+2*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*
EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/
2)
```

Rubi [A]
time = 0.08, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2771, 2748, 2721, 2719}

$$\frac{2(5a^2+2b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}-\frac{14ab(e\cos(c+dx))^{3/2}}{15de}-\frac{2b(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))}{5de}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-14*a*b*(e*Cos[c + d*x])^(3/2))/(15*d*e) + (2*(5*a^2 + 2*b^2)*Sqrt[e*Cos[c
 + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]) - (2*b*(e*Cos[
c + d*x])^(3/2)*(a + b*Sin[c + d*x]))/(5*d*e)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2771

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(
a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rubi steps

$$\int \sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))^2\,dx = -\frac{2b(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))}{5de}+\frac{2}{5}\int \sqrt{e\cos(c+dx}$$

$$= -\frac{14ab(e\cos(c+dx))^{3/2}}{15de}-\frac{2b(e\cos(c+dx))^{3/2}(a+b\sin(c+d}{5de}$$

$$= -\frac{14ab(e\cos(c+dx))^{3/2}}{15de}-\frac{2b(e\cos(c+dx))^{3/2}(a+b\sin(c+d}{5de}$$

$$= -\frac{14ab(e\cos(c+dx))^{3/2}}{15de}+\frac{2(5a^2+2b^2)\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(\right.}{5d\sqrt{\cos(c+dx)}}$$

Mathematica [**A**]
time = 0.34, size = 80, normalized size = 0.73

$$\frac{\sqrt{e\cos(c+dx)}\left(6(5a^2+2b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)-2b\cos^{\frac{3}{2}}(c+dx)(10a+3b\sin(c+dx))\right)}{15d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (Sqrt[e*Cos[c + d*x]]*(6*(5*a^2 + 2*b^2)*EllipticE[(c + d*x)/2, 2] - 2*b*Co
s[c + d*x]^(3/2)*(10*a + 3*b*Sin[c + d*x])))/(15*d*Sqrt[Cos[c + d*x]])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 250 vs. 2(121) = 242.
time = 4.52, size = 251, normalized size = 2.30

method	result
default	$\frac{2e\left(-24b^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-40ab\left(\sin^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+24b^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+15\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\,(\mathrm{s}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))^2*(e*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e*(-24*b^2*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-40*a*b*sin(1/2*d*x+1/2*c)^5+24*b^2*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2+6*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))*b^2+40*a*b*sin(1/2*d*x+1/2*c)^3-6*b^2*cos(1/2*d*x+1/2*c
)*sin(1/2*d*x+1/2*c)^2-10*a*b*sin(1/2*d*x+1/2*c))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^2*(e*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] e^(1/2)*integrate((b*sin(d*x + c) + a)^2*sqrt(cos(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 126, normalized size = 1.16

$$\frac{3i\sqrt{2}\,(5\,a^2+2\,b^2)e^{\frac{1}{2}}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\,\sin{(dx+c)}))-3i\sqrt{2}\,(5\,a^2+2\,b^2)e^{\frac{1}{2}}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos{(dx+c)}-i\,\sin{(dx+c)}))-2\left(3\,b^2\cos{(dx+c)}\,e^{\frac{1}{2}}\sin{(dx+c)}+10\,ab\cos{(dx+c)}\,e^{\frac{1}{2}}\right)\sqrt{\cos{(dx+c)}}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^2*(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*I*sqrt(2)*(5*a^2 + 2*b^2)*e^(1/2)*weierstrassZeta(-4, 0, weierstras
sPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*I*sqrt(2)*(5*a^2 + 2*b
^2)*e^(1/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c)
- I*sin(d*x + c))) - 2*(3*b^2*cos(d*x + c)*e^(1/2)*sin(d*x + c) + 10*a*b*co
s(d*x + c)*e^(1/2))*sqrt(cos(d*x + c)))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{e\cos{(c+dx)}}\,(a+b\sin{(c+dx)})^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))**2*(e*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(e*cos(c + d*x))*(a + b*sin(c + d*x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^2*(e*cos(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate((b*sin(d*x + c) + a)^2*sqrt(cos(d*x + c))*e^(1/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{e\,\cos\left(c+d\,x\right)}\,{\left(a+b\,\sin\left(c+d\,x\right)\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(1/2)*(a + b*sin(c + d*x))^2,x)
[Out] int((e*cos(c + d*x))^(1/2)*(a + b*sin(c + d*x))^2, x)
```

3.551 $\int \frac{(a+b\sin(c+dx))^2}{\sqrt{e\cos(c+dx)}}\,dx$

Optimal. Leaf size=109

$$-\frac{10ab\sqrt{e\cos(c+dx)}}{3de}+\frac{2(3a^2+2b^2)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{e\cos(c+dx)}}-\frac{2b\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))}{3de}$$

```
[Out] 2/3*(3*a^2+2*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF
(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(e*cos(d*x+c))^(1/2)-10/3*a
*b*(e*cos(d*x+c))^(1/2)/d/e-2/3*b*(a+b*sin(d*x+c))*(e*cos(d*x+c))^(1/2)/d/e
```

Rubi [A]
time = 0.09, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2771, 2748, 2721, 2720}

$$\frac{2(3a^2+2b^2)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{e\cos(c+dx)}}-\frac{10ab\sqrt{e\cos(c+dx)}}{3de}-\frac{2b\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))}{3de}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^2/Sqrt[e*Cos[c + d*x]],x]
```

```
[Out] (-10*a*b*Sqrt[e*Cos[c + d*x]])/(3*d*e) + (2*(3*a^2 + 2*b^2)*Sqrt[Cos[c + d*
x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[e*Cos[c + d*x]]) - (2*b*Sqrt[e*Cos
[c + d*x]]*(a + b*Sin[c + d*x]))/(3*d*e)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2771

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(
a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rubi steps

$$\int \frac{(a+b\sin(c+dx))^2}{\sqrt{e\cos(c+dx)}}\,dx = -\frac{2b\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))}{3de} + \frac{2}{3}\int \frac{\frac{3a^2}{2}+b^2+\frac{5}{2}ab\sin(c+dx)}{\sqrt{e\cos(c+dx)}}\,dx$$

$$= -\frac{10ab\sqrt{e\cos(c+dx)}}{3de} - \frac{2b\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))}{3de} + \frac{1}{3}(3a^2+2b$$

$$= -\frac{10ab\sqrt{e\cos(c+dx)}}{3de} - \frac{2b\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))}{3de} + \frac{((3a^2+2b}{}$$

$$= -\frac{10ab\sqrt{e\cos(c+dx)}}{3de} + \frac{2(3a^2+2b^2)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{e\cos(c+dx)}} - \frac{2b\sqrt{}}{}$$

Mathematica [A]
time = 0.42, size = 75, normalized size = 0.69

$$\frac{2(3a^2+2b^2)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)-2b\cos(c+dx)(6a+b\sin(c+dx))}{3d\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])^2/Sqrt[e*Cos[c + d*x]],x]
[Out] (2*(3*a^2 + 2*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] - 2*b*Cos[c
 + d*x]*(6*a + b*Sin[c + d*x]))/(3*d*Sqrt[e*Cos[c + d*x]])
```

Maple [A]
time = 3.47, size = 210, normalized size = 1.93

method	result
default	$-\frac{2\left(-4b^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)a^2}{3\sin(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))^2/(e*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*(-4*b^2*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2+2*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2))*b^2-12*a*b*sin(1/2*d*x+1/2*c)^3+2*b^2*cos(1/2*d*x+1/2*c)*si
n(1/2*d*x+1/2*c)^2+6*a*b*sin(1/2*d*x+1/2*c))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^2/(e*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] e^(-1/2)*integrate((b*sin(d*x + c) + a)^2/sqrt(cos(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 99, normalized size = 0.91

$$\frac{\left(\sqrt{2}\,(-3i\,a^2-2i\,b^2)\mathrm{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\,\sin{(dx+c)})+\sqrt{2}\,(3i\,a^2+2i\,b^2)\mathrm{weierstrassPInverse}(-4,0,\cos{(dx+c)}-i\,\sin{(dx+c)})-2\,(b^2\sin{(dx+c)}+6\,ab)\sqrt{\cos{(dx+c)}}\right)e^{\left(-\frac{1}{2}\right)}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^2/(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(sqrt(2)*(-3*I*a^2 - 2*I*b^2)*weierstrassPInverse(-4, 0, cos(d*x + c) +
 I*sin(d*x + c)) + sqrt(2)*(3*I*a^2 + 2*I*b^2)*weierstrassPInverse(-4, 0, c
os(d*x + c) - I*sin(d*x + c)) - 2*(b^2*sin(d*x + c) + 6*a*b)*sqrt(cos(d*x +
 c)))*e^(-1/2)/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(a+b\sin{(c+dx)})^2}{\sqrt{e\cos{(c+dx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))**2/(e*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((a + b*sin(c + d*x))**2/sqrt(e*cos(c + d*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^2/(e*cos(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate((b*sin(d*x + c) + a)^2*e^(-1/2)/sqrt(cos(d*x + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,\sin\left(c+d\,x\right)\right)}^2}{\sqrt{e\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^2/(e*cos(c + d*x))^(1/2),x)
[Out] int((a + b*sin(c + d*x))^2/(e*cos(c + d*x))^(1/2), x)
```

3.552 $\int \frac{(a+b\sin(c+dx))^2}{(e\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=113

$$\frac{2ab(e\cos(c+dx))^{3/2}}{de^3}-\frac{2(a^2+2b^2)\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{de^2\sqrt{\cos(c+dx)}}+\frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))}{de\sqrt{e\cos(c+dx)}}$$

```
[Out] 2*a*b*(e*cos(d*x+c))^(3/2)/d/e^3+2*(b+a*sin(d*x+c))*(a+b*sin(d*x+c))/d/e/(e
*cos(d*x+c))^(1/2)-2*(a^2+2*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1
/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/d/e^2/cos(
d*x+c)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2770, 2748, 2721, 2719}

$$-\frac{2(a^2+2b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{de^2\sqrt{\cos(c+dx)}}+\frac{2ab(e\cos(c+dx))^{3/2}}{de^3}+\frac{2(a\sin(c+dx)+b)(a+b\sin(c+dx))}{de\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^2/(e*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*a*b*(e*Cos[c + d*x])^(3/2))/(d*e^3) - (2*(a^2 + 2*b^2)*Sqrt[e*Cos[c + d*
x]]*EllipticE[(c + d*x)/2, 2])/(d*e^2*Sqrt[Cos[c + d*x]]) + (2*(b + a*Sin[c
 + d*x])*(a + b*Sin[c + d*x]))/(d*e*Sqrt[e*Cos[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2770

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x
])^(m - 1)*((b + a*Sin[e + f*x])/(f*g*(p + 1))), x] + Dist[1/(g^2*(p + 1)),
 Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a
^2*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g}
, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p
] || IntegerQ[m])
```

Rubi steps

$$\int \frac{(a+b\sin(c+dx))^2}{(e\cos(c+dx))^{3/2}}\,dx = \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))}{de\sqrt{e\cos(c+dx)}} - \frac{2\int \sqrt{e\cos(c+dx)}\left(\frac{a^2}{2}+b^2+\frac{3}{2}ab\,\mathrm{s}\right.}{e^2}$$

$$= \frac{2ab(e\cos(c+dx))^{3/2}}{de^3} + \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))}{de\sqrt{e\cos(c+dx)}} - \frac{(a^2+2b^2)\int}{}$$

$$= \frac{2ab(e\cos(c+dx))^{3/2}}{de^3} + \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))}{de\sqrt{e\cos(c+dx)}} - \frac{\left((a^2+2b^2)\right.}{}$$

$$= \frac{2ab(e\cos(c+dx))^{3/2}}{de^3} - \frac{2(a^2+2b^2)\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{de^2\sqrt{\cos(c+dx)}} + \frac{2(b+}{}$$

Mathematica [A]
time = 0.24, size = 71, normalized size = 0.63

$$\frac{4ab-2(a^2+2b^2)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+2(a^2+b^2)\sin(c+dx)}{de\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])^2/(e*Cos[c + d*x])^(3/2),x]
```

```
[Out] (4*a*b - 2*(a^2 + 2*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 2*(
a^2 + b^2)*Sin[c + d*x])/(d*e*Sqrt[e*Cos[c + d*x]])
```

Maple [A]
time = 5.94, size = 197, normalized size = 1.74

method	result

default	$-\frac{2\left(\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\operatorname{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)a^2+2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\right.}}{e\sqrt{-2\left(\sin^2\left(\frac{dx}{2}\right.\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))^2/(e*cos(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/e/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/sin(1/2*d*x+1/2*c)*((sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))*a^2+2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-2*a^2*cos(1/2*d*x+1/2*c)*sin(
1/2*d*x+1/2*c)^2-2*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-2*a*b*sin(1/
2*d*x+1/2*c))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^2/(e*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] e^(-3/2)*integrate((b*sin(d*x + c) + a)^2/cos(d*x + c)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 128, normalized size = 1.13

$$\frac{\left(\sqrt{2}\left(-i\,a^2-2i\,b^2\right)\cos\left(dx+c\right)\operatorname{weierstrassZeta}\left(-4,0,\operatorname{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)\right)+\sqrt{2}\left(i\,a^2+2i\,b^2\right)\cos\left(dx+c\right)\operatorname{weierstrassZeta}\left(-4,0,\operatorname{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)\right)+2\left(2\,ab+\left(a^2+b^2\right)\sin\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)}\right)e^{\left(-\frac{3}{2}\right)}}{d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^2/(e*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] (sqrt(2)*(-I*a^2 - 2*I*b^2)*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrass
PInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + sqrt(2)*(I*a^2 + 2*I*b^2)
*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c
) - I*sin(d*x + c))) + 2*(2*a*b + (a^2 + b^2)*sin(d*x + c))*sqrt(cos(d*x +
c)))*e^(-3/2)/(d*cos(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))**2/(e*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^2/(e*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^2*e^(-3/2)/cos(d*x + c)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\sin\left(c+d\,x\right))^2}{{\left(e\,\cos\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^2/(e*cos(c + d*x))^(3/2),x)
```

```
[Out] int((a + b*sin(c + d*x))^2/(e*cos(c + d*x))^(3/2), x)
```

3.553 $\int \frac{(a+b\sin(c+dx))^2}{(e\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=119

$$\frac{2ab\sqrt{e\cos(c+dx)}}{3de^3}+\frac{2(a^2-2b^2)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3de^2\sqrt{e\cos(c+dx)}}+\frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))}{3de(e\cos(c+dx))^{3/2}}$$

```
[Out] 2/3*(b+a*sin(d*x+c))*(a+b*sin(d*x+c))/d/e/(e*cos(d*x+c))^(3/2)+2/3*(a^2-2*b
^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1
/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/e^2/(e*cos(d*x+c))^(1/2)+2/3*a*b*(e*cos(d
*x+c))^(1/2)/d/e^3
```

Rubi [A]
time = 0.09, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2770, 2748, 2721, 2720}

$$\frac{2(a^2-2b^2)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3de^2\sqrt{e\cos(c+dx)}}+\frac{2ab\sqrt{e\cos(c+dx)}}{3de^3}+\frac{2(a\sin(c+dx)+b)(a+b\sin(c+dx))}{3de(e\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^2/(e*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*a*b*Sqrt[e*Cos[c + d*x]])/(3*d*e^3) + (2*(a^2 - 2*b^2)*Sqrt[Cos[c + d*x]
]*EllipticF[(c + d*x)/2, 2])/(3*d*e^2*Sqrt[e*Cos[c + d*x]]) + (2*(b + a*Sin
[c + d*x])*(a + b*Sin[c + d*x]))/(3*d*e*(e*Cos[c + d*x])^(3/2))
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2770

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x
])^(m - 1)*((b + a*Sin[e + f*x])/(f*g*(p + 1))), x] + Dist[1/(g^2*(p + 1)),
 Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a
^2*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g}
, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p
] || IntegerQ[m])
```

Rubi steps

$$\int \frac{(a+b\sin(c+dx))^2}{(e\cos(c+dx))^{5/2}}\,dx = \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))}{3de(e\cos(c+dx))^{3/2}} - \frac{2\int \frac{-\frac{a^2}{2}+b^2+\frac{1}{2}ab\sin(c+dx)}{\sqrt{e\cos(c+dx)}}\,dx}{3e^2}$$

$$= \frac{2ab\sqrt{e\cos(c+dx)}}{3de^3} + \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))}{3de(e\cos(c+dx))^{3/2}} + \frac{(a^2-2b^2)\int$$

$$= \frac{2ab\sqrt{e\cos(c+dx)}}{3de^3} + \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))}{3de(e\cos(c+dx))^{3/2}} + \frac{\left((a^2-2b^2)\right.$$

$$= \frac{2ab\sqrt{e\cos(c+dx)}}{3de^3} + \frac{2(a^2-2b^2)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3de^2\sqrt{e\cos(c+dx)}} + \frac{2(b+a\,\mathrm{s}$$

Mathematica [A]
time = 0.27, size = 72, normalized size = 0.61

$$\frac{2\left(2ab+(a^2-2b^2)\cos^{\frac{3}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)+(a^2+b^2)\sin(c+dx)\right)}{3de(e\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])^2/(e*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*(2*a*b + (a^2 - 2*b^2)*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + (a
^2 + b^2)*Sin[c + d*x]))/(3*d*e*(e*Cos[c + d*x])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 332 vs. 2(131) = 262.
time = 7.82, size = 333, normalized size = 2.80

method	result

default	$-\frac{2\left(2\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ \mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ a^2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4\sqrt{2\left(\sin^2\left(\frac{dx}{2}\right.\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))^2/(e*cos(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/(2*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2
*e+e)^(1/2)/e^2*(2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*sin(1/2*d*x+1/2*c)^2-4*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*b^2*sin(1/2*d*x+1/2*c)^2-(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2+
2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))*b^2+2*a^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^
2+2*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+2*a*b*sin(1/2*d*x+1/2*c))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^2/(e*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] e^(-5/2)*integrate((b*sin(d*x + c) + a)^2/cos(d*x + c)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 127, normalized size = 1.07

$$\frac{\left(\sqrt{2}\left(-i\,a^2+2i\,b^2\right)\cos\left(dx+c\right)^2\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)+\sqrt{2}\left(i\,a^2-2i\,b^2\right)\cos\left(dx+c\right)^2\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)+2\left(2\,ab+\left(a^2+b^2\right)\sin\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)}\right)e^{\left(-\frac{5}{2}\right)}}{3\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^2/(e*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(sqrt(2)*(-I*a^2 + 2*I*b^2)*cos(d*x + c)^2*weierstrassPInverse(-4, 0, c
os(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(I*a^2 - 2*I*b^2)*cos(d*x + c)^2*we
ierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*(2*a*b + (a^2 +
 b^2)*sin(d*x + c))*sqrt(cos(d*x + c)))*e^(-5/2)/(d*cos(d*x + c)^2)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))**2/(e*cos(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3881 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^2/(e*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^2*e^(-5/2)/cos(d*x + c)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,\sin\left(c+d\,x\right)\right)^2}{\left(e\,\cos\left(c+d\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^2/(e*cos(c + d*x))^(5/2),x)
```

```
[Out] int((a + b*sin(c + d*x))^2/(e*cos(c + d*x))^(5/2), x)
```

3.554 $\int \frac{(a+b\sin(c+dx))^2}{(e\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=160

$$\frac{2ab}{5de^3\sqrt{e\cos(c+dx)}}-\frac{2(3a^2-2b^2)\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5de^4\sqrt{\cos(c+dx)}}+\frac{2(3a^2-2b^2)\sin(c+dx)}{5de^3\sqrt{e\cos(c+dx)}}+\frac{2(b+a\sin($$

```
[Out] 2/5*(b+a*sin(d*x+c))*(a+b*sin(d*x+c))/d/e/(e*cos(d*x+c))^(5/2)+2/5*a*b/d/e^
3/(e*cos(d*x+c))^(1/2)+2/5*(3*a^2-2*b^2)*sin(d*x+c)/d/e^3/(e*cos(d*x+c))^(1
/2)-2/5*(3*a^2-2*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellip
ticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/d/e^4/cos(d*x+c)^(1/2
)
```

Rubi [A]
time = 0.12, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2770, 2748, 2716, 2721, 2719}

$$-\frac{2(3a^2-2b^2)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5de^4\sqrt{\cos(c+dx)}}+\frac{2(3a^2-2b^2)\sin(c+dx)}{5de^3\sqrt{e\cos(c+dx)}}+\frac{2ab}{5de^3\sqrt{e\cos(c+dx)}}+\frac{2(a\sin(c+dx)+b)(a+b\sin(c+dx))}{5de(e\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^2/(e*Cos[c + d*x])^(7/2),x]
```

```
[Out] (2*a*b)/(5*d*e^3*Sqrt[e*Cos[c + d*x]]) - (2*(3*a^2 - 2*b^2)*Sqrt[e*Cos[c +
d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*e^4*Sqrt[Cos[c + d*x]]) + (2*(3*a^2 -
 2*b^2)*Sin[c + d*x])/(5*d*e^3*Sqrt[e*Cos[c + d*x]]) + (2*(b + a*Sin[c + d*
x])*(a + b*Sin[c + d*x]))/(5*d*e*(e*Cos[c + d*x])^(5/2))
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2770

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x
])^(m - 1)*((b + a*Sin[e + f*x])/(f*g*(p + 1))), x] + Dist[1/(g^2*(p + 1)),
 Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a
^2*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g}
, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p
] || IntegerQ[m])
```

Rubi steps

$$\int \frac{(a+b\sin(c+dx))^2}{(e\cos(c+dx))^{7/2}}\,dx = \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))}{5de(e\cos(c+dx))^{5/2}} - \frac{2\int \frac{-\frac{3a^2}{2}+b^2-\frac{1}{2}ab\sin(c+dx)}{(e\cos(c+dx))^{3/2}}\,dx}{5e^2}$$

$$= \frac{2ab}{5de^3\sqrt{e\cos(c+dx)}} + \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))}{5de(e\cos(c+dx))^{5/2}} + \frac{(3a^2-2b^2)\ \ldots}{\ldots}$$

$$= \frac{2ab}{5de^3\sqrt{e\cos(c+dx)}} + \frac{2(3a^2-2b^2)\sin(c+dx)}{5de^3\sqrt{e\cos(c+dx)}} + \frac{2(b+a\sin(c+dx))(a+b\ldots}{5de(e\cos(c+dx)\ldots}$$

$$= \frac{2ab}{5de^3\sqrt{e\cos(c+dx)}} + \frac{2(3a^2-2b^2)\sin(c+dx)}{5de^3\sqrt{e\cos(c+dx)}} + \frac{2(b+a\sin(c+dx))(a+b\ldots}{5de(e\cos(c+dx)\ldots}$$

$$= \frac{2ab}{5de^3\sqrt{e\cos(c+dx)}} - \frac{2(3a^2-2b^2)\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5de^4\sqrt{\cos(c+dx)}} + \frac{2(3a\ldots}{5de\ldots}$$

Mathematica [A]
time = 0.57, size = 105, normalized size = 0.66

$$\frac{8ab-4(3a^2-2b^2)\cos^{\frac{5}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+(7a^2+2b^2)\sin(c+dx)+3a^2\sin(3(c+dx))-2b^2\sin(3(c+dx))}{10de(e\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])^2/(e*Cos[c + d*x])^(7/2),x]
```

[Out] (8*a*b - 4*(3*a^2 - 2*b^2)*Cos[c + d*x]^(5/2)*EllipticE[(c + d*x)/2, 2] + (7*a^2 + 2*b^2)*Sin[c + d*x] + 3*a^2*Sin[3*(c + d*x)] - 2*b^2*Sin[3*(c + d*x)])/(10*d*e*(e*Cos[c + d*x])^(5/2))

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 563 vs. $2(168) = 336$.
time = 14.66, size = 564, normalized size = 3.52

method	result
default	$-\frac{2\left(12\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,a^2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-8\,\mathrm{EllipticE}\left(\cos\left(\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sin(d*x+c))^2/(e*cos(d*x+c))^(7/2),x,method=_RETURNVERBOSE)

[Out] -2/5/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/e^3*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a^2*sin(1/2*d*x+1/2*c)^4-8*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*b^2*sin(1/2*d*x+1/2*c)^4-24*a^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+16*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a^2*sin(1/2*d*x+1/2*c)^2+8*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*b^2*sin(1/2*d*x+1/2*c)^2+24*a^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-16*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-8*a^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+2*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-2*a*b*sin(1/2*d*x+1/2*c))/d

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(d*x+c))^2/(e*cos(d*x+c))^(7/2),x, algorithm="maxima")

[Out] e^(-7/2)*integrate((b*sin(d*x + c) + a)^2/cos(d*x + c)^(7/2), x)

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 153, normalized size = 0.96

$$\frac{\left(\sqrt{2}\left(-3i\,a^2+2i\,b^2\right)\cos\left(dx+c\right)^3\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)))+\sqrt{2}\left(3i\,a^2-2i\,b^2\right)\cos\left(dx+c\right)^3\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)))+2\left(2\,ab+\left(\left(3\,a^2-2\,b^2\right)\cos\left(dx+c\right)^2+a^2+b^2\right)\sin\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)}\right)e^{\left(-\frac{7}{2}\right)}}{5\,d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^2/(e*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/5*(sqrt(2)*(-3*I*a^2 + 2*I*b^2)*cos(d*x + c)^3*weierstrassZeta(-4, 0, wei
erstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + sqrt(2)*(3*I*a^2
- 2*I*b^2)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0,
 cos(d*x + c) - I*sin(d*x + c))) + 2*(2*a*b + ((3*a^2 - 2*b^2)*cos(d*x + c)
^2 + a^2 + b^2)*sin(d*x + c))*sqrt(cos(d*x + c)))*e^(-7/2)/(d*cos(d*x + c)^
3)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))**2/(e*cos(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^2/(e*cos(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^2*e^(-7/2)/cos(d*x + c)^(7/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\sin\left(c+d\,x\right))^2}{(e\,\cos\left(c+d\,x\right))^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^2/(e*cos(c + d*x))^(7/2),x)
```

```
[Out] int((a + b*sin(c + d*x))^2/(e*cos(c + d*x))^(7/2), x)
```

3.555 $\int (e\cos(c+dx))^{7/2}(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=237

$$-\frac{2b(177a^2+44b^2)\,(e\cos(c+dx))^{9/2}}{1287de}+\frac{10a(11a^2+6b^2)\,e^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d\sqrt{e\cos(c+dx)}}+\frac{10a(11a^2+6b^2)\,e$$

```
[Out] -2/1287*b*(177*a^2+44*b^2)*(e*cos(d*x+c))^(9/2)/d/e+2/77*a*(11*a^2+6*b^2)*e
*(e*cos(d*x+c))^(5/2)*sin(d*x+c)/d-34/143*a*b*(e*cos(d*x+c))^(9/2)*(a+b*sin
(d*x+c))/d/e-2/13*b*(e*cos(d*x+c))^(9/2)*(a+b*sin(d*x+c))^2/d/e+10/231*a*(1
1*a^2+6*b^2)*e^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(
sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(e*cos(d*x+c))^(1/2)+10/231*
a*(11*a^2+6*b^2)*e^3*sin(d*x+c)*(e*cos(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.21, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2771, 2941, 2748, 2715, 2721, 2720}

$$\frac{10ae^4(11a^2+6b^2)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d\sqrt{e\cos(c+dx)}}+\frac{10ae^3(11a^2+6b^2)\sin(c+dx)\sqrt{e\cos(c+dx)}}{231d}-\frac{2b(177a^2+44b^2)\,(e\cos(c+dx))^{9/2}}{1287de}+\frac{2ae(11a^2+6b^2)\sin(c+dx)(e\cos(c+dx))^{5/2}}{77d}-\frac{2b(e\cos(c+dx))^{9/2}(a+b\sin(c+dx))^2}{13de}-\frac{34ab(e\cos(c+dx))^{9/2}(a+b\sin(c+dx))}{143de}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(7/2)*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-2*b*(177*a^2 + 44*b^2)*(e*Cos[c + d*x])^(9/2))/(1287*d*e) + (10*a*(11*a^2
 + 6*b^2)*e^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(231*d*Sqrt[e*C
os[c + d*x]]) + (10*a*(11*a^2 + 6*b^2)*e^3*Sqrt[e*Cos[c + d*x]]*Sin[c + d*x
])/(231*d) + (2*a*(11*a^2 + 6*b^2)*e*(e*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(
77*d) - (34*a*b*(e*Cos[c + d*x])^(9/2)*(a + b*Sin[c + d*x]))/(143*d*e) - (2
*b*(e*Cos[c + d*x])^(9/2)*(a + b*Sin[c + d*x])^2)/(13*d*e)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
```

```
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2771

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(
a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rule 2941

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp
[a*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x],
x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
&&  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Si
mplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\begin{aligned}
\int (e\cos(c+dx))^{7/2}(a+b\sin(c+dx))^3\,dx &= -\frac{2b(e\cos(c+dx))^{9/2}(a+b\sin(c+dx))^2}{13de}+\frac{2}{13}\int (e\cos(c+dx \\
&= -\frac{34ab(e\cos(c+dx))^{9/2}(a+b\sin(c+dx))}{143de}-\frac{2b(e\cos(c+dx))}{} \\
&= -\frac{2b(177a^2+44b^2)\,(e\cos(c+dx))^{9/2}}{1287de}-\frac{34ab(e\cos(c+dx))^{9/2}(}{143de} \\
&= -\frac{2b(177a^2+44b^2)\,(e\cos(c+dx))^{9/2}}{1287de}+\frac{2a(11a^2+6b^2)\,e(e\cos(}{77} \\
&= -\frac{2b(177a^2+44b^2)\,(e\cos(c+dx))^{9/2}}{1287de}+\frac{10a(11a^2+6b^2)\,e^3\sqrt{e\,c}}{23} \\
&= -\frac{2b(177a^2+44b^2)\,(e\cos(c+dx))^{9/2}}{1287de}+\frac{10a(11a^2+6b^2)\,e^3\sqrt{e\,c}}{23} \\
&= -\frac{2b(177a^2+44b^2)\,(e\cos(c+dx))^{9/2}}{1287de}+\frac{10a(11a^2+6b^2)\,e^4\sqrt{\mathrm{co}}}{231d\sqrt{e}}
\end{aligned}$$

Mathematica [**A**]
time = 2.04, size = 205, normalized size = 0.86

$$\frac{(e\cos(c+dx))^{7/2}\left(-154b(78a^2+11b^2)\sqrt{\cos(c+dx)}+2080(11a^3+6ab^2)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{1}{3}\sqrt{\cos(c+dx)}\,(-77b(624a^2+73b^2)\cos(2(c+dx))+154b(-78a^2+b^2)\cos(4(c+dx))+693b^3\cos(6(c+dx))+156a(506a^2+213b^2)\sin(c+dx)+234a(44a^2-39b^2)\sin(3(c+dx))-4914ab^2\sin(5(c+dx)))\right)}{48048d\cos^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(7/2)*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((e*Cos[c + d*x])^(7/2)*(-154*b*(78*a^2 + 11*b^2)*Sqrt[Cos[c + d*x]] + 2080
*(11*a^3 + 6*a*b^2)*EllipticF[(c + d*x)/2, 2] + (Sqrt[Cos[c + d*x]]*(-77*b*
(624*a^2 + 73*b^2)*Cos[2*(c + d*x)] + 154*b*(-78*a^2 + b^2)*Cos[4*(c + d*x)
] + 693*b^3*Cos[6*(c + d*x)] + 156*a*(506*a^2 + 213*b^2)*Sin[c + d*x] + 234
*a*(44*a^2 - 39*b^2)*Sin[3*(c + d*x)] - 4914*a*b^2*Sin[5*(c + d*x)]))/3))/(
48048*d*Cos[c + d*x]^(7/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 617 vs. 2(237) = 474.
time = 11.38, size = 618, normalized size = 2.61

method	result

default	$-\frac{2e^4\left(-240240a^2b\left(\sin^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+20592a^3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3003a^2b\sin\left(\frac{dx}{2}+\frac{c}{2}\right)-96096a^2b\left(\sin^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-157\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(7/2)*(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/9009/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^4*(-6864*a
^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-96096*a^2*b*sin(1/2*d*x+1/2*c)^1
1+240240*a^2*b*sin(1/2*d*x+1/2*c)^9-240240*a^2*b*sin(1/2*d*x+1/2*c)^7+12012
0*a^2*b*sin(1/2*d*x+1/2*c)^5-30030*a^2*b*sin(1/2*d*x+1/2*c)^3+3003*a^2*b*si
n(1/2*d*x+1/2*c)+20592*a^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-30888*a^
3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+24024*a^3*cos(1/2*d*x+1/2*c)*sin(
1/2*d*x+1/2*c)^4+1170*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2-310464*b^3*sin(1/2*d*x
+1/2*c)^13+433664*b^3*sin(1/2*d*x+1/2*c)^11+88704*b^3*sin(1/2*d*x+1/2*c)^15
-308000*b^3*sin(1/2*d*x+1/2*c)^9+113960*b^3*sin(1/2*d*x+1/2*c)^7-18172*b^3*
sin(1/2*d*x+1/2*c)^5-308*b^3*sin(1/2*d*x+1/2*c)^3+308*b^3*sin(1/2*d*x+1/2*c
)-381888*a*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+179712*a*b^2*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-36036*a*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x
+1/2*c)^4+1170*a*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+2145*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2))*a^3-157248*a*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12+3
93120*a*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] e^(7/2)*integrate((b*sin(d*x + c) + a)^3*cos(d*x + c)^(7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 193, normalized size = 0.81

$$\frac{-195i\sqrt{2}\,(11\,a^3+6\,ab^2)e^{\frac{7}{2}}\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+195i\sqrt{2}\,(11\,a^3+6\,ab^2)e^{\frac{7}{2}}\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c))+2\left(693\,b^3\cos(dx+c)^6\,e^{\frac{7}{2}}-1001\,(3\,a^2b+b^3)\cos(dx+c)^4\,e^{\frac{7}{2}}-39\left(63\,ab^2\cos(dx+c)^4\,e^{\frac{7}{2}}-3\,(11\,a^3+6\,ab^2)\cos(dx+c)^2\,e^{\frac{7}{2}}-5\,(11\,a^3+6\,ab^2)e^{\frac{7}{2}}\right)\sin(dx+c)\right)\sqrt{\cos(dx+c)}}{9009\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/9009*(-195*I*sqrt(2)*(11*a^3 + 6*a*b^2)*e^(7/2)*weierstrassPInverse(-4, 0
, cos(d*x + c) + I*sin(d*x + c)) + 195*I*sqrt(2)*(11*a^3 + 6*a*b^2)*e^(7/2)
*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*(693*b^3*cos
(d*x + c)^6*e^(7/2) - 1001*(3*a^2*b + b^3)*cos(d*x + c)^4*e^(7/2) - 39*(63*
a*b^2*cos(d*x + c)^4*e^(7/2) - 3*(11*a^3 + 6*a*b^2)*cos(d*x + c)^2*e^(7/2)
- 5*(11*a^3 + 6*a*b^2)*e^(7/2))*sin(d*x + c))*sqrt(cos(d*x + c)))/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(7/2)*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 7316 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^3*cos(d*x + c)^(7/2)*e^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (e\,\cos\left(c+d\,x\right))^{7/2}\,\left(a+b\,\sin\left(c+d\,x\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(7/2)*(a + b*sin(c + d*x))^3,x)
```

```
[Out] int((e*cos(c + d*x))^(7/2)*(a + b*sin(c + d*x))^3, x)
```

3.556 $\int (e\cos(c+dx))^{5/2}(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=197

$$-\frac{2b(43a^2+12b^2)\,(e\cos(c+dx))^{7/2}}{231de}+\frac{2a(3a^2+2b^2)\,e^2\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}}+\frac{2a(3a^2+2b^2)\,e(e\,\mathrm{c}}{}$$

```
[Out] -2/231*b*(43*a^2+12*b^2)*(e*cos(d*x+c))^(7/2)/d/e+2/15*a*(3*a^2+2*b^2)*e*(e
*cos(d*x+c))^(3/2)*sin(d*x+c)/d-10/33*a*b*(e*cos(d*x+c))^(7/2)*(a+b*sin(d*x
+c))/d/e-2/11*b*(e*cos(d*x+c))^(7/2)*(a+b*sin(d*x+c))^2/d/e+2/5*a*(3*a^2+2*
b^2)*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*
d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A]

time = 0.19, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2771, 2941, 2748, 2715, 2721, 2719}

$$\frac{2ae^2(3a^2+2b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}-\frac{2b(43a^2+12b^2)\,(e\cos(c+dx))^{7/2}}{231de}+\frac{2ae(3a^2+2b^2)\sin(c+dx)(e\cos(c+dx))^{3/2}}{15d}-\frac{2b(e\cos(c+dx))^{7/2}(a+b\sin(c+dx))^2}{11de}-\frac{10ab(e\cos(c+dx))^{7/2}(a+b\sin(c+dx))}{33de}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(5/2)*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-2*b*(43*a^2 + 12*b^2)*(e*Cos[c + d*x])^(7/2))/(231*d*e) + (2*a*(3*a^2 + 2
*b^2)*e^2*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c +
 d*x]]) + (2*a*(3*a^2 + 2*b^2)*e*(e*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(15*d
) - (10*a*b*(e*Cos[c + d*x])^(7/2)*(a + b*Sin[c + d*x]))/(33*d*e) - (2*b*(e
*Cos[c + d*x])^(7/2)*(a + b*Sin[c + d*x])^2)/(11*d*e)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2771

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(
a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rule 2941

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp
[a*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x],
x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
&&  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Si
mplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int (e\cos(c+dx))^{5/2}(a+b\sin(c+dx))^3\,dx = -\frac{2b(e\cos(c+dx))^{7/2}(a+b\sin(c+dx))^2}{11de}+\frac{2}{11}\int (e\cos(c+dx$$

$$= -\frac{10ab(e\cos(c+dx))^{7/2}(a+b\sin(c+dx))}{33de}-\frac{2b(e\cos(c+dx))}{}$$

$$= -\frac{2b(43a^2+12b^2)\,(e\cos(c+dx))^{7/2}}{231de}-\frac{10ab(e\cos(c+dx))^{7/2}(a}{33de}$$

$$= -\frac{2b(43a^2+12b^2)\,(e\cos(c+dx))^{7/2}}{231de}+\frac{2a(3a^2+2b^2)\,e(e\cos(c+}{15d}$$

$$= -\frac{2b(43a^2+12b^2)\,(e\cos(c+dx))^{7/2}}{231de}+\frac{2a(3a^2+2b^2)\,e(e\cos(c+}{15d}$$

$$= -\frac{2b(43a^2+12b^2)\,(e\cos(c+dx))^{7/2}}{231de}+\frac{2a(3a^2+2b^2)\,e^2\sqrt{e\cos(}}{5d\sqrt{\cos}}$$

Mathematica [A]
time = 1.42, size = 150, normalized size = 0.76

$$\frac{(e\cos(c+dx))^{5/2}\left(1848(3a^3+2ab^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+\cos^{\frac{3}{2}}(c+dx)\left(-1980a^2b-345b^3-60(33a^2b+4b^3)\cos(2(c+dx))+105b^3\cos(4(c+dx))+1848a^3\sin(c+dx)+462ab^2\sin(c+dx)-770ab^2\sin(3(c+dx))\right)\right)}{4620d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(5/2)*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((e*Cos[c + d*x])^(5/2)*(1848*(3*a^3 + 2*a*b^2)*EllipticE[(c + d*x)/2, 2] +
 Cos[c + d*x]^(3/2)*(-1980*a^2*b - 345*b^3 - 60*(33*a^2*b + 4*b^3)*Cos[2*(c
 + d*x)] + 105*b^3*Cos[4*(c + d*x)] + 1848*a^3*Sin[c + d*x] + 462*a*b^2*Sin
[c + d*x] - 770*a*b^2*Sin[3*(c + d*x)])))/(4620*d*Cos[c + d*x]^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 533 vs. 2(201) = 402.
time = 9.69, size = 534, normalized size = 2.71

method	result
default	$\frac{2e^3\left(6720b^3\left(\sin^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-12320a\,b^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-20160b^3\left(\sin^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-7920a^2b\left(\sin^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+24640\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(5/2)*(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/1155/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^3*(6720*b^3
*sin(1/2*d*x+1/2*c)^13-12320*a*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10
-20160*b^3*sin(1/2*d*x+1/2*c)^11-7920*a^2*b*sin(1/2*d*x+1/2*c)^9+24640*a*b^
2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+22560*b^3*sin(1/2*d*x+1/2*c)^9+18
48*a^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+15840*a^2*b*sin(1/2*d*x+1/2*
c)^7-17248*a*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-11520*b^3*sin(1/2*
d*x+1/2*c)^7-1848*a^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-11880*a^2*b*s
in(1/2*d*x+1/2*c)^5+4928*a*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+2340
*b^3*sin(1/2*d*x+1/2*c)^5+693*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3+462*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2
*c),2^(1/2))*a*b^2+462*a^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+3960*a^2
*b*sin(1/2*d*x+1/2*c)^3-462*a*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+6
0*b^3*sin(1/2*d*x+1/2*c)^3-495*a^2*b*sin(1/2*d*x+1/2*c)-60*b^3*sin(1/2*d*x+
1/2*c))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] e^(5/2)*integrate((b*sin(d*x + c) + a)^3*cos(d*x + c)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 181, normalized size = 0.92

$$\frac{231i\sqrt{2}\,(3\,a^3+2\,ab^2)e^{\frac{5}{2}}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)))-231i\sqrt{2}\,(3\,a^3+2\,ab^2)e^{\frac{5}{2}}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c)))+2\left(105\,b^3\cos(dx+c)^5\,e^{\frac{5}{2}}-165\,(3\,a^2b+b^3)\cos(dx+c)^3\,e^{\frac{5}{2}}-77\left(5\,ab^2\cos(dx+c)^3\,e^{\frac{5}{2}}-(3\,a^3+2\,ab^2)\cos(dx+c)\,e^{\frac{5}{2}}\right)\sin(dx+c)\right)\sqrt{\cos(dx+c)}}{1155\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/1155*(231*I*sqrt(2)*(3*a^3 + 2*a*b^2)*e^(5/2)*weierstrassZeta(-4, 0, weie
rstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 231*I*sqrt(2)*(3*a
^3 + 2*a*b^2)*e^(5/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos
(d*x + c) - I*sin(d*x + c))) + 2*(105*b^3*cos(d*x + c)^5*e^(5/2) - 165*(3*a
^2*b + b^3)*cos(d*x + c)^3*e^(5/2) - 77*(5*a*b^2*cos(d*x + c)^3*e^(5/2) - (
3*a^3 + 2*a*b^2)*cos(d*x + c)*e^(5/2))*sin(d*x + c))*sqrt(cos(d*x + c)))/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(5/2)*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3061 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^3*cos(d*x + c)^(5/2)*e^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (e\,\cos(c+d\,x))^{5/2}\,(a+b\,\sin(c+d\,x))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(5/2)*(a + b*sin(c + d*x))^3,x)
```

```
[Out] int((e*cos(c + d*x))^(5/2)*(a + b*sin(c + d*x))^3, x)
```

3.557 $\int (e\cos(c+dx))^{3/2}(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=197

$$-\frac{2b(89a^2+28b^2)\,(e\cos(c+dx))^{5/2}}{315de}+\frac{2a(7a^2+6b^2)\,e^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{e\cos(c+dx)}}+\frac{2a(7a^2+6b^2)\,e\sqrt{e\cos}}{}$$

```
[Out] -2/315*b*(89*a^2+28*b^2)*(e*cos(d*x+c))^(5/2)/d/e-26/63*a*b*(e*cos(d*x+c))^
(5/2)*(a+b*sin(d*x+c))/d/e-2/9*b*(e*cos(d*x+c))^(5/2)*(a+b*sin(d*x+c))^2/d/
e+2/21*a*(7*a^2+6*b^2)*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*
EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(e*cos(d*x+c))^(1/
2)+2/21*a*(7*a^2+6*b^2)*e*sin(d*x+c)*(e*cos(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.18, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2771, 2941, 2748, 2715, 2721, 2720}

$$\frac{2ae^2(7a^2+6b^2)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{e\cos(c+dx)}}-\frac{2b(89a^2+28b^2)\,(e\cos(c+dx))^{5/2}}{315de}+\frac{2ae(7a^2+6b^2)\sin(c+dx)\sqrt{e\cos(c+dx)}}{21d}-\frac{2b(e\cos(c+dx))^{5/2}(a+b\sin(c+dx))^2}{9de}-\frac{26ab(e\cos(c+dx))^{5/2}(a+b\sin(c+dx))}{63de}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-2*b*(89*a^2 + 28*b^2)*(e*Cos[c + d*x])^(5/2))/(315*d*e) + (2*a*(7*a^2 + 6
*b^2)*e^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[e*Cos[c
+ d*x]]) + (2*a*(7*a^2 + 6*b^2)*e*Sqrt[e*Cos[c + d*x]]*Sin[c + d*x])/(21*d)
 - (26*a*b*(e*Cos[c + d*x])^(5/2)*(a + b*Sin[c + d*x]))/(63*d*e) - (2*b*(e*
Cos[c + d*x])^(5/2)*(a + b*Sin[c + d*x])^2)/(9*d*e)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2771

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(
a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rule 2941

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp
[a*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x],
x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
&&  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Si
mplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int (e\cos(c+dx))^{3/2}(a+b\sin(c+dx))^3\,dx = -\frac{2b(e\cos(c+dx))^{5/2}(a+b\sin(c+dx))^2}{9de}+\frac{2}{9}\int (e\cos(c+dx)$$

$$= -\frac{26ab(e\cos(c+dx))^{5/2}(a+b\sin(c+dx))}{63de}-\frac{2b(e\cos(c+dx))}{}$$

$$= -\frac{2b(89a^2+28b^2)\,(e\cos(c+dx))^{5/2}}{315de}-\frac{26ab(e\cos(c+dx))^{5/2}(a}{63de}$$

$$= -\frac{2b(89a^2+28b^2)\,(e\cos(c+dx))^{5/2}}{315de}+\frac{2a(7a^2+6b^2)\,e\sqrt{e\cos(c}}{21d}$$

$$= -\frac{2b(89a^2+28b^2)\,(e\cos(c+dx))^{5/2}}{315de}+\frac{2a(7a^2+6b^2)\,e\sqrt{e\cos(c}}{21d}$$

$$= -\frac{2b(89a^2+28b^2)\,(e\cos(c+dx))^{5/2}}{315de}+\frac{2a(7a^2+6b^2)\,e^2\sqrt{\cos(c}}{21d\sqrt{e\cos}}$$

Mathematica [A]
time = 1.42, size = 153, normalized size = 0.78

$$\frac{(e\cos(c+dx))^{3/2}\left(80(7a^3+6ab^2)F\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{2}{3}\sqrt{\cos(c+dx)}\,(-756a^2b-147b^3-28(27a^2b+4b^3)\cos(2(c+dx))+35b^3\cos(4(c+dx))+840a^3\sin(c+dx)+450ab^2\sin(c+dx)-270ab^2\sin(3(c+dx)))\right)}{840d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((e*Cos[c + d*x])^(3/2)*(80*(7*a^3 + 6*a*b^2)*EllipticF[(c + d*x)/2, 2] + (
2*Sqrt[Cos[c + d*x]]*(-756*a^2*b - 147*b^3 - 28*(27*a^2*b + 4*b^3)*Cos[2*(c
 + d*x)] + 35*b^3*Cos[4*(c + d*x)] + 840*a^3*Sin[c + d*x] + 450*a*b^2*Sin[c
 + d*x] - 270*a*b^2*Sin[3*(c + d*x)]))/3))/(840*d*Cos[c + d*x]^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 449 vs. 2(201) = 402.
time = 8.31, size = 450, normalized size = 2.28

method	result
default	$-\frac{2e^2\left(1120b^3\left(\sin^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2160a\,b^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2800b^3\left(\sin^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1512a^2b\left(\sin^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3240a\,b^2\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(3/2)*(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/315/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^2*(1120*b^3
*sin(1/2*d*x+1/2*c)^11-2160*a*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-2
800*b^3*sin(1/2*d*x+1/2*c)^9-1512*a^2*b*sin(1/2*d*x+1/2*c)^7+3240*a*b^2*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+2296*b^3*sin(1/2*d*x+1/2*c)^7+420*a^3*
cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+2268*a^2*b*sin(1/2*d*x+1/2*c)^5-126
0*a*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-644*b^3*sin(1/2*d*x+1/2*c)^
5+105*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))*a^3+90*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2-210*a^
3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-1134*a^2*b*sin(1/2*d*x+1/2*c)^3+9
0*a*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-28*b^3*sin(1/2*d*x+1/2*c)^3
+189*a^2*b*sin(1/2*d*x+1/2*c)+28*b^3*sin(1/2*d*x+1/2*c))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] e^(3/2)*integrate((b*sin(d*x + c) + a)^3*cos(d*x + c)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 169, normalized size = 0.86

$$\frac{-15i\sqrt{2}\,(7\,a^3+6\,ab^2)e^{\frac{3}{2}}\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+15i\sqrt{2}\,(7\,a^3+6\,ab^2)e^{\frac{3}{2}}\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c))+2\left(35\,b^3\cos(dx+c)^4\,e^{\frac{3}{2}}-63\,(3\,a^2b+b^3)\cos(dx+c)^2\,e^{\frac{3}{2}}-15\left(9\,ab^2\cos(dx+c)^2\,e^{\frac{3}{2}}-(7\,a^3+6\,ab^2)e^{\frac{3}{2}}\right)\sin(dx+c)\right)\sqrt{\cos(dx+c)}}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/315*(-15*I*sqrt(2)*(7*a^3 + 6*a*b^2)*e^(3/2)*weierstrassPInverse(-4, 0, c
os(d*x + c) + I*sin(d*x + c)) + 15*I*sqrt(2)*(7*a^3 + 6*a*b^2)*e^(3/2)*weie
rstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*(35*b^3*cos(d*x +
 c)^4*e^(3/2) - 63*(3*a^2*b + b^3)*cos(d*x + c)^2*e^(3/2) - 15*(9*a*b^2*cos
(d*x + c)^2*e^(3/2) - (7*a^3 + 6*a*b^2)*e^(3/2))*sin(d*x + c))*sqrt(cos(d*x
 + c)))/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(3/2)*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4849 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^3*cos(d*x + c)^(3/2)*e^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(e\,\cos\left(c+d\,x\right)\right)}^{3/2}\,{\left(a+b\,\sin\left(c+d\,x\right)\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(3/2)*(a + b*sin(c + d*x))^3,x)
```

```
[Out] int((e*cos(c + d*x))^(3/2)*(a + b*sin(c + d*x))^3, x)
```

3.558 $\int \sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=156

$$-\frac{2b(57a^2+20b^2)\,(e\cos(c+dx))^{3/2}}{105de}+\frac{2a(5a^2+6b^2)\,\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5d\sqrt{\cos(c+dx)}}-\frac{22ab(e\cos(c+dx))^{3}}{35}$$

```
[Out] -2/105*b*(57*a^2+20*b^2)*(e*cos(d*x+c))^(3/2)/d/e-22/35*a*b*(e*cos(d*x+c))^
(3/2)*(a+b*sin(d*x+c))/d/e-2/7*b*(e*cos(d*x+c))^(3/2)*(a+b*sin(d*x+c))^2/d/
e+2/5*a*(5*a^2+6*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellip
ticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2771, 2941, 2748, 2721, 2719}

$$-\frac{2b(57a^2+20b^2)\,(e\cos(c+dx))^{3/2}}{105de}+\frac{2a(5a^2+6b^2)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)\sqrt{e\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}-\frac{2b(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))^2}{7de}-\frac{22ab(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))}{35de}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-2*b*(57*a^2 + 20*b^2)*(e*Cos[c + d*x])^(3/2))/(105*d*e) + (2*a*(5*a^2 + 6
*b^2)*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x
]]) - (22*a*b*(e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x]))/(35*d*e) - (2*b*
(e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x])^2)/(7*d*e)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2771

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(
a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rule 2941

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp
[a*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x],
x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
&&  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Si
mplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int \sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))^3\,dx = -\frac{2b(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))^2}{7de} + \frac{2}{7}\int \sqrt{e\cos(c+dx}$$

$$= -\frac{22ab(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))}{35de} - \frac{2b(e\cos(c+dx))^{3}}{}$$

$$= -\frac{2b(57a^2+20b^2)\,(e\cos(c+dx))^{3/2}}{105de} - \frac{22ab(e\cos(c+dx))^{3/2}(a}{35de}$$

$$= -\frac{2b(57a^2+20b^2)\,(e\cos(c+dx))^{3/2}}{105de} - \frac{22ab(e\cos(c+dx))^{3/2}(a}{35de}$$

$$= -\frac{2b(57a^2+20b^2)\,(e\cos(c+dx))^{3/2}}{105de} + \frac{2a(5a^2+6b^2)\,\sqrt{e\cos(c+}}{5d\sqrt{\cos(c}}$$

Mathematica [A]
time = 0.65, size = 101, normalized size = 0.65

$$\frac{\sqrt{e\cos(c+dx)}\left(42(5a^3+6ab^2)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)+b\cos^{\frac{3}{2}}(c+dx)\,(-210a^2-55b^2+15b^2\cos(2(c+dx))-126ab\sin(c+dx))\right)}{105d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (Sqrt[e*Cos[c + d*x]]*(42*(5*a^3 + 6*a*b^2)*EllipticE[(c + d*x)/2, 2] + b*C
os[c + d*x]^(3/2)*(-210*a^2 - 55*b^2 + 15*b^2*Cos[2*(c + d*x)] - 126*a*b*Si
n[c + d*x])))/(105*d*Sqrt[Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 338 vs. $2(164) = 328$.
time = 6.87, size = 339, normalized size = 2.17

method	result
default	$\frac{2e\left(240b^3\left(\sin^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-504a\,b^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-480b^3\left(\sin^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-420a^2b\left(\sin^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+504a\,b^2\cos\left(\frac{dx}{2}+\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))^3*(e*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/105/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e*(240*b^3*sin
(1/2*d*x+1/2*c)^9-504*a*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-480*b^3
*sin(1/2*d*x+1/2*c)^7-420*a^2*b*sin(1/2*d*x+1/2*c)^5+504*a*b^2*cos(1/2*d*x+
1/2*c)*sin(1/2*d*x+1/2*c)^4+220*b^3*sin(1/2*d*x+1/2*c)^5+105*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))*a^3+126*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2+420*a^2*b*sin(1/2*d*x+1/2
*c)^3-126*a*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+20*b^3*sin(1/2*d*x+
1/2*c)^3-105*a^2*b*sin(1/2*d*x+1/2*c)-20*b^3*sin(1/2*d*x+1/2*c))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^3*(e*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] e^(1/2)*integrate((b*sin(d*x + c) + a)^3*sqrt(cos(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 152, normalized size = 0.97

$$\frac{21i\sqrt{2}\,(5\,a^3+6\,ab^2)e^{\frac{1}{2}}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)))-21i\sqrt{2}\,(5\,a^3+6\,ab^2)e^{\frac{1}{2}}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c)))+2\left(15\,b^3\cos(dx+c)^3\,e^{\frac{1}{2}}-63\,ab^2\cos(dx+c)\,e^{\frac{1}{2}}\sin(dx+c)-35\,(3\,a^2b+b^3)\cos(dx+c)\,e^{\frac{1}{2}}\right)\sqrt{\cos(dx+c)}}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^3*(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/105*(21*I*sqrt(2)*(5*a^3 + 6*a*b^2)*e^(1/2)*weierstrassZeta(-4, 0, weiers
trassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 21*I*sqrt(2)*(5*a^3
+ 6*a*b^2)*e^(1/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*
x + c) - I*sin(d*x + c))) + 2*(15*b^3*cos(d*x + c)^3*e^(1/2) - 63*a*b^2*cos
(d*x + c)*e^(1/2)*sin(d*x + c) - 35*(3*a^2*b + b^3)*cos(d*x + c)*e^(1/2))*s
qrt(cos(d*x + c)))/d
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))**3*(e*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^3*(e*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^3*sqrt(cos(d*x + c))*e^(1/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{e\,\cos\left(c+d\,x\right)}\,{\left(a+b\,\sin\left(c+d\,x\right)\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(1/2)*(a + b*sin(c + d*x))^3,x)
```

```
[Out] int((e*cos(c + d*x))^(1/2)*(a + b*sin(c + d*x))^3, x)
```

3.559 $\int \frac{(a+b\sin(c+dx))^3}{\sqrt{e\cos(c+dx)}}\,dx$

Optimal. Leaf size=152

$$-\frac{2b(11a^2+4b^2)\sqrt{e\cos(c+dx)}}{5de}+\frac{2a(a^2+2b^2)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{e\cos(c+dx)}}-\frac{6ab\sqrt{e\cos(c+dx)}\,(a+}{5de}$$

```
[Out] 2*a*(a^2+2*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(s
in(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(e*cos(d*x+c))^(1/2)-2/5*b*(1
1*a^2+4*b^2)*(e*cos(d*x+c))^(1/2)/d/e-6/5*a*b*(a+b*sin(d*x+c))*(e*cos(d*x+c
))^(1/2)/d/e-2/5*b*(a+b*sin(d*x+c))^2*(e*cos(d*x+c))^(1/2)/d/e
```

Rubi [A]
time = 0.16, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2771, 2941, 2748, 2721, 2720}

$$-\frac{2b(11a^2+4b^2)\sqrt{e\cos(c+dx)}}{5de}+\frac{2a(a^2+2b^2)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{e\cos(c+dx)}}-\frac{2b\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))^2}{5de}-\frac{6ab\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))}{5de}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^3/Sqrt[e*Cos[c + d*x]],x]
```

```
[Out] (-2*b*(11*a^2 + 4*b^2)*Sqrt[e*Cos[c + d*x]])/(5*d*e) + (2*a*(a^2 + 2*b^2)*S
qrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*Sqrt[e*Cos[c + d*x]]) - (6*
a*b*Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x]))/(5*d*e) - (2*b*Sqrt[e*Cos[c
+ d*x]]*(a + b*Sin[c + d*x])^2)/(5*d*e)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2771

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(
a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rule 2941

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp
[a*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x],
x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
&&  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Si
mplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int \frac{(a+b\sin(c+dx))^3}{\sqrt{e\cos(c+dx)}}\,dx = -\frac{2b\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))^2}{5de}+\frac{2}{5}\int \frac{(a+b\sin(c+dx))\left(\frac{5a^2}{2}+2b^2+\right.}{\sqrt{e\cos(c+dx}}$$

$$= -\frac{6ab\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))}{5de}-\frac{2b\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx}{5de}$$

$$= -\frac{2b(11a^2+4b^2)\sqrt{e\cos(c+dx)}}{5de}-\frac{6ab\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))}{5de}-\frac{2}{}$$

$$= -\frac{2b(11a^2+4b^2)\sqrt{e\cos(c+dx)}}{5de}-\frac{6ab\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))}{5de}-\frac{2}{}$$

$$= -\frac{2b(11a^2+4b^2)\sqrt{e\cos(c+dx)}}{5de}+\frac{2a(a^2+2b^2)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right.}{d\sqrt{e\cos(c+dx)}}$$

Mathematica [A]
time = 0.82, size = 94, normalized size = 0.62

$$\frac{10a(a^2+2b^2)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+b\cos(c+dx)\left(-30a^2-9b^2+b^2\cos(2(c+dx))-10ab\sin(c+dx)\right)}{5d\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])^3/Sqrt[e*Cos[c + d*x]],x]
```

```
[Out] (10*a*(a^2 + 2*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + b*Cos[c
+ d*x]*(-30*a^2 - 9*b^2 + b^2*Cos[2*(c + d*x)] - 10*a*b*Sin[c + d*x]))/(5*d
*Sqrt[e*Cos[c + d*x]])
```

Maple [A]
time = 6.06, size = 279, normalized size = 1.84

method	result
default	$-\frac{2\left(8b^3\left(\sin^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-20a\,b^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-12b^3\left(\sin^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+5\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\right.\right.}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))^3/(e*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*(8*b^3*sin(1/2*
d*x+1/2*c)^7-20*a*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-12*b^3*sin(1/
2*d*x+1/2*c)^5+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^3+10*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a
*b^2-30*a^2*b*sin(1/2*d*x+1/2*c)^3+10*a*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+
1/2*c)^2-4*b^3*sin(1/2*d*x+1/2*c)^3+15*a^2*b*sin(1/2*d*x+1/2*c)+4*b^3*sin(1
/2*d*x+1/2*c))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^3/(e*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] e^(-1/2)*integrate((b*sin(d*x + c) + a)^3/sqrt(cos(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 124, normalized size = 0.82

$$-\frac{\left(5\sqrt{2}\left(i\,a^3+2i\,ab^2\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)+5\sqrt{2}\left(-i\,a^3-2i\,ab^2\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)-2\left(b^3\cos\left(dx+c\right)^2-5\,ab^2\sin\left(dx+c\right)-15\,a^2b-5\,b^3\right)\sqrt{\cos\left(dx+c\right)}\right)e^{\left(-\frac{1}{2}\right)}}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^3/(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/5*(5*sqrt(2)*(I*a^3 + 2*I*a*b^2)*weierstrassPInverse(-4, 0, cos(d*x + c)
 + I*sin(d*x + c)) + 5*sqrt(2)*(-I*a^3 - 2*I*a*b^2)*weierstrassPInverse(-4,
```

```
 0, cos(d*x + c) - I*sin(d*x + c)) - 2*(b^3*cos(d*x + c)^2 - 5*a*b^2*sin(d*
x + c) - 15*a^2*b - 5*b^3)*sqrt(cos(d*x + c)))*e^(-1/2)/d
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))**3/(e*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^3/(e*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^3*e^(-1/2)/sqrt(cos(d*x + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b \sin(c + d x))^3}{\sqrt{e \cos(c + d x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^3/(e*cos(c + d*x))^(1/2),x)
```

```
[Out] int((a + b*sin(c + d*x))^3/(e*cos(c + d*x))^(1/2), x)
```

3.560 $\int \frac{(a+b\sin(c+dx))^3}{(e\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=160

$$\frac{2b(3a^2+4b^2)\,(e\cos(c+dx))^{3/2}}{3de^3}-\frac{2a(a^2+6b^2)\,\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{de^2\sqrt{\cos(c+dx)}}+\frac{2ab(e\cos(c+dx))^{3/2}(a+}{de^3}$$

```
[Out] 2/3*b*(3*a^2+4*b^2)*(e*cos(d*x+c))^(3/2)/d/e^3+2*a*b*(e*cos(d*x+c))^(3/2)*(
a+b*sin(d*x+c))/d/e^3+2*(b+a*sin(d*x+c))*(a+b*sin(d*x+c))^2/d/e/(e*cos(d*x+
c))^(1/2)-2*a*(a^2+6*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*E
llipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/d/e^2/cos(d*x+c)^
(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2770, 2941, 2748, 2721, 2719}

$$\frac{2b(3a^2+4b^2)\,(e\cos(c+dx))^{3/2}}{3de^3}-\frac{2a(a^2+6b^2)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)\sqrt{e\cos(c+dx)}}{de^2\sqrt{\cos(c+dx)}}+\frac{2ab(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))}{de^3}+\frac{2(a\sin(c+dx)+b)(a+b\sin(c+dx))^2}{de\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^3/(e*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*b*(3*a^2 + 4*b^2)*(e*Cos[c + d*x])^(3/2))/(3*d*e^3) - (2*a*(a^2 + 6*b^2)
*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*e^2*Sqrt[Cos[c + d*x]])
 + (2*a*b*(e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x]))/(d*e^3) + (2*(b + a*
Sin[c + d*x])*(a + b*Sin[c + d*x])^2)/(d*e*Sqrt[e*Cos[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2770

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x
])^(m - 1)*((b + a*Sin[e + f*x])/(f*g*(p + 1))), x] + Dist[1/(g^2*(p + 1)),
 Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a
^2*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g}
, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p
] || IntegerQ[m])
```

Rule 2941

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp
[a*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x],
x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
&&  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Si
mplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int \frac{(a+b\sin(c+dx))^3}{(e\cos(c+dx))^{3/2}}\,dx = \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))^2}{de\sqrt{e\cos(c+dx)}} - \frac{2\int \sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx}{e^2}$$

$$= \frac{2ab(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))}{de^3} + \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx}{de\sqrt{e\cos(c+dx)}}$$

$$= \frac{2b(3a^2+4b^2)(e\cos(c+dx))^{3/2}}{3de^3} + \frac{2ab(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))}{de^3} + \frac{2(}{}$$

$$= \frac{2b(3a^2+4b^2)(e\cos(c+dx))^{3/2}}{3de^3} + \frac{2ab(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))}{de^3} + \frac{2(}{}$$

$$= \frac{2b(3a^2+4b^2)(e\cos(c+dx))^{3/2}}{3de^3} - \frac{2a(a^2+6b^2)\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{de^2\sqrt{\cos(c+dx)}}$$

Mathematica [A]
time = 0.44, size = 98, normalized size = 0.61

$$\frac{2b^3\cos^2(c+dx) - 6a(a^2+6b^2)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right) + 6(3a^2b+b^3+a(a^2+3b^2)\sin(c+dx))}{3de\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])^3/(e*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*b^3*Cos[c + d*x]^2 - 6*a*(a^2 + 6*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c +
 d*x)/2, 2] + 6*(3*a^2*b + b^3 + a*(a^2 + 3*b^2)*Sin[c + d*x]))/(3*d*e*Sqrt
[e*Cos[c + d*x]])
```

Maple [A]
time = 8.20, size = 248, normalized size = 1.55

method	result
default	$-\frac{2\left(-4b^3\left(\sin^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\;\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)a^3+18\sqrt{\frac{1}{2}-}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))^3/(e*cos(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/e/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/sin(1/2*d*x+1/2*c)*(-4*b^3*sin(1
/2*d*x+1/2*c)^5+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3+18*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*
a*b^2-6*a^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-18*a*b^2*cos(1/2*d*x+1/
2*c)*sin(1/2*d*x+1/2*c)^2+4*b^3*sin(1/2*d*x+1/2*c)^3-9*a^2*b*sin(1/2*d*x+1/
2*c)-4*b^3*sin(1/2*d*x+1/2*c))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^3/(e*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] e^(-3/2)*integrate((b*sin(d*x + c) + a)^3/cos(d*x + c)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 156, normalized size = 0.98

$$-\frac{\left(3\sqrt{2}\left(i\,a^3+6i\,ab^2\right)\cos\left(dx+c\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)))+3\sqrt{2}\left(-i\,a^3-6i\,ab^2\right)\cos\left(dx+c\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)))-2\left(b^3\cos\left(dx+c\right)^2+9\,a^2b+3\,b^3+3\left(a^3+3\,ab^2\right)\sin\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)}\right)e^{\left(-\frac{3}{2}\right)}}{3\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^3/(e*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/3*(3*sqrt(2)*(I*a^3 + 6*I*a*b^2)*cos(d*x + c)*weierstrassZeta(-4, 0, wei
erstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*sqrt(2)*(-I*a^3
 - 6*I*a*b^2)*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0
, cos(d*x + c) - I*sin(d*x + c))) - 2*(b^3*cos(d*x + c)^2 + 9*a^2*b + 3*b^3
 + 3*(a^3 + 3*a*b^2)*sin(d*x + c))*sqrt(cos(d*x + c)))*e^(-3/2)/(d*cos(d*x
+ c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))**3/(e*cos(d*x+c))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3065 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^3/(e*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^3*e^(-3/2)/cos(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,\sin\left(c+d\,x\right)\right)^3}{\left(e\,\cos\left(c+d\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^3/(e*cos(c + d*x))^(3/2),x)
```

```
[Out] int((a + b*sin(c + d*x))^3/(e*cos(c + d*x))^(3/2), x)
```

3.561 $\int \frac{(a+b\sin(c+dx))^3}{(e\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=164

$$\frac{2b(a^2+4b^2)\sqrt{e\cos(c+dx)}}{3de^3}+\frac{2a(a^2-6b^2)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3de^2\sqrt{e\cos(c+dx)}}+\frac{2ab\sqrt{e\cos(c+dx)}\,(a+b\sin}{3de^3}$$

```
[Out] 2/3*(b+a*sin(d*x+c))*(a+b*sin(d*x+c))^2/d/e/(e*cos(d*x+c))^(3/2)+2/3*a*(a^2
-6*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d
*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/e^2/(e*cos(d*x+c))^(1/2)+2/3*b*(a^2+4
*b^2)*(e*cos(d*x+c))^(1/2)/d/e^3+2/3*a*b*(a+b*sin(d*x+c))*(e*cos(d*x+c))^(1
/2)/d/e^3
```

Rubi [A]
time = 0.16, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2770, 2941, 2748, 2721, 2720}

$$\frac{2b(a^2+4b^2)\sqrt{e\cos(c+dx)}}{3de^3}+\frac{2a(a^2-6b^2)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3de^2\sqrt{e\cos(c+dx)}}+\frac{2ab\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))}{3de^3}+\frac{2(a\sin(c+dx)+b)(a+b\sin(c+dx))^2}{3de(e\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^3/(e*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*b*(a^2 + 4*b^2)*Sqrt[e*Cos[c + d*x]])/(3*d*e^3) + (2*a*(a^2 - 6*b^2)*Sqr
t[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*e^2*Sqrt[e*Cos[c + d*x]]) +
 (2*a*b*Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x]))/(3*d*e^3) + (2*(b + a*Si
n[c + d*x])*(a + b*Sin[c + d*x])^2)/(3*d*e*(e*Cos[c + d*x])^(3/2))
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2770

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x
])^(m - 1)*((b + a*Sin[e + f*x])/(f*g*(p + 1))), x] + Dist[1/(g^2*(p + 1)),
 Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a
^2*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g}
, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p
] || IntegerQ[m])
```

Rule 2941

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp
[a*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x],
x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
&&  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Si
mplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int \frac{(a+b\sin(c+dx))^3}{(e\cos(c+dx))^{5/2}}\,dx = \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))^2}{3de(e\cos(c+dx))^{3/2}} - \frac{2\int \frac{(a+b\sin(c+dx))\left(-\frac{a^2}{2}+2b^2+\frac{3}{2}ab\sin(c+dx)\right)}{\sqrt{e\cos(c+dx)}}}{3e^2}$$

$$= \frac{2ab\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))}{3de^3} + \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx)}{3de(e\cos(c+dx))^{3/2}}$$

$$= \frac{2b(a^2+4b^2)\sqrt{e\cos(c+dx)}}{3de^3} + \frac{2ab\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))}{3de^3} + \frac{2(b+}{}$$

$$= \frac{2b(a^2+4b^2)\sqrt{e\cos(c+dx)}}{3de^3} + \frac{2ab\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))}{3de^3} + \frac{2(b+}{}$$

$$= \frac{2b(a^2+4b^2)\sqrt{e\cos(c+dx)}}{3de^3} + \frac{2a(a^2-6b^2)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3de^2\sqrt{e\cos(c+dx)}} +$$

Mathematica [A]
time = 0.72, size = 103, normalized size = 0.63

$$\frac{6a^2b+5b^3+3b^3\cos(2(c+dx))+2a(a^2-6b^2)\cos^{\frac{3}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)+2a^3\sin(c+dx)+6ab^2\sin(c+dx)}{3de(e\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])^3/(e*Cos[c + d*x])^(5/2),x]
```

```
[Out] (6*a^2*b + 5*b^3 + 3*b^3*Cos[2*(c + d*x)] + 2*a*(a^2 - 6*b^2)*Cos[c + d*x]^
(3/2)*EllipticF[(c + d*x)/2, 2] + 2*a^3*Sin[c + d*x] + 6*a*b^2*Sin[c + d*x]
)/(3*d*e*(e*Cos[c + d*x])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 383 vs. $2(172) = 344$.
time = 8.68, size = 384, normalized size = 2.34

method	result
default	$-\frac{2\left(2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)a^3\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-12\sqrt{\frac{1}{2}-\frac{\cos(d}{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))^3/(e*cos(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/(2*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2
*e+e)^(1/2)/e^2*(2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^3*sin(1/2*d*x+1/2*c)^2-12*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2))*a*b^2*sin(1/2*d*x+1/2*c)^2+12*b^3*sin(1/2*d*x+1/2*c)^
5-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))*a^3+6*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2+2*a^3*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+6*a*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1
/2*c)^2-12*b^3*sin(1/2*d*x+1/2*c)^3+3*a^2*b*sin(1/2*d*x+1/2*c)+4*b^3*sin(1/
2*d*x+1/2*c))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^3/(e*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] e^(-5/2)*integrate((b*sin(d*x + c) + a)^3/cos(d*x + c)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 150, normalized size = 0.91

$$\frac{\left(\sqrt{2}\left(-i\,a^3+6i\,ab^2\right)\cos\left(dx+c\right)^2\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)+\sqrt{2}\left(i\,a^3-6i\,ab^2\right)\cos\left(dx+c\right)^2\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)+2\left(3\,b^3\cos\left(dx+c\right)^2+3\,a^2b+b^3+\left(a^3+3\,ab^2\right)\sin\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)}\right)e^{\left(-\frac{5}{2}\right)}}{3\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^3/(e*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(sqrt(2)*(-I*a^3 + 6*I*a*b^2)*cos(d*x + c)^2*weierstrassPInverse(-4, 0,
 cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(I*a^3 - 6*I*a*b^2)*cos(d*x + c)^
2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*(3*b^3*cos(
d*x + c)^2 + 3*a^2*b + b^3 + (a^3 + 3*a*b^2)*sin(d*x + c))*sqrt(cos(d*x + c
)))*e^(-5/2)/(d*cos(d*x + c)^2)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))**3/(e*cos(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5990 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^3/(e*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^3*e^(-5/2)/cos(d*x + c)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\,\sin(c + d\,x))^3}{(e\,\cos(c + d\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^3/(e*cos(c + d*x))^(5/2),x)
```

```
[Out] int((a + b*sin(c + d*x))^3/(e*cos(c + d*x))^(5/2), x)
```

3.562 $\int \frac{(a+b\sin(c+dx))^3}{(e\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=187

$$\frac{2b(3a^2-4b^2)\,(e\cos(c+dx))^{3/2}}{5de^5}-\frac{6a(a^2-2b^2)\,\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5de^4\sqrt{\cos(c+dx)}}+\frac{2(b+a\sin(c+dx))(a+b}{5de(e\cos(c+dx}$$

```
[Out] 2/5*b*(3*a^2-4*b^2)*(e*cos(d*x+c))^(3/2)/d/e^5+2/5*(b+a*sin(d*x+c))*(a+b*si
n(d*x+c))^2/d/e/(e*cos(d*x+c))^(5/2)-2/5*(a+b*sin(d*x+c))*(a*b-(3*a^2-4*b^2
)*sin(d*x+c))/d/e^3/(e*cos(d*x+c))^(1/2)-6/5*a*(a^2-2*b^2)*(cos(1/2*d*x+1/2
*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*co
s(d*x+c))^(1/2)/d/e^4/cos(d*x+c)^(1/2)
```

Rubi [A]

time = 0.17, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2770, 2940, 2748, 2721, 2719}

$$\frac{2b(3a^2-4b^2)\,(e\cos(c+dx))^{3/2}}{5de^5}-\frac{6a(a^2-2b^2)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)\sqrt{e\cos(c+dx)}}{5de^4\sqrt{\cos(c+dx)}}-\frac{2(ab-(3a^2-4b^2)\sin(c+dx))\,(a+b\sin(c+dx))}{5de^3\sqrt{e\cos(c+dx)}}+\frac{2(a\sin(c+dx)+b)(a+b\sin(c+dx))^2}{5de(e\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^3/(e*Cos[c + d*x])^(7/2),x]
```

```
[Out] (2*b*(3*a^2 - 4*b^2)*(e*Cos[c + d*x])^(3/2))/(5*d*e^5) - (6*a*(a^2 - 2*b^2)
*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*e^4*Sqrt[Cos[c + d*x]
]) + (2*(b + a*Sin[c + d*x])*(a + b*Sin[c + d*x])^2)/(5*d*e*(e*Cos[c + d*x]
)^(5/2)) - (2*(a + b*Sin[c + d*x])*(a*b - (3*a^2 - 4*b^2)*Sin[c + d*x]))/(5
*d*e^3*Sqrt[e*Cos[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
```

```
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2770

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x
])^(m - 1)*((b + a*Sin[e + f*x])/(f*g*(p + 1))), x] + Dist[1/(g^2*(p + 1)),
 Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a
^2*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g}
, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p
] || IntegerQ[m])
```

Rule 2940

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(g*
Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x])^m*((d + c*Sin[e + f*x])/(f*g*(p
 + 1))), x] + Dist[1/(g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin
[e + f*x])^(m - 1)*Simp[a*c*(p + 2) + b*d*m + b*c*(m + p + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && GtQ[
m, 0] && LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] &
& SimplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int \frac{(a+b\sin(c+dx))^3}{(e\cos(c+dx))^{7/2}}\,dx = \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))^2}{5de(e\cos(c+dx))^{5/2}} - \frac{2\int \frac{(a+b\sin(c+dx))\left(-\frac{3a^2}{2}+2b^2+\frac{1}{2}ab\sin(c+dx)\right)}{(e\cos(c+dx))^{3/2}}}{5e^2}$$

$$= \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))^2}{5de(e\cos(c+dx))^{5/2}} - \frac{2(a+b\sin(c+dx))\left(ab-(3a^2-4b^2\right.}{5de^3\sqrt{e\cos(c+dx)}}$$

$$= \frac{2b(3a^2-4b^2)(e\cos(c+dx))^{3/2}}{5de^5} + \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))^2}{5de(e\cos(c+dx))^{5/2}} - \frac{2(}{}$$

$$= \frac{2b(3a^2-4b^2)(e\cos(c+dx))^{3/2}}{5de^5} + \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))^2}{5de(e\cos(c+dx))^{5/2}} - \frac{2(}{}$$

$$= \frac{2b(3a^2-4b^2)(e\cos(c+dx))^{3/2}}{5de^5} - \frac{6a(a^2-2b^2)\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5de^4\sqrt{\cos(c+dx)}}$$

Mathematica [A]

time = 0.76, size = 126, normalized size = 0.67

$$\frac{2\left(-5b^3-3a(a^2-2b^2)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)+b(3a^2+b^2)\sec^2(c+dx)+3a^3\sin(c+dx)-6ab^2\sin(c+dx)+a(a^2+3b^2)\sec(c+dx)\tan(c+dx)\right)}{5de^3\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])^3/(e*Cos[c + d*x])^(7/2),x]
```

```
[Out] (2*(-5*b^3 - 3*a*(a^2 - 2*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]
 + b*(3*a^2 + b^2)*Sec[c + d*x]^2 + 3*a^3*Sin[c + d*x] - 6*a*b^2*Sin[c + d*
x] + a*(a^2 + 3*b^2)*Sec[c + d*x]*Tan[c + d*x]))/(5*d*e^3*Sqrt[e*Cos[c + d*
x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 617 vs. $2(195) = 390$.
time = 17.23, size = 618, normalized size = 3.30

method	result
default	$-\frac{2\left(12\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)a^3\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-24\sqrt{\frac{1}{2}-\frac{\cos(}{}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))^3/(e*cos(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)/(
-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/e^3*(12*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3*sin
(1/2*d*x+1/2*c)^4-24*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2*sin(1/2*d*x+1/2*c)^4-24
*a^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+48*a*b^2*cos(1/2*d*x+1/2*c)*si
n(1/2*d*x+1/2*c)^6-12*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3*sin(1/2*d*x+1/2*c)^2+24*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))*a*b^2*sin(1/2*d*x+1/2*c)^2+24*a^3*cos(1/2*d*x+1/2*
c)*sin(1/2*d*x+1/2*c)^4-48*a*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+20
*b^3*sin(1/2*d*x+1/2*c)^5+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3-6*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2))*a*b^2-8*a^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+6*a*b^2*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-20*b^3*sin(1/2*d*x+1/2*c)^3-3*a^2*b*sin(1
/2*d*x+1/2*c)+4*b^3*sin(1/2*d*x+1/2*c))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^3/(e*cos(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] e^(-7/2)*integrate((b*sin(d*x + c) + a)^3/cos(d*x + c)^(7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 181, normalized size = 0.97

$$-\frac{\left(3\sqrt{2}\left(i\,a^{3}-2i\,ab^{2}\right)\cos\left(dx+c\right)^{3}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)))+3\sqrt{2}\left(-i\,a^{3}+2i\,ab^{2}\right)\cos\left(dx+c\right)^{3}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)))+2\left(5\,b^{3}\cos\left(dx+c\right)^{2}-3\,a^{2}b-b^{3}-\left(a^{3}+3\,ab^{2}+3\left(a^{3}-2\,ab^{2}\right)\cos\left(dx+c\right)^{2}\right)\sin\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)}\right)e^{\left(-\frac{7}{2}\right)}}{5\,d\cos\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^3/(e*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] -1/5*(3*sqrt(2)*(I*a^3 - 2*I*a*b^2)*cos(d*x + c)^3*weierstrassZeta(-4, 0, w
eierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*sqrt(2)*(-I*a
^3 + 2*I*a*b^2)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-
4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(5*b^3*cos(d*x + c)^2 - 3*a^2*b -
 b^3 - (a^3 + 3*a*b^2 + 3*(a^3 - 2*a*b^2)*cos(d*x + c)^2)*sin(d*x + c))*sqr
t(cos(d*x + c)))*e^(-7/2)/(d*cos(d*x + c)^3)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))**3/(e*cos(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^3/(e*cos(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^3*e^(-7/2)/cos(d*x + c)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,\sin\left(c+d\,x\right)\right)}^3}{{\left(e\,\cos\left(c+d\,x\right)\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^3/(e*cos(c + d*x))^(7/2),x)
```

```
[Out] int((a + b*sin(c + d*x))^3/(e*cos(c + d*x))^(7/2), x)
```

3.563 $\int \frac{(a+b\sin(c+dx))^3}{(e\cos(c+dx))^{9/2}}\,dx$

Optimal. Leaf size=188

$$\frac{2b(5a^2-4b^2)\sqrt{e\cos(c+dx)}}{21de^5}+\frac{2a(5a^2-6b^2)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21de^4\sqrt{e\cos(c+dx)}}+\frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))^2}{7de(e\cos(c+dx))^{7/2}}$$

```
[Out] 2/7*(b+a*sin(d*x+c))*(a+b*sin(d*x+c))^2/d/e/(e*cos(d*x+c))^(7/2)+2/21*(a+b*
sin(d*x+c))*(a*b+(5*a^2-4*b^2)*sin(d*x+c))/d/e^3/(e*cos(d*x+c))^(3/2)+2/21*
a*(5*a^2-6*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(s
in(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/e^4/(e*cos(d*x+c))^(1/2)+2/21
*b*(5*a^2-4*b^2)*(e*cos(d*x+c))^(1/2)/d/e^5
```

Rubi [A]
time = 0.17, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2770, 2940, 2748, 2721, 2720}

$$\frac{2b(5a^2-4b^2)\sqrt{e\cos(c+dx)}}{21de^5}+\frac{2a(5a^2-6b^2)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21de^4\sqrt{e\cos(c+dx)}}+\frac{2((5a^2-4b^2)\sin(c+dx)+ab)(a+b\sin(c+dx))}{21de^3(e\cos(c+dx))^{3/2}}+\frac{2(a\sin(c+dx)+b)(a+b\sin(c+dx))^2}{7de(e\cos(c+dx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^3/(e*Cos[c + d*x])^(9/2),x]
```

```
[Out] (2*b*(5*a^2 - 4*b^2)*Sqrt[e*Cos[c + d*x]])/(21*d*e^5) + (2*a*(5*a^2 - 6*b^2
)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*e^4*Sqrt[e*Cos[c + d*
x]]) + (2*(b + a*Sin[c + d*x])*(a + b*Sin[c + d*x])^2)/(7*d*e*(e*Cos[c + d*
x])^(7/2)) + (2*(a + b*Sin[c + d*x])*(a*b + (5*a^2 - 4*b^2)*Sin[c + d*x]))/
(21*d*e^3*(e*Cos[c + d*x])^(3/2))
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
```

```
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2770

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x
])^(m - 1)*((b + a*Sin[e + f*x])/(f*g*(p + 1))), x] + Dist[1/(g^2*(p + 1)),
 Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a
^2*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g}
, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p
] || IntegerQ[m])
```

Rule 2940

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(g*
Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x])^m*((d + c*Sin[e + f*x])/(f*g*(p
 + 1))), x] + Dist[1/(g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin
[e + f*x])^(m - 1)*Simp[a*c*(p + 2) + b*d*m + b*c*(m + p + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && GtQ[
m, 0] && LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] &
& SimplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int \frac{(a+b\sin(c+dx))^3}{(e\cos(c+dx))^{9/2}}\,dx = \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))^2}{7de(e\cos(c+dx))^{7/2}} - \frac{2\int \frac{(a+b\sin(c+dx))\left(-\frac{5a^2}{2}+2b^2-\frac{1}{2}ab\sin(c+dx)\right.}{(e\cos(c+dx))^{5/2}}}{7e^2}$$

$$= \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))^2}{7de(e\cos(c+dx))^{7/2}} + \frac{2(a+b\sin(c+dx))\left(ab+(5a^2-4b^2\right.}{21de^3(e\cos(c+dx))^{3/2}}$$

$$= \frac{2b(5a^2-4b^2)\sqrt{e\cos(c+dx)}}{21de^5} + \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))^2}{7de(e\cos(c+dx))^{7/2}} + \frac{2(a}{}$$

$$= \frac{2b(5a^2-4b^2)\sqrt{e\cos(c+dx)}}{21de^5} + \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))^2}{7de(e\cos(c+dx))^{7/2}} + \frac{2(a}{}$$

$$= \frac{2b(5a^2-4b^2)\sqrt{e\cos(c+dx)}}{21de^5} + \frac{2a(5a^2-6b^2)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{21de^4\sqrt{e\cos(c+dx)}}$$

Mathematica [A]

time = 0.67, size = 140, normalized size = 0.74

$$\frac{\sqrt{e\cos(c+dx)}\,\sec^4(c+dx)\left(36a^2b-2b^3-14b^3\cos(2(c+dx))+4a(5a^2-6b^2)\cos^{\frac{7}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|\,2\right)+17a^3\sin(c+dx)+30ab^2\sin(c+dx)+5a^3\sin(3(c+dx))-6ab^2\sin(3(c+dx))\right)}{42de^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])^3/(e*Cos[c + d*x])^(9/2),x]
```

```
[Out] (Sqrt[e*Cos[c + d*x]]*Sec[c + d*x]^4*(36*a^2*b - 2*b^3 - 14*b^3*Cos[2*(c +
d*x)] + 4*a*(5*a^2 - 6*b^2)*Cos[c + d*x]^(7/2)*EllipticF[(c + d*x)/2, 2] +
17*a^3*Sin[c + d*x] + 30*a*b^2*Sin[c + d*x] + 5*a^3*Sin[3*(c + d*x)] - 6*a*
b^2*Sin[3*(c + d*x)]))/(42*d*e^5)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 749 vs. $2(196) = 392$.
time = 21.66, size = 750, normalized size = 3.99

method	result	size
default	Expression too large to display	750

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))^3/(e*cos(d*x+c))^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/21/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^
2-1)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/e^4*(40*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*a^3*sin(1/2*d*x+1/2*c)^6-48*EllipticF(cos(1/2*d*x+1/2*c),
2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a*b^
2*sin(1/2*d*x+1/2*c)^6-60*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a^3*sin(1/2*d*x+1/2*c)^4
+72*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*a*b^2*sin(1/2*d*x+1/2*c)^4+40*a^3*cos(1/2*d*x+
1/2*c)*sin(1/2*d*x+1/2*c)^6-48*a*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^
6+30*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))*a^3*sin(1/2*d*x+1/2*c)^2-36*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))*a*b^2*sin(1/2*d*x+1/2*c)^2-40*a^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/
2*c)^4+48*a*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-28*b^3*sin(1/2*d*x+
1/2*c)^5-5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^3+6*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2+16
*a^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+6*a*b^2*cos(1/2*d*x+1/2*c)*sin
(1/2*d*x+1/2*c)^2+28*b^3*sin(1/2*d*x+1/2*c)^3+9*a^2*b*sin(1/2*d*x+1/2*c)-4*
b^3*sin(1/2*d*x+1/2*c))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^3/(e*cos(d*x+c))^(9/2),x, algorithm="maxima")
```

```
[Out] e^(-9/2)*integrate((b*sin(d*x + c) + a)^3/cos(d*x + c)^(9/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 176, normalized size = 0.94

$$\frac{\left(\sqrt{2}\,(-5i\,a^{3}+6i\,ab^{2})\cos\left(dx+c\right)^{4}\,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+\sqrt{2}\,(5i\,a^{3}-6i\,ab^{2})\cos\left(dx+c\right)^{4}\,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right))-2\left(7\,b^{3}\cos\left(dx+c\right)^{2}-9\,a^{2}b-3\,b^{3}-\left(3\,a^{3}+9\,ab^{2}+(5\,a^{3}-6\,ab^{2})\cos\left(dx+c\right)^{2}\right)\sin\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)}\right)e^{\left(-\frac{9}{2}\right)}}{21\,d\cos\left(dx+c\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^3/(e*cos(d*x+c))^(9/2),x, algorithm="fricas")
```

```
[Out] 1/21*(sqrt(2)*(-5*I*a^3 + 6*I*a*b^2)*cos(d*x + c)^4*weierstrassPInverse(-4,
 0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(5*I*a^3 - 6*I*a*b^2)*cos(d*x
+ c)^4*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 2*(7*b^3
*cos(d*x + c)^2 - 9*a^2*b - 3*b^3 - (3*a^3 + 9*a*b^2 + (5*a^3 - 6*a*b^2)*co
s(d*x + c)^2)*sin(d*x + c))*sqrt(cos(d*x + c)))*e^(-9/2)/(d*cos(d*x + c)^4)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))**3/(e*cos(d*x+c))**(9/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4847 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^3/(e*cos(d*x+c))^(9/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^3*e^(-9/2)/cos(d*x + c)^(9/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,\sin\left(c+d\,x\right)\right)}^3}{{\left(e\,\cos\left(c+d\,x\right)\right)}^{9/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^3/(e*cos(c + d*x))^(9/2),x)
[Out] int((a + b*sin(c + d*x))^3/(e*cos(c + d*x))^(9/2), x)
```

3.564 $\int (e\cos(c+dx))^{7/2}(a+b\sin(c+dx))^4\,dx$

Optimal. Leaf size=305

$$-\frac{34ab(53a^2+38b^2)\,(e\cos(c+dx))^{9/2}}{6435de}+\frac{2(55a^4+60a^2b^2+4b^4)\,e^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d\sqrt{e\cos(c+dx)}}+\frac{2(55a^4+$$

```
[Out] -34/6435*a*b*(53*a^2+38*b^2)*(e*cos(d*x+c))^(9/2)/d/e+2/385*(55*a^4+60*a^2*
b^2+4*b^4)*e*(e*cos(d*x+c))^(5/2)*sin(d*x+c)/d-2/715*b*(93*a^2+26*b^2)*(e*c
os(d*x+c))^(9/2)*(a+b*sin(d*x+c))/d/e-14/65*a*b*(e*cos(d*x+c))^(9/2)*(a+b*s
in(d*x+c))^2/d/e-2/15*b*(e*cos(d*x+c))^(9/2)*(a+b*sin(d*x+c))^3/d/e+2/231*(
55*a^4+60*a^2*b^2+4*b^4)*e^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(e*cos(d*x+c))^(
1/2)+2/231*(55*a^4+60*a^2*b^2+4*b^4)*e^3*sin(d*x+c)*(e*cos(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.36, antiderivative size = 305, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2771, 2941, 2748, 2715, 2721, 2720}

$$-\frac{34ab(53a^2+38b^2)(e\cos(c+dx))^{9/2}}{6435de}-\frac{2b(93a^2+26b^2)(e\cos(c+dx))^{9/2}(a+b\sin(c+dx))}{715de}+\frac{2e^4(55a^4+60a^2b^2+4b^4)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d\sqrt{e\cos(c+dx)}}+\frac{2e^3(55a^4+60a^2b^2+4b^4)\sin(c+dx)\sqrt{e\cos(c+dx)}}{231d}+\frac{2e(55a^4+60a^2b^2+4b^4)\sin(c+dx)(e\cos(c+dx))^{5/2}}{385d}-\frac{2b(e\cos(c+dx))^{9/2}(a+b\sin(c+dx))^3}{15de}-\frac{14ab(e\cos(c+dx))^{9/2}(a+b\sin(c+dx))^2}{65de}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(7/2)*(a + b*Sin[c + d*x])^4,x]
```

```
[Out] (-34*a*b*(53*a^2 + 38*b^2)*(e*Cos[c + d*x])^(9/2))/(6435*d*e) + (2*(55*a^4
+ 60*a^2*b^2 + 4*b^4)*e^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(23
1*d*Sqrt[e*Cos[c + d*x]]) + (2*(55*a^4 + 60*a^2*b^2 + 4*b^4)*e^3*Sqrt[e*Cos
[c + d*x]]*Sin[c + d*x])/(231*d) + (2*(55*a^4 + 60*a^2*b^2 + 4*b^4)*e*(e*Co
s[c + d*x])^(5/2)*Sin[c + d*x])/(385*d) - (2*b*(93*a^2 + 26*b^2)*(e*Cos[c +
 d*x])^(9/2)*(a + b*Sin[c + d*x]))/(715*d*e) - (14*a*b*(e*Cos[c + d*x])^(9/
2)*(a + b*Sin[c + d*x])^2)/(65*d*e) - (2*b*(e*Cos[c + d*x])^(9/2)*(a + b*Si
n[c + d*x])^3)/(15*d*e)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2771

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(
a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rule 2941

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp
[a*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x],
x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
&&  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Si
mplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int (e\cos(c+dx))^{7/2}(a+b\sin(c+dx))^4\,dx = -\frac{2b(e\cos(c+dx))^{9/2}(a+b\sin(c+dx))^3}{15de}+\frac{2}{15}\int (e\cos(c+dx$$

$$= -\frac{14ab(e\cos(c+dx))^{9/2}(a+b\sin(c+dx))^2}{65de}-\frac{2b(e\cos(c+dx)}{}$$

$$= -\frac{2b(93a^2+26b^2)\,(e\cos(c+dx))^{9/2}(a+b\sin(c+dx))}{715de}-\frac{14ab(}{}$$

$$= -\frac{34ab(53a^2+38b^2)\,(e\cos(c+dx))^{9/2}}{6435de}-\frac{2b(93a^2+26b^2)\,(e\cos}{}$$

$$= -\frac{34ab(53a^2+38b^2)\,(e\cos(c+dx))^{9/2}}{6435de}+\frac{2(55a^4+60a^2b^2+4b^4}{}$$

$$= -\frac{34ab(53a^2+38b^2)\,(e\cos(c+dx))^{9/2}}{6435de}+\frac{2(55a^4+60a^2b^2+4b^4}{}$$

$$= -\frac{34ab(53a^2+38b^2)\,(e\cos(c+dx))^{9/2}}{6435de}+\frac{2(55a^4+60a^2b^2+4b^4}{}$$

$$= -\frac{34ab(53a^2+38b^2)\,(e\cos(c+dx))^{9/2}}{6435de}+\frac{2(55a^4+60a^2b^2+4b^4}{231}$$

Mathematica [A]
time = 4.56, size = 251, normalized size = 0.82

$$\frac{(e\cos(c+dx))^{7/2}\left(-154ab(26a^2+11b^2)\sqrt{\cos(c+dx)}+104(55a^4+60a^2b^2+4b^4)F\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{1}{120}\sqrt{\cos(c+dx)}\left(156(5720a^4+2460a^2b^2+87b^4)\sin(c+dx)+462b^3\cos(6(c+dx))(60a+13b\sin(c+dx))-28b\cos(4(c+dx))\left(220a(26a^2-b^2)+39b(180a^2+b^2)\sin(c+dx)\right)+\cos(2(c+dx))\left(-3080(208a^3b+73ab^3)+78(2640a^4-7200a^2b^2-557b^4)\sin(c+dx)\right)\right)\right)}{12012d\cos^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(7/2)*(a + b*Sin[c + d*x])^4,x]
```

```
[Out] ((e*Cos[c + d*x])^(7/2)*(-154*a*b*(26*a^2 + 11*b^2)*Sqrt[Cos[c + d*x]] + 10
4*(55*a^4 + 60*a^2*b^2 + 4*b^4)*EllipticF[(c + d*x)/2, 2] + (Sqrt[Cos[c + d
*x]]*(156*(5720*a^4 + 2460*a^2*b^2 + 87*b^4)*Sin[c + d*x] + 462*b^3*Cos[6*(
c + d*x)]*(60*a + 13*b*Sin[c + d*x]) - 28*b*Cos[4*(c + d*x)]*(220*a*(26*a^2
 - b^2) + 39*b*(180*a^2 + b^2)*Sin[c + d*x]) + Cos[2*(c + d*x)]*(-3080*(208
*a^3*b + 73*a*b^3) + 78*(2640*a^4 - 7200*a^2*b^2 - 557*b^4)*Sin[c + d*x]))
/120))/(12012*d*Cos[c + d*x]^(7/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 862 vs. $2(301) = 602$.
time = 12.36, size = 863, normalized size = 2.83

method	result	size

default	Expression too large to display	863

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(7/2)*(a+b*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] -2/45045/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^4*(867328
0*a*b^3*sin(1/2*d*x+1/2*c)^11-2690688*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/
2*c)^14+768768*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^16+3739008*b^4*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12-2620800*b^4*cos(1/2*d*x+1/2*c)*sin(1/
2*d*x+1/2*c)^10+102960*a^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+946608*b
^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-154440*a^4*cos(1/2*d*x+1/2*c)*si
n(1/2*d*x+1/2*c)^6+11700*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b^2+3931200*a^2*b^2*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10-3818880*a^2*b^2*cos(1/2*d*x+1/2*c)*
sin(1/2*d*x+1/2*c)^8+1797120*a^2*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^
6-360360*a^2*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+11700*a^2*b^2*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+10725*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^4+780*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))*b^4-1572480*a^2*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x
+1/2*c)^12-144456*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+120120*a^4*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-34320*a^4*cos(1/2*d*x+1/2*c)*sin(1/2*
d*x+1/2*c)^2+780*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-6209280*a*b^3*
sin(1/2*d*x+1/2*c)^13+1774080*a*b^3*sin(1/2*d*x+1/2*c)^15-640640*a^3*b*sin(
1/2*d*x+1/2*c)^11+1601600*a^3*b*sin(1/2*d*x+1/2*c)^9-6160000*a*b^3*sin(1/2*
d*x+1/2*c)^9-1601600*a^3*b*sin(1/2*d*x+1/2*c)^7+2279200*a*b^3*sin(1/2*d*x+1
/2*c)^7+800800*a^3*b*sin(1/2*d*x+1/2*c)^5-363440*a*b^3*sin(1/2*d*x+1/2*c)^5
-200200*a^3*b*sin(1/2*d*x+1/2*c)^3-6160*a*b^3*sin(1/2*d*x+1/2*c)^3+20020*a^
3*b*sin(1/2*d*x+1/2*c)+6160*a*b^3*sin(1/2*d*x+1/2*c))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)*(a+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] e^(7/2)*integrate((b*sin(d*x + c) + a)^4*cos(d*x + c)^(7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.16, size = 248, normalized size = 0.81

$$\frac{-195i\,\sqrt{2}\,(55\,a^4+60\,a^2b^2+4\,b^4)e^{\frac{7}{2}}\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+195i\,\sqrt{2}\,(55\,a^4+60\,a^2b^2+4\,b^4)e^{\frac{7}{2}}\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c))+2\left(13860\,ab^3\cos(dx+c)^6\,e^{\frac{7}{2}}-20020\,(a^3b+ab^3)\cos(dx+c)^4\,e^{\frac{7}{2}}+39\left(77\,b^4\cos(dx+c)^6\,e^{\frac{7}{2}}-7\,(90\,a^2b^2+17\,b^4)\cos(dx+c)^4\,e^{\frac{7}{2}}+3\,(55\,a^4+60\,a^2b^2+4\,b^4)\cos(dx+c)^2\,e^{\frac{7}{2}}+5\,(55\,a^4+60\,a^2b^2+4\,b^4)e^{\frac{7}{2}}\right)\sin(dx+c)\right)\sqrt{\cos(dx+c)}}{45045\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)*(a+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/45045*(-195*I*sqrt(2)*(55*a^4 + 60*a^2*b^2 + 4*b^4)*e^(7/2)*weierstrassPI
nverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 195*I*sqrt(2)*(55*a^4 + 60*a
^2*b^2 + 4*b^4)*e^(7/2)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x
 + c)) + 2*(13860*a*b^3*cos(d*x + c)^6*e^(7/2) - 20020*(a^3*b + a*b^3)*cos(
d*x + c)^4*e^(7/2) + 39*(77*b^4*cos(d*x + c)^6*e^(7/2) - 7*(90*a^2*b^2 + 17
*b^4)*cos(d*x + c)^4*e^(7/2) + 3*(55*a^4 + 60*a^2*b^2 + 4*b^4)*cos(d*x + c)
^2*e^(7/2) + 5*(55*a^4 + 60*a^2*b^2 + 4*b^4)*e^(7/2))*sin(d*x + c))*sqrt(co
s(d*x + c)))/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(7/2)*(a+b*sin(d*x+c))**4,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8856 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)*(a+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^4*cos(d*x + c)^(7/2)*e^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (e\,\cos\left(c+d\,x\right))^{7/2}\,\left(a+b\,\sin\left(c+d\,x\right)\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(7/2)*(a + b*sin(c + d*x))^4,x)
```

```
[Out] int((e*cos(c + d*x))^(7/2)*(a + b*sin(c + d*x))^4, x)
```

3.565 $\int (e\cos(c+dx))^{5/2}(a+b\sin(c+dx))^4\,dx$

Optimal. Leaf size=258

$$-\frac{10ab(115a^2+94b^2)\,(e\cos(c+dx))^{7/2}}{3003de}+\frac{2(39a^4+52a^2b^2+4b^4)\,e^2\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{65d\sqrt{\cos(c+dx)}}+\frac{2(39a$$

```
[Out] -10/3003*a*b*(115*a^2+94*b^2)*(e*cos(d*x+c))^(7/2)/d/e+2/195*(39*a^4+52*a^2
*b^2+4*b^4)*e*(e*cos(d*x+c))^(3/2)*sin(d*x+c)/d-2/429*b*(73*a^2+22*b^2)*(e*
cos(d*x+c))^(7/2)*(a+b*sin(d*x+c))/d/e-38/143*a*b*(e*cos(d*x+c))^(7/2)*(a+b
*sin(d*x+c))^2/d/e-2/13*b*(e*cos(d*x+c))^(7/2)*(a+b*sin(d*x+c))^3/d/e+2/65*
(39*a^4+52*a^2*b^2+4*b^4)*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*
c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/d/cos(d*x+c)^
(1/2)
```

Rubi [A]
time = 0.33, antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2771, 2941, 2748, 2715, 2721, 2719}

$$-\frac{10ab(115a^2+94b^2)\,(e\cos(c+dx))^{7/2}}{3003de}-\frac{2b(73a^2+22b^2)\,(e\cos(c+dx))^{7/2}(a+b\sin(c+dx))}{429de}+\frac{2e^2(39a^4+52a^2b^2+4b^4)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{65d\sqrt{\cos(c+dx)}}+\frac{2e(39a^4+52a^2b^2+4b^4)\sin(c+dx)(e\cos(c+dx))^{3/2}}{195d}-\frac{2b(e\cos(c+dx))^{7/2}(a+b\sin(c+dx))^3}{13de}-\frac{38ab(e\cos(c+dx))^{7/2}(a+b\sin(c+dx))^2}{143de}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(5/2)*(a + b*Sin[c + d*x])^4,x]
```

```
[Out] (-10*a*b*(115*a^2 + 94*b^2)*(e*Cos[c + d*x])^(7/2))/(3003*d*e) + (2*(39*a^4
 + 52*a^2*b^2 + 4*b^4)*e^2*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/
(65*d*Sqrt[Cos[c + d*x]]) + (2*(39*a^4 + 52*a^2*b^2 + 4*b^4)*e*(e*Cos[c + d
*x])^(3/2)*Sin[c + d*x])/(195*d) - (2*b*(73*a^2 + 22*b^2)*(e*Cos[c + d*x])^
(7/2)*(a + b*Sin[c + d*x]))/(429*d*e) - (38*a*b*(e*Cos[c + d*x])^(7/2)*(a +
 b*Sin[c + d*x])^2)/(143*d*e) - (2*b*(e*Cos[c + d*x])^(7/2)*(a + b*Sin[c +
d*x])^3)/(13*d*e)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2771

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(
a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rule 2941

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp
[a*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x],
x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
&&  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Si
mplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int (e\cos(c+dx))^{5/2}(a+b\sin(c+dx))^4\,dx = -\frac{2b(e\cos(c+dx))^{7/2}(a+b\sin(c+dx))^3}{13de}+\frac{2}{13}\int (e\cos(c+d$$

$$= -\frac{38ab(e\cos(c+dx))^{7/2}(a+b\sin(c+dx))^2}{143de}-\frac{2b(e\cos(c+dx$$

$$= -\frac{2b(73a^2+22b^2)\,(e\cos(c+dx))^{7/2}(a+b\sin(c+dx))}{429de}-\frac{38ab$$

$$= -\frac{10ab(115a^2+94b^2)\,(e\cos(c+dx))^{7/2}}{3003de}-\frac{2b(73a^2+22b^2)\,(e$$

$$= -\frac{10ab(115a^2+94b^2)\,(e\cos(c+dx))^{7/2}}{3003de}+\frac{2(39a^4+52a^2b^2+$$

$$= -\frac{10ab(115a^2+94b^2)\,(e\cos(c+dx))^{7/2}}{3003de}+\frac{2(39a^4+52a^2b^2+$$

$$= -\frac{10ab(115a^2+94b^2)\,(e\cos(c+dx))^{7/2}}{3003de}+\frac{2(39a^4+52a^2b^2+$$

Mathematica [A]
time = 2.09, size = 209, normalized size = 0.81

$$\frac{(e\cos(c+dx))^{5/2}\left(2(39a^4+52a^2b^2+4b^4)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+65\sqrt{\cos(c+dx)}\left(-\frac{1}{77}ab(66a^2+31b^2)\cos(c+dx)-\frac{1}{154}ab(44a^2+9b^2)\cos(3(c+dx))+\frac{1}{22}ab^3\cos(5(c+dx))+\frac{(624a^4-208a^2b^2-61b^4)\sin(2(c+dx))}{3120}-\frac{1}{78}b^2(13a^2+b^2)\sin(4(c+dx))+\frac{1}{208}b^4\sin(6(c+dx))\right)\right)}{65d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(5/2)*(a + b*Sin[c + d*x])^4,x]
```

```
[Out] ((e*Cos[c + d*x])^(5/2)*(2*(39*a^4 + 52*a^2*b^2 + 4*b^4)*EllipticE[(c + d*x
)/2, 2] + 65*Sqrt[Cos[c + d*x]]*(-1/77*(a*b*(66*a^2 + 31*b^2)*Cos[c + d*x])
 - (a*b*(44*a^2 + 9*b^2)*Cos[3*(c + d*x)])/154 + (a*b^3*Cos[5*(c + d*x)])/2
2 + ((624*a^4 - 208*a^2*b^2 - 61*b^4)*Sin[2*(c + d*x)])/3120 - (b^2*(13*a^2
 + b^2)*Sin[4*(c + d*x)])/78 + (b^4*Sin[6*(c + d*x)])/208)))/(65*d*Cos[c +
d*x]^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 775 vs. 2(258) = 516.
time = 11.02, size = 776, normalized size = 3.01

method	result	size
default	Expression too large to display	776

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(5/2)*(a+b*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 2/15015/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^3*(-104832
0*a*b^3*sin(1/2*d*x+1/2*c)^11+147840*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2
*c)^14-443520*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12+492800*b^4*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10-246400*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d
*x+1/2*c)^8+24024*a^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+9009*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2))*a^4+924*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^4-320320*a^2*b^2*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+640640*a^2*b^2*cos(1/2*d*x+1/2*c)*sin(1/
2*d*x+1/2*c)^8-448448*a^2*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+12812
8*a^2*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-12012*a^2*b^2*cos(1/2*d*x
+1/2*c)*sin(1/2*d*x+1/2*c)^2+616*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^
4+48664*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-24024*a^4*cos(1/2*d*x+1
/2*c)*sin(1/2*d*x+1/2*c)^4+6006*a^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2
-924*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+349440*a*b^3*sin(1/2*d*x+1
/2*c)^13-137280*a^3*b*sin(1/2*d*x+1/2*c)^9+1173120*a*b^3*sin(1/2*d*x+1/2*c)
^9+274560*a^3*b*sin(1/2*d*x+1/2*c)^7-599040*a*b^3*sin(1/2*d*x+1/2*c)^7-2059
20*a^3*b*sin(1/2*d*x+1/2*c)^5+121680*a*b^3*sin(1/2*d*x+1/2*c)^5+68640*a^3*b
*sin(1/2*d*x+1/2*c)^3+3120*a*b^3*sin(1/2*d*x+1/2*c)^3-8580*a^3*b*sin(1/2*d*
x+1/2*c)-3120*a*b^3*sin(1/2*d*x+1/2*c)+12012*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b
^2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] e^(5/2)*integrate((b*sin(d*x + c) + a)^4*cos(d*x + c)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.15, size = 228, normalized size = 0.88

$$\frac{231i\sqrt{2}\,(39\,a^4+52\,a^2b^2+4\,b^4)e^{\frac{5}{2}}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)))-231i\sqrt{2}\,(39\,a^4+52\,a^2b^2+4\,b^4)e^{\frac{5}{2}}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c)))+2\left(5460\,ab^3\cos(dx+c)^5\,e^{\frac{5}{2}}-8580\,(a^3b+ab^3)\cos(dx+c)^3\,e^{\frac{5}{2}}+77\left(15\,b^4\cos(dx+c)^5\,e^{\frac{5}{2}}-5\,(26\,a^2b^2+5\,b^4)\cos(dx+c)^3\,e^{\frac{5}{2}}+(39\,a^4+52\,a^2b^2+4\,b^4)\cos(dx+c)\,e^{\frac{5}{2}}\right)\sin(dx+c)\right)\sqrt{\cos(dx+c)}}{15015\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/15015*(231*I*sqrt(2)*(39*a^4 + 52*a^2*b^2 + 4*b^4)*e^(5/2)*weierstrassZet
a(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 231*I
*sqrt(2)*(39*a^4 + 52*a^2*b^2 + 4*b^4)*e^(5/2)*weierstrassZeta(-4, 0, weier
strassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(5460*a*b^3*cos(d
*x + c)^5*e^(5/2) - 8580*(a^3*b + a*b^3)*cos(d*x + c)^3*e^(5/2) + 77*(15*b^
```

```
4*cos(d*x + c)^5*e^(5/2) - 5*(26*a^2*b^2 + 5*b^4)*cos(d*x + c)^3*e^(5/2) +
(39*a^4 + 52*a^2*b^2 + 4*b^4)*cos(d*x + c)*e^(5/2))*sin(d*x + c))*sqrt(cos(
d*x + c)))/d
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(5/2)*(a+b*sin(d*x+c))**4,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3877 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+b*sin(d*x+c))^4,x, algorithm="giac")
[Out] integrate((b*sin(d*x + c) + a)^4*cos(d*x + c)^(5/2)*e^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (e\,\cos (c+d\,x))^{5/2}\,(a+b\,\sin (c+d\,x))^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(5/2)*(a + b*sin(c + d*x))^4,x)
[Out] int((e*cos(c + d*x))^(5/2)*(a + b*sin(c + d*x))^4, x)
```

3.566 $\int (e\cos(c+dx))^{3/2}(a+b\sin(c+dx))^4\,dx$

Optimal. Leaf size=258

$$-\frac{26ab(79a^2+74b^2)\,(e\cos(c+dx))^{5/2}}{3465de}+\frac{2(77a^4+132a^2b^2+12b^4)\,e^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{231d\sqrt{e\cos(c+dx)}}+\frac{2(77a^4}{}$$

```
[Out] -26/3465*a*b*(79*a^2+74*b^2)*(e*cos(d*x+c))^(5/2)/d/e-2/693*b*(167*a^2+54*b
^2)*(e*cos(d*x+c))^(5/2)*(a+b*sin(d*x+c))/d/e-34/99*a*b*(e*cos(d*x+c))^(5/2
)*(a+b*sin(d*x+c))^2/d/e-2/11*b*(e*cos(d*x+c))^(5/2)*(a+b*sin(d*x+c))^3/d/e
+2/231*(77*a^4+132*a^2*b^2+12*b^4)*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2
*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(e*cos
(d*x+c))^(1/2)+2/231*(77*a^4+132*a^2*b^2+12*b^4)*e*sin(d*x+c)*(e*cos(d*x+c)
)^(1/2)/d
```

Rubi [A]
time = 0.33, antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}=0.240$, Rules used = $\{2771, 2941, 2748, 2715, 2721, 2720\}$

$$-\frac{26ab(79a^2+74b^2)\,(e\cos(c+dx))^{5/2}}{3465de}-\frac{2b(167a^2+54b^2)\,(e\cos(c+dx))^{5/2}(a+b\sin(c+dx))}{693de}+\frac{2e^2(77a^4+132a^2b^2+12b^4)\,\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{231d\sqrt{e\cos(c+dx)}}+\frac{2e(77a^4+132a^2b^2+12b^4)\sin(c+dx)\sqrt{e\cos(c+dx)}}{231d}-\frac{2b(e\cos(c+dx))^{5/2}(a+b\sin(c+dx))^3}{11de}-\frac{34ab(e\cos(c+dx))^{5/2}(a+b\sin(c+dx))^2}{99de}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x])^4,x]
```

```
[Out] (-26*a*b*(79*a^2 + 74*b^2)*(e*Cos[c + d*x])^(5/2))/(3465*d*e) + (2*(77*a^4
+ 132*a^2*b^2 + 12*b^4)*e^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(
231*d*Sqrt[e*Cos[c + d*x]]) + (2*(77*a^4 + 132*a^2*b^2 + 12*b^4)*e*Sqrt[e*C
os[c + d*x]]*Sin[c + d*x])/(231*d) - (2*b*(167*a^2 + 54*b^2)*(e*Cos[c + d*x
])^(5/2)*(a + b*Sin[c + d*x]))/(693*d*e) - (34*a*b*(e*Cos[c + d*x])^(5/2)*(
a + b*Sin[c + d*x])^2)/(99*d*e) - (2*b*(e*Cos[c + d*x])^(5/2)*(a + b*Sin[c
+ d*x])^3)/(11*d*e)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2771

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(
a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rule 2941

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp
[a*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x],
x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
&&  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Si
mplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int (e\cos(c+dx))^{3/2}(a+b\sin(c+dx))^4\,dx = -\frac{2b(e\cos(c+dx))^{5/2}(a+b\sin(c+dx))^3}{11de}+\frac{2}{11}\int (e\cos(c+dx)$$

$$= -\frac{34ab(e\cos(c+dx))^{5/2}(a+b\sin(c+dx))^2}{99de}-\frac{2b(e\cos(c+dx)}{}$$

$$= -\frac{2b(167a^2+54b^2)\,(e\cos(c+dx))^{5/2}(a+b\sin(c+dx))}{693de}-\frac{34ab}{}$$

$$= -\frac{26ab(79a^2+74b^2)\,(e\cos(c+dx))^{5/2}}{3465de}-\frac{2b(167a^2+54b^2)\,(e\,\mathrm{co}}{}$$

$$= -\frac{26ab(79a^2+74b^2)\,(e\cos(c+dx))^{5/2}}{3465de}+\frac{2(77a^4+132a^2b^2+1}{}$$

$$= -\frac{26ab(79a^2+74b^2)\,(e\cos(c+dx))^{5/2}}{3465de}+\frac{2(77a^4+132a^2b^2+1}{}$$

$$= -\frac{26ab(79a^2+74b^2)\,(e\cos(c+dx))^{5/2}}{3465de}+\frac{2(77a^4+132a^2b^2+1}{231}$$

Mathematica [A]
time = 2.78, size = 189, normalized size = 0.73

$$\frac{(e\cos(c+dx))^{3/2}\left(240(77a^4+132a^2b^2+12b^4)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+\sqrt{\cos(c+dx)}\,(-1848b(12a^3+7ab^2)-2464(9a^3b+4ab^3)\cos(2(c+dx))+3080ab^3\cos(4(c+dx))+30(616a^4+660a^2b^2+39b^4)\sin(c+dx)-45b(264a^2b+31b^3)\sin(3(c+dx))+315b^4\sin(5(c+dx)))\right)}{27720d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x])^4,x]
```

```
[Out] ((e*Cos[c + d*x])^(3/2)*(240*(77*a^4 + 132*a^2*b^2 + 12*b^4)*EllipticF[(c +
 d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(-1848*b*(12*a^3 + 7*a*b^2) - 2464*(9*a^3*
b + 4*a*b^3)*Cos[2*(c + d*x)] + 3080*a*b^3*Cos[4*(c + d*x)] + 30*(616*a^4 +
 660*a^2*b^2 + 39*b^4)*Sin[c + d*x] - 45*b*(264*a^2*b + 31*b^3)*Sin[3*(c +
d*x)] + 315*b^4*Sin[5*(c + d*x)])))/(27720*d*Cos[c + d*x]^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 638 vs. $2(258) = 516$.
time = 9.99, size = 639, normalized size = 2.48

method	result
default	$-\frac{2e^2\left(20160b^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+49280a\,b^3\left(\sin^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-50400b^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-47520a^2b^2\cos\left(\frac{dx}{2}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(3/2)*(a+b*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] -2/3465/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^2*(20160*b
^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12+49280*a*b^3*sin(1/2*d*x+1/2*c)^
11-50400*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10-47520*a^2*b^2*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-123200*a*b^3*sin(1/2*d*x+1/2*c)^9+41040*b^
4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-22176*a^3*b*sin(1/2*d*x+1/2*c)^7+
71280*a^2*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+101024*a*b^3*sin(1/2*
d*x+1/2*c)^7-11160*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+4620*a^4*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+33264*a^3*b*sin(1/2*d*x+1/2*c)^5-27720
*a^2*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-28336*a*b^3*sin(1/2*d*x+1/
2*c)^5+1155*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^4+1980*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*
b^2+180*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^4-2310*a^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*
x+1/2*c)^2-16632*a^3*b*sin(1/2*d*x+1/2*c)^3+1980*a^2*b^2*cos(1/2*d*x+1/2*c)
*sin(1/2*d*x+1/2*c)^2-1232*a*b^3*sin(1/2*d*x+1/2*c)^3+180*b^4*cos(1/2*d*x+1
/2*c)*sin(1/2*d*x+1/2*c)^2+2772*a^3*b*sin(1/2*d*x+1/2*c)+1232*a*b^3*sin(1/2
*d*x+1/2*c))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] e^(3/2)*integrate((b*sin(d*x + c) + a)^4*cos(d*x + c)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 216, normalized size = 0.84

$$\frac{-15i\,\sqrt{2}\,(77\,a^4+132\,a^2b^2+12\,b^4)e^{\frac{3}{2}}\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+15i\,\sqrt{2}\,(77\,a^4+132\,a^2b^2+12\,b^4)e^{\frac{3}{2}}\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c))+2\left(1540\,ab^3\cos(dx+c)^4e^{\frac{3}{2}}-2772\,(a^3b+ab^3)\cos(dx+c)^2e^{\frac{3}{2}}+15\left(21\,b^4\cos(dx+c)^4e^{\frac{3}{2}}-3\,(66\,a^2b^2+13\,b^4)\cos(dx+c)^2e^{\frac{3}{2}}+(77\,a^4+132\,a^2b^2+12\,b^4)e^{\frac{3}{2}}\right)\sin(dx+c)\right)\sqrt{\cos(dx+c)}}{3465\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/3465*(-15*I*sqrt(2)*(77*a^4 + 132*a^2*b^2 + 12*b^4)*e^(3/2)*weierstrassPI
nverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 15*I*sqrt(2)*(77*a^4 + 132*a
^2*b^2 + 12*b^4)*e^(3/2)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*
```

```
x + c)) + 2*(1540*a*b^3*cos(d*x + c)^4*e^(3/2) - 2772*(a^3*b + a*b^3)*cos(d
*x + c)^2*e^(3/2) + 15*(21*b^4*cos(d*x + c)^4*e^(3/2) - 3*(66*a^2*b^2 + 13*
b^4)*cos(d*x + c)^2*e^(3/2) + (77*a^4 + 132*a^2*b^2 + 12*b^4)*e^(3/2))*sin(
d*x + c))*sqrt(cos(d*x + c)))/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(3/2)*(a+b*sin(d*x+c))**4,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 7319 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^4*cos(d*x + c)^(3/2)*e^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (e \cos (c + d x))^{3/2} (a + b \sin (c + d x))^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(3/2)*(a + b*sin(c + d*x))^4,x)
```

```
[Out] int((e*cos(c + d*x))^(3/2)*(a + b*sin(c + d*x))^4, x)
```

3.567 $\int \sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))^4\,dx$

Optimal. Leaf size=210

$$-\frac{22ab(17a^2+18b^2)\,(e\cos(c+dx))^{3/2}}{315de}+\frac{2(15a^4+36a^2b^2+4b^4)\,\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d\sqrt{\cos(c+dx)}}-\frac{2b(41a^2}{}$$

```
[Out] -22/315*a*b*(17*a^2+18*b^2)*(e*cos(d*x+c))^(3/2)/d/e-2/105*b*(41*a^2+14*b^2
)*(e*cos(d*x+c))^(3/2)*(a+b*sin(d*x+c))/d/e-10/21*a*b*(e*cos(d*x+c))^(3/2)*
(a+b*sin(d*x+c))^2/d/e-2/9*b*(e*cos(d*x+c))^(3/2)*(a+b*sin(d*x+c))^3/d/e+2/
15*(15*a^4+36*a^2*b^2+4*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(
1/2)
```

Rubi [A]
time = 0.28, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2771, 2941, 2748, 2721, 2719}

$$-\frac{22ab(17a^2+18b^2)\,(e\cos(c+dx))^{3/2}}{315de}-\frac{2b(41a^2+14b^2)\,(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))}{105de}+\frac{2(15a^4+36a^2b^2+4b^4)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{15d\sqrt{\cos(c+dx)}}-\frac{2b(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))^3}{9de}-\frac{10ab(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))^2}{21de}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x])^4,x]
```

```
[Out] (-22*a*b*(17*a^2 + 18*b^2)*(e*Cos[c + d*x])^(3/2))/(315*d*e) + (2*(15*a^4 +
 36*a^2*b^2 + 4*b^4)*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*d*
Sqrt[Cos[c + d*x]]) - (2*b*(41*a^2 + 14*b^2)*(e*Cos[c + d*x])^(3/2)*(a + b*
Sin[c + d*x]))/(105*d*e) - (10*a*b*(e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*
x])^2)/(21*d*e) - (2*b*(e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x])^3)/(9*d*
e)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
```

```
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2771

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(
a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rule 2941

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp
[a*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x],
x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
&&  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Si
mplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int \sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))^4\,dx = -\frac{2b(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))^3}{9de}+\frac{2}{9}\int \sqrt{e\cos(c+dx}$$

$$= -\frac{10ab(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))^2}{21de}-\frac{2b(e\cos(c+dx))}{}$$

$$= -\frac{2b(41a^2+14b^2)\,(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))}{105de}-\frac{10ab(e}{}$$

$$= -\frac{22ab(17a^2+18b^2)\,(e\cos(c+dx))^{3/2}}{315de}-\frac{2b(41a^2+14b^2)\,(e\cos(}{}$$

$$= -\frac{22ab(17a^2+18b^2)\,(e\cos(c+dx))^{3/2}}{315de}-\frac{2b(41a^2+14b^2)\,(e\cos(}{}$$

$$= -\frac{22ab(17a^2+18b^2)\,(e\cos(c+dx))^{3/2}}{315de}+\frac{2(15a^4+36a^2b^2+4b^4)}{15d}$$

Mathematica [A]

time = 1.12, size = 137, normalized size = 0.65

$$\frac{\sqrt{e\cos(c+dx)}\left(84(15a^4+36a^2b^2+4b^4)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)-b\cos^{\frac{3}{2}}(c+dx)\left(-360ab^2\cos(2(c+dx))+21b(72a^2+13b^2)\sin(c+dx)+5(336a^3+264ab^2-7b^3\sin(3(c+dx)))\right)\right)}{630d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x])^4,x]
```

```
[Out] (Sqrt[e*Cos[c + d*x]]*(84*(15*a^4 + 36*a^2*b^2 + 4*b^4)*EllipticE[(c + d*x)
/2, 2] - b*Cos[c + d*x]^(3/2)*(-360*a*b^2*Cos[2*(c + d*x)] + 21*b*(72*a^2 +
 13*b^2)*Sin[c + d*x] + 5*(336*a^3 + 264*a*b^2 - 7*b^3*Sin[3*(c + d*x)]))))
/(630*d*Sqrt[Cos[c + d*x]])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 524 vs. 2(214) = 428.
time = 8.57, size = 525, normalized size = 2.50

method	result
default	$\frac{2e\left(1120b^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2880a\,b^3\left(\sin^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2240b^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-3024a^2b^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\mathrm{si}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))^4*(e*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/315/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e*(1120*b^4*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+2880*a*b^3*sin(1/2*d*x+1/2*c)^9-2240
*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-3024*a^2*b^2*cos(1/2*d*x+1/2*c
)*sin(1/2*d*x+1/2*c)^6-5760*a*b^3*sin(1/2*d*x+1/2*c)^7+1064*b^4*cos(1/2*d*x
+1/2*c)*sin(1/2*d*x+1/2*c)^6-1680*a^3*b*sin(1/2*d*x+1/2*c)^5+3024*a^2*b^2*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+2640*a*b^3*sin(1/2*d*x+1/2*c)^5+56*b
^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+315*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^4
+756*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b^2+84*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^4+1680*
a^3*b*sin(1/2*d*x+1/2*c)^3-756*a^2*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c
)^2+240*a*b^3*sin(1/2*d*x+1/2*c)^3-84*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/
2*c)^2-420*a^3*b*sin(1/2*d*x+1/2*c)-240*a*b^3*sin(1/2*d*x+1/2*c))/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^4*(e*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] e^(1/2)*integrate((b*sin(d*x + c) + a)^4*sqrt(cos(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 196, normalized size = 0.93

$$\frac{21i\sqrt{2}\,(15\,a^4+36\,a^2b^2+4\,b^4)e^{\frac{1}{2}}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)))-21i\sqrt{2}\,(15\,a^4+36\,a^2b^2+4\,b^4)e^{\frac{1}{2}}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c)))+2\left(180\,ab^3\cos(dx+c)^3e^{\frac{1}{2}}-420\,(a^3b+ab^3)\cos(dx+c)e^{\frac{1}{2}}+7\left(5\,b^4\cos(dx+c)^3e^{\frac{1}{2}}-(54\,a^2b^2+11\,b^4)\cos(dx+c)e^{\frac{1}{2}}\right)\sin(dx+c)\right)\sqrt{\cos(dx+c)}}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^4*(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/315*(21*I*sqrt(2)*(15*a^4 + 36*a^2*b^2 + 4*b^4)*e^(1/2)*weierstrassZeta(-
4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 21*I*sqr
t(2)*(15*a^4 + 36*a^2*b^2 + 4*b^4)*e^(1/2)*weierstrassZeta(-4, 0, weierstra
ssPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(180*a*b^3*cos(d*x +
c)^3*e^(1/2) - 420*(a^3*b + a*b^3)*cos(d*x + c)*e^(1/2) + 7*(5*b^4*cos(d*x
+ c)^3*e^(1/2) - (54*a^2*b^2 + 11*b^4)*cos(d*x + c)*e^(1/2))*sin(d*x + c))*
sqrt(cos(d*x + c)))/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))**4*(e*cos(d*x+c))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3064 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^4*(e*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^4*sqrt(cos(d*x + c))*e^(1/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{e\,\cos(c+d\,x)}\,\left(a+b\,\sin(c+d\,x)\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(1/2)*(a + b*sin(c + d*x))^4,x)
```

```
[Out] int((e*cos(c + d*x))^(1/2)*(a + b*sin(c + d*x))^4, x)
```

3.568 $\int \frac{(a+b\sin(c+dx))^4}{\sqrt{e\cos(c+dx)}}\,dx$

Optimal. Leaf size=210

$$-\frac{6ab(31a^2+34b^2)\sqrt{e\cos(c+dx)}}{35de}+\frac{2(7a^4+28a^2b^2+4b^4)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7d\sqrt{e\cos(c+dx)}}-\frac{2b(29a^2+10b^2)\ldots}{}$$

```
[Out] 2/7*(7*a^4+28*a^2*b^2+4*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(e*cos(d*x+c))^(
1/2)-6/35*a*b*(31*a^2+34*b^2)*(e*cos(d*x+c))^(1/2)/d/e-2/35*b*(29*a^2+10*b^
2)*(a+b*sin(d*x+c))*(e*cos(d*x+c))^(1/2)/d/e-26/35*a*b*(a+b*sin(d*x+c))^2*(
e*cos(d*x+c))^(1/2)/d/e-2/7*b*(a+b*sin(d*x+c))^3*(e*cos(d*x+c))^(1/2)/d/e
```

Rubi [A]

time = 0.30, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2771, 2941, 2748, 2721, 2720}

$$-\frac{6ab(31a^2+34b^2)\sqrt{e\cos(c+dx)}}{35de}-\frac{2b(29a^2+10b^2)\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))}{35de}+\frac{2(7a^4+28a^2b^2+4b^4)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7d\sqrt{e\cos(c+dx)}}-\frac{2b\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))^3}{7de}-\frac{26ab\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))^2}{35de}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^4/Sqrt[e*Cos[c + d*x]],x]
```

```
[Out] (-6*a*b*(31*a^2 + 34*b^2)*Sqrt[e*Cos[c + d*x]])/(35*d*e) + (2*(7*a^4 + 28*a
^2*b^2 + 4*b^4)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(7*d*Sqrt[e*C
os[c + d*x]]) - (2*b*(29*a^2 + 10*b^2)*Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c +
d*x]))/(35*d*e) - (26*a*b*Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x])^2)/(35*
d*e) - (2*b*Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x])^3)/(7*d*e)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
```

```
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2771

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(
a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rule 2941

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp
[a*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x],
x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
&&  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Si
mplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int \frac{(a+b\sin(c+dx))^4}{\sqrt{e\cos(c+dx)}}\,dx = -\frac{2b\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))^3}{7de} + \frac{2}{7}\int \frac{(a+b\sin(c+dx))^2\left(\frac{7a^2}{2}+3b^2\right.}{\sqrt{e\cos(c+dx}}$$

$$= -\frac{26ab\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))^2}{35de} - \frac{2b\sqrt{e\cos(c+dx)}\,(a+b\sin(c+d}{7de}$$

$$= -\frac{2b(29a^2+10b^2)\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))}{35de} - \frac{26ab\sqrt{e\cos(c+dx)}\,(a}{35de}$$

$$= -\frac{6ab(31a^2+34b^2)\sqrt{e\cos(c+dx)}}{35de} - \frac{2b(29a^2+10b^2)\sqrt{e\cos(c+dx)}\,(a+b\sin}{35de}$$

$$= -\frac{6ab(31a^2+34b^2)\sqrt{e\cos(c+dx)}}{35de} - \frac{2b(29a^2+10b^2)\sqrt{e\cos(c+dx)}\,(a+b\sin}{35de}$$

$$= -\frac{6ab(31a^2+34b^2)\sqrt{e\cos(c+dx)}}{35de} + \frac{2(7a^4+28a^2b^2+4b^4)\sqrt{\cos(c+dx)}\,F(}{7d\sqrt{e\cos(c+dx)}}$$

Mathematica [A]

time = 1.18, size = 130, normalized size = 0.62

$$\frac{20(7a^4+28a^2b^2+4b^4)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)-b\cos(c+dx)\left(560a^3+504ab^2-56ab^2\cos(2(c+dx))+5b(56a^2+11b^2)\sin(c+dx)-5b^3\sin(3(c+dx))\right)}{70d\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])^4/Sqrt[e*Cos[c + d*x]],x]
```

```
[Out] (20*(7*a^4 + 28*a^2*b^2 + 4*b^4)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2,
2] - b*Cos[c + d*x]*(560*a^3 + 504*a*b^2 - 56*a*b^2*Cos[2*(c + d*x)] + 5*b*
(56*a^2 + 11*b^2)*Sin[c + d*x] - 5*b^3*Sin[3*(c + d*x)]))/(70*d*Sqrt[e*Cos[
c + d*x]])
```

Maple [A]
time = 6.46, size = 412, normalized size = 1.96

method	result
default	$-\frac{2\left(80b^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+224ab^3\left(\sin^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-120b^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-280a^2b^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{d}{2}\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))^4/(e*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/35/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*(80*b^4*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+224*a*b^3*sin(1/2*d*x+1/2*c)^7-120*b^4*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-280*a^2*b^2*cos(1/2*d*x+1/2*c)*sin(1/
2*d*x+1/2*c)^4-336*a*b^3*sin(1/2*d*x+1/2*c)^5+35*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a
^4+140*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b^2+20*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^4-280
*a^3*b*sin(1/2*d*x+1/2*c)^3+140*a^2*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*
c)^2-112*a*b^3*sin(1/2*d*x+1/2*c)^3+20*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1
/2*c)^2+140*a^3*b*sin(1/2*d*x+1/2*c)+112*a*b^3*sin(1/2*d*x+1/2*c))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^4/(e*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] e^(-1/2)*integrate((b*sin(d*x + c) + a)^4/sqrt(cos(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 163, normalized size = 0.78

$$-\frac{\left(5\sqrt{2}\left(7i\,a^4+28i\,a^2b^2+4i\,b^4\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)+5\sqrt{2}\left(-7i\,a^4-28i\,a^2b^2-4i\,b^4\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)-2\left(28\,ab^3\cos\left(dx+c\right)^2-140\,a^3b-140\,ab^3+5\left(b^4\cos\left(dx+c\right)^2-14\,a^2b^2-3\,b^4\right)\sin\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)}\right)e^{\left(-\frac{1}{2}\right)}}{35\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^4/(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/35*(5*sqrt(2)*(7*I*a^4 + 28*I*a^2*b^2 + 4*I*b^4)*weierstrassPInverse(-4,
 0, cos(d*x + c) + I*sin(d*x + c)) + 5*sqrt(2)*(-7*I*a^4 - 28*I*a^2*b^2 - 4
*I*b^4)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 2*(28*a
*b^3*cos(d*x + c)^2 - 140*a^3*b - 140*a*b^3 + 5*(b^4*cos(d*x + c)^2 - 14*a^
2*b^2 - 3*b^4)*sin(d*x + c))*sqrt(cos(d*x + c)))*e^(-1/2)/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))**4/(e*cos(d*x+c))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3065 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^4/(e*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^4*e^(-1/2)/sqrt(cos(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\sin\left(c+d\,x\right)\right)}^4}{\sqrt{e\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^4/(e*cos(c + d*x))^(1/2),x)
```

```
[Out] int((a + b*sin(c + d*x))^4/(e*cos(c + d*x))^(1/2), x)
```

3.569 $\int \frac{(a+b\sin(c+dx))^4}{(e\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=218

$$\frac{2ab(15a^2+62b^2)\,(e\cos(c+dx))^{3/2}}{15de^3}-\frac{2(5a^4+60a^2b^2+12b^4)\,\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5de^2\sqrt{\cos(c+dx)}}+\frac{2b(5a^2+6b$$

```
[Out] 2/15*a*b*(15*a^2+62*b^2)*(e*cos(d*x+c))^(3/2)/d/e^3+2/5*b*(5*a^2+6*b^2)*(e*
cos(d*x+c))^(3/2)*(a+b*sin(d*x+c))/d/e^3+2*a*b*(e*cos(d*x+c))^(3/2)*(a+b*si
n(d*x+c))^2/d/e^3+2*(b+a*sin(d*x+c))*(a+b*sin(d*x+c))^3/d/e/(e*cos(d*x+c))^
(1/2)-2/5*(5*a^4+60*a^2*b^2+12*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*
x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/d/e^2/c
os(d*x+c)^(1/2)
```

Rubi [A]
time = 0.29, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2770, 2941, 2748, 2721, 2719}

$$\frac{2ab(15a^2+62b^2)\,(e\cos(c+dx))^{3/2}}{15de^3}+\frac{2b(5a^2+6b^2)\,(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))}{5de^3}-\frac{2(5a^4+60a^2b^2+12b^4)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5de^2\sqrt{\cos(c+dx)}}+\frac{2ab(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))^2}{de^3}+\frac{2(a\sin(c+dx)+b)(a+b\sin(c+dx))^3}{de\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^4/(e*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*a*b*(15*a^2 + 62*b^2)*(e*Cos[c + d*x])^(3/2))/(15*d*e^3) - (2*(5*a^4 + 6
0*a^2*b^2 + 12*b^4)*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*e^
2*Sqrt[Cos[c + d*x]]) + (2*b*(5*a^2 + 6*b^2)*(e*Cos[c + d*x])^(3/2)*(a + b*
Sin[c + d*x]))/(5*d*e^3) + (2*a*b*(e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x
])^2)/(d*e^3) + (2*(b + a*Sin[c + d*x])*(a + b*Sin[c + d*x])^3)/(d*e*Sqrt[e
*Cos[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
```

```
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2770

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x
])^(m - 1)*((b + a*Sin[e + f*x])/(f*g*(p + 1))), x] + Dist[1/(g^2*(p + 1)),
 Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a
^2*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g}
, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p
] || IntegerQ[m])
```

Rule 2941

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp
[a*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x],
x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
&&  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Si
mplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int \frac{(a+b\sin(c+dx))^4}{(e\cos(c+dx))^{3/2}}\,dx = \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))^3}{de\sqrt{e\cos(c+dx)}} - \frac{2\int \sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx}{e^2}$$

$$= \frac{2ab(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))^2}{de^3} + \frac{2(b+a\sin(c+dx))(a+b\sin(c+d}{de\sqrt{e\cos(c+dx)}}$$

$$= \frac{2b(5a^2+6b^2)(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))}{5de^3} + \frac{2ab(e\cos(c+dx))^{3/2}(a+}{de^3}$$

$$= \frac{2ab(15a^2+62b^2)(e\cos(c+dx))^{3/2}}{15de^3} + \frac{2b(5a^2+6b^2)(e\cos(c+dx))^{3/2}(a+b\sin(}{5de^3}$$

$$= \frac{2ab(15a^2+62b^2)(e\cos(c+dx))^{3/2}}{15de^3} + \frac{2b(5a^2+6b^2)(e\cos(c+dx))^{3/2}(a+b\sin(}{5de^3}$$

$$= \frac{2ab(15a^2+62b^2)(e\cos(c+dx))^{3/2}}{15de^3} - \frac{2(5a^4+60a^2b^2+12b^4)\sqrt{e\cos(c+dx)}\,E}{5de^2\sqrt{\cos(c+dx)}}$$

Mathematica [A]
time = 0.63, size = 135, normalized size = 0.62

$$\frac{-6(5a^4+60a^2b^2+12b^4)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{1}{2}(240a^3b+280ab^3+40ab^3\cos(2(c+dx))+(60a^4+360a^2b^2+63b^4)\sin(c+dx)+3b^4\sin(3(c+dx)))}{15de\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])^4/(e*Cos[c + d*x])^(3/2),x]
```

```
[Out] (-6*(5*a^4 + 60*a^2*b^2 + 12*b^4)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2,
 2] + (240*a^3*b + 280*a*b^3 + 40*a*b^3*Cos[2*(c + d*x)] + (60*a^4 + 360*a^
2*b^2 + 63*b^4)*Sin[c + d*x] + 3*b^4*Sin[3*(c + d*x)])/2)/(15*d*e*Sqrt[e*Co
s[c + d*x]])
```

Maple [A]
time = 9.20, size = 378, normalized size = 1.73

method	result
default	$-\frac{2\left(-24b^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-80a\,b^3\left(\sin^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+24b^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+15\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))^4/(e*cos(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/15/e/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/sin(1/2*d*x+1/2*c)*(-24*b^4*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-80*a*b^3*sin(1/2*d*x+1/2*c)^5+24*b^4*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^4+180*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))*a^2*b^2+36*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^4-30*a^4*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-180*a^2*b^2*cos(1/2*d*x+1/2*c)*sin(1/2
*d*x+1/2*c)^2+80*a*b^3*sin(1/2*d*x+1/2*c)^3-36*b^4*cos(1/2*d*x+1/2*c)*sin(1
/2*d*x+1/2*c)^2-60*a^3*b*sin(1/2*d*x+1/2*c)-80*a*b^3*sin(1/2*d*x+1/2*c))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^4/(e*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] e^(-3/2)*integrate((b*sin(d*x + c) + a)^4/cos(d*x + c)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 194, normalized size = 0.89

$$-\frac{\left(3\,\sqrt{2}\,\left(5i\,a^{4}+60i\,a^{2}b^{2}+12i\,b^{4}\right)\cos\left(dx+c\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)\right)+3\,\sqrt{2}\,\left(-5i\,a^{4}-60i\,a^{2}b^{2}-12i\,b^{4}\right)\cos\left(dx+c\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)\right)-2\,\left(20\,ab^{3}\cos\left(dx+c\right)^{2}+60\,a^{3}b+60\,ab^{3}+3\,\left(b^{4}\cos\left(dx+c\right)^{2}+5\,a^{4}+30\,a^{2}b^{2}+5\,b^{4}\right)\sin\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)}\right)e^{\left(-\frac{3}{2}\right)}}{15\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^4/(e*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/15*(3*sqrt(2)*(5*I*a^4 + 60*I*a^2*b^2 + 12*I*b^4)*cos(d*x + c)*weierstra
ssZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) +
3*sqrt(2)*(-5*I*a^4 - 60*I*a^2*b^2 - 12*I*b^4)*cos(d*x + c)*weierstrassZeta
(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(20*
a*b^3*cos(d*x + c)^2 + 60*a^3*b + 60*a*b^3 + 3*(b^4*cos(d*x + c)^2 + 5*a^4
+ 30*a^2*b^2 + 5*b^4)*sin(d*x + c))*sqrt(cos(d*x + c)))*e^(-3/2)/(d*cos(d*x
 + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))**4/(e*cos(d*x+c))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4850 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^4/(e*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^4*e^(-3/2)/cos(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\sin\left(c+d\,x\right)\right)}^4}{{\left(e\,\cos\left(c+d\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^4/(e*cos(c + d*x))^(3/2),x)
```

```
[Out] int((a + b*sin(c + d*x))^4/(e*cos(c + d*x))^(3/2), x)
```

3.570 $\int \frac{(a+b\sin(c+dx))^4}{(e\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=216

$$\frac{2ab(a^2+14b^2)\sqrt{e\cos(c+dx)}}{3de^3}+\frac{2(a^4-12a^2b^2-4b^4)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3de^2\sqrt{e\cos(c+dx)}}+\frac{2b(a^2+2b^2)\sqrt{e\cos}}{}$$

```
[Out] 2/3*(b+a*sin(d*x+c))*(a+b*sin(d*x+c))^3/d/e/(e*cos(d*x+c))^(3/2)+2/3*(a^4-1
2*a^2*b^2-4*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(
sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/e^2/(e*cos(d*x+c))^(1/2)+2/3
*a*b*(a^2+14*b^2)*(e*cos(d*x+c))^(1/2)/d/e^3+2/3*b*(a^2+2*b^2)*(a+b*sin(d*x
+c))*(e*cos(d*x+c))^(1/2)/d/e^3+2/3*a*b*(a+b*sin(d*x+c))^2*(e*cos(d*x+c))^(
1/2)/d/e^3
```

Rubi [A]
time = 0.29, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2770, 2941, 2748, 2721, 2720}

$$\frac{2ab(a^2+14b^2)\sqrt{e\cos(c+dx)}}{3de^3}+\frac{2b(a^2+2b^2)\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))}{3de^3}+\frac{2(a^4-12a^2b^2-4b^4)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3de^2\sqrt{e\cos(c+dx)}}+\frac{2ab\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))^2}{3de^3}+\frac{2(a\sin(c+dx)+b)(a+b\sin(c+dx))^3}{3de(e\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^4/(e*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*a*b*(a^2 + 14*b^2)*Sqrt[e*Cos[c + d*x]])/(3*d*e^3) + (2*(a^4 - 12*a^2*b^
2 - 4*b^4)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*e^2*Sqrt[e*Co
s[c + d*x]]) + (2*b*(a^2 + 2*b^2)*Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x])
)/(3*d*e^3) + (2*a*b*Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x])^2)/(3*d*e^3)
 + (2*(b + a*Sin[c + d*x])*(a + b*Sin[c + d*x])^3)/(3*d*e*(e*Cos[c + d*x])^
(3/2))
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
```

```
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2770

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x
])^(m - 1)*((b + a*Sin[e + f*x])/(f*g*(p + 1))), x] + Dist[1/(g^2*(p + 1)),
 Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a
^2*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g}
, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p
] || IntegerQ[m])
```

Rule 2941

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp
[a*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x],
x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
&&  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Si
mplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int \frac{(a+b\sin(c+dx))^4}{(e\cos(c+dx))^{5/2}}\,dx = \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))^3}{3de(e\cos(c+dx))^{3/2}} - \frac{2\int \frac{(a+b\sin(c+dx))^2\left(-\frac{a^2}{2}+3b^2+\frac{5}{2}ab\sin(c+dx\right.}{\sqrt{e\cos(c+dx)}}}{3e^2}$$

$$= \frac{2ab\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))^2}{3de^3} + \frac{2(b+a\sin(c+dx))(a+b\sin(c+d}{3de(e\cos(c+dx))^{3/2}}$$

$$= \frac{2b(a^2+2b^2)\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))}{3de^3} + \frac{2ab\sqrt{e\cos(c+dx)}\,(a+b\,\mathrm{s}}{3de^3}$$

$$= \frac{2ab(a^2+14b^2)\sqrt{e\cos(c+dx)}}{3de^3} + \frac{2b(a^2+2b^2)\sqrt{e\cos(c+dx)}\,(a+b\sin(c+}{3de^3}$$

$$= \frac{2ab(a^2+14b^2)\sqrt{e\cos(c+dx)}}{3de^3} + \frac{2b(a^2+2b^2)\sqrt{e\cos(c+dx)}\,(a+b\sin(c+}{3de^3}$$

$$= \frac{2ab(a^2+14b^2)\sqrt{e\cos(c+dx)}}{3de^3} + \frac{2(a^4-12a^2b^2-4b^4)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c\right.}{3de^2\sqrt{e\cos(c+dx)}}$$

Mathematica [A]
time = 1.20, size = 137, normalized size = 0.63

$$\frac{16a^3b+40ab^3+24ab^3\cos(2(c+dx))+4(a^4-12a^2b^2-4b^4)\cos^{\frac{3}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)+4a^4\sin(c+dx)+24a^2b^2\sin(c+dx)+5b^4\sin(c+dx)+b^4\sin(3(c+dx))}{6de(e\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])^4/(e*Cos[c + d*x])^(5/2),x]
```

```
[Out] (16*a^3*b + 40*a*b^3 + 24*a*b^3*Cos[2*(c + d*x)] + 4*(a^4 - 12*a^2*b^2 - 4*
b^4)*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + 4*a^4*Sin[c + d*x] + 24
*a^2*b^2*Sin[c + d*x] + 5*b^4*Sin[c + d*x] + b^4*Sin[3*(c + d*x)])/(6*d*e*(
e*Cos[c + d*x])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 574 vs. 2(220) = 440.
time = 9.88, size = 575, normalized size = 2.66

method	result
default	$-\frac{2\left(8b^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)a^4\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))^4/(e*cos(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/(2*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2
*e+e)^(1/2)/e^2*(8*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+2*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))*a^4*sin(1/2*d*x+1/2*c)^2-24*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b^2
*sin(1/2*d*x+1/2*c)^2-8*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^4*sin(1/2*d*x+1/2*c)^2+4
8*a*b^3*sin(1/2*d*x+1/2*c)^5-8*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))*a^4+12*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b^2+4*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))*b^4+2*a^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+12*a^2*b
^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-48*a*b^3*sin(1/2*d*x+1/2*c)^3+4*
b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+4*a^3*b*sin(1/2*d*x+1/2*c)+16*a
*b^3*sin(1/2*d*x+1/2*c))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^4/(e*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] e^(-5/2)*integrate((b*sin(d*x + c) + a)^4/cos(d*x + c)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 185, normalized size = 0.86

$$\frac{\left(\sqrt{2}\left(-i\,a^4+12i\,a^2b^2+4i\,b^4\right)\cos\left(dx+c\right)^2\,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)+\sqrt{2}\left(i\,a^4-12i\,a^2b^2-4i\,b^4\right)\cos\left(dx+c\right)^2\,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)+2\left(12\,ab^3\cos\left(dx+c\right)^2+4\,a^3b+4\,ab^3+\left(b^4\cos\left(dx+c\right)^2+a^4+6\,a^2b^2+b^4\right)\sin\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)}\right)e^{\left(-\frac{5}{2}\right)}}{3\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^4/(e*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(sqrt(2)*(-I*a^4 + 12*I*a^2*b^2 + 4*I*b^4)*cos(d*x + c)^2*weierstrassPI
nverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(I*a^4 - 12*I*a^2*b^
2 - 4*I*b^4)*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin
(d*x + c)) + 2*(12*a*b^3*cos(d*x + c)^2 + 4*a^3*b + 4*a*b^3 + (b^4*cos(d*x
+ c)^2 + a^4 + 6*a^2*b^2 + b^4)*sin(d*x + c))*sqrt(cos(d*x + c)))*e^(-5/2)/
(d*cos(d*x + c)^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))**4/(e*cos(d*x+c))**(5/2),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^4/(e*cos(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((b*sin(d*x + c) + a)^4*e^(-5/2)/cos(d*x + c)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\sin\left(c+d\,x\right))^4}{(e\,\cos\left(c+d\,x\right))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^4/(e*cos(c + d*x))^(5/2),x)
[Out] int((a + b*sin(c + d*x))^4/(e*cos(c + d*x))^(5/2), x)
```

3.571 $\int \frac{(a+b\sin(c+dx))^4}{(e\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=237

$$\frac{2ab(3a^2-10b^2)\,(e\cos(c+dx))^{3/2}}{5de^5}-\frac{6(a^4-4a^2b^2-4b^4)\,\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5de^4\sqrt{\cos(c+dx)}}+\frac{6b(a^2-2b^2)\,(e\,\mathrm{co}}{}$$

```
[Out] 2/5*a*b*(3*a^2-10*b^2)*(e*cos(d*x+c))^(3/2)/d/e^5+6/5*b*(a^2-2*b^2)*(e*cos(
d*x+c))^(3/2)*(a+b*sin(d*x+c))/d/e^5+2/5*(b+a*sin(d*x+c))*(a+b*sin(d*x+c))^
3/d/e/(e*cos(d*x+c))^(5/2)-6/5*(a+b*sin(d*x+c))^2*(a*b-(a^2-2*b^2)*sin(d*x+
c))/d/e^3/(e*cos(d*x+c))^(1/2)-6/5*(a^4-4*a^2*b^2-4*b^4)*(cos(1/2*d*x+1/2*c
)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(
d*x+c))^(1/2)/d/e^4/cos(d*x+c)^(1/2)
```

Rubi [A]

time = 0.30, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2770, 2940, 2941, 2748, 2721, 2719}

$$\frac{2ab(3a^2-10b^2)\,(e\cos(c+dx))^{3/2}}{5de^5}+\frac{6b(a^2-2b^2)\,(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))}{5de^5}-\frac{6(ab-(a^2-2b^2)\sin(c+dx))\,(a+b\sin(c+dx))^2}{5de^3\sqrt{e\cos(c+dx)}}-\frac{6(a^4-4a^2b^2-4b^4)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)\sqrt{e\cos(c+dx)}}{5de^4\sqrt{\cos(c+dx)}}+\frac{2(a\sin(c+dx)+b)(a+b\sin(c+dx))^3}{5de(e\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^4/(e*Cos[c + d*x])^(7/2),x]
```

```
[Out] (2*a*b*(3*a^2 - 10*b^2)*(e*Cos[c + d*x])^(3/2))/(5*d*e^5) - (6*(a^4 - 4*a^2
*b^2 - 4*b^4)*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*e^4*Sqrt
[Cos[c + d*x]]) + (6*b*(a^2 - 2*b^2)*(e*Cos[c + d*x])^(3/2)*(a + b*Sin[c +
d*x]))/(5*d*e^5) + (2*(b + a*Sin[c + d*x])*(a + b*Sin[c + d*x])^3)/(5*d*e*(
e*Cos[c + d*x])^(5/2)) - (6*(a + b*Sin[c + d*x])^2*(a*b - (a^2 - 2*b^2)*Sin
[c + d*x]))/(5*d*e^3*Sqrt[e*Cos[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
```

```
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2770

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x
])^(m - 1)*((b + a*Sin[e + f*x])/(f*g*(p + 1))), x] + Dist[1/(g^2*(p + 1)),
 Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a
^2*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g}
, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p
] || IntegerQ[m])
```

Rule 2940

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(g*
Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x])^m*((d + c*Sin[e + f*x])/(f*g*(p
 + 1))), x] + Dist[1/(g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin
[e + f*x])^(m - 1)*Simp[a*c*(p + 2) + b*d*m + b*c*(m + p + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && GtQ[
m, 0] && LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] &
& SimplerQ[c + d*x, a + b*x])
```

Rule 2941

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp
[a*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x],
x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
&&  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Si
mplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int \frac{(a+b\sin(c+dx))^4}{(e\cos(c+dx))^{7/2}}\,dx = \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))^3}{5de(e\cos(c+dx))^{5/2}} - \frac{2\int \frac{(a+b\sin(c+dx))^2\left(-\frac{3a^2}{2}+3b^2+\frac{3}{2}ab\sin(c+dx\right.}{(e\cos(c+dx))^{3/2}}}{5e^2}$$

$$= \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))^3}{5de(e\cos(c+dx))^{5/2}} - \frac{6(a+b\sin(c+dx))^2\,(ab-(a^2-2b^2}{5de^3\sqrt{e\cos(c+dx)}}$$

$$= \frac{6b(a^2-2b^2)\,(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))}{5de^5} + \frac{2(b+a\sin(c+dx))(a+b\,\mathrm{si}}{5de(e\cos(c+dx))}$$

$$= \frac{2ab(3a^2-10b^2)\,(e\cos(c+dx))^{3/2}}{5de^5} + \frac{6b(a^2-2b^2)\,(e\cos(c+dx))^{3/2}(a+b\sin(c}{5de^5}$$

$$= \frac{2ab(3a^2-10b^2)\,(e\cos(c+dx))^{3/2}}{5de^5} + \frac{6b(a^2-2b^2)\,(e\cos(c+dx))^{3/2}(a+b\sin(c}{5de^5}$$

$$= \frac{2ab(3a^2-10b^2)\,(e\cos(c+dx))^{3/2}}{5de^5} - \frac{6(a^4-4a^2b^2-4b^4)\,\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c\right.}{5de^4\sqrt{\cos(c+dx)}}$$

Mathematica [A]
time = 0.61, size = 152, normalized size = 0.64

$$\frac{2\left(-20ab^3-3(a^4-4a^2b^2-4b^4)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+4ab(a^2+b^2)\sec^2(c+dx)+3a^4\sin(c+dx)-12a^2b^2\sin(c+dx)-7b^4\sin(c+dx)+(a^4+6a^2b^2+b^4)\sec(c+dx)\tan(c+dx)\right)}{5de^3\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])^4/(e*Cos[c + d*x])^(7/2),x]
```

```
[Out] (2*(-20*a*b^3 - 3*(a^4 - 4*a^2*b^2 - 4*b^4)*Sqrt[Cos[c + d*x]]*EllipticE[(c
 + d*x)/2, 2] + 4*a*b*(a^2 + b^2)*Sec[c + d*x]^2 + 3*a^4*Sin[c + d*x] - 12*
a^2*b^2*Sin[c + d*x] - 7*b^4*Sin[c + d*x] + (a^4 + 6*a^2*b^2 + b^4)*Sec[c +
 d*x]*Tan[c + d*x]))/(5*d*e^3*Sqrt[e*Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 873 vs. $2(241) = 482$.
time = 20.86, size = 874, normalized size = 3.69

method	result	size
default	Expression too large to display	874

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))^4/(e*cos(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)/(
-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/e^3*(12*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
```

```
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a^4*sin
(1/2*d*x+1/2*c)^4-48*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2
*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a^2*b^2*sin(1/2*d*x+1/2*c)^4-
48*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*b^4*sin(1/2*d*x+1/2*c)^4-24*a^4*cos(1/2*d*x+1/2
*c)*sin(1/2*d*x+1/2*c)^6+96*a^2*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6
+56*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*a^4*sin(1/2*d*x+1/2*c)^2+48*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a^2*b^2*sin(1/2*d*x+1
/2*c)^2+48*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*b^4*sin(1/2*d*x+1/2*c)^2+24*a^4*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-96*a^2*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+
1/2*c)^4+80*a*b^3*sin(1/2*d*x+1/2*c)^5-56*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*
x+1/2*c)^4+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^4-12*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b
^2-12*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))*b^4-8*a^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2
*c)^2+12*a^2*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-80*a*b^3*sin(1/2*d
*x+1/2*c)^3+12*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-4*a^3*b*sin(1/2*
d*x+1/2*c)+16*a*b^3*sin(1/2*d*x+1/2*c))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^4/(e*cos(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] e^(-7/2)*integrate((b*sin(d*x + c) + a)^4/cos(d*x + c)^(7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 210, normalized size = 0.89

$$-\frac{\left(3\sqrt{2}\left(i a^4-4 i a^2 b^2-4 i b^4\right)\cos\left(dx+c\right)^3 \mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)))+3\sqrt{2}\left(-i a^4+4 i a^2 b^2+4 i b^4\right)\cos\left(dx+c\right)^3 \mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)))+2\left(20 ab^3\cos\left(dx+c\right)^2-4a^3b-4ab^3-\left(a^4+6a^2b^2+b^4+\left(3a^4-12a^2b^2-7b^4\right)\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)}\right)e^{\left(-\frac{7}{2}\right)}}{5d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^4/(e*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] -1/5*(3*sqrt(2)*(I*a^4 - 4*I*a^2*b^2 - 4*I*b^4)*cos(d*x + c)^3*weierstrassZ
eta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*s
qrt(2)*(-I*a^4 + 4*I*a^2*b^2 + 4*I*b^4)*cos(d*x + c)^3*weierstrassZeta(-4,
0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(20*a*b^3
*cos(d*x + c)^2 - 4*a^3*b - 4*a*b^3 - (a^4 + 6*a^2*b^2 + b^4 + (3*a^4 - 12*
```

```
a^2*b^2 - 7*b^4)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(cos(d*x + c)))*e^(-7/2)
/(d*cos(d*x + c)^3)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))**4/(e*cos(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^4/(e*cos(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^4*e^(-7/2)/cos(d*x + c)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\sin(c+d\,x))^4}{(e\,\cos(c+d\,x))^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^4/(e*cos(c + d*x))^(7/2),x)
```

```
[Out] int((a + b*sin(c + d*x))^4/(e*cos(c + d*x))^(7/2), x)
```

3.572 $\int \frac{(a+b\sin(c+dx))^4}{(e\cos(c+dx))^{9/2}}\,dx$

Optimal. Leaf size=241

$$\frac{10ab(a^2-2b^2)\sqrt{e\cos(c+dx)}}{21de^5}+\frac{2(5a^4-12a^2b^2+12b^4)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21de^4\sqrt{e\cos(c+dx)}}+\frac{2b(5a^2-6b^2)\sqrt{}}{}$$

```
[Out] 2/7*(b+a*sin(d*x+c))*(a+b*sin(d*x+c))^3/d/e/(e*cos(d*x+c))^(7/2)-2/21*(a+b*
sin(d*x+c))^2*(a*b-(5*a^2-6*b^2)*sin(d*x+c))/d/e^3/(e*cos(d*x+c))^(3/2)+2/2
1*(5*a^4-12*a^2*b^2+12*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/e^4/(e*cos(d*x+c)
)^(1/2)+10/21*a*b*(a^2-2*b^2)*(e*cos(d*x+c))^(1/2)/d/e^5+2/21*b*(5*a^2-6*b^
2)*(a+b*sin(d*x+c))*(e*cos(d*x+c))^(1/2)/d/e^5
```

Rubi [A]
time = 0.30, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2770, 2940, 2941, 2748, 2721, 2720}

$$\frac{10ab(a^2-2b^2)\sqrt{e\cos(c+dx)}}{21de^5}+\frac{2b(5a^2-6b^2)\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))}{21de^5}-\frac{2(ab-(5a^2-6b^2)\sin(c+dx))\,(a+b\sin(c+dx))^2}{21de^3(e\cos(c+dx))^{3/2}}+\frac{2(5a^4-12a^2b^2+12b^4)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21de^4\sqrt{e\cos(c+dx)}}+\frac{2(a\sin(c+dx)+b)(a+b\sin(c+dx))^3}{7de(e\cos(c+dx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^4/(e*Cos[c + d*x])^(9/2),x]
```

```
[Out] (10*a*b*(a^2 - 2*b^2)*Sqrt[e*Cos[c + d*x]])/(21*d*e^5) + (2*(5*a^4 - 12*a^2
*b^2 + 12*b^4)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*e^4*Sqrt
[e*Cos[c + d*x]]) + (2*b*(5*a^2 - 6*b^2)*Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c
+ d*x]))/(21*d*e^5) + (2*(b + a*Sin[c + d*x])*(a + b*Sin[c + d*x])^3)/(7*d*
e*(e*Cos[c + d*x])^(7/2)) - (2*(a + b*Sin[c + d*x])^2*(a*b - (5*a^2 - 6*b^2
)*Sin[c + d*x]))/(21*d*e^3*(e*Cos[c + d*x])^(3/2))
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
```

```
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2770

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x
])^(m - 1)*((b + a*Sin[e + f*x])/(f*g*(p + 1))), x] + Dist[1/(g^2*(p + 1)),
 Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a
^2*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g}
, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p
] || IntegerQ[m])
```

Rule 2940

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(g*
Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x])^m*((d + c*Sin[e + f*x])/(f*g*(p
 + 1))), x] + Dist[1/(g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin
[e + f*x])^(m - 1)*Simp[a*c*(p + 2) + b*d*m + b*c*(m + p + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && GtQ[
m, 0] && LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] &
& SimplerQ[c + d*x, a + b*x])
```

Rule 2941

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp
[a*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x],
x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
&&  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Si
mplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int \frac{(a+b\sin(c+dx))^4}{(e\cos(c+dx))^{9/2}}\,dx = \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))^3}{7de(e\cos(c+dx))^{7/2}} - \frac{2\int \frac{(a+b\sin(c+dx))^2\left(-\frac{5a^2}{2}+3b^2+\frac{1}{2}ab\sin(c+c}{(e\cos(c+dx))^{5/2}}}{7e^2}$$

$$= \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))^3}{7de(e\cos(c+dx))^{7/2}} - \frac{2(a+b\sin(c+dx))^2\left(ab-(5a^2-6\right.}{21de^3(e\cos(c+dx))^3}$$

$$= \frac{2b(5a^2-6b^2)\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))}{21de^5} + \frac{2(b+a\sin(c+dx))(a+b}{7de(e\cos(c+dx}$$

$$= \frac{10ab(a^2-2b^2)\sqrt{e\cos(c+dx)}}{21de^5} + \frac{2b(5a^2-6b^2)\sqrt{e\cos(c+dx)}\,(a+b\sin(c+}{21de^5}$$

$$= \frac{10ab(a^2-2b^2)\sqrt{e\cos(c+dx)}}{21de^5} + \frac{2b(5a^2-6b^2)\sqrt{e\cos(c+dx)}\,(a+b\sin(c+}{21de^5}$$

$$= \frac{10ab(a^2-2b^2)\sqrt{e\cos(c+dx)}}{21de^5} + \frac{2(5a^4-12a^2b^2+12b^4)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}\right.}{21de^4\sqrt{e\cos(c+dx)}}$$

Mathematica [A]
time = 0.94, size = 177, normalized size = 0.73

$$\frac{\sqrt{e\cos(c+dx)}\,\sec^4(c+dx)\left(48a^3b-8ab^3-56ab^3\cos(2(c+dx))+4(5a^4-12a^2b^2+12b^4)\cos^{\frac{7}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)+17a^4\sin(c+dx)+60a^2b^2\sin(c+dx)+3b^4\sin(c+dx)+5a^4\sin(3(c+dx))-12a^2b^2\sin(3(c+dx))-9b^4\sin(3(c+dx))\right)}{42de^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])^4/(e*Cos[c + d*x])^(9/2),x]
```

```
[Out] (Sqrt[e*Cos[c + d*x]]*Sec[c + d*x]^4*(48*a^3*b - 8*a*b^3 - 56*a*b^3*Cos[2*(
c + d*x)] + 4*(5*a^4 - 12*a^2*b^2 + 12*b^4)*Cos[c + d*x]^(7/2)*EllipticF[(c
 + d*x)/2, 2] + 17*a^4*Sin[c + d*x] + 60*a^2*b^2*Sin[c + d*x] + 3*b^4*Sin[c
 + d*x] + 5*a^4*Sin[3*(c + d*x)] - 12*a^2*b^2*Sin[3*(c + d*x)] - 9*b^4*Sin[
3*(c + d*x)]))/(42*d*e^5)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1066 vs. 2(245) = 490.
time = 25.29, size = 1067, normalized size = 4.43

method	result	size
default	Expression too large to display	1067

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))^4/(e*cos(d*x+c))^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/21/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^
2-1)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/e^4*(40*a^4*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+30*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^4*sin(1/
2*d*x+1/2*c)^2+72*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^4*sin(1/2*d*x+1/2*c)^2+12*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2))*a^2*b^2-96*a^2*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*
c)^6+96*a^2*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+12*a^2*b^2*cos(1/2*
d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^4-12*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2))*b^4+72*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-72*b^4*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-40*a^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*
x+1/2*c)^4+16*a^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-12*b^4*cos(1/2*d*
x+1/2*c)*sin(1/2*d*x+1/2*c)^2-112*a*b^3*sin(1/2*d*x+1/2*c)^5+112*a*b^3*sin(
1/2*d*x+1/2*c)^3+12*a^3*b*sin(1/2*d*x+1/2*c)-16*a*b^3*sin(1/2*d*x+1/2*c)-72
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))*a^2*b^2*sin(1/2*d*x+1/2*c)^2-96*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*a^2*b^2*sin(1/2*d*x+1/2*c)^6+144*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*b^2*s
in(1/2*d*x+1/2*c)^4+40*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^4*sin(1/2*d*x+1/2*c)^6-14
4*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*b^4*sin(1/2*d*x+1/2*c)^4-60*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*a^4*sin(1/2*d*x+1/2*c)^4+96*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*b^4*sin(1/2*d*x+1
/2*c)^6)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^4/(e*cos(d*x+c))^(9/2),x, algorithm="maxima")
```

```
[Out] e^(-9/2)*integrate((b*sin(d*x + c) + a)^4/cos(d*x + c)^(9/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 206, normalized size = 0.85

$$\frac{\left(\sqrt{2}\,(-5i\,a^4+12i\,a^2b^2-12i\,b^4)\cos(dx+c)^4\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+\sqrt{2}\,(5i\,a^4-12i\,a^2b^2+12i\,b^4)\cos(dx+c)^4\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c))-2\left(28\,ab^3\cos(dx+c)^2-12\,a^3b-12\,ab^3-\left(3\,a^4+18\,a^2b^2+3\,b^4+(5\,a^4-12\,a^2b^2-9\,b^4)\cos(dx+c)^2\right)\sin(dx+c)\right)\sqrt{\cos(dx+c)}\right)e^{\left(-\frac{9}{2}\right)}}{21\,d\cos(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^4/(e*cos(d*x+c))^(9/2),x, algorithm="fricas")
```

```
[Out] 1/21*(sqrt(2)*(-5*I*a^4 + 12*I*a^2*b^2 - 12*I*b^4)*cos(d*x + c)^4*weierstra
ssPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(5*I*a^4 - 12*I*
a^2*b^2 + 12*I*b^4)*cos(d*x + c)^4*weierstrassPInverse(-4, 0, cos(d*x + c)
- I*sin(d*x + c)) - 2*(28*a*b^3*cos(d*x + c)^2 - 12*a^3*b - 12*a*b^3 - (3*a
^4 + 18*a^2*b^2 + 3*b^4 + (5*a^4 - 12*a^2*b^2 - 9*b^4)*cos(d*x + c)^2)*sin(
d*x + c))*sqrt(cos(d*x + c)))*e^(-9/2)/(d*cos(d*x + c)^4)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))**4/(e*cos(d*x+c))**(9/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4847 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^4/(e*cos(d*x+c))^(9/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^4*e^(-9/2)/cos(d*x + c)^(9/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\sin(c+d\,x))^4}{(e\cos(c+d\,x))^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^4/(e*cos(c + d*x))^(9/2),x)
```

```
[Out] int((a + b*sin(c + d*x))^4/(e*cos(c + d*x))^(9/2), x)
```

3.573 $\int \frac{(a+b\sin(c+dx))^4}{(e\cos(c+dx))^{11/2}}\,dx$

Optimal. Leaf size=264

$$\frac{2ab(21a^2-22b^2)(e\cos(c+dx))^{3/2}}{45de^7}-\frac{2(7a^4-12a^2b^2+4b^4)\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15de^6\sqrt{\cos(c+dx)}}+\frac{2(b+a\sin(c+\ldots}{9de(\ldots}$$

```
[Out] 2/45*a*b*(21*a^2-22*b^2)*(e*cos(d*x+c))^(3/2)/d/e^7+2/9*(b+a*sin(d*x+c))*(a
+b*sin(d*x+c))^3/d/e/(e*cos(d*x+c))^(9/2)+2/45*(a+b*sin(d*x+c))^2*(a*b+(7*a
^2-6*b^2)*sin(d*x+c))/d/e^3/(e*cos(d*x+c))^(5/2)-2/45*(a+b*sin(d*x+c))*(b*(
7*a^2-6*b^2)-a*(21*a^2-22*b^2)*sin(d*x+c))/d/e^5/(e*cos(d*x+c))^(1/2)-2/15*
(7*a^4-12*a^2*b^2+4*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*El
lipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/d/e^6/cos(d*x+c)^(
1/2)
```

Rubi [A]
time = 0.32, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2770, 2940, 2748, 2721, 2719}

$$\frac{2ab(21a^2-22b^2)(e\cos(c+dx))^{3/2}}{45de^7}-\frac{2(b(7a^2-6b^2)-a(21a^2-22b^2)\sin(c+dx))(a+b\sin(c+dx))}{45de^5\sqrt{e\cos(c+dx)}}+\frac{2((7a^2-6b^2)\sin(c+dx)+ab)(a+b\sin(c+dx))^2}{45de^3(e\cos(c+dx))^{5/2}}-\frac{2(7a^4-12a^2b^2+4b^4)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{15de^6\sqrt{\cos(c+dx)}}+\frac{2(a\sin(c+dx)+b)(a+b\sin(c+dx))^3}{9de(e\cos(c+dx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^4/(e*Cos[c + d*x])^(11/2),x]
```

```
[Out] (2*a*b*(21*a^2 - 22*b^2)*(e*Cos[c + d*x])^(3/2))/(45*d*e^7) - (2*(7*a^4 - 1
2*a^2*b^2 + 4*b^4)*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*d*e^
6*Sqrt[Cos[c + d*x]]) + (2*(b + a*Sin[c + d*x])*(a + b*Sin[c + d*x])^3)/(9*
d*e*(e*Cos[c + d*x])^(9/2)) - (2*(a + b*Sin[c + d*x])*(b*(7*a^2 - 6*b^2) -
a*(21*a^2 - 22*b^2)*Sin[c + d*x]))/(45*d*e^5*Sqrt[e*Cos[c + d*x]]) + (2*(a
+ b*Sin[c + d*x])^2*(a*b + (7*a^2 - 6*b^2)*Sin[c + d*x]))/(45*d*e^3*(e*Cos[
c + d*x])^(5/2))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2770

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x
])^(m - 1)*((b + a*Sin[e + f*x])/(f*g*(p + 1))), x] + Dist[1/(g^2*(p + 1)),
 Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a
^2*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g}
, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p
] || IntegerQ[m])
```

Rule 2940

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(g*
Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x])^m*((d + c*Sin[e + f*x])/(f*g*(p
 + 1))), x] + Dist[1/(g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin
[e + f*x])^(m - 1)*Simp[a*c*(p + 2) + b*d*m + b*c*(m + p + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && GtQ[
m, 0] && LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] &
& SimplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int \frac{(a+b\sin(c+dx))^4}{(e\cos(c+dx))^{11/2}}\,dx = \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))^3}{9de(e\cos(c+dx))^{9/2}} - \frac{2\int \frac{(a+b\sin(c+dx))^2\left(-\frac{7a^2}{2}+3b^2-\frac{1}{2}ab\sin(c+dx\right.}{(e\cos(c+dx))^{7/2}}}{9e^2}$$

$$= \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))^3}{9de(e\cos(c+dx))^{9/2}} + \frac{2(a+b\sin(c+dx))^2\,(ab+(7a^2-6b}{45de^3(e\cos(c+dx))^{5/2}}$$

$$= \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))^3}{9de(e\cos(c+dx))^{9/2}} - \frac{2(a+b\sin(c+dx))\,(b(7a^2-6b^2)-}{45de^5\sqrt{e\cos(c-}}$$

$$= \frac{2ab(21a^2-22b^2)\,(e\cos(c+dx))^{3/2}}{45de^7} + \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))^3}{9de(e\cos(c+dx))^{9/2}} -$$

$$= \frac{2ab(21a^2-22b^2)\,(e\cos(c+dx))^{3/2}}{45de^7} + \frac{2(b+a\sin(c+dx))(a+b\sin(c+dx))^3}{9de(e\cos(c+dx))^{9/2}} -$$

$$= \frac{2ab(21a^2-22b^2)\,(e\cos(c+dx))^{3/2}}{45de^7} - \frac{2(7a^4-12a^2b^2+4b^4)\sqrt{e\cos(c+dx)}\,E(}{15de^6\sqrt{\cos(c+dx)}}$$

Mathematica [A]
time = 1.61, size = 219, normalized size = 0.83

$$\frac{\sqrt{e\cos(c+dx)}\,\sec^5(c+dx)\left(320a^3b+32ab^3-288ab^3\cos(2(c+dx))-48(7a^4-12a^2b^2+4b^4)\cos^{\frac{9}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+150a^4\sin(c+dx)+360a^2b^2\sin(c+dx)+60b^4\sin(c+dx)+91a^4\sin(3(c+dx))-156a^2b^2\sin(3(c+dx))-8b^4\sin(3(c+dx))+21a^4\sin(5(c+dx))-36a^2b^2\sin(5(c+dx))+12b^4\sin(5(c+dx))\right)}{360de^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])^4/(e*Cos[c + d*x])^(11/2),x]
```

```
[Out] (Sqrt[e*Cos[c + d*x]]*Sec[c + d*x]^5*(320*a^3*b + 32*a*b^3 - 288*a*b^3*Cos[
2*(c + d*x)] - 48*(7*a^4 - 12*a^2*b^2 + 4*b^4)*Cos[c + d*x]^(9/2)*EllipticE
[(c + d*x)/2, 2] + 150*a^4*Sin[c + d*x] + 360*a^2*b^2*Sin[c + d*x] + 60*b^4
*Sin[c + d*x] + 91*a^4*Sin[3*(c + d*x)] - 156*a^2*b^2*Sin[3*(c + d*x)] - 8*
b^4*Sin[3*(c + d*x)] + 21*a^4*Sin[5*(c + d*x)] - 36*a^2*b^2*Sin[5*(c + d*x)
] + 12*b^4*Sin[5*(c + d*x)]))/(360*d*e^6)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1415 vs. 2(268) = 536.
time = 34.28, size = 1416, normalized size = 5.36

method	result	size
default	Expression too large to display	1416

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))^4/(e*cos(d*x+c))^(11/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/45/(16*sin(1/2*d*x+1/2*c)^8-32*sin(1/2*d*x+1/2*c)^6+24*sin(1/2*d*x+1/2*c
)^4-8*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e
+e)^(1/2)/e^5*(-384*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+1344*a^4*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+768*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d
*x+1/2*c)^8-1064*a^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+21*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))*a^4+12*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^4+1152*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2))*a^2*b^2*sin(1/2*d*x+1/2*c)^6-576*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b^2*sin(
1/2*d*x+1/2*c)^8+504*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2
*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a^4*sin(1/2*d*x+1/2*c)^4+288*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*b^4*sin(1/2*d*x+1/2*c)^4-168*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*a^4*sin(1/2*d*x+1/2*c)^2-96*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*b^4*sin(1/2*d*x+1/
2*c)^2+1152*a^2*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10-2304*a^2*b^2*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+1824*a^2*b^2*cos(1/2*d*x+1/2*c)*sin(
1/2*d*x+1/2*c)^6-672*a^2*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+36*a^2
*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+104*b^4*cos(1/2*d*x+1/2*c)*sin
(1/2*d*x+1/2*c)^4-488*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+392*a^4*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-66*a^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*
x+1/2*c)^2-12*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+144*a*b^3*sin(1/2
*d*x+1/2*c)^5-144*a*b^3*sin(1/2*d*x+1/2*c)^3-20*a^3*b*sin(1/2*d*x+1/2*c)+16
*a*b^3*sin(1/2*d*x+1/2*c)-36*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b^2+336*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2))*a^4*sin(1/2*d*x+1/2*c)^8-384*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^4*s
in(1/2*d*x+1/2*c)^6-672*a^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10-864*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*a^2*b^2*sin(1/2*d*x+1/2*c)^4+288*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*a^2*b^2*sin(1/2*d*x+1/2*c)^2+192*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^4*sin(1/2
*d*x+1/2*c)^8-672*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^4*sin(1/2*d*x+1/2*c)^6)/d
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^4/(e*cos(d*x+c))^(11/2),x, algorithm="maxima")
```

```
[Out] e^(-11/2)*integrate((b*sin(d*x + c) + a)^4/cos(d*x + c)^(11/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 243, normalized size = 0.92

$$-\frac{\left(3\sqrt{2}\left(7i\,a^4-12i\,a^2b^2+4i\,b^4\right)\cos\left(dx+c\right)^5\,\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)\right)+3\sqrt{2}\left(-7i\,a^4+12i\,a^2b^2-4i\,b^4\right)\cos\left(dx+c\right)^5\,\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)\right)+2\left(36\,ab^3\cos\left(dx+c\right)^2-20\,a^3b-20\,ab^3-\left(3\left(7\,a^4-12\,a^2b^2+4\,b^4\right)\cos\left(dx+c\right)^4+5\,a^4+30\,a^2b^2+5\,b^4+\left(7\,a^4-12\,a^2b^2-11\,b^4\right)\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)}\right)e^{\left(-\frac{11}{2}\right)}}{45\,d\cos\left(dx+c\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^4/(e*cos(d*x+c))^(11/2),x, algorithm="fricas")
```

```
[Out] -1/45*(3*sqrt(2)*(7*I*a^4 - 12*I*a^2*b^2 + 4*I*b^4)*cos(d*x + c)^5*weierstr
assZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) +
 3*sqrt(2)*(-7*I*a^4 + 12*I*a^2*b^2 - 4*I*b^4)*cos(d*x + c)^5*weierstrassZe
ta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(3
6*a*b^3*cos(d*x + c)^2 - 20*a^3*b - 20*a*b^3 - (3*(7*a^4 - 12*a^2*b^2 + 4*b
^4)*cos(d*x + c)^4 + 5*a^4 + 30*a^2*b^2 + 5*b^4 + (7*a^4 - 12*a^2*b^2 - 11*
b^4)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(cos(d*x + c)))*e^(-11/2)/(d*cos(d*x
 + c)^5)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))**4/(e*cos(d*x+c))**(11/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^4/(e*cos(d*x+c))^(11/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^4*e^(-11/2)/cos(d*x + c)^(11/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\sin\left(c+d\,x\right)\right)}^4}{{\left(e\,\cos\left(c+d\,x\right)\right)}^{11/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^4/(e*cos(c + d*x))^(11/2),x)
```

```
[Out] int((a + b*sin(c + d*x))^4/(e*cos(c + d*x))^(11/2), x)
```

3.574 $\int \frac{(e\cos(c+dx))^{11/2}}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=531

$$-\frac{(-a^2+b^2)^{9/4}\,e^{11/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{b^{11/2}d}-\frac{(-a^2+b^2)^{9/4}\,e^{11/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{b^{11/2}d}+2$$

```
[Out] -(-a^2+b^2)^(9/4)*e^(11/2)*arctan(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(
1/4)/e^(1/2))/b^(11/2)/d-(-a^2+b^2)^(9/4)*e^(11/2)*arctanh(b^(1/2)*(e*cos(d
*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(11/2)/d+2/9*e*(e*cos(d*x+c))^(9/2
)/b/d-2/35*e^3*(e*cos(d*x+c))^(5/2)*(7*a^2-7*b^2-5*a*b*sin(d*x+c))/b^3/d+2/
21*a*(21*a^4-49*a^2*b^2+33*b^4)*e^6*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*
x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/b^6/d/(e*co
s(d*x+c))^(1/2)-a*(a^2-b^2)^3*e^6*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+
1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*cos(
d*x+c)^(1/2)/b^6/d/(a^2-b*(b-(-a^2+b^2)^(1/2)))/(e*cos(d*x+c))^(1/2)-a*(a^2
-b^2)^3*e^6*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(
1/2*d*x+1/2*c),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(d*x+c)^(1/2)/b^6/d/(a^
2-b*(b+(-a^2+b^2)^(1/2)))/(e*cos(d*x+c))^(1/2)+2/21*e^5*(21*(a^2-b^2)^2-a*b
*(7*a^2-12*b^2)*sin(d*x+c))*(e*cos(d*x+c))^(1/2)/b^5/d
```

Rubi [A]
time = 1.27, antiderivative size = 531, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {2774, 2944, 2946, 2721, 2720, 2781, 2886, 2884, 335, 218, 214, 211}

$$-\frac{e^{11/2}(b^2-a^2)^{9/4}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{b^{11/2}d}-\frac{e^{11/2}(b^2-a^2)^{9/4}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{b^{11/2}d}-\frac{ae^6(a^2-b^2)^3\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{b^6d\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{e\cos(c+dx)}}-\frac{ae^6(a^2-b^2)^3\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{b^6d\left(a^2-b\left(\sqrt{b^2-a^2}+b\right)\right)\sqrt{e\cos(c+dx)}}+\frac{2e^5\sqrt{e\cos(c+dx)}\left(21(a^2-b^2)^2-ab(7a^2-12b^2)\sin(c+dx)\right)}{21b^5d}-\frac{2e^3(e\cos(c+dx))^{5/2}(7(a^2-b^2)-5ab\sin(c+dx))}{35b^3d}+\frac{2ae^6(21a^4-49a^2b^2+33b^4)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21b^6d\sqrt{e\cos(c+dx)}}+\frac{2e(e\cos(c+dx))^{9/2}}{9bd}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(11/2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -(((-a^2 + b^2)^(9/4)*e^(11/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2
 + b^2)^(1/4)*Sqrt[e])])/(b^(11/2)*d)) - ((-a^2 + b^2)^(9/4)*e^(11/2)*ArcTa
nh[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(b^(11/2)*
d) + (2*e*(e*Cos[c + d*x])^(9/2))/(9*b*d) + (2*a*(21*a^4 - 49*a^2*b^2 + 33*
b^4)*e^6*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*b^6*d*Sqrt[e*Cos
[c + d*x]]) - (a*(a^2 - b^2)^3*e^6*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b -
 Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(b^6*(a^2 - b*(b - Sqrt[-a^2 + b^2]))*
d*Sqrt[e*Cos[c + d*x]]) - (a*(a^2 - b^2)^3*e^6*Sqrt[Cos[c + d*x]]*EllipticP
i[(2*b)/(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(b^6*(a^2 - b*(b + Sqrt[-a
^2 + b^2]))*d*Sqrt[e*Cos[c + d*x]]) - (2*e^3*(e*Cos[c + d*x])^(5/2)*(7*(a^2
 - b^2) - 5*a*b*Sin[c + d*x]))/(35*b^3*d) + (2*e^5*Sqrt[e*Cos[c + d*x]]*(21
*(a^2 - b^2)^2 - a*b*(7*a^2 - 12*b^2)*Sin[c + d*x]))/(21*b^5*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2774

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(b*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2781

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[-a/(2*q), Int[1/(S
```

```
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[b*(g/f), Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2944

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Dist[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{11/2}}{a+b\sin(c+dx)}\,dx = \frac{2e(e\cos(c+dx))^{9/2}}{9bd} + \frac{e^2\int \frac{(e\cos(c+dx))^{7/2}(b+a\sin(c+dx))}{a+b\sin(c+dx)}dx}{b}$$

$$= \frac{2e(e\cos(c+dx))^{9/2}}{9bd} - \frac{2e^3(e\cos(c+dx))^{5/2}\left(7(a^2-b^2)-5ab\sin(c+dx)\right)}{35b^3d} + \frac{(2e$$

$$= \frac{2e(e\cos(c+dx))^{9/2}}{9bd} - \frac{2e^3(e\cos(c+dx))^{5/2}\left(7(a^2-b^2)-5ab\sin(c+dx)\right)}{35b^3d} + \frac{2e^5}{}$$

$$= \frac{2e(e\cos(c+dx))^{9/2}}{9bd} - \frac{2e^3(e\cos(c+dx))^{5/2}\left(7(a^2-b^2)-5ab\sin(c+dx)\right)}{35b^3d} + \frac{2e^5}{}$$

$$= \frac{2e(e\cos(c+dx))^{9/2}}{9bd} - \frac{2e^3(e\cos(c+dx))^{5/2}\left(7(a^2-b^2)-5ab\sin(c+dx)\right)}{35b^3d} + \frac{2e^5}{}$$

$$= \frac{2e(e\cos(c+dx))^{9/2}}{9bd} + \frac{2a(21a^4-49a^2b^2+33b^4)e^6\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right.}{21b^6d\sqrt{e\cos(c+dx)}}$$

$$= \frac{2e(e\cos(c+dx))^{9/2}}{9bd} + \frac{2a(21a^4-49a^2b^2+33b^4)e^6\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right.}{21b^6d\sqrt{e\cos(c+dx)}}$$

$$= -\frac{(-a^2+b^2)^{9/4}e^{11/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{b^{11/2}d} - \frac{(-a^2+b^2)^{9/4}e^{11/2}\tanh^{-1}}{b^{11/2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 57.66, size = 2035, normalized size = 3.83

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^(11/2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((e*Cos[c + d*x])^(11/2)*((-2*(280*a^4 - 636*a^2*b^2 + 721*b^4)*(a + b*Sqrt
[1 - Cos[c + d*x]^2])*((5*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[c +
d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[c + d*x]])/(Sqrt[1 - Co
s[c + d*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^
2*Cos[c + d*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[c
```

```
 + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4,
3/2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)])*Cos[c + d*
x]^2)*(a^2 + b^2*(-1 + Cos[c + d*x]^2))) - ((1/8 - I/8)*Sqrt[b]*(2*ArcTan[1
 - ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 +
((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 +
b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c +
d*x]] - Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[
c + d*x]] + I*b*Cos[c + d*x]]))/(-a^2 + b^2)^(3/4))*Sin[c + d*x])/(Sqrt[1 -
 Cos[c + d*x]^2]*(a + b*Sin[c + d*x])) + ((840*a^4 - 1764*a^2*b^2 + 959*b^4
)*(a + b*Sqrt[1 - Cos[c + d*x]^2])*Cos[2*(c + d*x)]*(((1/2 - I/2)*(-2*a^2 +
 b^2)*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)])/
(b^(3/2)*(-a^2 + b^2)^(3/4)) - ((1/2 - I/2)*(-2*a^2 + b^2)*ArcTan[1 + ((1 +
 I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)])/(b^(3/2)*(-a^2 + b^2)^
(3/4)) + (4*Sqrt[Cos[c + d*x]])/b - (4*a*AppellF1[5/4, 1/2, 1, 9/4, Cos[c +
 d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Cos[c + d*x]^(5/2))/(5*(a^2 - b
^2)) + (10*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Co
s[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[c + d*x]])/(Sqrt[1 - Cos[c + d*x]^2]*(
5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^
2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[c + d*x]^2, (b^2
*Cos[c + d*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Co
s[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)])*Cos[c + d*x]^2)*(a^2 + b^
2*(-1 + Cos[c + d*x]^2))) + ((1/4 - I/4)*(-2*a^2 + b^2)*Log[Sqrt[-a^2 + b^2
] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x
]])/(b^(3/2)*(-a^2 + b^2)^(3/4)) - ((1/4 - I/4)*(-2*a^2 + b^2)*Log[Sqrt[-a^
2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[
c + d*x]])/(b^(3/2)*(-a^2 + b^2)^(3/4)))*Sin[c + d*x])/(Sqrt[1 - Cos[c + d*
x]^2]*(-1 + 2*Cos[c + d*x]^2)*(a + b*Sin[c + d*x])) - (2*(-392*a^3*b + 722*
a*b^3)*(a + b*Sqrt[1 - Cos[c + d*x]^2])*((5*b*(a^2 - b^2)*AppellF1[1/4, -1/
2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[c +
d*x]]*Sqrt[1 - Cos[c + d*x]^2])/((-5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4
, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + 2*(2*b^2*AppellF1[5/
4, -1/2, 2, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + (a^2
- b^2)*AppellF1[5/4, 1/2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^
2 + b^2)])*Cos[c + d*x]^2)*(a^2 + b^2*(-1 + Cos[c + d*x]^2))) + (a*(-2*ArcT
an[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] + 2*ArcTan[1
 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 -
 b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*
x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c +
d*x]] + b*Cos[c + d*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)))*Sin[c + d*
x]^2)/((1 - Cos[c + d*x]^2)*(a + b*Sin[c + d*x]))))/(1680*b^4*d*Cos[c + d*x
]^(11/2)) + ((e*Cos[c + d*x])^(11/2)*Sec[c + d*x]^5*(((-9*a^2 + 14*b^2)*Cos
[2*(c + d*x)])/(45*b^3) + Cos[4*(c + d*x)]/(36*b) - (a*(28*a^2 - 51*b^2)*Si
n[c + d*x])/(42*b^4) + (a*Sin[3*(c + d*x)])/(14*b^2)))/d
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.

time = 14.97, size = 1807, normalized size = 3.40

method	result	size
default	Expression too large to display	1807

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*cos(d*x+c))^(11/2)/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)

[Out] (32/9*e^5/b*cos(1/2*d*x+1/2*c)^8*(2*cos(1/2*d*x+1/2*c)^2*e-e)^(1/2)-64/9*e^5/b*cos(1/2*d*x+1/2*c)^6*(2*cos(1/2*d*x+1/2*c)^2*e-e)^(1/2)+104/15*e^5/b*cos(1/2*d*x+1/2*c)^4*(2*cos(1/2*d*x+1/2*c)^2*e-e)^(1/2)-152/45*e^5/b*cos(1/2*d*x+1/2*c)^2*(2*cos(1/2*d*x+1/2*c)^2*e-e)^(1/2)-152/45*e^5/b*(2*cos(1/2*d*x+1/2*c)^2*e-e)^(1/2)-8/5*e^5/b^3*cos(1/2*d*x+1/2*c)^4*(2*cos(1/2*d*x+1/2*c)^2*e-e)^(1/2)*a^2+8/5*e^5/b^3*cos(1/2*d*x+1/2*c)^2*(2*cos(1/2*d*x+1/2*c)^2*e-e)^(1/2)*a^2+8/5*e^5/b^3*(2*cos(1/2*d*x+1/2*c)^2*e-e)^(1/2)*a^2+2*e^5/b^5*(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)*a^4-6*e^5/b^3*(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)*a^2+6*e^5/b*(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)-2*e^7/b^5*sum((_R^4+_R^2*e)/(_R^7*b^2-3*_R^5*b^2*e+8*_R^3*a^2*e^2-5*_R^3*b^2*e^2-_R*b^2*e^3)*ln((-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)-e^(1/2)*cos(1/2*d*x+1/2*c)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*e*_Z^6+(16*a^2*e^2-10*b^2*e^2)*_Z^4-4*b^2*e^3*_Z^2+b^2*e^4))*a^6+6*e^7/b^3*sum((_R^4+_R^2*e)/(_R^7*b^2-3*_R^5*b^2*e+8*_R^3*a^2*e^2-5*_R^3*b^2*e^2-_R*b^2*e^3)*ln((-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)-e^(1/2)*cos(1/2*d*x+1/2*c)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*e*_Z^6+(16*a^2*e^2-10*b^2*e^2)*_Z^4-4*b^2*e^3*_Z^2+b^2*e^4))*a^4-6*e^7/b*sum((_R^4+_R^2*e)/(_R^7*b^2-3*_R^5*b^2*e+8*_R^3*a^2*e^2-5*_R^3*b^2*e^2-_R*b^2*e^3)*ln((-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)-e^(1/2)*cos(1/2*d*x+1/2*c)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*e*_Z^6+(16*a^2*e^2-10*b^2*e^2)*_Z^4-4*b^2*e^3*_Z^2+b^2*e^4))*a^2+2*e^7*b*sum((_R^4+_R^2*e)/(_R^7*b^2-3*_R^5*b^2*e+8*_R^3*a^2*e^2-5*_R^3*b^2*e^2-_R*b^2*e^3)*ln((-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)-e^(1/2)*cos(1/2*d*x+1/2*c)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*e*_Z^6+(16*a^2*e^2-10*b^2*e^2)*_Z^4-4*b^2*e^3*_Z^2+b^2*e^4))-2*(e*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*e^6*a*(1/21/b^6/(-2*sin(1/2*d*x+1/2*c)^4*e+sin(1/2*d*x+1/2*c)^2*e)^(1/2)*(48*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-72*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-28*a^2*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+84*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^4-49*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b^2+33*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^4+14*a^2*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-30*b^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)+1/16*(-a^6+3*a^4*b^2-3*a^2*b^4+b^6)/b^8*sum(1/_alpha/(2*_alpha^2-1)*(2^(1/2)/(e*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*arctanh(1/2*e*(4*_alpha^2-3)/(4*a^2-3*b^2)*(4*cos(1/2*d*x+1/2*c)^2*a^2-3*b^2*cos(1/2*d*x+1/2*c)^2+b^2*_alpha^2-3*a^2+2*b^2)*2^(1/2)/(

```
e*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-e*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2
*d*x+1/2*c)^2))^(1/2))+8*b^2/a^2*_alpha*(_alpha^2-1)*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-sin(1/2*d*x+1/2*c)^2*e*(2*sin(1/
2*d*x+1/2*c)^2-1))^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-4*b^2/a^2*(_alpha^2
-1),2^(1/2))),_alpha=RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2)))/sin(1/2*d*x+1/2*c)
/(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2))/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(11/2)/(a+b*sin(d*x+c)),x, algorithm="maxima")
[Out] e^(11/2)*integrate(cos(d*x + c)^(11/2)/(b*sin(d*x + c) + a), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(11/2)/(a+b*sin(d*x+c)),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(11/2)/(a+b*sin(d*x+c)),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(11/2)/(a+b*sin(d*x+c)),x, algorithm="giac")
[Out] integrate(cos(d*x + c)^(11/2)*e^(11/2)/(b*sin(d*x + c) + a), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,\cos\left(c+d\,x\right))^{11/2}}{a+b\,\sin\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(11/2)/(a + b*sin(c + d*x)),x)
[Out] int((e*cos(c + d*x))^(11/2)/(a + b*sin(c + d*x)), x)
```

3.575 $\int \frac{(e\cos(c+dx))^{9/2}}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=446

$$\frac{(-a^2+b^2)^{7/4}\,e^{9/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{b^{9/2}d}-\frac{(-a^2+b^2)^{7/4}\,e^{9/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{b^{9/2}d}+\frac{2e(e$$

```
[Out] (-a^2+b^2)^(7/4)*e^(9/2)*arctan(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1/
4)/e^(1/2))/b^(9/2)/d-(-a^2+b^2)^(7/4)*e^(9/2)*arctanh(b^(1/2)*(e*cos(d*x+c
))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(9/2)/d+2/7*e*(e*cos(d*x+c))^(7/2)/b/d
-2/15*e^3*(e*cos(d*x+c))^(3/2)*(5*a^2-5*b^2-3*a*b*sin(d*x+c))/b^3/d+a*(a^2-
b^2)^2*e^5*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1
/2*d*x+1/2*c),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*cos(d*x+c)^(1/2)/b^5/d/(b-(
-a^2+b^2)^(1/2))/(e*cos(d*x+c))^(1/2)+a*(a^2-b^2)^2*e^5*(cos(1/2*d*x+1/2*c)
^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(b+(-a^2+b^2
)^(1/2)),2^(1/2))*cos(d*x+c)^(1/2)/b^5/d/(b+(-a^2+b^2)^(1/2))/(e*cos(d*x+c)
)^(1/2)-2/5*a*(5*a^2-8*b^2)*e^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/
2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/b^4/d/cos(d
*x+c)^(1/2)
```

Rubi [A]
time = 0.83, antiderivative size = 446, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {2774, 2944, 2946, 2721, 2719, 2780, 2886, 2884, 335, 304, 211, 214}

$$\frac{e^{9/2}(b^2-a^2)^{7/4}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{b^{9/2}d}-\frac{e^{9/2}(b^2-a^2)^{7/4}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{b^{9/2}d}+\frac{ae^5(a^2-b^2)^2\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{b^5d\left(b-\sqrt{b^2-a^2}\right)\sqrt{e\cos(c+dx)}}+\frac{ae^5(a^2-b^2)^2\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{b^5d\left(\sqrt{b^2-a^2}+b\right)\sqrt{e\cos(c+dx)}}-\frac{2ae^4(5a^2-8b^2)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5b^4d\sqrt{\cos(c+dx)}}-\frac{2e^3(e\cos(c+dx))^{3/2}(5(a^2-b^2)-3ab\sin(c+dx))}{15b^3d}+\frac{2e(e\cos(c+dx))^{7/2}}{7bd}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(9/2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((-a^2 + b^2)^(7/4)*e^(9/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 +
b^2)^(1/4)*Sqrt[e])])/(b^(9/2)*d) - ((-a^2 + b^2)^(7/4)*e^(9/2)*ArcTanh[(Sq
rt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(b^(9/2)*d) + (2
*e*(e*Cos[c + d*x])^(7/2))/(7*b*d) - (2*a*(5*a^2 - 8*b^2)*e^4*Sqrt[e*Cos[c
+ d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^4*d*Sqrt[Cos[c + d*x]]) + (a*(a^2 -
 b^2)^2*e^5*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c
+ d*x)/2, 2])/(b^5*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e*Cos[c + d*x]]) + (a*(a^2
 - b^2)^2*e^5*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (
c + d*x)/2, 2])/(b^5*(b + Sqrt[-a^2 + b^2])*d*Sqrt[e*Cos[c + d*x]]) - (2*e^
3*(e*Cos[c + d*x])^(3/2)*(5*(a^2 - b^2) - 3*a*b*Sin[c + d*x]))/(15*b^3*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2774

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(b*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2780

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[a*(g/(2*b)), Int[1/(Sq
```

```
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[a*(g/(2*b)), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[b*(g/f), Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2944

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Dist[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{9/2}}{a+b\sin(c+dx)}\,dx = \frac{2e(e\cos(c+dx))^{7/2}}{7bd} + \frac{e^2\int \frac{(e\cos(c+dx))^{5/2}(b+a\sin(c+dx))}{a+b\sin(c+dx)}\,dx}{b}$$

$$= \frac{2e(e\cos(c+dx))^{7/2}}{7bd} - \frac{2e^3(e\cos(c+dx))^{3/2}\left(5(a^2-b^2)-3ab\sin(c+dx)\right)}{15b^3d} + \frac{(2e^4 \cdots}{\cdots}$$

$$= \frac{2e(e\cos(c+dx))^{7/2}}{7bd} - \frac{2e^3(e\cos(c+dx))^{3/2}\left(5(a^2-b^2)-3ab\sin(c+dx)\right)}{15b^3d} - \frac{(a(5 \cdots}{\cdots}$$

$$= \frac{2e(e\cos(c+dx))^{7/2}}{7bd} - \frac{2e^3(e\cos(c+dx))^{3/2}\left(5(a^2-b^2)-3ab\sin(c+dx)\right)}{15b^3d} - \frac{\left(a(a \cdots\right.}{\cdots}$$

$$= \frac{2e(e\cos(c+dx))^{7/2}}{7bd} - \frac{2a(5a^2-8b^2)\,e^4\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^4d\sqrt{\cos(c+dx)}} - \frac{2e^3(e \cdots}{\cdots}$$

$$= \frac{2e(e\cos(c+dx))^{7/2}}{7bd} - \frac{2a(5a^2-8b^2)\,e^4\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^4d\sqrt{\cos(c+dx)}} + \frac{a(a^2 - \cdots}{\cdots}$$

$$= \frac{(-a^2+b^2)^{7/4}\,e^{9/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{b^{9/2}d} - \frac{(-a^2+b^2)^{7/4}\,e^{9/2}\tanh^{-1}\left(\frac{\sqrt{b \cdots}}{\cdots}\right.}{b^{9/2}d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 46.91, size = 834, normalized size = 1.87

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^(9/2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/5*((e*Cos[c + d*x])^(9/2)*((-2*(2*a^2*b - 5*b^3)*(a + b*Sqrt[1 - Cos[c +
 d*x]^2])*((a*AppellF1[3/4, 1/2, 1, 7/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^
2)/(-a^2 + b^2)]*Cos[c + d*x]^(3/2))/(3*(a^2 - b^2)) + ((1/8 + I/8)*(2*ArcT
an[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[
1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^
2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[
c + d*x]] + Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[
Cos[c + d*x]] + I*b*Cos[c + d*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)))*Sin[c + d
```

```
*x])/(Sqrt[1 - Cos[c + d*x]^2]*(a + b*Sin[c + d*x])) - ((5*a^3 - 8*a*b^2)*(
a + b*Sqrt[1 - Cos[c + d*x]^2])*(8*b^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Cos[
c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Cos[c + d*x]^(3/2) + 3*Sqrt[
2]*a*(a^2 - b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(
a^2 - b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2
- b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqr
t[Cos[c + d*x]] + b*Cos[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(
a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]]))*Sin[c + d*x]^2)/(12
*b^(3/2)*(-a^2 + b^2)*(1 - Cos[c + d*x]^2)*(a + b*Sin[c + d*x]))))/(b^3*d*C
os[c + d*x]^(9/2)) + ((e*Cos[c + d*x])^(9/2)*Sec[c + d*x]^4*(((-28*a^2 + 37
*b^2)*Cos[c + d*x])/(42*b^3) + Cos[3*(c + d*x)]/(14*b) + (a*Sin[2*(c + d*x)
])/(5*b^2)))/d
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 10.35, size = 1583, normalized size = 3.55

method	result	size
default	Expression too large to display	1583

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(9/2)/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] (16/7*e^4/b*cos(1/2*d*x+1/2*c)^6*(2*cos(1/2*d*x+1/2*c)^2*e-e)^(1/2)-24/7*e^
4/b*cos(1/2*d*x+1/2*c)^4*(2*cos(1/2*d*x+1/2*c)^2*e-e)^(1/2)+64/21*e^4/b*cos
(1/2*d*x+1/2*c)^2*(2*cos(1/2*d*x+1/2*c)^2*e-e)^(1/2)+64/21*e^4/b*(2*cos(1/2
*d*x+1/2*c)^2*e-e)^(1/2)-4/3*e^4/b^3*cos(1/2*d*x+1/2*c)^2*(2*cos(1/2*d*x+1/
2*c)^2*e-e)^(1/2)*a^2-4/3*e^4/b^3*(2*cos(1/2*d*x+1/2*c)^2*e-e)^(1/2)*a^2+2*
e^4/b^3*(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)*a^2-4*e^4/b*(e*(2*cos(1/2*d*x+
1/2*c)^2-1))^(1/2)+1/2*e^5/b^3*sum((_R^6-_R^4*e-_R^2*e^2+e^3)/(_R^7*b^2-3*_
R^5*b^2*e+8*_R^3*a^2*e^2-5*_R^3*b^2*e^2-_R*b^2*e^3)*ln((-2*sin(1/2*d*x+1/2*
c)^2*e+e)^(1/2)-e^(1/2)*cos(1/2*d*x+1/2*c)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4
*b^2*e*_Z^6+(16*a^2*e^2-10*b^2*e^2)*_Z^4-4*b^2*e^3*_Z^2+b^2*e^4))*a^4-e^5/b
*sum((_R^6-_R^4*e-_R^2*e^2+e^3)/(_R^7*b^2-3*_R^5*b^2*e+8*_R^3*a^2*e^2-5*_R^
3*b^2*e^2-_R*b^2*e^3)*ln((-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)-e^(1/2)*cos(1/
2*d*x+1/2*c)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*e*_Z^6+(16*a^2*e^2-10*b^2
*e^2)*_Z^4-4*b^2*e^3*_Z^2+b^2*e^4))*a^2+1/2*e^5*b*sum((_R^6-_R^4*e-_R^2*e^2
+e^3)/(_R^7*b^2-3*_R^5*b^2*e+8*_R^3*a^2*e^2-5*_R^3*b^2*e^2-_R*b^2*e^3)*ln((
-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)-e^(1/2)*cos(1/2*d*x+1/2*c)*2^(1/2)-_R),_
R=RootOf(b^2*_Z^8-4*b^2*e*_Z^6+(16*a^2*e^2-10*b^2*e^2)*_Z^4-4*b^2*e^3*_Z^2+
b^2*e^4))-1/40*(e*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*e^
5*(128*cos(1/2*d*x+1/2*c)^7*a^2*b^4-256*cos(1/2*d*x+1/2*c)^5*a^2*b^4+160*co
s(1/2*d*x+1/2*c)^3*a^2*b^4+80*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^4*b^2-128*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))*a^2*b^4-32*cos(1/2*d*x+1/2*c)*a^2*b^4+5*sum((a^4-2*a^2*b
```

```
^2+b^4)/_alpha*(8*(e*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c
),-4*b^2/a^2*(_alpha^2-1),2^(1/2))*_alpha^3*b^2-8*b^2*_alpha*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2
*c),-4*b^2/a^2*(_alpha^2-1),2^(1/2))*(e*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/
2)+a^2*2^(1/2)*arctanh(1/2*e*(4*_alpha^2-3)/(4*a^2-3*b^2)*(4*cos(1/2*d*x+1/
2*c)^2*a^2-3*b^2*cos(1/2*d*x+1/2*c)^2+b^2*_alpha^2-3*a^2+2*b^2)*2^(1/2)/(e*
(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-e*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d
*x+1/2*c)^2))^(1/2))*(-sin(1/2*d*x+1/2*c)^2*e*(2*sin(1/2*d*x+1/2*c)^2-1))^(
1/2))/(e*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-sin(1/2*d*x+1/2*c)^2*e*(2*
sin(1/2*d*x+1/2*c)^2-1))^(1/2),_alpha=RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2))*(-
e*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2))/a/b^6/(-e*(2*sin(1/
2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(e*(2*cos(1/
2*d*x+1/2*c)^2-1))^(1/2))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(9/2)/(a+b*sin(d*x+c)),x, algorithm="maxima")
[Out] e^(9/2)*integrate(cos(d*x + c)^(9/2)/(b*sin(d*x + c) + a), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(9/2)/(a+b*sin(d*x+c)),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(9/2)/(a+b*sin(d*x+c)),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 8857 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(9/2)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(9/2)*e^(9/2)/(b*sin(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(e\,\cos\left(c+d\,x\right)\right)^{9/2}}{a+b\,\sin\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(9/2)/(a + b*sin(c + d*x)),x)
```

```
[Out] int((e*cos(c + d*x))^(9/2)/(a + b*sin(c + d*x)), x)
```

3.576 $\int \frac{(e\cos(c+dx))^{7/2}}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=461

$$-\frac{(-a^2+b^2)^{5/4}\,e^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{b^{7/2}d}-\frac{(-a^2+b^2)^{5/4}\,e^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{b^{7/2}d}+\frac{2e(e$$

```
[Out] -(-a^2+b^2)^(5/4)*e^(7/2)*arctan(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1
/4)/e^(1/2))/b^(7/2)/d-(-a^2+b^2)^(5/4)*e^(7/2)*arctanh(b^(1/2)*(e*cos(d*x+
c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(7/2)/d+2/5*e*(e*cos(d*x+c))^(5/2)/b/
d-2/3*a*(3*a^2-4*b^2)*e^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*E
llipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/b^4/d/(e*cos(d*x+c))^
(1/2)+a*(a^2-b^2)^2*e^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ell
ipticPi(sin(1/2*d*x+1/2*c),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*cos(d*x+c)^(1/
2)/b^4/d/(a^2-b*(b-(-a^2+b^2)^(1/2)))/(e*cos(d*x+c))^(1/2)+a*(a^2-b^2)^2*e^
4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/
2*c),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(d*x+c)^(1/2)/b^4/d/(a^2-b*(b+(-a
^2+b^2)^(1/2)))/(e*cos(d*x+c))^(1/2)-2/3*e^3*(3*a^2-3*b^2-a*b*sin(d*x+c))*(
e*cos(d*x+c))^(1/2)/b^3/d
```

Rubi [A]
time = 0.87, antiderivative size = 461, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {2774, 2944, 2946, 2721, 2720, 2781, 2886, 2884, 335, 218, 214, 211}

$$-\frac{e^{7/2}(b^2-a^2)^{5/4}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{b^{7/2}d}-\frac{e^{7/2}(b^2-a^2)^{5/4}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{b^{7/2}d}-\frac{2ae^4(3a^2-4b^2)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^4d\sqrt{e\cos(c+dx)}}+\frac{ae^4(a^2-b^2)^2\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{b^4d\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{e\cos(c+dx)}}+\frac{ae^4(a^2-b^2)^2\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{b^4d\left(a^2-b\left(\sqrt{b^2-a^2}+b\right)\right)\sqrt{e\cos(c+dx)}}-\frac{2e^3\sqrt{e\cos(c+dx)}\,(3(a^2-b^2)-ab\sin(c+dx))}{3b^3d}+\frac{2e(e\cos(c+dx))^{5/2}}{5bd}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(7/2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -(((-a^2 + b^2)^(5/4)*e^(7/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2
+ b^2)^(1/4)*Sqrt[e])])/(b^(7/2)*d)) - ((-a^2 + b^2)^(5/4)*e^(7/2)*ArcTanh[
(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(b^(7/2)*d) +
 (2*e*(e*Cos[c + d*x])^(5/2))/(5*b*d) - (2*a*(3*a^2 - 4*b^2)*e^4*Sqrt[Cos[c
 + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^4*d*Sqrt[e*Cos[c + d*x]]) + (a*(a^
2 - b^2)^2*e^4*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]),
(c + d*x)/2, 2])/(b^4*(a^2 - b*(b - Sqrt[-a^2 + b^2]))*d*Sqrt[e*Cos[c + d*x
]]) + (a*(a^2 - b^2)^2*e^4*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a
^2 + b^2]), (c + d*x)/2, 2])/(b^4*(a^2 - b*(b + Sqrt[-a^2 + b^2]))*d*Sqrt[e
*Cos[c + d*x]]) - (2*e^3*Sqrt[e*Cos[c + d*x]]*(3*(a^2 - b^2) - a*b*Sin[c +
d*x]))/(3*b^3*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2774

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(b*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2781

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[-a/(2*q), Int[1/(S
```

```
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[b*(g/f), Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2944

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Dist[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{7/2}}{a+b\sin(c+dx)}\,dx = \frac{2e(e\cos(c+dx))^{5/2}}{5bd} + \frac{e^2\int \frac{(e\cos(c+dx))^{3/2}(b+a\sin(c+dx))}{a+b\sin(c+dx)}\,dx}{b}$$

$$= \frac{2e(e\cos(c+dx))^{5/2}}{5bd} - \frac{2e^3\sqrt{e\cos(c+dx)}\,(3(a^2-b^2)-ab\sin(c+dx))}{3b^3d} + \frac{(2e^4)\ \cdots}{\cdots}$$

$$= \frac{2e(e\cos(c+dx))^{5/2}}{5bd} - \frac{2e^3\sqrt{e\cos(c+dx)}\,(3(a^2-b^2)-ab\sin(c+dx))}{3b^3d} - \frac{(a(3a\ \cdots}{\cdots}$$

$$= \frac{2e(e\cos(c+dx))^{5/2}}{5bd} - \frac{2e^3\sqrt{e\cos(c+dx)}\,(3(a^2-b^2)-ab\sin(c+dx))}{3b^3d} - \frac{\left(a(-\ \cdots\right.}{\cdots}$$

$$= \frac{2e(e\cos(c+dx))^{5/2}}{5bd} - \frac{2a(3a^2-4b^2)\,e^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{3b^4d\sqrt{e\cos(c+dx)}} - \frac{2e^3\sqrt{e\ \cdots}}{\cdots}$$

$$= \frac{2e(e\cos(c+dx))^{5/2}}{5bd} - \frac{2a(3a^2-4b^2)\,e^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{3b^4d\sqrt{e\cos(c+dx)}} + \frac{a(-a^2\ \cdots}{\cdots}$$

$$= -\frac{(-a^2+b^2)^{5/4}\,e^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{b^{7/2}d} - \frac{(-a^2+b^2)^{5/4}\,e^{7/2}\tanh^{-1}\left(\cdots\right.}{b^{7/2}d\ \cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 58.42, size = 1955, normalized size = 4.24

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^(7/2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((e*Cos[c + d*x])^(7/2)*Sec[c + d*x]^3*(Cos[2*(c + d*x)]/(5*b) + (2*a*Sin[c
 + d*x])/(3*b^2)))/d - ((e*Cos[c + d*x])^(7/2)*((-2*(10*a^2 - 27*b^2)*(a +
b*Sqrt[1 - Cos[c + d*x]^2])*((5*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Co
s[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[c + d*x]])/(Sqrt[
1 - Cos[c + d*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^
2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4,
 Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1
[5/4, 3/2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)])*Cos[
```

```
c + d*x]^2)*(a^2 + b^2*(-1 + Cos[c + d*x]^2))) - ((1/8 - I/8)*Sqrt[b]*(2*Ar
cTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTa
n[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[-
a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Co
s[c + d*x]] - Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqr
t[Cos[c + d*x]] + I*b*Cos[c + d*x]]))/(-a^2 + b^2)^(3/4))*Sin[c + d*x])/(Sq
rt[1 - Cos[c + d*x]^2]*(a + b*Sin[c + d*x])) + ((30*a^2 - 33*b^2)*(a + b*Sq
rt[1 - Cos[c + d*x]^2])*Cos[2*(c + d*x)]*(((1/2 - I/2)*(-2*a^2 + b^2)*ArcTa
n[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)])/(b^(3/2)*(-
a^2 + b^2)^(3/4)) - ((1/2 - I/2)*(-2*a^2 + b^2)*ArcTan[1 + ((1 + I)*Sqrt[b]
*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)])/(b^(3/2)*(-a^2 + b^2)^(3/4)) + (4
*Sqrt[Cos[c + d*x]])/b - (4*a*AppellF1[5/4, 1/2, 1, 9/4, Cos[c + d*x]^2, (b
^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Cos[c + d*x]^(5/2))/(5*(a^2 - b^2)) + (10*
a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^
2)/(-a^2 + b^2)]*Sqrt[Cos[c + d*x]])/(Sqrt[1 - Cos[c + d*x]^2]*(5*(a^2 - b^
2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 +
b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*
x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[c + d*x]^
2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)])*Cos[c + d*x]^2)*(a^2 + b^2*(-1 + Cos
[c + d*x]^2))) + ((1/4 - I/4)*(-2*a^2 + b^2)*Log[Sqrt[-a^2 + b^2] - (1 + I)
*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]])/(b^(3/2
)*(-a^2 + b^2)^(3/4)) - ((1/4 - I/4)*(-2*a^2 + b^2)*Log[Sqrt[-a^2 + b^2] +
(1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]])/
(b^(3/2)*(-a^2 + b^2)^(3/4)))*Sin[c + d*x])/(Sqrt[1 - Cos[c + d*x]^2]*(-1 +
 2*Cos[c + d*x]^2)*(a + b*Sin[c + d*x])) + (28*a*b*(a + b*Sqrt[1 - Cos[c +
d*x]^2])*((5*b*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Cos[c + d*x]^2, (b^2
*Cos[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[c + d*x]]*Sqrt[1 - Cos[c + d*x]^2])
/((-5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c +
d*x]^2)/(-a^2 + b^2)] + 2*(2*b^2*AppellF1[5/4, -1/2, 2, 9/4, Cos[c + d*x]^2
, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + (a^2 - b^2)*AppellF1[5/4, 1/2, 1, 9/
4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)])*Cos[c + d*x]^2)*(a^2
 + b^2*(-1 + Cos[c + d*x]^2))) + (a*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Co
s[c + d*x]])/(a^2 - b^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c
+ d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 -
b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqr
t[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]]))/(4*Sq
rt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)))*Sin[c + d*x]^2)/((1 - Cos[c + d*x]^2)*(a
+ b*Sin[c + d*x]))))/(60*b^2*d*Cos[c + d*x]^(7/2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 11.99, size = 1246, normalized size = 2.70

method	result	size
default	Expression too large to display	1246

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(7/2)/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] (8/5*e^3/b*cos(1/2*d*x+1/2*c)^4*(2*cos(1/2*d*x+1/2*c)^2*e-e)^(1/2)-8/5*e^3/
b*cos(1/2*d*x+1/2*c)^2*(2*cos(1/2*d*x+1/2*c)^2*e-e)^(1/2)-8/5*e^3/b*(2*cos(
1/2*d*x+1/2*c)^2*e-e)^(1/2)-2*e^3/b^3*(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)*
a^2+4*e^3/b*(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)+2*e^5/b^3*sum((_R^4+_R^2*e
)/(_R^7*b^2-3*_R^5*b^2*e+8*_R^3*a^2*e^2-5*_R^3*b^2*e^2-_R*b^2*e^3)*ln((-2*s
in(1/2*d*x+1/2*c)^2*e+e)^(1/2)-e^(1/2)*cos(1/2*d*x+1/2*c)*2^(1/2)-_R),_R=Ro
otOf(b^2*_Z^8-4*b^2*e*_Z^6+(16*a^2*e^2-10*b^2*e^2)*_Z^4-4*b^2*e^3*_Z^2+b^2*
e^4))*a^4-4*e^5/b*sum((_R^4+_R^2*e)/(_R^7*b^2-3*_R^5*b^2*e+8*_R^3*a^2*e^2-5
*_R^3*b^2*e^2-_R*b^2*e^3)*ln((-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)-e^(1/2)*co
s(1/2*d*x+1/2*c)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*e*_Z^6+(16*a^2*e^2-10
*b^2*e^2)*_Z^4-4*b^2*e^3*_Z^2+b^2*e^4))*a^2+2*e^5*b*sum((_R^4+_R^2*e)/(_R^7
*b^2-3*_R^5*b^2*e+8*_R^3*a^2*e^2-5*_R^3*b^2*e^2-_R*b^2*e^3)*ln((-2*sin(1/2*
d*x+1/2*c)^2*e+e)^(1/2)-e^(1/2)*cos(1/2*d*x+1/2*c)*2^(1/2)-_R),_R=RootOf(b^
2*_Z^8-4*b^2*e*_Z^6+(16*a^2*e^2-10*b^2*e^2)*_Z^4-4*b^2*e^3*_Z^2+b^2*e^4))-2
*(e*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*e^4*a*(-1/3/b^4/
(-2*sin(1/2*d*x+1/2*c)^4*e+sin(1/2*d*x+1/2*c)^2*e)^(1/2)*(-4*b^2*cos(1/2*d*
x+1/2*c)*sin(1/2*d*x+1/2*c)^4+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-4*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))*b^2+2*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)+1/16*(a^4-2
*a^2*b^2+b^4)/b^6*sum(1/_alpha/(2*_alpha^2-1)*(2^(1/2)/(e*(2*_alpha^2*b^2+a
^2-2*b^2)/b^2)^(1/2)*arctanh(1/2*e*(4*_alpha^2-3)/(4*a^2-3*b^2)*(4*cos(1/2*
d*x+1/2*c)^2*a^2-3*b^2*cos(1/2*d*x+1/2*c)^2+b^2*_alpha^2-3*a^2+2*b^2)*2^(1/
2)/(e*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-e*(2*sin(1/2*d*x+1/2*c)^4-sin
(1/2*d*x+1/2*c)^2))^(1/2))+8*b^2/a^2*_alpha*(_alpha^2-1)*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-sin(1/2*d*x+1/2*c)^2*e*(2*si
n(1/2*d*x+1/2*c)^2-1))^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-4*b^2/a^2*(_alp
ha^2-1),2^(1/2))),_alpha=RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2)))/sin(1/2*d*x+1/
2*c)/(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2))/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] e^(7/2)*integrate(cos(d*x + c)^(7/2)/(b*sin(d*x + c) + a), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(7/2)/(a+b*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3878 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(7/2)*e^(7/2)/(b*sin(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(e\,\cos\left(c+d\,x\right)\right)}^{7/2}}{a+b\,\sin\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(7/2)/(a + b*sin(c + d*x)),x)
```

```
[Out] int((e*cos(c + d*x))^(7/2)/(a + b*sin(c + d*x)), x)
```

3.577 $\int \frac{(e\cos(c+dx))^{5/2}}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=384

$$\frac{(-a^2+b^2)^{3/4}\,e^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{b^{5/2}d}-\frac{(-a^2+b^2)^{3/4}\,e^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{b^{5/2}d}+\frac{2e(e$$

```
[Out] (-a^2+b^2)^(3/4)*e^(5/2)*arctan(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1/
4)/e^(1/2))/b^(5/2)/d-(-a^2+b^2)^(3/4)*e^(5/2)*arctanh(b^(1/2)*(e*cos(d*x+c
))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(5/2)/d+2/3*e*(e*cos(d*x+c))^(3/2)/b/d
-a*(a^2-b^2)*e^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi
(sin(1/2*d*x+1/2*c),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*cos(d*x+c)^(1/2)/b^3/
d/(b-(-a^2+b^2)^(1/2))/(e*cos(d*x+c))^(1/2)-a*(a^2-b^2)*e^3*(cos(1/2*d*x+1/
2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(b+(-a^2
+b^2)^(1/2)),2^(1/2))*cos(d*x+c)^(1/2)/b^3/d/(b+(-a^2+b^2)^(1/2))/(e*cos(d*
x+c))^(1/2)+2*a*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipti
cE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/b^2/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.56, antiderivative size = 384, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {2774, 2946, 2721, 2719, 2780, 2886, 2884, 335, 304, 211, 214}

$$\frac{e^{5/2}(b^2-a^2)^{3/4}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{b^{5/2}d}-\frac{e^{5/2}(b^2-a^2)^{3/4}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{b^{5/2}d}-\frac{ae^3(a^2-b^2)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{b^3d\left(b-\sqrt{b^2-a^2}\right)\sqrt{e\cos(c+dx)}}-\frac{ae^3(a^2-b^2)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{b^3d\left(\sqrt{b^2-a^2}+b\right)\sqrt{e\cos(c+dx)}}+\frac{2ae^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}+\frac{2e(e\cos(c+dx))^{3/2}}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(5/2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((-a^2 + b^2)^(3/4)*e^(5/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 +
b^2)^(1/4)*Sqrt[e])])/(b^(5/2)*d) - ((-a^2 + b^2)^(3/4)*e^(5/2)*ArcTanh[(Sq
rt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(b^(5/2)*d) + (2
*e*(e*Cos[c + d*x])^(3/2))/(3*b*d) + (2*a*e^2*Sqrt[e*Cos[c + d*x]]*Elliptic
E[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c + d*x]]) - (a*(a^2 - b^2)*e^3*Sqrt[Cos
[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(b^3*(
b - Sqrt[-a^2 + b^2])*d*Sqrt[e*Cos[c + d*x]]) - (a*(a^2 - b^2)*e^3*Sqrt[Cos
[c + d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(b^3*(
b + Sqrt[-a^2 + b^2])*d*Sqrt[e*Cos[c + d*x]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2774

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(b*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2780

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[a*(g/(2*b)), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[a*(g/(2*b)), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[b*(g/f), Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
```

```
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{5/2}}{a+b\sin(c+dx)}\,dx = \frac{2e(e\cos(c+dx))^{3/2}}{3bd} + \frac{e^2\int \frac{\sqrt{e\cos(c+dx)}\,(b+a\sin(c+dx))}{a+b\sin(c+dx)}\,dx}{b}$$

$$= \frac{2e(e\cos(c+dx))^{3/2}}{3bd} + \frac{(ae^2)\int \sqrt{e\cos(c+dx)}\,dx}{b^2} + \frac{((-a^2+b^2)\,e^2)\int \frac{\sqrt{e\cos(c+}}{a+b\sin(c+}}{b^2}$$

$$= \frac{2e(e\cos(c+dx))^{3/2}}{3bd} + \frac{(a(a^2-b^2)\,e^3)\int \frac{1}{\sqrt{e\cos(c+dx)}\left(\sqrt{-a^2+b^2}-b\cos(c+dx)\right)}}{2b^3}$$

$$= \frac{2e(e\cos(c+dx))^{3/2}}{3bd} + \frac{2ae^2\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{b^2d\sqrt{\cos(c+dx)}} - \frac{(2(a^2-b^2)\,e^3)\,\text{Sub}}{}$$

$$= \frac{2e(e\cos(c+dx))^{3/2}}{3bd} + \frac{2ae^2\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{b^2d\sqrt{\cos(c+dx)}} - \frac{a(a^2-b^2)\,e^3\sqrt{\cos}}{b^3\left(b-\right.}$$

$$= \frac{(-a^2+b^2)^{3/4}\,e^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{b^{5/2}d} - \frac{(-a^2+b^2)^{3/4}\,e^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}}{}\right.}{b^{5/2}d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 41.30, size = 709, normalized size = 1.85

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^(5/2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((e*Cos[c + d*x])^(5/2)*(2*Cos[c + d*x]^(3/2) - (a*(8*b^(5/2)*AppellF1[3/4,
 -1/2, 1, 7/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Cos[c + d
*x]^(3/2) + 3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sq
rt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[C
os[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a
^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]] + Log[Sqrt[a^2 - b^2]
+ Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]]))*
(a + b*Sqrt[Sin[c + d*x]^2]))/(4*b^(3/2)*(-a^2 + b^2)*(a + b*Sin[c + d*x]))
 - (6*b*((a*AppellF1[3/4, 1/2, 1, 7/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)
/(-a^2 + b^2)]*Cos[c + d*x]^(3/2))/(3*(a^2 - b^2)) + ((1/8 + I/8)*(2*ArcTan
```

```
[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1
+ ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2
+ b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c
+ d*x]] + Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Co
s[c + d*x]] + I*b*Cos[c + d*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)))*Sin[c + d*x
]*(a + b*Sqrt[Sin[c + d*x]^2]))/(Sqrt[Sin[c + d*x]^2]*(a + b*Sin[c + d*x]))
))/(3*b*d*Cos[c + d*x]^(5/2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 10.00, size = 1087, normalized size = 2.83

method	result	size
default	Expression too large to display	1087

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(5/2)/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] (4/3*e^2/b*cos(1/2*d*x+1/2*c)^2*(2*cos(1/2*d*x+1/2*c)^2*e-e)^(1/2)+4/3*e^2/
b*(2*cos(1/2*d*x+1/2*c)^2*e-e)^(1/2)-2*e^2/b*(e*(2*cos(1/2*d*x+1/2*c)^2-1))
^(1/2)-1/2*e^3/b*sum((_R^6-_R^4*e-_R^2*e^2+e^3)/(_R^7*b^2-3*_R^5*b^2*e+8*_R
^3*a^2*e^2-5*_R^3*b^2*e^2-_R*b^2*e^3)*ln((-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2
)-e^(1/2)*cos(1/2*d*x+1/2*c)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*e*_Z^6+(1
6*a^2*e^2-10*b^2*e^2)*_Z^4-4*b^2*e^3*_Z^2+b^2*e^4))*a^2+1/2*e^3*b*sum((_R^6
-_R^4*e-_R^2*e^2+e^3)/(_R^7*b^2-3*_R^5*b^2*e+8*_R^3*a^2*e^2-5*_R^3*b^2*e^2-
_R*b^2*e^3)*ln((-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)-e^(1/2)*cos(1/2*d*x+1/2*
c)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*e*_Z^6+(16*a^2*e^2-10*b^2*e^2)*_Z^4
-4*b^2*e^3*_Z^2+b^2*e^4))+1/8*(e*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2
*c)^2)^(1/2)*e^3/a*(16*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^2*a^2+sum((a^2-b^2)/_alp
ha*(8*(e*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-4*b^2/a^2
*(_alpha^2-1),2^(1/2))*_alpha^3*b^2-8*b^2*_alpha*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-4*b^2/a
^2*(_alpha^2-1),2^(1/2))*(e*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)+a^2*2^(1/
2)*arctanh(1/2*e*(4*_alpha^2-3)/(4*a^2-3*b^2)*(4*cos(1/2*d*x+1/2*c)^2*a^2-3
*b^2*cos(1/2*d*x+1/2*c)^2+b^2*_alpha^2-3*a^2+2*b^2)*2^(1/2)/(e*(2*_alpha^2*
b^2+a^2-2*b^2)/b^2)^(1/2)/(-e*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)
)^(1/2))*(-sin(1/2*d*x+1/2*c)^2*e*(2*sin(1/2*d*x+1/2*c)^2-1))^(1/2))/(e*(2*
_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-sin(1/2*d*x+1/2*c)^2*e*(2*sin(1/2*d*x+
1/2*c)^2-1))^(1/2),_alpha=RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2))*(-e*(2*sin(1/2
*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2))/b^4/(-e*(2*sin(1/2*d*x+1/2*c)^4
-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(e*(2*cos(1/2*d*x+1/2*c)^2
-1))^(1/2))/d
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+b*sin(d*x+c)),x, algorithm="maxima")
[Out] e^(5/2)*integrate(cos(d*x + c)^(5/2)/(b*sin(d*x + c) + a), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+b*sin(d*x+c)),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(5/2)/(a+b*sin(d*x+c)),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 4851 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+b*sin(d*x+c)),x, algorithm="giac")
[Out] integrate(cos(d*x + c)^(5/2)*e^(5/2)/(b*sin(d*x + c) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,\cos(c+d\,x))^{5/2}}{a+b\,\sin(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(5/2)/(a + b*sin(c + d*x)),x)
[Out] int((e*cos(c + d*x))^(5/2)/(a + b*sin(c + d*x)), x)
```

3.578 $\int \frac{(e\cos(c+dx))^{3/2}}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=397

$$-\frac{\sqrt[4]{-a^2+b^2}\,e^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{b^{3/2}d}-\frac{\sqrt[4]{-a^2+b^2}\,e^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{b^{3/2}d}+\frac{2e\sqrt{e\,\mathrm{c}}}{}$$

```
[Out] -(-a^2+b^2)^(1/4)*e^(3/2)*arctan(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1
/4)/e^(1/2))/b^(3/2)/d-(-a^2+b^2)^(1/4)*e^(3/2)*arctanh(b^(1/2)*(e*cos(d*x+
c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(3/2)/d+2*a*e^2*(cos(1/2*d*x+1/2*c)^2
)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)
^(1/2)/b^2/d/(e*cos(d*x+c))^(1/2)-a*(a^2-b^2)*e^2*(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(b-(-a^2+b^2)^(1/2
)),2^(1/2))*cos(d*x+c)^(1/2)/b^2/d/(a^2-b*(b-(-a^2+b^2)^(1/2)))/(e*cos(d*x+
c))^(1/2)-a*(a^2-b^2)*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*E
llipticPi(sin(1/2*d*x+1/2*c),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(d*x+c)^(
1/2)/b^2/d/(a^2-b*(b+(-a^2+b^2)^(1/2)))/(e*cos(d*x+c))^(1/2)+2*e*(e*cos(d*x
+c))^(1/2)/b/d
```

Rubi [A]

time = 0.57, antiderivative size = 397, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {2774, 2946, 2721, 2720, 2781, 2886, 2884, 335, 218, 214, 211}

$$-\frac{e^{3/2}\sqrt[4]{b^2-a^2}\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{b^{3/2}d}-\frac{ae^2(a^2-b^2)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{e\cos(c+dx)}}-\frac{ae^2(a^2-b^2)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\left(a^2-b\left(\sqrt{b^2-a^2}+b\right)\right)\sqrt{e\cos(c+dx)}}-\frac{e^{3/2}\sqrt[4]{b^2-a^2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{b^{3/2}d}+\frac{2ae^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{e\cos(c+dx)}}+\frac{2e\sqrt{e\cos(c+dx)}}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(3/2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -(((-a^2 + b^2)^(1/4)*e^(3/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2
+ b^2)^(1/4)*Sqrt[e])])/(b^(3/2)*d)) - ((-a^2 + b^2)^(1/4)*e^(3/2)*ArcTanh[
(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(b^(3/2)*d) +
 (2*e*Sqrt[e*Cos[c + d*x]])/(b*d) + (2*a*e^2*Sqrt[Cos[c + d*x]]*EllipticF[(
c + d*x)/2, 2])/(b^2*d*Sqrt[e*Cos[c + d*x]]) - (a*(a^2 - b^2)*e^2*Sqrt[Cos[
c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(b^2*(a
^2 - b*(b - Sqrt[-a^2 + b^2]))*d*Sqrt[e*Cos[c + d*x]]) - (a*(a^2 - b^2)*e^2
*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2
])/(b^2*(a^2 - b*(b + Sqrt[-a^2 + b^2]))*d*Sqrt[e*Cos[c + d*x]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2774

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(b*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2781

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[-a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[b*(g/f), Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
```

```
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{3/2}}{a+b\sin(c+dx)}\,dx = \frac{2e\sqrt{e\cos(c+dx)}}{bd} + \frac{e^2\int \frac{b+a\sin(c+dx)}{\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))}\,dx}{b}$$

$$= \frac{2e\sqrt{e\cos(c+dx)}}{bd} + \frac{(ae^2)\int \frac{1}{\sqrt{e\cos(c+dx)}}\,dx}{b^2} + \frac{((-a^2+b^2)\,e^2)\int \frac{}{\sqrt{e\cos(c+}}}{b^2}$$

$$= \frac{2e\sqrt{e\cos(c+dx)}}{bd} - \frac{\left(a\sqrt{-a^2+b^2}\,e^2\right)\int \frac{1}{\sqrt{e\cos(c+dx)}\left(\sqrt{-a^2+b^2}-b\cos(c+dx)\right.}}{2b^2}$$

$$= \frac{2e\sqrt{e\cos(c+dx)}}{bd} + \frac{2ae^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{b^2d\sqrt{e\cos(c+dx)}} - \frac{(2(a^2-b^2)\,e^3)\,\mathrm{Subst}}{}$$

$$= \frac{2e\sqrt{e\cos(c+dx)}}{bd} + \frac{2ae^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{b^2d\sqrt{e\cos(c+dx)}} + \frac{a\sqrt{-a^2+b^2}\,e^2\sqrt{\cos}}{b^2\left(b-\right.}$$

$$= -\frac{\sqrt[4]{-a^2+b^2}\,e^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{b^{3/2}d} - \frac{\sqrt[4]{-a^2+b^2}\,e^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt[4]{}}\right.}{b^{3/2}d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 26.74, size = 434, normalized size = 1.09

$$\frac{\left(\frac{1}{20}-\frac{i}{20}\right)(e\cos(c+dx))^{3/2}\left((4+4i)ab^{3/2}F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\cos^2(c+dx),\frac{b^2\cos^2(c+dx)}{-a^2+b^2}\right)\cos^{\frac{5}{2}}(c+dx)-5(a^2-b^2)\left(2\sqrt[4]{-a^2+b^2}\tan^{-1}\left(1-\frac{(1+i)\sqrt{b}\sqrt{\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}}\right)-2\sqrt[4]{-a^2+b^2}\tan^{-1}\left(1+\frac{(1+i)\sqrt{b}\sqrt{\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}}\right)+(4+4i)\sqrt{b}\sqrt{\cos(c+dx)}+\sqrt[4]{-a^2+b^2}\log\left(\sqrt{-a^2+b^2}-(1+i)\sqrt{b}\sqrt[4]{-a^2+b^2}\sqrt{\cos(c+dx)}+ib\cos(c+dx)\right)-\sqrt[4]{-a^2+b^2}\log\left(\sqrt{-a^2+b^2}+(1+i)\sqrt{b}\sqrt[4]{-a^2+b^2}\sqrt{\cos(c+dx)}+ib\cos(c+dx)\right)\right)\right)\sin(c+dx)\left(a+b\sqrt{\sin^2(c+dx)}\right)}{b^{3/2}(-a^2+b^2)\,d\cos^{\frac{3}{2}}(c+dx)\sqrt{\sin^2(c+dx)}\,(a+b\sin(c+dx))}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^(3/2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((1/20 - I/20)*(e*Cos[c + d*x])^(3/2)*((4 + 4*I)*a*b^(3/2)*AppellF1[5/4, 1/
2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Cos[c + d*x]^
(5/2) - 5*(a^2 - b^2)*(2*(-a^2 + b^2)^(1/4)*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqr
t[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*(-a^2 + b^2)^(1/4)*ArcTan[1 + ((1
+ I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] + (4 + 4*I)*Sqrt[b]*Sq
rt[Cos[c + d*x]] + (-a^2 + b^2)^(1/4)*Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b
]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]] - (-a^2 + b^2)^
(1/4)*Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c
+ d*x]] + I*b*Cos[c + d*x]]))*Sin[c + d*x]*(a + b*Sqrt[Sin[c + d*x]^2]))/(b
^(3/2)*(-a^2 + b^2)*d*Cos[c + d*x]^(3/2)*Sqrt[Sin[c + d*x]^2]*(a + b*Sin[c
+ d*x]))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 9.62, size = 828, normalized size = 2.09

method	result	size
default	Expression too large to display	828

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] (2*e/b*(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)-2*e^3/b*sum((_R^4+_R^2*e)/(_R^7
*b^2-3*_R^5*b^2*e+8*_R^3*a^2*e^2-5*_R^3*b^2*e^2-_R*b^2*e^3)*ln((-2*sin(1/2*
d*x+1/2*c)^2*e+e)^(1/2)-e^(1/2)*cos(1/2*d*x+1/2*c)*2^(1/2)-_R),_R=RootOf(b^
2*_Z^8-4*b^2*e*_Z^6+(16*a^2*e^2-10*b^2*e^2)*_Z^4-4*b^2*e^3*_Z^2+b^2*e^4))*a
^2+2*e^3*b*sum((_R^4+_R^2*e)/(_R^7*b^2-3*_R^5*b^2*e+8*_R^3*a^2*e^2-5*_R^3*b
^2*e^2-_R*b^2*e^3)*ln((-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)-e^(1/2)*cos(1/2*d
*x+1/2*c)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*e*_Z^6+(16*a^2*e^2-10*b^2*e^
2)*_Z^4-4*b^2*e^3*_Z^2+b^2*e^4))-2*(e*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*
x+1/2*c)^2)^(1/2)*e^2*a*(1/b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x
+1/2*c)^2+1)^(1/2)/(-e*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/16*(-a^2+b^2)/b^4*sum(1/_alpha/(2*
_alpha^2-1)*(2^(1/2)/(e*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*arctanh(1/2*e
*(4*_alpha^2-3)/(4*a^2-3*b^2)*(4*cos(1/2*d*x+1/2*c)^2*a^2-3*b^2*cos(1/2*d*x
+1/2*c)^2+b^2*_alpha^2-3*a^2+2*b^2)*2^(1/2)/(e*(2*_alpha^2*b^2+a^2-2*b^2)/b
^2)^(1/2)/(-e*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2))+8*b^2/a
^2*_alpha*(_alpha^2-1)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
2+1)^(1/2)/(-sin(1/2*d*x+1/2*c)^2*e*(2*sin(1/2*d*x+1/2*c)^2-1))^(1/2)*Ellip
ticPi(cos(1/2*d*x+1/2*c),-4*b^2/a^2*(_alpha^2-1),2^(1/2))),_alpha=RootOf(4*
_Z^4*b^2-4*_Z^2*b^2+a^2)))/sin(1/2*d*x+1/2*c)/(e*(2*cos(1/2*d*x+1/2*c)^2-1)
)^(1/2))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] e^(3/2)*integrate(cos(d*x + c)^(3/2)/(b*sin(d*x + c) + a), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(3/2)/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(3/2)*e^(3/2)/(b*sin(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,\cos(c+d\,x))^{3/2}}{a+b\,\sin(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(3/2)/(a + b*sin(c + d*x)),x)
```

```
[Out] int((e*cos(c + d*x))^(3/2)/(a + b*sin(c + d*x)), x)
```

3.579 $\int \frac{\sqrt{e\cos(c+dx)}}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=292

$$\frac{\sqrt{e}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{\sqrt{b}\,\sqrt[4]{-a^2+b^2}\,d}-\frac{\sqrt{e}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{\sqrt{b}\,\sqrt[4]{-a^2+b^2}\,d}+\frac{ae\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{-a^2+}}\right.}{b\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e}}$$

```
[Out] arctan(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))*e^(1/2)/(-a^2
+b^2)^(1/4)/d/b^(1/2)-arctanh(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)
/e^(1/2))*e^(1/2)/(-a^2+b^2)^(1/4)/d/b^(1/2)+a*e*(cos(1/2*d*x+1/2*c)^2)^(1/
2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(b-(-a^2+b^2)^(1/2)
),2^(1/2))*cos(d*x+c)^(1/2)/b/d/(b-(-a^2+b^2)^(1/2))/(e*cos(d*x+c))^(1/2)+a
*e*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1
/2*c),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(d*x+c)^(1/2)/b/d/(b+(-a^2+b^2)^
(1/2))/(e*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.37, antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {2780, 2886, 2884, 335, 304, 211, 214}

$$\frac{\sqrt{e}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{\sqrt{b}\,d\sqrt[4]{b^2-a^2}}-\frac{\sqrt{e}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{\sqrt{b}\,d\sqrt[4]{b^2-a^2}}+\frac{ae\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{bd\left(b-\sqrt{b^2-a^2}\right)\sqrt{e\cos(c+dx)}}+\frac{ae\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{bd\left(\sqrt{b^2-a^2}+b\right)\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cos[c + d*x]]/(a + b*Sin[c + d*x]),x]
```

```
[Out] (Sqrt[e]*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])
])/(Sqrt[b]*(-a^2 + b^2)^(1/4)*d) - (Sqrt[e]*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c
+ d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(Sqrt[b]*(-a^2 + b^2)^(1/4)*d) + (a
*e*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c + d*x)/2,
 2])/(b*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e*Cos[c + d*x]]) + (a*e*Sqrt[Cos[c +
d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(b*(b + Sqr
t[-a^2 + b^2])*d*Sqrt[e*Cos[c + d*x]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2780

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[a*(g/(2*b)), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[a*(g/(2*b)), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[b*(g/f), Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\sqrt{e\cos(c+dx)}}{a+b\sin(c+dx)}\,dx = -\frac{(ae)\int \frac{1}{\sqrt{e\cos(c+dx)}\left(\sqrt{-a^2+b^2}-b\cos(c+dx)\right)}\,dx}{2b} + \frac{(ae)\int \frac{}{\sqrt{e\cos(c+dx)}\,(\ldots}}{2\ldots}$$

$$= \frac{(2be)\text{Subst}\left(\int \frac{x^2}{(a^2-b^2)e^2+b^2x^4}\,dx, x, \sqrt{e\cos(c+dx)}\right)}{d} - \frac{\left(ae\sqrt{\cos(c+dx)}\right)\int \frac{}{\sqrt{c\ldots}}}{2b\ldots}$$

$$= \frac{ae\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{-a^2+b^2}};\frac{1}{2}(c+dx)\middle|2\right)}{b\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\cos(c+dx)}} + \frac{ae\sqrt{\cos(c+dx)}\,\Pi\left(\frac{}{b+\sqrt{-\ldots}}\ldots\right.}{b\left(b+\sqrt{-a^2+b^2}\right)\ldots}$$

$$= \frac{\sqrt{e}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{\sqrt{b}\sqrt[4]{-a^2+b^2}\,d} - \frac{\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{\sqrt{b}\sqrt[4]{-a^2+b^2}\,d} + \frac{ae\ldots}{\ldots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 25.80, size = 361, normalized size = 1.24

$$-\frac{2\sqrt{e\cos(c+dx)}\left(\frac{aF_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\cos^2(c+dx),\frac{b^2\cos^2(c+dx)}{-a^2+b^2}\right)\cos^{\frac{3}{2}}(c+dx)}{3(a^2-b^2)}+\frac{\left(\frac{1}{8}+\frac{i}{8}\right)\left(2\tan^{-1}\left(1-\frac{(1+i)\sqrt{b}\sqrt{\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}}\right)-2\tan^{-1}\left(1+\frac{(1+i)\sqrt{b}\sqrt{\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}}\right)-\log\left(\sqrt{-a^2+b^2}-(1+i)\sqrt{b}\sqrt[4]{-a^2+b^2}\sqrt{\cos(c+dx)}+ib\cos(c+dx)\right)+\log\left(\sqrt{-a^2+b^2}+(1+i)\sqrt{b}\sqrt[4]{-a^2+b^2}\sqrt{\cos(c+dx)}+ib\cos(c+dx)\right)\right)}{\sqrt{b}\sqrt[4]{-a^2+b^2}}\right)\sin(c+dx)\left(a+b\sqrt{\sin^2(c+dx)}\right)}{d\sqrt{\cos(c+dx)}\sqrt{\sin^2(c+dx)}\,(a+b\sin(c+dx))}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[e*Cos[c + d*x]]/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*Sqrt[e*Cos[c + d*x]]*((a*AppellF1[3/4, 1/2, 1, 7/4, Cos[c + d*x]^2, (b^
2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Cos[c + d*x]^(3/2))/(3*(a^2 - b^2)) + ((1/8
 + I/8)*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/
4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)]
 - Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d
*x]] + I*b*Cos[c + d*x]] + Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b
^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1
/4)))*Sin[c + d*x]*(a + b*Sqrt[Sin[c + d*x]^2]))/(d*Sqrt[Cos[c + d*x]]*Sqrt
[Sin[c + d*x]^2]*(a + b*Sin[c + d*x]))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 7.42, size = 680, normalized size = 2.33

method	result

default	$\frac{eb\left(\sum_{\_R=\mathrm{RootOf}\left(b^2\_Z^8-4b^2e\_Z^6+\left(16e^2a^2-10b^2e^2\right)\_Z^4-4b^2e^3\_Z^2+b^2e^4\right)}\frac{\left(\_R^6-\_R^4e-\_R^2e^2+e^3\right)\ln\left(\sqrt{-2\left(\sin^2\left(\frac{dx}{2}+\right.\right.}\right.}{\_R^7b^2-3\_R^5b^2e+8\_R^3a^2e^2-}\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*cos(d*x+c))^(1/2)/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)

[Out] (1/2*e*b*sum((_R^6-_R^4*e-_R^2*e^2+e^3)/(_R^7*b^2-3*_R^5*b^2*e+8*_R^3*a^2*e^2-5*_R^3*b^2*e^2-_R*b^2*e^3)*ln((-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)-e^(1/2)*cos(1/2*d*x+1/2*c)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*e*_Z^6+(16*a^2*e^2-10*b^2*e^2)*_Z^4-4*b^2*e^3*_Z^2+b^2*e^4))-1/8*(e*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*e/a/b^2*sum(1/_alpha*(8*(e*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-4*b^2/a^2*(_alpha^2-1),2^(1/2))*_alpha^3*b^2-8*b^2*_alpha*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-4*b^2/a^2*(_alpha^2-1),2^(1/2))*(e*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)+a^2*2^(1/2)*arctanh(1/2*e*(4*_alpha^2-3)/(4*a^2-3*b^2)*(4*cos(1/2*d*x+1/2*c)^2*a^2-3*b^2*cos(1/2*d*x+1/2*c)^2+b^2*_alpha^2-3*a^2+2*b^2)*2^(1/2)/(e*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-e*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2))*(-sin(1/2*d*x+1/2*c)^2*e*(2*sin(1/2*d*x+1/2*c)^2-1))^(1/2))/(e*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-sin(1/2*d*x+1/2*c)^2*e*(2*sin(1/2*d*x+1/2*c)^2-1))^(1/2),_alpha=RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2))/sin(1/2*d*x+1/2*c)/(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2))/d

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*cos(d*x+c))^(1/2)/(a+b*sin(d*x+c)),x, algorithm="maxima")

[Out] e^(1/2)*integrate(sqrt(cos(d*x + c))/(b*sin(d*x + c) + a), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(cos(d*x + c))*e^(1/2)/(b*sin(d*x + c) + a), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(1/2)/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(d*x + c))*e^(1/2)/(b*sin(d*x + c) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{e\,\cos\left(c+d\,x\right)}}{a+b\,\sin\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(1/2)/(a + b*sin(c + d*x)),x)
```

```
[Out] int((e*cos(c + d*x))^(1/2)/(a + b*sin(c + d*x)), x)
```

3.580 $\int \frac{1}{\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))}\,dx$

Optimal. Leaf size=299

$$-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{(-a^2+b^2)^{3/4}d\sqrt{e}}-\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{(-a^2+b^2)^{3/4}d\sqrt{e}}+\frac{a\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{-a^2+\ldots}}\right.}{\left(a^2-b\left(b-\sqrt{-a^2+b^2}\right)\right)d\ldots}$$

```
[Out] -arctan(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))*b^(1/2)/(-a^
2+b^2)^(3/4)/d/e^(1/2)-arctanh(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1/4
)/e^(1/2))*b^(1/2)/(-a^2+b^2)^(3/4)/d/e^(1/2)+a*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(b-(-a^2+b^2)^(1/2))
,2^(1/2))*cos(d*x+c)^(1/2)/d/(a^2-b*(b-(-a^2+b^2)^(1/2)))/(e*cos(d*x+c))^(1
/2)+a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*
x+1/2*c),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(d*x+c)^(1/2)/d/(a^2-b*(b+(-a
^2+b^2)^(1/2)))/(e*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.37, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {2781, 2886, 2884, 335, 218, 214, 211}

$$-\frac{\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{d\sqrt{e}\,(b^2-a^2)^{3/4}}-\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{d\sqrt{e}\,(b^2-a^2)^{3/4}}+\frac{a\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{d\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{e\cos(c+dx)}}+\frac{a\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{d\left(a^2-b\left(\sqrt{b^2-a^2}+b\right)\right)\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x])),x]
```

```
[Out] -((Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e
])])/((-a^2 + b^2)^(3/4)*d*Sqrt[e])) - (Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[e*Cos
[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/((-a^2 + b^2)^(3/4)*d*Sqrt[e]) +
 (a*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c + d*x)/2
, 2])/((a^2 - b*(b - Sqrt[-a^2 + b^2]))*d*Sqrt[e*Cos[c + d*x]]) + (a*Sqrt[C
os[c + d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/((a^
2 - b*(b + Sqrt[-a^2 + b^2]))*d*Sqrt[e*Cos[c + d*x]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2781

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[-a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[b*(g/f), Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))}\,dx = -\frac{a\int \frac{1}{\sqrt{e\cos(c+dx)}\left(\sqrt{-a^2+b^2}-b\cos(c+dx)\right)}\,dx}{2\sqrt{-a^2+b^2}} - \frac{a\int \frac{}{\sqrt{e\cos}}}{}$$

$$= \frac{(2be)\mathrm{Subst}\left(\int \frac{1}{(a^2-b^2)e^2+b^2x^4}\,dx, x, \sqrt{e\cos(c+dx)}\right)}{d} - \frac{\left(a\sqrt{\cos(}\right.}{}$$

$$= \frac{a\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{-a^2+b^2}};\frac{1}{2}(c+dx)\middle|2\right)}{\left(a^2-b\left(b-\sqrt{-a^2+b^2}\right)\right)d\sqrt{e\cos(c+dx)}} + \frac{a\sqrt{\cos(c+}}{\left(a^2-b\left(b\right.\right.}$$

$$= -\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{(-a^2+b^2)^{3/4}d\sqrt{e}} - \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos}}{\sqrt[4]{-a^2+}}\right.}{(-a^2+b^2)^{3/4}d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 26.31, size = 558, normalized size = 1.87

$$-\frac{2\sqrt{\cos(c+dx)}\sin(c+dx)\left(\frac{\left(-\frac{1}{8}+\frac{i}{8}\right)\sqrt{b}\left(2\tan^{-1}\left(1-\frac{(1+i)\sqrt{b}\sqrt{\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}}\right)-2\tan^{-1}\left(1+\frac{(1+i)\sqrt{b}\sqrt{\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}}\right)+\log\left(\sqrt{-a^2+b^2}-(1+i)\sqrt{b}\sqrt[4]{-a^2+b^2}\sqrt{\cos(c+dx)}+ib\cos(c+dx)\right)-\log\left(\sqrt{-a^2+b^2}+(1+i)\sqrt{b}\sqrt[4]{-a^2+b^2}\sqrt{\cos(c+dx)}+ib\cos(c+dx)\right)\right)}{(-a^2+b^2)^{3/4}}+\frac{5a(a^2-b^2)F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\cos^2(c+dx),\frac{b^2\cos^2(c+dx)}{-a^2+b^2}\right)\sqrt{\cos(c+dx)}}{(a^2-b^2+b^2\cos^2(c+dx))\left(5(a^2-b^2)F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\cos^2(c+dx),\frac{b^2\cos^2(c+dx)}{-a^2+b^2}\right)-2\left(2b^2F_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\cos^2(c+dx),\frac{b^2\cos^2(c+dx)}{-a^2+b^2}\right)+(-a^2+b^2)F_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};\cos^2(c+dx),\frac{b^2\cos^2(c+dx)}{-a^2+b^2}\right)\right)\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}\right)\left(a+b\sqrt{\sin^2(c+dx)}\right)}{d\sqrt{e\cos(c+dx)}\sqrt{\sin^2(c+dx)}(a+b\sin(c+dx))}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x])),x]
```

```
[Out] (-2*Sqrt[Cos[c + d*x]]*Sin[c + d*x]*(((-1/8 + I/8)*Sqrt[b]*(2*ArcTan[1 - ((
1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 +
 I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2]
- (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]]
 - Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d
*x]] + I*b*Cos[c + d*x]]))/(-a^2 + b^2)^(3/4) + (5*a*(a^2 - b^2)*AppellF1[1
/4, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Co
s[c + d*x]])/((a^2 - b^2 + b^2*Cos[c + d*x]^2)*(5*(a^2 - b^2)*AppellF1[1/4,
 1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] - 2*(2*b^2
*AppellF1[5/4, 1/2, 2, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^
2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos[c +
d*x]^2)/(-a^2 + b^2)])*Cos[c + d*x]^2)*Sqrt[Sin[c + d*x]^2]))*(a + b*Sqrt[S
in[c + d*x]^2]))/(d*Sqrt[e*Cos[c + d*x]]*Sqrt[Sin[c + d*x]^2]*(a + b*Sin[c
+ d*x]))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 8.36, size = 676, normalized size = 2.26

method	result
default	$\frac{2be\left(\sum_{\_R=\mathrm{RootOf}\left(b^2\_Z^8-4b^2e\_Z^6+\left(16e^2a^2-10b^2e^2\right)\_Z^4-4b^2e^3\_Z^2+b^2e^4\right)}\frac{\left(\_R^4+\_R^2e\right)\ln\left(\sqrt{-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)e+}\right.}{\_R^7b^2-3\_R^5b^2e+8\_R^3a^2e^2-5\_}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(d*x+c))/(e*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (2*b*e*sum((_R^4+_R^2*e)/(_R^7*b^2-3*_R^5*b^2*e+8*_R^3*a^2*e^2-5*_R^3*b^2*e
^2-_R*b^2*e^3)*ln((-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)-e^(1/2)*cos(1/2*d*x+1
/2*c)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*e*_Z^6+(16*a^2*e^2-10*b^2*e^2)*_
Z^4-4*b^2*e^3*_Z^2+b^2*e^4))-1/8*(e*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+
1/2*c)^2)^(1/2)/a/b^2*sum(1/_alpha/(2*_alpha^2-1)*(8*(e*(2*_alpha^2*b^2+a^2
-2*b^2)/b^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-4*b^2/a^2*(_alpha^2-1),2^(1/2))*_alph
a^3*b^2-8*b^2*_alpha*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+
1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-4*b^2/a^2*(_alpha^2-1),2^(1/2))*(e*
(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)+a^2*2^(1/2)*arctanh(1/2*e*(4*_alpha^2
-3)/(4*a^2-3*b^2)*(4*cos(1/2*d*x+1/2*c)^2*a^2-3*b^2*cos(1/2*d*x+1/2*c)^2+b^
2*_alpha^2-3*a^2+2*b^2)*2^(1/2)/(e*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-
e*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2))*(-sin(1/2*d*x+1/2*c
)^2*e*(2*sin(1/2*d*x+1/2*c)^2-1))^(1/2))/(e*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)
^(1/2)/(-sin(1/2*d*x+1/2*c)^2*e*(2*sin(1/2*d*x+1/2*c)^2-1))^(1/2),_alpha=Ro
otOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2))/sin(1/2*d*x+1/2*c)/(e*(2*cos(1/2*d*x+1/2*c
)^2-1))^(1/2))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c))/(e*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] e^(-1/2)*integrate(1/((b*sin(d*x + c) + a)*sqrt(cos(d*x + c))), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c))/(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c))/(e*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c))/(e*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(e^(-1/2)/((b*sin(d*x + c) + a)*sqrt(cos(d*x + c))), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{e\,\cos\left(c+d\,x\right)}\,\left(a+b\,\sin\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cos(c + d*x))^(1/2)*(a + b*sin(c + d*x))),x)
```

```
[Out] int(1/((e*cos(c + d*x))^(1/2)*(a + b*sin(c + d*x))), x)
```

3.581 $\int \frac{1}{(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))}\,dx$

Optimal. Leaf size=411

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{(-a^2+b^2)^{5/4}de^{3/2}}-\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{(-a^2+b^2)^{5/4}de^{3/2}}-\frac{2a\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\right|}{(a^2-b^2)\,de^2\sqrt{\cos(c+dx)}}$$

```
[Out] b^(3/2)*arctan(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/(-a^2
+b^2)^(5/4)/d/e^(3/2)-b^(3/2)*arctanh(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^
2)^(1/4)/e^(1/2))/(-a^2+b^2)^(5/4)/d/e^(3/2)-2*(b-a*sin(d*x+c))/(a^2-b^2)/d
/e/(e*cos(d*x+c))^(1/2)-a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*cos(d*x+c)
^(1/2)/(a^2-b^2)/d/e/(b-(-a^2+b^2)^(1/2))/(e*cos(d*x+c))^(1/2)-a*b*(cos(1/2
*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(
b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(d*x+c)^(1/2)/(a^2-b^2)/d/e/(b+(-a^2+b^2)^(
1/2))/(e*cos(d*x+c))^(1/2)-2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2
*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/(a^2-b^2)/d/
e^2/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.62, antiderivative size = 411, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {2775, 2946, 2721, 2719, 2780, 2886, 2884, 335, 304, 211, 214}

$$\frac{b^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{de^{3/2}(b^2-a^2)^{5/4}}-\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{de^2(a^2-b^2)\sqrt{\cos(c+dx)}}-\frac{2(b-a\sin(c+dx))}{de(a^2-b^2)\sqrt{e\cos(c+dx)}}-\frac{ab\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{de(a^2-b^2)\left(b-\sqrt{b^2-a^2}\right)\sqrt{e\cos(c+dx)}}-\frac{ab\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{de(a^2-b^2)\left(\sqrt{b^2-a^2}+b\right)\sqrt{e\cos(c+dx)}}-\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{de^{3/2}(b^2-a^2)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x])),x]
```

```
[Out] (b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])
])/((-a^2 + b^2)^(5/4)*d*e^(3/2)) - (b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c
+ d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/((-a^2 + b^2)^(5/4)*d*e^(3/2)) - (2
*a*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/((a^2 - b^2)*d*e^2*Sqrt[
Cos[c + d*x]]) - (a*b*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 +
b^2]), (c + d*x)/2, 2])/((a^2 - b^2)*(b - Sqrt[-a^2 + b^2])*d*e*Sqrt[e*Cos[
c + d*x]]) - (a*b*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]
), (c + d*x)/2, 2])/((a^2 - b^2)*(b + Sqrt[-a^2 + b^2])*d*e*Sqrt[e*Cos[c +
d*x]]) - (2*(b - a*Sin[c + d*x]))/((a^2 - b^2)*d*e*Sqrt[e*Cos[c + d*x]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2775

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^
(m + 1)*((b - a*Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rule 2780

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[a*(g/(2*b)), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[a*(g/(2*b)), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[b*(g/f), Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
```

```
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{1}{(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))}\,dx = -\frac{2(b-a\sin(c+dx))}{(a^2-b^2)\,de\sqrt{e\cos(c+dx)}} - \frac{2\int \frac{\sqrt{e\cos(c+dx)}\left(\frac{a^2}{2}+\frac{b^2}{2}+\frac{1}{2}a\right.}{a+b\sin(c+dx)}}{(a^2-b^2)\,e^2}$$

$$= -\frac{2(b-a\sin(c+dx))}{(a^2-b^2)\,de\sqrt{e\cos(c+dx)}} - \frac{a\int \sqrt{e\cos(c+dx)}\,dx}{(a^2-b^2)\,e^2} - \frac{b^2\int}{}$$

$$= -\frac{2(b-a\sin(c+dx))}{(a^2-b^2)\,de\sqrt{e\cos(c+dx)}} + \frac{(ab)\int \frac{1}{\sqrt{e\cos(c+dx)}\left(\sqrt{-a^2}\right.}}{2\left(a^2-b^2\right)}$$

$$= -\frac{2a\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{(a^2-b^2)\,de^2\sqrt{\cos(c+dx)}} - \frac{2(b-a\sin(c+dx)}{(a^2-b^2)\,de\sqrt{e\cos(c+}}$$

$$= -\frac{2a\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{(a^2-b^2)\,de^2\sqrt{\cos(c+dx)}} - \frac{ab\sqrt{\cos(c+dx)}\,\Pi\left(\frac{}{b-}\right.}{(a^2-b^2)\left(b-\sqrt{-a^2}\right.}$$

$$= \frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{(-a^2+b^2)^{5/4}\,de^{3/2}} - \frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(}}{\sqrt[4]{-a^2+b}}\right.}{(-a^2+b^2)^{5/4}\,de^3}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 42.63, size = 791, normalized size = 1.92

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x])),x]
```

```
[Out] (2*Cos[c + d*x]*(-b + a*Sin[c + d*x]))/((a^2 - b^2)*d*(e*Cos[c + d*x])^(3/2
)) - (Cos[c + d*x]^(3/2)*((-2*(a^2 + b^2)*(a + b*Sqrt[1 - Cos[c + d*x]^2])*
((a*AppellF1[3/4, 1/2, 1, 7/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 +
 b^2)]*Cos[c + d*x]^(3/2))/(3*(a^2 - b^2)) + ((1/8 + I/8)*(2*ArcTan[1 - ((1
 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 +
I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] -
 (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]]
+ Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*
x]] + I*b*Cos[c + d*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)))*Sin[c + d*x])/(Sqrt
[1 - Cos[c + d*x]^2]*(a + b*Sin[c + d*x])) - (a*(a + b*Sqrt[1 - Cos[c + d*x
```

```
]^2])*(8*b^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Cos[c + d*x]^2, (b^2*Cos[c + d
*x]^2)/(-a^2 + b^2)]*Cos[c + d*x]^(3/2) + 3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*(2*
ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcT
an[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a
^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c
+ d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[
c + d*x]] + b*Cos[c + d*x]]))*Sin[c + d*x]^2)/(12*Sqrt[b]*(-a^2 + b^2)*(1 -
 Cos[c + d*x]^2)*(a + b*Sin[c + d*x]))))/((a - b)*(a + b)*d*(e*Cos[c + d*x]
)^(3/2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 13.11, size = 1056, normalized size = 2.57

method	result	size
default	Expression too large to display	1056

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] (-1/2/e^2*b/(a^2-b^2)*2^(1/2)/(cos(1/2*d*x+1/2*c)-1/2*2^(1/2))*(-2*sin(1/2*
d*x+1/2*c)^2*e+e)^(1/2)+1/2/e^2*b/(a^2-b^2)*2^(1/2)/(cos(1/2*d*x+1/2*c)+1/2
*2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)-1/2/e*b^3/(a-b)/(a+b)*sum((_R
^6-_R^4*e-_R^2*e^2+e^3)/(_R^7*b^2-3*_R^5*b^2*e+8*_R^3*a^2*e^2-5*_R^3*b^2*e^
2-_R*b^2*e^3)*ln((-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)-e^(1/2)*cos(1/2*d*x+1/
2*c)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*e*_Z^6+(16*a^2*e^2-10*b^2*e^2)*_Z
^4-4*b^2*e^3*_Z^2+b^2*e^4))-1/8*(32*cos(1/2*d*x+1/2*c)^3*(-e*(2*sin(1/2*d*x
+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*a^2+16*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(e*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-32*cos(1/2*d*x+1
/2*c)*(-e*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*a^2-sum(1/_a
lpha*(8*(e*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-4*b^2/a
^2*(_alpha^2-1),2^(1/2))*_alpha^3*b^2-8*b^2*_alpha*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-4*b^2
/a^2*(_alpha^2-1),2^(1/2))*(e*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)+a^2*2^(
1/2)*arctanh(1/2*e*(4*_alpha^2-3)/(4*a^2-3*b^2)*(4*cos(1/2*d*x+1/2*c)^2*a^2
-3*b^2*cos(1/2*d*x+1/2*c)^2+b^2*_alpha^2-3*a^2+2*b^2)*2^(1/2)/(e*(2*_alpha^
2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-e*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^
2))^(1/2))*(-sin(1/2*d*x+1/2*c)^2*e*(2*sin(1/2*d*x+1/2*c)^2-1))^(1/2))/(e*(
2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-sin(1/2*d*x+1/2*c)^2*e*(2*sin(1/2*d*
x+1/2*c)^2-1))^(1/2),_alpha=RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2))*(e*(2*cos(1/
2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-e*(2*sin(1/2*d*x+1/2*c)^4-s
in(1/2*d*x+1/2*c)^2))^(1/2))/e/a/(a+b)/(a-b)/(-e*(2*sin(1/2*d*x+1/2*c)^4-si
n(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(e*(2*cos(1/2*d*x+1/2*c)^2-1)
)^(1/2))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c)),x, algorithm="maxima")
[Out] e^(-3/2)*integrate(1/((b*sin(d*x + c) + a)*cos(d*x + c)^(3/2)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c)),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(3/2)/(a+b*sin(d*x+c)),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c)),x, algorithm="giac")
[Out] integrate(e^(-3/2)/((b*sin(d*x + c) + a)*cos(d*x + c)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(e\,\cos\left(c+d\,x\right)\right)}^{3/2}\,\left(a+b\,\sin\left(c+d\,x\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cos(c + d*x))^(3/2)*(a + b*sin(c + d*x))),x)
[Out] int(1/((e*cos(c + d*x))^(3/2)*(a + b*sin(c + d*x))), x)
```

3.582 $\int \frac{1}{(e\cos(c+dx))^{5/2}(a+b\sin(c+dx))}\,dx$

Optimal. Leaf size=434

$$-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{\left(-a^2+b^2\right)^{7/4}de^{5/2}}-\frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{\left(-a^2+b^2\right)^{7/4}de^{5/2}}+\frac{2a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\right|}{3\left(a^2-b^2\right)de^2\sqrt{e\cos(c+dx}}$$

```
[Out] -b^(5/2)*arctan(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/(-a^
2+b^2)^(7/4)/d/e^(5/2)-b^(5/2)*arctanh(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b
^2)^(1/4)/e^(1/2))/(-a^2+b^2)^(7/4)/d/e^(5/2)-2/3*(b-a*sin(d*x+c))/(a^2-b^2
)/d/e/(e*cos(d*x+c))^(3/2)+2/3*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1
/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/(a^2-b^2)/d/e^
2/(e*cos(d*x+c))^(1/2)-a*b^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*cos(d*x+c
)^(1/2)/(a^2-b^2)/d/e^2/(a^2-b*(b-(-a^2+b^2)^(1/2)))/(e*cos(d*x+c))^(1/2)-a
*b^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x
+1/2*c),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(d*x+c)^(1/2)/(a^2-b^2)/d/e^2/
(a^2-b*(b+(-a^2+b^2)^(1/2)))/(e*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.67, antiderivative size = 434, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {2775, 2946, 2721, 2720, 2781, 2886, 2884, 335, 218, 214, 211}

$$-\frac{b^{5/2}\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{de^{5/2}\left(b^2-a^2\right)^{7/4}}+\frac{2a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3de^2\left(a^2-b^2\right)\sqrt{e\cos(c+dx)}}-\frac{ab^2\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{de^2\left(a^2-b^2\right)\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{e\cos(c+dx)}}-\frac{ab^2\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{de^2\left(a^2-b^2\right)\left(a^2-b\left(\sqrt{b^2-a^2}+b\right)\right)\sqrt{e\cos(c+dx)}}-\frac{2(b-a\sin(c+dx))}{3de\left(a^2-b^2\right)(e\cos(c+dx))^{3/2}}-\frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{de^{5/2}\left(b^2-a^2\right)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(5/2)*(a + b*Sin[c + d*x])),x]
```

```
[Out] -((b^(5/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e
])])/((-a^2 + b^2)^(7/4)*d*e^(5/2))) - (b^(5/2)*ArcTanh[(Sqrt[b]*Sqrt[e*Cos
[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/((-a^2 + b^2)^(7/4)*d*e^(5/2)) +
 (2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*(a^2 - b^2)*d*e^2*Sq
rt[e*Cos[c + d*x]]) - (a*b^2*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[
-a^2 + b^2]), (c + d*x)/2, 2])/((a^2 - b^2)*(a^2 - b*(b - Sqrt[-a^2 + b^2])
)*d*e^2*Sqrt[e*Cos[c + d*x]]) - (a*b^2*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/
(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/((a^2 - b^2)*(a^2 - b*(b + Sqrt[-a
^2 + b^2]))*d*e^2*Sqrt[e*Cos[c + d*x]]) - (2*(b - a*Sin[c + d*x]))/(3*(a^2
- b^2)*d*e*(e*Cos[c + d*x])^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2775

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^
(m + 1)*((b - a*Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rule 2781

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[-a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[b*(g/f), Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
```

```
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{1}{(e\cos(c+dx))^{5/2}(a+b\sin(c+dx))}\,dx = -\frac{2(b-a\sin(c+dx))}{3\,(a^2-b^2)\,de(e\cos(c+dx))^{3/2}} - \frac{2\int \frac{-\frac{a^2}{2}+\frac{3b^2}{2}-\frac{1}{2}ab\sin(c+dx)}{\sqrt{e\cos(c+dx)}\,(a+b\sin(}}{3\,(a^2-b^2)\,e^2}$$

$$= -\frac{2(b-a\sin(c+dx))}{3\,(a^2-b^2)\,de(e\cos(c+dx))^{3/2}} + \frac{a\int \frac{1}{\sqrt{e\cos(c+dx)}}\,dx}{3\,(a^2-b^2)\,e^2} - $$

$$= -\frac{2(b-a\sin(c+dx))}{3\,(a^2-b^2)\,de(e\cos(c+dx))^{3/2}} - \frac{(ab^2)\int \frac{}{\sqrt{e\cos(c+dx)}\,\left(\sqrt{}\right.}}{2\,(-a^2+b}$$

$$= \frac{2a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{3\,(a^2-b^2)\,de^2\sqrt{e\cos(c+dx)}} - \frac{2(b-a\sin(c+dx))}{3\,(a^2-b^2)\,de(e\cos(c+dx}$$

$$= \frac{2a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{3\,(a^2-b^2)\,de^2\sqrt{e\cos(c+dx)}} + \frac{ab^2\sqrt{\cos(c+dx)}\,\Pi\left(\frac{}{b-\sqrt{}}\right.}{(-a^2+b^2)^{3/2}\left(b-\sqrt{-a^2}\right.}$$

$$= -\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{(-a^2+b^2)^{7/4}\,de^{5/2}} - \frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos}}{\sqrt[4]{-a^2+}}\right.}{(-a^2+b^2)^{7/4}\,d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 44.45, size = 1192, normalized size = 2.75

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((e*Cos[c + d*x])^(5/2)*(a + b*Sin[c + d*x])),x]
```

```
[Out] (2*Cos[c + d*x]*(-b + a*Sin[c + d*x]))/(3*(a^2 - b^2)*d*(e*Cos[c + d*x])^(5
/2)) + (Cos[c + d*x]^(5/2)*((-2*(a^2 - 3*b^2)*(a + b*Sqrt[1 - Cos[c + d*x]^
2])*((5*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c
 + d*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[c + d*x]])/(Sqrt[1 - Cos[c + d*x]^2]*(5*(
a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/
(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[c + d*x]^2, (b^2*Co
s[c + d*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[c
 + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)])*Cos[c + d*x]^2)*(a^2 + b^2*(
-1 + Cos[c + d*x]^2))) - ((1/8 - I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 + I)*Sqrt[b
]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*S
```

```
qrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqr
t[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]] - Log[Sqrt[-
a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Co
s[c + d*x]]))/(-a^2 + b^2)^(3/4))*Sin[c + d*x])/(Sqrt[1 - Cos[c + d*x]^2]*(
a + b*Sin[c + d*x])) - (2*a*b*(a + b*Sqrt[1 - Cos[c + d*x]^2])*((5*b*(a^2 -
 b^2)*AppellF1[1/4, -1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^
2 + b^2)]*Sqrt[Cos[c + d*x]]*Sqrt[1 - Cos[c + d*x]^2])/((-5*(a^2 - b^2)*App
ellF1[1/4, -1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]
 + 2*(2*b^2*AppellF1[5/4, -1/2, 2, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2
)/(-a^2 + b^2)] + (a^2 - b^2)*AppellF1[5/4, 1/2, 1, 9/4, Cos[c + d*x]^2, (b
^2*Cos[c + d*x]^2)/(-a^2 + b^2)])*Cos[c + d*x]^2)*(a^2 + b^2*(-1 + Cos[c +
d*x]^2))) + (a*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b
^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^
(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c
 + d*x]] + b*Cos[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b
^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2 -
b^2)^(3/4)))*Sin[c + d*x]^2)/((1 - Cos[c + d*x]^2)*(a + b*Sin[c + d*x]))))/
(3*(a - b)*(a + b)*d*(e*Cos[c + d*x])^(5/2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 15.54, size = 932, normalized size = 2.15

method	result	size
default	Expression too large to display	932

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(5/2)/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] (-1/12/e^3*b/(a^2-b^2)/(cos(1/2*d*x+1/2*c)+1/2*2^(1/2))^2*(-2*sin(1/2*d*x+1
/2*c)^2*e+e)^(1/2)-1/12/e^3*b*2^(1/2)/(a^2-b^2)/(cos(1/2*d*x+1/2*c)+1/2*2^(
1/2))*(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)-2/e*b^3/(a-b)/(a+b)*sum((_R^4+_R^
2*e)/(_R^7*b^2-3*_R^5*b^2*e+8*_R^3*a^2*e^2-5*_R^3*b^2*e^2-_R*b^2*e^3)*ln((-
2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)-e^(1/2)*cos(1/2*d*x+1/2*c)*2^(1/2)-_R),_R
=RootOf(b^2*_Z^8-4*b^2*e*_Z^6+(16*a^2*e^2-10*b^2*e^2)*_Z^4-4*b^2*e^3*_Z^2+b
^2*e^4))-1/12/e^3*b/(a^2-b^2)/(cos(1/2*d*x+1/2*c)-1/2*2^(1/2))^2*(-2*sin(1/
2*d*x+1/2*c)^2*e+e)^(1/2)+1/12/e^3*b*2^(1/2)/(a^2-b^2)/(cos(1/2*d*x+1/2*c)-
1/2*2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)-2*(e*(2*cos(1/2*d*x+1/2*c)
^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a/e^2*(1/(a^2-b^2)*(-1/6*cos(1/2*d*x+1/2*
c)/e*(-e*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+
1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)/(-e*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2)))-1/16/(a-b)/(a+b)*sum(1/_alpha/(2*_alpha^2-1)*(
2^(1/2)/(e*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*arctanh(1/2*e*(4*_alpha^2-
3)/(4*a^2-3*b^2)*(4*cos(1/2*d*x+1/2*c)^2*a^2-3*b^2*cos(1/2*d*x+1/2*c)^2+b^2
*_alpha^2-3*a^2+2*b^2)*2^(1/2)/(e*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-e
```

```
*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2))+8*b^2/a^2*_alpha*(_a
lpha^2-1)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-
sin(1/2*d*x+1/2*c)^2*e*(2*sin(1/2*d*x+1/2*c)^2-1))^(1/2)*EllipticPi(cos(1/2
*d*x+1/2*c),-4*b^2/a^2*(_alpha^2-1),2^(1/2))),_alpha=RootOf(4*_Z^4*b^2-4*_Z
^2*b^2+a^2)))/sin(1/2*d*x+1/2*c)/(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2))/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(5/2)/(a+b*sin(d*x+c)),x, algorithm="maxima")
[Out] e^(-5/2)*integrate(1/((b*sin(d*x + c) + a)*cos(d*x + c)^(5/2)), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(5/2)/(a+b*sin(d*x+c)),x, algorithm="fricas")
[Out] integral(sqrt(cos(d*x + c))/(b*cos(d*x + c)^3*e^(5/2)*sin(d*x + c) + a*cos(
d*x + c)^3*e^(5/2)), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(5/2)/(a+b*sin(d*x+c)),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(5/2)/(a+b*sin(d*x+c)),x, algorithm="giac")
[Out] integrate(e^(-5/2)/((b*sin(d*x + c) + a)*cos(d*x + c)^(5/2)), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(e\,\cos\left(c+d\,x\right)\right)^{5/2}\,\left(a+b\,\sin\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cos(c + d*x))^(5/2)*(a + b*sin(c + d*x))),x)
[Out] int(1/((e*cos(c + d*x))^(5/2)*(a + b*sin(c + d*x))), x)
```

3.583 $\int \frac{1}{(e\cos(c+dx))^{7/2}(a+b\sin(c+dx))}\,dx$

Optimal. Leaf size=486

$$\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{(-a^2+b^2)^{9/4}\,de^{7/2}}-\frac{b^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{(-a^2+b^2)^{9/4}\,de^{7/2}}-\frac{2a(3a^2-8b^2)\sqrt{e\cos(c+dx)}\,E(}{5\,(a^2-b^2)^2\,de^4\sqrt{\cos(c+}}$$

```
[Out] b^(7/2)*arctan(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/(-a^2
+b^2)^(9/4)/d/e^(7/2)-b^(7/2)*arctanh(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^
2)^(1/4)/e^(1/2))/(-a^2+b^2)^(9/4)/d/e^(7/2)-2/5*(b-a*sin(d*x+c))/(a^2-b^2)
/d/e/(e*cos(d*x+c))^(5/2)+2/5*(5*b^3+a*(3*a^2-8*b^2)*sin(d*x+c))/(a^2-b^2)^
2/d/e^3/(e*cos(d*x+c))^(1/2)+a*b^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x
+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*cos
(d*x+c)^(1/2)/(a^2-b^2)^2/d/e^3/(b-(-a^2+b^2)^(1/2))/(e*cos(d*x+c))^(1/2)+a
*b^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x
+1/2*c),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(d*x+c)^(1/2)/(a^2-b^2)^2/d/e^
3/(b+(-a^2+b^2)^(1/2))/(e*cos(d*x+c))^(1/2)-2/5*a*(3*a^2-8*b^2)*(cos(1/2*d*
x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*
(e*cos(d*x+c))^(1/2)/(a^2-b^2)^2/d/e^4/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.86, antiderivative size = 486, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {2775, 2945, 2946, 2721, 2719, 2780, 2886, 2884, 335, 304, 211, 214}

$$\frac{b^{7/2}\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{de^{7/2}\,(b^2-a^2)^{9/4}}-\frac{2a(3a^2-8b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5de^4\,(a^2-b^2)^2\sqrt{\cos(c+dx)}}-\frac{2(b-a\sin(c+dx))}{5de\,(a^2-b^2)\,(e\cos(c+dx))^{5/2}}-\frac{b^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{de^{7/2}\,(b^2-a^2)^{9/4}}+\frac{2(a(3a^2-8b^2)\sin(c+dx)+5b^3)}{5de^3\,(a^2-b^2)^2\sqrt{e\cos(c+dx)}}+\frac{ab^3\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{de^3\,(a^2-b^2)^2\left(b-\sqrt{b^2-a^2}\right)\sqrt{e\cos(c+dx)}}+\frac{ab^3\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{de^3\,(a^2-b^2)^2\left(\sqrt{b^2-a^2}+b\right)\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(7/2)*(a + b*Sin[c + d*x])),x]
```

```
[Out] (b^(7/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])
])/((-a^2 + b^2)^(9/4)*d*e^(7/2)) - (b^(7/2)*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c
+ d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/((-a^2 + b^2)^(9/4)*d*e^(7/2)) - (2
*a*(3*a^2 - 8*b^2)*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*(a^2
- b^2)^2*d*e^4*Sqrt[Cos[c + d*x]]) + (a*b^3*Sqrt[Cos[c + d*x]]*EllipticPi[(
2*b)/(b - Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/((a^2 - b^2)^2*(b - Sqrt[-a^2
 + b^2])*d*e^3*Sqrt[e*Cos[c + d*x]]) + (a*b^3*Sqrt[Cos[c + d*x]]*EllipticPi
[(2*b)/(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/((a^2 - b^2)^2*(b + Sqrt[-a
^2 + b^2])*d*e^3*Sqrt[e*Cos[c + d*x]]) - (2*(b - a*Sin[c + d*x]))/(5*(a^2 -
 b^2)*d*e*(e*Cos[c + d*x])^(5/2)) + (2*(5*b^3 + a*(3*a^2 - 8*b^2)*Sin[c + d
*x]))/(5*(a^2 - b^2)^2*d*e^3*Sqrt[e*Cos[c + d*x]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2775

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^
(m + 1)*((b - a*Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rule 2780

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[a*(g/(2*b)), Int[1/(Sq
```

```
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[a*(g/(2*b)), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[b*(g/f), Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2945

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{1}{(e\cos(c+dx))^{7/2}(a+b\sin(c+dx))}\,dx = -\frac{2(b-a\sin(c+dx))}{5\left(a^2-b^2\right)de(e\cos(c+dx))^{5/2}} - \frac{2\int \frac{-\frac{3a^2}{2}+\frac{5b^2}{2}-\frac{3}{2}ab\sin(c+dx)}{(e\cos(c+dx))^{3/2}(a+b\sin(c+d}}{5\left(a^2-b^2\right)e^2}$$

$$= -\frac{2(b-a\sin(c+dx))}{5\left(a^2-b^2\right)de(e\cos(c+dx))^{5/2}} + \frac{2(5b^3+a(3a^2-8b^2)\sin(c}{5\left(a^2-b^2\right)^2 de^3\sqrt{e\cos(c}}$$

$$= -\frac{2(b-a\sin(c+dx))}{5\left(a^2-b^2\right)de(e\cos(c+dx))^{5/2}} + \frac{2(5b^3+a(3a^2-8b^2)\sin(c}{5\left(a^2-b^2\right)^2 de^3\sqrt{e\cos(c}}$$

$$= -\frac{2(b-a\sin(c+dx))}{5\left(a^2-b^2\right)de(e\cos(c+dx))^{5/2}} + \frac{2(5b^3+a(3a^2-8b^2)\sin(c}{5\left(a^2-b^2\right)^2 de^3\sqrt{e\cos(c}}$$

$$= -\frac{2a(3a^2-8b^2)\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5\left(a^2-b^2\right)^2 de^4\sqrt{\cos(c+dx)}} - \frac{2(b}{5\left(a^2-b^2\right.}$$

$$= -\frac{2a(3a^2-8b^2)\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5\left(a^2-b^2\right)^2 de^4\sqrt{\cos(c+dx)}} + \frac{ab^3\sqrt{\cos(}}{(a^2-b^2)}$$

$$= \frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{\left(-a^2+b^2\right)^{9/4}de^{7/2}} - \frac{b^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos}}{\sqrt[4]{-a^2+}}\right.}{\left(-a^2+b^2\right)^{9/4}d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 26.51, size = 881, normalized size = 1.81

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((e*Cos[c + d*x])^(7/2)*(a + b*Sin[c + d*x])),x]
```

```
[Out] -1/5*(Cos[c + d*x]^(7/2)*((-2*(3*a^4 - 8*a^2*b^2 - 5*b^4)*(a + b*Sqrt[1 - C
os[c + d*x]^2])*((a*AppellF1[3/4, 1/2, 1, 7/4, Cos[c + d*x]^2, (b^2*Cos[c +
 d*x]^2)/(-a^2 + b^2)]*Cos[c + d*x]^(3/2))/(3*(a^2 - b^2)) + ((1/8 + I/8)*(
2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*A
rcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sq
rt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*
b*Cos[c + d*x]] + Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)
```

```
*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)))*Sin
[c + d*x])/(Sqrt[1 - Cos[c + d*x]^2]*(a + b*Sin[c + d*x])) - ((3*a^3*b - 8*
a*b^3)*(a + b*Sqrt[1 - Cos[c + d*x]^2])*(8*b^(5/2)*AppellF1[3/4, -1/2, 1, 7
/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Cos[c + d*x]^(3/2) +
 3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c +
d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]
])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(
1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*S
qrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]]))*Sin[c + d*x
]^2)/(12*b^(3/2)*(-a^2 + b^2)*(1 - Cos[c + d*x]^2)*(a + b*Sin[c + d*x]))))/
((a - b)^2*(a + b)^2*d*(e*Cos[c + d*x])^(7/2)) + (Cos[c + d*x]^4*((2*Sec[c
+ d*x]^3*(-b + a*Sin[c + d*x]))/(5*(a^2 - b^2)) + (2*Sec[c + d*x]*(5*b^3 +
3*a^3*Sin[c + d*x] - 8*a*b^2*Sin[c + d*x]))/(5*(a^2 - b^2)^2)))/(d*(e*Cos[c
 + d*x])^(7/2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 22.54, size = 1510, normalized size = 3.11

method	result	size
default	Expression too large to display	1510

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(7/2)/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] (3/80/e^4*b/(a^2-b^2)/(cos(1/2*d*x+1/2*c)-1/2*2^(1/2))^2*(-2*sin(1/2*d*x+1/
2*c)^2*e+e)^(1/2)-3/80/e^4*b*2^(1/2)/(a^2-b^2)/(cos(1/2*d*x+1/2*c)-1/2*2^(1
/2))*(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)+1/2/e^4*b^3/(a^2-b^2)^2*2^(1/2)/(c
os(1/2*d*x+1/2*c)-1/2*2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)-1/2/e^4*
b^3/(a^2-b^2)^2*2^(1/2)/(cos(1/2*d*x+1/2*c)+1/2*2^(1/2))*(-2*sin(1/2*d*x+1/
2*c)^2*e+e)^(1/2)+3/80/e^4*b/(a^2-b^2)/(cos(1/2*d*x+1/2*c)+1/2*2^(1/2))^2*(
-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)+3/80/e^4*b*2^(1/2)/(a^2-b^2)/(cos(1/2*d*
x+1/2*c)+1/2*2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)-1/80/e^4*b/(a^2-b
^2)*2^(1/2)/(cos(1/2*d*x+1/2*c)-1/2*2^(1/2))^3*(-2*sin(1/2*d*x+1/2*c)^2*e+e
)^(1/2)+1/80/e^4*b/(a^2-b^2)*2^(1/2)/(cos(1/2*d*x+1/2*c)+1/2*2^(1/2))^3*(-2
*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)+1/2/e^3*b^5/(a-b)^2/(a+b)^2*sum((_R^6-_R^4
*e-_R^2*e^2+e^3)/(_R^7*b^2-3*_R^5*b^2*e+8*_R^3*a^2*e^2-5*_R^3*b^2*e^2-_R*b^
2*e^3)*ln((-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)-e^(1/2)*cos(1/2*d*x+1/2*c)*2^
(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*e*_Z^6+(16*a^2*e^2-10*b^2*e^2)*_Z^4-4*b^
2*e^3*_Z^2+b^2*e^4))-2*(e*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^
(1/2)*a/e^3*(-1/(a^2-b^2)^2*b^2*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4*e+sin(1/2*d*x+1/2*c)^2*e)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4*e+sin(1
/2*d*x+1/2*c)^2*e)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/e/sin(1/2
*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)-1/5/(a^2-b^2)/e/sin(1/2*d*x+1/2*c)
^2/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1
```

```
)*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6
*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin
(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-8*sin(1/2*d
*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4*e+sin(1/2*d*x+1/2*
c)^2*e)^(1/2)+1/16*b^2/(a-b)^2/(a+b)^2*sum(1/_alpha*(2^(1/2)/(e*(2*_alpha^2
*b^2+a^2-2*b^2)/b^2)^(1/2)*arctanh(1/2*e*(4*_alpha^2-3)/(4*a^2-3*b^2)*(4*co
s(1/2*d*x+1/2*c)^2*a^2-3*b^2*cos(1/2*d*x+1/2*c)^2+b^2*_alpha^2-3*a^2+2*b^2)
*2^(1/2)/(e*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-e*(2*sin(1/2*d*x+1/2*c)
^4-sin(1/2*d*x+1/2*c)^2))^(1/2))+8*b^2/a^2*_alpha*(_alpha^2-1)*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-sin(1/2*d*x+1/2*c)^2*e
*(2*sin(1/2*d*x+1/2*c)^2-1))^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-4*b^2/a^2
*(_alpha^2-1),2^(1/2))),_alpha=RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2)))/sin(1/2*
d*x+1/2*c)/(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(7/2)/(a+b*sin(d*x+c)),x, algorithm="maxima")
[Out] e^(-7/2)*integrate(1/((b*sin(d*x + c) + a)*cos(d*x + c)^(7/2)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(7/2)/(a+b*sin(d*x+c)),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(7/2)/(a+b*sin(d*x+c)),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 5992 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(7/2)/(a+b*sin(d*x+c)),x, algorithm="giac")
[Out] integrate(e^(-7/2)/((b*sin(d*x + c) + a)*cos(d*x + c)^(7/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(e\,\cos\left(c+d\,x\right)\right)^{7/2}\,\left(a+b\,\sin\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cos(c + d*x))^(7/2)*(a + b*sin(c + d*x))),x)
[Out] int(1/((e*cos(c + d*x))^(7/2)*(a + b*sin(c + d*x))), x)
```

3.584 $\int \frac{(e\cos(c+dx))^{11/2}}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=543

$$-\frac{9a(-a^2+b^2)^{5/4}e^{11/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2b^{11/2}d}-\frac{9a(-a^2+b^2)^{5/4}e^{11/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2b^{11/2}d}$$

```
[Out] -9/2*a*(-a^2+b^2)^(5/4)*e^(11/2)*arctan(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+
b^2)^(1/4)/e^(1/2))/b^(11/2)/d-9/2*a*(-a^2+b^2)^(5/4)*e^(11/2)*arctanh(b^(1
/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(11/2)/d+9/35*e^3*(e*c
os(d*x+c))^(5/2)*(7*a-5*b*sin(d*x+c))/b^3/d-e*(e*cos(d*x+c))^(9/2)/b/d/(a+b
*sin(d*x+c))-3/7*(21*a^4-28*a^2*b^2+5*b^4)*e^6*(cos(1/2*d*x+1/2*c)^2)^(1/2)
/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/
b^6/d/(e*cos(d*x+c))^(1/2)+9/2*a^2*(a^2-b^2)^2*e^6*(cos(1/2*d*x+1/2*c)^2)^(
1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(b-(-a^2+b^2)^(1/
2)),2^(1/2))*cos(d*x+c)^(1/2)/b^6/d/(a^2-b*(b-(-a^2+b^2)^(1/2)))/(e*cos(d*x
+c))^(1/2)+9/2*a^2*(a^2-b^2)^2*e^6*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x
+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos
(d*x+c)^(1/2)/b^6/d/(a^2-b*(b+(-a^2+b^2)^(1/2)))/(e*cos(d*x+c))^(1/2)-3/7*e
^5*(21*a*(a^2-b^2)-b*(7*a^2-5*b^2)*sin(d*x+c))*(e*cos(d*x+c))^(1/2)/b^5/d
```

Rubi [A]
time = 1.03, antiderivative size = 543, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {2772, 2944, 2946, 2721, 2720, 2781, 2886, 2884, 335, 218, 214, 211}

$$-\frac{9ae^{11/2}(b^2-a^2)^{5/4}\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2b^{11/2}d}-\frac{9ae^{11/2}(b^2-a^2)^{5/4}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2b^{11/2}d}+\frac{9a^2e^6(a^2-b^2)^2\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{2b^6d\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{e\cos(c+dx)}}+\frac{9a^2e^6(a^2-b^2)^2\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{2b^6d\left(a^2-b\left(\sqrt{b^2-a^2}+b\right)\right)\sqrt{e\cos(c+dx)}}-\frac{3e^5\sqrt{e\cos(c+dx)}\left(21a(a^2-b^2)-b(7a^2-5b^2)\sin(c+dx)\right)}{7b^5d}-\frac{3e^6(21a^4-28a^2b^2+5b^4)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7b^6d\sqrt{e\cos(c+dx)}}+\frac{9e^3(e\cos(c+dx))^{5/2}(7a-5b\sin(c+dx))}{35b^3d}-\frac{e(e\cos(c+dx))^{9/2}}{bd(a+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(11/2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-9*a*(-a^2 + b^2)^(5/4)*e^(11/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-
a^2 + b^2)^(1/4)*Sqrt[e])])/(2*b^(11/2)*d) - (9*a*(-a^2 + b^2)^(5/4)*e^(11/
2)*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(2
*b^(11/2)*d) - (3*(21*a^4 - 28*a^2*b^2 + 5*b^4)*e^6*Sqrt[Cos[c + d*x]]*Elli
pticF[(c + d*x)/2, 2])/(7*b^6*d*Sqrt[e*Cos[c + d*x]]) + (9*a^2*(a^2 - b^2)^
2*e^6*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c + d*x)
/2, 2])/(2*b^6*(a^2 - b*(b - Sqrt[-a^2 + b^2]))*d*Sqrt[e*Cos[c + d*x]]) + (
9*a^2*(a^2 - b^2)^2*e^6*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2
+ b^2]), (c + d*x)/2, 2])/(2*b^6*(a^2 - b*(b + Sqrt[-a^2 + b^2]))*d*Sqrt[e*
Cos[c + d*x]]) + (9*e^3*(e*Cos[c + d*x])^(5/2)*(7*a - 5*b*Sin[c + d*x]))/(3
5*b^3*d) - (e*(e*Cos[c + d*x])^(9/2))/(b*d*(a + b*Sin[c + d*x])) - (3*e^5*S
```

```
qrt[e*Cos[c + d*x]]*(21*a*(a^2 - b^2) - b*(7*a^2 - 5*b^2)*Sin[c + d*x]))/(7
*b^5*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2772

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Dist[g^2*((p - 1)/(b*(m + 1))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2781

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[-a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[b*(g/f), Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2944

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Dist[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{11/2}}{(a+b\sin(c+dx))^2}\,dx = -\frac{e(e\cos(c+dx))^{9/2}}{bd(a+b\sin(c+dx))} - \frac{(9e^2)\int \frac{(e\cos(c+dx))^{7/2}\sin(c+dx)}{a+b\sin(c+dx)}\,dx}{2b}$$

$$= \frac{9e^3(e\cos(c+dx))^{5/2}(7a-5b\sin(c+dx))}{35b^3d} - \frac{e(e\cos(c+dx))^{9/2}}{bd(a+b\sin(c+dx))} - \frac{(9e^4)\int \frac{(e\text{c}}{}}{}$$

$$= \frac{9e^3(e\cos(c+dx))^{5/2}(7a-5b\sin(c+dx))}{35b^3d} - \frac{e(e\cos(c+dx))^{9/2}}{bd(a+b\sin(c+dx))} - \frac{3e^5\sqrt{e\cos}}{}$$

$$= \frac{9e^3(e\cos(c+dx))^{5/2}(7a-5b\sin(c+dx))}{35b^3d} - \frac{e(e\cos(c+dx))^{9/2}}{bd(a+b\sin(c+dx))} - \frac{3e^5\sqrt{e\cos}}{}$$

$$= \frac{9e^3(e\cos(c+dx))^{5/2}(7a-5b\sin(c+dx))}{35b^3d} - \frac{e(e\cos(c+dx))^{9/2}}{bd(a+b\sin(c+dx))} - \frac{3e^5\sqrt{e\cos}}{}$$

$$= -\frac{3(21a^4-28a^2b^2+5b^4)\,e^6\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{7b^6d\sqrt{e\cos(c+dx)}} + \frac{9e^3(e\cos(c+dx)}{}$$

$$= -\frac{3(21a^4-28a^2b^2+5b^4)\,e^6\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{7b^6d\sqrt{e\cos(c+dx)}} + \frac{9a^2(-a^2+b^2)^{3/2}}{2b^6}$$

$$= -\frac{9a(-a^2+b^2)^{5/4}\,e^{11/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{2b^{11/2}d} - \frac{9a(-a^2+b^2)^{5/4}\,e^{11/2}\,\text{ta}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 57.49, size = 2030, normalized size = 3.74

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^(11/2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -1/70*((e*Cos[c + d*x])^(11/2)*((-2*(70*a^3*b - 93*a*b^3)*(a + b*Sqrt[1 - C
os[c + d*x]^2])*((5*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2
, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[c + d*x]])/(Sqrt[1 - Cos[c +
d*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[
c + d*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[c + d*x
```

```
]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1
, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)])*Cos[c + d*x]^2)*
(a^2 + b^2*(-1 + Cos[c + d*x]^2))) - ((1/8 - I/8)*Sqrt[b]*(2*ArcTan[1 - ((1
 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 +
I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] -
 (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]]
- Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*
x]] + I*b*Cos[c + d*x]]))/(-a^2 + b^2)^(3/4))*Sin[c + d*x])/(Sqrt[1 - Cos[c
 + d*x]^2]*(a + b*Sin[c + d*x])) + ((140*a^3*b - 147*a*b^3)*(a + b*Sqrt[1 -
 Cos[c + d*x]^2])*Cos[2*(c + d*x)]*(((1/2 - I/2)*(-2*a^2 + b^2)*ArcTan[1 -
((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)])/(b^(3/2)*(-a^2 +
b^2)^(3/4)) - ((1/2 - I/2)*(-2*a^2 + b^2)*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[
Cos[c + d*x]])/(-a^2 + b^2)^(1/4)])/(b^(3/2)*(-a^2 + b^2)^(3/4)) + (4*Sqrt[
Cos[c + d*x]])/b - (4*a*AppellF1[5/4, 1/2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos
[c + d*x]^2)/(-a^2 + b^2)]*Cos[c + d*x]^(5/2))/(5*(a^2 - b^2)) + (10*a*(a^2
 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a
^2 + b^2)]*Sqrt[Cos[c + d*x]])/(Sqrt[1 - Cos[c + d*x]^2]*(5*(a^2 - b^2)*App
ellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]
- 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/
(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[c + d*x]^2, (b^
2*Cos[c + d*x]^2)/(-a^2 + b^2)])*Cos[c + d*x]^2)*(a^2 + b^2*(-1 + Cos[c + d
*x]^2))) + ((1/4 - I/4)*(-2*a^2 + b^2)*Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[
b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]])/(b^(3/2)*(-a^
2 + b^2)^(3/4)) - ((1/4 - I/4)*(-2*a^2 + b^2)*Log[Sqrt[-a^2 + b^2] + (1 + I
)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]])/(b^(3/
2)*(-a^2 + b^2)^(3/4)))*Sin[c + d*x])/(Sqrt[1 - Cos[c + d*x]^2]*(-1 + 2*Cos
[c + d*x]^2)*(a + b*Sin[c + d*x])) - (2*(35*a^4 - 126*a^2*b^2 + 75*b^4)*(a
+ b*Sqrt[1 - Cos[c + d*x]^2])*((5*b*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4,
 Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[c + d*x]]*Sqrt
[1 - Cos[c + d*x]^2])/((-5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Cos[c +
d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + 2*(2*b^2*AppellF1[5/4, -1/2, 2
, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + (a^2 - b^2)*App
ellF1[5/4, 1/2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)])
*Cos[c + d*x]^2)*(a^2 + b^2*(-1 + Cos[c + d*x]^2))) + (a*(-2*ArcTan[1 - (Sq
rt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2
]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sq
rt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]] + Log[
Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*
Cos[c + d*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)))*Sin[c + d*x]^2)/((1
- Cos[c + d*x]^2)*(a + b*Sin[c + d*x]))))/(b^5*d*Cos[c + d*x]^(11/2)) + ((e
*Cos[c + d*x])^(11/2)*Sec[c + d*x]^5*((2*a*Cos[2*(c + d*x)])/(5*b^3) - ((-2
8*a^2 + 17*b^2)*Sin[c + d*x])/(14*b^4) - (-a^2 + b^2)^2/(b^5*(a + b*Sin[c +
 d*x])) - Sin[3*(c + d*x)]/(14*b^2)))/d
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.

time = 45.07, size = 15532, normalized size = 28.60

method	result	size
default	Expression too large to display	15532

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(11/2)/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(11/2)/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] e^(11/2)*integrate(cos(d*x + c)^(11/2)/(b*sin(d*x + c) + a)^2, x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(11/2)/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(11/2)/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(11/2)/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(11/2)*e^(11/2)/(b*sin(d*x + c) + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e \cos(c+dx))^{11/2}}{(a+b \sin(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(11/2)/(a + b*sin(c + d*x))^2,x)
```

```
[Out] int((e*cos(c + d*x))^(11/2)/(a + b*sin(c + d*x))^2, x)
```

3.585 $\int \frac{(e\cos(c+dx))^{9/2}}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=459

$$\frac{7a(-a^2+b^2)^{3/4}\,e^{9/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{2b^{9/2}d}-\frac{7a(-a^2+b^2)^{3/4}\,e^{9/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{2b^{9/2}d}+7$$

```
[Out] 7/2*a*(-a^2+b^2)^(3/4)*e^(9/2)*arctan(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^
2)^(1/4)/e^(1/2))/b^(9/2)/d-7/2*a*(-a^2+b^2)^(3/4)*e^(9/2)*arctanh(b^(1/2)*
(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(9/2)/d+7/15*e^3*(e*cos(d*
x+c))^(3/2)*(5*a-3*b*sin(d*x+c))/b^3/d-e*(e*cos(d*x+c))^(7/2)/b/d/(a+b*sin(
d*x+c))-7/2*a^2*(a^2-b^2)*e^5*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*
c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*cos(d*x+
c)^(1/2)/b^5/d/(b-(-a^2+b^2)^(1/2))/(e*cos(d*x+c))^(1/2)-7/2*a^2*(a^2-b^2)*
e^5*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+
1/2*c),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(d*x+c)^(1/2)/b^5/d/(b+(-a^2+b^
2)^(1/2))/(e*cos(d*x+c))^(1/2)+7/5*(5*a^2-3*b^2)*e^4*(cos(1/2*d*x+1/2*c)^2)
^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+
c))^(1/2)/b^4/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.73, antiderivative size = 459, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {2772, 2944, 2946, 2721, 2719, 2780, 2886, 2884, 335, 304, 211, 214}

$$\frac{7ae^{9/2}(b^2-a^2)^{3/4}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{2b^{9/2}d}-\frac{7ae^{9/2}(b^2-a^2)^{3/4}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{2b^{9/2}d}-\frac{7a^2e^5(a^2-b^2)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{2b^5d\left(b-\sqrt{b^2-a^2}\right)\sqrt{e\cos(c+dx)}}-\frac{7a^2e^5(a^2-b^2)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{2b^5d\left(\sqrt{b^2-a^2}+b\right)\sqrt{e\cos(c+dx)}}+\frac{7e^4(5a^2-3b^2)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5b^4d\sqrt{\cos(c+dx)}}+\frac{7e^3(e\cos(c+dx))^{3/2}(5a-3b\sin(c+dx))}{15b^3d}-\frac{e(e\cos(c+dx))^{7/2}}{bd(a+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(9/2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (7*a*(-a^2 + b^2)^(3/4)*e^(9/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^
2 + b^2)^(1/4)*Sqrt[e])])/(2*b^(9/2)*d) - (7*a*(-a^2 + b^2)^(3/4)*e^(9/2)*A
rcTanh[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(2*b^(
9/2)*d) + (7*(5*a^2 - 3*b^2)*e^4*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2
, 2])/(5*b^4*d*Sqrt[Cos[c + d*x]]) - (7*a^2*(a^2 - b^2)*e^5*Sqrt[Cos[c + d*
x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(2*b^5*(b - S
qrt[-a^2 + b^2])*d*Sqrt[e*Cos[c + d*x]]) - (7*a^2*(a^2 - b^2)*e^5*Sqrt[Cos[
c + d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(2*b^5*
(b + Sqrt[-a^2 + b^2])*d*Sqrt[e*Cos[c + d*x]]) + (7*e^3*(e*Cos[c + d*x])^(3
/2)*(5*a - 3*b*Sin[c + d*x]))/(15*b^3*d) - (e*(e*Cos[c + d*x])^(7/2))/(b*d*
(a + b*Sin[c + d*x]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2772

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Dist[g^2*((p - 1)/(b*(m + 1))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2780

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[a*(g/(2*b)), Int[1/(Sq
```

```
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[a*(g/(2*b)), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[b*(g/f), Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2944

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Dist[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{9/2}}{(a+b\sin(c+dx))^2}\,dx = -\frac{e(e\cos(c+dx))^{7/2}}{bd(a+b\sin(c+dx))} - \frac{(7e^2)\int \frac{(e\cos(c+dx))^{5/2}\sin(c+dx)}{a+b\sin(c+dx)}\,dx}{2b}$$

$$= \frac{7e^3(e\cos(c+dx))^{3/2}(5a-3b\sin(c+dx))}{15b^3d} - \frac{e(e\cos(c+dx))^{7/2}}{bd(a+b\sin(c+dx))} - \frac{(7e^4)\int \cdots}{\cdots}$$

$$= \frac{7e^3(e\cos(c+dx))^{3/2}(5a-3b\sin(c+dx))}{15b^3d} - \frac{e(e\cos(c+dx))^{7/2}}{bd(a+b\sin(c+dx))} + \frac{(7(5a^2-\cdots}{\cdots}$$

$$= \frac{7e^3(e\cos(c+dx))^{3/2}(5a-3b\sin(c+dx))}{15b^3d} - \frac{e(e\cos(c+dx))^{7/2}}{bd(a+b\sin(c+dx))} + \frac{(7a^2(a^2-\cdots}{\cdots}$$

$$= \frac{7(5a^2-3b^2)\,e^4\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^4d\sqrt{\cos(c+dx)}} + \frac{7e^3(e\cos(c+dx))^{3/2}(5a-\cdots}{15b^3d}$$

$$= \frac{7(5a^2-3b^2)\,e^4\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^4d\sqrt{\cos(c+dx)}} - \frac{7a^2(a^2-b^2)\,e^5\sqrt{\cos(c+d\cdots}}{2b^5\left(b-\sqrt{-a^2\cdots}\right.}$$

$$= \frac{7a(-a^2+b^2)^{3/4}\,e^{9/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{2b^{9/2}d} - \frac{7a(-a^2+b^2)^{3/4}\,e^{9/2}\tan\cdots}{2\cdots}$$

<u>Mathematica</u> [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

time = 46.42, size = 835, normalized size = 1.82

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^(9/2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (7*(e*Cos[c + d*x])^(9/2)*((-4*a*b*(a + b*Sqrt[1 - Cos[c + d*x]^2])*((a*App
ellF1[3/4, 1/2, 1, 7/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*
Cos[c + d*x]^(3/2))/(3*(a^2 - b^2)) + ((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*S
qrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt
[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - (1 + I
)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]] + Log[S
qrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I
```

```
*b*Cos[c + d*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)))*Sin[c + d*x])/(Sqrt[1 - Co
s[c + d*x]^2]*(a + b*Sin[c + d*x])) - ((5*a^2 - 3*b^2)*(a + b*Sqrt[1 - Cos[
c + d*x]^2])*(8*b^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Cos[c + d*x]^2, (b^2*Co
s[c + d*x]^2)/(-a^2 + b^2)]*Cos[c + d*x]^(3/2) + 3*Sqrt[2]*a*(a^2 - b^2)^(3
/4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] -
 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] - Log
[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b
*Cos[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sq
rt[Cos[c + d*x]] + b*Cos[c + d*x]]))*Sin[c + d*x]^2)/(12*b^(3/2)*(-a^2 + b^
2)*(1 - Cos[c + d*x]^2)*(a + b*Sin[c + d*x]))))/(10*b^3*d*Cos[c + d*x]^(9/2
)) + ((e*Cos[c + d*x])^(9/2)*Sec[c + d*x]^4*((4*a*Cos[c + d*x])/(3*b^3) + (
a^2*Cos[c + d*x] - b^2*Cos[c + d*x])/(b^3*(a + b*Sin[c + d*x])) - Sin[2*(c
+ d*x)]/(5*b^2)))/d
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 34.53, size = 19163, normalized size = 41.75

method	result	size
default	Expression too large to display	19163

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(9/2)/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(9/2)/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] e^(9/2)*integrate(cos(d*x + c)^(9/2)/(b*sin(d*x + c) + a)^2, x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(9/2)/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(9/2)/(a+b*sin(d*x+c))**2,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 8857 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(9/2)/(a+b*sin(d*x+c))^2,x, algorithm="giac")
[Out] integrate(cos(d*x + c)^(9/2)*e^(9/2)/(b*sin(d*x + c) + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,\cos\left(c+d\,x\right))^{9/2}}{{\left(a+b\,\sin\left(c+d\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(9/2)/(a + b*sin(c + d*x))^2,x)
[Out] int((e*cos(c + d*x))^(9/2)/(a + b*sin(c + d*x))^2, x)
```

3.586 $\int \frac{(e\cos(c+dx))^{7/2}}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=473

$$-\frac{5a\sqrt[4]{-a^2+b^2}\,e^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{2b^{7/2}d}-\frac{5a\sqrt[4]{-a^2+b^2}\,e^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{2b^{7/2}d}+\frac{5(3$$

```
[Out] -5/2*a*(-a^2+b^2)^(1/4)*e^(7/2)*arctan(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b
^2)^(1/4)/e^(1/2))/b^(7/2)/d-5/2*a*(-a^2+b^2)^(1/4)*e^(7/2)*arctanh(b^(1/2)
*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(7/2)/d-e*(e*cos(d*x+c))^
(5/2)/b/d/(a+b*sin(d*x+c))+5/3*(3*a^2-b^2)*e^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)
/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/
b^4/d/(e*cos(d*x+c))^(1/2)-5/2*a^2*(a^2-b^2)*e^4*(cos(1/2*d*x+1/2*c)^2)^(1/
2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(b-(-a^2+b^2)^(1/2)
),2^(1/2))*cos(d*x+c)^(1/2)/b^4/d/(a^2-b*(b-(-a^2+b^2)^(1/2)))/(e*cos(d*x+c
))^(1/2)-5/2*a^2*(a^2-b^2)*e^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2
*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(d*x
+c)^(1/2)/b^4/d/(a^2-b*(b+(-a^2+b^2)^(1/2)))/(e*cos(d*x+c))^(1/2)+5/3*e^3*(
3*a-b*sin(d*x+c))*(e*cos(d*x+c))^(1/2)/b^3/d
```

Rubi [A]
time = 0.74, antiderivative size = 473, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {2772, 2944, 2946, 2721, 2720, 2781, 2886, 2884, 335, 218, 214, 211}

$$-\frac{5ae^{7/2}\sqrt[4]{b^2-a^2}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{2b^{7/2}d}-\frac{5ae^{7/2}\sqrt[4]{b^2-a^2}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{2b^{7/2}d}+\frac{5e^4(3a^2-b^2)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^4d\sqrt{e\cos(c+dx)}}-\frac{5a^2e^4(a^2-b^2)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{2b^4d\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{e\cos(c+dx)}}-\frac{5a^2e^4(a^2-b^2)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{2b^4d\left(a^2-b\left(\sqrt{b^2-a^2}+b\right)\right)\sqrt{e\cos(c+dx)}}+\frac{5e^3\sqrt{e\cos(c+dx)}\,(3a-b\sin(c+dx))}{3b^3d}-\frac{e(e\cos(c+dx))^{5/2}}{bd(a+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(7/2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-5*a*(-a^2 + b^2)^(1/4)*e^(7/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a
^2 + b^2)^(1/4)*Sqrt[e])])/(2*b^(7/2)*d) - (5*a*(-a^2 + b^2)^(1/4)*e^(7/2)*
ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(2*b^
(7/2)*d) + (5*(3*a^2 - b^2)*e^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2
])/(3*b^4*d*Sqrt[e*Cos[c + d*x]]) - (5*a^2*(a^2 - b^2)*e^4*Sqrt[Cos[c + d*x
]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(2*b^4*(a^2 -
b*(b - Sqrt[-a^2 + b^2]))*d*Sqrt[e*Cos[c + d*x]]) - (5*a^2*(a^2 - b^2)*e^4*
Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2]
)/(2*b^4*(a^2 - b*(b + Sqrt[-a^2 + b^2]))*d*Sqrt[e*Cos[c + d*x]]) + (5*e^3*
Sqrt[e*Cos[c + d*x]]*(3*a - b*Sin[c + d*x]))/(3*b^3*d) - (e*(e*Cos[c + d*x]
)^(5/2))/(b*d*(a + b*Sin[c + d*x]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2772

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Dist[g^2*((p - 1)/(b*(m + 1))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2781

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[-a/(2*q), Int[1/(S
```

```
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[b*(g/f), Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2944

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Dist[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{7/2}}{(a+b\sin(c+dx))^2}\,dx = -\frac{e(e\cos(c+dx))^{5/2}}{bd(a+b\sin(c+dx))} - \frac{(5e^2)\int \frac{(e\cos(c+dx))^{3/2}\sin(c+dx)}{a+b\sin(c+dx)}\,dx}{2b}$$

$$= \frac{5e^3\sqrt{e\cos(c+dx)}\,(3a-b\sin(c+dx))}{3b^3d} - \frac{e(e\cos(c+dx))^{5/2}}{bd(a+b\sin(c+dx))} - \frac{(5e^4)\int \frac{\quad}{\sqrt{e}\cdots}}{\cdots}$$

$$= \frac{5e^3\sqrt{e\cos(c+dx)}\,(3a-b\sin(c+dx))}{3b^3d} - \frac{e(e\cos(c+dx))^{5/2}}{bd(a+b\sin(c+dx))} - \frac{(5a(a^2-b^2\cdots}{\cdots}$$

$$= \frac{5e^3\sqrt{e\cos(c+dx)}\,(3a-b\sin(c+dx))}{3b^3d} - \frac{e(e\cos(c+dx))^{5/2}}{bd(a+b\sin(c+dx))} - \frac{\left(5a^2\sqrt{-a^2\cdots}\right.}{\cdots}$$

$$= \frac{5(3a^2-b^2)\,e^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{3b^4d\sqrt{e\cos(c+dx)}} + \frac{5e^3\sqrt{e\cos(c+dx)}\,(3a-b\sin\cdots}{3b^3d}$$

$$= \frac{5(3a^2-b^2)\,e^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{3b^4d\sqrt{e\cos(c+dx)}} + \frac{5a^2\sqrt{-a^2+b^2}\,e^4\sqrt{\cos(c+d\cdots}}{2b^4\left(b-\sqrt{-a^2+\cdots}\right.}$$

$$= -\frac{5a\sqrt[4]{-a^2+b^2}\,e^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{2b^{7/2}d} - \frac{5a\sqrt[4]{-a^2+b^2}\,e^{7/2}\tanh^{-\cdots}}{2b\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 56.09, size = 1956, normalized size = 4.14

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^(7/2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((e*Cos[c + d*x])^(7/2)*Sec[c + d*x]^3*((-2*Sin[c + d*x])/(3*b^2) + (a^2 -
b^2)/(b^3*(a + b*Sin[c + d*x]))))/d + ((e*Cos[c + d*x])^(7/2)*((-8*a*b*(a +
 b*Sqrt[1 - Cos[c + d*x]^2])*((5*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, C
os[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[c + d*x]])/(Sqrt
[1 - Cos[c + d*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]
^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4
, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF
1[5/4, 3/2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)])*Cos
```

```
[c + d*x]^2)*(a^2 + b^2*(-1 + Cos[c + d*x]^2))) - ((1/8 - I/8)*Sqrt[b]*(2*A
rcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcT
an[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[
-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*C
os[c + d*x]] - Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sq
rt[Cos[c + d*x]] + I*b*Cos[c + d*x]]))/(-a^2 + b^2)^(3/4))*Sin[c + d*x])/(S
qrt[1 - Cos[c + d*x]^2]*(a + b*Sin[c + d*x])) + (6*a*b*(a + b*Sqrt[1 - Cos[
c + d*x]^2])*Cos[2*(c + d*x)]*(((1/2 - I/2)*(-2*a^2 + b^2)*ArcTan[1 - ((1 +
 I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)])/(b^(3/2)*(-a^2 + b^2)^
(3/4)) - ((1/2 - I/2)*(-2*a^2 + b^2)*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[c
 + d*x]])/(-a^2 + b^2)^(1/4)])/(b^(3/2)*(-a^2 + b^2)^(3/4)) + (4*Sqrt[Cos[c
 + d*x]])/b - (4*a*AppellF1[5/4, 1/2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos[c +
d*x]^2)/(-a^2 + b^2)]*Cos[c + d*x]^(5/2))/(5*(a^2 - b^2)) + (10*a*(a^2 - b^
2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 +
b^2)]*Sqrt[Cos[c + d*x]])/(Sqrt[1 - Cos[c + d*x]^2]*(5*(a^2 - b^2)*AppellF1
[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] - 2*(
2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2
 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos
[c + d*x]^2)/(-a^2 + b^2)])*Cos[c + d*x]^2)*(a^2 + b^2*(-1 + Cos[c + d*x]^2
))) + ((1/4 - I/4)*(-2*a^2 + b^2)*Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-
a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]])/(b^(3/2)*(-a^2 + b
^2)^(3/4)) - ((1/4 - I/4)*(-2*a^2 + b^2)*Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqr
t[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]])/(b^(3/2)*(-
a^2 + b^2)^(3/4)))*Sin[c + d*x])/(Sqrt[1 - Cos[c + d*x]^2]*(-1 + 2*Cos[c +
d*x]^2)*(a + b*Sin[c + d*x])) - (2*(3*a^2 - 5*b^2)*(a + b*Sqrt[1 - Cos[c +
d*x]^2])*((5*b*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Cos[c + d*x]^2, (b^2
*Cos[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[c + d*x]]*Sqrt[1 - Cos[c + d*x]^2])
/((-5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c +
d*x]^2)/(-a^2 + b^2)] + 2*(2*b^2*AppellF1[5/4, -1/2, 2, 9/4, Cos[c + d*x]^2
, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + (a^2 - b^2)*AppellF1[5/4, 1/2, 1, 9/
4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)])*Cos[c + d*x]^2)*(a^2
 + b^2*(-1 + Cos[c + d*x]^2))) + (a*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Co
s[c + d*x]])/(a^2 - b^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c
+ d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 -
b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqr
t[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]]))/(4*Sq
rt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)))*Sin[c + d*x]^2)/((1 - Cos[c + d*x]^2)*(a
+ b*Sin[c + d*x]))))/(6*b^3*d*Cos[c + d*x]^(7/2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 35.16, size = 11649, normalized size = 24.63

method	result	size
default	Expression too large to display	11649

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(7/2)/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] e^(7/2)*integrate(cos(d*x + c)^(7/2)/(b*sin(d*x + c) + a)^2, x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(7/2)/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3878 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(7/2)*e^(7/2)/(b*sin(d*x + c) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,\cos(c+d\,x))^{7/2}}{(a+b\,\sin(c+d\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(7/2)/(a + b*sin(c + d*x))^2,x)
```

```
[Out] int((e*cos(c + d*x))^(7/2)/(a + b*sin(c + d*x))^2, x)
```

3.587 $\int \frac{(e\cos(c+dx))^{5/2}}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=390

$$\frac{3ae^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2b^{5/2}\sqrt[4]{-a^2+b^2}\,d}-\frac{3ae^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2b^{5/2}\sqrt[4]{-a^2+b^2}\,d}-\frac{3e^2\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+\right.}{b^2d\sqrt{\cos(c+dx)}}$$

```
[Out] 3/2*a*e^(5/2)*arctan(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))
/b^(5/2)/(-a^2+b^2)^(1/4)/d-3/2*a*e^(5/2)*arctanh(b^(1/2)*(e*cos(d*x+c))^(1
/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(5/2)/(-a^2+b^2)^(1/4)/d-e*(e*cos(d*x+c))^(
3/2)/b/d/(a+b*sin(d*x+c))+3/2*a^2*e^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*
d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*
cos(d*x+c)^(1/2)/b^3/d/(b-(-a^2+b^2)^(1/2))/(e*cos(d*x+c))^(1/2)+3/2*a^2*e^
3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/
2*c),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(d*x+c)^(1/2)/b^3/d/(b+(-a^2+b^2)
^(1/2))/(e*cos(d*x+c))^(1/2)-3*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x
+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/b^2/d/co
s(d*x+c)^(1/2)
```

Rubi [A]

time = 0.54, antiderivative size = 390, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {2772, 2946, 2721, 2719, 2780, 2886, 2884, 335, 304, 211, 214}

$$\frac{3ae^{5/2}\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2b^{5/2}d\sqrt[4]{b^2-a^2}}-\frac{3ae^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2b^{5/2}d\sqrt[4]{b^2-a^2}}+\frac{3a^2e^3\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{2b^3d\left(b-\sqrt{b^2-a^2}\right)\sqrt{e\cos(c+dx)}}+\frac{3a^2e^3\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{2b^3d\left(\sqrt{b^2-a^2}+b\right)\sqrt{e\cos(c+dx)}}-\frac{e(e\cos(c+dx))^{3/2}}{bd(a+b\sin(c+dx))}-\frac{3e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(5/2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (3*a*e^(5/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt
[e])])/(2*b^(5/2)*(-a^2 + b^2)^(1/4)*d) - (3*a*e^(5/2)*ArcTanh[(Sqrt[b]*Sqr
t[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(2*b^(5/2)*(-a^2 + b^2)^(
1/4)*d) - (3*e^2*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqr
t[Cos[c + d*x]]) + (3*a^2*e^3*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt
[-a^2 + b^2]), (c + d*x)/2, 2])/(2*b^3*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e*Cos[
c + d*x]]) + (3*a^2*e^3*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2
+ b^2]), (c + d*x)/2, 2])/(2*b^3*(b + Sqrt[-a^2 + b^2])*d*Sqrt[e*Cos[c + d*
x]]) - (e*(e*Cos[c + d*x])^(3/2))/(b*d*(a + b*Sin[c + d*x]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2772

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Dist[g^2*((p - 1)/(b*(m + 1))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2780

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[a*(g/(2*b)), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[a*(g/(2*b)), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[b*(g/f), Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
```

```
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{5/2}}{(a+b\sin(c+dx))^2}\,dx = -\frac{e(e\cos(c+dx))^{3/2}}{bd(a+b\sin(c+dx))} - \frac{(3e^2)\int \frac{\sqrt{e\cos(c+dx)}\sin(c+dx)}{a+b\sin(c+dx)}\,dx}{2b}$$

$$= -\frac{e(e\cos(c+dx))^{3/2}}{bd(a+b\sin(c+dx))} - \frac{(3e^2)\int \sqrt{e\cos(c+dx)}\,dx}{2b^2} + \frac{(3ae^2)\int \frac{\sqrt{e\cos(c+a}}{a+b\sin(c+dx}}{2b^2}$$

$$= -\frac{e(e\cos(c+dx))^{3/2}}{bd(a+b\sin(c+dx))} - \frac{(3a^2e^3)\int \frac{1}{\sqrt{e\cos(c+dx)}\left(\sqrt{-a^2+b^2}-b\cos(c+dx)\right)}\,d}{4b^3}$$

$$= -\frac{3e^2\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{\cos(c+dx)}} - \frac{e(e\cos(c+dx))^{3/2}}{bd(a+b\sin(c+dx))} + \frac{(3ae^3)\,\mathrm{Subst}}{}$$

$$= -\frac{3e^2\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{\cos(c+dx)}} + \frac{3a^2e^3\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{-a^2+b^2}}\right.}{2b^3\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\cos}}$$

$$= \frac{3ae^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2b^{5/2}\sqrt[4]{-a^2+b^2}\,d} - \frac{3ae^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2b^{5/2}\sqrt[4]{-a^2+b^2}\,d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 86.90, size = 371, normalized size = 0.95

$$\frac{(e\cos(c+dx))^{5/2}\left(-8b^{3/2}\cos^{\frac{3}{2}}(c+dx) - \frac{\left(8b^{5/2}F_1\left(\frac{3}{4};-\frac{1}{2},1;\frac{7}{4};\cos^2(c+dx),\frac{b^2\cos^2(c+dx)}{-a^2+b^2}\right)\cos^{\frac{3}{2}}(c+dx)+3\sqrt{2}\,a(a^2-b^2)^{3/4}\left(2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{\cos(c+dx)}}{\sqrt[4]{a^2-b^2}}\right)-2\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{b}\sqrt{\cos(c+dx)}}{\sqrt[4]{a^2-b^2}}\right)-\log\left(\sqrt{a^2-b^2}-\sqrt{2}\sqrt{b}\sqrt[4]{a^2-b^2}\sqrt{\cos(c+dx)}+b\cos(c+dx)\right)+\log\left(\sqrt{a^2-b^2}+\sqrt{2}\sqrt{b}\sqrt[4]{a^2-b^2}\sqrt{\cos(c+dx)}+b\cos(c+dx)\right)\right)\right)\left(a+b\sqrt{\sin^2(c+dx)}\right)}{a^2-b^2}\right)}{8b^{5/2}d\cos^{\frac{5}{2}}(c+dx)(a+b\sin(c+dx))}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^(5/2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((e*Cos[c + d*x])^(5/2)*(-8*b^(3/2)*Cos[c + d*x]^(3/2) - ((8*b^(5/2)*Appell
F1[3/4, -1/2, 1, 7/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Co
s[c + d*x]^(3/2) + 3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sqr
t[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]
*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqr
t[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]] + Log[Sqrt[a^2
- b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d
*x]]))*(a + b*Sqrt[Sin[c + d*x]^2]))/(a^2 - b^2)))/(8*b^(5/2)*d*Cos[c + d*x
]^(5/2)*(a + b*Sin[c + d*x]))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 23.79, size = 12860, normalized size = 32.97

method	result	size
default	Expression too large to display	12860

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(5/2)/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] e^(5/2)*integrate(cos(d*x + c)^(5/2)/(b*sin(d*x + c) + a)^2, x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(5/2)/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(5/2)*e^(5/2)/(b*sin(d*x + c) + a)^2, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\cos(c+dx))^{5/2}}{(a+b\sin(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(5/2)/(a + b*sin(c + d*x))^2,x)
```

```
[Out] int((e*cos(c + d*x))^(5/2)/(a + b*sin(c + d*x))^2, x)
```

3.588 $\int \frac{(e\cos(c+dx))^{3/2}}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=404

$$-\frac{ae^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2b^{3/2}\left(-a^2+b^2\right)^{3/4}d}-\frac{ae^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2b^{3/2}\left(-a^2+b^2\right)^{3/4}d}-\frac{e^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx\right.}{b^2d\sqrt{e\cos(c+dx)}}$$

```
[Out] -1/2*a*e^(3/2)*arctan(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2)
)/b^(3/2)/(-a^2+b^2)^(3/4)/d-1/2*a*e^(3/2)*arctanh(b^(1/2)*(e*cos(d*x+c))^(
1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(3/2)/(-a^2+b^2)^(3/4)/d-e^2*(cos(1/2*d*x+
1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*co
s(d*x+c)^(1/2)/b^2/d/(e*cos(d*x+c))^(1/2)+1/2*a^2*e^2*(cos(1/2*d*x+1/2*c)^2
)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(b-(-a^2+b^2)^
(1/2)),2^(1/2))*cos(d*x+c)^(1/2)/b^2/d/(a^2-b*(b-(-a^2+b^2)^(1/2)))/(e*cos(
d*x+c))^(1/2)+1/2*a^2*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*E
llipticPi(sin(1/2*d*x+1/2*c),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(d*x+c)^(
1/2)/b^2/d/(a^2-b*(b+(-a^2+b^2)^(1/2)))/(e*cos(d*x+c))^(1/2)-e*(e*cos(d*x+c
))^(1/2)/b/d/(a+b*sin(d*x+c))
```

Rubi [A]
time = 0.60, antiderivative size = 404, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {2772, 2946, 2721, 2720, 2781, 2886, 2884, 335, 218, 214, 211}

$$-\frac{ae^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}d\left(b^2-a^2\right)^{3/4}}+\frac{a^2e^2\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{2b^2d\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{e\cos(c+dx)}}+\frac{a^2e^2\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{2b^2d\left(a^2-b\left(\sqrt{b^2-a^2}+b\right)\right)\sqrt{e\cos(c+dx)}}-\frac{ae^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}d\left(b^2-a^2\right)^{3/4}}-\frac{e\sqrt{e\cos(c+dx)}}{bd(a+b\sin(c+dx))}-\frac{e^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(3/2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -1/2*(a*e^(3/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*S
qrt[e])])/(b^(3/2)*(-a^2 + b^2)^(3/4)*d) - (a*e^(3/2)*ArcTanh[(Sqrt[b]*Sqrt
[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(2*b^(3/2)*(-a^2 + b^2)^(3
/4)*d) - (e^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(b^2*d*Sqrt[e*C
os[c + d*x]]) + (a^2*e^2*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2
 + b^2]), (c + d*x)/2, 2])/(2*b^2*(a^2 - b*(b - Sqrt[-a^2 + b^2]))*d*Sqrt[e
*Cos[c + d*x]]) + (a^2*e^2*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a
^2 + b^2]), (c + d*x)/2, 2])/(2*b^2*(a^2 - b*(b + Sqrt[-a^2 + b^2]))*d*Sqrt
[e*Cos[c + d*x]]) - (e*Sqrt[e*Cos[c + d*x]])/(b*d*(a + b*Sin[c + d*x]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2772

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Dist[g^2*((p - 1)/(b*(m + 1))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2781

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[-a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[b*(g/f), Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
```

```
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{3/2}}{(a+b\sin(c+dx))^2}\,dx = -\frac{e\sqrt{e\cos(c+dx)}}{bd(a+b\sin(c+dx))} - \frac{e^2\int \frac{\sin(c+dx)}{\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))}\,dx}{2b}$$

$$= -\frac{e\sqrt{e\cos(c+dx)}}{bd(a+b\sin(c+dx))} - \frac{e^2\int \frac{1}{\sqrt{e\cos(c+dx)}}\,dx}{2b^2} + \frac{(ae^2)\int \frac{1}{\sqrt{e\cos(c+dx)}\,(}}{2b^2}$$

$$= -\frac{e\sqrt{e\cos(c+dx)}}{bd(a+b\sin(c+dx))} - \frac{(a^2e^2)\int \frac{1}{\sqrt{e\cos(c+dx)}\left(\sqrt{-a^2+b^2}-b\cos(c+dx)\right)}\,dx}{4b^2\sqrt{-a^2+b^2}}$$

$$= -\frac{e^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{e\cos(c+dx)}} - \frac{e\sqrt{e\cos(c+dx)}}{bd(a+b\sin(c+dx))} + \frac{(ae^3)\,\mathrm{Subst}\left(\int\right.}{}$$

$$= -\frac{e^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{e\cos(c+dx)}} + \frac{a^2e^2\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{-a^2+b^2}};\frac{1}{2}\right.}{2b^2\left(a^2-b\left(b-\sqrt{-a^2+b^2}\right)\right)d\sqrt{e\,c}}$$

$$= -\frac{ae^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{2b^{3/2}\left(-a^2+b^2\right)^{3/4}d} - \frac{ae^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{2b^{3/2}\left(-a^2+b^2\right)^{3/4}d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 23.29, size = 614, normalized size = 1.52

$$-\frac{(e\cos(c+dx))^{3/2}\sec(c+dx)}{bd(a+b\sin(c+dx))} + \frac{(e\cos(c+dx))^{3/2}\left(a+b\sqrt{1-\cos^2(c+dx)}\right)\left(\frac{5b(a^2-b^2)F_1\left(\frac14;-\frac12,1;\frac54;\cos^2(c+dx),\frac{b^2\cos^2(c+dx)}{-a^2+b^2}\right)\sqrt{\cos(c+dx)}\sqrt{1-\cos^2(c+dx)}}{\left(-5(a^2-b^2)F_1\left(\frac14;-\frac12,1;\frac54;\cos^2(c+dx),\frac{b^2\cos^2(c+dx)}{-a^2+b^2}\right)+2\left(2b^2F_1\left(\frac54;-\frac12,2;\frac94;\cos^2(c+dx),\frac{b^2\cos^2(c+dx)}{-a^2+b^2}\right)+(a^2-b^2)F_1\left(\frac54;\frac12,1;\frac94;\cos^2(c+dx),\frac{b^2\cos^2(c+dx)}{-a^2+b^2}\right)\right)\cos^2(c+dx)\right)(a^2+b^2(-1+\cos^2(c+dx)))} + \frac{a\left(-2\tan^{-1}\left(1-\frac{\sqrt2\sqrt{b}\sqrt{\cos(c+dx)}}{\sqrt[4]{a^2-b^2}}\right)+2\tan^{-1}\left(1+\frac{\sqrt2\sqrt{b}\sqrt{\cos(c+dx)}}{\sqrt[4]{a^2-b^2}}\right)-\log\left(\sqrt{a^2-b^2}-\sqrt2\sqrt{b}\sqrt[4]{a^2-b^2}\sqrt{\cos(c+dx)}+b\cos(c+dx)\right)+\log\left(\sqrt{a^2-b^2}+\sqrt2\sqrt{b}\sqrt[4]{a^2-b^2}\sqrt{\cos(c+dx)}+b\cos(c+dx)\right)\right)}{4\sqrt2\sqrt{b}(a^2-b^2)^{3/4}}\right)\sin^2(c+dx)}{bd\cos^{\frac32}(c+dx)(1-\cos^2(c+dx))(a+b\sin(c+dx))}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^(3/2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -(((e*Cos[c + d*x])^(3/2)*Sec[c + d*x])/(b*d*(a + b*Sin[c + d*x]))) + ((e*C
os[c + d*x])^(3/2)*(a + b*Sqrt[1 - Cos[c + d*x]^2])*((5*b*(a^2 - b^2)*Appel
lF1[1/4, -1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*S
qrt[Cos[c + d*x]]*Sqrt[1 - Cos[c + d*x]^2])/((-5*(a^2 - b^2)*AppellF1[1/4,
-1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + 2*(2*b^2
*AppellF1[5/4, -1/2, 2, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b
^2)] + (a^2 - b^2)*AppellF1[5/4, 1/2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos[c +
d*x]^2)/(-a^2 + b^2)])*Cos[c + d*x]^2)*(a^2 + b^2*(-1 + Cos[c + d*x]^2))) +
 (a*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)]
+ 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] - Lo
g[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] +
```

```
b*Cos[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*S
qrt[Cos[c + d*x]] + b*Cos[c + d*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4))
)*Sin[c + d*x]^2)/(b*d*Cos[c + d*x]^(3/2)*(1 - Cos[c + d*x]^2)*(a + b*Sin[c
 + d*x]))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 24.52, size = 7936, normalized size = 19.64

method	result	size
default	Expression too large to display	7936

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] e^(3/2)*integrate(cos(d*x + c)^(3/2)/(b*sin(d*x + c) + a)^2, x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(3/2)/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3066 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c))^2,x, algorithm="giac")
[Out] integrate(cos(d*x + c)^(3/2)*e^(3/2)/(b*sin(d*x + c) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,\cos(c+d\,x))^{3/2}}{(a+b\,\sin(c+d\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(3/2)/(a + b*sin(c + d*x))^2,x)
[Out] int((e*cos(c + d*x))^(3/2)/(a + b*sin(c + d*x))^2, x)
```

3.589 $\int \frac{\sqrt{e\cos(c+dx)}}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=422

$$-\frac{a\sqrt{e}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{2\sqrt{b}\,(-a^2+b^2)^{5/4}\,d}+\frac{a\sqrt{e}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{2\sqrt{b}\,(-a^2+b^2)^{5/4}\,d}+\frac{\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx\right.}{(a^2-b^2)\,d\sqrt{\cos(c+dx)}}$$

```
[Out] b*(e*cos(d*x+c))^(3/2)/(a^2-b^2)/d/e/(a+b*sin(d*x+c))-1/2*a*arctan(b^(1/2)*
(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))*e^(1/2)/(-a^2+b^2)^(5/4)/d/b
^(1/2)+1/2*a*arctanh(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))
*e^(1/2)/(-a^2+b^2)^(5/4)/d/b^(1/2)+1/2*a^2*e*(cos(1/2*d*x+1/2*c)^2)^(1/2)/
cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(b-(-a^2+b^2)^(1/2)),2
^(1/2))*cos(d*x+c)^(1/2)/b/(a^2-b^2)/d/(b-(-a^2+b^2)^(1/2))/(e*cos(d*x+c))^
(1/2)+1/2*a^2*e*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(
sin(1/2*d*x+1/2*c),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(d*x+c)^(1/2)/b/(a^
2-b^2)/d/(b+(-a^2+b^2)^(1/2))/(e*cos(d*x+c))^(1/2)+(cos(1/2*d*x+1/2*c)^2)^(
1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c)
)^(1/2)/(a^2-b^2)/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.57, antiderivative size = 422, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {2773, 2946, 2721, 2719, 2780, 2886, 2884, 335, 304, 211, 214}

$$-\frac{a\sqrt{e}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}\,d\,(b^2-a^2)^{5/4}}+\frac{b(e\cos(c+dx))^{3/2}}{de\,(a^2-b^2)\,(a+b\sin(c+dx))}+\frac{a\sqrt{e}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}\,d\,(b^2-a^2)^{5/4}}+\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{d\,(a^2-b^2)\,\sqrt{\cos(c+dx)}}+\frac{a^2e\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{2bd\,(a^2-b^2)\left(b-\sqrt{b^2-a^2}\right)\sqrt{e\cos(c+dx)}}+\frac{a^2e\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{2bd\,(a^2-b^2)\left(\sqrt{b^2-a^2}+b\right)\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cos[c + d*x]]/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -1/2*(a*Sqrt[e]*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*S
qrt[e])])/(Sqrt[b]*(-a^2 + b^2)^(5/4)*d) + (a*Sqrt[e]*ArcTanh[(Sqrt[b]*Sqrt
[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(2*Sqrt[b]*(-a^2 + b^2)^(5
/4)*d) + (Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/((a^2 - b^2)*d*Sq
rt[Cos[c + d*x]]) + (a^2*e*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-a
^2 + b^2]), (c + d*x)/2, 2])/(2*b*(a^2 - b^2)*(b - Sqrt[-a^2 + b^2])*d*Sqrt
[e*Cos[c + d*x]]) + (a^2*e*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a
^2 + b^2]), (c + d*x)/2, 2])/(2*b*(a^2 - b^2)*(b + Sqrt[-a^2 + b^2])*d*Sqrt
[e*Cos[c + d*x]]) + (b*(e*Cos[c + d*x])^(3/2))/((a^2 - b^2)*d*e*(a + b*Sin[
c + d*x]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2773

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m + 1)/(f*g*(a^2 - b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1))
, Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p
 + 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^
2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2780

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[a*(g/(2*b)), Int[1/(Sq
```

```
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[a*(g/(2*b)), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[b*(g/f), Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{e\cos(c+dx)}}{(a+b\sin(c+dx))^2}\,dx = \frac{b(e\cos(c+dx))^{3/2}}{(a^2-b^2)\,de(a+b\sin(c+dx))} + \frac{\int \frac{\sqrt{e\cos(c+dx)}\left(-a-\frac{1}{2}b\sin(c+dx)\right)}{a+b\sin(c+dx)}\,dx}{-a^2+b^2}$$

$$= \frac{b(e\cos(c+dx))^{3/2}}{(a^2-b^2)\,de(a+b\sin(c+dx))} + \frac{\int \sqrt{e\cos(c+dx)}\,dx}{2(a^2-b^2)} + \frac{a\int \frac{\sqrt{e\cos(c+dx)} \ldots}{a+b\sin(c+dx)}}{2(a^2-b^2)}$$

$$= \frac{b(e\cos(c+dx))^{3/2}}{(a^2-b^2)\,de(a+b\sin(c+dx))} - \frac{(a^2e)\int \frac{1}{\sqrt{e\cos(c+dx)}\left(\sqrt{-a^2+b^2}-b\cos(c+\ldots\right.}}{4b(a^2-b^2)}$$

$$= \frac{\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{(a^2-b^2)\,d\sqrt{\cos(c+dx)}} + \frac{b(e\cos(c+dx))^{3/2}}{(a^2-b^2)\,de(a+b\sin(c+dx))} + \frac{(abe)\mathrm{Sub}\ldots}{\ldots}$$

$$= \frac{\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{(a^2-b^2)\,d\sqrt{\cos(c+dx)}} + \frac{a^2e\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{-a^2+b^2}};\frac{1}{2}(c-\ldots\right.}{2b(a^2-b^2)\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\cos(\ldots}}$$

$$= -\frac{a\sqrt{e}\,\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2\sqrt{b}\,(-a^2+b^2)^{5/4}\,d} + \frac{a\sqrt{e}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)\ldots}{2\sqrt{b}\,(-a^2+b^2)^{5/4}\,d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 36.13, size = 787, normalized size = 1.86

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[e*Cos[c + d*x]]/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -((b*Cos[c + d*x]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)*d*(a + b*Sin[c + d*x]
))) + (Sqrt[e*Cos[c + d*x]]*((-4*a*(a + b*Sqrt[1 - Cos[c + d*x]^2])*((a*App
ellF1[3/4, 1/2, 1, 7/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*
Cos[c + d*x]^(3/2))/(3*(a^2 - b^2)) + ((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*S
qrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt
[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - (1 + I
)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]] + Log[S
qrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I
*b*Cos[c + d*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)))*Sin[c + d*x])/(Sqrt[1 - Co
s[c + d*x]^2]*(a + b*Sin[c + d*x])) - ((a + b*Sqrt[1 - Cos[c + d*x]^2])*(8*
```

```
b^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-
a^2 + b^2)]*Cos[c + d*x]^(3/2) + 3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*(2*ArcTan[1
- (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + (S
qrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2]
 - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]] +
 Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]]
 + b*Cos[c + d*x]])*Sin[c + d*x]^2)/(12*Sqrt[b]*(-a^2 + b^2)*(1 - Cos[c +
d*x]^2)*(a + b*Sin[c + d*x]))))/(2*(a - b)*(a + b)*d*Sqrt[Cos[c + d*x]])
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 27.31, size = 9239, normalized size = 21.89

method	result	size
default	Expression too large to display	9239

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(1/2)/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] e^(1/2)*integrate(sqrt(cos(d*x + c))/(b*sin(d*x + c) + a)^2, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(-sqrt(cos(d*x + c))*e^(1/2)/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x +
c) - a^2 - b^2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(1/2)/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(d*x + c))*e^(1/2)/(b*sin(d*x + c) + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{e\,\cos\left(c+d\,x\right)}}{{\left(a+b\,\sin\left(c+d\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(1/2)/(a + b*sin(c + d*x))^2,x)
```

```
[Out] int((e*cos(c + d*x))^(1/2)/(a + b*sin(c + d*x))^2, x)
```

3.590 $\int \frac{1}{\sqrt{e \cos(c+dx)}\,(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=429

$$\frac{3a\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{2\,(-a^2+b^2)^{7/4}\,d\sqrt{e}}+\frac{3a\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{2\,(-a^2+b^2)^{7/4}\,d\sqrt{e}}-\frac{\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\right|}{(a^2-b^2)\,d\sqrt{e\cos(c+dx)}}$$

```
[Out] 3/2*a*arctan(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))*b^(1/2)
/(-a^2+b^2)^(7/4)/d/e^(1/2)+3/2*a*arctanh(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^
2+b^2)^(1/4)/e^(1/2))*b^(1/2)/(-a^2+b^2)^(7/4)/d/e^(1/2)-(cos(1/2*d*x+1/2*c
)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x
+c)^(1/2)/(a^2-b^2)/d/(e*cos(d*x+c))^(1/2)+3/2*a^2*(cos(1/2*d*x+1/2*c)^2)^(
1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(b-(-a^2+b^2)^(1/
2)),2^(1/2))*cos(d*x+c)^(1/2)/(a^2-b^2)/d/(a^2-b*(b-(-a^2+b^2)^(1/2)))/(e*c
os(d*x+c))^(1/2)+3/2*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*El
lipticPi(sin(1/2*d*x+1/2*c),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(d*x+c)^(1
/2)/(a^2-b^2)/d/(a^2-b*(b+(-a^2+b^2)^(1/2)))/(e*cos(d*x+c))^(1/2)+b*(e*cos(
d*x+c))^(1/2)/(a^2-b^2)/d/e/(a+b*sin(d*x+c))
```

Rubi [A]
time = 0.58, antiderivative size = 429, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {2773, 2946, 2721, 2720, 2781, 2886, 2884, 335, 218, 214, 211}

$$\frac{3a\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{2d\sqrt{e}\,(b^2-a^2)^{7/4}}+\frac{b\sqrt{e\cos(c+dx)}}{de\,(a^2-b^2)\,(a+b\sin(c+dx))}+\frac{3a\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{2d\sqrt{e}\,(b^2-a^2)^{7/4}}-\frac{\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\,(a^2-b^2)\,\sqrt{e\cos(c+dx)}}+\frac{3a^2\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{2d\,(a^2-b^2)\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{e\cos(c+dx)}}+\frac{3a^2\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{2d\,(a^2-b^2)\left(a^2-b\left(\sqrt{b^2-a^2}+b\right)\right)\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x])^2),x]
```

```
[Out] (3*a*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt
[e])])/(2*(-a^2 + b^2)^(7/4)*d*Sqrt[e]) + (3*a*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqr
t[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(2*(-a^2 + b^2)^(7/4)*d*S
qrt[e]) - (Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/((a^2 - b^2)*d*Sqr
t[e*Cos[c + d*x]]) + (3*a^2*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-
a^2 + b^2]), (c + d*x)/2, 2])/(2*(a^2 - b^2)*(a^2 - b*(b - Sqrt[-a^2 + b^2]
))*d*Sqrt[e*Cos[c + d*x]]) + (3*a^2*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b
+ Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(2*(a^2 - b^2)*(a^2 - b*(b + Sqrt[-a^
2 + b^2]))*d*Sqrt[e*Cos[c + d*x]]) + (b*Sqrt[e*Cos[c + d*x]])/((a^2 - b^2)*
d*e*(a + b*Sin[c + d*x]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2773

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m + 1)/(f*g*(a^2 - b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1))
, Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p
 + 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^
2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2781

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[-a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[b*(g/f), Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
```

```
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))^2}\,dx = \frac{b\sqrt{e\cos(c+dx)}}{(a^2-b^2)\,de(a+b\sin(c+dx))} + \frac{\int \frac{-a+\frac{1}{2}b\sin(c+dx)}{\sqrt{e\cos(c+dx)}\,(a+b\sin(c+d}}{-a^2+b^2}$$

$$= \frac{b\sqrt{e\cos(c+dx)}}{(a^2-b^2)\,de(a+b\sin(c+dx))} - \frac{\int \frac{1}{\sqrt{e\cos(c+dx)}}\,dx}{2\,(a^2-b^2)} + \frac{(3a}{}$$

$$= \frac{b\sqrt{e\cos(c+dx)}}{(a^2-b^2)\,de(a+b\sin(c+dx))} + \frac{(3a^2)\int \frac{1}{\sqrt{e\cos(c+dx)}\,\left(\sqrt{-\right.}}}{4\,(-a^2+b}$$

$$= -\frac{\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{(a^2-b^2)\,d\sqrt{e\cos(c+dx)}} + \frac{b\sqrt{e\cos(c+dx)}}{(a^2-b^2)\,de(a+b\sin(c+}$$

$$= -\frac{\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{(a^2-b^2)\,d\sqrt{e\cos(c+dx)}} - \frac{3a^2\sqrt{\cos(c+dx)}\,\Pi\left(\frac{}{b-}\right.}{2\,(-a^2+b^2)^{3/2}\left(b-\sqrt{-}\right.}$$

$$= \frac{3a\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{2\,(-a^2+b^2)^{7/4}\,d\sqrt{e}} + \frac{3a\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt[4]{-}}\right.}{2\,(-a^2+b^2)}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 43.50, size = 1181, normalized size = 2.75

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x])^2),x]
```

```
[Out] (b*Cos[c + d*x])/((a^2 - b^2)*d*Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x]))
+ (Sqrt[Cos[c + d*x]]*((-4*a*(a + b*Sqrt[1 - Cos[c + d*x]^2])*((5*a*(a^2 -
b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2
+ b^2)]*Sqrt[Cos[c + d*x]])/(Sqrt[1 - Cos[c + d*x]^2]*(5*(a^2 - b^2)*Appell
F1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] - 2
*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a
^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[c + d*x]^2, (b^2*C
os[c + d*x]^2)/(-a^2 + b^2)])*Cos[c + d*x]^2)*(a^2 + b^2*(-1 + Cos[c + d*x]
^2))) - ((1/8 - I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*
x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]]
```

```
)/(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)
^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]] - Log[Sqrt[-a^2 + b^2] + (1 +
 I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]]))/(-a
^2 + b^2)^(3/4))*Sin[c + d*x])/(Sqrt[1 - Cos[c + d*x]^2]*(a + b*Sin[c + d*x
])) + (2*b*(a + b*Sqrt[1 - Cos[c + d*x]^2])*((5*b*(a^2 - b^2)*AppellF1[1/4,
 -1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[
c + d*x]]*Sqrt[1 - Cos[c + d*x]^2])/((-5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1,
 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + 2*(2*b^2*AppellF
1[5/4, -1/2, 2, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + (
a^2 - b^2)*AppellF1[5/4, 1/2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/
(-a^2 + b^2)])*Cos[c + d*x]^2)*(a^2 + b^2*(-1 + Cos[c + d*x]^2))) + (a*(-2*
ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] + 2*ArcT
an[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a
^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c
+ d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[
c + d*x]] + b*Cos[c + d*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)))*Sin[c
+ d*x]^2)/((1 - Cos[c + d*x]^2)*(a + b*Sin[c + d*x]))))/(2*(a - b)*(a + b)*
d*Sqrt[e*Cos[c + d*x]])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 27.17, size = 5088, normalized size = 11.86

method	result	size
default	Expression too large to display	5088

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(d*x+c))^2/(e*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c))^2/(e*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] e^(-1/2)*integrate(1/((b*sin(d*x + c) + a)^2*sqrt(cos(d*x + c))), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c))^2/(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c))**2/(e*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c))^2/(e*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(e^(-1/2)/((b*sin(d*x + c) + a)^2*sqrt(cos(d*x + c))), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{e\,\cos\left(c+d\,x\right)}\,{\left(a+b\,\sin\left(c+d\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cos(c + d*x))^(1/2)*(a + b*sin(c + d*x))^2),x)
```

```
[Out] int(1/((e*cos(c + d*x))^(1/2)*(a + b*sin(c + d*x))^2), x)
```

3.591 $\int \frac{1}{(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=492

$$-\frac{5ab^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{2\left(-a^2+b^2\right)^{9/4}de^{3/2}}+\frac{5ab^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{2\left(-a^2+b^2\right)^{9/4}de^{3/2}}-\frac{(2a^2+3b^2)\sqrt{e\cos(c+dx)}}{\left(a^2-b^2\right)^2de^2\sqrt{\cos}}$$

```
[Out] -5/2*a*b^(3/2)*arctan(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2)
)/(-a^2+b^2)^(9/4)/d/e^(3/2)+5/2*a*b^(3/2)*arctanh(b^(1/2)*(e*cos(d*x+c))^(
1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/(-a^2+b^2)^(9/4)/d/e^(3/2)+b/(a^2-b^2)/d/e/(
a+b*sin(d*x+c))/(e*cos(d*x+c))^(1/2)+(-5*a*b+(2*a^2+3*b^2)*sin(d*x+c))/(a^2
-b^2)^2/d/e/(e*cos(d*x+c))^(1/2)-5/2*a^2*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos
(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(b-(-a^2+b^2)^(1/2)),2^(1
/2))*cos(d*x+c)^(1/2)/(a^2-b^2)^2/d/e/(b-(-a^2+b^2)^(1/2))/(e*cos(d*x+c))^(
1/2)-5/2*a^2*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(s
in(1/2*d*x+1/2*c),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(d*x+c)^(1/2)/(a^2-b
^2)^2/d/e/(b+(-a^2+b^2)^(1/2))/(e*cos(d*x+c))^(1/2)-(2*a^2+3*b^2)*(cos(1/2*
d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)
)*(e*cos(d*x+c))^(1/2)/(a^2-b^2)^2/d/e^2/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.81, antiderivative size = 492, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {2773, 2945, 2946, 2721, 2719, 2780, 2886, 2884, 335, 304, 211, 214}

$$-\frac{5ab^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{2de^{3/2}\left(b^2-a^2\right)^{9/4}}-\frac{(2a^2+3b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{de^2\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}}-\frac{5ab-(2a^2+3b^2)\sin(c+dx)}{de\left(a^2-b^2\right)^2\sqrt{e\cos(c+dx)}}+\frac{b}{de\left(a^2-b^2\right)\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))}-\frac{5a^2b\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{2de\left(a^2-b^2\right)^2\left(b-\sqrt{b^2-a^2}\right)\sqrt{e\cos(c+dx)}}-\frac{5a^2b\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{2de\left(a^2-b^2\right)^2\left(\sqrt{b^2-a^2}+b\right)\sqrt{e\cos(c+dx)}}+\frac{5ab^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{2de^{3/2}\left(b^2-a^2\right)^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x])^2),x]
```

```
[Out] (-5*a*b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqr
t[e])])/(2*(-a^2 + b^2)^(9/4)*d*e^(3/2)) + (5*a*b^(3/2)*ArcTanh[(Sqrt[b]*Sq
rt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(2*(-a^2 + b^2)^(9/4)*d*
e^(3/2)) - ((2*a^2 + 3*b^2)*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])
/((a^2 - b^2)^2*d*e^2*Sqrt[Cos[c + d*x]]) - (5*a^2*b*Sqrt[Cos[c + d*x]]*Ell
ipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(2*(a^2 - b^2)^2*(b
- Sqrt[-a^2 + b^2])*d*e*Sqrt[e*Cos[c + d*x]]) - (5*a^2*b*Sqrt[Cos[c + d*x]]
*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(2*(a^2 - b^2)^2
*(b + Sqrt[-a^2 + b^2])*d*e*Sqrt[e*Cos[c + d*x]]) + b/((a^2 - b^2)*d*e*Sqrt
[e*Cos[c + d*x]]*(a + b*Sin[c + d*x])) - (5*a*b - (2*a^2 + 3*b^2)*Sin[c + d
*x])/((a^2 - b^2)^2*d*e*Sqrt[e*Cos[c + d*x]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2773

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m + 1)/(f*g*(a^2 - b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1))
, Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p
 + 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^
2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2780

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[a*(g/(2*b)), Int[1/(Sq
```

```
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[a*(g/(2*b)), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[b*(g/f), Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2945

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{1}{(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))^2}\,dx = \frac{b}{(a^2-b^2)\,de\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))} + \frac{\int \frac{-a+}{(e\cos(c+dx}}{}$$

$$= \frac{b}{(a^2-b^2)\,de\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))} - \frac{5ab-(2a^2}{(a^2-b^2)^2}$$

$$= \frac{b}{(a^2-b^2)\,de\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))} - \frac{5ab-(2a^2}{(a^2-b^2)^2}$$

$$= \frac{b}{(a^2-b^2)\,de\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))} - \frac{5ab-(2a^2}{(a^2-b^2)^2}$$

$$= -\frac{(2a^2+3b^2)\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{(a^2-b^2)^2\,de^2\sqrt{\cos(c+dx)}} + \frac{}{(a^2-b^2)\,de}$$

$$= -\frac{(2a^2+3b^2)\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{(a^2-b^2)^2\,de^2\sqrt{\cos(c+dx)}} - \frac{5a^2b\sqrt{\cos(}}{2(a^2-b^2}$$

$$= -\frac{5ab^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2(-a^2+b^2)^{9/4}\,de^{3/2}} + \frac{5ab^{3/2}\tanh^{-1}\left(\sqrt{b}\right.}{2(-a^2+}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

time = 26.42, size = 777, normalized size = 1.58

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x])^2),x]
```

```
[Out] (Cos[c + d*x]^(3/2)*((12*(-6*a^2*b + b^3 - (2*a^2*b + 3*b^3)*Cos[2*(c + d*x
)] + 4*a*(a^2 - b^2)*Sin[c + d*x]))/((a^2 - b^2)^2*Sqrt[Cos[c + d*x]]) - (S
in[c + d*x]*(-(((2*a^2 + 3*b^2)*Csc[c + d*x]*(8*b^(5/2)*AppellF1[3/4, -1/2,
 1, 7/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Cos[c + d*x]^(3
/2) + 3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos
```

```
[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c +
 d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b
^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt
[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]]))/(Sqrt
[b]*(-a^2 + b^2))) - (48*a*(a^2 + 4*b^2)*((a*AppellF1[3/4, 1/2, 1, 7/4, Cos
[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Cos[c + d*x]^(3/2))/(3*(a^2
 - b^2)) + ((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/
(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a
^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4
)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]] + Log[Sqrt[-a^2 + b^2] + (1 + I)*S
qrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]]))/(Sqrt[b]
*(-a^2 + b^2)^(1/4))))/Sqrt[Sin[c + d*x]^2])*(a + b*Sqrt[Sin[c + d*x]^2]))/
((a - b)^2*(a + b)^2)))/(24*d*(e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x]))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 40.78, size = 7528, normalized size = 15.30

method	result	size
default	Expression too large to display	7528

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] e^(-3/2)*integrate(1/((b*sin(d*x + c) + a)^2*cos(d*x + c)^(3/2)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(3/2)/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(e^(-3/2)/((b*sin(d*x + c) + a)^2*cos(d*x + c)^(3/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(e\,\cos\left(c+d\,x\right)\right)^{3/2}\,\left(a+b\,\sin\left(c+d\,x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cos(c + d*x))^(3/2)*(a + b*sin(c + d*x))^2),x)
```

```
[Out] int(1/((e*cos(c + d*x))^(3/2)*(a + b*sin(c + d*x))^2), x)
```

3.592 $\int \frac{1}{(e\cos(c+dx))^{5/2}(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=514

$$\frac{7ab^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{2\left(-a^2+b^2\right)^{11/4}de^{5/2}}+\frac{7ab^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{2\left(-a^2+b^2\right)^{11/4}de^{5/2}}+\frac{(2a^2+5b^2)\sqrt{\cos(c+dx)}\,F(}{3\left(a^2-b^2\right)^2de^2\sqrt{e\cos(}}$$

```
[Out] 7/2*a*b^(5/2)*arctan(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))
/(-a^2+b^2)^(11/4)/d/e^(5/2)+7/2*a*b^(5/2)*arctanh(b^(1/2)*(e*cos(d*x+c))^(
1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/(-a^2+b^2)^(11/4)/d/e^(5/2)+b/(a^2-b^2)/d/e/
(e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c))+1/3*(-7*a*b+(2*a^2+5*b^2)*sin(d*x+c))
/(a^2-b^2)^2/d/e/(e*cos(d*x+c))^(3/2)+1/3*(2*a^2+5*b^2)*(cos(1/2*d*x+1/2*c)
^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+
c)^(1/2)/(a^2-b^2)^2/d/e^2/(e*cos(d*x+c))^(1/2)-7/2*a^2*b^2*(cos(1/2*d*x+1/
2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(b-(-a^2
+b^2)^(1/2)),2^(1/2))*cos(d*x+c)^(1/2)/(a^2-b^2)^2/d/e^2/(a^2-b*(b-(-a^2+b^
2)^(1/2)))/(e*cos(d*x+c))^(1/2)-7/2*a^2*b^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/co
s(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(b+(-a^2+b^2)^(1/2)),2^(
1/2))*cos(d*x+c)^(1/2)/(a^2-b^2)^2/d/e^2/(a^2-b*(b+(-a^2+b^2)^(1/2)))/(e*co
s(d*x+c))^(1/2)
```

Rubi [A]
time = 0.90, antiderivative size = 514, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {2773, 2945, 2946, 2721, 2720, 2781, 2886, 2884, 335, 218, 214, 211}

$$\frac{7ab^{5/2}\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{2de^{5/2}\left(b^2-a^2\right)^{11/4}}+\frac{(2a^2+5b^2)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3de^2\left(a^2-b^2\right)^2\sqrt{e\cos(c+dx)}}-\frac{7a^2b^2\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{2de^2\left(a^2-b^2\right)^2\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{e\cos(c+dx)}}-\frac{7a^2b^2\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{2de^2\left(a^2-b^2\right)^2\left(a^2-b\left(\sqrt{b^2-a^2}+b\right)\right)\sqrt{e\cos(c+dx)}}+\frac{b}{de\left(a^2-b^2\right)(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))}-\frac{7ab-(2a^2+5b^2)\sin(c+dx)}{3de\left(a^2-b^2\right)^2(e\cos(c+dx))^{3/2}}+\frac{7ab^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{2de^{5/2}\left(b^2-a^2\right)^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(5/2)*(a + b*Sin[c + d*x])^2),x]
```

```
[Out] (7*a*b^(5/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt
[e])])/(2*(-a^2 + b^2)^(11/4)*d*e^(5/2)) + (7*a*b^(5/2)*ArcTanh[(Sqrt[b]*Sq
rt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(2*(-a^2 + b^2)^(11/4)*d
*e^(5/2)) + ((2*a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/
(3*(a^2 - b^2)^2*d*e^2*Sqrt[e*Cos[c + d*x]]) - (7*a^2*b^2*Sqrt[Cos[c + d*x]
]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(2*(a^2 - b^2)^
2*(a^2 - b*(b - Sqrt[-a^2 + b^2]))*d*e^2*Sqrt[e*Cos[c + d*x]]) - (7*a^2*b^2
*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2
])/(2*(a^2 - b^2)^2*(a^2 - b*(b + Sqrt[-a^2 + b^2]))*d*e^2*Sqrt[e*Cos[c + d
*x]]) + b/((a^2 - b^2)*d*e*(e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x])) - (
7*a*b - (2*a^2 + 5*b^2)*Sin[c + d*x])/(3*(a^2 - b^2)^2*d*e*(e*Cos[c + d*x])
^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2773

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m + 1)/(f*g*(a^2 - b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1))
, Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p
 + 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^
2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2781

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[-a/(2*q), Int[1/(S
```

```
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[b*(g/f), Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2945

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{1}{(e\cos(c+dx))^{5/2}(a+b\sin(c+dx))^2}\,dx = \frac{b}{(a^2-b^2)\,de(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))} + \frac{\int \frac{-a}{(e\cos(c+a}}{}$$

$$= \frac{b}{(a^2-b^2)\,de(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))} - \frac{7ab-(2}{3\,(a^2-b^2}$$

$$= \frac{b}{(a^2-b^2)\,de(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))} - \frac{7ab-(2}{3\,(a^2-b^2}$$

$$= \frac{b}{(a^2-b^2)\,de(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))} - \frac{7ab-(2}{3\,(a^2-b^2}$$

$$= \frac{(2a^2+5b^2)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{3\,(a^2-b^2)^2\,de^2\sqrt{e\cos(c+dx)}} + \frac{}{(a^2-b^2)\,de(e}$$

$$= \frac{(2a^2+5b^2)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{3\,(a^2-b^2)^2\,de^2\sqrt{e\cos(c+dx)}} - \frac{7a^2b^2\sqrt{\cos(c}}{2\,(-a^2+b^2)^{5/}}$$

$$= \frac{7ab^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{2\,(-a^2+b^2)^{11/4}\,de^{5/2}} + \frac{7ab^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt[4]{}}\right.}{2\,(-a^2+b^2)}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

time = 44.25, size = 1258, normalized size = 2.45

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((e*Cos[c + d*x])^(5/2)*(a + b*Sin[c + d*x])^2),x]
```

```
[Out] (Cos[c + d*x]^(5/2)*((-2*(2*a^3 - 16*a*b^2)*(a + b*Sqrt[1 - Cos[c + d*x]^2]
)*((5*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c +
 d*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[c + d*x]])/(Sqrt[1 - Cos[c + d*x]^2]*(5*(a^
2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-
a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[c + d*x]^2, (b^2*Cos[
c + d*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[c +
 d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)])*Cos[c + d*x]^2)*(a^2 + b^2*(-1
```

```
 + Cos[c + d*x]^2))) - ((1/8 - I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*
Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqr
t[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[
b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]] - Log[Sqrt[-a^
2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[
c + d*x]]))/(-a^2 + b^2)^(3/4))*Sin[c + d*x])/(Sqrt[1 - Cos[c + d*x]^2]*(a
+ b*Sin[c + d*x])) - (2*(2*a^2*b + 5*b^3)*(a + b*Sqrt[1 - Cos[c + d*x]^2])*
((5*b*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c +
d*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[c + d*x]]*Sqrt[1 - Cos[c + d*x]^2])/((-5*(a^
2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(
-a^2 + b^2)] + 2*(2*b^2*AppellF1[5/4, -1/2, 2, 9/4, Cos[c + d*x]^2, (b^2*Co
s[c + d*x]^2)/(-a^2 + b^2)] + (a^2 - b^2)*AppellF1[5/4, 1/2, 1, 9/4, Cos[c
+ d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)])*Cos[c + d*x]^2)*(a^2 + b^2*(-
1 + Cos[c + d*x]^2))) + (a*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x
]])/(a^2 - b^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/
(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4
)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt
[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]]))/(4*Sqrt[2]*Sqr
t[b]*(a^2 - b^2)^(3/4)))*Sin[c + d*x]^2)/((1 - Cos[c + d*x]^2)*(a + b*Sin[c
 + d*x]))))/(6*(a - b)^2*(a + b)^2*d*(e*Cos[c + d*x])^(5/2)) + (Cos[c + d*x
]^3*(-(b^3/((a^2 - b^2)^2*(a + b*Sin[c + d*x]))) + (2*Sec[c + d*x]^2*(-2*a*
b + a^2*Sin[c + d*x] + b^2*Sin[c + d*x]))/(3*(a^2 - b^2)^2)))/(d*(e*Cos[c +
 d*x])^(5/2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 54.50, size = 4976, normalized size = 9.68

method	result	size
default	Expression too large to display	4976

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(5/2)/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-1/6/e^3*a*b/(a^2-b^2)^2/(cos(1/2*d*x+1/2*c)+1/2*2^(1/2))^2*(-2*sin(1/2*d*
x+1/2*c)^2*e+e)^(1/2)-1/6/e^3*a*b*2^(1/2)/(a^2-b^2)^2/(cos(1/2*d*x+1/2*c)+1
/2*2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)-1/6/e^3*a*b/(a^2-b^2)^2/(co
s(1/2*d*x+1/2*c)-1/2*2^(1/2))^2*(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)+1/6/e^3
*a*b*2^(1/2)/(a^2-b^2)^2/(cos(1/2*d*x+1/2*c)-1/2*2^(1/2))*(-2*sin(1/2*d*x+1
/2*c)^2*e+e)^(1/2)+512*e^(3/2)*a*b^3/(a-b)/(a+b)/(1024*e^(7/2)*2^(1/2)*(-2*
sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6
-1792*e^(7/2)*2^(1/2)*(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*b^2*sin(1/2*d*x+1
/2*c)^4*cos(1/2*d*x+1/2*c)+256*e^(7/2)*2^(1/2)*(-2*sin(1/2*d*x+1/2*c)^2*e+e
)^(1/2)*a^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+768*e^(7/2)*2^(1/2)*(-2
*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^
2+2048*b^2*e^4*sin(1/2*d*x+1/2*c)^8-192*e^(7/2)*2^(1/2)*(-2*sin(1/2*d*x+1/2
```

*c)^2*e+e)^(1/2)*a^2*cos(1/2*d*x+1/2*c)-5120*b^2*e^4*sin(1/2*d*x+1/2*c)^6+5
12*a^2*e^4*sin(1/2*d*x+1/2*c)^4+4160*b^2*e^4*sin(1/2*d*x+1/2*c)^4-768*a^2*e
^4*sin(1/2*d*x+1/2*c)^2-1088*b^2*e^4*sin(1/2*d*x+1/2*c)^2+272*e^4*a^2)/(a^2
-b^2)*2^(1/2)*cos(1/2*d*x+1/2*c)^5-160*e*a*b^3/(a-b)/(a+b)/(1024*e^(7/2)*2^
(1/2)*(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*
x+1/2*c)^6-1792*e^(7/2)*2^(1/2)*(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*b^2*sin
(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+256*e^(7/2)*2^(1/2)*(-2*sin(1/2*d*x+1/
2*c)^2*e+e)^(1/2)*a^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+768*e^(7/2)*2
^(1/2)*(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d
*x+1/2*c)^2+2048*b^2*e^4*sin(1/2*d*x+1/2*c)^8-192*e^(7/2)*2^(1/2)*(-2*sin(1
/2*d*x+1/2*c)^2*e+e)^(1/2)*a^2*cos(1/2*d*x+1/2*c)-5120*b^2*e^4*sin(1/2*d*x+
1/2*c)^6+512*a^2*e^4*sin(1/2*d*x+1/2*c)^4+4160*b^2*e^4*sin(1/2*d*x+1/2*c)^4
-768*a^2*e^4*sin(1/2*d*x+1/2*c)^2-1088*b^2*e^4*sin(1/2*d*x+1/2*c)^2+272*e^4
a^2)/(a^2-b^2)(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)*cos(1/2*d*x+1/2*c)^4-3
84*e^(3/2)*a*b^3/(a-b)/(a+b)/(1024*e^(7/2)*2^(1/2)*(-2*sin(1/2*d*x+1/2*c)^2
*e+e)^(1/2)*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-1792*e^(7/2)*2^(1/2
)*(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*b^2*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+
1/2*c)+256*e^(7/2)*2^(1/2)*(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*a^2*cos(1/2*
d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+768*e^(7/2)*2^(1/2)*(-2*sin(1/2*d*x+1/2*c)^
2*e+e)^(1/2)*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+2048*b^2*e^4*sin(1
/2*d*x+1/2*c)^8-192*e^(7/2)*2^(1/2)*(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*a^2
*cos(1/2*d*x+1/2*c)-5120*b^2*e^4*sin(1/2*d*x+1/2*c)^6+512*a^2*e^4*sin(1/2*d
*x+1/2*c)^4+4160*b^2*e^4*sin(1/2*d*x+1/2*c)^4-768*a^2*e^4*sin(1/2*d*x+1/2*c
)^2-1088*b^2*e^4*sin(1/2*d*x+1/2*c)^2+272*e^4*a^2)/(a^2-b^2)*2^(1/2)*cos(1/
2*d*x+1/2*c)^3-160*a*b^3/(a-b)/(a+b)/(1024*e^(7/2)*2^(1/2)*(-2*sin(1/2*d*x+
1/2*c)^2*e+e)^(1/2)*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-1792*e^(7/2
)*2^(1/2)*(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*b^2*sin(1/2*d*x+1/2*c)^4*cos(
1/2*d*x+1/2*c)+256*e^(7/2)*2^(1/2)*(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*a^2*
cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+768*e^(7/2)*2^(1/2)*(-2*sin(1/2*d*x
+1/2*c)^2*e+e)^(1/2)*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+2048*b^2*e
^4*sin(1/2*d*x+1/2*c)^8-192*e^(7/2)*2^(1/2)*(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(
1/2)*a^2*cos(1/2*d*x+1/2*c)-5120*b^2*e^4*sin(1/2*d*x+1/2*c)^6+512*a^2*e^4*s
in(1/2*d*x+1/2*c)^4+4160*b^2*e^4*sin(1/2*d*x+1/2*c)^4-768*a^2*e^4*sin(1/2*d
*x+1/2*c)^2-1088*b^2*e^4*sin(1/2*d*x+1/2*c)^2+272*e^4*a^2)/(a^2-b^2)*(e*(2*
cos(1/2*d*x+1/2*c)^2-1))^(3/2)*cos(1/2*d*x+1/2*c)^2+64*e^(3/2)*a*b^3/(a-b)/
(a+b)/(1024*e^(7/2)*2^(1/2)*(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*b^2*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-1792*e^(7/2)*2^(1/2)*(-2*sin(1/2*d*x+1/2*c
)^2*e+e)^(1/2)*b^2*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+256*e^(7/2)*2^(1
/2)*(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*a^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+
1/2*c)^2+768*e^(7/2)*2^(1/2)*(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*b^2*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+2048*b^2*e^4*sin(1/2*d*x+1/2*c)^8-192*e^(
7/2)*2^(1/2)*(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*a^2*cos(1/2*d*x+1/2*c)-512
0*b^2*e^4*sin(1/2*d*x+1/2*c)^6+512*a^2*e^4*sin(1/2*d*x+1/2*c)^4+4160*b^2*e^
4*sin(1/2*d*x+1/2*c)^4-768*a^2*e^4*sin(1/2*d*x+1/2*c)^2-1088*b^2*e^4*sin(1/
2*d*x+1/2*c)^2+272*e^4*a^2)/(a^2-b^2)*2^(1/2)*cos(1/2*d*x+1/2*c)-8/e*a*b^3/

```
(a-b)/(a+b)/(1024*e^(7/2)*2^(1/2)*(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*b^2*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-1792*e^(7/2)*2^(1/2)*(-2*sin(1/2*d*x
+1/2*c)^2*e+e)^(1/2)*b^2*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+256*e^(7/2
)*2^(1/2)*(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*a^2*cos(1/2*d*x+1/2*c)*sin(1/
2*d*x+1/2*c)^2+768*e^(7/2)*2^(1/2)*(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*b^2*
cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+2048*b^2*e^4*sin(1/2*d*x+1/2*c)^8-1
92*e^(7/2)*2^(1/2)*(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*a^2*cos(1/2*d*x+1/2*
c)-5120*b^2*e^4*sin(1/2*d*x+1/2*c)^6+512*a^2*e^4*sin(1/2*d*x+1/2*c)^4+4160*
b^2*e^4*sin(1/2*d*x+1/2*c)^4-768*a^2*e^4*sin(1/2*d*x+1/2*c)^2-1088*b^2*e^4*
sin(1/2*d*x+1/2*c)^2+272*e^4*a^2)/(a^2-b^2)*(e*...
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(5/2)/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
[Out] e^(-5/2)*integrate(1/((b*sin(d*x + c) + a)^2*cos(d*x + c)^(5/2)), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(5/2)/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
[Out] integral(-sqrt(cos(d*x + c))/(b^2*cos(d*x + c)^5*e^(5/2) - 2*a*b*cos(d*x +
c)^3*e^(5/2)*sin(d*x + c) - (a^2 + b^2)*cos(d*x + c)^3*e^(5/2)), x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(5/2)/(a+b*sin(d*x+c))**2,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3883 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(5/2)/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(e^(-5/2)/((b*sin(d*x + c) + a)^2*cos(d*x + c)^(5/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(e\,\cos\left(c+d\,x\right)\right)^{5/2}\,\left(a+b\,\sin\left(c+d\,x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cos(c + d*x))^(5/2)*(a + b*sin(c + d*x))^2),x)
```

```
[Out] int(1/((e*cos(c + d*x))^(5/2)*(a + b*sin(c + d*x))^2), x)
```

3.593 $\int \frac{1}{(e\cos(c+dx))^{7/2}(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=574

$$-\frac{9ab^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{2\left(-a^2+b^2\right)^{13/4}de^{7/2}}+\frac{9ab^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{2\left(-a^2+b^2\right)^{13/4}de^{7/2}}-\frac{3(2a^4-10a^2b^2-7b^4)\,\sqrt{e\cos}}{5\left(a^2-b^2\right)^3de^4}$$

```
[Out] -9/2*a*b^(7/2)*arctan(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2)
)/(-a^2+b^2)^(13/4)/d/e^(7/2)+9/2*a*b^(7/2)*arctanh(b^(1/2)*(e*cos(d*x+c))^
(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/(-a^2+b^2)^(13/4)/d/e^(7/2)+b/(a^2-b^2)/d/e
/(e*cos(d*x+c))^(5/2)/(a+b*sin(d*x+c))+1/5*(-9*a*b+(2*a^2+7*b^2)*sin(d*x+c)
)/(a^2-b^2)^2/d/e/(e*cos(d*x+c))^(5/2)+3/5*(15*a*b^3+(2*a^4-10*a^2*b^2-7*b^
4)*sin(d*x+c))/(a^2-b^2)^3/d/e^3/(e*cos(d*x+c))^(1/2)+9/2*a^2*b^3*(cos(1/2*
d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(b
-(-a^2+b^2)^(1/2)),2^(1/2))*cos(d*x+c)^(1/2)/(a^2-b^2)^3/d/e^3/(b-(-a^2+b^2
)^(1/2))/(e*cos(d*x+c))^(1/2)+9/2*a^2*b^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(
1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/
2))*cos(d*x+c)^(1/2)/(a^2-b^2)^3/d/e^3/(b+(-a^2+b^2)^(1/2))/(e*cos(d*x+c))^
(1/2)-3/5*(2*a^4-10*a^2*b^2-7*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x
+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/(a^2-b^2
)^3/d/e^4/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 1.09, antiderivative size = 574, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {2773, 2945, 2946, 2721, 2719, 2780, 2886, 2884, 335, 304, 211, 214}

$$-\frac{9ab^{7/2}\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{2de^{7/2}\left(b^2-a^2\right)^{13/4}}+\frac{b}{de\left(a^2-b^2\right)(e\cos(c+dx))^{5/2}(a+b\sin(c+dx))}-\frac{9ab-(2a^2+7b^2)\sin(c+dx)}{5de\left(a^2-b^2\right)^2(e\cos(c+dx))^{5/2}}+\frac{9ab^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{2de^{7/2}\left(b^2-a^2\right)^{13/4}}+\frac{9a^2b^3\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{2de^3\left(a^2-b^2\right)^3\left(b-\sqrt{b^2-a^2}\right)\sqrt{e\cos(c+dx)}}+\frac{9a^2b^3\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{2de^3\left(a^2-b^2\right)^3\left(\sqrt{b^2-a^2}+b\right)\sqrt{e\cos(c+dx)}}-\frac{3(2a^4-10a^2b^2-7b^4)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{5de^4\left(a^2-b^2\right)^3\sqrt{\cos(c+dx)}}+\frac{3((2a^4-10a^2b^2-7b^4)\sin(c+dx)+15ab^3)}{5de^3\left(a^2-b^2\right)^3\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(7/2)*(a + b*Sin[c + d*x])^2),x]
```

```
[Out] (-9*a*b^(7/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqr
t[e])])/(2*(-a^2 + b^2)^(13/4)*d*e^(7/2)) + (9*a*b^(7/2)*ArcTanh[(Sqrt[b]*S
qrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(2*(-a^2 + b^2)^(13/4)*
d*e^(7/2)) - (3*(2*a^4 - 10*a^2*b^2 - 7*b^4)*Sqrt[e*Cos[c + d*x]]*EllipticE
[(c + d*x)/2, 2])/(5*(a^2 - b^2)^3*d*e^4*Sqrt[Cos[c + d*x]]) + (9*a^2*b^3*S
qrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])
/(2*(a^2 - b^2)^3*(b - Sqrt[-a^2 + b^2])*d*e^3*Sqrt[e*Cos[c + d*x]]) + (9*a
^2*b^3*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c + d*x
)/2, 2])/(2*(a^2 - b^2)^3*(b + Sqrt[-a^2 + b^2])*d*e^3*Sqrt[e*Cos[c + d*x]]
) + b/((a^2 - b^2)*d*e*(e*Cos[c + d*x])^(5/2)*(a + b*Sin[c + d*x])) - (9*a*
b - (2*a^2 + 7*b^2)*Sin[c + d*x])/(5*(a^2 - b^2)^2*d*e*(e*Cos[c + d*x])^(5/
```

```
2)) + (3*(15*a*b^3 + (2*a^4 - 10*a^2*b^2 - 7*b^4)*Sin[c + d*x]))/(5*(a^2 -
b^2)^3*d*e^3*Sqrt[e*Cos[c + d*x]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2773

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m + 1)/(f*g*(a^2 - b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1))
, Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p
 + 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^
2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2780

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[a*(g/(2*b)), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[a*(g/(2*b)), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[b*(g/f), Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2945

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{1}{(e\cos(c+dx))^{7/2}(a+b\sin(c+dx))^2}\,dx = \frac{b}{(a^2-b^2)\,de(e\cos(c+dx))^{5/2}(a+b\sin(c+dx))} + \frac{\int \frac{-a}{(e\cos(c+d}}{}$$

$$= \frac{b}{(a^2-b^2)\,de(e\cos(c+dx))^{5/2}(a+b\sin(c+dx))} - \frac{9ab-(2}{5\,(a^2-b^2}$$

$$= \frac{b}{(a^2-b^2)\,de(e\cos(c+dx))^{5/2}(a+b\sin(c+dx))} - \frac{9ab-(2}{5\,(a^2-b^2}$$

$$= \frac{b}{(a^2-b^2)\,de(e\cos(c+dx))^{5/2}(a+b\sin(c+dx))} - \frac{9ab-(2}{5\,(a^2-b^2}$$

$$= \frac{b}{(a^2-b^2)\,de(e\cos(c+dx))^{5/2}(a+b\sin(c+dx))} - \frac{9ab-(2}{5\,(a^2-b^2}$$

$$= -\frac{3(2a^4-10a^2b^2-7b^4)\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5\,(a^2-b^2)^3\,de^4\sqrt{\cos(c+dx)}} + $$

$$= -\frac{3(2a^4-10a^2b^2-7b^4)\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5\,(a^2-b^2)^3\,de^4\sqrt{\cos(c+dx)}} + $$

$$= -\frac{9ab^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{2\,(-a^2+b^2)^{13/4}\,de^{7/2}} + \frac{9ab^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}}{}\right.}{2\,(-a^2+b}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 26.63, size = 949, normalized size = 1.65

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((e*Cos[c + d*x])^(7/2)*(a + b*Sin[c + d*x])^2),x]
```

```
[Out] (-3*Cos[c + d*x]^(7/2)*((-2*(2*a^5 - 10*a^3*b^2 - 22*a*b^4)*(a + b*Sqrt[1 -
 Cos[c + d*x]^2])*((a*AppellF1[3/4, 1/2, 1, 7/4, Cos[c + d*x]^2, (b^2*Cos[c
 + d*x]^2)/(-a^2 + b^2)]*Cos[c + d*x]^(3/2))/(3*(a^2 - b^2)) + ((1/8 + I/8)
*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2
```

```
*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[
Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] +
I*b*Cos[c + d*x]] + Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/
4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)))*S
in[c + d*x])/(Sqrt[1 - Cos[c + d*x]^2]*(a + b*Sin[c + d*x])) - ((2*a^4*b -
10*a^2*b^3 - 7*b^5)*(a + b*Sqrt[1 - Cos[c + d*x]^2])*(8*b^(5/2)*AppellF1[3/
4, -1/2, 1, 7/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Cos[c +
 d*x]^(3/2) + 3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*
Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt
[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*
(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]] + Log[Sqrt[a^2 - b^2
] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]])
)*Sin[c + d*x]^2)/(12*b^(3/2)*(-a^2 + b^2)*(1 - Cos[c + d*x]^2)*(a + b*Sin[
c + d*x]))))/(10*(a - b)^3*(a + b)^3*d*(e*Cos[c + d*x])^(7/2)) + (Cos[c + d
*x]^4*((b^5*Cos[c + d*x])/((a^2 - b^2)^3*(a + b*Sin[c + d*x])) + (2*Sec[c +
 d*x]^3*(-2*a*b + a^2*Sin[c + d*x] + b^2*Sin[c + d*x]))/(5*(a^2 - b^2)^2) +
 (2*Sec[c + d*x]*(20*a*b^3 + 3*a^4*Sin[c + d*x] - 15*a^2*b^2*Sin[c + d*x] -
 8*b^4*Sin[c + d*x]))/(5*(a^2 - b^2)^3)))/(d*(e*Cos[c + d*x])^(7/2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 82.80, size = 8202, normalized size = 14.29

method	result	size
default	Expression too large to display	8202

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(7/2)/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(7/2)/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] e^(-7/2)*integrate(1/((b*sin(d*x + c) + a)^2*cos(d*x + c)^(7/2)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(7/2)/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(7/2)/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3063 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(7/2)/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(e^(-7/2)/((b*sin(d*x + c) + a)^2*cos(d*x + c)^(7/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(e\,\cos\left(c+d\,x\right)\right)}^{7/2}\,{\left(a+b\,\sin\left(c+d\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cos(c + d*x))^(7/2)*(a + b*sin(c + d*x))^2),x)
```

```
[Out] int(1/((e*cos(c + d*x))^(7/2)*(a + b*sin(c + d*x))^2), x)
```

3.594 $\int \frac{(e\cos(c+dx))^{13/2}}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=575

$$-\frac{11(9a^4-11a^2b^2+2b^4)\,e^{13/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{8b^{13/2}\sqrt[4]{-a^2+b^2}\,d}+\frac{11(9a^4-11a^2b^2+2b^4)\,e^{13/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e}}{\sqrt[4]{-a^2}}\right.}{8b^{13/2}\sqrt[4]{-a^2+b^2}\,d}$$

```
[Out] -11/8*(9*a^4-11*a^2*b^2+2*b^4)*e^(13/2)*arctan(b^(1/2)*(e*cos(d*x+c))^(1/2)
/(-a^2+b^2)^(1/4)/e^(1/2))/b^(13/2)/(-a^2+b^2)^(1/4)/d+11/8*(9*a^4-11*a^2*b
^2+2*b^4)*e^(13/2)*arctanh(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^
(1/2))/b^(13/2)/(-a^2+b^2)^(1/4)/d-1/2*e*(e*cos(d*x+c))^(11/2)/b/d/(a+b*sin
(d*x+c))^2-11/28*e^3*(e*cos(d*x+c))^(7/2)*(9*a+2*b*sin(d*x+c))/b^3/d/(a+b*s
in(d*x+c))+11/60*e^5*(e*cos(d*x+c))^(3/2)*(45*a^2-10*b^2-27*a*b*sin(d*x+c))
/b^5/d-11/8*a*(9*a^4-11*a^2*b^2+2*b^4)*e^7*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos
(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(b-(-a^2+b^2)^(1/2)),2^(1
/2))*cos(d*x+c)^(1/2)/b^7/d/(b-(-a^2+b^2)^(1/2))/(e*cos(d*x+c))^(1/2)-11/8*
a*(9*a^4-11*a^2*b^2+2*b^4)*e^7*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2
*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(d*x
+c)^(1/2)/b^7/d/(b+(-a^2+b^2)^(1/2))/(e*cos(d*x+c))^(1/2)+11/20*a*(45*a^2-3
7*b^2)*e^6*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/
2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/b^6/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.94, antiderivative size = 575, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {2772, 2942, 2944, 2946, 2721, 2719, 2780, 2886, 2884, 335, 304, 211, 214}

$$\frac{11ae^6(45a^2-37b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{20b^6d\sqrt{\cos(c+dx)}}+\frac{11e^5(e\cos(c+dx))^{3/2}\left(5(9a^2-2b^2)-27ab\sin(c+dx)\right)}{60b^5d}-\frac{11e^{13/2}(9a^4-11a^2b^2+2b^4)\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{8b^{13/2}d\sqrt[4]{b^2-a^2}}+\frac{11e^{13/2}(9a^4-11a^2b^2+2b^4)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{8b^{13/2}d\sqrt[4]{b^2-a^2}}-\frac{11ae^7(9a^4-11a^2b^2+2b^4)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{8b^7d\left(b-\sqrt{b^2-a^2}\right)\sqrt{e\cos(c+dx)}}-\frac{11ae^7(9a^4-11a^2b^2+2b^4)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{8b^7d\left(\sqrt{b^2-a^2}+b\right)\sqrt{e\cos(c+dx)}}-\frac{11e^3(e\cos(c+dx))^{7/2}(9a+2b\sin(c+dx))}{28b^3d(a+b\sin(c+dx))}-\frac{e(e\cos(c+dx))^{11/2}}{2bd(a+b\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(13/2)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-11*(9*a^4 - 11*a^2*b^2 + 2*b^4)*e^(13/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d
*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(8*b^(13/2)*(-a^2 + b^2)^(1/4)*d) + (1
1*(9*a^4 - 11*a^2*b^2 + 2*b^4)*e^(13/2)*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c + d*x
]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(8*b^(13/2)*(-a^2 + b^2)^(1/4)*d) + (11*
a*(45*a^2 - 37*b^2)*e^6*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(20
*b^6*d*Sqrt[Cos[c + d*x]]) - (11*a*(9*a^4 - 11*a^2*b^2 + 2*b^4)*e^7*Sqrt[Co
s[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(8*b^
7*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e*Cos[c + d*x]]) - (11*a*(9*a^4 - 11*a^2*b^
2 + 2*b^4)*e^7*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]),
(c + d*x)/2, 2])/(8*b^7*(b + Sqrt[-a^2 + b^2])*d*Sqrt[e*Cos[c + d*x]]) - (e
```

```
*(e*Cos[c + d*x])^(11/2))/(2*b*d*(a + b*Sin[c + d*x])^2) - (11*e^3*(e*Cos[c
 + d*x])^(7/2)*(9*a + 2*b*Sin[c + d*x]))/(28*b^3*d*(a + b*Sin[c + d*x])) +
(11*e^5*(e*Cos[c + d*x])^(3/2)*(5*(9*a^2 - 2*b^2) - 27*a*b*Sin[c + d*x]))/(
60*b^5*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2772

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Dist[g^2*((p - 1)/(b*(m + 1))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
```

```
tegersQ[2*m, 2*p]
```

Rule 2780

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[a*(g/(2*b)), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[a*(g/(2*b)), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[b*(g/f), Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2942

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*C
os[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x])/(b^2*f*(m + 1)*(m + p + 1))), x] + Dist[g^2*((
p - 1)/(b^2*(m + 1)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rule 2944

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Dist[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
```

```
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{13/2}}{(a+b\sin(c+dx))^3}\,dx = -\frac{e(e\cos(c+dx))^{11/2}}{2bd(a+b\sin(c+dx))^2} - \frac{(11e^2)\int \frac{(e\cos(c+dx))^{9/2}\sin(c+dx)}{(a+b\sin(c+dx))^2}\,dx}{4b}$$

$$= -\frac{e(e\cos(c+dx))^{11/2}}{2bd(a+b\sin(c+dx))^2} - \frac{11e^3(e\cos(c+dx))^{7/2}(9a+2b\sin(c+dx))}{28b^3d(a+b\sin(c+dx))} + \frac{(11e}{}$$

$$= -\frac{e(e\cos(c+dx))^{11/2}}{2bd(a+b\sin(c+dx))^2} - \frac{11e^3(e\cos(c+dx))^{7/2}(9a+2b\sin(c+dx))}{28b^3d(a+b\sin(c+dx))} + \frac{11e}{}$$

$$= -\frac{e(e\cos(c+dx))^{11/2}}{2bd(a+b\sin(c+dx))^2} - \frac{11e^3(e\cos(c+dx))^{7/2}(9a+2b\sin(c+dx))}{28b^3d(a+b\sin(c+dx))} + \frac{11e}{}$$

$$= -\frac{e(e\cos(c+dx))^{11/2}}{2bd(a+b\sin(c+dx))^2} - \frac{11e^3(e\cos(c+dx))^{7/2}(9a+2b\sin(c+dx))}{28b^3d(a+b\sin(c+dx))} + \frac{11e}{}$$

$$= \frac{11a(45a^2-37b^2)\,e^6\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{20b^6d\sqrt{\cos(c+dx)}} - \frac{e(e\cos(c+dx))^{11/2}}{2bd(a+b\sin(c+dx))}$$

$$= \frac{11a(45a^2-37b^2)\,e^6\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{20b^6d\sqrt{\cos(c+dx)}} - \frac{11a(9a^4-11a^2b^2+2b}{8b^7(}$$

$$= -\frac{11(9a^4-11a^2b^2+2b^4)\,e^{13/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8b^{13/2}\sqrt[4]{-a^2+b^2}\,d} + \frac{11(9a^4-11a^2}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

time = 40.82, size = 932, normalized size = 1.62

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^(13/2)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (11*(e*Cos[c + d*x])^(13/2)*((-2*(18*a^2*b - 10*b^3)*(a + b*Sqrt[1 - Cos[c
+ d*x]^2])*((a*AppellF1[3/4, 1/2, 1, 7/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]
^2)/(-a^2 + b^2)]*Cos[c + d*x]^(3/2))/(3*(a^2 - b^2)) + ((1/8 + I/8)*(2*Arc
Tan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan
[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a
^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos
[c + d*x]] + Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt
[Cos[c + d*x]] + I*b*Cos[c + d*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)))*Sin[c +
d*x])/(Sqrt[1 - Cos[c + d*x]^2]*(a + b*Sin[c + d*x])) - ((45*a^3 - 37*a*b^2
)*(a + b*Sqrt[1 - Cos[c + d*x]^2])*(8*b^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, C
os[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Cos[c + d*x]^(3/2) + 3*Sq
rt[2]*a*(a^2 - b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]]
)/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a
^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*
Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b
]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]]))*Sin[c + d*x]^2)/
(12*b^(3/2)*(-a^2 + b^2)*(1 - Cos[c + d*x]^2)*(a + b*Sin[c + d*x]))))/(40*b
^5*d*Cos[c + d*x]^(13/2)) + ((e*Cos[c + d*x])^(13/2)*Sec[c + d*x]^6*(-1/42*
((-168*a^2 + 65*b^2)*Cos[c + d*x])/b^5 - Cos[3*(c + d*x)]/(14*b^3) + (-(a^4
*Cos[c + d*x]) + 2*a^2*b^2*Cos[c + d*x] - b^4*Cos[c + d*x])/(2*b^5*(a + b*S
in[c + d*x])^2) + (19*(a^3*Cos[c + d*x] - a*b^2*Cos[c + d*x]))/(4*b^5*(a +
b*Sin[c + d*x])) - (3*a*Sin[2*(c + d*x)])/(5*b^4)))/d
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 112.65, size = 109325, normalized size = 190.13

method	result	size
default	Expression too large to display	109325

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(13/2)/(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(13/2)/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] e^(13/2)*integrate(cos(d*x + c)^(13/2)/(b*sin(d*x + c) + a)^3, x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(13/2)/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(13/2)/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(13/2)/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\cos(c+dx))^{13/2}}{(a+b\sin(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(13/2)/(a + b*sin(c + d*x))^3,x)
```

```
[Out] int((e*cos(c + d*x))^(13/2)/(a + b*sin(c + d*x))^3, x)
```

3.595 $\int \frac{(e\cos(c+dx))^{11/2}}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=589

$$\frac{9(7a^4-9a^2b^2+2b^4)\,e^{11/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{8b^{11/2}\left(-a^2+b^2\right)^{3/4}d}+\frac{9(7a^4-9a^2b^2+2b^4)\,e^{11/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c-}}{\sqrt[4]{-a^2+b^2}}\right.}{8b^{11/2}\left(-a^2+b^2\right)^{3/4}d}$$

```
[Out] 9/8*(7*a^4-9*a^2*b^2+2*b^4)*e^(11/2)*arctan(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-
a^2+b^2)^(1/4)/e^(1/2))/b^(11/2)/(-a^2+b^2)^(3/4)/d+9/8*(7*a^4-9*a^2*b^2+2*
b^4)*e^(11/2)*arctanh(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2)
)/b^(11/2)/(-a^2+b^2)^(3/4)/d-1/2*e*(e*cos(d*x+c))^(9/2)/b/d/(a+b*sin(d*x+c
))^2-9/20*e^3*(e*cos(d*x+c))^(5/2)*(7*a+2*b*sin(d*x+c))/b^3/d/(a+b*sin(d*x+
c))+3/4*a*(21*a^2-13*b^2)*e^6*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*
c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/b^6/d/(e*cos(d*x+
c))^(1/2)-9/8*a*(7*a^4-9*a^2*b^2+2*b^4)*e^6*(cos(1/2*d*x+1/2*c)^2)^(1/2)/co
s(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(b-(-a^2+b^2)^(1/2)),2^(
1/2))*cos(d*x+c)^(1/2)/b^6/d/(a^2-b*(b-(-a^2+b^2)^(1/2)))/(e*cos(d*x+c))^(1
/2)-9/8*a*(7*a^4-9*a^2*b^2+2*b^4)*e^6*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*
d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*
cos(d*x+c)^(1/2)/b^6/d/(a^2-b*(b+(-a^2+b^2)^(1/2)))/(e*cos(d*x+c))^(1/2)+3/
4*e^5*(21*a^2-6*b^2-7*a*b*sin(d*x+c))*(e*cos(d*x+c))^(1/2)/b^5/d
```

Rubi [A]
time = 1.03, antiderivative size = 589, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$,
Rules used = {2772, 2942, 2944, 2946, 2721, 2720, 2781, 2886, 2884, 335, 218, 214, 211}

$$\frac{3ae^6(21a^2-13b^2)\sqrt{\cos(c+dx)}\,F\left(\left.\frac{1}{2}(c+dx)\right|2\right)}{4b^6d\sqrt{e\cos(c+dx)}}+\frac{3e^5\sqrt{e\cos(c+dx)}\,(3(7a^2-2b^2)-7ab\sin(c+dx))}{4b^5d}+\frac{9e^{11/2}(7a^4-9a^2b^2+2b^4)\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{8b^{11/2}d\left(b^2-a^2\right)^{3/4}}+\frac{9e^{11/2}(7a^4-9a^2b^2+2b^4)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{8b^{11/2}d\left(b^2-a^2\right)^{3/4}}-\frac{9ae^6(7a^4-9a^2b^2+2b^4)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\left.\frac{1}{2}(c+dx)\right|2\right)}{8b^6d\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{e\cos(c+dx)}}-\frac{9ae^6(7a^4-9a^2b^2+2b^4)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\left.\frac{1}{2}(c+dx)\right|2\right)}{8b^6d\left(a^2-b\left(\sqrt{b^2-a^2}+b\right)\right)\sqrt{e\cos(c+dx)}}-\frac{9e^3(e\cos(c+dx))^{5/2}(7a+2b\sin(c+dx))}{20b^3d(a+b\sin(c+dx))}-\frac{e(e\cos(c+dx))^{9/2}}{2bd(a+b\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(11/2)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (9*(7*a^4 - 9*a^2*b^2 + 2*b^4)*e^(11/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]
])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(8*b^(11/2)*(-a^2 + b^2)^(3/4)*d) + (9*(7
*a^4 - 9*a^2*b^2 + 2*b^4)*e^(11/2)*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/(
(-a^2 + b^2)^(1/4)*Sqrt[e])])/(8*b^(11/2)*(-a^2 + b^2)^(3/4)*d) + (3*a*(21*
a^2 - 13*b^2)*e^6*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(4*b^6*d*Sq
rt[e*Cos[c + d*x]]) - (9*a*(7*a^4 - 9*a^2*b^2 + 2*b^4)*e^6*Sqrt[Cos[c + d*x
]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(8*b^6*(a^2 -
b*(b - Sqrt[-a^2 + b^2]))*d*Sqrt[e*Cos[c + d*x]]) - (9*a*(7*a^4 - 9*a^2*b^2
 + 2*b^4)*e^6*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (
c + d*x)/2, 2])/(8*b^6*(a^2 - b*(b + Sqrt[-a^2 + b^2]))*d*Sqrt[e*Cos[c + d*
```

```
x]]) - (e*(e*Cos[c + d*x])^(9/2))/(2*b*d*(a + b*Sin[c + d*x])^2) - (9*e^3*(
e*Cos[c + d*x])^(5/2)*(7*a + 2*b*Sin[c + d*x]))/(20*b^3*d*(a + b*Sin[c + d*
x])) + (3*e^5*Sqrt[e*Cos[c + d*x]]*(3*(7*a^2 - 2*b^2) - 7*a*b*Sin[c + d*x])
)/(4*b^5*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2772

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Dist[g^2*((p - 1)/(b*(m + 1))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
```

```
tegersQ[2*m, 2*p]
```

Rule 2781

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[-a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[b*(g/f), Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2942

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*C
os[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x])/(b^2*f*(m + 1)*(m + p + 1))), x] + Dist[g^2*((
p - 1)/(b^2*(m + 1)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rule 2944

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Dist[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
```

```
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{11/2}}{(a+b\sin(c+dx))^3}\,dx = -\frac{e(e\cos(c+dx))^{9/2}}{2bd(a+b\sin(c+dx))^2} - \frac{(9e^2)\int \frac{(e\cos(c+dx))^{7/2}\sin(c+dx)}{(a+b\sin(c+dx))^2}\,dx}{4b}$$

$$= -\frac{e(e\cos(c+dx))^{9/2}}{2bd(a+b\sin(c+dx))^2} - \frac{9e^3(e\cos(c+dx))^{5/2}(7a+2b\sin(c+dx))}{20b^3d(a+b\sin(c+dx))} + \frac{(9e^4)}{}$$

$$= -\frac{e(e\cos(c+dx))^{9/2}}{2bd(a+b\sin(c+dx))^2} - \frac{9e^3(e\cos(c+dx))^{5/2}(7a+2b\sin(c+dx))}{20b^3d(a+b\sin(c+dx))} + \frac{3e^5\sqrt{}}{}$$

$$= -\frac{e(e\cos(c+dx))^{9/2}}{2bd(a+b\sin(c+dx))^2} - \frac{9e^3(e\cos(c+dx))^{5/2}(7a+2b\sin(c+dx))}{20b^3d(a+b\sin(c+dx))} + \frac{3e^5\sqrt{}}{}$$

$$= -\frac{e(e\cos(c+dx))^{9/2}}{2bd(a+b\sin(c+dx))^2} - \frac{9e^3(e\cos(c+dx))^{5/2}(7a+2b\sin(c+dx))}{20b^3d(a+b\sin(c+dx))} + \frac{3e^5\sqrt{}}{}$$

$$= \frac{3a(21a^2-13b^2)\,e^6\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^6d\sqrt{e\cos(c+dx)}} - \frac{e(e\cos(c+dx))^{9/2}}{2bd(a+b\sin(c+dx))^2} -$$

$$= \frac{3a(21a^2-13b^2)\,e^6\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^6d\sqrt{e\cos(c+dx)}} - \frac{9a(7a^4-9a^2b^2+2b^4)\,e^6}{8b^6\left(a^2-b\left(b\right.\right.}$$

$$= \frac{9(7a^4-9a^2b^2+2b^4)\,e^{11/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{8b^{11/2}\left(-a^2+b^2\right)^{3/4}d} + \frac{9(7a^4-9a^2b^2+2b}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

time = 54.33, size = 2024, normalized size = 3.44

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^(11/2)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((e*Cos[c + d*x])^(11/2)*Sec[c + d*x]^5*(-1/5*Cos[2*(c + d*x)]/b^3 - (2*a*S
in[c + d*x])/b^4 - (-a^2 + b^2)^2/(2*b^5*(a + b*Sin[c + d*x])^2) + (17*a*(a
^2 - b^2))/(4*b^5*(a + b*Sin[c + d*x]))))/d + (3*(e*Cos[c + d*x])^(11/2)*((
-2*(30*a^2*b - 16*b^3)*(a + b*Sqrt[1 - Cos[c + d*x]^2])*((5*a*(a^2 - b^2)*A
ppellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)
]*Sqrt[Cos[c + d*x]])/(Sqrt[1 - Cos[c + d*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4
, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] - 2*(2*b^
2*AppellF1[5/4, 1/2, 2, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b
^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos[c +
 d*x]^2)/(-a^2 + b^2)])*Cos[c + d*x]^2)*(a^2 + b^2*(-1 + Cos[c + d*x]^2)))
- ((1/8 - I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(
-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^
2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)
*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]] - Log[Sqrt[-a^2 + b^2] + (1 + I)*Sq
rt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]]))/(-a^2 + b
^2)^(3/4))*Sin[c + d*x])/(Sqrt[1 - Cos[c + d*x]^2]*(a + b*Sin[c + d*x])) +
((40*a^2*b - 14*b^3)*(a + b*Sqrt[1 - Cos[c + d*x]^2])*Cos[2*(c + d*x)]*(((1
/2 - I/2)*(-2*a^2 + b^2)*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-
a^2 + b^2)^(1/4)])/(b^(3/2)*(-a^2 + b^2)^(3/4)) - ((1/2 - I/2)*(-2*a^2 + b^
2)*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)])/(b^
(3/2)*(-a^2 + b^2)^(3/4)) + (4*Sqrt[Cos[c + d*x]])/b - (4*a*AppellF1[5/4, 1
/2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Cos[c + d*x]
^(5/2))/(5*(a^2 - b^2)) + (10*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[
c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[c + d*x]])/(Sqrt[1
- Cos[c + d*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2,
 (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, C
os[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5
/4, 3/2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)])*Cos[c
+ d*x]^2)*(a^2 + b^2*(-1 + Cos[c + d*x]^2))) + ((1/4 - I/4)*(-2*a^2 + b^2)*
Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]
] + I*b*Cos[c + d*x]])/(b^(3/2)*(-a^2 + b^2)^(3/4)) - ((1/4 - I/4)*(-2*a^2
+ b^2)*Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c
 + d*x]] + I*b*Cos[c + d*x]])/(b^(3/2)*(-a^2 + b^2)^(3/4)))*Sin[c + d*x])/(
Sqrt[1 - Cos[c + d*x]^2]*(-1 + 2*Cos[c + d*x]^2)*(a + b*Sin[c + d*x])) - (2
*(25*a^3 - 37*a*b^2)*(a + b*Sqrt[1 - Cos[c + d*x]^2])*((5*b*(a^2 - b^2)*App
ellF1[1/4, -1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]
*Sqrt[Cos[c + d*x]]*Sqrt[1 - Cos[c + d*x]^2])/((-5*(a^2 - b^2)*AppellF1[1/4
, -1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + 2*(2*b
```

```
^2*AppellF1[5/4, -1/2, 2, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 +
 b^2)] + (a^2 - b^2)*AppellF1[5/4, 1/2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos[c
+ d*x]^2)/(-a^2 + b^2)])*Cos[c + d*x]^2)*(a^2 + b^2*(-1 + Cos[c + d*x]^2)))
 + (a*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)
] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] -
Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]]
+ b*Cos[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)
*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4
)))*Sin[c + d*x]^2)/((1 - Cos[c + d*x]^2)*(a + b*Sin[c + d*x]))))/(40*b^5*d
*Cos[c + d*x]^(11/2))
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 116.91, size = 78449, normalized size = 133.19

method	result	size
default	Expression too large to display	78449

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(11/2)/(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(11/2)/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] e^(11/2)*integrate(cos(d*x + c)^(11/2)/(b*sin(d*x + c) + a)^3, x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(11/2)/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(11/2)/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(11/2)/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(11/2)*e^(11/2)/(b*sin(d*x + c) + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,\cos\left(c+d\,x\right))^{11/2}}{(a+b\,\sin\left(c+d\,x\right))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(11/2)/(a + b*sin(c + d*x))^3,x)
```

```
[Out] int((e*cos(c + d*x))^(11/2)/(a + b*sin(c + d*x))^3, x)
```

3.596 $\int \frac{(e\cos(c+dx))^{9/2}}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=483

$$\frac{7(5a^2-2b^2)\,e^{9/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{8b^{9/2}\sqrt[4]{-a^2+b^2}\,d}-\frac{7(5a^2-2b^2)\,e^{9/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{8b^{9/2}\sqrt[4]{-a^2+b^2}\,d}-\frac{35ae^4}{}$$

```
[Out] 7/8*(5*a^2-2*b^2)*e^(9/2)*arctan(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1
/4)/e^(1/2))/b^(9/2)/(-a^2+b^2)^(1/4)/d-7/8*(5*a^2-2*b^2)*e^(9/2)*arctanh(b
^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(9/2)/(-a^2+b^2)^(1
/4)/d-1/2*e*(e*cos(d*x+c))^(7/2)/b/d/(a+b*sin(d*x+c))^2-7/12*e^3*(e*cos(d*x
+c))^(3/2)*(5*a+2*b*sin(d*x+c))/b^3/d/(a+b*sin(d*x+c))+7/8*a*(5*a^2-2*b^2)*
e^5*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+
1/2*c),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*cos(d*x+c)^(1/2)/b^5/d/(b-(-a^2+b^
2)^(1/2))/(e*cos(d*x+c))^(1/2)+7/8*a*(5*a^2-2*b^2)*e^5*(cos(1/2*d*x+1/2*c)^
2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(b+(-a^2+b^2)
^(1/2)),2^(1/2))*cos(d*x+c)^(1/2)/b^5/d/(b+(-a^2+b^2)^(1/2))/(e*cos(d*x+c))
^(1/2)-35/4*a*e^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE
(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/b^4/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.68, antiderivative size = 483, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {2772, 2942, 2946, 2721, 2719, 2780, 2886, 2884, 335, 304, 211, 214}

$$\frac{7e^{9/2}(5a^2-2b^2)\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{8b^{9/2}d\sqrt[4]{b^2-a^2}}-\frac{7e^{9/2}(5a^2-2b^2)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{8b^{9/2}d\sqrt[4]{b^2-a^2}}+\frac{7ae^5(5a^2-2b^2)\,\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{8b^5d\left(b-\sqrt{b^2-a^2}\right)\sqrt{e\cos(c+dx)}}+\frac{7ae^5(5a^2-2b^2)\,\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{8b^5d\left(\sqrt{b^2-a^2}+b\right)\sqrt{e\cos(c+dx)}}-\frac{35ae^4E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{4b^4d\sqrt{\cos(c+dx)}}-\frac{7e^3(e\cos(c+dx))^{3/2}(5a+2b\sin(c+dx))}{12b^3d(a+b\sin(c+dx))}-\frac{e(e\cos(c+dx))^{7/2}}{2bd(a+b\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(9/2)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (7*(5*a^2 - 2*b^2)*e^(9/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b
^2)^(1/4)*Sqrt[e])])/(8*b^(9/2)*(-a^2 + b^2)^(1/4)*d) - (7*(5*a^2 - 2*b^2)*
e^(9/2)*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])
])/(8*b^(9/2)*(-a^2 + b^2)^(1/4)*d) - (35*a*e^4*Sqrt[e*Cos[c + d*x]]*Ellipt
icE[(c + d*x)/2, 2])/(4*b^4*d*Sqrt[Cos[c + d*x]]) + (7*a*(5*a^2 - 2*b^2)*e^
5*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c + d*x)/2,
2])/(8*b^5*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e*Cos[c + d*x]]) + (7*a*(5*a^2 - 2
*b^2)*e^5*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c +
d*x)/2, 2])/(8*b^5*(b + Sqrt[-a^2 + b^2])*d*Sqrt[e*Cos[c + d*x]]) - (e*(e*C
os[c + d*x])^(7/2))/(2*b*d*(a + b*Sin[c + d*x])^2) - (7*e^3*(e*Cos[c + d*x]
)^(3/2)*(5*a + 2*b*Sin[c + d*x]))/(12*b^3*d*(a + b*Sin[c + d*x]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2772

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Dist[g^2*((p - 1)/(b*(m + 1))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2780

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[a*(g/(2*b)), Int[1/(Sq
```

```
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[a*(g/(2*b)), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[b*(g/f), Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2942

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*C
os[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x])/(b^2*f*(m + 1)*(m + p + 1))), x] + Dist[g^2*((
p - 1)/(b^2*(m + 1)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{9/2}}{(a+b\sin(c+dx))^3}\,dx = -\frac{e(e\cos(c+dx))^{7/2}}{2bd(a+b\sin(c+dx))^2} - \frac{(7e^2)\int \frac{(e\cos(c+dx))^{5/2}\sin(c+dx)}{(a+b\sin(c+dx))^2}\,dx}{4b}$$

$$= -\frac{e(e\cos(c+dx))^{7/2}}{2bd(a+b\sin(c+dx))^2} - \frac{7e^3(e\cos(c+dx))^{3/2}(5a+2b\sin(c+dx))}{12b^3d(a+b\sin(c+dx))} + \frac{(7e^4)\int \cdots}{\cdots}$$

$$= -\frac{e(e\cos(c+dx))^{7/2}}{2bd(a+b\sin(c+dx))^2} - \frac{7e^3(e\cos(c+dx))^{3/2}(5a+2b\sin(c+dx))}{12b^3d(a+b\sin(c+dx))} - \frac{(35ae^4 \cdots}{\cdots}$$

$$= -\frac{e(e\cos(c+dx))^{7/2}}{2bd(a+b\sin(c+dx))^2} - \frac{7e^3(e\cos(c+dx))^{3/2}(5a+2b\sin(c+dx))}{12b^3d(a+b\sin(c+dx))} - \frac{(7a(5a \cdots}{\cdots}$$

$$= -\frac{35ae^4\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^4d\sqrt{\cos(c+dx)}} - \frac{e(e\cos(c+dx))^{7/2}}{2bd(a+b\sin(c+dx))^2} - \frac{7e^3(e\cos(\cdots}{1 \cdots}$$

$$= -\frac{35ae^4\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^4d\sqrt{\cos(c+dx)}} + \frac{7a(5a^2-2b^2)\,e^5\sqrt{\cos(c+dx)}\,\Pi\Big(\cdots}{8b^5\left(b-\sqrt{-a^2+b^2}\right)d \cdots}$$

$$= \frac{7(5a^2-2b^2)\,e^{9/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{8b^{9/2}\sqrt[4]{-a^2+b^2}\,d} - \frac{7(5a^2-2b^2)\,e^{9/2}\tanh^{-1}\Big(\cdots}{8b^{9/2}\sqrt[4]{-a^2+ \cdots}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 28.17, size = 777, normalized size = 1.61

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^(9/2)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((e*Cos[c + d*x])^(9/2)*((-16*Cos[c + d*x]^(3/2))/(3*b^3) + (4*(a^2 - b^2)*
Cos[c + d*x]^(3/2))/(b^3*(a + b*Sin[c + d*x])^2) - (22*a*Cos[c + d*x]^(3/2)
)/(b^3*(a + b*Sin[c + d*x])) + (35*a*(8*b^(5/2)*AppellF1[3/4, -1/2, 1, 7/4,
 Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Cos[c + d*x]^(3/2) + 3*
Sqrt[2]*a*(a^2 - b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x
]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/
(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4
```

```
)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt
[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]]))*(a + b*Sqrt[Si
n[c + d*x]^2]))/(12*b^(9/2)*(-a^2 + b^2)*(a + b*Sin[c + d*x])) + (28*((a*Ap
pellF1[3/4, 1/2, 1, 7/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]
*Cos[c + d*x]^(3/2))/(3*(a^2 - b^2)) + ((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*
Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqr
t[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - (1 +
I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]] + Log[
Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] +
I*b*Cos[c + d*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)))*Sin[c + d*x]*(a + b*Sqrt[
Sin[c + d*x]^2]))/(b^2*Sqrt[Sin[c + d*x]^2]*(a + b*Sin[c + d*x]))))/(8*d*Co
s[c + d*x]^(9/2))
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 91.50, size = 84670, normalized size = 175.30

method	result	size
default	Expression too large to display	84670

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(9/2)/(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(9/2)/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
[Out] e^(9/2)*integrate(cos(d*x + c)^(9/2)/(b*sin(d*x + c) + a)^3, x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(9/2)/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(9/2)/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8857 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(9/2)/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(9/2)*e^(9/2)/(b*sin(d*x + c) + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,\cos(c+d\,x))^{9/2}}{(a+b\,\sin(c+d\,x))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(9/2)/(a + b*sin(c + d*x))^3,x)
```

```
[Out] int((e*cos(c + d*x))^(9/2)/(a + b*sin(c + d*x))^3, x)
```

3.597 $\int \frac{(e\cos(c+dx))^{7/2}}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=497

$$-\frac{5(3a^2-2b^2)\,e^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{8b^{7/2}\left(-a^2+b^2\right)^{3/4}d}-\frac{5(3a^2-2b^2)\,e^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{8b^{7/2}\left(-a^2+b^2\right)^{3/4}d}-\frac{15ae}{}$$

```
[Out] -5/8*(3*a^2-2*b^2)*e^(7/2)*arctan(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(
1/4)/e^(1/2))/b^(7/2)/(-a^2+b^2)^(3/4)/d-5/8*(3*a^2-2*b^2)*e^(7/2)*arctanh(
b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(7/2)/(-a^2+b^2)^(
3/4)/d-1/2*e*(e*cos(d*x+c))^(5/2)/b/d/(a+b*sin(d*x+c))^2-15/4*a*e^4*(cos(1/
2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/
2))*cos(d*x+c)^(1/2)/b^4/d/(e*cos(d*x+c))^(1/2)+5/8*a*(3*a^2-2*b^2)*e^4*(co
s(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),
2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*cos(d*x+c)^(1/2)/b^4/d/(a^2-b*(b-(-a^2+b^
2)^(1/2)))/(e*cos(d*x+c))^(1/2)+5/8*a*(3*a^2-2*b^2)*e^4*(cos(1/2*d*x+1/2*c)
^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(b+(-a^2+b^2
)^(1/2)),2^(1/2))*cos(d*x+c)^(1/2)/b^4/d/(a^2-b*(b+(-a^2+b^2)^(1/2)))/(e*co
s(d*x+c))^(1/2)-5/4*e^3*(3*a+2*b*sin(d*x+c))*(e*cos(d*x+c))^(1/2)/b^3/d/(a+
b*sin(d*x+c))
```

Rubi [A]

time = 0.74, antiderivative size = 497, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {2772, 2942, 2946, 2721, 2720, 2781, 2886, 2884, 335, 218, 214, 211}

$$-\frac{5e^{7/2}(3a^2-2b^2)\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{8b^{7/2}d\left(b^2-a^2\right)^{3/4}}-\frac{5e^{7/2}(3a^2-2b^2)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{8b^{7/2}d\left(b^2-a^2\right)^{3/4}}+\frac{5ae^4(3a^2-2b^2)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{8b^4d\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{e\cos(c+dx)}}+\frac{5ae^4(3a^2-2b^2)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{8b^4d\left(a^2-b\left(\sqrt{b^2-a^2}+b\right)\right)\sqrt{e\cos(c+dx)}}-\frac{15ae^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^4d\sqrt{e\cos(c+dx)}}-\frac{5e^3\sqrt{e\cos(c+dx)}\,(3a+2b\sin(c+dx))}{4b^3d(a+b\sin(c+dx))}-\frac{e(e\cos(c+dx))^{5/2}}{2bd(a+b\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(7/2)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-5*(3*a^2 - 2*b^2)*e^(7/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 +
b^2)^(1/4)*Sqrt[e])])/(8*b^(7/2)*(-a^2 + b^2)^(3/4)*d) - (5*(3*a^2 - 2*b^2)
*e^(7/2)*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e]
)])/(8*b^(7/2)*(-a^2 + b^2)^(3/4)*d) - (15*a*e^4*Sqrt[Cos[c + d*x]]*Ellipti
cF[(c + d*x)/2, 2])/(4*b^4*d*Sqrt[e*Cos[c + d*x]]) + (5*a*(3*a^2 - 2*b^2)*e
^4*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c + d*x)/2,
 2])/(8*b^4*(a^2 - b*(b - Sqrt[-a^2 + b^2]))*d*Sqrt[e*Cos[c + d*x]]) + (5*a
*(3*a^2 - 2*b^2)*e^4*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b
^2]), (c + d*x)/2, 2])/(8*b^4*(a^2 - b*(b + Sqrt[-a^2 + b^2]))*d*Sqrt[e*Cos
[c + d*x]]) - (e*(e*Cos[c + d*x])^(5/2))/(2*b*d*(a + b*Sin[c + d*x])^2) - (
5*e^3*Sqrt[e*Cos[c + d*x]]*(3*a + 2*b*Sin[c + d*x]))/(4*b^3*d*(a + b*Sin[c
+ d*x]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2772

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Dist[g^2*((p - 1)/(b*(m + 1))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2781

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[-a/(2*q), Int[1/(S
```

```
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[b*(g/f), Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2942

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*C
os[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x])/(b^2*f*(m + 1)*(m + p + 1))), x] + Dist[g^2*((
p - 1)/(b^2*(m + 1)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{7/2}}{(a+b\sin(c+dx))^3}\,dx = -\frac{e(e\cos(c+dx))^{5/2}}{2bd(a+b\sin(c+dx))^2} - \frac{(5e^2)\int \frac{(e\cos(c+dx))^{3/2}\sin(c+dx)}{(a+b\sin(c+dx))^2}\,dx}{4b}$$

$$= -\frac{e(e\cos(c+dx))^{5/2}}{2bd(a+b\sin(c+dx))^2} - \frac{5e^3\sqrt{e\cos(c+dx)}\,(3a+2b\sin(c+dx))}{4b^3d(a+b\sin(c+dx))} + \frac{(5e^4)\int}{}$$

$$= -\frac{e(e\cos(c+dx))^{5/2}}{2bd(a+b\sin(c+dx))^2} - \frac{5e^3\sqrt{e\cos(c+dx)}\,(3a+2b\sin(c+dx))}{4b^3d(a+b\sin(c+dx))} - \frac{(15ae^4)}{}$$

$$= -\frac{e(e\cos(c+dx))^{5/2}}{2bd(a+b\sin(c+dx))^2} - \frac{5e^3\sqrt{e\cos(c+dx)}\,(3a+2b\sin(c+dx))}{4b^3d(a+b\sin(c+dx))} - \frac{(5a(3a^2}{}$$

$$= -\frac{15ae^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{4b^4d\sqrt{e\cos(c+dx)}} - \frac{e(e\cos(c+dx))^{5/2}}{2bd(a+b\sin(c+dx))^2} - \frac{5e^3\sqrt{e\cos(}}{4b}$$

$$= -\frac{15ae^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{4b^4d\sqrt{e\cos(c+dx)}} + \frac{5a(3a^2-2b^2)\,e^4\sqrt{\cos(c+dx)}\,\Pi\left(\frac{}{b-}\right.}{8b^4\left(a^2-b\left(b-\sqrt{-a^2+b^2}\right)\right.}$$

$$= -\frac{5(3a^2-2b^2)\,e^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{8b^{7/2}\left(-a^2+b^2\right)^{3/4}d} - \frac{5(3a^2-2b^2)\,e^{7/2}\tanh^{-1}\left(\right.}{8b^{7/2}\left(-a^2+\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 51.09, size = 1954, normalized size = 3.93

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^(7/2)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((e*Cos[c + d*x])^(7/2)*Sec[c + d*x]^3*((a^2 - b^2)/(2*b^3*(a + b*Sin[c + d
*x])^2) - (9*a)/(4*b^3*(a + b*Sin[c + d*x]))))/d - ((e*Cos[c + d*x])^(7/2)*
((-12*b*(a + b*Sqrt[1 - Cos[c + d*x]^2])*((5*a*(a^2 - b^2)*AppellF1[1/4, 1/
2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[c +
d*x]])/(Sqrt[1 - Cos[c + d*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4,
Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4,
 1/2, 2, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + (-a^2 +
b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2
```

```
+ b^2)])*Cos[c + d*x]^2)*(a^2 + b^2*(-1 + Cos[c + d*x]^2))) - ((1/8 - I/8)*
Sqrt[b]*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/
4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)]
 + Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d
*x]] + I*b*Cos[c + d*x]] - Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b
^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]]))/(-a^2 + b^2)^(3/4))*Sin[
c + d*x])/(Sqrt[1 - Cos[c + d*x]^2]*(a + b*Sin[c + d*x])) + (4*b*(a + b*Sqr
t[1 - Cos[c + d*x]^2])*Cos[2*(c + d*x)]*(((1/2 - I/2)*(-2*a^2 + b^2)*ArcTan
[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)])/(b^(3/2)*(-a
^2 + b^2)^(3/4)) - ((1/2 - I/2)*(-2*a^2 + b^2)*ArcTan[1 + ((1 + I)*Sqrt[b]*
Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)])/(b^(3/2)*(-a^2 + b^2)^(3/4)) + (4*
Sqrt[Cos[c + d*x]])/b - (4*a*AppellF1[5/4, 1/2, 1, 9/4, Cos[c + d*x]^2, (b^
2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Cos[c + d*x]^(5/2))/(5*(a^2 - b^2)) + (10*a
*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2
)/(-a^2 + b^2)]*Sqrt[Cos[c + d*x]])/(Sqrt[1 - Cos[c + d*x]^2]*(5*(a^2 - b^2
)*AppellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b
^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x
]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[c + d*x]^2
, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)])*Cos[c + d*x]^2)*(a^2 + b^2*(-1 + Cos[
c + d*x]^2))) + ((1/4 - I/4)*(-2*a^2 + b^2)*Log[Sqrt[-a^2 + b^2] - (1 + I)*
Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]])/(b^(3/2)
*(-a^2 + b^2)^(3/4)) - ((1/4 - I/4)*(-2*a^2 + b^2)*Log[Sqrt[-a^2 + b^2] + (
1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]])/(
b^(3/2)*(-a^2 + b^2)^(3/4)))*Sin[c + d*x])/(Sqrt[1 - Cos[c + d*x]^2]*(-1 +
2*Cos[c + d*x]^2)*(a + b*Sin[c + d*x])) - (14*a*(a + b*Sqrt[1 - Cos[c + d*x
]^2])*((5*b*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Co
s[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[c + d*x]]*Sqrt[1 - Cos[c + d*x]^2])/((
-5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x
]^2)/(-a^2 + b^2)] + 2*(2*b^2*AppellF1[5/4, -1/2, 2, 9/4, Cos[c + d*x]^2, (
b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + (a^2 - b^2)*AppellF1[5/4, 1/2, 1, 9/4,
Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)])*Cos[c + d*x]^2)*(a^2 +
b^2*(-1 + Cos[c + d*x]^2))) + (a*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c
 + d*x]])/(a^2 - b^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d
*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2
)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2
]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]]))/(4*Sqrt[
2]*Sqrt[b]*(a^2 - b^2)^(3/4)))*Sin[c + d*x]^2)/((1 - Cos[c + d*x]^2)*(a + b
*Sin[c + d*x]))))/(8*b^3*d*Cos[c + d*x]^(7/2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 90.85, size = 60431, normalized size = 121.59

method	result	size
default	Expression too large to display	60431

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(7/2)/(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
[Out] e^(7/2)*integrate(cos(d*x + c)^(7/2)/(b*sin(d*x + c) + a)^3, x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(7/2)/(a+b*sin(d*x+c))**3,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3878 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+b*sin(d*x+c))^3,x, algorithm="giac")
[Out] integrate(cos(d*x + c)^(7/2)*e^(7/2)/(b*sin(d*x + c) + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,\cos\left(c+d\,x\right))^{7/2}}{\left(a+b\,\sin\left(c+d\,x\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(7/2)/(a + b*sin(c + d*x))^3,x)
[Out] int((e*cos(c + d*x))^(7/2)/(a + b*sin(c + d*x))^3, x)
```

3.598 $\int \frac{(e\cos(c+dx))^{5/2}}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=505

$$\frac{3(a^2-2b^2)\,e^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{8b^{5/2}\left(-a^2+b^2\right)^{5/4}d}-\frac{3(a^2-2b^2)\,e^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{8b^{5/2}\left(-a^2+b^2\right)^{5/4}d}+\frac{3ae^2\sqrt{e\,c}}{4b^2\,(}$$

```
[Out] 3/8*(a^2-2*b^2)*e^(5/2)*arctan(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1/4
)/e^(1/2))/b^(5/2)/(-a^2+b^2)^(5/4)/d-3/8*(a^2-2*b^2)*e^(5/2)*arctanh(b^(1/
2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(5/2)/(-a^2+b^2)^(5/4)/
d-1/2*e*(e*cos(d*x+c))^(3/2)/b/d/(a+b*sin(d*x+c))^2+3/4*a*e*(e*cos(d*x+c))^
(3/2)/b/(a^2-b^2)/d/(a+b*sin(d*x+c))-3/8*a*(a^2-2*b^2)*e^3*(cos(1/2*d*x+1/2
*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(b-(-a^2+
b^2)^(1/2)),2^(1/2))*cos(d*x+c)^(1/2)/b^3/(a^2-b^2)/d/(b-(-a^2+b^2)^(1/2))/
(e*cos(d*x+c))^(1/2)-3/8*a*(a^2-2*b^2)*e^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos
(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(b+(-a^2+b^2)^(1/2)),2^(1
/2))*cos(d*x+c)^(1/2)/b^3/(a^2-b^2)/d/(b+(-a^2+b^2)^(1/2))/(e*cos(d*x+c))^(
1/2)+3/4*a*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(si
n(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/b^2/(a^2-b^2)/d/cos(d*x+c)^(
1/2)
```

Rubi [A]
time = 0.73, antiderivative size = 505, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {2772, 2943, 2946, 2721, 2719, 2780, 2886, 2884, 335, 304, 211, 214}

$$\frac{3e^{5/2}(a^2-2b^2)\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{8b^{5/2}d\,(b^2-a^2)^{5/4}}+\frac{3ae^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{4b^2d\,(a^2-b^2)\sqrt{\cos(c+dx)}}+\frac{3ae(e\cos(c+dx))^{3/2}}{4bd\,(a^2-b^2)\,(a+b\sin(c+dx))}-\frac{3e^{5/2}(a^2-2b^2)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{8b^{5/2}d\,(b^2-a^2)^{5/4}}-\frac{3ae^3(a^2-2b^2)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{8b^3d\,(a^2-b^2)\left(b-\sqrt{b^2-a^2}\right)\sqrt{e\cos(c+dx)}}-\frac{3ae^3(a^2-2b^2)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{8b^3d\,(a^2-b^2)\left(\sqrt{b^2-a^2}+b\right)\sqrt{e\cos(c+dx)}}-\frac{e(e\cos(c+dx))^{3/2}}{2bd(a+b\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(5/2)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (3*(a^2 - 2*b^2)*e^(5/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2
)^(1/4)*Sqrt[e])])/(8*b^(5/2)*(-a^2 + b^2)^(5/4)*d) - (3*(a^2 - 2*b^2)*e^(5
/2)*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(
8*b^(5/2)*(-a^2 + b^2)^(5/4)*d) + (3*a*e^2*Sqrt[e*Cos[c + d*x]]*EllipticE[(
c + d*x)/2, 2])/(4*b^2*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]) - (3*a*(a^2 - 2*b^
2)*e^3*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c + d*x
)/2, 2])/(8*b^3*(a^2 - b^2)*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e*Cos[c + d*x]])
- (3*a*(a^2 - 2*b^2)*e^3*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2
 + b^2]), (c + d*x)/2, 2])/(8*b^3*(a^2 - b^2)*(b + Sqrt[-a^2 + b^2])*d*Sqrt
[e*Cos[c + d*x]]) - (e*(e*Cos[c + d*x])^(3/2))/(2*b*d*(a + b*Sin[c + d*x])^
2) + (3*a*e*(e*Cos[c + d*x])^(3/2))/(4*b*(a^2 - b^2)*d*(a + b*Sin[c + d*x])
)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2772

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Dist[g^2*((p - 1)/(b*(m + 1))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2780

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[a*(g/(2*b)), Int[1/(Sq
```

```
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[a*(g/(2*b)), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[b*(g/f), Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2943

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c
 - a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^(m + 1)/(f*g*(a^2 -
 b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*
(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p
+ 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[
a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{5/2}}{(a+b\sin(c+dx))^3}\,dx = -\frac{e(e\cos(c+dx))^{3/2}}{2bd(a+b\sin(c+dx))^2} - \frac{(3e^2)\int \frac{\sqrt{e\cos(c+dx)}\sin(c+dx)}{(a+b\sin(c+dx))^2}\,dx}{4b}$$

$$= -\frac{e(e\cos(c+dx))^{3/2}}{2bd(a+b\sin(c+dx))^2} + \frac{3ae(e\cos(c+dx))^{3/2}}{4b(a^2-b^2)\,d(a+b\sin(c+dx))} + \frac{(3e^2)\int \sqrt{e\cos(c}}{4b}$$

$$= -\frac{e(e\cos(c+dx))^{3/2}}{2bd(a+b\sin(c+dx))^2} + \frac{3ae(e\cos(c+dx))^{3/2}}{4b(a^2-b^2)\,d(a+b\sin(c+dx))} + \frac{(3ae^2)\int \sqrt{e\cos(}}{8b^2(a^2-}$$

$$= -\frac{e(e\cos(c+dx))^{3/2}}{2bd(a+b\sin(c+dx))^2} + \frac{3ae(e\cos(c+dx))^{3/2}}{4b(a^2-b^2)\,d(a+b\sin(c+dx))} + \frac{(3a(a^2-2b^2)\,e^3}{}$$

$$= \frac{3ae^2\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^2(a^2-b^2)\,d\sqrt{\cos(c+dx)}} - \frac{e(e\cos(c+dx))^{3/2}}{2bd(a+b\sin(c+dx))^2} + \frac{3ae(e\,c}{4b(a^2-b^2)\,c}$$

$$= \frac{3ae^2\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^2(a^2-b^2)\,d\sqrt{\cos(c+dx)}} - \frac{3a(a^2-2b^2)\,e^3\sqrt{\cos(c+dx)}\,\Pi\left(\frac{}{b-\sqrt{-}}\right.}{8b^3(a^2-b^2)\left(b-\sqrt{-a^2+b^2}\right)}$$

$$= \frac{3(a^2-2b^2)\,e^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{8b^{5/2}(-a^2+b^2)^{5/4}\,d} - \frac{3(a^2-2b^2)\,e^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt[4]{-}}\right.}{8b^{5/2}(-a^2+b^2)^{5}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 30.71, size = 831, normalized size = 1.65

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^(5/2)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((e*Cos[c + d*x])^(5/2)*Sec[c + d*x]^2*(-1/2*Cos[c + d*x]/(b*(a + b*Sin[c +
 d*x])^2) - (3*a*Cos[c + d*x])/(4*b*(-a^2 + b^2)*(a + b*Sin[c + d*x]))))/d
+ (3*(e*Cos[c + d*x])^(5/2)*((-4*b*(a + b*Sqrt[1 - Cos[c + d*x]^2])*((a*App
ellF1[3/4, 1/2, 1, 7/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*
Cos[c + d*x]^(3/2))/(3*(a^2 - b^2)) + ((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*S
qrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt
[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - (1 + I
```

```
)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]] + Log[S
qrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I
*b*Cos[c + d*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)))*Sin[c + d*x])/(Sqrt[1 - Co
s[c + d*x]^2]*(a + b*Sin[c + d*x])) - (a*(a + b*Sqrt[1 - Cos[c + d*x]^2])*(
8*b^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/
(-a^2 + b^2)]*Cos[c + d*x]^(3/2) + 3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*(2*ArcTan[
1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 +
(Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^
2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]]
 + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x
]] + b*Cos[c + d*x]]))*Sin[c + d*x]^2)/(12*b^(3/2)*(-a^2 + b^2)*(1 - Cos[c
+ d*x]^2)*(a + b*Sin[c + d*x]))))/(8*(a - b)*b*(a + b)*d*Cos[c + d*x]^(5/2)
)
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 93.46, size = 60162, normalized size = 119.13

method	result	size
default	Expression too large to display	60162

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(5/2)/(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] e^(5/2)*integrate(cos(d*x + c)^(5/2)/(b*sin(d*x + c) + a)^3, x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(5/2)/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(5/2)*e^(5/2)/(b*sin(d*x + c) + a)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,\cos\left(c+d\,x\right))^{5/2}}{\left(a+b\,\sin\left(c+d\,x\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(5/2)/(a + b*sin(c + d*x))^3,x)
```

```
[Out] int((e*cos(c + d*x))^(5/2)/(a + b*sin(c + d*x))^3, x)
```

3.599 $\int \frac{(e\cos(c+dx))^{3/2}}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=519

$$\frac{(a^2+2b^2)\,e^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{8b^{3/2}\left(-a^2+b^2\right)^{7/4}d}+\frac{(a^2+2b^2)\,e^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{8b^{3/2}\left(-a^2+b^2\right)^{7/4}d}-\frac{ae^2\sqrt{\cos(c}}{4b^2\,(a^2-}$$

```
[Out] 1/8*(a^2+2*b^2)*e^(3/2)*arctan(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1/4
)/e^(1/2))/b^(3/2)/(-a^2+b^2)^(7/4)/d+1/8*(a^2+2*b^2)*e^(3/2)*arctanh(b^(1/
2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(3/2)/(-a^2+b^2)^(7/4)/
d-1/4*a*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1
/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/b^2/(a^2-b^2)/d/(e*cos(d*x+c))^(1/2
)+1/8*a*(a^2+2*b^2)*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ell
ipticPi(sin(1/2*d*x+1/2*c),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*cos(d*x+c)^(1/
2)/b^2/(a^2-b^2)/d/(a^2-b*(b-(-a^2+b^2)^(1/2)))/(e*cos(d*x+c))^(1/2)+1/8*a*
(a^2+2*b^2)*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(
sin(1/2*d*x+1/2*c),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(d*x+c)^(1/2)/b^2/(
a^2-b^2)/d/(a^2-b*(b+(-a^2+b^2)^(1/2)))/(e*cos(d*x+c))^(1/2)-1/2*e*(e*cos(d
*x+c))^(1/2)/b/d/(a+b*sin(d*x+c))^2+1/4*a*e*(e*cos(d*x+c))^(1/2)/b/(a^2-b^2
)/d/(a+b*sin(d*x+c))
```

Rubi [A]
time = 0.77, antiderivative size = 519, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {2772, 2943, 2946, 2721, 2720, 2781, 2886, 2884, 335, 218, 214, 211}

$$\frac{e^{3/2}(a^2+2b^2)\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{8b^{3/2}d\left(b^2-a^2\right)^{7/4}}-\frac{ae^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^2d\left(a^2-b^2\right)\sqrt{e\cos(c+dx)}}+\frac{ae^2(a^2+2b^2)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{8b^2d\left(a^2-b^2\right)\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{e\cos(c+dx)}}+\frac{ae^2(a^2+2b^2)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{8b^2d\left(a^2-b^2\right)\left(a^2-b\left(\sqrt{b^2-a^2}+b\right)\right)\sqrt{e\cos(c+dx)}}+\frac{ae\sqrt{e\cos(c+dx)}}{4bd\left(a^2-b^2\right)(a+b\sin(c+dx))}+\frac{e^{3/2}(a^2+2b^2)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{8b^{3/2}d\left(b^2-a^2\right)^{7/4}}-\frac{e\sqrt{e\cos(c+dx)}}{2bd(a+b\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(3/2)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((a^2 + 2*b^2)*e^(3/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^
(1/4)*Sqrt[e])])/(8*b^(3/2)*(-a^2 + b^2)^(7/4)*d) + ((a^2 + 2*b^2)*e^(3/2)*
ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(8*b^
(3/2)*(-a^2 + b^2)^(7/4)*d) - (a*e^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)
/2, 2])/(4*b^2*(a^2 - b^2)*d*Sqrt[e*Cos[c + d*x]]) + (a*(a^2 + 2*b^2)*e^2*S
qrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])
/(8*b^2*(a^2 - b^2)*(a^2 - b*(b - Sqrt[-a^2 + b^2]))*d*Sqrt[e*Cos[c + d*x]]
) + (a*(a^2 + 2*b^2)*e^2*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2
 + b^2]), (c + d*x)/2, 2])/(8*b^2*(a^2 - b^2)*(a^2 - b*(b + Sqrt[-a^2 + b^2
]))*d*Sqrt[e*Cos[c + d*x]]) - (e*Sqrt[e*Cos[c + d*x]])/(2*b*d*(a + b*Sin[c
+ d*x])^2) + (a*e*Sqrt[e*Cos[c + d*x]])/(4*b*(a^2 - b^2)*d*(a + b*Sin[c + d
*x]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2772

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Dist[g^2*((p - 1)/(b*(m + 1))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2781

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[-a/(2*q), Int[1/(S
```

```
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[b*(g/f), Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2943

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c
 - a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^(m + 1)/(f*g*(a^2 -
 b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*
(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p
+ 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[
a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{3/2}}{(a+b\sin(c+dx))^3}\,dx = -\frac{e\sqrt{e\cos(c+dx)}}{2bd(a+b\sin(c+dx))^2} - \frac{e^2\int \frac{\sin(c+dx)}{\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))^2}\,dx}{4b}$$

$$= -\frac{e\sqrt{e\cos(c+dx)}}{2bd(a+b\sin(c+dx))^2} + \frac{ae\sqrt{e\cos(c+dx)}}{4b\,(a^2-b^2)\,d(a+b\sin(c+dx))} + \frac{e^2\int \frac{b-\frac{1}{2}}{\sqrt{e\cos(c+}}}{4b\,(}$$

$$= -\frac{e\sqrt{e\cos(c+dx)}}{2bd(a+b\sin(c+dx))^2} + \frac{ae\sqrt{e\cos(c+dx)}}{4b\,(a^2-b^2)\,d(a+b\sin(c+dx))} - \frac{(ae^2)\int \frac{1}{\sqrt{e\cos(}}}{8b^2\,(a^2-}$$

$$= -\frac{e\sqrt{e\cos(c+dx)}}{2bd(a+b\sin(c+dx))^2} + \frac{ae\sqrt{e\cos(c+dx)}}{4b\,(a^2-b^2)\,d(a+b\sin(c+dx))} + \frac{(a(a^2+2b^2)\,e^2)}{}$$

$$= -\frac{ae^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^2\,(a^2-b^2)\,d\sqrt{e\cos(c+dx)}} - \frac{e\sqrt{e\cos(c+dx)}}{2bd(a+b\sin(c+dx))^2} + \frac{ae\sqrt{e\,c}}{4b\,(a^2-b^2)\,d}$$

$$= -\frac{ae^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^2\,(a^2-b^2)\,d\sqrt{e\cos(c+dx)}} - \frac{a(a^2+2b^2)\,e^2\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{-a}}\right.}{8b^2\,(-a^2+b^2)^{3/2}\left(b-\sqrt{-a^2+b^2}\right)}$$

$$= \frac{(a^2+2b^2)\,e^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{8b^{3/2}\,(-a^2+b^2)^{7/4}\,d} + \frac{(a^2+2b^2)\,e^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e}}{\sqrt[4]{-a}}\right.}{8b^{3/2}\,(-a^2+b^2)^{7/4}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 41.82, size = 1211, normalized size = 2.33

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^(3/2)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((e*Cos[c + d*x])^(3/2)*Sec[c + d*x]*(-1/2*1/(b*(a + b*Sin[c + d*x])^2) - a
/(4*b*(-a^2 + b^2)*(a + b*Sin[c + d*x]))))/d - ((e*Cos[c + d*x])^(3/2)*((4*
b*(a + b*Sqrt[1 - Cos[c + d*x]^2])*((5*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1,
5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[c + d*x]])
/(Sqrt[1 - Cos[c + d*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[c
+ d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2,
2, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*A
```

```
ppellF1[5/4, 3/2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)
])*Cos[c + d*x]^2)*(a^2 + b^2*(-1 + Cos[c + d*x]^2))) - ((1/8 - I/8)*Sqrt[b
]*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] -
2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] + Log
[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] +
 I*b*Cos[c + d*x]] - Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1
/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]]))/(-a^2 + b^2)^(3/4))*Sin[c + d*
x])/(Sqrt[1 - Cos[c + d*x]^2]*(a + b*Sin[c + d*x])) - (2*a*(a + b*Sqrt[1 -
Cos[c + d*x]^2])*((5*b*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Cos[c + d*x]
^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[c + d*x]]*Sqrt[1 - Cos[c +
d*x]^2])/((-5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*
Cos[c + d*x]^2)/(-a^2 + b^2)] + 2*(2*b^2*AppellF1[5/4, -1/2, 2, 9/4, Cos[c
+ d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + (a^2 - b^2)*AppellF1[5/4, 1/
2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)])*Cos[c + d*x]
^2)*(a^2 + b^2*(-1 + Cos[c + d*x]^2))) + (a*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]
*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqr
t[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]
*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]] + Log[Sqrt[a^2 - b^
2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]]
))/(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)))*Sin[c + d*x]^2)/((1 - Cos[c + d*x
]^2)*(a + b*Sin[c + d*x]))))/(8*(a - b)*b*(a + b)*d*Cos[c + d*x]^(3/2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 94.08, size = 42544, normalized size = 81.97

method	result	size
default	Expression too large to display	42544

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
[Out] e^(3/2)*integrate(cos(d*x + c)^(3/2)/(b*sin(d*x + c) + a)^3, x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(3/2)/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4851 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(3/2)*e^(3/2)/(b*sin(d*x + c) + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(e\,\cos\left(c+d\,x\right)\right)}^{3/2}}{{\left(a+b\,\sin\left(c+d\,x\right)\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(3/2)/(a + b*sin(c + d*x))^3,x)
```

```
[Out] int((e*cos(c + d*x))^(3/2)/(a + b*sin(c + d*x))^3, x)
```

3.600 $\int \frac{\sqrt{e\cos(c+dx)}}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=514

$$\frac{(3a^2+2b^2)\sqrt{e}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{8\sqrt{b}\,(-a^2+b^2)^{9/4}\,d}-\frac{(3a^2+2b^2)\sqrt{e}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{8\sqrt{b}\,(-a^2+b^2)^{9/4}\,d}+\frac{5a\sqrt{e\,\mathrm{co}}}{4\,(a^2}$$

```
[Out] 1/2*b*(e*cos(d*x+c))^(3/2)/(a^2-b^2)/d/e/(a+b*sin(d*x+c))^2+5/4*a*b*(e*cos(
d*x+c))^(3/2)/(a^2-b^2)^2/d/e/(a+b*sin(d*x+c))+1/8*(3*a^2+2*b^2)*arctan(b^(
1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))*e^(1/2)/(-a^2+b^2)^(9/4
)/d/b^(1/2)-1/8*(3*a^2+2*b^2)*arctanh(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^
2)^(1/4)/e^(1/2))*e^(1/2)/(-a^2+b^2)^(9/4)/d/b^(1/2)+1/8*a*(3*a^2+2*b^2)*e*
(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*
c),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*cos(d*x+c)^(1/2)/b/(a^2-b^2)^2/d/(b-(-
a^2+b^2)^(1/2))/(e*cos(d*x+c))^(1/2)+1/8*a*(3*a^2+2*b^2)*e*(cos(1/2*d*x+1/2
*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(b+(-a^2+
b^2)^(1/2)),2^(1/2))*cos(d*x+c)^(1/2)/b/(a^2-b^2)^2/d/(b+(-a^2+b^2)^(1/2))/
(e*cos(d*x+c))^(1/2)+5/4*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*
EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/(a^2-b^2)^2/d/co
s(d*x+c)^(1/2)
```

Rubi [A]

time = 0.78, antiderivative size = 514, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {2773, 2943, 2946, 2721, 2719, 2780, 2886, 2884, 335, 304, 211, 214}

$$\frac{\sqrt{e}\,(3a^2+2b^2)\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{8\sqrt{b}\,d\,(b^2-a^2)^{9/4}}+\frac{5ab(e\cos(c+dx))^{3/2}}{4de\,(a^2-b^2)^2\,(a+b\sin(c+dx))}+\frac{b(e\cos(c+dx))^{3/2}}{2de\,(a^2-b^2)\,(a+b\sin(c+dx))^2}-\frac{\sqrt{e}\,(3a^2+2b^2)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{8\sqrt{b}\,d\,(b^2-a^2)^{9/4}}+\frac{5aE\left(\frac{1}{2}(c+dx)|2\right)\sqrt{e\cos(c+dx)}}{4d\,(a^2-b^2)^2\sqrt{\cos(c+dx)}}+\frac{ae(3a^2+2b^2)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)|2\right)}{8bd\,(a^2-b^2)^2\left(b-\sqrt{b^2-a^2}\right)\sqrt{e\cos(c+dx)}}+\frac{ae(3a^2+2b^2)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)|2\right)}{8bd\,(a^2-b^2)^2\left(\sqrt{b^2-a^2}+b\right)\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cos[c + d*x]]/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((3*a^2 + 2*b^2)*Sqrt[e]*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2
)^(1/4)*Sqrt[e])])/(8*Sqrt[b]*(-a^2 + b^2)^(9/4)*d) - ((3*a^2 + 2*b^2)*Sqrt
[e]*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(
8*Sqrt[b]*(-a^2 + b^2)^(9/4)*d) + (5*a*Sqrt[e*Cos[c + d*x]]*EllipticE[(c +
d*x)/2, 2])/(4*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]) + (a*(3*a^2 + 2*b^2)*e*S
qrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])
/(8*b*(a^2 - b^2)^2*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e*Cos[c + d*x]]) + (a*(3*
a^2 + 2*b^2)*e*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]),
(c + d*x)/2, 2])/(8*b*(a^2 - b^2)^2*(b + Sqrt[-a^2 + b^2])*d*Sqrt[e*Cos[c +
 d*x]]) + (b*(e*Cos[c + d*x])^(3/2))/(2*(a^2 - b^2)*d*e*(a + b*Sin[c + d*x]
)^2) + (5*a*b*(e*Cos[c + d*x])^(3/2))/(4*(a^2 - b^2)^2*d*e*(a + b*Sin[c + d
*x]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2773

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m + 1)/(f*g*(a^2 - b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1))
, Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p
 + 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^
2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2780

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[a*(g/(2*b)), Int[1/(Sq
```

```
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[a*(g/(2*b)), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[b*(g/f), Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2943

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c
 - a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^(m + 1)/(f*g*(a^2 -
 b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*
(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p
+ 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[
a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{e\cos(c+dx)}}{(a+b\sin(c+dx))^3}\,dx = \frac{b(e\cos(c+dx))^{3/2}}{2(a^2-b^2)\,de(a+b\sin(c+dx))^2} - \frac{\int \frac{\sqrt{e\cos(c+dx)}\left(-2a+\frac{1}{2}b\sin(c+dx)\right)}{(a+b\sin(c+dx))^2}\,dx}{2(a^2-b^2)}$$

$$= \frac{b(e\cos(c+dx))^{3/2}}{2(a^2-b^2)\,de(a+b\sin(c+dx))^2} + \frac{5ab(e\cos(c+dx))^{3/2}}{4(a^2-b^2)^2\,de(a+b\sin(c+dx))} + \frac{\int \sqrt{e\,c}\ \cdots}{\cdots}$$

$$= \frac{b(e\cos(c+dx))^{3/2}}{2(a^2-b^2)\,de(a+b\sin(c+dx))^2} + \frac{5ab(e\cos(c+dx))^{3/2}}{4(a^2-b^2)^2\,de(a+b\sin(c+dx))} + \frac{(5a)\int \cdots}{\cdots}$$

$$= \frac{b(e\cos(c+dx))^{3/2}}{2(a^2-b^2)\,de(a+b\sin(c+dx))^2} + \frac{5ab(e\cos(c+dx))^{3/2}}{4(a^2-b^2)^2\,de(a+b\sin(c+dx))} - \frac{(a(3a^2\cdots}{\cdots}$$

$$= \frac{5a\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4(a^2-b^2)^2\,d\sqrt{\cos(c+dx)}} + \frac{b(e\cos(c+dx))^{3/2}}{2(a^2-b^2)\,de(a+b\sin(c+dx))^2} + \frac{\cdots}{4(a^2\cdots}$$

$$= \frac{5a\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4(a^2-b^2)^2\,d\sqrt{\cos(c+dx)}} + \frac{a(3a^2+2b^2)\,e\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{-a^2\cdots}}\cdots\right.}{8b(a^2-b^2)^2\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{\cdots}}$$

$$= \frac{(3a^2+2b^2)\sqrt{e}\,\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8\sqrt{b}\,(-a^2+b^2)^{9/4}\,d} - \frac{(3a^2+2b^2)\sqrt{e}\,\tanh^{-1}\left(\frac{\sqrt{b}\cdots}{\sqrt[4]{\cdots}}\right.}{8\sqrt{b}\,(-a^2+b^2)\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 30.14, size = 748, normalized size = 1.46

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[e*Cos[c + d*x]]/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (Sqrt[e*Cos[c + d*x]]*((2*b*Cos[c + d*x]*(7*a^2 - 2*b^2 + 5*a*b*Sin[c + d*x
]))/((a^2 - b^2)^2*(a + b*Sin[c + d*x])^2) + (Sin[c + d*x]*((-5*a*Csc[c + d
*x]*(8*b^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x
]^2)/(-a^2 + b^2)]*Cos[c + d*x]^(3/2) + 3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*(2*Ar
cTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan
```

```
[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2
 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c +
d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c
+ d*x]] + b*Cos[c + d*x]])))/(Sqrt[b]*(-a^2 + b^2)) - (48*(4*a^2 + b^2)*((a
*AppellF1[3/4, 1/2, 1, 7/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^
2)]*Cos[c + d*x]^(3/2))/(3*(a^2 - b^2)) + ((1/8 + I/8)*(2*ArcTan[1 - ((1 +
I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*
Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - (1
 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]] + L
og[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]]
 + I*b*Cos[c + d*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4))))/Sqrt[Sin[c + d*x]^2])
*(a + b*Sqrt[Sin[c + d*x]^2]))/(12*(a - b)^2*(a + b)^2*Sqrt[Cos[c + d*x]]*(
a + b*Sin[c + d*x]))))/(8*d)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 93.59, size = 35480, normalized size = 69.03

method	result	size
default	Expression too large to display	35480

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(1/2)/(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] e^(1/2)*integrate(sqrt(cos(d*x + c))/(b*sin(d*x + c) + a)^3, x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(1/2)/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(d*x + c))*e^(1/2)/(b*sin(d*x + c) + a)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{e\,\cos\left(c+d\,x\right)}}{{\left(a+b\,\sin\left(c+d\,x\right)\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(1/2)/(a + b*sin(c + d*x))^3,x)
```

```
[Out] int((e*cos(c + d*x))^(1/2)/(a + b*sin(c + d*x))^3, x)
```

3.601 $\int \frac{1}{\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=520

$$-\frac{3\sqrt{b}\,(5a^2+2b^2)\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{8\,(-a^2+b^2)^{11/4}\,d\sqrt{e}}-\frac{3\sqrt{b}\,(5a^2+2b^2)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{8\,(-a^2+b^2)^{11/4}\,d\sqrt{e}}-\frac{7a\sqrt{}}{4\,(}$$

```
[Out] -3/8*(5*a^2+2*b^2)*arctan(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(
1/2))*b^(1/2)/(-a^2+b^2)^(11/4)/d/e^(1/2)-3/8*(5*a^2+2*b^2)*arctanh(b^(1/2)
*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))*b^(1/2)/(-a^2+b^2)^(11/4)/d
/e^(1/2)-7/4*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(si
n(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/(a^2-b^2)^2/d/(e*cos(d*x+c))^(1/
2)+3/8*a*(5*a^2+2*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticPi(sin(1/2*d*x+1/2*c),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*cos(d*x+c)^(1/2
)/(a^2-b^2)^2/d/(a^2-b*(b-(-a^2+b^2)^(1/2)))/(e*cos(d*x+c))^(1/2)+3/8*a*(5*
a^2+2*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1
/2*d*x+1/2*c),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(d*x+c)^(1/2)/(a^2-b^2)^
2/d/(a^2-b*(b+(-a^2+b^2)^(1/2)))/(e*cos(d*x+c))^(1/2)+1/2*b*(e*cos(d*x+c))^
(1/2)/(a^2-b^2)/d/e/(a+b*sin(d*x+c))^2+7/4*a*b*(e*cos(d*x+c))^(1/2)/(a^2-b^
2)^2/d/e/(a+b*sin(d*x+c))
```

Rubi [A]

time = 0.81, antiderivative size = 520, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {2773, 2943, 2946, 2721, 2720, 2781, 2886, 2884, 335, 218, 214, 211}

$$-\frac{3\sqrt{b}\,(5a^2+2b^2)\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{8d\sqrt{e}\,(b^2-a^2)^{11/4}}+\frac{7ab\sqrt{e\cos(c+dx)}}{4de\,(a^2-b^2)^2\,(a+b\sin(c+dx))}+\frac{b\sqrt{e\cos(c+dx)}}{2de\,(a^2-b^2)\,(a+b\sin(c+dx))^2}-\frac{3\sqrt{b}\,(5a^2+2b^2)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{8d\sqrt{e}\,(b^2-a^2)^{11/4}}-\frac{7a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4d\,(a^2-b^2)^2\,\sqrt{e\cos(c+dx)}}+\frac{3a(5a^2+2b^2)\,\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{8d\,(a^2-b^2)^2\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{e\cos(c+dx)}}+\frac{3a(5a^2+2b^2)\,\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{8d\,(a^2-b^2)^2\left(a^2-b\left(\sqrt{b^2-a^2}+b\right)\right)\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x])^3),x]
```

```
[Out] (-3*Sqrt[b]*(5*a^2 + 2*b^2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 +
b^2)^(1/4)*Sqrt[e])])/(8*(-a^2 + b^2)^(11/4)*d*Sqrt[e]) - (3*Sqrt[b]*(5*a^2
 + 2*b^2)*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e
])])/(8*(-a^2 + b^2)^(11/4)*d*Sqrt[e]) - (7*a*Sqrt[Cos[c + d*x]]*EllipticF[
(c + d*x)/2, 2])/(4*(a^2 - b^2)^2*d*Sqrt[e*Cos[c + d*x]]) + (3*a*(5*a^2 + 2
*b^2)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c + d*x)
/2, 2])/(8*(a^2 - b^2)^2*(a^2 - b*(b - Sqrt[-a^2 + b^2]))*d*Sqrt[e*Cos[c +
d*x]]) + (3*a*(5*a^2 + 2*b^2)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b + Sqrt
[-a^2 + b^2]), (c + d*x)/2, 2])/(8*(a^2 - b^2)^2*(a^2 - b*(b + Sqrt[-a^2 +
b^2]))*d*Sqrt[e*Cos[c + d*x]]) + (b*Sqrt[e*Cos[c + d*x]])/(2*(a^2 - b^2)*d*
e*(a + b*Sin[c + d*x])^2) + (7*a*b*Sqrt[e*Cos[c + d*x]])/(4*(a^2 - b^2)^2*d
*e*(a + b*Sin[c + d*x]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2773

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m + 1)/(f*g*(a^2 - b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1))
, Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p
 + 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^
2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2781

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[-a/(2*q), Int[1/(S
```

```
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[b*(g/f), Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2943

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c
 - a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^(m + 1)/(f*g*(a^2 -
 b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*
(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p
+ 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[
a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))^3}\,dx = \frac{b\sqrt{e\cos(c+dx)}}{2(a^2-b^2)\,de(a+b\sin(c+dx))^2} - \frac{\int \frac{-2a+\frac{3}{2}b\sin(c+dx)}{\sqrt{e\cos(c+dx)}\,(a+b\sin(c+}}{2(a^2-b^2)}$$

$$= \frac{b\sqrt{e\cos(c+dx)}}{2(a^2-b^2)\,de(a+b\sin(c+dx))^2} + \frac{7ab\sqrt{e\cos(c+dx)}}{4(a^2-b^2)^2\,de(a+b\sin(c+}$$

$$= \frac{b\sqrt{e\cos(c+dx)}}{2(a^2-b^2)\,de(a+b\sin(c+dx))^2} + \frac{7ab\sqrt{e\cos(c+dx)}}{4(a^2-b^2)^2\,de(a+b\sin(c+}$$

$$= \frac{b\sqrt{e\cos(c+dx)}}{2(a^2-b^2)\,de(a+b\sin(c+dx))^2} + \frac{7ab\sqrt{e\cos(c+dx)}}{4(a^2-b^2)^2\,de(a+b\sin(c+}$$

$$= -\frac{7a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{4(a^2-b^2)^2\,d\sqrt{e\cos(c+dx)}} + \frac{b\sqrt{e\cos(c+dx)}}{2(a^2-b^2)\,de(a+b\sin(c}$$

$$= -\frac{7a\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{4(a^2-b^2)^2\,d\sqrt{e\cos(c+dx)}} + \frac{3a(5a^2+2b^2)\,\sqrt{\cos(c+}}{8(-a^2+b^2)^{5/2}\left(b-\right.}$$

$$= -\frac{3\sqrt{b}\,(5a^2+2b^2)\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{8(-a^2+b^2)^{11/4}\,d\sqrt{e}} - \frac{3\sqrt{b}\,(5a^2}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

time = 42.44, size = 1226, normalized size = 2.36

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x])^3),x]
```

```
[Out] (Cos[c + d*x]*(b/(2*(a^2 - b^2)*(a + b*Sin[c + d*x])^2) + (7*a*b)/(4*(a^2 -
 b^2)^2*(a + b*Sin[c + d*x]))))/(d*Sqrt[e*Cos[c + d*x]]) + (Sqrt[Cos[c + d*
x]]*((-2*(8*a^2 + 6*b^2)*(a + b*Sqrt[1 - Cos[c + d*x]^2])*((5*a*(a^2 - b^2)
*AppellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^
2)]*Sqrt[Cos[c + d*x]])/(Sqrt[1 - Cos[c + d*x]^2]*(5*(a^2 - b^2)*AppellF1[1
/4, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] - 2*(2*
b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 +
```

```
 b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos[c
 + d*x]^2)/(-a^2 + b^2)])*Cos[c + d*x]^2)*(a^2 + b^2*(-1 + Cos[c + d*x]^2))
) - ((1/8 - I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])
/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-
a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/
4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]] - Log[Sqrt[-a^2 + b^2] + (1 + I)*
Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]]))/(-a^2 +
 b^2)^(3/4))*Sin[c + d*x])/(Sqrt[1 - Cos[c + d*x]^2]*(a + b*Sin[c + d*x]))
+ (14*a*b*(a + b*Sqrt[1 - Cos[c + d*x]^2])*((5*b*(a^2 - b^2)*AppellF1[1/4,
-1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[c
 + d*x]]*Sqrt[1 - Cos[c + d*x]^2])/((-5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1,
5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + 2*(2*b^2*AppellF1
[5/4, -1/2, 2, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + (a
^2 - b^2)*AppellF1[5/4, 1/2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(
-a^2 + b^2)])*Cos[c + d*x]^2)*(a^2 + b^2*(-1 + Cos[c + d*x]^2))) + (a*(-2*A
rcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] + 2*ArcTa
n[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^
2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c +
 d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c
 + d*x]] + b*Cos[c + d*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)))*Sin[c +
 d*x]^2)/((1 - Cos[c + d*x]^2)*(a + b*Sin[c + d*x]))))/(8*(a - b)^2*(a + b)
^2*d*Sqrt[e*Cos[c + d*x]])
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 91.42, size = 24488, normalized size = 47.09

method	result	size
default	Expression too large to display	24488

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(d*x+c))^3/(e*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c))^3/(e*cos(d*x+c))^(1/2),x, algorithm="maxima")
[Out] e^(-1/2)*integrate(1/((b*sin(d*x + c) + a)^3*sqrt(cos(d*x + c))), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c))^3/(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c))**3/(e*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c))^3/(e*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(e^(-1/2)/((b*sin(d*x + c) + a)^3*sqrt(cos(d*x + c))), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{e\,\cos\left(c+d\,x\right)}\,{\left(a+b\,\sin\left(c+d\,x\right)\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cos(c + d*x))^(1/2)*(a + b*sin(c + d*x))^3),x)
```

```
[Out] int(1/((e*cos(c + d*x))^(1/2)*(a + b*sin(c + d*x))^3), x)
```

3.602 $\int \frac{1}{(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=596

$$\frac{5b^{3/2}(7a^2+2b^2)\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{8\left(-a^2+b^2\right)^{13/4}de^{3/2}}-\frac{5b^{3/2}(7a^2+2b^2)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{8\left(-a^2+b^2\right)^{13/4}de^{3/2}}-\frac{a(8a^2+}{}$$

```
[Out] 5/8*b^(3/2)*(7*a^2+2*b^2)*arctan(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1
/4)/e^(1/2))/(-a^2+b^2)^(13/4)/d/e^(3/2)-5/8*b^(3/2)*(7*a^2+2*b^2)*arctanh(
b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/(-a^2+b^2)^(13/4)/d/
e^(3/2)+1/2*b/(a^2-b^2)/d/e/(a+b*sin(d*x+c))^2/(e*cos(d*x+c))^(1/2)+9/4*a*b
/(a^2-b^2)^2/d/e/(a+b*sin(d*x+c))/(e*cos(d*x+c))^(1/2)+1/4*(-5*b*(7*a^2+2*b
^2)+a*(8*a^2+37*b^2)*sin(d*x+c))/(a^2-b^2)^3/d/e/(e*cos(d*x+c))^(1/2)-5/8*a
*b*(7*a^2+2*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi
(sin(1/2*d*x+1/2*c),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*cos(d*x+c)^(1/2)/(a^2
-b^2)^3/d/e/(b-(-a^2+b^2)^(1/2))/(e*cos(d*x+c))^(1/2)-5/8*a*b*(7*a^2+2*b^2)
*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2
*c),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(d*x+c)^(1/2)/(a^2-b^2)^3/d/e/(b+(
-a^2+b^2)^(1/2))/(e*cos(d*x+c))^(1/2)-1/4*a*(8*a^2+37*b^2)*(cos(1/2*d*x+1/2
*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*co
s(d*x+c))^(1/2)/(a^2-b^2)^3/d/e^2/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 1.05, antiderivative size = 596, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {2773, 2943, 2945, 2946, 2721, 2719, 2780, 2886, 2884, 335, 304, 211, 214}

$$\frac{5b^{3/2}(7a^2+2b^2)\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{8de^{3/2}\left(b^2-a^2\right)^{13/4}}-\frac{a(8a^2+37b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{4de^2\left(a^2-b^2\right)^3\sqrt{\cos(c+dx)}}+\frac{9ab}{4de\left(a^2-b^2\right)^2\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))}+\frac{b}{2de\left(a^2-b^2\right)\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))^2}-\frac{5b(7a^2+2b^2)-a(8a^2+37b^2)\sin(c+dx)}{4de\left(a^2-b^2\right)^3\sqrt{e\cos(c+dx)}}-\frac{5ab(7a^2+2b^2)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{8de\left(a^2-b^2\right)^3\left(b-\sqrt{b^2-a^2}\right)\sqrt{e\cos(c+dx)}}-\frac{5ab(7a^2+2b^2)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{8de\left(a^2-b^2\right)^3\left(\sqrt{b^2-a^2}+b\right)\sqrt{e\cos(c+dx)}}-\frac{5b^{3/2}(7a^2+2b^2)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{8de^{3/2}\left(b^2-a^2\right)^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x])^3),x]
```

```
[Out] (5*b^(3/2)*(7*a^2 + 2*b^2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b
^2)^(1/4)*Sqrt[e])])/(8*(-a^2 + b^2)^(13/4)*d*e^(3/2)) - (5*b^(3/2)*(7*a^2
+ 2*b^2)*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e]
)])/(8*(-a^2 + b^2)^(13/4)*d*e^(3/2)) - (a*(8*a^2 + 37*b^2)*Sqrt[e*Cos[c +
d*x]]*EllipticE[(c + d*x)/2, 2])/(4*(a^2 - b^2)^3*d*e^2*Sqrt[Cos[c + d*x]])
 - (5*a*b*(7*a^2 + 2*b^2)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^
2 + b^2]), (c + d*x)/2, 2])/(8*(a^2 - b^2)^3*(b - Sqrt[-a^2 + b^2])*d*e*Sqr
t[e*Cos[c + d*x]]) - (5*a*b*(7*a^2 + 2*b^2)*Sqrt[Cos[c + d*x]]*EllipticPi[(
2*b)/(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(8*(a^2 - b^2)^3*(b + Sqrt[-a
^2 + b^2])*d*e*Sqrt[e*Cos[c + d*x]]) + b/(2*(a^2 - b^2)*d*e*Sqrt[e*Cos[c +
d*x]]*(a + b*Sin[c + d*x])^2) + (9*a*b)/(4*(a^2 - b^2)^2*d*e*Sqrt[e*Cos[c +
```

```
 d*x]]*(a + b*Sin[c + d*x])) - (5*b*(7*a^2 + 2*b^2) - a*(8*a^2 + 37*b^2)*Si
n[c + d*x])/(4*(a^2 - b^2)^3*d*e*Sqrt[e*Cos[c + d*x]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2773

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m + 1)/(f*g*(a^2 - b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1))
, Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p
 + 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^
2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2780

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[a*(g/(2*b)), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[a*(g/(2*b)), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[b*(g/f), Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2943

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c
 - a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^(m + 1)/(f*g*(a^2 -
 b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*
(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p
+ 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[
a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2945

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{1}{(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))^3}\,dx = \frac{b}{2\left(a^2-b^2\right)de\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))^2} - \frac{\int \frac{-2}{(e\cos(c+d}}{}$$

$$= \frac{b}{2\left(a^2-b^2\right)de\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))^2} + \frac{}{4\left(a^2-b^2\right.}$$

$$= \frac{b}{2\left(a^2-b^2\right)de\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))^2} + \frac{}{4\left(a^2-b^2\right.}$$

$$= \frac{b}{2\left(a^2-b^2\right)de\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))^2} + \frac{}{4\left(a^2-b^2\right.}$$

$$= \frac{b}{2\left(a^2-b^2\right)de\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))^2} + \frac{}{4\left(a^2-b^2\right.}$$

$$= -\frac{a(8a^2+37b^2)\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4\left(a^2-b^2\right)^3de^2\sqrt{\cos(c+dx)}} + \frac{}{2\left(a^2-b^2\right.}$$

$$= -\frac{a(8a^2+37b^2)\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4\left(a^2-b^2\right)^3de^2\sqrt{\cos(c+dx)}} - \frac{5ab(7a^2+}{8(a}$$

$$= \frac{5b^{3/2}(7a^2+2b^2)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8\left(-a^2+b^2\right)^{13/4}de^{3/2}} - \frac{5b^{3/2}(7a^2+}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 16.37, size = 922, normalized size = 1.55

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x])^3),x]
```

```
[Out] -1/8*(Cos[c + d*x]^(3/2)*((-2*(8*a^4 + 72*a^2*b^2 + 10*b^4)*(a + b*Sqrt[1 -
 Cos[c + d*x]^2])*((a*AppellF1[3/4, 1/2, 1, 7/4, Cos[c + d*x]^2, (b^2*Cos[c
 + d*x]^2)/(-a^2 + b^2)]*Cos[c + d*x]^(3/2))/(3*(a^2 - b^2)) + ((1/8 + I/8)
*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2
*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[
Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] +
I*b*Cos[c + d*x]] + Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/
4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)))*S
in[c + d*x])/(Sqrt[1 - Cos[c + d*x]^2]*(a + b*Sin[c + d*x])) - ((8*a^3*b +
37*a*b^3)*(a + b*Sqrt[1 - Cos[c + d*x]^2])*(8*b^(5/2)*AppellF1[3/4, -1/2, 1
, 7/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Cos[c + d*x]^(3/2
) + 3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c
 + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d
*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2
)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2
]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]]))*Sin[c +
d*x]^2)/(12*b^(3/2)*(-a^2 + b^2)*(1 - Cos[c + d*x]^2)*(a + b*Sin[c + d*x]))
))/((a - b)^3*(a + b)^3*d*(e*Cos[c + d*x])^(3/2)) + (Cos[c + d*x]^2*(-1/2*(
b^3*Cos[c + d*x])/((a^2 - b^2)^2*(a + b*Sin[c + d*x])^2) - (13*a*b^3*Cos[c
+ d*x])/(4*(a^2 - b^2)^3*(a + b*Sin[c + d*x])) + (2*Sec[c + d*x]*(-3*a^2*b
- b^3 + a^3*Sin[c + d*x] + 3*a*b^2*Sin[c + d*x]))/(a^2 - b^2)^3))/(d*(e*Cos
[c + d*x])^(3/2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 170.83, size = 43749, normalized size = 73.40

method	result	size
default	Expression too large to display	43749

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(3/2)/(a+b*sin(d*x+c))**3,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3067 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c))^3,x, algorithm="giac")
[Out] integrate(e^(-3/2)/((b*sin(d*x + c) + a)^3*cos(d*x + c)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(e\,\cos\left(c+d\,x\right)\right)}^{3/2}\,{\left(a+b\,\sin\left(c+d\,x\right)\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cos(c + d*x))^(3/2)*(a + b*sin(c + d*x))^3),x)
[Out] int(1/((e*cos(c + d*x))^(3/2)*(a + b*sin(c + d*x))^3), x)
```

3.603 $\int \frac{1}{(e\cos(c+dx))^{5/2}(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=614

$$-\frac{7b^{5/2}(9a^2+2b^2)\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{8\left(-a^2+b^2\right)^{15/4}de^{5/2}}-\frac{7b^{5/2}(9a^2+2b^2)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{8\left(-a^2+b^2\right)^{15/4}de^{5/2}}+\frac{a(8a^2}{}$$

```
[Out] -7/8*b^(5/2)*(9*a^2+2*b^2)*arctan(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(
1/4)/e^(1/2))/(-a^2+b^2)^(15/4)/d/e^(5/2)-7/8*b^(5/2)*(9*a^2+2*b^2)*arctanh
(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/(-a^2+b^2)^(15/4)/d
/e^(5/2)+1/2*b/(a^2-b^2)/d/e/(e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c))^2+11/4*a
*b/(a^2-b^2)^2/d/e/(e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c))+1/12*(-7*b*(9*a^2+
2*b^2)+a*(8*a^2+69*b^2)*sin(d*x+c))/(a^2-b^2)^3/d/e/(e*cos(d*x+c))^(3/2)+1/
12*a*(8*a^2+69*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipti
cF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/(a^2-b^2)^3/d/e^2/(e*cos(d*
x+c))^(1/2)-7/8*a*b^2*(9*a^2+2*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*
x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*co
s(d*x+c)^(1/2)/(a^2-b^2)^3/d/e^2/(a^2-b*(b-(-a^2+b^2)^(1/2)))/(e*cos(d*x+c)
)^(1/2)-7/8*a*b^2*(9*a^2+2*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/
2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(d*
x+c)^(1/2)/(a^2-b^2)^3/d/e^2/(a^2-b*(b+(-a^2+b^2)^(1/2)))/(e*cos(d*x+c))^(1
/2)
```

Rubi [A]
time = 1.18, antiderivative size = 614, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {2773, 2943, 2945, 2946, 2721, 2720, 2781, 2886, 2884, 335, 218, 214, 211}

$$-\frac{7b^{5/2}(9a^2+2b^2)\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{8de^{5/2}\left(b^2-a^2\right)^{15/4}}+\frac{a(8a^2+69b^2)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{12de^2\left(a^2-b^2\right)^3\sqrt{e\cos(c+dx)}}-\frac{7ab^2(9a^2+2b^2)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{8de^2\left(a^2-b^2\right)^3\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{e\cos(c+dx)}}-\frac{7ab^2(9a^2+2b^2)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{8de^2\left(a^2-b^2\right)^3\left(a^2-b\left(\sqrt{b^2-a^2}+b\right)\right)\sqrt{e\cos(c+dx)}}+\frac{11ab}{4de\left(a^2-b^2\right)^2(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))}+\frac{b}{2de\left(a^2-b^2\right)(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))^2}-\frac{7b(9a^2+2b^2)-a(8a^2+69b^2)\sin(c+dx)}{12de\left(a^2-b^2\right)^3(e\cos(c+dx))^{3/2}}-\frac{7b^{5/2}(9a^2+2b^2)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{8de^{5/2}\left(b^2-a^2\right)^{15/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(5/2)*(a + b*Sin[c + d*x])^3),x]
```

```
[Out] (-7*b^(5/2)*(9*a^2 + 2*b^2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 +
b^2)^(1/4)*Sqrt[e])])/(8*(-a^2 + b^2)^(15/4)*d*e^(5/2)) - (7*b^(5/2)*(9*a^2
 + 2*b^2)*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e
])])/(8*(-a^2 + b^2)^(15/4)*d*e^(5/2)) + (a*(8*a^2 + 69*b^2)*Sqrt[Cos[c + d
*x]]*EllipticF[(c + d*x)/2, 2])/(12*(a^2 - b^2)^3*d*e^2*Sqrt[e*Cos[c + d*x]
]) - (7*a*b^2*(9*a^2 + 2*b^2)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt
[-a^2 + b^2]), (c + d*x)/2, 2])/(8*(a^2 - b^2)^3*(a^2 - b*(b - Sqrt[-a^2 +
b^2]))*d*e^2*Sqrt[e*Cos[c + d*x]]) - (7*a*b^2*(9*a^2 + 2*b^2)*Sqrt[Cos[c +
d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(8*(a^2 - b
^2)^3*(a^2 - b*(b + Sqrt[-a^2 + b^2]))*d*e^2*Sqrt[e*Cos[c + d*x]]) + b/(2*(
```

a^2 - b^2)*d*e*(e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x])^2) + (11*a*b)/(4*(a^2 - b^2)^2*d*e*(e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x])) - (7*b*(9*a^2 + 2*b^2) - a*(8*a^2 + 69*b^2)*Sin[c + d*x])/(12*(a^2 - b^2)^3*d*e*(e*Cos[c + d*x])^(3/2))

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2773

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m + 1)/(f*g*(a^2 - b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1))
, Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p
 + 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^
```

```
2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2781

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[-a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[b*(g/f), Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2943

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c
 - a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^(m + 1)/(f*g*(a^2 -
 b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*
(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p
+ 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[
a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2945

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
```

```
[p, -1] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{1}{(e\cos(c+dx))^{5/2}(a+b\sin(c+dx))^3}\,dx = \frac{b}{2\left(a^2-b^2\right)de(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))^2} - \frac{\int \frac{-}{(e\cos(c+}}{}$$

$$= \frac{b}{2\left(a^2-b^2\right)de(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))^2} + \frac{}{4\left(a^2-b\right.}$$

$$= \frac{b}{2\left(a^2-b^2\right)de(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))^2} + \frac{}{4\left(a^2-b\right.}$$

$$= \frac{b}{2\left(a^2-b^2\right)de(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))^2} + \frac{}{4\left(a^2-b\right.}$$

$$= \frac{b}{2\left(a^2-b^2\right)de(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))^2} + \frac{}{4\left(a^2-b\right.}$$

$$= \frac{a(8a^2+69b^2)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{12\left(a^2-b^2\right)^3de^2\sqrt{e\cos(c+dx)}} + \frac{}{2\left(a^2-b^2\right)de}$$

$$= \frac{a(8a^2+69b^2)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{12\left(a^2-b^2\right)^3de^2\sqrt{e\cos(c+dx)}} + \frac{7ab^2(9a^2+2b}{8\left(-a^2+\right.}$$

$$= -\frac{7b^{5/2}(9a^2+2b^2)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8\left(-a^2+b^2\right)^{15/4}de^{5/2}} - \frac{7b^{5/2}(9a^2}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

time = 41.87, size = 1308, normalized size = 2.13

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((e*Cos[c + d*x])^(5/2)*(a + b*Sin[c + d*x])^3),x]
```

```
[Out] (Cos[c + d*x]^(5/2)*((-2*(8*a^4 - 120*a^2*b^2 - 42*b^4)*(a + b*Sqrt[1 - Cos
[c + d*x]^2])*((5*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2,
(b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[c + d*x]])/(Sqrt[1 - Cos[c + d*
x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c
+ d*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[c + d*x]^
2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1,
9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)])*Cos[c + d*x]^2)*(a
^2 + b^2*(-1 + Cos[c + d*x]^2))) - ((1/8 - I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 +
 I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)
*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - (
1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]] -
Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]
] + I*b*Cos[c + d*x]]))/(-a^2 + b^2)^(3/4))*Sin[c + d*x])/(Sqrt[1 - Cos[c +
 d*x]^2]*(a + b*Sin[c + d*x])) - (2*(8*a^3*b + 69*a*b^3)*(a + b*Sqrt[1 - Co
s[c + d*x]^2])*((5*b*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Cos[c + d*x]^2
, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[c + d*x]]*Sqrt[1 - Cos[c + d*
x]^2])/((-5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Co
s[c + d*x]^2)/(-a^2 + b^2)] + 2*(2*b^2*AppellF1[5/4, -1/2, 2, 9/4, Cos[c +
d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + (a^2 - b^2)*AppellF1[5/4, 1/2,
 1, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)])*Cos[c + d*x]^2
)*(a^2 + b^2*(-1 + Cos[c + d*x]^2))) + (a*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*S
qrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[
Cos[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(
a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]] + Log[Sqrt[a^2 - b^2]
 + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]]))
/(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)))*Sin[c + d*x]^2)/((1 - Cos[c + d*x]^
2)*(a + b*Sin[c + d*x]))))/(24*(a - b)^3*(a + b)^3*d*(e*Cos[c + d*x])^(5/2)
) + (Cos[c + d*x]^3*(-1/2*b^3/((a^2 - b^2)^2*(a + b*Sin[c + d*x])^2) - (15*
a*b^3)/(4*(a^2 - b^2)^3*(a + b*Sin[c + d*x])) + (2*Sec[c + d*x]^2*(-3*a^2*b
 - b^3 + a^3*Sin[c + d*x] + 3*a*b^2*Sin[c + d*x]))/(3*(a^2 - b^2)^3)))/(d*(
e*Cos[c + d*x])^(5/2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 228.92, size = 30052, normalized size = 48.94

method	result	size
default	Expression too large to display	30052

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(5/2)/(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(5/2)/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(5/2)/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(5/2)/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(5/2)/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(e^(-5/2)/((b*sin(d*x + c) + a)^3*cos(d*x + c)^(5/2)), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(e\cos(c+dx))^{5/2}\,(a+b\sin(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cos(c + d*x))^(5/2)*(a + b*sin(c + d*x))^3),x)
[Out] int(1/((e*cos(c + d*x))^(5/2)*(a + b*sin(c + d*x))^3), x)
```

3.604 $\int \frac{1}{(e\cos(c+dx))^{7/2}(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=685

$$\frac{9b^{7/2}(11a^2+2b^2)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8\left(-a^2+b^2\right)^{17/4}de^{7/2}}-\frac{9b^{7/2}(11a^2+2b^2)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8\left(-a^2+b^2\right)^{17/4}de^{7/2}}-\frac{3a(8a^4}{}$$

```
[Out] 9/8*b^(7/2)*(11*a^2+2*b^2)*arctan(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(
1/4)/e^(1/2))/(-a^2+b^2)^(17/4)/d/e^(7/2)-9/8*b^(7/2)*(11*a^2+2*b^2)*arctan
h(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/(-a^2+b^2)^(17/4)/
d/e^(7/2)+1/2*b/(a^2-b^2)/d/e/(e*cos(d*x+c))^(5/2)/(a+b*sin(d*x+c))^2+13/4*
a*b/(a^2-b^2)^2/d/e/(e*cos(d*x+c))^(5/2)/(a+b*sin(d*x+c))+1/20*(-9*b*(11*a^
2+2*b^2)+a*(8*a^2+109*b^2)*sin(d*x+c))/(a^2-b^2)^3/d/e/(e*cos(d*x+c))^(5/2)
+3/20*(15*b^3*(11*a^2+2*b^2)+a*(8*a^4-64*a^2*b^2-139*b^4)*sin(d*x+c))/(a^2-
b^2)^4/d/e^3/(e*cos(d*x+c))^(1/2)+9/8*a*b^3*(11*a^2+2*b^2)*(cos(1/2*d*x+1/2
*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(b-(-a^2+
b^2)^(1/2)),2^(1/2))*cos(d*x+c)^(1/2)/(a^2-b^2)^4/d/e^3/(b-(-a^2+b^2)^(1/2)
)/(e*cos(d*x+c))^(1/2)+9/8*a*b^3*(11*a^2+2*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(b+(-a^2+b^2)^(1/2))
,2^(1/2))*cos(d*x+c)^(1/2)/(a^2-b^2)^4/d/e^3/(b+(-a^2+b^2)^(1/2))/(e*cos(d*
x+c))^(1/2)-3/20*a*(8*a^4-64*a^2*b^2-139*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/
cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/
2)/(a^2-b^2)^4/d/e^4/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 1.34, antiderivative size = 685, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {2773, 2943, 2945, 2946, 2721, 2719, 2780, 2886, 2884, 335, 304, 211, 214}

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(7/2)*(a + b*Sin[c + d*x])^3),x]
```

```
[Out] (9*b^(7/2)*(11*a^2 + 2*b^2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 +
b^2)^(1/4)*Sqrt[e])])/(8*(-a^2 + b^2)^(17/4)*d*e^(7/2)) - (9*b^(7/2)*(11*a^
2 + 2*b^2)*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[
e])])/(8*(-a^2 + b^2)^(17/4)*d*e^(7/2)) - (3*a*(8*a^4 - 64*a^2*b^2 - 139*b^
4)*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(20*(a^2 - b^2)^4*d*e^4*
Sqrt[Cos[c + d*x]]) + (9*a*b^3*(11*a^2 + 2*b^2)*Sqrt[Cos[c + d*x]]*Elliptic
Pi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(8*(a^2 - b^2)^4*(b - Sqr
t[-a^2 + b^2])*d*e^3*Sqrt[e*Cos[c + d*x]]) + (9*a*b^3*(11*a^2 + 2*b^2)*Sqrt
[Cos[c + d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(8
```

```
*(a^2 - b^2)^4*(b + Sqrt[-a^2 + b^2])*d*e^3*Sqrt[e*Cos[c + d*x]]) + b/(2*(a
^2 - b^2)*d*e*(e*Cos[c + d*x])^(5/2)*(a + b*Sin[c + d*x])^2) + (13*a*b)/(4*
(a^2 - b^2)^2*d*e*(e*Cos[c + d*x])^(5/2)*(a + b*Sin[c + d*x])) - (9*b*(11*a
^2 + 2*b^2) - a*(8*a^2 + 109*b^2)*Sin[c + d*x])/(20*(a^2 - b^2)^3*d*e*(e*Co
s[c + d*x])^(5/2)) + (3*(15*b^3*(11*a^2 + 2*b^2) + a*(8*a^4 - 64*a^2*b^2 -
139*b^4)*Sin[c + d*x]))/(20*(a^2 - b^2)^4*d*e^3*Sqrt[e*Cos[c + d*x]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2773

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m + 1)/(f*g*(a^2 - b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1))
```

```
, Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p
 + 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^
2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2780

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[a*(g/(2*b)), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[a*(g/(2*b)), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[b*(g/f), Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2943

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c
 - a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^(m + 1)/(f*g*(a^2 -
 b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*
(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p
+ 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[
a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2945

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
```

```
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{1}{(e\cos(c+dx))^{7/2}(a+b\sin(c+dx))^3}\,dx = \frac{b}{2\,(a^2-b^2)\,de(e\cos(c+dx))^{5/2}(a+b\sin(c+dx))^2} - \frac{\int \frac{-}{(e\cos(c+}}{}$$

$$= \frac{b}{2\,(a^2-b^2)\,de(e\cos(c+dx))^{5/2}(a+b\sin(c+dx))^2} + \frac{}{4\,(a^2-b}$$

$$= \frac{b}{2\,(a^2-b^2)\,de(e\cos(c+dx))^{5/2}(a+b\sin(c+dx))^2} + \frac{}{4\,(a^2-b}$$

$$= \frac{b}{2\,(a^2-b^2)\,de(e\cos(c+dx))^{5/2}(a+b\sin(c+dx))^2} + \frac{}{4\,(a^2-b}$$

$$= \frac{b}{2\,(a^2-b^2)\,de(e\cos(c+dx))^{5/2}(a+b\sin(c+dx))^2} + \frac{}{4\,(a^2-b}$$

$$= \frac{b}{2\,(a^2-b^2)\,de(e\cos(c+dx))^{5/2}(a+b\sin(c+dx))^2} + \frac{}{4\,(a^2-b}$$

$$= -\frac{3a(8a^4-64a^2b^2-139b^4)\,\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{20\,(a^2-b^2)^4\,de^4\sqrt{\cos(c+dx)}}$$

$$= -\frac{3a(8a^4-64a^2b^2-139b^4)\,\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{20\,(a^2-b^2)^4\,de^4\sqrt{\cos(c+dx)}}$$

$$= \frac{9b^{7/2}(11a^2+2b^2)\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{8\,(-a^2+b^2)^{17/4}\,de^{7/2}} - \frac{9b^{7/2}(11a^2}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 16.47, size = 1014, normalized size = 1.48

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((e*Cos[c + d*x])^(7/2)*(a + b*Sin[c + d*x])^3),x]
```

```
[Out] (-3*Cos[c + d*x]^(7/2)*((-2*(8*a^6 - 64*a^4*b^2 - 304*a^2*b^4 - 30*b^6)*(a
+ b*Sqrt[1 - Cos[c + d*x]^2])*((a*AppellF1[3/4, 1/2, 1, 7/4, Cos[c + d*x]^2
```

```
, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Cos[c + d*x]^(3/2))/(3*(a^2 - b^2)) +
((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2
)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(
1/4)] - Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[
c + d*x]] + I*b*Cos[c + d*x]] + Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^
2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]]))/(Sqrt[b]*(-a^2 + b^
2)^(1/4)))*Sin[c + d*x])/(Sqrt[1 - Cos[c + d*x]^2]*(a + b*Sin[c + d*x])) -
((8*a^5*b - 64*a^3*b^3 - 139*a*b^5)*(a + b*Sqrt[1 - Cos[c + d*x]^2])*(8*b^(
5/2)*AppellF1[3/4, -1/2, 1, 7/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2
 + b^2)]*Cos[c + d*x]^(3/2) + 3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*(2*ArcTan[1 - (
Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt
[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] -
Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]] + Lo
g[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] +
b*Cos[c + d*x]]))*Sin[c + d*x]^2)/(12*b^(3/2)*(-a^2 + b^2)*(1 - Cos[c + d*x
]^2)*(a + b*Sin[c + d*x]))))/(40*(a - b)^4*(a + b)^4*d*(e*Cos[c + d*x])^(7/
2)) + (Cos[c + d*x]^4*((b^5*Cos[c + d*x])/(2*(a^2 - b^2)^3*(a + b*Sin[c + d
*x])^2) + (21*a*b^5*Cos[c + d*x])/(4*(a^2 - b^2)^4*(a + b*Sin[c + d*x])) +
(2*Sec[c + d*x]^3*(-3*a^2*b - b^3 + a^3*Sin[c + d*x] + 3*a*b^2*Sin[c + d*x]
))/(5*(a^2 - b^2)^3) + (2*Sec[c + d*x]*(50*a^2*b^3 + 10*b^5 + 3*a^5*Sin[c +
 d*x] - 24*a^3*b^2*Sin[c + d*x] - 39*a*b^4*Sin[c + d*x]))/(5*(a^2 - b^2)^4)
))/(d*(e*Cos[c + d*x])^(7/2))
```

Maple [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(e\cos\left(dx+c\right)\right)^{\frac{7}{2}}\left(a+b\sin\left(dx+c\right)\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(7/2)/(a+b*sin(d*x+c))^3,x)
[Out] int(1/(e*cos(d*x+c))^(7/2)/(a+b*sin(d*x+c))^3,x)
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(7/2)/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
[Out] Timed out
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(7/2)/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(7/2)/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3879 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(7/2)/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(e^(-7/2)/((b*sin(d*x + c) + a)^3*cos(d*x + c)^(7/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(e\,\cos\left(c+d\,x\right)\right)^{7/2}\,{\left(a+b\,\sin\left(c+d\,x\right)\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cos(c + d*x))^(7/2)*(a + b*sin(c + d*x))^3),x)
```

```
[Out] int(1/((e*cos(c + d*x))^(7/2)*(a + b*sin(c + d*x))^3), x)
```

3.605 $\int \frac{(e\cos(c+dx))^{15/2}}{(a+b\sin(c+dx))^4}\,dx$

Optimal. Leaf size=671

$$\frac{39a(11a^4-17a^2b^2+6b^4)\,e^{15/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{16b^{15/2}\left(-a^2+b^2\right)^{3/4}d}+\frac{39a(11a^4-17a^2b^2+6b^4)\,e^{15/2}\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt[4]{}}\right.}{16b^{15/2}\left(-a^2+b^2\right)^{3/4}d}$$

```
[Out] 39/16*a*(11*a^4-17*a^2*b^2+6*b^4)*e^(15/2)*arctan(b^(1/2)*(e*cos(d*x+c))^(1
/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(15/2)/(-a^2+b^2)^(3/4)/d+39/16*a*(11*a^4-1
7*a^2*b^2+6*b^4)*e^(15/2)*arctanh(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(
1/4)/e^(1/2))/b^(15/2)/(-a^2+b^2)^(3/4)/d-1/3*e*(e*cos(d*x+c))^(13/2)/b/d/(
a+b*sin(d*x+c))^3-13/84*e^3*(e*cos(d*x+c))^(9/2)*(11*a+4*b*sin(d*x+c))/b^3/
d/(a+b*sin(d*x+c))^2-39/280*e^5*(e*cos(d*x+c))^(5/2)*(77*a^2-20*b^2+22*a*b*
sin(d*x+c))/b^5/d/(a+b*sin(d*x+c))+13/56*(231*a^4-203*a^2*b^2+20*b^4)*e^8*(
cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c)
,2^(1/2))*cos(d*x+c)^(1/2)/b^8/d/(e*cos(d*x+c))^(1/2)-39/16*a^2*(11*a^4-17*
a^2*b^2+6*b^4)*e^8*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
Pi(sin(1/2*d*x+1/2*c),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*cos(d*x+c)^(1/2)/b^
8/d/(a^2-b*(b-(-a^2+b^2)^(1/2)))/(e*cos(d*x+c))^(1/2)-39/16*a^2*(11*a^4-17*
a^2*b^2+6*b^4)*e^8*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
Pi(sin(1/2*d*x+1/2*c),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(d*x+c)^(1/2)/b^
8/d/(a^2-b*(b+(-a^2+b^2)^(1/2)))/(e*cos(d*x+c))^(1/2)+13/56*e^7*(21*a*(11*a
^2-6*b^2)-b*(77*a^2-20*b^2)*sin(d*x+c))*(e*cos(d*x+c))^(1/2)/b^7/d
```

Rubi [A]

time = 1.22, antiderivative size = 671, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {2772, 2942, 2944, 2946, 2721, 2720, 2781, 2886, 2884, 335, 218, 214, 211}

$$\frac{13e^7\sqrt{e\cos(c+dx)}\,(21a(11a^2-6b^2)-b(77a^2-20b^2)\sin(c+dx))}{56b^7d}-\frac{39e^5(e\cos(c+dx))^{5/2}(77a^2+22ab\sin(c+dx)-20b^2)}{280b^5d(a+b\sin(c+dx))}+\frac{39ae^{15/2}(11a^4-17a^2b^2+6b^4)\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{16b^{15/2}d\,(b^2-a^2)^{3/4}}+\frac{39ae^{15/2}(11a^4-17a^2b^2+6b^4)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{16b^{15/2}d\,(b^2-a^2)^{3/4}}+\frac{13e^8(231a^4-203a^2b^2+20b^4)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)|2\right)}{56b^8d\sqrt{e\cos(c+dx)}}-\frac{39a^2e^8(11a^4-17a^2b^2+6b^4)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)|2\right)}{16b^8d\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{e\cos(c+dx)}}-\frac{39a^2e^8(11a^4-17a^2b^2+6b^4)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)|2\right)}{16b^8d\left(a^2-b\left(\sqrt{b^2-a^2}+b\right)\right)\sqrt{e\cos(c+dx)}}-\frac{13e^3(e\cos(c+dx))^{9/2}(11a+4b\sin(c+dx))}{84b^3d(a+b\sin(c+dx))^2}-\frac{e(e\cos(c+dx))^{13/2}}{3bd(a+b\sin(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(15/2)/(a + b*Sin[c + d*x])^4,x]
```

```
[Out] (39*a*(11*a^4 - 17*a^2*b^2 + 6*b^4)*e^(15/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c +
 d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(16*b^(15/2)*(-a^2 + b^2)^(3/4)*d) +
 (39*a*(11*a^4 - 17*a^2*b^2 + 6*b^4)*e^(15/2)*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c
 + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(16*b^(15/2)*(-a^2 + b^2)^(3/4)*d)
 + (13*(231*a^4 - 203*a^2*b^2 + 20*b^4)*e^8*Sqrt[Cos[c + d*x]]*EllipticF[(c
 + d*x)/2, 2])/(56*b^8*d*Sqrt[e*Cos[c + d*x]]) - (39*a^2*(11*a^4 - 17*a^2*b
^2 + 6*b^4)*e^8*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]),
 (c + d*x)/2, 2])/(16*b^8*(a^2 - b*(b - Sqrt[-a^2 + b^2]))*d*Sqrt[e*Cos[c +
```

```
 d*x]]) - (39*a^2*(11*a^4 - 17*a^2*b^2 + 6*b^4)*e^8*Sqrt[Cos[c + d*x]]*Elli
pticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(16*b^8*(a^2 - b*(b +
 Sqrt[-a^2 + b^2]))*d*Sqrt[e*Cos[c + d*x]]) - (e*(e*Cos[c + d*x])^(13/2))/(
3*b*d*(a + b*Sin[c + d*x])^3) - (13*e^3*(e*Cos[c + d*x])^(9/2)*(11*a + 4*b*
Sin[c + d*x]))/(84*b^3*d*(a + b*Sin[c + d*x])^2) - (39*e^5*(e*Cos[c + d*x])
^(5/2)*(77*a^2 - 20*b^2 + 22*a*b*Sin[c + d*x]))/(280*b^5*d*(a + b*Sin[c + d
*x])) + (13*e^7*Sqrt[e*Cos[c + d*x]]*(21*a*(11*a^2 - 6*b^2) - b*(77*a^2 - 2
0*b^2)*Sin[c + d*x]))/(56*b^7*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2772

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
```

```
])^(m + 1)/(b*f*(m + 1))), x] + Dist[g^2*((p - 1)/(b*(m + 1))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2781

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[-a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[b*(g/f), Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2942

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*C
os[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x])/(b^2*f*(m + 1)*(m + p + 1))), x] + Dist[g^2*((
p - 1)/(b^2*(m + 1)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rule 2944

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
```

```
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Dist[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{15/2}}{(a+b\sin(c+dx))^4}\,dx = -\frac{e(e\cos(c+dx))^{13/2}}{3bd(a+b\sin(c+dx))^3} - \frac{(13e^2)\int \frac{(e\cos(c+dx))^{11/2}\sin(c+dx)}{(a+b\sin(c+dx))^3}\,dx}{6b}$$

$$= -\frac{e(e\cos(c+dx))^{13/2}}{3bd(a+b\sin(c+dx))^3} - \frac{13e^3(e\cos(c+dx))^{9/2}(11a+4b\sin(c+dx))}{84b^3d(a+b\sin(c+dx))^2} + \frac{(39}{}$$

$$= -\frac{e(e\cos(c+dx))^{13/2}}{3bd(a+b\sin(c+dx))^3} - \frac{13e^3(e\cos(c+dx))^{9/2}(11a+4b\sin(c+dx))}{84b^3d(a+b\sin(c+dx))^2} - \frac{39}{}$$

$$= -\frac{e(e\cos(c+dx))^{13/2}}{3bd(a+b\sin(c+dx))^3} - \frac{13e^3(e\cos(c+dx))^{9/2}(11a+4b\sin(c+dx))}{84b^3d(a+b\sin(c+dx))^2} - \frac{39}{}$$

$$= -\frac{e(e\cos(c+dx))^{13/2}}{3bd(a+b\sin(c+dx))^3} - \frac{13e^3(e\cos(c+dx))^{9/2}(11a+4b\sin(c+dx))}{84b^3d(a+b\sin(c+dx))^2} - \frac{39}{}$$

$$= -\frac{e(e\cos(c+dx))^{13/2}}{3bd(a+b\sin(c+dx))^3} - \frac{13e^3(e\cos(c+dx))^{9/2}(11a+4b\sin(c+dx))}{84b^3d(a+b\sin(c+dx))^2} - \frac{39}{}$$

$$= \frac{13(231a^4-203a^2b^2+20b^4)\,e^8\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{56b^8d\sqrt{e\cos(c+dx)}} - \frac{e(e\cos(c+}{3bd(a+b\sin(}$$

$$= \frac{13(231a^4-203a^2b^2+20b^4)\,e^8\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{56b^8d\sqrt{e\cos(c+dx)}} - \frac{39a^2(11a^4-}{16}$$

$$= \frac{39a(11a^4-17a^2b^2+6b^4)\,e^{15/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{16b^{15/2}\left(-a^2+b^2\right)^{3/4}d} + \frac{39a(11a^4-1}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 55.43, size = 2102, normalized size = 3.13

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^(15/2)/(a + b*Sin[c + d*x])^4,x]
```

```
[Out] ((e*Cos[c + d*x])^(15/2)*((-2*(4410*a^3*b - 3418*a*b^3)*(a + b*Sqrt[1 - Cos
[c + d*x]^2])*((5*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2,
```

```
(b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[c + d*x]])/(Sqrt[1 - Cos[c + d*
x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c
+ d*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[c + d*x]^
2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1,
9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)])*Cos[c + d*x]^2)*(a
^2 + b^2*(-1 + Cos[c + d*x]^2))) - ((1/8 - I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 +
 I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)
*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - (
1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]] -
Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]
] + I*b*Cos[c + d*x]]))/(-a^2 + b^2)^(3/4))*Sin[c + d*x])/(Sqrt[1 - Cos[c +
 d*x]^2]*(a + b*Sin[c + d*x])) + ((5600*a^3*b - 3472*a*b^3)*(a + b*Sqrt[1 -
 Cos[c + d*x]^2])*Cos[2*(c + d*x)]*(((1/2 - I/2)*(-2*a^2 + b^2)*ArcTan[1 -
((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)])/(b^(3/2)*(-a^2 +
b^2)^(3/4)) - ((1/2 - I/2)*(-2*a^2 + b^2)*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[
Cos[c + d*x]])/(-a^2 + b^2)^(1/4)])/(b^(3/2)*(-a^2 + b^2)^(3/4)) + (4*Sqrt[
Cos[c + d*x]])/b - (4*a*AppellF1[5/4, 1/2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos
[c + d*x]^2)/(-a^2 + b^2)]*Cos[c + d*x]^(5/2))/(5*(a^2 - b^2)) + (10*a*(a^2
 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a
^2 + b^2)]*Sqrt[Cos[c + d*x]])/(Sqrt[1 - Cos[c + d*x]^2]*(5*(a^2 - b^2)*App
ellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]
- 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/
(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[c + d*x]^2, (b^
2*Cos[c + d*x]^2)/(-a^2 + b^2)])*Cos[c + d*x]^2)*(a^2 + b^2*(-1 + Cos[c + d
*x]^2))) + ((1/4 - I/4)*(-2*a^2 + b^2)*Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[
b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]])/(b^(3/2)*(-a^
2 + b^2)^(3/4)) - ((1/4 - I/4)*(-2*a^2 + b^2)*Log[Sqrt[-a^2 + b^2] + (1 + I
)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]])/(b^(3/
2)*(-a^2 + b^2)^(3/4)))*Sin[c + d*x])/(Sqrt[1 - Cos[c + d*x]^2]*(-1 + 2*Cos
[c + d*x]^2)*(a + b*Sin[c + d*x])) - (2*(3815*a^4 - 6251*a^2*b^2 + 1300*b^4
)*(a + b*Sqrt[1 - Cos[c + d*x]^2])*((5*b*(a^2 - b^2)*AppellF1[1/4, -1/2, 1,
 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[c + d*x]]
*Sqrt[1 - Cos[c + d*x]^2])/((-5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Cos
[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + 2*(2*b^2*AppellF1[5/4, -1
/2, 2, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + (a^2 - b^2
)*AppellF1[5/4, 1/2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b
^2)])*Cos[c + d*x]^2)*(a^2 + b^2*(-1 + Cos[c + d*x]^2))) + (a*(-2*ArcTan[1
- (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] + 2*ArcTan[1 + (S
qrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2]
 - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]] +
 Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]]
 + b*Cos[c + d*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)))*Sin[c + d*x]^2)
/((1 - Cos[c + d*x]^2)*(a + b*Sin[c + d*x]))))/(560*b^7*d*Cos[c + d*x]^(15/
2)) + ((e*Cos[c + d*x])^(15/2)*Sec[c + d*x]^7*((-4*a*Cos[2*(c + d*x)])/(5*b
^5) + ((-280*a^2 + 79*b^2)*Sin[c + d*x])/(42*b^6) - (-a^2 + b^2)^3/(3*b^7*(
```

```
a + b*Sin[c + d*x])^3) - (37*a*(a^2 - b^2)^2)/(12*b^7*(a + b*Sin[c + d*x])^
2) + ((-a^2 + b^2)*(-393*a^2 + 76*b^2))/(24*b^7*(a + b*Sin[c + d*x])) + Sin
[3*(c + d*x)]/(14*b^4)))/d
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos (dx+c))^{\frac{15}{2}}}{(a+b\sin (dx+c))^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(15/2)/(a+b*sin(d*x+c))^4,x)
[Out] int((e*cos(d*x+c))^(15/2)/(a+b*sin(d*x+c))^4,x)
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(15/2)/(a+b*sin(d*x+c))^4,x, algorithm="maxima")
[Out] Timed out
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(15/2)/(a+b*sin(d*x+c))^4,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(15/2)/(a+b*sin(d*x+c))**4,x)
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(15/2)/(a+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\cos(c+dx))^{15/2}}{(a+b\sin(c+dx))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(15/2)/(a + b*sin(c + d*x))^4,x)
```

```
[Out] int((e*cos(c + d*x))^(15/2)/(a + b*sin(c + d*x))^4, x)
```

3.606 $\int \frac{(e\cos(c+dx))^{13/2}}{(a+b\sin(c+dx))^4}\,dx$

Optimal. Leaf size=557

$$\frac{77a(3a^2-2b^2)\,e^{13/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{16b^{13/2}\sqrt[4]{-a^2+b^2}\,d}-\frac{77a(3a^2-2b^2)\,e^{13/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{16b^{13/2}\sqrt[4]{-a^2+b^2}\,d}-$$

```
[Out] 77/16*a*(3*a^2-2*b^2)*e^(13/2)*arctan(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^
2)^(1/4)/e^(1/2))/b^(13/2)/(-a^2+b^2)^(1/4)/d-77/16*a*(3*a^2-2*b^2)*e^(13/2
)*arctanh(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(13/2)/(
-a^2+b^2)^(1/4)/d-1/3*e*(e*cos(d*x+c))^(11/2)/b/d/(a+b*sin(d*x+c))^3-11/60*
e^3*(e*cos(d*x+c))^(7/2)*(9*a+4*b*sin(d*x+c))/b^3/d/(a+b*sin(d*x+c))^2-77/1
20*e^5*(e*cos(d*x+c))^(3/2)*(15*a^2-4*b^2+6*a*b*sin(d*x+c))/b^5/d/(a+b*sin(
d*x+c))+77/16*a^2*(3*a^2-2*b^2)*e^7*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*
x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*co
s(d*x+c)^(1/2)/b^7/d/(b-(-a^2+b^2)^(1/2))/(e*cos(d*x+c))^(1/2)+77/16*a^2*(3
*a^2-2*b^2)*e^7*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(
sin(1/2*d*x+1/2*c),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(d*x+c)^(1/2)/b^7/d
/(b+(-a^2+b^2)^(1/2))/(e*cos(d*x+c))^(1/2)-77/40*(15*a^2-4*b^2)*e^6*(cos(1/
2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/
2))*(e*cos(d*x+c))^(1/2)/b^6/d/cos(d*x+c)^(1/2)
```

Rubi [A]

time = 0.87, antiderivative size = 557, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {2772, 2942, 2946, 2721, 2719, 2780, 2886, 2884, 335, 304, 211, 214}

$$\frac{77ae^{13/2}(3a^2-2b^2)\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{16b^{13/2}d\sqrt[4]{b^2-a^2}}-\frac{77ae^{13/2}(3a^2-2b^2)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{16b^{13/2}d\sqrt[4]{b^2-a^2}}+\frac{77a^2e^7(3a^2-2b^2)\,\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{16b^7d\left(b-\sqrt{b^2-a^2}\right)\sqrt{e\cos(c+dx)}}+\frac{77a^2e^7(3a^2-2b^2)\,\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{16b^7d\left(\sqrt{b^2-a^2}+b\right)\sqrt{e\cos(c+dx)}}-\frac{77e^6(15a^2-4b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{40b^6d\sqrt{\cos(c+dx)}}-\frac{77e^5(e\cos(c+dx))^{3/2}\,(15a^2+6ab\sin(c+dx)-4b^2)}{120b^5d(a+b\sin(c+dx))}-\frac{11e^3(e\cos(c+dx))^{7/2}(9a+4b\sin(c+dx))}{60b^3d(a+b\sin(c+dx))^2}-\frac{e(e\cos(c+dx))^{11/2}}{3bd(a+b\sin(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(13/2)/(a + b*Sin[c + d*x])^4,x]
```

```
[Out] (77*a*(3*a^2 - 2*b^2)*e^(13/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2
 + b^2)^(1/4)*Sqrt[e])])/(16*b^(13/2)*(-a^2 + b^2)^(1/4)*d) - (77*a*(3*a^2
- 2*b^2)*e^(13/2)*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4
)*Sqrt[e])])/(16*b^(13/2)*(-a^2 + b^2)^(1/4)*d) - (77*(15*a^2 - 4*b^2)*e^6*
Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(40*b^6*d*Sqrt[Cos[c + d*x]
]) + (77*a^2*(3*a^2 - 2*b^2)*e^7*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - S
qrt[-a^2 + b^2]), (c + d*x)/2, 2])/(16*b^7*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e*
Cos[c + d*x]]) + (77*a^2*(3*a^2 - 2*b^2)*e^7*Sqrt[Cos[c + d*x]]*EllipticPi[
(2*b)/(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(16*b^7*(b + Sqrt[-a^2 + b^2
])*d*Sqrt[e*Cos[c + d*x]]) - (e*(e*Cos[c + d*x])^(11/2))/(3*b*d*(a + b*Sin[
```

```
c + d*x])^3) - (11*e^3*(e*Cos[c + d*x])^(7/2)*(9*a + 4*b*Sin[c + d*x]))/(60
*b^3*d*(a + b*Sin[c + d*x])^2) - (77*e^5*(e*Cos[c + d*x])^(3/2)*(15*a^2 - 4
*b^2 + 6*a*b*Sin[c + d*x]))/(120*b^5*d*(a + b*Sin[c + d*x]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2772

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Dist[g^2*((p - 1)/(b*(m + 1))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2780

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[a*(g/(2*b)), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[a*(g/(2*b)), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[b*(g/f), Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2942

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*C
os[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x])/(b^2*f*(m + 1)*(m + p + 1))), x] + Dist[g^2*((
p - 1)/(b^2*(m + 1)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{13/2}}{(a+b\sin(c+dx))^4}\,dx = -\frac{e(e\cos(c+dx))^{11/2}}{3bd(a+b\sin(c+dx))^3} - \frac{(11e^2)\int \frac{(e\cos(c+dx))^{9/2}\sin(c+dx)}{(a+b\sin(c+dx))^3}\,dx}{6b}$$

$$= -\frac{e(e\cos(c+dx))^{11/2}}{3bd(a+b\sin(c+dx))^3} - \frac{11e^3(e\cos(c+dx))^{7/2}(9a+4b\sin(c+dx))}{60b^3d(a+b\sin(c+dx))^2} + \frac{(77e^4 \cdots}{\cdots}$$

$$= -\frac{e(e\cos(c+dx))^{11/2}}{3bd(a+b\sin(c+dx))^3} - \frac{11e^3(e\cos(c+dx))^{7/2}(9a+4b\sin(c+dx))}{60b^3d(a+b\sin(c+dx))^2} - \frac{77e^5(\cdots}{\cdots}$$

$$= -\frac{e(e\cos(c+dx))^{11/2}}{3bd(a+b\sin(c+dx))^3} - \frac{11e^3(e\cos(c+dx))^{7/2}(9a+4b\sin(c+dx))}{60b^3d(a+b\sin(c+dx))^2} - \frac{77e^5(\cdots}{\cdots}$$

$$= -\frac{e(e\cos(c+dx))^{11/2}}{3bd(a+b\sin(c+dx))^3} - \frac{11e^3(e\cos(c+dx))^{7/2}(9a+4b\sin(c+dx))}{60b^3d(a+b\sin(c+dx))^2} - \frac{77e^5(\cdots}{\cdots}$$

$$= -\frac{77(15a^2-4b^2)\,e^6\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{40b^6d\sqrt{\cos(c+dx)}} - \frac{e(e\cos(c+dx))^{11/2}}{3bd(a+b\sin(c+dx))^3} \cdots$$

$$= -\frac{77(15a^2-4b^2)\,e^6\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{40b^6d\sqrt{\cos(c+dx)}} + \frac{77a^2(3a^2-2b^2)\,e^7\sqrt{\cos \cdots}}{16b^7\left(b-\sqrt{\cdots}\right.}$$

$$= \frac{77a(3a^2-2b^2)\,e^{13/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{16b^{13/2}\sqrt[4]{-a^2+b^2}\,d} - \frac{77a(3a^2-2b^2)\,e^{13/2}\tanh^{\cdots}}{16b^{13/2}\sqrt[4]{\cdots}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 33.78, size = 937, normalized size = 1.68

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^(13/2)/(a + b*Sin[c + d*x])^4,x]
```

```
[Out] (-77*(e*Cos[c + d*x])^(13/2)*((-12*a*b*(a + b*Sqrt[1 - Cos[c + d*x]^2])*((a
*AppellF1[3/4, 1/2, 1, 7/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^
2)]*Cos[c + d*x]^(3/2))/(3*(a^2 - b^2)) + ((1/8 + I/8)*(2*ArcTan[1 - ((1 +
I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*
Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - (1
```

```
 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]] + L
og[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]]
 + I*b*Cos[c + d*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)))*Sin[c + d*x])/(Sqrt[1
- Cos[c + d*x]^2]*(a + b*Sin[c + d*x])) - ((15*a^2 - 4*b^2)*(a + b*Sqrt[1 -
 Cos[c + d*x]^2])*(8*b^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Cos[c + d*x]^2, (b
^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Cos[c + d*x]^(3/2) + 3*Sqrt[2]*a*(a^2 - b^
2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/
4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)]
- Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]
] + b*Cos[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/
4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]]))*Sin[c + d*x]^2)/(12*b^(3/2)*(-a^2
 + b^2)*(1 - Cos[c + d*x]^2)*(a + b*Sin[c + d*x]))))/(80*b^5*d*Cos[c + d*x]
^(13/2)) + ((e*Cos[c + d*x])^(13/2)*Sec[c + d*x]^6*((-8*a*Cos[c + d*x])/(3*
b^5) + (-(a^4*Cos[c + d*x]) + 2*a^2*b^2*Cos[c + d*x] - b^4*Cos[c + d*x])/(3
*b^5*(a + b*Sin[c + d*x])^3) + (9*(a^3*Cos[c + d*x] - a*b^2*Cos[c + d*x]))/
(4*b^5*(a + b*Sin[c + d*x])^2) + (-71*a^2*Cos[c + d*x] + 20*b^2*Cos[c + d*x
])/(8*b^5*(a + b*Sin[c + d*x])) + Sin[2*(c + d*x)]/(5*b^4)))/d
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 284.54, size = 177735, normalized size = 319.09

method	result	size
default	Expression too large to display	177735

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(13/2)/(a+b*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(13/2)/(a+b*sin(d*x+c))^4,x, algorithm="maxima")
[Out] Timed out
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(13/2)/(a+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(13/2)/(a+b*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(13/2)/(a+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,\cos\left(c+d\,x\right))^{13/2}}{(a+b\,\sin\left(c+d\,x\right))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(13/2)/(a + b*sin(c + d*x))^4,x)
```

```
[Out] int((e*cos(c + d*x))^(13/2)/(a + b*sin(c + d*x))^4, x)
```

3.607 $\int \frac{(e\cos(c+dx))^{11/2}}{(a+b\sin(c+dx))^4}\,dx$

Optimal. Leaf size=571

$$-\frac{15a(7a^2-6b^2)\,e^{11/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{16b^{11/2}\left(-a^2+b^2\right)^{3/4}d}-\frac{15a(7a^2-6b^2)\,e^{11/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{16b^{11/2}\left(-a^2+b^2\right)^{3/4}d}$$

```
[Out] -15/16*a*(7*a^2-6*b^2)*e^(11/2)*arctan(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b
^2)^(1/4)/e^(1/2))/b^(11/2)/(-a^2+b^2)^(3/4)/d-15/16*a*(7*a^2-6*b^2)*e^(11/
2)*arctanh(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(11/2)/
(-a^2+b^2)^(3/4)/d-1/3*e*(e*cos(d*x+c))^(9/2)/b/d/(a+b*sin(d*x+c))^3-1/4*e^
3*(e*cos(d*x+c))^(5/2)*(7*a+4*b*sin(d*x+c))/b^3/d/(a+b*sin(d*x+c))^2-5/8*(2
1*a^2-4*b^2)*e^6*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(
sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/b^6/d/(e*cos(d*x+c))^(1/2)+15/
16*a^2*(7*a^2-6*b^2)*e^6*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*El
lipticPi(sin(1/2*d*x+1/2*c),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*cos(d*x+c)^(1
/2)/b^6/d/(a^2-b*(b-(-a^2+b^2)^(1/2)))/(e*cos(d*x+c))^(1/2)+15/16*a^2*(7*a^
2-6*b^2)*e^6*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin
(1/2*d*x+1/2*c),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(d*x+c)^(1/2)/b^6/d/(a
^2-b*(b+(-a^2+b^2)^(1/2)))/(e*cos(d*x+c))^(1/2)-5/8*e^5*(21*a^2-4*b^2+14*a*
b*sin(d*x+c))*(e*cos(d*x+c))^(1/2)/b^5/d/(a+b*sin(d*x+c))
```

Rubi [A]
time = 0.95, antiderivative size = 571, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {2772, 2942, 2946, 2721, 2720, 2781, 2886, 2884, 335, 218, 214, 211}

$$-\frac{15ae^{11/2}(7a^2-6b^2)\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{16b^{11/2}d\left(b^2-a^2\right)^{3/4}}-\frac{15ae^{11/2}(7a^2-6b^2)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{16b^{11/2}d\left(b^2-a^2\right)^{3/4}}-\frac{5e^6(21a^2-4b^2)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)|2\right)}{8b^6d\sqrt{e\cos(c+dx)}}+\frac{15a^2e^6(7a^2-6b^2)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)|2\right)}{16b^6d\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{e\cos(c+dx)}}+\frac{15a^2e^6(7a^2-6b^2)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)|2\right)}{16b^6d\left(a^2-b\left(\sqrt{b^2-a^2}+b\right)\right)\sqrt{e\cos(c+dx)}}-\frac{5e^5\sqrt{e\cos(c+dx)}\,(21a^2+14ab\sin(c+dx)-4b^2)}{8b^5d(a+b\sin(c+dx))}-\frac{e^3(e\cos(c+dx))^{5/2}(7a+4b\sin(c+dx))}{4b^3d(a+b\sin(c+dx))^2}-\frac{e(e\cos(c+dx))^{9/2}}{3bd(a+b\sin(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(11/2)/(a + b*Sin[c + d*x])^4,x]
```

```
[Out] (-15*a*(7*a^2 - 6*b^2)*e^(11/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^
2 + b^2)^(1/4)*Sqrt[e])])/(16*b^(11/2)*(-a^2 + b^2)^(3/4)*d) - (15*a*(7*a^2
 - 6*b^2)*e^(11/2)*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/
4)*Sqrt[e])])/(16*b^(11/2)*(-a^2 + b^2)^(3/4)*d) - (5*(21*a^2 - 4*b^2)*e^6*
Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(8*b^6*d*Sqrt[e*Cos[c + d*x]]
) + (15*a^2*(7*a^2 - 6*b^2)*e^6*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sq
rt[-a^2 + b^2]), (c + d*x)/2, 2])/(16*b^6*(a^2 - b*(b - Sqrt[-a^2 + b^2]))*
d*Sqrt[e*Cos[c + d*x]]) + (15*a^2*(7*a^2 - 6*b^2)*e^6*Sqrt[Cos[c + d*x]]*El
lipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(16*b^6*(a^2 - b*(b
 + Sqrt[-a^2 + b^2]))*d*Sqrt[e*Cos[c + d*x]]) - (e*(e*Cos[c + d*x])^(9/2))/
```

```
(3*b*d*(a + b*Sin[c + d*x])^3) - (e^3*(e*Cos[c + d*x])^(5/2)*(7*a + 4*b*Sin
[c + d*x]))/(4*b^3*d*(a + b*Sin[c + d*x])^2) - (5*e^5*Sqrt[e*Cos[c + d*x]]*
(21*a^2 - 4*b^2 + 14*a*b*Sin[c + d*x]))/(8*b^5*d*(a + b*Sin[c + d*x]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2772

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Dist[g^2*((p - 1)/(b*(m + 1))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2781

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[-a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[b*(g/f), Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2942

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*C
os[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x])/(b^2*f*(m + 1)*(m + p + 1))), x] + Dist[g^2*((
p - 1)/(b^2*(m + 1)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{11/2}}{(a+b\sin(c+dx))^4}\,dx = -\frac{e(e\cos(c+dx))^{9/2}}{3bd(a+b\sin(c+dx))^3} - \frac{(3e^2)\int \frac{(e\cos(c+dx))^{7/2}\sin(c+dx)}{(a+b\sin(c+dx))^3}\,dx}{2b}$$

$$= -\frac{e(e\cos(c+dx))^{9/2}}{3bd(a+b\sin(c+dx))^3} - \frac{e^3(e\cos(c+dx))^{5/2}(7a+4b\sin(c+dx))}{4b^3d(a+b\sin(c+dx))^2} + \frac{(5e^4)\int}{}$$

$$= -\frac{e(e\cos(c+dx))^{9/2}}{3bd(a+b\sin(c+dx))^3} - \frac{e^3(e\cos(c+dx))^{5/2}(7a+4b\sin(c+dx))}{4b^3d(a+b\sin(c+dx))^2} - \frac{5e^5\sqrt{e}}{}$$

$$= -\frac{e(e\cos(c+dx))^{9/2}}{3bd(a+b\sin(c+dx))^3} - \frac{e^3(e\cos(c+dx))^{5/2}(7a+4b\sin(c+dx))}{4b^3d(a+b\sin(c+dx))^2} - \frac{5e^5\sqrt{e}}{}$$

$$= -\frac{e(e\cos(c+dx))^{9/2}}{3bd(a+b\sin(c+dx))^3} - \frac{e^3(e\cos(c+dx))^{5/2}(7a+4b\sin(c+dx))}{4b^3d(a+b\sin(c+dx))^2} - \frac{5e^5\sqrt{e}}{}$$

$$= -\frac{5(21a^2-4b^2)\,e^6\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{8b^6d\sqrt{e\cos(c+dx)}} - \frac{e(e\cos(c+dx))^{9/2}}{3bd(a+b\sin(c+dx))^3} - \frac{e}{}$$

$$= -\frac{5(21a^2-4b^2)\,e^6\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{8b^6d\sqrt{e\cos(c+dx)}} + \frac{15a^2(7a^2-6b^2)\,e^6\sqrt{\cos(c-}}{16b^6\left(a^2-b\left(b-\sqrt{}\right.\right.}$$

$$= -\frac{15a(7a^2-6b^2)\,e^{11/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{16b^{11/2}\left(-a^2+b^2\right)^{3/4}d} - \frac{15a(7a^2-6b^2)\,e^{11/2}\tan}{16b^{11/2}(-}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 52.02, size = 2020, normalized size = 3.54

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^(11/2)/(a + b*Sin[c + d*x])^4,x]
```

```
[Out] ((e*Cos[c + d*x])^(11/2)*Sec[c + d*x]^5*((2*Sin[c + d*x])/(3*b^4) - (-a^2 +
 b^2)^2/(3*b^5*(a + b*Sin[c + d*x])^3) + (25*a*(a^2 - b^2))/(12*b^5*(a + b*
Sin[c + d*x])^2) + (-165*a^2 + 52*b^2)/(24*b^5*(a + b*Sin[c + d*x]))))/d -
((e*Cos[c + d*x])^(11/2)*((-76*a*b*(a + b*Sqrt[1 - Cos[c + d*x]^2])*((5*a*(
a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/
```

```
(-a^2 + b^2)]*Sqrt[Cos[c + d*x]])/(Sqrt[1 - Cos[c + d*x]^2]*(5*(a^2 - b^2)*
AppellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2
)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^
2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[c + d*x]^2,
(b^2*Cos[c + d*x]^2)/(-a^2 + b^2)])*Cos[c + d*x]^2)*(a^2 + b^2*(-1 + Cos[c
+ d*x]^2))) - ((1/8 - I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[
c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[c +
 d*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2
+ b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]] - Log[Sqrt[-a^2 + b^2]
+ (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]]
))/(-a^2 + b^2)^(3/4))*Sin[c + d*x])/(Sqrt[1 - Cos[c + d*x]^2]*(a + b*Sin[c
 + d*x])) + (32*a*b*(a + b*Sqrt[1 - Cos[c + d*x]^2])*Cos[2*(c + d*x)]*(((1/
2 - I/2)*(-2*a^2 + b^2)*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a
^2 + b^2)^(1/4)])/(b^(3/2)*(-a^2 + b^2)^(3/4)) - ((1/2 - I/2)*(-2*a^2 + b^2
)*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)])/(b^(
3/2)*(-a^2 + b^2)^(3/4)) + (4*Sqrt[Cos[c + d*x]])/b - (4*a*AppellF1[5/4, 1/
2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Cos[c + d*x]^
(5/2))/(5*(a^2 - b^2)) + (10*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[c
 + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[c + d*x]])/(Sqrt[1 -
 Cos[c + d*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2,
(b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Co
s[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/
4, 3/2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)])*Cos[c +
 d*x]^2)*(a^2 + b^2*(-1 + Cos[c + d*x]^2))) + ((1/4 - I/4)*(-2*a^2 + b^2)*L
og[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]]
 + I*b*Cos[c + d*x]])/(b^(3/2)*(-a^2 + b^2)^(3/4)) - ((1/4 - I/4)*(-2*a^2 +
 b^2)*Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c
+ d*x]] + I*b*Cos[c + d*x]])/(b^(3/2)*(-a^2 + b^2)^(3/4)))*Sin[c + d*x])/(S
qrt[1 - Cos[c + d*x]^2]*(-1 + 2*Cos[c + d*x]^2)*(a + b*Sin[c + d*x])) - (2*
(41*a^2 - 20*b^2)*(a + b*Sqrt[1 - Cos[c + d*x]^2])*((5*b*(a^2 - b^2)*Appell
F1[1/4, -1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Sq
rt[Cos[c + d*x]]*Sqrt[1 - Cos[c + d*x]^2])/((-5*(a^2 - b^2)*AppellF1[1/4, -
1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + 2*(2*b^2*
AppellF1[5/4, -1/2, 2, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^
2)] + (a^2 - b^2)*AppellF1[5/4, 1/2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d
*x]^2)/(-a^2 + b^2)])*Cos[c + d*x]^2)*(a^2 + b^2*(-1 + Cos[c + d*x]^2))) +
(a*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] +
 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] - Log
[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b
*Cos[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sq
rt[Cos[c + d*x]] + b*Cos[c + d*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)))
*Sin[c + d*x]^2)/((1 - Cos[c + d*x]^2)*(a + b*Sin[c + d*x]))))/(16*b^5*d*Co
s[c + d*x]^(11/2))
```

Maple [F]

time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^{\frac{11}{2}}}{(a+b\sin(dx+c))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(11/2)/(a+b*sin(d*x+c))^4,x)
[Out] int((e*cos(d*x+c))^(11/2)/(a+b*sin(d*x+c))^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(11/2)/(a+b*sin(d*x+c))^4,x, algorithm="maxima")
[Out] e^(11/2)*integrate(cos(d*x + c)^(11/2)/(b*sin(d*x + c) + a)^4, x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(11/2)/(a+b*sin(d*x+c))^4,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(11/2)/(a+b*sin(d*x+c))**4,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(11/2)/(a+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(11/2)/(a + b*sin(c + d*x))^4,x)
```

```
[Out] \text{Hanged}
```

3.608 $\int \frac{(e\cos(c+dx))^{9/2}}{(a+b\sin(c+dx))^4}\,dx$

Optimal. Leaf size=591

$$\frac{7a(5a^2-6b^2)\,e^{9/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{16b^{9/2}\left(-a^2+b^2\right)^{5/4}d}-\frac{7a(5a^2-6b^2)\,e^{9/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{16b^{9/2}\left(-a^2+b^2\right)^{5/4}d}+\frac{7(5a^2}{}$$

```
[Out] 7/16*a*(5*a^2-6*b^2)*e^(9/2)*arctan(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)
^(1/4)/e^(1/2))/b^(9/2)/(-a^2+b^2)^(5/4)/d-7/16*a*(5*a^2-6*b^2)*e^(9/2)*arc
tanh(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(9/2)/(-a^2+b
^2)^(5/4)/d-1/3*e*(e*cos(d*x+c))^(7/2)/b/d/(a+b*sin(d*x+c))^3+7/8*(5*a^2-4*
b^2)*e^3*(e*cos(d*x+c))^(3/2)/b^3/(a^2-b^2)/d/(a+b*sin(d*x+c))-7/12*e^3*(e*
cos(d*x+c))^(3/2)*(5*a+4*b*sin(d*x+c))/b^3/d/(a+b*sin(d*x+c))^2-7/16*a^2*(5
*a^2-6*b^2)*e^5*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(
sin(1/2*d*x+1/2*c),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*cos(d*x+c)^(1/2)/b^5/(
a^2-b^2)/d/(b-(-a^2+b^2)^(1/2))/(e*cos(d*x+c))^(1/2)-7/16*a^2*(5*a^2-6*b^2)
*e^5*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x
+1/2*c),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(d*x+c)^(1/2)/b^5/(a^2-b^2)/d/
(b+(-a^2+b^2)^(1/2))/(e*cos(d*x+c))^(1/2)+7/8*(5*a^2-4*b^2)*e^4*(cos(1/2*d*
x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*
(e*cos(d*x+c))^(1/2)/b^4/(a^2-b^2)/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.98, antiderivative size = 591, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {2772, 2942, 2943, 2946, 2721, 2719, 2780, 2886, 2884, 335, 304, 211, 214}

$$\frac{7ae^{9/2}(5a^2-6b^2)\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{16b^{9/2}d\left(b^2-a^2\right)^{5/4}}-\frac{7ae^{9/2}(5a^2-6b^2)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{16b^{9/2}d\left(b^2-a^2\right)^{5/4}}-\frac{7a^2e^5(5a^2-6b^2)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{16b^5d\left(a^2-b^2\right)\left(b-\sqrt{b^2-a^2}\right)\sqrt{e\cos(c+dx)}}-\frac{7a^2e^5(5a^2-6b^2)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{16b^5d\left(a^2-b^2\right)\left(\sqrt{b^2-a^2}+b\right)\sqrt{e\cos(c+dx)}}+\frac{7e^4(5a^2-4b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{8b^4d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}}+\frac{7e^3(5a^2-4b^2)(e\cos(c+dx))^{3/2}}{8b^3d\left(a^2-b^2\right)(a+b\sin(c+dx))}-\frac{7e^3(e\cos(c+dx))^{3/2}(5a+4b\sin(c+dx))}{12b^3d(a+b\sin(c+dx))^2}-\frac{e(e\cos(c+dx))^{7/2}}{3bd(a+b\sin(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(9/2)/(a + b*Sin[c + d*x])^4,x]
```

```
[Out] (7*a*(5*a^2 - 6*b^2)*e^(9/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 +
 b^2)^(1/4)*Sqrt[e])])/(16*b^(9/2)*(-a^2 + b^2)^(5/4)*d) - (7*a*(5*a^2 - 6*
b^2)*e^(9/2)*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqr
t[e])])/(16*b^(9/2)*(-a^2 + b^2)^(5/4)*d) + (7*(5*a^2 - 4*b^2)*e^4*Sqrt[e*C
os[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(8*b^4*(a^2 - b^2)*d*Sqrt[Cos[c + d
*x]]) - (7*a^2*(5*a^2 - 6*b^2)*e^5*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b -
 Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(16*b^5*(a^2 - b^2)*(b - Sqrt[-a^2 + b
^2])*d*Sqrt[e*Cos[c + d*x]]) - (7*a^2*(5*a^2 - 6*b^2)*e^5*Sqrt[Cos[c + d*x]
]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(16*b^5*(a^2 -
b^2)*(b + Sqrt[-a^2 + b^2])*d*Sqrt[e*Cos[c + d*x]]) - (e*(e*Cos[c + d*x])^(
```

```
7/2))/(3*b*d*(a + b*Sin[c + d*x])^3) + (7*(5*a^2 - 4*b^2)*e^3*(e*Cos[c + d*
x])^(3/2))/(8*b^3*(a^2 - b^2)*d*(a + b*Sin[c + d*x])) - (7*e^3*(e*Cos[c + d
*x])^(3/2)*(5*a + 4*b*Sin[c + d*x]))/(12*b^3*d*(a + b*Sin[c + d*x])^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2772

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Dist[g^2*((p - 1)/(b*(m + 1))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2780

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[a*(g/(2*b)), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[a*(g/(2*b)), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[b*(g/f), Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2942

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*C
os[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x])/(b^2*f*(m + 1)*(m + p + 1))), x] + Dist[g^2*((
p - 1)/(b^2*(m + 1)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rule 2943

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c
 - a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^(m + 1)/(f*g*(a^2 -
 b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*
(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p
+ 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[
a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{9/2}}{(a+b\sin(c+dx))^4}\,dx = -\frac{e(e\cos(c+dx))^{7/2}}{3bd(a+b\sin(c+dx))^3} - \frac{(7e^2)\int \frac{(e\cos(c+dx))^{5/2}\sin(c+dx)}{(a+b\sin(c+dx))^3}\,dx}{6b}$$

$$= -\frac{e(e\cos(c+dx))^{7/2}}{3bd(a+b\sin(c+dx))^3} - \frac{7e^3(e\cos(c+dx))^{3/2}(5a+4b\sin(c+dx))}{12b^3d(a+b\sin(c+dx))^2} + \frac{(7e^4)\cdots}{\cdots}$$

$$= -\frac{e(e\cos(c+dx))^{7/2}}{3bd(a+b\sin(c+dx))^3} + \frac{7(5a^2-4b^2)\,e^3(e\cos(c+dx))^{3/2}}{8b^3\,(a^2-b^2)\,d(a+b\sin(c+dx))} - \frac{7e^3(e\cos(c+\cdots}{12b^3d\cdots}$$

$$= -\frac{e(e\cos(c+dx))^{7/2}}{3bd(a+b\sin(c+dx))^3} + \frac{7(5a^2-4b^2)\,e^3(e\cos(c+dx))^{3/2}}{8b^3\,(a^2-b^2)\,d(a+b\sin(c+dx))} - \frac{7e^3(e\cos(c+\cdots}{12b^3d\cdots}$$

$$= -\frac{e(e\cos(c+dx))^{7/2}}{3bd(a+b\sin(c+dx))^3} + \frac{7(5a^2-4b^2)\,e^3(e\cos(c+dx))^{3/2}}{8b^3\,(a^2-b^2)\,d(a+b\sin(c+dx))} - \frac{7e^3(e\cos(c+\cdots}{12b^3d\cdots}$$

$$= \frac{7(5a^2-4b^2)\,e^4\sqrt{e\cos(c+dx)}\,E\left(\tfrac{1}{2}(c+dx)\middle|\,2\right)}{8b^4\,(a^2-b^2)\,d\sqrt{\cos(c+dx)}} - \frac{e(e\cos(c+dx))^{7/2}}{3bd(a+b\sin(c+dx))^3} + \cdots$$

$$= \frac{7(5a^2-4b^2)\,e^4\sqrt{e\cos(c+dx)}\,E\left(\tfrac{1}{2}(c+dx)\middle|\,2\right)}{8b^4\,(a^2-b^2)\,d\sqrt{\cos(c+dx)}} - \frac{7a^2(5a^2-6b^2)\,e^5\sqrt{\cos(c+\cdots}}{16b^5\,(a^2-b^2)\left(b-\cdots\right.}$$

$$= \frac{7a(5a^2-6b^2)\,e^{9/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{16b^{9/2}\left(-a^2+b^2\right)^{5/4}d} - \frac{7a(5a^2-6b^2)\,e^{9/2}\tanh^{-1}\cdots}{16b^{9/2}\left(-a^2\cdots\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 33.53, size = 900, normalized size = 1.52

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^(9/2)/(a + b*Sin[c + d*x])^4,x]
```

```
[Out] ((e*Cos[c + d*x])^(9/2)*Sec[c + d*x]^4*((a^2*Cos[c + d*x] - b^2*Cos[c + d*x
])/(3*b^3*(a + b*Sin[c + d*x])^3) - (5*a*Cos[c + d*x])/(4*b^3*(a + b*Sin[c
+ d*x])^2) + (-19*a^2*Cos[c + d*x] + 12*b^2*Cos[c + d*x])/(8*b^3*(-a^2 + b^
2)*(a + b*Sin[c + d*x]))))/d + (7*(e*Cos[c + d*x])^(9/2)*((-4*a*b*(a + b*Sq
rt[1 - Cos[c + d*x]^2])*((a*AppellF1[3/4, 1/2, 1, 7/4, Cos[c + d*x]^2, (b^2
*Cos[c + d*x]^2)/(-a^2 + b^2)]*Cos[c + d*x]^(3/2))/(3*(a^2 - b^2)) + ((1/8
+ I/8)*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4
)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)]
- Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*
x]] + I*b*Cos[c + d*x]] + Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^
2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/
4)))*Sin[c + d*x])/(Sqrt[1 - Cos[c + d*x]^2]*(a + b*Sin[c + d*x])) - ((5*a^
2 - 4*b^2)*(a + b*Sqrt[1 - Cos[c + d*x]^2])*(8*b^(5/2)*AppellF1[3/4, -1/2,
1, 7/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Cos[c + d*x]^(3/
2) + 3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[
c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c +
d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^
2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[
2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]]))*Sin[c +
 d*x]^2)/(12*b^(3/2)*(-a^2 + b^2)*(1 - Cos[c + d*x]^2)*(a + b*Sin[c + d*x])
)))/(16*(a - b)*b^3*(a + b)*d*Cos[c + d*x]^(9/2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 277.04, size = 234955, normalized size = 397.55

method	result	size
default	Expression too large to display	234955

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(9/2)/(a+b*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(9/2)/(a+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] e^(9/2)*integrate(cos(d*x + c)^(9/2)/(b*sin(d*x + c) + a)^4, x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(9/2)/(a+b*sin(d*x+c))^4,x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(9/2)/(a+b*sin(d*x+c))**4,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 8857 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(9/2)/(a+b*sin(d*x+c))^4,x, algorithm="giac")
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,\cos\left(c+d\,x\right))^{9/2}}{\left(a+b\,\sin\left(c+d\,x\right)\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(9/2)/(a + b*sin(c + d*x))^4,x)
[Out] int((e*cos(c + d*x))^(9/2)/(a + b*sin(c + d*x))^4, x)
```

3.609 $\int \frac{(e\cos(c+dx))^{7/2}}{(a+b\sin(c+dx))^4}\,dx$

Optimal. Leaf size=597

$$-\frac{5a(a^2-2b^2)\,e^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{16b^{7/2}\left(-a^2+b^2\right)^{7/4}d}-\frac{5a(a^2-2b^2)\,e^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{16b^{7/2}\left(-a^2+b^2\right)^{7/4}d}+\frac{5(3a^2}{}$$

```
[Out] -5/16*a*(a^2-2*b^2)*e^(7/2)*arctan(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^
(1/4)/e^(1/2))/b^(7/2)/(-a^2+b^2)^(7/4)/d-5/16*a*(a^2-2*b^2)*e^(7/2)*arctan
h(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(7/2)/(-a^2+b^2)
^(7/4)/d-1/3*e*(e*cos(d*x+c))^(5/2)/b/d/(a+b*sin(d*x+c))^3+5/24*(3*a^2-4*b^
2)*e^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*
x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/b^4/(a^2-b^2)/d/(e*cos(d*x+c))^(1/2)-5/1
6*a^2*(a^2-2*b^2)*e^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellip
ticPi(sin(1/2*d*x+1/2*c),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*cos(d*x+c)^(1/2)
/b^4/(a^2-b^2)/d/(a^2-b*(b-(-a^2+b^2)^(1/2)))/(e*cos(d*x+c))^(1/2)-5/16*a^2
*(a^2-2*b^2)*e^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi
(sin(1/2*d*x+1/2*c),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(d*x+c)^(1/2)/b^4/
(a^2-b^2)/d/(a^2-b*(b+(-a^2+b^2)^(1/2)))/(e*cos(d*x+c))^(1/2)-5/24*(3*a^2-4
*b^2)*e^3*(e*cos(d*x+c))^(1/2)/b^3/(a^2-b^2)/d/(a+b*sin(d*x+c))+5/12*e^3*(3
*a+4*b*sin(d*x+c))*(e*cos(d*x+c))^(1/2)/b^3/d/(a+b*sin(d*x+c))^2
```

Rubi [A]
time = 1.02, antiderivative size = 597, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {2772, 2942, 2943, 2946, 2721, 2720, 2781, 2886, 2884, 335, 218, 214, 211}

$$-\frac{5ae^{7/2}(a^2-2b^2)\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{16b^{7/2}d\left(b^2-a^2\right)^{7/4}}-\frac{5ae^{7/2}(a^2-2b^2)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{16b^{7/2}d\left(b^2-a^2\right)^{7/4}}+\frac{5e^4(3a^2-4b^2)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{24b^4d\left(a^2-b^2\right)\sqrt{e\cos(c+dx)}}-\frac{5a^2e^4(a^2-2b^2)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{16b^4d\left(a^2-b^2\right)\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{e\cos(c+dx)}}-\frac{5a^2e^4(a^2-2b^2)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{16b^4d\left(a^2-b^2\right)\left(a^2-b\left(\sqrt{b^2-a^2}+b\right)\right)\sqrt{e\cos(c+dx)}}-\frac{5e^3(3a^2-4b^2)\sqrt{e\cos(c+dx)}}{24b^3d\left(a^2-b^2\right)(a+b\sin(c+dx))}+\frac{5e^3\sqrt{e\cos(c+dx)}\,(3a+4b\sin(c+dx))}{12b^3d(a+b\sin(c+dx))^2}-\frac{e(e\cos(c+dx))^{5/2}}{3bd(a+b\sin(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(7/2)/(a + b*Sin[c + d*x])^4,x]
```

```
[Out] (-5*a*(a^2 - 2*b^2)*e^(7/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 +
b^2)^(1/4)*Sqrt[e])])/(16*b^(7/2)*(-a^2 + b^2)^(7/4)*d) - (5*a*(a^2 - 2*b^2
)*e^(7/2)*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e
])])/(16*b^(7/2)*(-a^2 + b^2)^(7/4)*d) + (5*(3*a^2 - 4*b^2)*e^4*Sqrt[Cos[c
+ d*x]]*EllipticF[(c + d*x)/2, 2])/(24*b^4*(a^2 - b^2)*d*Sqrt[e*Cos[c + d*x
]]) - (5*a^2*(a^2 - 2*b^2)*e^4*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqr
t[-a^2 + b^2]), (c + d*x)/2, 2])/(16*b^4*(a^2 - b^2)*(a^2 - b*(b - Sqrt[-a^
2 + b^2]))*d*Sqrt[e*Cos[c + d*x]]) - (5*a^2*(a^2 - 2*b^2)*e^4*Sqrt[Cos[c +
d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(16*b^4*(a^
2 - b^2)*(a^2 - b*(b + Sqrt[-a^2 + b^2]))*d*Sqrt[e*Cos[c + d*x]]) - (e*(e*C
```

```
os[c + d*x])^(5/2))/(3*b*d*(a + b*Sin[c + d*x])^3) - (5*(3*a^2 - 4*b^2)*e^3
*Sqrt[e*Cos[c + d*x]])/(24*b^3*(a^2 - b^2)*d*(a + b*Sin[c + d*x])) + (5*e^3
*Sqrt[e*Cos[c + d*x]]*(3*a + 4*b*Sin[c + d*x]))/(12*b^3*d*(a + b*Sin[c + d*
x])^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2772

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Dist[g^2*((p - 1)/(b*(m + 1))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
```

```
tegersQ[2*m, 2*p]
```

Rule 2781

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[-a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[b*(g/f), Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2942

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*C
os[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x])/(b^2*f*(m + 1)*(m + p + 1))), x] + Dist[g^2*((
p - 1)/(b^2*(m + 1)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rule 2943

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c
 - a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^(m + 1)/(f*g*(a^2 -
 b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*
(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p
+ 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[
```

```
a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{7/2}}{(a+b\sin(c+dx))^4}\,dx = -\frac{e(e\cos(c+dx))^{5/2}}{3bd(a+b\sin(c+dx))^3} - \frac{(5e^2)\int \frac{(e\cos(c+dx))^{3/2}\sin(c+dx)}{(a+b\sin(c+dx))^3}\,dx}{6b}$$

$$= -\frac{e(e\cos(c+dx))^{5/2}}{3bd(a+b\sin(c+dx))^3} + \frac{5e^3\sqrt{e\cos(c+dx)}\,(3a+4b\sin(c+dx))}{12b^3d(a+b\sin(c+dx))^2} - \frac{(5e^4)\ \ldots}{\ldots}$$

$$= -\frac{e(e\cos(c+dx))^{5/2}}{3bd(a+b\sin(c+dx))^3} - \frac{5(3a^2-4b^2)\,e^3\sqrt{e\cos(c+dx)}}{24b^3\,(a^2-b^2)\,d(a+b\sin(c+dx))} + \frac{5e^3\sqrt{e\cos(c}{12b^3\ldots}$$

$$= -\frac{e(e\cos(c+dx))^{5/2}}{3bd(a+b\sin(c+dx))^3} - \frac{5(3a^2-4b^2)\,e^3\sqrt{e\cos(c+dx)}}{24b^3\,(a^2-b^2)\,d(a+b\sin(c+dx))} + \frac{5e^3\sqrt{e\cos(c}{12b^3\ldots}$$

$$= -\frac{e(e\cos(c+dx))^{5/2}}{3bd(a+b\sin(c+dx))^3} - \frac{5(3a^2-4b^2)\,e^3\sqrt{e\cos(c+dx)}}{24b^3\,(a^2-b^2)\,d(a+b\sin(c+dx))} + \frac{5e^3\sqrt{e\cos(c}{12b^3\ldots}$$

$$= \frac{5(3a^2-4b^2)\,e^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{24b^4\,(a^2-b^2)\,d\sqrt{e\cos(c+dx)}} - \frac{e(e\cos(c+dx))^{5/2}}{3bd(a+b\sin(c+dx))^3} - \frac{5\ldots}{24\ldots}$$

$$= \frac{5(3a^2-4b^2)\,e^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{24b^4\,(a^2-b^2)\,d\sqrt{e\cos(c+dx)}} + \frac{5a^2(a^2-2b^2)\,e^4\sqrt{\cos(c+dx}}{16b^4\,(-a^2+b^2)^{3/2}\left(b-\ldots\right.}$$

$$= -\frac{5a(a^2-2b^2)\,e^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{16b^{7/2}\,(-a^2+b^2)^{7/4}\,d} - \frac{5a(a^2-2b^2)\,e^{7/2}\tanh^{-1}\left(\ldots\right.}{16b^{7/2}\,(-a^2\ldots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

time = 42.34, size = 1263, normalized size = 2.12

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^(7/2)/(a + b*Sin[c + d*x])^4,x]
```

```
[Out] ((e*Cos[c + d*x])^(7/2)*Sec[c + d*x]^3*((a^2 - b^2)/(3*b^3*(a + b*Sin[c + d
*x])^3) - (13*a)/(12*b^3*(a + b*Sin[c + d*x])^2) + (-33*a^2 + 28*b^2)/(24*b
^3*(-a^2 + b^2)*(a + b*Sin[c + d*x]))))/d + (5*(e*Cos[c + d*x])^(7/2)*((-4*
a*b*(a + b*Sqrt[1 - Cos[c + d*x]^2])*((5*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1
, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[c + d*x]
])/(Sqrt[1 - Cos[c + d*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[
c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2
, 2, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)
*AppellF1[5/4, 3/2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^
2)])*Cos[c + d*x]^2)*(a^2 + b^2*(-1 + Cos[c + d*x]^2))) - ((1/8 - I/8)*Sqrt
[b]*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)]
- 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] + L
og[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]]
 + I*b*Cos[c + d*x]] - Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^
(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]]))/(-a^2 + b^2)^(3/4))*Sin[c +
d*x])/(Sqrt[1 - Cos[c + d*x]^2]*(a + b*Sin[c + d*x])) - (2*(3*a^2 - 4*b^2)*
(a + b*Sqrt[1 - Cos[c + d*x]^2])*((5*b*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5
/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[c + d*x]]*S
qrt[1 - Cos[c + d*x]^2])/((-5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Cos[c
 + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + 2*(2*b^2*AppellF1[5/4, -1/2
, 2, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + (a^2 - b^2)*
AppellF1[5/4, 1/2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2
)])*Cos[c + d*x]^2)*(a^2 + b^2*(-1 + Cos[c + d*x]^2))) + (a*(-2*ArcTan[1 -
(Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] + 2*ArcTan[1 + (Sqr
t[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] -
 Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]] + L
og[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] +
 b*Cos[c + d*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)))*Sin[c + d*x]^2)/(
(1 - Cos[c + d*x]^2)*(a + b*Sin[c + d*x]))))/(48*(a - b)*b^3*(a + b)*d*Cos[
c + d*x]^(7/2))
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^{\frac{7}{2}}}{(a+b\sin(dx+c))^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(7/2)/(a+b*sin(d*x+c))^4,x)
```

```
[Out] int((e*cos(d*x+c))^(7/2)/(a+b*sin(d*x+c))^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] e^(7/2)*integrate(cos(d*x + c)^(7/2)/(b*sin(d*x + c) + a)^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral(cos(d*x + c)^(7/2)*e^(7/2)/(b^4*cos(d*x + c)^4 + a^4 + 6*a^2*b^2 +
 b^4 - 2*(3*a^2*b^2 + b^4)*cos(d*x + c)^2 - 4*(a*b^3*cos(d*x + c)^2 - a^3*b
 - a*b^3)*sin(d*x + c)), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(7/2)/(a+b*sin(d*x+c))**4,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3878 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(7/2)*e^(7/2)/(b*sin(d*x + c) + a)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\cos(c+dx))^{7/2}}{(a+b\sin(c+dx))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(7/2)/(a + b*sin(c + d*x))^4,x)
[Out] int((e*cos(c + d*x))^(7/2)/(a + b*sin(c + d*x))^4, x)
```

3.610 $\int \frac{(e\cos(c+dx))^{5/2}}{(a+b\sin(c+dx))^4}\,dx$

Optimal. Leaf size=574

$$-\frac{a(a^2-6b^2)\,e^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{16b^{5/2}\left(-a^2+b^2\right)^{9/4}d}+\frac{a(a^2-6b^2)\,e^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{16b^{5/2}\left(-a^2+b^2\right)^{9/4}d}+\frac{(a^2+4}{}$$

```
[Out] -1/16*a*(a^2-6*b^2)*e^(5/2)*arctan(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^
(1/4)/e^(1/2))/b^(5/2)/(-a^2+b^2)^(9/4)/d+1/16*a*(a^2-6*b^2)*e^(5/2)*arctan
h(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(5/2)/(-a^2+b^2)
^(9/4)/d-1/3*e*(e*cos(d*x+c))^(3/2)/b/d/(a+b*sin(d*x+c))^3+1/4*a*e*(e*cos(d
*x+c))^(3/2)/b/(a^2-b^2)/d/(a+b*sin(d*x+c))^2+1/8*(a^2+4*b^2)*e*(e*cos(d*x+
c))^(3/2)/b/(a^2-b^2)^2/d/(a+b*sin(d*x+c))-1/16*a^2*(a^2-6*b^2)*e^3*(cos(1/
2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/
(b-(-a^2+b^2)^(1/2)),2^(1/2))*cos(d*x+c)^(1/2)/b^3/(a^2-b^2)^2/d/(b-(-a^2+b
^2)^(1/2))/(e*cos(d*x+c))^(1/2)-1/16*a^2*(a^2-6*b^2)*e^3*(cos(1/2*d*x+1/2*c
)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(b+(-a^2+b^
2)^(1/2)),2^(1/2))*cos(d*x+c)^(1/2)/b^3/(a^2-b^2)^2/d/(b+(-a^2+b^2)^(1/2))/
(e*cos(d*x+c))^(1/2)+1/8*(a^2+4*b^2)*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1
/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/b^
2/(a^2-b^2)^2/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.95, antiderivative size = 574, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {2772, 2943, 2946, 2721, 2719, 2780, 2886, 2884, 335, 304, 211, 214}

$$-\frac{ae^{5/2}(a^2-6b^2)\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{16b^{5/2}d\left(b^2-a^2\right)^{9/4}}+\frac{e^2(a^2+4b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{8b^2d\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}}+\frac{e(a^2+4b^2)\,(e\cos(c+dx))^{3/2}}{8bd\left(a^2-b^2\right)^2(a+b\sin(c+dx))}+\frac{ae(e\cos(c+dx))^{3/2}}{4bd\left(a^2-b^2\right)(a+b\sin(c+dx))^2}+\frac{ae^{5/2}(a^2-6b^2)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{16b^{5/2}d\left(b^2-a^2\right)^{9/4}}-\frac{a^2e^3(a^2-6b^2)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{16b^3d\left(a^2-b^2\right)^2\left(b-\sqrt{b^2-a^2}\right)\sqrt{e\cos(c+dx)}}-\frac{a^2e^3(a^2-6b^2)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{16b^3d\left(a^2-b^2\right)^2\left(\sqrt{b^2-a^2}+b\right)\sqrt{e\cos(c+dx)}}-\frac{e(e\cos(c+dx))^{3/2}}{3bd(a+b\sin(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(5/2)/(a + b*Sin[c + d*x])^4,x]
```

```
[Out] -1/16*(a*(a^2 - 6*b^2)*e^(5/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2
 + b^2)^(1/4)*Sqrt[e])])/(b^(5/2)*(-a^2 + b^2)^(9/4)*d) + (a*(a^2 - 6*b^2)*
e^(5/2)*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])
])/(16*b^(5/2)*(-a^2 + b^2)^(9/4)*d) + ((a^2 + 4*b^2)*e^2*Sqrt[e*Cos[c + d*
x]]*EllipticE[(c + d*x)/2, 2])/(8*b^2*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]) -
 (a^2*(a^2 - 6*b^2)*e^3*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2
+ b^2]), (c + d*x)/2, 2])/(16*b^3*(a^2 - b^2)^2*(b - Sqrt[-a^2 + b^2])*d*Sq
rt[e*Cos[c + d*x]]) - (a^2*(a^2 - 6*b^2)*e^3*Sqrt[Cos[c + d*x]]*EllipticPi[
(2*b)/(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(16*b^3*(a^2 - b^2)^2*(b + S
qrt[-a^2 + b^2])*d*Sqrt[e*Cos[c + d*x]]) - (e*(e*Cos[c + d*x])^(3/2))/(3*b*
```

```
d*(a + b*Sin[c + d*x])^3) + (a*e*(e*Cos[c + d*x])^(3/2))/(4*b*(a^2 - b^2)*d
*(a + b*Sin[c + d*x])^2) + ((a^2 + 4*b^2)*e*(e*Cos[c + d*x])^(3/2))/(8*b*(a
^2 - b^2)^2*d*(a + b*Sin[c + d*x]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2772

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Dist[g^2*((p - 1)/(b*(m + 1))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2780

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[a*(g/(2*b)), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[a*(g/(2*b)), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[b*(g/f), Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2943

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c
 - a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^(m + 1)/(f*g*(a^2 -
 b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*
(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p
+ 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[
a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{5/2}}{(a+b\sin(c+dx))^4}\,dx = -\frac{e(e\cos(c+dx))^{3/2}}{3bd(a+b\sin(c+dx))^3} - \frac{e^2\int \frac{\sqrt{e\cos(c+dx)}\sin(c+dx)}{(a+b\sin(c+dx))^3}dx}{2b}$$

$$= -\frac{e(e\cos(c+dx))^{3/2}}{3bd(a+b\sin(c+dx))^3} + \frac{ae(e\cos(c+dx))^{3/2}}{4b(a^2-b^2)d(a+b\sin(c+dx))^2} + \frac{e^2\int \frac{\sqrt{e\cos(c+}}{(a}}{4b}$$

$$= -\frac{e(e\cos(c+dx))^{3/2}}{3bd(a+b\sin(c+dx))^3} + \frac{ae(e\cos(c+dx))^{3/2}}{4b(a^2-b^2)d(a+b\sin(c+dx))^2} + \frac{(a^2+4b^2)e(e}{8b(a^2-b^2)^2d(}$$

$$= -\frac{e(e\cos(c+dx))^{3/2}}{3bd(a+b\sin(c+dx))^3} + \frac{ae(e\cos(c+dx))^{3/2}}{4b(a^2-b^2)d(a+b\sin(c+dx))^2} + \frac{(a^2+4b^2)e(e}{8b(a^2-b^2)^2d(}$$

$$= -\frac{e(e\cos(c+dx))^{3/2}}{3bd(a+b\sin(c+dx))^3} + \frac{ae(e\cos(c+dx))^{3/2}}{4b(a^2-b^2)d(a+b\sin(c+dx))^2} + \frac{(a^2+4b^2)e(e}{8b(a^2-b^2)^2d(}$$

$$= \frac{(a^2+4b^2)e^2\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{8b^2(a^2-b^2)^2d\sqrt{\cos(c+dx)}} - \frac{e(e\cos(c+dx))^{3/2}}{3bd(a+b\sin(c+dx))^3} + \frac{}{4b(a}$$

$$= \frac{(a^2+4b^2)e^2\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{8b^2(a^2-b^2)^2d\sqrt{\cos(c+dx)}} - \frac{a^2(a^2-6b^2)e^3\sqrt{\cos(c+dx)}\,\Pi}{16b^3(a^2-b^2)^2\left(b-\sqrt{-a}\right.}$$

$$= -\frac{a(a^2-6b^2)e^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{16b^{5/2}(-a^2+b^2)^{9/4}d} + \frac{a(a^2-6b^2)e^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}}{}\right.}{16b^{5/2}(-a^2+b}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

time = 33.64, size = 892, normalized size = 1.55

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^(5/2)/(a + b*Sin[c + d*x])^4,x]
```

```
[Out] ((e*Cos[c + d*x])^(5/2)*Sec[c + d*x]^2*(-1/3*Cos[c + d*x]/(b*(a + b*Sin[c +
 d*x])^3) - (a*Cos[c + d*x])/(4*b*(-a^2 + b^2)*(a + b*Sin[c + d*x])^2) + (a
^2*Cos[c + d*x] + 4*b^2*Cos[c + d*x])/(8*b*(-a^2 + b^2)^2*(a + b*Sin[c + d*
x]))))/d + ((e*Cos[c + d*x])^(5/2)*((-20*a*b*(a + b*Sqrt[1 - Cos[c + d*x]^2
```

```
])*((a*AppellF1[3/4, 1/2, 1, 7/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^
2 + b^2)]*Cos[c + d*x]^(3/2))/(3*(a^2 - b^2)) + ((1/8 + I/8)*(2*ArcTan[1 -
((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1
 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2
] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x
]] + Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c +
 d*x]] + I*b*Cos[c + d*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)))*Sin[c + d*x])/(S
qrt[1 - Cos[c + d*x]^2]*(a + b*Sin[c + d*x])) - ((a^2 + 4*b^2)*(a + b*Sqrt[
1 - Cos[c + d*x]^2])*(8*b^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Cos[c + d*x]^2,
 (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Cos[c + d*x]^(3/2) + 3*Sqrt[2]*a*(a^2 -
 b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^
(1/4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4
)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d
*x]] + b*Cos[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^
(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]]))*Sin[c + d*x]^2)/(12*b^(3/2)*(-
a^2 + b^2)*(1 - Cos[c + d*x]^2)*(a + b*Sin[c + d*x]))))/(16*(a - b)^2*b*(a
+ b)^2*d*Cos[c + d*x]^(5/2))
```

Maple [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(e \cos \left(dx +c \right)\right)^{\frac{5}{2}}}{\left(a +b \sin \left(dx +c \right)\right)^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(5/2)/(a+b*sin(d*x+c))^4,x)
[Out] int((e*cos(d*x+c))^(5/2)/(a+b*sin(d*x+c))^4,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+b*sin(d*x+c))^4,x, algorithm="maxima")
[Out] e^(5/2)*integrate(cos(d*x + c)^(5/2)/(b*sin(d*x + c) + a)^4, x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(5/2)/(a+b*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(5/2)*e^(5/2)/(b*sin(d*x + c) + a)^4, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(e\,\cos\left(c+d\,x\right)\right)^{5/2}}{\left(a+b\,\sin\left(c+d\,x\right)\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(5/2)/(a + b*sin(c + d*x))^4,x)
```

```
[Out] int((e*cos(c + d*x))^(5/2)/(a + b*sin(c + d*x))^4, x)
```

3.611 $\int \frac{(e\cos(c+dx))^{3/2}}{(a+b\sin(c+dx))^4}\,dx$

Optimal. Leaf size=592

$$-\frac{a(a^2+6b^2)\,e^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{16b^{3/2}\left(-a^2+b^2\right)^{11/4}d}-\frac{a(a^2+6b^2)\,e^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{16b^{3/2}\left(-a^2+b^2\right)^{11/4}d}-\frac{(3a^2+}{2}$$

```
[Out] -1/16*a*(a^2+6*b^2)*e^(3/2)*arctan(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^
(1/4)/e^(1/2))/b^(3/2)/(-a^2+b^2)^(11/4)/d-1/16*a*(a^2+6*b^2)*e^(3/2)*arcta
nh(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(3/2)/(-a^2+b^2
)^(11/4)/d-1/24*(3*a^2+4*b^2)*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+
1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/b^2/(a^2-b^2)
^2/d/(e*cos(d*x+c))^(1/2)+1/16*a^2*(a^2+6*b^2)*e^2*(cos(1/2*d*x+1/2*c)^2)^(
1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(b-(-a^2+b^2)^(1/
2)),2^(1/2))*cos(d*x+c)^(1/2)/b^2/(a^2-b^2)^2/d/(a^2-b*(b-(-a^2+b^2)^(1/2))
)/(e*cos(d*x+c))^(1/2)+1/16*a^2*(a^2+6*b^2)*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(b+(-a^2+b^2)^(1/2))
,2^(1/2))*cos(d*x+c)^(1/2)/b^2/(a^2-b^2)^2/d/(a^2-b*(b+(-a^2+b^2)^(1/2)))/(
e*cos(d*x+c))^(1/2)-1/3*e*(e*cos(d*x+c))^(1/2)/b/d/(a+b*sin(d*x+c))^3+1/12*
a*e*(e*cos(d*x+c))^(1/2)/b/(a^2-b^2)/d/(a+b*sin(d*x+c))^2+1/24*(3*a^2+4*b^2
)*e*(e*cos(d*x+c))^(1/2)/b/(a^2-b^2)^2/d/(a+b*sin(d*x+c))
```

Rubi [A]
time = 1.00, antiderivative size = 592, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {2772, 2943, 2946, 2721, 2720, 2781, 2886, 2884, 335, 218, 214, 211}

$$-\frac{ae^{3/2}(a^2+6b^2)\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{16b^{3/2}d\left(b^2-a^2\right)^{11/4}}-\frac{e^2(3a^2+4b^2)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)|2\right)}{24b^2d\left(a^2-b^2\right)^2\sqrt{e\cos(c+dx)}}+\frac{a^2e^2(a^2+6b^2)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)|2\right)}{16b^2d\left(a^2-b^2\right)^2\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{e\cos(c+dx)}}+\frac{a^2e^2(a^2+6b^2)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)|2\right)}{16b^2d\left(a^2-b^2\right)^2\left(a^2-b\left(\sqrt{b^2-a^2}+b\right)\right)\sqrt{e\cos(c+dx)}}+\frac{e(3a^2+4b^2)\sqrt{e\cos(c+dx)}}{24bd\left(a^2-b^2\right)^2(a+b\sin(c+dx))}+\frac{ae\sqrt{e\cos(c+dx)}}{12bd\left(a^2-b^2\right)(a+b\sin(c+dx))^2}-\frac{ae^{3/2}(a^2+6b^2)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{16b^{3/2}d\left(b^2-a^2\right)^{11/4}}-\frac{e\sqrt{e\cos(c+dx)}}{3bd(a+b\sin(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(3/2)/(a + b*Sin[c + d*x])^4,x]
```

```
[Out] -1/16*(a*(a^2 + 6*b^2)*e^(3/2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2
 + b^2)^(1/4)*Sqrt[e])])/(b^(3/2)*(-a^2 + b^2)^(11/4)*d) - (a*(a^2 + 6*b^2)
*e^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e]
)])/(16*b^(3/2)*(-a^2 + b^2)^(11/4)*d) - ((3*a^2 + 4*b^2)*e^2*Sqrt[Cos[c +
d*x]]*EllipticF[(c + d*x)/2, 2])/(24*b^2*(a^2 - b^2)^2*d*Sqrt[e*Cos[c + d*x
]]) + (a^2*(a^2 + 6*b^2)*e^2*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[
-a^2 + b^2]), (c + d*x)/2, 2])/(16*b^2*(a^2 - b^2)^2*(a^2 - b*(b - Sqrt[-a^
2 + b^2]))*d*Sqrt[e*Cos[c + d*x]]) + (a^2*(a^2 + 6*b^2)*e^2*Sqrt[Cos[c + d*
x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(16*b^2*(a^2
- b^2)^2*(a^2 - b*(b + Sqrt[-a^2 + b^2]))*d*Sqrt[e*Cos[c + d*x]]) - (e*Sqrt
```

```
[e*Cos[c + d*x]])/(3*b*d*(a + b*Sin[c + d*x])^3) + (a*e*Sqrt[e*Cos[c + d*x]
])/(12*b*(a^2 - b^2)*d*(a + b*Sin[c + d*x])^2) + ((3*a^2 + 4*b^2)*e*Sqrt[e*
Cos[c + d*x]])/(24*b*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2772

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Dist[g^2*((p - 1)/(b*(m + 1))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2781

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[-a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[b*(g/f), Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2943

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c
 - a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^(m + 1)/(f*g*(a^2 -
 b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*
(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p
+ 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[
a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{3/2}}{(a+b\sin(c+dx))^4}\,dx = -\frac{e\sqrt{e\cos(c+dx)}}{3bd(a+b\sin(c+dx))^3} - \frac{e^2\int \frac{\sin(c+dx)}{\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))^3}dx}{6b}$$

$$= -\frac{e\sqrt{e\cos(c+dx)}}{3bd(a+b\sin(c+dx))^3} + \frac{ae\sqrt{e\cos(c+dx)}}{12b\,(a^2-b^2)\,d(a+b\sin(c+dx))^2} + \frac{e^2\int \frac{2b}{\sqrt{e\cos(c}}}{12}$$

$$= -\frac{e\sqrt{e\cos(c+dx)}}{3bd(a+b\sin(c+dx))^3} + \frac{ae\sqrt{e\cos(c+dx)}}{12b\,(a^2-b^2)\,d(a+b\sin(c+dx))^2} + \frac{(3a^2+4b^2)}{24b\,(a^2-b^2)^2}$$

$$= -\frac{e\sqrt{e\cos(c+dx)}}{3bd(a+b\sin(c+dx))^3} + \frac{ae\sqrt{e\cos(c+dx)}}{12b\,(a^2-b^2)\,d(a+b\sin(c+dx))^2} + \frac{(3a^2+4b^2)}{24b\,(a^2-b^2)^2}$$

$$= -\frac{e\sqrt{e\cos(c+dx)}}{3bd(a+b\sin(c+dx))^3} + \frac{ae\sqrt{e\cos(c+dx)}}{12b\,(a^2-b^2)\,d(a+b\sin(c+dx))^2} + \frac{(3a^2+4b^2)}{24b\,(a^2-b^2)^2}$$

$$= -\frac{(3a^2+4b^2)\,e^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{24b^2\,(a^2-b^2)^2\,d\sqrt{e\cos(c+dx)}} - \frac{e\sqrt{e\cos(c+dx)}}{3bd(a+b\sin(c+dx))^3} + \frac{}{12b}$$

$$= -\frac{(3a^2+4b^2)\,e^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{24b^2\,(a^2-b^2)^2\,d\sqrt{e\cos(c+dx)}} + \frac{a^2(a^2+6b^2)\,e^2\sqrt{\cos(c+dx)}}{16b^2\,(-a^2+b^2)^{5/2}\left(b-\sqrt{\right.}}$$

$$= -\frac{a(a^2+6b^2)\,e^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{16b^{3/2}\,(-a^2+b^2)^{11/4}\,d} - \frac{a(a^2+6b^2)\,e^{3/2}\tanh^{-1}\left(\sqrt{b}\right.}{16b^{3/2}\,(-a^2+b^2}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 41.94, size = 1263, normalized size = 2.13

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^(3/2)/(a + b*Sin[c + d*x])^4,x]
```

```
[Out] ((e*Cos[c + d*x])^(3/2)*Sec[c + d*x]*(-1/3*1/(b*(a + b*Sin[c + d*x])^3) - a
/(12*b*(-a^2 + b^2)*(a + b*Sin[c + d*x])^2) + (3*a^2 + 4*b^2)/(24*b*(-a^2 +
 b^2)^2*(a + b*Sin[c + d*x]))))/d - ((e*Cos[c + d*x])^(3/2)*((28*a*b*(a + b
*Sqrt[1 - Cos[c + d*x]^2])*((5*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos
```

```
[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[c + d*x]])/(Sqrt[1
 - Cos[c + d*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2
, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4,
Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[
5/4, 3/2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)])*Cos[c
 + d*x]^2)*(a^2 + b^2*(-1 + Cos[c + d*x]^2))) - ((1/8 - I/8)*Sqrt[b]*(2*Arc
Tan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan
[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a
^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos
[c + d*x]] - Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt
[Cos[c + d*x]] + I*b*Cos[c + d*x]]))/(-a^2 + b^2)^(3/4))*Sin[c + d*x])/(Sqr
t[1 - Cos[c + d*x]^2]*(a + b*Sin[c + d*x])) - (2*(3*a^2 + 4*b^2)*(a + b*Sqr
t[1 - Cos[c + d*x]^2])*((5*b*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Cos[c
+ d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[c + d*x]]*Sqrt[1 - Co
s[c + d*x]^2])/((-5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Cos[c + d*x]^2,
 (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + 2*(2*b^2*AppellF1[5/4, -1/2, 2, 9/4,
Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + (a^2 - b^2)*AppellF1[5
/4, 1/2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)])*Cos[c
+ d*x]^2)*(a^2 + b^2*(-1 + Cos[c + d*x]^2))) + (a*(-2*ArcTan[1 - (Sqrt[2]*S
qrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[
b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*S
qrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]] + Log[Sqrt[a^
2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c +
 d*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)))*Sin[c + d*x]^2)/((1 - Cos[c
 + d*x]^2)*(a + b*Sin[c + d*x]))))/(48*(a - b)^2*b*(a + b)^2*d*Cos[c + d*x]
^(3/2))
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(e\cos\left(dx+c\right)\right)^{\frac{3}{2}}}{\left(a+b\sin\left(dx+c\right)\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c))^4,x)
```

```
[Out] int((e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c))^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] e^(3/2)*integrate(cos(d*x + c)^(3/2)/(b*sin(d*x + c) + a)^4, x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c))^4,x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(3/2)/(a+b*sin(d*x+c))**4,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 7321 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c))^4,x, algorithm="giac")
[Out] integrate(cos(d*x + c)^(3/2)*e^(3/2)/(b*sin(d*x + c) + a)^4, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(e\,\cos\left(c+d\,x\right)\right)}^{3/2}}{{\left(a+b\,\sin\left(c+d\,x\right)\right)}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(3/2)/(a + b*sin(c + d*x))^4,x)
[Out] int((e*cos(c + d*x))^(3/2)/(a + b*sin(c + d*x))^4, x)
```

3.612 $\int \frac{\sqrt{e\cos(c+dx)}}{(a+b\sin(c+dx))^4}\,dx$

Optimal. Leaf size=579

$$-\frac{5a(a^2+2b^2)\sqrt{e}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{16\sqrt{b}\,(-a^2+b^2)^{13/4}\,d}+\frac{5a(a^2+2b^2)\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{16\sqrt{b}\,(-a^2+b^2)^{13/4}\,d}+\frac{(11a}{}$$

```
[Out] 1/3*b*(e*cos(d*x+c))^(3/2)/(a^2-b^2)/d/e/(a+b*sin(d*x+c))^3+3/4*a*b*(e*cos(
d*x+c))^(3/2)/(a^2-b^2)^2/d/e/(a+b*sin(d*x+c))^2+1/8*b*(11*a^2+4*b^2)*(e*co
s(d*x+c))^(3/2)/(a^2-b^2)^3/d/e/(a+b*sin(d*x+c))-5/16*a*(a^2+2*b^2)*arctan(
b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))*e^(1/2)/(-a^2+b^2)^(
13/4)/d/b^(1/2)+5/16*a*(a^2+2*b^2)*arctanh(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a
^2+b^2)^(1/4)/e^(1/2))*e^(1/2)/(-a^2+b^2)^(13/4)/d/b^(1/2)+5/16*a^2*(a^2+2*
b^2)*e*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d
*x+1/2*c),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*cos(d*x+c)^(1/2)/b/(a^2-b^2)^3/
d/(b-(-a^2+b^2)^(1/2))/(e*cos(d*x+c))^(1/2)+5/16*a^2*(a^2+2*b^2)*e*(cos(1/2
*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(
b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(d*x+c)^(1/2)/b/(a^2-b^2)^3/d/(b+(-a^2+b^2)
^(1/2))/(e*cos(d*x+c))^(1/2)+1/8*(11*a^2+4*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(
1/2)/(a^2-b^2)^3/d/cos(d*x+c)^(1/2)
```

Rubi [A]

time = 1.00, antiderivative size = 579, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {2773, 2943, 2946, 2721, 2719, 2780, 2886, 2884, 335, 304, 211, 214}

$$-\frac{5a\sqrt{e}(a^2+2b^2)\,\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{16\sqrt{b}\,d\,(b^2-a^2)^{13/4}}+\frac{3ab(e\cos(c+dx))^{3/2}}{4de\,(a^2-b^2)^2\,(a+b\sin(c+dx))^2}+\frac{b(11a^2+4b^2)\,(e\cos(c+dx))^{3/2}}{8de\,(a^2-b^2)^3\,(a+b\sin(c+dx))}+\frac{b(e\cos(c+dx))^{3/2}}{3de\,(a^2-b^2)\,(a+b\sin(c+dx))^3}+\frac{5a\sqrt{e}(a^2+2b^2)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{16\sqrt{b}\,d\,(b^2-a^2)^{13/4}}+\frac{(11a^2+4b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{8d\,(a^2-b^2)^3\sqrt{\cos(c+dx)}}+\frac{5a^2e(a^2+2b^2)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{16bd\,(a^2-b^2)^3\left(b-\sqrt{b^2-a^2}\right)\sqrt{e\cos(c+dx)}}+\frac{5a^2e(a^2+2b^2)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{16bd\,(a^2-b^2)^3\left(\sqrt{b^2-a^2}+b\right)\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cos[c + d*x]]/(a + b*Sin[c + d*x])^4,x]
```

```
[Out] (-5*a*(a^2 + 2*b^2)*Sqrt[e]*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 +
b^2)^(1/4)*Sqrt[e])])/(16*Sqrt[b]*(-a^2 + b^2)^(13/4)*d) + (5*a*(a^2 + 2*b^
2)*Sqrt[e]*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[
e])])/(16*Sqrt[b]*(-a^2 + b^2)^(13/4)*d) + ((11*a^2 + 4*b^2)*Sqrt[e*Cos[c +
 d*x]]*EllipticE[(c + d*x)/2, 2])/(8*(a^2 - b^2)^3*d*Sqrt[Cos[c + d*x]]) +
(5*a^2*(a^2 + 2*b^2)*e*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 +
 b^2]), (c + d*x)/2, 2])/(16*b*(a^2 - b^2)^3*(b - Sqrt[-a^2 + b^2])*d*Sqrt[
e*Cos[c + d*x]]) + (5*a^2*(a^2 + 2*b^2)*e*Sqrt[Cos[c + d*x]]*EllipticPi[(2*
b)/(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(16*b*(a^2 - b^2)^3*(b + Sqrt[-
a^2 + b^2])*d*Sqrt[e*Cos[c + d*x]]) + (b*(e*Cos[c + d*x])^(3/2))/(3*(a^2 -
```

```
b^2)*d*e*(a + b*Sin[c + d*x])^3) + (3*a*b*(e*Cos[c + d*x])^(3/2))/(4*(a^2 -
 b^2)^2*d*e*(a + b*Sin[c + d*x])^2) + (b*(11*a^2 + 4*b^2)*(e*Cos[c + d*x])^
(3/2))/(8*(a^2 - b^2)^3*d*e*(a + b*Sin[c + d*x]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2773

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m + 1)/(f*g*(a^2 - b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1))
, Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p
 + 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^
2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2780

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[a*(g/(2*b)), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[a*(g/(2*b)), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[b*(g/f), Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2943

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c
 - a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^(m + 1)/(f*g*(a^2 -
 b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*
(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p
+ 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[
a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{e\cos(c+dx)}}{(a+b\sin(c+dx))^4}\,dx = \frac{b(e\cos(c+dx))^{3/2}}{3\,(a^2-b^2)\,de(a+b\sin(c+dx))^3} - \frac{\int \frac{\sqrt{e\cos(c+dx)}\,\left(-3a+\frac{3}{2}b\sin(c+dx)\right)}{(a+b\sin(c+dx))^3}\,dx}{3\,(a^2-b^2)}$$

$$= \frac{b(e\cos(c+dx))^{3/2}}{3\,(a^2-b^2)\,de(a+b\sin(c+dx))^3} + \frac{3ab(e\cos(c+dx))^{3/2}}{4\,(a^2-b^2)^2\,de(a+b\sin(c+dx))^2} + \frac{\int \sqrt{e}}{}$$

$$= \frac{b(e\cos(c+dx))^{3/2}}{3\,(a^2-b^2)\,de(a+b\sin(c+dx))^3} + \frac{3ab(e\cos(c+dx))^{3/2}}{4\,(a^2-b^2)^2\,de(a+b\sin(c+dx))^2} + \frac{b(11a}{8\,(a^2 -}$$

$$= \frac{b(e\cos(c+dx))^{3/2}}{3\,(a^2-b^2)\,de(a+b\sin(c+dx))^3} + \frac{3ab(e\cos(c+dx))^{3/2}}{4\,(a^2-b^2)^2\,de(a+b\sin(c+dx))^2} + \frac{b(11a}{8\,(a^2 -}$$

$$= \frac{b(e\cos(c+dx))^{3/2}}{3\,(a^2-b^2)\,de(a+b\sin(c+dx))^3} + \frac{3ab(e\cos(c+dx))^{3/2}}{4\,(a^2-b^2)^2\,de(a+b\sin(c+dx))^2} + \frac{b(11a}{8\,(a^2 -}$$

$$= \frac{(11a^2+4b^2)\,\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{8\,(a^2-b^2)^3\,d\sqrt{\cos(c+dx)}} + \frac{b(e\cos(c+dx))^{3/2}}{3\,(a^2-b^2)\,de(a+b\sin(c+dx)}$$

$$= \frac{(11a^2+4b^2)\,\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{8\,(a^2-b^2)^3\,d\sqrt{\cos(c+dx)}} + \frac{5a^2(a^2+2b^2)\,e\sqrt{\cos(c+dx)}\,\Gamma}{16b\,(a^2-b^2)^3\,\left(b-\sqrt{-a}\right.}$$

$$= -\frac{5a(a^2+2b^2)\,\sqrt{e}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{16\sqrt{b}\,(-a^2+b^2)^{13/4}\,d} + \frac{5a(a^2+2b^2)\,\sqrt{e}\,\tanh^{-1}\left(\right.}{16\sqrt{b}\,(-a^2+}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

time = 16.31, size = 900, normalized size = 1.55

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[e*Cos[c + d*x]]/(a + b*Sin[c + d*x])^4,x]
```

```
[Out] (Sqrt[e*Cos[c + d*x]]*((b*Cos[c + d*x])/(3*(a^2 - b^2)*(a + b*Sin[c + d*x])
^3) + (3*a*b*Cos[c + d*x])/(4*(a^2 - b^2)^2*(a + b*Sin[c + d*x])^2) - (-11*
a^2*b*Cos[c + d*x] - 4*b^3*Cos[c + d*x])/(8*(a^2 - b^2)^3*(a + b*Sin[c + d*
x]))))/d + (Sqrt[e*Cos[c + d*x]]*((-2*(16*a^3 + 14*a*b^2)*(a + b*Sqrt[1 - C
```

```
os[c + d*x]^2])*((a*AppellF1[3/4, 1/2, 1, 7/4, Cos[c + d*x]^2, (b^2*Cos[c +
 d*x]^2)/(-a^2 + b^2)]*Cos[c + d*x]^(3/2))/(3*(a^2 - b^2)) + ((1/8 + I/8)*(
2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*A
rcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sq
rt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*
b*Cos[c + d*x]] + Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)
*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)))*Sin
[c + d*x])/(Sqrt[1 - Cos[c + d*x]^2]*(a + b*Sin[c + d*x])) - ((11*a^2*b + 4
*b^3)*(a + b*Sqrt[1 - Cos[c + d*x]^2])*(8*b^(5/2)*AppellF1[3/4, -1/2, 1, 7/
4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)]*Cos[c + d*x]^(3/2) +
3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d
*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]]
)/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1
/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sq
rt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]]))*Sin[c + d*x]
^2)/(12*b^(3/2)*(-a^2 + b^2)*(1 - Cos[c + d*x]^2)*(a + b*Sin[c + d*x]))))/(
16*(a - b)^3*(a + b)^3*d*Sqrt[Cos[c + d*x]])
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 298.14, size = 109490, normalized size = 189.10

method	result	size
default	Expression too large to display	109490

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(1/2)/(a+b*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] e^(1/2)*integrate(sqrt(cos(d*x + c))/(b*sin(d*x + c) + a)^4, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral(sqrt(cos(d*x + c))*e^(1/2)/(b^4*cos(d*x + c)^4 + a^4 + 6*a^2*b^2 +
 b^4 - 2*(3*a^2*b^2 + b^4)*cos(d*x + c)^2 - 4*(a*b^3*cos(d*x + c)^2 - a^3*b
 - a*b^3)*sin(d*x + c)), x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(1/2)/(a+b*sin(d*x+c))**4,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3066 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(d*x + c))*e^(1/2)/(b*sin(d*x + c) + a)^4, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{e\,\cos\left(c+d\,x\right)}}{{\left(a+b\,\sin\left(c+d\,x\right)\right)}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(1/2)/(a + b*sin(c + d*x))^4,x)
```

```
[Out] int((e*cos(c + d*x))^(1/2)/(a + b*sin(c + d*x))^4, x)
```

3.613 $\int \frac{1}{\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))^4}\,dx$

Optimal. Leaf size=593

$$\frac{7a\sqrt{b}\,(5a^2+6b^2)\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{16\,(-a^2+b^2)^{15/4}\,d\sqrt{e}}+\frac{7a\sqrt{b}\,(5a^2+6b^2)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{16\,(-a^2+b^2)^{15/4}\,d\sqrt{e}}-\frac{(57a}{}$$

```
[Out] 7/16*a*(5*a^2+6*b^2)*arctan(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e
^(1/2))*b^(1/2)/(-a^2+b^2)^(15/4)/d/e^(1/2)+7/16*a*(5*a^2+6*b^2)*arctanh(b^
(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))*b^(1/2)/(-a^2+b^2)^(15
/4)/d/e^(1/2)-1/24*(57*a^2+20*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x
+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/(a^2-b^2)^3/
d/(e*cos(d*x+c))^(1/2)+7/16*a^2*(5*a^2+6*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/
cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(b-(-a^2+b^2)^(1/2)),2
^(1/2))*cos(d*x+c)^(1/2)/(a^2-b^2)^3/d/(a^2-b*(b-(-a^2+b^2)^(1/2)))/(e*cos(
d*x+c))^(1/2)+7/16*a^2*(5*a^2+6*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*c
os(d*x+c)^(1/2)/(a^2-b^2)^3/d/(a^2-b*(b+(-a^2+b^2)^(1/2)))/(e*cos(d*x+c))^(
1/2)+1/3*b*(e*cos(d*x+c))^(1/2)/(a^2-b^2)/d/e/(a+b*sin(d*x+c))^3+11/12*a*b*
(e*cos(d*x+c))^(1/2)/(a^2-b^2)^2/d/e/(a+b*sin(d*x+c))^2+1/24*b*(57*a^2+20*b
^2)*(e*cos(d*x+c))^(1/2)/(a^2-b^2)^3/d/e/(a+b*sin(d*x+c))
```

Rubi [A]
time = 1.06, antiderivative size = 593, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {2773, 2943, 2946, 2721, 2720, 2781, 2886, 2884, 335, 218, 214, 211}

$$\frac{7a\sqrt{b}\,(5a^2+6b^2)\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{16d\sqrt{e}\,(b^2-a^2)^{15/4}}+\frac{11ab\sqrt{e\cos(c+dx)}}{12de\,(a^2-b^2)^2\,(a+b\sin(c+dx))^2}+\frac{b(57a^2+20b^2)\,\sqrt{e\cos(c+dx)}}{24de\,(a^2-b^2)^3\,(a+b\sin(c+dx))}+\frac{b\sqrt{e\cos(c+dx)}}{3de\,(a^2-b^2)\,(a+b\sin(c+dx))^3}+\frac{7a\sqrt{b}\,(5a^2+6b^2)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{16d\sqrt{e}\,(b^2-a^2)^{15/4}}-\frac{(57a^2+20b^2)\,\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{24d\,(a^2-b^2)^3\,\sqrt{e\cos(c+dx)}}+\frac{7a^2(5a^2+6b^2)\,\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{16d\,(a^2-b^2)^3\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{e\cos(c+dx)}}+\frac{7a^2(5a^2+6b^2)\,\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{16d\,(a^2-b^2)^3\left(a^2-b\left(\sqrt{b^2-a^2}+b\right)\right)\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x])^4),x]
```

```
[Out] (7*a*Sqrt[b]*(5*a^2 + 6*b^2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 +
 b^2)^(1/4)*Sqrt[e])])/(16*(-a^2 + b^2)^(15/4)*d*Sqrt[e]) + (7*a*Sqrt[b]*(5
*a^2 + 6*b^2)*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)*Sq
rt[e])])/(16*(-a^2 + b^2)^(15/4)*d*Sqrt[e]) - ((57*a^2 + 20*b^2)*Sqrt[Cos[c
 + d*x]]*EllipticF[(c + d*x)/2, 2])/(24*(a^2 - b^2)^3*d*Sqrt[e*Cos[c + d*x]
]) + (7*a^2*(5*a^2 + 6*b^2)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(b - Sqrt[-
a^2 + b^2]), (c + d*x)/2, 2])/(16*(a^2 - b^2)^3*(a^2 - b*(b - Sqrt[-a^2 + b
^2]))*d*Sqrt[e*Cos[c + d*x]]) + (7*a^2*(5*a^2 + 6*b^2)*Sqrt[Cos[c + d*x]]*E
llipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(16*(a^2 - b^2)^3*
(a^2 - b*(b + Sqrt[-a^2 + b^2]))*d*Sqrt[e*Cos[c + d*x]]) + (b*Sqrt[e*Cos[c
```

```
+ d*x]])/(3*(a^2 - b^2)*d*e*(a + b*Sin[c + d*x])^3) + (11*a*b*Sqrt[e*Cos[c
+ d*x]])/(12*(a^2 - b^2)^2*d*e*(a + b*Sin[c + d*x])^2) + (b*(57*a^2 + 20*b^
2)*Sqrt[e*Cos[c + d*x]])/(24*(a^2 - b^2)^3*d*e*(a + b*Sin[c + d*x]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2773

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m + 1)/(f*g*(a^2 - b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1))
, Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p
 + 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^
2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2781

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[-a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[b*(g/f), Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2943

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c
 - a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^(m + 1)/(f*g*(a^2 -
 b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*
(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p
+ 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[
a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))^4}\,dx = \frac{b\sqrt{e\cos(c+dx)}}{3(a^2-b^2)\,de(a+b\sin(c+dx))^3} - \frac{\int \frac{-3a+\frac{5}{2}b\sin(c+dx)}{\sqrt{e\cos(c+dx)}\,(a+b\sin(c+}}{3(a^2-b^2)}$$

$$= \frac{b\sqrt{e\cos(c+dx)}}{3(a^2-b^2)\,de(a+b\sin(c+dx))^3} + \frac{11ab\sqrt{e\cos(c+dx)}}{12(a^2-b^2)^2\,de(a+b\sin(c}$$

$$= \frac{b\sqrt{e\cos(c+dx)}}{3(a^2-b^2)\,de(a+b\sin(c+dx))^3} + \frac{11ab\sqrt{e\cos(c+dx)}}{12(a^2-b^2)^2\,de(a+b\sin(c}$$

$$= \frac{b\sqrt{e\cos(c+dx)}}{3(a^2-b^2)\,de(a+b\sin(c+dx))^3} + \frac{11ab\sqrt{e\cos(c+dx)}}{12(a^2-b^2)^2\,de(a+b\sin(c}$$

$$= \frac{b\sqrt{e\cos(c+dx)}}{3(a^2-b^2)\,de(a+b\sin(c+dx))^3} + \frac{11ab\sqrt{e\cos(c+dx)}}{12(a^2-b^2)^2\,de(a+b\sin(c}$$

$$= -\frac{(57a^2+20b^2)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{24(a^2-b^2)^3\,d\sqrt{e\cos(c+dx)}} + \frac{b\sqrt{e}}{3(a^2-b^2)\,de}$$

$$= -\frac{(57a^2+20b^2)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{24(a^2-b^2)^3\,d\sqrt{e\cos(c+dx)}} - \frac{7a^2(5a^2+6b}{16(-a^2}$$

$$= \frac{7a\sqrt{b}\,(5a^2+6b^2)\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{16(-a^2+b^2)^{15/4}\,d\sqrt{e}} + \frac{7a\sqrt{b}\,(5a^2}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 42.32, size = 1276, normalized size = 2.15

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x])^4),x]
```

```
[Out] (Cos[c + d*x]*(b/(3*(a^2 - b^2)*(a + b*Sin[c + d*x])^3) + (11*a*b)/(12*(a^2
 - b^2)^2*(a + b*Sin[c + d*x])^2) + (b*(57*a^2 + 20*b^2))/(24*(a^2 - b^2)^3
*(a + b*Sin[c + d*x]))))/(d*Sqrt[e*Cos[c + d*x]]) + (Sqrt[Cos[c + d*x]]*((-
2*(48*a^3 + 106*a*b^2)*(a + b*Sqrt[1 - Cos[c + d*x]^2])*((5*a*(a^2 - b^2)*A
```

```
ppellF1[1/4, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)
]*Sqrt[Cos[c + d*x]])/(Sqrt[1 - Cos[c + d*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4
, 1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] - 2*(2*b^
2*AppellF1[5/4, 1/2, 2, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b
^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos[c +
 d*x]^2)/(-a^2 + b^2)])*Cos[c + d*x]^2)*(a^2 + b^2*(-1 + Cos[c + d*x]^2)))
- ((1/8 - I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(
-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^
2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)
*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]] - Log[Sqrt[-a^2 + b^2] + (1 + I)*Sq
rt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]]))/(-a^2 + b
^2)^(3/4))*Sin[c + d*x])/(Sqrt[1 - Cos[c + d*x]^2]*(a + b*Sin[c + d*x])) -
(2*(-57*a^2*b - 20*b^3)*(a + b*Sqrt[1 - Cos[c + d*x]^2])*((5*b*(a^2 - b^2)*
AppellF1[1/4, -1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^
2)]*Sqrt[Cos[c + d*x]]*Sqrt[1 - Cos[c + d*x]^2])/((-5*(a^2 - b^2)*AppellF1[
1/4, -1/2, 1, 5/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)] + 2*(
2*b^2*AppellF1[5/4, -1/2, 2, 9/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^
2 + b^2)] + (a^2 - b^2)*AppellF1[5/4, 1/2, 1, 9/4, Cos[c + d*x]^2, (b^2*Cos
[c + d*x]^2)/(-a^2 + b^2)])*Cos[c + d*x]^2)*(a^2 + b^2*(-1 + Cos[c + d*x]^2
))) + (a*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1
/4)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)]
 - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x
]] + b*Cos[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1
/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(
3/4)))*Sin[c + d*x]^2)/((1 - Cos[c + d*x]^2)*(a + b*Sin[c + d*x]))))/(48*(a
 - b)^3*(a + b)^3*d*Sqrt[e*Cos[c + d*x]])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 293.57, size = 82867, normalized size = 139.74

method	result	size
default	Expression too large to display	82867

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(d*x+c))^4/(e*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c))^4/(e*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c))^4/(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c))**4/(e*cos(d*x+c))**(1/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3067 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c))^4/(e*cos(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(e^(-1/2)/((b*sin(d*x + c) + a)^4*sqrt(cos(d*x + c))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{e\,\cos\left(c+d\,x\right)}\,{\left(a+b\,\sin\left(c+d\,x\right)\right)}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cos(c + d*x))^(1/2)*(a + b*sin(c + d*x))^4),x)
[Out] int(1/((e*cos(c + d*x))^(1/2)*(a + b*sin(c + d*x))^4), x)
```

3.614 $\int \frac{1}{(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))^4}\,dx$

Optimal. Leaf size=674

$$-\frac{15ab^{3/2}(7a^2+6b^2)\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{16\left(-a^2+b^2\right)^{17/4}de^{3/2}}+\frac{15ab^{3/2}(7a^2+6b^2)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{16\left(-a^2+b^2\right)^{17/4}de^{3/2}}-\cdots$$

```
[Out] -15/16*a*b^(3/2)*(7*a^2+6*b^2)*arctan(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^
2)^(1/4)/e^(1/2))/(-a^2+b^2)^(17/4)/d/e^(3/2)+15/16*a*b^(3/2)*(7*a^2+6*b^2)
*arctanh(b^(1/2)*(e*cos(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/(-a^2+b^2)^
(17/4)/d/e^(3/2)+1/3*b/(a^2-b^2)/d/e/(a+b*sin(d*x+c))^3/(e*cos(d*x+c))^(1/2
)+13/12*a*b/(a^2-b^2)^2/d/e/(a+b*sin(d*x+c))^2/(e*cos(d*x+c))^(1/2)+1/24*b*
(89*a^2+28*b^2)/(a^2-b^2)^3/d/e/(a+b*sin(d*x+c))/(e*cos(d*x+c))^(1/2)+1/8*(
-15*a*b*(7*a^2+6*b^2)+(16*a^4+151*a^2*b^2+28*b^4)*sin(d*x+c))/(a^2-b^2)^4/d
/e/(e*cos(d*x+c))^(1/2)-15/16*a^2*b*(7*a^2+6*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(b-(-a^2+b^2)^(1/2
)),2^(1/2))*cos(d*x+c)^(1/2)/(a^2-b^2)^4/d/e/(b-(-a^2+b^2)^(1/2))/(e*cos(d*
x+c))^(1/2)-15/16*a^2*b*(7*a^2+6*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*
d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*
cos(d*x+c)^(1/2)/(a^2-b^2)^4/d/e/(b+(-a^2+b^2)^(1/2))/(e*cos(d*x+c))^(1/2)-
1/8*(16*a^4+151*a^2*b^2+28*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/
2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/(a^2-b^2)^4
/d/e^2/cos(d*x+c)^(1/2)
```

Rubi [A]

time = 1.30, antiderivative size = 674, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {2773, 2943, 2945, 2946, 2721, 2719, 2780, 2886, 2884, 335, 304, 211, 214}

$$-\frac{15ab^{3/2}(7a^2+6b^2)\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{16de^{3/2}\left(b^2-a^2\right)^{17/4}}+\frac{13ab}{12de\left(a^2-b^2\right)^2\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))^2}+\frac{b(89a^2+28b^2)}{24de\left(a^2-b^2\right)^3\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))}+\frac{b}{3de\left(a^2-b^2\right)\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))^3}-\frac{15a^2b(7a^2+6b^2)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{16de\left(a^2-b^2\right)^4\left(b-\sqrt{b^2-a^2}\right)\sqrt{e\cos(c+dx)}}-\frac{15a^2b(7a^2+6b^2)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}(c+dx)\middle|2\right)}{16de\left(a^2-b^2\right)^4\left(\sqrt{b^2-a^2}+b\right)\sqrt{e\cos(c+dx)}}+\frac{15ab^{3/2}(7a^2+6b^2)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\cos(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{16de^{3/2}\left(b^2-a^2\right)^{17/4}}-\frac{(16a^4+151a^2b^2+28b^4)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{8de^2\left(a^2-b^2\right)^4\sqrt{\cos(c+dx)}}-\frac{15ab(7a^2+6b^2)-(16a^4+151a^2b^2+28b^4)\sin(c+dx)}{8de\left(a^2-b^2\right)^4\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x])^4),x]
```

```
[Out] (-15*a*b^(3/2)*(7*a^2 + 6*b^2)*ArcTan[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2
 + b^2)^(1/4)*Sqrt[e])])/(16*(-a^2 + b^2)^(17/4)*d*e^(3/2)) + (15*a*b^(3/2)
*(7*a^2 + 6*b^2)*ArcTanh[(Sqrt[b]*Sqrt[e*Cos[c + d*x]])/((-a^2 + b^2)^(1/4)
*Sqrt[e])])/(16*(-a^2 + b^2)^(17/4)*d*e^(3/2)) - ((16*a^4 + 151*a^2*b^2 + 2
8*b^4)*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(8*(a^2 - b^2)^4*d*e
^2*Sqrt[Cos[c + d*x]]) - (15*a^2*b*(7*a^2 + 6*b^2)*Sqrt[Cos[c + d*x]]*Ellip
ticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/(16*(a^2 - b^2)^4*(b -
 Sqrt[-a^2 + b^2])*d*e*Sqrt[e*Cos[c + d*x]]) - (15*a^2*b*(7*a^2 + 6*b^2)*Sq
rt[Cos[c + d*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c + d*x)/2, 2])/
```

```
(16*(a^2 - b^2)^4*(b + Sqrt[-a^2 + b^2])*d*e*Sqrt[e*Cos[c + d*x]]) + b/(3*(
a^2 - b^2)*d*e*Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x])^3) + (13*a*b)/(12*
(a^2 - b^2)^2*d*e*Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x])^2) + (b*(89*a^2
 + 28*b^2))/(24*(a^2 - b^2)^3*d*e*Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x])
) - (15*a*b*(7*a^2 + 6*b^2) - (16*a^4 + 151*a^2*b^2 + 28*b^4)*Sin[c + d*x])
/(8*(a^2 - b^2)^4*d*e*Sqrt[e*Cos[c + d*x]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2773

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m + 1)/(f*g*(a^2 - b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1))
```

```
, Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p
 + 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^
2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2780

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[a*(g/(2*b)), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[a*(g/(2*b)), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[b*(g/f), Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2943

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c
 - a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^(m + 1)/(f*g*(a^2 -
 b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*
(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p
+ 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[
a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2945

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
```

```
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{1}{(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))^4}\,dx = \frac{b}{3\left(a^2-b^2\right)de\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))^3} - \frac{\int \frac{}{(e\cos(c\cdots}}{}$$

$$= \frac{b}{3\left(a^2-b^2\right)de\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))^3} + \frac{}{12\left(a^2-\cdots\right.}$$

$$= \frac{b}{3\left(a^2-b^2\right)de\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))^3} + \frac{}{12\left(a^2-\cdots\right.}$$

$$= \frac{b}{3\left(a^2-b^2\right)de\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))^3} + \frac{}{12\left(a^2-\cdots\right.}$$

$$= \frac{b}{3\left(a^2-b^2\right)de\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))^3} + \frac{}{12\left(a^2-\cdots\right.}$$

$$= \frac{b}{3\left(a^2-b^2\right)de\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))^3} + \frac{}{12\left(a^2-\cdots\right.}$$

$$= -\frac{\left(16a^4+151a^2b^2+28b^4\right)\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{8\left(a^2-b^2\right)^4de^2\sqrt{\cos(c+dx)}} \cdots$$

$$= -\frac{\left(16a^4+151a^2b^2+28b^4\right)\sqrt{e\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{8\left(a^2-b^2\right)^4de^2\sqrt{\cos(c+dx)}} \cdots$$

$$= -\frac{15ab^{3/2}(7a^2+6b^2)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\cos(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{16\left(-a^2+b^2\right)^{17/4}de^{3/2}} + \frac{15ab^{3/\cdots}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 16.43, size = 996, normalized size = 1.48

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x])^4),x]
```

```
[Out] -1/16*(Cos[c + d*x]^(3/2)*((-2*(16*a^5 + 256*a^3*b^2 + 118*a*b^4)*(a + b*Sq
rt[1 - Cos[c + d*x]^2])*((a*AppellF1[3/4, 1/2, 1, 7/4, Cos[c + d*x]^2, (b^2
```

```
*Cos[c + d*x]^2)/(-a^2 + b^2)]*Cos[c + d*x]^(3/2))/(3*(a^2 - b^2)) + ((1/8
+ I/8)*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4
)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[c + d*x]])/(-a^2 + b^2)^(1/4)]
- Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[c + d*
x]] + I*b*Cos[c + d*x]] + Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^
2)^(1/4)*Sqrt[Cos[c + d*x]] + I*b*Cos[c + d*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/
4)))*Sin[c + d*x])/(Sqrt[1 - Cos[c + d*x]^2]*(a + b*Sin[c + d*x])) - ((16*a
^4*b + 151*a^2*b^3 + 28*b^5)*(a + b*Sqrt[1 - Cos[c + d*x]^2])*(8*b^(5/2)*Ap
pellF1[3/4, -1/2, 1, 7/4, Cos[c + d*x]^2, (b^2*Cos[c + d*x]^2)/(-a^2 + b^2)
]*Cos[c + d*x]^(3/2) + 3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]
*Sqrt[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*Sqr
t[b]*Sqrt[Cos[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]
*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c + d*x]] + Log[Sqrt[
a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[c + d*x]] + b*Cos[c
 + d*x]]))*Sin[c + d*x]^2)/(12*b^(3/2)*(-a^2 + b^2)*(1 - Cos[c + d*x]^2)*(a
 + b*Sin[c + d*x]))))/((a - b)^4*(a + b)^4*d*(e*Cos[c + d*x])^(3/2)) + (Cos
[c + d*x]^2*(-1/3*(b^3*Cos[c + d*x])/((a^2 - b^2)^2*(a + b*Sin[c + d*x])^3)
 - (7*a*b^3*Cos[c + d*x])/(4*(a^2 - b^2)^3*(a + b*Sin[c + d*x])^2) + (-55*a
^2*b^3*Cos[c + d*x] - 12*b^5*Cos[c + d*x])/(8*(a^2 - b^2)^4*(a + b*Sin[c +
d*x])) + (2*Sec[c + d*x]*(-4*a^3*b - 4*a*b^3 + a^4*Sin[c + d*x] + 6*a^2*b^2
*Sin[c + d*x] + b^4*Sin[c + d*x]))/(a^2 - b^2)^4))/(d*(e*Cos[c + d*x])^(3/2
))
```

Maple [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(e\cos\left(dx+c\right)\right)^{\frac{3}{2}}\left(a+b\sin\left(dx+c\right)\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c))^4,x)
[Out] int(1/(e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c))^4,x)
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c))^4,x, algorithm="maxima")
[Out] Timed out
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(3/2)/(a+b*sin(d*x+c))**4,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4852 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate(e^(-3/2)/((b*sin(d*x + c) + a)^4*cos(d*x + c)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(e\,\cos\left(c+d\,x\right)\right)^{3/2}\,\left(a+b\,\sin\left(c+d\,x\right)\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*cos(c + d*x))^(3/2)*(a + b*sin(c + d*x))^4),x)
```

```
[Out] int(1/((e*cos(c + d*x))^(3/2)*(a + b*sin(c + d*x))^4), x)
```

3.615 $\int \frac{1}{\sqrt{c\cos(e+fx)}\sqrt{a+b\sin(e+fx)}}\,dx$

Optimal. Leaf size=183

$$\frac{2\sqrt{2}\sqrt[4]{-a+b}\sqrt{c\cos(e+fx)}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{a+b}\sqrt{\frac{1+\cos(e+fx)+\sin(e+fx)}{1+\cos(e+fx)-\sin(e+fx)}}}{\sqrt[4]{-a+b}}\right)\middle|-1\right)\sqrt{\frac{a+b\sin(e+fx)}{(a-b)(1-\sin(e+fx))}}}{\sqrt[4]{a+b}\,cf\sqrt{\frac{1+\cos(e+fx)+\sin(e+fx)}{1+\cos(e+fx)-\sin(e+fx)}}\sqrt{a+b\sin(e+fx)}}$$

```
[Out] 2*(-a+b)^(1/4)*EllipticF((a+b)^(1/4)*((1+cos(f*x+e)+sin(f*x+e))/(1+cos(f*x+
e)-sin(f*x+e)))^(1/2)/(-a+b)^(1/4),I)*2^(1/2)*(c*cos(f*x+e))^(1/2)*((a+b*si
n(f*x+e))/(a-b)/(1-sin(f*x+e)))^(1/2)/(a+b)^(1/4)/c/f/((1+cos(f*x+e)+sin(f*
x+e))/(1+cos(f*x+e)-sin(f*x+e)))^(1/2)/(a+b*sin(f*x+e))^(1/2)
```

Rubi [**B**] Leaf count is larger than twice the leaf count of optimal. 374 vs. $2(183) = 366$.
time = 0.31, antiderivative size = 374, normalized size of antiderivative = 2.04, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2776, 226}

$$\frac{\sqrt{2}\sqrt[4]{a-b}\sqrt{c\cos(e+fx)}\sqrt{\frac{a+b\sin(e+fx)}{(a-b)(1-\sin(e+fx))}}\sqrt{\frac{a+b\sin(e+fx)}{(a-b)(\sin(e)(-\cos(fx))-\cos(e)\sin(fx)+1)\left(\frac{\sqrt{a+b}(\sin(e+fx)+\cos(e+fx)+1)}{\sqrt{a-b}(-\sin(e+fx)+\cos(e+fx)+1)}+1\right)^2}}\left(\frac{\sqrt{a+b}(\sin(e+fx)+\cos(e+fx)+1)}{\sqrt{a-b}(-\sin(e+fx)+\cos(e+fx)+1)}+1\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{a+b}\sqrt{\frac{\cos(e+fx)+\sin(e+fx)+1}{\cos(e+fx)-\sin(e+fx)+1}}}{\sqrt[4]{a-b}}\right)\middle|\frac{1}{2}\right)}{cf\sqrt[4]{a+b}\sqrt{\frac{\sin(e+fx)+\cos(e+fx)+1}{-\sin(e+fx)+\cos(e+fx)+1}}\sqrt{a+b\sin(e+fx)}\sqrt{\frac{a+b\sin(e+fx)}{(a-b)(\sin(e)(-\cos(fx))-\cos(e)\sin(fx)+1)}}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[1/(Sqrt[c*Cos[e + f*x]]*Sqrt[a + b*Sin[e + f*x]]),x]
```

```
[Out] (Sqrt[2]*(a - b)^(1/4)*Sqrt[c*Cos[e + f*x]]*EllipticF[2*ArcTan[((a + b)^(1/
4)*Sqrt[(1 + Cos[e + f*x] + Sin[e + f*x])/(1 + Cos[e + f*x] - Sin[e + f*x])
])/(a - b)^(1/4)], 1/2]*Sqrt[(a + b*Sin[e + f*x])/((a - b)*(1 - Sin[e + f*x
]))]*Sqrt[(a + b*Sin[e + f*x])/((a - b)*(1 - Cos[f*x]*Sin[e] - Cos[e]*Sin[f
*x])*(1 + (Sqrt[a + b]*(1 + Cos[e + f*x] + Sin[e + f*x]))/(Sqrt[a - b]*(1 +
 Cos[e + f*x] - Sin[e + f*x])))^2)]*(1 + (Sqrt[a + b]*(1 + Cos[e + f*x] + S
in[e + f*x]))/(Sqrt[a - b]*(1 + Cos[e + f*x] - Sin[e + f*x]))))/((a + b)^(1
/4)*c*f*Sqrt[(1 + Cos[e + f*x] + Sin[e + f*x])/(1 + Cos[e + f*x] - Sin[e +
f*x])]*Sqrt[a + b*Sin[e + f*x]]*Sqrt[(a + b*Sin[e + f*x])/((a - b)*(1 - Cos
[f*x]*Sin[e] - Cos[e]*Sin[f*x]))])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
```

```
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 2776

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Dist[2*Sqrt[2]*Sqrt[g*Cos[e + f*x]]*(Sqrt[(a + b*
Sin[e + f*x])/((a - b)*(1 - Sin[e + f*x]))]/(f*g*Sqrt[a + b*Sin[e + f*x]]*S
qrt[(1 + Cos[e + f*x] + Sin[e + f*x])/(1 + Cos[e + f*x] - Sin[e + f*x])])),
 Subst[Int[1/Sqrt[1 + (a + b)*(x^4/(a - b))], x], x, Sqrt[(1 + Cos[e + f*x]
 + Sin[e + f*x])/(1 + Cos[e + f*x] - Sin[e + f*x])]], x] /; FreeQ[{a, b, e,
 f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{c\cos(e+fx)}\sqrt{a+b\sin(e+fx)}}\,dx = \frac{\left(2\sqrt{2}\sqrt{c\cos(e+fx)}\sqrt{\frac{a+b\sin(e+fx)}{(a-b)(1-\sin(e+fx))}}\right)\text{Subst}}{cf\sqrt{\frac{1+\cos(e+fx)+\sin}{1+\cos(e+fx)-\sin}}}$$

$$= \frac{\sqrt{2}\sqrt[4]{a-b}\sqrt{c\cos(e+fx)}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a+b}\sqrt{\frac{1+\cos}{1+\cos}}}{\sqrt{}}\right.\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.22, size = 117, normalized size = 0.64

$$-\frac{2c\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};-\frac{2(a+b\sin(e+fx))}{(a-b)(-1+\sin(e+fx))}\right)(-1+\sin(e+fx))\left(\frac{(a+b)(1+\sin(e+fx))}{(a-b)(-1+\sin(e+fx))}\right)^{3/4}\sqrt{a+b\sin(e+fx)}}{(a+b)f(c\cos(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[c*Cos[e + f*x]]*Sqrt[a + b*Sin[e + f*x]]),x]
```

```
[Out] (-2*c*Hypergeometric2F1[1/2, 3/4, 3/2, (-2*(a + b*Sin[e + f*x]))/((a - b)*(
-1 + Sin[e + f*x]))]*(-1 + Sin[e + f*x])*(((a + b)*(1 + Sin[e + f*x]))/((a
```

```
- b)*(-1 + Sin[e + f*x])))^(3/4)*Sqrt[a + b*Sin[e + f*x]])/((a + b)*f*(c*Co
s[e + f*x])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 750 vs. $2(163) = 326$.
time = 11.44, size = 751, normalized size = 4.10

method	result
default	$\frac{4\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(\cos(fx+e)\sqrt{-a^2+b^2}-a\sin(fx+e)-b\cos(fx+e)+\sqrt{-a^2+b^2}-b\right)\left(-a+b+\sqrt{-a^2+b^2}\right)}{\left(\cos(fx+e)\sqrt{-a^2+b^2}+a\sin(fx+e)+b\cos(fx+e)+\sqrt{-a^2+b^2}+b\right)\left(a-b+\sqrt{-a^2+b^2}\right)}},\sqrt{\frac{(b}{(-a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 4/f*EllipticF(((cos(f*x+e)*(-a^2+b^2)^(1/2)-a*sin(f*x+e)-b*cos(f*x+e)+(-a^2
+b^2)^(1/2)-b)/(cos(f*x+e)*(-a^2+b^2)^(1/2)+a*sin(f*x+e)+b*cos(f*x+e)+(-a^2
+b^2)^(1/2)+b)*(-a+b+(-a^2+b^2)^(1/2))/(a-b+(-a^2+b^2)^(1/2)))^(1/2),((b+(-
a^2+b^2)^(1/2)+a)*(a-b+(-a^2+b^2)^(1/2))/(-a-b+(-a^2+b^2)^(1/2))/(-a+b+(-a^
2+b^2)^(1/2)))^(1/2))*((cos(f*x+e)*(-a^2+b^2)^(1/2)-a*sin(f*x+e)-b*cos(f*x+
e)+(-a^2+b^2)^(1/2)-b)/(cos(f*x+e)*(-a^2+b^2)^(1/2)+a*sin(f*x+e)+b*cos(f*x+
e)+(-a^2+b^2)^(1/2)+b)*(-a+b+(-a^2+b^2)^(1/2))/(a-b+(-a^2+b^2)^(1/2)))^(1/2
)*((1+cos(f*x+e)+sin(f*x+e))/(cos(f*x+e)*(-a^2+b^2)^(1/2)+a*sin(f*x+e)+b*co
s(f*x+e)+(-a^2+b^2)^(1/2)+b)*(-a^2+b^2)^(1/2)*a/(a-b+(-a^2+b^2)^(1/2)))^(1/
2)*(-(1+cos(f*x+e)-sin(f*x+e))/(cos(f*x+e)*(-a^2+b^2)^(1/2)+a*sin(f*x+e)+b*
cos(f*x+e)+(-a^2+b^2)^(1/2)+b)*(-a^2+b^2)^(1/2)*a/(-a-b+(-a^2+b^2)^(1/2)))^
(1/2)*(cos(f*x+e)+1)^2*(-1+cos(f*x+e))^2*((-a^2+b^2)^(1/2)*cos(f*x+e)*a-(-a
^2+b^2)^(1/2)*cos(f*x+e)*b-a*(-a^2+b^2)^(1/2)*sin(f*x+e)+cos(f*x+e)*a^2-cos
(f*x+e)*b^2+a^2*sin(f*x+e)-a*b*sin(f*x+e)-b*(-a^2+b^2)^(1/2)+a*b-b^2)/(a+b*
sin(f*x+e))^(1/2)/sin(f*x+e)^4/(c*cos(f*x+e))^(1/2)/(-a^2+b^2)^(1/2)/(-a+b+
(-a^2+b^2)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(1/(sqrt(c*cos(f*x + e))*sqrt(b*sin(f*x + e) + a)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(1/2),x, algorithm="frica
s")
```

```
[Out] integral(sqrt(c*cos(f*x + e))*sqrt(b*sin(f*x + e) + a)/(b*c*cos(f*x + e)*si
n(f*x + e) + a*c*cos(f*x + e)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{c\cos{(e+fx)}}\sqrt{a+b\sin{(e+fx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cos(f*x+e))**(1/2)/(a+b*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(c*cos(e + f*x))*sqrt(a + b*sin(e + f*x))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(1/(sqrt(c*cos(f*x + e))*sqrt(b*sin(f*x + e) + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{c\,\cos\left(e+f\,x\right)}\,\sqrt{a+b\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*cos(e + f*x))^(1/2)*(a + b*sin(e + f*x))^(1/2)),x)
```

```
[Out] int(1/((c*cos(e + f*x))^(1/2)*(a + b*sin(e + f*x))^(1/2)), x)
```

3.616 $\int (e\cos(c+dx))^p(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=229

$$-\frac{b(2b^2(2+p)+a^2(11+6p+p^2))\,(e\cos(c+dx))^{1+p}}{de(1+p)(2+p)(3+p)}-\frac{a(3b^2+a^2(2+p))\,(e\cos(c+dx))^{1+p}\,{}_2F_1\left(\frac{1}{2},\frac{1+p}{2};\frac{3+p}{2};\right.}{de(1+p)(2+p)\sqrt{\sin^2(c+dx}}$$

```
[Out] -b*(2*b^2*(2+p)+a^2*(p^2+6*p+11))*(e*cos(d*x+c))^(1+p)/d/e/(3+p)/(p^2+3*p+2
)-a*b*(5+p)*(e*cos(d*x+c))^(1+p)*(a+b*sin(d*x+c))/d/e/(2+p)/(3+p)-b*(e*cos(
d*x+c))^(1+p)*(a+b*sin(d*x+c))^2/d/e/(3+p)-a*(3*b^2+a^2*(2+p))*(e*cos(d*x+c
))^(1+p)*hypergeom([1/2, 1/2+1/2*p],[3/2+1/2*p],cos(d*x+c)^2)*sin(d*x+c)/d/
e/(1+p)/(2+p)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.26, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2771, 2941, 2748, 2722}

$$-\frac{a(a^2(p+2)+3b^2)\sin(c+dx)(e\cos(c+dx))^{p+1}\,{}_2F_1\left(\frac{1}{2},\frac{p+1}{2};\frac{p+3}{2};\cos^2(c+dx)\right)}{de(p+1)(p+2)\sqrt{\sin^2(c+dx)}}-\frac{b(a^2(p^2+6p+11)+2b^2(p+2))\,(e\cos(c+dx))^{p+1}}{de(p+1)(p+2)(p+3)}-\frac{b(a+b\sin(c+dx))^2(e\cos(c+dx))^{p+1}}{de(p+3)}-\frac{ab(p+5)(a+b\sin(c+dx))(e\cos(c+dx))^{p+1}}{de(p+2)(p+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^p*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -((b*(2*b^2*(2 + p) + a^2*(11 + 6*p + p^2))*(e*Cos[c + d*x])^(1 + p))/(d*e*
(1 + p)*(2 + p)*(3 + p))) - (a*(3*b^2 + a^2*(2 + p))*(e*Cos[c + d*x])^(1 +
p)*Hypergeometric2F1[1/2, (1 + p)/2, (3 + p)/2, Cos[c + d*x]^2]*Sin[c + d*x
])/(d*e*(1 + p)*(2 + p)*Sqrt[Sin[c + d*x]^2]) - (a*b*(5 + p)*(e*Cos[c + d*x
])^(1 + p)*(a + b*Sin[c + d*x]))/(d*e*(2 + p)*(3 + p)) - (b*(e*Cos[c + d*x]
)^(1 + p)*(a + b*Sin[c + d*x])^2)/(d*e*(3 + p))
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2771

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(
a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rule 2941

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp
[a*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x],
x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
&&  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Si
mplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int (e\cos(c+dx))^p(a+b\sin(c+dx))^3\,dx = -\frac{b(e\cos(c+dx))^{1+p}(a+b\sin(c+dx))^2}{de(3+p)} + \frac{\int (e\cos(c+dx))^p(a$$

$$= -\frac{ab(5+p)(e\cos(c+dx))^{1+p}(a+b\sin(c+dx))}{de(2+p)(3+p)} - \frac{b(e\cos(c+d}{}$$

$$= -\frac{b(2b^2(2+p)+a^2(11+6p+p^2))\,(e\cos(c+dx))^{1+p}}{de(1+p)\,(6+5p+p^2)} - \frac{ab(5+}{}$$

$$= -\frac{b(2b^2(2+p)+a^2(11+6p+p^2))\,(e\cos(c+dx))^{1+p}}{de(1+p)\,(6+5p+p^2)} - \frac{a\Big(a^2+}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 55.71, size = 17703, normalized size = 77.31

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^p*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] Result too large to show
```

Maple [F]
time = 0.46, size = 0, normalized size = 0.00

$$\int \left(e\cos\left(dx+c\right)\right)^{p} \left(a+b\sin\left(dx+c\right)\right)^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^p*(a+b*sin(d*x+c))^3,x)
```

```
[Out] int((e*cos(d*x+c))^p*(a+b*sin(d*x+c))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^3*(cos(d*x + c)*e)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(-(3*a*b^2*cos(d*x + c)^2 - a^3 - 3*a*b^2 + (b^3*cos(d*x + c)^2 - 3
*a^2*b - b^3)*sin(d*x + c))*(cos(d*x + c)*e)^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(e\cos\left(c+dx\right)\right)^{p} \left(a+b\sin\left(c+dx\right)\right)^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**p*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral((e*cos(c + d*x))**p*(a + b*sin(c + d*x))**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^3*(cos(d*x + c)*e)^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (e\cos(c+d\,x))^p\,(a+b\sin(c+d\,x))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^p*(a + b*sin(c + d*x))^3,x)
```

```
[Out] int((e*cos(c + d*x))^p*(a + b*sin(c + d*x))^3, x)
```

3.617 $\int (e\cos(c+dx))^p(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=157

$$-\frac{ab(3+p)(e\cos(c+dx))^{1+p}}{de(1+p)(2+p)}-\frac{(b^2+a^2(2+p))\,(e\cos(c+dx))^{1+p}\,{}_2F_1\left(\frac{1}{2},\frac{1+p}{2};\frac{3+p}{2};\cos^2(c+dx)\right)\sin(c+dx)}{de(1+p)(2+p)\sqrt{\sin^2(c+dx)}}-$$

```
[Out] -a*b*(3+p)*(e*cos(d*x+c))^(1+p)/d/e/(1+p)/(2+p)-b*(e*cos(d*x+c))^(1+p)*(a+b
*sin(d*x+c))/d/e/(2+p)-(b^2+a^2*(2+p))*(e*cos(d*x+c))^(1+p)*hypergeom([1/2,
 1/2+1/2*p],[3/2+1/2*p],cos(d*x+c)^2)*sin(d*x+c)/d/e/(1+p)/(2+p)/(sin(d*x+c
)^2)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2771, 2748, 2722}

$$-\frac{(a^2(p+2)+b^2)\sin(c+dx)(e\cos(c+dx))^{p+1}\,{}_2F_1\left(\frac{1}{2},\frac{p+1}{2};\frac{p+3}{2};\cos^2(c+dx)\right)}{de(p+1)(p+2)\sqrt{\sin^2(c+dx)}}-\frac{ab(p+3)(e\cos(c+dx))^{p+1}}{de(p+1)(p+2)}-\frac{b(a+b\sin(c+dx))(e\cos(c+dx))^{p+1}}{de(p+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^p*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -((a*b*(3 + p)*(e*Cos[c + d*x])^(1 + p))/(d*e*(1 + p)*(2 + p))) - ((b^2 + a
^2*(2 + p))*(e*Cos[c + d*x])^(1 + p)*Hypergeometric2F1[1/2, (1 + p)/2, (3 +
 p)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*e*(1 + p)*(2 + p)*Sqrt[Sin[c + d*x]
^2]) - (b*(e*Cos[c + d*x])^(1 + p)*(a + b*Sin[c + d*x]))/(d*e*(2 + p))
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2771

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
```

```
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(
a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rubi steps

$$\begin{aligned}\int (e\cos(c+dx))^p(a+b\sin(c+dx))^2\,dx &= -\frac{b(e\cos(c+dx))^{1+p}(a+b\sin(c+dx))}{de(2+p)}+\frac{\int (e\cos(c+dx))^p\,(b^2\\ &= -\frac{ab(3+p)(e\cos(c+dx))^{1+p}}{de(1+p)(2+p)}-\frac{b(e\cos(c+dx))^{1+p}(a+b\sin(c}{de(2+p)}\\ &= -\frac{ab(3+p)(e\cos(c+dx))^{1+p}}{de(1+p)(2+p)}-\frac{(b^2+a^2(2+p))\,(e\cos(c+dx)}{de(1+p}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.70, size = 285, normalized size = 1.82

$$-\frac{(e\cos(c+dx))^p\left(2^{-p}ab\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^p\left(1+e^{2i(c+dx)}\right)\cos^{-p}(c+dx)\left(-e^{-i(c+dx)}(-1+p)\,{}_2F_1\left(1,\frac{1+p}{2};\frac{1-p}{2};-e^{2i(c+dx)}\right)+e^{i(c+dx)}(1+p)\,{}_2F_1\left(1,\frac{3+p}{2};\frac{3-p}{2};-e^{2i(c+dx)}\right)\right)\sqrt{\sin^2(c+dx)}-\frac{1}{2}b^2(-1+p)\,{}_2F_1\left(-\frac{1}{2},\frac{1+p}{2};\frac{3+p}{2};\cos^2(c+dx)\right)\sin(2(c+dx))-\frac{1}{2}a^2(-1+p)\,{}_2F_1\left(\frac{1}{2},\frac{1+p}{2};\frac{3+p}{2};\cos^2(c+dx)\right)\sin(2(c+dx))\right)}{(d-dp^2)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^p*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -(((e*Cos[c + d*x])^p*((a*b*(E^((-I)*(c + d*x)) + E^(I*(c + d*x)))^p*(1 + E
^((2*I)*(c + d*x)))*(-(((-1 + p)*Hypergeometric2F1[1, (1 + p)/2, (1 - p)/2,
 -E^((2*I)*(c + d*x))])/E^(I*(c + d*x))) + E^(I*(c + d*x))*(1 + p)*Hypergeo
metric2F1[1, (3 + p)/2, (3 - p)/2, -E^((2*I)*(c + d*x))])*Sqrt[Sin[c + d*x]
^2])/(2^p*Cos[c + d*x]^p) - (b^2*(-1 + p)*Hypergeometric2F1[-1/2, (1 + p)/2
, (3 + p)/2, Cos[c + d*x]^2]*Sin[2*(c + d*x)])/2 - (a^2*(-1 + p)*Hypergeome
tric2F1[1/2, (1 + p)/2, (3 + p)/2, Cos[c + d*x]^2]*Sin[2*(c + d*x)])/2))/((
d - d*p^2)*Sqrt[Sin[c + d*x]^2]))
```

Maple [F]
time = 0.70, size = 0, normalized size = 0.00

$$\int \left(e\cos\left(dx+c\right)\right)^{p}\left(a+b\sin\left(dx+c\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^p*(a+b*sin(d*x+c))^2,x)
```

```
[Out] int((e*cos(d*x+c))^p*(a+b*sin(d*x+c))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
[Out] integrate((b*sin(d*x + c) + a)^2*(cos(d*x + c)*e)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
[Out] integral(-(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)*(cos(d*x +
c)*e)^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos{(c+dx)})^p\,(a+b\sin{(c+dx)})^2\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**p*(a+b*sin(d*x+c))**2,x)
[Out] Integral((e*cos(c + d*x))**p*(a + b*sin(c + d*x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+b*sin(d*x+c))^2,x, algorithm="giac")
[Out] integrate((b*sin(d*x + c) + a)^2*(cos(d*x + c)*e)^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(e\,\cos\left(c+d\,x\right)\right)}^p\,{\left(a+b\,\sin\left(c+d\,x\right)\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^p*(a + b*sin(c + d*x))^2,x)
[Out] int((e*cos(c + d*x))^p*(a + b*sin(c + d*x))^2, x)
```

3.618 $\int (e\cos(c+dx))^p(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=97

$$-\frac{b(e\cos(c+dx))^{1+p}}{de(1+p)}-\frac{a(e\cos(c+dx))^{1+p}\,{}_2F_1\left(\frac{1}{2},\frac{1+p}{2};\frac{3+p}{2};\cos^2(c+dx)\right)\sin(c+dx)}{de(1+p)\sqrt{\sin^2(c+dx)}}$$

```
[Out] -b*(e*cos(d*x+c))^(1+p)/d/e/(1+p)-a*(e*cos(d*x+c))^(1+p)*hypergeom([1/2, 1/
2+1/2*p],[3/2+1/2*p],cos(d*x+c)^2)*sin(d*x+c)/d/e/(1+p)/(sin(d*x+c)^2)^(1/2
)
```

Rubi [A]
time = 0.04, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2748, 2722}

$$-\frac{a\sin(c+dx)(e\cos(c+dx))^{p+1}\,{}_2F_1\left(\frac{1}{2},\frac{p+1}{2};\frac{p+3}{2};\cos^2(c+dx)\right)}{de(p+1)\sqrt{\sin^2(c+dx)}}-\frac{b(e\cos(c+dx))^{p+1}}{de(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^p*(a + b*Sin[c + d*x]),x]
```

```
[Out] -((b*(e*Cos[c + d*x])^(1 + p))/(d*e*(1 + p))) - (a*(e*Cos[c + d*x])^(1 + p)
*Hypergeometric2F1[1/2, (1 + p)/2, (3 + p)/2, Cos[c + d*x]^2]*Sin[c + d*x])
/(d*e*(1 + p)*Sqrt[Sin[c + d*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\int (e\cos(c+dx))^p(a+b\sin(c+dx))\,dx = -\frac{b(e\cos(c+dx))^{1+p}}{de(1+p)} + a\int (e\cos(c+dx))^p\,dx$$
$$= -\frac{b(e\cos(c+dx))^{1+p}}{de(1+p)} - \frac{a(e\cos(c+dx))^{1+p}\,{}_2F_1\left(\frac{1}{2},\frac{1+p}{2};\frac{3+p}{2};\cos^2(c\right.}{de(1+p)\sqrt{\sin^2(c+dx}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.65, size = 240, normalized size = 2.47

$$-\frac{(e\cos(c+dx))^p\left(2^{-1-p}b\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^p\left(1+e^{2i(c+dx)}\right)\cos^{-p}(c+dx)\left(-e^{-i(c+dx)}(-1+p)\,{}_2F_1\left(1,\frac{1+p}{2};\frac{1-p}{2};-e^{2i(c+dx)}\right)+e^{i(c+dx)}(1+p)\,{}_2F_1\left(1,\frac{3+p}{2};\frac{3-p}{2};-e^{2i(c+dx)}\right)\right)\sqrt{\sin^2(c+dx)}-\frac{1}{2}a(-1+p)\,{}_2F_1\left(\frac{1}{2},\frac{1+p}{2};\frac{3+p}{2};\cos^2(c+dx)\right)\sin(2(c+dx))\right)}{(d-dp^2)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^p*(a + b*Sin[c + d*x]),x]
```

```
[Out] -(((e*Cos[c + d*x])^p*((2^(-1 - p)*b*(E^((-I)*(c + d*x)) + E^(I*(c + d*x)))
^p*(1 + E^((2*I)*(c + d*x)))*(-(((-1 + p)*Hypergeometric2F1[1, (1 + p)/2, (
1 - p)/2, -E^((2*I)*(c + d*x))])/E^(I*(c + d*x))) + E^(I*(c + d*x))*(1 + p)
*Hypergeometric2F1[1, (3 + p)/2, (3 - p)/2, -E^((2*I)*(c + d*x))])*Sqrt[Sin
[c + d*x]^2])/Cos[c + d*x]^p - (a*(-1 + p)*Hypergeometric2F1[1/2, (1 + p)/2
, (3 + p)/2, Cos[c + d*x]^2]*Sin[2*(c + d*x)])/2))/((d - d*p^2)*Sqrt[Sin[c
+ d*x]^2]))
```

Maple [F]
time = 0.15, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^p\,(a+b\sin(dx+c))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^p*(a+b*sin(d*x+c)),x)
```

```
[Out] int((e*cos(d*x+c))^p*(a+b*sin(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)*(cos(d*x + c)*e)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+b*sin(d*x+c)),x, algorithm="fricas")
[Out] integral((b*sin(d*x + c) + a)*(cos(d*x + c)*e)^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(c+dx))^p (a+b\sin(c+dx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**p*(a+b*sin(d*x+c)),x)
[Out] Integral((e*cos(c + d*x))**p*(a + b*sin(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+b*sin(d*x+c)),x, algorithm="giac")
[Out] integrate((b*sin(d*x + c) + a)*(cos(d*x + c)*e)^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (e\,\cos(c+d\,x))^p\,(a+b\,\sin(c+d\,x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^p*(a + b*sin(c + d*x)),x)
[Out] int((e*cos(c + d*x))^p*(a + b*sin(c + d*x)), x)
```

3.619 $\int \frac{(e\cos(c+dx))^p}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=158

$$-\frac{eF_1\left(1-p;\frac{1-p}{2},\frac{1-p}{2};2-p;\frac{a+b}{a+b\sin(c+dx)},\frac{a-b}{a+b\sin(c+dx)}\right)(e\cos(c+dx))^{-1+p}\left(-\frac{b(1-\sin(c+dx))}{a+b\sin(c+dx)}\right)^{\frac{1-p}{2}}\left(\frac{b(1+\sin(c+dx))}{a+b\sin(c+dx)}\right)}{bd(1-p)}$$

```
[Out] -e*AppellF1(1-p,1/2-1/2*p,1/2-1/2*p,2-p,(a-b)/(a+b*sin(d*x+c)),(a+b)/(a+b*s
in(d*x+c)))*(e*cos(d*x+c))^(-1+p)*(-b*(1-sin(d*x+c))/(a+b*sin(d*x+c)))^(1/2
-1/2*p)*(b*(1+sin(d*x+c))/(a+b*sin(d*x+c)))^(1/2-1/2*p)/b/d/(1-p)
```

Rubi [A]
time = 0.06, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {2782}

$$-\frac{e(e\cos(c+dx))^{p-1}\left(-\frac{b(1-\sin(c+dx))}{a+b\sin(c+dx)}\right)^{\frac{1-p}{2}}\left(\frac{b(\sin(c+dx)+1)}{a+b\sin(c+dx)}\right)^{\frac{1-p}{2}}F_1\left(1-p;\frac{1-p}{2},\frac{1-p}{2};2-p;\frac{a+b}{a+b\sin(c+dx)},\frac{a-b}{a+b\sin(c+dx)}\right)}{bd(1-p)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^p/(a + b*Sin[c + d*x]),x]
```

```
[Out] -((e*AppellF1[1 - p, (1 - p)/2, (1 - p)/2, 2 - p, (a + b)/(a + b*Sin[c + d*
x]), (a - b)/(a + b*Sin[c + d*x])]*(e*Cos[c + d*x])^(-1 + p)*(-((b*(1 - Sin
[c + d*x]))/(a + b*Sin[c + d*x])))^((1 - p)/2)*((b*(1 + Sin[c + d*x]))/(a +
 b*Sin[c + d*x]))^((1 - p)/2))/(b*d*(1 - p)))
```

Rule 2782

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p)*((-b)*((1 - Sin[e + f*x])/(a + b*Sin[e + f*x])))^((
p - 1)/2)*(b*((1 + Sin[e + f*x])/(a + b*Sin[e + f*x])))^((p - 1)/2)))*Appel
lF1[-p - m, (1 - p)/2, (1 - p)/2, 1 - p - m, (a + b)/(a + b*Sin[e + f*x]),
(a - b)/(a + b*Sin[e + f*x])], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^
2 - b^2, 0] && ILtQ[m, 0] &&  !IGtQ[m + p + 1, 0]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^p}{a+b\sin(c+dx)}\,dx = -\frac{eF_1\left(1-p;\frac{1-p}{2},\frac{1-p}{2};2-p;\frac{a+b}{a+b\sin(c+dx)},\frac{a-b}{a+b\sin(c+dx)}\right)(e\cos(c+dx))^{-1+p}\left(-\frac{b(1-\sin}{a+b\sin}\right.}{bd(1-p)}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 3815 vs. $2(158) = 316$.
time = 19.16, size = 3815, normalized size = 24.15

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^p/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((e*Cos[c + d*x])^p*Tan[c + d*x]*(a*Sqrt[Sec[c + d*x]^2] + b*Tan[c + d*x])*
(-(b*AppellF1[1, (1 + p)/2, 1, 2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d
*x]^2]*Tan[c + d*x]) - (6*a^5*AppellF1[1/2, p/2, 1, 3/2, -Tan[c + d*x]^2, (
(-a^2 + b^2)*Tan[c + d*x]^2)/a^2])/((Sec[c + d*x]^2)^(p/2)*(a^2 + (a^2 - b^
2)*Tan[c + d*x]^2)*(-3*a^2*AppellF1[1/2, p/2, 1, 3/2, -Tan[c + d*x]^2, (-1
+ b^2/a^2)*Tan[c + d*x]^2] + (2*(a^2 - b^2)*AppellF1[3/2, p/2, 2, 5/2, -Tan
[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2] + a^2*p*AppellF1[3/2, (2 + p)/2
, 1, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2])*Tan[c + d*x]^2))
))/(2*a^2*d*Sqrt[Sec[c + d*x]^2]*(a + b*Sin[c + d*x])*(a + (b*Tan[c + d*x])
/Sqrt[Sec[c + d*x]^2])*((Sqrt[Sec[c + d*x]^2]*(a*Sqrt[Sec[c + d*x]^2] + b*T
an[c + d*x])*(-(b*AppellF1[1, (1 + p)/2, 1, 2, -Tan[c + d*x]^2, (-1 + b^2/a
^2)*Tan[c + d*x]^2]*Tan[c + d*x]) - (6*a^5*AppellF1[1/2, p/2, 1, 3/2, -Tan[
c + d*x]^2, ((-a^2 + b^2)*Tan[c + d*x]^2)/a^2])/((Sec[c + d*x]^2)^(p/2)*(a^
2 + (a^2 - b^2)*Tan[c + d*x]^2)*(-3*a^2*AppellF1[1/2, p/2, 1, 3/2, -Tan[c +
 d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2] + (2*(a^2 - b^2)*AppellF1[3/2, p/2,
 2, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2] + a^2*p*AppellF1[3
/2, (2 + p)/2, 1, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2])*Tan
[c + d*x]^2))))/(2*a^2*(a + (b*Tan[c + d*x])/Sqrt[Sec[c + d*x]^2])) - (Tan[
c + d*x]^2*(a*Sqrt[Sec[c + d*x]^2] + b*Tan[c + d*x])*(-(b*AppellF1[1, (1 +
p)/2, 1, 2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2]*Tan[c + d*x]) -
 (6*a^5*AppellF1[1/2, p/2, 1, 3/2, -Tan[c + d*x]^2, ((-a^2 + b^2)*Tan[c + d
*x]^2)/a^2])/((Sec[c + d*x]^2)^(p/2)*(a^2 + (a^2 - b^2)*Tan[c + d*x]^2)*(-3
*a^2*AppellF1[1/2, p/2, 1, 3/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x
]^2] + (2*(a^2 - b^2)*AppellF1[3/2, p/2, 2, 5/2, -Tan[c + d*x]^2, (-1 + b^2
/a^2)*Tan[c + d*x]^2] + a^2*p*AppellF1[3/2, (2 + p)/2, 1, 5/2, -Tan[c + d*x
]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2])*Tan[c + d*x]^2))))/(2*a^2*Sqrt[Sec[c +
 d*x]^2]*(a + (b*Tan[c + d*x])/Sqrt[Sec[c + d*x]^2])) + (Tan[c + d*x]*(b*Se
c[c + d*x]^2 + a*Sqrt[Sec[c + d*x]^2]*Tan[c + d*x])*(-(b*AppellF1[1, (1 + p
)/2, 1, 2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2]*Tan[c + d*x]) -
(6*a^5*AppellF1[1/2, p/2, 1, 3/2, -Tan[c + d*x]^2, ((-a^2 + b^2)*Tan[c + d*
x]^2)/a^2])/((Sec[c + d*x]^2)^(p/2)*(a^2 + (a^2 - b^2)*Tan[c + d*x]^2)*(-3*
a^2*AppellF1[1/2, p/2, 1, 3/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]
^2] + (2*(a^2 - b^2)*AppellF1[3/2, p/2, 2, 5/2, -Tan[c + d*x]^2, (-1 + b^2/
a^2)*Tan[c + d*x]^2] + a^2*p*AppellF1[3/2, (2 + p)/2, 1, 5/2, -Tan[c + d*x]
^2, (-1 + b^2/a^2)*Tan[c + d*x]^2])*Tan[c + d*x]^2))))/(2*a^2*Sqrt[Sec[c +
d*x]^2]*(a + (b*Tan[c + d*x])/Sqrt[Sec[c + d*x]^2])) - (Tan[c + d*x]*(a*Sqr
```

```
t[Sec[c + d*x]^2] + b*Tan[c + d*x])*(b*Sqrt[Sec[c + d*x]^2] - (b*Tan[c + d*
x]^2)/Sqrt[Sec[c + d*x]^2])*(-(b*AppellF1[1, (1 + p)/2, 1, 2, -Tan[c + d*x]
^2, (-1 + b^2/a^2)*Tan[c + d*x]^2]*Tan[c + d*x]) - (6*a^5*AppellF1[1/2, p/2
, 1, 3/2, -Tan[c + d*x]^2, ((-a^2 + b^2)*Tan[c + d*x]^2)/a^2])/((Sec[c + d*
x]^2)^(p/2)*(a^2 + (a^2 - b^2)*Tan[c + d*x]^2)*(-3*a^2*AppellF1[1/2, p/2, 1
, 3/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2] + (2*(a^2 - b^2)*App
ellF1[3/2, p/2, 2, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2] + a
^2*p*AppellF1[3/2, (2 + p)/2, 1, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c
 + d*x]^2])*Tan[c + d*x]^2))))/(2*a^2*Sqrt[Sec[c + d*x]^2]*(a + (b*Tan[c +
d*x])/Sqrt[Sec[c + d*x]^2])^2) + (Tan[c + d*x]*(a*Sqrt[Sec[c + d*x]^2] + b*
Tan[c + d*x])*(-(b*AppellF1[1, (1 + p)/2, 1, 2, -Tan[c + d*x]^2, (-1 + b^2/
a^2)*Tan[c + d*x]^2]*Sec[c + d*x]^2) - b*Tan[c + d*x]*((-1 + b^2/a^2)*Appel
lF1[2, (1 + p)/2, 2, 3, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2]*Sec
[c + d*x]^2*Tan[c + d*x] - ((1 + p)*AppellF1[2, 1 + (1 + p)/2, 1, 3, -Tan[c
 + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2]*Sec[c + d*x]^2*Tan[c + d*x])/2) +
 (12*a^5*(a^2 - b^2)*AppellF1[1/2, p/2, 1, 3/2, -Tan[c + d*x]^2, ((-a^2 + b
^2)*Tan[c + d*x]^2)/a^2]*(Sec[c + d*x]^2)^(1 - p/2)*Tan[c + d*x])/((a^2 + (
a^2 - b^2)*Tan[c + d*x]^2)^2*(-3*a^2*AppellF1[1/2, p/2, 1, 3/2, -Tan[c + d*
x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2] + (2*(a^2 - b^2)*AppellF1[3/2, p/2, 2,
 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2] + a^2*p*AppellF1[3/2,
 (2 + p)/2, 1, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2])*Tan[c
+ d*x]^2)) + (6*a^5*p*AppellF1[1/2, p/2, 1, 3/2, -Tan[c + d*x]^2, ((-a^2 +
b^2)*Tan[c + d*x]^2)/a^2]*Tan[c + d*x])/((Sec[c + d*x]^2)^(p/2)*(a^2 + (a^2
 - b^2)*Tan[c + d*x]^2)*(-3*a^2*AppellF1[1/2, p/2, 1, 3/2, -Tan[c + d*x]^2,
 (-1 + b^2/a^2)*Tan[c + d*x]^2] + (2*(a^2 - b^2)*AppellF1[3/2, p/2, 2, 5/2,
 -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2] + a^2*p*AppellF1[3/2, (2 +
 p)/2, 1, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2])*Tan[c + d*x
]^2)) - (6*a^5*(-1/3*(p*AppellF1[3/2, 1 + p/2, 1, 5/2, -Tan[c + d*x]^2, ((-
a^2 + b^2)*Tan[c + d*x]^2)/a^2]*Sec[c + d*x]^2*Tan[c + d*x]) + (2*(-a^2 + b
^2)*AppellF1[3/2, p/2, 2, 5/2, -Tan[c + d*x]^2, ((-a^2 + b^2)*Tan[c + d*x]^
2)/a^2]*Sec[c + d*x]^2*Tan[c + d*x])/(3*a^2)))/...
```

Maple [F]
time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^p}{a+b\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^p/(a+b*sin(d*x+c)),x)
[Out] int((e*cos(d*x+c))^p/(a+b*sin(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((cos(d*x + c)*e)^p/(b*sin(d*x + c) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((cos(d*x + c)*e)^p/(b*sin(d*x + c) + a), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**p/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((cos(d*x + c)*e)^p/(b*sin(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,\cos\left(c+d\,x\right))^p}{a+b\,\sin\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^p/(a + b*sin(c + d*x)),x)
```

```
[Out] int((e*cos(c + d*x))^p/(a + b*sin(c + d*x)), x)
```

3.620 $\int \frac{(e\cos(c+dx))^p}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=170

$$-\frac{eF_1\left(2-p;\frac{1-p}{2},\frac{1-p}{2};3-p;\frac{a+b}{a+b\sin(c+dx)},\frac{a-b}{a+b\sin(c+dx)}\right)(e\cos(c+dx))^{-1+p}\left(-\frac{b(1-\sin(c+dx))}{a+b\sin(c+dx)}\right)^{\frac{1-p}{2}}\left(\frac{b(1+\sin(c+dx))}{a+b\sin(c+dx)}\right)}{bd(2-p)(a+b\sin(c+dx))}$$

```
[Out] -e*AppellF1(2-p,1/2-1/2*p,1/2-1/2*p,3-p,(a-b)/(a+b*sin(d*x+c)),(a+b)/(a+b*s
in(d*x+c)))*(e*cos(d*x+c))^(-1+p)*(-b*(1-sin(d*x+c))/(a+b*sin(d*x+c)))^(1/2
-1/2*p)*(b*(1+sin(d*x+c))/(a+b*sin(d*x+c)))^(1/2-1/2*p)/b/d/(2-p)/(a+b*sin(
d*x+c))
```

Rubi [A]
time = 0.05, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {2782}

$$-\frac{e(e\cos(c+dx))^{p-1}\left(-\frac{b(1-\sin(c+dx))}{a+b\sin(c+dx)}\right)^{\frac{1-p}{2}}\left(\frac{b(\sin(c+dx)+1)}{a+b\sin(c+dx)}\right)^{\frac{1-p}{2}}F_1\left(2-p;\frac{1-p}{2},\frac{1-p}{2};3-p;\frac{a+b}{a+b\sin(c+dx)},\frac{a-b}{a+b\sin(c+dx)}\right)}{bd(2-p)(a+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^p/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -((e*AppellF1[2 - p, (1 - p)/2, (1 - p)/2, 3 - p, (a + b)/(a + b*Sin[c + d*
x]), (a - b)/(a + b*Sin[c + d*x])]*(e*Cos[c + d*x])^(-1 + p)*(-((b*(1 - Sin
[c + d*x]))/(a + b*Sin[c + d*x])))^((1 - p)/2)*((b*(1 + Sin[c + d*x]))/(a +
 b*Sin[c + d*x]))^((1 - p)/2))/(b*d*(2 - p)*(a + b*Sin[c + d*x])))
```

Rule 2782

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p)*((-b)*((1 - Sin[e + f*x])/(a + b*Sin[e + f*x])))^((
p - 1)/2)*(b*((1 + Sin[e + f*x])/(a + b*Sin[e + f*x])))^((p - 1)/2)))*Appel
lF1[-p - m, (1 - p)/2, (1 - p)/2, 1 - p - m, (a + b)/(a + b*Sin[e + f*x]),
(a - b)/(a + b*Sin[e + f*x])], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^
2 - b^2, 0] && ILtQ[m, 0] &&  !IGtQ[m + p + 1, 0]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^p}{(a+b\sin(c+dx))^2}\,dx = -\frac{eF_1\left(2-p;\frac{1-p}{2},\frac{1-p}{2};3-p;\frac{a+b}{a+b\sin(c+dx)},\frac{a-b}{a+b\sin(c+dx)}\right)(e\cos(c+dx))^{-1+p}\left(-\frac{b(1-\right.}{a+\right.}}{bd(2-p)(a+b\sin(c+dx))}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 4727 vs. $2(170) = 340$.

time = 24.30, size = 4727, normalized size = 27.81

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^p/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((e*Cos[c + d*x])^p*Tan[c + d*x]*(b*(a^2 - b^2)*AppellF1[1, (-1 + p)/2, 2,
2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2]*Tan[c + d*x] + (3*a^5*((
-2*a^2*b^2*AppellF1[1/2, p/2, 2, 3/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c
 + d*x]^2])/((-3*a^2*AppellF1[1/2, p/2, 2, 3/2, -Tan[c + d*x]^2, (-1 + b^2/
a^2)*Tan[c + d*x]^2] + (4*(a^2 - b^2)*AppellF1[3/2, p/2, 3, 5/2, -Tan[c + d
*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2] + a^2*p*AppellF1[3/2, (2 + p)/2, 2, 5
/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2])*Tan[c + d*x]^2)*(b^2*T
an[c + d*x]^2 - a^2*(1 + Tan[c + d*x]^2))^2) + ((a^2 + b^2)*AppellF1[1/2, p
/2, 1, 3/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2])/((-3*a^2*Appel
lF1[1/2, p/2, 1, 3/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2] + (2*
(a^2 - b^2)*AppellF1[3/2, p/2, 2, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[
c + d*x]^2] + a^2*p*AppellF1[3/2, (2 + p)/2, 1, 5/2, -Tan[c + d*x]^2, (-1 +
 b^2/a^2)*Tan[c + d*x]^2])*Tan[c + d*x]^2)*(-(b^2*Tan[c + d*x]^2) + a^2*(1
+ Tan[c + d*x]^2)))))/(1 + Tan[c + d*x]^2)^(p/2)))/(a^3*(-a^2 + b^2)*d*(a +
 b*Sin[c + d*x])^2*((Sec[c + d*x]^2*(b*(a^2 - b^2)*AppellF1[1, (-1 + p)/2,
2, 2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2]*Tan[c + d*x] + (3*a^5
*((-2*a^2*b^2*AppellF1[1/2, p/2, 2, 3/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Ta
n[c + d*x]^2])/((-3*a^2*AppellF1[1/2, p/2, 2, 3/2, -Tan[c + d*x]^2, (-1 + b
^2/a^2)*Tan[c + d*x]^2] + (4*(a^2 - b^2)*AppellF1[3/2, p/2, 3, 5/2, -Tan[c
+ d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2] + a^2*p*AppellF1[3/2, (2 + p)/2, 2
, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2])*Tan[c + d*x]^2)*(b^
2*Tan[c + d*x]^2 - a^2*(1 + Tan[c + d*x]^2))^2) + ((a^2 + b^2)*AppellF1[1/2
, p/2, 1, 3/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2])/((-3*a^2*Ap
pellF1[1/2, p/2, 1, 3/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2] +
(2*(a^2 - b^2)*AppellF1[3/2, p/2, 2, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*T
an[c + d*x]^2] + a^2*p*AppellF1[3/2, (2 + p)/2, 1, 5/2, -Tan[c + d*x]^2, (-
1 + b^2/a^2)*Tan[c + d*x]^2])*Tan[c + d*x]^2)*(-(b^2*Tan[c + d*x]^2) + a^2*
(1 + Tan[c + d*x]^2)))))/(1 + Tan[c + d*x]^2)^(p/2)))/(a^3*(-a^2 + b^2)) +
(Tan[c + d*x]*(b*(a^2 - b^2)*AppellF1[1, (-1 + p)/2, 2, 2, -Tan[c + d*x]^2,
 (-1 + b^2/a^2)*Tan[c + d*x]^2]*Sec[c + d*x]^2 + b*(a^2 - b^2)*Tan[c + d*x]
*(-1/2*((-1 + p)*AppellF1[2, 1 + (-1 + p)/2, 2, 3, -Tan[c + d*x]^2, (-1 + b
^2/a^2)*Tan[c + d*x]^2]*Sec[c + d*x]^2*Tan[c + d*x]) + 2*(-1 + b^2/a^2)*App
ellF1[2, (-1 + p)/2, 3, 3, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2]*
Sec[c + d*x]^2*Tan[c + d*x]) - 3*a^5*p*Sec[c + d*x]^2*Tan[c + d*x]*(1 + Tan
[c + d*x]^2)^(-1 - p/2)*((-2*a^2*b^2*AppellF1[1/2, p/2, 2, 3/2, -Tan[c + d*
x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2])/((-3*a^2*AppellF1[1/2, p/2, 2, 3/2, -
```

```
Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2] + (4*(a^2 - b^2)*AppellF1[3/
2, p/2, 3, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2] + a^2*p*App
ellF1[3/2, (2 + p)/2, 2, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^
2])*Tan[c + d*x]^2)*(b^2*Tan[c + d*x]^2 - a^2*(1 + Tan[c + d*x]^2))^2) + ((
a^2 + b^2)*AppellF1[1/2, p/2, 1, 3/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c
 + d*x]^2])/((-3*a^2*AppellF1[1/2, p/2, 1, 3/2, -Tan[c + d*x]^2, (-1 + b^2/
a^2)*Tan[c + d*x]^2] + (2*(a^2 - b^2)*AppellF1[3/2, p/2, 2, 5/2, -Tan[c + d
*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2] + a^2*p*AppellF1[3/2, (2 + p)/2, 1, 5
/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2])*Tan[c + d*x]^2)*(-(b^2
*Tan[c + d*x]^2) + a^2*(1 + Tan[c + d*x]^2)))) + (3*a^5*((4*a^2*b^2*AppellF
1[1/2, p/2, 2, 3/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2]*(-2*a^2
*Sec[c + d*x]^2*Tan[c + d*x] + 2*b^2*Sec[c + d*x]^2*Tan[c + d*x]))/((-3*a^2
*AppellF1[1/2, p/2, 2, 3/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2]
 + (4*(a^2 - b^2)*AppellF1[3/2, p/2, 3, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2
)*Tan[c + d*x]^2] + a^2*p*AppellF1[3/2, (2 + p)/2, 2, 5/2, -Tan[c + d*x]^2,
 (-1 + b^2/a^2)*Tan[c + d*x]^2])*Tan[c + d*x]^2)*(b^2*Tan[c + d*x]^2 - a^2*
(1 + Tan[c + d*x]^2))^3) - (2*a^2*b^2*(-1/3*(p*AppellF1[3/2, 1 + p/2, 2, 5/
2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2]*Sec[c + d*x]^2*Tan[c + d
*x]) + (4*(-1 + b^2/a^2)*AppellF1[3/2, p/2, 3, 5/2, -Tan[c + d*x]^2, (-1 +
b^2/a^2)*Tan[c + d*x]^2]*Sec[c + d*x]^2*Tan[c + d*x])/3))/((-3*a^2*AppellF1
[1/2, p/2, 2, 3/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2] + (4*(a^
2 - b^2)*AppellF1[3/2, p/2, 3, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c +
 d*x]^2] + a^2*p*AppellF1[3/2, (2 + p)/2, 2, 5/2, -Tan[c + d*x]^2, (-1 + b^
2/a^2)*Tan[c + d*x]^2])*Tan[c + d*x]^2)*(b^2*Tan[c + d*x]^2 - a^2*(1 + Tan[
c + d*x]^2))^2) - ((a^2 + b^2)*AppellF1[1/2, p/2, 1, 3/2, -Tan[c + d*x]^2,
(-1 + b^2/a^2)*Tan[c + d*x]^2]*(2*a^2*Sec[c + d*x]^2*Tan[c + d*x] - 2*b^2*S
ec[c + d*x]^2*Tan[c + d*x]))/((-3*a^2*AppellF1[1/2, p/2, 1, 3/2, -Tan[c + d
*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2] + (2*(a^2 - b^2)*AppellF1[3/2, p/2, 2
, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2] + a^2*p*AppellF1[3/2
, (2 + p)/2, 1, 5/2, -Tan[c + d*x]^2, (-1 + b^2/a^2)*Tan[c + d*x]^2])*Tan[c
 + d*x]^2)*(-(b^2*Tan[c + d*x]^2) + a^2*(1 + Ta...
```

Maple [F]
time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^p}{(a+b\sin(dx+c))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^p/(a+b*sin(d*x+c))^2,x)
[Out] int((e*cos(d*x+c))^p/(a+b*sin(d*x+c))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((cos(d*x + c)*e)^p/(b*sin(d*x + c) + a)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(-(cos(d*x + c)*e)^p/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2
 - b^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**p/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((cos(d*x + c)*e)^p/(b*sin(d*x + c) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,\cos\left(c+d\,x\right))^p}{{\left(a+b\,\sin\left(c+d\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^p/(a + b*sin(c + d*x))^2,x)
```

```
[Out] int((e*cos(c + d*x))^p/(a + b*sin(c + d*x))^2, x)
```

3.621 $\int \frac{(e\cos(c+dx))^p}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=170

$$-\frac{eF_1\left(3-p;\frac{1-p}{2},\frac{1-p}{2};4-p;\frac{a+b}{a+b\sin(c+dx)},\frac{a-b}{a+b\sin(c+dx)}\right)(e\cos(c+dx))^{-1+p}\left(-\frac{b(1-\sin(c+dx))}{a+b\sin(c+dx)}\right)^{\frac{1-p}{2}}\left(\frac{b(1+\sin(c+dx))}{a+b\sin(c+dx)}\right)}{bd(3-p)(a+b\sin(c+dx))^2}$$

```
[Out] -e*AppellF1(3-p,1/2-1/2*p,1/2-1/2*p,4-p,(a-b)/(a+b*sin(d*x+c)),(a+b)/(a+b*s
in(d*x+c)))*(e*cos(d*x+c))^(-1+p)*(-b*(1-sin(d*x+c))/(a+b*sin(d*x+c)))^(1/2
-1/2*p)*(b*(1+sin(d*x+c))/(a+b*sin(d*x+c)))^(1/2-1/2*p)/b/d/(3-p)/(a+b*sin(
d*x+c))^2
```

Rubi [A]
time = 0.05, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {2782}

$$-\frac{e(e\cos(c+dx))^{p-1}\left(-\frac{b(1-\sin(c+dx))}{a+b\sin(c+dx)}\right)^{\frac{1-p}{2}}\left(\frac{b(\sin(c+dx)+1)}{a+b\sin(c+dx)}\right)^{\frac{1-p}{2}}F_1\left(3-p;\frac{1-p}{2},\frac{1-p}{2};4-p;\frac{a+b}{a+b\sin(c+dx)},\frac{a-b}{a+b\sin(c+dx)}\right)}{bd(3-p)(a+b\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^p/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -((e*AppellF1[3 - p, (1 - p)/2, (1 - p)/2, 4 - p, (a + b)/(a + b*Sin[c + d*
x]), (a - b)/(a + b*Sin[c + d*x])]*(e*Cos[c + d*x])^(-1 + p)*(-((b*(1 - Sin
[c + d*x]))/(a + b*Sin[c + d*x])))^((1 - p)/2)*((b*(1 + Sin[c + d*x]))/(a +
 b*Sin[c + d*x]))^((1 - p)/2))/(b*d*(3 - p)*(a + b*Sin[c + d*x])^2))
```

Rule 2782

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p)*((-b)*((1 - Sin[e + f*x])/(a + b*Sin[e + f*x])))^((
p - 1)/2)*(b*((1 + Sin[e + f*x])/(a + b*Sin[e + f*x])))^((p - 1)/2)))*Appel
lF1[-p - m, (1 - p)/2, (1 - p)/2, 1 - p - m, (a + b)/(a + b*Sin[e + f*x]),
(a - b)/(a + b*Sin[e + f*x])], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^
2 - b^2, 0] && ILtQ[m, 0] &&  !IGtQ[m + p + 1, 0]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^p}{(a+b\sin(c+dx))^3}\,dx = -\frac{eF_1\left(3-p;\frac{1-p}{2},\frac{1-p}{2};4-p;\frac{a+b}{a+b\sin(c+dx)},\frac{a-b}{a+b\sin(c+dx)}\right)(e\cos(c+dx))^{-1+p}\left(-\frac{b(1-}{a+}\right.}{bd(3-p)(a+b\sin(c+dx))^2}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 7781 vs. $2(170) = 340$.
time = 25.70, size = 7781, normalized size = 45.77

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^p/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] Result too large to show
```

Maple [F]
time = 0.45, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^p}{(a+b\sin(dx+c))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^p/(a+b*sin(d*x+c))^3,x)
```

```
[Out] int((e*cos(d*x+c))^p/(a+b*sin(d*x+c))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate((cos(d*x + c)*e)^p/(b*sin(d*x + c) + a)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(-(cos(d*x + c)*e)^p/(3*a*b^2*cos(d*x + c)^2 - a^3 - 3*a*b^2 + (b^3
*cos(d*x + c)^2 - 3*a^2*b - b^3)*sin(d*x + c)), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**p/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((cos(d*x + c)*e)^p/(b*sin(d*x + c) + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,\cos\left(c+d\,x\right))^p}{{\left(a+b\,\sin\left(c+d\,x\right)\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^p/(a + b*sin(c + d*x))^3,x)
```

```
[Out] int((e*cos(c + d*x))^p/(a + b*sin(c + d*x))^3, x)
```

3.622 $\int \frac{(e\cos(c+dx))^p}{(a+b\sin(c+dx))^8}\,dx$

Optimal. Leaf size=170

$$-\frac{eF_1\left(8-p;\frac{1-p}{2},\frac{1-p}{2};9-p;\frac{a+b}{a+b\sin(c+dx)},\frac{a-b}{a+b\sin(c+dx)}\right)(e\cos(c+dx))^{-1+p}\left(-\frac{b(1-\sin(c+dx))}{a+b\sin(c+dx)}\right)^{\frac{1-p}{2}}\left(\frac{b(1+\sin(c+dx))}{a+b\sin(c+dx)}\right.}{bd(8-p)(a+b\sin(c+dx))^7}$$

```
[Out] -e*AppellF1(8-p,1/2-1/2*p,1/2-1/2*p,9-p,(a-b)/(a+b*sin(d*x+c)),(a+b)/(a+b*s
in(d*x+c)))*(e*cos(d*x+c))^(-1+p)*(-b*(1-sin(d*x+c))/(a+b*sin(d*x+c)))^(1/2
-1/2*p)*(b*(1+sin(d*x+c))/(a+b*sin(d*x+c)))^(1/2-1/2*p)/b/d/(8-p)/(a+b*sin(
d*x+c))^7
```

Rubi [A]
time = 0.05, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {2782}

$$-\frac{e(e\cos(c+dx))^{p-1}\left(-\frac{b(1-\sin(c+dx))}{a+b\sin(c+dx)}\right)^{\frac{1-p}{2}}\left(\frac{b(\sin(c+dx)+1)}{a+b\sin(c+dx)}\right)^{\frac{1-p}{2}}F_1\left(8-p;\frac{1-p}{2},\frac{1-p}{2};9-p;\frac{a+b}{a+b\sin(c+dx)},\frac{a-b}{a+b\sin(c+dx)}\right)}{bd(8-p)(a+b\sin(c+dx))^7}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^p/(a + b*Sin[c + d*x])^8,x]
```

```
[Out] -((e*AppellF1[8 - p, (1 - p)/2, (1 - p)/2, 9 - p, (a + b)/(a + b*Sin[c + d*
x]), (a - b)/(a + b*Sin[c + d*x])]*(e*Cos[c + d*x])^(-1 + p)*(-((b*(1 - Sin
[c + d*x]))/(a + b*Sin[c + d*x])))^((1 - p)/2)*((b*(1 + Sin[c + d*x]))/(a +
 b*Sin[c + d*x]))^((1 - p)/2))/(b*d*(8 - p)*(a + b*Sin[c + d*x])^7))
```

Rule 2782

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p)*((-b)*((1 - Sin[e + f*x])/(a + b*Sin[e + f*x])))^((
p - 1)/2)*(b*((1 + Sin[e + f*x])/(a + b*Sin[e + f*x])))^((p - 1)/2)))*Appel
lF1[-p - m, (1 - p)/2, (1 - p)/2, 1 - p - m, (a + b)/(a + b*Sin[e + f*x]),
(a - b)/(a + b*Sin[e + f*x])], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^
2 - b^2, 0] && ILtQ[m, 0] &&  !IGtQ[m + p + 1, 0]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^p}{(a+b\sin(c+dx))^8}\,dx = -\frac{eF_1\left(8-p;\frac{1-p}{2},\frac{1-p}{2};9-p;\frac{a+b}{a+b\sin(c+dx)},\frac{a-b}{a+b\sin(c+dx)}\right)(e\cos(c+dx))^{-1+p}\left(-\frac{b(\right.}{bd(8-p)(a+b\sin(c+dx))^7}$$

Mathematica [F]
time = 46.11, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(c+dx))^p}{(a+b\sin(c+dx))^8}\,dx$$

Verification is not applicable to the result.

[In] Integrate[(e*Cos[c + d*x])^p/(a + b*Sin[c + d*x])^8,x]

[Out] Integrate[(e*Cos[c + d*x])^p/(a + b*Sin[c + d*x])^8, x]

Maple [F]
time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^p}{(a+b\sin(dx+c))^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*cos(d*x+c))^p/(a+b*sin(d*x+c))^8,x)

[Out] int((e*cos(d*x+c))^p/(a+b*sin(d*x+c))^8,x)

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*cos(d*x+c))^p/(a+b*sin(d*x+c))^8,x, algorithm="maxima")

[Out] Timed out

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*cos(d*x+c))^p/(a+b*sin(d*x+c))^8,x, algorithm="fricas")

[Out] integral((cos(d*x + c)*e)^p/(b^8*cos(d*x + c)^8 + a^8 + 28*a^6*b^2 + 70*a^4
*b^4 + 28*a^2*b^6 + b^8 - 4*(7*a^2*b^6 + b^8)*cos(d*x + c)^6 + 2*(35*a^4*b^
4 + 42*a^2*b^6 + 3*b^8)*cos(d*x + c)^4 - 4*(7*a^6*b^2 + 35*a^4*b^4 + 21*a^2
*b^6 + b^8)*cos(d*x + c)^2 - 8*(a*b^7*cos(d*x + c)^6 - a^7*b - 7*a^5*b^3 -
7*a^3*b^5 - a*b^7 - (7*a^3*b^5 + 3*a*b^7)*cos(d*x + c)^4 + (7*a^5*b^3 + 14*
a^3*b^5 + 3*a*b^7)*cos(d*x + c)^2)*sin(d*x + c)), x)

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**p/(a+b*sin(d*x+c))**8,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+b*sin(d*x+c))^8,x, algorithm="giac")
```

```
[Out] integrate((cos(d*x + c)*e)^p/(b*sin(d*x + c) + a)^8, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,\cos\left(c+d\,x\right))^p}{(a+b\,\sin\left(c+d\,x\right))^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^p/(a + b*sin(c + d*x))^8,x)
```

```
[Out] int((e*cos(c + d*x))^p/(a + b*sin(c + d*x))^8, x)
```

3.623 $\int (e\cos(c+dx))^p(a+b\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=156

$$\frac{2eF_1\left(\frac{7}{2};\frac{1-p}{2},\frac{1-p}{2};\frac{9}{2};\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)(e\cos(c+dx))^{-1+p}(a+b\sin(c+dx))^{7/2}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{\frac{1-p}{2}}\left(\right.}{7bd}$$

```
[Out] 2/7*e*AppellF1(7/2,1/2-1/2*p,1/2-1/2*p,9/2,(a+b*sin(d*x+c))/(a-b),(a+b*sin(
d*x+c))/(a+b))*(e*cos(d*x+c))^(-1+p)*(a+b*sin(d*x+c))^(7/2)*(1+(-a-b*sin(d*
x+c))/(a-b))^(1/2-1/2*p)*(1+(-a-b*sin(d*x+c))/(a+b))^(1/2-1/2*p)/b/d
```

Rubi [A]
time = 0.09, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2783, 143}

$$\frac{2e(a+b\sin(c+dx))^{7/2}(e\cos(c+dx))^{p-1}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{\frac{1-p}{2}}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{\frac{1-p}{2}}F_1\left(\frac{7}{2};\frac{1-p}{2},\frac{1-p}{2};\frac{9}{2};\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)}{7bd}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^p*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (2*e*AppellF1[7/2, (1 - p)/2, (1 - p)/2, 9/2, (a + b*Sin[c + d*x])/(a - b),
 (a + b*Sin[c + d*x])/(a + b)]*(e*Cos[c + d*x])^(-1 + p)*(a + b*Sin[c + d*x
])^(7/2)*(1 - (a + b*Sin[c + d*x])/(a - b))^((1 - p)/2)*(1 - (a + b*Sin[c +
 d*x])/(a + b))^((1 - p)/2))/(7*b*d)
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 2783

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[g*((g*Cos[e + f*x])^(p - 1)/(f*(1 - (a + b*Sin
[e + f*x])/(a - b))^((p - 1)/2)*(1 - (a + b*Sin[e + f*x])/(a + b))^((p - 1)
/2))), Subst[Int[(-b/(a - b) - b*(x/(a - b)))^((p - 1)/2)*(b/(a + b) - b*(x
/(a + b)))^((p - 1)/2)*(a + b*x)^m, x], x, Sin[e + f*x]], x] /; FreeQ[{a, b
```

```
, e, f, g, m, p}, x] && NeQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int (e\cos(c+dx))^p(a+b\sin(c+dx))^{5/2}\,dx = \frac{\left(e(e\cos(c+dx))^{-1+p}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{\frac{1-p}{2}}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right.\right.}{}$$

$$= \frac{2eF_1\left(\frac{7}{2};\frac{1-p}{2},\frac{1-p}{2};\frac{9}{2};\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)(e\cos(c+dx))^{-}}{}$$

Mathematica [A]
time = 10.28, size = 187, normalized size = 1.20

$$\frac{2eF_1\left(\frac{7}{2};\frac{1-p}{2},\frac{1-p}{2};\frac{9}{2};\frac{a+b\sin(c+dx)}{a-\sqrt{b^2}},\frac{a+b\sin(c+dx)}{a+\sqrt{b^2}}\right)(e\cos(c+dx))^{-1+p}\left(\frac{\sqrt{b^2}-b\sin(c+dx)}{a+\sqrt{b^2}}\right)^{\frac{1-p}{2}}(a+b\sin(c+dx))^{7/2}\left(\frac{\sqrt{b^2}+b\sin(c+dx)}{-a+\sqrt{b^2}}\right)^{\frac{1-p}{2}}}{7bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^p*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (2*e*AppellF1[7/2, (1 - p)/2, (1 - p)/2, 9/2, (a + b*Sin[c + d*x])/(a - Sqr
t[b^2]), (a + b*Sin[c + d*x])/(a + Sqrt[b^2])]*(e*Cos[c + d*x])^(-1 + p)*((
Sqrt[b^2] - b*Sin[c + d*x])/(a + Sqrt[b^2]))^((1 - p)/2)*(a + b*Sin[c + d*x
])^(7/2)*((Sqrt[b^2] + b*Sin[c + d*x])/(-a + Sqrt[b^2]))^((1 - p)/2))/(7*b*
d)
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^p\,(a+b\sin(dx+c))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^p*(a+b*sin(d*x+c))^(5/2),x)
```

```
[Out] int((e*cos(d*x+c))^p*(a+b*sin(d*x+c))^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+b*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(5/2)*(cos(d*x + c)*e)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)*sqrt(b*sin(
d*x + c) + a)*(cos(d*x + c)*e)^p, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**p*(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6189 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+b*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(5/2)*(cos(d*x + c)*e)^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (e\,\cos\left(c+d\,x\right))^p\,{\left(a+b\,\sin\left(c+d\,x\right)\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^p*(a + b*sin(c + d*x))^(5/2),x)
```

```
[Out] int((e*cos(c + d*x))^p*(a + b*sin(c + d*x))^(5/2), x)
```

3.624 $\int (e\cos(c+dx))^p(a+b\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=156

$$\frac{2eF_1\left(\frac{5}{2};\frac{1-p}{2},\frac{1-p}{2};\frac{7}{2};\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)(e\cos(c+dx))^{-1+p}(a+b\sin(c+dx))^{5/2}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{\frac{1-p}{2}}}{5bd}$$

```
[Out] 2/5*e*AppellF1(5/2,1/2-1/2*p,1/2-1/2*p,7/2,(a+b*sin(d*x+c))/(a-b),(a+b*sin(
d*x+c))/(a+b))*(e*cos(d*x+c))^(-1+p)*(a+b*sin(d*x+c))^(5/2)*(1+(-a-b*sin(d*
x+c))/(a-b))^(1/2-1/2*p)*(1+(-a-b*sin(d*x+c))/(a+b))^(1/2-1/2*p)/b/d
```

Rubi [A]
time = 0.08, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2783, 143}

$$\frac{2e(a+b\sin(c+dx))^{5/2}(e\cos(c+dx))^{p-1}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{\frac{1-p}{2}}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{\frac{1-p}{2}}F_1\left(\frac{5}{2};\frac{1-p}{2},\frac{1-p}{2};\frac{7}{2};\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)}{5bd}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^p*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (2*e*AppellF1[5/2, (1 - p)/2, (1 - p)/2, 7/2, (a + b*Sin[c + d*x])/(a - b),
 (a + b*Sin[c + d*x])/(a + b)]*(e*Cos[c + d*x])^(-1 + p)*(a + b*Sin[c + d*x
])^(5/2)*(1 - (a + b*Sin[c + d*x])/(a - b))^((1 - p)/2)*(1 - (a + b*Sin[c +
 d*x])/(a + b))^((1 - p)/2))/(5*b*d)
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 2783

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[g*((g*Cos[e + f*x])^(p - 1)/(f*(1 - (a + b*Sin
[e + f*x])/(a - b))^((p - 1)/2)*(1 - (a + b*Sin[e + f*x])/(a + b))^((p - 1)
/2))), Subst[Int[(-b/(a - b) - b*(x/(a - b)))^((p - 1)/2)*(b/(a + b) - b*(x
/(a + b)))^((p - 1)/2)*(a + b*x)^m, x], x, Sin[e + f*x]], x] /; FreeQ[{a, b
```

```
, e, f, g, m, p}, x] && NeQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int (e\cos(c+dx))^p(a+b\sin(c+dx))^{3/2}\,dx = \frac{\left(e(e\cos(c+dx))^{-1+p}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{\frac{1-p}{2}}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{\frac{1}{}}\right.}{}$$

$$= \frac{2eF_1\left(\frac{5}{2};\frac{1-p}{2},\frac{1-p}{2};\frac{7}{2};\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)(e\cos(c+dx))^{-1+}}{}$$

Mathematica [A]
time = 2.06, size = 187, normalized size = 1.20

$$\frac{2eF_1\left(\frac{5}{2};\frac{1-p}{2},\frac{1-p}{2};\frac{7}{2};\frac{a+b\sin(c+dx)}{a-\sqrt{b^2}},\frac{a+b\sin(c+dx)}{a+\sqrt{b^2}}\right)(e\cos(c+dx))^{-1+p}\left(\frac{\sqrt{b^2}-b\sin(c+dx)}{a+\sqrt{b^2}}\right)^{\frac{1-p}{2}}(a+b\sin(c+dx))^{5/2}\left(\frac{\sqrt{b^2}+b\sin(c+dx)}{-a+\sqrt{b^2}}\right)^{\frac{1-p}{2}}}{5bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^p*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (2*e*AppellF1[5/2, (1 - p)/2, (1 - p)/2, 7/2, (a + b*Sin[c + d*x])/(a - Sqr
t[b^2]), (a + b*Sin[c + d*x])/(a + Sqrt[b^2])]*(e*Cos[c + d*x])^(-1 + p)*((
Sqrt[b^2] - b*Sin[c + d*x])/(a + Sqrt[b^2]))^((1 - p)/2)*(a + b*Sin[c + d*x
])^(5/2)*((Sqrt[b^2] + b*Sin[c + d*x])/(-a + Sqrt[b^2]))^((1 - p)/2))/(5*b*
d)
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^p\,(a+b\sin(dx+c))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^p*(a+b*sin(d*x+c))^(3/2),x)
```

```
[Out] int((e*cos(d*x+c))^p*(a+b*sin(d*x+c))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+b*sin(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((b*sin(d*x + c) + a)^(3/2)*(cos(d*x + c)*e)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
[Out] integral((b*sin(d*x + c) + a)^(3/2)*(cos(d*x + c)*e)^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(c+dx))^p (a+b\sin(c+dx))^{\frac{3}{2}}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**p*(a+b*sin(d*x+c))**(3/2),x)
[Out] Integral((e*cos(c + d*x))**p*(a + b*sin(c + d*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+b*sin(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((b*sin(d*x + c) + a)^(3/2)*(cos(d*x + c)*e)^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (e\,\cos(c+d\,x))^p\,(a+b\,\sin(c+d\,x))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^p*(a + b*sin(c + d*x))^(3/2),x)
[Out] int((e*cos(c + d*x))^p*(a + b*sin(c + d*x))^(3/2), x)
```

3.625 $\int (e\cos(c+dx))^p \sqrt{a+b\sin(c+dx)}\, dx$

Optimal. Leaf size=156

$$\frac{2eF_1\left(\frac{3}{2};\frac{1-p}{2},\frac{1-p}{2};\frac{5}{2};\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)(e\cos(c+dx))^{-1+p}(a+b\sin(c+dx))^{3/2}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{\frac{1-p}{2}}\left(\right.}{3bd}$$

```
[Out] 2/3*e*AppellF1(3/2,1/2-1/2*p,1/2-1/2*p,5/2,(a+b*sin(d*x+c))/(a-b),(a+b*sin(
d*x+c))/(a+b))*(e*cos(d*x+c))^(-1+p)*(a+b*sin(d*x+c))^(3/2)*(1+(-a-b*sin(d*
x+c))/(a-b))^(1/2-1/2*p)*(1+(-a-b*sin(d*x+c))/(a+b))^(1/2-1/2*p)/b/d
```

Rubi [A]
time = 0.07, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2783, 143}

$$\frac{2e(a+b\sin(c+dx))^{3/2}(e\cos(c+dx))^{p-1}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{\frac{1-p}{2}}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{\frac{1-p}{2}}F_1\left(\frac{3}{2};\frac{1-p}{2},\frac{1-p}{2};\frac{5}{2};\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^p*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (2*e*AppellF1[3/2, (1 - p)/2, (1 - p)/2, 5/2, (a + b*Sin[c + d*x])/(a - b),
 (a + b*Sin[c + d*x])/(a + b)]*(e*Cos[c + d*x])^(-1 + p)*(a + b*Sin[c + d*x
])^(3/2)*(1 - (a + b*Sin[c + d*x])/(a - b))^((1 - p)/2)*(1 - (a + b*Sin[c +
 d*x])/(a + b))^((1 - p)/2))/(3*b*d)
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 2783

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[g*((g*Cos[e + f*x])^(p - 1)/(f*(1 - (a + b*Sin
[e + f*x])/(a - b))^((p - 1)/2)*(1 - (a + b*Sin[e + f*x])/(a + b))^((p - 1)
/2))), Subst[Int[(-b/(a - b) - b*(x/(a - b)))^((p - 1)/2)*(b/(a + b) - b*(x
/(a + b)))^((p - 1)/2)*(a + b*x)^m, x], x, Sin[e + f*x]], x] /; FreeQ[{a, b
```

```
, e, f, g, m, p}, x] && NeQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int (e\cos(c+dx))^p\sqrt{a+b\sin(c+dx)}\,dx = \frac{\left(e(e\cos(c+dx))^{-1+p}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{\frac{1-p}{2}}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)\right.}{}$$

$$= \frac{2eF_1\left(\frac{3}{2};\frac{1-p}{2},\frac{1-p}{2};\frac{5}{2};\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)(e\cos(c+dx))^{-1-}}{}$$

Mathematica [A]
time = 0.68, size = 187, normalized size = 1.20

$$\frac{2eF_1\left(\frac{3}{2};\frac{1-p}{2},\frac{1-p}{2};\frac{5}{2};\frac{a+b\sin(c+dx)}{a-\sqrt{b^2}},\frac{a+b\sin(c+dx)}{a+\sqrt{b^2}}\right)(e\cos(c+dx))^{-1+p}\left(\frac{\sqrt{b^2}-b\sin(c+dx)}{a+\sqrt{b^2}}\right)^{\frac{1-p}{2}}(a+b\sin(c+dx))^{3/2}\left(\frac{\sqrt{b^2}+b\sin(c+dx)}{-a+\sqrt{b^2}}\right)^{\frac{1-p}{2}}}{3bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^p*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (2*e*AppellF1[3/2, (1 - p)/2, (1 - p)/2, 5/2, (a + b*Sin[c + d*x])/(a - Sqr
t[b^2]), (a + b*Sin[c + d*x])/(a + Sqrt[b^2])]*(e*Cos[c + d*x])^(-1 + p)*((
Sqrt[b^2] - b*Sin[c + d*x])/(a + Sqrt[b^2]))^((1 - p)/2)*(a + b*Sin[c + d*x
])^(3/2)*((Sqrt[b^2] + b*Sin[c + d*x])/(-a + Sqrt[b^2]))^((1 - p)/2))/(3*b*
d)
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^p\sqrt{a+b\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^p*(a+b*sin(d*x+c))^(1/2),x)
```

```
[Out] int((e*cos(d*x+c))^p*(a+b*sin(d*x+c))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+b*sin(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sin(d*x + c) + a)*(cos(d*x + c)*e)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sin(d*x + c) + a)*(cos(d*x + c)*e)^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos{(c+dx)})^p\sqrt{a+b\sin{(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**p*(a+b*sin(d*x+c))**(1/2),x)
[Out] Integral((e*cos(c + d*x))**p*sqrt(a + b*sin(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+b*sin(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sin(d*x + c) + a)*(cos(d*x + c)*e)^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(e\,\cos\left(c+d\,x\right)\right)}^p\,\sqrt{a+b\,\sin\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^p*(a + b*sin(c + d*x))^(1/2),x)
[Out] int((e*cos(c + d*x))^p*(a + b*sin(c + d*x))^(1/2), x)
```

3.626 $\int \frac{(e\cos(c+dx))^p}{\sqrt{a+b\sin(c+dx)}}\,dx$

Optimal. Leaf size=154

$$\frac{2eF_1\left(\frac{1}{2};\frac{1-p}{2},\frac{1-p}{2};\frac{3}{2};\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)(e\cos(c+dx))^{-1+p}\sqrt{a+b\sin(c+dx)}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{\frac{1-p}{2}}\left(\right.}{bd}$$

```
[Out] 2*e*AppellF1(1/2,1/2-1/2*p,1/2-1/2*p,3/2,(a+b*sin(d*x+c))/(a-b),(a+b*sin(d*
x+c))/(a+b))*(e*cos(d*x+c))^(-1+p)*(1+(-a-b*sin(d*x+c))/(a-b))^(1/2-1/2*p)*
(1+(-a-b*sin(d*x+c))/(a+b))^(1/2-1/2*p)*(a+b*sin(d*x+c))^(1/2)/b/d
```

Rubi [A]

time = 0.08, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2783, 143}

$$\frac{2e\sqrt{a+b\sin(c+dx)}\,(e\cos(c+dx))^{p-1}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{\frac{1-p}{2}}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{\frac{1-p}{2}}F_1\left(\frac{1}{2};\frac{1-p}{2},\frac{1-p}{2};\frac{3}{2};\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^p/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (2*e*AppellF1[1/2, (1 - p)/2, (1 - p)/2, 3/2, (a + b*Sin[c + d*x])/(a - b),
 (a + b*Sin[c + d*x])/(a + b)]*(e*Cos[c + d*x])^(-1 + p)*Sqrt[a + b*Sin[c +
 d*x]]*(1 - (a + b*Sin[c + d*x])/(a - b))^((1 - p)/2)*(1 - (a + b*Sin[c + d
*x])/(a + b))^((1 - p)/2))/(b*d)
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 2783

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[g*((g*Cos[e + f*x])^(p - 1)/(f*(1 - (a + b*Sin
[e + f*x])/(a - b))^((p - 1)/2)*(1 - (a + b*Sin[e + f*x])/(a + b))^((p - 1)
/2))), Subst[Int[(-b/(a - b) - b*(x/(a - b)))^((p - 1)/2)*(b/(a + b) - b*(x
/(a + b)))^((p - 1)/2)*(a + b*x)^m, x], x, Sin[e + f*x]], x] /; FreeQ[{a, b
```

```
, e, f, g, m, p}, x] && NeQ[a^2 - b^2, 0] && !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^p}{\sqrt{a+b\sin(c+dx)}}\,dx = \frac{\left(e(e\cos(c+dx))^{-1+p}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{\frac{1-p}{2}}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{\frac{1-p}{2}}\right)\text{Subst}\left(\int \frac{(}{}\right.}{d}$$

$$= \frac{2eF_1\left(\frac{1}{2};\frac{1-p}{2},\frac{1-p}{2};\frac{3}{2};\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)(e\cos(c+dx))^{-1+p}\sqrt{a+b\sin(c+}}{bd}$$

Mathematica [**A**]
time = 2.05, size = 185, normalized size = 1.20

$$\frac{2eF_1\left(\frac{1}{2};\frac{1-p}{2},\frac{1-p}{2};\frac{3}{2};\frac{a+b\sin(c+dx)}{a-\sqrt{b^2}},\frac{a+b\sin(c+dx)}{a+\sqrt{b^2}}\right)(e\cos(c+dx))^{-1+p}\left(\frac{\sqrt{b^2}-b\sin(c+dx)}{a+\sqrt{b^2}}\right)^{\frac{1-p}{2}}\sqrt{a+b\sin(c+dx)}\left(\frac{\sqrt{b^2}+b\sin(c+dx)}{-a+\sqrt{b^2}}\right)^{\frac{1-p}{2}}}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^p/Sqrt[a + b*Sin[c + d*x]],x]
[Out] (2*e*AppellF1[1/2, (1 - p)/2, (1 - p)/2, 3/2, (a + b*Sin[c + d*x])/(a - Sqr
t[b^2]), (a + b*Sin[c + d*x])/(a + Sqrt[b^2])]*(e*Cos[c + d*x])^(-1 + p)*((
Sqrt[b^2] - b*Sin[c + d*x])/(a + Sqrt[b^2]))^((1 - p)/2)*Sqrt[a + b*Sin[c +
 d*x]]*((Sqrt[b^2] + b*Sin[c + d*x])/(-a + Sqrt[b^2]))^((1 - p)/2))/(b*d)
```

Maple [**F**]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^p}{\sqrt{a+b\sin(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^p/(a+b*sin(d*x+c))^(1/2),x)
[Out] int((e*cos(d*x+c))^p/(a+b*sin(d*x+c))^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+b*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((cos(d*x + c)*e)^p/sqrt(b*sin(d*x + c) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((cos(d*x + c)*e)^p/sqrt(b*sin(d*x + c) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(c+dx))^p}{\sqrt{a+b\sin(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**p/(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral((e*cos(c + d*x))**p/sqrt(a + b*sin(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+b*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((cos(d*x + c)*e)^p/sqrt(b*sin(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,\cos(c+d\,x))^p}{\sqrt{a+b\,\sin(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^p/(a + b*sin(c + d*x))^(1/2),x)
```

```
[Out] int((e*cos(c + d*x))^p/(a + b*sin(c + d*x))^(1/2), x)
```

3.627 $\int \frac{(e\cos(c+dx))^p}{(a+b\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=154

$$-\frac{2eF_1\left(-\frac{1}{2};\frac{1-p}{2},\frac{1-p}{2};\frac{1}{2};\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)(e\cos(c+dx))^{-1+p}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{\frac{1-p}{2}}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)}{bd\sqrt{a+b\sin(c+dx)}}$$

```
[Out] -2*e*AppellF1(-1/2,1/2-1/2*p,1/2-1/2*p,1/2,(a+b*sin(d*x+c))/(a-b),(a+b*sin(
d*x+c))/(a+b))*(e*cos(d*x+c))^(-1+p)*(1+(-a-b*sin(d*x+c))/(a-b))^(1/2-1/2*p
)*(1+(-a-b*sin(d*x+c))/(a+b))^(1/2-1/2*p)/b/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2783, 143}

$$-\frac{2e(e\cos(c+dx))^{p-1}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{\frac{1-p}{2}}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{\frac{1-p}{2}}F_1\left(-\frac{1}{2};\frac{1-p}{2},\frac{1-p}{2};\frac{1}{2};\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)}{bd\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^p/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*e*AppellF1[-1/2, (1 - p)/2, (1 - p)/2, 1/2, (a + b*Sin[c + d*x])/(a - b
), (a + b*Sin[c + d*x])/(a + b)]*(e*Cos[c + d*x])^(-1 + p)*(1 - (a + b*Sin[
c + d*x])/(a - b))^((1 - p)/2)*(1 - (a + b*Sin[c + d*x])/(a + b))^((1 - p)/
2))/(b*d*Sqrt[a + b*Sin[c + d*x]])
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 2783

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[g*((g*Cos[e + f*x])^(p - 1)/(f*(1 - (a + b*Sin
[e + f*x])/(a - b))^((p - 1)/2)*(1 - (a + b*Sin[e + f*x])/(a + b))^((p - 1)
/2))), Subst[Int[(-b/(a - b) - b*(x/(a - b)))^((p - 1)/2)*(b/(a + b) - b*(x
```

```
/(a + b)))^((p - 1)/2)*(a + b*x)^m, x], x, Sin[e + f*x]], x] /; FreeQ[{a, b
, e, f, g, m, p}, x] && NeQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^p}{(a+b\sin(c+dx))^{3/2}}\,dx = \frac{\left(e(e\cos(c+dx))^{-1+p}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{\frac{1-p}{2}}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{\frac{1-p}{2}}\right)\text{Subst}\Big(\int}{d}$$

$$= -\frac{2eF_1\left(-\frac{1}{2};\frac{1-p}{2},\frac{1-p}{2};\frac{1}{2};\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)(e\cos(c+dx))^{-1+p}\Big(1-\frac{a+}{}}{bd\sqrt{a+b\sin(c+dx)}}$$

Mathematica [A]
time = 2.35, size = 185, normalized size = 1.20

$$-\frac{2eF_1\left(-\frac{1}{2};\frac{1-p}{2},\frac{1-p}{2};\frac{1}{2};\frac{a+b\sin(c+dx)}{a-\sqrt{b^2}},\frac{a+b\sin(c+dx)}{a+\sqrt{b^2}}\right)(e\cos(c+dx))^{-1+p}\left(\frac{\sqrt{b^2}-b\sin(c+dx)}{a+\sqrt{b^2}}\right)^{\frac{1-p}{2}}\left(\frac{\sqrt{b^2}+b\sin(c+dx)}{-a+\sqrt{b^2}}\right)^{\frac{1-p}{2}}}{bd\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^p/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*e*AppellF1[-1/2, (1 - p)/2, (1 - p)/2, 1/2, (a + b*Sin[c + d*x])/(a - S
qrt[b^2]), (a + b*Sin[c + d*x])/(a + Sqrt[b^2])]*(e*Cos[c + d*x])^(-1 + p)*
((Sqrt[b^2] - b*Sin[c + d*x])/(a + Sqrt[b^2]))^((1 - p)/2)*((Sqrt[b^2] + b*
Sin[c + d*x])/(-a + Sqrt[b^2]))^((1 - p)/2))/(b*d*Sqrt[a + b*Sin[c + d*x]])
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^p}{(a+b\sin(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^p/(a+b*sin(d*x+c))^(3/2),x)
```

```
[Out] int((e*cos(d*x+c))^p/(a+b*sin(d*x+c))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+b*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((cos(d*x + c)*e)^p/(b*sin(d*x + c) + a)^(3/2), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(b*sin(d*x + c) + a)*(cos(d*x + c)*e)^p/(b^2*cos(d*x + c)^2 -
 2*a*b*sin(d*x + c) - a^2 - b^2), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(c+dx))^p}{(a+b\sin(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**p/(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral((e*cos(c + d*x))**p/(a + b*sin(c + d*x))**(3/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+b*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,\cos(c+d\,x))^p}{(a+b\,\sin(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^p/(a + b*sin(c + d*x))^(3/2),x)
```

```
[Out] int((e*cos(c + d*x))^p/(a + b*sin(c + d*x))^(3/2), x)
```

3.628 $\int \frac{(e\cos(c+dx))^p}{(a+b\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=156

$$-\frac{2eF_1\left(-\frac{3}{2};\frac{1-p}{2},\frac{1-p}{2};-\frac{1}{2};\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)(e\cos(c+dx))^{-1+p}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{\frac{1-p}{2}}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{\frac{1-p}{2}}}{3bd(a+b\sin(c+dx))^{3/2}}$$

```
[Out] -2/3*e*AppellF1(-3/2,1/2-1/2*p,1/2-1/2*p,-1/2,(a+b*sin(d*x+c))/(a-b),(a+b*s
in(d*x+c))/(a+b))*(e*cos(d*x+c))^(-1+p)*(1+(-a-b*sin(d*x+c))/(a-b))^(1/2-1/
2*p)*(1+(-a-b*sin(d*x+c))/(a+b))^(1/2-1/2*p)/b/d/(a+b*sin(d*x+c))^(3/2)
```

Rubi [A]
time = 0.08, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2783, 143}

$$-\frac{2e(e\cos(c+dx))^{p-1}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{\frac{1-p}{2}}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{\frac{1-p}{2}}F_1\left(-\frac{3}{2};\frac{1-p}{2},\frac{1-p}{2};-\frac{1}{2};\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)}{3bd(a+b\sin(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^p/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*e*AppellF1[-3/2, (1 - p)/2, (1 - p)/2, -1/2, (a + b*Sin[c + d*x])/(a -
b), (a + b*Sin[c + d*x])/(a + b)]*(e*Cos[c + d*x])^(-1 + p)*(1 - (a + b*Sin
[c + d*x])/(a - b))^((1 - p)/2)*(1 - (a + b*Sin[c + d*x])/(a + b))^((1 - p)
/2))/(3*b*d*(a + b*Sin[c + d*x])^(3/2))
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 2783

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[g*((g*Cos[e + f*x])^(p - 1)/(f*(1 - (a + b*Sin
[e + f*x])/(a - b))^((p - 1)/2)*(1 - (a + b*Sin[e + f*x])/(a + b))^((p - 1)
/2))), Subst[Int[(-b/(a - b) - b*(x/(a - b)))^((p - 1)/2)*(b/(a + b) - b*(x
/(a + b)))^((p - 1)/2)*(a + b*x)^m, x], x, Sin[e + f*x]], x] /; FreeQ[{a, b
```

```
, e, f, g, m, p}, x] && NeQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^p}{(a+b\sin(c+dx))^{5/2}}\,dx = \frac{\left(e(e\cos(c+dx))^{-1+p}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{\frac{1-p}{2}}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{\frac{1-p}{2}}\right)\text{Subst}\Big(\int}{d}$$

$$= -\frac{2eF_1\left(-\frac{3}{2};\frac{1-p}{2},\frac{1-p}{2};-\frac{1}{2};\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)(e\cos(c+dx))^{-1+p}\Big(1-\frac{a+}{}}{3bd(a+b\sin(c+dx))^{3/2}}$$

Mathematica [A]
time = 2.64, size = 187, normalized size = 1.20

$$-\frac{2eF_1\left(-\frac{3}{2};\frac{1-p}{2},\frac{1-p}{2};-\frac{1}{2};\frac{a+b\sin(c+dx)}{a-\sqrt{b^2}},\frac{a+b\sin(c+dx)}{a+\sqrt{b^2}}\right)(e\cos(c+dx))^{-1+p}\left(\frac{\sqrt{b^2}-b\sin(c+dx)}{a+\sqrt{b^2}}\right)^{\frac{1-p}{2}}\left(\frac{\sqrt{b^2}+b\sin(c+dx)}{-a+\sqrt{b^2}}\right)^{\frac{1-p}{2}}}{3bd(a+b\sin(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^p/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*e*AppellF1[-3/2, (1 - p)/2, (1 - p)/2, -1/2, (a + b*Sin[c + d*x])/(a -
Sqrt[b^2]), (a + b*Sin[c + d*x])/(a + Sqrt[b^2])]*(e*Cos[c + d*x])^(-1 + p)
*((Sqrt[b^2] - b*Sin[c + d*x])/(a + Sqrt[b^2]))^((1 - p)/2)*((Sqrt[b^2] + b
*Sin[c + d*x])/(-a + Sqrt[b^2]))^((1 - p)/2))/(3*b*d*(a + b*Sin[c + d*x])^(
3/2))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(dx+c))^p}{(a+b\sin(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^p/(a+b*sin(d*x+c))^(5/2),x)
```

```
[Out] int((e*cos(d*x+c))^p/(a+b*sin(d*x+c))^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+b*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((cos(d*x + c)*e)^p/(b*sin(d*x + c) + a)^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(b*sin(d*x + c) + a)*(cos(d*x + c)*e)^p/(3*a*b^2*cos(d*x + c)
^2 - a^3 - 3*a*b^2 + (b^3*cos(d*x + c)^2 - 3*a^2*b - b^3)*sin(d*x + c)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\cos(c+dx))^p}{(a+b\sin(c+dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**p/(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Integral((e*cos(c + d*x))**p/(a + b*sin(c + d*x))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p/(a+b*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((cos(d*x + c)*e)^p/(b*sin(d*x + c) + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,\cos(c+d\,x))^p}{(a+b\,\sin(c+d\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^p/(a + b*sin(c + d*x))^(5/2),x)
```

```
[Out] int((e*cos(c + d*x))^p/(a + b*sin(c + d*x))^(5/2), x)
```

3.629 $\int (e\cos(c+dx))^p(a+b\sin(c+dx))^m\,dx$

Optimal. Leaf size=158

$$\frac{eF_1\left(1+m;\frac{1-p}{2},\frac{1-p}{2};2+m;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)(e\cos(c+dx))^{-1+p}(a+b\sin(c+dx))^{1+m}\left(1-\frac{a+b\sin(c}{a-b}\right.}{bd(1+m)}$$

```
[Out] e*AppellF1(1+m,1/2-1/2*p,1/2-1/2*p,2+m,(a+b*sin(d*x+c))/(a-b),(a+b*sin(d*x+
c))/(a+b))*(e*cos(d*x+c))^(-1+p)*(a+b*sin(d*x+c))^(1+m)*(1+(-a-b*sin(d*x+c)
)/(a-b))^(1/2-1/2*p)*(1+(-a-b*sin(d*x+c))/(a+b))^(1/2-1/2*p)/b/d/(1+m)
```

Rubi [A]
time = 0.07, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2783, 143}

$$\frac{e(e\cos(c+dx))^{p-1}(a+b\sin(c+dx))^{m+1}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{\frac{1-p}{2}}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{\frac{1-p}{2}}F_1\left(m+1;\frac{1-p}{2},\frac{1-p}{2};m+2;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)}{bd(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^p*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] (e*AppellF1[1 + m, (1 - p)/2, (1 - p)/2, 2 + m, (a + b*Sin[c + d*x])/(a - b
), (a + b*Sin[c + d*x])/(a + b)]*(e*Cos[c + d*x])^(-1 + p)*(a + b*Sin[c + d
*x])^(1 + m)*(1 - (a + b*Sin[c + d*x])/(a - b))^((1 - p)/2)*(1 - (a + b*Sin
[c + d*x])/(a + b))^((1 - p)/2))/(b*d*(1 + m))
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 2783

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[g*((g*Cos[e + f*x])^(p - 1)/(f*(1 - (a + b*Sin
[e + f*x])/(a - b))^((p - 1)/2)*(1 - (a + b*Sin[e + f*x])/(a + b))^((p - 1)
/2))), Subst[Int[(-b/(a - b) - b*(x/(a - b)))^((p - 1)/2)*(b/(a + b) - b*(x
/(a + b)))^((p - 1)/2)*(a + b*x)^m, x], x, Sin[e + f*x]], x] /; FreeQ[{a, b
```

```
, e, f, g, m, p}, x] && NeQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int (e\cos(c+dx))^p(a+b\sin(c+dx))^m\,dx = \frac{\left(e(e\cos(c+dx))^{-1+p}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{\frac{1-p}{2}}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)\right.}{}$$

$$= \frac{eF_1\left(1+m;\frac{1-p}{2},\frac{1-p}{2};2+m;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)(e\cos(c+}{}$$

Mathematica [F]
time = 1.57, size = 0, normalized size = 0.00

$$\int (e\cos(c+dx))^p(a+b\sin(c+dx))^m\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(e*Cos[c + d*x])^p*(a + b*Sin[c + d*x])^m,x]
[Out] Integrate[(e*Cos[c + d*x])^p*(a + b*Sin[c + d*x])^m, x]
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int \left(e\cos\left(dx+c\right)\right)^{p}\left(a+b\sin\left(dx+c\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^p*(a+b*sin(d*x+c))^m,x)
[Out] int((e*cos(d*x+c))^p*(a+b*sin(d*x+c))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+b*sin(d*x+c))^m,x, algorithm="maxima")
[Out] integrate((cos(d*x + c)*e)^p*(b*sin(d*x + c) + a)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+b*sin(d*x+c))^m,x, algorithm="fricas")
[Out] integral((cos(d*x + c)*e)^p*(b*sin(d*x + c) + a)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos{(c+dx)})^p (a+b\sin{(c+dx)})^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**p*(a+b*sin(d*x+c))**m,x)
[Out] Integral((e*cos(c + d*x))**p*(a + b*sin(c + d*x))**m, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^p*(a+b*sin(d*x+c))^m,x, algorithm="giac")
[Out] integrate((cos(d*x + c)*e)^p*(b*sin(d*x + c) + a)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(e\,\cos\left(c+d\,x\right)\right)}^p\,{\left(a+b\,\sin\left(c+d\,x\right)\right)}^m \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^p*(a + b*sin(c + d*x))^m,x)
[Out] int((e*cos(c + d*x))^p*(a + b*sin(c + d*x))^m, x)
```

3.630 $\int \cos^7(c+dx)(a+b\sin(c+dx))^m \, dx$

Optimal. Leaf size=254

$$-\frac{(a^2-b^2)^3(a+b\sin(c+dx))^{1+m}}{b^7d(1+m)}+\frac{6a(a^2-b^2)^2(a+b\sin(c+dx))^{2+m}}{b^7d(2+m)}-\frac{3(5a^4-6a^2b^2+b^4)(a+b\sin(c+
$$

```
[Out] -(a^2-b^2)^3*(a+b*sin(d*x+c))^(1+m)/b^7/d/(1+m)+6*a*(a^2-b^2)^2*(a+b*sin(d*
x+c))^(2+m)/b^7/d/(2+m)-3*(5*a^4-6*a^2*b^2+b^4)*(a+b*sin(d*x+c))^(3+m)/b^7/
d/(3+m)+4*a*(5*a^2-3*b^2)*(a+b*sin(d*x+c))^(4+m)/b^7/d/(4+m)-3*(5*a^2-b^2)*
(a+b*sin(d*x+c))^(5+m)/b^7/d/(5+m)+6*a*(a+b*sin(d*x+c))^(6+m)/b^7/d/(6+m)-(
a+b*sin(d*x+c))^(7+m)/b^7/d/(7+m)
```

Rubi [A]

time = 0.12, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2747, 711}

$$-\frac{(a^2-b^2)^3(a+b\sin(c+dx))^{m+1}}{b^7d(m+1)}+\frac{6a(a^2-b^2)^2(a+b\sin(c+dx))^{m+2}}{b^7d(m+2)}+\frac{4a(5a^2-3b^2)(a+b\sin(c+dx))^{m+4}}{b^7d(m+4)}-\frac{3(5a^2-b^2)(a+b\sin(c+dx))^{m+5}}{b^7d(m+5)}-\frac{3(5a^4-6a^2b^2+b^4)(a+b\sin(c+dx))^{m+3}}{b^7d(m+3)}+\frac{6a(a+b\sin(c+dx))^{m+6}}{b^7d(m+6)}-\frac{(a+b\sin(c+dx))^{m+7}}{b^7d(m+7)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] -(((a^2 - b^2)^3*(a + b*Sin[c + d*x])^(1 + m))/(b^7*d*(1 + m))) + (6*a*(a^2
 - b^2)^2*(a + b*Sin[c + d*x])^(2 + m))/(b^7*d*(2 + m)) - (3*(5*a^4 - 6*a^2
*b^2 + b^4)*(a + b*Sin[c + d*x])^(3 + m))/(b^7*d*(3 + m)) + (4*a*(5*a^2 - 3
*b^2)*(a + b*Sin[c + d*x])^(4 + m))/(b^7*d*(4 + m)) - (3*(5*a^2 - b^2)*(a +
 b*Sin[c + d*x])^(5 + m))/(b^7*d*(5 + m)) + (6*a*(a + b*Sin[c + d*x])^(6 +
m))/(b^7*d*(6 + m)) - (a + b*Sin[c + d*x])^(7 + m)/(b^7*d*(7 + m))
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^7(c+dx)(a+b\sin(c+dx))^m\,dx = \frac{\text{Subst}\left(\int (a+x)^m\left(b^2-x^2\right)^3\,dx, x, b\sin(c+dx)\right)}{b^7d}$$

$$= \frac{\text{Subst}\left(\int \left(-(a^2-b^2)^3(a+x)^m + 6a(a^2-b^2)^2(a+x)^{1+m} - 3(5a^4\right.\right.}{}$$

$$= -\frac{(a^2-b^2)^3(a+b\sin(c+dx))^{1+m}}{b^7d(1+m)} + \frac{6a(a^2-b^2)^2(a+b\sin(c+dx}{b^7d(2+m)}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 854 vs. 2(254) = 508.

time = 3.08, size = 854, normalized size = 3.36

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] ((a + b*Sin[c + d*x])^(1 + m)*(-23040*a^6 + 85248*a^4*b^2 - 121536*a^2*b^4
+ 109728*b^6 + 12672*a^4*b^2*m - 52944*a^2*b^4*m + 143880*b^6*m - 3456*a^4*
b^2*m^2 + 840*a^2*b^4*m^2 + 74896*b^6*m^2 + 1392*a^2*b^4*m^3 + 20718*b^6*m^
3 + 24*a^2*b^4*m^4 + 3286*b^6*m^4 + 282*b^6*m^5 + 10*b^6*m^6 + 3*b^2*(2 + 3
*m + m^2)*(1920*a^4 - 32*a^2*b^2*(192 + 43*m + m^2) + b^4*(7176 + 4718*m +
1139*m^2 + 122*m^3 + 5*m^4))*Cos[2*(c + d*x)] + 6*b^4*(24 + 50*m + 35*m^2 +
 10*m^3 + m^4)*(-20*a^2 + b^2*(54 + 15*m + m^2))*Cos[4*(c + d*x)] + 720*b^6
*Cos[6*(c + d*x)] + 1764*b^6*m*Cos[6*(c + d*x)] + 1624*b^6*m^2*Cos[6*(c + d
*x)] + 735*b^6*m^3*Cos[6*(c + d*x)] + 175*b^6*m^4*Cos[6*(c + d*x)] + 21*b^6
*m^5*Cos[6*(c + d*x)] + b^6*m^6*Cos[6*(c + d*x)] + 23040*a^5*b*Sin[c + d*x]
 - 79488*a^3*b^3*Sin[c + d*x] + 103104*a*b^5*Sin[c + d*x] + 23040*a^5*b*m*S
in[c + d*x] - 95040*a^3*b^3*m*Sin[c + d*x] + 161136*a*b^5*m*Sin[c + d*x] -
14976*a^3*b^3*m^2*Sin[c + d*x] + 68376*a*b^5*m^2*Sin[c + d*x] + 576*a^3*b^3
*m^3*Sin[c + d*x] + 11064*a*b^5*m^3*Sin[c + d*x] + 744*a*b^5*m^4*Sin[c + d*
x] + 24*a*b^5*m^5*Sin[c + d*x] - 5760*a^3*b^3*Sin[3*(c + d*x)] + 16992*a*b^
5*Sin[3*(c + d*x)] - 10560*a^3*b^3*m*Sin[3*(c + d*x)] + 35400*a*b^5*m*Sin[3
*(c + d*x)] - 5760*a^3*b^3*m^2*Sin[3*(c + d*x)] + 24996*a*b^5*m^2*Sin[3*(c
+ d*x)] - 960*a^3*b^3*m^3*Sin[3*(c + d*x)] + 7476*a*b^5*m^3*Sin[3*(c + d*x)
] + 924*a*b^5*m^4*Sin[3*(c + d*x)] + 36*a*b^5*m^5*Sin[3*(c + d*x)] + 1440*a
*b^5*Sin[5*(c + d*x)] + 3288*a*b^5*m*Sin[5*(c + d*x)] + 2700*a*b^5*m^2*Sin[
5*(c + d*x)] + 1020*a*b^5*m^3*Sin[5*(c + d*x)] + 180*a*b^5*m^4*Sin[5*(c + d
*x)] + 12*a*b^5*m^5*Sin[5*(c + d*x)]))/(32*b^7*d*(1 + m)*(2 + m)*(3 + m)*(4
 + m)*(5 + m)*(6 + m)*(7 + m))
```

Maple [F]
time = 0.17, size = 0, normalized size = 0.00

$$\int \left(\cos^{7}\left(dx+c\right)\right)\left(a+b\sin\left(dx+c\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*(a+b*sin(d*x+c))^m,x)
```

```
[Out] int(cos(d*x+c)^7*(a+b*sin(d*x+c))^m,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 558 vs. $2(254) = 508$.
time = 0.31, size = 558, normalized size = 2.20

$$\frac{\frac{\left(b\sin\left(dx+c\right)+a\right)^{m+1}}{b\left(m+1\right)}-\frac{3\left(\left(m^{2}+3m+2\right)b^{3}\sin\left(dx+c\right)^{3}+\left(m^{2}+m\right)ab^{2}\sin\left(dx+c\right)^{2}-2a^{2}bm\sin\left(dx+c\right)+2a^{3}\right)\left(b\sin\left(dx+c\right)+a\right)^{m}}{\left(m^{3}+6m^{2}+11m+6\right)b^{3}}+\frac{3\left(\left(m^{4}+10m^{3}+35m^{2}+50m+24\right)b^{5}\sin\left(dx+c\right)^{5}+\left(m^{4}+6m^{3}+11m^{2}+6m\right)ab^{4}\sin\left(dx+c\right)^{4}-4\left(m^{3}+3m^{2}+2m\right)a^{2}b^{3}\sin\left(dx+c\right)^{3}+12\left(m^{2}+m\right)a^{3}b^{2}\sin\left(dx+c\right)^{2}-24a^{4}bm\sin\left(dx+c\right)+24a^{5}\right)\left(b\sin\left(dx+c\right)+a\right)^{m}}{\left(m^{5}+15m^{4}+85m^{3}+225m^{2}+274m+120\right)b^{5}}-\frac{\left(\left(m^{6}+21m^{5}+175m^{4}+735m^{3}+1624m^{2}+1764m+720\right)b^{7}\sin\left(dx+c\right)^{7}+\left(m^{6}+15m^{5}+85m^{4}+225m^{3}+274m^{2}+120m\right)ab^{6}\sin\left(dx+c\right)^{6}-6\left(m^{5}+10m^{4}+35m^{3}+50m^{2}+24m\right)a^{2}b^{5}\sin\left(dx+c\right)^{5}+30\left(m^{4}+6m^{3}+11m^{2}+6m\right)a^{3}b^{4}\sin\left(dx+c\right)^{4}-120\left(m^{3}+3m^{2}+2m\right)a^{4}b^{3}\sin\left(dx+c\right)^{3}+360\left(m^{2}+m\right)a^{5}b^{2}\sin\left(dx+c\right)^{2}-720a^{6}bm\sin\left(dx+c\right)+720a^{7}\right)\left(b\sin\left(dx+c\right)+a\right)^{m}}{\left(m^{7}+28m^{6}+322m^{5}+1960m^{4}+6769m^{3}+13132m^{2}+13068m+5040\right)b^{7}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+b*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] ((b*sin(d*x + c) + a)^(m + 1)/(b*(m + 1)) - 3*((m^2 + 3*m + 2)*b^3*sin(d*x
+ c)^3 + (m^2 + m)*a*b^2*sin(d*x + c)^2 - 2*a^2*b*m*sin(d*x + c) + 2*a^3)*(
b*sin(d*x + c) + a)^m/((m^3 + 6*m^2 + 11*m + 6)*b^3) + 3*((m^4 + 10*m^3 + 3
5*m^2 + 50*m + 24)*b^5*sin(d*x + c)^5 + (m^4 + 6*m^3 + 11*m^2 + 6*m)*a*b^4*
sin(d*x + c)^4 - 4*(m^3 + 3*m^2 + 2*m)*a^2*b^3*sin(d*x + c)^3 + 12*(m^2 + m
)*a^3*b^2*sin(d*x + c)^2 - 24*a^4*b*m*sin(d*x + c) + 24*a^5)*(b*sin(d*x + c
) + a)^m/((m^5 + 15*m^4 + 85*m^3 + 225*m^2 + 274*m + 120)*b^5) - ((m^6 + 21
*m^5 + 175*m^4 + 735*m^3 + 1624*m^2 + 1764*m + 720)*b^7*sin(d*x + c)^7 + (m
^6 + 15*m^5 + 85*m^4 + 225*m^3 + 274*m^2 + 120*m)*a*b^6*sin(d*x + c)^6 - 6*
(m^5 + 10*m^4 + 35*m^3 + 50*m^2 + 24*m)*a^2*b^5*sin(d*x + c)^5 + 30*(m^4 +
6*m^3 + 11*m^2 + 6*m)*a^3*b^4*sin(d*x + c)^4 - 120*(m^3 + 3*m^2 + 2*m)*a^4*
b^3*sin(d*x + c)^3 + 360*(m^2 + m)*a^5*b^2*sin(d*x + c)^2 - 720*a^6*b*m*sin
(d*x + c) + 720*a^7)*(b*sin(d*x + c) + a)^m/((m^7 + 28*m^6 + 322*m^5 + 1960
*m^4 + 6769*m^3 + 13132*m^2 + 13068*m + 5040)*b^7))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 814 vs. $2(254) = 508$.
time = 0.45, size = 814, normalized size = 3.20

$$-\frac{\left(720a^{7}-3024a^{5}b^{2}+5040a^{3}b^{4}-5040ab^{6}-\left(ab^{6}m^{6}+15ab^{6}m^{5}+85ab^{6}m^{4}+225ab^{6}m^{3}+274ab^{6}m^{2}+120ab^{6}m\right)\cos\left(dx+c\right)^{6}-6\left(2ab^{6}m^{5}-\left(5a^{3}b^{4}-23ab^{6}\right)m^{4}-2\left(15a^{3}b^{4}-44ab^{6}\right)m^{3}-\left(55a^{3}b^{4}-133ab^{6}\right)m^{2}-6\left(5a^{3}b^{4}-11ab^{6}\right)m\right)\cos\left(dx+c\right)^{4}-192\left(a^{3}b^{4}+ab^{6}\right)m^{3}+288\left(a^{5}b^{2}-2a^{3}b^{4}-7ab^{6}\right)m^{2}-24\left(\left(a^{3}b^{4}+3ab^{6}\right)m^{4}-6\left(a^{3}b^{4}-5ab^{6}\right)m^{3}+\left(15a^{5}b^{2}-55a^{3}b^{4}+84ab^{6}\right)m^{2}+3\left(5a^{5}b^{2}-16a^{3}b^{4}+19ab^{6}\right)m\right)\cos\left(dx+c\right)^{2}-192\left(3a^{5}b^{2}-13a^{3}b^{4}+32ab^{6}\right)m-\left(2304b^{7}+\left(b^{7}m^{6}+21b^{7}m^{5}+175b^{7}m^{4}+735b^{7}m^{3}+1624b^{7}m^{2}+1764b^{7}m+720b^{7}\right)\cos\left(dx+c\right)^{6}+6\left(144b^{7}+\left(a^{2}b^{5}+b^{7}\right)m^{5}+2\left(5a^{2}b^{5}+8b^{7}\right)m^{4}+5\left(7a^{2}b^{5}+19b^{7}\right)m^{3}+10\left(5a^{2}b^{5}+26b^{7}\right)m^{2}+12\left(2a^{2}b^{5}+27b^{7}\right)m\right)\cos\left(dx+c\right)^{4}+48\left(a^{4}b^{3}+6a^{2}b^{5}+b^{7}\right)m^{3}-576\left(a^{4}b^{3}-4a^{2}b^{5}-b^{7}\right)m^{2}+24\left(48b^{7}+\left(3a^{2}b^{5}+b^{7}\right)m^{4}-\left(5a^{4}b^{3}-24a^{2}b^{5}-13b^{7}\right)m^{3}-\left(15a^{4}b^{3}-51a^{2}b^{5}-56b^{7}\right)m^{2}-2\left(5a^{4}b^{3}-15a^{2}b^{5}-46b^{7}\right)m\right)\cos\left(dx+c\right)^{2}+48\left(15a^{6}b-58a^{4}b^{3}+87a^{2}b^{5}+44b^{7}\right)m\right)\sin\left(dx+c\right)\right)\left(b\sin\left(dx+c\right)+a\right)^{m}}{b^{7}dm^{7}+28b^{7}dm^{6}+322b^{7}dm^{5}+1960b^{7}dm^{4}+6769b^{7}dm^{3}+13132b^{7}dm^{2}+13068b^{7}dm+5040b^{7}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+b*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] -(720*a^7 - 3024*a^5*b^2 + 5040*a^3*b^4 - 5040*a*b^6 - (a*b^6*m^6 + 15*a*b^
6*m^5 + 85*a*b^6*m^4 + 225*a*b^6*m^3 + 274*a*b^6*m^2 + 120*a*b^6*m)*cos(d*x
```

```
 + c)^6 - 6*(2*a*b^6*m^5 - (5*a^3*b^4 - 23*a*b^6)*m^4 - 2*(15*a^3*b^4 - 44*
a*b^6)*m^3 - (55*a^3*b^4 - 133*a*b^6)*m^2 - 6*(5*a^3*b^4 - 11*a*b^6)*m)*cos
(d*x + c)^4 - 192*(a^3*b^4 + a*b^6)*m^3 + 288*(a^5*b^2 - 2*a^3*b^4 - 7*a*b^
6)*m^2 - 24*((a^3*b^4 + 3*a*b^6)*m^4 - 6*(a^3*b^4 - 5*a*b^6)*m^3 + (15*a^5*
b^2 - 55*a^3*b^4 + 84*a*b^6)*m^2 + 3*(5*a^5*b^2 - 16*a^3*b^4 + 19*a*b^6)*m)
*cos(d*x + c)^2 - 192*(3*a^5*b^2 - 13*a^3*b^4 + 32*a*b^6)*m - (2304*b^7 + (
b^7*m^6 + 21*b^7*m^5 + 175*b^7*m^4 + 735*b^7*m^3 + 1624*b^7*m^2 + 1764*b^7*
m + 720*b^7)*cos(d*x + c)^6 + 6*(144*b^7 + (a^2*b^5 + b^7)*m^5 + 2*(5*a^2*b
^5 + 8*b^7)*m^4 + 5*(7*a^2*b^5 + 19*b^7)*m^3 + 10*(5*a^2*b^5 + 26*b^7)*m^2
+ 12*(2*a^2*b^5 + 27*b^7)*m)*cos(d*x + c)^4 + 48*(a^4*b^3 + 6*a^2*b^5 + b^7
)*m^3 - 576*(a^4*b^3 - 4*a^2*b^5 - b^7)*m^2 + 24*(48*b^7 + (3*a^2*b^5 + b^7
)*m^4 - (5*a^4*b^3 - 24*a^2*b^5 - 13*b^7)*m^3 - (15*a^4*b^3 - 51*a^2*b^5 -
56*b^7)*m^2 - 2*(5*a^4*b^3 - 15*a^2*b^5 - 46*b^7)*m)*cos(d*x + c)^2 + 48*(1
5*a^6*b - 58*a^4*b^3 + 87*a^2*b^5 + 44*b^7)*m)*sin(d*x + c))*(b*sin(d*x + c
) + a)^m/(b^7*d*m^7 + 28*b^7*d*m^6 + 322*b^7*d*m^5 + 1960*b^7*d*m^4 + 6769*
b^7*d*m^3 + 13132*b^7*d*m^2 + 13068*b^7*d*m + 5040*b^7*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*(a+b*sin(d*x+c))**m,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 3579 vs. $2(254) = 508$.
time = 4.22, size = 3579, normalized size = 14.09

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+b*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] -((b*sin(d*x + c) + a)^7*(b*sin(d*x + c) + a)^m*m^6 - 6*(b*sin(d*x + c) + a
)^6*(b*sin(d*x + c) + a)^m*a*m^6 + 15*(b*sin(d*x + c) + a)^5*(b*sin(d*x + c
) + a)^m*a^2*m^6 - 20*(b*sin(d*x + c) + a)^4*(b*sin(d*x + c) + a)^m*a^3*m^6
 + 15*(b*sin(d*x + c) + a)^3*(b*sin(d*x + c) + a)^m*a^4*m^6 - 6*(b*sin(d*x
+ c) + a)^2*(b*sin(d*x + c) + a)^m*a^5*m^6 + (b*sin(d*x + c) + a)*(b*sin(d*
x + c) + a)^m*a^6*m^6 - 3*(b*sin(d*x + c) + a)^5*(b*sin(d*x + c) + a)^m*b^2
*m^6 + 12*(b*sin(d*x + c) + a)^4*(b*sin(d*x + c) + a)^m*a*b^2*m^6 - 18*(b*s
in(d*x + c) + a)^3*(b*sin(d*x + c) + a)^m*a^2*b^2*m^6 + 12*(b*sin(d*x + c)
+ a)^2*(b*sin(d*x + c) + a)^m*a^3*b^2*m^6 - 3*(b*sin(d*x + c) + a)*(b*sin(d
*x + c) + a)^m*a^4*b^2*m^6 + 3*(b*sin(d*x + c) + a)^3*(b*sin(d*x + c) + a)^
```

m*b^4*m^6 - 6*(b*sin(d*x + c) + a)^2*(b*sin(d*x + c) + a)^m*a*b^4*m^6 + 3*(
b*sin(d*x + c) + a)*(b*sin(d*x + c) + a)^m*a^2*b^4*m^6 - (b*sin(d*x + c) +
a)*(b*sin(d*x + c) + a)^m*b^6*m^6 + 21*(b*sin(d*x + c) + a)^7*(b*sin(d*x +
c) + a)^m*m^5 - 132*(b*sin(d*x + c) + a)^6*(b*sin(d*x + c) + a)^m*a*m^5 + 3
45*(b*sin(d*x + c) + a)^5*(b*sin(d*x + c) + a)^m*a^2*m^5 - 480*(b*sin(d*x +
 c) + a)^4*(b*sin(d*x + c) + a)^m*a^3*m^5 + 375*(b*sin(d*x + c) + a)^3*(b*s
in(d*x + c) + a)^m*a^4*m^5 - 156*(b*sin(d*x + c) + a)^2*(b*sin(d*x + c) + a
)^m*a^5*m^5 + 27*(b*sin(d*x + c) + a)*(b*sin(d*x + c) + a)^m*a^6*m^5 - 69*(
b*sin(d*x + c) + a)^5*(b*sin(d*x + c) + a)^m*b^2*m^5 + 288*(b*sin(d*x + c)
+ a)^4*(b*sin(d*x + c) + a)^m*a*b^2*m^5 - 450*(b*sin(d*x + c) + a)^3*(b*sin
(d*x + c) + a)^m*a^2*b^2*m^5 + 312*(b*sin(d*x + c) + a)^2*(b*sin(d*x + c) +
 a)^m*a^3*b^2*m^5 - 81*(b*sin(d*x + c) + a)*(b*sin(d*x + c) + a)^m*a^4*b^2*
m^5 + 75*(b*sin(d*x + c) + a)^3*(b*sin(d*x + c) + a)^m*b^4*m^5 - 156*(b*sin
(d*x + c) + a)^2*(b*sin(d*x + c) + a)^m*a*b^4*m^5 + 81*(b*sin(d*x + c) + a)
*(b*sin(d*x + c) + a)^m*a^2*b^4*m^5 - 27*(b*sin(d*x + c) + a)*(b*sin(d*x +
c) + a)^m*b^6*m^5 + 175*(b*sin(d*x + c) + a)^7*(b*sin(d*x + c) + a)^m*m^4 -
 1140*(b*sin(d*x + c) + a)^6*(b*sin(d*x + c) + a)^m*a*m^4 + 3105*(b*sin(d*x
 + c) + a)^5*(b*sin(d*x + c) + a)^m*a^2*m^4 - 4520*(b*sin(d*x + c) + a)^4*(
b*sin(d*x + c) + a)^m*a^3*m^4 + 3705*(b*sin(d*x + c) + a)^3*(b*sin(d*x + c)
 + a)^m*a^4*m^4 - 1620*(b*sin(d*x + c) + a)^2*(b*sin(d*x + c) + a)^m*a^5*m^
4 + 295*(b*sin(d*x + c) + a)*(b*sin(d*x + c) + a)^m*a^6*m^4 - 621*(b*sin(d*
x + c) + a)^5*(b*sin(d*x + c) + a)^m*b^2*m^4 + 2712*(b*sin(d*x + c) + a)^4*
(b*sin(d*x + c) + a)^m*a*b^2*m^4 - 4446*(b*sin(d*x + c) + a)^3*(b*sin(d*x +
 c) + a)^m*a^2*b^2*m^4 + 3240*(b*sin(d*x + c) + a)^2*(b*sin(d*x + c) + a)^m
*a^3*b^2*m^4 - 885*(b*sin(d*x + c) + a)*(b*sin(d*x + c) + a)^m*a^4*b^2*m^4
+ 741*(b*sin(d*x + c) + a)^3*(b*sin(d*x + c) + a)^m*b^4*m^4 - 1620*(b*sin(d
x + c) + a)^2(b*sin(d*x + c) + a)^m*a*b^4*m^4 + 885*(b*sin(d*x + c) + a)*
(b*sin(d*x + c) + a)^m*a^2*b^4*m^4 - 295*(b*sin(d*x + c) + a)*(b*sin(d*x +
c) + a)^m*b^6*m^4 + 735*(b*sin(d*x + c) + a)^7*(b*sin(d*x + c) + a)^m*m^3 -
 4920*(b*sin(d*x + c) + a)^6*(b*sin(d*x + c) + a)^m*a*m^3 + 13875*(b*sin(d*
x + c) + a)^5*(b*sin(d*x + c) + a)^m*a^2*m^3 - 21120*(b*sin(d*x + c) + a)^4
*(b*sin(d*x + c) + a)^m*a^3*m^3 + 18285*(b*sin(d*x + c) + a)^3*(b*sin(d*x +
 c) + a)^m*a^4*m^3 - 8520*(b*sin(d*x + c) + a)^2*(b*sin(d*x + c) + a)^m*a^5
m^3 + 1665(b*sin(d*x + c) + a)*(b*sin(d*x + c) + a)^m*a^6*m^3 - 2775*(b*s
in(d*x + c) + a)^5*(b*sin(d*x + c) + a)^m*b^2*m^3 + 12672*(b*sin(d*x + c) +
 a)^4*(b*sin(d*x + c) + a)^m*a*b^2*m^3 - 21942*(b*sin(d*x + c) + a)^3*(b*si
n(d*x + c) + a)^m*a^2*b^2*m^3 + 17040*(b*sin(d*x + c) + a)^2*(b*sin(d*x + c
) + a)^m*a^3*b^2*m^3 - 4995*(b*sin(d*x + c) + a)*(b*sin(d*x + c) + a)^m*a^4
*b^2*m^3 + 3657*(b*sin(d*x + c) + a)^3*(b*sin(d*x + c) + a)^m*b^4*m^3 - 852
0*(b*sin(d*x + c) + a)^2*(b*sin(d*x + c) + a)^m*a*b^4*m^3 + 4995*(b*sin(d*x
 + c) + a)*(b*sin(d*x + c) + a)^m*a^2*b^4*m^3 - 1665*(b*sin(d*x + c) + a)*(
b*sin(d*x + c) + a)^m*b^6*m^3 + 1624*(b*sin(d*x + c) + a)^7*(b*sin(d*x + c)
 + a)^m*m^2 - 11094*(b*sin(d*x + c) + a)^6*(b*sin(d*x + c) + a)^m*a*m^2 + 3
2160*(b*sin(d*x + c) + a)^5*(b*sin(d*x + c) + a)^m*a^2*m^2 - 50900*(b*sin(d
x + c) + a)^4(b*sin(d*x + c) + a)^m*a^3*m^2 + 46680*(b*sin(d*x + c) + a)^

```
3*(b*sin(d*x + c) + a)^m*a^4*m^2 - 23574*(b*sin(d*x + c) + a)^2*(b*sin(d*x
+ c) + a)^m*a^5*m^2 + 5104*(b*sin(d*x + c) + a)*(b*sin(d*x + c) + a)^m*a^6*
m^2 - 6432*(b*sin(d*x + c) + a)^5*(b*sin(d*x + c) + a)^m*b^2*m^2 + 30540*(b
*sin(d*x + c) + a)^4*(b*sin(d*x + c) + a)^m*a*b^2*m^2 - 56016*(b*sin(d*x +
c) + a)^3*(b*sin(d*x + c) + a)^m*a^2*b^2*m^2 + 47148*(b*sin(d*x + c) + a)^2
*(b*sin(d*x + c) + a)^m*a^3*b^2*m^2 - 15312*(b*sin(d*x + c) + a)*(b*sin(d*x
 + c) + a)^m*a^4*b^2*m^2 + 9336*(b*sin(d*x + c) + a)^3*(b*sin(d*x + c) + a)
^m*b^4*m^2 - 23574*(b*sin(d*x + c) + a)^2*(b*sin(d*x + c) + a)^m*a*b^4*m^2
+ 15312*(b*sin(d*x + c) + a)*(b*sin(d*x + c) + a)^m*a^2*b^4*m^2 - 5104*(b*s
in(d*x + c) + a)*(b*sin(d*x + c) + a)^m*b^6*m^2 + 1764*(b*sin(d*x + c) + a)
^7*(b*sin(d*x + c) + a)^m*m - 12228*(b*sin(d*x + c) + a)^6*(b*sin(d*x + c)
+ a)^m*a*m + 36180*(b*sin(d*x + c) + a)^5*(b*si...
```

Mupad [B]
time = 19.09, size = 1196, normalized size = 4.71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7*(a + b*sin(c + d*x))^m,x)
```

```
[Out] ((a + b*sin(c + d*x))^m*(a*b^6*645120i - a^7*92160i - a^3*b^4*645120i + a^5
*b^2*387072i - a^3*b^4*m*401856i + a^5*b^2*m*96768i + a*b^6*m^2*436336i + a
*b^6*m^3*105000i + a*b^6*m^4*14632i + a*b^6*m^5*1176i + a*b^6*m^6*40i - a^3
*b^4*m^2*26592i - a^5*b^2*m^2*13824i + a^3*b^4*m^3*6720i + a^3*b^4*m^4*96i
+ a*b^6*m*897792i))/(128*b^7*d*(m*13068i + m^2*13132i + m^3*6769i + m^4*196
0i + m^5*322i + m^6*28i + m^7*1i + 5040i)) + (sin(7*c + 7*d*x)*(a + b*sin(c
 + d*x))^m*(1764*m + 1624*m^2 + 735*m^3 + 175*m^4 + 21*m^5 + m^6 + 720)*1i)
/(64*d*(m*13068i + m^2*13132i + m^3*6769i + m^4*1960i + m^5*322i + m^6*28i
+ m^7*1i + 5040i)) + (sin(c + d*x)*(a + b*sin(c + d*x))^m*(194868*b^7*m + 1
76400*b^7 + 78968*b^7*m^2 + 16299*b^7*m^3 + 2027*b^7*m^4 + 153*b^7*m^5 + 5*
b^7*m^6 + 279936*a^2*b^5*m - 182016*a^4*b^3*m + 169440*a^2*b^5*m^2 - 42624*
a^4*b^3*m^2 + 29328*a^2*b^5*m^3 + 1152*a^4*b^3*m^3 + 1632*a^2*b^5*m^4 + 48*
a^2*b^5*m^5 + 46080*a^6*b*m)*1i)/(64*b^7*d*(m*13068i + m^2*13132i + m^3*676
9i + m^4*1960i + m^5*322i + m^6*28i + m^7*1i + 5040i)) + (sin(3*c + 3*d*x)*
(a + b*sin(c + d*x))^m*(3*m + m^2 + 2)*(3602*b^4*m - 640*a^4*m + 5880*b^4 +
 797*b^4*m^2 + 78*b^4*m^3 + 3*b^4*m^4 + 2208*a^2*b^2*m + 552*a^2*b^2*m^2 +
24*a^2*b^2*m^3)*3i)/(64*b^4*d*(m*13068i + m^2*13132i + m^3*6769i + m^4*1960
i + m^5*322i + m^6*28i + m^7*1i + 5040i)) + (sin(5*c + 5*d*x)*(a + b*sin(c
+ d*x))^m*(24*a^2*m + 79*b^2*m + 294*b^2 + 5*b^2*m^2)*(50*m + 35*m^2 + 10*m
^3 + m^4 + 24)*1i)/(64*b^2*d*(m*13068i + m^2*13132i + m^3*6769i + m^4*1960i
 + m^5*322i + m^6*28i + m^7*1i + 5040i)) + (a*m*cos(6*c + 6*d*x)*(a + b*sin
(c + d*x))^m*(m*274i + m^2*225i + m^3*85i + m^4*15i + m^5*1i + 120i))/(32*b
*d*(m*13068i + m^2*13132i + m^3*6769i + m^4*1960i + m^5*322i + m^6*28i + m^
7*1i + 5040i)) + (3*a*m*cos(4*c + 4*d*x)*(a + b*sin(c + d*x))^m*(b^2*m*17i
```

```
- a^2*20i + b^2*64i + b^2*m^2*1i)*(11*m + 6*m^2 + m^3 + 6))/(16*b^3*d*(m*13
068i + m^2*13132i + m^3*6769i + m^4*1960i + m^5*322i + m^6*28i + m^7*1i + 5
040i)) + (3*a*m*cos(2*c + 2*d*x)*(m + 1)*(a + b*sin(c + d*x))^m*(b^4*m*6370
i + a^4*1920i + b^4*10008i - a^2*b^2*7104i + b^4*m^2*1411i + b^4*m^3*134i +
 b^4*m^4*5i - a^2*b^2*m*1696i - a^2*b^2*m^2*32i))/(32*b^5*d*(m*13068i + m^2
*13132i + m^3*6769i + m^4*1960i + m^5*322i + m^6*28i + m^7*1i + 5040i))
```

3.631 $\int \cos^5(c+dx)(a+b\sin(c+dx))^m\,dx$

Optimal. Leaf size=167

$$\frac{(a^2-b^2)^2\,(a+b\sin(c+dx))^{1+m}}{b^5d(1+m)}-\frac{4a(a^2-b^2)\,(a+b\sin(c+dx))^{2+m}}{b^5d(2+m)}+\frac{2(3a^2-b^2)\,(a+b\sin(c+dx))^{3+m}}{b^5d(3+m)}-$$

```
[Out] (a^2-b^2)^2*(a+b*sin(d*x+c))^(1+m)/b^5/d/(1+m)-4*a*(a^2-b^2)*(a+b*sin(d*x+c
))^(2+m)/b^5/d/(2+m)+2*(3*a^2-b^2)*(a+b*sin(d*x+c))^(3+m)/b^5/d/(3+m)-4*a*(
a+b*sin(d*x+c))^(4+m)/b^5/d/(4+m)+(a+b*sin(d*x+c))^(5+m)/b^5/d/(5+m)
```

Rubi [A]
time = 0.08, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2747, 711}

$$\frac{(a^2-b^2)^2\,(a+b\sin(c+dx))^{m+1}}{b^5d(m+1)}-\frac{4a(a^2-b^2)\,(a+b\sin(c+dx))^{m+2}}{b^5d(m+2)}+\frac{2(3a^2-b^2)\,(a+b\sin(c+dx))^{m+3}}{b^5d(m+3)}-\frac{4a(a+b\sin(c+dx))^{m+4}}{b^5d(m+4)}+\frac{(a+b\sin(c+dx))^{m+5}}{b^5d(m+5)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] ((a^2 - b^2)^2*(a + b*Sin[c + d*x])^(1 + m))/(b^5*d*(1 + m)) - (4*a*(a^2 -
b^2)*(a + b*Sin[c + d*x])^(2 + m))/(b^5*d*(2 + m)) + (2*(3*a^2 - b^2)*(a +
b*Sin[c + d*x])^(3 + m))/(b^5*d*(3 + m)) - (4*a*(a + b*Sin[c + d*x])^(4 + m
))/(b^5*d*(4 + m)) + (a + b*Sin[c + d*x])^(5 + m)/(b^5*d*(5 + m))
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^5(c+dx)(a+b\sin(c+dx))^m\,dx = \frac{\text{Subst}\left(\int (a+x)^m\left(b^2-x^2\right)^2\,dx, x, b\sin(c+dx)\right)}{b^5d}$$

$$= \frac{\text{Subst}\left(\int \left(\left(a^2-b^2\right)^2(a+x)^m - 4\left(a^3-ab^2\right)(a+x)^{1+m} + 2(3a^2 -\right.\right.}{}$$

$$= \frac{\left(a^2-b^2\right)^2(a+b\sin(c+dx))^{1+m}}{b^5d(1+m)} - \frac{4a\left(a^2-b^2\right)(a+b\sin(c+dx)}{b^5d(2+m)}$$

Mathematica [A]
time = 0.95, size = 330, normalized size = 1.98

$$\frac{(a+b\sin(c+dx))^{1+m}\left(192a^4-544a^2b^2+712b^4-144a^2b^2m+758b^4m+16a^2b^2m^2+281b^4m^2+46b^4m^3+3b^4m^4+4b^2(2+3m+m^2)\left(-12a^2+b^2(28+11m+m^2)\right)\cos(2(c+dx))+b^4(24+50m+35m^2+10m^3+m^4)\cos(4(c+dx))-192a^3b\sin(c+dx)+496ab^3\sin(c+dx)-192a^3bm\sin(c+dx)+664ab^3m\sin(c+dx)+176ab^3m^2\sin(c+dx)+8ab^3m^3\sin(c+dx)+48ab^3\sin(3(c+dx))+88ab^3m\sin(3(c+dx))+48ab^3m^2\sin(3(c+dx))+8ab^3m^3\sin(3(c+dx))\right)}{8b^5d(1+m)(2+m)(3+m)(4+m)(5+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] ((a + b*Sin[c + d*x])^(1 + m)*(192*a^4 - 544*a^2*b^2 + 712*b^4 - 144*a^2*b^
2*m + 758*b^4*m + 16*a^2*b^2*m^2 + 281*b^4*m^2 + 46*b^4*m^3 + 3*b^4*m^4 + 4
*b^2*(2 + 3*m + m^2)*(-12*a^2 + b^2*(28 + 11*m + m^2))*Cos[2*(c + d*x)] + b
^4*(24 + 50*m + 35*m^2 + 10*m^3 + m^4)*Cos[4*(c + d*x)] - 192*a^3*b*Sin[c +
 d*x] + 496*a*b^3*Sin[c + d*x] - 192*a^3*b*m*Sin[c + d*x] + 664*a*b^3*m*Sin
[c + d*x] + 176*a*b^3*m^2*Sin[c + d*x] + 8*a*b^3*m^3*Sin[c + d*x] + 48*a*b^
3*Sin[3*(c + d*x)] + 88*a*b^3*m*Sin[3*(c + d*x)] + 48*a*b^3*m^2*Sin[3*(c +
d*x)] + 8*a*b^3*m^3*Sin[3*(c + d*x)]))/(8*b^5*d*(1 + m)*(2 + m)*(3 + m)*(4
+ m)*(5 + m))
```

Maple [F]
time = 0.15, size = 0, normalized size = 0.00

$$\int \left(\cos^5(dx+c)\right)(a+b\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+b*sin(d*x+c))^m,x)
```

```
[Out] int(cos(d*x+c)^5*(a+b*sin(d*x+c))^m,x)
```

Maxima [A]
time = 0.30, size = 286, normalized size = 1.71

$$\frac{\frac{(b\sin(dx+c)+a)^{m+1}}{b(m+1)} - \frac{2\left((m^2+3m+2)b^3\sin(dx+c)^3+(m^2+m)ab^2\sin(dx+c)^2-2a^2bm\sin(dx+c)+2a^3\right)(b\sin(dx+c)+a)^m}{(m^3+6m^2+11m+6)b^3} + \frac{\left((m^4+10m^3+35m^2+50m+24)b^5\sin(dx+c)^5+(m^4+6m^3+11m^2+6m)ab^4\sin(dx+c)^4-4(m^3+3m^2+2m)a^2b^3\sin(dx+c)^3+12(m^2+m)a^3b^2\sin(dx+c)^2-24a^4bm\sin(dx+c)+24a^5\right)(b\sin(dx+c)+a)^m}{(m^5+15m^4+85m^3+225m^2+274m+120)b^5}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] ((b*sin(d*x + c) + a)^(m + 1)/(b*(m + 1)) - 2*((m^2 + 3*m + 2)*b^3*sin(d*x
+ c)^3 + (m^2 + m)*a*b^2*sin(d*x + c)^2 - 2*a^2*b*m*sin(d*x + c) + 2*a^3)*(
b*sin(d*x + c) + a)^m/((m^3 + 6*m^2 + 11*m + 6)*b^3) + ((m^4 + 10*m^3 + 35*
m^2 + 50*m + 24)*b^5*sin(d*x + c)^5 + (m^4 + 6*m^3 + 11*m^2 + 6*m)*a*b^4*si
n(d*x + c)^4 - 4*(m^3 + 3*m^2 + 2*m)*a^2*b^3*sin(d*x + c)^3 + 12*(m^2 + m)*
a^3*b^2*sin(d*x + c)^2 - 24*a^4*b*m*sin(d*x + c) + 24*a^5)*(b*sin(d*x + c)
+ a)^m/((m^5 + 15*m^4 + 85*m^3 + 225*m^2 + 274*m + 120)*b^5))/d
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 381 vs. $2(167) = 334$.
time = 0.41, size = 381, normalized size = 2.28

$$\frac{\left(24\,a^5-80\,a^3b^2+120\,ab^4+\left(ab^4m^4+6\,ab^4m^3+11\,ab^4m^2+6\,ab^4m\right)\cos\left(dx+c\right)^4+8\left(a^3b^2+3\,ab^4\right)m^2+4\left(2\,ab^4m^3-3\left(a^3b^2-3\,ab^4\right)m^2-\left(3\,a^3b^2-7\,ab^4\right)m\right)\cos\left(dx+c\right)^2-24\left(a^3b^2-5\,ab^4\right)m+\left(64\,b^5+\left(b^5m^4+10\,b^5m^3+35\,b^5m^2+50\,b^5m+24\,b^5\right)\cos\left(dx+c\right)^4+8\left(3\,a^2b^3+b^5\right)m^2+4\left(8\,b^5+\left(a^2b^3+b^5\right)m^3+\left(3\,a^2b^3+7\,b^5\right)m^2+2\left(a^2b^3+7\,b^5\right)m\right)\cos\left(dx+c\right)^2-24\left(a^4b-3\,a^2b^3-2\,b^5\right)m\right)\sin\left(dx+c\right)\right)\left(b\sin\left(dx+c\right)+a\right)^m}{b^5dm^5+15\,b^5dm^4+85\,b^5dm^3+225\,b^5dm^2+274\,b^5dm+120\,b^5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] (24*a^5 - 80*a^3*b^2 + 120*a*b^4 + (a*b^4*m^4 + 6*a*b^4*m^3 + 11*a*b^4*m^2
+ 6*a*b^4*m)*cos(d*x + c)^4 + 8*(a^3*b^2 + 3*a*b^4)*m^2 + 4*(2*a*b^4*m^3 -
3*(a^3*b^2 - 3*a*b^4)*m^2 - (3*a^3*b^2 - 7*a*b^4)*m)*cos(d*x + c)^2 - 24*(a
^3*b^2 - 5*a*b^4)*m + (64*b^5 + (b^5*m^4 + 10*b^5*m^3 + 35*b^5*m^2 + 50*b^5
*m + 24*b^5)*cos(d*x + c)^4 + 8*(3*a^2*b^3 + b^5)*m^2 + 4*(8*b^5 + (a^2*b^3
 + b^5)*m^3 + (3*a^2*b^3 + 7*b^5)*m^2 + 2*(a^2*b^3 + 7*b^5)*m)*cos(d*x + c)
^2 - 24*(a^4*b - 3*a^2*b^3 - 2*b^5)*m)*sin(d*x + c))*(b*sin(d*x + c) + a)^m
/(b^5*d*m^5 + 15*b^5*d*m^4 + 85*b^5*d*m^3 + 225*b^5*d*m^2 + 274*b^5*d*m + 1
20*b^5*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+b*sin(d*x+c))**m,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1410 vs. $2(167) = 334$.
time = 4.55, size = 1410, normalized size = 8.44

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^5*(a+b*sin(d*x+c))^m,x, algorithm="giac")

[Out] ((b*sin(d*x + c) + a)^5*(b*sin(d*x + c) + a)^m*m^4 - 4*(b*sin(d*x + c) + a)
^4*(b*sin(d*x + c) + a)^m*a*m^4 + 6*(b*sin(d*x + c) + a)^3*(b*sin(d*x + c)
+ a)^m*a^2*m^4 - 4*(b*sin(d*x + c) + a)^2*(b*sin(d*x + c) + a)^m*a^3*m^4 +
(b*sin(d*x + c) + a)*(b*sin(d*x + c) + a)^m*a^4*m^4 - 2*(b*sin(d*x + c) + a
)^3*(b*sin(d*x + c) + a)^m*b^2*m^4 + 4*(b*sin(d*x + c) + a)^2*(b*sin(d*x +
c) + a)^m*a*b^2*m^4 - 2*(b*sin(d*x + c) + a)*(b*sin(d*x + c) + a)^m*a^2*b^2
*m^4 + (b*sin(d*x + c) + a)*(b*sin(d*x + c) + a)^m*b^4*m^4 + 10*(b*sin(d*x
+ c) + a)^5*(b*sin(d*x + c) + a)^m*m^3 - 44*(b*sin(d*x + c) + a)^4*(b*sin(d
*x + c) + a)^m*a*m^3 + 72*(b*sin(d*x + c) + a)^3*(b*sin(d*x + c) + a)^m*a^2
m^3 - 52(b*sin(d*x + c) + a)^2*(b*sin(d*x + c) + a)^m*a^3*m^3 + 14*(b*sin
(d*x + c) + a)*(b*sin(d*x + c) + a)^m*a^4*m^3 - 24*(b*sin(d*x + c) + a)^3*(
b*sin(d*x + c) + a)^m*b^2*m^3 + 52*(b*sin(d*x + c) + a)^2*(b*sin(d*x + c) +
a)^m*a*b^2*m^3 - 28*(b*sin(d*x + c) + a)*(b*sin(d*x + c) + a)^m*a^2*b^2*m^
3 + 14*(b*sin(d*x + c) + a)*(b*sin(d*x + c) + a)^m*b^4*m^3 + 35*(b*sin(d*x
+ c) + a)^5*(b*sin(d*x + c) + a)^m*m^2 - 164*(b*sin(d*x + c) + a)^4*(b*sin(
d*x + c) + a)^m*a*m^2 + 294*(b*sin(d*x + c) + a)^3*(b*sin(d*x + c) + a)^m*a
^2*m^2 - 236*(b*sin(d*x + c) + a)^2*(b*sin(d*x + c) + a)^m*a^3*m^2 + 71*(b*
sin(d*x + c) + a)*(b*sin(d*x + c) + a)^m*a^4*m^2 - 98*(b*sin(d*x + c) + a)^
3*(b*sin(d*x + c) + a)^m*b^2*m^2 + 236*(b*sin(d*x + c) + a)^2*(b*sin(d*x +
c) + a)^m*a*b^2*m^2 - 142*(b*sin(d*x + c) + a)*(b*sin(d*x + c) + a)^m*a^2*b
^2*m^2 + 71*(b*sin(d*x + c) + a)*(b*sin(d*x + c) + a)^m*b^4*m^2 + 50*(b*sin
(d*x + c) + a)^5*(b*sin(d*x + c) + a)^m*m - 244*(b*sin(d*x + c) + a)^4*(b*s
in(d*x + c) + a)^m*a*m + 468*(b*sin(d*x + c) + a)^3*(b*sin(d*x + c) + a)^m*
a^2*m - 428*(b*sin(d*x + c) + a)^2*(b*sin(d*x + c) + a)^m*a^3*m + 154*(b*si
n(d*x + c) + a)*(b*sin(d*x + c) + a)^m*a^4*m - 156*(b*sin(d*x + c) + a)^3*(
b*sin(d*x + c) + a)^m*b^2*m + 428*(b*sin(d*x + c) + a)^2*(b*sin(d*x + c) +
a)^m*a*b^2*m - 308*(b*sin(d*x + c) + a)*(b*sin(d*x + c) + a)^m*a^2*b^2*m +
154*(b*sin(d*x + c) + a)*(b*sin(d*x + c) + a)^m*b^4*m + 24*(b*sin(d*x + c)
+ a)^5*(b*sin(d*x + c) + a)^m - 120*(b*sin(d*x + c) + a)^4*(b*sin(d*x + c)
+ a)^m*a + 240*(b*sin(d*x + c) + a)^3*(b*sin(d*x + c) + a)^m*a^2 - 240*(b*s
in(d*x + c) + a)^2*(b*sin(d*x + c) + a)^m*a^3 + 120*(b*sin(d*x + c) + a)*(b
*sin(d*x + c) + a)^m*a^4 - 80*(b*sin(d*x + c) + a)^3*(b*sin(d*x + c) + a)^m
b^2 + 240(b*sin(d*x + c) + a)^2*(b*sin(d*x + c) + a)^m*a*b^2 - 240*(b*sin
(d*x + c) + a)*(b*sin(d*x + c) + a)^m*a^2*b^2 + 120*(b*sin(d*x + c) + a)*(b
*sin(d*x + c) + a)^m*b^4)/((b^4*m^5 + 15*b^4*m^4 + 85*b^4*m^3 + 225*b^4*m^2
+ 274*b^4*m + 120*b^4)*b*d)

Mupad [B]
time = 11.62, size = 641, normalized size = 3.84

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^5*(a + b*sin(c + d*x))^m,x)

[Out] ((a + b*sin(c + d*x))^m*(1920*a*b^4 + 1200*b^5*sin(c + d*x) + 384*a^5 - 128
0*a^3*b^2 + 200*b^5*sin(3*c + 3*d*x) + 24*b^5*sin(5*c + 5*d*x) - 480*a^3*b^
2*m + 738*a*b^4*m^2 + 100*a*b^4*m^3 + 6*a*b^4*m^4 + 374*b^5*m*sin(3*c + 3*d
*x) + 50*b^5*m*sin(5*c + 5*d*x) + 310*b^5*m^2*sin(c + d*x) + 36*b^5*m^3*sin
(c + d*x) + 2*b^5*m^4*sin(c + d*x) + 32*a^3*b^2*m^2 + 217*b^5*m^2*sin(3*c +
 3*d*x) + 46*b^5*m^3*sin(3*c + 3*d*x) + 3*b^5*m^4*sin(3*c + 3*d*x) + 35*b^5
*m^2*sin(5*c + 5*d*x) + 10*b^5*m^3*sin(5*c + 5*d*x) + b^5*m^4*sin(5*c + 5*d
*x) + 2180*a*b^4*m + 1092*b^5*m*sin(c + d*x) - 96*a^3*b^2*m*cos(2*c + 2*d*x
) + 376*a*b^4*m^2*cos(2*c + 2*d*x) + 112*a*b^4*m^3*cos(2*c + 2*d*x) + 8*a*b
^4*m^4*cos(2*c + 2*d*x) + 22*a*b^4*m^2*cos(4*c + 4*d*x) + 12*a*b^4*m^3*cos(
4*c + 4*d*x) + 2*a*b^4*m^4*cos(4*c + 4*d*x) + 32*a^2*b^3*m*sin(3*c + 3*d*x)
 + 432*a^2*b^3*m^2*sin(c + d*x) + 16*a^2*b^3*m^3*sin(c + d*x) - 384*a^4*b*m
*sin(c + d*x) - 96*a^3*b^2*m^2*cos(2*c + 2*d*x) + 48*a^2*b^3*m^2*sin(3*c +
3*d*x) + 16*a^2*b^3*m^3*sin(3*c + 3*d*x) + 272*a*b^4*m*cos(2*c + 2*d*x) + 1
2*a*b^4*m*cos(4*c + 4*d*x) + 1184*a^2*b^3*m*sin(c + d*x)))/(16*b^5*d*(274*m
 + 225*m^2 + 85*m^3 + 15*m^4 + m^5 + 120))

3.632 $\int \cos^3(c+dx)(a+b\sin(c+dx))^m\,dx$

Optimal. Leaf size=92

$$-\frac{(a^2-b^2)\,(a+b\sin(c+dx))^{1+m}}{b^3d(1+m)}+\frac{2a(a+b\sin(c+dx))^{2+m}}{b^3d(2+m)}-\frac{(a+b\sin(c+dx))^{3+m}}{b^3d(3+m)}$$

```
[Out] -(a^2-b^2)*(a+b*sin(d*x+c))^(1+m)/b^3/d/(1+m)+2*a*(a+b*sin(d*x+c))^(2+m)/b^
3/d/(2+m)-(a+b*sin(d*x+c))^(3+m)/b^3/d/(3+m)
```

Rubi [A]
time = 0.06, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2747, 711}

$$-\frac{(a^2-b^2)\,(a+b\sin(c+dx))^{m+1}}{b^3d(m+1)}+\frac{2a(a+b\sin(c+dx))^{m+2}}{b^3d(m+2)}-\frac{(a+b\sin(c+dx))^{m+3}}{b^3d(m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] -(((a^2 - b^2)*(a + b*Sin[c + d*x])^(1 + m))/(b^3*d*(1 + m))) + (2*a*(a + b
*Sin[c + d*x])^(2 + m))/(b^3*d*(2 + m)) - (a + b*Sin[c + d*x])^(3 + m)/(b^3
*d*(3 + m))
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^3(c+dx)(a+b\sin(c+dx))^m\,dx &= \frac{\text{Subst}\left(\int (a+x)^m\,(b^2-x^2)\,dx,x,b\sin(c+dx)\right)}{b^3d}\\ &= \frac{\text{Subst}\left(\int \left((-a^2+b^2)\,(a+x)^m+2a(a+x)^{1+m}-(a+x)^{2+m}\right)\,dx,\right.}{b^3d}\\ &= -\frac{(a^2-b^2)\,(a+b\sin(c+dx))^{1+m}}{b^3d(1+m)}+\frac{2a(a+b\sin(c+dx))^{2+m}}{b^3d(2+m)}-\end{aligned}$$

Mathematica [A]
time = 0.27, size = 96, normalized size = 1.04

$$\frac{(a+b\sin(c+dx))^{1+m}\left(-4a^2+10b^2+7b^2m+b^2m^2+b^2(2+3m+m^2)\cos(2(c+dx))+4ab(1+m)\sin(c+dx)\right)}{2b^3d(1+m)(2+m)(3+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] ((a + b*Sin[c + d*x])^(1 + m)*(-4*a^2 + 10*b^2 + 7*b^2*m + b^2*m^2 + b^2*(2
 + 3*m + m^2)*Cos[2*(c + d*x)] + 4*a*b*(1 + m)*Sin[c + d*x]))/(2*b^3*d*(1 +
 m)*(2 + m)*(3 + m))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \left(\cos^3(dx+c)\right)(a+b\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*sin(d*x+c))^m,x)
```

```
[Out] int(cos(d*x+c)^3*(a+b*sin(d*x+c))^m,x)
```

Maxima [A]
time = 0.29, size = 117, normalized size = 1.27

$$\frac{\frac{(b\sin(dx+c)+a)^{m+1}}{b(m+1)}-\frac{\left((m^2+3m+2)b^3\sin(dx+c)^3+(m^2+m)ab^2\sin(dx+c)^2-2a^2bm\sin(dx+c)+2a^3\right)(b\sin(dx+c)+a)^m}{(m^3+6m^2+11m+6)b^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] ((b*sin(d*x + c) + a)^(m + 1)/(b*(m + 1)) - ((m^2 + 3*m + 2)*b^3*sin(d*x +
c)^3 + (m^2 + m)*a*b^2*sin(d*x + c)^2 - 2*a^2*b*m*sin(d*x + c) + 2*a^3)*(b*
sin(d*x + c) + a)^m/((m^3 + 6*m^2 + 11*m + 6)*b^3))/d
```

Fricas [A]
time = 0.36, size = 142, normalized size = 1.54

$$\frac{\left(4\,ab^2m-2\,a^3+6\,ab^2+(ab^2m^2+ab^2m)\cos(dx+c)^2+\left(4\,b^3+(b^3m^2+3\,b^3m+2\,b^3)\cos(dx+c)^2+2\,(a^2b+b^3)m\right)\sin(dx+c)\right)(b\sin(dx+c)+a)^m}{b^3dm^3+6\,b^3dm^2+11\,b^3dm+6\,b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] (4*a*b^2*m - 2*a^3 + 6*a*b^2 + (a*b^2*m^2 + a*b^2*m)*cos(d*x + c)^2 + (4*b^
3 + (b^3*m^2 + 3*b^3*m + 2*b^3)*cos(d*x + c)^2 + 2*(a^2*b + b^3)*m)*sin(d*x
```

```
 + c))*(b*sin(d*x + c) + a)^m/(b^3*d*m^3 + 6*b^3*d*m^2 + 11*b^3*d*m + 6*b^3
*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 2837 vs. $2(75) = 150$.
time = 204.46, size = 2837, normalized size = 30.84

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*sin(d*x+c))**m,x)
```

```
[Out] Piecewise((a**m*(2*sin(c + d*x)**3/(3*d) + sin(c + d*x)*cos(c + d*x)**2/d),
 Eq(b, 0)), (x*(a + b*sin(c))**m*cos(c)**3, Eq(d, 0)), (-2*a**2*log(a/b + s
in(c + d*x))/(2*a**2*b**3*d + 4*a*b**4*d*sin(c + d*x) + 2*b**5*d*sin(c + d*
x)**2) - 2*a**2/(2*a**2*b**3*d + 4*a*b**4*d*sin(c + d*x) + 2*b**5*d*sin(c +
 d*x)**2) - 4*a*b*log(a/b + sin(c + d*x))*sin(c + d*x)/(2*a**2*b**3*d + 4*a
*b**4*d*sin(c + d*x) + 2*b**5*d*sin(c + d*x)**2) - 2*a*b*sin(c + d*x)/(2*a*
*2*b**3*d + 4*a*b**4*d*sin(c + d*x) + 2*b**5*d*sin(c + d*x)**2) - 2*b**2*lo
g(a/b + sin(c + d*x))*sin(c + d*x)**2/(2*a**2*b**3*d + 4*a*b**4*d*sin(c + d
*x) + 2*b**5*d*sin(c + d*x)**2) - b**2*cos(c + d*x)**2/(2*a**2*b**3*d + 4*a
*b**4*d*sin(c + d*x) + 2*b**5*d*sin(c + d*x)**2), Eq(m, -3)), (2*a**2*log(a
/b + sin(c + d*x))/(a*b**3*d + b**4*d*sin(c + d*x)) + 2*a**2/(a*b**3*d + b*
*4*d*sin(c + d*x)) + 2*a*b*log(a/b + sin(c + d*x))*sin(c + d*x)/(a*b**3*d +
 b**4*d*sin(c + d*x)) - 2*b**2*sin(c + d*x)**2/(a*b**3*d + b**4*d*sin(c + d
*x)) - b**2*cos(c + d*x)**2/(a*b**3*d + b**4*d*sin(c + d*x)), Eq(m, -2)), (
a**2*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2)**4/(b**3*d*tan(c/2 + d*x
/2)**4 + 2*b**3*d*tan(c/2 + d*x/2)**2 + b**3*d) + 2*a**2*log(tan(c/2 + d*x/
2)**2 + 1)*tan(c/2 + d*x/2)**2/(b**3*d*tan(c/2 + d*x/2)**4 + 2*b**3*d*tan(c
/2 + d*x/2)**2 + b**3*d) + a**2*log(tan(c/2 + d*x/2)**2 + 1)/(b**3*d*tan(c/
2 + d*x/2)**4 + 2*b**3*d*tan(c/2 + d*x/2)**2 + b**3*d) - a**2*log(tan(c/2 +
 d*x/2) + b/a - sqrt(-a**2 + b**2)/a)*tan(c/2 + d*x/2)**4/(b**3*d*tan(c/2 +
 d*x/2)**4 + 2*b**3*d*tan(c/2 + d*x/2)**2 + b**3*d) - 2*a**2*log(tan(c/2 +
d*x/2) + b/a - sqrt(-a**2 + b**2)/a)*tan(c/2 + d*x/2)**2/(b**3*d*tan(c/2 +
d*x/2)**4 + 2*b**3*d*tan(c/2 + d*x/2)**2 + b**3*d) - a**2*log(tan(c/2 + d*x
/2) + b/a - sqrt(-a**2 + b**2)/a)/(b**3*d*tan(c/2 + d*x/2)**4 + 2*b**3*d*ta
n(c/2 + d*x/2)**2 + b**3*d) - a**2*log(tan(c/2 + d*x/2) + b/a + sqrt(-a**2
+ b**2)/a)*tan(c/2 + d*x/2)**4/(b**3*d*tan(c/2 + d*x/2)**4 + 2*b**3*d*tan(c
/2 + d*x/2)**2 + b**3*d) - 2*a**2*log(tan(c/2 + d*x/2) + b/a + sqrt(-a**2 +
 b**2)/a)*tan(c/2 + d*x/2)**2/(b**3*d*tan(c/2 + d*x/2)**4 + 2*b**3*d*tan(c/
2 + d*x/2)**2 + b**3*d) - a**2*log(tan(c/2 + d*x/2) + b/a + sqrt(-a**2 + b*
*2)/a)/(b**3*d*tan(c/2 + d*x/2)**4 + 2*b**3*d*tan(c/2 + d*x/2)**2 + b**3*d)
 + 2*a*b*tan(c/2 + d*x/2)**3/(b**3*d*tan(c/2 + d*x/2)**4 + 2*b**3*d*tan(c/2
 + d*x/2)**2 + b**3*d) + 2*a*b*tan(c/2 + d*x/2)/(b**3*d*tan(c/2 + d*x/2)**4
 + 2*b**3*d*tan(c/2 + d*x/2)**2 + b**3*d) - b**2*log(tan(c/2 + d*x/2)**2 +
```

```
1)*tan(c/2 + d*x/2)**4/(b**3*d*tan(c/2 + d*x/2)**4 + 2*b**3*d*tan(c/2 + d*x
/2)**2 + b**3*d) - 2*b**2*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2)**2/
(b**3*d*tan(c/2 + d*x/2)**4 + 2*b**3*d*tan(c/2 + d*x/2)**2 + b**3*d) - b**2
*log(tan(c/2 + d*x/2)**2 + 1)/(b**3*d*tan(c/2 + d*x/2)**4 + 2*b**3*d*tan(c/
2 + d*x/2)**2 + b**3*d) + b**2*log(tan(c/2 + d*x/2) + b/a - sqrt(-a**2 + b*
*2)/a)*tan(c/2 + d*x/2)**4/(b**3*d*tan(c/2 + d*x/2)**4 + 2*b**3*d*tan(c/2 +
 d*x/2)**2 + b**3*d) + 2*b**2*log(tan(c/2 + d*x/2) + b/a - sqrt(-a**2 + b**
2)/a)*tan(c/2 + d*x/2)**2/(b**3*d*tan(c/2 + d*x/2)**4 + 2*b**3*d*tan(c/2 +
d*x/2)**2 + b**3*d) + b**2*log(tan(c/2 + d*x/2) + b/a - sqrt(-a**2 + b**2)/
a)/(b**3*d*tan(c/2 + d*x/2)**4 + 2*b**3*d*tan(c/2 + d*x/2)**2 + b**3*d) + b
**2*log(tan(c/2 + d*x/2) + b/a + sqrt(-a**2 + b**2)/a)*tan(c/2 + d*x/2)**4/
(b**3*d*tan(c/2 + d*x/2)**4 + 2*b**3*d*tan(c/2 + d*x/2)**2 + b**3*d) + 2*b*
*2*log(tan(c/2 + d*x/2) + b/a + sqrt(-a**2 + b**2)/a)*tan(c/2 + d*x/2)**2/(
b**3*d*tan(c/2 + d*x/2)**4 + 2*b**3*d*tan(c/2 + d*x/2)**2 + b**3*d) + b**2*
log(tan(c/2 + d*x/2) + b/a + sqrt(-a**2 + b**2)/a)/(b**3*d*tan(c/2 + d*x/2)
**4 + 2*b**3*d*tan(c/2 + d*x/2)**2 + b**3*d) - 2*b**2*tan(c/2 + d*x/2)**2/(
b**3*d*tan(c/2 + d*x/2)**4 + 2*b**3*d*tan(c/2 + d*x/2)**2 + b**3*d), Eq(m,
-1)), (-2*a**3*(a + b*sin(c + d*x))**m/(b**3*d*m**3 + 6*b**3*d*m**2 + 11*b*
*3*d*m + 6*b**3*d) + 2*a**2*b*m*(a + b*sin(c + d*x))**m*sin(c + d*x)/(b**3*
d*m**3 + 6*b**3*d*m**2 + 11*b**3*d*m + 6*b**3*d) + a*b**2*m**2*(a + b*sin(c
 + d*x))**m*cos(c + d*x)**2/(b**3*d*m**3 + 6*b**3*d*m**2 + 11*b**3*d*m + 6*
b**3*d) + 4*a*b**2*m*(a + b*sin(c + d*x))**m*sin(c + d*x)**2/(b**3*d*m**3 +
 6*b**3*d*m**2 + 11*b**3*d*m + 6*b**3*d) + 5*a*b**2*m*(a + b*sin(c + d*x))*
*m*cos(c + d*x)**2/(b**3*d*m**3 + 6*b**3*d*m**2 + 11*b**3*d*m + 6*b**3*d) +
 6*a*b**2*(a + b*sin(c + d*x))**m*sin(c + d*x)**2/(b**3*d*m**3 + 6*b**3*d*m
**2 + 11*b**3*d*m + 6*b**3*d) + 6*a*b**2*(a + b*sin(c + d*x))**m*cos(c + d*
x)**2/(b**3*d*m**3 + 6*b**3*d*m**2 + 11*b**3*d*m + 6*b**3*d) + b**3*m**2*(a
 + b*sin(c + d*x))**m*sin(c + d*x)*cos(c + d*x)**2/(b**3*d*m**3 + 6*b**3*d*
m**2 + 11*b**3*d*m + 6*b**3*d) + 2*b**3*m*(a + b*sin(c + d*x))**m*sin(c + d
*x)**3/(b**3*d*m**3 + 6*b**3*d*m**2 + 11*b**3*d*m + 6*b**3*d) + 5*b**3*m*(a
 + b*sin(c + d*x))**m*sin(c + d*x)*cos(c + d*x)**2/(b**3*d*m**3 + 6*b**3*d*
m**2 + 11*b**3*d*m + 6*b**3*d) + 4*b**3*(a + b*sin(c + d*x))**m*sin(c + d*x
)**3/(b**3*d*m**3 + 6*b**3*d*m**2 + 11*b**3*d*m + 6*b**3*d) + 6*b**3*(a + b
*sin(c + d*x))**m*sin(c + d*x)*cos(c + d*x)**2/...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 373 vs. 2(92) = 184.
time = 5.01, size = 373, normalized size = 4.05

$$-\frac{(b\sin(dx+c)+a)^3(b\sin(dx+c)+a)^m m^2 - 2(b\sin(dx+c)+a)^2(b\sin(dx+c)+a)^m am^2 + (b\sin(dx+c)+a)(b\sin(dx+c)+a)^m a^2m^2 - (b\sin(dx+c)+a)(b\sin(dx+c)+a)^m b^2m^2 + 3(b\sin(dx+c)+a)^3(b\sin(dx+c)+a)^m m - 8(b\sin(dx+c)+a)^2(b\sin(dx+c)+a)^m am + 5(b\sin(dx+c)+a)(b\sin(dx+c)+a)^m a^2m - 5(b\sin(dx+c)+a)(b\sin(dx+c)+a)^m b^2m + 2(b\sin(dx+c)+a)^3(b\sin(dx+c)+a)^m - 6(b\sin(dx+c)+a)^2(b\sin(dx+c)+a)^m a + 6(b\sin(dx+c)+a)(b\sin(dx+c)+a)^m a^2 - 6(b\sin(dx+c)+a)(b\sin(dx+c)+a)^m b^2}{(b^2m^3+6b^2m^2+11b^2m+6b^2)bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] -((b*sin(d*x + c) + a)^3*(b*sin(d*x + c) + a)^m*m^2 - 2*(b*sin(d*x + c) + a
)^2*(b*sin(d*x + c) + a)^m*a*m^2 + (b*sin(d*x + c) + a)*(b*sin(d*x + c) + a
```

```
)^m*a^2*m^2 - (b*sin(d*x + c) + a)*(b*sin(d*x + c) + a)^m*b^2*m^2 + 3*(b*si
n(d*x + c) + a)^3*(b*sin(d*x + c) + a)^m*m - 8*(b*sin(d*x + c) + a)^2*(b*si
n(d*x + c) + a)^m*a*m + 5*(b*sin(d*x + c) + a)*(b*sin(d*x + c) + a)^m*a^2*m
 - 5*(b*sin(d*x + c) + a)*(b*sin(d*x + c) + a)^m*b^2*m + 2*(b*sin(d*x + c)
+ a)^3*(b*sin(d*x + c) + a)^m - 6*(b*sin(d*x + c) + a)^2*(b*sin(d*x + c) +
a)^m*a + 6*(b*sin(d*x + c) + a)*(b*sin(d*x + c) + a)^m*a^2 - 6*(b*sin(d*x +
 c) + a)*(b*sin(d*x + c) + a)^m*b^2)/((b^2*m^3 + 6*b^2*m^2 + 11*b^2*m + 6*b
^2)*b*d)
```

Mupad [B]
time = 7.51, size = 197, normalized size = 2.14

$$\frac{(a+b\sin(c+dx))^m\left(24ab^2+18b^3\sin(c+dx)-8a^3+2b^3\sin(3c+3dx)+2ab^2m^2+3b^3m\sin(3c+3dx)+b^3m^2\sin(c+dx)+b^3m^2\sin(3c+3dx)+18ab^2m+11b^3m\sin(c+dx)+8a^2bm\sin(c+dx)-2ab^2m\left(2\sin(c+dx)^2-1\right)-2ab^2m^2\left(2\sin(c+dx)^2-1\right)\right)}{4b^3d\left(m^3+6m^2+11m+6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a + b*sin(c + d*x))^m,x)
```

```
[Out] ((a + b*sin(c + d*x))^m*(24*a*b^2 + 18*b^3*sin(c + d*x) - 8*a^3 + 2*b^3*sin
(3*c + 3*d*x) + 2*a*b^2*m^2 + 3*b^3*m*sin(3*c + 3*d*x) + b^3*m^2*sin(c + d*
x) + b^3*m^2*sin(3*c + 3*d*x) + 18*a*b^2*m + 11*b^3*m*sin(c + d*x) + 8*a^2*
b*m*sin(c + d*x) - 2*a*b^2*m*(2*sin(c + d*x)^2 - 1) - 2*a*b^2*m^2*(2*sin(c
+ d*x)^2 - 1)))/(4*b^3*d*(11*m + 6*m^2 + m^3 + 6))
```

3.633 $\int \cos(c+dx)(a+b\sin(c+dx))^m\,dx$

Optimal. Leaf size=26

$$\frac{(a+b\sin(c+dx))^{1+m}}{bd(1+m)}$$

```
[Out] (a+b*sin(d*x+c))^(1+m)/b/d/(1+m)
```

Rubi [A]
time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = $\{2747, 32\}$

$$\frac{(a+b\sin(c+dx))^{m+1}}{bd(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] (a + b*Sin[c + d*x])^(1 + m)/(b*d*(1 + m))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a+b\sin(c+dx))^m\,dx &= \frac{\text{Subst}\left(\int (a+x)^m\,dx, x, b\sin(c+dx)\right)}{bd}\\ &= \frac{(a+b\sin(c+dx))^{1+m}}{bd(1+m)}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 26, normalized size = 1.00

$$\frac{(a+b\sin(c+dx))^{1+m}}{bd(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] (a + b*Sin[c + d*x])^(1 + m)/(b*d*(1 + m))
```

Maple [A]
time = 0.12, size = 27, normalized size = 1.04

method	result
derivativedivides	$\frac{(a+b\sin(dx+c))^{1+m}}{bd(1+m)}$
default	$\frac{(a+b\sin(dx+c))^{1+m}}{bd(1+m)}$
norman	$\frac{\frac{a\,{\mathrm e}^{m\ln\left(a+\frac{2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)}}{bd(1+m)}+\frac{a\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right){\mathrm e}^{m\ln\left(a+\frac{2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)}}{bd(1+m)}+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right){\mathrm e}^{m\ln\left(a+\frac{2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)}}{d(1+m)}}{1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*sin(d*x+c))^m,x,method=_RETURNVERBOSE)
```

```
[Out] (a+b*sin(d*x+c))^(1+m)/b/d/(1+m)
```

Maxima [A]
time = 0.28, size = 26, normalized size = 1.00

$$\frac{(b\sin(dx+c)+a)^{m+1}}{bd(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] (b*sin(d*x + c) + a)^(m + 1)/(b*d*(m + 1))
```

Fricas [A]
time = 0.35, size = 33, normalized size = 1.27

$$\frac{(b\sin(dx+c)+a)(b\sin(dx+c)+a)^{m}}{bdm+bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] (b*sin(d*x + c) + a)*(b*sin(d*x + c) + a)^m/(b*d*m + b*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(19) = 38$.

time = 0.53, size = 99, normalized size = 3.81

$$\begin{cases} \frac{x\cos(c)}{a} & \text{for } b = 0 \wedge d = 0 \wedge m = -1 \\ \frac{a^m \sin(c+dx)}{d} & \text{for } b = 0 \\ x(a+b\sin(c))^m \cos(c) & \text{for } d = 0 \\ \frac{\log\left(\frac{a}{b}+\sin(c+dx)\right)}{bd} & \text{for } m = -1 \\ \frac{a(a+b\sin(c+dx))^m}{bdm+bd} + \frac{b(a+b\sin(c+dx))^m \sin(c+dx)}{bdm+bd} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))**m,x)
```

```
[Out] Piecewise((x*cos(c)/a, Eq(b, 0) & Eq(d, 0) & Eq(m, -1)), (a**m*sin(c + d*x)
/d, Eq(b, 0)), (x*(a + b*sin(c))**m*cos(c), Eq(d, 0)), (log(a/b + sin(c + d
*x))/(b*d), Eq(m, -1)), (a*(a + b*sin(c + d*x))**m/(b*d*m + b*d) + b*(a + b
*sin(c + d*x))**m*sin(c + d*x)/(b*d*m + b*d), True))
```

Giac [A]

time = 3.94, size = 26, normalized size = 1.00

$$\frac{(b\sin(dx+c)+a)^{m+1}}{bd(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] (b*sin(d*x + c) + a)^(m + 1)/(b*d*(m + 1))
```

Mupad [B]

time = 6.32, size = 26, normalized size = 1.00

$$\frac{(a+b\sin(c+dx))^{m+1}}{b\,d\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a + b*sin(c + d*x))^m,x)
```

```
[Out] (a + b*sin(c + d*x))^(m + 1)/(b*d*(m + 1))
```

3.634 $\int \sec(c+dx)(a+b\sin(c+dx))^m\,dx$

Optimal. Leaf size=115

$$-\frac{{}_2F_1\left(1,1+m;2+m;\frac{a+b\sin(c+dx)}{a-b}\right)(a+b\sin(c+dx))^{1+m}}{2(a-b)d(1+m)}+\frac{{}_2F_1\left(1,1+m;2+m;\frac{a+b\sin(c+dx)}{a+b}\right)(a+b\sin(c+dx))^{1+m}}{2(a+b)d(1+m)}$$

```
[Out] -1/2*hypergeom([1, 1+m],[2+m],(a+b*sin(d*x+c))/(a-b))*(a+b*sin(d*x+c))^(1+m
)/(a-b)/d/(1+m)+1/2*hypergeom([1, 1+m],[2+m],(a+b*sin(d*x+c))/(a+b))*(a+b*s
in(d*x+c))^(1+m)/(a+b)/d/(1+m)
```

Rubi [A]

time = 0.08, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2747, 726, 70}

$$\frac{(a+b\sin(c+dx))^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{a+b\sin(c+dx)}{a+b}\right)}{2d(m+1)(a+b)}-\frac{(a+b\sin(c+dx))^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{a+b\sin(c+dx)}{a-b}\right)}{2d(m+1)(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] -1/2*(Hypergeometric2F1[1, 1 + m, 2 + m, (a + b*Sin[c + d*x])/(a - b)]*(a +
 b*Sin[c + d*x])^(1 + m))/((a - b)*d*(1 + m)) + (Hypergeometric2F1[1, 1 + m
, 2 + m, (a + b*Sin[c + d*x])/(a + b)]*(a + b*Sin[c + d*x])^(1 + m))/(2*(a
+ b)*d*(1 + m))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 726

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(d + e*x)^m, 1/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m}, x] &
& NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[m]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec(c+dx)(a+b\sin(c+dx))^m\,dx = \frac{b\text{Subst}\left(\int \frac{(a+x)^m}{b^2-x^2}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{b\text{Subst}\left(\int \left(\frac{(a+x)^m}{2b(b-x)}+\frac{(a+x)^m}{2b(b+x)}\right)\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a+x)^m}{b-x}\,dx, x, b\sin(c+dx)\right)}{2d}+\frac{\text{Subst}\left(\int \frac{(a+x)^m}{b+x}\,dx, x, b\sin(c\right.}{2d}$$
$$= -\frac{{}_2F_1\left(1,1+m;2+m;\frac{a+b\sin(c+dx)}{a-b}\right)(a+b\sin(c+dx))^{1+m}}{2(a-b)d(1+m)}+\frac{{}_2F_1\left(\right.}{}$$

Mathematica [A]
time = 0.08, size = 99, normalized size = 0.86

$$-\frac{\left((a+b)\,{}_2F_1\left(1,1+m;2+m;\frac{a+b\sin(c+dx)}{a-b}\right)+(-a+b)\,{}_2F_1\left(1,1+m;2+m;\frac{a+b\sin(c+dx)}{a+b}\right)\right)(a+b\sin(c+dx))^{1+m}}{2(a-b)(a+b)d(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sin[c + d*x])^m,x]
[Out] -1/2*(((a + b)*Hypergeometric2F1[1, 1 + m, 2 + m, (a + b*Sin[c + d*x])/(a -
 b)] + (-a + b)*Hypergeometric2F1[1, 1 + m, 2 + m, (a + b*Sin[c + d*x])/(a
+ b)])*(a + b*Sin[c + d*x])^(1 + m))/((a - b)*(a + b)*d*(1 + m))
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \sec\left(dx+c\right)\left(a+b\sin\left(dx+c\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*sin(d*x+c))^m,x)
[Out] int(sec(d*x+c)*(a+b*sin(d*x+c))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^m*sec(d*x + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((b*sin(d*x + c) + a)^m*sec(d*x + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \sin (c + dx))^m \sec (c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))**m,x)
```

```
[Out] Integral((a + b*sin(c + d*x))**m*sec(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^m*sec(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\, \sin (c + d\, x))^m}{\cos (c + d\, x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^m/cos(c + d*x),x)
```

```
[Out] int((a + b*sin(c + d*x))^m/cos(c + d*x), x)
```

3.635 $\int \sec^3(c+dx)(a+b\sin(c+dx))^m\,dx$

Optimal. Leaf size=183

$$-\frac{(a-b(1-m))\,{}_2F_1\left(1,1+m;2+m;\frac{a+b\sin(c+dx)}{a-b}\right)(a+b\sin(c+dx))^{1+m}}{4(a-b)^2d(1+m)}+\frac{(a+b-bm)\,{}_2F_1\left(1,1+m;2+\cdots\right.}{4(a+\cdots}$$

```
[Out] -1/4*(a-b*(1-m))*hypergeom([1, 1+m],[2+m],(a+b*sin(d*x+c))/(a-b))*(a+b*sin(
d*x+c))^(1+m)/(a-b)^2/d/(1+m)+1/4*(-b*m+a+b)*hypergeom([1, 1+m],[2+m],(a+b*
sin(d*x+c))/(a+b))*(a+b*sin(d*x+c))^(1+m)/(a+b)^2/d/(1+m)-1/2*sec(d*x+c)^2*
(b-a*sin(d*x+c))*(a+b*sin(d*x+c))^(1+m)/(a^2-b^2)/d
```

Rubi [A]
time = 0.17, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2747, 755, 845, 70}

$$-\frac{\sec^2(c+dx)(b-a\sin(c+dx))(a+b\sin(c+dx))^{m+1}}{2d\left(a^2-b^2\right)}-\frac{(a-b(1-m))(a+b\sin(c+dx))^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{a+b\sin(c+dx)}{a-b}\right)}{4d(m+1)(a-b)^2}+\frac{(a-bm+b)(a+b\sin(c+dx))^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{a+b\sin(c+dx)}{a+b}\right)}{4d(m+1)(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] -1/4*((a - b*(1 - m))*Hypergeometric2F1[1, 1 + m, 2 + m, (a + b*Sin[c + d*x
])/(a - b)]*(a + b*Sin[c + d*x])^(1 + m))/((a - b)^2*d*(1 + m)) + ((a + b -
 b*m)*Hypergeometric2F1[1, 1 + m, 2 + m, (a + b*Sin[c + d*x])/(a + b)]*(a +
 b*Sin[c + d*x])^(1 + m))/(4*(a + b)^2*d*(1 + m)) - (Sec[c + d*x]^2*(b - a*
Sin[c + d*x])*(a + b*Sin[c + d*x])^(1 + m))/(2*(a^2 - b^2)*d)
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 845

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a + c*x^2), x], x
] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !RationalQ[m
]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^3(c+dx)(a+b\sin(c+dx))^m\,dx = \frac{b^3\text{Subst}\left(\int \frac{(a+x)^m}{(b^2-x^2)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= -\frac{\sec^2(c+dx)(b-a\sin(c+dx))(a+b\sin(c+dx))^{1+m}}{2(a^2-b^2)\,d} + \frac{b\text{Subst}}{}$$

$$= -\frac{\sec^2(c+dx)(b-a\sin(c+dx))(a+b\sin(c+dx))^{1+m}}{2(a^2-b^2)\,d} + \frac{b\text{Subst}}{}$$

$$= -\frac{\sec^2(c+dx)(b-a\sin(c+dx))(a+b\sin(c+dx))^{1+m}}{2(a^2-b^2)\,d} + \frac{((a+b}{}$$

$$= -\frac{(a-b(1-m))\,{}_2F_1\left(1,1+m;2+m;\frac{a+b\sin(c+dx)}{a-b}\right)(a+b\sin(c+}{4(a-b)^2d(1+m)}$$

Mathematica [A]

time = 0.43, size = 157, normalized size = 0.86

$$\frac{(a+b\sin(c+dx))^{1+m}\left(\frac{b\left((a+b)^2(a+b(-1+m))\,{}_2F_1\left(1,1+m;2+m;\frac{a+b\sin(c+dx)}{a-b}\right)-(a-b)^2(a+b-bm)\,{}_2F_1\left(1,1+m;2+m;\frac{a+b\sin(c+dx)}{a+b}\right)\right)}{(a-b)(a+b)(1+m)}+2b\sec^2(c+dx)(b-a\sin(c+dx))\right)}{4b\,(-a^2+b^2)\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] ((a + b*Sin[c + d*x])^(1 + m)*((b*((a + b)^2*(a + b*(-1 + m))*Hypergeometri
c2F1[1, 1 + m, 2 + m, (a + b*Sin[c + d*x])/(a - b)] - (a - b)^2*(a + b - b*
m)*Hypergeometric2F1[1, 1 + m, 2 + m, (a + b*Sin[c + d*x])/(a + b)]))/((a -
 b)*(a + b)*(1 + m)) + 2*b*Sec[c + d*x]^2*(b - a*Sin[c + d*x])))/(4*b*(-a^2
 + b^2)*d)
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \left(\sec^3\left(dx+c\right)\right)\left(a+b\sin\left(dx+c\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+b*sin(d*x+c))^m,x)
[Out] int(sec(d*x+c)^3*(a+b*sin(d*x+c))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sin(d*x+c))^m,x, algorithm="maxima")
[Out] integrate((b*sin(d*x + c) + a)^m*sec(d*x + c)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sin(d*x+c))^m,x, algorithm="fricas")
[Out] integral((b*sin(d*x + c) + a)^m*sec(d*x + c)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sin\left(c+dx\right)\right)^m\sec^3\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+b*sin(d*x+c))**m,x)
[Out] Integral((a + b*sin(c + d*x))**m*sec(c + d*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^m*sec(d*x + c)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\sin\left(c+d\,x\right))^m}{\cos\left(c+d\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^m/cos(c + d*x)^3,x)
```

```
[Out] int((a + b*sin(c + d*x))^m/cos(c + d*x)^3, x)
```

3.636 $\int \sec^5(c+dx)(a+b\sin(c+dx))^m\,dx$

Optimal. Leaf size=305

$$-\frac{(3a^2-3ab(2-m)+b^2(3-4m+m^2))\ {}_2F_1\left(1,1+m;2+m;\frac{a+b\sin(c+dx)}{a-b}\right)(a+b\sin(c+dx))^{1+m}}{16(a-b)^3d(1+m)}+\frac{(3a^2+$$

```
[Out] -1/16*(3*a^2-3*a*b*(2-m)+b^2*(m^2-4*m+3))*hypergeom([1, 1+m],[2+m],(a+b*sin
(d*x+c))/(a-b))*(a+b*sin(d*x+c))^(1+m)/(a-b)^3/d/(1+m)+1/16*(3*a^2+3*a*b*(2
-m)+b^2*(m^2-4*m+3))*hypergeom([1, 1+m],[2+m],(a+b*sin(d*x+c))/(a+b))*(a+b*
sin(d*x+c))^(1+m)/(a+b)^3/d/(1+m)-1/4*sec(d*x+c)^4*(b-a*sin(d*x+c))*(a+b*si
n(d*x+c))^(1+m)/(a^2-b^2)/d+1/8*sec(d*x+c)^2*(a+b*sin(d*x+c))^(1+m)*(b*(b^2
*(3-m)-a^2*(1+m))+a*(3*a^2-b^2*(5-2*m))*sin(d*x+c))/(a^2-b^2)^2/d
```

Rubi [A]
time = 0.30, antiderivative size = 305, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2747, 755, 837, 845, 70}

$$-\frac{(3a^2-3ab(2-m)+b^2(m^2-4m+3))(a+b\sin(c+dx))^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{a+b\sin(c+dx)}{a-b}\right)}{16d(m+1)(a-b)^3}+\frac{(3a^2+3ab(2-m)+b^2(m^2-4m+3))(a+b\sin(c+dx))^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{a+b\sin(c+dx)}{a+b}\right)}{16d(m+1)(a+b)^3}-\frac{\sec^4(c+dx)(b-a\sin(c+dx))(a+b\sin(c+dx))^{m+1}}{4d(a^2-b^2)}+\frac{\sec^2(c+dx)\left(a(3a^2-b^2(5-2m))\sin(c+dx)+b(b^2(3-m)-a^2(m+1))\right)(a+b\sin(c+dx))^{m+1}}{8d(a^2-b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] -1/16*((3*a^2 - 3*a*b*(2 - m) + b^2*(3 - 4*m + m^2))*Hypergeometric2F1[1, 1
 + m, 2 + m, (a + b*Sin[c + d*x])/(a - b)]*(a + b*Sin[c + d*x])^(1 + m))/((
a - b)^3*d*(1 + m)) + ((3*a^2 + 3*a*b*(2 - m) + b^2*(3 - 4*m + m^2))*Hyperg
eometric2F1[1, 1 + m, 2 + m, (a + b*Sin[c + d*x])/(a + b)]*(a + b*Sin[c + d
*x])^(1 + m))/(16*(a + b)^3*d*(1 + m)) - (Sec[c + d*x]^4*(b - a*Sin[c + d*x
])*(a + b*Sin[c + d*x])^(1 + m))/(4*(a^2 - b^2)*d) + (Sec[c + d*x]^2*(a + b
*Sin[c + d*x])^(1 + m)*(b*(b^2*(3 - m) - a^2*(1 + m)) + a*(3*a^2 - b^2*(5 -
 2*m))*Sin[c + d*x]))/(8*(a^2 - b^2)^2*d)
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
```

```
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 845

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a + c*x^2), x], x
] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !RationalQ[m
]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^5(c+dx)(a+b\sin(c+dx))^m\,dx = \frac{b^5\text{Subst}\left(\int \frac{(a+x)^m}{(b^2-x^2)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= -\frac{\sec^4(c+dx)(b-a\sin(c+dx))(a+b\sin(c+dx))^{1+m}}{4(a^2-b^2)\,d} + \frac{b^3\text{Subst}\left(\right.}{}$$

$$= -\frac{\sec^4(c+dx)(b-a\sin(c+dx))(a+b\sin(c+dx))^{1+m}}{4(a^2-b^2)\,d} + \frac{\sec^2(c+}{}$$

$$= -\frac{\sec^4(c+dx)(b-a\sin(c+dx))(a+b\sin(c+dx))^{1+m}}{4(a^2-b^2)\,d} + \frac{\sec^2(c+}{}$$

$$= -\frac{\sec^4(c+dx)(b-a\sin(c+dx))(a+b\sin(c+dx))^{1+m}}{4(a^2-b^2)\,d} + \frac{\sec^2(c+}{}$$

$$= -\frac{(3a^2-3ab(2-m)+b^2(3-4m+m^2))\ {}_2F_1\left(1,1+m;2+m;\frac{a+b}{}\right.}{16(a-b)^3d(1+m)}$$

Mathematica [A]
time = 2.57, size = 260, normalized size = 0.85

$$\frac{(a+b\sin(c+dx))^{1+m}\left(\frac{(a+b)^3(3a^2+3ab(-2+m)+b^2(3-4m+m^2))\ {}_2F_1\left(1,1+m;2+m;\frac{a+b\sin(c+dx)}{a-b}\right)-(a-b)^3(3a^2-3ab(-2+m)+b^2(3-4m+m^2))\ {}_2F_1\left(1,1+m;2+m;\frac{a+b\sin(c+dx)}{a+b}\right)}{(a-b)(a+b)(a^2-b^2)(1+m)}+4\sec^4(c+dx)(b-a\sin(c+dx))+\frac{2\sec^2(c+dx)(b^3(-3+m)+a^2b(1+m)-a(3a^2+b^2(-5+2m))\sin(c+dx))}{a^2-b^2}\right)}{16(-a^2+b^2)\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] ((a + b*Sin[c + d*x])^(1 + m)*(((a + b)^3*(3*a^2 + 3*a*b*(-2 + m) + b^2*(3
- 4*m + m^2))*Hypergeometric2F1[1, 1 + m, 2 + m, (a + b*Sin[c + d*x])/(a -
b)] - (a - b)^3*(3*a^2 - 3*a*b*(-2 + m) + b^2*(3 - 4*m + m^2))*Hypergeometr
ic2F1[1, 1 + m, 2 + m, (a + b*Sin[c + d*x])/(a + b)])/((a - b)*(a + b)*(a^2
 - b^2)*(1 + m)) + 4*Sec[c + d*x]^4*(b - a*Sin[c + d*x]) + (2*Sec[c + d*x]^
2*(b^3*(-3 + m) + a^2*b*(1 + m) - a*(3*a^2 + b^2*(-5 + 2*m))*Sin[c + d*x]))
/(a^2 - b^2)))/(16*(-a^2 + b^2)*d)
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \left(\sec^5(dx+c)\right)(a+b\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(a+b*sin(d*x+c))^m,x)
```

```
[Out] int(sec(d*x+c)^5*(a+b*sin(d*x+c))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*sin(d*x+c))^m,x, algorithm="maxima")
[Out] integrate((b*sin(d*x + c) + a)^m*sec(d*x + c)^5, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*sin(d*x+c))^m,x, algorithm="fricas")
[Out] integral((b*sin(d*x + c) + a)^m*sec(d*x + c)^5, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a+b*sin(d*x+c))**m,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*sin(d*x+c))^m,x, algorithm="giac")
[Out] integrate((b*sin(d*x + c) + a)^m*sec(d*x + c)^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\sin(c + d\,x))^m}{\cos(c + d\,x)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^m/cos(c + d*x)^5,x)
[Out] int((a + b*sin(c + d*x))^m/cos(c + d*x)^5, x)
```

3.637 $\int \cos^4(c+dx)(a+b\sin(c+dx))^m\,dx$

Optimal. Leaf size=129

$$\frac{F_1\left(1+m;-\frac{3}{2},-\frac{3}{2};2+m;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)\cos^3(c+dx)(a+b\sin(c+dx))^{1+m}}{bd(1+m)\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{3/2}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{3/2}}$$

```
[Out] AppellF1(1+m,-3/2,-3/2,2+m,(a+b*sin(d*x+c))/(a-b),(a+b*sin(d*x+c))/(a+b))*c
os(d*x+c)^3*(a+b*sin(d*x+c))^(1+m)/b/d/(1+m)/(1+(-a-b*sin(d*x+c))/(a-b))^(3
/2)/(1+(-a-b*sin(d*x+c))/(a+b))^(3/2)
```

Rubi [A]

time = 0.07, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2783, 143}

$$\frac{\cos^3(c+dx)(a+b\sin(c+dx))^{m+1}F_1\left(m+1;-\frac{3}{2},-\frac{3}{2};m+2;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)}{bd(m+1)\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{3/2}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] (AppellF1[1 + m, -3/2, -3/2, 2 + m, (a + b*Sin[c + d*x])/(a - b), (a + b*Si
n[c + d*x])/(a + b)]*Cos[c + d*x]^3*(a + b*Sin[c + d*x])^(1 + m))/(b*d*(1 +
 m)*(1 - (a + b*Sin[c + d*x])/(a - b))^(3/2)*(1 - (a + b*Sin[c + d*x])/(a +
 b))^(3/2))
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 2783

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[g*((g*Cos[e + f*x])^(p - 1)/(f*(1 - (a + b*Sin
[e + f*x])/(a - b))^((p - 1)/2)*(1 - (a + b*Sin[e + f*x])/(a + b))^((p - 1)
/2))), Subst[Int[(-b/(a - b) - b*(x/(a - b)))^((p - 1)/2)*(b/(a + b) - b*(x
```

```
/(a + b)))^((p - 1)/2)*(a + b*x)^m, x], x, Sin[e + f*x]], x] /; FreeQ[{a, b
, e, f, g, m, p}, x] && NeQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \cos^4(c+dx)(a+b\sin(c+dx))^m\,dx = \frac{\cos^3(c+dx)\text{Subst}\left(\int (a+bx)^m\left(-\frac{b}{a-b}-\frac{bx}{a-b}\right)^{3/2}\left(\frac{b}{a+b}-\frac{bx}{a+b}\right)^{3/2}d}{d\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{3/2}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{3/2}}$$

$$= \frac{F_1\left(1+m;-\frac{3}{2},-\frac{3}{2};2+m;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)\cos^3(c+dx)(}{bd(1+m)\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{3/2}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right.}$$

Mathematica [F]
time = 1.49, size = 0, normalized size = 0.00

$$\int \cos^4(c+dx)(a+b\sin(c+dx))^m\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cos[c + d*x]^4*(a + b*Sin[c + d*x])^m,x]
[Out] Integrate[Cos[c + d*x]^4*(a + b*Sin[c + d*x])^m, x]
```

Maple [F]
time = 0.18, size = 0, normalized size = 0.00

$$\int \left(\cos^4(dx+c)\right)(a+b\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+b*sin(d*x+c))^m,x)
[Out] int(cos(d*x+c)^4*(a+b*sin(d*x+c))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^m*cos(d*x + c)^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sin(d*x+c))^m,x, algorithm="fricas")
[Out] integral((b*sin(d*x + c) + a)^m*cos(d*x + c)^4, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+b*sin(d*x+c))**m,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sin(d*x+c))^m,x, algorithm="giac")
[Out] integrate((b*sin(d*x + c) + a)^m*cos(d*x + c)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^4\,{\left(a+b\,\sin\left(c+d\,x\right)\right)}^m \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(a + b*sin(c + d*x))^m,x)
[Out] int(cos(c + d*x)^4*(a + b*sin(c + d*x))^m, x)
```

3.638 $\int \cos^2(c+dx)(a+b\sin(c+dx))^m\,dx$

Optimal. Leaf size=127

$$\frac{F_1\left(1+m;-\frac{1}{2},-\frac{1}{2};2+m;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)\cos(c+dx)(a+b\sin(c+dx))^{1+m}}{bd(1+m)\sqrt{1-\frac{a+b\sin(c+dx)}{a-b}}\sqrt{1-\frac{a+b\sin(c+dx)}{a+b}}}$$

```
[Out] AppellF1(1+m,-1/2,-1/2,2+m,(a+b*sin(d*x+c))/(a-b),(a+b*sin(d*x+c))/(a+b))*c
os(d*x+c)*(a+b*sin(d*x+c))^(1+m)/b/d/(1+m)/(1+(-a-b*sin(d*x+c))/(a-b))^(1/2
)/(1+(-a-b*sin(d*x+c))/(a+b))^(1/2)
```

Rubi [A]

time = 0.06, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2783, 143}

$$\frac{\cos(c+dx)(a+b\sin(c+dx))^{m+1}F_1\left(m+1;-\frac{1}{2},-\frac{1}{2};m+2;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)}{bd(m+1)\sqrt{1-\frac{a+b\sin(c+dx)}{a-b}}\sqrt{1-\frac{a+b\sin(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] (AppellF1[1 + m, -1/2, -1/2, 2 + m, (a + b*Sin[c + d*x])/(a - b), (a + b*Si
n[c + d*x])/(a + b)]*Cos[c + d*x]*(a + b*Sin[c + d*x])^(1 + m))/(b*d*(1 + m
)*Sqrt[1 - (a + b*Sin[c + d*x])/(a - b)]*Sqrt[1 - (a + b*Sin[c + d*x])/(a +
 b)])
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 2783

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[g*((g*Cos[e + f*x])^(p - 1)/(f*(1 - (a + b*Sin
[e + f*x])/(a - b))^((p - 1)/2)*(1 - (a + b*Sin[e + f*x])/(a + b))^((p - 1)
```

```
/2))), Subst[Int[(-b/(a - b) - b*(x/(a - b)))^((p - 1)/2)*(b/(a + b) - b*(x
/(a + b)))^((p - 1)/2)*(a + b*x)^m, x], x, Sin[e + f*x]], x] /; FreeQ[{a, b
, e, f, g, m, p}, x] && NeQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \cos^2(c+dx)(a+b\sin(c+dx))^m\,dx = \frac{\cos(c+dx)\text{Subst}\left(\int (a+bx)^m\sqrt{-\frac{b}{a-b}-\frac{bx}{a-b}}\sqrt{\frac{b}{a+b}-\frac{bx}{a+b}}\right.}{d\sqrt{1-\frac{a+b\sin(c+dx)}{a-b}}\sqrt{1-\frac{a+b\sin(c+dx)}{a+b}}}$$

$$= \frac{F_1\left(1+m;-\frac{1}{2},-\frac{1}{2};2+m;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)\cos(c+dx)(a+}{bd(1+m)\sqrt{1-\frac{a+b\sin(c+dx)}{a-b}}\sqrt{1-\frac{a+b\sin(c+}{a+b}}}$$

Mathematica [F]
time = 2.76, size = 0, normalized size = 0.00

$$\int \cos^2(c+dx)(a+b\sin(c+dx))^m\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Sin[c + d*x])^m,x]
[Out] Integrate[Cos[c + d*x]^2*(a + b*Sin[c + d*x])^m, x]
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int \left(\cos^2\left(dx+c\right)\right)\left(a+b\sin\left(dx+c\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*sin(d*x+c))^m,x)
[Out] int(cos(d*x+c)^2*(a+b*sin(d*x+c))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sin(d*x+c))^m,x, algorithm="maxima")
[Out] integrate((b*sin(d*x + c) + a)^m*cos(d*x + c)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sin(d*x+c))^m,x, algorithm="fricas")
[Out] integral((b*sin(d*x + c) + a)^m*cos(d*x + c)^2, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*sin(d*x+c))**m,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sin(d*x+c))^m,x, algorithm="giac")
[Out] integrate((b*sin(d*x + c) + a)^m*cos(d*x + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^2\,{\left(a+b\,\sin\left(c+d\,x\right)\right)}^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a + b*sin(c + d*x))^m,x)
[Out] int(cos(c + d*x)^2*(a + b*sin(c + d*x))^m, x)
```

3.639 $\int \sec^2(c+dx)(a+b\sin(c+dx))^m\,dx$

Optimal. Leaf size=129

$$\frac{F_1\left(1+m;\frac{3}{2},\frac{3}{2};2+m;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)\sec^3(c+dx)(a+b\sin(c+dx))^{1+m}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{3/2}\left(1-\right.}{bd(1+m)}$$

```
[Out] AppellF1(1+m,3/2,3/2,2+m,(a+b*sin(d*x+c))/(a-b),(a+b*sin(d*x+c))/(a+b))*sec
(d*x+c)^3*(a+b*sin(d*x+c))^(1+m)*(1+(-a-b*sin(d*x+c))/(a-b))^(3/2)*(1+(-a-b
*sin(d*x+c))/(a+b))^(3/2)/b/d/(1+m)
```

Rubi [A]
time = 0.06, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2783, 143}

$$\frac{\sec^3(c+dx)\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{3/2}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{3/2}(a+b\sin(c+dx))^{m+1}F_1\left(m+1;\frac{3}{2},\frac{3}{2};m+2;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)}{bd(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] (AppellF1[1 + m, 3/2, 3/2, 2 + m, (a + b*Sin[c + d*x])/(a - b), (a + b*Sin[
c + d*x])/(a + b)]*Sec[c + d*x]^3*(a + b*Sin[c + d*x])^(1 + m)*(1 - (a + b*
Sin[c + d*x])/(a - b))^(3/2)*(1 - (a + b*Sin[c + d*x])/(a + b))^(3/2))/(b*d
*(1 + m))
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 2783

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[g*((g*Cos[e + f*x])^(p - 1)/(f*(1 - (a + b*Sin
[e + f*x])/(a - b))^((p - 1)/2)*(1 - (a + b*Sin[e + f*x])/(a + b))^((p - 1)
/2))), Subst[Int[(-b/(a - b) - b*(x/(a - b)))^((p - 1)/2)*(b/(a + b) - b*(x
/(a + b)))^((p - 1)/2)*(a + b*x)^m, x], x, Sin[e + f*x]], x] /; FreeQ[{a, b
```

```
, e, f, g, m, p}, x] && NeQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \sec^2(c+dx)(a+b\sin(c+dx))^m\,dx = \frac{\left(\sec^3(c+dx)\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{3/2}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{3/2}\right)\text{Subst}\Big(}{d}$$

$$= \frac{F_1\left(1+m;\frac{3}{2},\frac{3}{2};2+m;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)\sec^3(c+dx)(a+}{bd(1+m}$$

Mathematica [**F**]
time = 1.35, size = 0, normalized size = 0.00

$$\int \sec^2(c+dx)(a+b\sin(c+dx))^m\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Sin[c + d*x])^m,x]
[Out] Integrate[Sec[c + d*x]^2*(a + b*Sin[c + d*x])^m, x]
```

Maple [**F**]
time = 0.06, size = 0, normalized size = 0.00

$$\int \left(\sec^2(dx+c)\right)(a+b\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+b*sin(d*x+c))^m,x)
[Out] int(sec(d*x+c)^2*(a+b*sin(d*x+c))^m,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sin(d*x+c))^m,x, algorithm="maxima")
[Out] integrate((b*sin(d*x + c) + a)^m*sec(d*x + c)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sin(d*x+c))^m,x, algorithm="fricas")
[Out] integral((b*sin(d*x + c) + a)^m*sec(d*x + c)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sin(c+dx))^m \sec^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*sin(d*x+c))**m,x)
[Out] Integral((a + b*sin(c + d*x))**m*sec(c + d*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sin(d*x+c))^m,x, algorithm="giac")
[Out] integrate((b*sin(d*x + c) + a)^m*sec(d*x + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\sin(c+d\,x))^m}{\cos(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^m/cos(c + d*x)^2,x)
[Out] int((a + b*sin(c + d*x))^m/cos(c + d*x)^2, x)
```

3.640 $\int \sec^4(c+dx)(a+b\sin(c+dx))^m\,dx$

Optimal. Leaf size=129

$$\frac{F_1\left(1+m;\frac{5}{2},\frac{5}{2};2+m;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)\sec^5(c+dx)(a+b\sin(c+dx))^{1+m}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{5/2}\left(1\right.}{bd(1+m)}$$

```
[Out] AppellF1(1+m,5/2,5/2,2+m,(a+b*sin(d*x+c))/(a-b),(a+b*sin(d*x+c))/(a+b))*sec
(d*x+c)^5*(a+b*sin(d*x+c))^(1+m)*(1+(-a-b*sin(d*x+c))/(a-b))^(5/2)*(1+(-a-b
*sin(d*x+c))/(a+b))^(5/2)/b/d/(1+m)
```

Rubi [A]

time = 0.07, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2783, 143}

$$\frac{\sec^5(c+dx)\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{5/2}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{5/2}(a+b\sin(c+dx))^{m+1}F_1\left(m+1;\frac{5}{2},\frac{5}{2};m+2;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)}{bd(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] (AppellF1[1 + m, 5/2, 5/2, 2 + m, (a + b*Sin[c + d*x])/(a - b), (a + b*Sin[
c + d*x])/(a + b)]*Sec[c + d*x]^5*(a + b*Sin[c + d*x])^(1 + m)*(1 - (a + b*
Sin[c + d*x])/(a - b))^(5/2)*(1 - (a + b*Sin[c + d*x])/(a + b))^(5/2))/(b*d
*(1 + m))
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 2783

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[g*((g*Cos[e + f*x])^(p - 1)/(f*(1 - (a + b*Sin
[e + f*x])/(a - b))^((p - 1)/2)*(1 - (a + b*Sin[e + f*x])/(a + b))^((p - 1)
/2))), Subst[Int[(-b/(a - b) - b*(x/(a - b)))^((p - 1)/2)*(b/(a + b) - b*(x
/(a + b)))^((p - 1)/2)*(a + b*x)^m, x], x, Sin[e + f*x]], x] /; FreeQ[{a, b
```

```
, e, f, g, m, p}, x] && NeQ[a^2 - b^2, 0] && !IGtQ[m, 0]
```

Rubi steps

$$\int \sec^4(c+dx)(a+b\sin(c+dx))^m\,dx = \frac{\left(\sec^5(c+dx)\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{5/2}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{5/2}\right)\text{Subst}\Big(\int}{d}$$

$$= \frac{F_1\left(1+m;\frac{5}{2},\frac{5}{2};2+m;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)\sec^5(c+dx)(a+b\,s}{bd(1+m)}$$

Mathematica [F]
time = 3.06, size = 0, normalized size = 0.00

$$\int \sec^4(c+dx)(a+b\sin(c+dx))^m\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^4*(a + b*Sin[c + d*x])^m,x]
[Out] Integrate[Sec[c + d*x]^4*(a + b*Sin[c + d*x])^m, x]
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \left(\sec^4(dx+c)\right)(a+b\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+b*sin(d*x+c))^m,x)
[Out] int(sec(d*x+c)^4*(a+b*sin(d*x+c))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sin(d*x+c))^m,x, algorithm="maxima")
[Out] integrate((b*sin(d*x + c) + a)^m*sec(d*x + c)^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sin(d*x+c))^m,x, algorithm="fricas")
[Out] integral((b*sin(d*x + c) + a)^m*sec(d*x + c)^4, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+b*sin(d*x+c))**m,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sin(d*x+c))^m,x, algorithm="giac")
[Out] integrate((b*sin(d*x + c) + a)^m*sec(d*x + c)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\sin(c+dx))^m}{\cos(c+dx)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^m/cos(c + d*x)^4,x)
[Out] int((a + b*sin(c + d*x))^m/cos(c + d*x)^4, x)
```

3.641 $\int (e\cos(c+dx))^{5/2}(a+b\sin(c+dx))^m\,dx$

Optimal. Leaf size=134

$$\frac{eF_1\left(1+m;-\frac{3}{4},-\frac{3}{4};2+m;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))^{1+m}}{bd(1+m)\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{3/4}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{3/4}}$$

```
[Out] e*AppellF1(1+m,-3/4,-3/4,2+m,(a+b*sin(d*x+c))/(a-b),(a+b*sin(d*x+c))/(a+b))
*(e*cos(d*x+c))^(3/2)*(a+b*sin(d*x+c))^(1+m)/b/d/(1+m)/(1+(-a-b*sin(d*x+c))
/(a-b))^(3/4)/(1+(-a-b*sin(d*x+c))/(a+b))^(3/4)
```

Rubi [A]
time = 0.08, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2783, 143}

$$\frac{e(e\cos(c+dx))^{3/2}(a+b\sin(c+dx))^{m+1}F_1\left(m+1;-\frac{3}{4},-\frac{3}{4};m+2;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)}{bd(m+1)\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{3/4}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(5/2)*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] (e*AppellF1[1 + m, -3/4, -3/4, 2 + m, (a + b*Sin[c + d*x])/(a - b), (a + b*
Sin[c + d*x])/(a + b)]*(e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x])^(1 + m))
/(b*d*(1 + m)*(1 - (a + b*Sin[c + d*x])/(a - b))^(3/4)*(1 - (a + b*Sin[c +
d*x])/(a + b))^(3/4))
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 2783

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[g*((g*Cos[e + f*x])^(p - 1)/(f*(1 - (a + b*Sin
[e + f*x])/(a - b))^((p - 1)/2)*(1 - (a + b*Sin[e + f*x])/(a + b))^((p - 1)
/2))), Subst[Int[(-b/(a - b) - b*(x/(a - b)))^((p - 1)/2)*(b/(a + b) - b*(x
```

```
/(a + b)))^((p - 1)/2)*(a + b*x)^m, x], x, Sin[e + f*x]], x] /; FreeQ[{a, b
, e, f, g, m, p}, x] && NeQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int (e\cos(c+dx))^{5/2}(a+b\sin(c+dx))^m\,dx = \frac{\left(e(e\cos(c+dx))^{3/2}\right)\operatorname{Subst}\left(\int (a+bx)^m\left(-\frac{b}{a-b}-\frac{bx}{a-b}\right)^{3/4}\left(\frac{b}{a+\cdots}\right.\right.}{d\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{3/4}\left(1-\frac{a+b\sin(c+\cdots}{a+b}\right.}$$

$$= \frac{eF_1\left(1+m;-\frac{3}{4},-\frac{3}{4};2+m;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)(e\cos(c\cdots}{bd(1+m)\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{3/4}\left(1-\frac{a+b\cdots}{\cdots}\right.}$$

Mathematica [F]
time = 37.24, size = 0, normalized size = 0.00

$$\int (e\cos(c+dx))^{5/2}(a+b\sin(c+dx))^m\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(e*Cos[c + d*x])^(5/2)*(a + b*Sin[c + d*x])^m,x]
[Out] Integrate[(e*Cos[c + d*x])^(5/2)*(a + b*Sin[c + d*x])^m, x]
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \left(e\cos\left(dx+c\right)\right)^{\frac{5}{2}}\left(a+b\sin\left(dx+c\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(5/2)*(a+b*sin(d*x+c))^m,x)
[Out] int((e*cos(d*x+c))^(5/2)*(a+b*sin(d*x+c))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+b*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] e^(5/2)*integrate((b*sin(d*x + c) + a)^m*cos(d*x + c)^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+b*sin(d*x+c))^m,x, algorithm="fricas")
[Out] integral((b*sin(d*x + c) + a)^m*cos(d*x + c)^(5/2)*e^(5/2), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(5/2)*(a+b*sin(d*x+c))**m,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 8570 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+b*sin(d*x+c))^m,x, algorithm="giac")
[Out] integrate((b*sin(d*x + c) + a)^m*cos(d*x + c)^(5/2)*e^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (e\cos(c+d\,x))^{5/2}\,(a+b\sin(c+d\,x))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(5/2)*(a + b*sin(c + d*x))^m,x)
[Out] int((e*cos(c + d*x))^(5/2)*(a + b*sin(c + d*x))^m, x)
```

3.642 $\int (e\cos(c+dx))^{3/2}(a+b\sin(c+dx))^m\,dx$

Optimal. Leaf size=134

$$\frac{eF_1\left(1+m;-\frac{1}{4},-\frac{1}{4};2+m;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))^{1+m}}{bd(1+m)\sqrt[4]{1-\frac{a+b\sin(c+dx)}{a-b}}\sqrt[4]{1-\frac{a+b\sin(c+dx)}{a+b}}}$$

```
[Out] e*AppellF1(1+m,-1/4,-1/4,2+m,(a+b*sin(d*x+c))/(a-b),(a+b*sin(d*x+c))/(a+b))
*(a+b*sin(d*x+c))^(1+m)*(e*cos(d*x+c))^(1/2)/b/d/(1+m)/(1+(-a-b*sin(d*x+c))
/(a-b))^(1/4)/(1+(-a-b*sin(d*x+c))/(a+b))^(1/4)
```

Rubi [A]
time = 0.08, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2783, 143}

$$\frac{e\sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))^{m+1}F_1\left(m+1;-\frac{1}{4},-\frac{1}{4};m+2;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)}{bd(m+1)\sqrt[4]{1-\frac{a+b\sin(c+dx)}{a-b}}\sqrt[4]{1-\frac{a+b\sin(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] (e*AppellF1[1 + m, -1/4, -1/4, 2 + m, (a + b*Sin[c + d*x])/(a - b), (a + b*
Sin[c + d*x])/(a + b)]*Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x])^(1 + m))/(
b*d*(1 + m)*(1 - (a + b*Sin[c + d*x])/(a - b))^(1/4)*(1 - (a + b*Sin[c + d*
x])/(a + b))^(1/4))
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 2783

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[g*((g*Cos[e + f*x])^(p - 1)/(f*(1 - (a + b*Sin
[e + f*x])/(a - b))^((p - 1)/2)*(1 - (a + b*Sin[e + f*x])/(a + b))^((p - 1)
```

```
/2))), Subst[Int[(-b/(a - b) - b*(x/(a - b)))^((p - 1)/2)*(b/(a + b) - b*(x
/(a + b)))^((p - 1)/2)*(a + b*x)^m, x], x, Sin[e + f*x]], x] /; FreeQ[{a, b
, e, f, g, m, p}, x] && NeQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int (e\cos(c+dx))^{3/2}(a+b\sin(c+dx))^m\,dx = \frac{\left(e\sqrt{e\cos(c+dx)}\right)\text{Subst}\left(\int (a+bx)^m\sqrt[4]{-\frac{b}{a-b}-\frac{bx}{a-b}}\,\sqrt[4]{\cdots}\right.}{d\sqrt[4]{1-\frac{a+b\sin(c+dx)}{a-b}}\,\sqrt[4]{1-\frac{a+b\sin\cdots}{\cdots}}}$$

$$= \frac{eF_1\left(1+m;-\frac{1}{4},-\frac{1}{4};2+m;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)\sqrt{e\cos(c\cdots}}{bd(1+m)\sqrt[4]{1-\frac{a+b\sin(c+dx)}{a-b}}\,\sqrt[4]{1-\frac{a+\cdots}{\cdots}}}$$

Mathematica [F]
time = 5.65, size = 0, normalized size = 0.00

$$\int (e\cos(c+dx))^{3/2}(a+b\sin(c+dx))^m\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x])^m,x]
[Out] Integrate[(e*Cos[c + d*x])^(3/2)*(a + b*Sin[c + d*x])^m, x]
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \left(e\cos\left(dx+c\right)\right)^{\frac{3}{2}}\left(a+b\sin\left(dx+c\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(3/2)*(a+b*sin(d*x+c))^m,x)
[Out] int((e*cos(d*x+c))^(3/2)*(a+b*sin(d*x+c))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+b*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] e^(3/2)*integrate((b*sin(d*x + c) + a)^m*cos(d*x + c)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+b*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((b*sin(d*x + c) + a)^m*cos(d*x + c)^(3/2)*e^(3/2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(3/2)*(a+b*sin(d*x+c))**m,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+b*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^m*cos(d*x + c)^(3/2)*e^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (e\,\cos\left(c+d\,x\right))^{3/2}\,\left(a+b\,\sin\left(c+d\,x\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(3/2)*(a + b*sin(c + d*x))^m,x)
```

```
[Out] int((e*cos(c + d*x))^(3/2)*(a + b*sin(c + d*x))^m, x)
```

3.643 $\int \sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))^m\,dx$

Optimal. Leaf size=134

$$\frac{eF_1\left(1+m;\frac{1}{4},\frac{1}{4};2+m;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)(a+b\sin(c+dx))^{1+m}\sqrt[4]{1-\frac{a+b\sin(c+dx)}{a-b}}\sqrt[4]{1-\frac{a+b}{}}}{bd(1+m)\sqrt{e\cos(c+dx)}}$$

```
[Out] e*AppellF1(1+m,1/4,1/4,2+m,(a+b*sin(d*x+c))/(a-b),(a+b*sin(d*x+c))/(a+b))*(
a+b*sin(d*x+c))^(1+m)*(1+(-a-b*sin(d*x+c))/(a-b))^(1/4)*(1+(-a-b*sin(d*x+c)
)/(a+b))^(1/4)/b/d/(1+m)/(e*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2783, 143}

$$\frac{e\sqrt[4]{1-\frac{a+b\sin(c+dx)}{a-b}}\sqrt[4]{1-\frac{a+b\sin(c+dx)}{a+b}}(a+b\sin(c+dx))^{m+1}F_1\left(m+1;\frac{1}{4},\frac{1}{4};m+2;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)}{bd(m+1)\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] (e*AppellF1[1 + m, 1/4, 1/4, 2 + m, (a + b*Sin[c + d*x])/(a - b), (a + b*Si
n[c + d*x])/(a + b)]*(a + b*Sin[c + d*x])^(1 + m)*(1 - (a + b*Sin[c + d*x])
/(a - b))^(1/4)*(1 - (a + b*Sin[c + d*x])/(a + b))^(1/4))/(b*d*(1 + m)*Sqrt
[e*Cos[c + d*x]])
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 2783

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[g*((g*Cos[e + f*x])^(p - 1)/(f*(1 - (a + b*Sin
[e + f*x])/(a - b))^((p - 1)/2)*(1 - (a + b*Sin[e + f*x])/(a + b))^((p - 1)
/2))), Subst[Int[(-b/(a - b) - b*(x/(a - b)))^((p - 1)/2)*(b/(a + b) - b*(x
/(a + b)))^((p - 1)/2)*(a + b*x)^m, x], x, Sin[e + f*x]], x] /; FreeQ[{a, b
```

```
, e, f, g, m, p}, x] && NeQ[a^2 - b^2, 0] && !IGtQ[m, 0]
```

Rubi steps

$$\int \sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))^m\,dx = \frac{\left(e\sqrt[4]{1-\frac{a+b\sin(c+dx)}{a-b}}\,\sqrt[4]{1-\frac{a+b\sin(c+dx)}{a+b}}\right)\text{Subst}\,(}{d\sqrt{e\cos(}}$$

$$= \frac{eF_1\left(1+m;\frac{1}{4},\frac{1}{4};2+m;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)(a+b\sin(c}{bd(1+m)\sqrt{e}}$$

Mathematica [F]
time = 1.30, size = 0, normalized size = 0.00

$$\int \sqrt{e\cos(c+dx)}\,(a+b\sin(c+dx))^m\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x])^m,x]
[Out] Integrate[Sqrt[e*Cos[c + d*x]]*(a + b*Sin[c + d*x])^m, x]
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \sqrt{e\cos(dx+c)}\,(a+b\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(1/2)*(a+b*sin(d*x+c))^m,x)
[Out] int((e*cos(d*x+c))^(1/2)*(a+b*sin(d*x+c))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)*(a+b*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] e^(1/2)*integrate((b*sin(d*x + c) + a)^m*sqrt(cos(d*x + c)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)*(a+b*sin(d*x+c))^m,x, algorithm="fricas")
[Out] integral((b*sin(d*x + c) + a)^m*sqrt(cos(d*x + c))*e^(1/2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{e\cos{(c+dx)}}\,(a+b\sin{(c+dx)})^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(1/2)*(a+b*sin(d*x+c))**m,x)
[Out] Integral(sqrt(e*cos(c + d*x))*(a + b*sin(c + d*x))**m, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)*(a+b*sin(d*x+c))^m,x, algorithm="giac")
[Out] integrate((b*sin(d*x + c) + a)^m*sqrt(cos(d*x + c))*e^(1/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{e\,\cos\left(c+d\,x\right)}\,{\left(a+b\,\sin\left(c+d\,x\right)\right)}^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(1/2)*(a + b*sin(c + d*x))^m,x)
[Out] int((e*cos(c + d*x))^(1/2)*(a + b*sin(c + d*x))^m, x)
```

3.644 $\int \frac{(a+b\sin(c+dx))^m}{\sqrt{e\cos(c+dx)}}\,dx$

Optimal. Leaf size=134

$$\frac{eF_1\left(1+m;\frac{3}{4},\frac{3}{4};2+m;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)(a+b\sin(c+dx))^{1+m}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{3/4}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{3/4}}{bd(1+m)(e\cos(c+dx))^{3/2}}$$

```
[Out] e*AppellF1(1+m,3/4,3/4,2+m,(a+b*sin(d*x+c))/(a-b),(a+b*sin(d*x+c))/(a+b))*(
a+b*sin(d*x+c))^(1+m)*(1+(-a-b*sin(d*x+c))/(a-b))^(3/4)*(1+(-a-b*sin(d*x+c)
)/(a+b))^(3/4)/b/d/(1+m)/(e*cos(d*x+c))^(3/2)
```

Rubi [A]
time = 0.07, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2783, 143}

$$\frac{e\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{3/4}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{3/4}(a+b\sin(c+dx))^{m+1}F_1\left(m+1;\frac{3}{4},\frac{3}{4};m+2;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)}{bd(m+1)(e\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^m/Sqrt[e*Cos[c + d*x]],x]
```

```
[Out] (e*AppellF1[1 + m, 3/4, 3/4, 2 + m, (a + b*Sin[c + d*x])/(a - b), (a + b*Si
n[c + d*x])/(a + b)]*(a + b*Sin[c + d*x])^(1 + m)*(1 - (a + b*Sin[c + d*x])
/(a - b))^(3/4)*(1 - (a + b*Sin[c + d*x])/(a + b))^(3/4))/(b*d*(1 + m)*(e*C
os[c + d*x])^(3/2))
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 2783

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[g*((g*Cos[e + f*x])^(p - 1)/(f*(1 - (a + b*Sin
[e + f*x])/(a - b))^((p - 1)/2)*(1 - (a + b*Sin[e + f*x])/(a + b))^((p - 1)
/2))), Subst[Int[(-b/(a - b) - b*(x/(a - b)))^((p - 1)/2)*(b/(a + b) - b*(x
```

```
/(a + b)))^((p - 1)/2)*(a + b*x)^m, x], x, Sin[e + f*x]], x] /; FreeQ[{a, b
, e, f, g, m, p}, x] && NeQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(a+b\sin(c+dx))^m}{\sqrt{e\cos(c+dx)}}\,dx = \frac{\left(e\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{3/4}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{3/4}\right)\operatorname{Subst}\left(\int \frac{(a+bx)^m}{\left(-\frac{b}{a-b}-\frac{bx}{a-b}\right)^{3/4}\left(\frac{b}{a+b}-\frac{bx}{a+b}\right.}\right.}{d(e\cos(c+dx))^{3/2}}$$

$$= \frac{eF_1\left(1+m;\frac{3}{4},\frac{3}{4};2+m;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)(a+b\sin(c+dx))^{1+m}\left(1-\frac{a}{}\right.}{bd(1+m)(e\cos(c+dx))^{3/2}}$$

Mathematica [F]
time = 1.45, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sin(c+dx))^m}{\sqrt{e\cos(c+dx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Sin[c + d*x])^m/Sqrt[e*Cos[c + d*x]],x]
[Out] Integrate[(a + b*Sin[c + d*x])^m/Sqrt[e*Cos[c + d*x]], x]
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sin(dx+c))^m}{\sqrt{e\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))^m/(e*cos(d*x+c))^(1/2),x)
[Out] int((a+b*sin(d*x+c))^m/(e*cos(d*x+c))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^m/(e*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] e^(-1/2)*integrate((b*sin(d*x + c) + a)^m/sqrt(cos(d*x + c)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^m/(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral((b*sin(d*x + c) + a)^m*e^(-1/2)/sqrt(cos(d*x + c)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \sin(c + dx))^m}{\sqrt{e \cos(c + dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))**m/(e*cos(d*x+c))**(1/2),x)
[Out] Integral((a + b*sin(c + d*x))**m/sqrt(e*cos(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^m/(e*cos(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate((b*sin(d*x + c) + a)^m*e^(-1/2)/sqrt(cos(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,\sin\left(c+d\,x\right)\right)}^m}{\sqrt{e\,\cos\left(c+d\,x\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^m/(e*cos(c + d*x))^(1/2),x)
[Out] int((a + b*sin(c + d*x))^m/(e*cos(c + d*x))^(1/2), x)
```

3.645 $\int \frac{(a+b\sin(c+dx))^m}{(e\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=134

$$\frac{eF_1\left(1+m;\frac{5}{4},\frac{5}{4};2+m;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)(a+b\sin(c+dx))^{1+m}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{5/4}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{5/4}}{bd(1+m)(e\cos(c+dx))^{5/2}}$$

```
[Out] e*AppellF1(1+m,5/4,5/4,2+m,(a+b*sin(d*x+c))/(a-b),(a+b*sin(d*x+c))/(a+b))*(
a+b*sin(d*x+c))^(1+m)*(1+(-a-b*sin(d*x+c))/(a-b))^(5/4)*(1+(-a-b*sin(d*x+c)
)/(a+b))^(5/4)/b/d/(1+m)/(e*cos(d*x+c))^(5/2)
```

Rubi [A]
time = 0.07, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2783, 143}

$$\frac{e\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{5/4}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{5/4}(a+b\sin(c+dx))^{m+1}F_1\left(m+1;\frac{5}{4},\frac{5}{4};m+2;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)}{bd(m+1)(e\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^m/(e*Cos[c + d*x])^(3/2),x]
```

```
[Out] (e*AppellF1[1 + m, 5/4, 5/4, 2 + m, (a + b*Sin[c + d*x])/(a - b), (a + b*Si
n[c + d*x])/(a + b)]*(a + b*Sin[c + d*x])^(1 + m)*(1 - (a + b*Sin[c + d*x])
/(a - b))^(5/4)*(1 - (a + b*Sin[c + d*x])/(a + b))^(5/4))/(b*d*(1 + m)*(e*C
os[c + d*x])^(5/2))
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 2783

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[g*((g*Cos[e + f*x])^(p - 1)/(f*(1 - (a + b*Sin
[e + f*x])/(a - b))^((p - 1)/2)*(1 - (a + b*Sin[e + f*x])/(a + b))^((p - 1)
/2))), Subst[Int[(-b/(a - b) - b*(x/(a - b)))^((p - 1)/2)*(b/(a + b) - b*(x
/(a + b)))^((p - 1)/2)*(a + b*x)^m, x], x, Sin[e + f*x]], x] /; FreeQ[{a, b
```

```
, e, f, g, m, p}, x] && NeQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(a+b\sin(c+dx))^m}{(e\cos(c+dx))^{3/2}}\,dx = \frac{\left(e\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{5/4}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{5/4}\right)\text{Subst}\left(\int \frac{(a+bx)^m}{\left(-\frac{b}{a-b}-\frac{bx}{a-b}\right)^{5/4}\left(\frac{b}{a+b}-\frac{bx}{a+}\right.}\right.}{d(e\cos(c+dx))^{5/2}}$$

$$= \frac{eF_1\left(1+m;\frac{5}{4},\frac{5}{4};2+m;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)(a+b\sin(c+dx))^{1+m}\left(1-\right.}{bd(1+m)(e\cos(c+dx))^{5/2}}$$

Mathematica [F]
time = 1.67, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sin(c+dx))^m}{(e\cos(c+dx))^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Sin[c + d*x])^m/(e*Cos[c + d*x])^(3/2),x]
[Out] Integrate[(a + b*Sin[c + d*x])^m/(e*Cos[c + d*x])^(3/2), x]
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sin(dx+c))^m}{(e\cos(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))^m/(e*cos(d*x+c))^(3/2),x)
[Out] int((a+b*sin(d*x+c))^m/(e*cos(d*x+c))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^m/(e*cos(d*x+c))^(3/2),x, algorithm="maxima")
[Out] e^(-3/2)*integrate((b*sin(d*x + c) + a)^m/cos(d*x + c)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^m/(e*cos(d*x+c))^(3/2),x, algorithm="fricas")
[Out] integral((b*sin(d*x + c) + a)^m*e^(-3/2)/cos(d*x + c)^(3/2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sin(c+dx))^m}{(e\cos(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))**m/(e*cos(d*x+c))**(3/2),x)
[Out] Integral((a + b*sin(c + d*x))**m/(e*cos(c + d*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^m/(e*cos(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((b*sin(d*x + c) + a)^m*e^(-3/2)/cos(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\sin(c+d\,x))^m}{(e\,\cos(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^m/(e*cos(c + d*x))^(3/2),x)
[Out] int((a + b*sin(c + d*x))^m/(e*cos(c + d*x))^(3/2), x)
```

3.646 $\int \frac{(a+b\sin(c+dx))^m}{(e\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=134

$$\frac{eF_1\left(1+m;\frac{7}{4},\frac{7}{4};2+m;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)(a+b\sin(c+dx))^{1+m}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{7/4}\left(1-\frac{a+b\sin(c+}{a+b}\right.}{bd(1+m)(e\cos(c+dx))^{7/2}}$$

```
[Out] e*AppellF1(1+m,7/4,7/4,2+m,(a+b*sin(d*x+c))/(a-b),(a+b*sin(d*x+c))/(a+b))*(
a+b*sin(d*x+c))^(1+m)*(1+(-a-b*sin(d*x+c))/(a-b))^(7/4)*(1+(-a-b*sin(d*x+c)
)/(a+b))^(7/4)/b/d/(1+m)/(e*cos(d*x+c))^(7/2)
```

Rubi [A]
time = 0.07, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2783, 143}

$$\frac{e\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{7/4}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{7/4}(a+b\sin(c+dx))^{m+1}F_1\left(m+1;\frac{7}{4},\frac{7}{4};m+2;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)}{bd(m+1)(e\cos(c+dx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^m/(e*Cos[c + d*x])^(5/2),x]
```

```
[Out] (e*AppellF1[1 + m, 7/4, 7/4, 2 + m, (a + b*Sin[c + d*x])/(a - b), (a + b*Si
n[c + d*x])/(a + b)]*(a + b*Sin[c + d*x])^(1 + m)*(1 - (a + b*Sin[c + d*x])
/(a - b))^(7/4)*(1 - (a + b*Sin[c + d*x])/(a + b))^(7/4))/(b*d*(1 + m)*(e*C
os[c + d*x])^(7/2))
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 2783

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[g*((g*Cos[e + f*x])^(p - 1)/(f*(1 - (a + b*Sin
[e + f*x])/(a - b))^((p - 1)/2)*(1 - (a + b*Sin[e + f*x])/(a + b))^((p - 1)
/2))), Subst[Int[(-b/(a - b) - b*(x/(a - b)))^((p - 1)/2)*(b/(a + b) - b*(x
/(a + b)))^((p - 1)/2)*(a + b*x)^m, x], x, Sin[e + f*x]], x] /; FreeQ[{a, b
```

```
, e, f, g, m, p}, x] && NeQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(a+b\sin(c+dx))^m}{(e\cos(c+dx))^{5/2}}\,dx = \frac{\left(e\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{7/4}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{7/4}\right)\text{Subst}\left(\int \frac{(a+bx)^m}{\left(-\frac{b}{a-b}-\frac{bx}{a-b}\right)^{7/4}\left(\frac{b}{a+b}-\frac{bx}{a+b}\right.}\right.}{d(e\cos(c+dx))^{7/2}}$$

$$= \frac{eF_1\left(1+m;\frac{7}{4},\frac{7}{4};2+m;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)(a+b\sin(c+dx))^{1+m}\left(1-\frac{a}{}\right.}{bd(1+m)(e\cos(c+dx))^{7/2}}$$

Mathematica [F]
time = 1.87, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sin(c+dx))^m}{(e\cos(c+dx))^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Sin[c + d*x])^m/(e*Cos[c + d*x])^(5/2),x]
[Out] Integrate[(a + b*Sin[c + d*x])^m/(e*Cos[c + d*x])^(5/2), x]
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sin(dx+c))^m}{(e\cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))^m/(e*cos(d*x+c))^(5/2),x)
[Out] int((a+b*sin(d*x+c))^m/(e*cos(d*x+c))^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^m/(e*cos(d*x+c))^(5/2),x, algorithm="maxima")
[Out] e^(-5/2)*integrate((b*sin(d*x + c) + a)^m/cos(d*x + c)^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^m/(e*cos(d*x+c))^(5/2),x, algorithm="fricas")
[Out] integral((b*sin(d*x + c) + a)^m*e^(-5/2)/cos(d*x + c)^(5/2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))**m/(e*cos(d*x+c))**(5/2),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^m/(e*cos(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((b*sin(d*x + c) + a)^m*e^(-5/2)/cos(d*x + c)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,\sin\left(c+d\,x\right)\right)}^m}{{\left(e\,\cos\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^m/(e*cos(c + d*x))^(5/2),x)
[Out] int((a + b*sin(c + d*x))^m/(e*cos(c + d*x))^(5/2), x)
```

3.647 $\int (e\cos(c+dx))^{-4-m}(a+b\sin(c+dx))^m\,dx$

Optimal. Leaf size=598

$$-\frac{(e\cos(c+dx))^{-3-m}(a+b\sin(c+dx))^{1+m}}{(a-b)de(3+m)}+\frac{2b(e\cos(c+dx))^{-1-m}(a+b\sin(c+dx))^{1+m}}{(a-b)^2de^3(1+m)(3+m)}+\frac{a(e\cos(c+dx)}{}$$

```
[Out] -(e*cos(d*x+c))^(-3-m)*(a+b*sin(d*x+c))^(1+m)/(a-b)/d/e/(3+m)+2*b*(e*cos(d*
x+c))^(-1-m)*(a+b*sin(d*x+c))^(1+m)/(a-b)^2/d/e^3/(1+m)/(3+m)+a*(e*cos(d*x+
c))^(-3-m)*(1+sin(d*x+c))*(a+b*sin(d*x+c))^(1+m)/(a^2-b^2)/d/e/(3+m)+a*(3*b
+a*(2+m))*(e*cos(d*x+c))^(-3-m)*(1-sin(d*x+c))*(1+sin(d*x+c))*(a+b*sin(d*x+
c))^(1+m)/(a-b)/(a+b)^2/d/e/(1+m)/(3+m)-2^(3/2-1/2*m)*a*b*(e*cos(d*x+c))^(-
1-m)*hypergeom([-1/2-1/2*m, 1/2+1/2*m],[1/2-1/2*m],1/2*(a-b)*(1-sin(d*x+c))
/(a+b*sin(d*x+c)))*((a+b)*(1+sin(d*x+c))/(a+b*sin(d*x+c)))^(1/2+1/2*m)*(a+b
*sin(d*x+c))^(1+m)/(a-b)^2/(a+b)/d/e^3/(m^2+4*m+3)-2^(-1/2-1/2*m)*a*(2*a*b-
b^2+a^2*(2+m))*(e*cos(d*x+c))^(-3-m)*hypergeom([1/2-1/2*m, 3/2+1/2*m],[3/2-
1/2*m],1/2*(a-b)*(1-sin(d*x+c))/(a+b*sin(d*x+c)))*(1-sin(d*x+c))^2*((a+b)*(
1+sin(d*x+c))/(a+b*sin(d*x+c)))^(3/2+1/2*m)*(a+b*sin(d*x+c))^(1+m)/(a-b)/(a
+b)^3/d/e/(1-m)/(3+m)
```

Rubi [A]

time = 0.71, antiderivative size = 598, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2779, 2778, 2999, 134, 136, 160, 12}

$$-\frac{a2^{-\frac{m}{2}-\frac{1}{2}}\left(a^2(m+2)+2ab-b^2\right)(1-\sin(c+dx))^2(e\cos(c+dx))^{-m-3}\left(\frac{(a+b)(\sin(c+dx)+1)}{a+b\sin(c+dx)}\right)^{\frac{m+3}{2}}(a+b\sin(c+dx))^{m+1}{}_2F_1\left(\frac{1-m}{2},\frac{m+3}{2};\frac{3-m}{2};\frac{(a-b)(1-\sin(c+dx))}{2(a+b\sin(c+dx))}\right)}{de(1-m)(m+3)(a-b)(a+b)^3}+\frac{a(\sin(c+dx)+1)(e\cos(c+dx))^{-m-3}(a+b\sin(c+dx))^{m+1}}{de(m+3)\left(a^2-b^2\right)}-\frac{ab2^{\frac{3}{2}-\frac{m}{2}}(e\cos(c+dx))^{-m-1}\left(\frac{(a+b)(\sin(c+dx)+1)}{a+b\sin(c+dx)}\right)^{\frac{m+1}{2}}(a+b\sin(c+dx))^{m+1}{}_2F_1\left(\frac{1}{2}(-m-1),\frac{m+1}{2};\frac{1-m}{2};\frac{(a-b)(1-\sin(c+dx))}{2(a+b\sin(c+dx))}\right)}{de^3(m+1)(m+3)(a-b)^2(a+b)}+\frac{2b(e\cos(c+dx))^{-m-1}(a+b\sin(c+dx))^{m+1}}{de^3(m+1)(m+3)(a-b)^2}+\frac{a(a(m+2)+3b)(1-\sin(c+dx))(\sin(c+dx)+1)(e\cos(c+dx))^{-m-3}(a+b\sin(c+dx))^{m+1}}{de(m+1)(m+3)(a-b)(a+b)^2}-\frac{(e\cos(c+dx))^{-m-3}(a+b\sin(c+dx))^{m+1}}{de(m+3)(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(-4 - m)*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] -(((e*Cos[c + d*x])^(-3 - m)*(a + b*Sin[c + d*x])^(1 + m))/((a - b)*d*e*(3
+ m))) + (2*b*(e*Cos[c + d*x])^(-1 - m)*(a + b*Sin[c + d*x])^(1 + m))/((a -
 b)^2*d*e^3*(1 + m)*(3 + m)) + (a*(e*Cos[c + d*x])^(-3 - m)*(1 + Sin[c + d*
x])*(a + b*Sin[c + d*x])^(1 + m))/((a^2 - b^2)*d*e*(3 + m)) + (a*(3*b + a*(
2 + m))*(e*Cos[c + d*x])^(-3 - m)*(1 - Sin[c + d*x])*(1 + Sin[c + d*x])*(a
+ b*Sin[c + d*x])^(1 + m))/((a - b)*(a + b)^2*d*e*(1 + m)*(3 + m)) - (2^(3/
2 - m/2)*a*b*(e*Cos[c + d*x])^(-1 - m)*Hypergeometric2F1[(-1 - m)/2, (1 + m
)/2, (1 - m)/2, ((a - b)*(1 - Sin[c + d*x]))/(2*(a + b*Sin[c + d*x]))]*(((a
 + b)*(1 + Sin[c + d*x]))/(a + b*Sin[c + d*x]))^((1 + m)/2)*(a + b*Sin[c +
d*x])^(1 + m))/((a - b)^2*(a + b)*d*e^3*(1 + m)*(3 + m)) - (2^(-1/2 - m/2)*
a*(2*a*b - b^2 + a^2*(2 + m))*(e*Cos[c + d*x])^(-3 - m)*Hypergeometric2F1[(
1 - m)/2, (3 + m)/2, (3 - m)/2, ((a - b)*(1 - Sin[c + d*x]))/(2*(a + b*Sin[
c + d*x]))]*(1 - Sin[c + d*x])^2*(((a + b)*(1 + Sin[c + d*x]))/(a + b*Sin[c
 + d*x]))^((3 + m)/2)*(a + b*Sin[c + d*x])^(1 + m))/((a - b)*(a + b)^3*d*e*
(1 - m)*(3 + m))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 134

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)
/((b*e - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*
((a + b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*
(e + f*x))))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 136

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && ILtQ[m + n + p + 2,
0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !SumSimplerQ[n, 1] &&  !SumSimp
lerQ[p, 1]))
```

Rule 160

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 2778

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])
^(m + 1)/(f*g*(a - b)*(p + 1))), x] + Dist[a/(g^2*(a - b)), Int[(g*Cos[e +
f*x])^(p + 2)*((a + b*Sin[e + f*x])^m/(1 - Sin[e + f*x])), x], x] /; FreeQ[
{a, b, e, f, g, m, p}, x] && NeQ[a^2 - b^2, 0] && EqQ[m + p + 2, 0]
```

Rule 2779

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])
^(m + 1)/(f*g*(a - b)*(p + 1))), x] + (-Dist[b*((m + p + 2)/(g^2*(a - b)*(p
 + 1))), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m, x], x] + Dist
[a/(g^2*(a - b)), Int[(g*Cos[e + f*x])^(p + 2)*((a + b*Sin[e + f*x])^m/(1 -
 Sin[e + f*x])), x], x]) /; FreeQ[{a, b, e, f, g, m, p}, x] && NeQ[a^2 - b^
2, 0] && ILtQ[m + p + 2, 0]
```

Rule 2999

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^
m*g*((g*Cos[e + f*x])^(p - 1)/(f*(1 + Sin[e + f*x])^((p - 1)/2)*(1 - Sin[e
+ f*x])^((p - 1)/2))), Subst[Int[(1 + (b/a)*x)^(m + (p - 1)/2)*(1 - (b/a)*x
)^((p - 1)/2)*(c + d*x)^n, x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e
, f, n, p}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int (e\cos(c+dx))^{-4-m}(a+b\sin(c+dx))^m\,dx = -\frac{(e\cos(c+dx))^{-3-m}(a+b\sin(c+dx))^{1+m}}{(a-b)de(3+m)} + \frac{a\int \frac{(e\cos(c+dx}{\cdots}}{\cdots}$$

$$= -\frac{(e\cos(c+dx))^{-3-m}(a+b\sin(c+dx))^{1+m}}{(a-b)de(3+m)} + \frac{2b(e\cos(c+}{(a-}$$

$$= -\frac{(e\cos(c+dx))^{-3-m}(a+b\sin(c+dx))^{1+m}}{(a-b)de(3+m)} + \frac{2b(e\cos(c+}{(a-}$$

$$= -\frac{(e\cos(c+dx))^{-3-m}(a+b\sin(c+dx))^{1+m}}{(a-b)de(3+m)} + \frac{2b(e\cos(c+}{(a-}$$

$$= -\frac{(e\cos(c+dx))^{-3-m}(a+b\sin(c+dx))^{1+m}}{(a-b)de(3+m)} + \frac{2b(e\cos(c+}{(a-}$$

$$= -\frac{(e\cos(c+dx))^{-3-m}(a+b\sin(c+dx))^{1+m}}{(a-b)de(3+m)} + \frac{2b(e\cos(c+}{(a-}$$

Mathematica [A]
time = 6.07, size = 826, normalized size = 1.38

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(-4 - m)*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] (Cos[c + d*x]*(e*Cos[c + d*x])^(-4 - m)*(a + b*Sin[c + d*x])^(1 + m))/((a -
 b)*d*(-3 - m)) + (2*b*Cos[c + d*x]^(4 + m)*(e*Cos[c + d*x])^(-4 - m)*((Cos
[c + d*x]^(-1 - m)*(a + b*Sin[c + d*x])^(1 + m))/((a - b)*d*(-1 - m)) + (2^
(1 + (-1 - m)/2)*a*Cos[c + d*x]^(-1 - m)*Hypergeometric2F1[(-1 - m)/2, (1 +
 m)/2, 1 + (-1 - m)/2, -1/2*((-a + b)*(1 - Sin[c + d*x]))/(a + b*Sin[c + d*
x])]*(1 - Sin[c + d*x])^((-1 - m)/2 + (1 + m)/2)*(1 + Sin[c + d*x])^((-1 -
m)/2 + (1 + m)/2)*(-(((-a - b)*(1 + Sin[c + d*x]))/(a + b*Sin[c + d*x])))^(
(1 + m)/2)*(a + b*Sin[c + d*x])^(1 + m))/((-a - b)*(a - b)*d*(-1 - m))))/((
a - b)*(-3 - m)) + (a*Cos[c + d*x]*(e*Cos[c + d*x])^(-4 - m)*(1 - Sin[c + d
*x])^((3 + m)/2)*(1 + Sin[c + d*x])^((3 + m)/2)*(((1 - Sin[c + d*x])^((-3 -
 m)/2)*(1 + Sin[c + d*x])^(1 + (-3 - m)/2)*(a + b*Sin[c + d*x])^(1 + m))/((
-a - b)*(-3 - m)) - (-1/2*((3*b + a*(2 + m))*(1 - Sin[c + d*x])^(1 + (-3 -
m)/2)*(1 + Sin[c + d*x])^(1 + (-3 - m)/2)*(a + b*Sin[c + d*x])^(1 + m))/((-
a - b)*(1 + (-3 - m)/2)) - (2^(-1 + (-3 - m)/2)*(1 + m)*(2*a*b - b^2 + a^2*
(2 + m))*Hypergeometric2F1[2 + (-3 - m)/2, (3 + m)/2, 3 + (-3 - m)/2, -1/2*
((-a + b)*(1 - Sin[c + d*x]))/(a + b*Sin[c + d*x])]*(1 - Sin[c + d*x])^(2 +
 (-3 - m)/2)*(1 + Sin[c + d*x])^((-3 - m)/2)*(-(((-a - b)*(1 + Sin[c + d*x]
))/(a + b*Sin[c + d*x])))^((3 + m)/2)*(a + b*Sin[c + d*x])^(1 + m))/((-a -
b)^2*(1 + (-3 - m)/2)*(2 + (-3 - m)/2)))/((-a - b)*(-3 - m))))/((a - b)*d)
```

Maple [F]
time = 0.17, size = 0, normalized size = 0.00

$$\int \left(e \cos \left(dx+c\right)\right)^{-4-m} \left(a+b \sin \left(dx+c\right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(-4-m)*(a+b*sin(d*x+c))^m,x)
```

```
[Out] int((e*cos(d*x+c))^(-4-m)*(a+b*sin(d*x+c))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-4-m)*(a+b*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((cos(d*x + c)*e)^(-m - 4)*(b*sin(d*x + c) + a)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-4-m)*(a+b*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((cos(d*x + c)*e)^(-m - 4)*(b*sin(d*x + c) + a)^m, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(-4-m)*(a+b*sin(d*x+c))**m,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5007 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-4-m)*(a+b*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((cos(d*x + c)*e)^(-m - 4)*(b*sin(d*x + c) + a)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\sin\left(c+d\,x\right)\right)}^m}{{\left(e\,\cos\left(c+d\,x\right)\right)}^{m+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^m/(e*cos(c + d*x))^(m + 4),x)
```

```
[Out] int((a + b*sin(c + d*x))^m/(e*cos(c + d*x))^(m + 4), x)
```

3.648 $\int (e\cos(c+dx))^{-3-m}(a+b\sin(c+dx))^m\,dx$

Optimal. Leaf size=311

$$\frac{(e\cos(c+dx))^{-m}\sec^4(c+dx)(-1+\sin(c+dx))(1+\sin(c+dx))(a+b\sin(c+dx))^{1+m}}{(a-b)de^3(2+m)}+\frac{(-2b+a(2+m$$

```
[Out] sec(d*x+c)^4*(sin(d*x+c)-1)*(1+sin(d*x+c))*(a+b*sin(d*x+c))^(1+m)/(a-b)/d/e
^3/(2+m)/((e*cos(d*x+c))^m)+(-2*b+a*(2+m))*sec(d*x+c)^4*(sin(d*x+c)-1)*(1+s
in(d*x+c))^2*(a+b*sin(d*x+c))^(1+m)/(a-b)^2/d/e^3/m/(2+m)/((e*cos(d*x+c))^m
)-(-b^2+a^2*(1+m))*hypergeom([1+m, 1/2*m],[2+m],-2*(a+b*sin(d*x+c))/(a-b)/(
sin(d*x+c)-1))*sec(d*x+c)^4*(1+sin(d*x+c))^3*((a+b)*(1+sin(d*x+c))/(a-b)/(s
in(d*x+c)-1))^(-1+1/2*m)*(a+b*sin(d*x+c))^(1+m)/(a-b)^3/d/e^3/m/(1+m)/((e*c
os(d*x+c))^m)
```

Rubi [A]

time = 0.35, antiderivative size = 420, normalized size of antiderivative = 1.35, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {2779, 2777, 2999, 98, 134}

$$-\frac{b(1-\sin(c+dx))(e\cos(c+dx))^{-m-2}\left(-\frac{(a-b)(1-\sin(c+dx))}{(a+b)(\sin(c+dx)+1)}\right)^{m/2}(a+b\sin(c+dx))^{m+1}\,{}_2F_1\left(m+1,\frac{m+2}{2};m+2;\frac{2(a+b\sin(c+dx))}{(a+b)(\sin(c+dx)+1)}\right)}{de(m+1)(m+2)\left(a^2-b^2\right)}+\frac{a(\sin(c+dx)+1)(e\cos(c+dx))^{-m-2}(a+b\sin(c+dx))^{m+1}}{de(m+2)\left(a^2-b^2\right)}+\frac{a2^{-m/2}(am+a+b)(1-\sin(c+dx))(e\cos(c+dx))^{-m-2}\left(\frac{(a+b)(\sin(c+dx)+1)}{a+b\sin(c+dx)}\right)^{\frac{m+2}{2}}(a+b\sin(c+dx))^{m+1}\,{}_2F_1\left(-\frac{m}{2},\frac{m+2}{2};\frac{2-m}{2};\frac{(a-b)(1-\sin(c+dx))}{2(a+b\sin(c+dx))}\right)}{dem(m+2)(a-b)(a+b)^2}-\frac{(e\cos(c+dx))^{-m-2}(a+b\sin(c+dx))^{m+1}}{de(m+2)(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(-3 - m)*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] -(((e*Cos[c + d*x])^(-2 - m)*(a + b*Sin[c + d*x])^(1 + m))/((a - b)*d*e*(2
+ m))) - (b*(e*Cos[c + d*x])^(-2 - m)*Hypergeometric2F1[1 + m, (2 + m)/2, 2
 + m, (2*(a + b*Sin[c + d*x]))/((a + b)*(1 + Sin[c + d*x]))]*(1 - Sin[c + d
*x])*(-(((a - b)*(1 - Sin[c + d*x]))/((a + b)*(1 + Sin[c + d*x]))))^(m/2)*(
a + b*Sin[c + d*x])^(1 + m))/((a^2 - b^2)*d*e*(1 + m)*(2 + m)) + (a*(e*Cos[
c + d*x])^(-2 - m)*(1 + Sin[c + d*x])*(a + b*Sin[c + d*x])^(1 + m))/((a^2 -
 b^2)*d*e*(2 + m)) + (a*(a + b + a*m)*(e*Cos[c + d*x])^(-2 - m)*Hypergeomet
ric2F1[-1/2*m, (2 + m)/2, (2 - m)/2, ((a - b)*(1 - Sin[c + d*x]))/(2*(a + b
*Sin[c + d*x]))]*(1 - Sin[c + d*x])*(((a + b)*(1 + Sin[c + d*x]))/(a + b*Si
n[c + d*x]))^((2 + m)/2)*(a + b*Sin[c + d*x])^(1 + m))/(2^(m/2)*(a - b)*(a
+ b)^2*d*e*m*(2 + m))
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
```

```
, 1])
```

Rule 134

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)
/((b*e - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*
((a + b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*
(e + f*x))))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 2777

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*(1 - Sin[e + f*x])*
(a + b*Sin[e + f*x])^(m + 1)*(((-(a - b))*((1 - Sin[e + f*x])/((a + b)*(1 +
 Sin[e + f*x]))))^(m/2)/(f*(a + b)*(m + 1)))*Hypergeometric2F1[m + 1, m/2 +
 1, m + 2, 2*((a + b*Sin[e + f*x])/((a + b)*(1 + Sin[e + f*x])))], x] /; Fr
eeQ[{a, b, e, f, g, m, p}, x] && NeQ[a^2 - b^2, 0] && EqQ[m + p + 1, 0]
```

Rule 2779

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])
^(m + 1)/(f*g*(a - b)*(p + 1))), x] + (-Dist[b*((m + p + 2)/(g^2*(a - b)*(p
 + 1))), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m, x], x] + Dist
[a/(g^2*(a - b)), Int[(g*Cos[e + f*x])^(p + 2)*((a + b*Sin[e + f*x])^m/(1 -
 Sin[e + f*x])), x], x]) /; FreeQ[{a, b, e, f, g, m, p}, x] && NeQ[a^2 - b^
2, 0] && ILtQ[m + p + 2, 0]
```

Rule 2999

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^
m*g*((g*Cos[e + f*x])^(p - 1)/(f*(1 + Sin[e + f*x])^((p - 1)/2)*(1 - Sin[e
+ f*x])^((p - 1)/2))), Subst[Int[(1 + (b/a)*x)^(m + (p - 1)/2)*(1 - (b/a)*x
)^((p - 1)/2)*(c + d*x)^n, x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e
, f, n, p}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int (e\cos(c+dx))^{-3-m}(a+b\sin(c+dx))^m\,dx = -\frac{(e\cos(c+dx))^{-2-m}(a+b\sin(c+dx))^{1+m}}{(a-b)de(2+m)} + \frac{a\int (e\cos(c+\cdots}{\cdots}$$

$$= -\frac{(e\cos(c+dx))^{-2-m}(a+b\sin(c+dx))^{1+m}}{(a-b)de(2+m)} - \frac{b(e\cos(c+\cdots}{\cdots}$$

$$= -\frac{(e\cos(c+dx))^{-2-m}(a+b\sin(c+dx))^{1+m}}{(a-b)de(2+m)} - \frac{b(e\cos(c+\cdots}{\cdots}$$

$$= -\frac{(e\cos(c+dx))^{-2-m}(a+b\sin(c+dx))^{1+m}}{(a-b)de(2+m)} - \frac{b(e\cos(c+\cdots}{\cdots}$$

Mathematica [A]
time = 3.25, size = 319, normalized size = 1.03

$$\frac{(e\cos(c+dx))^{-m}\sec^2(c+dx)(a+b\sin(c+dx))^m\left(-a-b\sin(c+dx)+\frac{b\,{}_2F_1\left(1+m,\frac{2+m}{2};2+m;-\frac{2(a+b\sin(c+dx))}{(a-b)(-1+\sin(c+dx))}\right)(1+\sin(c+dx))\left(\frac{(a+b)(1+\sin(c+dx))}{(a-b)(-1+\sin(c+dx))}\right)^{m/2}(a+b\sin(c+dx))}{(a-b)(1+m)}+\frac{a(1-\sin(c+dx))(1+\sin(c+dx))\left(2^{-m/2}(a+b+am)\,{}_2F_1\left(-\frac{m}{2},\frac{2+m}{2};1-\frac{m}{2};-\frac{(a-b)(-1+\sin(c+dx))}{2(a+b\sin(c+dx))}\right)\left(\frac{(a+b)(1+\sin(c+dx))}{a+b\sin(c+dx)}\right)^{m/2}-\frac{m(a+b\sin(c+dx))}{-1+\sin(c+dx)}\right)}{(a+b)m}\right)}{(a-b)de^3(2+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(-3 - m)*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] (Sec[c + d*x]^2*(a + b*Sin[c + d*x])^m*(-a - b*Sin[c + d*x] + (b*Hypergeome
tric2F1[1 + m, (2 + m)/2, 2 + m, (-2*(a + b*Sin[c + d*x]))/((a - b)*(-1 + S
in[c + d*x]))]*(1 + Sin[c + d*x])*(((a + b)*(1 + Sin[c + d*x]))/((a - b)*(-
1 + Sin[c + d*x])))^(m/2)*(a + b*Sin[c + d*x]))/((a - b)*(1 + m)) + (a*(1 -
 Sin[c + d*x])*(1 + Sin[c + d*x])*(((a + b + a*m)*Hypergeometric2F1[-1/2*m,
 (2 + m)/2, 1 - m/2, -1/2*((a - b)*(-1 + Sin[c + d*x]))/(a + b*Sin[c + d*x]
)]*(((a + b)*(1 + Sin[c + d*x]))/(a + b*Sin[c + d*x]))^(m/2))/2^(m/2) - (m*
(a + b*Sin[c + d*x]))/(-1 + Sin[c + d*x])))/((a + b)*m)))/((a - b)*d*e^3*(2
 + m)*(e*Cos[c + d*x])^m)
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int \left(e\cos\left(dx+c\right)\right)^{-3-m}\left(a+b\sin\left(dx+c\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(-3-m)*(a+b*sin(d*x+c))^m,x)
```

```
[Out] int((e*cos(d*x+c))^(-3-m)*(a+b*sin(d*x+c))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-3-m)*(a+b*sin(d*x+c))^m,x, algorithm="maxima")
[Out] integrate((cos(d*x + c)*e)^(-m - 3)*(b*sin(d*x + c) + a)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-3-m)*(a+b*sin(d*x+c))^m,x, algorithm="fricas")
[Out] integral((cos(d*x + c)*e)^(-m - 3)*(b*sin(d*x + c) + a)^m, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(-3-m)*(a+b*sin(d*x+c))**m,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-3-m)*(a+b*sin(d*x+c))^m,x, algorithm="giac")
[Out] integrate((cos(d*x + c)*e)^(-m - 3)*(b*sin(d*x + c) + a)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\sin(c+d\,x))^m}{(e\cos(c+d\,x))^{m+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^m/(e*cos(c + d*x))^(m + 3),x)
[Out] int((a + b*sin(c + d*x))^m/(e*cos(c + d*x))^(m + 3), x)
```

3.649 $\int (e\cos(c+dx))^{-2-m}(a+b\sin(c+dx))^m\,dx$

Optimal. Leaf size=201

$$-\frac{(e\cos(c+dx))^{-1-m}(a+b\sin(c+dx))^{1+m}}{(a-b)de(1+m)}+\frac{2^{\frac{1}{2}-\frac{m}{2}}a(e\cos(c+dx))^{-1-m}\,{}_2F_1\left(\frac{1}{2}(-1-m),\frac{1+m}{2};\frac{1-m}{2};\frac{(a-b)(1}{2(a+b}\right.}{(a^2-b^2)\,de}$$

```
[Out] -(e*cos(d*x+c))^(-1-m)*(a+b*sin(d*x+c))^(1+m)/(a-b)/d/e/(1+m)+2^(1/2-1/2*m)
*a*(e*cos(d*x+c))^(-1-m)*hypergeom([-1/2-1/2*m, 1/2+1/2*m],[1/2-1/2*m],1/2*
(a-b)*(1-sin(d*x+c))/(a+b*sin(d*x+c)))*((a+b)*(1+sin(d*x+c))/(a+b*sin(d*x+c
)))^(1/2+1/2*m)*(a+b*sin(d*x+c))^(1+m)/(a^2-b^2)/d/e/(1+m)
```

Rubi [A]
time = 0.20, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2778, 2999, 134}

$$\frac{a2^{\frac{1}{2}-\frac{m}{2}}(e\cos(c+dx))^{-m-1}\left(\frac{(a+b)(\sin(c+dx)+1)}{a+b\sin(c+dx)}\right)^{\frac{m+1}{2}}(a+b\sin(c+dx))^{m+1}\,{}_2F_1\left(\frac{1}{2}(-m-1),\frac{m+1}{2};\frac{1-m}{2};\frac{(a-b)(1-\sin(c+dx))}{2(a+b\sin(c+dx))}\right)}{de(m+1)\left(a^2-b^2\right)}-\frac{(e\cos(c+dx))^{-m-1}(a+b\sin(c+dx))^{m+1}}{de(m+1)(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(-2 - m)*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] -(((e*Cos[c + d*x])^(-1 - m)*(a + b*Sin[c + d*x])^(1 + m))/((a - b)*d*e*(1
+ m))) + (2^(1/2 - m/2)*a*(e*Cos[c + d*x])^(-1 - m)*Hypergeometric2F1[(-1 -
 m)/2, (1 + m)/2, (1 - m)/2, ((a - b)*(1 - Sin[c + d*x]))/(2*(a + b*Sin[c +
 d*x]))]*(((a + b)*(1 + Sin[c + d*x]))/(a + b*Sin[c + d*x]))^((1 + m)/2)*(a
 + b*Sin[c + d*x])^(1 + m))/((a^2 - b^2)*d*e*(1 + m))
```

Rule 134

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)
/((b*e - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*
((a + b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*
(e + f*x))))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 2778

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])
^(m + 1)/(f*g*(a - b)*(p + 1))), x] + Dist[a/(g^2*(a - b)), Int[(g*Cos[e +
f*x])^(p + 2)*((a + b*Sin[e + f*x])^m/(1 - Sin[e + f*x])), x], x] /; FreeQ[
{a, b, e, f, g, m, p}, x] && NeQ[a^2 - b^2, 0] && EqQ[m + p + 2, 0]
```

Rule 2999

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^
m*g*((g*Cos[e + f*x])^(p - 1)/(f*(1 + Sin[e + f*x])^((p - 1)/2)*(1 - Sin[e
+ f*x])^((p - 1)/2))), Subst[Int[(1 + (b/a)*x)^(m + (p - 1)/2)*(1 - (b/a)*x
)^((p - 1)/2)*(c + d*x)^n, x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e
, f, n, p}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int (e\cos(c+dx))^{-2-m}(a+b\sin(c+dx))^m\,dx = -\frac{(e\cos(c+dx))^{-1-m}(a+b\sin(c+dx))^{1+m}}{(a-b)de(1+m)} + \frac{a\int \frac{(e\cos(c+dx}{}}{}$$

$$= -\frac{(e\cos(c+dx))^{-1-m}(a+b\sin(c+dx))^{1+m}}{(a-b)de(1+m)} + \frac{\Big(a(e\cos(c+}{}$$

$$= -\frac{(e\cos(c+dx))^{-1-m}(a+b\sin(c+dx))^{1+m}}{(a-b)de(1+m)} + \frac{2^{\frac{1}{2}-\frac{m}{2}}a(e\cos}{}$$

Mathematica [A]
time = 0.62, size = 168, normalized size = 0.84

$$-\frac{2^{\frac{1}{2}(-1-m)}(e\cos(c+dx))^{-1-m}(a+b\sin(c+dx))^{1+m}\left(2^{\frac{1+m}{2}}(a+b)-2a\,{}_2F_1\left(\frac{1}{2}(-1-m),\frac{1+m}{2};\frac{1-m}{2};-\frac{(a-b)(-1+\sin(c+dx))}{2(a+b\sin(c+dx))}\right)\left(\frac{(a+b)(1+\sin(c+dx))}{a+b\sin(c+dx)}\right)^{\frac{1+m}{2}}\right)}{(a-b)(a+b)de(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(-2 - m)*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] -((2^((-1 - m)/2)*(e*Cos[c + d*x])^(-1 - m)*(a + b*Sin[c + d*x])^(1 + m)*(2
^((1 + m)/2)*(a + b) - 2*a*Hypergeometric2F1[(-1 - m)/2, (1 + m)/2, (1 - m)
/2, -1/2*((a - b)*(-1 + Sin[c + d*x]))/(a + b*Sin[c + d*x])]*(((a + b)*(1 +
 Sin[c + d*x]))/(a + b*Sin[c + d*x]))^((1 + m)/2)))/((a - b)*(a + b)*d*e*(1
 + m)))
```

Maple [F]
time = 0.14, size = 0, normalized size = 0.00

$$\int \left(e\cos\left(dx+c\right)\right)^{-2-m}\left(a+b\sin\left(dx+c\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(-2-m)*(a+b*sin(d*x+c))^m,x)
```

```
[Out] int((e*cos(d*x+c))^(-2-m)*(a+b*sin(d*x+c))^m,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-2-m)*(a+b*sin(d*x+c))^m,x, algorithm="maxima")
[Out] integrate((cos(d*x + c)*e)^(-m - 2)*(b*sin(d*x + c) + a)^m, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-2-m)*(a+b*sin(d*x+c))^m,x, algorithm="fricas")
[Out] integral((cos(d*x + c)*e)^(-m - 2)*(b*sin(d*x + c) + a)^m, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(-2-m)*(a+b*sin(d*x+c))**m,x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-2-m)*(a+b*sin(d*x+c))^m,x, algorithm="giac")
[Out] integrate((cos(d*x + c)*e)^(-m - 2)*(b*sin(d*x + c) + a)^m, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\sin(c+d\,x))^m}{(e\cos(c+d\,x))^{m+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^m/(e*cos(c + d*x))^(m + 2),x)
[Out] int((a + b*sin(c + d*x))^m/(e*cos(c + d*x))^(m + 2), x)
```

3.650 $\int (e\cos(c+dx))^{-1-m}(a+b\sin(c+dx))^m\,dx$

Optimal. Leaf size=132

$$\frac{e(e\cos(c+dx))^{-2-m}\,{}_2F_1\left(1+m,\frac{2+m}{2};2+m;\frac{2(a+b\sin(c+dx))}{(a+b)(1+\sin(c+dx))}\right)(1-\sin(c+dx))\left(-\frac{(a-b)(1-\sin(c+dx))}{(a+b)(1+\sin(c+dx))}\right)^{m/2}(a\ -}{(a+b)d(1+m)}$$

```
[Out] e*(e*cos(d*x+c))^(-2-m)*hypergeom([1+m, 1+1/2*m],[2+m],2*(a+b*sin(d*x+c))/(
a+b)/(1+sin(d*x+c)))*(1-sin(d*x+c))*(-(a-b)*(1-sin(d*x+c))/(a+b)/(1+sin(d*x
+c)))^(1/2*m)*(a+b*sin(d*x+c))^(1+m)/(a+b)/d/(1+m)
```

Rubi [A]
time = 0.05, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$, Rules used = {2777}

$$\frac{e(1-\sin(c+dx))(e\cos(c+dx))^{-m-2}\left(-\frac{(a-b)(1-\sin(c+dx))}{(a+b)(\sin(c+dx)+1)}\right)^{m/2}(a+b\sin(c+dx))^{m+1}\,{}_2F_1\left(m+1,\frac{m+2}{2};m+2;\frac{2(a+b\sin(c+dx))}{(a+b)(\sin(c+dx)+1)}\right)}{d(m+1)(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(-1 - m)*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] (e*(e*Cos[c + d*x])^(-2 - m)*Hypergeometric2F1[1 + m, (2 + m)/2, 2 + m, (2*
(a + b*Sin[c + d*x]))/((a + b)*(1 + Sin[c + d*x]))]*(1 - Sin[c + d*x])*(-((
(a - b)*(1 - Sin[c + d*x]))/((a + b)*(1 + Sin[c + d*x]))))^(m/2)*(a + b*Sin
[c + d*x])^(1 + m))/((a + b)*d*(1 + m))
```

Rule 2777

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*(1 - Sin[e + f*x])*
(a + b*Sin[e + f*x])^(m + 1)*(((-(a - b))*((1 - Sin[e + f*x])/((a + b)*(1 +
 Sin[e + f*x]))))^(m/2)/(f*(a + b)*(m + 1)))*Hypergeometric2F1[m + 1, m/2 +
 1, m + 2, 2*((a + b*Sin[e + f*x])/((a + b)*(1 + Sin[e + f*x])))], x] /; Fr
eeQ[{a, b, e, f, g, m, p}, x] && NeQ[a^2 - b^2, 0] && EqQ[m + p + 1, 0]
```

Rubi steps

$$\int (e\cos(c+dx))^{-1-m}(a+b\sin(c+dx))^m\,dx = \frac{e(e\cos(c+dx))^{-2-m}\,{}_2F_1\left(1+m,\frac{2+m}{2};2+m;\frac{2(a+b\sin(c+dx))}{(a+b)(1+\sin(c+dx}\right.}{(a+}$$

Mathematica [A]
time = 0.27, size = 132, normalized size = 1.00

$$-\frac{e(e\cos(c+dx))^{-2-m}\,{}_2F_1\left(1+m,\frac{2+m}{2};2+m;-\frac{2(a+b\sin(c+dx))}{(a-b)(-1+\sin(c+dx))}\right)(1+\sin(c+dx))\left(\frac{(a+b)(1+\sin(c+dx))}{(a-b)(-1+\sin(c+dx))}\right)^{m/2}(a+b\sin(c+dx))^{1+m}}{(a-b)d(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(-1 - m)*(a + b*Sin[c + d*x])^m,x]
[Out] -((e*(e*Cos[c + d*x])^(-2 - m)*Hypergeometric2F1[1 + m, (2 + m)/2, 2 + m, (
-2*(a + b*Sin[c + d*x]))/((a - b)*(-1 + Sin[c + d*x]))]*(1 + Sin[c + d*x])*
(((a + b)*(1 + Sin[c + d*x]))/((a - b)*(-1 + Sin[c + d*x])))^(m/2)*(a + b*S
in[c + d*x])^(1 + m))/((a - b)*d*(1 + m)))
```

Maple [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{-1-m}(a+b\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(-1-m)*(a+b*sin(d*x+c))^m,x)
[Out] int((e*cos(d*x+c))^(-1-m)*(a+b*sin(d*x+c))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-1-m)*(a+b*sin(d*x+c))^m,x, algorithm="maxima")
[Out] integrate((cos(d*x + c)*e)^(-m - 1)*(b*sin(d*x + c) + a)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-1-m)*(a+b*sin(d*x+c))^m,x, algorithm="fricas")
[Out] integral((cos(d*x + c)*e)^(-m - 1)*(b*sin(d*x + c) + a)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(c+dx))^{-m-1}(a+b\sin(c+dx))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(-1-m)*(a+b*sin(d*x+c))**m,x)
[Out] Integral((e*cos(c + d*x))**(-m - 1)*(a + b*sin(c + d*x))**m, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(-1-m)*(a+b*sin(d*x+c))^m,x, algorithm="giac")
[Out] integrate((cos(d*x + c)*e)^(-m - 1)*(b*sin(d*x + c) + a)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\sin(c+d\,x))^m}{(e\,\cos(c+d\,x))^{m+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^m/(e*cos(c + d*x))^(m + 1),x)
[Out] int((a + b*sin(c + d*x))^m/(e*cos(c + d*x))^(m + 1), x)
```

3.651 $\int (e\cos(c+dx))^{-m}(a+b\sin(c+dx))^m\,dx$

Optimal. Leaf size=152

$$\frac{eF_1\left(1+m;\frac{1+m}{2},\frac{1+m}{2};2+m;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)(e\cos(c+dx))^{-1-m}(a+b\sin(c+dx))^{1+m}\left(1-\frac{a+b\,\mathrm{s}\ldots}{}\right.}{bd(1+m)}$$

```
[Out] e*AppellF1(1+m,1/2+1/2*m,1/2+1/2*m,2+m,(a+b*sin(d*x+c))/(a-b),(a+b*sin(d*x+
c))/(a+b))*(e*cos(d*x+c))^(-1-m)*(a+b*sin(d*x+c))^(1+m)*(1+(-a-b*sin(d*x+c)
)/(a-b))^(1/2+1/2*m)*(1+(-a-b*sin(d*x+c))/(a+b))^(1/2+1/2*m)/b/d/(1+m)
```

Rubi [A]

time = 0.08, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2783, 143}

$$\frac{e(e\cos(c+dx))^{-m-1}(a+b\sin(c+dx))^{m+1}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{\frac{m+1}{2}}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{\frac{m+1}{2}}F_1\left(m+1;\frac{m+1}{2},\frac{m+1}{2};m+2;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)}{bd(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^m/(e*Cos[c + d*x])^m,x]
```

```
[Out] (e*AppellF1[1 + m, (1 + m)/2, (1 + m)/2, 2 + m, (a + b*Sin[c + d*x])/(a - b
), (a + b*Sin[c + d*x])/(a + b)]*(e*Cos[c + d*x])^(-1 - m)*(a + b*Sin[c + d
*x])^(1 + m)*(1 - (a + b*Sin[c + d*x])/(a - b))^((1 + m)/2)*(1 - (a + b*Sin
[c + d*x])/(a + b))^((1 + m)/2))/(b*d*(1 + m))
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 2783

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[g*((g*Cos[e + f*x])^(p - 1)/(f*(1 - (a + b*Sin
[e + f*x])/(a - b))^((p - 1)/2)*(1 - (a + b*Sin[e + f*x])/(a + b))^((p - 1)
/2))), Subst[Int[(-b/(a - b) - b*(x/(a - b)))^((p - 1)/2)*(b/(a + b) - b*(x
/(a + b)))^((p - 1)/2)*(a + b*x)^m, x], x, Sin[e + f*x]], x] /; FreeQ[{a, b
```

```
, e, f, g, m, p}, x] && NeQ[a^2 - b^2, 0] && !IGtQ[m, 0]
```

Rubi steps

$$\int (e\cos(c+dx))^{-m}(a+b\sin(c+dx))^m\,dx = \frac{\left(e(e\cos(c+dx))^{-1-m}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{\frac{1+m}{2}}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right.\right.}{}$$

$$= \frac{eF_1\left(1+m;\frac{1+m}{2},\frac{1+m}{2};2+m;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)(e\cos(c}{}$$

Mathematica [F]
time = 1.12, size = 0, normalized size = 0.00

$$\int (e\cos(c+dx))^{-m}(a+b\sin(c+dx))^m\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Sin[c + d*x])^m/(e*Cos[c + d*x])^m,x]
[Out] Integrate[(a + b*Sin[c + d*x])^m/(e*Cos[c + d*x])^m, x]
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int \left(e\cos\left(dx+c\right)\right)^{-m}\left(a+b\sin\left(dx+c\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))^m/((e*cos(d*x+c))^m),x)
[Out] int((a+b*sin(d*x+c))^m/((e*cos(d*x+c))^m),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^m/((e*cos(d*x+c))^m),x, algorithm="maxima")
[Out] integrate((b*sin(d*x + c) + a)^m/(cos(d*x + c)*e)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^m/((e*cos(d*x+c))^m),x, algorithm="fricas")
```

```
[Out] integral((b*sin(d*x + c) + a)^m/(cos(d*x + c)*e)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos(c+dx))^{-m}(a+b\sin(c+dx))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))**m/((e*cos(d*x+c))**m),x)
```

```
[Out] Integral((a + b*sin(c + d*x))**m/(e*cos(c + d*x))**m, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^m/((e*cos(d*x+c))^m),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^m/(cos(d*x + c)*e)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\sin(c+d\,x))^m}{(e\,\cos(c+d\,x))^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^m/(e*cos(c + d*x))^m,x)
```

```
[Out] int((a + b*sin(c + d*x))^m/(e*cos(c + d*x))^m, x)
```

3.652 $\int (e\cos(c+dx))^{1-m}(a+b\sin(c+dx))^m\,dx$

Optimal. Leaf size=142

$$\frac{eF_1\left(1+m;\frac{m}{2},\frac{m}{2};2+m;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)(e\cos(c+dx))^{-m}(a+b\sin(c+dx))^{1+m}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right.}{bd(1+m)}$$

```
[Out] e*AppellF1(1+m,1/2*m,1/2*m,2+m,(a+b*sin(d*x+c))/(a-b),(a+b*sin(d*x+c))/(a+b
))*(a+b*sin(d*x+c))^(1+m)*(1+(-a-b*sin(d*x+c))/(a-b))^(1/2*m)*(1+(-a-b*sin(
d*x+c))/(a+b))^(1/2*m)/b/d/(1+m)/((e*cos(d*x+c))^m)
```

Rubi [A]
time = 0.07, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2783, 143}

$$\frac{e(e\cos(c+dx))^{-m}(a+b\sin(c+dx))^{m+1}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{m/2}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{m/2}F_1\left(m+1;\frac{m}{2},\frac{m}{2};m+2;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)}{bd(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(1 - m)*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] (e*AppellF1[1 + m, m/2, m/2, 2 + m, (a + b*Sin[c + d*x])/(a - b), (a + b*Si
n[c + d*x])/(a + b)]*(a + b*Sin[c + d*x])^(1 + m)*(1 - (a + b*Sin[c + d*x])
/(a - b))^(m/2)*(1 - (a + b*Sin[c + d*x])/(a + b))^(m/2))/(b*d*(1 + m)*(e*C
os[c + d*x])^m)
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 2783

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[g*((g*Cos[e + f*x])^(p - 1)/(f*(1 - (a + b*Sin
[e + f*x])/(a - b))^((p - 1)/2)*(1 - (a + b*Sin[e + f*x])/(a + b))^((p - 1)
/2))), Subst[Int[(-b/(a - b) - b*(x/(a - b)))^((p - 1)/2)*(b/(a + b) - b*(x
/(a + b)))^((p - 1)/2)*(a + b*x)^m, x], x, Sin[e + f*x]], x] /; FreeQ[{a, b
```

```
, e, f, g, m, p}, x] && NeQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int (e\cos(c+dx))^{1-m}(a+b\sin(c+dx))^m\,dx = \frac{\left(e(e\cos(c+dx))^{-m}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{m/2}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right.\right.}{}$$

$$= \frac{eF_1\left(1+m;\frac{m}{2},\frac{m}{2};2+m;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)(e\cos(c+}{}$$

Mathematica [F]
time = 3.60, size = 0, normalized size = 0.00

$$\int (e\cos(c+dx))^{1-m}(a+b\sin(c+dx))^m\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(e*Cos[c + d*x])^(1 - m)*(a + b*Sin[c + d*x])^m,x]
[Out] Integrate[(e*Cos[c + d*x])^(1 - m)*(a + b*Sin[c + d*x])^m, x]
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int (e\cos(dx+c))^{1-m}(a+b\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(1-m)*(a+b*sin(d*x+c))^m,x)
[Out] int((e*cos(d*x+c))^(1-m)*(a+b*sin(d*x+c))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1-m)*(a+b*sin(d*x+c))^m,x, algorithm="maxima")
[Out] integrate((cos(d*x + c)*e)^(-m + 1)*(b*sin(d*x + c) + a)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1-m)*(a+b*sin(d*x+c))^m,x, algorithm="fricas")
[Out] integral((cos(d*x + c)*e)^(-m + 1)*(b*sin(d*x + c) + a)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e \cos(c + dx))^{1-m} (a + b \sin(c + dx))^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(1-m)*(a+b*sin(d*x+c))**m,x)
[Out] Integral((e*cos(c + d*x))**(1 - m)*(a + b*sin(c + d*x))**m, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1-m)*(a+b*sin(d*x+c))^m,x, algorithm="giac")
[Out] integrate((cos(d*x + c)*e)^(-m + 1)*(b*sin(d*x + c) + a)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (e\,\cos(c + d\,x))^{1-m}\,(a + b\,\sin(c + d\,x))^m \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(1 - m)*(a + b*sin(c + d*x))^m,x)
[Out] int((e*cos(c + d*x))^(1 - m)*(a + b*sin(c + d*x))^m, x)
```

3.653 $\int (e\cos(c+dx))^{2-m}(a+b\sin(c+dx))^m\,dx$

Optimal. Leaf size=152

$$\frac{eF_1\left(1+m;\frac{1}{2}(-1+m),\frac{1}{2}(-1+m);2+m;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)(e\cos(c+dx))^{1-m}(a+b\sin(c+dx))^{1\cdots}}{bd(1+m)}$$

```
[Out] e*AppellF1(1+m,-1/2+1/2*m,-1/2+1/2*m,2+m,(a+b*sin(d*x+c))/(a-b),(a+b*sin(d*
x+c))/(a+b))*(e*cos(d*x+c))^(1-m)*(a+b*sin(d*x+c))^(1+m)*(1+(-a-b*sin(d*x+c
))/(a-b))^(-1/2+1/2*m)*(1+(-a-b*sin(d*x+c))/(a+b))^(-1/2+1/2*m)/b/d/(1+m)
```

Rubi [A]
time = 0.07, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2783, 143}

$$\frac{e(e\cos(c+dx))^{1-m}(a+b\sin(c+dx))^{m+1}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{\frac{m-1}{2}}\left(1-\frac{a+b\sin(c+dx)}{a+b}\right)^{\frac{m-1}{2}}F_1\left(m+1;\frac{m-1}{2},\frac{m-1}{2};m+2;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin(c+dx)}{a+b}\right)}{bd(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(2 - m)*(a + b*Sin[c + d*x])^m,x]
```

```
[Out] (e*AppellF1[1 + m, (-1 + m)/2, (-1 + m)/2, 2 + m, (a + b*Sin[c + d*x])/(a -
 b), (a + b*Sin[c + d*x])/(a + b)]*(e*Cos[c + d*x])^(1 - m)*(a + b*Sin[c +
d*x])^(1 + m)*(1 - (a + b*Sin[c + d*x])/(a - b))^((-1 + m)/2)*(1 - (a + b*S
in[c + d*x])/(a + b))^((-1 + m)/2))/(b*d*(1 + m))
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 2783

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[g*((g*Cos[e + f*x])^(p - 1)/(f*(1 - (a + b*Sin
[e + f*x])/(a - b))^((p - 1)/2)*(1 - (a + b*Sin[e + f*x])/(a + b))^((p - 1)
/2))), Subst[Int[(-b/(a - b) - b*(x/(a - b)))^((p - 1)/2)*(b/(a + b) - b*(x
/(a + b)))^((p - 1)/2)*(a + b*x)^m, x], x, Sin[e + f*x]], x] /; FreeQ[{a, b
```

```
, e, f, g, m, p}, x] && NeQ[a^2 - b^2, 0] && !IGtQ[m, 0]
```

Rubi steps

$$\int (e\cos(c+dx))^{2-m}(a+b\sin(c+dx))^m\,dx = \frac{\left(e(e\cos(c+dx))^{1-m}\left(1-\frac{a+b\sin(c+dx)}{a-b}\right)^{\frac{1}{2}(-1+m)}\left(1-\frac{a+b\sin(c+}{a+b}\right.\right.}{}$$

$$= \frac{eF_1\left(1+m;\frac{1}{2}(-1+m),\frac{1}{2}(-1+m);2+m;\frac{a+b\sin(c+dx)}{a-b},\frac{a+b\sin}{a}\right.}{}$$

Mathematica [F]
time = 2.85, size = 0, normalized size = 0.00

$$\int (e\cos(c+dx))^{2-m}(a+b\sin(c+dx))^m\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(e*Cos[c + d*x])^(2 - m)*(a + b*Sin[c + d*x])^m,x]
[Out] Integrate[(e*Cos[c + d*x])^(2 - m)*(a + b*Sin[c + d*x])^m, x]
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int \left(e\cos\left(dx+c\right)\right)^{2-m}\left(a+b\sin\left(dx+c\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(2-m)*(a+b*sin(d*x+c))^m,x)
[Out] int((e*cos(d*x+c))^(2-m)*(a+b*sin(d*x+c))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(2-m)*(a+b*sin(d*x+c))^m,x, algorithm="maxima")
[Out] integrate((cos(d*x + c)*e)^(-m + 2)*(b*sin(d*x + c) + a)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(2-m)*(a+b*sin(d*x+c))^m,x, algorithm="fricas")
[Out] integral((cos(d*x + c)*e)^(-m + 2)*(b*sin(d*x + c) + a)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e\cos{(c+dx)})^{2-m} (a+b\sin{(c+dx)})^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(2-m)*(a+b*sin(d*x+c))**m,x)
[Out] Integral((e*cos(c + d*x))**(2 - m)*(a + b*sin(c + d*x))**m, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(2-m)*(a+b*sin(d*x+c))^m,x, algorithm="giac")
[Out] integrate((cos(d*x + c)*e)^(-m + 2)*(b*sin(d*x + c) + a)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(e\,\cos\left(c+d\,x\right)\right)}^{2-m}\,{\left(a+b\,\sin\left(c+d\,x\right)\right)}^m \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(c + d*x))^(2 - m)*(a + b*sin(c + d*x))^m,x)
[Out] int((e*cos(c + d*x))^(2 - m)*(a + b*sin(c + d*x))^m, x)
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                  convert(leaf_count_result,string)," vs. $2 (",
                                  convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
          asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
          asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
           fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
           gamma,loggamma,digamma,zeta,polylog,LambertW,
           elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#             Albert Rich to use with Sagemath. This is used to
#             grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#             'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#             issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
   #debug=False
   m = func.name() in ['exp','log','ln',
          'sin','cos','tan','cot','sec','csc',
          'arcsin','arccos','arctan','arccot','arcsec','arccsc',
          'sinh','cosh','tanh','coth','sech','csch',
          'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
      'arctan2','floor','abs'
      ]
   if debug:
      if m:
         print ("func ", func , " is elementary_function")
      else:
         print ("func ", func , " is NOT elementary_function")


   return m

def is_special_function(func):
   #debug=False
   if debug:
      print ("type(func)=", type(func))

   m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
         'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
         'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
         'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
         'elliptic_pi','exp_integral_e','log_integral']

   if debug:
      print ("m=",m)
      if m:
         print ("func ", func ," is special_function")
      else:
         print ("func ", func ," is NOT special_function")


   return m


def is_hypergeometric_function(func):
   return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
   return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).___name___ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```